PHYS 121: Cycle 2 Review Sheet, Part 2

October 10, 2020

Universal Gravity:

Newton showed that there is an attractive force that is between any two bodies that is proportional to their mass and inversely proportional to the distance squared. He called this the force of Universal Gravity:

$$F_{UG} = \frac{GMm}{r^2}$$

Note that the force of *Universal Gravity* is a non-contact force. Note also that it is a *known* force in that given the masses, you know the force. Also, you might notice that "Weight" and "Universal Gravity" are two forms of the same thing (right)? We'll do more with this in Cycle 2.

Note that the force of Universal Gravity is always **attractive**. This means that the force vector on one body always points toward the other body.

Spring force:

An ideal spring can exert either a push or a pull. The magnitude of the force is given by:

$$F_{sp} = k|x|$$

Where k is the spring constant and indicates how stiff the spring is. The position x represents the displacement position for the end of the spring with respect to the equilibrium location. This expression is known as **Hooke's Law**. Note that x can be positive or negative and that the direction of the force is in the opposite direction to the displacement of the spring. We will do more with springs and oscillations in Cycle 3.

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A Force Menu for Cycle 2:

We now have added new forces to our Force Menu which will be (almost) all that you should be most problems that require dealing with Newton's Second Law: Remember that the only forces that should appear on your Free Body Diagrams should be these forces listed below along with any "given applied forces".

Force	Symbol	Contact?	Details
Weight	$\vec{W}$	No	Downward force due to gravity near surface of the earth, Magnitude $W = mg$ (this is *known* force) **This force is Conservative.**
Normal	$\vec{N}$	Yes	A *push* perpendicular to a surface, the magnitudes is generally *unknown*.
Tension	$\vec{T}$	Yes	A *pull* usually due to a rope or cable, may be "re-routed" by pulleys, etc. the magnitudes is usually *unknown*.
Friction	$\vec{f}$	Yes	A force applied between two surfaces parallel to the surface and in opposition to the motion of one surface *relative* to the other, comes in **two flavors**: (1) *static* friction, where the magnitude is generally *unknown* and (2) *kinetic* friction (sliding) where is *known* to be $f_k = \mu_k N$ where μ_k is called the "coefficient of kinetic friction".
Spring Force	$\vec{F_{sp}}$	Yes	A push or pull according to Hooke's Law: $F_{sp} = k\|x\|$ (*known*). The direction is toward the equilibrium point $x = 0$, in opposition to the displacement. **This force is Conservative.**
Universal Gravity	$\vec{F_{UG}}$	No	Newton's universal re-casting of gravity as a force between any two bodies. Weight is a particular manifestation of this force. **This force is Conservative.**

Generally, your FBD for Newton's Second Law should *only* include forces from the above menu. If it is not on the menu, do not put it on your FBD. In particular do not put the net for or any sort of "centripetal force" on the FBD.

There is is (sort of) one exception to the rule. If you are given *explicitly* an "applied force" with some particular magnitude and direction you can put this on your FBD without further specification – although you can pretty much count on this applied force being one (or more) of the forces on the menu.

Work-Energy Relation:

Don't forget the Classical Work Energy theorem:

$$\Delta K = W_{tot}$$

where

$$K = \frac{1}{2}mv^2$$

This means that if a net force F does total work W_{tot} on a body, the change in kinetic energy $\Delta(\frac{1}{2}mv^2)$ is the result of the total work.

Conservative Forces:

We say a force is *em conservative* if the work done to move a particle from point P to point Q is *independent* of the path we actually take.

This is equivalent to saying that for a conservative force, the work done by moving the particle on any closed loop path is zero.

Important Fact: Note that in *general* forces are *not* conservative. This means that unless you know for sure that the force is conservative, the path integral for work around some loop is not expected to be zero.

Potential Energy:

If we have a force $\vec{F}$ that is conservative then we can construct a Potential Energy U for that force. The potential energy is calculated by taking the integral:

$$U(P) = -\int_{P_0}^{P} \vec{F} \cdot d\vec{r} + U(P_0)$$

For any potential energy scheme, we are free to choose any value for the potential energy scheme $U(P_0)$ at some particular single point P_0. Often we set $U(P_0 = 0) = 0$ but the decision as to where to evaluate $U(P_0)$ at what value of P_0 is arbitrary and should be chosen to make the problem most convenient.

Here are three form of potential energy that we are concerned with:

Potential Energy (U) due to Weight near the surface of the earth: The potential energy we obtain by lifting a mass m to a height h is just the work done, i.e., the force times the distance:

$$U_{weight} = mgy$$

Potential Energy (U) due to a spring: We can demonstrate empirically that the force associated with displacement of a spring from equilibrium position $x = 0$ is given by:

$$F_{sp} = k|\vec{x}|$$

which is called Hooke's Law. We can define the potential energy that we obtain when we do work against this spring as:

$$U_{sp} = \tfrac{1}{2}kx^2$$

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Potential Energy (U) due to Universal Gravity: When we consider moving an object over a large distance, we can no longer treat the force of gravity at the surface of the earth as a constant force. Instead, we must consider the equation for Universal Gravity:

$$F_{UG} = G\frac{Mm}{r^2}$$

which tells us that the force of gravity decreases as we move out to larger distances r. If we define the potential energy as zero when a body is placed at an infinite distance (where the force is zero) then the expression for the gravitational potential energy is:

$$U_{UG} = -G\frac{Mm}{r}$$

From this we can derive the expression for the *escape velocity* by determining the kinetic energy required to match the potential energy due to gravity. We can show that for a body of mass M the velocity to escape from a surface at radius R to infinity is:

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

To escape from the surface of the earth, for example, $v_{esc} = 1.12 \times 10^4$ m/s.