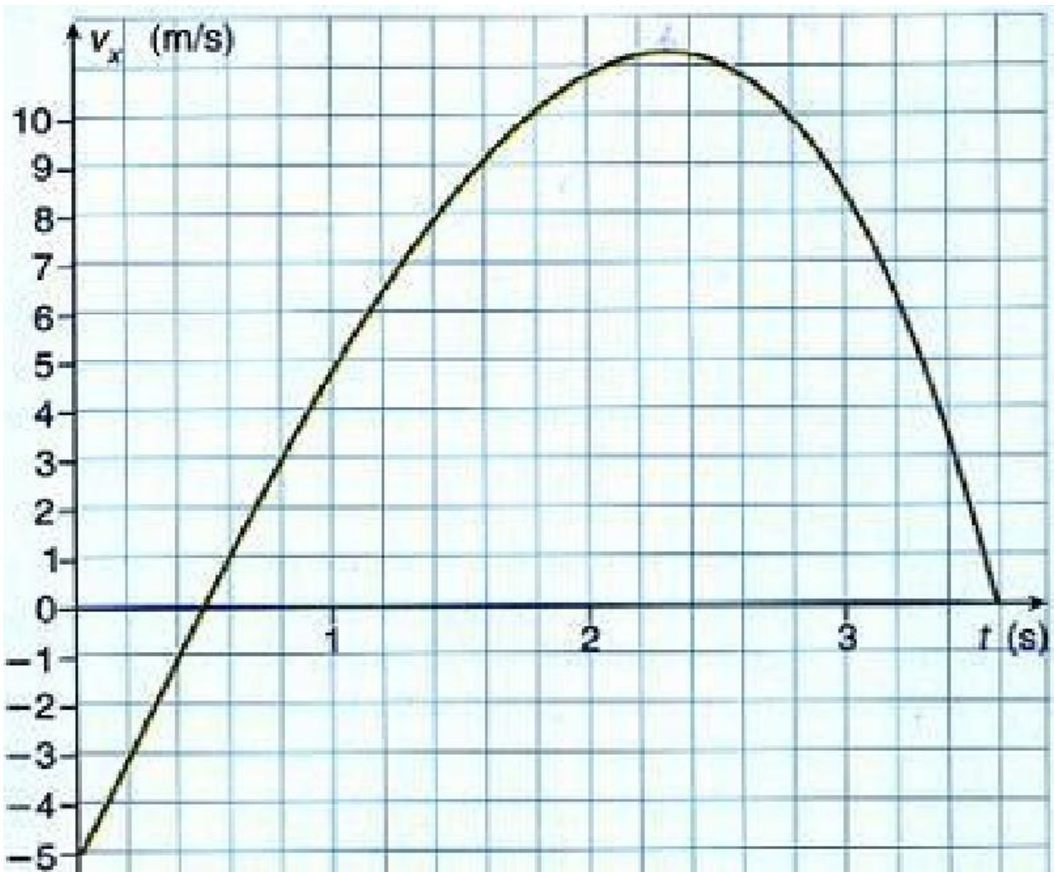
PHYS 121: Solutions to Written Homework #01
January 29, 2024

Note: this homework is worth **15 points** as follows:

Problem 1	5 points	One Point Per Part, do not grade part (g).
Problem 2	0 points	Do Not Grade.
Problem 3	6 points	3 Points Per Part.
Problem 4	0 points	Do Not Grade
Problem 5	0 points	Do Not Grade
Problem 6	4 points	Grade Parts (a,c,d and f)
Problem 7	0 points	Do Not Grade

Problem 1. The 1-D velocity of a particle is graphed in the figure below as a function of time. Answer each of these questions. **Explain carefully and completely** your answer for each part. Important: You are **obligated to explain** how and when you use detailed information from the graph. For example, if you you use the slope of the graph at some point, you need to explain which point(s) you are measuring the slope at and you need to show your **calculations** for determining the value of the slope.



Important: *You must* **not** *assume any specific analytical functional form for this graph.* Maybe this curve sort of looks like a parabola perhaps. Maybe not. But you cannot rely on this being true to solve this problem. This means that you must use graphical – and not analytical – methods to solve this problem.

Problem 1 continues....

Please answer **all** of the following questions regarding the graph shown on the previous page. **Important! You must explain your answers with a sentence or two telling the grader how and why you got the answer you have. Also, if you use information from the graph to answer the question, you must show a sketch or a marked copy of the graph that indicates how you used the graph. For example, if you calculate a slope, you need to show us a sketch that indicates how you measured (ideally with a ruler) the quantities that go into your calculation for the slope.**

- **Part (a):** Three students make statements:

  Student 1: *"At time $t = 0$ we know the particle has zero velocity because particles must start at rest."*

  Student 2: *"No, you are wrong. In fact at time $t = 0$ we know the particle is already moving in the negative direction. In fact the particle is slowing down."*

  Student 3: *"No, you are both wrong. At time $t = 0$ the particle is speeding up. We know this because the slope is positive."*

  Only one of these three students is correct. Which student is correct and why? Explain your answer.

- **Part (b):** Is the acceleration of the particle constant during this the entire interval shown here? How do you know this? Explain your answer.

- **Part (c):** Consider the following equation:

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

  In principle, can we apply this equation to describe the motion of the particle during this entire interval? Yes or No? How do you know this? Explain your answer.

- **Part (d):** Are there one or more instants in time that corresponds to **zero acceleration** of the particle? If so, indicate the time(s) (in seconds) and explain: How do you know that the indicates time(s) corresponds to zero acceleration? Very important: Explain your answer.

- **Part (e):** Is there any single instant in time that corresponds to the **minimum position** of the particle? If so, what time is this and how do you know that this is the minimum position? Explain your answer.

- **Part (f):** Estimate the acceleration of the particle at the instant corresponding to $t = 1.0$ seconds. Important: You **must** explicitly show your numerical calculations in a complete way and you must explicitly reference the graph to explain **how in terms of some graphical method** you used the graph to calculate this. Be careful with the units!

- **Part (g):** Harder and more time consuming: Estimate the *final position* of the particle $x(t)$ at time $t = 3.6$ seconds relative to the initial position $x = 0$ at $t = 0$ seconds. There are several approximately correct graphical ways to do this. As long as you are consistent, any method that gives reasonable accurate results is acceptable. Again, you must use words and **use the graph** to explain **how in terms of some graphical method** you you calculate this. Clearly demonstrate to the grader how you obtained your answer. Be careful with the units!

(Homework assignment continues next page...)

SOLUTION TO PROBLEM 1:

Part (a): Let's look at these statements one at a time:

> Student 1: *"At time $t = 0$ we know the particle has zero velocity because particles must start at rest."*

This just is **not correct**. Indeed, as we can see from the plot, at time $t = 0$ corresponding to the left edge of the plot, the value of v is about -5 m/s. So not at rest.

> Student 2: *"No, you are wrong. In fact at time $t = 0$ we know the particle is already moving in the negative direction. In fact the particle is slowing down."*

As we just said, indeed the particle is moving in the negative direction. It is also true that the ***speed*** of the particle (the absolute value of the velocity) is decreasing toward zero. So, yes, the particle is "slowing down". This statement looks to be **correct**.

> Student 3: *"No, you are both wrong. At time $t = 0$ the particle is speeding up. We know this because the slope is positive."*

The slope is positive, which means that ***velocity*** is increasing. But when we use words like "slowing down" and "speeding up" these usually apply to the speed, not the velocity. So this statement is **not (quite) correct**.

So in summary **Only Student 2 is Correct.**

Part (b): The **definition of acceleration** is given by:

$$a(t) \equiv \frac{dv}{dt}$$

This means the acceleration, which is the time-derivative of the velocity corresponds to the *slope* of the curve of $v(t)$ vs. t. If the acceleration is constant we would expect the plot of velocity vs. time to be ***linear*** as a function of time. However, in this plot, we can see that the graph is *curved everywhere* which means that the slope is always changing and therefore The acceleration is not constant for any finite interval shown on the plot. [1]

Part (c): The equation:

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

is one of the three equations that corresponds to **constant acceleration**. As we have already concluded in Part (b), the acceleration is **not** constant.

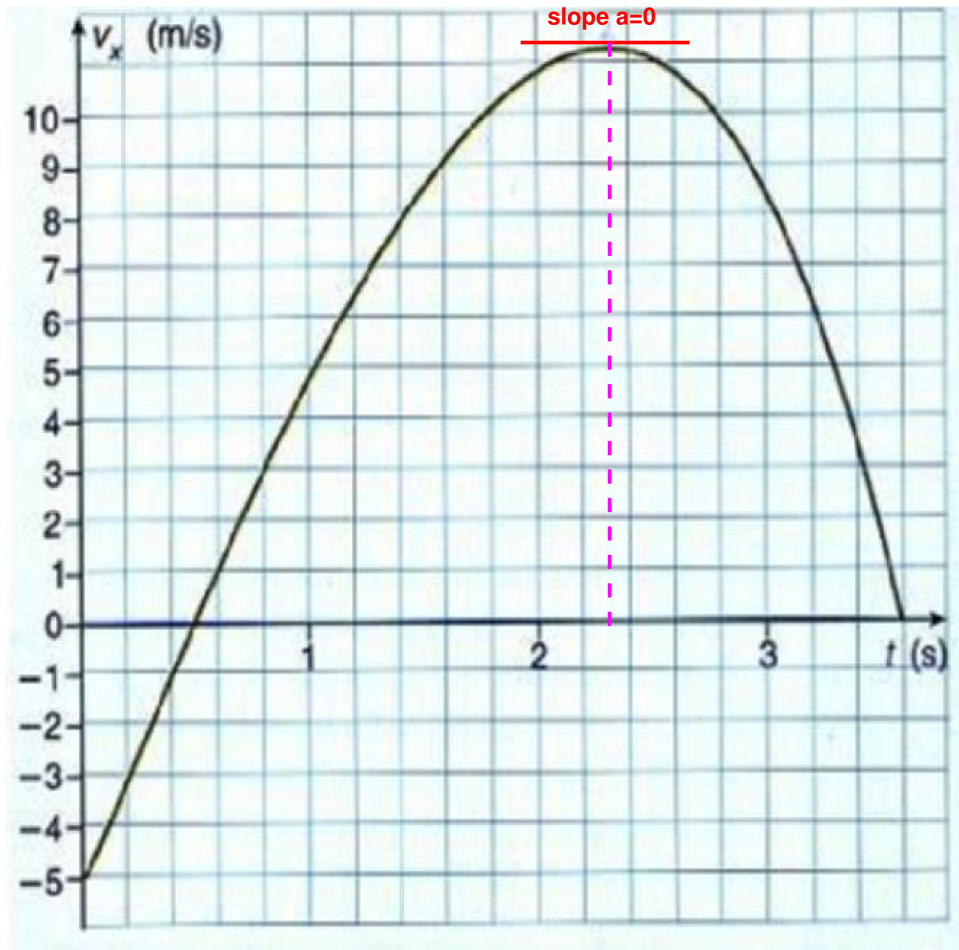
Therefore, even in principle, **this equation does not apply here.**

Solution to Problem 1 continues next page....

