Chapter 2++

More on Constant Acceleration

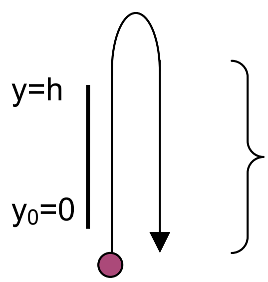
Revisit:	Understand:
• "Throw-up" problems • "Catch-up" problems	• Symmetry of gravity trajectories • Which variables we need • Drawing intersecting trajectories

- **1D Throw-Up — One More Time**

Let's revisit a catch-up problem by solving an example like the one in Chapter 2+:

EXAMPLE: A girl watches from inside a window as a stone goes sailing vertically up and then a little later falls down again past her view. She doesn't see it during the time it is above the top of her window (or below the bottom, either). Suppose the window is 6.0 ft from bottom to top and she sees the stone for a TOTAL of 1.0 s.

a) Quickie! What is the time she **sees** the stone on the **upward** part of its motion? (As you know, if there's no air drag, these up and down motions are **symmetric** between the corresponding portions, as we show on the next page.)



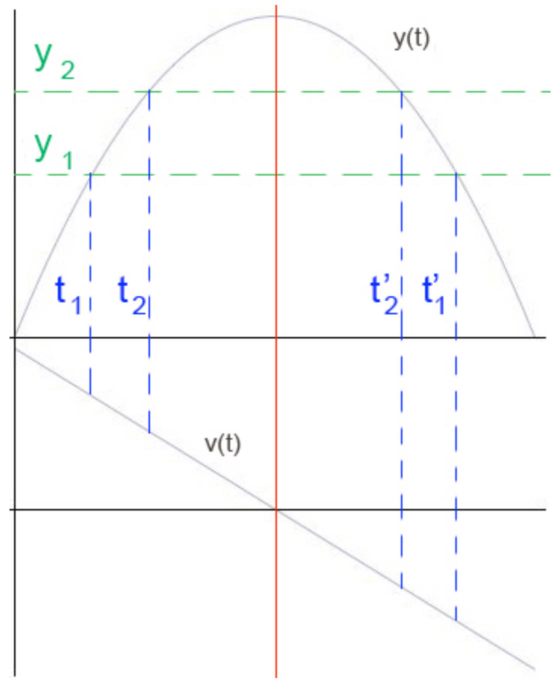
sees up and down for 1 sec.

$\therefore \boxed{\frac{1}{2}\text{ sec}}$

b) What is the speed with which the stone was moving upwards just when it passed the **bottom** of her window? We know how long it took to go from the bottom to the top, and how far it went, so we can use y(t) to find out what the initial speed (speed at the bottom). Inserting $t = \frac{1}{2}$ s, $g = 32$ ft/s^2, $y - y_0 = 6$ ft into $y - y_0 = v_0 t - \frac{1}{2}gt^2$, we have

$$6 = v_0\frac{1}{2} - \frac{1}{2}32\left(\frac{1}{2}\right)^2,\ \text{and solving for } v_0,\ \text{we find } v_0 = \frac{6+4}{1/2} = \boxed{20\text{ ft/s}}.$$

c) How high above the **top** of the window does the stone rise? Noting that we now have $v_0 = 20$ ft / s from (b), we know v = 0 at the peak, and we want y, we should use v(y). Putting those numbers into $v^2 - v_0^2 = -2g(y - y_0)$, we have $\frac{0 - 20^2}{-2(32)} = y - y_0$ or $y - y_0 = 6.25$ ft, the distance from the bottom to the peak. Answer: $\boxed{1/4\text{ ft}}$ above the top.



Symmetry proof: And now for the official symmetry proof, folks! We have already proven this symmetry in Ch. 3 but let's review it and extend the story a little.

What is obvious from the y(t) graph shown, the time going up (t_2 - t_1) on one side of the trajectory, from height y_1 to y_2 , will be the same as the time coming down ($t'_1 - t'_2$), on the other side of the trajectory, from height y_2 to y_1.

