Cycle 1 Review Sheet, Part 1

16 January 2024

From time to time this semester we will hand out a "review sheet" which is a condensed summary of what I consider some of the more important topics and key concepts from the lectures. Remember in this class, the lectures define the range of content that you are responsible for, not the text. This is especially pertinent because of the cyclical nature of the class. If we indicate a point here in a review sheet this means that this is something I want to emphasize.

1-D Kinematics:

Definitions:

$$v_{ave} \equiv \frac{\Delta x}{\Delta t}$$ Definition of *average velocity*, defined over an interval.

$$\boxed{v \equiv \frac{dx}{dt} \qquad \text{Definition of instantaneous velocity.}}$$

$$a_{ave} \equiv \frac{\Delta v}{\Delta t}$$ Definition of *average acceleration*, defined over an interval.

$$\boxed{a \equiv \frac{dv}{dt} = \frac{d^2x}{dt^2} \qquad \text{Definition of instantaneous acceleration.}}$$

Note that the above definitions can be applied to *any* kind of 1-D motion. **These four definitions are always true.**

Working Backwards:

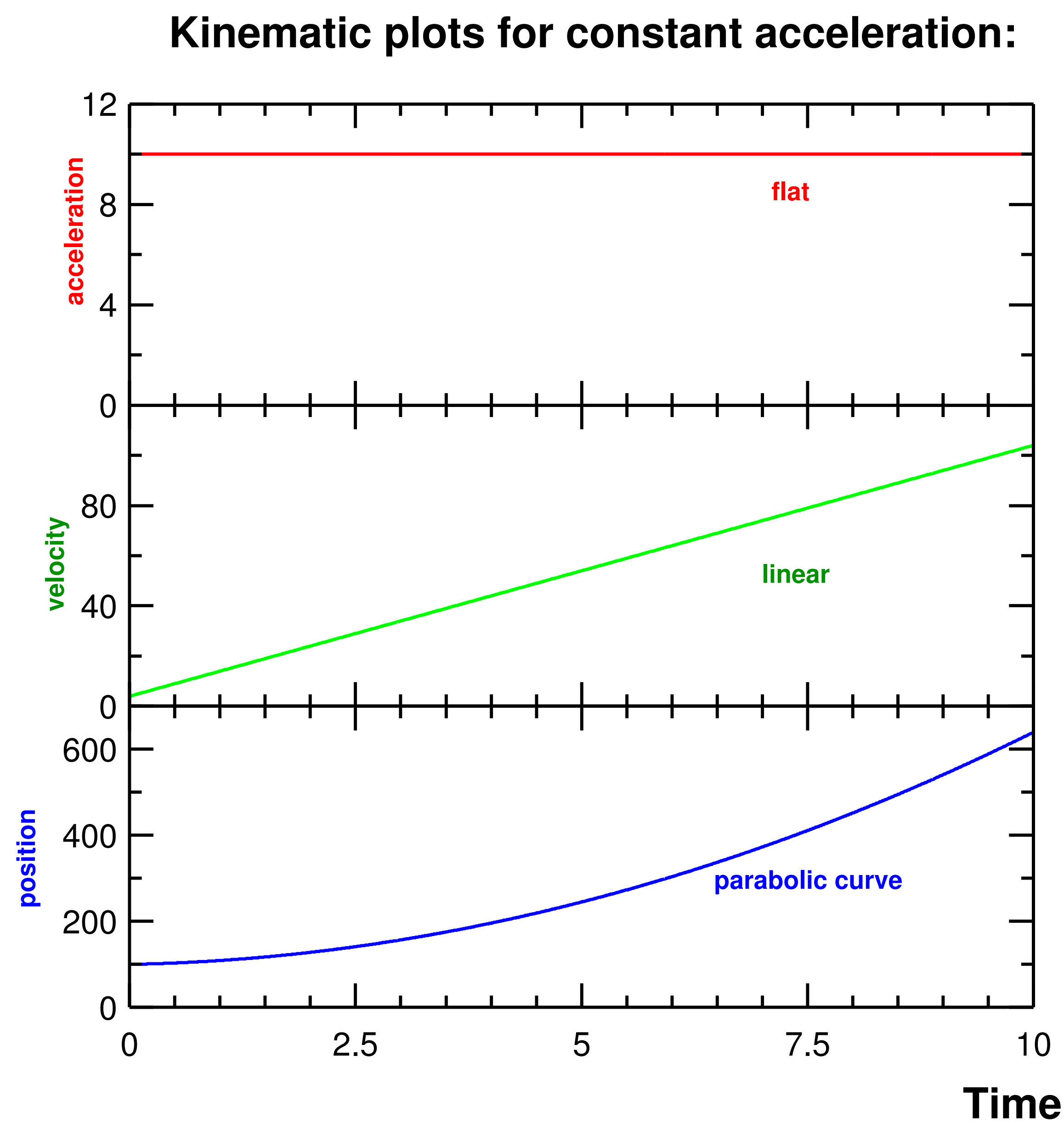
Since we can always determine velocity and acceleration by taking time *derivatives* of the position, we can go the other way to get velocity and position by taking time integrals:

