Chapter 2

Mostly Constant Force in One-Dimension (1D)

- **Velocity and Acceleration: Average and Instantaneous**
- **Constant Force Implies Constant Acceleration**
- **Examples of Constant Acceleration in 1D**

PRELIMINARIES: What is Acceleration?

On the way to discussing force, we need to discuss acceleration. But before discussing acceleration, it will help to look some more at velocity – and officially to define “speed.” It may help to blurt out immediately that speed is the magnitude of velocity, remembering that velocity is perfectly described as a mathematical vector with both magnitude and direction.

Average velocity and average speed:

For any kind of 1D motion, the average velocity for a given "trip" from t_1 to t_2 is defined to be

$$v_{av} = \frac{\text{displacement during that time interval}}{\text{time interval}} = \frac{x_2 - x_1}{t_2 - t_1}$$

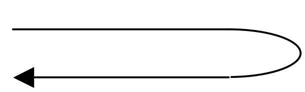
where we see

- by average we mean "time" average (i.e., the mean or average over a duration of time)
- v_{av} is positive, zero, or negative (this is the x-component and the only component in 1D : $v_{av} \equiv (v_{av})_x$)
- when the average was the same for all intervals, we had a uniform velocity in Ch. 1

The average speed is defined to be

$$\text{speed}_{av} = \frac{\text{total distance gone disregarding direction}}{\text{time interval}}$$

and is always positive (it’s not a component). But we can have v_{av} negative or vanish. For example, along the following hair-pin path:



Here, v_{av}=0 because the total displacement is zero, but the total distance gone is not zero so the $\text{speed}_{av} \neq 0$

Instantaneous velocity (v) is the instantaneous rate of change of position with time or, simply, the velocity. (Again, we drop the subscript x: $v_x \equiv v$)

$$\boxed{\begin{aligned}&\text{velocity in the limiting neighborhood of time } t_1 = \\ &= \lim_{t_2 \to t_1} \frac{x_2 - x_1}{t_2 - t_1} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}(t_1) \equiv v(t_1)\end{aligned}}$$

where $x_2 = x_1 + \Delta x$, $t_2 = t_1 + \Delta t$. Of course, all this talk about continuous motion brings up calculus (limits and derivatives) very naturally!

GRAPHICALLY: From the above, we see that the instantaneous velocity at any time is the slope of the function x(t) at that time. The average velocity involves the slope of the straight line between the two different positions:

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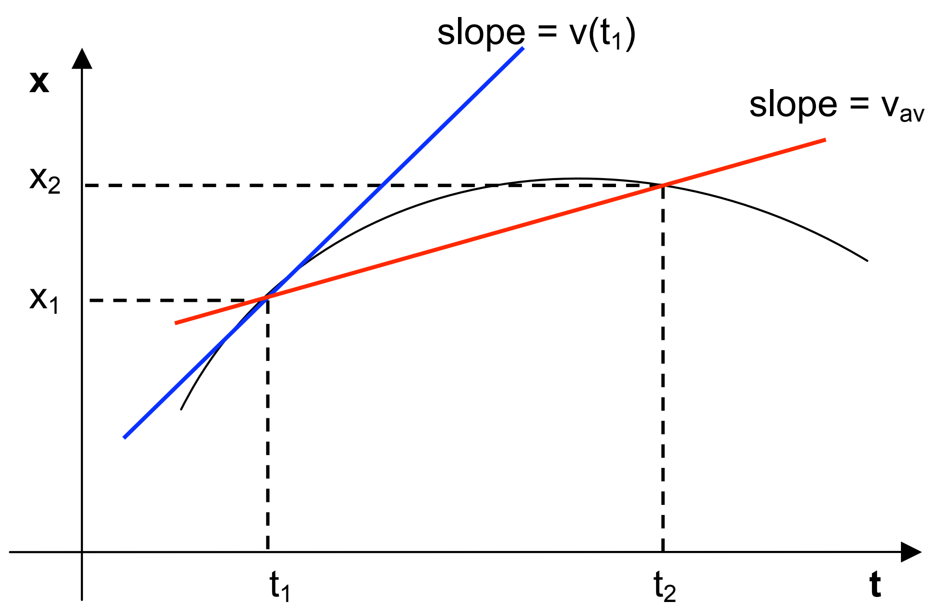


