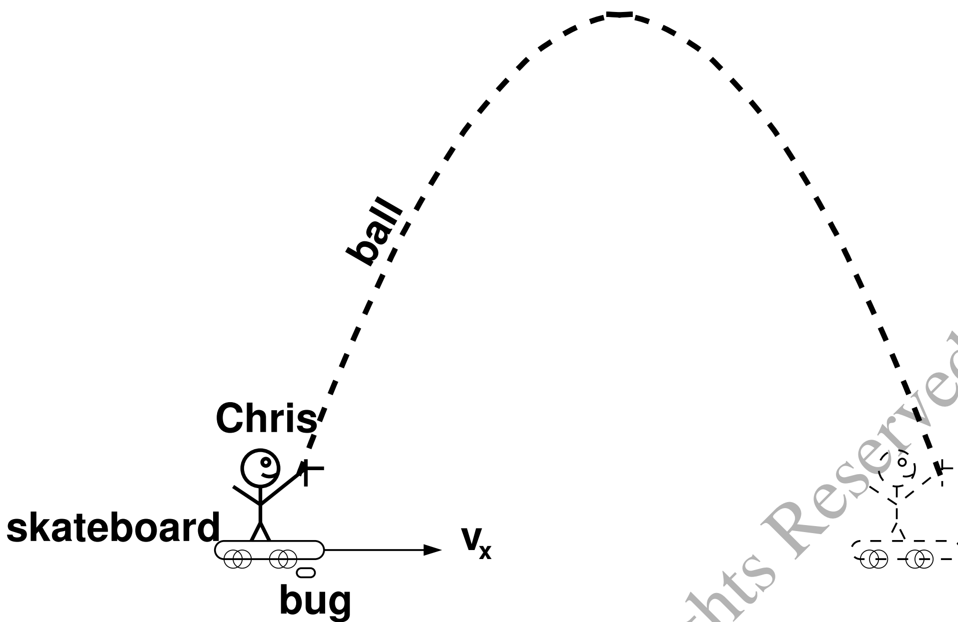
PHYS 121: Solutions to Written Homework #04
February 27, 2024

Note: this homework is worth **15 points** as follows:

___	Problem 1	0 points	Do not grade
___	Problem 2	5 points	5 pts total
___	Problem 3	5 Points	5 pts total
___	Problem 4	0 points	Do not grade
___	Problem 5	5 points	5 pts total

Problem 1:



Chris is on a skateboard that is traveling at a constant horizontal speed $v_x = 4.35$ m/s. They hold a ball in their hand at a height of $h = 1.22$ meters above the ground and then they toss the ball "up" (as seen in their frame) so that they catch it back in their hand exactly 2.85 seconds later as show above (figure *not* shown to scale!)

a) ***Relative to Chris' hand***, what is the speed of the ball as it it travels upward? Explain your answer.

b) Relative to the fixed ground, what is the maximum height of the ball in the air? Explain your answer.

c) Suppose there is a tiny motionless bug on the ground immediately below Chris' hand at the instant when they toss the ball upward. Suppose we define standard unit vectors $\hat{\imath}$ and $\hat{\jmath}$ corresponding to the frame of reference for the bug. Write down *vector expressions* for the position, velocity, and acceleration of the ball ($\vec{r}$, $\vec{v}$, and $\vec{a}$) as a function of time, *t as seen by the bug* at the instant of time just before the ball is caught by Chris. Explain your answer.

Solution to Problem 1 on next page....

SOLUTION TO PROBLEM 1:

$\boxed{\textbf{Part (a):}}$

Chris' hand has the *same horizontal velocity* as the ball. So the *relative* velocity only concerns the vertical component:

We know that the total flight time is given ($t = 2.85$ seconds). So we use normal **free-fall constant acceleration** kinematics to determine the initial vertical speed:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

The initial and final vertical positions are zero (relative to the hand). We solve for the velocity

$$0 = 0 + v_{y0}t - \frac{1}{2}gt^2$$

$$\boxed{v_{y0} = \frac{1}{2}gt}$$

Plugging in numbers:

$$v_{y0} = \frac{1}{2}(9.81 \text{ m/s}^2)(2.85 \text{ s})$$

$$\boxed{v_{y0} = 14.0 \text{ m/s}}$$

$\boxed{\textbf{Part (b):}}$

We can solve this at least two different ways. The easiest way is to use *symmetry* to argue that the time to get to the top is half the time to go up and then come back down. So now $t = 2.85/2$ seconds. We calculate the height:

$$\boxed{y = y_0 + v_{y0}t - \frac{1}{2}gt^2}$$

Plugging in:

$$y = (1.22 \text{ m} + (14.01 \text{ m/s})\left[\left(\frac{2.85}{2}\right) \text{ s}\right] - \frac{1}{2}(9.81 \text{ m/s}^2)\left[\left(\frac{2.85}{2}\right) \text{ s}\right]^2$$

$$\boxed{y = 11.2 \text{ meters}}$$

Part (c):

