PHYS 121: Second Hour Exam Announcement, Reading Assignment, and Practice Problems

Please note: The Second Hour Exam is Friday, March 22 at 11:40 AM EDT in Strosacker). Same rules as last time. You are allowed *two pages* (front and back) for your own hand-written notes. Handwritten notes only – no note created by electronic means of any kind. Also a calculator is allowed but you will not need it.

Because of the exam, there is *no homework* assignment until after the exam.

Reading Assignment continues next page....

Nine Practice Problems for PHYS 121 Second Hour Exam:

- **Problems X01, X02, and X03 from Fall 2013 and Fall 2014 PHYS 121 Second Hour Exam**
- **Problems Y01, Y02, and Y03 from Fall 2012 and Fall 2015 Second Hour Exams.**
- **Problems Z01, Z02, and Z03 from Fall 2016 and Spring 2023 Second Hour Exams.**

Important: These problems for Practice Only. These problems will not be collected and will not be graded. Solutions are found after each set.

Guidelines for working Practice Problems on next page....

Important: Do not look at solutions! Read First:

Practice problems can be useful for students. However, there are pitfalls to using practice problems and so these general guidelines might be helpful: students:

- First and foremost, all students are encouraged to attempt to work all problems "under exam conditions". In other words, ***work each problem without notes or books, and without looking at the solutions.*** Ideally, give yourself about 15 to 20 minutes and try to complete each problem. Doing the practice problems under exam conditions will tell you if you are "well-prepared" to do well on the exam as it will actually be given.

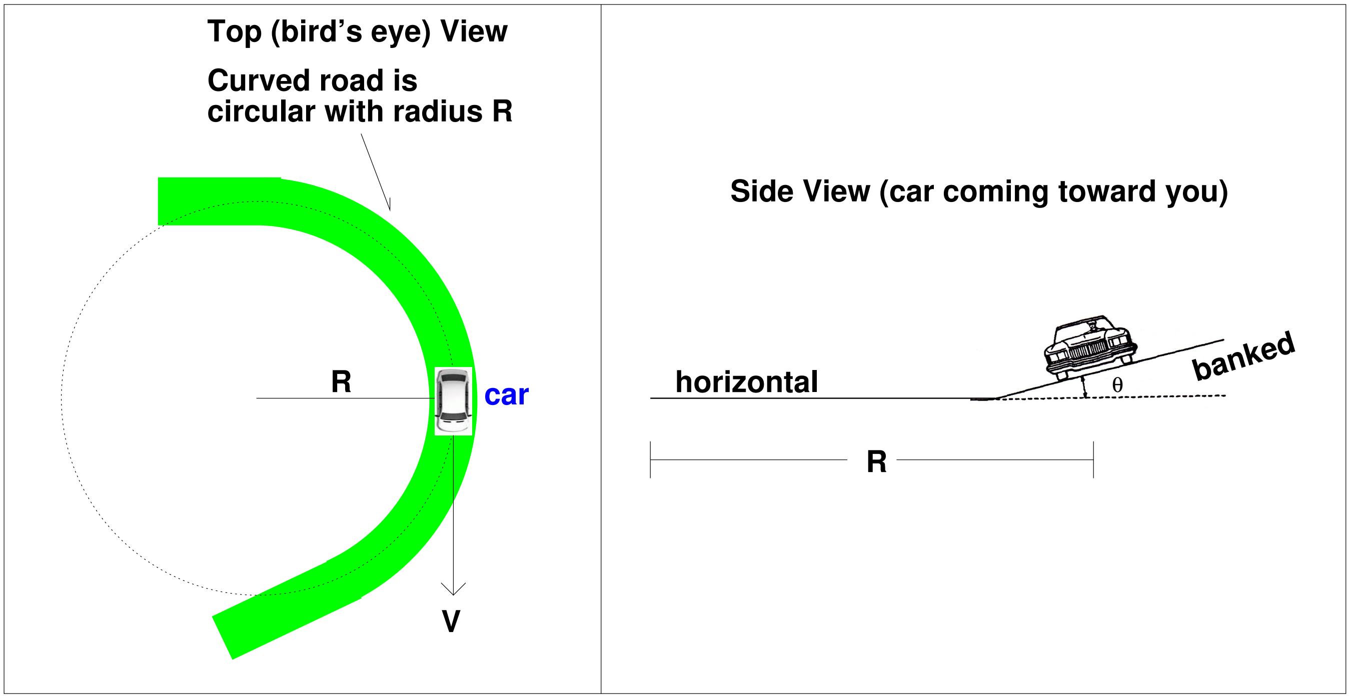
- Again, *resist any temptation to look at the solutions before you have made a decent effort to solve the problem on your own.*. This is really important. Student who quickly look at the solutions before they actually work the problems risk fooling themselves into thinking they know how to solve the problem. **Physics is learned by doing, not observing.** Simply looking at solutions before you try the problems as preparation for an exam is like trying to learn how to ride a bicycle by watching someone else riding. You tell yourself "Oh I understand that. I can do that." But you are fooling yourself. The only way to know if you have mastered a problem is to demonstrate that you have actually done the problem on your own.

- So give yourself a fixed period of time to work one or more problems. Use the whole time to work the problem even if you are stuck. At the end of the time, look at the solution. First, make sure that you are using the correct ***Main Physics Concept*** for any given part of a problem. This is most important. If you pick the wrong Main Physics Concept you will get nowhere. Study the solution. Make sure you understand it. Maye sure you understand where you went wrong. Then *put the problem away for at least 12 to 24 hours.* Now, say, the next day, try to solve the problem – again without looking at the solution. Maybe you will get further. Repeat until you have mastered the problem and you can solve it quickly without looking at the solution at all.

- If you are struggling with a Main Concept, better to make sure you get some help with that Concept instead of doing many practice problems. Every student has different learning preferences. Some students do best by studying the materials. Others do well with practice problems. You want to develop a study strategy that is best for you.

- Remember the goal of the course is to have students *demonstrate that they understand the Main Physics Concept by successfully solving problems.* We are not looking for "the right answer". We are looking for you to show us that you understand the main concepts and can apply these correctly to solve a wide variety of problems. The problems on the exam will be *different* from the practice problems, guaranteed. Do not "memorize solutions" to problems. Instead, make sure you understand the problems so that you can tackle new problems using the same Physics Concepts – problems that may seem both familiar and unfamiliar. Mastery means being ready to apply the Main Physics Concepts to new problems that you have not yet encountered, working your way methodically towards the correct solution quickly and with confidence.

Problem X01: 2014 Second Hour Exam: Problem 1: A Banked Curved Road: (40 points)



Curved roads are often **banked**, that is to say tilted by a small bank angle relative to the horizontal toward the inside of the curve, to reduce the force of static friction required to keep the car on the road.

Consider a car of given mass M traveling with a given constant speed V along a road that is curved into a circular path of given radius R. The road is banked so that **the force of static friction on the car is exactly zero** as the car goes around the curve. Important: the car maintains a *constant* vertical position. The angle θ is **not** a given parameter.

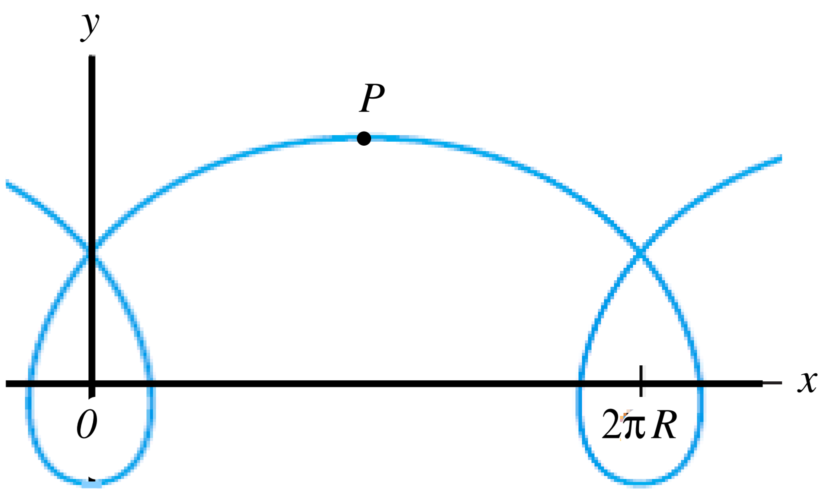
Part a) (5 points) – What is the magnitude of the **acceleration** of the car? Explain your work. Present your answer in terms of the given parameters. Hint: this is simple.

Part b) (5 points) – What is the magnitude of the **net force** on the car? Explain your work. Present your answer in terms of the given parameters. Hint: this is simple.

Part c) (5 points) – Draw a **Free Body Diagram (FBD)** indicating the directions of all of the forces on the car. Be sure to include an appropriate **coordinate system**, that is, one that is lined up with the direction of acceleration.

Part d) (25 points) – Using the results from parts (b) and (c) above, calculate the value of the **bank angle** θ so that the force of static friction on the car is **exactly zero** as the car goes around the curve. Present your answer in terms of the given parameters. Explain your work. Present your answer in terms of the given parameters.

Problem X02: 2-D Kinematics – (15 points)



A **prolate cycloid** is the shape of curve that describes the path of motion of a particle with specified coordinates as a function of time as follows:

$$x(t) = R\omega t - C\sin(\omega t)$$
$$y(t) = R - C\cos(\omega t)$$

where R, C and ω are given positive parameters with appropriate units.

