Simple Harmonic Motion

(SHM)

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1 Abstract

I have tested the spring constant, torsion constant, and torsion modulus for two independent simple harmonic motion apparatuses by using principles of linear and rotational harmonic motion. Using Newton's Second Law, I recorded an angular frequency ω for an apparatus with a spring to be $\omega = 7.673 \pm 0.003\ s^{-1}$. Using this value, I calculated a spring constant of $k_s = 2.94 \pm 0.06\ \frac{N}{m}$. I then used a similar expression based off similar principles to determine the angular frequency for the same apparatus with added weight. The calculated value of $\omega_p = 5.72 \pm 0.07 s^{-1}$ does not align with the experimentally determined value as a result of minor systematic errors in the spring. Since the spring is not always properly taken care of, the expected spring constant is not going to necessarily align with the experiment itself, impacted the value of ω_p. I then determined a value for the torsion constant of a translational pendulum using the angular analogue of Newton's Second Law to be $k_t = 2.14 \pm 0.05\ N \cdot m$. Using this value, I calculated the torsion modulus for the rod in the apparatus to be $n = 5.46 \times 10^{10} \pm 0.52 \times 10^{10} \frac{N}{m^2}$. This value does not align with the given moduli for possible materials for the rod, so I concluded that the rod must be some Copper-Iron alloy.The discrepancy here is most likely due to misuse of the apparatus over time combined with improper measurement techniques in the lab. Systematic errors plagued the calculations and experiments done in this lab, preventing sound and confident conclusions.

2 Introduction and Theory

Translational, or linear, harmonic motion can be analyzed and calculated using springs and Newtons Second Law. When a spring with given spring constant k_s experiences a shift x from its equilibrium position x_0, it exerts a restoring force according to:

$$F = -k_s(x - x_0) \tag{1}$$

According to Newton's Second Law $F = ma$, we can rewrite Equation 1 as:

$$a = \frac{F}{m} = \frac{-k_s(x - x_0)}{m} \tag{2}$$

We can then set up a Second Order Differential Equation using the fact that $a = \frac{d^2x}{dt^2}$:

$$-k_s(x - x_0) = m\frac{d^2x}{dt^2} \tag{3}$$

Using the principles of Differential Equations, we can then derive the general solution:

$$x = x_m \sin(\omega t + \phi) + x_0 \tag{4}$$

where

$$\omega = \sqrt{\frac{k_s}{m}} \quad \text{and} \quad k_s = \omega^2 m \tag{5}$$

For the above relationships, x_m is given as the oscillation amplitude, x_0 is the spring's equilibrium position, ϕ is the phase angle where the wave crosses its equilibrium with a positive slope for the first time, and ω is the angular frequency which can be expressed as seen in Equation 5 or as:

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{6}$$

By analogue, angular, or rotational harmonic motion can be analyzed using an elastic rod with a torsion constant k_t . When twisted at an angle θ, the restoring torque is:

$$\tau = -k_t\theta \tag{7}$$

According to the rotational analogue of Newton's Second Law $\tau = I\alpha$, we can rewrite Equation 7 as:

$$-k_t\theta = I\alpha \tag{8}$$

We can set up a Second Order Differential Equation similar to that in Equation 3:

$$-k_t\theta = I\frac{d^2\theta}{dt^2} \tag{9}$$

Using the principles of Differential Equations, we can then derive the general solution:

$$\theta = \theta_m \sin(\omega t + \phi) \tag{10}$$

The above relationships contain nearly identical contents when compared to Equation 4, though ω can be determined through the following expression:

$$\omega = \sqrt{\frac{k_t}{I}} \tag{11}$$

The fact that our apparatus uses a solid cylindrical plate allows us to derive the expression for I:

$$I = \frac{1}{2}MR^2 \tag{12}$$

We can then combine Equations 6, 11, and 12 to derive an expression for k_t as follows:

$$\omega^2 = \frac{k_t}{I} \quad \therefore \quad k_t = I\omega^2 = \frac{1}{2}MR^2(\frac{2\pi}{T})^2 = \frac{M(\frac{D}{2})^2}{2} \cdot \frac{4\pi^2}{T^2}$$
$$\rightarrow k_t = \frac{\pi^2 M D^2}{2T^2} \tag{13}$$

