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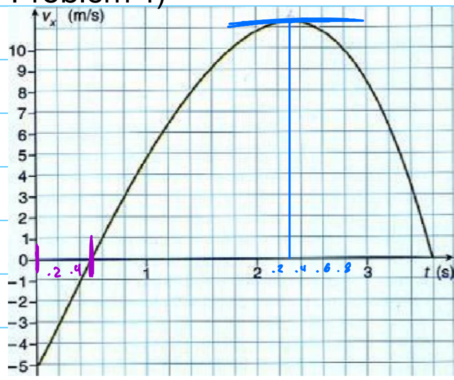
PHYS121 - HW1

Problem 1)



a) 1D Kinematics At $t = 0$, the particles velocity, as shown by the graph is $v(0) = -5$ m/s. Student 2 is correct because the particles velocity starts negative (moving in neg. direction) and also becoming more positive over the first couple seconds. A negative velocity becoming more positive means slowing down towards $v = 0$.

b) 1D Kinematics: Special Case - Constant a.

The acceleration of a particle is only constant when its velocity is linear. Since the graph provided shows velocity in a clearly non-linear relationship with time, acceleration cannot be constant

c) 1D Kinematics: Special Case - Constant a.

The equation $x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$ describes the position of a particle with constant acceleration. As stated in part (B) The acceleration is not constant, so this model cannot be used.

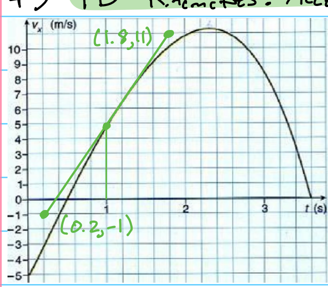
d) 1D Kinematics: Definition of acceleration

Acceleration is defined as the derivative of velocity. According to the graph, 0 acceleration would be at a time t s.t. the slope of the velocity is 0, represented by a horizontal slope. This is indicated by the blue line on the graph on the top of this page. It corresponds to about $t = 2.3$ seconds. The slope of velocity is $\neq 0$ anywhere else on the interval.

e) 1D Kinematics: Negative Velocity

The particles velocity is negative on the region marked by the purple lines on the graph atop this page. This means, by definition, the particle is moving in the negative direction. After this interval, the particle moves strictly positively, so the minimum position corresponds to the second vertical purple line on the graph. This occurs around the $t = 0.5$ s mark.

f) 1D Kinematics: Acceleration Definition



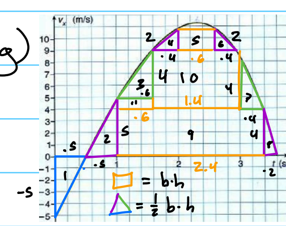
Again, accel. is the derivative, or slope, of velocity at a given t.

The green line represents the given time t.

I drew an extended tangent line to get "nice" points to calculate the slope at time $t = 1.0$ seconds

$$a = \frac{11\text{ m/s} - (-1\text{ m/s})}{1.8\text{ s} - 0.2\text{ s}} = \frac{12\text{ m/s}}{1.6\text{ s}} = \boxed{7.5\text{ m/s}^2}$$

g)



1D Kinematics: Integral Relation of x and v

To estimate the final position, I added the signed area under the curve together and added it to the initial pos.

This can be done using a series of rectangles and triangles.

$x_{fin} = x_0 + \sum \text{area}$, area is represented by colored regions

Triangles: $\frac{1}{2}\left((0.5 \cdot -5) + (0.5 \cdot 5) + (0.6 \cdot 4) + 2(0.4 \cdot 2) + (0.4 \cdot 4) + (0.2 \cdot 4)\right) = 3.2$

Rectangles: $(0.6 \cdot 1) + (1.4 \cdot 5) + (0.6 \cdot 2) + (2.4 \cdot 4) = 18.4$

$x_{fin} = 0\text{ m} + 3.2\text{ m} + 18.4\text{ m} = \boxed{21.6\text{ m}}$

Problem 2) A heavy package of given mass M is released at rest to fall straight down from a hovering helicopter located at a given height H meters from the ground.