[1] In principle, of course, we could zoom in on some small interval of time Δt and during that interval, the plot of $v(t)$ vs. t will be *approximately* linear, but this doesn't change the fact that 'constant acceleration' does not describe the motion of the particle during the whole time interval corresponding to the plot.

Solution to Problem 1 continues....

Part (d): As we indicated in Part (a), the acceleration is the derivative of the velocity corresponding to the **instantaneous slope** of the curve of $v(t)$ vs. t. So we know that the acceleration is zero at the point on the curve corresponding to **zero slope**. This means we expect the tangent line to be **flat, horizontal**. As we can see, this happens at one point on the curve:

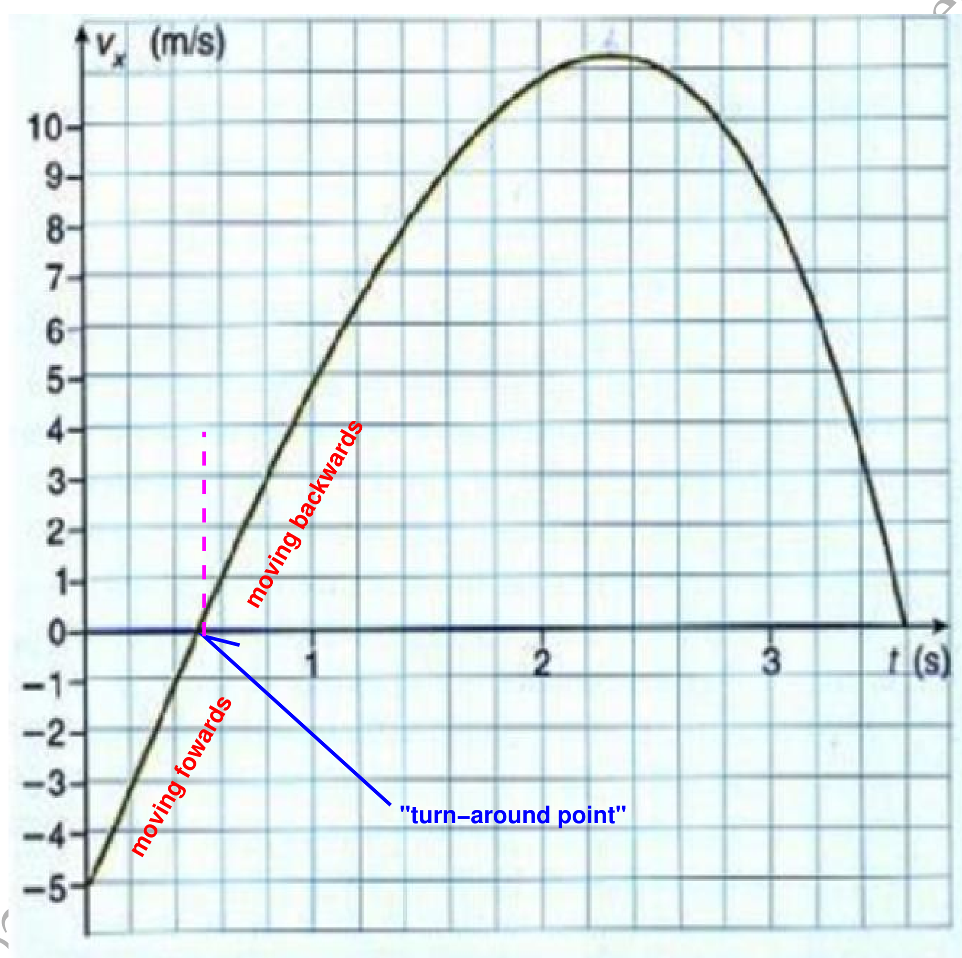


This corresponds to the point of maximum velocity where the slope is zero. As we can see from the plot this happens at a time of approximately $\boxed{t = 2.3 \text{ seconds.}}$

Solution to Problem 1 continues next page....

Solution to Problem 1 continues....

Part (e): By definition, the "minimum position" corresponds to the minimum x-value. Note that we are already moving in the *negative* direction at time $t = 0$ and even as our speed is decreasing, we continue to move in the negative direction until the curve crossed the x-axis corresponding to $v = 0$ where the velocity changes from *negative* to positive. For the remainder of the trip, the particle retains a positive velocity, which means that the position continues to increase from this point.

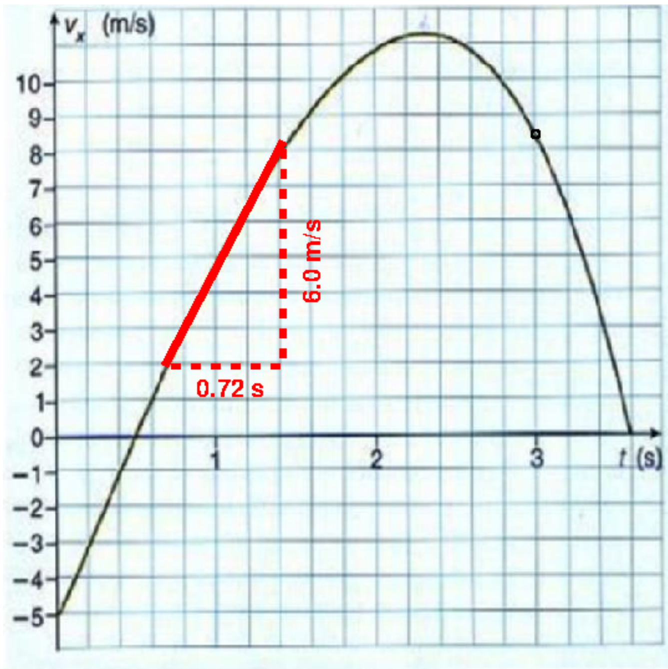


Thus the point of minimum position corresponds to the point of zero velocity. As we can see from the plot this happens at a time of approximately $\boxed{t = 0.5 \text{ seconds.}}$

Solution to Problem 1 continues next page....

Solution to Problem 1 continues....

Part (f): We can determine the instantaneous acceleration by measuring the slope of the curve of $v(t)$ vs. t at $t = 1.0$ seconds.



This is shown above. We draw a tangent line and then estimate the lengths of the triangle to get the slope:

$$a(t = 1.0s) = \text{slope} = \frac{\Delta v}{\Delta t}$$

$$a(t = 1.0s) = \frac{6.0 \text{ m/s}}{0.72 \text{ s}}$$

$$\boxed{a(t = 1.0s) \approx +8.3 \text{ m/s}^2}$$

Note the positive sign indicating a upward slope. Note that these numbers do not just appear out of "thin air". You can't just stare at the plot and guess the slope. You need to show your work. You need to to draw a triangle or something like to this to graphically infer the slope. Note also that you must use the **units** correctly here. Each 'box' tick in the vertical direction corresponds to 1 m/s velocity, but the ticks in the horizontal direction are only 0.2 seconds in time.

Solution to Problem 1 continues next page....

Solution to Problem 1 continues....

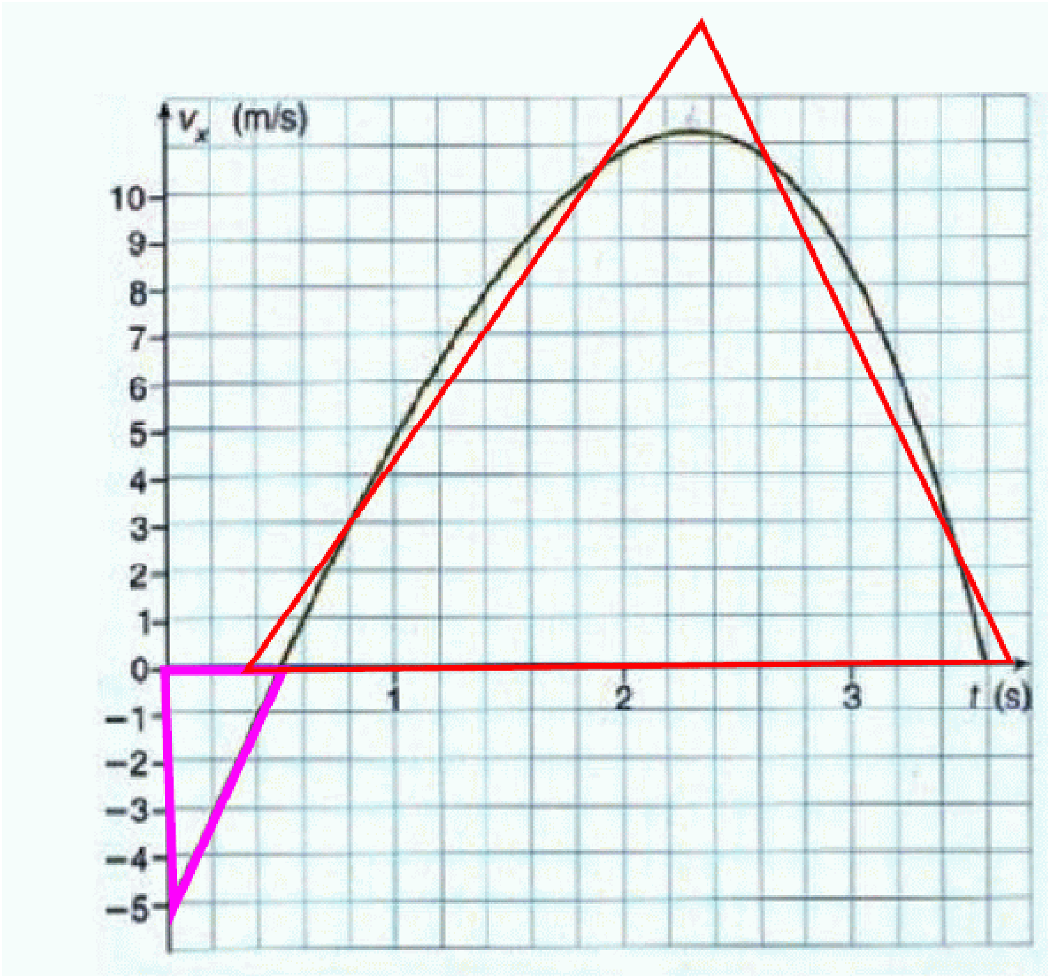
Part (g): We know that we can relate position to velocity by taking the "anti-derivative" as follows:

$$x(t) = x_0 + \int_0^t v(t')dt'$$

In this particular problem we see that $x(t) = 0$ for $t = 0$ so this tells us immediate that $x_0 = 0$. So all we need to do is graphically integrate the function from $t = 0$ to $t = 3.6$ seconds. I can thinks of two ways to do this. First, to integrate means to find the total area "under the curve".

Any time we are "graphically integrating" like this there is always the question of how accurate an estimate we are looking for. As a first "order or magnitude estimate" we can approximate each of the two regions as triangles, one with "negative" area (magenta) and one with "positive" area (red) as shown:

Solution to Problem 1 continues next page....



Here we estimate the integral as the sum of the area of each triangles keeping track of the signs:

$$x(t = 3.6 \text{ s}) = -(\text{magenta triangle area}) + (\text{red triangle area})$$

$$x(t = 3.6 \text{ s}) = -\frac{1}{2} b_{magenta} h_{magenta} + \frac{1}{2} b_{red} h_{red}$$

$$x(t = 3.6 \text{ s}) = -\frac{1}{2}(0.5 \text{ s})(5.5 \text{ m/s}) + \frac{1}{2}(3.4 \text{ s})(14 \text{ m/s})$$

$$x(t = 3.6 \text{ s}) = -1.4 \text{ m} + 23.8 \text{ m}$$

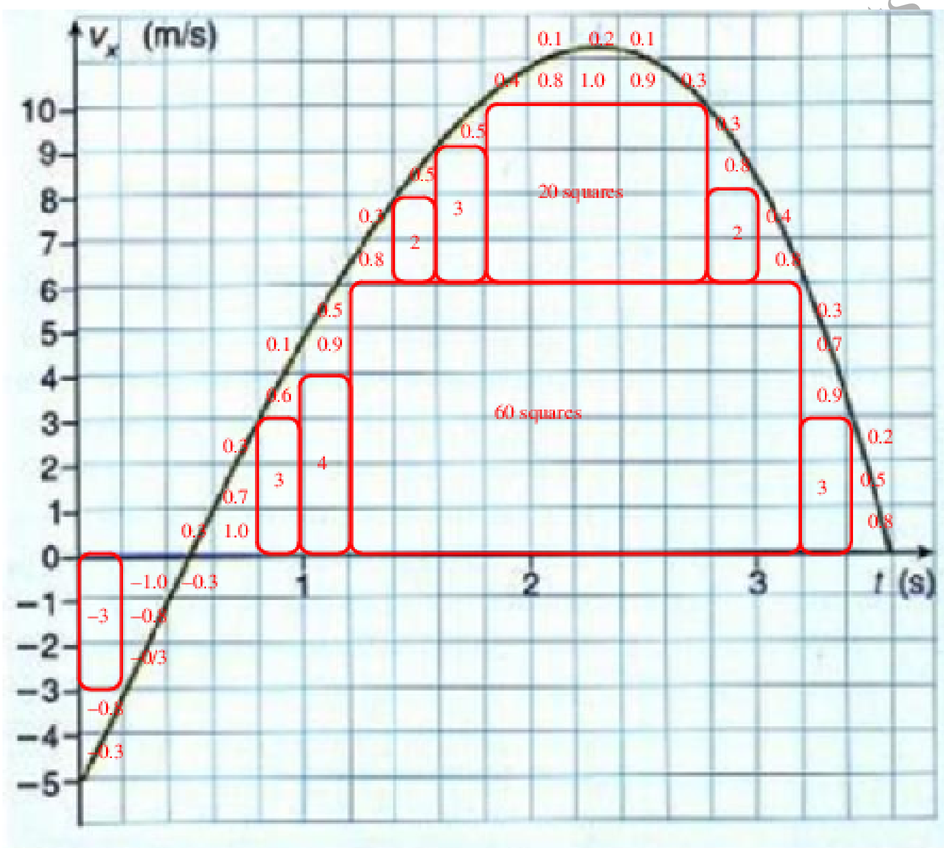
$$x(t = 3.6 \text{ s}) = 22 \text{ m}$$

Because we are using triangles as pretty rough approximations for the shape of the function, we probably do not expect this result to be better than one or two digits of precision.

Solution to Problem 1 continues next page....

Solution to Problem 1 continues ...

A more accurate approach would be to take advantage of the graph paper here and “count squares” to get the area, keeping in mind that squares that are below the zero line count as negative. The following figure shows my “method” for counting squares and fractional squares:



Here’s my table of totaled values. I basically start adding from top down left-to-right....

Counted Squares	Running Total
-3.0	-3
-0.8	-3.8
-0.3	-4.1
-1.0	-5.1
-0.8	-5.9
-0.3	-6.2
+0.3	-5.9
-0.3	-6.2
+0.3	-5.9
+0.7	-5.2
+1.0	-4.2
+0.1	-4.1
+0.6	-3.5
+3.0	-0.5
+0.5	0
+0.9	0.9
+4.0	4.9
+0.3	5.2
+0.8	6
+60.0	66
+0.5	66.5
+2.0	68.5
+0.5	69
+3.0	72
+0.4	72.4
+20.0	92.4
+0.1	92.5
+0.8	93.3
+0.2	93.5
+1.0	94.5
+0.1	94.6
+0.9	95.5
+0.3	95.8
+0.3	96.1
+0.8	96.9
+2.0	98.9
+0.4	99.3
+0.8	100.1
+0.3	100.4
+0.7	101.1
+0.9	102
+3.0	105
+0.2	105.2
+0.5	105.7
+0.8	**106.5**

Solution to Problem 1 continues next page...

Solution to Problem 1 continues ...

So this give an integral of 106.5 squares with each square corresponding to an area of **0.2 seconds** times **1.0 meters/second** = 0.2 meters. So the total distance traveled is given by:

$$(106.5 \text{ squares}) \left(\frac{0.2 \text{ meters}}{\text{square}} \right) = \boxed{21.3 \text{ meters}}$$

Based our approach I would guess that this answer is good to at least two digits of precision, but probably not three digits.

This is pretty close to our first much rougher approximation. If accuracy were very important, we could use standard methods of numerical integration such as the *trapezoidal rule* which would be probably more accurate and perhaps a little more tedious than our "counting the squares" calculation.

Problem 2.

A heavy package of given mass M is released at rest to fall straight down from a hovering helicopter located at a given height H meters from the ground.

- **Part (a): –** How much time t will pass from when the package is released to when it hits the ground? Important: use the equations of Free-Fall Kinematics to solve this problem *symbolically* in terms of the given parameters. Here the given parameters are M and H. You can also always assume that any physical constants such as g are also given). Assume you an ignore air resistant. Explain what you are doing. Show your work.
  $\boxed{\text{Put a box around your final symbolic answer.}}$

- **Part (b): –** How fast will it be moving at the instant just before it hits? Again, use the equations of Free-Fall Kinematics to solve this problem *symbolically* in terms of the given parameters. Explain your work.
  $\boxed{\text{Put a box around your final symbolic answer.}}$

- **Parts (c): –** Now suppose you are told that the mass of the heavy box is specified as M=47.84 kilograms and the release height of the helicopter is given as $H = 52.8$ meters. Plug these numbers into the symbolic answer you just obtained above to calculate the speed of the box *numerically*. Be sure to take care with significant digits and physical units.
  $\boxed{\text{Please put a box around your numerical answer.}}$

Important! The method outlined here is how we want you to solve every "story problem" in this course. Specifically:

- Use *symbolic variables* (such as M and h to represent given parameters. Always solve in terms of symbolic variables. Do not plug in numbers. Do this even if you are only given numerical values.

- Solve the problem in terms of given symbolic parameters and any known physical constants, such as g. Your answer should be an equation where time is given in terms of the given symbolic parameters only: $t = f(M, h, g)$. Put a $\boxed{\text{box}}$ around your symbolic solution. Note that it is possible that the final solution will not explicitly depend on all of the given parameters.

(Problem 2 continues next page...)

(Problem 2 continues:)

- Once you have solved the problem symbolically, *then and* **only** *then* should you go ahead and plug in numerical values for the given parameters. Be sure to include appropriate units, and present your answer in terms of an appropriate number of significant digits. Put a $\boxed{\text{box}}$ around your final numerical solution (with units).

Very important Reminder: Put a $\boxed{\textbf{box}}$ around your symbolic solutions. Put a $\boxed{\textbf{box}}$ around your final numerical solutions (with appropriate SI units). You must always put a box around both your final symbolic and/or numeric solutions.

