Chapter 8

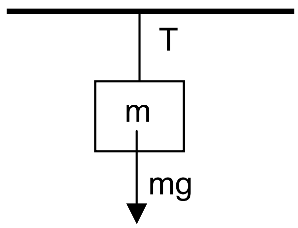
Pulley Systems

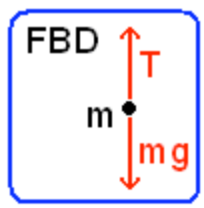
- **Preliminary: Blocks Suspended by Ideal Ropes**
- **Ideal Pulleys for Changing the Direction of a Force**

Blocks Suspended by Light Ropes

Before discussing pulleys, we learn how to handle weights hanging from ropes, since pulleys often are used to lift weights. We'll continue to ignore the rope mass. In the examples below, the FBD's will help us remember all the forces, which are then shoved into the second law:

- **An example with zero acceleration using a weight hanging from the ceiling:**

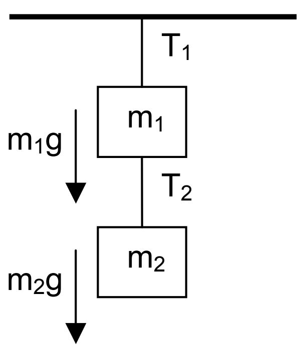


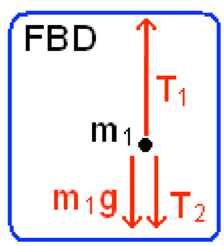


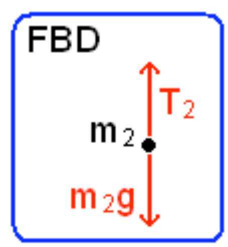
Call up the positive y-direction, and write the second law in terms of magnitudes and explicit signs here and throughout this chapter:

$$\left.\begin{aligned} a &= 0 \\ F_{\text{net, on m}} &= T - mg = ma = 0 \end{aligned}\right\} \quad \boxed{T = mg}$$

- **Another example with zero acceleration but now with two weights -**







Call up positive and write the two second laws:

$$a_1 = a_2 = 0$$

$$m_2: \quad F_{\text{net, 2}} = T_2 - m_2 g = 0 \quad \Rightarrow \quad \boxed{T_2 = m_2 g}$$

$$m_1: \quad F_{\text{net, on 1}} = T_1 - T_2 - m_1 g = 0 \quad \Rightarrow \quad T_1 = T_2 + m_1 g$$

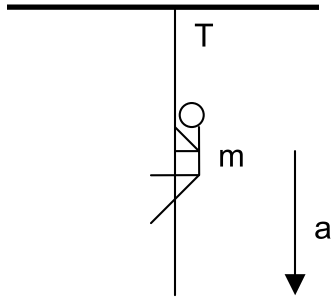
$$\text{with } T_2 = m_2 g \quad \Rightarrow \quad \boxed{T_1 = (m_1 + m_2)\, g}$$

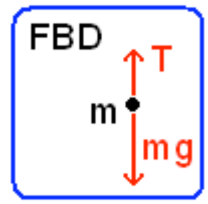
Notice $T_1 > T_2$, as expected, since the upper rope has to support both masses.

Alternatively, we can consider the whole system together (and its FBD) and come up with the equation $T_1 - (m_1 + m_2)\, g = 0$ as a quicker derivation of the $T_1 = (m_1 + m_2)\, g$ result.

• **Another example but with nonzero acceleration:** A child sliding down a rope with acceleration a (a is defined as positive if it is downward) -

now call down positive





With down the positive y-direction, we write the second law:

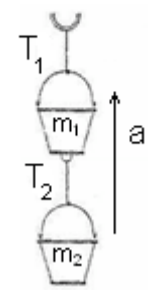
$$mg - T = ma \quad \Rightarrow \quad \boxed{T = mg - ma}$$

Comment: Notice that when a = g, we indeed get zero tension corresponding to "free fall."

Next, we consider pulling two weights upward with a nonzero acceleration in the first problem, and we have fun in the second problem with the difference between a steady pull and a jerk!

