Hour Exam 1

Problem 1)

a) Acceleration $= \frac{d}{dt}$(velocity)

= slope of velocity

a_a: (T, V_1), $(0,0)$ $= \frac{V_1 - 0}{T - 0}$ $\boxed{a_a = \frac{V_1}{T}}$

a_b: Constant velocity = 0 acceleration

$\boxed{a_b = 0}$

a_c: $(0, V_1)$, $(T, 0)$ $= \frac{V_1 - 0}{0 - T}$ $\boxed{a_c = -\frac{V_1}{T}}$

b) position $= x_0 + \int v\,dt$

= initial + area under curve

All areas are equal

$$\int_0^T V_a\,dt = \int_0^T V_b\,dt = \int_0^T V_c\,dt = \frac{1}{2}TV_1$$

$$\boxed{\begin{aligned} x_a &= \tfrac{1}{2}TV_1 \\ x_b &= \tfrac{1}{2}TV_1 \\ x_c &= \tfrac{1}{2}TV_1 \end{aligned}}$$

c) N2L Particle a

$F_{Net_a} = m_A a_a$

$F_{Net} = m_A \frac{V_1}{T}$

$$\boxed{F_{Net} = m_A \frac{V_1}{T}}$$

Definition of Work

$W_{\vec{F}} = \int \vec{F} \cdot d\vec{r}$

$= \int_0^{\frac{1}{2}TV_1} m_A \frac{V_1}{T}\,dx$

$W_{\vec{F}} = \frac{1}{2}\cancel{T}V_1\left(m_A \frac{V_1}{\cancel{T}}\right)$

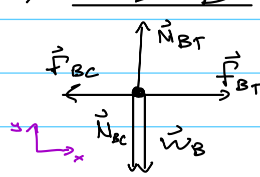
$W_{\vec{F}} = \frac{1}{2} m_A V_1^2$

$m_A = 3M$

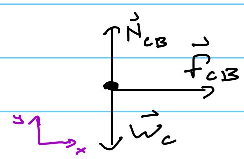
$$\boxed{W_{\vec{F}} = \frac{3MV_1^2}{2}}$$

Problem 2)

a) FBD B



FBD C



b) N2L C x-dir

$F_{Net_x} = ma_x$

$\vec{f}_{CB} = ma_x$

$\mu_k N_{CB} = ma_x$

$a_x = \frac{\mu_k N_{CB}}{m_c}$ stop!

$\rightarrow \boxed{a_{c_x} = \mu_k g}$

N2L C_y-dir

$F_{Net_y} = m\cancel{a_y}$ 0

$N_{CB} - W_c = 0$

$N_{CB} = m_c g$

c) N2L B x-dir

$F_{Net_x} = ma_x$ $a_x = A$ Unknown Constant

$f_{BT} - f_{BC} = mA$

$f_{BT} = mA + f_{BC}$ $\boxed{f_{BT} = mA + \mu_k g}$

N3L $f_{BC} = f_{CB}$

$f_{BC} = \mu_k N_{CB}$

$= \mu_k m_c g$

$$\boxed{f_{BT} = m_B A + \mu_k m_c g}$$

Double check what you plug into the equations you're using!

Problem 3)

a) C of ME conditions: Neglect friction, magnetis conservative, normal force does no work.

Before = After

$E_{Tot} = E'_{Tot}$

$U + K = U' + K'$

$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$

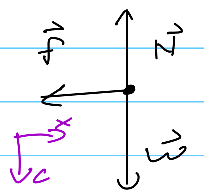
$y = 0,\ v = V_0,\ y' = H,\ v' = v$?