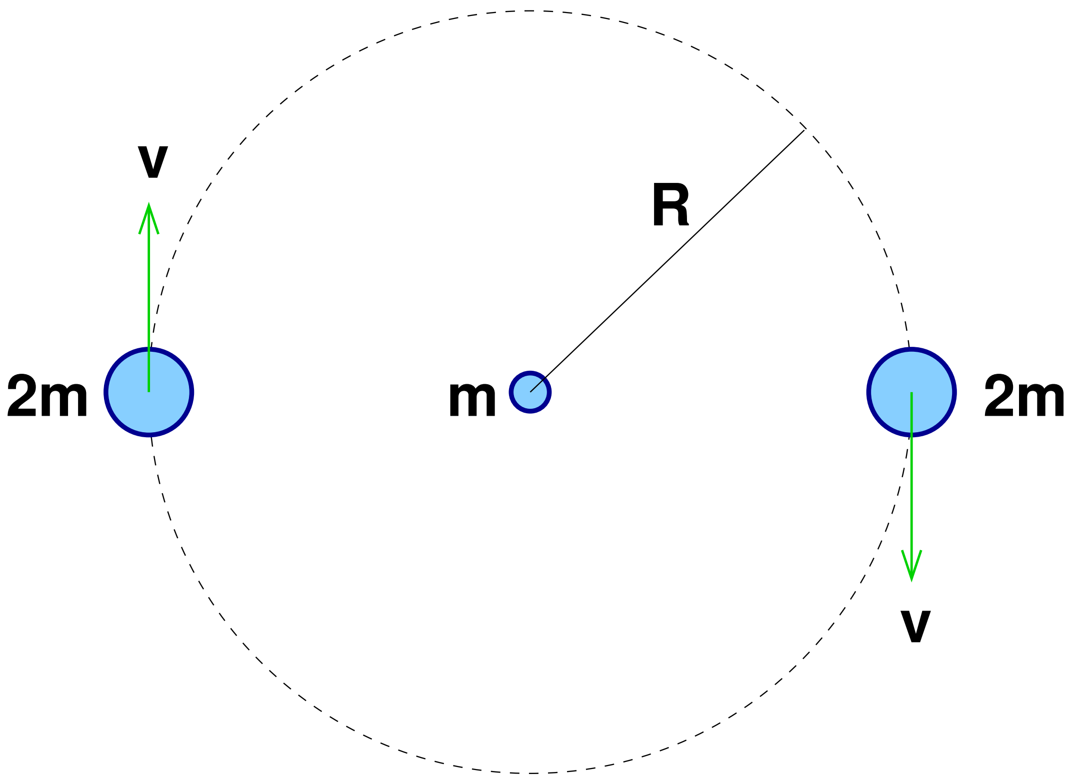
NOTE: The point P corresponds to the position of the particle at the time $t = \frac{\pi}{\omega}$

Part (a) – (5 points): ***Use the equations given above*** for $x(t)$ and $y(t)$ to determine the **vector expression** for $\vec{r}$ at time $t = \frac{\pi}{\omega}$ corresponding to the position vector of the particle at point P. Your answer should be presented in terms of given parameters and Cartesian basis unit vectors such as $\hat{\imath}$ and $\hat{\jmath}$. Explain your work. Hint: Your answer should not have t as a parameter.

Part (b) – (5 points): Determine the **speed** of the particle at point P. Express your answer in terms of the given parameters. Explain your work.

Part (c) – (10 points): Determine the **radius of curvature** of the cycloid path corresponding to point P. Express your answer in terms of the given parameters. Explain your work.

Problem X03: Newton's Laws and Universal Gravity (40 points)



Three stars are arranged in a special symmetric configuration as shown. Two outer stars, each with given mass $M_{outer} = 2m$ follow a circular orbit of given radius R around a smaller central star of given mass $M_{central} = m$.

Part a) – (10 points): Draw a neat and complete Free Body Diagram for **each** of the three stars showing all of the forces on each star.

Part b) – (10 points): In terms of the given parameters m and R, what is the *net force* on the *central* star. Explain how you know this.

Part c) – (10 points): In terms of the given parameters m and R, what is the *net force* on either one of the *outer* stars? Explain how you know this.

Part d)– (10 points): In terms of the given parameters m and R, what is the speed v of each outer star? Note that each of the outer stars has the same speed. Explain.

SOLUTION to Problem X01 on next page....

Physics 121 Fall 2014 – Second Hour Exam Name: SOLUTIONS NetID:________

YOUR ANSWER FOR PROBLEM 1 STARTS HERE

PROBLEM 1 ONLY!

PART (a)
→ Curved Path radius R
→ Constant Speed V
→ Uniform Circular Motion
→ Centripetal Acceleration:

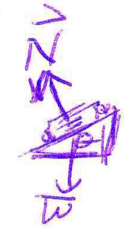
$$\boxed{a_c = \frac{V^2}{R}}$$ (5 points)

PART (b) Apply: Newton's Second Law

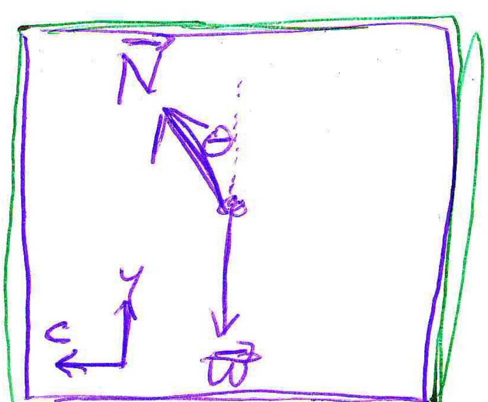
$F_{NET} = ma$, where $a = a_c = \frac{V^2}{R}$ so

$$\boxed{F_{NET} = \frac{mV^2}{R}}$$ (5 points)

part (c) Free Body Diagram:
→ Weight of car: $\vec{W}$
→ Normal force due to road $\vec{N}$
→ GIVEN: NO FRICTION



2 forces only $\vec{N}$ and $\vec{W}$



5 points

Coord system is aligned with HORIZONTAL DIRECTION corresponding to CENTRIPETAL ACCELERATION

⟵ This way to center

Problem 1
40
out of 40

PROBLEM 1 CONTINUES HERE

PROBLEM 1 ONLY!

PART (d) Apply Newton's 2nd Law

Y-coord:

$F_y = m a_y$, $a_y = 0$

$N\cos\theta - W = 0$

$N\cos\theta = mg$ ①

~~Ncosθ~~

C-coord:

$F_c = m a_c$

$N\sin\theta = \frac{mV^2}{R}$ ②

2 equations
2 unknowns, N and θ

~~8 Ret~~

ALGEBRA

Start with ②
solve for N

$\rightarrow N = \frac{mV^2}{R\sin\theta}$

plug into ①

$\left(\frac{mV^2}{R\sin\theta}\right)\cos\theta = mg$

$\frac{\cos\theta}{\sin\theta} = \frac{gR}{V^2}$

$\frac{\sin\theta}{\cos\theta} = \frac{V^2}{gR}$

$\tan\theta = \frac{V^2}{R}$

$\boxed{\theta = \arctan\left[\frac{V^2}{gR}\right]}$

or

$\boxed{\theta = \text{arccot}\left[\frac{gR}{V^2}\right]}$

(25 points)

Solution to Problem X02: (25 points)

Problem X02: Part (a) – (5 points):

We use the **Definition of Vector Position (2-D Kinematics)**:

$$\vec{r} \equiv x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$$

Here we know x and y:

$$x(t) = R\omega t - C\sin(\omega t)$$
$$y(t) = R - C\cos(\omega t)$$

So:

$$\vec{r} = [R\omega t - C\sin(\omega t)]\,\hat{\boldsymbol{\imath}} + [R - C\cos(\omega t)]\,\hat{\boldsymbol{\jmath}}$$

Plugging in the time corresponding to point P which is given by $t = \frac{\pi}{\omega}$ we get:

$$\vec{r} = \left\{R\omega\left(\frac{\pi}{\omega}\right) - C\sin\left[\omega\left(\frac{\pi}{\omega}\right)\right]\right\}\,\hat{\boldsymbol{\imath}} + \left\{R - C\cos\left[\omega\left(\frac{\pi}{\omega}\right)\right]\right\}\,\hat{\boldsymbol{\jmath}}$$

Simplifying:

$$\vec{r} = [R\pi - C\sin(\pi)]\,\hat{\boldsymbol{\imath}} + [R - C\cos(\pi)]\,\hat{\boldsymbol{\jmath}}$$

We note here the simple values of sine and cosine of pi:

$$\sin\pi = 0$$
$$\cos\pi = -1$$

We plug these in and continue to simplify:

$$\vec{r} = [R\pi - 0]\,\hat{\boldsymbol{\imath}} + [R - C(-1)]\,\hat{\boldsymbol{\jmath}}$$

$$\boxed{\vec{r} = \pi R\,\hat{\boldsymbol{\imath}} + (R + C)\,\hat{\boldsymbol{\jmath}}}$$

This value is satisfyingly consistent with the location of point P as indicated on the figure for this problem for C somewhat greater than R.

Solution to Problem X02 Continues....:

Problem X02: Part (b) – (5 points):

***Speed* is defined as the *magnitude* of the velocity.** We apply the **Definition of Vector Velocity** which tells us that the velocity is calculated by taking the *derivative with respect to time* of the position vector:

$$\vec{v} \equiv \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d}{dt}\left\{[R\omega t - C\sin(\omega t)]\,\hat{\boldsymbol{\imath}} + [R - C\cos(\omega t)]\ \hat{\boldsymbol{\jmath}}\right\}$$

$$\vec{v} = [R\omega - \omega C\cos(\omega t)]\ \hat{\boldsymbol{\imath}} + [C\omega\sin(\omega t)]\ \hat{\boldsymbol{\jmath}}$$

Plugging in $t = \frac{\pi}{\omega}$ for point P and noting the same values for sine and cosine as for the previous part, we get:

$$\vec{v} = [R\omega + C\omega]\ \hat{\boldsymbol{\imath}} + 0\,\hat{\boldsymbol{\jmath}}$$

$$\vec{v} = (R + C)\,\omega\,\hat{\boldsymbol{\imath}}$$

The vector $\vec{v}$ is non-zero along the x-axis (i-hat) only (which is what we expect since the velocity vector is tangential to the path). In this case, the speed is just the magnitude of the x-component of the velocity:

$$\boxed{\text{speed} = |\vec{v}| = v = (R + C)\omega}$$

Note that the speed of the cycloid particle more than twice the speed of the wheel as a whole. In other words, the top of wheel is moving at more than twice the speed of the center of the wheel.