Package Mass $= M$ Height $= H$ meters from ground ignore air resistance

a) F. F Kinematics

$+y$ constant acceleration

$g = 9.81\ m/s^2$

$a_y = g$, $v_0 = 0$, $y_0 = H$, $y = 0$

F.F. Kinematics: $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$\therefore 0 = H + 0 - \frac{1}{2} g t^2$

$\frac{1}{2} g t^2 = H$

$t^2 = 2H/g$

$\boxed{t = \sqrt{\frac{2H}{g}}}$

b) F. F. Kinematics

$g = 9.81\ m/s^2$ $v_0 = 0$ $t = \sqrt{\frac{2H}{g}}$

F.F. Kinematics: $v_y = v_0 - g t$

$v_y = 0 - g\left(\sqrt{\frac{2H}{g}}\right)$

$v_y = -\sqrt{2Hg}$

$|v_y| = \sqrt{2Hg}$

How fast = speed $= |v|$

$\boxed{V_y = \sqrt{2Hg}}$

c) Let... $H = 52.8\ m$

$M = 47.84\ kg$

$g = 9.81\ m/s^2$

Speed $= \sqrt{2Hg} = \sqrt{2(52.8m)(9.81m/s^2)}$

$= \boxed{32.2\ m/s}$

F. F. Kinematics with symbolic equations from (a) and (b)

Question 3) Free Fall Kinematics: $y = y_0 + v_0 t - \frac{1}{2} g t^2$

$g = 9.81\ m/s^2$, $V_0 = 23.7\ m/s$, $t =$ time (s), $y_0 = 0$

$y = 0 + v_0 t - \frac{1}{2} g t^2$

$y = v_0 t - \frac{1}{2} g t^2$ max @ $v = 0$

$v = v_0 - g t$

max height

$0 = v_0 - g t$

$t = \frac{v_0}{g}$

$y = v_0\left(\frac{v_0}{g}\right) - \frac{1}{2} g \left(\frac{v_0}{g}\right)^2$

$y = \frac{v_0^2}{g} - \frac{v_0^2}{2g} = \frac{v_0^2}{2g}$

$\boxed{y = \frac{V_0^2}{2g}}$

time elapsed

$y = V_0 t - \frac{1}{2} g t^2$ $y = 0$ at catch

$0 = v_0 t - \frac{1}{2} g t^2$

$0 = t\left(v_0 - \frac{1}{2} g t\right)$

$\cancel{t = 0}$ or $v_0 - \frac{1}{2} g t = 0$

Instance of Bat Strike

$v_0 = \frac{1}{2} g t$

$\boxed{t = \frac{2 v_0}{g}}$

numerical: max height

$$y = \frac{V_0^2}{2g}\Bigg|_{\substack{v_0 = 23.7 m/s \\ g = 9.81 m/s^2}} = \frac{(23.7 m/s)^2}{2(9.81 m/s^2)} = \boxed{28.63 m}$$

time elapsed

$$t = \frac{2 v_0}{g}\Bigg|_{\substack{v_0 = 23.7 m/s \\ g = 9.81 m/s^2}} = \frac{2(23.7 m/s)}{9.81 m/s^2} = \boxed{4.83 s}$$

Question 4) 1D Kinematics Special Case: Constant Acceleration

a) $A = 0.238\ m/s^2 \quad a_1 = A, \quad a_2 = -A, \quad D = 85\ AU \times \frac{1.5\times10^{11} m}{1 AU} = 1.27\times10^{13}\ m$

$x = x_0 + v_0 t + \frac{1}{2}at^2$ Double time of half trip

$x_0 = 0,\ v_0 = 0,\ a = A,\ x = \frac{1}{2}D$

$x = 0 + 0t + \frac{1}{2}At^2$

$\frac{1}{2}D = \frac{1}{2}At^2$

$D = At^2 \qquad t = \sqrt{\frac{D}{A}}$

$D/A = t^2 \qquad \boxed{t = 2\sqrt{\frac{D}{A}}}$

where D is in meters

Total Time is Double the expression by symmetry

$t = 2\sqrt{\frac{1.27\times10^{13}\ m}{0.238\ m/s^2}} = 14609769.96\ s$

$14609769.96\ s \times \frac{1\ year}{31536000\ sec.} = \boxed{0.463\ years}$

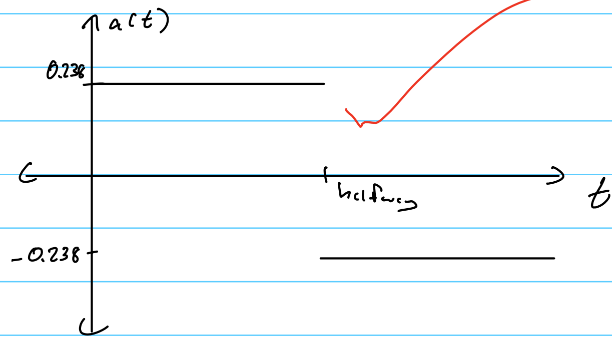
b) $v = v_0 + at,\ v_0 = 0,\ a = A,\ t = \sqrt{\frac{D}{A}}$ ← half trip time