$0 + \frac{1}{2}mV_0^2 = mgH + \frac{1}{2}mV^2$

$\cancel{m}V_0^2 - 2\cancel{m}gH = \cancel{m}V^2$

$$\boxed{V = \sqrt{V_0^2 - 2gH}}$$

b) FBD Top



N2L c-dir

$F_{Net} = ma_c$

$W - N = ma_c$

$W - N = m\frac{v^2}{R}$

$N = mg - m\frac{v_0^2 - 2gH}{R}$

$$\boxed{N = m\left(g - \frac{V_0^2 - 2gH}{R}\right)}$$

~~c)~~ Work Energy Thm

$W_{Tot} = \Delta K$

$W_w + W_f = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$

$W_f = \frac{1}{2}mV^2 - \frac{1}{2}mV_f^2 - (mgy - mgy')$

$= \frac{1}{2}m(V_0^2 - 2gH) - \frac{1}{2}mV_f^2 - mgH$

$= \frac{1}{2}m(V_0^2 - 2gH - V_f^2 - 2gH)$

$= \frac{1}{2}m(V_0^2 - V_f^2 - 4gH)$

$$\cancel{\boxed{W_f = \frac{1}{2}m(V_0^2 - V_f^2 - 4gH)}}$$

No, this problem must considered with entire path to analyze total work done

Option 1

$W_{Tot} = \Delta K$

$W_w + W_N + W_f = \Delta K$

over whole path, w does no work as $y = y'$, and N does no work as argued in (a)

$\therefore W_f = \frac{1}{2}mV_f^2 - \frac{1}{2}mV_0^2$

$= \frac{1}{2}m(V_f^2 - V_0^2)$

$$\boxed{W_f = \frac{1}{2}m(V_f^2 - V_0^2)}$$

That box says you can do it as above but ΔK is wrong!

$W_{Tot} = \Delta K$

$W_w + W_f = \frac{1}{2}mV_f^2 - \frac{1}{2}mV_0^2$

$W_f = \frac{1}{2}mV_f^2 - \frac{1}{2}m(V_0^2 - 2gH) - W_w$

$= \frac{1}{2}m(V_f^2 - V_0^2 + 2gH - 2gH)$

$$\boxed{W_w = \frac{1}{2}m(V_f^2 - V_0^2)}$$

$W_w = -\Delta U_w$

$= mgy - mgy'$

$= mgH - 0$

Option 2

Hour Exam 2

Problem 1) $r(t) = C\cos(\omega t)\hat{i} - 2C\sin(\omega t)\hat{j}$

a) speed $= |v|$, $\vec{v}(t) = \frac{d}{dt}(\vec{r}(t))$

$\vec{v}(t) = -C\omega\sin(\omega t)\hat{i} - 2C\omega\cos(\omega t)\hat{j}$

speed $= \sqrt{(-C\omega\sin(\omega t))^2 + (-2C\omega\cos(\omega t))^2}$

$= \sqrt{C^2\omega^2\sin^2(\omega t) + 4C^2\omega^2\cos^2(\omega t)}$

$|\vec{v}(0)| = \sqrt{0 + 4C^2\omega^2}$

$= \sqrt{4C^2\omega^2}$

$= 2C\omega$

$\boxed{\text{speed} = 2C\omega}$

b) $\vec{a}(t) = \frac{d}{dt}(\vec{v}(t))$

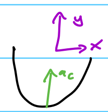
$\vec{a}(t) = -C\omega^2\cos(\omega t)\hat{i} + 2C\omega^2\sin(\omega t)\hat{j}$

N2L $F_{Net} = m\vec{a}$ Point P: $\vec{a}(0) = -C\omega^2\hat{i} + 0$

$\boxed{F_{Net} = -mC\omega^2\hat{i}}$ $\vec{a}(0) = -C\omega^2\hat{i}$

c) $a_c = \frac{v^2}{r}$ $\vec{a} = -C\omega^2\cos(\omega t)\hat{i} + 2C\omega^2\sin(\omega t)\hat{j}$

Point Q $\vec{a}_{Tot} = \vec{a}_{\omega} + \vec{a}_c$ $\vec{a}\left(\frac{\pi}{2\omega}\right) = -C\omega^2\cos\left(\frac{\pi}{2}\right)\hat{i} + 2C\omega^2\sin\left(\frac{\pi}{2}\right)\hat{j}$



