PHYS 121: Third Hour Exam Announcement and Practice Problems WITH SOLUTIONS

Please note: The Third Hour Exam is Friday, April 19, 2024 at 11:40 AM EDT Same rules as last time. You are allowed *three pages* (front and back) for your own *hand-written notes*. Also a calculator is allowed but you will not need it.

Because of the exam, there is *no homework* assignment until after the exam.

NINE Practice Problems for PHYS 121 THIRD Hour Exam:

- **Problems X01, X02, and X03 from Fall 2015 PHYS 121 Third Hour Exam**
- **Problems Y01, Y02, and Y03 from Fall 2013 and 2017 Third Hour Exams.**
- **Problems Z01, Z02, and Z03 from Fall 2016 and 2019 Third Hour Exams.**

Important: These problems for Practice Only. These problems will not be collected and will not be graded. Solutions are found after each set.

Guidelines for working Practice Problems on next page....

Important: Do not look at solutions! Read First:

Practice problems can be useful for students. However, there are pitfalls to using practice problems and so these general guidelines might be helpful: students:

- First and foremost, all students are encouraged to attempt to work all problems "under exam conditions". In other words, ***work each problem without notes or books, and without looking at the solutions.*** Ideally, give yourself about 15 to 20 minutes and try to complete each problem. Doing the practice problems under exam conditions will tell you if you are "well-prepared" to do well on the exam as it will actually be given.

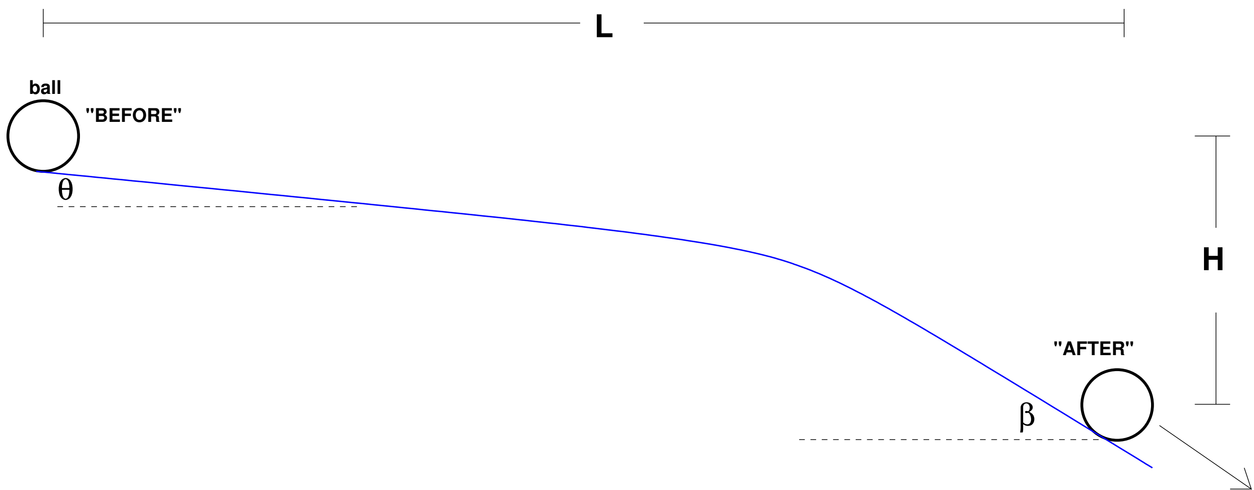
- Again, *resist any temptation to look at the solutions before you have made a decent effort to solve the problem on your own.*. This is really important. Student who quickly look at the solutions before they actually work the problems risk fooling themselves into thinking they know how to solve the problem. **Physics is learned by doing, not observing.** Simply looking at solutions before you try the problems as preparation for an exam is like trying to learn how to ride a bicycle by watching someone else riding. You tell yourself "Oh I understand that. I can do that." But you are fooling yourself. The only way to know if you have mastered a problem is to demonstrate that you have actually done the problem on your own.

- So give yourself a fixed period of time to work one or more problems. Use the whole time to work the problem even if you are stuck. At the end of the time, look at the solution. First, make sure that you are using the correct ***Main Physics Concept*** for any given part of a problem. This is most important. If you pick the wrong Main Physics Concept you will get nowhere. Study the solution. Make sure you understand it. Maybe sure you understand where you went wrong. Then *put the problem away for at least 12 to 24 hours.* Now, say, the next day, try to solve the problem – again without looking at the solution. Maybe you will get further. Repeat until you have mastered the problem and you can solve it quickly without looking at the solution at all.

- If you are struggling with a Main Concept, better to make sure you get some help with that Concept instead of doing many practice problems. Every student has different learning preferences. Some students do best by studying the materials. Others do well with practice problems. You want to develop a study strategy that is best for you.

- Remember the goal of the course is to have students *demonstrate that they understand the Main Physics Concept by successfully solving problems.* We are not looking for "the right answer". We are looking for you to show us that you understand the main concepts and can apply these correctly to solve a wide variety of problems. The problems on the exam will be *different* from the practice problems, guaranteed. Do not "memorize solutions" to problems. Instead, make sure you understand the problems so that you can tackle new problems using the same Physics Concepts – problems that may seem both familiar and unfamiliar. Mastery means being ready to apply the Main Physics Concepts to new problems that you have not yet encountered, working your way methodically towards the correct solution quickly and with confidence.

Problem X01: Hollow Spherical Ball Rolling Down a Hill (30 points)

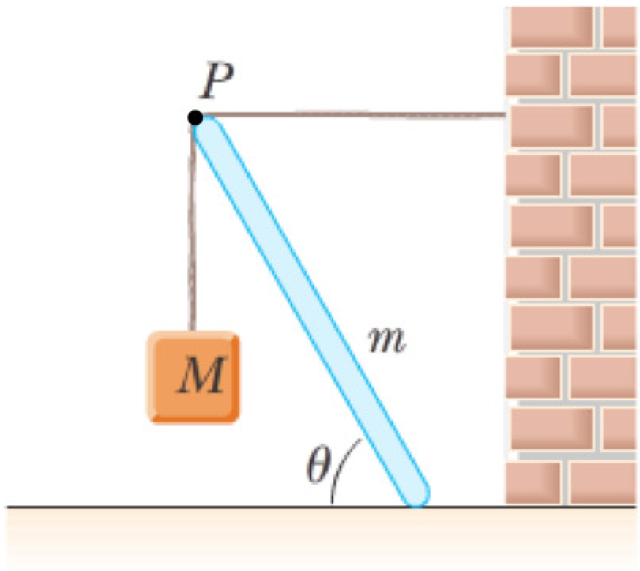


A ***hollow spherical ball*** of given mass m and radius R is released at rest near the top of a hill as shown above. The initial tilt angle of the surface of the hill with respect to the horizontal is given as θ. However, the hollow ball eventually rolls down to a part of the hill where the angle with respect to the horizontal is steeper, given as β. Assume that the hollow ball *always* rolls with no slipping, no sliding.

Part (a) – (15 points) : Consider the moving hollow ball at the instant when it is in the position corresponding to the position labeled **“AFTER”**. The total horizontal displacement to the right is given as L and the total vertical displacement downward is given as H as shown above. Calculate the ***linear speed*** of the ball at this instant. Express your answer in terms of the given parameters. Explain what you are doing. Show your work.

Part (b) – (15 points) : Calculate the magnitude of the ***linear acceleration*** of the hollow ball in the direction of motion at this same instant, corresponding to the position labeled **“AFTER”**. Express your answer in terms of the given parameters. Explain what you are doing. Show your work.

Problem X02: Rod held in place (40 points)



A uniform beam of given mass m is inclined at some angle relative the floor. A single piece of ideal rope is connected horizontally to a wall and then is is diverted from the horizontal to the vertical around the top end of the beam (corresponding to Point P), down to a block of given mass M. The bottom end of the board is held in place on the floor by static friction. The beam is tipped at an angle so that the whole system is in equilibrium and does not move at all. There is no friction at all between the rope and the beam.

Part (a) – (10 points): Draw both a Free-Body-Diagram (FBD) and an Extended Free Body Diagram (XFBD) indicating all of the forces on the beam.

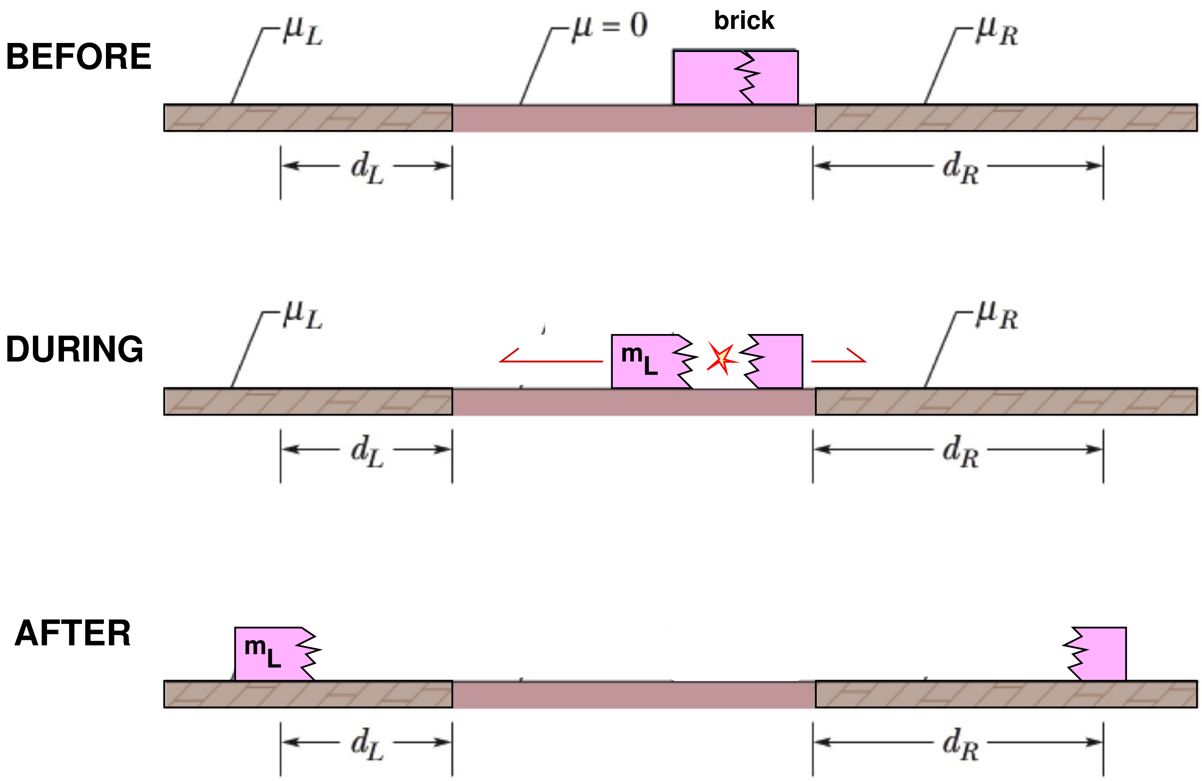
Part (b) – (15 points): Calculate the magnitude of every force on the beam. Express your answers in terms of given parameters. Explain your work.

Part (c) – (10 points): Calculate the value of the angle θ such that the system remains motionless. Express your answers in terms of given parameters. Explain your work.

Part (d) – (5 points): What is the constraint on μ_s, the coefficient of static friction between the beam and the floor, so as to ensure that the board is held in place and does not slip or slide? Express your answer as in inequality applied to μ_s in terms of the given parameters. Explain your work.

Important Hint: Because the rope is ideal and because there is zero friction between the rope and the beam, the end of the beam at Point P behaves as if it were a very small ideal pulley with respect to the rope. In other words, the rope applies both a horizontal *and* vertical force of Tension to the beam at Point P.

Problem X03: Exploding Brick: (30 points)



An explosive charge is placed inside a brick which starts at rest on a table. The brick explodes horizontally into just two fragments. The left fragment has a given mass m_L and moves left and eventually encounters a surface region with a given coefficient of kinetic friction μ_L. The left fragment comes to rest in this after sliding a given distance d_L. The right fragment likewise encounters a region with a given coefficient of kinetic friction μ_R and eventually comes to rest after sliding a given distance d_R.

What was the **total mass** M of the original brick? Give your answer in terms of the given parameters of the problem. **Important:** Note that the mass of the **right** fragment is **not** a given parameter in this problem. Explain your work.

