☐PHYS115 ☐PHYS121 ☐PHYS123
☐PHYS116 ☐PHYS122 ☐PHYS124
Lab Cover Letter

Author (*You*) Trevor Swan   Signature: Trevor Swan

I declare that this assignment is original and has not been submitted for assessment elsewhere, and acknowledge that the assessor of this assignment may, for the purpose of assessing this assignment: (1) reproduce this assignment and provide a copy to another member of faculty; and/or (2) communicate a copy of this assignment to a plagiarism checking service (which may then retain a copy of this assignment on its database for the purpose of future plagiarism checking).

Lab Partner(s) Adi Malik

Date Performed January 31, 2024   Date Submitted February 6, 2024

Lab (*such as #1: UNC*) #2: 1P

TA: Philip Dudones

GRADE (*to be filled in by your TA*) See your TA for detailed feedback.
An '*x*' next to a subcategory means you need to improve this aspect of your work.

Paper Subtotals (points)

( ) ***General* (6)**
_____ Sig. figs.
_____ Units
_____ Clarity of Presentation
_____ Format

( ) ***Abstract* (4)**
_____ Quantity or principle
_____ How measurement was made
_____ Numerical Results
_____ Conclusion

( ) ***Intro & Theory* (9)**
_____ Basic principle
_____ Main equations to be used
_____ Apparatus
_____ What will be plotted
_____ Fitting parameters related

( ) ***Exp. Procedures* (15)**
_____ Description
_____ Stating and justifying uncertainties
_____ Data Record
_____ Quality of Lab Work

( ) ***Analysis & Error Analysis* (20)**
_____ Discussion
_____ Equations & Calculations
_____ Presentation inc. Graphs, Tables
_____ Results Reported & Reasonable
_____ Underlined items addressed

( ) ***Discussion & Conclusions* (6)**
_____ Numerical comparison of results
_____ Logical conclusions
_____ Discussion of pos. errors
_____ Suggestions to reduce errors

( ) **Paper Total (60 points)**
(30 points for CME or EPF)

( ) **Notebook (10 points)**
_____ Format (*proper style, following directions*)
_____ Apparatus (*brief description of equipment, including sketches*)
_____ Data (*including computer file names and manually recorded data*)
_____ Experimental Technique (*describing your procedures; stating & justifying uncerts.*)
_____ Analysis (*results and errors*)

( ) **Worksheet(s)/Fill-in-the-Blank-Report (30 points)** *if applicable*

( ) **Adjustments** – late submissions, improper procedures, etc. – or bonus points for exceptional work.

( ) Total Grade

Graded by ___________ *(TA's initial)*

Inclined Plane

Trevor C. Swan
Department of Physics, Case Western Reserve University
Cleveland, OH 44016-7079

Abstract:

I have tested the theory of Newton's Second Law of Motion with a system of a cart on an inclined plane connected to a counterweight by a string over a pulley. After releasing the system from rest, I measured the velocity as a function of time. According to Newton's theory, the velocity should vary *linearly* with time. The data that I have collected does not support a linear dependence between velocity and time to within the uncertainties on the data points. I have also measured the average acceleration of the system as a_{meas} = 0.33 ± 0.02 m/s². Newton's Second Law predicts that the acceleration of the system should be a_{pred} = 0.362 ± 0.002 m/s². I find that the measured acceleration is not consistent with the predicted acceleration. Although friction played a role in the discrepancy of our measurements, I can safely conclude that our experiment is a decent model for predicting Newton's Second Law.

(*If your measured velocity supports a linear model and/or your accelerations are consistent with the predicted acceleration, cross out the "not's" in the above abstract paragraph. Give a one-sentence conclusion about the lab.*)

Theory and Background:

One can determine the acceleration of the system depicted in Figure 1 by using Newton's Second Law to analyze the motion. Assuming that the frictional force $\vec{f}$ is negligible and that the pulley is massless and frictionless, the acceleration a of the system is[1]

$$a = \frac{g(m_1 - m_2 \sin\theta)}{m_1 + m_2}. \quad (1)$$

where m_1 is the mass of the weight, m_2 is the mass of the cart, and θ is the incline angle.