Purely Centripidal $= 2C\omega^2\hat{j}$

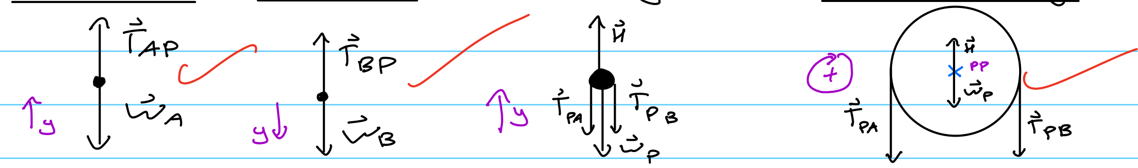
$\vec{v} = -C\omega\sin(\omega t)\hat{i} - 2C\omega\cos(\omega t)\hat{j}$

$\rightarrow \vec{a}_{Tot} = \vec{a}_c$ $\vec{v}\left(\frac{\pi}{2\omega}\right) = -C\omega\hat{i}$

$a_c = \frac{v^2}{r}$ $r = \frac{v^2}{a_c} = \frac{C^{\cancel{2}}\cancel{\omega^2}}{2\cancel{C}\cancel{\omega^2}}$ $\boxed{R_c = \frac{C}{2}}$

Problem 2)

a) FBD A — FBD B — FBD Pully 'P' — × FBD Pully



b) NZL A y-dir

$$F_{Net} = ma_y$$
$$T_{AP} - W_A = ma_y$$
$$T_{AP} - W_A = ma_y$$

Stop!

NZL B y-dir

$$F_{Net} = ma_y$$
$$W_B - T_{BP} = ma_y$$
$$2mg - T_{BP} = 2ma_y$$

Stop!

NZL Rot Pully

$$\tau_{Net} = I\alpha$$
$$\cancel{\tau_W}^{0} + \cancel{\tau_H}^{0} + \tau_{T_A} + \tau_{T_B} = \tfrac{1}{2}MR^2\alpha$$
$$-\cancel{R}T_A + \cancel{R}T_B = \tfrac{1}{2}M\cancel{R^2}\left(\tfrac{a}{\cancel{R}}\right)$$
$$T_B - T_A = \tfrac{1}{2}(6m)a$$
$$3ma = T_B - T_A$$

Stop!

Rolly Constraint

$$\alpha = \frac{a}{r}$$

Kinematic Constraint $a = a_y = a_y = a_A$ move 'together'

$$\begin{cases} T_A - W_A = ma_A \\ 2mg - T_B = 2ma_A \\ 3ma_A = T_B - T_A \end{cases}$$

$$T_B = 3ma_A + T_A$$

$$\begin{cases} 2mg - 3ma_A - T_A = 2ma_A \\ T_A = mg + ma_A \end{cases}$$

$$2mg - 3ma_A - mg - ma_A = 2ma_A$$
$$mg - 4ma_A = ma_A$$
$$\cancel{m}g = 6\cancel{m}a_A$$

$$\boxed{a_A = \frac{1}{6}g}$$

Watch for masses!

c) $T_A - W_A = m\left(\frac{g}{6}\right)$

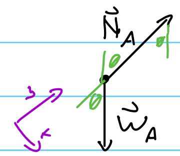
$$T_A = \frac{1}{6}mg + mg$$
$$\boxed{T_A = \frac{7}{6}mg}$$

d) $2mg - T_B = 2ma_A$

$$T_B = 2m(g - a_A)$$
$$T_B = 2m\left(\frac{5}{6}g\right)$$
$$\boxed{T_B = \frac{5}{3}mg}$$

Problem 3)

a) FBD A



N2L x-dir

$F_{Net} = m a_x$

$W\sin\theta = m a_x$

$\boxed{a_x = g\sin\theta}$

b) C of ME conditions: frictionless, normal is ⊥ to motion, weight is conservative

