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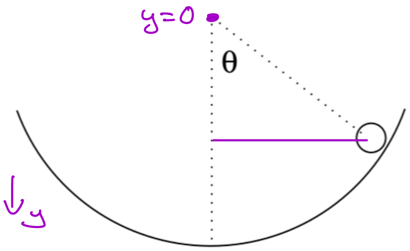
PHYS121 - Homework 7

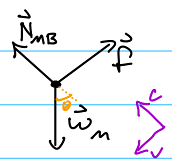
Problem 1)



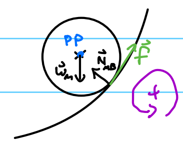
Given

- Marble of radius r and mass m
- Marble released @ rest w/ given angle θ_0
- Bowl has Radius of curvature R and $r << R$
- Friction, no slip no slide

a) FBD Marble



b) ~~x~~ FBD Marble



c) Determine Net Torque based on given

$\tau_{Net} = \tau_f + \tau_N + \tau_\omega$

$\tau_{Net} = r f \sin 90° + 0 + 0$

$\tau_{Net} = r f$

$\boxed{\therefore \tau_{Net} = mgr\sin\theta}$

N2L $F_v = m a_v$

$f - W\cos\theta = 0$ released @ rest

$f = mg\sin\theta$

$\boxed{\tau_{Net} = \frac{2}{7} m R g \sin\theta}$

d) N2L c-dir

$F_c = m a_c$ UCM $a_c = \frac{v_0^2}{r}$

$\vec{N}_{MB} - W_c = m \frac{\cancel{v_0^2}}{R}$ 0 @ rest

$\vec{N}_{MB} - W\cos\theta = 0$

$\vec{N}_{MB} = mg\cos\theta$

N2L v-dir

$F_v = m a_v$

$W_v - f = m a_v$

$W\sin\theta - f = m a_v$

Stop! $mg\sin\theta - f = m a_v$

N2L Rot

$\tau_{Net} = I\alpha$

$\tau_N + \tau_\omega + \tau_f = \frac{2}{5} m r^2 \alpha$ R.C $\alpha = \frac{a}{r}$

$0 \cdot W + rN\sin 180° + rf\sin 90° = \frac{2}{5} m r^2 \left(\frac{a_v}{r}\right)$

$\cancel{r} f = \frac{2}{5} m \cancel{r}^2 \left(\frac{a_v}{\cancel{r}}\right)$

$f = \frac{2}{5} m a_v$ Stop!

Two Eqs, Two Unknowns

$$\begin{cases} mg\sin\theta - f = m a_v \\ f = \frac{2}{5} m a_v \end{cases}$$

$\cancel{m} g\sin\theta - \frac{2}{5} \cancel{m} a_v = \cancel{m} a_v$

$g\sin\theta = \frac{7}{5} a_v$

$\therefore \boxed{a_v = \frac{5}{7} g \sin\theta}$

e) C of ME

Conditions: Weight is conservative, Normal Force $\perp$ → no work, friction does no work @ point of contact.

Before = After

$E_{Tot} = E_{Tot}'$

$U + K_R + K_T = U' + K_R' + K_T'$

$mgy + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = mgy' + \frac{1}{2} m v'^2 + \frac{1}{2} I \omega'^2$

Rolling Constraint: $\omega = \frac{v}{r}$

$y = -R\cos\theta,\ y' = R,\ v = 0,\ v' = ?\ \omega = 0,\ \omega' = \frac{v'}{r}$

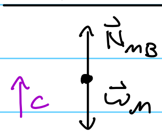
$-mgR\cos\theta + 0 + 0 = mgR + \frac{1}{2} m v_f^2 + \frac{1}{2}\left(\frac{2}{5} m \cancel{r}^2\right)\left(\frac{v_f}{\cancel{r}}\right)^2$

$-\cancel{m} gR\cos\theta = -\cancel{m} gR + \frac{1}{2} \cancel{m} v_f^2 + \frac{1}{5} \cancel{m} v_f^2$

$gR - gR\cos\theta = v_f^2 \left(\frac{1}{2} + \frac{1}{5}\right) \rightarrow gR(1 - \cos\theta) = \frac{7}{10} v_f^2$