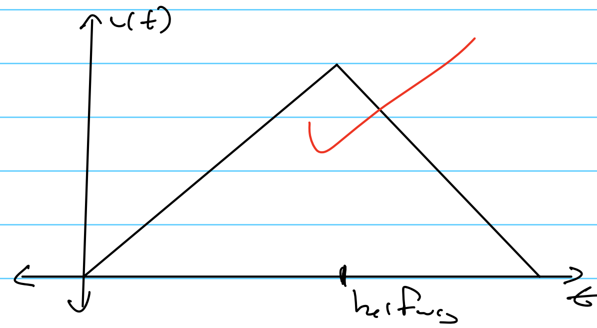
$v = A\left(\sqrt{\frac{D}{A}}\right) = \sqrt{AD}$ $\quad \boxed{V = \sqrt{AD}\text{, where D is in meters}}$

greatest speed at halfway point

$v = \sqrt{(0.238\ m/s^2)(1.27\times10^{13}\ m)} = \boxed{1.74\times10^{6}\ m/s}$

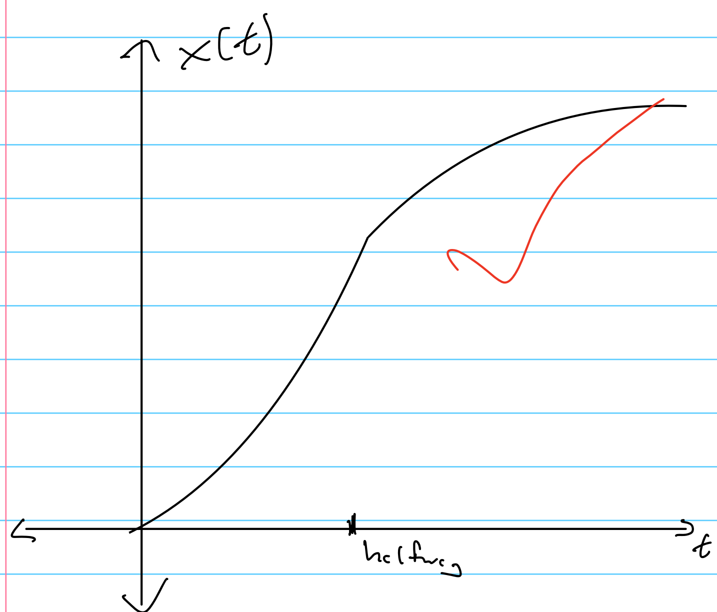
c)





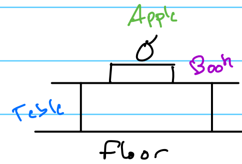
Acceleration is positive at 0.238 m/s^2 for half the trip, then negative at the same value for the second half of the trip

Since acceleration is piece-wise, the velocity graph is linear, with opposite slopes at the halfway point



The position of the ship increases quadratically to the halfway point, then it decreases quadratically at the same, but opposite rate, on the latter half of the trip. This is supported by the opposite slopes of the velocity graph.

Question 5)



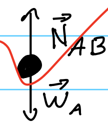
apple's mass: $m_A = 76g$

book's mass: $m_B = 1.07kg$ $a = 0$

table's mass: $m_T = 13.4kg$

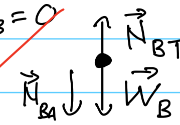
a) N2L The book has 0 acceleration, so by N2L, the Net Force on the book must be 0 as well. This is by the relation: $F = ma$ where $a = 0$

b) Apple FBD: $a_A = 0$ ↑Y



← On apple due to book

Book FBD: $a_B = 0$ ↑Y



c) W=mg is known, N2L, F=ma

$m_A = 76g = 0.076kg$, $m_B = 1.07kg$, $m_T = 13.4kg$, $a_y = 0$

Apple: $F_{A_y} = m_A a_y$, $a_y = 0$

$N_{AB} - W_A = 0$, Weight force is neg.

