PHYS 121: Homework #04 and Reading Assignment

February 19, 2024

This Homework is due Monday, February 26 at 11 PM and must be submitted online as a PDF file via Canvas.

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Physics 121: Fall 2023: Week 6 Reading assignment:

REQUIRED: ***Physics 121 Online Class Notes*** **Cycle 2, Chapters 01+ through 04+, posted on the Course Website, as follows:**

- **Skim Chapter 01+**: We will take a closer look at Relative Velocity in multiple dimensions during Cycle 3.
- **Read Chapter Chapter 02+**: This is essentially a review of 1-D Acceleration with some cute examples. Be sure you are able to do the first "throw up" example (starting on page 2-10) and the "catch up" example starting on page 2-12.
- **Read Chapter 03+**: Time to get serious about projectile motion kinematics.
- **Read Chapter 04+**: You want to be sure to have a sense of what Universal Gravity is and how the notion of conservation laws extends to this.
- **Read Chapter 12+**: but for now ***only pages 12-12 to 12-14***. Here your textbook introduces Cartesian Unit Vectors. Yes, we are jumping around a little bit in the textbook during Cycle 2.

OPTIONAL BUT RECOMMENDED: **Read** ***Physics for Scientist and Engineers*** **by Ohanian and Markert** as follows:

- **Read Chapter 3, Selected Sections as follows:**
  - **Review Sections 3.1 through 3.4** This is all the things you should have learned about vectors in your previous math classes. The notion of the "Cartesian basis unit-vectors" (so-called i-hat, j-hat, and k-hat) is something you really want to master.
- **Read Chapter 4, Selected Sections as follows:**
  - **Read Section 4.2** This shows you how to treat vector kinematics component-by-component using unit-vectors. If you are not familiar with applying unit vectors in problems, this is a good introduction.
  - **Read Section 4.3** Compare and contrast this "standard" presentation of projectile motion kinematics with the way this is presented in the Online Notes. The key idea here is that we can break projectile motion into two 1-D components of motion.

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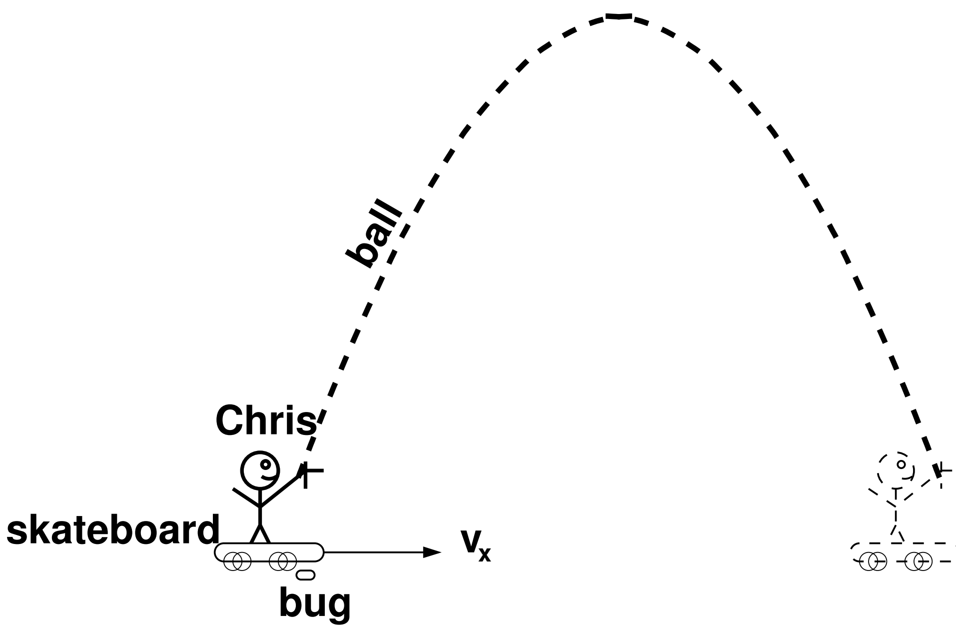
Physics 121: Fall 2024: Homework #04

This homework due just outside of Rock 207: 11 PM Sharp, MONDAY, February 26, 2024