Solution to Problem X02 Continues....:

Problem X02: Part (c) – (10 points):

To get the radius of curvature, we will need to calculate the acceleration of the particle. We use the **Definition of Vector Acceleration**: which tells us that the acceleration is the *derivative with respect to time* of the velocity vector:

$$\vec{a} \equiv \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d}{dt}\left\{[R\omega - C\omega\cos(\omega t)]\ \hat{\boldsymbol{\imath}} + [C\omega\sin(\omega t)]\ \hat{\boldsymbol{\jmath}}\right\}$$

$$\vec{a} = \left[C\omega^2\sin(\omega t)\right]\ \hat{\boldsymbol{\imath}} + \left[C\omega^2\cos(\omega t)\right]\ \hat{\boldsymbol{\jmath}}$$

Again, plugging in $t = \frac{\pi}{\omega}$ for point P:

$$\vec{a} = 0\ \hat{\boldsymbol{\imath}} + C\omega^2(-1)]\ \hat{\boldsymbol{\jmath}}$$

$$\vec{a} = -C\omega^2\ \hat{\boldsymbol{\jmath}}$$

In general we know that we break acceleration into both tangential and centripetal components:

$$\vec{a} = \vec{a}_{tangential} + \vec{a}_{centripetal}$$

However, in this case we see that $\vec{a}$ points down (negative j-hat) and is therefore **purely centripetal** (since acceleration points perpendicular to the path of the particle). Therefore the magnitude of the (total) acceleration (centripetal) is just the magnitude of the y-component of the vector acceleration:

$$|\vec{a}| = |a_y| = a = a_{total} = a_c = C\omega^2$$

Now we know that the expression for **Centripetal Acceleration** is given by:

$$a_c = \frac{v^2}{R_{curve}}$$

where R_{curve} is the radius of curvature. Rearranging to get the radius of curvature:

$$R_{curve} = \frac{v^2}{a_c}$$

Plugging in our speed from Part (b) and the magnitude of the (centripetal) acceleration we have calculated:

$$R_{curve} = \frac{[(R+C)\omega]^2}{C\omega^2}$$

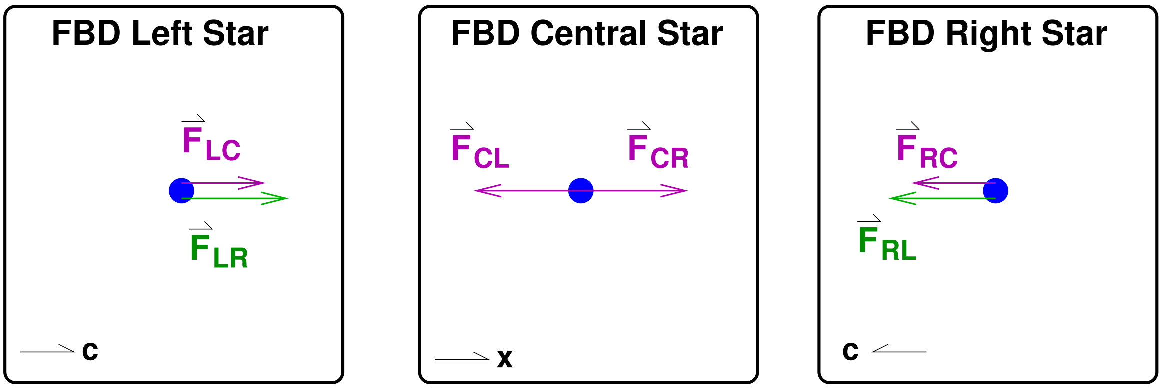
$$R_{curve} = \frac{(R+C)^2\omega^2}{C\omega^2}$$

$$\boxed{R_{curve} = \frac{(R+C)^2}{C}}$$

For $(R+C)$ somewhat larger than twice C, this gives a radius of curvature of somewhat larger than $4R$ – a value is satisfyingly consistent with the shape of the path as shown in the figure.

Solution to Problem X03:

Part a) – : Since we are in deep space and no bodies are in contact, the only forces that can possibly apply are the forces of **Universal Gravity**. We use subscripts to indicate forces arise on/due to each pairing:



Explicitly:

- Force F_{LC} is the force of Universal Gravity on the *left* star due to the *central* star.
- Force F_{LR} is the force of Universal Gravity on the *left* star due to the *right* star.
- Force F_{CL} is the force of Universal Gravity on the *central* star due to the *left* star.
- Force F_{CR} is the force of Universal Gravity on the *central* star due to the *right* star.
- Force F_{RC} is the force of Universal Gravity on the *right* star due to the *central* star.
- Force F_{RL} is the force of Universal Gravity on the *right* star due to the *left* star.

Problem X03 solution continues next page....

Part b) – : We see **by symmetry** that F_{CL}, the force applied on the central start due to the left star, is balanced by F_{CR}, the force applied on the central start due to the right star. Therefore, $\boxed{\text{the net force on the central star is zero}}$. Note: this is a result of the *symmetry* of the particular configuration, and *not* a result of Newton's Third Law. If we had not selected two stars of equal mass then the net force on the central star would not be zero. Note also that this result implies that the central star does not accelerate. In other words, it stays put.

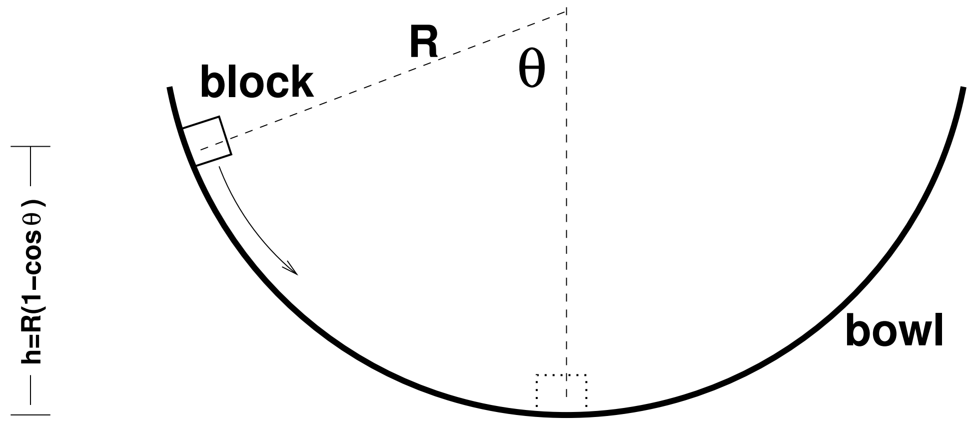
Part c) – : Let's calculate the net force on the left star. Since we have not (yet) calculated the speed of the star in the orbit, we do not know the acceleration, therefore we **cannot use Newton's Second Law** to calculate the net force. However, since the magnitude of the force due to **Universal Gravity** is known, we can just add up these forces. We define "positive" forces in the "centripetal" direction. The net force on the left star is the same as the total force in the centripetal component on the left star which we can call F_{Lc}. Calculating this:

$$F_{Lc} = F_{LC} + F_{LR}$$
$$F_{Lc} = \frac{GM_LM_C}{r_{LC}^2} + \frac{GM_LM_C}{r_{LR}^2}$$
$$F_{Lc} = \frac{G(2m)m}{R^2} + \frac{G(2m)(2m)}{(2R)^2}$$
$$F_{Lc} = \frac{2Gm^2}{R^2} + \frac{4Gm^2}{4R^2}$$
$$F_{Lc} = \frac{2Gm^2}{R^2} + \frac{Gm^2}{R^2}$$
$$\boxed{F_{Lc} = \frac{3Gm^2}{R^2}}$$

Part (d) – : Now we apply **Newton's Second Law** to calculate the speed, using what we learned in Part (b) regarding the net force:

$$F_{Lc} = m_La_c$$
$$\frac{3Gm^2}{R^2} = m_La_c$$
$$\frac{3Gm^2}{R^2} = (2m)\left(\frac{v^2}{R}\right)$$
$$\frac{v^2}{R} = \frac{3Gm}{2R^2}$$
$$v^2 = \frac{3Gm}{2R}$$
$$\boxed{v = \sqrt{\frac{3Gm}{2R}}}$$

Problem Y01: Circular Motion, Force, and Conservation (30 points)



A block of given mass m is placed on the inside surface of a fixed bowl and released at rest. Note that the height h of the block above the bottom of the bowl can be calculated by trigonometry to be $h = R(1 - \cos\theta)$ where R is the given radius of curvature of the bowl and θ is the given angle to the block relative to the vertical. After release, the block slides without friction on the surface of the bowl.

Part (a) 5 pts – Draw a careful ***Free Body Diagram*** showing all of the forces on the Block at the instant immediately after it is released. Be sure to include an appropriate coordinate system.

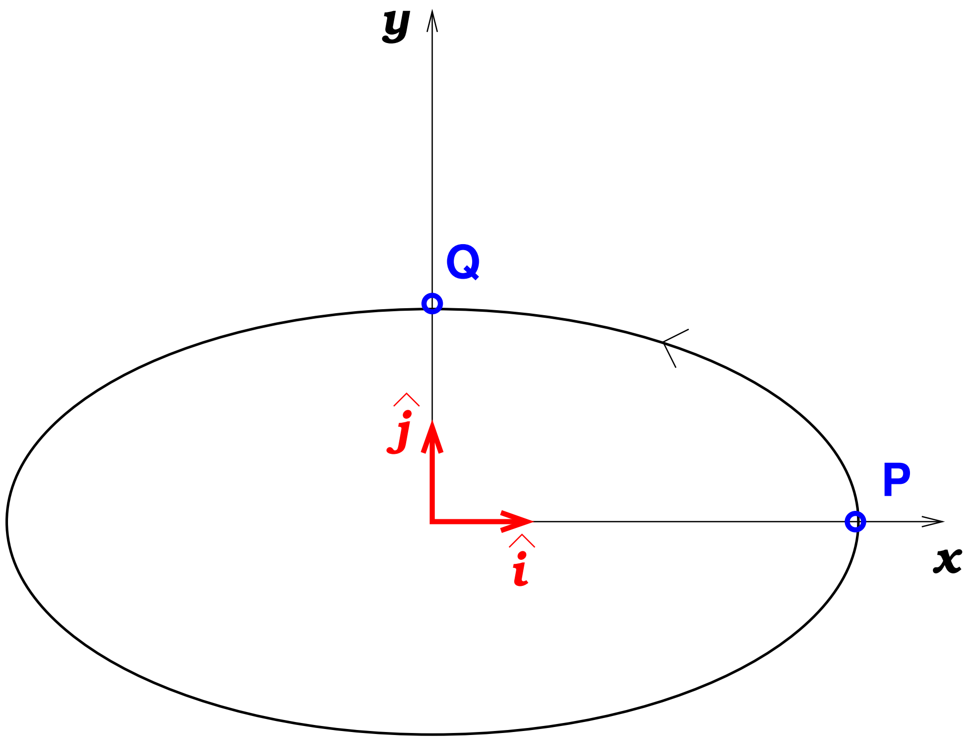
Part (b) 5 pts –Use **Newton's Second Law** to calculate the magnitude the ***Normal force*** of the block immediately after the block is released. Give your answers in terms of the given parameters. Explain your work.