We can also present the torsion constant k_t as an expression in terms of the rod's parameters. For the expression below, A is the cross-sectional area, n is the torsion modulus of the material, and L is the length of the rod. The expression is as follows:

$$k_t = \frac{nA^2}{2\pi L} \tag{14}$$

Since the rod is a cylinder, it has circular cross sections that have an area that can be presented as $A = \pi(\frac{D}{2})^2$, where D is the rod's diameter. Using this expression and Equation 14, we can solve for the torsion modulus n as follows:

$$n = \frac{2\pi L k_t}{A^2} = \frac{2\pi L k_t}{(\pi(\frac{D}{2})^2)^2} = \frac{2\pi L k_t}{\pi^2 \frac{D^4}{16}} = \frac{32 L k_t}{\pi D^4} \tag{15}$$

It should also be noted that creating a histogram of the distribution of the periods of the pendulum should roughly follow a Gaussian distribution if the calculations and experiments are performed properly.

Finally, we can safely assume that the moment of inertia in the rod is negligible when compared to the cylinder because of the noticeable differences between the bodies masses and radii.

3 Experimental Procedure

3.1 Translational SHM

We started our experiment with the Apparatus seen in Figure 1a. We first measured the weight our hanging mass to be $0.050 \pm 0.001\ kg$. We decided on this uncertainty as it aligns with the precision of our labs scale. We then attached the hanging mass to a ring stand positioned directly above a sonic motion detector, which would send position information to *Logger Pro*. Before running any experiments, we ensured that our detector was functioning properly by running *Logger Pro* with a lab notebook moving over the detector.

To conduct the experiment itself, we lowered the spring a distance of about $4cm$ from its equilibrium position and released it when *Logger Pro* was ready to collect data. We used *Logger Pro's* fitting software to record the values of the sinusoidal oscillations seen in Equation 4. We repeated this 3 more times to ensure that our data was consistent and could be used to calculate uncertainties. Then, we conducted a similar experiment, altering the distance we pulled the spring from its equilibrium to about 10 cm. Finally, we returned to the 4 cm position, but added a measured mass of $0.040 \pm 0.001\ kg$ to the hanger to conduct this test. For both of these alterations, we recorded the constants *Logger Pro* provided us. Again, the derivation of our chosen uncertainty is from the precision of our lab's scale ($\frac{1}{10}$ g).

3.2 Rotational SHM

The second experiment we conducted made use of the apparatus seen in Figure 1b. We started by measuring the length of the rod and diameters of the plate and rod. To measure the length of the rod, we measured from its connection to the plate to the rod's connection to the table. This resulted in a measurement of 0.8571 ± 0.0001 m. We decided on this uncertainty because the graduations on our meter stick went to the nearest millimeter, so we could estimate one decimal place after that. To measure the rod's diameter, we used *Vernier* Calipers. We measured a diameter of 0.0043 ± 0.0001 m. Our reasoning for the uncertainty in this measurement is similar to that of the rod's length, as the calipers are only so precise. Finally, we measured a plate diameter of 0.2555 ± 0.0005 m. Determining an accurate measurement of the diameter of the plate proved to be difficult because of the rods placement, so we accounted for this by increasing the uncertainty from the assumed 0.0001 m produced by our meter sticks. We were also given the mass plate as we were unable to remove it in lab. Our plate was the Gray Iron plate, which had a mass of 4.6 ± 0.1 kg.