Important note: The homework will be graded on a scale of 0 to 15 points. **Not all problems will be graded.**

Problem 1:



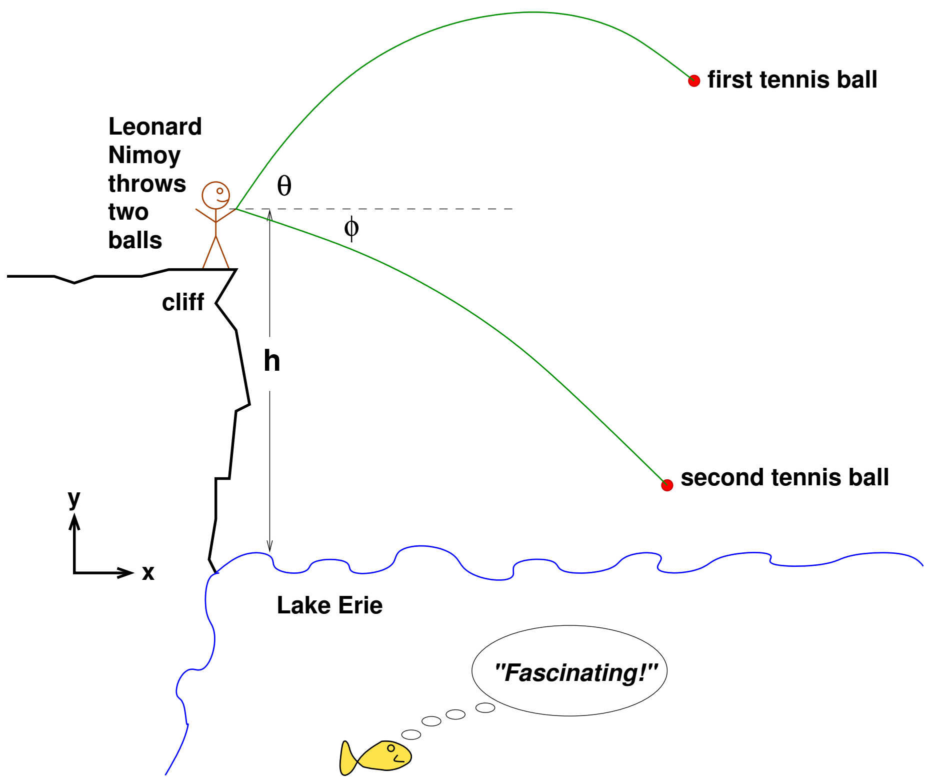
Chris is on a skateboard that is traveling at a constant horizontal speed $v_x = 4.35$ m/s. They hold a ball in their hand at a height of $h = 1.22$ meters above the ground and then they toss the ball "up" (as seen in their frame) so that they catch it back in their hand exactly 2.85 seconds later as show above (figure *not* shown to scale!)

a) ***Relative to Chris' hand***, what is the speed of the ball as it it travels upward? Explain your answer.

b) Relative to the fixed ground, what is the maximum height of the ball in the air? Explain your answer.

c) Suppose there is a tiny motionless bug on the ground immediately below Chris' hand at the instant when they toss the ball upward. Suppose we define standard unit vectors $\hat{\imath}$ and $\hat{\jmath}$ corresponding to the frame of reference for the bug. Write down *vector expressions* for the position, velocity, and acceleration of the ball ($\vec{r}$, $\vec{v}$, and $\vec{a}$) as a function of time, t *as seen by the bug* at the instant of time just before the ball is caught by Chris. Explain your answer.

Problem 2: (from a previous exam)



Leonard Nimoy stands at the edge of a cliff at a given elevation h above the still waters of Lake Erie as shown in the figure. He throws a tennis ball at a given speed v_0 at a given angle of θ above the horizontal direction towards the Lake. Then he throws a second tennis ball at the *same* speed v_0 but downwards at an given angle ϕ below the horizontal. Here assume $\theta > \phi$.