Part (c) 5 pts – Use **Newton's Second Law** to calculate the magnitude the ***acceleration*** of the block immediately after the block is released. Give your answers in terms of the given parameters. Explain your work.

Part (d) 5 pts – What is the **Work done on the block by the Weight force** as the block moves from its starting position to the bottom of the bowl? Give your answer in terms of given parameters. Explain how you determined this.

Part (e) 10 pts – What is the ***magnitude*** and ***direction*** of the ***Normal force*** on the ***bowl*** due to the ***block*** when the block reaches the bottom of the bowl? Give your answer in terms of given parameters. Explain how you determined this.

Problem Y02: 2013 Second Hour Exam: Problem 1: Kinematics on an Oval Path (30 points)



A particle of given mass m moves counterclockwise around a horizontal oval path as shown above according to the following equation for the position vector as a function of time:

$$\vec{r}(t) = 2A\cos(\omega t)\,\hat{\imath} + A\sin(\omega t)\,\hat{\jmath}$$

where A and ω are given positive parameters and $\hat{\imath}$ and $\hat{\jmath}$ are standard Cartesian unit vectors.

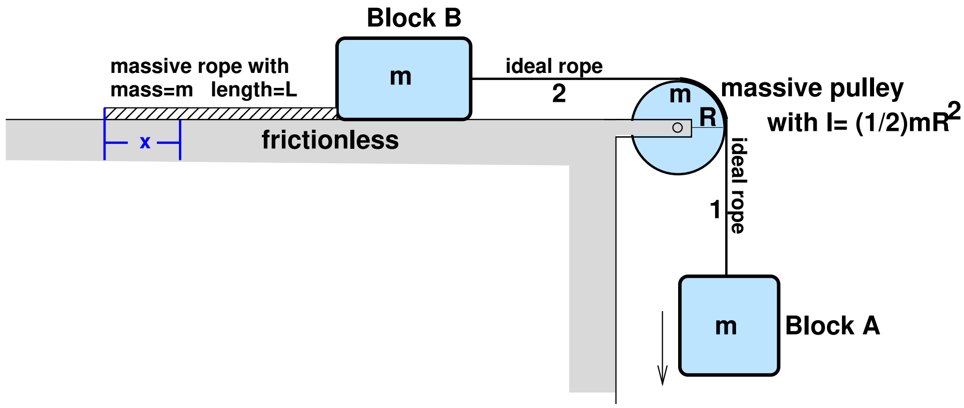
Part (a) 10 pts: – Point P corresponds to the position of the particle at time $t = 0$. What is the speed of the particle at point P? Give your answer in terms of the given parameters. Explain your work.

Part (b) 10 pts: – Write down a *vector expression* that gives the ***net force*** on the particle when it is at point P in terms of the given parameters and the unit vectors as indicated. Explain how you know this. Show your work.

Part (c) 10 pts – Point Q corresponds to the position of the particle at time $t = \frac{\pi}{2\omega}$. What is the **radius of curvature** when the particle is at point Q as shown? Give your answer in terms of the given parameters. Explain how you know this.

Some possibly useful trigonometric facts: $\sin(0) = 0$, $\cos(0) = 1$, $\sin(\frac{\pi}{2}) = 1$, and $\cos(\frac{\pi}{2}) = 0$.

Problem Y03: 2015 2nd Hour Exam Prob 1: Massive Pulley & Massive Rope: (40 points)



An arrangement of two blocks, ropes, and a pulley are assembled as shown above and released at rest. Block A and Block B each both have a given mass m. These two blocks are connected by an *ideal* rope that is partially wrapped around a *massive* pulley with given mass also m and given radius R. The rotational inertia of the massive pulley is given as $I = \frac{1}{2}mR^2$. Block B sits on a frictionless surface. A massive rope of given mass also m and given length L is attached to Block B as shown.

Part (a) – (30 points):

- Determine the acceleration a_B of Block B to the right.
- Determine the magnitude of the tension T_1 in the piece of ideal rope that is attached to Block A.
- Determine magnitude of the tension T_2 in the piece of ideal rope that is attached to Block B.

Important: Show your work. Give your answers as expressions in terms of the given parameters: m and R. Explain.

Part (b) – (10 points): Determine $T(x)$ the tension in the massive rope as a function of position x as measured from the left edge of the rope as shown. Show your work. Give your answers as expressions in terms of the given parameters Explain.

Possibly useful Hints for Part (a):

– For Part (a) only, it might be helpful to treat Block B and the piece of massive rope together as one system.

– Because of the massive pulley, the Tension T_1 is *not* equal to the tension T_2.

– Do not worry about the direction and magnitude of any "hinge force" on the pulley here. You do not need to calculate the hinge force and this will not impact your answers.

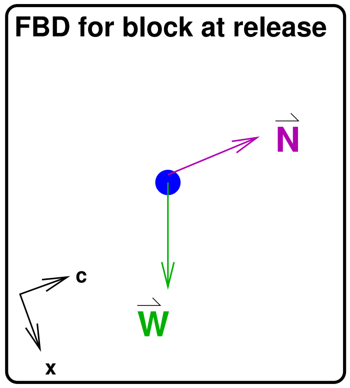
– You *will* need to consider the torques on the pulley.

– You *will* need to apply the Rolling Constraint.

– You will need to work some algebra to solve for "N equations with N unknowns."

Solution to Problem Y01:

Part (a): (5 points)



The figure above shows the **Free-Body Diagram** at the release point. There are two forces: Weight and Normal. We choose a **tilted coordinate system** so that one of the coordinates (here called "x") points in the direction of tangential acceleration, and the other coordinate (here called "c") points toward the center of the circular path corresponding to the direction of centripetal acceleration. We note that at the position of release the **speed is zero** and therefore the centripetal component of the acceleration which is given by $a_c = v^2/R$ is also zero.

Part (b): (5 points)

Since we are looking for the Normal Force, we better apply **Newton's Second Law**. We note that the Normal Force is aligned with the centripetal coordinate direction, so we start with this:

$$F_c = ma_c$$
$$F_c = m\frac{v^2}{R}$$

Since we release at rest, the speed is zero:

$$F_c = 0$$

Looking at the Free-Body Diagram we see that we get the cosine component of Weight in the centripetal direction:

$$N - W\cos\theta = 0$$
$$N = W\cos\theta$$
$$\boxed{N = mg\cos\theta}$$

Part (c): (5 points)

Since we are looking for the acceleration, we better apply **Newton's Second Law**. We note that the acceleration is is aligned with the the "x" coordinate direction:

$$F_x = ma_x$$

Looking at the Free-Body Diagram we see that we get the sine component of Weight in the tangential direction:

$$W \sin\theta = ma_x$$
$$mg \sin\theta = ma_x$$
$$\boxed{a_x = g \sin\theta}$$

Part (d): (5 points)

We note that *weight* is a **Conservative Force**. That is to say, the work done by *Weight* in going from one point to another point is **path-independent**.

This means that the work done by Weight on the block as it moves from the edge of the bowl to the bottom of the bowl is the same as the work that would be done if the block simply moved in a path straight down. In this case, the work is very simple to calculate. We can use the expression for work a constant force that is aligned to the direction of straight line motion:

$$\text{Work}_{(Weight)} = F_{(parallel)} \cdot d$$
$$\text{Work}_W = Wh$$
$$\text{Work}_W = mgh$$

We note the trigonometric connection in the figure above $h = R(1 - \cos\theta)$:

$$\boxed{\text{Work}_W = mgR(1 - \cos\theta)}$$

Note that the work done is **positive** since the direction of the motion and the force of Weight are aligned (both down).

Part (e): (10 points)

This requires two steps. Since we are dealing with centripetal acceleration, we will need to calculate the **speed** at the bottom of the bowl. So we want to consider possibly applying Conservation of Energy. But we need to check if we are allowed to do this first:

There are two forces: *Weight* and *Normal*. We know that **Weight is Conservative**, and that **the Normal force does no work in this problem.** Therefore we have **met the conditions** to apply **Conservation of Energy**.

We define the "Before" as the block at the top edge of the bowl as shown in the figure. We define the "After" as the block at the bottom of the bowl.

We write down **Conservation of Energy:** in symbolic form:

$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

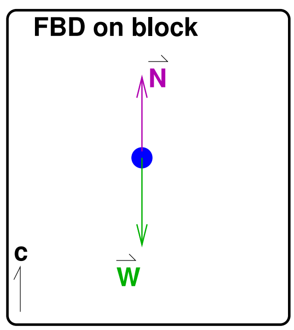
We know that in the "before" case, the y-position is the height h and the velocity is zero. We know that in the "after" case, the y position is zero. We put these conditions into our equation:

$$mgh = 0 = 0 + \frac{1}{2}mv'^2$$

Now we solve for v':

$$\frac{1}{2}mv'^2 = mgh$$
$$v'^2 = 2gh$$

Now, since this is asking us for a *force* we are going to want to consider applying **Newton's Laws**. First, let's write down a **Free-Body-Diagram** for the block at the bottom of the bowl:



We note that since the block is moving in a circular path, it is experiencing **centripetal acceleration**. The coordinate that points to the center of the circle is labeled "c" (up) in the diagram.