$N_{AB} = W_A$, $W_A = m_A g$ is known

$\boxed{N_{AB} = m_A g \text{ and } W_A = m_A g}$

$N_{AB} = W_A = .076\ kg(9.81 m/s^2)$

$= 0.746\ kg\frac{m}{s^2}$

$\boxed{N_{AB} = .746\ kg\frac{m}{s^2} \text{ and } W_A = .746\ kg\frac{m}{s^2}}$

Book: $F_{B_y} = m_B a_y$, $a_y = 0$

$N_{BT} - N_{BA} - W_B = 0$, $W_B = m_B g$

$N_{BT} - N_{BA} - m_B g = 0$

N3L $N_{BA} = N_{AB}$ ← opposite directions

$N_{BT} - m_A g - m_B g = 0$

$\boxed{N_{BT} = (m_A + m_B)g}$ $\boxed{N_{BA} = m_A g,\ W_B = m_B g}$

$N_{BA} = m_A g = .076\ kg(9.81 m/s^2) = \boxed{0.746\ kg \cdot \frac{m}{s^2}}$

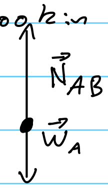
$N_{BT} = (m_A + m_B)g = (.076kg + 1.07kg)(9.81 m/s^2)$

$= \boxed{11.24\ kg \cdot \frac{m}{s^2}}$

$W_B = m_B g = 1.07kg(9.81 m/s^2) = \boxed{10.50\ kg \cdot \frac{m}{s^2}}$

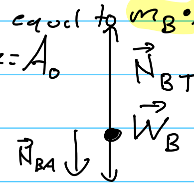
d) N2L: The Net force on the book is equal to $F_{Net} = m_B \cdot A_0$. This means that the sum of the forces on the book in an elevator will accel $= A_0$ is equal to $m_B \cdot A_0$

e) Apple FBD: $a = A_0$ ↑Y



Normal Force must be >W due to positive acceleration

Book FBD: $a = A_0$ ↑Y



f) $m_A = 0.076$, $m_B = 1.07kg$, $A_0 = 6.96 m/s^2$, $W = mg$ ← defined force N2L

Apple: $F_{A_y} = m_A a$, $m = m_A$, $a = A_0$, $W = mg$

$N_{AB} - W_A = m_A \cdot A_0$

$N_{AB} - m_A g = m_A \cdot A_0$

$\boxed{N_{AB} = m_A(g + A_0)}$ and $\boxed{W_A = m_A g}$

$N_{AB} = 0.076kg(9.81 m/s^2 + 6.96 m/s^2) = \boxed{1.27\ kg \cdot \frac{m}{s^2}}$

$W_A = 0.076 kg(9.81 m/s^2) = \boxed{0.746\ m/s^2}$

Book: $F_{B_y} = m_B a$, $m = m_B$, $a = A_0$, $W = mg$

$N_{BT} - N_{BA} - W_B = m_B \cdot A_0$

N3L $N_{BA} = N_{AB}$ ∴ $N_{BT} - (g + A_0)m_A - m_B g = m_B \cdot A_0$

$N_{BT} = m_B(g + A_0) + m_A(g + A_0)$

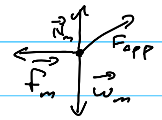
$\boxed{N_{BT} = (m_B + m_A)(g + A_0),\ W_B = m_B g,\ N_{BA} = m_A(g + A_0)}$

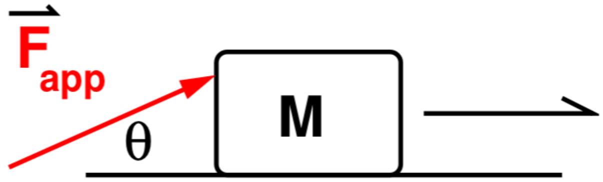
$N_{BT} = (1.07kg + .076kg)(9.81 m/s^2 + 6.96 m/s^2) = \boxed{14.22\ kg \cdot \frac{m}{s^2}}$

$W_B = (1.07kg)(9.81 m/s^2) = \boxed{10.50\ kg \cdot \frac{m}{s^2}}$

$N_{BA} = 0.076\ kg(9.81 m/s^2 + 6.96 m/s^2) = \boxed{1.27\ kg \cdot \frac{m}{s^2}}$

Question 6)





Block Mass: M
Distance Traveled: D
Friction: f
Angle to Horizontal: θ

a) Normal Force's Work

θ is given in relation to the horizon, so the normal force points upward 90° from θ. By the formula $\vec{W} = Fd\cos\theta$, $W = 0$ when $\theta = 90°$. The Normal Force does no work. $\boxed{\vec{W} = 0}$

b) Definition of Work

Again, θ is in relation to the horizon. The weight force point straight down, making a $-90°$ angle with the horizon. By $\vec{W} = Fd\cos\theta$, we know that $\vec{W} = 0$ when $\theta = 90°$ or $\theta = -90°$. Therefore, the weight force does no work. $\boxed{\vec{W} = 0}$

c) By Definition, work is force applied over a distance. We are given parameters θ, D, and F_{app} for angle, distance, and Applied Force, respectively. Plugging these values into the aforementioned work formula yields $\boxed{\vec{W}_{app} = F_{app} \cdot D \cdot \cos\theta}$