****

Problem 8-1 *One paint bucket with mass* m_1 *is hanging by a massless cord from another paint bucket with mass* m_2, *as shown. The two are being pulled by a massless cord attached to the upper bucket upward by a moving support with a given acceleration of* a.



a) *Guess which of the tensions* T_1 *and* T_2 *is larger - or are they equal? As always in these initial guesses, any honest attempt is given full credit.*

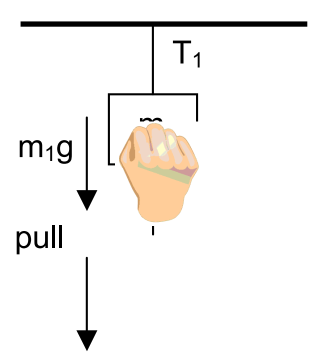
b) *Now calculate the tensions* T_1 *and* T_2 *in lbs. for the two cords by drawing* ***FBD's for each mass, then using the second law to get an equation from each*** *FBD, and finally solving the two equations for the two unknowns* T_1 *and* T_2 *in terms of* a, g, m_1 *and* m_2.

c) *Compare your answer in (b) to your guess in (a). If your guess was wrong, please examine your previous reasoning to try to improve it.*

****

****

Problem 8-2 *A mass* m *hangs from a rope section attached to a ceiling as shown. You grab a hold of another rope section hanging down from* m *with your hand, also as shown. The two rope sections come from the same original rope, which was really a light and rather weak string.*



a) *If you were to gently and gradually pull harder and harder, which section of rope would break first, or would it just be up to chance? Explain your answer.*

b) *If you were to pull downward with a powerful sudden jerk, which section of rope would break first, or would it just be a matter of random chance? Explain your answer.*

Try to do this experiment to verify your answers.

****

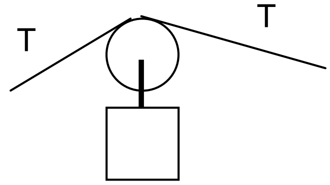
Ideal Pulleys for Changing the Direction of a Force

Recall that ideal ropes (i.e., massless ropes) have the same tension throughout their length.

Now define: $\boxed{\text{Ideal pulleys} \equiv \text{massless, nonslipping pulleys with frictionless pivots}}$

Then we have the following wonderful result. For an ideal rope wound around an ideal pulley, we have the "law" of constant T:

If the tension is T somewhere, it's T everywhere!!

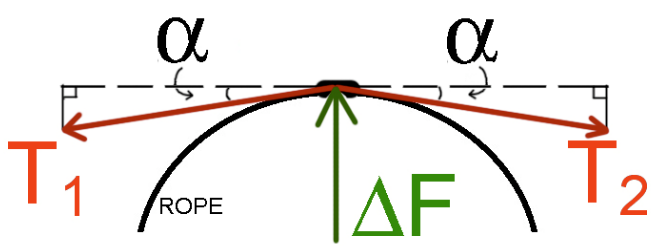


arbitrary angle

Proof:

1) Each little piece of rope feels a perpendicular contact force due to the frictionless pulley (recall that's what frictionless means: the surface can only exert a normal force on whatever is sliding on it). See the force analysis below to help see this:

   Consider a little piece of the rope (its thickness is exaggerated in the figure for convenience of viewing) that is supported by the pulley through a small normal force ΔF as shown. (We've also chosen the little piece to be horizontal for simplicity.)



   The little piece is pulled to the left and right by the tension forces T_1 and T_2 , respectively. The symmetry of the pulley means the angle that T_1 makes with the horizontal is the same as the angle that T_2 makes with the horizontal. So we call this common angle α.

   From the respective golden right triangles, the magnitudes of the horizontal components are $T_1\cos\alpha$ and $T_2 \cos\alpha$. Since ΔF does not have a horizontal component, the horizontal components of the tensions must balance each other: $T_1\cos\alpha = T_2 \cos\alpha$, so $T_1 = T_2$, and we see that the tension is unchanged. This story repeats itself all around the pulley. Thus the direction, but not the magnitude, of the tension is changed.

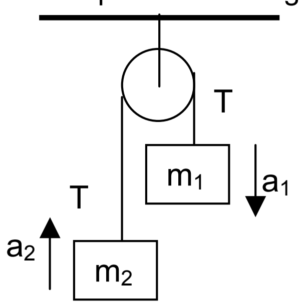
2) And a massless pulley doesn't "eat up" any force as part of an accelerated system. See Ch. 14 when we include mass, and hence when we include "rotational inertia", which leads to a change in tension.

Even in a real pulley situation it is not such a bad thing to consider a pulley as ideal. First, its friction in its pivot is usually small, and so is any rope slippage that might take place. Second, we can neglect its "rotational inertia," or what we'll more commonly call its "moment of inertia," because its mass is usually small compared to the other masses in the system.