We write down **Newton's Second Law** in the centripetal coordinate:

$$F_c = ma_c$$
$$N - W = ma_c$$
$$N - W = m\frac{v^2}{R}$$

Now we use algebra to solve for the force normal on the block N:

$$N = m\frac{v^2}{R} + W$$
$$N = m\frac{v^2}{R} + mg$$

$$N = m\left(\frac{v^2}{R} + g\right)$$

$$N = mg\left(\frac{2h}{R} + 1\right)$$

where here we have used our calculation of v^2. We also plug in our expression for h:

$$N = mg\left[\frac{2R(1-\cos\theta)}{R} + 1\right]$$

$$N = mg\left[2(1-\cos\theta) + 1\right]$$

$$N = mg\left(2 - 2\cos\theta + 1\right)$$

$$N = mg\left(3 - 2\cos\theta\right)$$

This has the form we expect. As θ approaches zero, the speed at the bottom goes to zero and the normal force has the same value as the weight.

Finally we note that the problem asks for the *Normal force on the bowl*, whereas here we have calculated the *Normal force on the block*. We apply **Newton's Third Law** to argue that the force on the bowl is the *same* as the force on the block ($N_{(bowl} = N_{(block)}$) but applied in the opposite direction (down instead of up):

$$\boxed{N_{(bowl)} = mg\left(3 - 2\cos\theta\right) \;\; \textbf{downward}}$$

Solution to Problem Y02:

Part (a): (10 points)

To get a handle on the speed, which is the **magnitude of the velocity** we want to write down the vector expression for velocity by taking the **time derivative of the position vector:**

$$\vec{v}(t) \equiv \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[2A\cos(\omega t)\,\hat{\boldsymbol{\imath}} + A\sin(\omega t)\,\hat{\boldsymbol{\jmath}}\right]$$
$$\vec{v}(t) = -2A\omega\sin(\omega t)\,\hat{\boldsymbol{\imath}} + A\omega\cos(\omega t)\,\hat{\boldsymbol{\jmath}} \tag{1}$$

Now we plug in $t = 0$. The sine term goes to zero, the cosine term goes to one:

$$\vec{v}(t) = -2A\omega\sin(0)\,\hat{\boldsymbol{\imath}} + A\omega\cos(0)\,\hat{\boldsymbol{\jmath}}$$
$$\vec{v}(t) = -2A\omega(0)\,\hat{\boldsymbol{\imath}} + A\omega(1)\,\hat{\boldsymbol{\jmath}}$$
$$\vec{v}_P = \vec{v}(t=0) = 0\hat{\boldsymbol{\imath}} + A\omega\hat{\boldsymbol{\jmath}}$$
$$\vec{v}_P = A\omega\hat{\boldsymbol{\jmath}}$$

This is a velocity vector that points along the y-direction with a magnitude of $A\omega$, corresponding to the speed:

$$v_P \equiv |\vec{v}_P|$$
$$\boxed{v_P = A\omega}$$

Part (b): (10 points)

In this problem, we have no idea what the actual specific forces are on the particle, so trying to calculate the net force by totally up specific forces will not work.

To get the net force we need to consider **Newton's Second Law** in vector form:

$$\vec{F}_{net} = m\vec{a}$$

We are given the mass so all we need to do is calculate the kinematic vector acceleration. To get the acceleration we just take the **time derivative of the velocity vector** which we calculated above (Equation 1):

$$\vec{a}(t) \equiv \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[-2A\omega\sin(\omega t)\,\hat{\boldsymbol{\imath}} + A\omega\cos(\omega t)\,\hat{\boldsymbol{\jmath}}\right]$$
$$\vec{a}(t) = -2A\omega^2\cos(\omega t)\,\hat{\boldsymbol{\imath}} - A\omega^2\sin(\omega t)\,\hat{\boldsymbol{\jmath}} \tag{2}$$

Again, we plug in $t = 0$:

$$\vec{a}(t=0) = -2A\omega^2\cos(0)\,\hat{\boldsymbol{\imath}} - A\omega^2\sin(0)\,\hat{\boldsymbol{\jmath}}$$
$$\vec{a} = -2A\omega^2(1)\,\hat{\boldsymbol{\imath}} - A\omega^2(0)\,\hat{\boldsymbol{\jmath}}$$

$$\vec{a} = -2A\omega^2\hat{\boldsymbol{\imath}} - 0\hat{\boldsymbol{\jmath}}$$
$$\vec{a} = -2A\omega^2\hat{\boldsymbol{\imath}}$$

Finally we plug this into Newton's Second Law:

$$\vec{F}_{net} = m\vec{a}$$
$$\boxed{\vec{F}_{net} = -2mA\omega^2\hat{\boldsymbol{\imath}}}$$

Part (c): (10 points)

The radius of curvature is kinematically connected to the **centripetal component of the acceleration**:

$$a_c = \frac{v^2}{R_c}$$

This means we need to find the acceleration and the velocity at Point Q. So we use our results from Parts (a) and (b), plugging in $t = \frac{\pi}{2\omega}$. Starting with Equation (1) for velocity:

$$\begin{aligned}
\vec{v}(t) &= -2A\omega \sin(\omega t)\ \hat{\boldsymbol{\imath}} + A\omega \cos(\omega t)\ \hat{\boldsymbol{\jmath}} \\
\vec{v}(t = \frac{\pi}{2\omega}) &= -2A\omega \sin(\frac{\pi}{2})\ \hat{\boldsymbol{\imath}} + A\omega \cos(\frac{\pi}{2\omega})\ \hat{\boldsymbol{\jmath}} \\
\vec{v}_Q &= -2A\omega(1)\ \hat{\boldsymbol{\imath}} + A\omega(0)\ \hat{\boldsymbol{\jmath}} \\
\vec{v}_Q &= -2A\omega\ \hat{\boldsymbol{\imath}} \\
v_Q \equiv |\vec{v}_Q| &= 2A\omega
\end{aligned}$$

Likewise, using Equation (2) from Part (b) for acceleration:

$$\begin{aligned}
\vec{a}(t) &= -2A\omega^2 \cos(\omega t)\ \hat{\boldsymbol{\imath}} - A\omega^2 \sin(\omega t)\ \hat{\boldsymbol{\jmath}} \\
\vec{a}(t = \frac{\pi}{2\omega}) &= -2A\omega^2 \cos(\frac{\pi}{2})\ \hat{\boldsymbol{\imath}} - A\omega^2 \sin(\frac{\pi}{2})\ \hat{\boldsymbol{\jmath}} \\
\vec{a}_Q &= 2A\omega^2(0)\ \hat{\boldsymbol{\imath}} - A\omega^2(1)\ \hat{\boldsymbol{\jmath}} \\
\vec{a}_Q &= -A\omega^2\ \hat{\boldsymbol{\jmath}}
\end{aligned}$$

We note that the **acceleration** at point Q points **perpendicular to the velocity** at point Q. In other words if we write down the expression for total acceleration:

$$\vec{a}_{tot} = \vec{a}_c + \vec{a}_v$$

at point Q, we see that there is no component of acceleration (which points along the negative y-direction) in the tangential direction (negative x-direction). Therefore **the acceleration must be purely centripetal**:

$$\vec{a} = \vec{a}_c$$

$$a_c = |\vec{a}| = A\omega^2$$

We then use the expression for **centripetal acceleration** along with our calculation of the speed to get the radius of curvature:

$$\begin{aligned}
a_c = \frac{v^2}{R_c} &= A\omega^2 \\
\frac{(2A\omega)^2}{R_c} &= A\omega^2
\end{aligned}$$

$$\frac{4A^2\omega^2}{R_c} = A\omega^2$$

$$A^2\omega^2 = (4A\omega^2)(R_c)$$

$$A^2\omega^2 R_c = 4A^2\omega^2$$

$$\boxed{R_c = 4A}$$

Solution to Problem Y03:

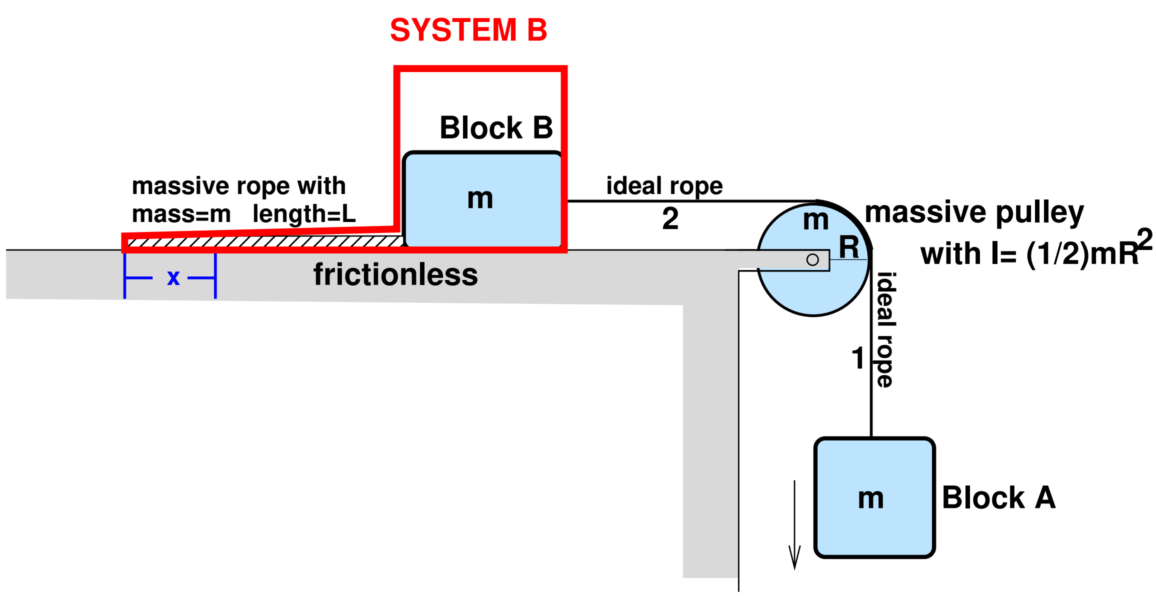
Part (a): (30 points)

So this is a problem involving the application of **Newton's Laws**. We want to follow the "standard recipie" for solving these sorts of problems?