(Homework assignment continues next page...)

SOLUTION TO PROBLEM 2:

Part (a): – This is a **kinematics problem** with **Free-fall** motion (Constant Acceleration downward). We define a coordinate system with one Cartesian coordinate y that points *up*. The three equations that govern this motion are:

$$a_y = -g$$
$$v_y = v_{y0} - gt$$
$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

We consider the given parameters corresponding to known initial and final conditions:

$$y_0 = H$$

where H is the given height above ground of the package when it is released.

The final position is also specified, since we have defined the ground as zero position:

$$y = 0$$

Finally, since we have released the box "at rest" we know that it has zero initial velocity:

$$v_{y0} = v_0 = 0$$

We are looking to solve for t, the time of impact. We need to use the third equation to determine t:

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$
$$0 = H + 0 - \frac{1}{2}gt^2$$
$$\frac{1}{2}gt^2 = H$$
$$gt^2 = 2H$$
$$t^2 = \frac{2H}{g}$$
$$\boxed{t = \sqrt{\frac{2H}{g}}}$$

Solutions to Problem 2 continues next page...

Solutions to Problem 2 continues...

Part (b): – To get the speed we work the second equation:

$$v_y = v_{y0} - gt$$

Plugging in our symbolic solution from Part (a):

$$v_y = 0 - g\left(\sqrt{\frac{2H}{g}}\right)$$

$$\boxed{v_y = -\sqrt{2Hg}}$$

Here, the negative sign indicates the box is moving in the negative y-direction when it hits. Since we asked "how fast" the box is moving, this implies we are asking for the speed, which is the defined as the absolute value of the velocity:

$$\boxed{\text{speed} = |v_y| = \sqrt{2Hg}}$$

Either of the above boxed answers are acceptable.[2]

Part (c):

- First (just for fun) let's find the numerical value for time. We are told $H = 52.8$ meters. So we right down our symbolic solution for time and plug in:

$$t = \sqrt{\frac{2H}{g}}$$

$$t = \sqrt{\frac{2(52.8\text{ m})}{9.81\text{ m/s}^2}}$$

$$t = \sqrt{4.6075\text{ s}^2}$$

$$\boxed{t = 2.15\text{ s}}$$

[2] **Important note: It is not correct to place SI units on symbolic answers.** When we give a *symbolic* answer such as $\boxed{v_y = -\sqrt{2Hg}}$, it is ***not correct*** to append units such as "meters/sec" to our answer. This is because the symbols themselves represent not just numbers but values with the "appropriate units" built in. The equation $\boxed{v_y = -\sqrt{2Hg}}$ applies in *any* system of units, SI, Imperial, whatever, as long as we all agree on the system of units and use appropriate values for physical constants in those units. So, no, do **not** put units next to your symbolic answers. They only belong with your numerical answers.

- So likewise plugging in numbers to get the velocity:

$$v_y = -\sqrt{2Hg}$$

$$v_y = -\sqrt{2(52.8\text{ m})(9.81\text{ m/s}^2)}$$

$$v_y = -\sqrt{1035.94\text{ m}^2/\text{s}^2}$$

$$\boxed{v_y = -31.2\text{ m/s}}$$

or equally acceptable is the speed:

$$\boxed{\text{speed} = |v_y| = 31.2\text{ m/s}}$$

Note the the value of the mass of the box does not impact the kinematics (free fall) of the box. In other words, the mass of the box is extraneous information.

Problem 3. José Ramírez swings the bat, but hits a pop-up. The baseball has a vertical upward velocity of $V_0 = 23.7$ meters per second when the ball leaves his bat. Relative to his bat, how high in the air will the baseball travel? How much total time will elapse before the ball is caught by the opposing catcher? Assume the ball is caught at the same vertical location that it was hit. Ignore air resistance.

- As always, **first** solve the problem symbolically. You want expressions for both the height and the time in terms of the given parameters (here just v_0) and and relevant physical constants (here just g). Put a $\boxed{\text{box}}$ around your symbolic solution.
- Now that you have solved the problem symbolically, *then and* **only** *then* should you go ahead and plug in numerical values (with appropriate units) for the given parameters and constants. Put a $\boxed{\text{box}}$ around your numerical solution.

As always, **explain your work.**

Very important Reminder: Put a $\boxed{\text{box}}$ around your symbolic solutions. Put a $\boxed{\text{box}}$ around your final numerical solutions (with appropriate SI units). You must always put a box around both your final symbolic and/or numeric solutions.

(Homework assignment continues next page...)

SOLUTION TO PROBLEM 3:

This is a simple problem in vertical **Free Fall Kinematics**. As soon as the ball leave the bat in the vertical direction, it is subject to the three fundamental kinematic laws of motion with **Constant Acceleration** (here we neglect air resistance):[3]

$$a_y(t) = -g \qquad \text{(a constant!)}$$
$$v_y(t) = v_{y0}t - gt$$
$$y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

Although we are asked for the height of the ball relative to the bat, we can't really determine this until we figure out the ***time*** at the top of the path. At the highest position, we are at the **turn around point** so $v_y(t) = 0$ at this point: For clarity we note that $v_{y0} = V_0$, the initial ball speed, which is a given parameter.

One more clarification of detail. We are asked to find the height **relative to the bat.** Since we are not given a position of the bat in any absolute sense, we are *free to position our coordinate system* so that by definition the starting position (relative to the bat) is zero. In other words $y(t = 0) = t_0 = 0$. So we continue with our **Kinematic Equations for Free Fall**:[4]

- **First, we solve the problem symbolically:**

$$v_y(t) = v_0 - gt$$
$$0 = V_0 - gt$$

Solve for time:

$$t = \frac{V_0}{g}$$

Now we go to the third equation:

$$y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

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[3]Neglecting air resistance in this specific example turns out to be a wildly inaccurate assumption. Anyone who has played baseball knows that the air and wind have significant impact on the path of a baseball moving at speed over any distance. We neglect this here for the purposes of exploring an idealized problem and mastering the method of solving it.

[4]Some students raised the issue that we do not clearly indicate that the position of the bat and the position of the catchers glove are the same. Since we do not spell this out in the problem, it is entirely reasonable to assume that this is approximately true, certainly to within a handful of centimeters which is definitely negligible compared to the total height that the ball will rise. Again, (as in the case of the air resistance) this is a reasonable abstraction for the purposes of doing the problem.

We know that the initial position is given by $y_0 = 0$ (the ball position relative to the bat is by definition zero at time t=0 here). Plugging this zero value in and also plugging in our expression for time:

$$y = 0 + V_0\left(\frac{V_0}{g}\right) - \frac{1}{2}g\left(\frac{V_0}{g}\right)^2$$

$$y = \frac{V_0^2}{g} - \frac{1}{2}\left(\frac{V_0^2}{g}\right)$$

$$\boxed{y = \frac{V_0^2}{2g}}$$

- **Second, we plug in numbers to get our numerical solution:**

$$y = \frac{(23.7\text{ m/s})^2}{2(9.81\text{ m/s}^2)}$$

$$\boxed{y = 28.6\text{ meters}}$$

Finally, we already have the time for to get to the top of the jump. **By symmetry** it takes just as long to go from the bat to the top as it does to go from the top back down into the glove.[5] So the total time of the jump is just twice the jump-to-top time (here given **symbolically**):

$$\boxed{t_{total} = 2t_{top} = \frac{2V_0}{g}}$$

Finally, we plug in numbers to get our **numerical solution:**

$$\boxed{t_{total} == \frac{2(23.7\text{ m/s})}{(9.81\text{ m/s}^2)} = 4.83\text{ seconds}}$$

So we are talking about a pop-up that is skied over 90 feet into the air and returns to the catcher within five seconds. These are plausible sounding numbers.

[5] A perfectly acceptable alternate method is to set the final position to $y = 0$ and solve for time. You will get the same answer with only slightly more algebra.

Problem 4. The dwarf planet Sedna is at a distance $D = 85$ AU (Astronomical Units) from the Earth. You want to fly a space-ship to Sedna. Suppose you have a space-ship that is able to linearly accelerate in either a positive or a negative direction at exactly $A = 0.238$ m/s^2. You plan a trip where you move with constant *positive* acceleration in a straight line toward Sedna, speeding up the whole time, and then, at precisely the half-way point, you turn the ship around and accelerate in the opposite direction *e.g., with negative acceleration* of the same magnitude so that you are now slowing down. **By symmetry**, if you reverse acceleration at the half-way point, you should arrive at your destination with zero velocity (right?). Note: in this problem we are completely ignoring the practical reality of how one might build a spaceship that can sustain constant acceleration for an arbitrary period of time. We are also ignoring the motion of the Earth and of Sedna during the trip.

- **Part (a): –** How long will the trip take? Important: You must given a **symbolic** answer in terms of the given parameters (here a and D corresponding to the acceleration of the spaceship and the distance traveled to Sedna. $\boxed{\text{Box}}$ your symbolic answer. Also give a numerical answer in terms of seconds. Be sure that the answer has an appropriate number of significant digits. $\boxed{\text{Box}}$ your numerical answer. Also convert your answer in seconds to an answer in units that are most appropriate to actual length of the trip (days, years, etc.) Again box your answer. Show your work. Hint: you will need to convert from "AU" to meters.

- **Part (b): –** What is the maximum velocity of the spaceship during the trip? Again, present your answer symbolically in terms of given parameters, $\boxed{\text{box}}$ this, and then plug in to get a numerical answer in terms of appropriate SI units, and then $\boxed{\text{box}}$ this. Be sure to explain your work.

- **Part (c): –** Draw a qualitative but clear **sketch** showing plots of the acceleration, the velocity and the position of the spacecraft as a function of time during the entire trip from Earth To Sedna. For each plot, explain in a sentence or two how you determined the shape of the plot.

(Homework assignment continues next page...)

