PHYS 121: Third Hour Exam (Exam #3)

April 19, 2024

Do not open this exam until instructed to do so.

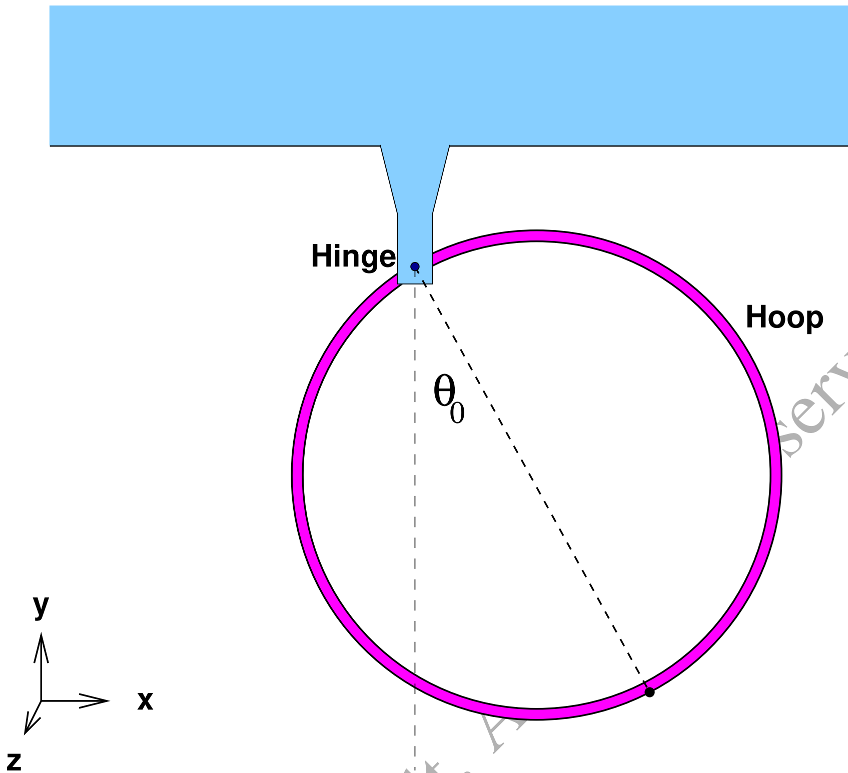
Important: Only these vector forces allowed on your Free-Body-Diagrams:

- $\vec{W}$ = Weight Force
- $\vec{N}$ = Normal Force
- $\vec{T}$ = Tension Force
- $\vec{f}$ = Friction Force
- $\vec{F}_{app}$ = Applied force
- $\vec{F}_{sp}$ = Spring force
- $\vec{F}_{ug}$ = Force of Universal Gravity
- $\vec{H}$ = Hinge Force (force due to an axle or pin)

Example Notation for force subscripts:

- $\vec{T}$ = "The Tension Force."
- $\vec{W}_A$ = "The Weight Force *on* Body A."
- $\vec{N}_{BT}$ = "The Normal Force *on* Body B *due* to the Table."
- f_{AB} = "The magnitude of the Friction Force *on* Body A *due* to Body B."
- F_c = "The magnitude of the *net* force in the *centripetal* direction."
- F_{Ax} = "The magnitude of the *net* force on Body A in the x-direction."

Problem 1: A Different Pivot Point (15 points)



A hula-hoop of given radius R and given mass M is attached to an ideal hinge as shown in the figure above. The hoop is positioned at a given angle θ_0 relative to the vertical and released at rest. ***Your answers below should be given in term of given parameters*** R***,*** M***, and*** θ_0***. Be sure to explain your work.***

Part (a): – [5 points] Determine the magnitude of the **net torque** on the hoop calculated about the hinge point immediately after it is released. (***Hint:*** the answer is not zero.)

Part (b): – [5 points] What is the magnitude of the **angular acceleration** α of the hoop in the instant immediately after the hoops has been released? Explain your answer. ***Hint:*** To determine the Rotational Inertia of the hoop as calculated about the hinge point, you can use **the Parallel Axis Theorem** which says that:

$$I = I_{cm} + MD^2$$

where I_{cm} is the rotational inertia as calculated about the center-of-mass point and D is the displacement between the center-of-mass point and the point of rotation (pivot point) for the rigid body.