First: What can we say about the acceleration? Well we can say the following:

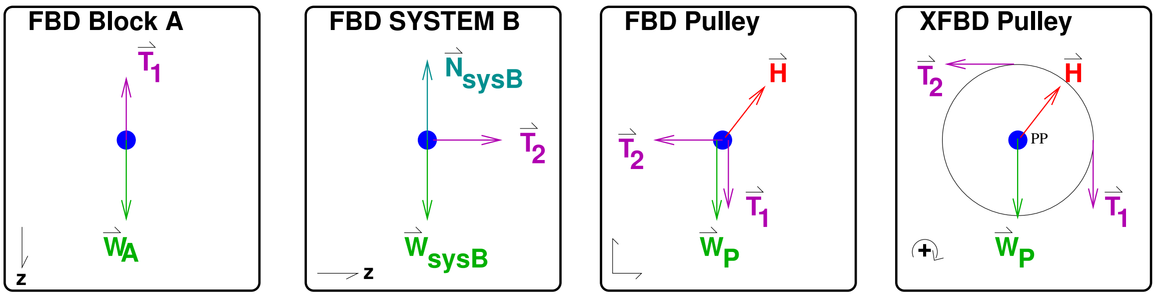
- The acceleration is not zero, and it's not given. Indeed we are asked to calculate the acceleration of Block B.
- The *rightward* linear acceleration of the rope and the Block are the same and this is also the *downward* accleration of Block A. So we say $a_z \equiv_{rope} = a_B = a_A$ corresponing to a **kinematic constraint** that applies to the motion of the rope and blocks. For simplicity, we will define the coordinate "z" as pointing in the direction of positive acceleration for each body.
- Since there is no slipping or sliding, the **Rolling Constaint** relates the linear acceleration of the Blocks to the rotational acceleration of the pulley.

Next, before we set up our Free-Body Diagrams, *We want to combine the massive rope and Block B into once system, like this:*



Problem solution continues next page...

The three FBDs (and the XFBD for the massive pulley) should look something like this:



We note that the even though we have placed forces on the massive pulley – including an unknown "Hinge force" – we are not asked to calculate the direction or magnitude of this force. Note also that the vertical forces on Block B and the massive rope are not relevant here.

We start by applying **Newton's Second Law** to **Body A** taking care that we have defined the direction of positive acceleration downward in our FBD:

$$F_{Az} = m_A a_z$$
$$W_A - T_1 = m a_z$$
$$mg - T_1 = m a_z \tag{3}$$

That's as far as we can get. Both T_1 and a_z are unknown.

Next we apply **Newton's Second Law** to **System B**

$$F_{Bsys,z} = m_{Bsys} a_z$$

Here $m_{Bsys} = m_{rope} + m_B = 2m$ and therefore:

$$T_2 = 2m a_z \tag{4}$$

Here both T_2 and a_z are unknown.

Now, finally we must contend with the **rotational motion** of the massive pulley. We will apply **Newton's Second Law in Rotational Form:**

$$\tau_{net} = I\alpha$$

The left side is the vector sum of the torques due to the four individual forces. For the right side, we are given I here.

$$\tau_H + \tau_W + \tau_{T1} + \tau_{T2} + = \left(\frac{1}{2} m R^2\right) \alpha$$

The **definition of torque** that results from a specific force $\vec{F}$ is as follows:

$$\vec{\tau}_{\vec{F}} \equiv \vec{r} \times \vec{F}$$
$$\tau_F \equiv rF \sin\phi$$

Here $\vec{r}$ points from the pivot point to the location where the force is applied, and phi is the angle between the vector $\vec{r}$ and the vector $\vec{F}$.

Looking at our XFBD we see that the torques due to the Weight and Hinge forces are both ***zero*** because the vector $\vec{r}$ has zero length (the forces are applied at the pivot point). For the two tension forces, we can see that the angle ϕ is 90 degrees. In accordance with our coordinate system, the tension T_1 will provide positive torque whilst the tension T_2 provides negative torque:

$$0 + 0 + RT_1 - RT_2 = \left(\frac{1}{2}mR^2\right)\alpha$$
$$RT_1 - RT_2 = \left(\frac{1}{2}mR^2\right)\alpha$$

We now apply the **Rolling Constraint** that tells us that $a = r\alpha$ so that $\alpha = a_z/R$:

$$RT_1 - RT_2 = \left(\frac{1}{2}mR^2\right)\left(\frac{a_z}{R}\right)$$
$$T_1 - T_2 = \frac{ma_z}{2} \tag{5}$$

Okay, that's three equations and three unknowns: a_z, T_1, andT_2. **The rest is algebra:**

For example, for Equation (1) we can re-arrange to express the tensions terms of the acceleration:

$$T_1 = mg - ma_z$$

and then we can use this result together Equation (2) and substitute for the tensions in Equation (3).

$$mg - ma_z - 2ma_z = \frac{ma_z}{2}$$
$$g - 3a_z = \frac{a_z}{2}$$
$$2g - 6a_z = a_z$$
$$7a_z = 2g$$
$$\boxed{a_B = a_z = \frac{2g}{7}}$$

Now it's easy to plug this in to get the tensions:

$$T_1 = mg - m\left(\frac{2g}{7}\right)$$

$$\boxed{T_1 = \frac{5mg}{7}}$$

and

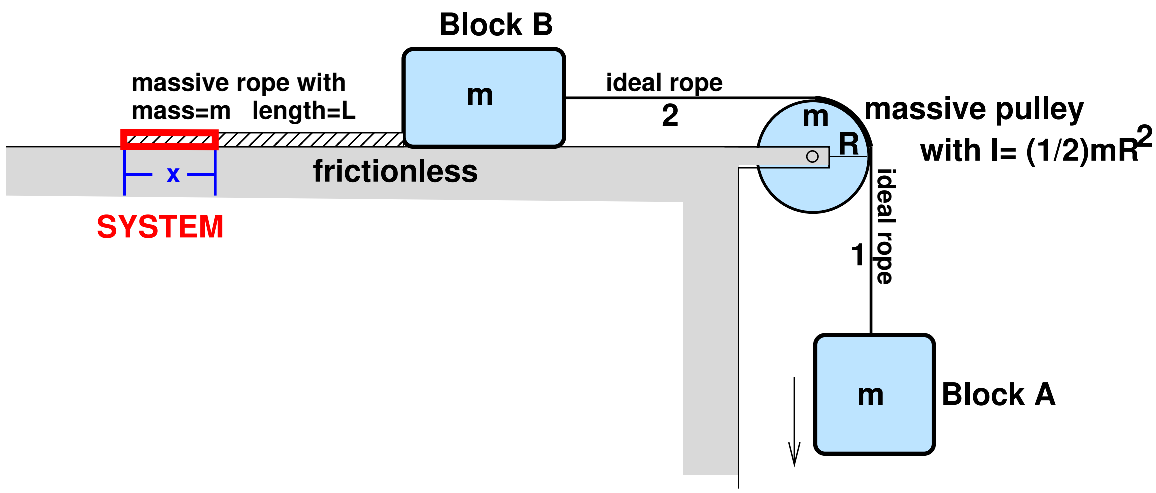
$$T_2 = 2m(\left(\frac{2g}{7}\right)$$

$$\boxed{T_2 = \frac{4mg}{7}}$$

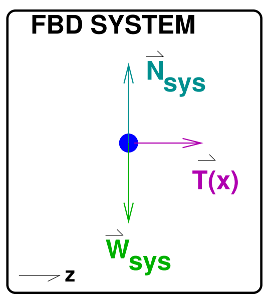
That's it. Phew!

Part (b): (10 points)

For this part we just define a ***system*** corresponding to the left-most piece of the string of length x.



The FBD for this system is very simple:



We apply **Newton's Second Law:**

$$F_{sys,rope} = (m_{sys,rope})a_z$$
$$T(x) = [m(x)]\ a_z$$

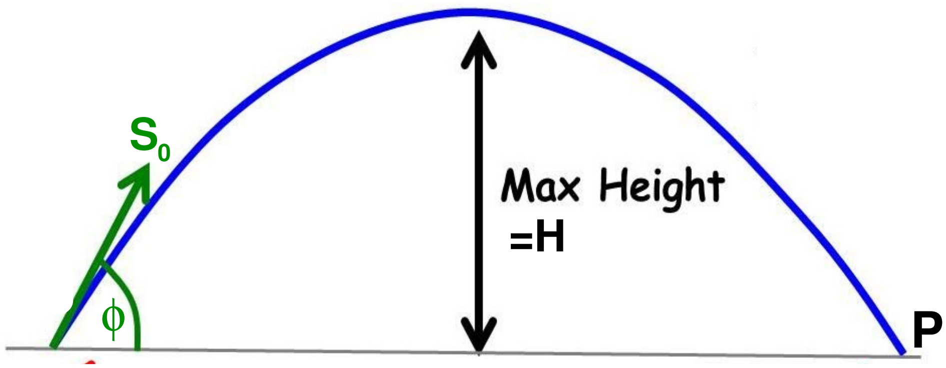
Here $m(x)$ means the mass of the little piece of rope of length x as a function of x. This is just the mass of the rope times the fraction of the rope contained in the system up to position x:

$$m(x) = m_{rope}\left(\frac{x}{L}\right)$$
$$m(x) = \frac{mx}{L}$$

and so:

$$T(x) = \left(\frac{mx}{L}\right)\left(\frac{2g}{7}\right)$$
$$\boxed{T(x) = \frac{2mxg}{7L}}$$

Problem Z01: Projectile Motion (15 points)



A projectile is launched from the ground at an initial **given** speed of S_0 at an initial **given** angle ϕ relative to the horizontal as shown above.