(*Write down the appropriate equation; define all variables that haven't been defined yet.*)

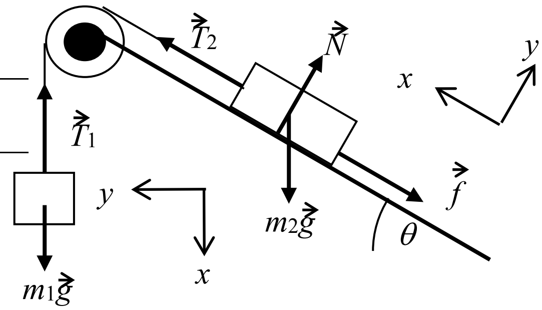


Figure 1: Schematic of Forces in Experiment. Courtesy Driscoll, (year).

One can find the sine of the angle of the incline using Eq. 1. If we adjust m_1 and m_2 so that the acceleration is zero and call this hanging mass the balancing mass m_b, then

$$0 = \frac{g(m_b - m_2 \sin\theta)}{m_b + m_2} \quad (2)$$

$$\Rightarrow 0 = m_b - m_2 \sin\theta \quad (3)$$

$$\Rightarrow \sin\theta = \frac{m_b}{m_2}. \quad (4)$$

(*Eq. 2 should be Eq. 1 with $a = 0$; Eq. 3 should be an intermediate algebra step; Eq. 4 should be $\sin\theta$ in terms of m_b and m_2.*)

Equation 1 also implies that the acceleration of the system will be constant, so the velocity as a function of time will be

$$v(t) = \frac{g(m_b - m_2 \sin\theta)}{m_b + m_2} \cdot t \quad (5)$$

where t is the time in seconds and m_b is the balance weight.

(*For equation 5 write down the expression for velocity in terms of time and other variables. Explain any new variables you introduce.*) From Eq. 5, we can see that if we fit a straight line to a plot of v vs. t, the slope of the line will be the acceleration and the intercept will be the velocity at time zero.

Procedure:

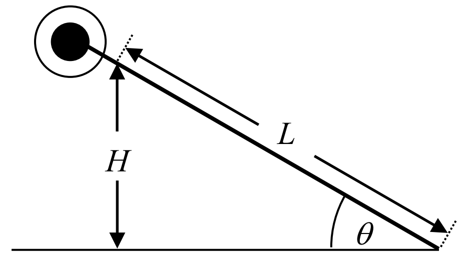


Figure 2: Estimating the angle. Courtesy Driscoll, (year).

To get an estimate of the angle of the incline I estimated the length L and height H of the incline as in Figure 2. I used a meter stick to measure H = 42.08 cm and L = 120.65 cm. Getting accurate and precise measurements of H and L was difficult because the IP was slightly elevated.

I compensated for these difficulties by measuring the Height of the IP by going from the table to the bottom edge of the ramp.

Because of these issues, I estimate that my uncertainty in H is δ_H = 0.01 cm and my uncertainty in L is δ_L = 0.01 cm.

My first estimate of $\sin\theta$ is then

$$\sin\theta = \frac{H}{L} \tag{6}$$

$$\Rightarrow \sin\theta = \frac{42.08}{120.65} = .3322$$

(*Put the appropriate variables in the first line; put your actual measurements and final value for* $\sin\theta$ *in the second line.*)

The uncertainty in $\sin\theta$ is

$$\delta_{\sin\theta} = \sqrt{\delta^2_{\sin\theta,H} + \delta^2_{\sin\theta,L}} \tag{7}$$

where $\delta_{\sin\theta,H}$ is the uncertainty in $\sin\theta$ due to δ_H and $\delta_{\sin\theta,L}$ is the uncertainty in $\sin\theta$ due to δ_L.