Okay so to get the position, we need to know the displacement in the *horizontal* direction. We have **constant velocity:**

$$x = x_0 + v_{x0}t$$

$$x = 0 + (4.35 \text{ m/s})(2.85 \text{ s})$$

$$x = 0 + (12.40 \text{ m})$$

And the vertical direction corresponds to the height of the hand as seen by the bug: $y = 1.22$ meters.

So the position vector is given by:

$$\boxed{\vec{r} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}}$$

$$\boxed{\vec{r} = (12.40 \text{ m})\hat{\boldsymbol{\imath}} + (1.22 \text{ m})\hat{\boldsymbol{\jmath}}}$$

The velocity components we already know from the previous parts. Note that the downward vertical velocity component is just equal and opposite to the upward component when the ball is first thrown:

$$\boxed{\vec{v} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}}}$$

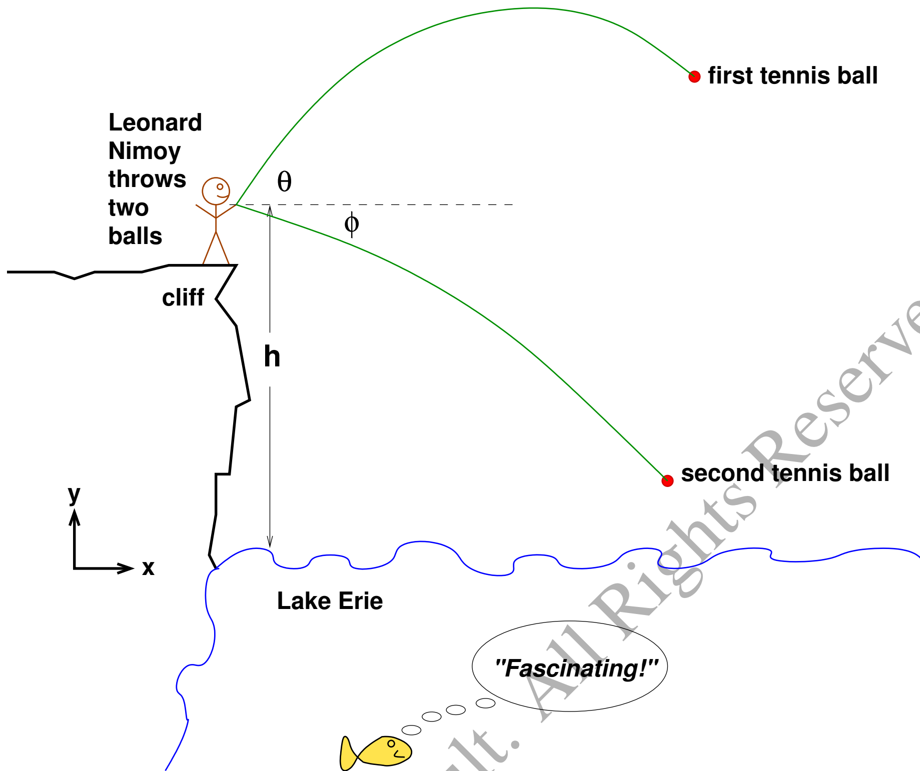
$$\boxed{\vec{v} = (4.35\text{m/s})\hat{\boldsymbol{\imath}} - (14.0\text{m/s})\hat{\boldsymbol{\jmath}}}$$

Finally, we have free-fall kinematics, so we know the acceleration automatically:

$$\boxed{\vec{a} = -g\hat{\boldsymbol{\jmath}}}$$

$$\boxed{\vec{a} = -(9.81\text{m/s}^2)\hat{\boldsymbol{\jmath}}}$$

Problem 2: (from a previous exam)



Leonard Nimoy stands at the edge of a cliff at a given elevation h above the still waters of Lake Erie as shown in the figure. He throws a tennis ball at a given speed v_0 at a given angle of θ above the horizontal direction towards the Lake. Then he throws a second tennis ball at the *same* speed v_0 but downwards at an given angle ϕ below the horizontal. Here assume $\theta > \phi$.