Part (c): – [5 points] What is the the **maximum angular speed of hoop** ω_{max} as calculated for rotation about the hinge at some time after the hoop has been released?

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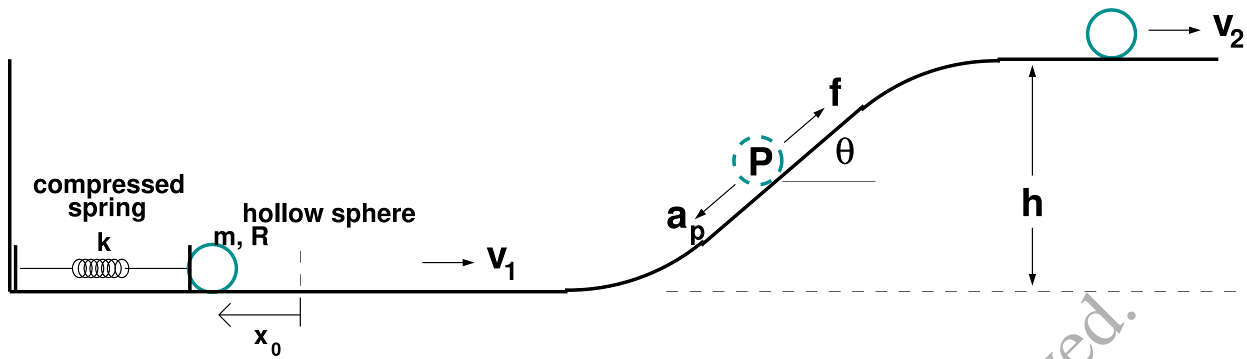
Problem 2: Spring pushes rolling ball up hill (20 points)



A hollow sphere of given mass m and radius R is placed against a spring of given constant k. The spring is compressed by a given displacement distance x_0. At this point the ball is released at rest. The spring pushes the ball so that it rolls without slipping or sliding with respect to the the horizontal surface. The ball subsequently rolls without slipping up a hill of given height h as shown.

Assume that there is zero friction between the pad of the spring and the ball.

Part (a) (5 points) – Calculate the **speed** v_1 of the hollow sphere as it rolls along the lower horizontal surface before it encounters the hill. Give your answers in terms of the given parameters. Explain your work.

Part (b) (5 points) – Calculate the magnitude of the **linear acceleration** a_p of the ball at it rolls past the point labeled "**P**" where the track is tipped at a given angle θ relative to the horizontal. Give your answer in terms of given parameters. Explain your work. ***Hint:*** Since the ball is slowing down here, it is reasonable to assume that the acceleration is pointed down and to the left along the ramp, and it is also reasonable to assume that the force of friction on the ball is pointed up and to the right along the ramp, as shown above.

Part (c) (5 points) – **Also**, calculate the magnitude of the force of **friction** on the ball at this same point "**P**". Give your answer in terms of given parameters. Explain your work.

Part (d) (5 points) – Calculate the **final speed** v_2 of the ball when it is rolling along the upper horizontal surface as shown. Give your answer in terms of given parameters. Explain your work.

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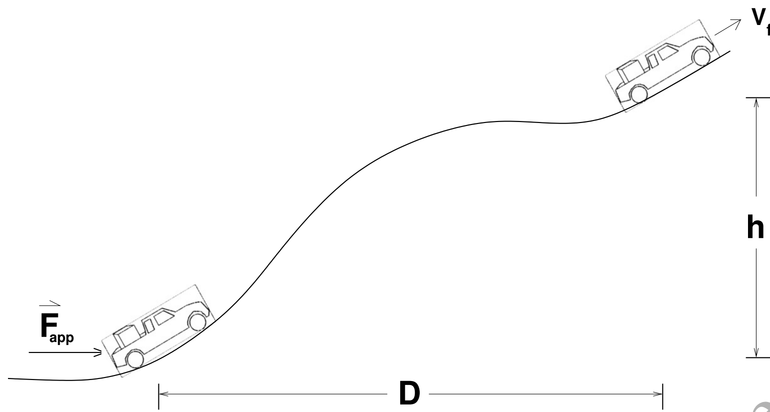
Problem 3: Work, Work, Work – (15 points)



