RKE

Rotational Kinetic Energy

revised March 14, 2018

Learning Objectives:

During this lab, you will

1. communicate scientific results in writing.
2. estimate the uncertainty in a quantity that is calculated from quantities that are uncertain.
3. be introduced to the Monte Carlo Simulation method.
4. test a physical law experimentally.

A. Introduction

You will use the principle of conservation of energy to determine the moment of inertia of a system of four identical masses symmetrically located on the circumference of a wheel which is rotating about its axis (Figure 1). You will compare the value you measure to the value you calculate from theory.

Because the wheel has a complicated shape, you must measure rather than calculate its moment of inertia. You can also measure the combined moment of the wheel and the mass loads and then calculate the moment of inertia of the loads alone as the difference between these two measurements.

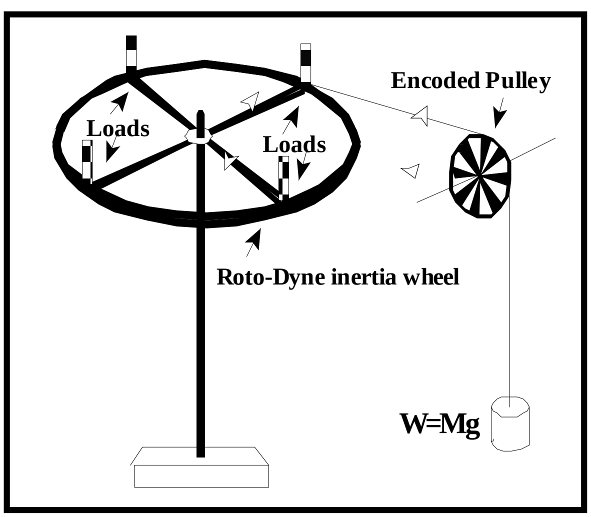

Figure 1: Experimental Arrangement

This experiment also includes an introduction to a numerical method that may be new to you - Monte Carlo computer simulations. Monte Carlo simulations are used to study phenomena where after many random trials, “out” data should start to converge to the value of real data. Randomness is also a feature of the experimental errors that one often encounters. You will use a Monte Carlo simulation for the RKE experiment to create data for debugging and testing your analysis routine. You will then apply this analysis routine to experimental data collected from the inertia wheel and mass loads.

You must write a FULL paper for this lab worth 60 points.

B. Apparatus

Neighboring groups will share a single setup, but each group must acquire and analyze their own set of data. It takes very little time to take the data you need for this experiment so sharing a single setup should not be a problem. Before taking any data, be certain that the cable from the encoded pulley is plugged into the jack on your own computer station.

The key piece of apparatus is a *Roto-Dyne* inertia wheel [*mass* M_R *= 1.5 kg, radius r =* *0.200±0.002 m*] on which you can mount four cylindrical mass loads [M_L *= 0.225±0.002 kg each*]. The torque necessary to rotate this wheel will be supplied by gravity acting on a hanging mass [*M = 60 g*] and some paper clip masses, *m*. You will also use a meter stick, an encoded pulley and a computer running the *Logger Pro* program.

The experimental setup is illustrated in Figure 1. The hanging mass *M* is tied to a string hanging over an encoded pulley and wrapped around the grooved circumference of the wheel. The encoded pulley records time and distance intervals (Δs *= 0.015 m*) as the weight falls and the wheel rotates. *Logger Pro* measures the time intervals between light pulses from the encoded pulley and automatically calculates positions and velocities.

The two radii, r and k, required for this experiment are illustrated in Figure 2.

- r the hanging weight is wrapped on the wheel at the radius r. The weight has fallen a total distance $y = i\Delta s$ after the encoder has recorded i measurements. The distance $y = r\Delta\theta$ corresponds to angle $\Delta\theta$ through which the wheel turns as the weight falls.
- k the wheel loads can be installed at selected radii k.

The outer radius of the wheel plays no part in the calculation.

