Cycle 1 Review Sheet, Part 3

06 February 2024

Rotational Motion Analogy Table:

We introduce the idea of *rotational* (spinning) motion, and how this motion could be described in a way that is analogous to the *translational* motion we have already talked about in the class. Here is a table indicating some of these important analogies:

Quantity of Motion	Translational Motion	Rotational Motion
"Position"	x (distance)	θ (angle)
"Velocity"	$v = \frac{dx}{dt}$ (linear velocity)	$\omega = \frac{d\theta}{dt}$ (angular velocity) (also called angular frequency)
"Acceleration"	$a = \frac{dv}{dt}$ (linear acceleration)	$\alpha = \frac{d\omega}{dt}$ (angular acceleration)
Newtons 2nd Law	$F = ma$ (force law)	$\tau = I\alpha$ (torque law)
Momentum	$p = mv$ (linear momentum)	$L = I\omega$ (angular momentum)
Kinetic Energy	$K_{linear} = \frac{1}{2}mv^2$	$K_{rot} = \frac{1}{2}I\omega^2$

Angular Velocity

Angular velocity ω is the rate of change of angle:

$$\omega = \frac{d\theta}{dt}$$

The units of ω are radians/second. Sometime we call ω the angular frequency.

In the *special case where ω is constant* we can define a few other useful quantities: Just as for uniform circular motion, we can relate ω to a rotational frequency:

$$f = \frac{\omega}{2\pi}$$

and the period of rotation:

$$P = 1/f = \frac{2\pi}{\omega}$$

Angular Frequency

This term is completely synonymous with the term Angular Velocity.

Angular Acceleration

Angular acceleration is the rate of change of angular velocity:

$$\alpha = \frac{d\omega}{dt}$$

Note that since ω is really the magnitude of a vector, $\vec{\omega}$, then angular acceleration is also a vector:

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

The Definition of Torque

Torque is the quantity that when applied to a body results in angular acceleration. In other words, torque is applied to get something spinning.

The torque is defined as being proportional to both the Force and the radius. Here "radius" means the distance from the pivot point (axis of rotation) to where the force is applied. Torque depends only on the component of force that is perpendicular to the radius line. In the special case that the force F is applied perpendicular to the radius, then

$$\tau \equiv Fr \qquad \text{when } \vec{F} \perp \vec{r}$$

If the force is not applied perpendicularly, then the torque is determined by the component of the force that is perpendicular:

$$\tau \equiv F_{\perp} r$$
$$\tau \equiv Fr \sin\theta$$

where θ is the angle between the "radius vector" (the line from the point of rotation to the point of force application) and the force vector.

Note also that vector operations give us a more compact definition of torque as a vector quantity. The torque due to a given force can be described as a *vector cross-product*:

$$\vec{\tau}_{\vec{F}} \equiv \vec{r} \times \vec{F}$$

where $\vec{r}$ is the *position vector* from the "pivot point" to the place where the force is applied to the body. You are free to define the pivot point at any place you want.

Newton's Second Law applied to Rotational Motion:

The analog of $F_{net} = ma$ is:

$$\tau_{net} = I\alpha$$

where I is the analogy to inertial mass, called the *rotational inertia*. Your textbook calls this quantity the *moment of inertia*. As a rule the moment of inertia is proportional to some factor times MR^2. The factor depends on the shape of the body.

The Vector Nature of Rotational Quantities

Note that all rotational quantities like Torque are really a vector quantities. They point “along the axis of rotation” in accordance with the Right-Hand-Rule. We can write:

$$\vec{\tau}_{net} = I\vec{\alpha}$$

where both $\vec{\tau}$ and $I\vec{\alpha}$ point along the axis of rotation.

Angular momentum

By analogy we can construct the quantity of angular momentum:

$$L = I\omega$$

The law of conservation of angular momentum says that in an isolated system, the total angular momentum of all bodies is the same at all times:

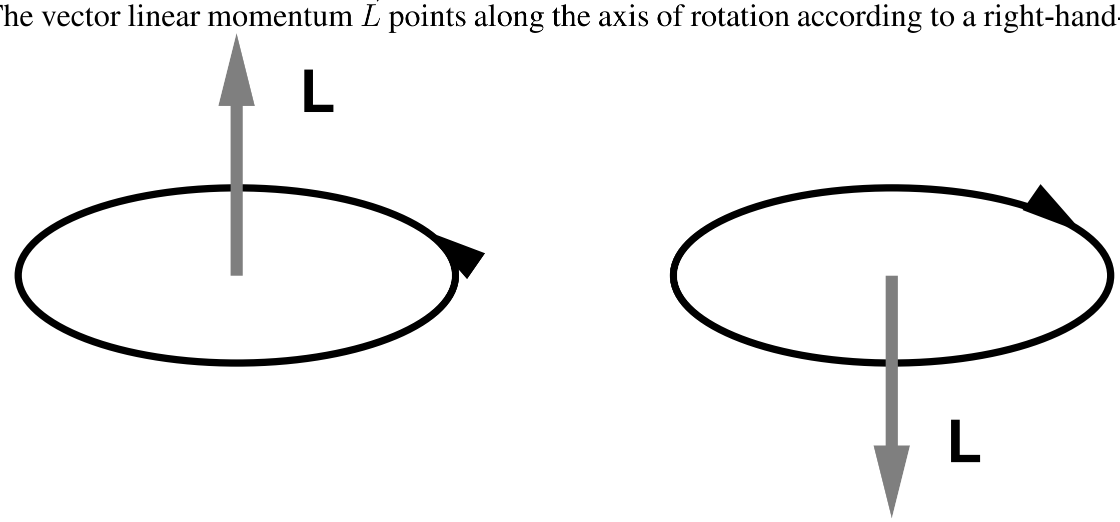
$$BEFORE = AFTER$$

$$L_{tot} = L'_{tot}$$

We note that, like linear momentum, angular momentum is a *vector* quantity. In other words, we can write:

$$\vec{L} = I\vec{\omega}$$

The vector linear momentum $\vec{L}$ points along the axis of rotation according to a right-hand-rule:



Extended Free-Body Diagrams:

In order to apply **Newton's Second Law for Rotational Systems** to actual rigid bodies we *extend* the recipe that we developed for applying Newton's Second Law as follows:

1. First, before considering rotations, follow the usual recipe for applying Newton's Laws for Translational motion. In other words, for each body, consider the translational acceleration, and for each body make sure that you have a correct and proper **Free Body Diagram**. Then go ahead and apply **Newton's Second Law** for each body, one coordinate at a time in the usual way. I strongly recommend that students first work through all of the issues related to the bodies' translational motion before even considering the rotational motion.

2. For any bodies that rotate (instead of and/or in addition to translating), you should also draw an **Extended Free Body Diagram** (sometimes called an XFBD).

3. Your XFBD should include an outline indicating the shape of the of the rigid body. You should place each of the forces that are **already on your regular FBD** onto your XFBD. There should be a one-to-one correspondence between the forces on your FBD and your XFBD. The tail of each force should be placed on the point of application (point of contact) on the object. For the force of Weight, the application point is the Center-of-Mass. For ropes on a pulley, the application points are those points where the rope comes into contact with the pulley.

4. You should mark the location of the **pivot point** that you will choose for this problem. You are free to choose any pivot point you like. However, if the system is actually rotating about a fixed axis, choose this point. If the system is static (not moving) you can choice any point you like. However, if you choose a point that is co-located with application point of one or more unknown forces, then you might simplify the algebra.

5. You also want to define a "rotation-direction". For bodies that are rotating on a fixed axis, we can usually just define either "clockwise" or "counter-clockwise" as the direction of positive torque. If the object has angular acceleration, it is usually a good idea to define the direction of angular acceleration as positive. If the object is static then we are free to choose, but the standard convention is to choose counter-clockwise as positive.

6. Now write down Newton's Second Law in rotational form:

$$\tau_{net} = I\alpha$$

7. If α is known or zero then plug that in.

8. Now **use the XFBD to calculate the torque due to each force.** Be sure to work out whether the torque is positive or negative. Add up the terms to obtain the left-hand side of the equation.

Statics:

Statics refers to a class of physics problems where the body is not moving at all. In this case, we can immediately apply **Newton's Second Law** for both linear (translational) and angular (rotational) motion, so that we can say that both the net Force and and net Torque are zero.

$$\vec{F}_{net} = \Sigma \vec{F}_i = 0$$
$$\vec{\tau}_{net} = \Sigma \vec{\tau}_i = 0$$

To solve these problems we follow the recipe above two steps. First we use a regular *Free Body Diagram* (FBD) and we write down the net forces in each component. This allows us to apply Newton's Laws to the rigid body system. If the body is static the acceleration in any direction is zero. Next we write down an *Extended Free Body Diagram* (XFBD) which indicates the positions on the body where the forces are applied. We also pick a point of rotation (also called pivot point or rotational axis). Based on this, we work on the torque due to each force. If the body is static, the total of all torques is zero. The good news is that for a static problem, we do not need to know the mass or the rotational inertia of the body.

Note that the general problem in statics usually boils down to two equations for each FBD (Newton's Second Law in each of two Cartesian coordinates for each body) and one equation for each of your XFBD (Newton's Law for Rotations). The result will be a series of N equations and N unknowns. Keeping track of your knowns and unknowns is essential.