Chapter 10+

Collisions and Momentum Conservation

Revisit:	To-Do:
• Momentum conservation when there's zero net external force	• Elastic 1D collisions • Simple 2D fusion/fission reactions

BACK TO THE FUTURE: Another Sequel

We revisit **momentum conservation**, another tool with which we could predict the future. Recall that we have overall momentum conservation for a system if the **net external force on the system is zero** (or, at least, negligible during the time we are talking about, such as collisions where the violent contact forces are internal AND much larger than outside forces such as gravity). In particular, for two bodies, whatever they are doing to each other, as long as there is no external interaction, we must have their total momentum, $\vec{p}_1 + \vec{p}_2$, the same before and after. If unprimed refers to before and primed after, then

$$\text{momentum before} \equiv \vec{p}_1 + \vec{p}_2 = \vec{p}'_1 + \vec{p}'_2 \equiv \text{momentum after}$$

Let's look at two-body collisions in 1D now.

Elastic 1D Collisions

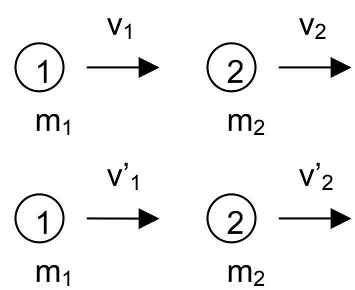
The previous "unstuck" and "stuck" cases were "inelastic," meaning that kinetic energy was not conserved. Now consider 1D collisions (head-on collisions where the bodies move only straight forward or straight backward) between two masses that are **"elastic,"** where **kinetic energy, as well as momentum, is conserved** from the initial state to the final state (as before, we can consider the changes in any potential energies due to any forces before and after collisions to be zero).

e.g. billiards or elementary particles

1D collision:

initial (before collision):

final (after collision):



where we define all v's as positive if going to the right (i.e., all are x-components if the horizontal axis is the x-axis). **E.g., $v_2 < 0$ means that ball 2 is initially going to the left.** This convention will avoid having to remember where to put in minus signs.

Given the initial velocities, the two unknowns (given the initial situation) are v'_1 and v'_2, but we completely determine them by momentum and energy conservation on the next page.

Finding the final velocities: We solve for v'_1 and v'_2**:**

Momentum conservation: $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$ **A**

rewrite this as $m_1(v_1 - v_1') = m_2(v_2' - v_2)$ **A'**

Kinetic energy conservation: $\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$ **B**

Rewrite **B** as $\frac{1}{2}m_1(v_1^2 - v_1'^2) = \frac{1}{2}m_2(v_2'^2 - v_2^2)$ and then rewrite that in factored form

$$\frac{1}{2}m_1(v_1 - v_1')(v_1 + v_1') = \frac{1}{2}m_2(v_2' - v_2)(v_2' + v_2) \qquad \textbf{C}$$

Now, divide **C** by **A'** $\Rightarrow$ masses cancel and velocity difference factors cancel, leaving

$$v_1 + v_1' = v_2 + v_2' \qquad \textbf{D}$$

If we rewrite **D,** we can show that the RELATIVE VELOCITY BETWEEN 1 AND 2 MAINTAINS THE SAME MAGNITUDE BUT CHANGES SIGN AFTER COLLISION:

$$\underbrace{v_1 - v_2}_{\text{v of 1 wrt 2}} = - \underbrace{(v_1' - v_2')}_{\text{v of 1' wrt 2'}} \qquad \textbf{D'}$$

Why do we call it relative velocity? Use our relative velocity notation to check that the left-hand-side, e.g., is indeed the relative velocity:

$$v_{1/2} = v_{1/ground} + v_{ground/2} = v_1 - v_{2/ground} = v_1 - v_2 \qquad \text{Yes! (Etc. for } v_{1'/2'}\text{)}$$

So **A** and **D'** give us the

1D ELASTIC COLLISION MASTER EQUATIONS

$$\boxed{\begin{array}{c} m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \text{(use only for 1D collisions!)} \\ v_1 - v_2 = v_2' - v_1' \quad \text{(use only for 1D elastic collisions!!)} \end{array}} \qquad \textbf{E}$$

Crucial comments:

1) *USE THESE ONLY FOR* ***COLLISIONS THAT STAY IN A STRAIGHT LINE***
2) *the first is just 1D momentum conservation and is OK for any 1D collision, but use the* ***second one only for elastic 1D collisions*** *(i.e., where KINETIC ENERGY IS ALSO CONSERVED)*

(We will come back later to 2D collisions where the bodies can go off at angles!!)