A Basic Pulley System

(This is sometimes called a simple "Atwood machine.")

Consider weights on both ends of a rope hung over a pulley. The picture and the individual FBD's are like the child sliding or climbing a rope; the movement of the rope here does not change the force pictures. Using the same T on both sides (!), we have



If we call up positive for a_2 and down positive for a_1 , then the constraint of the whole system moving together is $a_1 = a_2 \equiv a$ (if we had defined them both upward, for example, we would have $a_1 = -a_2$, and one would have to be a negative number – so our choice is for convenience). Notice please that even though we ostensibly start out with magnitudes, we may sometimes get an negative answer for a given quantity. This means simply that that force or acceleration or whatever is pointing opposite to what we assumed.

We will also find it convenient to use the same conventions for the components in the second law equations (i.e., up positive for m_2 and down positive for m_1). Hereafter, we promise not to belabor these convention choices any more than absolutely necessary!

Second-law equations for each mass:

Down positive: $m_1 g - T = m_1 a$

Up positive: $T - m_2 g = m_2 a$

Solve for a by adding:

$$\begin{array}{rrcl} & m_1 g - \cancel{T} & = & m_1 a \\ + & -m_2 g + \cancel{T} & = & m_2 a \\ \hline & (m_1 - m_2)g & = & (m_1 + m_2)a \end{array}$$

(notice we could have gotten this right away by using a whole-system FBD)

Hence: $$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2} g}$$

We can substitute the expression for a into either one of the second-law equations expression to find T in terms of the given parameters. Let's choose the m_1 equation:

$$T = m_1 g - m_1 a = m_1 g - m_1 \frac{m_1 - m_2}{m_1 + m_2} g$$

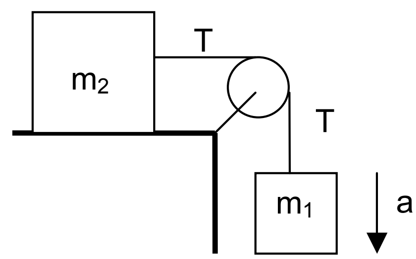
With the common denominator $(m_1 + m_2)$: $$T = m_1 g \left(\frac{m_1 + m_2}{m_1 + m_2} \right) - m_1 g \frac{m_1 - m_2}{m_1 + m_2}$$

Now combine:

$$T = \frac{m_1 (\cancel{m_1} + m_2) - m_1 (\cancel{m_1} - m_2)}{m_1 + m_2} g \quad \Rightarrow \quad \boxed{T = \frac{2 m_1 m_2}{m_1 + m_2} g}$$

****

Problem 8-3



One block with mass m_2 is *pulled across a frictionless table by a massless cord, with tension* T, *draped over a pulley and attached to another block with mass* m_1. *As a result, both blocks are accelerated with the same acceleration* a.

a) *Draw FBD's for both masses.*

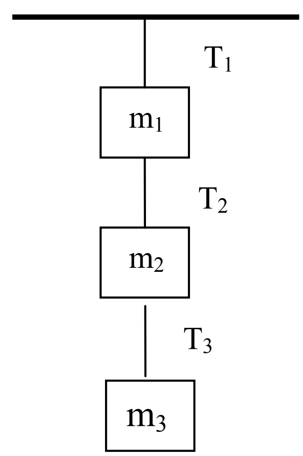
b) *Now apply the second law involving the acceleration* a *to each block and write down two equations for the two unknowns*, a *and* T.

c) *Solve the two equations for the two unknowns.*

****

****

Problem 8-4



Three weights, with masses m_1, m_2 , m_3 , are hung down from a ceiling due to constant gravity g as shown (one directly, the next hung from it, and the third hung from the second). The three rope sections have tensions T_1, T_2 , T_3 , respectively, and negligible masses.

Now you could derive the tensions by drawing FBD's and using the second law for each mass. But that's too much like what we have been doing. So let's do something different.

Continued on the next page

a) If the top rope is cut, guess what happens to the tensions T_2 , T_3 .

Now let's check your guess by careful FBD and second-law steps, using the given parameters m_1, m_2 , m_3 and g, as follows:

b) You may assume that all three masses have the same acceleration a (call up positive). By an FBD for the whole system and a second law, derive this common acceleration a.

c) Now draw an FBD for the system made up of the bottom two masses and derive T_2 from a second law.

d) Finally draw an FBD for the system made up of the bottom mass and derive T_3 from a second law.

How did you do on guessing?

****