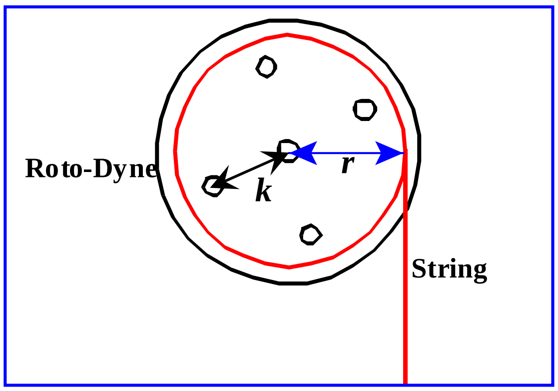

Figure 2: Top view of *Roto-Dyne* wheel, showing two radii, r and k.

C. Theory

The moment of inertia of a rotating object depends upon the mass and shape of the object. For a point object of mass M rotating in a circle of radius R, the moment of inertia I is given by

$$I = MR^2 \qquad (1)$$

The moment of inertia of an extended object can be calculated by conceptually breaking it up into infinitesimally small pieces of mass dM, each at its own distance R from the axis of rotation, and integrating over the volume of the object.

$$I = \int R^2 dM \qquad (1a)$$

However, the moment of inertia can be quite difficult to calculate explicitly for objects that don't have very simple shapes.

In this experiment you determine the moment of inertia of a mass-loaded, spoked wheel experimentally by measuring its rotational kinetic energy and invoking the principle of conservation of mechanical energy.

C.1. Conservation of energy

The kinetic energy associated with the translational speed v of an object of mass M is given by

$$K_T = ½Mv^2 \qquad (2)$$

The kinetic energy of a rotating object with angular speed ω is given by the expression

$$K_R = ½I\omega^2 \qquad (3)$$

where I is the moment of inertia of the object about the axis of rotation.

When the hanging mass in Figure 1 falls a distance y from rest, its decrease in potential energy ΔU_W is transformed into an increase in total kinetic energy K of the system plus a small energy loss W_f to friction. For any position of the falling weight we can write

$$\Delta U_W + K_T + K_R = W_f \qquad (4)$$

or

$$\Delta U_W + ½Mv^2 + ½I\omega^2 = W_f \qquad (5)$$

Note that ΔU_W and W_f are negative while the other quantities are positive.

C.2. A Working Equation

We initially hang just enough mass m on the string to overcome friction so that we can eliminate W_f from Equation 5 *i.e.*, we separate the potential energy into two parts,

$$\Delta U_W = \Delta U_m + \Delta U_M = mgy + Mgy \qquad (6)$$

where m is the small mass necessary to overcome friction (*so that* $\Delta U_m = W_f$) and Mg is the weight that accelerates the wheel. Equation 5 then reduces to

$$\Delta U_M + ½(M+m)v^2 + ½I\omega^2 = 0 \qquad (7)$$

If for simplicity we take y to be positive in the downward direction, we have

$$-Mgy + ½Mv^2 + ½I\omega^2 = 0, \qquad (8)$$

where we should in principle incorporate the small mass m into M for the kinetic energy term but will in fact ignore m altogether since it should be negligible compared to M.

Substituting $\omega = v/r$ into Equation 8, we obtain

$$-Mgy + ½Mv^2 + ½I\,(v/r)^2 = 0\,, \qquad (9)$$

$$\Rightarrow Mgy = ½[M + I/r^2]v^2, \qquad (10)$$

or

$$gy = ½[1 + I/Mr^2]v^2 . \qquad (11)$$

(remember that M is the hanging mass, and I is the moment of inertia of the wheel)

Considering that *Logger Pro* will actually only collect data at discrete times and distances described as Δt and Δs, with the measurements numbered by an index i,

$$v \approx \Delta s/\Delta t , \qquad (12)$$

(Note: *Logger Pro actually uses a more sophisticated calculation to determine the velocity, but we shall use Equation 12 in these calculations*.)