Example:

A 1-kg glider *A* moving at 50 cm/s on an air track collides **elastically** with a 2-kg glider *B* initially at rest (the glider track forces this to be 1D). Starting with the conservation relations, find the velocity of each glider after the collision:

initial: (A) $\xrightarrow{0.5}$ (B)

1 kg 2 kg

The two master equations give

$$1(0.5) + 0 = 1\, v_1' + 2\, v_2'$$

and

$$0.5 - 0 = v_2' - v_1'$$

$$\text{add them} \;\Rightarrow\; 1.0 = 3v_2' \quad => \quad v_2' = 0.33 \text{ m/s}$$

and plugging this into the second master equation result gives

$$v_1' = v_2' - 0.5 \quad => \quad v_1' = -\,0.17 \text{ m/s}$$

The implication for the final picture is

← (A) (B) →

- 0.17 m/s + 0.33 m/s

General Solutions to 1D ELASTIC COLLISION MASTER EQUATIONS

We can solve the master equations once and for all. In the problem that follows, you can practice your algebra skills and show that the velocities after the **one-dimensional elastic collisions** are given by

$$v_1' = v_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right) + v_2\left(\frac{2m_2}{m_1 + m_2}\right)$$

F

$$v_2' = v_1\left(\frac{2m_1}{m_1 + m_2}\right) + v_2\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$

Remember the "right-go" + sign conventions when you use any of these formulas!!!

(and for the umpteenth time, don't you dare use these for two-dimensional collisions when the balls go off at angles in two dimensions!!!!!)

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Problem 10-5

a) Solve once and for all the master equations **E** *on p. 10-12 for* v_1' *and* v_2'*. (This is good algebra practice!) That is, in general, for any head-on, one-dimensional, elastic collision, show that the velocities after the collision are*

$$v_1' = v_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right) + v_2\left(\frac{2m_2}{m_1 + m_2}\right)$$

and **F**

$$v_2' = v_1\left(\frac{2m_1}{m_1 + m_2}\right) + v_2\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$

Remember these are all x*-components (the only components of 1D velocity vectors).*

b) Check these formulas against the answers in the glider example on p. 10-13. We note, however, that it is SAFER to remember the master equations or even the original momentum and energy conservation pair of equations and solve them in a given problem than to try to remember these somewhat awkward formulas!

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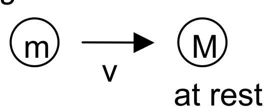
Problem 10-6

Now let's have some fun. Consider a mass m *coming in with velocity* v *hitting head-on into another mass* M *at rest. Consider the collision to be* ***elastic and 1D****. There's no friction or any other external force for that matter and we consider only straight ahead or straight backward motion (you could imagine this as sliding gliders with different masses going straight at each other and colliding on a frictionless 1D track):*



1) Suppose the masses are equal: $m = M$ *.*

a) Guess what happens afterwards – that is, guess the values of both v_1' *and* v_2' *(just use your intuition or pool ball experience) – all answers are given full credit, as usual*

b) Check your answers by using the solutions **F***.*

2) Suppose the second mass is huge: $m \ll M$ *.*

a) Guess what happens afterwards (think of hitting a wall)

b) Check your answers by using the solutions **F** *with* $m=0,\ M \neq 0$ *.*

3) Suppose the second mass is tiny: $m \gg M$ *.*

a) Guess what happens afterwards (think of a moving wall hitting a resting marble and view the collision from the wall's reference frame and then take your result back to the original reference frame)

b) Check your answers by using the solutions **F** *with* $M=0,\ m \neq 0$ *.*

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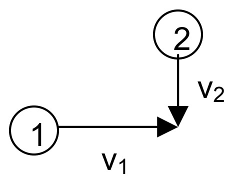
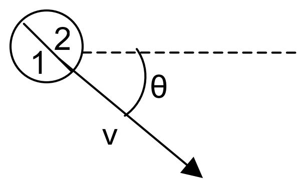
Simple Inelastic 2D Collisions