Before = After

$E_{Tot} = E_{Tot}'$

$U_A + K_A = U_A' + K_A'$

$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$

$y = -R\cos\theta,\ v = 0,\ y' = -R,\ v' = ?$

$-\cancel{m}gR\cos\theta + 0 = -\cancel{m}gR + \frac{1}{2}\cancel{m}v'^2$

$gR - gR\cos\theta = \frac{1}{2}v'^2$

$2gR(1-\cos\theta) = v'^2$

$$\boxed{v' = \sqrt{2gR(1-\cos\theta)}}$$

c) $v_{cm} = \dfrac{M\left(\sqrt{2gR(1-\cos\theta)}\right)}{M + 4M}$

$v_{cm} = \frac{1}{4}\sqrt{2gR(1-\cos\theta)}$

$= \frac{1}{4}v_A$

	Before	After
Lab Frame	v_A [A] 0 [B]	$-\frac{1}{2}v_A$ [A] $\frac{1}{2}v_A$ [B]
CM Frame	$\frac{3v_A}{4}$ [A] $-\frac{v_A}{4}$ [B]	$-\frac{3v_A}{4}$ [A] $\frac{v_A}{4}$ [B]

$-v_{cm}$

$+v_{cm}$

Bounce

$v_B' = \frac{1}{2}\sqrt{2gR(1-\cos\theta)}$

C of ME conditions: Weight Conservative, Normal ⊥

Before = After

$E_{Tot} = E_{Tot}'$

$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$

$-\cancel{m}\cancel{g}R + \frac{1}{2}\cancel{m}\left(\frac{1}{4}(2\cancel{g}R(1-\cos\theta))\right) = -\cancel{m}\cancel{g}R\cos\phi$

$-1 + \frac{1}{4}(1-\cos\theta) = -\cos\phi$

$\cos\phi = 1 - \frac{1}{4} + \frac{\cos\theta}{4}$

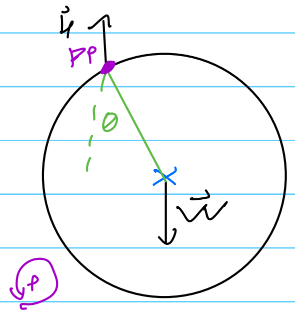
$\cos\phi = \frac{3}{4} + \frac{\cos\theta}{4}$

$$\boxed{\phi = \cos^{-1}\left(\frac{3+\cos\theta}{4}\right)}$$

Hour Exam 3

Problem 1)

a) xFBD



$\tau_{Net} = \tau_H + \tau_W$

$= 0 + RW\sin\theta_0$

$= MgR\sin\theta_0$

$$\boxed{\tau_{Net} = MgR\sin\theta_0}$$

b) $I_{Hoop} = I_{cm} + MR^2$

$= MR^2 + MR^2$

$= 2MR^2$

$\tau_{Net} = I\alpha$

$\cancel{M}gR\sin\theta_0 = 2\cancel{M}R^{\cancel{2}}\alpha$

$$\boxed{\alpha = \frac{g\sin\theta_0}{2R}}$$

c) C of ME Conditions: isolated, Hinge no work on pp, weight cons

Before = After

$E_{Tot} = E_{Tot}'$

$mgy + \frac{1}{2}I\omega^2 = mgy' + \frac{1}{2}I\omega'^2$

$mg(-R\cos\theta_0) + 0 = mg(-R) + \frac{1}{2}(2MR^2)\omega'^2$

$\cancel{mg}R - \cancel{mg}R\cos\theta_0 = \cancel{M}R^{\cancel{2}}\omega'^2$

$g(1-\cos\theta_0) = R\omega'^2$

$$\boxed{\omega' = \sqrt{\frac{g(1-\cos\theta_0)}{R}}}$$

Problem 2)

a) CofME   Conditions: frictionless, Normal ⊥, weight cons!

$$\text{Before} = \text{After}$$

$$E_{Tot} = E'_{Tot}$$

$$mgy + \tfrac{1}{2}Kx^2 + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgy' + \tfrac{1}{2}mv'^2 + \tfrac{1}{2}I\omega'^2$$

$$\tfrac{1}{2}Kx_0^2 = \tfrac{1}{2}mv'^2 + \tfrac{1}{\cancel{2}}\left(\tfrac{\cancel{2}}{3}m\cancel{R^2}\right)\left(\tfrac{v}{\cancel{R}}\right)^2$$

$$\tfrac{1}{2}Kx_0^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{3}mv^2$$