D. Monte Carlo Simulation

The "Monte Carlo" (MC) technique is based on the use of random-number-generation algorithms to simulate the randomness of events in real world experiments. Large, complex experiments, including many performed in particle physics, may make extensive use of Monte Carlo programs to estimate experimental yields and uncertainties and to provide simulated (*fake*) data for debugging analysis programs.

You will use a Monte Carlo calculation in *Origin* to create simulated data for a system for which you know the answer in advance. Your simulation will include an artificial "smearing" of the raw data to allow you to take into account the measurement uncertainties in your analysis. You will then write a routine to analyze the simulated data. You can tell that your program is debugged when the result it gives you for the moment of inertia is consistent with the value you chose as the known input to your simulation. After the routine has been debugged with the simulated data, you will use your routine to analyze your actual experimental data.

D.1. Generating Simulated Data

In order to simulate an experiment realistically, you must first estimate the value of the quantity you wish to measure. <u>**Make a rough estimate (*J*) of the moment of inertia of the wheel**</u>**.** (*We use "J" rather than "I" in Origin because the program reserves "I" for row numbers*.) Remember, $I_{disk} = ½M_{disk}R^2$ and $I_{ring} = M_{ring}R^2$ so the moment of inertia of the wheel is probably somewhere between these two limits. Don't spend much time on this; an accurate estimate is not required. You won't need to estimate the uncertainty in J.

D.1.a. Generating time data without random errors.

From the FILE / OPEN menu in *Origin*, open the *program file rke_mc.opj* which you should be able to find on the Wertsrv P: drive or on your local hard disk. **Select WINDOW / SCRIPT WINDOW to pop up a window labeled *Script Window.* From within *Script Window*, use FILE / OPEN to open *rke_mc.ogs*** (*which you may also find on the server or on your local hard disk*). This script is a short program that generates data for successive time intervals, *dt0*(Y), given the known displacements for each measurement ($Y(x) = i\Delta s = 0.015\ i$).

Expand this window and examine the equations for y and Δt_0 for columns COL(Y) and COL(DT0) of the *Origin* worksheet. **Copy the equations into your notebook.** <u>**In your paper, you should derive the equations used in this script to calculate *dt0*.**</u> (HINT: *Examine Eqs. 10-12 as well as the equation entered in the script window*).

To run the script program, use the mouse to select everything below the dashed line and hit ENTER. You will be asked to type in several numbers to initialize the calculation; these are your birth date in MMDD format (*i.e, if your birthday is July 17th, type in 0717; this number is used as an input 'seed' for the random number generator*), **your rough estimate of the moment of inertia, *J*, of the *Roto-Dyne* wheel, and the value *delT = 0.0002*** which corresponds to an estimate of the uncertainty of the timing accuracy of the *Logger Pro* hardware and software. Be sure to record in your notebook the numbers you use for this step. (*For more information on the use of the Script Window in Origin, refer to Appendix IV, Sect. D.*)

Before continuing, you should **rename and save your *Origin* project into your own folder. To protect against losing your work, save it again after completing each important step.**

D.1.b Simulating random errors

In this step you are "smearing" each of the calculated values with an artificial uncertainty, as illustrated in Figure 3. You simulate the measurement uncertainties in the time intervals by *randomizing* Δt ; *i.e.*, adding to the calculated values Δt_0 a random number Δt_G drawn from a Gaussian distribution G with mean 0 and standard deviation equal to the estimated measuring uncertainty $\delta_{\Delta t}$ to create Δt_R. With a good random number routine, each "*measurement*" has a different "*error*" added to it just as each real measurement differs from the "*true*" value by a random amount.

To perform the calculation of the randomized Δt column you will use the formula *dtr = dt0+delT*grnd*(), where *grnd* is a Gaussian random number generating function. **Right click on the top of the third column, *dtr*(*Y*) and select SET COLUMN VALUES.** You should see that the necessary formula for performing this calculation was already entered when you loaded the files specified earlier in this write-up. The value *delT* = 0.0002 is the uncertainty $\delta_{\Delta t}$ in Δt which you typed in when you ran the Script Window program.

