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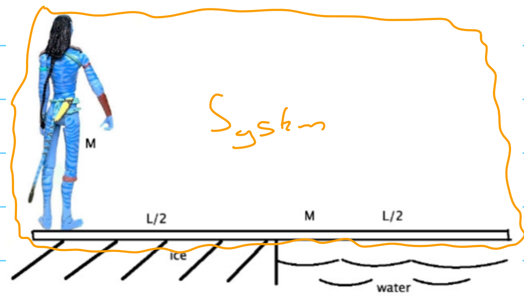
PHYS121 - Homework 8

Problem 1)



Given

- Uniform Board of length L
- Neytiri half is on slippery half
- Other half is over water
- Neytiri and board both mass M

a) Let $x=0$ be @ Neytiri End, $x=L$ be end of board

Define the system as Neytiri and the Board

Neytiri @ $x=0$, mass M

Board @ $\frac{0+L}{2}=\frac{L}{2}$, mass M

$$x_{cm} = \frac{M(0)+M(\frac{L}{2})}{M+M} = \frac{LM}{4M} = \frac{L}{4}$$

$\boxed{\text{CoM of System is } \frac{L}{4}}$

b) System is isolated, so $F_{\substack{Net\\Sys\\Ext}}=0$ means $\vec{a}_{cm}=0$, therefore $\vec{v}_{cm}=\text{const}$ by definition. The board must move 'against' Neytiri to ensure that this is true. This means by the time Neytiri makes her way to the end of the board, the board will have moved the same amount behind her. This will result in her falling off the board. If the board was not slipping, then she would fall as a result of the board not having a normal force applied by the water.

d) $F_{\substack{Net\\Sys\\Ext}}=0$, so $v_{cm}=\text{const}$ to maintain this fact.

System Isolated CofLM applies

$$P_{tot} = P'_{tot}$$

$$mv_N + mv_B = mv'_N + mv'_B$$

$$0+0 = Mv + Mv'_B$$

$$-Mv = Mv'_B$$

$$\boxed{v'_B = -v}$$

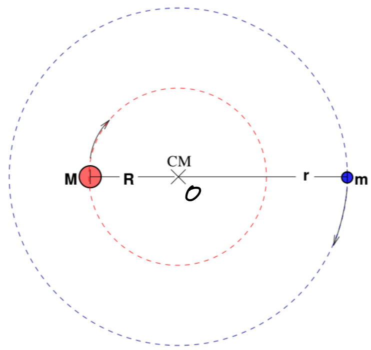
c) $v_{cm} = \frac{m_1v_1+m_2v_2}{m_1+m_2}$ $\quad v_1 = v \quad m_1=m_2=M$

$v_2 = -v$

$$v_{cm} = \frac{Mv - Mv}{2m} = 0$$

$$\boxed{v_{cm}=0}$$

Problem 2)



Given

- Both stars in circular orbit around CoM
- Mass of red star is M
- Orbit R is not given
- Mass of blue star is m
- Orbit r is given

Define x_{cm} as 0

$$x_{cm} = \frac{M(-R)+m(r)}{M+m} = 0$$

$$\rightarrow -MR + mr = 0$$ Expression for radius

$$\frac{R}{r} = \frac{m}{M} \quad \therefore \quad R = \frac{m}{M} r$$

Determine linear velocity of blue star mass m

N2L $\quad F_{Net} = m a_c$

$$F_{UG} = m\left(\frac{v_m^2}{r}\right)$$

Distance btwn stars

$$\frac{GMm}{r_{dist}^2} = \frac{m v_m^2}{r}$$

Distance from CoM to Star

$$\frac{GMm}{(R+r)^2} = \frac{m v_m^2}{r}$$

Plug in expression:

$$\frac{GM\cancel{m}}{\left(\frac{m}{M}r + r\right)^2} = \frac{\cancel{m} v_m^2}{r}$$

$$\frac{GM}{\left(\frac{r(M+m)}{M}\right)^2} = \frac{v_m^2}{r}$$

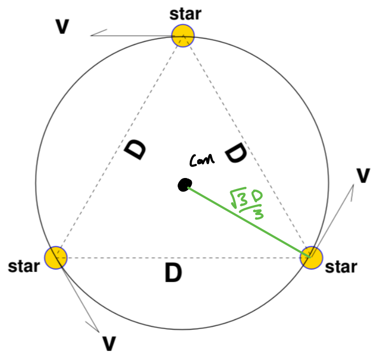
$$\frac{GM}{\frac{r^2}{M^2}(M+m)^2} = \frac{v_m^2}{r}$$

$$\rightarrow \frac{GM^3}{r^{\cancel{2}}(M+m)^2} = \frac{v_m^2}{\cancel{r}}$$

$$v_m^2 = \frac{GM^3}{r(M+m)^2}$$

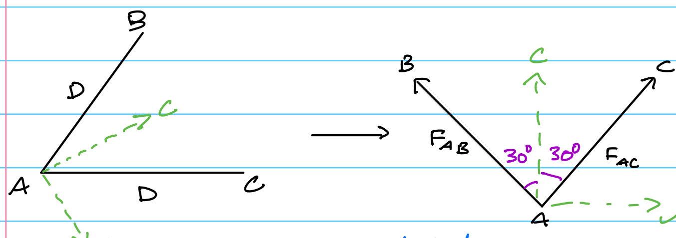
$$\boxed{v_m = \sqrt{\frac{GM^3}{r(M+m)^2}}}$$

Problem 3)



Given

- All stars of mass m
- Positioned in an equilateral triangle
- Distance between any two stars is D
- Circular Orbit of all 3 stars
- Orbit with radius $\frac{\sqrt{3}D}{3}$



Must determine resultant vector

$F_{AB} = F_{AC} = \frac{GMm}{r^2} = \frac{Gmm}{D^2}$ all same mass, distance D apart

N2L c-dir

$F_{Net,c} = m a_c$ UCM $\frac{v_A^2}{r}$

$F_{AB_c} + F_{AC_c} = m a_c$

$F_{AB}\cos 30 + F_{AC}\cos -30° = m\frac{v_A^2}{r}$ $\quad r = \frac{\sqrt{3}D}{3}$

$$\frac{Gmm}{D^2}\left(\frac{\sqrt{3}}{2}\right) + \frac{Gmm}{D^2}\left(\frac{\sqrt{3}}{2}\right) = \frac{3mv_A^2}{\sqrt{3}D}$$

$$\frac{Gmm}{D^2}\sqrt{3} = \frac{3mv_A^2}{\sqrt{3}D}$$

$$\frac{Gm^2}{D^2}\cdot\cancel{3} = \frac{\cancel{3}mv_A^2}{D}$$

$$\frac{Gm^{\cancel{2}}}{D^{\cancel{2}}} = \frac{\cancel{m}v_A^2}{\cancel{D}}$$

$$\frac{Gm}{D} = v_A^2$$

$$\boxed{v_A = \sqrt{\frac{Gm}{D}}}$$