P121 Exam 2 Summary Session - Answers

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1 Question 1: Spring Launcher

1.1 Work Done By Normal Force

The work done by normal force is $\boxed{0.}$ Since the Normal Force is always pointed perpendicular to the path, and the definition of work is $W = Fd\cos\theta$, the work done is zero.

1.2 Speed At Peak

As we said in part a, the normal force does no work. The other two forces, Spring Force and Weight Force, are conservative. So, we're free to use **Conservation of Mechanical Energy**

$$"Before" = "After'$$

$$E_{TOT} = E'_{TOT}$$

...Let's be careful, though. We need to make sure to include the potential energy from both weight and the spring.

$$U_W + U_{Sp} + K = U'_W + U'_{Sp} + K'$$

$$mgy + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}kx'^2 + \frac{1}{2}mv'^2$$

Let's set $y = 0$ to be at the track. We also know that the block starts at rest, and that the spring is at its relaxed position when the block is released. This let's us remove a few terms.

Let's also go ahead and sub in $y' = H$, since that's the height we're interested about.

$$\frac{1}{2}kx^2 = mgH + \frac{1}{2}mv'^2$$

Then, all you need to do is rearrange

$$\boxed{v' = \sqrt{\frac{kx^2}{m} - 2gH}}$$

1.3 Minimum Length of Track

There are two ways to do this problem, and both should get you the right answer. You can either use the **Work-Energy Theorem** or **1-D Kinematics**.

Here, we're gonna use **Work-Energy Theorem**, as it's a bit more straightforward:

$$W_{TOT} = \Delta K$$

$$W_{TOT} = K' - K$$

Since we are not allowed to include the height H in our answer, we need to think about how to set our K and K'. One method is to say that K is the kinetic energy before the spring launches the block, and K' is the kinetic energy once the block has stopped. This means that K and K' both equal zero. It also means that we need to consider the work done by Weight Force and Spring Force, as well as the work done by the friction on the leftmost bit of track.

$$W_W + W_{Sp} + W_f = 0$$

We know that the work done by a conservative force is equal to the negative change in potential energy. We also know that the work done by friction in this case is equal to the Force of friction times the distance times $\cos 180$, which equals -1.

$$mgy - mgy' + \frac{1}{2}kx^2 - \frac{1}{2}kx'^2 - \mu_k mgL = 0$$

Here, I've subbed in $\mu_k mg$ as the force due to kinetic friction (see further down for details) and L as the distance (L of course being the thing we want to solve for).

Now, let's look at our terms here. The block starts and ends on the track, so both y and y' equal zero. Also, x' also equals zero because the spring stretches to its relaxed length. Finally, x is given.

So,

$$\frac{1}{2}kx^2 = \mu_k mgL$$

$$\boxed{L = \frac{x^2 k}{2\mu_k mg}}$$

You get the same result if you do this with kinematics. So let's do it with kinematics (for fun :D):

Newton's Second Law tells us that $\Sigma F = ma$. Working in the x-direction, we find:

$$\Sigma F = f$$

$$f = \mu_k N$$

$$\mu_k N = ma$$

$$a = \frac{\mu_k N}{m}$$

We need to find the magnitude of N, so let's work N2L in the y-direction (this also applies to the explanation for the Work-Energy Theorem method):

$$\Sigma F = ma$$
$$\Sigma F = N - W$$
$$W = mg$$
$$N = mg$$

Now we can put the acceleration in terms of given parameters.

$$a = \frac{\mu_k mg}{m}$$
$$a = \mu_k g$$

This acceleration is constant, so we can use the 1-D Kinematics Constant Acceleration equations.

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

We know the acceleration, but we don't know the initial velocity. To find this, we can use Conservation of Mechanical Energy up until the moment *before* friction is applied:

$$E_{tot} = E'_{tot}$$
$$U_{tot} + Ktot = U'_{tot} + K'_{tot}$$
$$U_{sp} + U_W + K = U'_{sp} + U'_W + K'$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{kx^2}{m}}$$

This will be our initial velocity for our Constant A equations. From there, the rest is algebra. (Hint: use the velocity equation to solve for t first, then plug t into the position equation.)

2 Question 2: Way Too Many Collisions

2.1 Inelastic Collision

Since there is no friction or net forces applied to the system of the boxes, we're allowed to use **Conservation of Linear Momentum** to solve.

$$"Before" = "After"$$

$$P_{TOT} = P'_{TOT}$$

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

We have one equation and two unknowns! We need another piece of information. Thankfully, the collision in part a is totally inelastic, which means that the blocks stick together after the collision. This tells us that the final velocities of both blocks are the same.

$$m_A v_A + m_B v_B = (m_A + m_B)v'$$

$$Mv_0 = (4M)v'$$

$$\boxed{\frac{v_0}{4} = v'}$$

2.2 Elastic Collision

Okay, this is the same deal as before. We're still gonna use **CofLM** here, so let's write that down:

$$"Before" = "After"$$

$$P_{TOT} = P'_{TOT}$$

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

This time, though, we have a totally elastic collision! In totally elastic collisions, kinetic energy is conserved. This gives us another equation we can use:

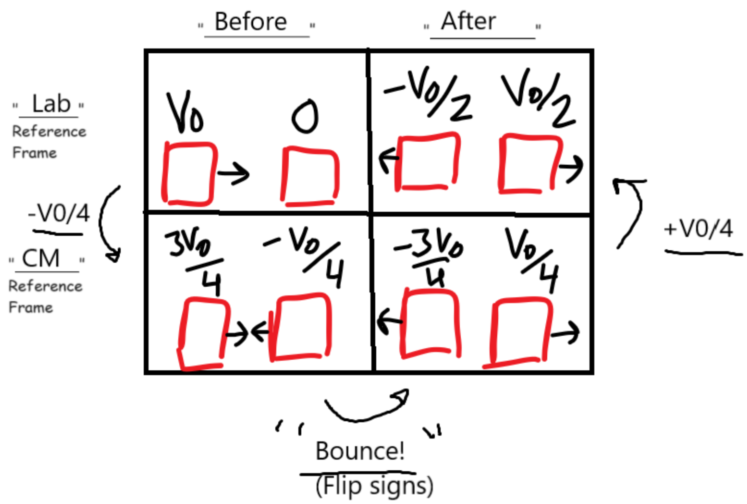
$$v_a - v_b = v'_b - v'_a$$

If you'd like, you can do a system of equations to solve for the answer. However, the **Four-Box Method** also works. First, let's calculate the velocity of the center of mass so that we can go into that reference frame.

$$v_{cm} = \frac{m_A v_a + m_B v_B}{m_A + m_B}$$

$$= \frac{Mv_0 + 0}{4M} = \frac{v_0}{4}$$

Then we draw our 4 boxes and put in our values. *Put the Four Box Method on your crib sheet if you haven't!!*



We get that $\boxed{v_A' = \frac{-v_0}{2}}$ and $\boxed{v_B' = \frac{v_0}{2}}$

2.3 Neither Kind of Collision

Don't panic! This is just a classic **CofLM** problem!

$$"Before" = "After"$$

$$P_{TOT} = P'_{TOT}$$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

We have all the info we need, so we can just "plug'n'chug"

$$M v_0 + 0 = 0 + 3M v_B'$$

$$\boxed{\frac{v_0}{3} = v_B'}$$

2.4 V_{cm} as Function of Time

In this part, the system is *not* isolated since there's an external force. Since the magnitude of the force is given, and it's the only force external to the system, let's go ahead and use **N2L**:

$$F^{net}_{\substack{sys\\ext}} = M_{sys} a_{cm}$$

$$F_{app} = M_{sys} a_{cm}$$

We know the mass of our system, and we know how to go from acceleration to velocity, so let's solve for a_{cm}:

$$a_{cm} = \frac{F_{app}}{4M}$$

So all we need to do to get our velocity is take an antiderivative like so:

$$v_{cm} = \frac{F_{app}}{4M} t + v_0$$

But what's v_0? It must be the velocity of the system right before the force is applied, since the force is applied at $t = 0$. What is this velocity? This is the same as the v_{cm} we calculated in part b. So,

$$\boxed{v_{cm} = \frac{F_{app}}{4M} t + \frac{v_0}{4}}$$

3 Question 3: Elliptical Pendulum

3.1 Speed at Point P

We recall that speed is defined as the magnitude of velocity, where velocity is the time derivative of position. Since we are given a vector expression for position, we need only to take the derivative with respect to time:

$$\vec{v}(t) = -3A\omega \sin(\omega t)\hat{i} + 2A\omega \cos(\omega t)\hat{j}$$

Knowing that sin(0)=0 and cos(0)=1, the velocity at time t=0 simplifies to

$$\vec{v}(0) = 2A\omega\hat{j}$$

If the velocity vector were in two or more directions, we would have the add each component in quadrature to find the magnitude of the velocity (i.e. the speed). However, since it is in only one direction, finding the magnitude is as easy as taking the absolute value and removing the unit vector:

$$\boxed{v = 2A\omega}$$

3.2 Net Force at Point P

***Note: The original copy of the problem sheet does not include mass as a given parameter. Please consider the mass of the pendulum to be a given positive constant* m.**

Newton's Second Law defines the net force on a body as the product of its mass and acceleration, $\Sigma\vec{F} = m\vec{a}$.

We know that acceleration is defined as the time derivative of velocity, so we can write a vector expression for a as follows:

$$\vec{a}(t) = -3A\omega^2 \cos(\omega t)\hat{\imath} - 2A\omega^2 \sin(\omega t)\hat{\jmath}$$

Now plugging in t=0:

$$\vec{a}(0) = -3A\omega^2\hat{\imath}$$

Since the mass is given, we need only plug these two values into Newton's Second Law.

$$\Sigma\vec{F} = m(-3A\omega^2\hat{\imath})$$

$$\boxed{\Sigma\vec{F} = -3A\omega^2 m\hat{\imath}}$$

3.3 Radius of Curvature at Point Q

To find the radius of curvature, we can use the equation $a_c = \frac{v^2}{R}$. Note that this applies even if the entire path of the bob is not circular (it is elliptical for this problem) - we're merely treating a section of its path as circular to find the *radius of curvature*.

This calls for the magnitude of the acceleration and velocity, so first we have to plug $t = \frac{\pi}{2\omega}$ into the vector expressions for velocity and acceleration:

$$\vec{v}(\frac{\pi}{2\omega}) = -3A\omega\hat{\imath}$$

$$\vec{a}(\frac{\pi}{2\omega}) = -2A\omega^2\hat{\jmath}$$

Again, since each of these vector expressions has only one component, finding the magnitude is quite simple.

$$v = 3A\omega$$

$$a = 2A\omega^2$$

With these values, we can solve the equation $a_c = \frac{v^2}{R}$ to find the radius.

$$\boxed{R = \frac{9}{2}A}$$