SOLUTION TO PROBLEM 4:

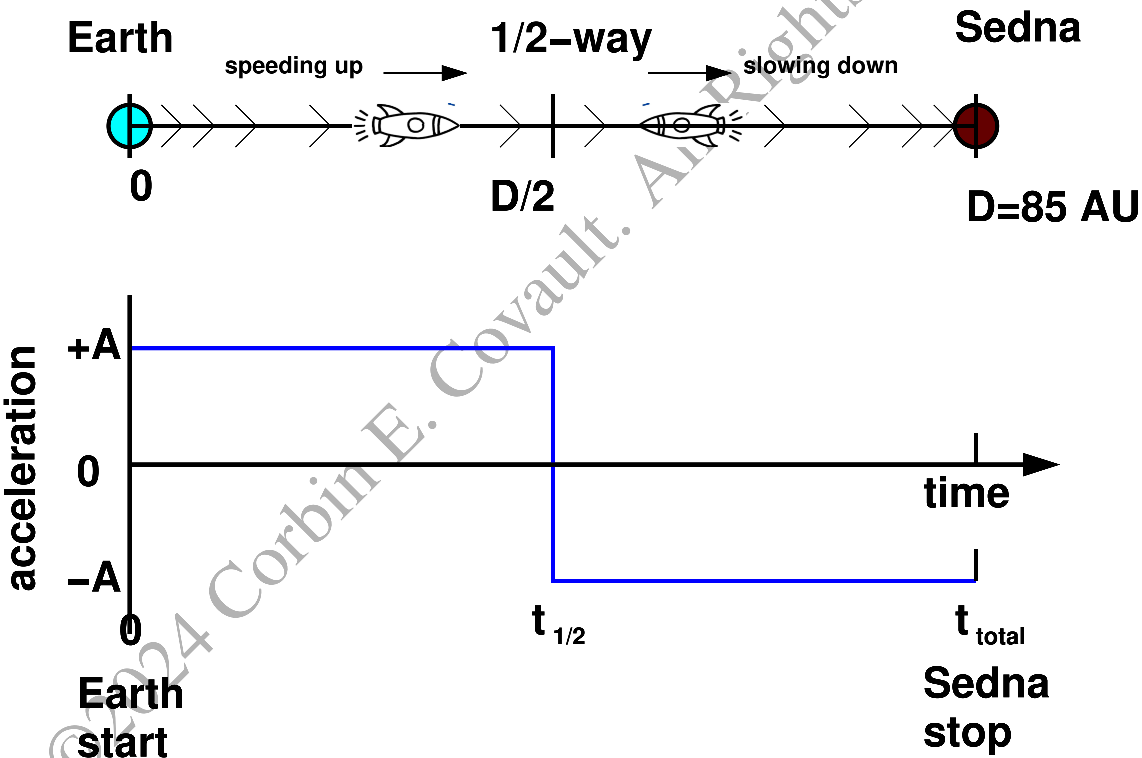
Let's make a little diagram showing what is going on here. The trip includes two parts, one with constant acceleration in the positive direction and one with constant acceleration in the negative direction. In other words, for the first half of the trip we get:

$$a_x = +A = +0.238 \text{ m/s}^2$$

And for the second half of the trip we have:

$$a_x = -A = -0.238 \text{ m/s}^2$$

The rocket is always moving forward (velocity is positive, the position is increasing toward Sedna) but the rocket is speeding up (thrust forward) to the half-way point and then slowing down (thrust backwards) after the half-way point. Here's what this looks like on a plot: [6]



Part (a) – The key to this problem is to recognize that although acceleration is *not* constant over the entire 85 AU distance, the acceleration *is* constant over each of the two 42.5 AU intervals as shown in the plot of acceleration vs. time above. Furthermore, ***by symmetry*** we can argue that the

[6]Note that the idea of a ship that maintains constant acceleration over the entire trip even at these relatively low values (corresponding to about 1/4 of a gee) is not actually realistic. This would require spacecraft power sources well in excess of those that would normally be associated with any current technology envisioned for the foreseeable future. So the whole problem falls into the category of something that could only happen in some very distant science fiction future. But at least conceptually, it's a simple idea: accelerate until you get to the half-way point, and then decelerate at the same rate to arrive at rest at your desired location.

entire trip will take precisely ***twice as long*** as the trip to the "half-way-point". So the key to this problem is recognizing that ***we really only need to solve one problem:*** *What are the kinematics of the trip to the half-way point at* **constant acceleration?** We use the standard equation for **constant acceleration**:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Here we know about certain initial conditions: $x_0 = 0$, $v_0 = 0$, and $a = A$. We are also given the distance to Sedna $D = 85$ AU, so we can also specify the distance to the half-way point as $x = x_{\frac{1}{2}} = D/2$ where D corresponds to a **given parameter**. Plugging these terms into our equation for position at the half-way time $t_{halfway}$, we get:

$$x_{\frac{1}{2}} = \frac{1}{2}At^2_{halfway}$$

$$\frac{D}{2} = \frac{1}{2}At^2_{halfway}$$

Solving for the unknown time $t_{halfway}$ in terms of the given parameters:

$$t^2_{halfway} = \frac{2\left(\frac{D}{2}\right)}{A}$$

$$t^2_{halfway} = \frac{D}{A}$$

$$t_{halfway} = \sqrt{\frac{D}{A}}$$

Now, one last detail, which is that I asked for the total trip time which is precisely **twice** the time for the travel to the half-way-point:

$$\boxed{t_{total} = 2t_{halfway} = 2\sqrt{\frac{D}{A}}}$$

This is our **symbolic answer** for the total time given in terms of given parameters.

Finally, plugging in numbers, and keeping in mind that the correct distance D here is the distance to Sedna as given:[7]

$$t_{total} = 2\sqrt{\frac{\left[(85\text{ AU})\left(\frac{1.496\times10^{11}\text{m}}{\text{AU}}\right)\right]}{\left(0.238\text{ m/s}^2\right)}}$$

$$t_{total} = 2(7,310,000\text{ s})$$

$$\boxed{t_{total} = 14,600,000\text{ seconds}}$$

[7] You can find the conversion factor from "AU" to meters by asking Google.

Converting units and rounding to two or maybe three significant digits of accuracy (corresponding to 85 AU):[8]

$$t_{total} = (14,600,000 \text{ seconds}) \left(\frac{1 \text{ day}}{3600 \times 24 \text{ seconds}} \right)$$

$$\boxed{t_{total} = 170 \text{ days}}$$

In other words, not quite six months.

Part (b) –

We can get the speed likewise:

$$v = v_0 + at$$

$$v = 0 + At_{halfway}$$

$$v = A\sqrt{\frac{D}{A}}$$

$$\boxed{v = \sqrt{AD}}$$

Again: this is our **symbolic answer**.

We get the numbers by plugging in the given parameters (rounding to two digits):

$$v = \sqrt{\left(0.238 \, \text{m/s}^2\right) (85 \, \text{AU}) \left(\frac{1.496 \times 10^{11} \text{m}}{\text{AU}} \right)}$$

$$\boxed{v = 1.7 \times 10^6 \text{ m/s} = 1,700 \, \text{km/s}}$$

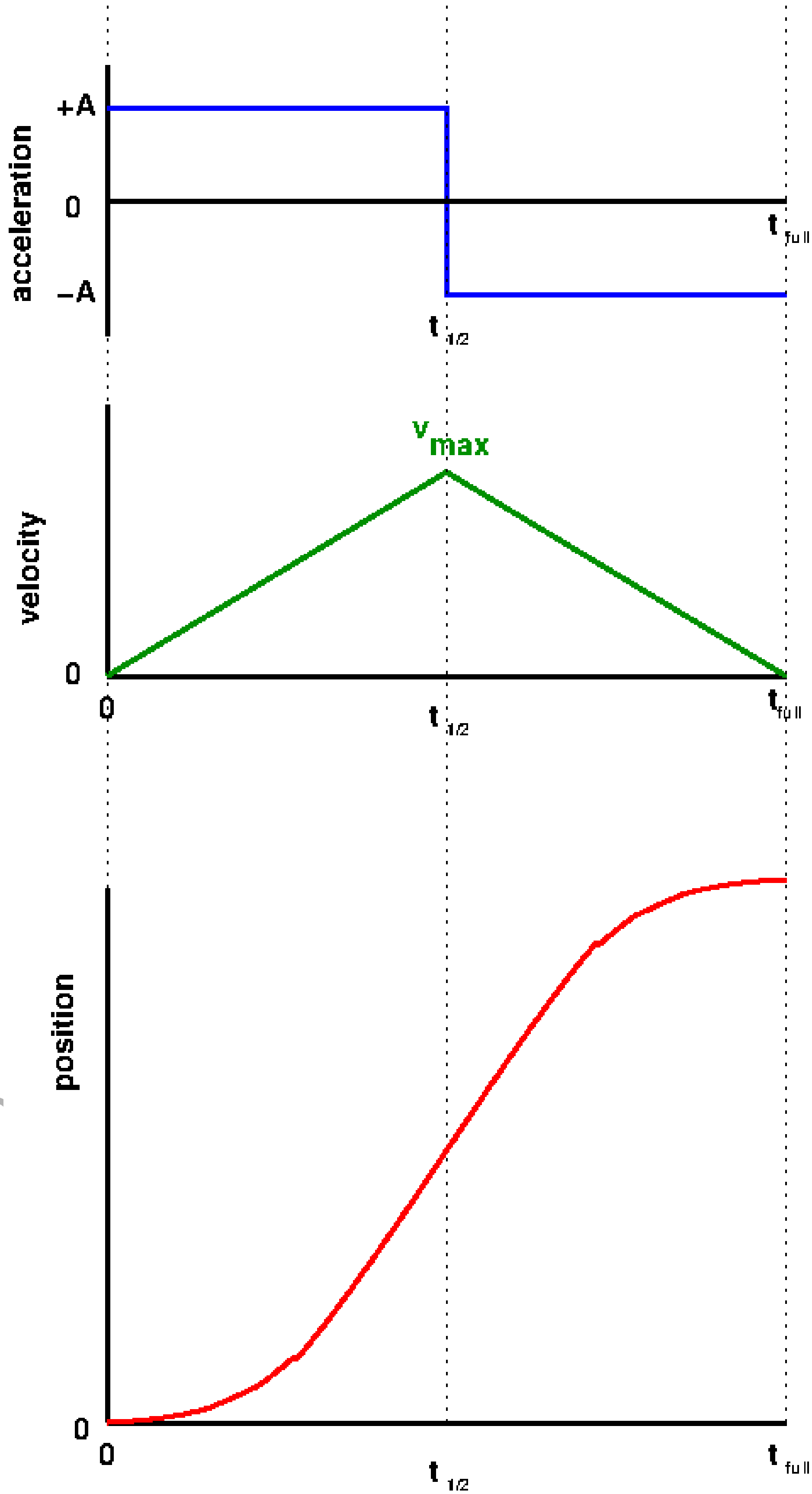
It's worth noticing that this speed is about half-a-percent of the speed of light: pretty wicked fast.[9] In fact, using current technologies, the maximum speed that been obtained by any self-propelled human-created spacecraft is about 16,000 meters per second, which is about 250 times slower. This is the fastest speed of NASA's *New Horizons* spaceprobe which is indeed on it's way into deep space having been launched in 2006 and then arrived to a close encounter with Pluto during July of 2015. See http://pluto.jhuapl.edu/Mission/Where-is-New-Horizons/index.php