Solution to Problem X01:

Part (a)– (15 points):

For this problem, we have a ball that rolls without slipping. Weight is **conservative** while the forces of Normal and Friction **do no work here** so we have ***met the conditions*** for the application of **Conservation of Mechanical Energy**. The "Before" situation corresponds to the ball at the left side, with zero velocity. The "After" situation corresponds to the ball at the indicated position on the right side of the hill. Because are a moving and rolling at the same time, the kinetic energy term must include both translational and rotational kinetic energy:

$$\text{“}Before'' = \text{“}After''$$
$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgy' + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$$

We start at $y = 0$, $v = 0$, $\omega = 0$. We end at $y = -H$:

$$0 + 0 + 0 = -mgH + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$$
$$\frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2 = mgH$$

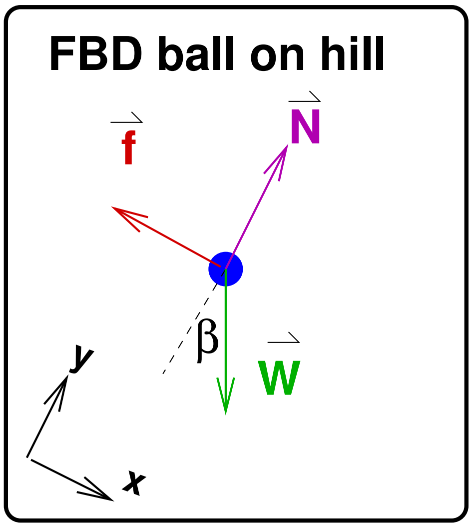
Then we use $I = \frac{2}{3}mR^2$ corresponding to a **hollow sphere** and the **Rolling Constraint:** $v = \omega R$:

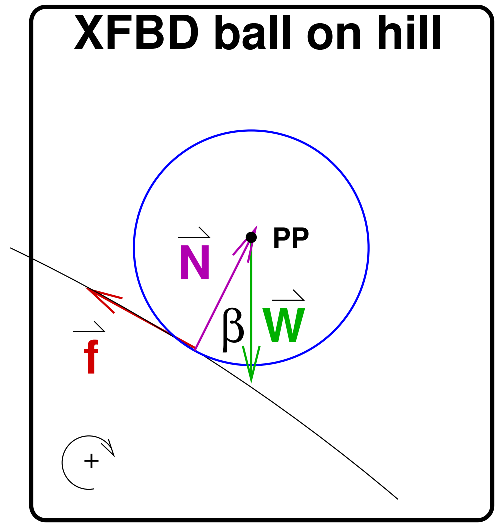
$$\frac{1}{2}mv'^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v'}{R}\right)^2 = mgH$$
$$v'^2 + \left(\frac{2}{3}R^2\right)\left(\frac{v'}{R}\right)^2 = 2gH$$
$$v'^2 + \left(\frac{2}{3}\right)v'^2 = 2gH$$
$$\frac{5v'^2}{3} = 2gH$$
$$v'^2 = \frac{6gH}{5}$$
$$\boxed{v' = \sqrt{\frac{6gH}{5}}}$$

Solution Continues next page...

Part (b)– (15 points):

For this part, we are interested in the acceleration. Therefore we want to apply **Newton's Laws**. We generate a **Free-Body-Diagram** and an **Extended Free-Body Diagram:**





Note that we choose a **tipped coordinate system** so that the "x" direction is lined up with the direction of acceleration. We start by applying **Newton's Second Law** in this coordinate:

$$F_x = ma_x$$
$$W \sin\beta - f = ma_x$$
$$mg \sin\beta - f = ma_x$$

This is as far as we can go for now. We have one equation, but two unknowns, f and a_x. (Note that applying Newton's Second Law to the "y"-coordinate is not helpful here).

We then consider **Newton' Second Law in Rotational Form**. As shown in the XFBD, we define the direction down the hill (clockwise) as positive and choose the pivot point at the center-of-mass of the sphere. Then we can write down:

$$\tau_{net} = I\alpha$$

We use **the Definition of Torque** to infer the torque due to each particular force:

$$\tau_{\vec{F}} \equiv rF \sin\varphi$$

where r is the displacement vector *from* the Pivot Point *to* the location where the force is applied and φ is the angle between the vectors $\vec{r}$ and $\vec{F}$.

Solution continues next page...

We choose the Pivot Point at the center of the sphere[1]

The torque due to **Weight** force calculated about the Pivot Point at the center of the sphere is **zero** because the force is applied at the Pivot Point, so $\vec{r}$ has zero length for this force.

The torque due to **Normal** force is **zero** because the Normal force is applied in a direction parallel to the radius vector and so $\sin\varphi = 0$ for this force.

The torque due to the **Friction** force is **not zero**. In this case, Friction is applied at a right angle relative to the displacement vector $\vec{r}$ which has a length R. Therefore $\tau_{\vec{f}} = Rf$ and so:

$$Rf = I\alpha$$

$$Rf = \frac{2}{3}mR^2\alpha$$

We apply the **Rolling Constraint** again, but for acceleration: $a_x = \alpha R$:

$$Rf = \frac{2}{3}mR^2\left(\frac{a_x}{R}\right)$$

$$f = \frac{2}{3}(ma_x)$$

So we plug this into our previous equation:

$$mg\sin\beta - \frac{2}{3}(ma_x) = ma_x$$

Solving for a_x:

$$mg\sin\beta = ma_x + \frac{2}{3}(ma_x)$$

$$a_x + \frac{2}{3}a_x = g\sin\beta$$

[1]Note: Some students selected a Pivot Point at the location corresponding to the point of contact between the sphere and the surface of the hill. This is in principle acceptable. However, if you do this then it is no longer true that $I = \frac{2}{3}mR^2$ for this problem. Indeed, when we say $I = \frac{2}{3}mR^2$ for a hollow sphere, we mean the moment of inertia about the **center-of-mass** point. If you want to change the Pivot Point to be a location that is offset from the center-of-mass point, then you ***must*** use the **Parallel Axis Theorem** to compute the new rotational inertia:

$$I_{PP} = I_{cm} + MD^2 = \frac{2}{3}mR^2 + mR^2 = \frac{5}{3}mR^2$$

With this correct rotational inertia, the torque at this point is due only to Weight and is easy to calculate:

$$\tau_{\vec{W}} = I_{PP}\alpha$$

$$mgR\sin\beta = \left(\frac{5}{3}\right)mR^2\left(\frac{a}{R}\right)$$

$$\boxed{a = \frac{3g\sin\beta}{5}}$$

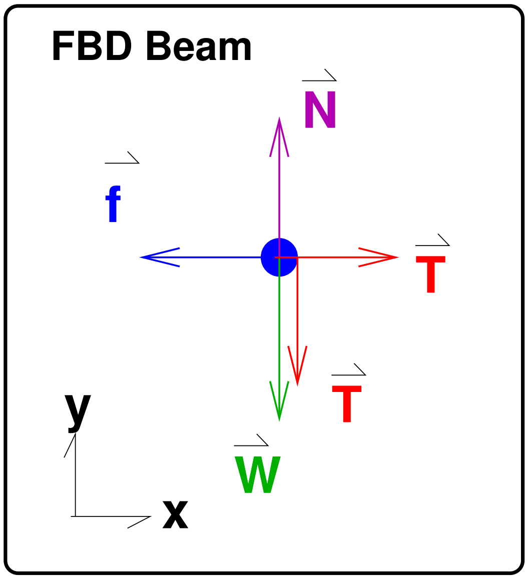
Which is the correct answer, elegantly derived. But you only get the correct answer if you properly use the Parallel Axis Theorem to re-calculate the rotational inertia of the hollow sphere.

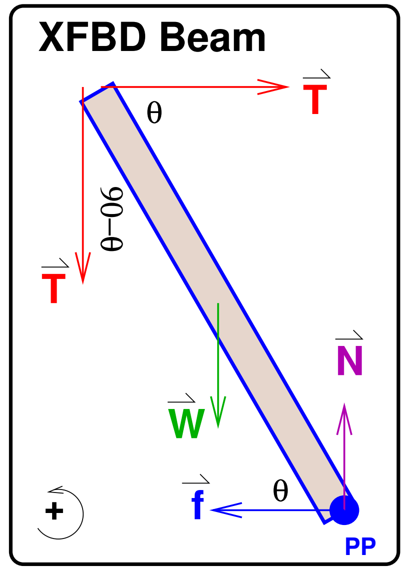
$$\frac{5}{3}a_x = g\sin\beta$$

$$\boxed{a_x = \frac{3g\sin\beta}{5}}$$

Solution to Problem X02:

Part (a) – 10 points:





Part (b) – 15 points:

Note that the only given parameters in this problem are m and M.

Calculating the tension on the rope is done simply noticing that everything is at rest. This means by **Newton's Second Law** that the net force on Block M must be zero, so therefore the Tension on Block M must exactly balance the Weight of Block M. This instantly gives us the Tension:[2]

$$F_{net,BlockM} = Ma_{BlockM}$$
$$T - Mg = 0$$
$$\boxed{T = Mg}$$

We also know the Weight of the beam:

$$\boxed{W = mg}$$

[2]Note: Many students asserted that the vertical Tension force and the horizontal Tension force should be considered as two different forces with two different magnitudes. *This is incorrect.* As clearly indicated in the hint at the bottom of the problem:

> "Because the rope is ideal and because there is zero friction between the rope and the beam, the end of the beam at Point P behaves as if it were a very small ideal pulley with respect to the rope. In other words, the rope applies both a horizontal *and* vertical force of Tension to the beam at Point P."

The rope is ***ideal***. The corner of the beam acts like an ***ideal pulley***. Therefore, the Tension in both pieces of the rope is the ***same***. Ideal rope, ideal pulley, same Tension. There is only one value of Tension T here, not two.

We apply **Newton's Second Law** for the beam in the vertical direction:

$$N - T - mg = 0$$

$$N - Mg - mg = 0$$

$$\boxed{N = (M + m)g}$$

Finally we apply **Newton's Second Law** for the beam in the horizontal direction:

$$T - f = 0$$

$$f = T$$

$$\boxed{f = Mg}$$

That's it. *Note in particular that applying Newton's Second Law in Rotational form for is at best a useless waste of time for Part (b). None of the forces depend on the angle theta. You can find the values of all forces on the beam by* only *considering the linear form of Newton's Second Law.*[3]

[3]Note: Several students failed to notice that all of the answers for Part (b) are given by the application of Newton's Second Law in linear form only. Some students applied Newton's Second Law in Rotational Form for Part (b) to calculate either/both the Tension and/or the Normal force in terms of the tip angle θ. While such efforts correspond to correct physics, in fact **the angle θ is *not* a given parameter in this problem** and must be calculated in Part (c) using forces that are completely determined in Part (b). Students who applied **Newton's Second Law in Rotational Form** inappropriately for Part (b) were generally given credit if this work resulted in a result that could be used in Part (c).

Part (c) – 10 points:

To find the angle θ we must next consider the XFBD and torque on beam. You will get the right answer if you choose any pivot point but for the purposes of demonstrating a solution, we choose the Pivot point at the lower end as shown in our XFBD. As a result, torques due to Friction and Normal force are zero because these forces are applied at the pivot point.

Torque due to Horizontal Tension: Use the **Definition of Torque** $\tau_{\vec{F}} \equiv rF\sin\phi$ where r is the vector that points *from* the Pivot Point *to* the location on the body where the force is applied: Here $r = L$ the (unknown) length of the beam, and the force is the (now known) Horizontal Tension:

$$\tau_{TH} = LT\sin\theta$$

Likewise the torque due to Vertical Tension is given by:

$$\tau_{TV} = LT\sin(90^\circ - \theta)$$
$$\tau_{TV} = LT\cos\theta$$

Torque due to (vertical) Weight is a similar calculation but the distance r from the Pivot Point to the Center-of-Mass point is have the length of the beam:

$$\tau_W = \left(\frac{Lmg}{2}\right)\cos\theta$$

We apply **Newton's Second Law for Rotations:** we know that the beam is **static** so zero acceleration:

$$\tau_{net} = I\alpha = 0$$

$$LT\sin\theta - LT\cos\theta - \left(\frac{Lmg}{2}\right)\cos\theta = 0$$
$$Mg\sin\theta - Mg\cos\theta - \left(\frac{mg}{2}\right)\cos\theta = 0$$
$$\sin\theta - \cos\theta - \left(\frac{m}{2M}\right)\cos\theta = 0$$
$$\tan\theta - 1 - \left(\frac{m}{2M}\right) = 0$$
$$\tan\theta = 1 + \frac{m}{2M}$$
$$\boxed{\theta = \arctan\left(1 + \frac{m}{2M}\right)}$$

Note that (happily) this result does *not* depend on the (unknown) value of the length of the beam L. It only depends on the given masses m and M. Note also that in the limit that $M >> m$, θ =45 degrees corresponding to a symmetric torque balance between vertical and horizontal tension with negligible torque due to the mass of the beam. Finally note that as the mass of the beam m grows larger in comparison to the mass of the block M, the angle θ increases toward 90 degrees – in other words, the heavier the beam relative to the block, the more vertical the beam will need to be to maintain torque equilibrium – just as we would expect.

Part (d) – 5 points:

Constraint for Static Friction:

$$f_s \leq \mu_s N$$

All we need to do is plug values in from Part (b) and separate out the coefficient of friction:

$$Mg \leq \mu_s(M+m)g$$

$$\mu_s(M+m)g \geq Mg$$

$$\boxed{\mu_s \geq \frac{M}{M+m}}$$

Solution to Problem X03:

Of central importance is that this represents a **compound problem** where the student must break the problem into ***two distinct sub-parts*** as follows:

- **Part "A"** corresponding to **the explosion** including **just *before* the explosion** and **just *after* the explosion**. For this part, the central physics concept is **Conservation of Linear Momentum**, and;
- **Part "B"** corresponding to **the interaction of each brick piece with the surface via kinetic friction**. For this part, the central physics concept is **The Work-Energy Theorem**.