$$v(t) = v_0 + \int_0^t a(t')dt'$$ Going from acceleration to velocity.

$$x(t) = x_0 + \int_0^t v(t')dt'$$ Going from velocity to position.

Graphical Representations of 1-D Kinematics:

You should be comfortable with the idea of looking at plots of position vs. time (x vs. t) to describe the motion of a particular body. Such plots are called *kinematic plots* or **worldlines**. Based on the plot of position vs. time, you should be able to identify and possibly generate plausible plots of velocity vs. time and acceleration vs. time plots that are all self-consistent. You should be able to immediately recognize the graphical properties of motion with **Constant Velocity** and motion with **Constant Acceleration**. Based on these plots you should be able to say something about the velocity of the body at any time during the motion. (Is the velocity zero? Is it constant? Is it positive, negative, or zero? Is it increasing or decreasing?) Based on the plot, you should be able to say something about the acceleration at any time during the the motion (Is the acceleration zero?, constant? positive? negative?) You should be able to describe a "scenario of motion" which is to say that you should be able to describe in very well-structured terms how a body moves based on examining any of these plots. And (vice versa) if a description of motion of a body is given to you, in words, you should be able to identify and possibly generate a plausible set of plots of position, velocity, and acceleration vs. time plots for that particular description.



Constant Acceleration:

$a = $ (constant) acceleration
$v = v_0 + at$ velocity
$x = x_0 + v_0 t + \frac{1}{2}at^2$ position

You should understand that these above equations arise naturally by *integration*.

You should be so familiar with the equations listed above that you have them committed to memory. **These are the only fundamental equations for Constant Acceleration that you need in this class.** You should be able to solve any given 1-D kinematics problem in this class using these equations *only*.

Free Fall Kinematics:

Constant Acceleration due to gravity:

Remember this is just a special case of constant acceleration with $a = -g$ where we empirically observe that $g = +9.81 m/s^2$ when y is defined as the coordinate *upward*.