Click on OK and note that *Origin* calculates a set of values for this column. Record the first three of these values; then repeat this process, making sure "recalculate" is set to "none", right clicking on the column, picking SET COLUMN VALUES and pressing OK. Again, record the first three values; observing that they have changed. Repeat two more times so that you **have recorded the first three values a total of four times.** Note that every time you click *OK*, you get a different set of numbers *dtr*, just as every time you repeat an experiment you get different results.

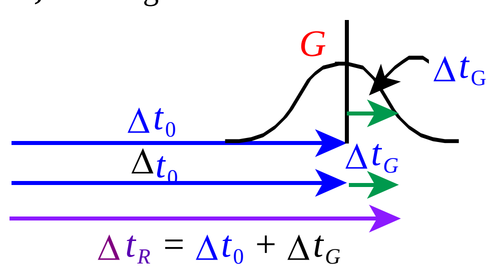

Figure 3. Addition of the random error term Δt_G to the value of Δt_0 calculated from Eq. 13. Δt_G represents a number drawn at random from the Gaussian distribution G with mean 0 and standard deviation $\delta_{\Delta t}$.

D.2. RKE MC Analysis

The basic idea is to **plot v^2 versus y, fit a straight line to the plot and calculate the moment of inertia I from the slope.** The columns you need for the next two steps have been created for you in the *Origin* project file. However you must enter the formulae that *Origin* requires to make the necessary calculations. You can enter the formulae by right clicking on each column in turn and using the SET COLUMN VALUES option.

Calculate in *Origin* a column of squared velocities v [Equation 12]. The formula you need for this calculation is

(0.015/col(dtr))^2 .

Next, calculate a column that contains the uncertainties (δ_{v^2}) in v^2 resulting from the uncertainties $\delta_{\Delta t}$ in Δt. Use the derivative method to write the equation you use for calculating these uncertainties (*and provide this equation in your paper*). (P115 students *may* use the computation method to write this equation. The Lab Director thinks the derivative method will be easier.) You should assume that the errors in the displacements are negligible while the errors in the time are 0.0002 sec (*which you will later change to* 0.00005 *sec for your experimental data*). **Plot v^2 (ordinate) vs. y (abscissa) with uncertainties in v^2 and fit a straight line ($v^2 = A + By$) to the data** as illustrated in Figure 4. **Clean up the plot** (*reduce the datapoint size to about 3, paste in the fit parameters with proper significant figures and units and label the axes appropriately*) before saving and printing it.

To calculate the moment of inertia *I* of the *Roto-Dyne* from this data, **you must first rewrite Eq. 11 to express v^2** (*dependent variable*) **as a function of *y*** (*independent variable*). Then **write an expression for the slope *B* of your fitted line in terms of the constants *I, M (the hanging mass)*, and *r*, and solve to obtain an expression for *I* in terms of *B, M,* and *r*. Show your derivation in your paper**; the result should be

$$I = Mr^2\left[\frac{2g}{B} - 1\right]. \qquad (13)$$

<u>**Calculate *I* and the uncertainty in *I* resulting only from the uncertainty in the slope *B*. Compare your calculated value of *I* to the value *J* used as input to the Monte Carlo calculation.**</u> If your calculations are correct, the two values will probably agree to 1 or 2 standard deviations. If they disagree by more than about 3 standard deviations, your analysis routine probably has an error. Correct the error before proceeding with the experiment.

The MC generation and analysis must be completed and initialed by a TA before you begin to analyze your experimental data. [*Once you have written and debugged the MC routine with simulated data, it should be a simple matter to analyze both sets of experimental data*.]