Using the "computational method" to determine $\delta_{\sin\theta,H}$ and $\delta_{\sin\theta,L}$, I obtain

$$\delta_{\sin\theta,H} = \frac{H+\delta_H}{L} - \frac{H}{L} = \frac{\delta_H}{L} \tag{8}$$

$$\Rightarrow \delta_{\sin\theta,H} = \frac{0.01}{120.65} = 8.3\times10^{-5}$$

and

$$\delta_{\sin\theta,L} = \frac{H}{L+\delta_L} - \frac{H}{L} \tag{9}$$

$$\delta_{\sin\theta,L} = \frac{42.08}{120.65+.01} - \frac{42.08}{120.65} = \underline{2.9\times10^{-5}},$$

yielding $\delta_{\sin\theta} = \sqrt{\left(\underline{8.3\times10^{-5}}\right)^2 + \left(\underline{2.9\times10^{-5}}\right)^2} = \underline{8.8\times10^{-5}}$, or $\sin\theta = \underline{.33210} \pm \underline{8.8\times10^{-5}}$. Since $\theta = \sin^{-1}(\sin\theta)$, the uncertainty in θ is

$$\delta_\theta = \underline{\sin^{-1}(\sin\theta + \delta_{\sin\theta})} - \underline{\sin^{-1}(\sin\theta)} \tag{10}$$

$$\Rightarrow \delta_\theta = \underline{\sin^{-1}(.3321 + 8.8\times10^{-5})} - \underline{\sin^{-1}(.3321)} = \underline{0.005}\ \underline{\ ^\circ\ },$$

or $\theta = \underline{20.410} \pm \underline{0.005}\ \underline{\ ^\circ\ }$.

I measured m_2 with an electronic balance and determined that $\underline{m_2} = \underline{490.7} \pm \underline{0.1}\ \underline{g}$. I estimated the uncertainty δ_{m2} as $\underline{0.1}\ \underline{g}$ because The scale only goes to the tenths place so we can only be that certain.

I now used the first estimate of θ to determine an estimate of the mass m_b required to balance the system by taking Eq. 4 and solving for m_b:

$$m_b = \underline{m_2 \sin\theta} \tag{11}$$

$$\Rightarrow m_b = \underline{490.7(\sin(20.41))} = \underline{171.1}\ \underline{g}.$$

(*Solve Eq. 4 for m_b, substitute in the appropriate numbers, and solve.*)

I then set the mass of m_1 to $\underline{171.1}$ by adding masses to the hanger. After releasing the cart, the system was not in balance; the system tended towards the hanger, pulling the cart.

(*If the system was balanced, cross out "not." Describe the system's motion in the blank.*)

I then found the minimum and maximum masses that lead to zero acceleration (m_{min} and m_{max}) by adding and removing mass from m_1 in order to obtain a better estimate of the angle of the incline and to account for the small amount of friction in the system. I tested zero acceleration by a stationary cart because of the ease of use. While constant velocity would account for friction more accurately, it is much harder to determine constant velocity in our lab. Stationary measurements were taken as a result of this.

(*State if you tested zero acceleration by a stationary cart or cart moving with constant velocity. If you used the constant velocity test, also state how you determined the cart was moving at constant speed. Write a sentence about your reasons for choosing your methods, i.e., the advantages and disadvantages of your methods over other choices.*)

Using the procedure above, I determined that m_{min} = 157.9 g and m_{max} = 164.2 g, each with negligible uncertainty. I then set the average of m_{min} and m_{max} to be the "balancing mass" m_b and half the difference between m_{min} and m_{max} as the uncertainty δ_{mb}, so m_b = 161.1 ± 3.2 g. Substituting m_b into Eq. 4, we see that sine of the angle of the incline is

$$\sin\theta = \frac{161.1}{490.7} = .3283 .$$

The uncertainty in $\sin\theta$ is

$$\delta_{\sin\theta} = \sqrt{\delta^2_{\sin\theta,m_b} + \delta^2_{\sin\theta,m_2}} \tag{12}$$

where $\delta_{\sin\theta,m_b}$ is the uncertainty in $\sin\theta$ due to δ_{mb} and $\delta_{\sin\theta,m_2}$ is the uncertainty in $\sin\theta$ due to δ_{m2}.