Part (a) – : What is the maximum vertical distance relative to the surface of the Lake that the first tennis ball will achieve? Give you answer in terms of the given parameters (not time). Explain how you got you answer in one or two sentences.

Part (b) – : Write down three *vector expressions* – one for the *position*, one for the *velocity* and one for the *acceleration* – corresponding to the motion of the first tennis ball just at the instant it achieves maximum vertical distance relative to the surface of the Lake. Your answer should be expressed in terms of given parameters only; note that if you have an expression that is a function of time t you are not done. Your expression should include relevant **Cartesian basis unit-vectors.** Explain how you got you answer in one or two sentences.

Part (c) – : The first tennis ball makes a *parabolic curved path* as it travels through the air. Calculate the *radius of curvature* on that path at the single point corresponding to when the tennis ball has maximum vertical distance relative to the surface of the Lake. Your answer should be expressed in terms of given parameters only; note that if you have an expression of time t you are not done. Explain how you got your answer in one or two sentences.

Part (d) – : Which ball (the first ball or the second ball) will have a greater *speed* in the instant just prior to impact with the Lake? Explain your answer in one or two sentences.

Solution to Problem 2:

Part (a):

This is a problem of **Projectile Motion Kinematics**. The *horizontal component* of the motion (along the x-direction) corresponds to to **Constant Velocity:**

$$a_x = 0$$
$$v_x = v_{x0} \quad \text{(a constant)}$$
$$x = x_0 + v_{x0}t$$

The *vertical* component of the motion (along the y-direction) corresponds to **Constant Acceleration due to Free Fall:**

$$a_y = -g$$
$$v_y = v_{y0} - gt$$
$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

The initial conditions for the first tennis ball correspond to those for standard launched projectiles inferred from the figure:

$$x_0 = 0$$
$$y_0 = h$$
$$v_{x0} = v_0 \cos\theta$$
$$v_{y0} = v_0 \sin\theta$$

To get the height above the Lake, we need only consider the **vertical motion**. We know that the y-velocity will be zero at the instant when the tennis ball arrives to its maximum height: We therefore use this fact to find the *time* at this instant and then use this to get the *y-position*:

$$v_y = v_{y0} - gt$$
$$0 = v_{y0} - gt$$
$$t = \frac{v_{y0}}{g}$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$
$$y = h + v_{y0}\left(\frac{v_{y0}}{g}\right) - \frac{1}{2}g\left(\frac{v_{y0}}{g}\right)^2$$
$$y = h + \frac{1}{2}\left(\frac{v_{y0}^2}{g}\right)$$

$$y = h + \frac{1}{2}\left[\frac{(v_0 \sin\theta)^2}{g}\right]$$

$$\boxed{y = h + \frac{v_0^2 \sin^2\theta}{2g}}$$

Part (b):

At maximum vertical distance, the **y-velocity is zero**, so the first tennis ball only retains the (constant) x-velocity it originally had when it was launched:

$$\vec{v} = v_{x0}\hat{\boldsymbol{\imath}} + 0\,\hat{\boldsymbol{\jmath}}$$

$$\boxed{\vec{v} = v_0 \cos\theta\,\hat{\boldsymbol{\imath}}}$$

And since we are in **free fall** with only constant vertical acceleration, we know that:

$$\vec{a} = 0\hat{\boldsymbol{\imath}} - g\,\hat{\boldsymbol{\jmath}}$$

$$\boxed{\vec{a} = -g\,\hat{\boldsymbol{\jmath}}}$$

Part (c):

On any curved path, the acceleration can be decomposed into **centripetal** and **tangential** components. At the position of maximum distance above the Lake, the first tennis ball has constant horizontal velocity, and therefore the tangential component of acceleration is zero. This means that the acceleration is purely centripetal and we can use the expression for the value of the centripetal acceleration in terms of the radius of curvature:

$$a_c = \frac{v^2}{R_c}$$

Here $a_c = -a_y = g$ and $v = v_{x0} = v_o \cos\theta$ and so:

$$g = \frac{v_0^2 \cos^2\theta}{R_c}$$

$$\boxed{R_c = \frac{v_0^2 \cos^2\theta}{g}}$$

Solution continues next page....