$a_y = -g$
$v_y = v_{y_0} - gt$
$y = y_0 + v_{y_0} t - \frac{1}{2}gt^2$

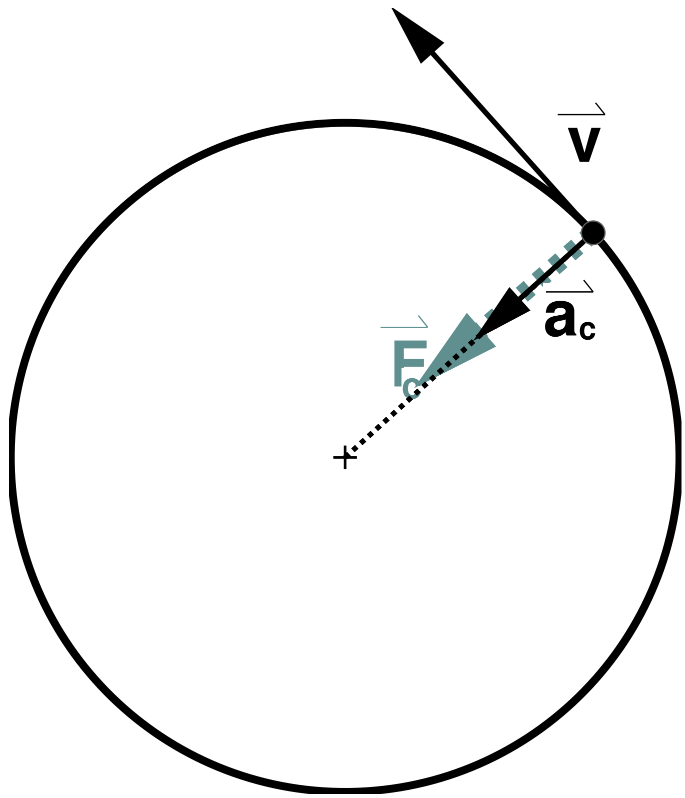
(Memorize this! You should be ready to apply it to many problems. Always be consistent with your definition of positive (up), acceleration (negative down) and the value of g (positive 9.81 m/s^2).

Constant Velocity:

This is just Constant Acceleration but with $a = 0$. In this case the position is linear as a function of time:

$a = 0$ acceleration
$v = v_0$ constant velocity
$x = x_0 + v_0 t$ position

Uniform Circular Motion:



- Particle moves around in circle of radius r at a constant speed v.
- Note that while speed v is constant, the velocity $\vec{v}$ is *not* constant. While the magnitude of $\vec{v}$ is constant, the direction of $\vec{v}$ is changing. The direction of $\vec{v}$ is a tangent to the circle, perpendicular to the radius vector, and is rotating as the particle rotates around the circle.
- The acceleration $\vec{a}_c$is perpendicular to velocity.
- The acceleration is always towards the center of the circle. This is called *centripetal acceleration*. Centripetal acceleration *always* points towards the center of the circle.
- One way to think about this is to imagine the path that the particle would take if the velocity were constants and there were no centripetal acceleration: it would go off in a straight line tangent to the circle. The centripetal acceleration represents a change in the direction of the velocity vector "towards the center of the circle" relative to the path it would have taken.
- The centripetal acceleration always acts *perpendicular* to the motion of the particle. Therefore, the speed of the particle does not change. Only the direction of the velocity changes.
- Since the speed is constant, the magnitude of the acceleration is also constant.

- The magnitude of the centripetal acceleration is given by

$$a_c = \frac{v^2}{r}$$

- We can calculate the *period* of rotation P by noting that the speed of the particle must be the the distance around the divided by the period:

$$v = 2\pi r/P \qquad \text{so} \qquad P = 2\pi r/v$$

- We define the *frequency* f as the inverse of the period:

$$f = 1/P$$

  We define the units of frequency (1/s) as Hertz (Hz). One Hz is one revolution per second.

- We define the *angular frequency* ω as the number of radians swept per unit time interval:

$$\omega = 2\pi f$$

- We also note that the relationship between angular velocity and linear velocity around a circle is given by the "Rolling Constraint" which is:

$$v = \omega r$$

- Therefore we can also write down the centripetal acceleration in terms of the angular frequency:

$$a_c = \omega^2 r$$

Newton's First Law:

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> *Law of inertia: A body has has a constant velocity unless there is a non-zero net force acting upon it.*

This is a statement about what the *Natural Motion* of bodies is about. Newton is saying that in the absence of "external influences" the natural motion of a body is to move with constant velocity, that is constant speed and direction. Newton is calling the impact of "external influence" the *net force* on a body.

The First Law also implicitly asserts the existence of *inertial frames of reference*. An inertial frame of reference is any defined coordinate system that moves with *constant velocity*.