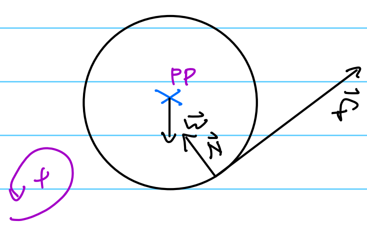
$$\tfrac{1}{2}Kx_0^2 = \tfrac{5}{6}mv^2$$

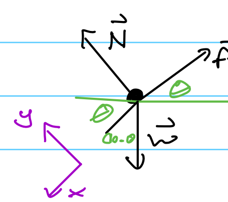
$$\tfrac{3}{5}Kx_0^2 = mv^2$$

$$v'^2 = \frac{3Kx_0^2}{5m}$$

$$\boxed{v' = \sqrt{\frac{3Kx_0^2}{5m}}}$$

b) xFBD Ball   FBD Ball   N2L Ball x-dir





$$F_{Net} = ma_x$$

$$W\cos(90-\theta) - f = ma_x$$

$$mg\sin\theta - f = ma_x$$

N2L Rot

$$\tau_{Net} = I\alpha \qquad \text{R.C} \quad \alpha = \frac{a}{R}$$

$$\tau_w + \tau_N + \tau_f = \tfrac{2}{3}mR^2\alpha$$

$$\cancel{R}f = \tfrac{2}{3}m\cancel{R^2}\left(\tfrac{a}{\cancel{R}}\right)$$

$$f = \tfrac{2}{3}ma$$

$$\cancel{m}g\sin\theta - \tfrac{2}{3}\cancel{m}a_x = \cancel{m}a_x$$

$$g\sin\theta = \tfrac{5}{3}a_x$$

$$\boxed{a_x = \tfrac{3}{5}g\sin\theta}$$

c) $f = \tfrac{2}{3}ma_x, \; a_x = \tfrac{3}{5}g\sin\theta$

$\therefore$

$$\boxed{f = \tfrac{2}{5}mg\sin\theta}$$

d) CofME   Conditions: frictionless, Normal ⊥, weight cons!

$$\text{Before} = \text{After}$$

$$E_{Tot} = E'_{Tot}$$

$$U_{sp} + U_B + K_B + K_{Rot}$$

$$U_{sp} = U'_B + K'_B + K'_{Rot}$$

$$\tfrac{1}{2}Kx^2 = mgy' + \tfrac{1}{2}mv'^2 + \tfrac{1}{2}I\omega'^2$$

$$Kx^2 = 2mgh + mv'^2 + \left(\tfrac{2}{3}m\cancel{R^2}\right)\left(\tfrac{v'}{\cancel{R}}\right)^2$$

$$Kx^2 = 2mgh + mv'^2 + \tfrac{2}{3}mv'^2$$

$$Kx^2 - 2mgh = \tfrac{5}{3}mv'^2$$

$$v'^2 = \tfrac{3}{5}\left(\frac{Kx^2}{m} - 2gh\right)$$

$$\boxed{v' = \sqrt{\tfrac{3}{5}\left(\frac{Kx_0^2}{m} - 2gh\right)}}$$

Problem 3) $F_{app} = B(D^2 - x^2)\hat{i}$

a) $W_{\vec{w}} = -\Delta U_{\vec{w}}$

$= -(mgy' - mgy)$

$= mgy - mgy'$

$\boxed{W_{\vec{w}} = -mgh}$

b) $W_{\vec{F}} = \int \vec{F} \cdot d\vec{r}$

$W_{F_{app}} = \int_0^D F_{app}\,dx$

$= \int_0^D BD^2 - Bx^2\,dx$

$= \left[BD^2x - \frac{B}{3}x^3\right]_0^D$

$= BD^3 - \frac{1}{3}BD^3$

$\boxed{W_{F_{app}} = \frac{2}{3}BD^3}$

c) Work Eng Thm.

$W_{Tot} = \Delta K$

$W_f + W_w + W_{F_{app}} = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2} \; 0$

$W_f = \frac{1}{2}mv_f^2 - W_w - W_{F_{app}}$

$W_f = \frac{1}{2}mv_f^2 + mgh - \frac{2}{3}BD^2$

$\boxed{W_{\vec{f}} = \frac{1}{2}mv_f^2 + mgh - \frac{2}{3}BD^3}$