Part (d):

Although we have used Kinematics for parts (a), (b), and (c), using **Conservation of Energy** gives us the quickest answer here. Since the only force on each tennis ball is Weight (which is a **Conservative Force**) we have *met the condition for Conservation of Energy*. The "Before" corresponds to leaving Nimoy's hands, the "After" corresponds to just prior to impact with the Lake:

$$E_{tot} = E_{tot}{}'$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

We note that for **both** tennis balls, the following are true:

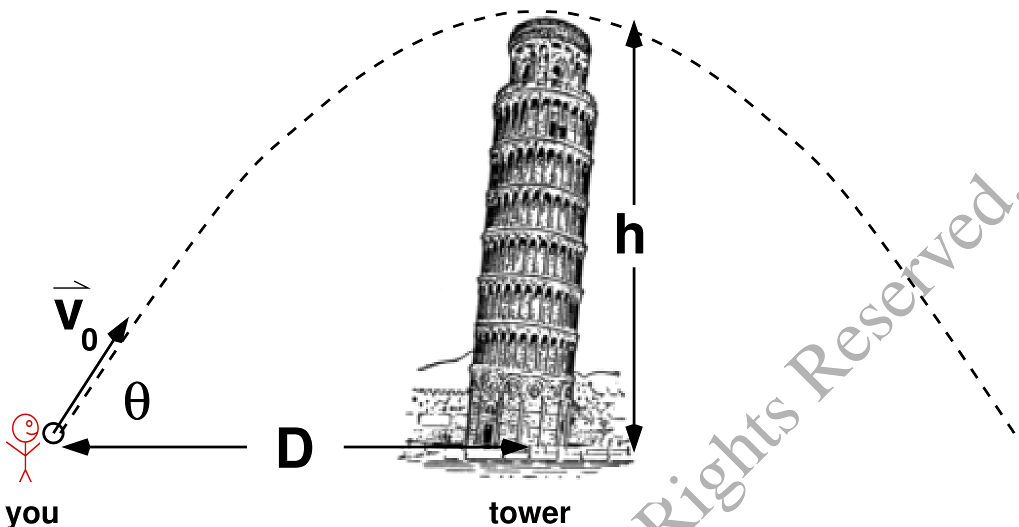
$$y = h$$
$$y' = 0$$
$$v = v_0$$

$$mgh + \frac{1}{2}mv_0^2 = 0 + \frac{1}{2}mv'^2$$
$$v'^2 = 2gh + v_0^2$$
$$v' = \sqrt{2gh + v_0^2}$$

Each tennis ball starts with the same speed v_0 and the same height h. So they end with the same final speed v'. In other words, the final speed of each ball depends on neither m nor the angles θ and ϕ. Only the launch speed and the falling distance matter, and these are the same for each of the two tennis balls. Therefore:

$\boxed{\text{The two balls will hit the Lake with equal speeds.}}$

Problem 3: Projectile Motion Kinematics (from 2013 Final Exam)



You are standing a given horizontal distance D from the base of the Leaning Tower of Pisa which has a given height h. Assume that your height and the diameter of the tower and the deflection of the tower are all negligible compared to the height of the tower. Neglect air resistance.

At what speed v_0 and at what angle θ relative to the horizontal should you throw the ball so that the ball just barely clears the top of the tower at its maximum height, as shown? Give your answers in terms of given parameters only. Explain your work.

Important Hints: See if you can come up with an expression for the time that the ball takes to get to the top, and then see if you can use this to calculate first v_{y0} and then v_{x0} corresponding to the vertical and horizontal components of the initial velocity vector. Once you have these, work out the magnitude and direction of the initial velocity $\vec{v}_0$.

Solution to Problem 3:

This is a problem of **Projectile Motion Kinematics**. The *horizontal component* of the motion (along the x-direction) corresponds to to **Constant Velocity:**

$$a_x = 0$$
$$v_x = v_{x0} \quad \text{(a constant)}$$
$$x = x_0 + v_{x0}t$$

The *vertical* component of the motion (along the y-direction) corresponds to **Constant Acceleration due to Free Fall:**

$$a_y = -g$$
$$v_y = v_{y0} - gt$$
$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

The initial conditions for the ball correspond to those for standard launched projectiles inferred from the figure:

$$x_0 = 0$$
$$y_0 = 0$$
$$v_{x0} = v_0 \cos\theta$$
$$v_{y0} = v_0 \sin\theta$$

To get the ball up to a height h, we need only consider the **vertical motion**. We know that the y-velocity will be zero at the instant when the tennis ball arrives to its maximum height: We therefore use this fact to find the *time* at this instant::

$$v_y = v_{y0} - gt$$
$$0 = v_{y0} - gt$$
$$t = \frac{v_{y0}}{g}$$

Now we use the position equation and solve for the unknown initial vertical velocity v_{y0}:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$
$$h = 0 + v_{y0}\left(\frac{v_{y0}}{g}\right) - \frac{1}{2}g\left(\frac{v_{y0}}{g}\right)^2$$
$$h = \left(\frac{v_{y0}^2}{g}\right) - \frac{1}{2}\left(\frac{v_{y0}^2}{g}\right)$$

$$h = \frac{1}{2}\left(\frac{v_{y0}^2}{g}\right)$$
$$\frac{1}{2}\left(\frac{v_{y0}^2}{g}\right) = h$$
$$v_{y0}^2 = 2gh$$
$$v_{y0} = \sqrt{2gh}$$