A toy truck of mass m placed at rest on a small hill and then is pushed up the hill as a result of a *horizontal* applied force $\vec{F}_{app}$. The horizontal applied force is defined as a function of horizontal position x according to this expression:

$$\vec{F}_{app} \equiv B(D^2 - x^2)\,\hat{\imath}$$

where B is a constant with appropriate units and D is the given total horizontal displacement as shown.

Furthermore the wheels on the truck are quite sticky so that there is considerable irregular friction (sometimes sliding, sometimes rolling) between the wheels and the hill as the truck is pushed up the hill. The final horizontal displacement of the truck is D and the final vertical displacement is h. Assume that the truck starts at rest and that it arrives to the top of the hill with final forward speed of V_f. Assume that m, B, D, V_f and h are given. Assume that the shape of the path and the length of the path and any coefficients of friction can be *neither* measured *nor* determined and are therefore unknown and *not* given.

Part a) (5 points) – Calculate the **Work done by the force of Weight** on the truck as it moves from the bottom of the hill to the top as shown. Explain your work. Give your answer in terms of the given parameters.

Part b) (5 points) – Calculate the **Work done by the applied force** $\vec{F}_{app}$ on the truck as it moves from the bottom of the hill to the top as shown. Explain your work. Give your answer in terms of the given parameters.

Part c) (5 points) – More Difficult: Calculate the **Work done by the force of friction** on the truck as it moves from the bottom of the hill to the top as shown. Important: Explain your work. Give your answer in terms of the given parameters.

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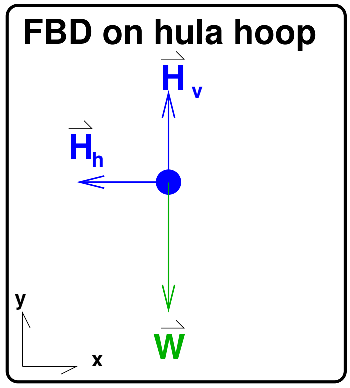
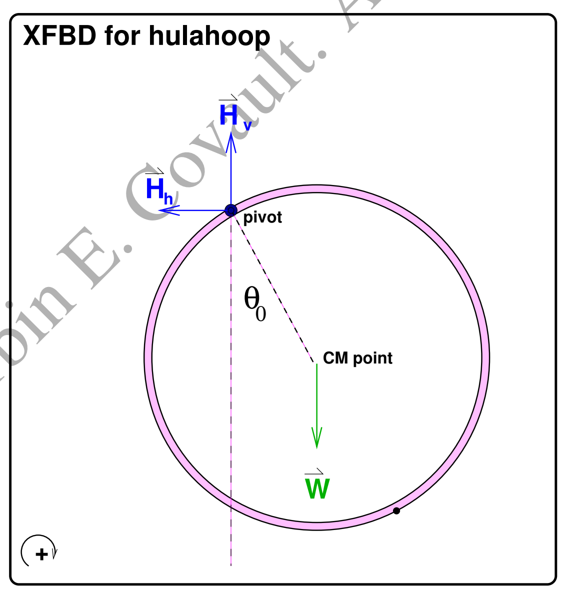
Solution to Problem 1: **Part (a):** (5 points)

In order to consider the torques on the hula-hoop we have to first consider the forces. The forces on the hoop can be articulated in a Free Body Diagram for the pulley:



What are the forces on the hula-hoop? Well the hoop has weight. And the only thing in contact with the hoop is the hinge. We do not know either the direction or the magnitude of the Hinge force but we can just assign vertical and horizontal components.