E. Measurements

<u>**Start by measuring the radii, *k*, of the wheel loads** (*and stating your method of measurement and justification for your estimate of uncertainty*)</u>, which should be installed at their outermost positions. Check that the cable from the encoded pulley is plugged into the jack on your workstation. <u>**Each group must record, save, and analyze 2 separate sets of measurements, one with and one without the 4 mass loads.**</u>

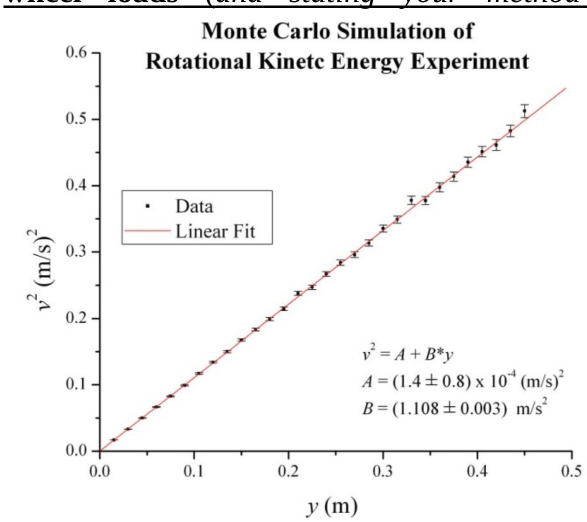

Figure 4: Monte Carlo Data, delT=0.0002 s.

Start *Logger Pro* and open the file *P:Logger Pro 3__Mech Labs\RKE*. To compensate for the effect of kinetic friction, **add small hanging weights until you find that the wheel rotates at constant speed when you give it a gentle push**. You may use *Logger Pro* to monitor the velocity. The required mass *m* is very small - **use paper clips for masses** in this part. When you have this adjusted optimally, the change in potential energy of these falling masses is equal to the work required to overcome friction. These masses will remain in use throughout the experiment. <u>**Record these masses**</u>**, but do not use them in the calculations**. (*There is a small flaw in this procedure - the paper clip masses will gain kinetic energy if they accelerate while falling. Fortunately, you should find that these clips are much less massive than the M = 60 g mass that you will also hang on the string. You may ignore this error*.)

Hang an additional mass *M* of 60 g on the string, release the mass and record the fall, trying to stop the wheel and *Logger Pro* before the mass hits the ground. Repeat a few times, examining the *Logger Pro* plots after each trial until you are sure that you are getting good data. **Save a good set of data to a file and export it to ASCII for analysis in *Origin*.**

If the loads are not on the wheel, mount them at the outermost positions, radius *k* (see Figure 2); if they are already on the wheel, remove them. **Repeat the measurement**, <u>first checking to see if you need to vary the number of hanging paper clips when the loads are removed or installed.</u>

F. Analysis

You should now apply your Monte Carlo *Origin* analysis procedures to your experimental data, but first be sure you've saved your MC worksheet so as not to lose your work. Then **open up a new *Origin* project and import your time intervals for one of your experimental runs from *Logger Pro.* Delete any obviously bad data at the beginning and end of the worksheet, then add new columns for v^2 and *del*v^2. Set the values in these columns using the appropriate formulae, but decrease the uncertainty in the time intervals to δ_t = 0.00005 s.** (*For the Monte Carlo calculation, we exaggerated this uncertainty to display better its effect on the scatter of the "measured" points.*)

Plot v^2 vs. *sDist(y)* with error bars for v^2. Note that your LoggerPro data does not specifically give you the time intervals, Δt, which means that you cannot use the same equation for the error in v^2 that you calculated in the Monte Carlo simulation. There are various ways to go around this little setback. You can either have Origin calculate the Δt's for you in a new column, or use the uncertainty equation from the Monte Carlo simulation and perform a little algebraic trick on it:

$$\delta v^2{}_t = 2\frac{\Delta s^2}{\Delta t^3}\delta t = 2\frac{\Delta s^2}{\Delta t^3}\frac{\Delta s}{\Delta s}\delta t = 2\frac{\Delta v^3}{\Delta s}\delta t\ .$$

Fit a straight-line (v^2 =*A* +*By*) to the data points. Be sure to use ANALYSIS/ FITTING/LINEAR FIT and *not* TOOLS/LINEAR FIT; the two procedures are not equivalent. Check the value of the standard deviation *SD* of the fit in the *Result Window*. It should be approximately 1, indicating that the uncertainties in v^2 are consistent with the scatter of the data points. Save this *Origin* project using a new filename.