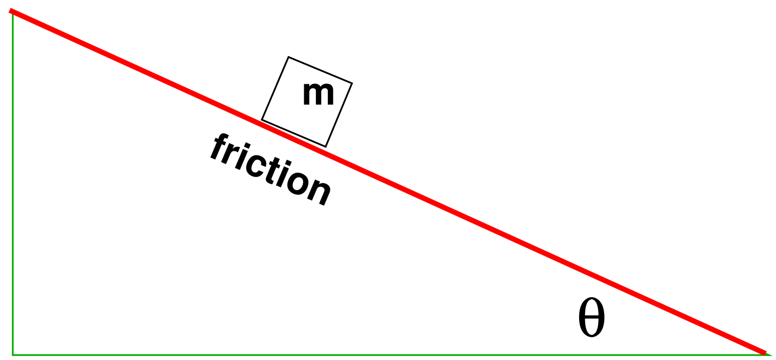
Part (a) (5 points) – Calculate the **maximum height** H of the projectile.

Part (b) (5 points) – Calculate the **Radius of Curvature** R_c of the path at the point corresponding to when the projectile reaches its ***maximum height***. Hint: R_c is ***not*** equal to H.

Part (c) (5 points) – Write an expression for the **vector velocity** $\vec{v}_p$ corresponding to the instant in time just ***before*** the projectile hits the ground at the point labeled P. Your answer must be expressed using ***vector representation*** including appropriate Cartesian-basis Unit Vectors (such as $\hat{\imath}$ and $\hat{\jmath}$).

Important: You must show and explain your work. The correct answer alone with no work shown is worth zero points. You must show how your answer can be calculated using the fundamental kinematic equations of Projectile Motion. Your answers must be expressed in terms of the given parameters of the problem which are S_0 and ϕ. Hint: the unknown variable t, corresponding to time, should *not* be included in any of your final answers. Ignore air resistance

Problem Z02: Block on a Ramp (30 points)

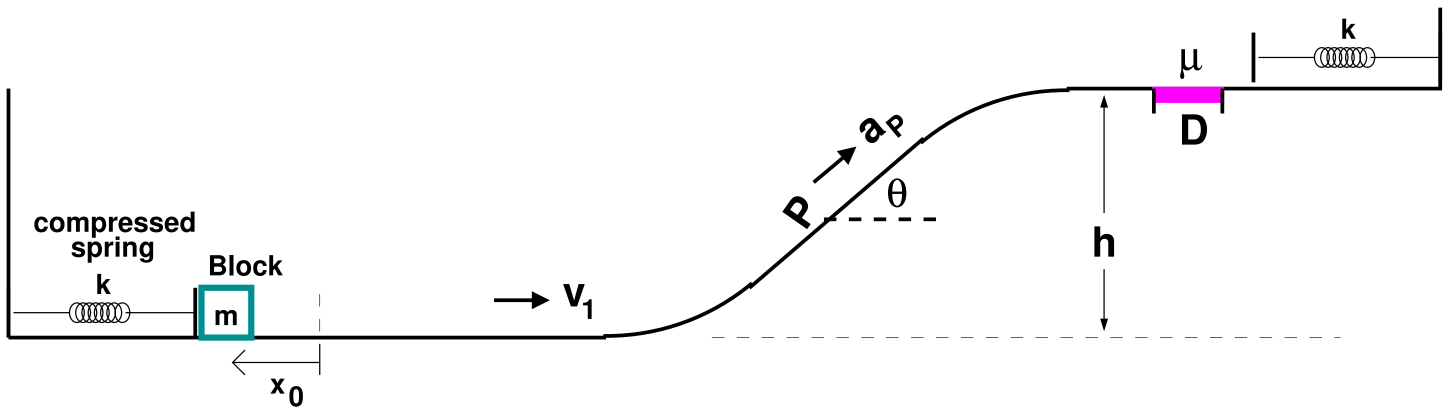


A block of given mass m is positioned on a fixed ramp with a given angle θ relative to the horizontal as shown above. For this problem, assume that there is ***friction*** between the block and the surface of the ramp. Assume that the coefficient of static friction μ_s and the coefficient of kinetic (sliding) friction μ_k are both *given* parameters.

Part (a) – (15 points): *First* assume that the block is released on the ramp and that it subsequently remains ***at rest***. Calculate the *magnitude* of ***each and every force*** on the block. Show your work. Give your answer in terms of the given parameters.

Part (b) – (15 points): Now ***change the problem as follows***: Assume that the block is released on the ramp and that it immediately begins to *accelerate down the ramp.* Calculate the *magnitude* of ***each and every force*** on the block. Also calculate the magnitude of the ***acceleration*** of the block. Show your work. Give your answer in terms of the given parameters.

Problem Z03: Looks More Complicated Than It Is: (15 points)



A block of given mass m is placed at rest on a frictionless track. The block is released while in contact with a compressed ideal spring as shown above. The spring constant is given as k and the block is released when the spring has an initial compression given as x_0. The block travels to the right, up a ramp, to a level at a given height h and then across a small patch of horizontal track of given length D that has a non-zero coefficient of kinetic friction given as μ. The block then collides with another ideal spring at the right end of the track, and then rebounds to travel in the opposite direction where it eventually collides with the original spring at the left end of the track.

Part (a) (5 points) – Calculate v_1 which is the **speed of the block** after the block loses contact with the spring but before it begins to travels up the ramp.

Part (b) (5 points) – A position on the ramp labeled P is a location where the angle of the track relative to the horizontal is given as θ. Calculate a_p which is the (1-D) **acceleration of the block** at point P:

Part (c) (5 points) – More Difficult: Calculate x_{max} which is the **maximum compression of the left spring** when the block collides with it after the entire round-trip.

Important: You must show and explain your work. The correct answer alone with no work shown is worth zero points. Present your work in terms of the given parameters.

Problem Z01: Part (a) – (5 points):

This is a problem of **Projectile Motion Kinematics**. The *horizontal component* of the motion (along the x-direction) corresponds to to **Constant Velocity:**

$$a_x = 0$$
$$v_x = v_{x0} \quad \text{(a constant)}$$
$$x = x_0 + v_{x0}t$$

The *vertical* component of the motion (along the y-direction) corresponds to **Constant Acceleration due to Free Fall:**

$$a_y = -g$$
$$v_y = v_{y0} - gt$$
$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

The initial conditions are inferred from the figure. Here the speed is seen to be S_0 and the launch angle is ϕ. We launch from the ground so we define the launch position as the origin.

$$x_0 = 0$$
$$y_0 = 0$$
$$v_{x0} = S_0 \cos\phi$$
$$v_{y0} = S_0 \sin\phi$$

To get the height H above the ground, we need only consider the **vertical motion kinematics**. We know that the *y-velocity will be zero* at the instant when the projectile arrives to its maximum height corresponding to

$$y = H$$
$$v_y = 0$$

We therefore use this fact to find the *time* at this instant with the velocity equation:

$$v_y = v_{y0} - gt$$
$$0 = v_{y0} - gt$$
$$t = \frac{v_{y0}}{g}$$

and then we use the position equation:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$H = 0 + v_{y0}\left(\frac{v_{y0}}{g}\right) - \frac{1}{2}g\left(\frac{v_{y0}}{g}\right)^2$$

$$H = \left(\frac{v_{y0}^2}{g}\right) - \frac{1}{2}\left(\frac{v_{y0}^2}{g}\right)$$

$$H = \frac{1}{2}\left(\frac{v_{y0}^2}{g}\right)$$

$$H = \frac{1}{2}\left[\frac{(S_0 \sin\phi)^2}{g}\right]$$

$$\boxed{H = \frac{S_0^2 \sin^2\phi}{2g}}$$

Problem Z01: Part (b) – (5 points):

On any curved path, the acceleration can be decomposed into **centripetal** and **tangential** components. At the position of maximum distance above the Lake, the first tennis ball has constant horizontal velocity, and therefore the tangential component of acceleration is zero. This means that the acceleration is purely centripetal and we can use the expression for the value of the centripetal acceleration in terms of the radius of curvature:

$$a_c = \frac{v^2}{R_c}$$
$$R_c = \frac{v^2}{a_c}$$

Here $a_c = -a_y = g$ and $v = v_x = v_{x0} = S_o \cos\phi$ (at the top, the vertical velocity is zero). The acceleration is purely centripetal and have the value g (projectile in free-fall) and so:

$$\boxed{R_c = \frac{S_0^2 \cos^2\phi}{g}}$$

Problem Z01: Part (c) – (5 points):

So we are asked to write down an expression for the vector velocity. In general this is a expression with velocity as a function of time broken out in two components, one in x and one in y.

$$\vec{v}(t) = v_x(t)\hat{\boldsymbol{\imath}} + v_y(t)\hat{\boldsymbol{\jmath}}$$

So we need expressions for x_x and v_y. There are two equally acceptable methods to approach this problem.

Method 1: Use Kinematic Equations:

The velocity in the x-direction in constant:

$$v_x(t) = v_{x0} = S_0 \cos(\phi)$$

However, velocity in the y-direction depends on time:

$$v_y(t) = v_{y0} - gt$$

So we need to find the time t_P corresponding to just-before-impact at point P. There are ***two ways to get the time:*** The quickest way is to invoke **symmetry** and the expectation is that the time at point P_P is *twice* the time corresponding to maximum height which we already calculated in Part (a):

$$t_P = 2t_{max-height} = 2\left(\frac{v_{y0}}{g}\right)$$

Alternatively we can find the time at point P directly by solving vertical position kinematic equation where we know $y = 0$:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

Setting $y = 0$, and also $y_0 = 0$ as in Part (a):

$$0 = 0 + v_{y0}t_P - \frac{1}{2}gt_P^2$$

$$\frac{1}{2}gt_P^2 = v_{y0}t_P$$

$$\frac{1}{2}gt_P = v_{y0}$$

$$t_P = 2\left(\frac{v_{y0}}{g}\right)$$

Once we have the time at Point P, We put this result into the vertical velocity equation:

$$v_y(t) = v_{y0} - g\left(2\frac{v_{y0}}{g}\right)$$
$$v_y(t) = -v_{y0}$$
$$v_y(t) = -S_0\sin(\phi)$$

Plugging this into the vector expression:

$$\boxed{\vec{v}(t) = S_0\cos(\phi)\hat{\boldsymbol{\imath}} - S_0\sin(\phi)\hat{\boldsymbol{\jmath}}}$$

Method 2: Use Symmetry Argument:

Given the physical **symmetry** of the problem as stated, we can argue that while the horizontal velocity is constant. However, the vertical velocity changes linearly and crosses zero at the half-way point. In other words, the projectile should be landing with the same vertical vertical speed, just in the opposite direction. velocity at launch:

$$v_y(t_p) = -v_y(0) = -S_0\sin(\phi)$$

Plugging this into the vector expression:

$$\boxed{\vec{v}(t) = S_0\cos(\phi)\hat{\boldsymbol{\imath}} - S_0\sin(\phi)\hat{\boldsymbol{\jmath}}}$$

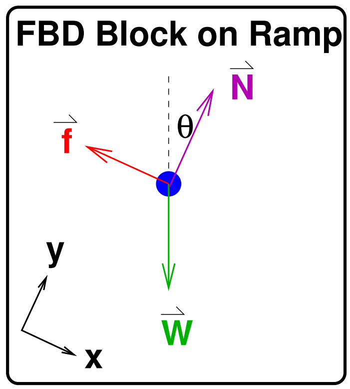
Solution to Problem Z02:

For this whole problem the central idea is applying **Newton's Laws**.