Newton's Second Law:

Law of Forces: $\vec{F}_{net} = m\vec{a}$

Important point: **The Net Force is *not* a force**. The Net Force is a vector sum of all of the real physical forces on a given body or system:

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...\vec{F}_N$$

Note that we have written this in *vector* notation to make it clear that force has a a magnitude (value) and a direction. For this part of the course we will apply Newton's Law in **one dimension at a time only**. We will generally use the Cartesian subscripts "x" or "y" to indicate the relevant dimension we are considering.

For example when we want to talk about the component of the net force in the x-direction, we represent this as the vector component F_x. Note that we never put little arrows on components. We can just write the 1-D version of Newton's Second Law:

$$F_x = ma_x$$

Here $F_x$is the net force in the x-direct and a_x is the acceleration defined positive in the x-direction.

Finally note that the variable m what is called "mass" refers to the *inertia* of a body. In other words m tells you how hard it is to acceleration something. It's harder to get a freight train to accelerate than it is to get a mosquito to accelerate. We always call m the mass, not the weight.

Newton's Third Law:

Law of Action-Reaction: If there exists a force on body A due to body B: $\vec{F}_{AB}$ then the body A must also exert a force $\vec{F}_{BA}$ on body B so that these two forces have the same magnitude:

$$F_{AB} = F_{BA}$$

It it also true that these two forces are directed in *opposite directions*. Newton's Third Law is *powerful* and *dangerous*.

Note in particular that these two forces for the Third Law always act on *two different bodies*. Note that in principle *all forces* are the result of interactions between two bodies so whenever you have a force of any kind on one body you should be able to identify a "reaction force" on another body. You should be able to identify "action-reaction pairs" of forces.

Working Problems with Newton's Laws:

Here's sort of a "recipe" for working problems with forces and Newton's Laws:

1. Sketch the problem at hand carefully, indicating the relevant bodies. If you are given numerical values, recast these as symbolic parameters. Note any physical constraints on the problem. Make a list of all of the given parameters. Also make a list of the quantities you are asked to calculate.

2. Before you start, consider the acceleration. Is it zero? Then note that $a = 0$. Is it another constant? Then $a =$ another number. Make note of this, keep it in mind, and be ready to use it later.

3. Draw a *separate* Free-Body Diagram (FBD) for *each* relevant body in a problem, and represent each body as a *point only* with forces acting upon it.

   - Avoid the habit of drawing FBDs on top of your sketch (a bad habit that is nonetheless widely practiced in some physics texts. Draw each FBD *separately*. This helps you because it is important to think about each body as an "isolated point" with forces acting on it. You should be ignoring everything in a problem *except* for the forces acting in each particular FBD. That's why we call it a "free-body". Free! Get it?
   - Include each of the forces as a vector (arrow) on your free body diagram. It is important to get the direction written down correctly on the diagram. Don't worry about the magnitude. I like to draw one arrow for each individual force.
   - Put only *real* physical forces on your FBD. Use your **Force Menu** for reference (see the next page), and *only* use forces that you find on the menu. Is the net force on your list? No, it is not. So never draw it on your FBD. Is the velocity vector or the acceleration vector on the force menu? No, they are not even forces, so don't draw them on your FBD. The whole point of the FBD is to act as a bookkeeping device so you can be very clear about all of the forces before you try to apply Newton's Second Law. Don't draw anything that is not an actual physical force on your FBD. Don't make up your own forces. Use forces from the menu only.
   - Remember the difference between *contact* and *non-contact* forces. Weight is the only non-contact force on your Force Menu, meaning that Weight applies to any body that is located or near the surface of the earth. The other forces on your Force Menu are contact forces, meaning that they require physical contact. Is there physical contact? Draw the force on your FBD. Is there no physical contact? Don't draw the force on your FBD; it doesn't exist.