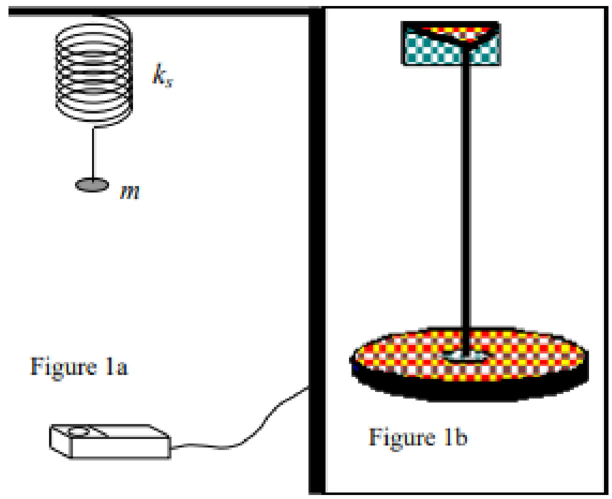

Figure 1: Experimental Apparatuses

To conduct the experiment itself, we twisted the plate just under 40° to prevent damage to the apparatus. Using *Logger Pro*, we recorded about 25 oscillations after releasing the plate in such a way that allowed a vertical rod to pass through a Photogate senor. We repeated this experiment until we had data that appeared to oscillate around a fixed value.

4 Results and Analysis

4.1 Translational SHM

First, we determined an accurate measurement of the angular frequency ω but finding the average value, $\bar{\omega}$, of our 4 trials using the second column in Table 1:

$$\bar{\omega} = \frac{7.672 + 7.681 + 7.670 + 7.670}{4} = 7.673$$

Although $\bar{\omega} = 7.673$ s^{-1} is the correct notation for this quantity, I will refer to it simplify as ω.

Then, we determined the estimated uncertainty for a single measurement σ, by using the below formula and technology to handle the calculation itself as it is too long and redundant to include in this report.

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}} \tag{16}$$

In this expression, N is the number of samples in the dataset, in this case 4, and x_i is the indexed value of the data in the set. For our data, we calculated $\sigma = 0.0052599$.

Finally, we determined the error in the mean according to the following equation. N represents the same quantity that is the number of values in the data set.

$$\delta = \frac{\sigma}{\sqrt{N}} \tag{17}$$

For our data, we calculated $\delta_\omega = 0.00263$.

Using this data we can propose a value for ω with its uncertainty which is $w = 7.673 \pm 0.003\ s^{-1}$. Further analyzing Table 1, we compared this value with the measured ω value for the trial with increased amplitude. We observed that the large amplitude value of $\omega = 7.672 \pm 0.001\ s^{-1}$ lies within the uncertainty of our calculated value above, showing that angular frequency is independent of amplitude.

We can then use Equation 5 with this measured value for ω as well as a hanging mass weight $m = 0.050\ kg$ to determine a value for the spring constant k_s:

$$k_s = (7.673\ s^{-1})^2(0.050\ kg) = 2.944\frac{N}{m}$$

We can also use Equation 5 to determine the uncertainty in k_s using the expression below:

$$\delta_{k_s} = \sqrt{\delta_{k_s,\omega}^2 + \delta_{k_s,m}^2} \tag{18}$$

We can derive expressions for $\delta_{k_s,\omega}$ and $\delta_{k_s,m}$ using the derivative method and Equation 5 as follows:

$$\begin{aligned} \delta_{k_s,\omega} = \frac{\partial k_s}{\partial \omega} * \delta_\omega &= \frac{\partial}{\partial \omega}(\omega^2 m) * \delta_\omega \\ &= 2\omega m * \delta_\omega \end{aligned} \tag{19}$$

$$\begin{aligned} \delta_{k_s,m} = \frac{\partial k_s}{\partial m} * \delta_\omega &= \frac{\partial}{\partial m}(\omega^2 m) * \delta_m \\ &= \omega^2 * \delta_m \end{aligned} \tag{20}$$

We can then compute the values of these uncertainties based on the values stated above. Here, we will use our measured values used to calculate k_s, and uncertainties $\delta_\omega = 0.003\ s^{-1}$ and $\delta_m = 0.001\ kg$. Substituting these values into Equations 19 and 20 yields:

$$\delta_{k_s,\omega} = 2(7.673)(0.050) * 0.003 = 0.0023019$$

$$\delta_{k_s,m} = (7.673)^2 * (0.001) = 0.05887$$

We can then plug these values into Equation 18 to yield the following value of δ_{k_s}:

$$\delta_{k_s} = \sqrt{0.0023019^2 + 0.05887^2} = 0.0589\ \frac{N}{m}$$

With these two values, we can rewrite k_s with its uncertainties as $k_s = 2.94 \pm 0.06\ \frac{N}{m}$.