[8] So we are not going to quibble about the difference between two or three significant digits here. All we ask is that you get this approximately right. Either two digits or three digits is fine. Four digits is really not justified but okay. One digit is of course really too few. All 9 digits from you calculator is Way Too Many Digits!

[9] Note that while the idea of a constant acceleration ship is beyond current rocket technology, it is not completely beyond the realm of possibility. One idea that has been around for a while time is the Bussard Ram Jet that could in principle use some kind of magnetic scoop to grab and compress the very thinly spread hydrogen atoms out of space as the ship travels. So far, no one has really figured out a way to make this work. So for now it's just a science fiction idea.

Part (c) – It is worth reflecting just a little on the graphical kinematics of this problem which are shown on the next page. Note the following physically intuitive results:

- The plot of **acceleration** vs. time is **constant** (flat) over two equal time intervals, so that the total time speeding up is matched by the total time slowing down.
- The plot of **velocity** vs. time is **linear** over each time interval, symmetric with the positive slope equal and opposite the negative slope. Note that we start and end the trip with zero velocity. Note that although the plot of acceleration vs. time is discontinuous, the plot of velocity vs. time must be continuous.
- The plot of **position** vs. time is also symmetric. Note that we are always moving forward on this trip, never backwards (because the velocity is always positive even though the acceleration is not). Note that the plot of position vs. time is both continuous and smooth at the turnover point. (The position plot must be smooth if the velocity plot is continuous, due to basic calculus. Right?)



acceleration
+A
0
–A
t full
t 1/2
velocity
v max
0
0
t 1/2
t full
position
0
0
t 1/2
t full

Problem 5. An apple sits at rest on top of a book which in turn sits on a table that sits on the floor. The mass of the apple is given as $m_A = 76$ grams and the mass of the book is $m_B = 1.07$ kg and the mass of the table is $m_T = 13.4$ kg.

- **Part (a): –** Determine the **net force** on the book. Hint. This is a *conceptual question*. Literally: no calculations are required, whatsoever. Just apply **Newton's Second Law.**
- **Part (b): –** Draw complete and proper **Free Body Diagrams** for both the apple and the book. Include only forces that are listed on the "Force Menu" (see Course Document #04, page 7) Each and every force on each object must be consistently and properly labeled. For example, if you want to indicate the force of **Weight** on the apple then you should label such a force as $\vec{W}_A$ which means "the Weight force on Body A." Likewise if you want to label the Normal force on the book *due to* the apple you would label this $\vec{N}_{BA}$.
- **Part (c): –** Apply **Newton's Second Law** together with your two **Free-Body Diagrams** to calculate the magnitude of each and every individual force on each of the two object (apple and book). Ignore air resistance and friction. Explain your work. Note that you must invoke and then apply Newton's Second Law to receive proper credit. Note: you **must** first set up and solve this problem ***symbolically***, not numerically. In other words, give your answers in terms of the given parameters m_A, m_B and perhaps m_T, and the physical constant g. Only after you have done this, should you plug in the numbers to your answer in terms numbers with appropriate units at the very end. ***Answers that are solved numerically and not symbolically will not receive full credit.***

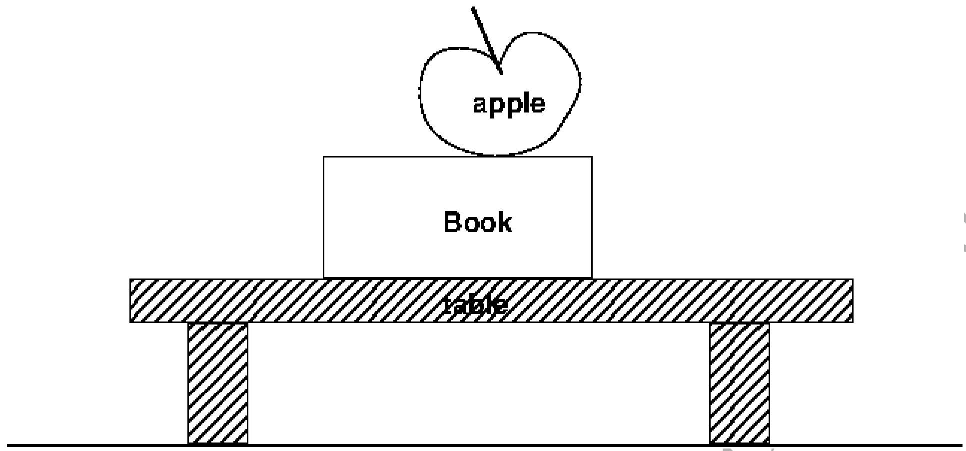
(Problem 5 continues next page...)

(Problem 5 continues:)

- **Parts (d), (e) and (f)): –** Repeat parts (a), (b) and (c) for Problem 5 above except now assume that table is sitting in an elevator that is accelerating *upward* with a given acceleration of precisely $A_0 = 6.96$ m/s^2. Be sure to explain your work. Note that you must explicitly invoke and then apply Newton's Second Law to receive proper credit. Note that students are specifically disallowed from invoking "fictitious" forces of any kind. Students are disallowed from "changing the value of g" or anything like this. Students **must** solve the problem using **Newton's Second Law** and they must solve the problem using kinematic variables as defined in an *inertial reference frame* only. Note: again you **must** set up and solve this problem *symbolically*, first, not numerically. **Plug in numbers only at the very end.**

SOLUTION TO PROBLEM 5:

Here's a diagram showing what the system looks like:



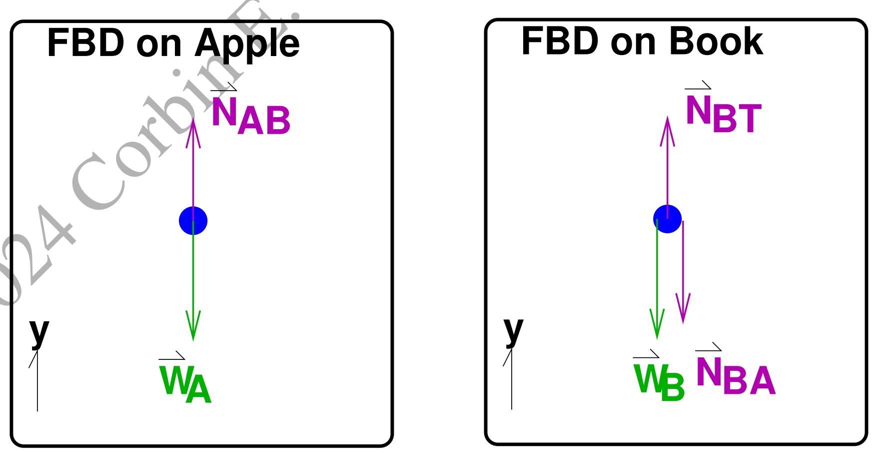
Part (a):

First, let's immediately answer the question about the *Net Force* on the book. We do *not* need to add up all of the forces to get this. All we need to do is notice that the acceleration of each book is ***exactly zero***. This immediately tells us by **Newton's Second Law** that

the net force on the book is exactly zero.

Part (b):

To calculate the actual forces on the apple and the book we want to write down a **Free Body Diagram** for each object:



Problem solution continues next page....

Part (c):

These free body diagrams explicitly give us the direction of every force. All we need to do is get the magnitudes. We charge ahead applying **Newton's Second Law** one body at a time, keeping in mind that the acceleration of each body is zero.

N's 2nd, Apple, y-direction:

$$F_{Ay} = m_A a_{Ay}$$
$$N_{AB} - W_A = m_A(0)$$
$$N_{AB} - W_A = 0$$
$$N_{AB} = W_A$$

And we use the known fact that $\boxed{W_A = m_A g}$:

$$\boxed{N_{AB} = m_A g}$$

Plugging in numbers (and using the conversion that 76 grams equals 0.076 kg):

$$\boxed{N_{AB} = W_A = m_A g = (0.076\text{ kg})(9.81\text{ m/s}^2) = 0.75\text{ Newtons}}$$

A "good rule of thumb" is that the weight force of an apple is about one Newton.