Once we have the forces we can put them down on an **Extended Free Body Diagram** with the same force.[1]



Here we have defined positive torque clockwise and the pivot point at the hinge. Torque applied to the hula hoop can be calculated from the **definition of torque:**

$$\tau_{\vec{F}} \equiv rF\sin\phi$$

Here, r corresponds to the length of the radial vector *from* the pivot point *to* the point of application of the force. We know that the Weight force is applied at the **center-of-mass** point for the hoop. The Weight always acts as if it is applied at the center-of-mass point even if in fact that point does

[1] Note to grader. Although the solution here shows both Free-Body-Diagram (FBD) and an Extended-Free-Body-Diagram (XFBD), the problem as posted does not explicitly ask the student to provide these. If the student does not write down either a regular FBD and/or an XFBD, this does not automatically count for lost points. If the student calculates the torque correctly and also *explains* that work in a correct, clear, and complete way, then the student should get full credit, even if the FBD and/or the XFBD is omitted.

not corresponds to a physically solid part of the body[2] We see immediately that the Hinge Forces (both components) on the hula hoop contribute *zero* torque because the hinge forces are applied at the pivot point so that $r = 0$. The only force that applies a non-zero torque is the Weight force. The cross product gives a magnitude for the torque $\tau_F = rF \sin\phi$ where $r = R$ the radius of the hoop, $F = W = Mg$ the weight and ϕ is the angle between the radius and the vertical which is ϕ_0.

$$\tau_{\vec{W}} = RMg\sin\theta_0$$

Since there are no other torques, this s the magnitude of the net torque.

$$\boxed{\tau_{net} = RMg\sin\theta_0}$$

[2] That's right. The center-of-mass of a doughnut is in the hole.

Solution to Problem 1: **Part (b):** (5 points)

We need to consider I. For a hoop, the rotational inertia about the center-of-mass point is given by:

$$I_{CM} = MR^2$$

However, in this case the pivot point is not located at the center-of-mass of the hoop, but is offsets by a distance R in the direction at the hinge. We use the **Parallel Axis Theorem** to determine the correct rotational inertia of the hoop for the pivot point at the hinge, here $D = R$:

$$I_{hoop} = I_{CM} + MD^2$$

We know $I_{CM} = MR^2$ for a hoop, and in this case the displacement of the pivot point is $D = R$ the radius:

$$I_{hoop} = MR^2 + MR^2$$
$$I_{hoop} = 2MR^2$$

So now we use **Newton's Second Law for Rotational Motion:**[3]

$$\tau_{net} = I\alpha$$
$$\alpha = \frac{\tau_{net}}{I}$$
$$\alpha = \frac{RMg\sin\theta_0}{2MR^2}$$
$$\boxed{\alpha = \frac{g\sin\theta_0}{2R}}$$

Note that if you fail to consider that the pivot point is off the center-of-mass point here, you will get this answer wrong by a factor of two.

[3]Note to the grader: if the student defines the direction of angular acceleration as *negative* instead of positive, this is entirely acceptable. No points should be taken away.

Solution to Problem 1: Part (c): (5 points)

Since we are asking for a speed we want to consider the application of **Conservation of Energy**. Do we meet the condition? Certainly the weight force is conservative. The hinge forces are definitely *not* conservative but in this problem because that part of the hoop is pinned in place, the hinge forces are not applied over any distance and so **the hinge forces do no work.** So we *meet the condition for Conservation of Energy:*

$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$

At this point we want to consider the potential and kinetic energies.

- The potential energy is just the position energy due to gravity: $U = Mgy$ where y is the vertical coordinate of the *center of mass* of the hoop. If we choose the pivot point as $y = 0$ this will simplify our calculations.
- In general kinetic energy for rigid bodies contains both rotational and translational terms. However, in this case because the whole hoop is pivoting about the hinge, if we use the position at the hinge as the reference point, we can define the motion as *purely rotational*. $K = K_{rot} = \frac{1}{2}I\omega^2$.