Now for acceleration:

In examples like the curve above, the velocity (slope) changes in time, so the idea of the rate of change of the velocity, defined as **acceleration** (or the rate of change of the rate of change of the position) becomes useful. **And acceleration is what Newton uses in his famous equations.**

Average acceleration (a_{av}) This again involves only the "end points:"

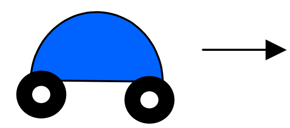
$$a_{av} = \frac{\text{net change in velocity in a given time interval}}{\text{given time interval}} = \frac{v_2 - v_1}{t_2 - t_1}$$

Instantaneous acceleration (a) = instantaneous rate of change of velocity with time (or, simply, acceleration), so, like the instantaneous velocity, we have

$$\boxed{\begin{array}{l}\text{acceleration in the limiting neighborhood of time } t_1 = \\ \qquad = \lim\limits_{t_2 \to t_1} \dfrac{v_2 - v_1}{t_2 - t_1} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} \equiv \dfrac{dv}{dt}(t_1) = \dfrac{d^2x}{dt^2}(t_1) \equiv a(t_1)\end{array}}$$

Acceleration example: Suppose a Porsche goes from 0 to 60 MPH in 3 s:

$$a_{av} = \frac{60\,\text{miles/hr} - 0}{3\text{s}} = \frac{88\,\text{ft/s} - 0}{3\text{s}} \cong 30\,{}^{\text{ft}}\!/\!{}_{\text{s}^2} \cong 9\,{}^{\text{m}}\!/\!{}_{\text{s}^2}$$



with 1 m = 3.28 ft

Comments

1. We will understand later why this number is close to the famous earth's surface gravitational acceleration g. Hint: friction, which is necessary for the Porsche to move, is related to the Porsche's weight here!
2. We promised "JIT" units discussion! Here is one:

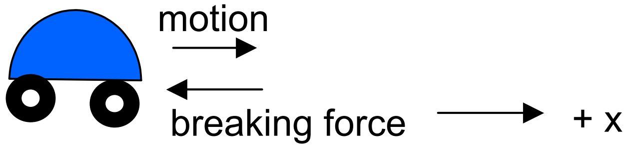
$$60\text{ miles/hour} = 60\,{}^{\text{miles}}\!/\!{}_{\text{hour}} \bullet \frac{5280\,{}^{\text{ft}}\!/\!{}_{\text{mile}}}{3600\,{}^{\text{s}}\!/\!{}_{\text{hour}}} = 88\,{}^{\text{ft}}\!/\!{}_{\text{sec}}$$

Deceleration example: Suppose that the brakes on your car are capable of giving you at best an **average** deceleration of 15 ft/s^2. (This is about half of the famous earth's surface gravitational acceleration g – see Chapter 3.) You're really cruising down the highway when you see a police car. You are able to slow down to 55 MPH in just 3 seconds. How fast were you originally going, in MPH?

Before we solve for this speed, let's be careful with sign conventions. Define what is your positive direction and stick with it! We assume the Porsche is going in a straight line and we are free to call that our x-axis, and we are free to call the direction the Porsche is moving, the positive x-direction. Thus v > 0 (remember $v \equiv v_x$ and also remember a vector in 1D has just one component) since x is increasing in time.

Then the acceleration vector (again, $a \equiv a_x$, another vector in 1D, and we drop the x for convenience) should be negative in this example, since v is decreasing in time. Let's check it on the next page.