Likewise we use the expression for the horizontal position where $x = D$ and solve for the unknown vertical velocity v_{x0}.

$$x = x_0 + v_{x0}t$$
$$D = 0 + v_{x0}t$$
$$v_{x0}\left(\frac{v_{y0}}{g}\right) = D$$
$$v_{x0} = \left(\frac{Dg}{v_{y0}}\right)$$

To get the total speed we take the square-root of the sum of the squares for each component:

$$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2}$$
$$v_0 = \sqrt{\frac{D^2g^2}{2gh} + 2gh}$$
$$\boxed{v_0 = \sqrt{\frac{D^2g}{2h} + 2gh}}$$

To get the angle, we calculate the inverse tangent of the ratio of velocities:

$$\tan\theta = \frac{v_{y0}}{v_{x0}}$$
$$\tan\theta = \frac{v_{y0}}{\left(\frac{Dg}{v_{y0}}\right)}$$
$$\tan\theta = \frac{v_{y0}^2}{Dg}$$
$$\tan\theta = \frac{2gh}{Dg}$$
$$\tan\theta = \frac{2h}{D}$$
$$\boxed{\theta = \arctan\left(\frac{2h}{D}\right)}$$

Problem 4: A Special Kind of Motion

Suppose a particle has a position which is defined by a **position vector** expression as follows:

$$\vec{r}(t) \equiv R\cos(\omega t)\hat{\boldsymbol{\imath}} + R\sin(\omega t)\hat{\boldsymbol{\jmath}}$$

Prove that this represents **Uniform Circular Motion**. In other words, prove that the following three things are true:

- Prove that this particle travels on a path that is a *circle* by demonstrating that the particle is always found at a distance R from the origin. Hint: Trig identity: $\sin^2(\theta) + \cos^2(\theta) = 1$ for *any* angle θ.
- Prove that this particle has a *constant speed* v, where $v = \omega R$.
- Prove that the acceleration vector always has fixed magnitude v^2/R and points toward the center of the circular path.

Solution to Problem 4 on next page....

SOLUTION TO PROBLEM 4:

Prove that this particle travels on a path that is a *circle* by demonstrating that the particle is always found at a distance R from the origin.

PROOF:

We write down the *vector expression:*

$$\vec{r}(t) \equiv R\cos(\omega t)\hat{\imath} + R\sin(\omega t)\hat{\jmath}$$

Now this is a *position vector*. We can calculate the length of this vector using Pythagorean Theorem:

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2}$$
$$|\vec{r}| = \sqrt{[R\cos(\omega t)]^2 + [R\sin(\omega t)]^2}$$
$$|\vec{r}| = \sqrt{R^2[\cos^2(\omega t) + \sin^2(\omega t)]}$$
$$|\vec{r}| = \sqrt{R^2}$$
$$|\vec{r}| = R$$

This means that the length of the position vector is a constant R. This means that the all points on the path are constant distance R from the origin. This is a circle. $\boxed{QED}$.

Prove that this particle has a *constant speed* where $v = \omega R$.

PROOF:

Let's take the time derivative to get the velocity vector:

$$\vec{v} \equiv \frac{d}{dt}\vec{r}$$
$$\vec{v} = -R\omega\sin(\omega t)\hat{\imath} + R\omega\cos(\omega t)\hat{\jmath}$$

Again, the length of the vector $\vec{v}$:

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$
$$|\vec{v}| = \sqrt{[-R\omega\sin(\omega t)]^2 + [R\omega\cos(\omega t)]^2}$$
$$|\vec{v}| = \sqrt{R^2\omega^2[\sin^2(\omega t) + \cos^2(\omega t)]}$$
$$|\vec{v}| = \sqrt{R^2\omega^2}$$
$$|\vec{v}| = \omega R$$

$\boxed{QED}$.

Prove that the acceleration vector always has magnitude v^2/R and points toward the center of the circular path.