So we carry on:

$$U + K = U' + K'$$
$$Mgy + \frac{1}{2}I\omega^2 = Mgy' + \frac{1}{2}I\omega'^2$$

We see that the initial conditions correspond to $y = -R\cos\theta_0$ and $\omega = 0$ (released at rest). We see that in the final condition we expect the maximum speed when the hoop is at the lowest position on the track so that $y' = -R$:

$$Mg(-R\cos\theta_0) + 0 = Mg(-R) + \frac{1}{2}I\omega'^2$$

$$\frac{1}{2}I\omega'^2 = MgR(1 - \cos\theta_0)$$

$$\omega'^2 = \frac{2MgR(1 - \cos\theta_0)}{2MR^2}$$

$$\omega'^2 = \frac{g(1 - \cos\theta_0)}{R}$$

$$\omega' = \sqrt{\frac{g(1 - \cos\theta_0)}{R}}$$

$$\boxed{\omega_{max} = \sqrt{\frac{g(1 - \cos\theta_0)}{R}}}$$

Solution to Problem 2: **Part (a):** (5 points)

In this problem the forces on the ball are all either **Conservative** (Weight and Spring force) or **Do No Work** (Normal and no-slip-Friction). We note in particular that no non-conservative work is done by the friction force because at the point-of-contact, the force is not being applied over a displacement. The ball is (instantaneously) not moving at the point of contact.

So we define "Before" at the point of release, and then "After" is once we are on top of the hill. We have to contend with two different kinds of potential energy (potential energy due to Spring Force and potential energy due to Weight) and two different forms of kinetic energy (translational and rotational):

$$E_{tot} = E'_{tot}$$
$$U_{tot} + K_{tot} = U'_{tot} + K'_{tot}$$
$$U_{sp} + U_W + K_{rot} + K_{tran} = U'_{sp} + U'_W + K'_{tran} + K'_{rot}$$

For Part (a) the ball is not changing vertical position at all, so we can ignore the (unchanging) potential energy due to weight:

$$U_{sp} + K_{tran} + K_{rot} = U'_{sp} + U'_W + K'_{tran} + K'_{rot}$$

We plug in expressions for the potential and kinetic energy terms:

$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx'^2 + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$$

Okay so for "Before" we only have spring potential energy: $x = x_0$, $v = 0$, and $\omega = 0$. For "After" we have $x = 0$, $v' = v_1$ and $\omega' = \omega_1$:

$$\frac{1}{2}kx^2 + 0 + 0 = 0 + \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

Finally we plug in $I = \left(\frac{2}{3}\right)mR^2$ for a hollow sphere and then also apply the **Rolling Constraint** that tells us $\omega_1 = \frac{v_1}{R}$ so:

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}\right)mR^2\left(\frac{v_1}{R}\right)^2$$

The Rest is Algebra. Solving for v_1

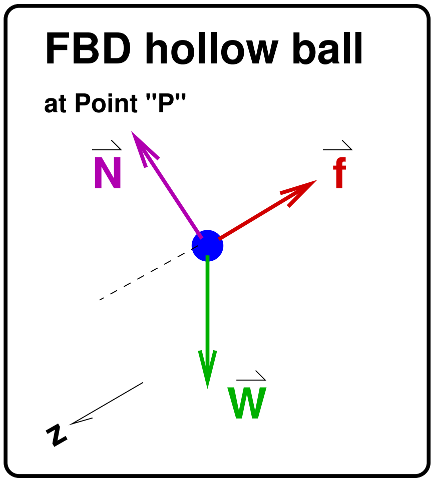
$$kx_0^2 = mv_1^2 + \left(\frac{2}{3}\right)mv_1^2$$
$$kx_0^2 = mv_1^2\left(1 + \frac{2}{3}\right)$$
$$kx_0^2 = mv_1^2\left(\frac{5}{3}\right)$$
$$mv_1^2\left(\frac{5}{3}\right) = kx_0^2$$
$$v_1^2 = \frac{3}{5}\left(\frac{kx_0^2}{m}\right)$$
$$\boxed{v_1 = \sqrt{\frac{3}{5}\left(\frac{kx_0^2}{m}\right)}}$$