We can then use this value in conjunction with Equation 5 to determine a theoretical value for ω_p, the angular frequency of oscillations with an increased mass. Since we added a total of $0.040\ kg$ to the hanger, we can represent the mass as $m = 0.090 \pm 0.001\ kg$. We can use this equation to make the following predictive calculation:

$$\omega_p = \sqrt{\frac{2.944}{0.090}} = 5.719\ s^{-1}$$

We can also use Equation 5 to determine the uncertainty in ω_p using the expression below:

$$\delta_{\omega_p} = \sqrt{\delta^2_{\omega_p,k_s} + \delta^2_{\omega_p,m}} \tag{21}$$

We can derive expressions for δ_{ω_p,k_s} and $\delta_{\omega_p,m}$ using the derivative method and Equation 5 as follows:

$$\begin{aligned} \delta_{\omega_p,k_s} = \frac{\partial \omega_p}{\partial k_s} * \delta_{k_s} &= \frac{\partial}{\partial k_s}(\sqrt{\frac{k_s}{m}}) * \delta_{k_s} \\ &= \frac{\sqrt{\frac{k_s}{m}}}{2k_s} * \delta_{k_s} \end{aligned} \tag{22}$$

$$\begin{aligned} \delta_{\omega_p,m} = \frac{\partial \omega_p}{\partial m} * \delta_m &= \frac{\partial}{\partial m}(\sqrt{\frac{k_s}{m}}) * \delta_m \\ &= -\frac{\sqrt{\frac{k_s}{m}}}{2m} * \delta_m \end{aligned} \tag{23}$$

We can then compute the values of these uncertainties based on the values stated above. Here, we will use our measured values used to calculate ω_p, and uncertainties $\delta_{k_s} = 0.06\ \frac{N}{m}$ and $\delta_m = 0.001\ kg$. Substituting these values into Equations 22 and 23 yields:

$$\delta_{\omega_p,k_s} = \frac{\sqrt{\frac{2.944}{0.090}}}{2(2.944)} * 0.06 = 0.05828$$

$$\delta_{\omega_p,m} = -\frac{\sqrt{\frac{2.944}{0.090}}}{2(0.090)} * 0.001 = -0.03177$$

We can then plug these values into Equation 21 to yield the following value of δ_{ω_p}:

$$\delta_{\omega_p} = \sqrt{0.05828^2 + (-0.03177)^2} = 0.06638\ s^{-1}$$

With these two values, we can rewrite ω_p with its uncertainties as $\omega_p = 5.72 \pm 0.07\ s^{-1}$. We can then compare this calculation with the experimentally determined ω_3 value for our trial with added mass. From Table 1, we can see that our measured value is $\omega_3 = 5.815 \pm 0.001\ s^{-1}$. These values do not agree within their uncertainties as the measured value ω_3 is about 2 standard deviations away from the predicted value ω_p.

4.2 Rotational SHM

To analyze our data from the second part of our experiment, we made use of *Origin's* fitting software to create a histogram of the Periods experienced by the pendulum over time. We then analyzed the distribution of these periods by fitting a Gaussian curve to the data as seen in Figure 3. The Gaussian curve is mostly accurate with respect to our data, though it fails to account for a slight right skew. Our data may have been skewed right as a result of poorly positioning the Photogate with respect to the vertical rod on the plate. This would've recorded more time between passes on one side of the gate, and shorter times on the other leading to skewed data. Overall, our data wasn't necessarily perfect, but the Gaussian curve captures a significant percentage of our data correctly.