It is also clear for the v(t) graph shown just below the y(t) graph, the velocity going up at any point will be equal and opposite the velocity going down at the symmetric point on the other side. For example, $v(t'_1) = -v(t_1)$, and this is true for any two points placed symmetrically around the peak of y(t).

To repeat our Ch. 3 story, we can prove this fast if you'll allow us to put our origin, **y=0, t=0, at the peak** so that times on the left of the **center vertical** line are negative. Then, if we can show **y(-t) = y(t)** and **v(-t) = - v(t)**, for any t, we are done! With y=0, t=0 at the peak, then $y(0) \equiv y_o = 0$, $v(0) \equiv v_o = 0$, and we have $y(t) =-1/2gt^2$ and $v(t) = -gt$. We immediately see $y(-t) = y(t)$ and $v(-t) = - v(t)$. Q.E.D.

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Problem 2-8 A basketball player, about to dunk the ball, jumps 76 cm vertically. How much time does the player spend

a) during the whole jump (up and down)

b) in the top 15 cm of this jump (up and down).

c) in the bottom 15 cm (up and down).

This problem helps explain why such players seem to hang in the air at the top of their jumps.
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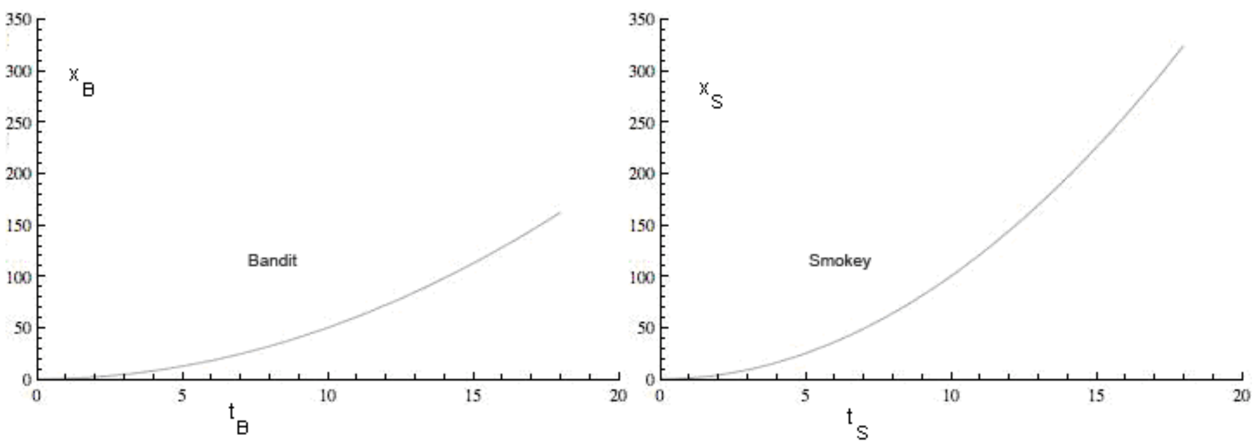
- **1D Catch-Up — One More Time**

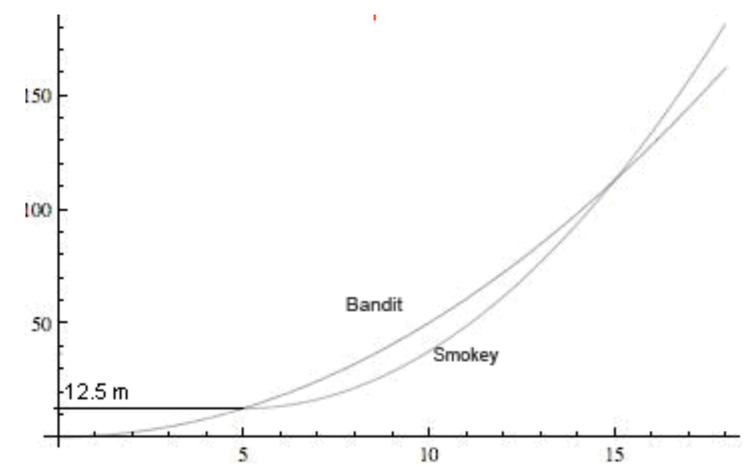
Example: SMOKEY AND THE BANDIT **-** The Bandit accelerates down a straight road starting from rest at a constant acceleration of 1.0 m/s^2, and 5.0 seconds later, he passes Smokey who was parked by the side of the road. Because Smokey saw him coming, at the instant the Bandit passed him Smokey immediately was able to start out from rest with twice the constant acceleration, 2.0 m/s^2. If the Bandit is oblivious to Smokey and doesn't change his acceleration, how many seconds **after the Bandit began accelerating** does Smokey catch up to him?