$\boxed{\textbf{PROOF:}}$

Let's write down the acceleration:

$$\vec{a} \equiv \frac{d}{dt}\vec{v}$$
$$\vec{a} = -R\omega^2 \cos(\omega t)\hat{\boldsymbol{\imath}} - R\omega^2 \sin(\omega t)\hat{\boldsymbol{\jmath}}$$

By inspection we see that this vector is precisely the same as the position vector multiplied by a the scalar term $-\omega^2$. In other words while the position vector points away from the origin, the acceleration vector points in the *opposite* direction toward the origin.

Again, the length of the vector $\vec{a}$ is found the same way:

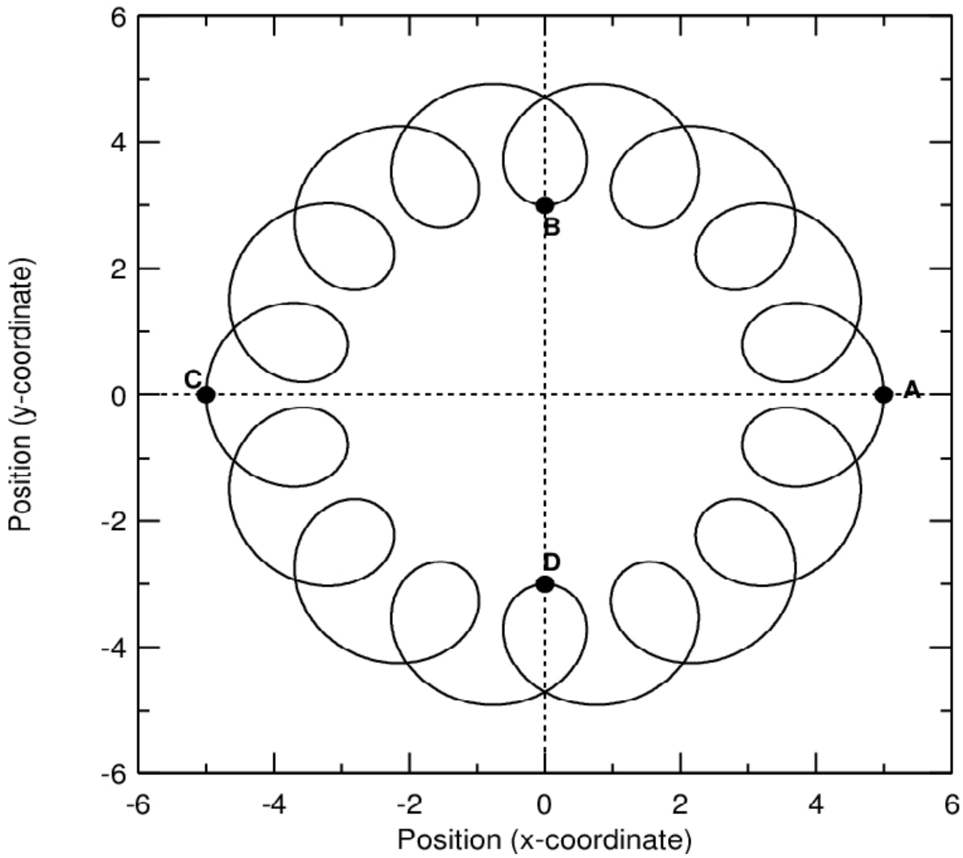
$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$
$$|\vec{a}| = \sqrt{[-R\omega^2 \cos(\omega t)]^2 + [-R\omega^2 \sin(\omega t)]^2}$$
$$|\vec{a}| = \sqrt{R^2\omega^4[\cos^2(\omega t) + \sin^2(\omega t)]}$$
$$|\vec{a}| = \sqrt{R^2\omega^4}$$
$$|\vec{a}| = \omega^2 R$$

Using result from previous proof: $v = \omega R$ which tells us $\omega = \frac{v}{R}$:

$$|\vec{a}| = \frac{v^2}{R}$$

$\boxed{\textit{QED}}$. (Quite Easily Done).