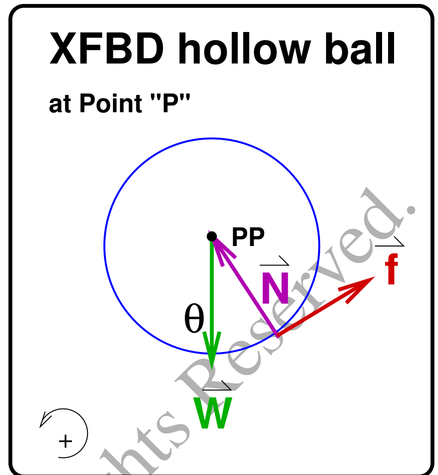
Guidelines for graders:

- **Perfect Work with proper invocation of Conservation of Energy including consideration and explanation of Conditions for Conservation of Energy = 5 points.**
- **Correct answer but failure to correctly consider whether Conditions for Conservation of Energy are met: 4 points.**
- **Forgot one or more important energy terms, either potential or kinetic energy = 2 or 3 points.**
- **Correct answer but but missing substitutions to give final answer in terms of given parameters: 2 or 3 points.**
- **Incorrect or inappropriate physics ideas: Not more than 1 or 2 points.**

Solution to Problem 2: **Part (b):** (5 points)

Since we want an **acceleration** here we are motivated to consider applying **Newton's Laws**. We start with a **Free-Body-Diagram** as shown on the left figure below:





We note in particular that there is a **force of friction that points up and to the left as indicated as a hint in the statement of the problem.** We are also told that the ball is slowing down at Point **P** so that the acceleration is down and to the left. In the FBD we indicate this with a coordinate system z where the z-direction corresponds to the direction of acceleration.

Note that this friction is not *kinetic friction and is therefore unknown prior to applying Newton's Laws.*

We apply **Newton's Second Law** in the z-component:

$$F_z = ma_z$$

Using the FBD we take the component of the Weight that is along the z-axis:

$$W\sin\theta - f = ma_z$$

$$mg\sin\theta - f = ma_z \qquad (1)$$

This is as far as we can go with the algebra since we have *two unknowns*: f and a_z.

So next we consider the torques. We will apply **Newton's Second Law in Rotational Form:**

$$\tau_{net} = I\alpha$$

We can work out the torques by writing up an **Extended Free-Body-Diagram** as shown in the right figure above. We have three forces and we need to sum up the torques due to each force:

$$\tau_{\vec{W}} + \tau_{\vec{N}} + \tau_{\vec{f}} = I\alpha$$

We use the definition of torque:

$$\tau_{\vec{F}} = rF\sin\phi$$

and apply this in turn to each of the forces. Here $\vec{r}$ is the radial displacement vector *from* the pivot point (center of the ball) *to* the point of application of the force:

- $\tau_{\vec{W}}$: The Weight force is applied at the pivot point, so r is zero, so $\tau_{\vec{W}} = 0$.

- $\tau_{\vec{N}}$: The Normal force applied by the floor to the ball is applied in a direction that is *anti-parallel* to the radial vector $\vec{r}$, and so $\sin\phi = 0$ and so $\tau_{\vec{N}} = 0$.
- $\tau_{\vec{f}}$: The Friction Force is applied at a distance $r = R$ from the central pivot point and also applied at a right angle so that $\sin\phi = 1$, and so $\tau_{\vec{f}} = Rf$.

In other words, all of the forces except the friction contribute zero torque. So plugging these all back into our expression for Newton's Second Law:

$$0 + 0 + Rf = I\alpha$$
$$Rf = I\alpha$$

Then we plug in $I = \left(\frac{2}{3}\right) mR^2$ for a hollow sphere and the **Rolling Constraint** that tells us $\alpha = \frac{a_x}{R}$ so:

$$Rf = \left(\frac{2}{3}\right) mR^2 \left(\frac{a_z}{R}\right)$$
$$f = \left(\frac{2}{3}\right) ma_z \qquad (2)$$

So now we have two equations, two unknowns. Plug Equation (2) into Equation (1):

$$mg\sin\theta - \left(\frac{2}{3}\right) ma_z = ma_z$$
$$mg\sin\theta = \frac{5}{3} ma_z$$
$$\frac{5}{3} ma_z = mg\sin\theta$$
$$\boxed{a_z = \frac{3g}{5}\sin\theta}$$