$\therefore \boxed{v_f = \sqrt{\frac{10}{7} g R (1 - \cos\theta)}}$

f) FBD Bottom



N2L c-dir

$F_c = m a_c$ UCM $a_c = \frac{v^2}{r}$

$N_{MB} - W_m = m \frac{v^2}{R}$

$N_{MB} = W_m + m \frac{v_f^2}{R}$

$= mg + m \frac{\cancel{\sqrt{\frac{10}{7} g \cancel{R} (1 - \cos\theta)}}^{\cancel{2}}}{\cancel{R}}$

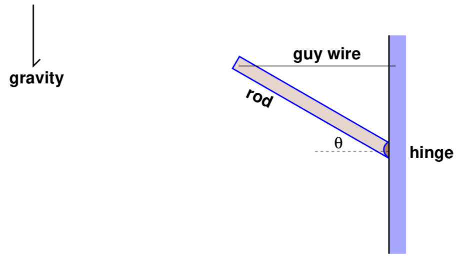
$N_{MB} = mg + \frac{10}{7} mg (1 - \cos\theta)$

$= mg\left(1 + \frac{10}{7} - \frac{10}{7}\cos\theta\right)$

$\therefore N_{MB} = mg\left(\frac{17}{7} - \frac{10}{7}\cos\theta\right)$

$\boxed{N_{MB} = \frac{1}{7} mg (17 - 10\cos\theta)}$

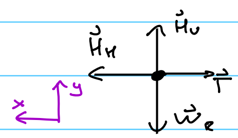
Problem 2)



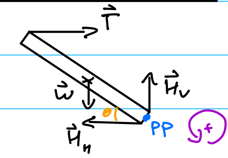
Given

- Rod of mass m and length l
- Rod has $I = \frac{1}{3}ml^2$
- Rod held in place @ θ
- Rod held by ideal String

a) FBD Rod



× FBD Rot



b) N2L Rod y-dir

$F_y = m\cancel{a_y}^{0}$

$H_v - W_R = 0$

$\boxed{H_v = mg}$

N2L Rod x-dir

$F_x = m\cancel{a_x}^{0}$

$H_H - T = 0$ Stop!

$H_H - \frac{1}{2}mg\tan\theta = 0$

$\therefore \boxed{H_H = \frac{1}{2}mg\,\cancel{\tan}\,\theta}$ cot

N2L Rot

$\tau_{Net} = I\cancel{\alpha}^{0}$

$\tau_H + \tau_v + \tau_T + \tau_w = 0$

$0 + 0 - lT\sin(90-\theta) + \frac{l}{2}W\sin\theta = 0$

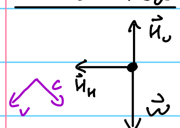
$\frac{\cancel{l}}{2}mg\sin\theta - \cancel{l}T\cos\theta = 0$

$\frac{1}{2}mg\sin\theta = T\cos\theta$

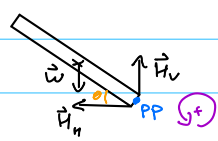
$T = \frac{1}{2}mg\tan\theta$

c) Rod starts @ rest!

FBD Rod



× FBD Rod



N2L Rot

$\tau_{Net} = I\alpha$ R.C. $\alpha = \frac{a_v}{r}$ $r = \frac{l}{2}$ for CoM

$\tau_H + \tau_v + \tau_w = \frac{1}{3}ml^2\left(\frac{a_v}{l/2}\right)$

$0 + 0 + \frac{\cancel{l}}{2}mg\sin\theta = \frac{1}{3}m\cancel{l^2}\left(\frac{2a_v}{\cancel{l}}\right)$

$\frac{1}{2}\cancel{m}g\sin\theta = \frac{2}{3}\cancel{m}a_v$

$\therefore \boxed{a_v = \frac{3}{4}g\,\cancel{\sin}\,\theta}$ cos

d) CofME

Conditions: Weights conservative

Hinge does no work at contact.