To analyze the torsion constant and modulus, we first started by calculating the moment of inertia of the plate according to Equation 12. In the following expression M is the mass of the plate, and D is the measured diameter of the Plate.

$$I_P = \frac{1}{2}M(\frac{D}{2})^2 = \frac{1}{2}(4.6\ kg)(\frac{0.2555\ m}{2})^2 = 0.0375\ kg \cdot m^2$$

We can then use Equation 13, an expression of k_t simplified from an expression in terms of I_P and T, to calculate the torsion constant based off of the plate's diameter D, it's mass M, and its period T. Here, $T = 0.832\ s$ as it is the mean period exhibited by the pendulum as confirmed by Figure 3. Plugging the necessary values into this Equation yields the following calculation:

$$k_t = \frac{\pi^2(4.6\ kg)(0.2555\ m)^2}{2(0.832\ s)^2} = 2.141\ N \cdot m \tag{24}$$

We can also use Equation 13 to determine the uncertainty in k_t using the expression below:

$$\delta_{k_t} = \sqrt{\delta_{k_t,M}^2 + \delta_{k_t,D}^2 + \delta_{k_t,T}^2} \tag{25}$$

We can derive expressions for $\delta_{k_t,M}$, $\delta_{k_t,D}$, and $\delta_{k_t,T}$ using the derivative method and Equation 13 as follows:

$$\begin{aligned} \delta_{k_t,M} = \frac{\partial k_t}{\partial M} * \delta_M &= \frac{\partial}{\partial M}(\frac{\pi^2 MD^2}{2T^2}) * \delta_M \\ &= \frac{\pi^2 D^2}{2T^2} * \delta_M \end{aligned} \tag{26}$$

$$\begin{aligned} \delta_{k_t,D} = \frac{\partial k_t}{\partial D} * \delta_D &= \frac{\partial}{\partial D}(\frac{\pi^2 MD^2}{2T^2}) * \delta_D \\ &= \frac{\pi^2 MD}{T^2} * \delta_D \end{aligned} \tag{27}$$

$$\begin{aligned} \delta_{k_t,T} = \frac{\partial k_t}{\partial T} * \delta_T &= \frac{\partial}{\partial T}(\frac{\pi^2 MD^2}{2T^2}) * \delta_T \\ &= -\frac{\pi^2 MD^2}{T^3} * \delta_T \end{aligned} \tag{28}$$

We can then substitute the same values used to calculate k_t into Equations 26, 27, and 28 along with uncertainty values of $\delta_M = 0.1\ kg$, $\delta_D = 0.0005\ m$, and $\delta_T = 0.003\ s$ (from Figure 3) to yield the following calculations:

$$\delta_{k_t,M} = \frac{\pi^2(0.2555)^2}{2(0.832)^2} * 0.1 = 0.0465$$

$$\delta_{k_t,D} = \frac{\pi^2(4.6)(0.2555)}{(0.832)^2} * 0.0005 = 0.00838$$

$$\delta_{k_t,T} = -\frac{\pi^2(4.6)(0.2555)^2}{(0.832)^3} * 0.003 = -0.0154$$

We can then plug these values into Equation 25 to yield the following value of δ_{k_t}:

$$\delta_{k_t} = \sqrt{0.0465^2 + 0.00838^2 + (-0.0154)^2} = 0.0497\ N \cdot m$$

We can then rewrite k_t with its uncertainty as $k_t = 2.14 \pm 0.05\ N \cdot m$. This value is crucial for determining the uncertainty in n, then torsion modulus of the rod. To calculate this value, we can use Equation 15 in conjunction with previously used and calculated constants, with parameters being in terms of the rod this time. We will use our measured length of the rod $L = 0.8571\ m$ and diameter $D = 0.0043\ m$ with the calculated value of k_t to perform this as follows:

$$n = \frac{32(0.8571)(2.14)}{\pi(0.0043)^4} = 5.46 \times 10^{10} \ \frac{N}{m^2}$$

We can also use Equation 15 to determine the uncertainty in n using the expression:

$$\delta_n = \sqrt{\delta_{n,L}^2 + \delta_{n,k_t}^2 + \delta_{n,D}^2} \tag{29}$$