Problem 5: More Fun with 2-D Kinematics



A particle moves on a 'spirograph' pattern as shown above which is defined by the following parametrization:

$$\vec{r} = [P\cos(\Omega t) + Q\cos(\omega t)]\hat{\boldsymbol{\imath}} + [P\sin(\Omega t) + Q\sin(\omega t)]\hat{\boldsymbol{\jmath}}$$

where $P = 4$ meters, $Q = 1$ meter, $\Omega = 2\pi$ radians per second, and $\omega = 30\pi$ radians per second. The plot shown corresponds to the path of the particle for $0 < t < 1$ second. Note that Point **A** as shown corresponds to the position at both $t = 0$ and $t = 1$ second.

a) What is value of the time t corresponding to the position of the particle at point **B** as shown above? Explain or justify your answer. Hint: you cannot "solve for time" here. A better strategy is to ***guess*** the answer and then ***prove*** your guess is correct by calculating the position at that time.

b) What is the *speed* of the particle at point **B** above? Explain your answer.

c) Write down a vector expression for the *total acceleration* of the particle at at point **B**. Explain your answer.

d) Use your answers to parts (b) and (c) above to calculate the **radius of curvature** of the path at point **B**. Also try to estimate the radius of curvature directly from the graph. Do you

SOLUTION TO PROBLEM 5:

Part (a): The easiest way to find the time t corresponding to point **B** above is to make an educated guess and then prove that our guess is correct.

To guess, we note that the time required to get around the whole path is **one second**. The point **B** corresponds to approximately one-fourth the way around, at least in terms of angle. So we $t = 0.25$ **seconds as our guess**. We plug this time into the expressions for the position:

$$\vec{r} = [P\cos(\Omega t) + Q\cos(\omega t)]\hat{\boldsymbol{\imath}} + [P\sin(\Omega t) + Q\sin(\omega t)]\hat{\boldsymbol{\jmath}}$$

Plugging in all of the numbers:

$$\vec{r} = \left\{(4\text{ m})\cos\left[(2\pi\text{ s}^{-1})\left(\frac{1}{4}\text{ s}\right)\right] + (1\text{ m})\cos\left[(30\pi\text{ s}^{-1})\left(\frac{1}{4}\text{ s}\right)\right]\right\}\hat{\boldsymbol{\imath}}$$
$$+\left\{(4\text{ m})\sin\left[(2\pi\text{ s}^{-1})\left(\frac{1}{4}\text{ s}\right)\right] + (1\text{ m})\sin\left[(30\pi\text{ s}^{-1})\left(\frac{1}{4}\text{ s}\right)\right]\right\}\hat{\boldsymbol{\jmath}}$$

Simplifying expressions inside the sine and cosine functions:

$$\vec{r} = \left[(4\text{ m})\cos\left(\frac{\pi}{2}\right) + (1\text{ m})\cos\left(\frac{15\pi}{2}\right)\right]\hat{\boldsymbol{\imath}} + \left[(4\text{ m})\sin\left(\frac{\pi}{2}\right) + (1\text{ m})\sin\left(\frac{15\pi}{2}\right)\right]\hat{\boldsymbol{\jmath}}$$

We note that $\frac{15\pi}{2} = 6\pi + \frac{3\pi}{2}$ so we can re-write:

$$\vec{r} = \left[(4\text{ m})\cos\left(\frac{\pi}{2}\right) + (1\text{ m})\cos\left(\frac{3\pi}{2}\right)\right]\hat{\boldsymbol{\imath}} + \left[(4\text{ m})\sin\left(\frac{\pi}{2}\right) + (1\text{ m})\sin\left(\frac{3\pi}{2}\right)\right]\hat{\boldsymbol{\jmath}}$$

The Cosine terms are zero. The Sine terms are just +1 and -1....

$$\vec{r} = [(4\text{ m})(0) + (1\text{ m})(0)]\,\hat{\boldsymbol{\imath}} + [(4\text{ m})(1) + (1\text{ m})(-1)]\,\hat{\boldsymbol{\jmath}}$$

$$\boxed{\vec{r} = (0\text{ m})\hat{\boldsymbol{\imath}} + (3\text{ m})\hat{\boldsymbol{\jmath}}}$$

This is indeed the position vector corresponding to Point **B** and therefore we have proved that this point corresponds to $\boxed{t = \frac{1}{4}\text{ seconds}}$.

Part (b): We take the time derivative:

$$\vec{v} = \frac{d}{dt}\vec{r}$$

$$\vec{v} = \frac{d}{dt}[P\cos(\Omega t) + Q\cos(\omega t)]\hat{\boldsymbol{\imath}} + [P\sin(\Omega t) + Q\sin(\omega t)]\hat{\boldsymbol{\jmath}}$$

$$\boxed{\vec{v} = [-P\Omega\sin(\Omega t) - Q\omega\sin(\omega t)]\hat{\boldsymbol{\imath}} + [P\Omega\cos(\Omega t) + Q\omega\cos(\omega t)]\hat{\boldsymbol{\jmath}}}$$