Steps to solve:

1) We first know that, after Smokey begins to accelerate, the Bandit and Smokey will be somewhere on their respective parabolic trajectories. What we haven't made clear yet is how to compare their time and position axes (t_B with t_S, x_B with x_S).



2) We put Smokey's curve on top of Bandit's curve in the figure below such that the **beginning** of Smokey's curve intersects the Bandit's position five seconds after the Bandit started. We immediately see that Smokey catches the Bandit $\boxed{\text{15 seconds}}$ after the Bandit began accelerating.



NOTE that we had to calculate the common position when they first passed each other and we did it using $x_B = ½\, a_B t_B^2 = 12.5$ m at five seconds for the distance the Bandit went relative to the Bandit's initial position. And this was Smokey's original position where he initially parked (we added the horizontal line at 12.5 m to show his original "trajectory.")

3) Finally, we want to show how to calculate with formulas the intersection points found above graphically. These points are given by the two solutions to the time quadratic equation that comes from setting $x_B = x_S$ only AFTER Smokey has begun to accelerate:

$$x_B = ½\, a_B t_B^2 = 12.5 + ½\, a_S (t_B - 5)^2 = x_S$$

$$\Rightarrow \quad 0 = 0.5\, t_B^2 - 10\, t_B + 37.5 \quad \Rightarrow \quad t_B = 5 \text{ s},\ 15 \text{ s}$$

Here, we used the fact that, at $t_B = 5$ s, Smokey's starting position is 12.5 m, his starting velocity is 0, and he begins to **accelerate**. Note that $½\, a_S (t_B - 5)^2$ time dependence arises as explained on p. 2-10 and corresponds to $t_S = t_B - 5$ in Smokey's top right graph of the previous page.

You could have started your clock differently and used the t_S we just mentioned so that Smokey begins to accelerate at $t_S = 0$. Also, use $x_S(0) = 0$ at that instant. This is like laying Bandit's graph on top of Smokey's graph and using the time variable t_S shown on Smokey's graph. Then $x^S(t_S) = 0 + 0 + 1.0\, t_S^2$. For the Bandit, $x^B(t) = 0 + 5\, t_S + 0.5\, t_S^2$, since at $t_S = 0$ the Bandit has the same position as Smokey, but a velocity of $a_B \cdot 5\text{ s} = 5$ m/s at $t_S = 0$. Now $x_B = x_S$ yields

$$5\, t_S + 0.5\, t_S^2 = 1.0\, t_S^2 \quad \Rightarrow t_S = 0 \text{ s},\ 10 \text{ s}$$

which agree with $t_B = t_S + 5 = 5$ s, 15 s

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Problem 2-9

Mary is driving at a constant speed of 90.0 km/h along a straight highway and she passes an old friend, Jerry, who is traveling in the same direction at a constant speed of 72.0 km/h. Mary and Jerry finally realize simultaneously what has happened 2.00 seconds later from which point Jerry speeds up with constant acceleration of 2.00 m/s^2 and Mary slows down with a constant deceleration of -5.00 m/s^2.

a) If x =0 is the position where they first pass each other and if t=0 is the time when Jerry begins to accelerate and Mary begins to decelerate, sketch roughly a picture x(t) of their trajectories showing both intersections.

b) How much time passes ***from the moment they first passed each other*** *before their cars are back together again?*

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