Although the sub-parts can be tackled in any order, here we ***work backwards*** since **Part "B"** will give us an expression for the speeds of each fragment, which we can then use as input into **Part "A"** to get the masses.

Part "B" corresponding to **the interaction of each brick piece with the surface via kinetic friction**:

After the explosion, each fragment of the brick experiences only one horizontal force, and that is the force of **kinetic (sliding) friction**. We can use the sliding distance to infer the initial velocity of the fragment. There are several ways to do this, but probably the most straight-forward is to apply the **Work-Energy Theorem** which tells us that the total work done on a body is equal to the change in kinetic energy:

$$\mathcal{W}_{tot} = \Delta K$$

Let's apply this first to the left fragment. The only work done is due to the force of sliding (kinetic) friction. In this case, the force of friction is quite easily calculated from the vertical normal force:

$$f_s = \mu_k N$$

The **Definition of Work** corresponding to a constant force applied over a straight-line path is:

$$\mathcal{W}_{\vec{f}_s} \equiv \vec{f}_s \cdot \vec{d}$$

where $\vec{d}$ is the straight-line vector displacement. Since the forces are anti-parallel (friction acting in the opposite direction to the displacement) the dot-product just yields negative work:

$$\mathcal{W}_{\vec{f}_s} = -f_s d = -\mu_k mgd$$

For the left fragment the given values for the coefficient, mass, and distance can be plugged in. This is just:

$$\mathcal{W}_{tot} = \mathcal{W}_{\vec{f}_s} = -\mu_L m_L g d_L$$

The change in kinetic energy is just:

$$\Delta K = K_f - K_i = -\frac{1}{2}mv^2$$

So we just put this result in the **Work-Energy Theorem** using terms for the left fragment:

$$-\mu_L g d_L = -\frac{1}{2}m_L v_L^2$$

Solving for the *speed* (not velocity) v_L:

$$v_L^2 = 2\mu_L g d_L$$
$$v_L = \sqrt{2\mu_L g d_L} \tag{1}$$

We now have an expression for the speed v_L in terms of purely given parameters.

Identical logic allow us to derive an expression for the *speed* of the right fragment:

$$v_R = \sqrt{2\mu_R g d_R} \tag{2}$$

Part "A" corresponding to **the explosion**.

Since the system is **isolated** during and *immediately after* the explosion, we can apply **Conservation of Linear Momentum** to place constraints on the speeds of each piece. Before the brick explodes, ***The initial momentum of the brick is zero***, and therefore each brick fragment must have equal (and oppositely directed) linear momentum. Again v_R and v_L represent the ***speeds*** of the two fragments:

$$P_{tot} = P'_{tot}$$
$$0 = m_L(-v_L) + m_R v_R$$
$$m_L v_L = m_R v_R$$

Plugging in our results from "Part B" Equations (1) and (2) we get:

$$m_L\sqrt{2\mu_L g d_L} = m_R\sqrt{2\mu_R g d_R}$$

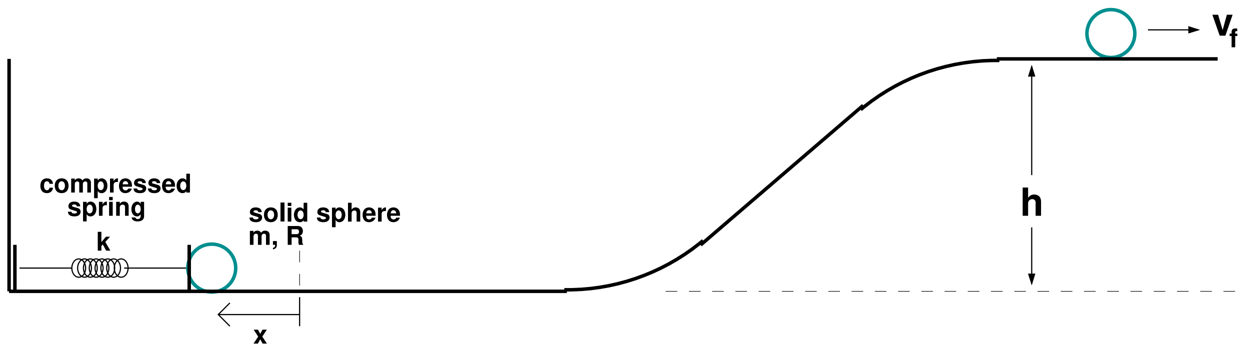
Solving for one the unknown: m_R:

$$m_R = m_L\sqrt{\frac{\mu_L d_L}{\mu_R d_R}}$$

And finality to get an expression for the total mass M we have:

$$M = m_R + m_L$$
$$M = m_L\sqrt{\frac{\mu_L d_L}{\mu_R d_R}} + m_L$$
$$\boxed{M = m_L\left(1 + \sqrt{\frac{\mu_L d_L}{\mu_R d_R}}\right)}$$

Problem Y01: Spring pushes rolling ball up hill (30 points)



A solid sphere of given mass m and radius R is placed against a spring of given constant k. The spring is compressed by a given displacement distance x. At this point the ball is released at rest. The spring pushes the ball so that it rolls without slipping or sliding with respect to the the horizontal surface. The ball subsequently rolls without slipping up a hill of given height h as shown.

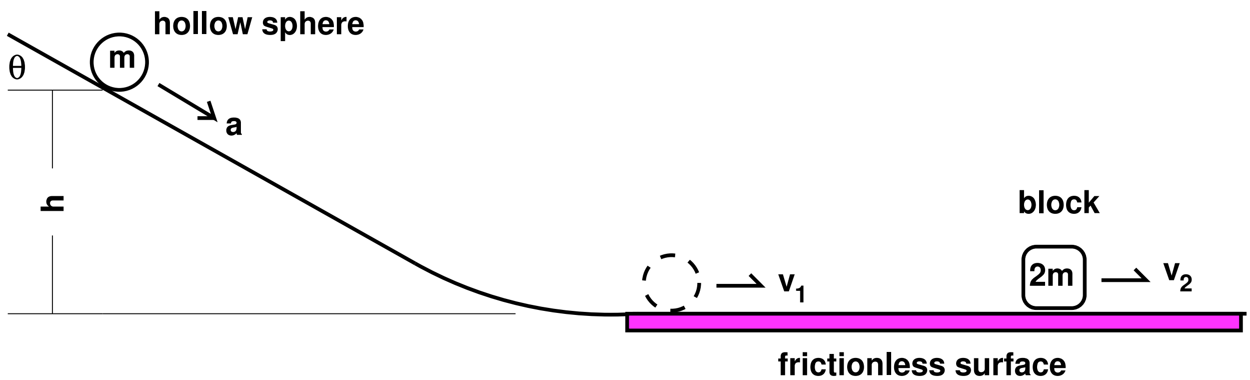
Assume that there is zero friction between the pad of the spring and the ball.

Part (a) (15 points) – Calculate the instantaneous linear acceleration of the ball at the instant immediately after release. Give your answer in terms of given parameters. Explain your work.

Part (b) (15 points) – Calculate the final speed v_f of the ball when it is rolling along the horizontal surface as shown. Give your answer in terms of given parameters. Explain your work.

Problem Y02: Hollow Sphere, Elastic Collision – (30 points)

A **hollow sphere** with given radius R and given mass m is released at rest on a ramp at a given angle θ and a given height h relative to the horizontal as shown. The sphere rolls without slipping or sliding until it reaches a ***frictionless*** horizontal surface. The sphere then collides with a block of given mass $2m$. The surface of the block is also frictionless. The collision is **totally elastic**. The block is at rest before the collision.

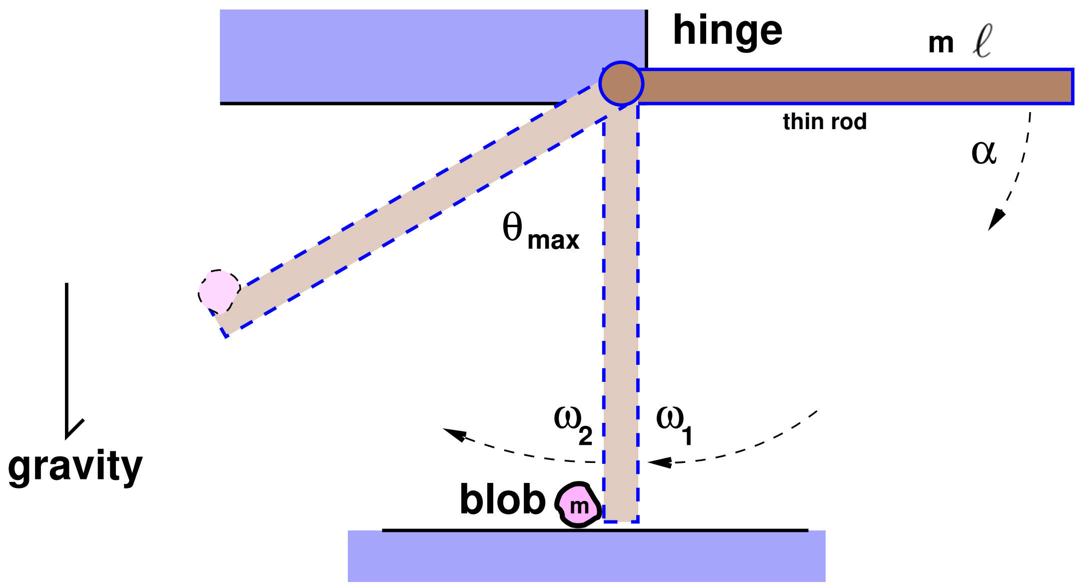


Part (a) – (10 points): What is the **linear acceleration**, $\boldsymbol{a}$, of the hollow sphere as it rolls down the ramp? Present your answer in terms of given parameters. Explain your work.

Part (b) – (10 points): What is the **linear speed**, $\boldsymbol{v_1}$, of the hollow sphere once it arrives to the horizontal surface? Present your answer in terms of given parameters. Explain your work.

Part (c) – (10 points): What is the **linear speed**, $\boldsymbol{v_2}$, of the block **after** the collision? Present your answer in terms of given parameters. Explain your work.

Problem Y03: Thin Rod and Clay Blob, Inelastic Collision



A thin **rod** of given length ℓ and given mass m is attached to an ideal hinge and released at rest in the horizontal position as shown. The rod swings down and the far end of the rod collides with a small **blob** of clay, also with given mass m, which was sitting at rest on a frictionless horizontal surface. The blob ***sticks*** to the rod. Assume that the size of the blob is small compared to the length of the rod.

Part (a): Calculate the **angular acceleration**, $\boldsymbol{\alpha}$, of the rod at the instant immediately after it is released. Present your answer in terms of given parameters. Explain your work. Hint: you do **not** need to calculate the value of the "Hinge Force" to work this problem.

Part (b): Calculate the **angular speed**, $\boldsymbol{\omega_1}$, of the thin rod in the instant ***just before*** the rod collides with the clay blob. Present your answer in terms of given parameters. Explain your work.

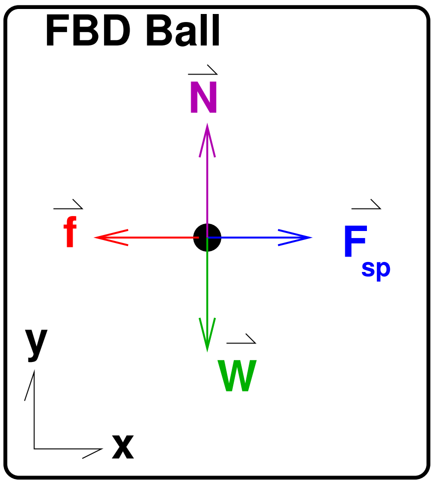
Part (c): Calculate the **angular speed**, $\boldsymbol{\omega_2}$, of the thin rod in the instant ***just after*** the rod collides with the clay blob. Present your answer in terms of given parameters. Explain your work.

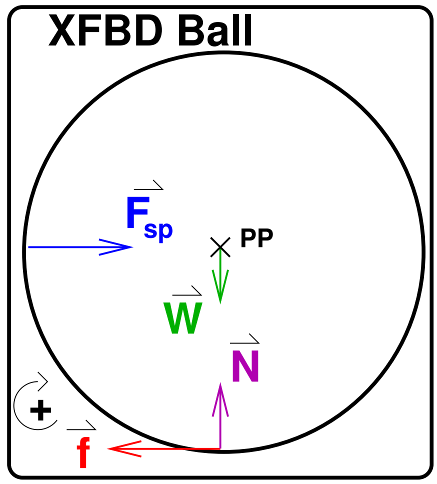
Part (d): Harder: Calculate the angle, $\boldsymbol{\theta_{max}}$, corresponding to the maximum angle that the rod reaches swinging up after the collision. Present your answer in terms of given parameters. Explain your work.