Z02 Part (a): (15 points)

Acceleration: Here we are told that the block remains at rest so **the acceleration is zero**.

Free Body Diagram: of the block on the on the ramp. For convenience, we chose a ***"tipped coordinate system"*** where we define the x-direction as "parallel to the surface of the ramp".[1]



Now we apply **Newton's Second Law** one component at a time:

N's 2nd on Block, y-component:

$$F_y = ma_y$$
$$F_y = 0$$
$$N - W\cos\theta = 0$$
$$N = W\cos\theta$$
$$\boxed{N = mg\cos\theta}$$

Here we have made used of the fact that the Weight force is known: $\boxed{W = mg}$.

N's 2nd on Block, x-component:

$$F_x = ma_x$$
$$F_x = 0$$
$$W\sin\theta - f = 0$$

[1]Note: For Part (a), since the acceleration is zero, students can also profitably choose a **non-tipped** coordinate system. Students will get the same answer. Full credit if the work is done correctly.

$$mg\sin\theta = f$$
$$\boxed{f = mg\sin\theta}$$

There you go! Note that the solution to Part (a) *does not* depend in *any* way on the value of the coefficient of static friction, μ_s.

Guidelines for graders for Problem 2, Part (a):

Final boxed answers for Z02 Part (a):

$$\boxed{N = mg\cos\theta}$$
$$\boxed{W = mg}$$
$$\boxed{f = mg\sin\theta}$$

- **3 points:** Students should have a clear and correctly labeled Free-Body Diagram.
- **2 points:** Students should have a clear coordinate system for conducting their work.
- **4 points:** Students should correctly apply Newton's Second Law in the "y-coordinate".
- **6 points:** Students should correctly apply Newton's Second Law in the "x-coordinate".

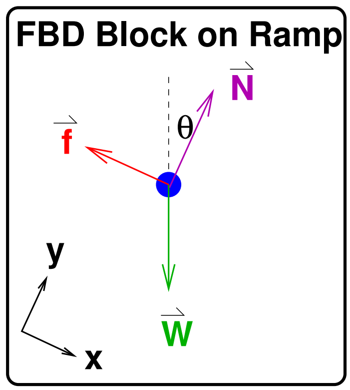
Common error: students incorrectly assert that the static friction is given by $f_s = \mu_s N$ This is a *serious conceptual error*. Student who assert this and/or use this to determine the value of other forces can earn at most 11 points on this part.

Solution continues next page...

Z02 Part (b): (15 points)

Acceleration: Given that we are sliding (accelerating) down the ramp and that sliding (kinetic) friction is in play here, we know they "y-component" of the acceleration is still zero but the x-component is unknown.

Free Body Diagram: This remains **unchanged**. Friction is friction as far as the FBD is concerned:



We apply **Newton's Second Law** in the usual manner, one component at a time:

First we remind ourselves that the Normal force does not depend on friction:

N's 2nd on Block, y-component:

$$F_y = ma_y$$
$$N - W\cos\theta = 0$$

Same as before:

$$\boxed{N = mg\cos\theta}$$

N's 2nd on Block, x-component:

$$F_x = ma_x$$
$$W\sin\theta - f_k = ma_x$$
$$W\sin\theta - \mu_k N = ma_x$$
$$W\sin\theta - \mu_k mg\cos\theta = ma_x$$
$$mg\sin\theta - \mu_k mg\cos\theta = ma_x$$
$$a_x = g\sin\theta - \mu_k g\cos\theta$$
$$\boxed{a_x = g(\sin\theta - \mu_k\cos\theta)}$$

Here we have made use of the fact that the Weight force is known $\boxed{W = mg}$ and the force of *sliding friction* just depends on the magnitude of the Normal force: $\boxed{f = \mu_k N = \mu_k mg\cos\theta}$.

Guidelines for graders for Problem Z02, Part (b):

Final boxed answers for Part (b):

$$\boxed{N = mg\cos\theta}$$

$$\boxed{W = mg}$$

$$\boxed{f = \mu_k mg\cos\theta}$$

$$\boxed{a_x = g(\sin\theta - \mu_k\cos\theta)}$$

- **3 points:** Students should have a clear and correctly labeled Free-Body Diagram and/or should argue (either implicitly or explicitly) that the FBD they used for Part (a) also applies to Part (b).
- **2 points:** Students should have a clear coordinate system for conducting their work and/or should argue (either implicitly or explicitly) that the FBD they used for Part (a) also applies to Part (b).
- **4 points:** Students should correctly apply Newton's Second Law in the "y-coordinate" and/or should argue (either implicitly or explicitly) that the result they obtained for Part (a) also applies to Part (b).
- **6 points:** Students should correctly apply Newton's Second Law in the "x-coordinate". Students need to invoke the rule for **sliding friction** (also called "kinetic friction") which states that $f_k = \mu_k N$.

Problem Z03: Part (a) – (5 points):

Since the spring force is Conservative and there are no friction forces, we meet the *Condition* for applying **Conservation of Mechanical Energy**:

$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$

The potential energy of a spring is given by $U = \frac{1}{2}kx^2$ (where x is the displacement of the spring from its relaxed length). The kinetic energy is given by $K = \frac{1}{2}mv^2$:

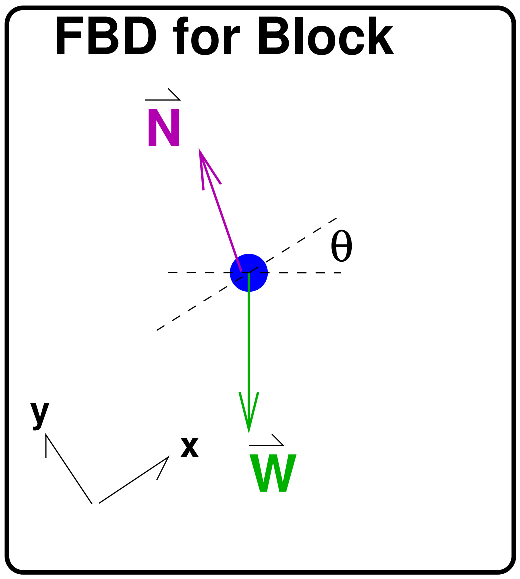
$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx'^2 + \frac{1}{2}mv'^2$$

In this case, we start with the spring compressed, released at rest, so $x = X_0$ and $v = 0$, Once the block has move away from the spring, $x' = 0$ and $v' = v_1$ which is what we want to solve for:

$$\frac{1}{2}kx_0^2 + 0 = 0 + \frac{1}{2}mv_1^2$$
$$\frac{1}{2}mv_1^2 = \frac{1}{2}kx_0^2$$
$$mv_1^2 = kx_0^2$$
$$v_1^2 = \frac{kx_0^2}{m}$$
$$v_1 = \sqrt{\frac{kx_0^2}{m}}$$
$$\boxed{v_1 = x_0\sqrt{\frac{k}{m}}}$$

Problem Z03: Part (b) – (5 points):

So point P corresponds to being on a ramp a "ramp" with a given tilt angle of θ relative the horizontal. We are asking for acceleration, so we just need to use **Newton's Second Law**. The **Free Body Diagram** for the block on the ramp looks like this:



We are asking for the acceleration a_P which on the diagram is indicated in the uphill direction. However, the component of the Weight force along the tipped x-direction is negative:

$$-W_x = ma_p$$

$$-mg\sin\theta = ma_p$$

$$\boxed{a_p = -g\sin\theta}$$

Note that the speed and/or vertical position of the block at point P is not relevant to finding the acceleration.

Problem Z03: Part (c) – (5 points):

So by far the simplest way to contend with this problem is in the context of the **Classical Work-Energy Relation:**

$$\mathcal{W}_{tot} = \Delta K$$

We have a ***round trip*** which means that the work done by every conservative force on the trip from the left to the right end of the track is canceled by the negative of that work from the right to the left. For example, work done by the force of gravity going up the hill is precisely the negative of the work going down the hill. Likewise for the work done by the right spring. The only non-conservative forces applied during the round trip is the force the friction, and we can quickly calculate the work done:

$$\mathcal{W}_{friction} = \text{"force"} \times \text{"distance"}$$
$$\mathcal{W}_{friction} = (\mu mg) \times (-2D)$$
$$\mathcal{W}_{friction} = -2\mu mgD$$

Note that the work is ***negative*** (force applied against the motion) and the distance is $2D$ since we lose energy going both right and left.

The change in potential energy in the left spring is determined by the change in the kinetic energy at the spring which is just the lost energy as indicated above:

$$\Delta U = -2\mu mgD$$
$$\frac{1}{2}kx_{max}^2 - \frac{1}{2}kx_0^2 = -2\mu mgD$$
$$\frac{1}{2}kx_{max}^2 = \frac{1}{2}kx_0^2 - 2\mu mgD$$
$$x_{max}^2 = x_0^2 - \frac{4\mu mgD}{k}$$
$$\boxed{x_{max} = \sqrt{x_0^2 - \frac{4\mu mgD}{k}}}$$