Again we can evaluate this expression by plugging in numbers. We note from the last time that the cosine terms are all zero and the sine terms are either +1 or -1 accordingly:

$$\vec{v} = [-P\Omega(1) - Q\omega(-1)]\hat{\boldsymbol{\imath}} + (0\text{ m/s})\hat{\boldsymbol{\jmath}}$$
$$\vec{v} = [-P\Omega + Q\omega]\hat{\boldsymbol{\imath}} + (0\text{ m/s})\hat{\boldsymbol{\jmath}}$$
$$\vec{v} = [-(4\text{ m})(2\pi\text{ s}^{-1}) + (1\text{ m})(30\pi\text{ s}^{-1})]\hat{\boldsymbol{\imath}} + (0\text{ m/s})\hat{\boldsymbol{\jmath}}$$
$$\vec{v} = [(-8\pi\text{ m/s}) + (30\pi\text{ m/s})]\hat{\boldsymbol{\imath}} + (0\text{ m/s})\hat{\boldsymbol{\jmath}}$$
$$\vec{v} = (22\pi\text{ m/s})\hat{\boldsymbol{\imath}} + (0\text{ m/s})\hat{\boldsymbol{\jmath}}$$

Note that this vector points in the positive-x direction as we expect by looking at the graph.

The speed is just the magnitude of this vector:

$$\boxed{v = 22\pi\text{ m/s} = 69.12\text{ m/s}}$$

Part (c): We take the time derivative again: This pulls out another power in each omega and the sines and cosines change again. Watch the signs...

$$\vec{a} = \frac{d}{dt}\vec{v}$$

$$\vec{a} = \frac{d}{dt}[-P\Omega\sin(\Omega t) - Q\omega\sin(\omega t)]\hat{\boldsymbol{\imath}} + [P\Omega\cos(\Omega t) + Q\omega\cos(\omega t)]\hat{\boldsymbol{\jmath}}$$

$$\boxed{\vec{a} = [-P\Omega^2\cos(\Omega t) - Q\omega^2\cos(\omega t)]\hat{\boldsymbol{\imath}} + [-P\Omega^2\sin(\Omega t) - Q\omega^2\sin(\omega t)]\hat{\boldsymbol{\jmath}}}$$

Again we can evaluate this expression. The horizontal term is zero this time:

$$\vec{a} = (0 \text{ m/s}^2)\hat{\boldsymbol{\imath}} + [-P\Omega^2(1) - Q\omega^2(-1)]\hat{\boldsymbol{\jmath}}$$
$$\vec{a} = (0 \text{ m/s}^2)\hat{\boldsymbol{\imath}} + [-P\Omega^2 + Q\omega^2)]\hat{\boldsymbol{\jmath}}$$
$$\vec{a} = (0 \text{ m/s}^2)\hat{\boldsymbol{\imath}} + [-(4 \text{ m})(2\pi \text{ s}^{-1})^2 + (1 \text{ m})(30\pi \text{ s}^{-1})^2]\hat{\boldsymbol{\jmath}}$$
$$\vec{a} = (0 \text{ m/s}^2)\hat{\boldsymbol{\imath}} + [(-16\pi^2 \text{ m/s}^2) + (900\pi^2 \text{ m/s}^2)]\hat{\boldsymbol{\jmath}}$$
$$\boxed{\vec{a} = (0 \text{ m/s}^2)\hat{\boldsymbol{\imath}} + (884\pi^2 \text{ m/s}^2)\hat{\boldsymbol{\jmath}}}$$

In decimal representation: $884\pi^2$ m/s^2 $= 8700$ m/s^2 to two significant digits or roughly 900 Gees of acceleration. That little particle really whips around!

Part (d): We note that at **Point B** the Cartesian coordinate system and the "velocity" coordinate system are lined up with each other. The direction corresponding to "positive-y" also corresponds to the direction of centripetal acceleration. In this case since the horizontal term is zero, there is no tangential acceleration. So we just set the centripetal acceleration equal to the total acceleration in the y-direction and solve for the radius of curvature:

$$a_{\hat{\boldsymbol{c}}} = a_y$$
$$\frac{v^2}{R_c} = a_y$$
$$\boxed{R_c = \frac{v^2}{a_y}}$$

Now we plug in the results from Parts (b) and (c)

$$R_c = \frac{(22\pi \text{ m/s})^2}{884\pi^2 \text{ m/s}^2}$$
$$R_c = \frac{(484\pi^2 \text{ m/s})^2}{884\pi^2 \text{ m/s}^2}$$
$$\boxed{R_c = \frac{121}{221} \text{ m} = 0.548 \text{ meters}}$$

Happily, this is consistent with what we see on the graph.