Problem solving advice: Let the direction to the right be the positive x-direction (the direction the Porsche is moving). Now let's carefully check the direction of the acceleration if the Porsche is slowing down (it better be negative):



$$\Rightarrow a_{av} = \frac{v_f - v_i}{\Delta t} < 0 \quad (\text{since } v_f < v_i)$$

Now with careful signs and unit conversions, write all velocities in MPH and then solve for the initial velocity (which was the question to be answered):

$$a_{av} = \frac{v_f - v_i}{\Delta t} = -15\text{ft}/\text{s}^2$$
$$\Rightarrow v_i = v_f - a_{av}\Delta t = 55\text{MPH} - \left(-15\text{ft}/\text{s}^2\right)\left(3\text{s}\right) = 55\text{MPH} + 45\text{ft}/\text{s}$$
$$= 55\text{MPH} + \left(\frac{45\text{ft}/\text{s}}{88\text{ft}/\text{s}}\right)60\text{MPH} = 55\text{MPH} + 30.7\text{MPH} \cong 86\text{MPH}$$

A CLOSER LOOK AT THE SECOND LAW

Now we're ready to think about the forces that cause these accelerations and the basic equation that lets us find the acceleration, given the force:

Recall Newton's Second Law: if there is a nonzero acceleration, then the net force, which is proportional to that acceleration according to Newton's second law, **cannot be zero.** The proportionality constant measures how hard it is to move the given body, and indeed this constant is the "inertial mass," and putting it in gives us the full law:

$$\boxed{\vec{F}_{net} = M\vec{a}} \qquad \text{where M is the mass.}$$

Comments:

a) We have the same coefficient M for all three components (all three equations) that this vector equation represents

$$\boxed{F_{net,x} = Ma_x\,,\ F_{net,y} = Ma_y\,,\ F_{net,z} = Ma_z\,,}$$

b) **Larger M ⇒ smaller acceleration for a given force** and vice versa. Therefore the same interaction (force) accelerates different masses differently. This gives us a way to measure M – see the next problem.

c) REMEMBER: In Newton's second law, everything should be measured relative to an "inertial frame" (see the appendix, Ch. 1).

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Problem 2-1 *Let's think some more about Newton's second law.*

a) If you could apply the same horizontal force to two different blocks sitting on some surface with very little friction (e.g., different sized books sliding on ice), then how might you determine the mass of one block if you knew the other mass? Assume that you can somehow find out (measure) the acceleration at any time. You will probably be most comfortable referring to an equation to answer this.

b) A seal with a mass 15 kg is sliding down a straight "inclined" board and its speed increases by 1 m/s in a time interval of 1 s. Note that for any given board angle, this is a 1D problem and you can choose the board direction to be along the x axis. (You are absolutely allowed to choose your x axis along a direction that's not horizontal!) Now suppose that the acceleration along the board is constant. What is the net force on the seal in N?*

**We will see in coming chapters that, for a given board angle, both the component of the gravity force along the board and the opposing sliding friction force along the board are constant.*

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Newton's Second Law for a Constant Force in 1D:

The easiest, most familiar (you saw it in high school), and most famous application of Newton's 2nd law (it is used for earth's surface gravity) corresponds to a constant force. This means the acceleration is constant, for constant mass. For constant $a \equiv a_0$, we remember from high school:

$$\boxed{\begin{aligned} x &= x_0 + v_0 t + \frac{1}{2} a_0 t^2 \\ v &= v_0 + a_0 t \\ a &\equiv a_0 \end{aligned}}$$

• and, if you eliminate t, you get $\boxed{v^2 = v_0^2 + 2a_0(x-x_0)}$ (see the problem)

• or, if you eliminate a_0, you get $\boxed{x = x_0 + v_{av} t}$

with $\boxed{v_{av} = \frac{1}{2}(v+v_0)}$ (see the problem)

where x_0 is the position and v_0 is the velocity at $t = 0$. But don't worry, we'll remind you how everything is proved on the next page.

Proof of Constant Acceleration Equations in the Box on Previous Page.