d) The work done by Friction is calculated with an incidence angle opposite the direction of motion. The block is moving right (180°), so the friction force is $-180°$. Plugging this into our work formula yields $\vec{W} = f \cdot D\cos(-180°)$ which simplifies to $\boxed{\vec{W} = -f \cdot D}$

e) The work done on the block is the sum of the individual work vectors acting on the block. This is represented by $\boxed{\vec{W}_{Tot} = F_{app} \cdot D \cdot \cos\theta - f \cdot D}$

f) Work Energy Relation: $W_T = \Delta K$

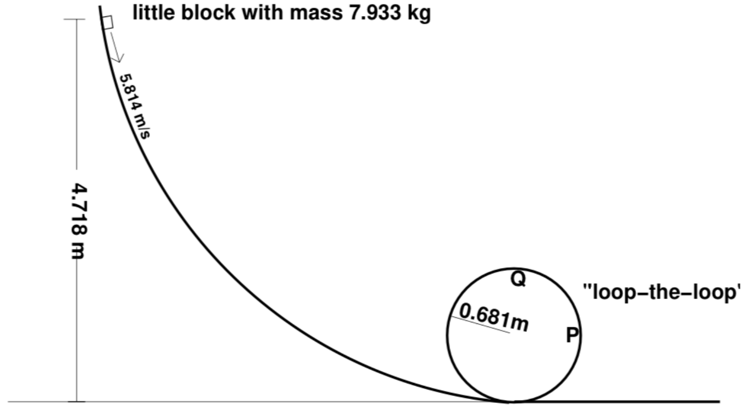
$W_{Tot} = K_f - K_i$ $\qquad W_{Tot} = F_{app} \cdot D \cdot \cos\theta - f \cdot D$, $m = M$,

$W_{Tot} = \frac{1}{2}mv^2 - K_i$, $K_i = 0$ as block starts at rest

$W_{Tot} = \frac{1}{2} M \cdot v^2 - 0$

$2 \cdot \frac{W_{Tot}}{M} = v^2$ $\qquad v = \sqrt{\frac{2 \cdot W_{Tot}}{M}}$ $\Rightarrow$ $\boxed{v = \sqrt{\frac{2(F_{app} \cdot D \cdot \cos\theta - f \cdot D)}{M}}}$

Question 7)



mass = m_b = 7.933 kg

int. speed = v_i = 5.814 m/s

height = h = 4.718 m

Radius = R = 0.681 m

frictionless surface

$g = 9.81\ m/s$

a) Normal Force Definition

The Work due to the Normal Force, N, is 0 as it is always perpendicular to the direction of motion. $W = Fd\cos\theta$, so $\perp$ forces have $\theta = 90°$, so $Fd\cos(90) = 0$. N = 0

b) Work done by the weight force

$W_{\vec{w}} = -mg\Delta y$, $y_i = h$, $y_f = R$, $m = m_b$

$W_{\vec{w}} = -m_b g(y_f - y_i)$

$W_{\vec{w}} = -m_b g(R-h)$

$\boxed{W_{\vec{w}} = -m_b g(R-h)}$

This makes sense because our coordinate system points in the direction of gravity, so the weight force should do positive work.

$W_{\vec{w}} = -(7.933\ kg)(9.81\ m/s^2)(0.681\ m - 4.718\ m)$

$\boxed{W_{\vec{w}} = 314.17\ \frac{kg \cdot m^2}{s^2}}$

c) Conservation Laws

check conditions: ⓐ Force is conservative
or ⓑ Force does 0 work

$\vec{N}$ does 0 work, proved in (a)

$\vec{w}$ is always conservative

$E_{total} = E'_{total}$

$U_i + K_i = U'_f + K'_f$

$\cancel{m}gy + \frac{1}{2}\cancel{m}v^2 = \cancel{m}gy' + \frac{1}{2}\cancel{m}v'^2$ $\left\{ \begin{array}{l} y = h,\ y' = R \\ v = v_i,\ v' = v_f \end{array} \right.$

$gh + \frac{1}{2}v_i^2 = gR + \frac{1}{2}v_f^2$

$gh - gR + \frac{1}{2}v_i^2 = \frac{1}{2}v_f^2$

$2g(h-R) + v_i^2 = v_f^2$

$\boxed{v_f = \sqrt{2g(h-R) + v_i^2}}$

Because conditions are met for conservation law of energy, it can be applied to this scenario

$v_f = \sqrt{2(9.81\ m/s^2)(4.718\ m - 0.681\ m) + (5.814\ m/s)^2}$

$\therefore$ $\boxed{v_f = 10.63\ m/s}$