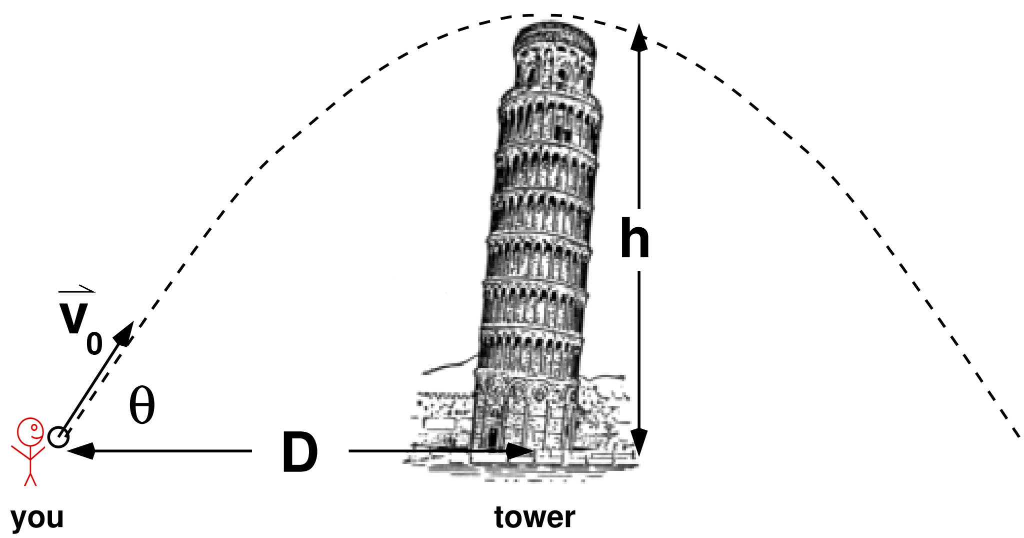
Part (a) – : What is the maximum vertical distance relative to the surface of the Lake that the first tennis ball will achieve? Give you answer in terms of the given parameters (not time). Explain how you got you answer in one or two sentences.

Part (b) – : Write down three *vector expressions* – one for the *position*, one for the *velocity* and one for the *acceleration* – corresponding to the motion of the first tennis ball just at the instant it achieves maximum vertical distance relative to the surface of the Lake. Your answer should be expressed in terms of given parameters only; note that if you have an expression that is a function of time t you are not done. Your expression should include relevant **Cartesian basis unit-vectors.** Explain how you got you answer in one or two sentences.

Part (c) – : The first tennis ball makes a *parabolic curved path* as it travels through the air. Calculate the *radius of curvature* on that path at the single point corresponding to when the tennis ball has maximum vertical distance relative to the surface of the Lake. Your answer should be expressed in terms of given parameters only; note that if you have an expression of time t you are not done. Explain how you got your answer in one or two sentences.

Part (d) – : Which ball (the first ball or the second ball) will have a greater *speed* in the instant just prior to impact with the Lake? Explain your answer in one or two sentences.

Problem 3: Projectile Motion Kinematics (from 2013 Final Exam)



You are standing a given horizontal distance D from the base of the Leaning Tower of Pisa which has a given height h. Assume that your height and the diameter of the tower and the deflection of the tower are all negligible compared to the height of the tower. Neglect air resistance.

At what speed v_0 and at what angle θ relative to the horizontal should you throw the ball so that the ball just barely clears the top of the tower at its maximum height, as shown? Give your answers in terms of given parameters only. Explain your work.

Important Hints: See if you can come up with an expression for the time that the ball takes to get to the top, and then see if you can use this to calculate first v_{y0} and then v_{x0} corresponding to the vertical and horizontal components of the initial velocity vector. Once you have these, work out the magnitude and direction of the initial velocity $\vec{v}_0$.