Using the "computational method" to determine $\delta_{\sin\theta,m_b}$ and $\delta_{\sin\theta,m_2}$, I obtain

$$\delta_{\sin\theta,m_b} = \frac{m_b + \delta_{mb}}{m_2} - \frac{m_b}{m_2} = \frac{\delta_{mb}}{m_2} \tag{13}$$

$$\Rightarrow \delta_{\sin\theta,m_b} = \frac{0.1}{490.7} = 2\times10^{-4}$$

and

Trevor Swan

$$\delta_{\sin\theta,m_2} = \frac{m_b}{m_2+\delta_{m_2}} - \frac{m_b}{m_2} \tag{14}$$

$$\delta_{\sin\theta,m_2} = \left|\frac{161.1}{490.7+3.2} - \frac{161.1}{490.7}\right| = \underline{0.002},$$

yielding $\delta_{\sin\theta} = \sqrt{\left(\underline{2\times10^{-4}}\right)^2 + \left(\underline{0.002}\right)^2} = \underline{0.002}$, or sin θ = 0.328 ± 0.002. Compared with my previous value of 0.33210±8.8x10^-5, this value is less certain and smaller in magnitude.

(*Compare this value of sinθ with the value from direct measurement of L and H.*)

I will adopt this value for sinθ.

I then set the counterweight m_1 to a value of 186.1 g to allow the cart to accelerate up the plane. I will refer to this value of m_1 as the experiment value m_e. I recorded the motion of the cart using an encoded pulley and *Logger Pro* software.[2] I subsequently exported the data from *Logger Pro* to *Origin* for a more complete analysis. Specifically, I plotted velocity vs. time to determine if the velocity has a linear dependence and to measure the slope. Previous experimenters[3] using this equipment have determined that the uncertainty in v has a value of 0.008 m/s; we adopted this value in our analysis.

Results:

The average acceleration recorded directly by *Logger Pro* statistics software was a_{meas1} = 0.33 ± 0.02 m/s². Figure 3 shows a plot of velocity vs. time and a best linear fit using the *Origin* software. (*Attach a copy of your v vs. t graph labeled "Figure 3" to the end of the report.*) For this plot, vertical error bars are assigned based on an estimated uncertainty of the velocity measurements of ± 0.008 m/s for each point, where this value was determined by previous measurements done by the laboratory staff. As can be seen in the plot, the since ~~not~~ every data point lies within about one error bar of the best linear fit we conclude that these data are ~~not~~ consistent with Newton's model. (*If the data points fit the line to within about one error bar then delete the words "not" above. If the data are not consistent be sure to address this in your conclusion. Is Newton wrong? Or might there be systematic error in your data?*)

The slope of the graph as determined by *Origin's* fitting software is a_{meas2} = 0.348 ± 0.002 m/s². In comparing this value to the value obtained directly from *Logger Pro* I note that these two values do not agree (*agree/do not agree to within their uncertainties/are exactly the same*). We expect that a_{meas2} should be more accurate because the software outputted a very high r^2 value of 0.999 and I will adopt it as the measured value, a_{meas}.

By substituting in known values into Eq. 1, one can determine a theoretical value for the acceleration of the system, a_{pred}. Since I did not measure θ directly, I will substitute Eq. 4 into Eq. 1 to obtain:

$$a_{pred} = \frac{g(m_e - m_b)}{m_e + m_2} \tag{15}$$

$$\Rightarrow a_{pred} = \frac{9.81(186.1 - 161.1)}{186.1 + 490.7} = .362 \text{ m/s}^2.$$

Error Analysis:

To find the uncertainty in a_{pred}, $\delta_{a_{pred}}$, I must find the contribution to $\delta_{a_{pred}}$ for each of the quantities in Eq. 15 and add them in quadrature. The uncertainties in m_e and g are negligible compared to the other quantities because m_e is an additive quantity expressed by $m_b + 2S_g$, and g is a known value, and testing it is not relevant here.