Solution to Problem Y01:

Part (a): (15 points)

Since we want an **acceleration** here we are motivated to consider applying **Newton's Laws**. We start with a **Free-Body-Diagram** as shown on the left figure below:





We note in particular that there is a **force of friction that points to the left.** We know such a force must exist for two reasons (1) this the the only force that will get the ball rolling and/or (2) we know that the ball is accelerating linearly (under the spring force). If the ball is rolling-without-slipping then it must also be accelerating rotationally as well. Note that this friction is *not* kinetic friction and is therefore unknown prior to applying Newton's Laws.

We apply **Newton's Second Law** in the horizontal component (note that the vertical component does nothing for us here):

$$F_x = ma_x$$

Using the FBD we have:

$$F_{sp} - f = ma_x$$

The Spring Force is known and given by Hooke's Law: $F_{sp} = kx$

$$kx - f = ma_x \tag{3}$$

This is as far as we can go with the algebra since we have *two unknowns*: f and a_x.

So next we consider the torques. We apply **Newton's Second Law in Rotational Form:**

$$\tau_{net} = I\alpha$$

We can work out the torques by writing up an **Extended Free-Body-Diagram** as shown in the right figure above. We have four forces and we need to sum up the torques due to each force:

$$\tau_{\vec{W}} + \tau_{\vec{N}} + \tau_{\vec{F_{sp}}} + \tau_{\vec{f}} = I\alpha$$

We use the definition of torque:

$$\tau_{\vec{F}} = rF\sin\phi$$

and apply this in turn to each of the four forces. Here $\vec{r}$ is the radial displacement vector *from* the pivot point (center of the ball) *to* the point of application of the force:

- $\tau_{\vec{W}}$: The Weight force is applied at the pivot point, so r is zero, so $\tau_{\vec{W}} = 0$.
- $\tau_{\vec{N}}$: The Normal force applied by the floor to the ball is applied in a direction that is *anti-parallel* to the radial vector $\vec{r}$, and so $\sin\phi = 0$ and so $\tau_{\vec{N}} = 0$.
- $\tau_{\vec{F_{sp}}}$: The Spring force applied to the ball is applied in a direction that is *anti-parallel* to the radial vector $\vec{r}$, and so $\sin\phi = 0$ and so $\tau_{\vec{F_{sp}}} = 0$.
- $\tau_{\vec{f}}$: The Friction Force is applied at a distance $r = R$ from the central pivot point and also applied at a right angle so that $\sin\phi = 1$, and so $\tau_{\vec{f}} = Rf$.

In other words, all of the forces except the friction contribute zero torque. So plugging these all back into our expression for Newton's Second Law:

$$0 + 0 + 0 + Rf = I\alpha$$

Then we plug in $I = \left(\frac{2}{5}\right) mR^2$ for a solid sphere and the **Rolling Constraint** that tells us $\alpha = \frac{a_x}{R}$ so:

$$Rf = \left(\frac{2}{5}\right) mR^2 \left(\frac{a_x}{R}\right)$$

$$f = \left(\frac{2}{5}\right) ma_x \tag{4}$$

So now we have two equations, two unknowns. Plug Equation (2) into Equation (1):

$$kx - \left(\frac{2}{5}\right) ma_x = ma_x$$

$$ma_x + \frac{2}{5} ma_x = kx$$

$$ma_x \left(1 + \frac{2}{5}\right) = kx$$

$$\boxed{a_x = \frac{5}{7}\left(\frac{kx}{m}\right)}$$

Guidelines for graders:

- **Perfect Work with proper invocation of Newton's Laws = 15 points.**
- **Physics correct but answers given in terms of r or f or other terms that do not corresponds to given parameters: 7 to 9 points.**
- **Correct application of Newton's Second Law in linear form but missing/incorrect application of torque: 5 to 7 points.**
- **Correct assertion of Newton's Second Law in both linear and rotational form, but incorrect calculation of individual torques: 8 to 12 points.**
- **Correct physics but algebra failure to calculate acceleration: 12 to 13 points.**
- **Incorrect or missing application of Rolling Constraint: 12 to 13 points.**
- **Incorrect or inappropriate physics ideas: Not more than 3 points.**

Solution to Problem Y01 Continues...

Part (b): (15 points)

In this problem the forces on the ball are all either **Conservative** (Weight and Spring force) or **Do No Work** (Normal and Rolling Friction). We note in particular that no non-conservative work is done by the friction force because at the point-of-contact, the force is not being applied over a displacement. The ball is (instantaneously) not moving at the point of contact.

So we define "Before" at the point of release, and then "After" is once we are on top of the hill. We have to contend with two different kinds of potential energy (potential energy due to Spring Force and potential energy due to Weight) and two different forms of kinetic energy (translational and rotational):

$$E_{tot} = E'_{tot}$$
$$U_{tot} + K_{tot} = U'_{tot} + K'_{tot}$$
$$U_{sp} + U_W + K_{tran} + K_{tran} = U'_{sp} + U'_W + K'_{tran} + K'_{tran}$$
$$\frac{1}{2}kx^2 + mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx'^2 + mgy' + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$$

Okay so for "Before" we only have spring potential energy: $x = x$, $y = 0$, $v = 0$, $\omega = 0$. For "After" we have no spring potential energy but weight potential energy: $y' = h$, $v' = v_f$ and $\omega' = \omega_f$:

$$\frac{1}{2}kx^2 + 0 + 0 + 0 = 0 + mgh + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

Finally we plug in $I = \left(\frac{2}{5}\right)mR^2$ for a solid sphere and the **Rolling Constraint** that tells us $\omega_f = \frac{v_f}{R}$ so:

$$\frac{1}{2}kx^2 = mgh + \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}\right)mR^2\left(\frac{v_f}{R}\right)^2$$

Algebra:

$$kx^2 = 2mgh + mv_f^2 + \left(\frac{2}{5}\right)mv_f^2$$
$$kx^2 = 2mgh + mv_f^2\left(1 + \frac{2}{5}\right)$$
$$kx^2 = 2mgh + mv_f^2\left(\frac{7}{5}\right)$$
$$mv_f^2\left(\frac{7}{5}\right) = kx^2 = 2mgh$$
$$v_f^2 = \frac{5}{7}\left(\frac{kx^2}{m} - 2gh\right)$$

$$\boxed{v_f = \sqrt{\frac{5}{7}\left(\frac{kx^2}{m} - 2gh\right)}}$$

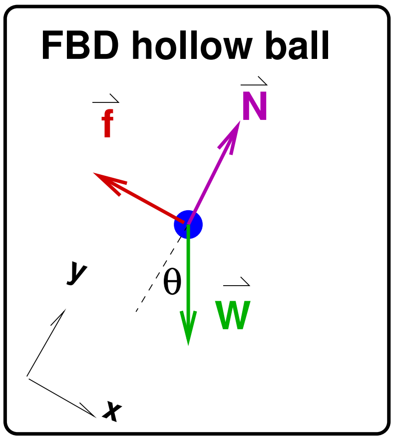
Guidelines for graders:

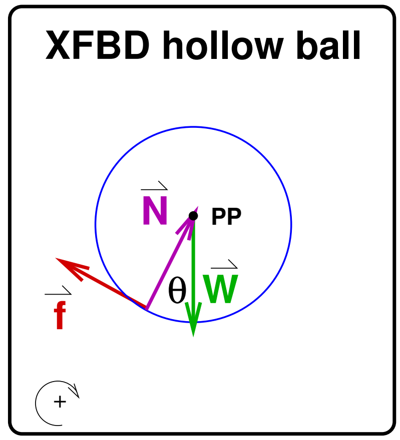
- **Perfect Work with proper invocation of Conservation of Energy including consideration and explanation of Conditions for Conservation of Energy = 15 points.**
- **Perfect Work with proper invocation of Conservation of Energy = 15 points.**
- **Correct answer but failure to correctly consider whether Conditions for Conservation of Energy are met: 13 points.**
- **Forgot one or more important energy terms, either potential or kinetic energy = 5 to 10 points.**
- **Correct answer but but missing substitutions to give final answer in terms of given parameters: 10 to 12 points.**
- **Incorrect or inappropriate physics ideas: Not more than 3 points.**

Solution to Problem Y02: (30 points)

Part (a) – (10 points):

So we want **Acceleration** so we will need to apply **Newton's Laws**. We know the acceleration is *down the ramp* but otherwise it remains unknown. We write down both **Free Body Diagram** and Extended Free Body Diagram for the hollow sphere:





Note that have a ***tilted coordinate system*** for our FBD, lined up with the linear acceleration. Note that there are ***three forces***, Weight, Normal, and Friction on both diagrams. Note that we choose a pivot point at the center-of-mass point.

We start with **Newton's Second Law** in the "x"-coordinate:

$$F_x = ma_x$$
$$W \sin\theta - f = ma_x$$
$$mg \sin\theta - f = ma_x \tag{5}$$

This is as far as we can go with this, since both f and a_x are unknown.

Next, we apply **Newton's Second Law for Rotation:**

$$\tau_{net} = I\alpha$$

There are three forces we need to consider:

$$\tau_W + \tau_N + \tau_f = I\alpha$$

We use the XFBD and the **Definition of Torque:** $\tau_F = rF\sin\phi$ to calculate the torques:

- $\tau_W = 0$ since the Weight is applied at the Pivot Point ($r = 0$).
- $\tau_N = 0$ since the Normal force is applied parallel to the radial vector ($\sin\phi = 0$).
- $\tau_f = Rf$ since the friction is applied at a right angle relative to the radial vector.

Therefore:

$$Rf = I\alpha$$

We plug in I for a hollow sphere $I = \frac{2}{3}mR^2$ and we use the **Rolling Constraint** to say that $\alpha = \frac{a_x}{R}$ and so:

$$Rf = \left(\frac{2}{3}mR^2\right)\left(\frac{a_x}{R}\right)$$

$$f = \frac{2ma_x}{3} \tag{6}$$

We now have ***two equations and two unknowns. The rest is algebra.*** Plugging Equation (3) into (2) we get:

$$mg\sin\theta - \frac{2ma_x}{3} = ma_x$$

$$\frac{5ma_x}{3} = mg\sin\theta$$

$$\boxed{a_x = \frac{3g\sin\theta}{5}}$$

Guidelines for graders for Part (a) (worth 10 points):

- Perfect score with explanation including Newton's Second Law (N2L, N2LRot) and appropriate Free Body Diagrams (FBD, XFBD) = 10 points.
- Friction backwards and not caught/fixed/noted by the end, but otherwise correct = 8 points.
- Correct concepts (N2L,N2LRot) applied but incorrect handling Rolling Constraint = 8 points.
- Getting to Correct Equation (2) and (3) (or equivalent) but not completing algebra and/or making some algebra mistakes = 8 points
- Missing friction and/or failure to consider friction = at most 5 points.
- Correctly handing N2L $F = ma$ but mishandling N2LRot $\tau = I\alpha$, between 4 and 6 points, depending on how torques are mis-calculated.
- Incorrect physics concept (e.g., Energy, Angular Momentum or something else, at most 2 points.

Part (b) – (10 points):

Here we are asked for the linear speed, so we want to consider applying **Conservation of Mechanical Energy**.[4] The three forces on the hollow sphere are **Weight *which is Conservative***, and **The Normal Force which *does zero work***, and **static friction** which at the point of contact also ***does zero work***, so the **condition is met**. Here we choose the reference position for kinetic energy as center-of-mass point, which means we need to describe the motion of the sphere in terms of both *translational* and *rotational* kinetic energy.

So the "Before" corresponds to release at rest, the sphere at $y = y$. The "After" corresponds to the sphere at the bottom of the hill, so $y' = 0$. We write down **Conservation of Mechanical Energy**.

$$E_{tot} = E'_{tot}$$
$$U + K_{trans} + K_{rot} = U' + K'_{trans} + K'_{rot}$$
$$mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgy' + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$$

Plugging in:

$$mgh + 0 + 0 = 0 + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$$

Using the known value for I and the **Rolling Constraint** to tell us $\omega' = \frac{v'}{R}$:

$$mgh = \frac{1}{2}mv'^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v'}{R}\right)^2$$
$$\frac{1}{2}mv'^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v'}{R}\right)^2 = mgh$$
$$mv'^2 + \left(\frac{2}{3}mR^2\right)\left(\frac{v'}{R}\right)^2 = 2mgh$$
$$mv'^2 + \left(\frac{2}{3}\right)mv'^2 = 2mgh$$
$$\left(\frac{5}{3}\right)v'^2 = 2gh$$
$$v'^2 = \frac{6gh}{5}$$
$$\boxed{v_1 = v' = \sqrt{\frac{6gh}{5}}}$$

Solution to Part (b) continues next page...

[4]**Important Completely Acceptable Alternate Solution: Apply *Constant Acceleration Kinematics*.** Given acceleration from Part (a), use 1-D kinematics of constant acceleration applied over a fixed distance $D = \frac{h}{\sin\theta}$. You will get the same final answer.