1) Simple calculus proof of boxed equations (involving simple anti-derivatives):

Start with $\frac{dv}{dt} = a_0$ and integrate to get $v = a_0 t + C$ with C an integration constant but $v = \frac{dx}{dt} = a_0 t + C$ so integrate again to get $x = \frac{1}{2} a_0 t^2 + Ct + D$ with another integration constant D and the integration constants C, D are clearly $C = v(0), D = x(0)$. (Why clearly? Well, just put t=0 in the v(t) and x(t) equations, and see what you get!) Of course, you can quickly check that velocity v and acceleration a follow from x by taking time derivatives if you haven't had the anti-derivative practice yet in your calculus class corresponding to "integration."

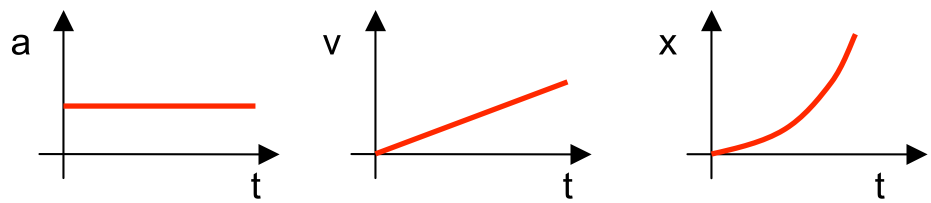
2) Proof without calculus. Recalling Chapter 1 and (1.1), we can say

$$\frac{v_2 - v_1}{t_2 - t_1} = \text{constant} = a_x^c \equiv a_0$$

because now it's constant acceleration instead of constant velocity. Let $v_2 \equiv v$ $v_1 \equiv v_0$, $t_2 \equiv t$, and $t_1 \equiv 0$. Then $\frac{v - v_0}{t} = a_0$ or $\boxed{v = v_0 + a_0 t}$. So far so good, but what about x? Well, we know the average velocity over the time interval from t=0 to t=t (see comment below*) is $v_{av} = \frac{x - x_0}{t - 0}$, but we also know the wonderful fact that when a velocity grows with time at a constant rate, its average velocity is just the average of the initial and final velocity: $v_{av} = \frac{1}{2}(v_2 + v_1) \equiv \frac{1}{2}(v + v_0)$ (true only for constant acceleration $\Rightarrow$ constant slope of v). See the homework. If we set these two expressions for v_{av} equal to each other, and substitute into them the $v = v_0 + a_0 t$ expression, we get $(x - x_0)/t = \frac{1}{2}(v_0 + a_0 t + v_0)$ or, finally, $\boxed{x = x_0 + v_0 t + \frac{1}{2} a_0 t^2}$.

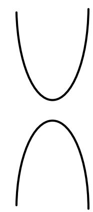
* People often say t = t which sounds stupid. It's just a short-cut to saying, for example, t = t_2, and then dropping the subscript 2 for convenience, and label the final time t to simplify your equations.

GRAPHS: The time graphs (constant a, v with constant slope, and parabolic x) are



v(t) is a part of a straight line

x(t) is a part of a parabola if $a_0 \neq 0$



Comments on graphs:

1) For constant acceleration, we always have horizontal lines $a(t) = a_0$ where above , on, and below the time-axis refers to $a_0>0$, $a_0=0$, $a_0<0$, respectively.

2) For constant acceleration, velocity plots v(t) are always straight lines with positive, zero, and negative slopes for $a_0>0$, $a_0=0$, $a_0<0$, respectively.

3) For constant acceleration, position plots x(t) always lie on some part of a parabola (i.e, it is generally quadratic in t). The parabola is concave upward if the acceleration is positive ($a_0>0$) and concave downward if the acceleration is negative ($a_0<0$). If the acceleration is zero ($a_0=0$), x(t) lies along straight lines, of course, like in Ch. 1.