$E_{Tot} = E'_{Tot}$

$U + K_T + K_R = U' + K_T' + K_R'$

$mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgy' + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$ R.C

$y = \frac{l}{2}\sin\theta,\ y' = -\frac{l}{2},\ v = 0,\ v_f = \omega_f r = \frac{\omega_f l}{2},\ \omega = 0,\ \omega' = \omega_f$

$mg\left(\frac{l}{2}\sin\theta\right) + 0 + 0 = mg\left(-\frac{l}{2}\right) + \frac{1}{2}m\left(\frac{\omega_f l}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega_f^2$

$\frac{1}{2}\cancel{m}g\cancel{l}\sin\theta = -\frac{1}{2}\cancel{m}g\cancel{l} + \frac{1}{8}\cancel{m}l^{\cancel{2}}\omega_f^2 + \frac{1}{6}\cancel{m}l^{\cancel{2}}\omega_f^2$

$\frac{1}{2}g\sin\theta = -\frac{1}{2}g + \frac{1}{8}l\omega_f^2 + \frac{1}{6}l\omega_f^2$

$\frac{1}{2}g(\sin\theta + 1) = \omega_f^2\left(\frac{1}{8}l + \frac{1}{6}l\right)$

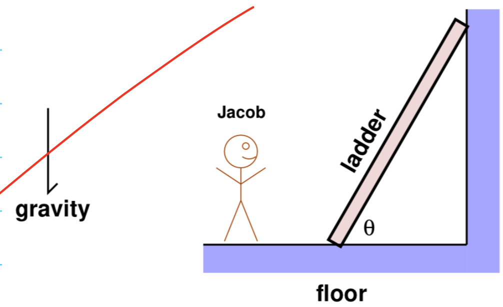
$\frac{1}{2}g(\sin\theta + 1) = \omega_f^2\left(\frac{7}{24}l\right)$

$\omega_f^2 = \frac{12}{7l}g(\sin\theta + 1)$

$\therefore \boxed{\omega_{hit} = \sqrt{\frac{12g}{7l}(\sin\theta + 1)}}$

$= \sqrt{\frac{3g(1 + \sin\theta)}{l}}$

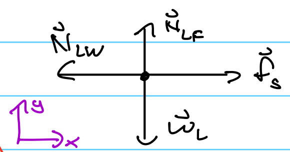
Problem 3)



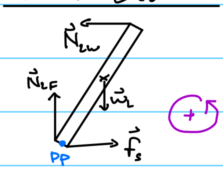
<u>Given</u>

- Ladder with length L, mass M
- Ladder @ rest ← statics
- Given angle θ with wall
- No friction with wall
- Friction with floor

a) <u>FBD Ladder</u>



<u>xFBD Ladder</u>



4 forces total :)

b) N2L y-dir

$F_y = m\cancel{a_y}^{0}$

$N_{LF} - W_L = 0$

① $\boxed{W_L = Mg}$

② $\boxed{N_{LF} = Mg}$

N2L x-dir

$F_x = m\cancel{a_x}^{0}$

$f_s - N_{LW} = 0$

Stop!

N2L Rot

$\tau_{Net} = I\cancel{\alpha}^{0}$

$\tau_{N_{LF}} + \tau_{f_s} + \tau_W + \tau_{N_{LW}} = 0$

$0 + 0 - \frac{1}{2}W\sin\theta + \cancel{L}N_{LW}\sin(90-\theta) = 0$

$N_{LW}\cos\theta - \frac{1}{2}Mg\sin\theta = 0$

$N_{LW}\cos\theta = \frac{1}{2}Mg\sin\theta$

$\therefore \boxed{N_{LW} = \frac{1}{2}Mg\tan\theta}$ ③

$\begin{cases} f_s - N_{LW} = 0 \\ N_{LW} = \frac{1}{2}Mg\tan\theta \end{cases}$ → $f_s = N_{LW}$ $\therefore \boxed{f_s = \frac{1}{2}Mg\tan\theta}$ ④

c) Given μ_s and μ_k for static and kinetic friction, respectively

Constraint: $f_s \le \mu_s N_{LF}$ Plug ② and ④ from (b)

$\frac{1}{2}Mg\tan\theta \le \mu_s(Mg)$

$\mu_s \ge \frac{1}{2}\tan\theta$ $\therefore \boxed{\theta \le \arctan(2\mu_s)}$