Guidelines for graders for Part (b) (worth 10 points):

- Perfect score with explanation including Conservation of Mechanical Energy and also consideration of Condition for CoME = 10 points
- Perfect work but no Consideration of Condition for CoME = 9 points
- Not correctly contending with **both** translational **and** rotational forms of kinetic energy = at most 7 points
- Correct concept (CoME) but problems with with algebra or other manipulations that result in a final answer that does not have the correct units, between 6 and 8 points.
- Correct concept (CoME) but incorrect set up, mixing up linear and rotational speeds, incorrectly applying Rolling Constraint, etc. at most between 5 and 6 points.
- Incorrect physics concept (e.g., Newton's laws, Angular Momentum, and/or the "Rolling Constraint" or something else, at most 2 points.

Part (c) – (10 points):

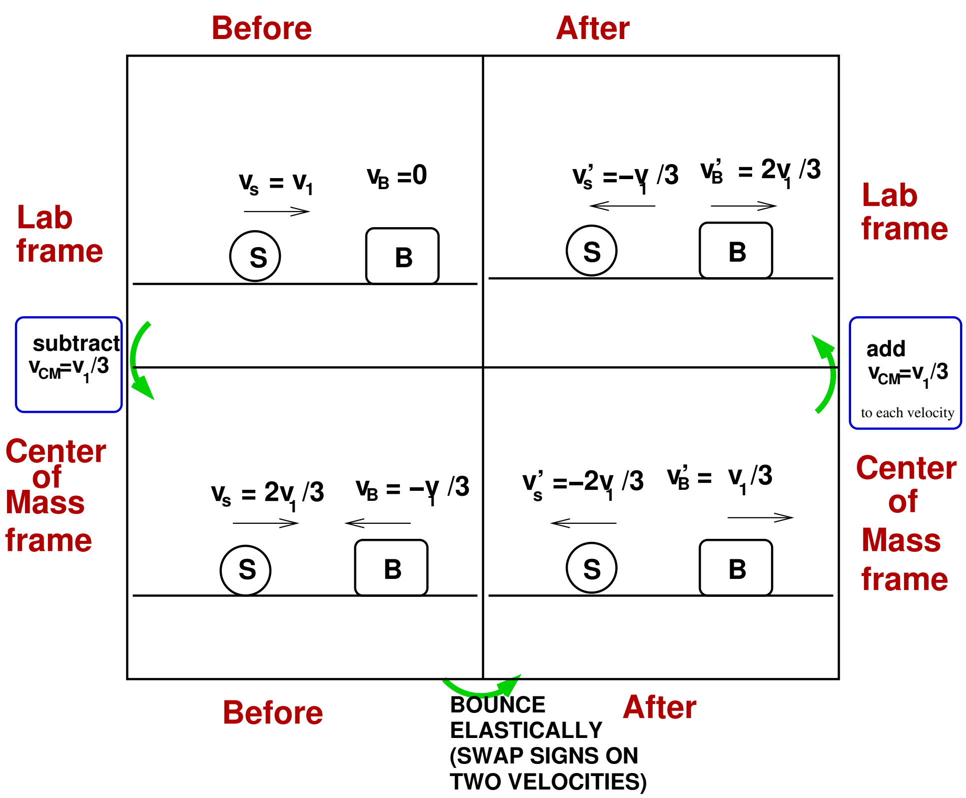
This is a **Totally Elastic Collision** which means that **Kinetic Energy is Conserved**.

Note that since there is **zero friction**, the collision cannot impart a **torque** on the sphere, and so ***the rotational kinetic energy of the sphere cannot change***. This means we only need consider the linear (translational) kinetic energies.

There are many methods for contending with the **Totally Elastic collision**. It is completely acceptable to use the so-called "Master equations" which include $v_1 - v_2 = v_2' - v_1'$ or **any other physically valid method that conserves kinetic energy.** Here we apply the "4-box method" where we calculate the **center-of-mass velocity**:

$$v_{cm} \equiv \frac{m_a v_a + m_b v_b}{m_a + m_b} = \frac{mv_1 + 0}{m + 2m} = \frac{v_1}{3}$$

We apply the **4-box method as follows:**



This shows that the final velocity of the block (as measured in the lab frame) is given by:

$$v_2 = v_B = \frac{2v_1}{3}$$

Plugging in our result from Part (b) we get:

$$\boxed{v_2 = v_B = \frac{2}{3}\sqrt{\frac{6gh}{5}} = \sqrt{\frac{8gh}{15}}}$$

Guidelines for graders for Part (c) (worth 10 points):

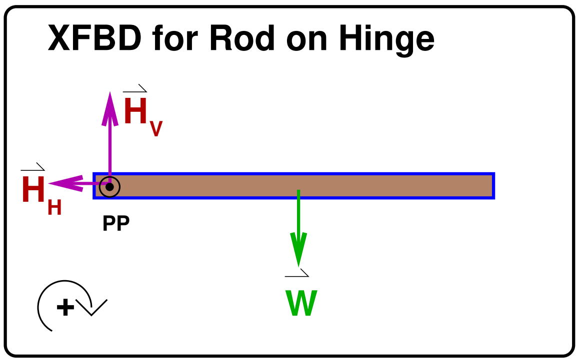
- Perfect score with explanation making it clear that **Kinetic Energy Must be Conserved** = 10 points
- Incorrectly asserting that rotational kinetic energy is converted into linear kinetic energy here: = at most 8 points
- Correct concept (CoKE) but problems with with algebra or other manipulations that result in a final answer that does not have the correct units, between 6 and 8 points.
- Correct concept (CoKE) but incorrect set up, mixing up different forms of energy/work/forces/ etc. 5 and 6 points.
- Incorrect physics concept (e.g., Newton's laws, Angular Momentum, and/or the "Rolling Constraint" or something else, at most 2 points.

Solution to Problem Y03:

Solution to Problem Y03 Part (a):

Since we are asked for angular acceleration, we want to apply **Newton's Second Law for Rotations:**

First, we set up am **Extended Free Body Diagram** (XFBD) for the rod just after it is released:



We start with the equation for **Newton's Second Law for Rotations:**

$$\tau_{net} = I\alpha$$

The net torque can be written out in terms of the torque associated with each force:

$$\tau_{H_H} + \tau_{H_V} + \tau_W = I\alpha$$

We use the **Definition of Torque** to calculate the terms:

$$\tau_F \equiv rF\sin\phi$$

Since the pivot point is on the hinge, ***the torque due to the Hinge forces is zero.***

This means the only non-zero torque is due to Weight. Here $F = mg$, $r = \frac{\ell}{2}$ and $\phi = 90$ degrees, so:

$$\frac{mg\ell\sin 90^\circ}{2} = I\alpha$$

Here we know I for a thin rod with the pivot point about one end:

$$I_{rod,end} = \frac{1}{3}m\ell^2$$

So:

$$\frac{mg\ell\sin 90^\circ}{2} = \frac{1}{3}m\ell^2\alpha$$

Solving for α

$$\frac{1}{3}m\ell^2\alpha = \frac{mg\ell}{2}$$
$$\alpha = \frac{3mg\ell}{2m\ell^2}$$
$$\boxed{\alpha = \frac{3g}{2\ell}}$$

Solution to Problem Y03 Part (b):

Here we are asked for the angular speed, so we want to consider applying **Conservation of Mechanical Energy**. The only two forces on the rod are **Weight *which is Conservative***, and **The Hinge Force(s) which *does zero work***, so the **condition is met**. Here we choose the reference position for kinetic energy as the point of the rod that is in contact with the hinge, which means we can describe the motion of the rod as *purely rotational*.

So the "Before" corresponds to release at rest, rod in the horizontal position, the "After" corresponds to the rod in the vertical position just before it hits the clay blob. We write down **Conservation of Mechanical Energy**.

$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy_{cm} + \frac{1}{2}I\omega^2 = mgy'_{cm} + \frac{1}{2}I\omega'^2$$

We have to choose a ***zero point reference*** for the Potential Energy: the ***usual convention is to define*** $y = 0$ ***at the hinge pivot point***.

So the **Before** situation corresponds to $y_{cm} = 0$ and $\omega = 0$. The **After** situation corresponds to $y_{cm} = -\frac{\ell}{2}$ and $\omega = \omega_1$. Plugging these in:

$$0 + 0 = mg\left(-\frac{\ell}{2}\right) + \frac{1}{2}I\omega_1^2$$

Plugging in for I and doing algebra, solving for ω_1 we get:

$$\frac{1}{2}I\omega_1^2 = mg\left(\frac{\ell}{2}\right)$$
$$\omega_1^2 = \frac{mg\ell}{I}$$
$$\omega_1^2 = \frac{mg\ell}{\frac{1}{3}m\ell^2}$$
$$\omega_1^2 = \frac{3g}{\ell}$$
$$\boxed{\omega_1 = \sqrt{\frac{3g}{\ell}}}$$

Solution to Problem Y03 Part (c):

Now we have a **collision**. In this case we can apply **neither** Conservation of Mechanical Energy (because the collision is totally inelastic) **nor** Conservation of Linear Momentum (because the system is not isolated during the collision: there is an unknown Hinge force applied). Applying either Conservation of Energy *and/or* Conservation of Linear Momentum corresponds to a major conceptual error for this problem.

However, if we select the ***Hinge as the pivot point*** then (by definition) the torque due to the Hinge force is zero and the torque due to any vertical forces are also zero (because such forces are parallel to the radial vector). Therefore ***we meet the Condition for the application of*** **Conservation of Angular Momentum:**

$$L_{tot} = L'_{tot}$$

Two objects in the collision, the **rod** and the **clay blob**:

$$L_r + L_c = L'_r + L'_c$$

Before the collision $L_r = I\omega_1$ and $L_c = 0$ (at rest). **After** the collision $L_r = I\omega_2$ and $L_c = m\ell^2\omega_2$ (this is because the clay blob now moves with ***Uniform Circular Motion***). Plugging these in along with the rotational inertia for the rod, we get:

$$I\omega_1 + 0 = I\omega_2 + m\ell^2\omega_2$$

Solving for ω_2:

$$I\omega_2 + m\ell^2\omega_2 = I\omega_1$$

$$\omega_2(I + m\ell^2) = I\omega_1$$

$$\omega_2 = \frac{I\omega_1}{I + m\ell^2}$$

$$\omega_2 = \omega_1\left(\frac{\frac{1}{3}m\ell^2}{\frac{1}{3}m\ell^2 + m\ell^2}\right)$$

$$\omega_2 = \omega_1\left(\frac{\frac{1}{3}}{\frac{4}{3}}\right)$$

$$\omega_2 = \frac{\omega_1}{4}$$

$$\boxed{\omega_2 = \frac{1}{4}\sqrt{\frac{3g}{\ell}}}$$

Solution to Problem Y03 Part (d):

Finally, after the collision, we can go back to **Conservation of Mechanical Energy** (same conditions apply again). Now, however, both the Potential and Kinetic energies needs to include both the clay blob and the rod:

$$E_{tot} = E'_{tot}$$

A convenient "short cut" is to note that since the rod and the blob move together as one *rigid body* system we can write down the Potential energy in terms of the center-of-mass of the whole system:

$$y_{sys} = \frac{m_r y_r + m_c y_c}{m_r + m_c} = \frac{y_r + y_c}{2}$$

Similarly, kinetic energy for the system as a whole as purely rotational, with a rotational inertia given as the sum of the that for the rod and blob:

$$I_{sys} = I_r + I_c = \frac{1}{3}m\ell^2 + m\ell^2 = \frac{4}{3}m\ell^2$$

So:[5]

$$U_{sys} + K_{sys} = U'_{sys} + K'_{sys}$$

$$m_{sys} g y_{sys} + \frac{1}{2} I_{sys}\omega^2 = m_{sys} g y'_{sys} + \frac{1}{2} I_{sys}\omega'^2$$

For the **Before** situation,

$$y_{sys} = \frac{y_r + y_c}{2} = \frac{-\left(\frac{\ell}{2}\right) + (-\ell)}{2} = \frac{\frac{-3}{2}\ell}{2} = -\frac{3\ell}{4}$$

The **Before** angular speed is ω_2 which we have already calculated.

For the **After** situation, the position the same but times $\cos\theta_{max}$:

$$y'_{sys} = -\frac{3\ell}{4}\cos\theta_{max}$$

The **After** angular speed is **zero** (because we have reached the maximum height).

Part (d) Solution continues next page....

[5] A completely acceptable alternative calculation is to determine the Potential and Kinetic energies for both the rod and the blob separately:

$$U_r + U_c + K_r + K_c = U'_r + U'_c + K'_r + K'_c$$

If done correctly, this will lead to the same answer.