Solution to Problem 2: **Part (b):** (5 points)

Once we have the acceleration, calculating the friction is simple. We plug back into Equation (2):

$$f = \left(\frac{2}{3}\right) m \left(frac3g5\sin\theta\right)$$
$$f = \boxed{\frac{2mg\sin\theta}{5}}$$

Guidelines for graders for Problem 2 parts (a) and (b) :

- **Perfect Work with proper invocation of Newton's Laws and Rotational Dynamics = 5 points for each part.**
- **Physics correct but answers given in terms of r or f or other terms that do not corresponds to given parameters: 3 or 4 points for each part.**
- **Correct application of Newton's Second Law in linear form but missing/incorrect application of torque: at most 2 or 3 points per part.**
- **Correct assertion of Newton's Second Law in both linear and rotational form, but incorrect calculation of individual torques: perhaps 3 pf 4 points per part.**
- **Correct physics but algebra failure to calculate acceleration: minus one or to points.**
- **Incorrect or missing application of Rolling Constraint: minus one or two points.**
- **Incorrect or inappropriate physics ideas: Not more than 1 or 2 points per part.**

Solution to Problem 2: **Part (c):** (5 points)

Happily,since the ball rolls without slipping or sliding, we can still use **Conservation of Mechanical Energy** to find the speed here. The easiest path is simply to set the final potential and kinetic energies corresponds to the position at v_2 equal to the original potential energy of the spring when the whole problem starts:

$$U_{sp} = U''_W + K''_{tran} + K''_{rot}$$
$$\frac{1}{2}kx_0^2 = mgy'' + \frac{1}{2}mv''^2 + \frac{1}{2}I\omega''^2$$

Here the double-prime corresponds to the position on the upper horizontal track. The rest follows in an algebra that is almost identical to that for Part (a) except for the extra term mgh:

$$\frac{1}{2}kx_0^2 = mgh + \frac{1}{2}v_2^2 + \frac{1}{2}\left(\frac{2}{3}\right)mv_2^2$$
$$kx_0^2 = 2mgh + mv_2^2 + \left(\frac{2}{3}\right)mv_2^2$$
$$\left(\frac{5}{3}\right)mv_2^2 = kx_0^2 - 2mgh$$
$$mv_2^2 = \left(\frac{3}{5}\right)(kx_0^2 - 2mgh)$$
$$\boxed{v_2 = \sqrt{\frac{3}{5}\left(\frac{kx_0^2 - 2mgh}{m}\right)}}$$

or

$$\boxed{v_2 = \sqrt{\frac{3}{5}\left(\frac{kx_0^2}{m} - 2gh\right)}}$$

Solution to Problem 3: (15 points)

Part (a)– (5 points):

Quickest way: **The Work done by a Conservative Force is the negative of the change in Potential Energy for that Force.** For Weight we have:

$$\text{Work}_{\vec{W}} = -\Delta U_{\vec{W}}$$

$$\text{Work}_{\vec{W}} = -[mgy]_{y=0}^{y=h}$$

$$\boxed{\text{Work}_{\vec{W}} = -mgh}$$

Alternate Solution to Part (a)– (5 points):

We can calculate the work explicitly by invoking the **Definition of Work:**

$$\mathcal{W}_{\vec{F}}(P_1 \to P_2) \equiv \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy$$

where F_x is the x-component of the force and F_y is the y-component of the force.

Here for the Weight Force we have:

$$F_x = 0$$

and

$$F_y = -mg$$

So we just need to calculate the one integral along the y-direction:

$$\text{Work}_{\vec{W}} = \int_{y=0}^{y=h} (-mg)\, dy$$

$$\text{Work}_{\vec{W}} = (-mg)\, [y]_{y=0}^{y=h}$$

$$\boxed{\text{Work}_{\vec{W}} = -mgh}$$

Part (b)– (5 points):

Note: Since the force is not constant, we cannot simply define Work at as a force times a distance. In this case, we have no choice but to complete the path integral.