We can derive expressions for $\delta_{n,L}$, δ_{n,k_t}, and $\delta_{n,D}$ using the derivative method and Equation 15 as follows:

$$\begin{aligned} \delta_{n,L} = \frac{\partial n}{\partial L} * \delta_L &= \frac{\partial}{\partial L}(\frac{32Lk_t}{\pi D^4}) * \delta_L \\ &= \frac{32k_t}{\pi D^4} * \delta_L \end{aligned} \tag{30}$$

$$\begin{aligned} \delta_{n,k_t} = \frac{\partial n}{\partial k_t} * \delta_{k_t} &= \frac{\partial}{\partial k_t}(\frac{32Lk_t}{\pi D^4}) * \delta_{k_t} \\ &= \frac{32L}{\pi D^4} * \delta_{k_t} \end{aligned} \tag{31}$$

$$\begin{aligned} \delta_{n,D} = \frac{\partial n}{\partial D} * \delta_D &= \frac{\partial}{\partial D}(\frac{32Lk_t}{\pi D^4}) * \delta_D \\ &= -\frac{128Lk_t}{\pi D^5} * \delta_D \end{aligned} \tag{32}$$

We can then substitute the same values used to calculate n into Equations 30, 31, and 32 along with uncertainty values of $\delta_L = 0.0001\ m$, $\delta_{k_t} = 0.05\ \frac{N}{m^2}$, and $\delta_D = 0.0001\ m$ to yield the following calculations:

$$\delta_{n,L} = \frac{32(2.14)}{\pi(0.0043)^4} * 0.0001 = 6.38 \times 10^6$$

$$\delta_{n,k_t} = \frac{32(0.8571)}{\pi(0.0043)^4} * 0.05 = 1.28 \times 10^9$$

$$\delta_{n,D} = -\frac{128(0.8571)(2.14)}{\pi(0.0043)^5} * 0.0001 = -5.08 \times 10^9$$

We can then plug these values into Equation 29 to yield the following value of δ_n:

$$\delta_n = \sqrt{(6.38 \times 10^6)^2 + (1.28 \times 10^9)^2 + (-5.08 \times 10^9)^2} = 5.24 \times 10^9 \ \frac{N}{m^2}$$

These calculations allow us to determine the torsion modulus with its uncertainty is $n = 5.46 \times 10^{10} \pm 5.24 \times 10^9\ \frac{N}{m^2}$, which can be written as $n = 5.46 \times 10^{10} \pm 0.52 \times 10^{10}\ \frac{N}{m^2}$ for readability.

To show the importance of a good diameter measurement, we increased our diameter measurement for the rod by 1%, which can be written as:

$$D_{new} = 1.01(D_{old}) = 1.01(0.0043) = 0.004343$$

Redoing our original calculation of n, which uses Equation 15, but using $D = 0.004343$ instead yields:

$$n = \frac{32(0.8571)(2.14)}{\pi(0.004343)^4} = 5.25 \times 10^{10}$$

Setting up a ratio of the two values as follows reveals the percent change seen in D:

$$\frac{5.46 \times 10^{10} - 5.25 \times 10^{10}}{5.46 \times 10^{10}} \times 100 = 3.85 \approx 4\%$$

We observed a large change in the values when only a small change was applied, which shows the necessity of proper measurement techniques in the lab.

Based on our calculated torsion modulus (without the percentage increase) and its uncertainty, we can conclude that our rod is most likely a Copper-Iron alloy based on the Torsion Modulus values given in Figure 4. We do not feel confident in this conclusion because our calculated value itself does not directly align with any of the values seen in the table. We cannot safely say that the value $n = 5.46 \times 10^{10} \pm 0.52 \times 10^{10}\ \frac{N}{m^2}$ properly aligns itself with any of the values in the table as none of the values are captured within a single standard error of our value.