Part (d) Solution continues here:

Plugging all of this in (not forgetting that $m_{sys} = 2m$):

$$(2m)g\left(-\frac{3\ell}{4}\right) + \frac{1}{2}\left(\frac{4}{3}m\ell^2\right)\omega_2^2 = (2m)g\left(-\frac{3\ell}{4}\cos\theta_{max}\right) + 0 \tag{7}$$

Solving for θ_{max} and doing an ***annoying amount of algebra:***

$$2mg\left(-\frac{3\ell}{4}\right)\cos\theta_{max} = 2mg\left(-\frac{3\ell}{4}\right) + \frac{1}{2}\left(\frac{4}{3}m\ell^2\right)\omega_2^2$$

$$2mg\left(\frac{3\ell}{4}\right) - 2mg\left(\frac{3\ell}{4}\right)\cos\theta_{max} = \frac{1}{2}\left(\frac{4}{3}m\ell^2\right)\omega_2^2$$

$$2mg\left(\frac{3\ell}{4}\right)(1-\cos\theta_{max}) = \frac{1}{2}\left(\frac{4}{3}m\ell^2\right)\omega_2^2$$

$$\frac{3mg\ell}{2}(1-\cos\theta_{max}) = \frac{2m\ell^2}{3}\omega_2^2$$

$$(1-\cos\theta_{max}) = \frac{\frac{2m\ell^2}{3}\omega_2^2}{\frac{3mg\ell}{2}}$$

$$(1-\cos\theta_{max}) = \frac{4\ell\omega_2^2}{9g}$$

$$(1-\cos\theta_{max}) = \frac{4\ell\left(\frac{1}{4}\sqrt{\frac{3g}{\ell}}\right)^2}{9g}$$

$$(1-\cos\theta_{max}) = \frac{4\ell\left(\frac{3g}{16\ell}\right)}{9g}$$

$$(1-\cos\theta_{max}) = \frac{1}{12}$$

$$\cos\theta_{max} = 1 - \frac{1}{12}$$

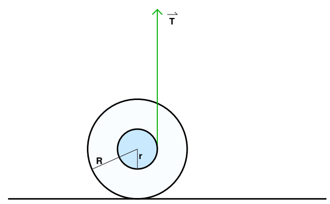
$$\boxed{\theta_{max} = \arccos\left(\frac{11}{12}\right) = 23.6 \text{ degrees}}$$

Phew!

Z01: Pulling a Spool of Rope

It is possible to do an impressive demonstration where a string is wound up on a spool as shown. The spool axle has radius r and the flanges have radius R as shown. The spool has mass m. The rotational inertia of the entire spool about its center-of-mass is given as I_s.

Part (a)

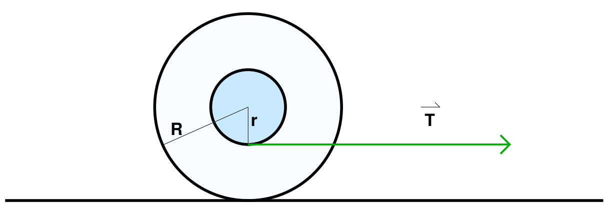


Part (a)

Suppose a given tension force $\vec{T}$ is applied to the string vertically as shown. Assume that the force $\vec{T}$ is small so that the spool not does jump up and does not skid across the floor but instead the spool *rolls without slipping*.

(a) Draw a Free Body Diagram (FBD) and *also* draw an Extended Free Body Diagram (XFBD) for the spool. Carefully indicate all of the forces on each diagram. Calculate the value of the translational acceleration of the spool along the floor. Which way does the spool move? Does it wind up or unwind?

Part (b)

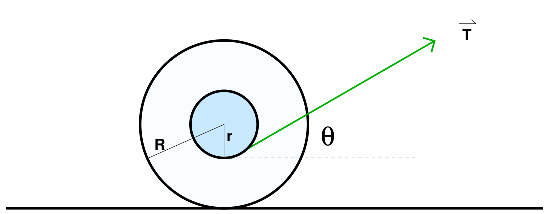


Part (b)

Now suppose a given tension force $\vec{T}$ is applied to the string to the right as shown. Assume that the force $\vec{T}$ is small enough so that the spool does not skid across the floor but instead it *rolls without slipping*.

(b) Draw a Free Body Diagram (FBD) and *also* draw an Extended Free Body Diagram (XFBD) for the spool. Carefully indicate all of the forces on each diagram. Calculate the value of the translational acceleration of the spool along the floor. Which way does the spool move now? Does it wind up or unwind?

Part (c)

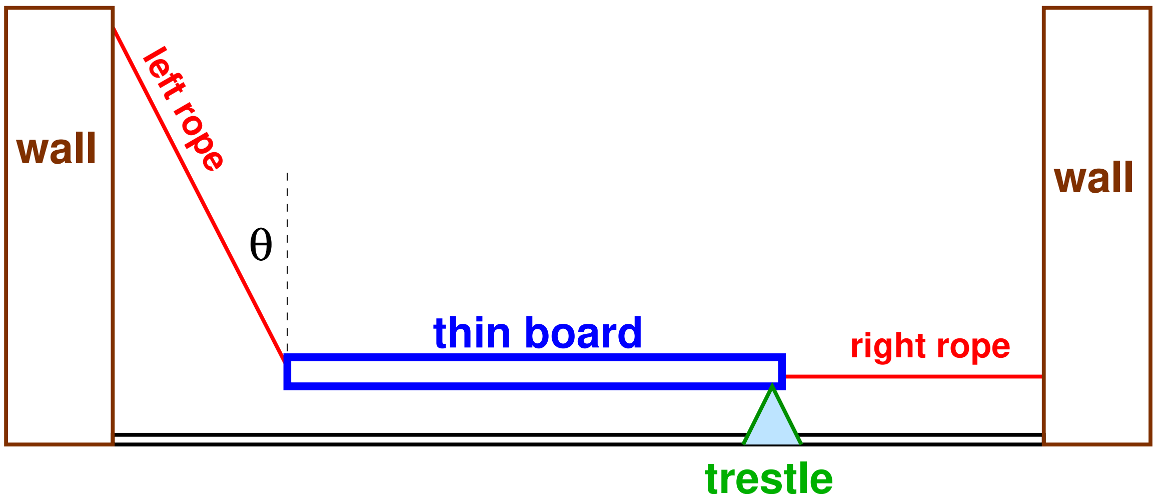


Part (c)

Now suppose a given tension force $\vec{T}$ is applied to the string at some special angle θ. Assume that the force $\vec{T}$ is small enough so that the spool does not slip or skid on the table.

(c) What is the special angle θ that we should choose so that the spool *neither* winds nor *unwinds*? In other words, what angle corresponds to having the spool stay motionless in one place regardless of the tension T applied? Hint: there is a very elegant way to solve this problem by choosing a pivot point that is not the center of the spool.

Problem Z02: A Statics Problem (30 points)



A thin board of given length L and given mass M is suspended at rest on two ideal ropes and an ideal trestle as shown above. The left rope is attached to the board at a given angle θ as shown. The right rope is horizontal. The trestle applies a vertical force to the board at the point just (barely) onto the right end of the board. Assume that there is no friction.

Determine the magnitude of each and every individual force applied to the board. Be sure to draw a complete and properly labeled Free-Body-Diagram (FBD) and also a complete and properly labeled Extended-Free-Body-Diagram (XFBD) for the board. Express your answer in terms of the given parameters only. Explain your answer. Important: you must explain and justify any physics concepts that you are using here. Hint: your choice of a good pivot point will make the problem much easier.

Problem Z03: Apply a Bridge Concept – (30 points)

Consider a simple glass window pane installed on the front of a house which is vulnerable to breakage according to the following rule:

> **If any external force applied to the center of the glass window pane exceeds a certain given threshold force, then the glass will invariably break.**

Suppose we consider two situations where a ball with a given horizontal speed and a given mass collides squarely with the middle of the window pane:

- **Case I:** The ball collides totally ***inelastically*** with the window pane.
- **Case II:** The ball collides very nearly totally ***elastically*** with the window pane.

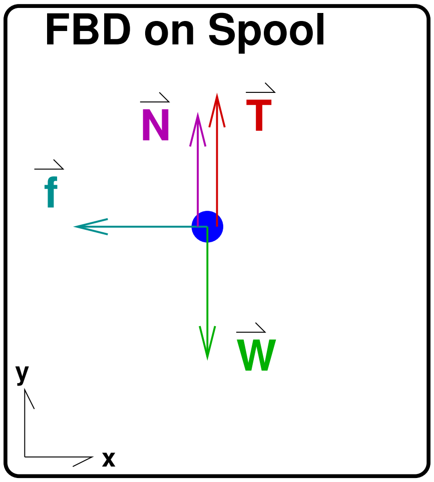
Which of these two collisions, Case I or Case II, is *more likely* to break the glass window pane? Explain your answer.

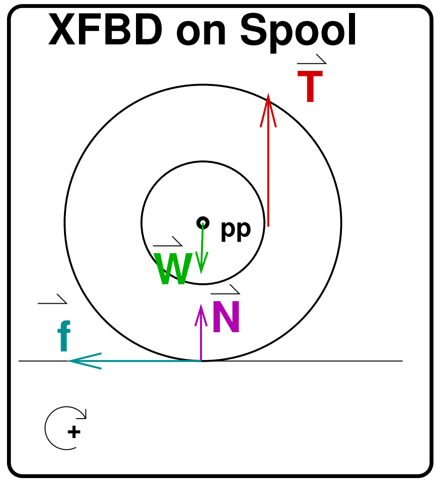
Please note: In each case the mass of the ball, m, is the same, the incoming speed of the ball, v, is the same, and most importantly **the duration of the collision, Δt, is the same.** The only difference between the two cases is that the first collision is *totally inelastic* while the second collision is *very nearly totally elastic*. When we say "very nearly" what we mean is that you should assume the energy lost in barely breaking/cracking the glass is small compared to kinetic energy of the ball. Assume that the glass window pane is *firmly held in place* by a surrounding wooden frame so that both during and after the collision the window pane does not move or bend significantly.

Important: This is a *Conceptual Question*. You *absolutely must* make a Coherent Logical Physics Argument based on the correct Physics Concept here to justify your answer. Simply guessing and/or providing an explanation that is not clearly based on one or more specific physics concepts is not sufficient. You **must** explain your reasoning and you must use **Physics Concepts** (and any applicable associated mathematical equations/relations). **Anything that approaches "just randomly guessing" here will earn zero points.**

Solution to Problem Z01:

Solution to Problem 5.





Part (a):

We start by considering the FBD and the XFBD for the spool. Here we choose the center of the spool as the "pivot point" and we define positive rotation clockwise, to match positive translation to the right. We note that the friction must point to the left because if there were no friction the wheel would respond to the tension by spinning in the counter-clockwise direction which would result in the surface of the spool moving to the right with respect to the table.

We start with **Newton's Second Law** in the x-component:

$$F_x = ma_x$$
$$-f = ma_x$$

This is as far as we can go. The friction force and the acceleration are both unknown.

Note that we could do the vertical component of Newton's Second Law but this does not help us much here because it just add two more unknowns (Normal and Tension) and one more equation.

Next we write down **Newton's Second Law for Torques** working in the "clockwise" component:

$$\tau_{net} = I\alpha$$

We note that the forces of Weight and Normal do not contribute to the torque as calculated at the pivot point (labeled PP). So the torques are due only to friction and tension. Both forces apply perpendicular to the radius vector relative to the pivot point: In our definition here, the torque due to tension is negative and the torque due to friction is positive:

$$-rT + Rf = I\alpha$$

We apply the rolling constraint $a_x = R\alpha$ to eliminate α:

$$-rT + Rf = \frac{Ia_x}{R}$$

$$-rRT + R^2 f = Ia_x$$

And plug in our result from the regular FBD to eliminate the unknown friction:

$$-rRT + R^2(-ma_x) = Ia_x$$

$$-rRT + -mR^2 a_x = Ia_x$$

$$Ia_x + mR^2 a_x = -rRT$$

$$a_x(I + mR^2) = -rRT$$

$$\boxed{a_x = -\frac{rRT}{I + mR^2}}$$

The spool has negative acceleration corresponding to motion to the $\boxed{\textbf{left}}$. The spool **unwinds**.

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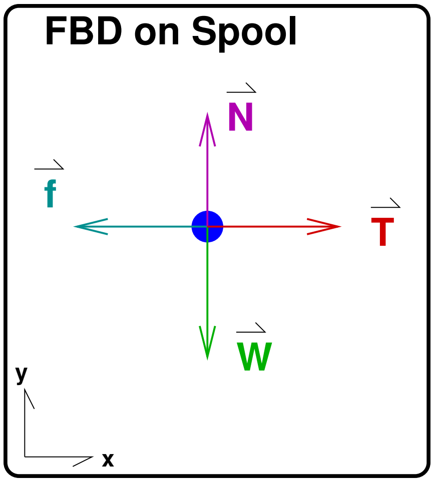
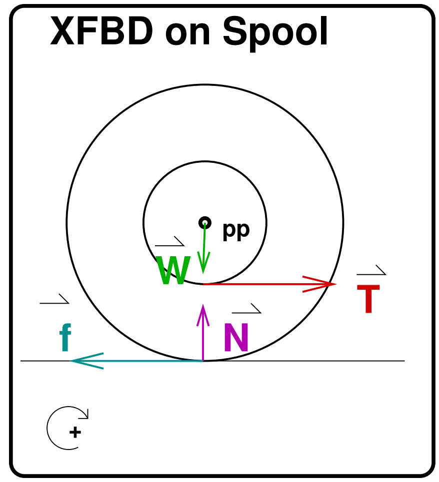
Part (b)





We start with **Newton's Second Law** in the x-component:

$$F_x = ma_x$$
$$T - f = ma_x$$

This is as far as we can go. The force T is given, but the friction force and the acceleration are both unknown.