<u>Repeat this process for your other set of experimental data and include both graphs, with fits and fit parameters, in your paper.</u>

Calculate separately I_1 (*from your measurement with mass loads*) **and I_2** (*for the wheel without mass loads).* Note that only *B* is different in Equation 13 for the two measurements of *I*. **<u>Calculate the experimental value of the moment of inertia of the mass loads alone</u>**

$$I_E = I_1 - I_2. \qquad (14)$$

F.1. Uncertainty in I_E

Refer to Equation 13. For the analysis of the Monte Carlo data, we considered only the uncertainty in *B*. For the analysis of the measured data, we are interested in determining the uncertainty in I_E produced by the uncertainties in all the measured variables (*B and r; assume negligible uncertainty in M*).

Rewrite Equation 14, substituting the full expressions from Equation 13 for I_1 and I_2 as functions of B_1 and B_2. <u>Find the uncertainty in I_E from this expression directly</u><u>, not by finding the uncertainties separately in I_1 and I_2</u>. This procedure must be followed because I_1 and I_2 include the common variable *r*.

[*When doing error calculations, it is important to watch out for correlations among the terms. Often the only way to be certain that the procedure is correct, is to combine all terms into a single expression or routine before calculating the final uncertainty. As a trivial example, consider the calculation of y=2+r-r, where we have r =3±1. Find y and its uncertainty. Clearly, y=2 and has no uncertainty since it is not a function of r. However, if we were to apply the error propagation equation blindly to each term we would conclude that y = 2± √2, which is clearly wrong*.]

F.2. Comparison with Predicted Value

<u>Assume that the four loads can be treated as point masses and calculate a predicted total moment of inertia I_P from Equation 1. Find the uncertainty in I_P from your estimate of the uncertainties in the masses of the four loads and in their radial position *k*.</u>

Now that you have values for I_E and I_p and for their uncertainties, **<u>compare these two quantities</u>.** Do they agree within estimated errors? (*With more sophisticated methods of statistical analysis, it is possible to give 'odds' on whether these two values really do describe the same physical system. This is true whether or not the values agree within their error bars.*) <u>Remember:</u>

You must show all equations and derivations in your paper.

G. Report Hints

Unlike with the previous lab report, you must write up the full paper this time. In addition to the Abstract and the Results and Conclusions that you also had to write for the previous lab, that includes the Introduction/Theory, Procedure with apparatus diagrams, and Analysis/Error Analysis sections. You may split these up based on which part of the experiment you are doing, or split within the listed sections.

G.1. Introduction/Theory

The Introduction/Theory section is perhaps the easiest section to write. You must show your starting equations and how you get to the final ones that you use for your analysis. You need not show all of the math, but you must at least explain your method. This is also the section to show your experimental procedures and Apparatus. Data collected can be put into another section. Make sure you properly reference your graphs, where appropriate.

G.2. Analysis

In the Analysis section, you must show all of your work necessary to reach a conclusion. All the equations, derivations, and additional manipulations of equations have to be included here, and have to be your own work. Make sure you show all of your Error Analysis in this section. You should also use the end of this section to put in a small amount of your conclusions based on values you just found.

G.3. Additional Information

You must submit the entire written report to TURNITIN.COM through Canvas. Make sure that you properly acknowledge all sources that you used to reach your conclusion, including the lab manual. Partners may share graphs and data, but must write their own report, with their own derivations and calculations. See Appendix II for more hints and samples.

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