Chapter 6

Force Components

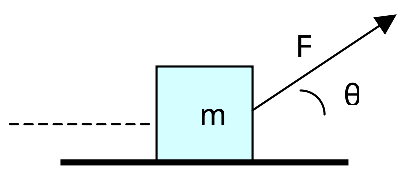
- **Force Components in 2D**
- **Newton's Second Law in Both Dimensions**
- **Appendix: Vector Primer**

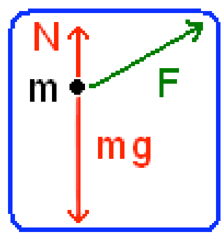
Force-at-an-Angle: Force Components

We consider forces that are not just parallel or perpendicular to the motion but at an arbitrary angle to that motion.

Blocks pulled or pushed at an angle without friction:

Example: Pulling a block at an angle to the horizontal corresponds to the following FBD (we still ignore friction):





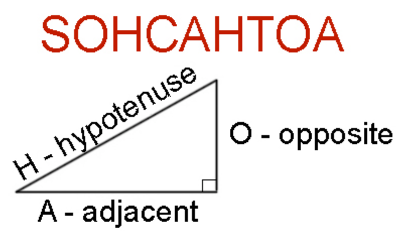
The force in the horizontal direction is less than F, and we call this the horizontal component of F. This component is what goes into the second law for finding the horizontal acceleration.

So how do we calculate this component?

First of all, recall the picture of a 2D force vector is an arrow whose direction is the force direction and whose length is the force magnitude.

GOLDEN RULE:

- Make a right triangle whose hypotenuse is the force vector and whose legs are in the directions you need (the components you need).
- See if an angle in the right triangle is related to any angle given in the problem.
- Calculate the lengths of the legs (the components) using good old SOHCAHTOA.



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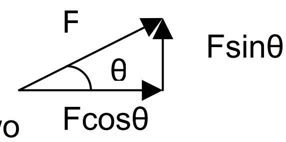
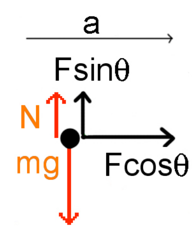
- The directions of the components are along the legs and follow from the fact that they must add up, as "vector addition" to the total force vector (directed along the hypotenuse) – see the appendix.

Newton's Second Law in Example Using Components:



The right triangle of interest for the previous force-on-block situation is shown on the right. (Since this is needed for our Golden Rule, we might call it our **GOLDEN** triangle.) The angle related to θ is obvious. The force vector is the result of adding the two vectors (the up vertical vector with magnitude Fsinθ and the right horizontal vector with magnitude Fcosθ) corresponding to the two components.



We replace F by its force components in the FBD from which we can write the two second-law equations for the vertical and horizontal directions. If the horizontal acceleration is called a, the horizontal second law is

$$F\cos\theta = ma$$

and the vertical second law is reduced to an equilibrium condition if F is not large enough to produce any vertical motion (so certainly the vertical acceleration is zero),

$$F\sin\theta + N - mg = 0$$

For a given m, F and θ, the results for the acceleration and the support force on the block by the table ($F_{\substack{\text{on block}\\ \text{by table}}} \equiv N$) are therefore found by solving the above two equations for a and N, respectively:

$$\boxed{a = \frac{F\cos\theta}{m}} \quad \text{and} \quad \boxed{N = mg - F\sin\theta}$$

Comments:

1) We see that this is an example where N is not simply mg. **This example warns us NOT to blithely write N = mg !!** Draw the FBD and find out what N really is from the second law.

2) Recall a footnote in Ch. 5. When there's **no** friction, then the force due to any surface in contact with an object is normal (i.e., perpendicular). By definition, if there is a component due to the surface that is parallel to the surface, then that's FRICTION! See Ch. 7.

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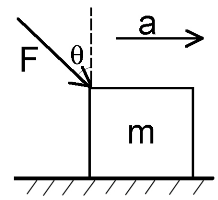
Problem 6-1



A block of mass m i*s PUSHED along a horizontal* ***frictionless*** *table by a force* F *at an angle* θ *with the vertical, as shown in the figure. (If we showed* F *in Newtons in the figure, we would have Fig Newtons!)*

(a) *Draw an FBD for this block.*
(b) *Guess whether the normal support force* N *by the table on the block is bigger, equal to, or smaller than the weight of the block.*
(c) *Find, but do not solve, two separate second-law equations, one involving the horizontal acceleration* a *and the other involving* N, *given the parameters* m, F, θ, *as well as* g.
(d) *Solve the two equations in (c) for the horizontal acceleration* a, *and for the normal support force* N, *in terms of the given parameters.*
(e) *Find the normal support force in Newtons if* $m = 5.0$ kg , $F = 12$ N, $\theta = 65^o$. *Compare it to the block's weight and thus compare it to your answer in (b).*

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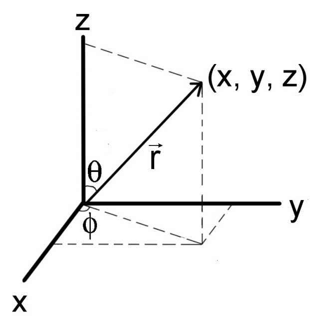
Appendix: A Little Vector Primer

Let's do a mini-course on the use of vectors. Instead of three rectangular coordinates relative to some origin, we can describe, for example, a position by a vector $\vec{r}$:

$\vec{r}$ = arrow from the origin to the position,
with the head at the position, the tail at the origin.