Now we consider rotation. In this configuration, the XFBD is re-written with the result that the tension is applied to the right and now contributes a negative torque instead of a positive one. We work the **2nd Law for Torques**:

$$-rT + Rf = I\alpha$$
$$-rT + Rf = \frac{Ia_x}{R}$$
$$-rRT + R^2 f = Ia_x$$
$$-rRT + R^2(T - ma_x) = Ia_x$$
$$T(R^2 - rT) - R^2 ma_x = Ia_x$$
$$Ia_x + R^2 ma_x = T(R^2 - rR)$$
$$a_x(I + mR^2) = T(R^2 - rR)$$
$$\boxed{a_x = \frac{T(R^2 - rR)}{I + mR^2}}$$

Since for a spool we have the condition that $r < R$ it must be true that $R^2 - rR > 0$, this means the spool has positive acceleration and will move to the $\boxed{\textbf{right}}$. The spool **winds up**.

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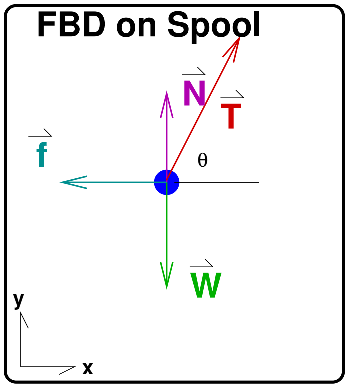
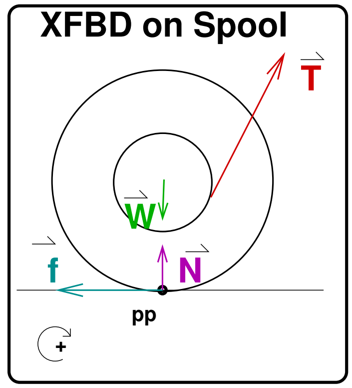
Part (c)





There are several ways to tackle this problem. One approach that is completely acceptable is to proceed as we did in Parts (a) and (b), to work out forces and torques with respect to a pivot point selected at the center-of-mass point of the spool. However, there is a simple and elegant solution we can find quickly if we select the **pivot point as the point on the spool that is in contact with the floor**. This is perfectly valid pivot point, especially given that the spool will not be rotating. This means that all of the torques and all of the forces must total to zero.

Consider the XFBD diagram above showing the four forces applied to the spool. We ask the question: given the pivot point as shown, what is the torque due to each force. Force-by-force:

- For the **Weight** force, the torque is defined as $\vec{\tau}_{\vec{W}} = \vec{r} \times \vec{W}$ where here the vector $\vec{r}$ runs from the pivot point to locate where the Weight is applied (at the position of the center of the spool). Since both $\vec{r}$ and $\vec{W}$ are directed vertically (they are parallel) the cross product is zero. Therefore **the torque due to Weight is zero.**

- The **Normal** force is applied *at the pivot point*. Therefore $\vec{\tau}_{\vec{N}} = \vec{r} \times \vec{N}$ where $\vec{r}$ has zero length. Therefore **the torque due to the Normal force is zero.**

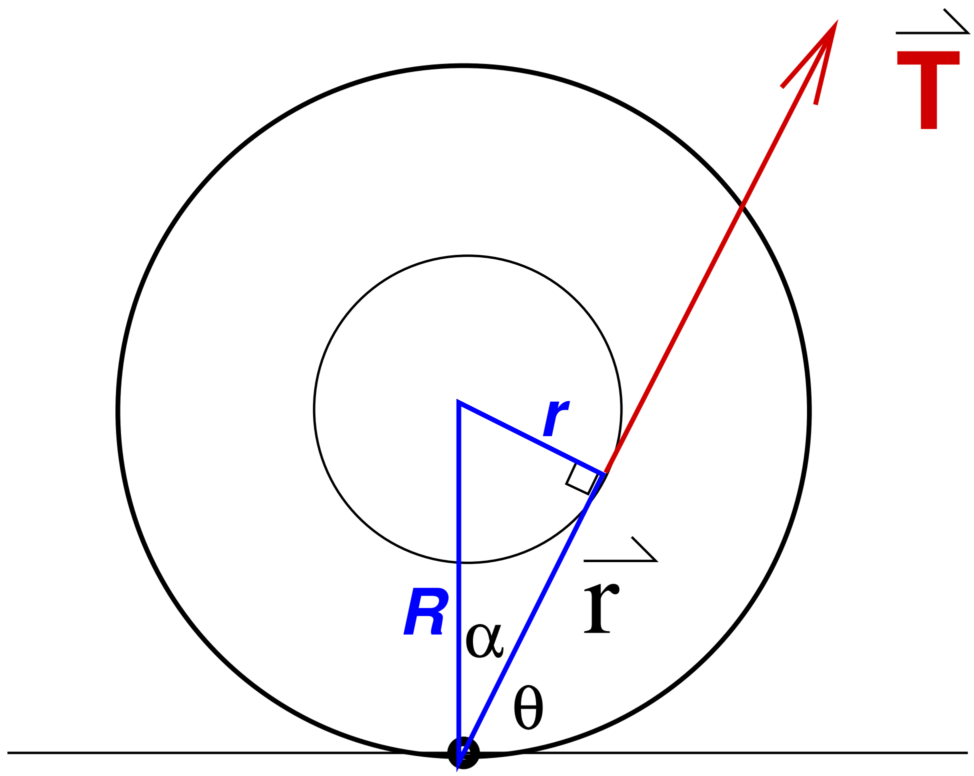
- The **Friction** force is applied *at the pivot point*. Therefore $\vec{\tau}_{\vec{f}} = \vec{r} \times \vec{f}$ where $\vec{r}$ has zero length. Therefore **the torque due to the Friction force is zero.**

So we conclude that the torque due to Weight, Normal, and Friction is zero. Given that the total torque must be zero, this means that the torque due to the **Tension** must be zero. But how do we ensure that this is true? Well we need to select a particular angle θ so that this is true.

In order to have zero torque, the tension vector $\vec{T}$ must be applied at an angle that is **parallel** to the angle of the radius angle $\vec{r}$. This is show in the XFBD. Any other angle will result in a non-zero cross-product and therefore a non-zero net torque on the spool.

Solution continues next page...

To determine the angle θ that makes $\vec{T}$ parallel to $\vec{\mathrm{r}}$, we zoom on geometry of the spool:



We notice the right-triangle here, with

$$\sin\alpha = \frac{r}{R}$$

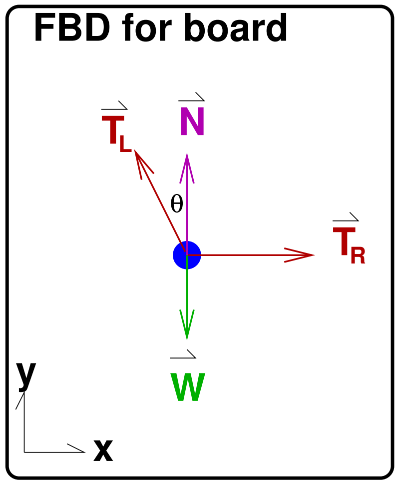
$$\cos(90^\circ - \alpha) = \frac{r}{R}$$

$$\cos\theta = \frac{r}{R}$$

$$\boxed{\theta = \arccos\left(\frac{r}{R}\right)}$$

Solution to Problem Z02:

This is a **statics problem** which mean that the net force and the net torque are both zero. Let's start with the **Free Body Diagram** for the board:



We see there are precised ***four forces*** on the board, the Weight force (down), the Normal force (up) due to the trestle and two Tension forces.[6] Here we have labeled the Tension force applied to the Left end of the board as $\vec{T}_L$ and the Tension force applied to the right end of the board as $\vec{T}_R$. We have chosen a standard coordinate system that is aligned with most of the forces.

We now apply **Newton's Second Law** one component at a time starting with the vertical coordinate:

$$F_y = Ma_y$$
$$T\cos\theta + N - W = 0$$
$$T\cos\theta + N - Mg = 0 \tag{8}$$

This is as far as we can go with this expression. One equation, two unknowns (N and T_L).

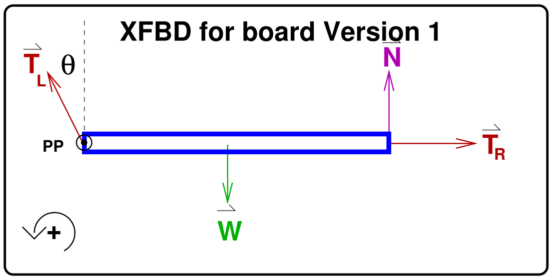
For the horizontal coordinate we get:

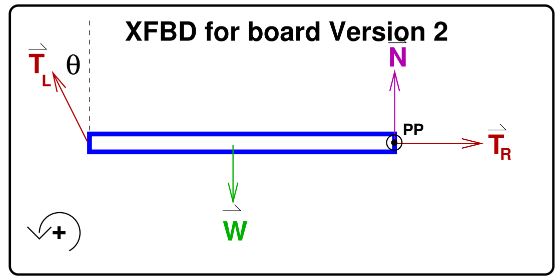
$$F_x = Ma_x$$
$$T_R - T_L\sin\theta = 0 \tag{9}$$

Again other equation with two unknowns T_R and T_L.

[6]Several students used "complicated" indices for the forces indicated. For example, the Normal force was often labeled $\vec{N}_{BT}$ instead of just $\vec{N}$. Or the left Tension force was labeled something like: $\vec{T}_{BLR}$ which stands for "Tension on Board due to Left Rope" or something. While including all of these indices is not incorrect, it is a big waste of time and effort. There is only one Normal force in this problem and only one body that it applies to, so simply labeling it $\vec{N}$ is completely sufficient. Likewise there are only two tension forces, and a single index is adequate to distinguish these. Of course using for example $\vec{T}_1$ and $\vec{T}_1$ instead of $\vec{T}_L$ and $\vec{T}_R$ is completely acceptable.

So now we need to contend with the rotational quantities. We start with an **Extended Free Body Diagram**. Here are two versions of the same XFBD – both are correct. The only difference is the choice of Pivot Point





Here we have placed the four forces where they are applied. In Version 1 one the left we have chosen a pivot point on the left end of the board. This is probably the best choice in terms of simple algebra and trigonometry. Version 2 shows the FBD with a pivot point on the right end of the board. This is also a perfectly acceptable choice. We will work the solution for both choices here.[7]

XFBD Version 1 Solution: We apply **Newton's Second Law in Rotational Form:**

$$\tau_{net} = I\alpha$$

Since the board is static, the angular acceleration is zero:

$$\tau_{\vec{N}} + \tau_{\vec{T}_R} + \tau_{\vec{T}_L} + \tau_{\vec{W}} = 0$$

We use the Version 1 XFBD and our **Definition of Torque** ($\tau_{\vec{F}} \equiv rF\sin\phi$) to calculate each of the four torques.

- Since we have chosen a pivot point on the left end, the radial distance to point of application of the left tension force is zero so the left rope contributes zero torque.
- The radial distance from the Pivot Point to the right tension force is the length of the board. The angle ϕ here, however, is the angle between the radial vector and the tension vector which is $\phi = 0°$ so $\sin\phi = \sin(0°) = 0$. So the torque due to the right tension force is zero.
- The radial distance from the Pivot Point to the Normal force applied by the trestle is the length of the board. The angle ϕ here is the angle between the radial vector and the Normal vector which is $\phi = 90°$ so $\sin\phi = \sin(= 90° = 1$. So the torque due to the Normal force is $\tau_{\vec{N}} = LN$. This torque is directed in the *counter-clockwise* direction which according to our choice of coordinate systems is *positive*.

[7] A choice of the pivot point at the center-of-mass point of the board is also completely acceptable although this requires more algebra. In this case the torques will be different but the final values for every force will be the same as shown here.

- For the Weight force the radial distance from the pivot point is $L/2$ and for this force, the angle ϕ is a right angle so again $\sin\phi = 1$. This torque is directed in the *clockwise* direction which according to our choice of coordinate systems is *positive*.

Putting it together and substituting $\boxed{W = mg}$ we get:

$$LN + 0 + 0 - MgL/2 = 0$$

$$\boxed{N = \frac{Mg}{2}}$$

Now that we have this expression, ***the rest is algebra.*** Plugging this result into Equation (1) gives us:

$$T_L\cos\theta + \frac{Mg}{2} - Mg = 0$$

$$T_L\cos\theta = \frac{Mg}{2}$$

$$\boxed{T_L = \frac{Mg}{2\cos\theta}}$$

Plugging this result into Equation (2) gives us:

$$T_R - \left(\frac{mg}{2\cos\theta}\right)\sin\theta = 0$$

$$\boxed{T_R = \frac{mg\tan\theta}{2}}$$

XFBD Version 2 Solution: We apply **Newton's Second Law in Rotational Form:**

$$\tau_{net} = I\alpha$$

Since the board is static, the angular acceleration is zero:

$$\tau_{\vec{N}} + \tau_{\vec{T}_R} + \tau_{\vec{T}_L} + \tau_{\vec{W}} = 0$$

We use the XFBD and our **Definition of Torque** ($\tau_{\vec{F}} \equiv rF\sin\phi$) to calculate each of the four torques.