- Never include the "net force" on your FBD. Never include the "centripetal force" on your FBD. Never. Ever. These are not real physical forces.
- The Weight force should only appear once on each FBD. I usually draw it first before I draw any other forces. Never ever draw more than one Weight force on a single FBD. Never. Ever.
- Your FBD should have vectors that are labeled in a consistent way. If there is only one body and/or only one of any particular kind of force then you can just use letters such as W, T, f, etc. However, if you have more than one body and if you have forces between two bodies, you will need to use a consistent scheme of labeling subscripts. I strongly recommend something like this:
  - F_A – This means the force *on* Body "A".
  - F_{AB} – This means the force *on* Body "A" *due to* Body "B".
- Also, before you start drawing forces on your point body, you might want to write down in the corner a little coordinate system to make it clear which directions you are talking about. If your motion is 1-D then just write a little "x" or "y" vector to indicate clearly which way is defined as positive. If you have options for choosing coordinates, then always choose a coordinate system so that at least one of the coordinates is lined up with the acceleration (assuming it is non-zero).

4. Solve Newton's Second Law.

- Set the net force equal to the mass times acceleration in the equation, working one dimension at a time for each body. Actually start by **writing down the Second Law**. Don't skip this step. Write something like like $F_x = ma_x$ (this means "The net force on the body in the x-direction equals its mass times it's acceleration in the x-direction") or like this: $F_{By} = m_B a_y$ (this means "The net force on Body B in the y-direction equals the mass of Body B times its acceleration in the y-direction"). Do you already know the acceleration? If it is zero, then now you can Plug It In.
- Important: Actually *use the FBD* to deal with the left-hand-side of the equation. Check your coordinate system. Add each force that contributes in the positive direction. Subtract each force that contributes in the negative direction. Do not include forces that are perpendicular to the dimension you are working with.
- Use any constraints that you have on the acceleration or the forces at this point to connect the bodies. Is the acceleration in one body related to the acceleration in another body? Use it.
- Think about applying Newton's Third Law if there is more than one body in the problem. Remember always that the Third Law always applies between two forces on two *different* FBDs. It never applies to two forces on the *same* FBD. If you use my notation for labeling forces, using Newton's Third Law is easy: Just switch the subscript labels and set the forces equal. For example: $N_{AB} = N_{BA}$.
- Once you have done these things, "the rest is algebra". Use algebra to solve for the unknown quantities that you are asked to find in terms of the given (known) symbolic

parameters. Do not plug in numerical values. But a box around your answer. If you are asked to present a numerical answer, then plug in numbers at the end, only.

You should soundly understand and regularly use Free-Body Diagrams. If you are not actually *using* them to get your solution, then you are not doing them correctly. If you are uncomfortable with them, I *strongly* recommend that you seek help *before*, rather than *after*, the test.

Force Menu for Physics 121 for Cycle 1:

Here is a Force Menu which should be all you need for any problem in this class that requires dealing with Newton's Second Law through this part of the course.

Force	Symbol	Contact?	Details
Weight	$\vec{W}$	No	A *downward* force due to earth's gravity applies to all bodies near surface of the earth, Magnitude is *known*: $W = mg$.
Normal	$\vec{N}$	Yes	A *push* perpendicular to a surface; the magnitude is generally *unknown*.
Tension	$\vec{T}$	Yes	A *pull* usually due to a rope or cable, may be "re-routed" by pulleys, etc; the magnitude is generally *unknown*.
Friction	$\vec{f}$	Yes	A force applied between two surfaces parallel to the surface and in opposition to the motion of one surface relative to the other; the magnitude is generally *unknown*; However, only in the case of *sliding friction*, (also called kinetic friction) we can say $f = \mu_k N$ where μ_k is the coefficient of kinetic friction.

Your FBD for Newton's Second Law should *only* include forces from the above menu. If it is not on the menu, do not put it on your FBD. In particular never put the net force or any sort of "centripetal force" on the FBD.

There is is (sort of) one exception to the rule. If you are given *explicitly* an "applied force" with some particular magnitude and direction you can put this on your FBD without further specification – although you can pretty much count on this applied force being one (or more) of the forces on the menu. I usually label such a known specified (given) applied forces as $\vec{F}_{app}$.