∴ A vector may be defined in terms of its magnitude and direction.

Comments:

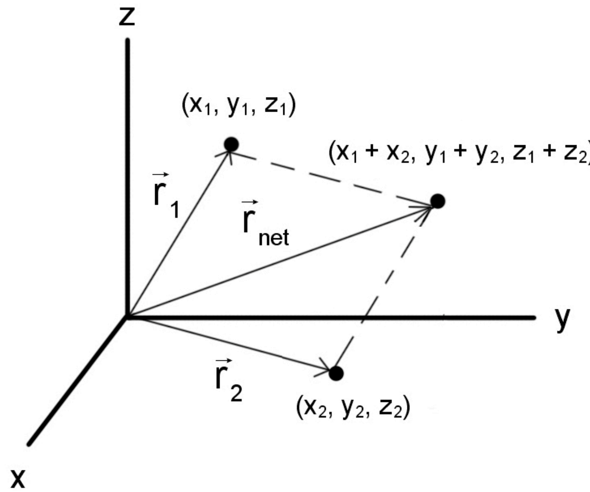
1) It is very convenient to write equations of motion, like Newton's second law, as vector equations. Displacements, velocities, accelerations, forces, momenta, angular momenta, and more are represented by vectors.

2) So, the vector is a short-hand notation for three numbers (the usual choices for these three numbers are the Cartesian coordinates x, y, z or spherical coordinates r, θ, ϕ – beware of the different conventions used by the mathematics world and the physical sciences world).

3) The magnitude of $\vec{r}$ is $\sqrt{x^2 + y^2 + z^2} \equiv |\vec{r}| \equiv r$

4) By the way, time, mass, and temperature are represented by single numbers ("scalars").

Writing vectors in terms of rectangular coordinates:
We often write: $\vec{r} = (x, y, z)$. To see the usefulness of this representation, consider a rock (why not?) to be moved a displacement corresponding to $\vec{r}_1 = (x_1, y_1, z_1)$ and then it is moved again by a subsequent displacement $\vec{r}_2 = (x_2, y_2, z_2)$. Draw arrows for displacements and for the net overall displacement as shown.



We can think of the resultant as due to a parallelogram construction or by tail-to-tip construction or by Cartesian addition. All are equivalent.

Rules:

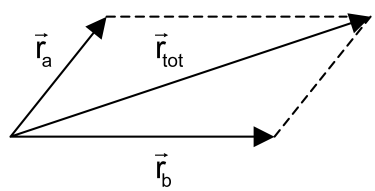
1) Call any two vectors equivalent if they have the same magnitude and if they are parallel. (They do not have to lie on top of each other; two things displaced with equal displacement vectors may be miles apart.)

2) Find sums (or differences) such as $\vec{r}_a + \vec{r}_b$ in three equivalent ways:

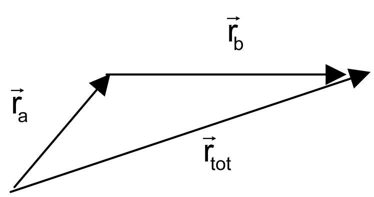
a. Coordinate or Component

$$\text{If } \vec{r}_a = (a_1, a_2, a_3) \text{ and } \vec{r}_b = (b_1, b_2, b_3),$$
$$\text{Then } \vec{r}_{tot} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

b. Parallelogram



c. Tail-to-Tip



3) The negative of $\vec{r} = (x, y, z)$ is $-\vec{r} = (-x, -y, -z)$ and subtracting $\vec{r}$ is equivalent to adding $-\vec{r}$.

EXAMPLE:

$$\vec{c} = (7.4, -3.8, -6.1)$$
$$\vec{d} = (4.4, -2.0, 3.3)$$
$$\therefore \vec{c} + \vec{d} = (11.8, -5.8, -2.8)$$
$$\therefore \left|\vec{c} + \vec{d}\right| = \sqrt{(11.8)^2 + (5.8)^2 + (2.8)^2} = 13.4$$

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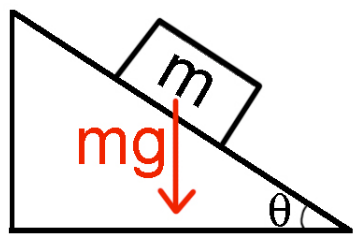
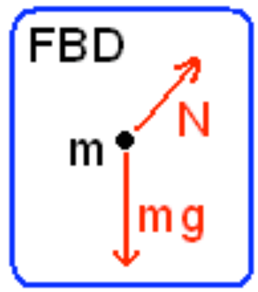
Problem 6-2 Quickies!

a) Just to make sure we're on the right page (namely, this one ☺), find the x,y,z *components and the magnitude of the vector* $-\vec{c}+\vec{d}$ *for* $\vec{c}$ *and* $\vec{d}$ *given above.*

b) *Let's discover something new in anticipating some results in the second cycle. A block of mass* m *is on a hill at an angle* θ *and subject to gravity as shown in the figure. The only forces are gravity and the normal support force from the slippery hill, as shown in the FBD:*





Think about the directions normal and parallel to the hill. If we tell you one of them has a sinθ factor and the other has a cosθ factor, guess what the normal and parallel components of the force of gravity (i.e., the weight mg*) are as a function of* θ? *Check yourself by finding out if your answers agree with what you expect them to be at* θ=0? *And at* θ=90°?

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