- Since we have chosen a pivot point on the right end, the radial distance to both the Normal and $\vec{T}_L$ forces is zero so they contribute zero torque.

- The radial distance from the Pivot Point to the left tension force is the length of the board. The angle ϕ here is the angle between the radial vector and the tension vector which is $\phi = \theta + 90°$ so $\sin\phi = \sin(\theta + 90°) = \cos\theta$. So the torque due to the left tension is $\tau_{\vec{T}_L} = LT_L\cos\theta$. This torque is directed in the *clockwise* direction which according to our choice of coordinate systems is *negative*.

- For the Weight force the radial distance from the pivot point is $L/2$ and for this force, the angle ϕ is a right angle so $\sin\phi = 1$. This torque is directed in the *counter-clockwise* direction which according to our choice of coordinate systems is *positive*.

Putting it together and substituting $\boxed{W = mg}$ we get:

$$0 + 0 - LT_L \cos\theta + mgL/2 = 0$$
$$LT_L \cos\theta = mgL/2$$
$$T_L \cos\theta = mg/2$$
$$\boxed{T_L = \frac{mg}{2\cos\theta}}$$

Now that we have this expression, ***the rest is algebra.*** Plugging this result into Equation (22) gives us:

$$T_R - \left(\frac{mg}{2\cos\theta}\right)\sin\theta = 0$$
$$\boxed{T_R = \frac{mg\tan\theta}{2}}$$

Finally plugging what we know into Equation (1):

$$\left(\frac{mg}{2\cos\theta}\right)\cos\theta + N - W = 0$$
$$N = mg - \left(\frac{mg}{2}\right)$$
$$\boxed{N = \frac{mg}{2}}$$

Guidelines for grader:

- **30 pts:** Correct answer for all four forces includes technically complete and correct **FBD** and **XFBD** (including appropriate choices of coordinate systems and pivot point), clear explanation of **Newton's Second Law**, **Newton's Second Law in Rotational Form**, and the **Definition of Torque**, along with clear, coherent organization of method and accurate algebra.
- **at least 25 pts:** Correct and accurate with explanations all the way up to point of *"the rest is algebra..."*.
- **27 pts:** Correct except for incorrectly setting torque due to left tension force equal to $T_L L \sin\theta$ instead of $T_L L \sin(90 - \theta) = T_L L \cos\theta$.
- **at most 27 pts:** wrong sign on torque and/or associated negative values propagating without being caught.
- **at most 27 pts:** Failure to explain or introduce the concept of the Definition of Torque and/or failure to explain how torque is calculated for each force.
- **at most 15 pts:** Contending with either Linear or Rotational form of Newton's Second Law but not both.
- **at most 3 pts:** Some other incorrect physics ideas.

Solution to Problem Z03: (30 points)

According to the explicit model given for breakage, ***the glass pane will break if a force greater than some threshold force is applied.*** This means we need to contend with the force on the glass. Any other approach does not address our explicit model for breakage.

First want to get a handle on what determines the **force applied** to the glass pane during the collision. Important: **Any approach that does not contend with FORCE is not correct.**

Our first step is to note that **in accordance with Newton's Third Law:** *the force applied to the glass pane is exactly equal in magnitude to that applied to the ball.*

$$F_{on-glass-due-to-ball} = F_{on-ball-due-to-glass}$$

Yes, many students implied this but did not explicitly state this. Students who forgot to explicitly invoke **Newton's Third Law** to state that the force on the glass pane and the force on the ball must be the of equal magnitude **lost 2 or 3 points.**

So how do we get a handle on the force on the ball? The correct method is to apply the **Force-Impulse** relation: (sometimes called the **Impulse-Momentum** relation:

$$\int F\,dt = \Delta p$$

or

$$F_{ave}\Delta t = \Delta p$$

This means that the **force** can be expressed as:

$$F_{ave} = \frac{\Delta p}{\Delta t}$$

We are ***told explicitly*** that the value of Δt for each of the two collisions is the same, so to assess the average force applied to the ball, we need only consider the **Impulse** which is defined as *the change in linear momentum that results during a collision* in each case.

$$\text{Impulse} \equiv \Delta p = p_{final} - p_{initial}$$

For **Case I: the inelastic collision** the ball sticks to the glass – or at least it comes to a complete stop after the collision. So in this case, the impulse applied to the ball is given by:

$$\Delta p_{inelastic} = p_{final} - p_{initial} = 0 - (-mv)$$

$$\boxed{\Delta p_{inelastic} = mv}$$

Solution to Problem Z03 continues next page...

On the other hand, for **Case II: the (very nearly) elastic collision**, since the glass pane is held in place and does not move at all, at any time before, during, or after the collision, **Conservation of Kinetic Energy** means that ball (alone) retains it's total kinetic energy:

$$K_{ball} = K'_{ball}$$
$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2$$
$$v^2 = v'^2$$
$$v' = -v$$

In other words the outbound speed of the ball from the glass must be (very nearly) the inbound speed. Therefore the impulse applied to the ball for the elastic collision is given by:

$$\Delta p_{elastic} = p_{final} - p_{initial} = (mv) - (-mv)$$
$$\boxed{\Delta p_{elastic} = 2mv}$$

Here we define "positive" as the outward direction of the ball after it bounces.[8] Note that we need to keep track of positive vs. negative values of momentum.

The impulse due to the elastic collision is twice the impulse due to the inelastic collision.

Therefore, **by the Force-Impulse Relation** the elastic collision results in **twice the average force** on the window pane relative to the inelastic collision[9] which means that

Case II: the elastic collision is much more likely to break the glass pane.

Solution to Problem Z03 continues next page...

[8]Of course you will get the same answer in the end if you define the directions of positive and negative oppositely.

[9]If this confuses you, think of it this way. In the *inelastic* collision the change in momentum simply corresponds to stopping the ball. In the *elastic collision* however, we have to stop and then also reverse the ball's direction. This corresponds to twice as much change.

Some examples of *incorrect* answers to Problem 3:

- Several students tried to apply Conservation of Linear Momentum to the problem. *This does not work because a window pane attached to the house therefore does* **not** *belong to part of an isolated system.* This means that ***the Condition for Conservation of Linear Momentum fails here.***[10]

- Some student went a little further and tried to address this by arguing that since the window pane is not moving, the net force on the window pane must be zero and therefore it is isolated. This argument sounds oddly compelling, and it is true for the time *before* the ball comes into contact with the glass pane. But it is *not at all true* during the collision. During the collision the **wooden frame which is connected to the house** applies a contact force to the window pane that acts to keep the glass pane in place even while the ball applies a force the other way. As a rule, if a body is physically attached to another body that is outside the system during a collision, that body is *not* isolated. And so **Conservation of Linear Momentum does not apply to this problem at all.**

- Many students having argued incorrectly that the system is isolated then went on to calculate the velocity of the glass plane after the collision. **But you are explicitly told the glass pane is held firmly in place and does not move.** This is hard to overstate: **YOU ARE TOLD THAT THE WINDOW PANE DOES NOT MOVE. THEREFORE THE GLASS WINDOW PANE HAS ZERO VELOCITY BEFORE, DURING AND AFTER THE COLLISION.** Any calculation that involves the motion of the glass window pane is therefore conceptually flawed.

- Again, many many students have the misconception that if the collision is either elastic or inelastic, Linear Momentum must be Conserved. That's just not true. You absolutely must check that the Condition is Satisfied before you apply any Conservation Law. If you proceeded to incorrectly apply Conservation of Linear Momentum to the problem, you have made a **Serious Conceptual Error** and you might expect to earn ***at most 15 to 20 points*** even if you then subsequently solve the problem otherwise perfectly using a different technique.

- By the way, several students went ahead and assigned some mass to the glass pane and then worked out the details for both elastic and inelastic collisions assuming incorrectly that the system was isolated. *This is conceptually incorrect and represents a great deal of wasted work.* Sometimes this involved the four-box calculation, other times lots of algebra. Why all of the calculation? We tell you explicitly the following statement: ***Important: this is***

[10]Several students argued *forcefully and confidently* that the system *is* isolated or that we can ignore gravity and thereby approximate this as an isolated system. They often simply asserted this as obviously true. But it's not at all true. **The window pane is firmly attached to the house via a wooden frame as can be seen in the photo. You are explicitly told this in the problem. This means the window pane and ball together are not an isolated system.** Not convinced? Let's unpack: Okay so ignoring gravity, the only force on the ball is the force due to the window pane, an internal force. So the ball part of the system seems to be isolated, agreed. But the window pane is connected to the house via the wooden frame. You are told this. There could be all kinds of forces on the window pane that arise from being connected to the house during the collision. You know those forces are there because in their absence the whole window pane would go flying through the air as a result of the collision this does not happen.

a conceptual question! In other words, please do *not* do lots of calculation. How can we make it more clear? If you are lucky when you get done with the calculations, you find that the change in velocity of the "window" and/or the velocity of the ball is greater for the elastic collision relative to the same for the inelastic collision and if you are still on the ball at this point you can apply the **Force- Impulse/Momentum Relation** to argue that the force is greater for the elastic collision to earn reasonable partial credit. But if we tell you that this is a conceptual question it must be true that you can arrive at an answer with very little algebra. So obviously whatever motivates you to do all that algebra is at best a waste of time and effort and at worst a path to the wrong answer.

- Some student incorrectly applied Conservation of Momentum and then calculated that the final velocity of the window pane in the elastic case is twice that for the inelastic case. Based on this, a claim is made that the elastic collision is more likely to break the window. This is (at least) the right answer but it is logically flawed in three ways: **(a)** it depends on the incorrect application of Conservation of Linear Momentum, **(b)** there is no justification as to a physics reason for why having a greater final velocity in the calculation means that the glass is more likely to break, and **(c)** the result is in direct contradiction to the stated fact that the glass pane does not move. So this is the correct answer for the wrong reason. But sort of buried in the calculation is the idea that the momentum transfer is twice that for the elastic as for the inelastic collision, so we awarded ***between 15 and 20 points*** for this kind of calculation and claim.

- Another idea that several students described was some notion that because the ball *sticks* to the window pane during an inelastic collision, and because the ball has to be moving, the window pane must also move, and therefore the glass must break. *This is not a physics argument.* There is no principle of physics that says the ball must continue to move after colliding with the window. Indeed, the glass pane is attached to the wooden frame and the house which is motionless, and therefore after the inelastic collision, the ball must also be at rest. If you made an argument that somehow the ball must move "though the glass" then this is worth ***perhaps 4 to 6 points at most.*** Such an argument is not physics.

- Several students argued that because the collision was "inelastic" this meant that kinetic energy was "lost" and therefore had to "go somewhere" which is to say it "transferred" into the glass so therefore the inelastic collision is more likely to break the glass. This is really not a good physics argument. ***The notion that the energy (or any other quantity) must be transferred from the ball to the window is a major misconception.*** Inelastic has nothing to say about where the energy goes. Inelastic just means that the energy goes into another form: like the deformation of the ball for example. Failure to consider that the energy could have other forms besides doing glass-breaking work is a failure of imagination. Not convinced? Consider a sticky marshmallow vs. a hard elastic golf-ball: Same mass, same velocity, would you not agree intuitively that the "soft" marshmallow will dramatically deform to absorb the energy and would you not agree that the hard golf ball is rather more likely to break the glass than the soft marshmallow? [11]

[11] By the way, this is why bumpers and guard rails are designed to "crumple" upon a strong collision. You want to reduce the impulse and increase the time of the collision so as to reduce the maximum force which in turn reduces the

- Consider the "inverse" argument, that because no energy is lost, an *elastic* collision has more energy to "give" to the glass and therefore is more likely to break it. This is also flawed Indeed, ***any discussion that boils down to how much energy is being "transferred" to the glass is not right.*** In the case that energy (and not force) is invoked, this is a major conceptual error and is worth ***5 to 7 points at most.*** The issue is **not** energy. The issue is the maximum force applied and whether that force exceeds the threshold. For example, if the collision results in a force that just *barely* cracks the glass, then there will be virtually no displacement of the glass and no work done and very little energy transferred from ball into glass. The elastic ball will bounce off, pretty much elastically. We tell you this, we point out that the window does not move. But the glass will now be cracked. Any idea that you have to penetrate the glass to break it is not physics. Impact ad bounce will break glass just fine. This happens all of the time when a small rock hits a windshield, for example. Try it some time if you don't believe me. An (elastic) pebble ***much*** more likely to break your car windshield than an (inelastic) bug. In neither case is momentum conserved. In neither case does the object come smashing through the glass to the other side.

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deadly acceleration.