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Problem 2-2

Suppose you have a wood block moving without rotating in one-dimensional motion, an initial position x_0 *(of, say, any point painted on it), and an initial velocity* v_0 *(initial means time* $t = 0$*). For constant acceleration* a_0*, show*

*a) when you eliminate** *the (constant) acceleration, you get* $\boxed{x = x_0 + \frac{1}{2}(v+v_0)t}$

b) when you eliminate the time t, you get $\boxed{v^2 = v_0^2 + 2a_0(x-x_0)}$

** "Eliminate" means solve for it from one equation and substitute that answer into another equation.*

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A SMALL NUMERICAL EXAMPLE FOR CONSTANT ACCELERATION USING THE RESULT OF THE PREVIOUS HOMEWORK

A Jaguar sports car skids to a stop on the famous M1 roadway in England, making skid marks 276 m long! Suppose that its deceleration is roughly the same as the constant gravitational acceleration 9.8 m/s^2 (see Chapter 3), which, as in the Porsche example, will make more sense when we connect friction to the weight of a sliding body in Chapter 7. What was the initial speed?

SOLUTION: Well, we want to relate speed and distance directly and bypass time. And we have the direct relation between speed and distance from the previous homework:

$$v^2 = v_0^2 + 2a_0(x-x_0) \text{ with the final speed } v=0\text{ , } x-x_0=276\text{ m, and } a_0 = -9.8\text{ m/s}^2.$$

(The acceleration is negative – we use the convention that we are moving in the positive x direction.) So we can solve for v_0 :

$$\therefore\ v_0 = \sqrt{+2(9.8)276} = 74\text{ m/s} = (74/26.8)60 \quad \text{(converting to MPH)} \quad = 166\text{ MPH !!}$$

(We can testify that people really do travel that fast on the M1. In 1986, Doc Brown drove for 6 hours down from Scotland to London with his elderly parents crouched in the back seat frightened out of their minds. It was wonderful.)

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Problem 2-3 *A manufacturer of a certain sports car claims that the car will accelerate from 0 to 90 MPH in 10 s. You and a friend set out to determine whether or not this is true. Here is your plan: you will find a straight and empty stretch of road, which is one mile long. You will start the car, and your friend will get into the passenger seat with a stopwatch. When you yell "Go!" you will hit the gas pedal and at the same time your friend will start the stopwatch. You will continue to hold down the accelerator until you hit 90 MPH, at which point your friend will stop the watch and tell you how much time elapsed. You will then coast for a short while, and then you will apply the brakes evenly to stop.*

(a) One mile is approximately 1600 m. What is the speed of 90 MPH, approximately, in units of m/s?

(b) Assume that while you hold down the gas pedal, you are accelerating constantly. If the manufacturer's claim is true, what is your acceleration in units of m/s^2 ?

(c) If the manufacturer's claim is true, what distance (in m) will you have

covered when you hit 90 MPH? Use the first equation in the earlier boxed formulas.

(d) Redo (c) but now use the first equation in Problem 2-2 (part a).

(e) Redo (c) but now use the second equation in Problem 2-2 (part b).

(f) Suppose that at the point that you reach 90 MPH, your friend also checks the odometer, and tells you that you've traveled something less than two tenths of a mile. Is this what you expect?

(g) Assume that once you reach 90 MPH, you will take precisely 4 seconds to move your foot from the gas pedal to the brake. Assume that during this four seconds, you coast at a constant velocity of 90 MPH (we ignore any frictional slowdown). How far (in meters) do you travel while you are coasting?

(h) Assume further that the manufacturer claims that the car can brake from 90 MPH to zero in 15 seconds. Assume that when stopping, the brakes apply a constant deceleration (i.e. a constant negative acceleration) during the whole braking time. If the manufacturer's claim is true, what is the braking distance (in meters) required to stop the car?

(i) Using your answers obtained thus far, what is the total distance (in m) that the car will have traveled when you have completed the entire road test, assuming that all of the manufacturer's claims are true? Is a one mile stretch of road adequate to do the complete test?