An **IN**ELASTIC example ("fusion") where kinetic energy is not conserved:

before:  a right-angle example (special case)

after: 

$$\vec{p}_{total,before} = m_1\vec{v}_1 + m_2\vec{v}_2 \quad \text{must equal} \quad \vec{p}_{total,after} = (m_1 + m_2)\vec{v}$$

NOTE: v_i and v are magnitudes (speeds) now! So we need to put in signs by hand.

equating horizontal components: $m_1v_1 = (m_1 + m_2)\, v \cos\theta$

equating vertical components: $- m_2v_2 = - (m_1 + m_2)\, v \sin\theta$

$$\text{divide} \Rightarrow \tan\theta = \frac{m_2v_2}{m_1v_1}$$

$$\text{square and add} \Rightarrow (m_1v_1)^2 + (m_2v_2)^2 = (m_1 + m_2)^2 v^2(\cos^2\theta + \sin^2\theta)$$

$$\text{so} \quad (m_1v_1)^2 + (m_2v_2)^2 = (m_1 + m_2)^2 v^2$$

$$\text{thus we can solve for } \theta, v \text{ given } v_1, v_2, m_1, m_2$$

We see that kinetic energy is not conserved:

$$\text{final KE} = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}\frac{1}{m_1 + m_2}(m_1 + m_2)^2 v^2 =$$

$$= \frac{1}{2}\frac{1}{m_1 + m_2}[(m_1v_1)^2 + (m_2v_2)^2] =$$

$$= \frac{m_1}{m_1 + m_2}\frac{1}{2}m_1{v_1}^2 + \frac{m_2}{m_1 + m_2}\frac{1}{2}m_2{v_2}^2 < \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2$$

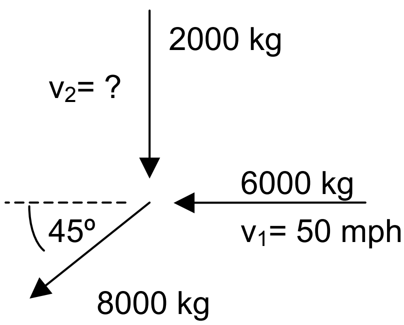
$$\Rightarrow \text{heating loss}$$

An example of this "fusion: coming up on the next page.

Example: In a certain road accident (this is based on an actual case) a car of mass 2000 kg, traveling south, collided in the middle of an intersection with a truck of mass 6000 kg, traveling west. The vehicles locked together and skidded off the road along a line pointing almost exactly southwest. A witness claimed that the truck had entered the intersection at 50 mph.

DO YOU BELIEVE THE WITNESS?

Let us calculate the speed of the car implied by the claim of the witness.



The equations on the previous page apply :

$$\tan 45^\circ = \frac{2000\,v_2}{6000(50)} \Rightarrow v_2 = 150\ \text{mph}$$

which is simply too large! The truck speed v_2 must have been much less!

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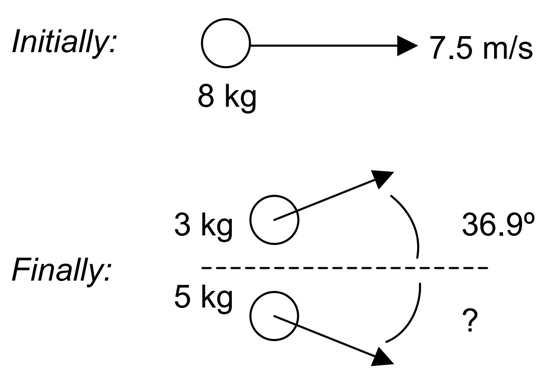
Problem 10-7 *A mass slides without friction on a flat surface with a velocity of 7.5 m/s in the horizontal direction. It is blown apart into two fragments of masses 3-kg and 5-kg. SEE BELOW. After the explosion, the fragments slide along the surface, with the 3-kg mass having a velocity of 15m/s at an angle of 36.9° (chosen for fun to use once again the famous right triangle whose sines and cosines are simply 3/5 and 4/5, respectively) with the +x-axis. Find the magnitude and direction of the velocity of the 5-kg mass.*

*Comment: You can guess that an explosion means there is additional chemical (or nuclear fission) energy added so that mechanical energy is not conserved. The frictionless surface means the mass won't slow down before it blows apart. Even though energy is not conserved during the explosion, the linear momentum along the surface is conserved with no **external** forces present parallel to the surface. And that is robust enough to tell you everything that happens!*



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