(*Identify any quantities that you will treat as having negligible uncertainty and justify your treatment. Then show your work in estimating the uncertainty in a_{pred} on the next page.*)

Because $m_e = 25_g + m_b$, we can substitute this expression into Eq.15

This leaves us with: $a = \frac{25 \cdot g}{m_b + m_2 + 25}$ and $\delta_{a_{pred}} = \sqrt{\delta_{a_{pred_{m_b}}}^2 + \delta_{a_{pred_{m_2}}}^2}$

From here we can calculate δ_a using the derivative method:

$$\delta_{a_{pred_{m_b}}} = \frac{\partial}{\partial m_b}(a) \cdot \delta_{m_b} = \frac{25 \cdot g}{(m_b + m_2 + 25)^2} \cdot \delta_{m_b} = \frac{25_g(9.81\,m/s^2)}{(161.1_g + 400.7_g + 25_g)^2} \cdot (3.2) = 0.00171$$

$$\delta_{a_{pred_{m_2}}} = \frac{\partial}{\partial m_2}(a) \cdot \delta_{m_2} = \frac{25 \cdot g}{(m_b + m_2 + 25)^2} \cdot \delta_{m_2} = \frac{25_g(9.81\,m/s^2)}{(161.1_g + 400.7_g + 25_g)^2} \cdot (0.1) = 5.35 \times 10^{-5}$$

$$\therefore \delta_{a_{pred}} = \sqrt{(0.00171)^2 + (5.35 \times 10^{-5})^2} = 0.00171\ m/s^2$$

So a_{pred} = 0.362 ± 0.002 m/s^2.

Conclusions:

The predicted value for the acceleration was a_{pred} = 0.362 ± 0.002 m/s^2 and the measured value for the acceleration of the system was a_{meas} = 0.348 ± 0.002 m/s^2. The acceleration do not agree as they do not lie within their uncertainties. This is a result of systematic error due to friction. Not accounting for friction in our prediction simplified our work, but created inaccurate results. The real world is not frictionless, so we could account for this error by involving a friction coefficient, or something similar, to our equations.

(*State whether or not your values agree within their uncertainties. If they do not agree, suggest at least one source of systematic error that were not adequately accounted for and suggest a way to reduce the effect of this error. If the two values do agree, suggest at least one source of random error and suggest a way to reduce the effect of this error. Make a quantitative statement about the effect friction should have had on this experiment. Make a conclusion about Newton's Second Law, especially regarding whether or not the data points support a linear model. If the model does not show a linear dependence give at least one reason why this might not be.*)

Acknowledgements:

I would like to thank Adi Mallik, Case Department of Physics, for his help in obtaining the experimental data and preparing the figures.

(*Thank your lab partner(s).* ***If they or anyone else gave you additional assistance, say who they were and specifically what their assistance was***.)

References:

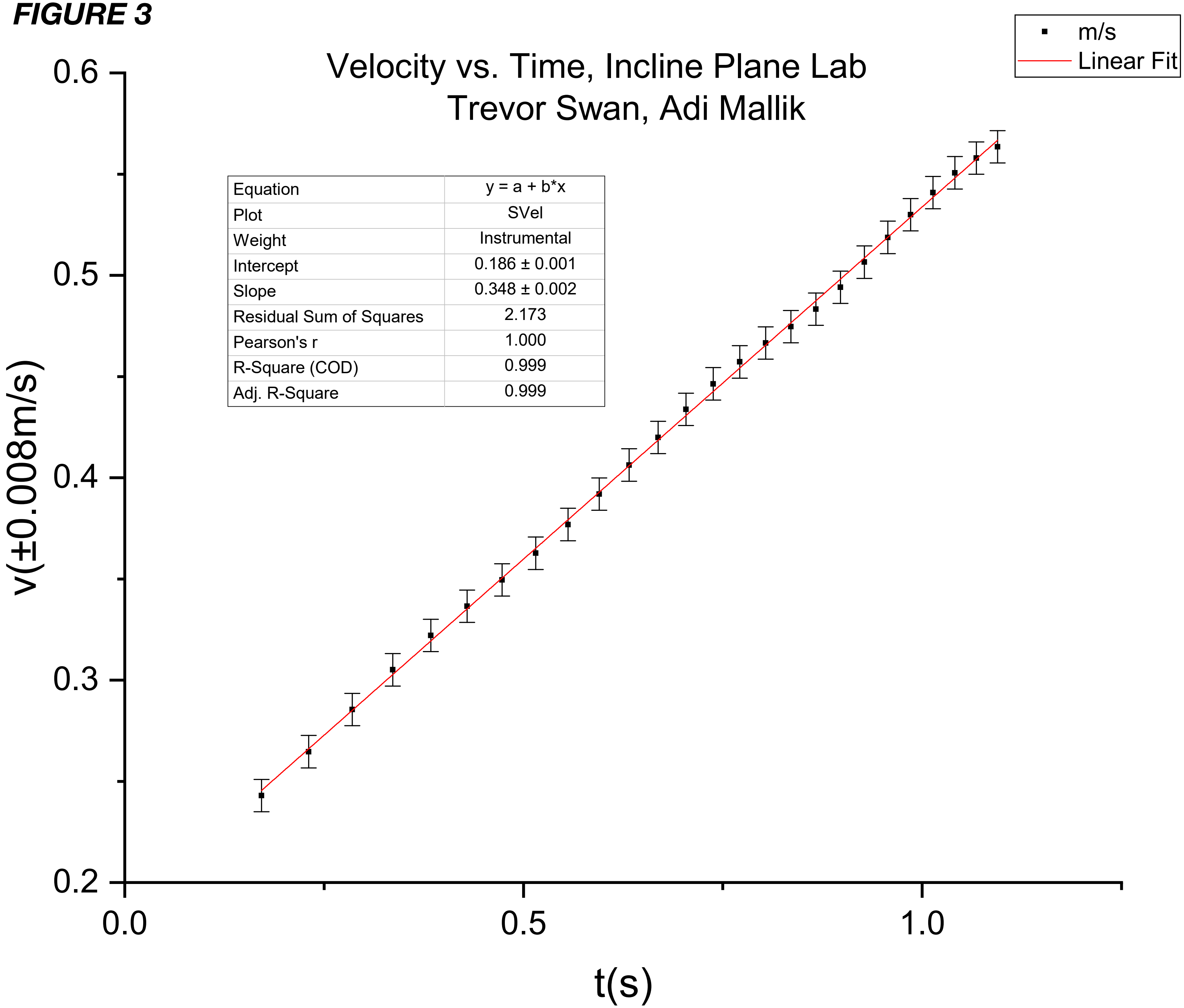
(*If you have any additional references, list them below. Make sure to indicate with an endnote where in the report you referred to the reference.*)

1. Driscoll, D., *General Physics I: Mechanics Lab Manual*, “Inclined Plane,” CWRU Bookstore, 2014.

2.

End Notes:
[1] Driscoll, D., p. 2.
[2] Driscoll, D., p. 3, describes the encoded pulley.
[3] Driscoll, D., p. 5

FIGURE 3


Velocity vs. Time, Incline Plane Lab
Trevor Swan, Adi Mallik
m/s
Linear Fit
Equation y = a + b*x
Plot SVel
Weight Instrumental
Intercept 0.186 ± 0.001
Slope 0.348 ± 0.002
Residual Sum of Squares 2.173
Pearson's r 1.000
R-Square (COD) 0.999
Adj. R-Square 0.999
v(±0.008m/s)
t(s)
0.6
0.5
0.4
0.3
0.2
0.0
0.5
1.0