Problem 4: A Special Kind of Motion

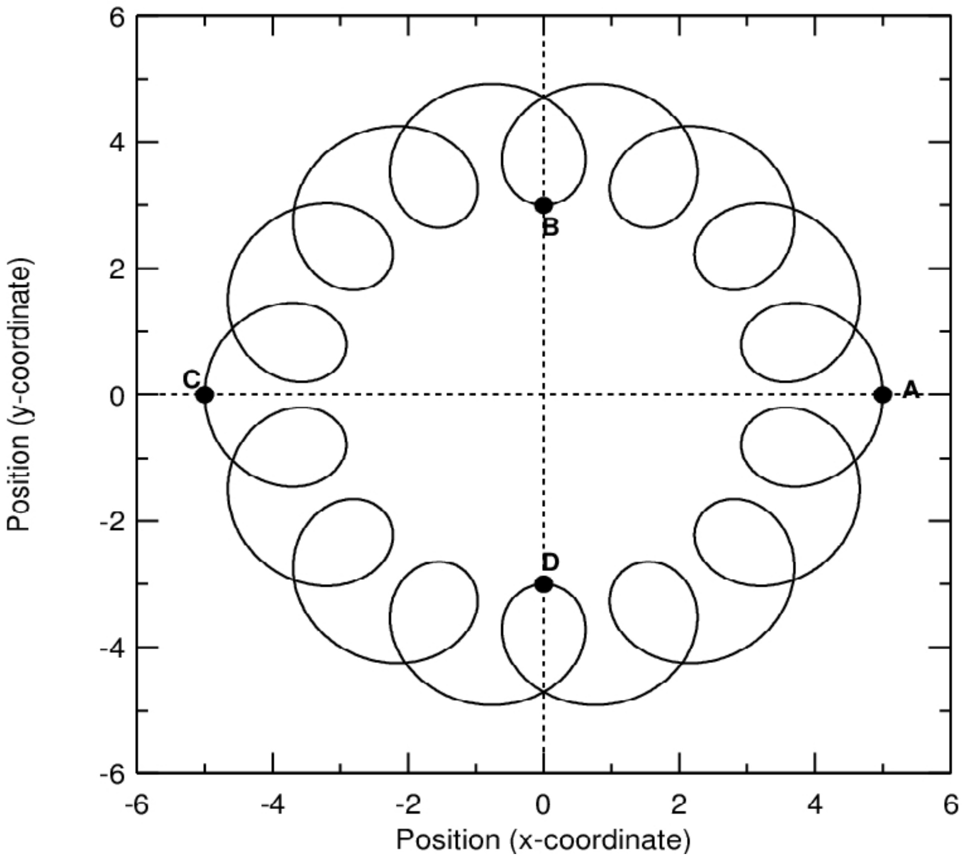
Suppose a particle has a position which is defined by a **position vector** expression as follows:

$$\vec{r}(t) \equiv R\cos(\omega t)\hat{\boldsymbol{\imath}} + R\sin(\omega t)\hat{\boldsymbol{\jmath}}$$

Prove that this represents **Uniform Circular Motion**. In other words, prove that the following three things are true:

- **Prove** that this particle travels on a path that is a *circle* by demonstrating that the particle is always found at a distance R from the origin. Hint: Trig identity: $\sin^2(\theta) + \cos^2(\theta) = 1$ for *any* angle θ.
- **Prove** that this particle has a *constant speed* v, where $v = \omega R$.
- **Prove** that the acceleration vector always has fixed magnitude v^2/R and points toward the center of the circular path.

Problem 5: More Fun with 2-D Kinematics



A particle moves on a 'spirograph' pattern as shown above which is defined by the following parameterization:

$$\vec{r} = [P\cos(\Omega t) + Q\cos(\omega t)]\hat{\imath} + [P\sin(\Omega t) + Q\sin(\omega t)]\hat{\jmath}$$

where $P = 4$ meters, $Q = 1$ meter, $\Omega = 2\pi$ radians per second, and $\omega = 30\pi$ radians per second. The plot shown corresponds to the path of the particle for $0 < t < 1$ second. Note that Point **A** as shown corresponds to the position at both $t = 0$ and $t = 1$ second.

a) What is value of the time t corresponding to the position of the particle at point **B** as shown above? Explain or justify your answer. Hint: you cannot "solve for time" here. A better strategy is to ***guess*** the answer and then ***prove*** your guess is correct by calculating the position at that time.

b) What is the *speed* of the particle at point **B** above? Explain your answer.

c) Write down a vector expression for the *total acceleration* of the particle at at point **B**. Explain your answer.

d) Use your answers to parts (b) and (c) above to calculate the **radius of curvature** of the path at point **B**. Also try to estimate the radius of curvature directly from the graph. Do you get the same approximate result? Explain.