5 Conclusion

We calculated the spring constant for a common Translational Simple Harmonic Motion Apparatus to be $k_s = 2.94 \pm 0.06\ \frac{N}{m}$. We were then able to predict the angular frequency of the same apparatus with increased mass with moderate success. The value we calculated was $\omega_p = 5.72 \pm 0.07\ s^{-1}$, which deviated from the experimentally determined value of $\omega_3 = 5.815 \pm 0.001\ s^{-1}$ by about 2 standard deviations. The reasoning for this discrepancy is most likely because of the Spring itself experiencing wear and tear over time. For example, someone may have left the hanging mass on the Spring over night, which would've altered the spring's actual spring constant which, in turn, would severely impact our calculations for ω_p. That source of error is most likely the main contributing source of systematic error in this half of the experiment.

We then calculated the torsion constant for a common translational pendulum to be $k_t = 2.14 \pm 0.05\ N \cdot m$. We used this value and its uncertainty to calculate the torsion modulus of the rod used on the pendulum to be $n = 5.46 \times 10^{10} \pm 0.52 \times 10^{10}\ \frac{N}{m^2}$. This value does not align with any of the values presented in Figure 4, though we can conclude that the material that the rod is made of is some sort of Copper-Iron Alloy. The reason our value doesn't align with any of the values is likely due to two reasons. First, the rod may have experienced some significant wear and tear as well, with some students likely twisting the rod past the allowed 45° limit. Second, the apparatus was very difficult to control. The apparatus experienced some slight horizontal motion that we ultimately couldn't prevent.

These two factors collided to lead to a large amount of systematic error, which prevented our calculations from aligning with the given values.

5.1 Acknowledgments

I would like to thank Adi Mallik, CWRU Department of Physics, for his help in obtaining the experimental data, preparing the figures, and checking my calculations.