N's 2nd, Book, y-direction:

$$F_{By} = m_B a_{By}$$
$$N_{BT} - N_{BA} - W_B = m_B(0)$$
$$N_{BT} - N_{BA} - W_B = 0$$
$$N_{BT} = N_{BA} + W_B$$

We use the known fact that $\boxed{W_B = m_B g}$, and we also use **Newton's Third Law** together with the previous part of this problem, to tells us that $\boxed{N_{BA} = N_{AB} = m_A g}$. Plugging these in to solve for the one remaining unknown force:

$$N_{BT} = m_A g + m_B g$$

$$\boxed{N_{BT} = (m_A + m_B)g}$$

Plugging in numbers:

$$W_B = (1.07\text{kg})(9.81\text{ m/s}^2)$$
$$\boxed{W_B = 10.5\text{ Newtons}}$$

and

$$N_{BT} = (0.076\text{ kg} + 1.07\text{ kg})(9.81\text{ m/s}^2)$$

$$\boxed{N_{BT} = 11.2 \text{ Newtons}}$$

Re-articulating all of the previous calculations:

$$\boxed{\begin{gathered} W_A = m_A g = 0.75 \text{ Newtons} \\ \\ W_B = m_B g = 11.2 \text{ Newtons} \\ \\ N_{AB} = N_{BA} = m_A g = 10.5 \text{ Newtons} \\ \\ N_{BT} = N_{TB} = (m_A + m_B) g = 11.2 \text{ Newtons} \end{gathered}}$$

Part (d):

First, to calculate the *net force* on each object, we use **Newton's Second Law** which states that $F_{net} = ma$. In this case we are given the acceleration of the elevator A_0. We know it's value and we know that since the book is on the elevator and the apple is on the book, then all three objects move together. This means that ***All three objects have the same acceleration***. This is what we call a **"kinematic constraint:"**

$$a_{Ay} = a_{By} = A_0 = 6.96 \text{ m/s}^2$$

Since for each body we *know* the mass and we *know* the acceleration, we can just write down the **net force in accordance with Newton's Second Law** without any consideration whatsoever of what the actual individual forces on each body really are:

$$\text{Net force on Block B} = F_{By} = m_B aBy = \boxed{m_B A_0}$$

Numerically:

$$F_{By} = (1.07 \text{ kg})(6.96 \text{ m/s}^2) = \boxed{7.44 \text{ Newtons}}$$

Part (e):

Now we go on to calculate the individual forces. The key concept is this: If we change only the acceleration in a given problem, then this changes the *right-hand side* of Newton's Law. Changing the acceleration does *not change the term on the left-hand side.* **We therefore still use the same FBDs**, just now we solve with a different acceleration. In this problem, we now have constant acceleration, in the y-direction: $A = a_y = 6.96$ m/s^2.

N's 2nd, Apple, y-direction:

$$F_{Ay} = m_A a_{Ay}$$

$$N_{AB} - W_A = m_A A_0$$

$$N_{AB} - W_A = m_A A$$

$$N_{AB} = W_A + m_A A$$

And we use the known fact that $\boxed{W_A = m_A g}$:

$$N_{AB} = m_A g + m_A A$$

$$\boxed{N_{AB} = m_A(g + A)}$$

Plugging in numbers:

$$N_{AB} = (0.076 \text{ kg})(9.81 \text{ m/s}^2 + 6.96 \text{ m/s}^2)$$

$$\boxed{N_{AB} = 1.27 \text{ Newtons}}$$

Part (f):

$\boxed{\textbf{N's 2nd, Book, y-direction:}}$

$$F_{By} = m_B a_{By}$$
$$N_{BT} - N_{BA} - W_B = m_B A$$
$$N_{BT} = N_{BA} + W_B + m_B A$$

We use the known fact that $\boxed{W_B = m_B g}$, and we also use **Newton's Third Law** together with the previous part of this problem, to tells us that $\boxed{N_{BA} = N_{AB} = m_A(g + A)}$. Plugging these in to solve for the one remaining unknown force:

$$N_{BT} = m_A(g + a) + m_B g + m_B A$$

$$\boxed{N_{BT} = (m_A + m_B)(g + A)}$$

Plugging in numbers:

$$N_{BT} = (0.076 \text{ kg} + 1.07 \text{ kg})(9.81 \text{ m/s}^2 + 6.96 \text{ m/s}^2)$$

$$\boxed{N_{BT} = 19.2 \text{ Newtons}}$$

Note that since the constant acceleration A_0 is positive, the normal forces on the bodies in the elevator are **greater** than they would be if the elevator were not accelerating. If the elevator were accelerating downward, the acceleration would be negative and the normal forces on the bodies in the elevator would be **less** than they would be if the elevator were not accelerating.

Note that several students tried to solve this problem by arguing that since the elevator is accelerating, that this acceleration needs to be "added to g". Sometimes when this is done, the student manages to get the right numerical answer. This can be explained in the context of the idea of what happens in an accelerating reference frame. Either by intuition or previous training, students who tackled the problem this way are taking advantage of the fact that the effects of being in an accelerating reference frame are similar to changing the way gravity works.

The problem with this approach, however, is that it puts the cart before the horse. Newton's First Law says in a fundamental way that the Laws of Physics only work when applied in an *inertial reference frame* (e.g., not accelerating!). So the problem must be solved from the point of view of somewhat standing outside the elevator. In this non-accelerating frame, there must be real forces applied to the objects to make them accelerate, and the weight forces on each object are constant regardless of their motion.

Trying to solve the problem by "adding the acceleration to g" at this point in the course is not sticking to first principles. It argues an effect as cause and it leads to fuzzy logic and ill-structured solution methods. A student might be lucky and get the right answer, but the student is also likely to get confused and make sign errors, or add the same effect twice. Even if technically a student has arrived at the right answer, the method if fictitious forces is far more complicated an subtle than is called for here.

Therefore, until we get to the point in the course where we actually set up the explicit rules for trying to do physics in an accelerating reference frame (e.g., the very last week of the semester), I will **insist** that students solve all problem in an *inertial* (non-accelerating) reference frame. In such a frame, all forces are real physical forces, and all of the Newton's Law's work correctly. Every problem we give in class can be solved within a standard inertial reference frame. This is the method we expect all students to be able to demonstrate that they can use. If you can reliably and consistently apply Newtons Second Law to a wide variety of force problems then you have really learned something central to what this course is all about. This is what is means to say that you have *mastered* the material: If you are not actually explicitly using Newton's Second Law to solve these problems then you have not mastered the material.

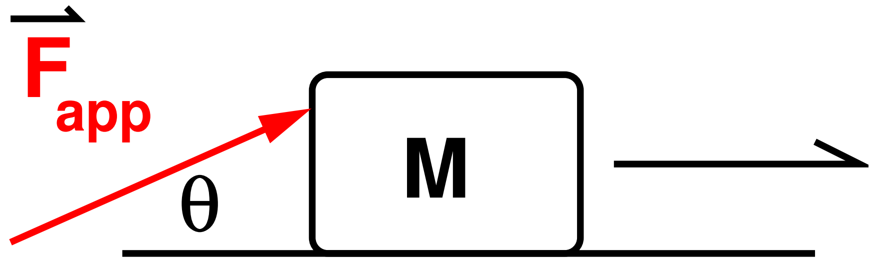
Re-articulating all of the previous calculations for parts (e) and (f):

$$W_A = m_A g = 0.075 \text{ Newtons (same as Parts (b), and (c)}$$

$$W_B = m_B g = 11.2 \text{ Newtons (same as Parts (b), and (c)}$$

$$N_{AB} = N_{BA} = m_A(g + A) = 1.27 \text{ Newtons}$$

$$N_{BT} = N_{TB} = (m_A + m_B)(g + A) = 19.2 \text{ Newtons}$$



Problem 6. An force of given magnitude F_{app} is applied to a block of a given mass M. The force is applied to the right and upward at a given angle θ relative to the horizontal as shown. There is a given force of friction with magnitude f that acts on the block to the left. As a result of these forces, the block is observed to move to the right a given distance D.

Important: For each question: give your answer in terms of the **given parameters**:

Part (a) – What is the **Work** done by the *Normal* force on the block? Explain your answer.

Part (b) – What is the **Work** done by the *Weight* force on the block? Explain your answer.

Part (c) – What is the **Work** done by the *Applied Force* force on the block? Explain your answer.

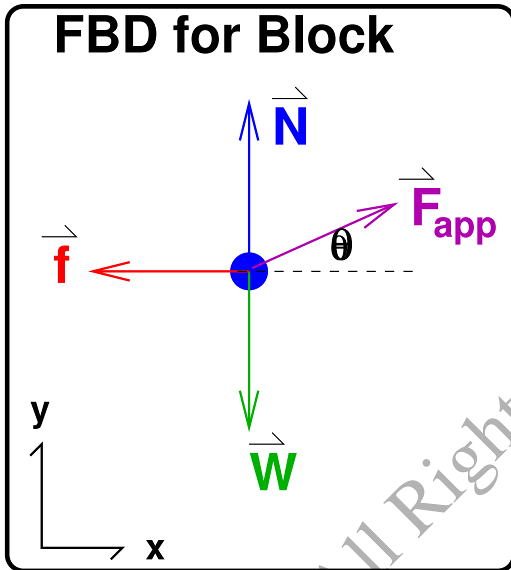
Part (d) – What is the **Work** done by the *Friction Force* force on the block?

Part (e) – What is the **Total Work** done on the block? Explain your answer.

Part (f) – Assuming that the block starts at rest, what is the **final speed** block when it has traveled a distance D?

SOLUTION TO PROBLEM 6:

So to make things clear, even though we have not been asked to do so explicitly, let's make a **Free Body Diagram** so we can keep track of all of the forces on the block:



We note that there are **four forces** here. Note that the forces labeled F_{app} and f are **given forces**. This is very important to keep track of as we go.

We consider the answers to each of the questions in turn:

Part (a) – What is the **Work** done by the *Normal* force on the block?