We use the **Definition of Work** where the Work done by any Force going from Point P_1 to Point P_2 given by:

$$\mathcal{W}_{\vec{F}}(P_1 \to P_2) \equiv \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy$$

where F_x is the x-component of the force and F_y is the y-component of the force.

Here for $\vec{F}_{app}$ we have:

$$F_x = B(D^2 - x^2)$$

and

$$F_y = 0$$

So we just need to calculate the one integral along the x-direction:

$$\text{Work}_{\vec{F}_{app}} = \int_{x=0}^{x=D} B(D^2 - x^2)\, dx$$

$$\text{Work}_{\vec{F}_{app}} = B \int_{x=0}^{x=D} (D^2 - x^2)\, dx$$

These are simple anti-derivatives:

$$\text{Work}_{\vec{F}_{app}} = B \left[D^2 x - \frac{x^3}{3} \right]_{x=0}^{x=D}$$

$$\text{Work}_{\vec{F}_{app}} = B \left(D^2(D) - \frac{D^3}{3} \right)$$

$$\text{Work}_{\vec{F}_{app}} = B \left(D^3 - \frac{D^3}{3} \right)$$

$$\boxed{\text{Work}_{\vec{F}_{app}} = \frac{2BD^3}{3}}$$

Solution continues next page....

Part (c)– (5 points):

Since there are no other forces on the toy truck we can use the **Classical Work-Energy Relation** (sometimes called the Work-Energy Theorem), to get the Work due to Friction we proceed as follows:

The truck starts at rest so the change in kinetic energy is just the same as the final kinetic energy:

$$\text{Work}_{tot} = \Delta K = K_F - K_i = \frac{1}{2} m V_f^2$$

The total Work is by definition the sum of the Work done by each of the four forces on the toy truck:

$$\text{Work}_{\vec{W}} + \text{Work}_{\vec{N}} + \text{Work}_{\vec{F}_{app}} + \text{Work}_{friction} = \frac{1}{2} m v_F^2$$

Re-arranging so that we have an expression for the Work due to Friction:

$$\text{Work}_{friction} = \frac{1}{2} m v_f^2 - \text{Work}_{\vec{W}} - \text{Work}_{\vec{N}} - \text{Work}_{\vec{F}_{app}}$$

We know that the Work due to the Normal Force here is zero (because the force is applied perpendicular to the path). The Work due to the other forces we have already calculated in the previous parts. We plug these in:

$$\text{Work}_{friction} = \frac{1}{2} m V_f^2 - (-mgh) - 0 - \frac{2BD^3}{3}$$

$$\boxed{\text{Work}_{friction} = \frac{1}{2} m V_f^2 + mgh - \frac{2BD^3}{3}}$$

Solutions Continue Next Page....

Guidelines for graders for Problem 3 (worth 15 points):

- Each part is worth 5 points for a total of 15 points.
- For Part (a) (5 points) the student must invoke either the Definition of Work or the idea that *The Work done by a Conservative Force is the negative of the change in potential energy* to get full credit. It is also acceptable to argue that since the force is constant, the work done is the (negative) force ($= mg$) times the distance/displacement ($= h$). But there needs to be some explanation. Simply writing down "Work = mgh" with no explanation is worth at most 4 points. Having the sign wrong on Part (a) is a one-point error.
- For Part (b) the student needs to invoke the Definition of Work and needs to calculate the integral.
  - Simply multiplying the force F_{app} times the distance D is a major conceptual error. Award at most 3 points.
  - Bad calculus is a 1 or 2 point error depending on whether the results are sensible and/or have consistent units with the correct answer.
- For Part (c) the **only** path to the answer is invoking the Classical Work Energy Relation (sometimes simply called the Work-Energy Theorem).
  - If kinetic and/or static friction forces are "calculated" using N2L and/or invoking a coefficient of friction, etc. This is a major conceptual error and is worth perhaps 1 or maybe points. There is no mechanism here for calculating the work done by friction directly.
  - Failure to contend with and argue that the Normal force does zero work is a conceptual error and will cost a point.
  - Sign errors here will cost (at most) one point.