5.2 References

1. Driscoll, D., General Physics I: Mechanics Lab Manual, “Simple Harmonic Motion”, CWRU Bookstore, 2014.

6 Appendix

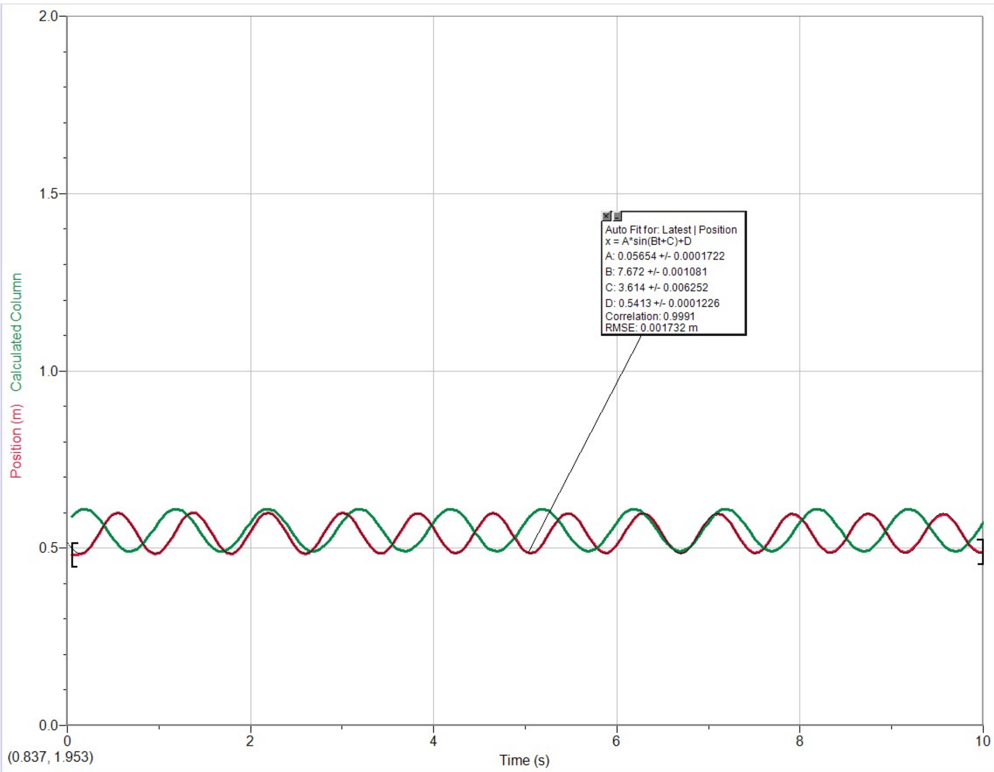

Figure 2: Harmonic Motion of Hanging Mass

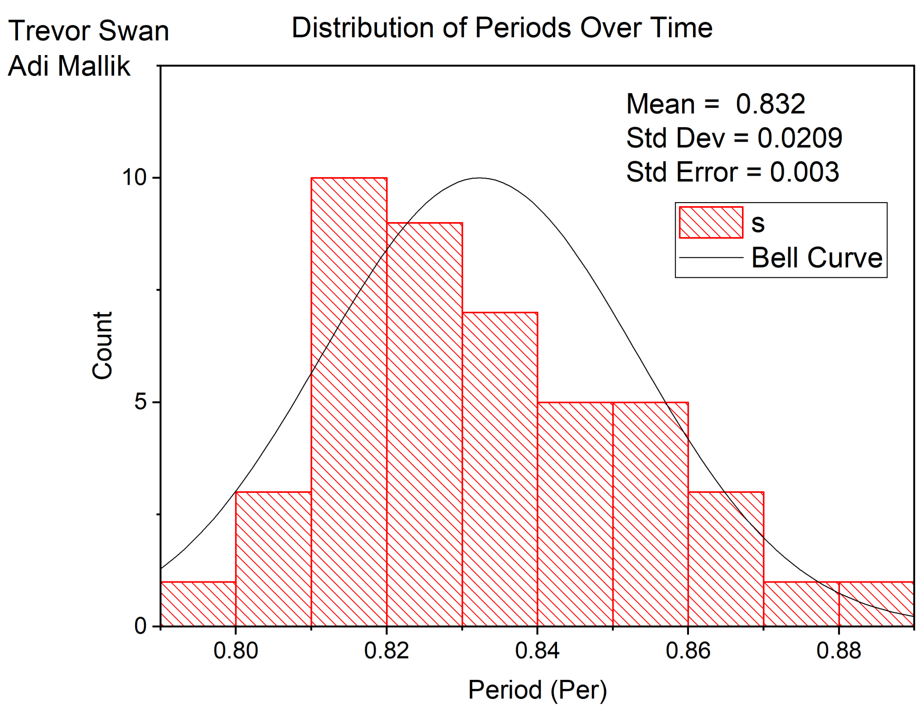

Figure 3: Pendulum Distribution

Lead	0.54×10^{10} N·m^{-2}
Magnesium	1.67×10^{10} N·m^{-2}
Aluminum	2.37×10^{10} N·m^{-2}
Brass	3.53×10^{10} N·m^{-2}
Copper	4.24×10^{10} N·m^{-2}
Iron	7.0×10^{10} N·m^{-2}
Nickel	7.0-7.6×10^{10} N·m^{-2}
Steel	7.8×10^{10} N·m^{-2}
Molybdenum	14.7×10^{10} N·m^{-2}
Tungsten	14.8×10^{10} N·m^{-2}

Figure 4: Torsion Moduli

Trial	x_m	ω	ϕ	x_0
Hanger 1	0.05654 ± 0.000172	7.672 ± 0.001081	3.614 ± 0.006252	0.5413 ± 0.0001226
Hanger 2	0.06036 ± 0.0004562	7.681 ± 0.003385	0.3487 ± 0.02188	0.5280 ± 0.0003250
Hanger 3	0.06553 ± 0.0001749	7.670 ± 0.0009447	1.065 ± 0.005451	0.5490 ± 0.0001245
Hanger 4	0.6596 ± 0.0004576	7.670 ± 0.002452	1.105 ± 0.01415	0.5445 ± 0.0001627
Large Amp	0.1066 ± 0.0004402	7.672 ± 0.001407	5.861 ± 0.008181	0.5481 ± 0.0003101
Large Mass	0.2193 ± 0.0007210	5.815 ± 0.001119	4.824 ± 0.006505	0.4264 ± 0.0005079

Table 1: Translational Motion Trials
Not rounded for calculation accuracy