We make use of the **Definition of Work** which say that for constant forces applied during a displacement in a straight light, the Work due to a force is the ***dot product of the force and that displacement vector***.

$$\mathcal{W}_{\vec{F}} \equiv \vec{F} \cdot \vec{d} = (Fd)\cos\theta$$

where $theta$ is the angle between the force and the displacement. In this case the force is the ***Normal force*** which points up. The displacement vector $\vec{D}$ is a vector with a magnitude D pointing to the right. So the dot product can be written as:

$$\mathcal{W}_{\vec{F}} = \vec{N} \cdot \vec{D}$$

Since **the Normal vector is perpendicular Displacement vector**, **the Work done by the Normal force is zero.** In other words, the cosine of ninety degree in the dot product is zero, so the Work is zero. This is true regardless of the magnitude of the Normal force or the displacement.

Solution to Problem 6 continues...

Part (b) – What is the **Work** done by the *Weight* force on the block? Explain your answer.

By precisely the same logic, since the Weight force is applied perpendicular to the displacement, **the Work done by the Weight force is zero.**

Part (c) – What is the **Work** done by the *Applied Force* force on the block? Explain your answer.

The applied for force $\vec{F}_{app}$ is ***given*** here. We use the same **Definition of Work**. This time the force is the applied force and the angle applied is the given angle θ:

$$\mathcal{W}_{\vec{F}_{app}} = \vec{F}_{app} \cdot \vec{D}$$

$$\boxed{\mathcal{W}_{\vec{F}_{app}} = F_{app} D \cos\theta}$$

Part (d) – What is the **Work** done by the *Friction Force* force on the block?

The friction force $\vec{f}$ is ***given*** here. We use the same **Definition of Work**. This time the force is applied in the opposite direction of displacement. Therefore the Work done is negative. (In other words, the angle between the friction force vector and the displacement vector is 180 degrees and the cosine of this is negative one.)

$$\mathcal{W}_{\vec{f}} = \vec{f} \cdot \vec{D}$$

$$\boxed{\mathcal{W}_{\vec{f}} = -fD}$$

Part (e) – What is the **Total Work** done on the block? Explain your answer.

The Total Work is just the sum of all of the Works done by all of the forces:

$$\mathcal{W}_{total} = \mathcal{W}_{\vec{N}} + \mathcal{W}_{\vec{W}} + \mathcal{W}_{\vec{F}_{app}} + \mathcal{W}_{\vec{f}}$$

$$\mathcal{W}_{total} = \mathcal{W}_{total} = 0 + 0 + F_{app} D \cos\theta - fD$$

$$\boxed{\mathcal{W}_{total} = D(F_{app} \cos\theta - f)}$$

Solution to Problem 6 continues...

Part (f) – Assuming that the block starts at rest, what is the **final speed** block when it has traveled a distance D?

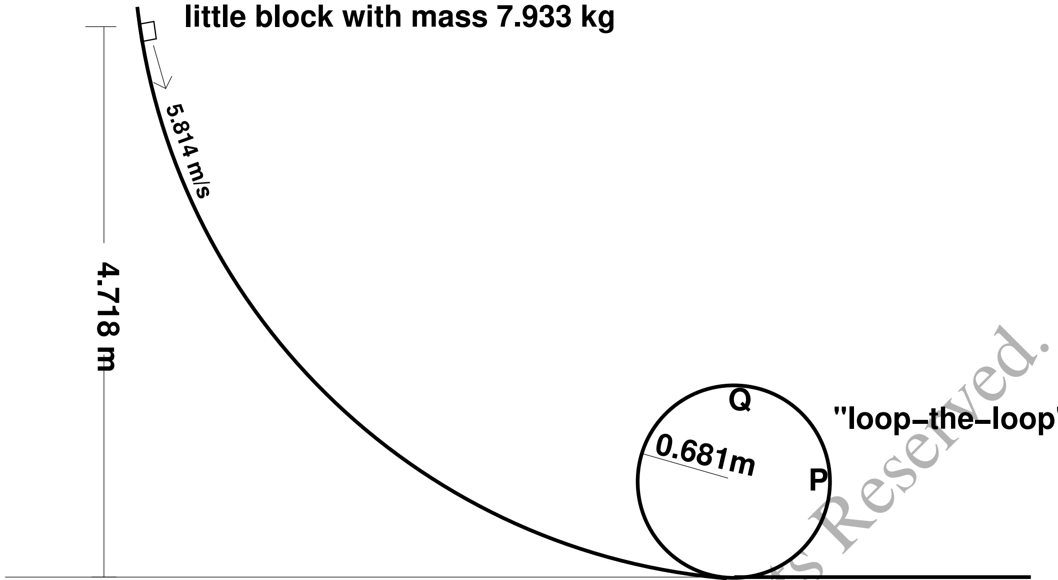
We use the **Classical Work-Energy Theorem** to answer this problem:

$$\mathcal{W}_{total} = \Delta K$$
$$\mathcal{W}_{total} = K_{final} - K_{initial}$$

We are told that the block starts at rest, so the kinetic energy $K_{initial} = 0$. We use our result from Part (e) together with the **Definition of Kinetic Energy** to find the final velocity of the block:

$$D(F_{app}\cos\theta - f) = \frac{1}{2}Mv_{final}^2$$
$$\frac{1}{2}Mv_{final}^2 = D(F_{app}\cos\theta - f)$$
$$v_{final}^2 = \frac{2D}{M}(F_{app}\cos\theta - f)$$
$$\boxed{v_{final} = \sqrt{\frac{2D}{M}(F_{app}\cos\theta - f)}}$$

Let's check and make sure this answer ***makes sense***. We expect the speed to be larger when the applied force is larger relative to the friction, so that works. The units here are correct: Forces are in units of (mass)(distance)/(time)^2 and this is multiplied by a (distance) and divided by a (mass). The result is a (mass)^2/(time)^2 and when we take the square-root we get (mass)/(time) which are the correct units of speed.



Problem 7. A small block of metal with a given mass of $m_b = 7.933$ kg is provided given initial speed $V_i = 5.814$ at the top of a curved **frictionless** surface as shown above. The block moves down the track and then does a "loop-the-loop". The height of the track h and the radius of the loop R are given as shown in the figure.

Important: you must work the following problems symbolically first, (giving your answer in terms of given parameters) and then numerically, part-by-part:

Part (a) – What is the **Work** done by the *Normal* force on the block due to the track as it moves from the top of the track to point P (precisely half-way up the "loop-the-loop")? Explain your answer.

Part (b) – What is the **Work** done by the force of *Weight* on the block as it moves from the top of the track to point P? Is this positive work or negative work? Explain your answer.

Part (c) – What is the ***speed*** of the block at point P? Show how you determined this. Explain what physics concept(s) and/or Laws you used to determine this. Explain your work.

SOLUTION TO PROBLEM 7:

Set-up: Before we get into the details, let's make a list of the **given parameters** and define **symbolic variables** to represent these:

$m = 7.399$ kg (the mass of the block.)

$h = 4.718$ meters (the initial vertical position of the block.)

$V_0 = 5.814$ m/s (the initial speed of the block.)

$R = 0.681$ m (the radius of the loop.)

Part (a) We use the **Definition of Work:** as follows: Work associated with a particular force in going from point P to point Q is defined as follows:

$$\mathcal{W}_{\vec{F}} \equiv \int_P^Q \vec{F} \cdot d\vec{r}$$

In other words, the Work results from the integral of the component of the force aligned parallel to the direction of the path of the body.

This means that $\boxed{\textbf{the work done by the Normal Force is zero}}$ because the Normal Force is ***always*** applied **perpendicular to the direction of motion along the path** in this problem..

Part (b) The Work done by the force of Weight is easily calculated because the Weight force is **Conservative** and therefore path-independent. So we need only consider the vertical component of the motion:

$$\mathcal{W}_{\vec{W}} = \int_y^{y'} W dy$$

$$\mathcal{W}_{\vec{W}} = \int_y^{y'} (-mg) dy$$

$$\mathcal{W}_{\vec{W}} = -mg(y' - y)$$

In this problem the initial position is given by $y = h$ and the "final" position at the right side of the loop (point P) is given by $y' = R$. So we have:

$$\mathcal{W}_{\vec{W}} = -mg(R - h)$$

$$\boxed{\mathcal{W}_{\vec{W}} = mg(h - R)}$$

Note also that the work done by a **Conservative Force** is equal to the negative of the change in **Potential Energy**, as expected. Note also that the Work is **positive** because the force of Weight (downward) is applied in the same direction as the vertical displacement (downward).

Plugging in numbers:

$$\mathcal{W}_{\vec{W}} = (7.399\,\text{kg})(9.81\,\text{m/s}^2)[(4.718\,\text{m}) - (0.681\,\text{m})]$$

$$\boxed{\mathcal{W}_{\vec{W}} = 293\,\text{Joules}}$$

Part (c) There are only two forces on the block for this whole problem: Weight and Normal. Because the Weight Force is ***Conservative*** and the Normal Force does ***zero work***, the **Conditions are Satisfied** for applying **Conservation of Mechanical Energy**:

$$\text{“BEFORE”} = \text{“AFTER”}$$

$$E_{tot} = E'_{tot}$$

$$U + K = U' + K'$$

$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

In this problem, the “Before” corresponds to the top of the ramp where $y = h$ and $v = V_0$. The “After” corresponds to the position at point P corresponding to $y' = R$. We plug these values in and solve for the unknown final velocity $V_P = v'$.

$$mgh + \frac{1}{2}mV_0^2 = mgR + \frac{1}{2}mV_P^2$$

Solving for V_P

$$mgR + \frac{1}{2}mV_P^2 = mgh + \frac{1}{2}mV_0^2$$

$$\frac{1}{2}mV_P^2 = mgh + \frac{1}{2}mV_0^2 - mgR$$

$$V_P^2 = 2gh + V_0^2 - 2gR$$

$$V_P^2 = 2g(h - R) + V_0^2$$

$$\boxed{V_P = \sqrt{2g(h - R) + V_0^2}}$$

Plugging in:

$$v_P = \sqrt{2(9.81\,\text{m/s}^2)\,[(4.718\,\text{m}) - (0.681\,\text{m})] + (5.814\,\text{m/s})^2}$$

$$\boxed{v_P = 10.6\,\text{m/s}}$$