

Chapter 5

Contact Forces: Free Body Diagrams and Ideal Ropes

- Pushes and Pulls in 1D, and Newton's Second Law Neglecting Friction
- Free Body Diagrams
- Tension Along Ideal Ropes (i.e., Massless Ropes)
- Newton's Third Law

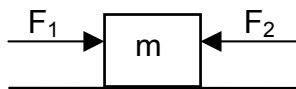
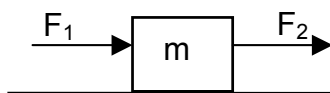
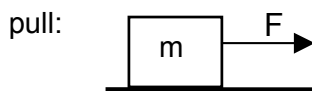
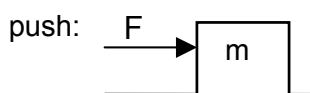
Bodies in Contact as Blocks or ... Points

- BODIES in CONTACT: Consider bodies in contact with each other and the outside world. CONTACTS mean *interactions* and interactions mean *forces*. The BODIES are books, air molecules, horses, trains, etc., and we focus on their 1D motion in this chapter. We won't worry about their being twisted, rotated, squished or stretched — yet.
- BLOCKS: So we will simply view these bodies as BLOCKS. Blocks have mass and the rigid shape of boxes. Analyzing blocks is easy but still very practical and relevant.
- POINTS: Instead of a horse, we have argued that we should use a block picture (which is certainly easier to draw!). Well, for present purposes, we could just as well represent a horse by a POINT (which is really easier to draw) and forget the block altogether!

Single Block Sliding on a Table (Friction Neglected)

Consider several examples of different combinations of forces on a single block, their associated Newton second-law equations, and their fancy-shmancy “free-body diagrams.” All forces are parallel or anti-parallel to the direction of the acceleration a , so this is truly 1D. Our convention often is to write equations in terms of the force magnitudes $F > 0$ rather than the force components F_x which can be either positive or negative. So if a force is in the negative direction, we put the minus sign in explicitly in front of the magnitude.

Plain Vanilla Diagrams (only horizontal forces shown)



Newton's Second Law In terms of force magnitudes

$$F = ma \quad (F_x = F > 0)$$

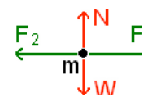
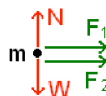
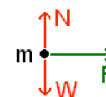
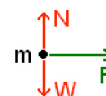
$$F = ma \quad (F_x = F > 0)$$

$$F_1 + F_2 = ma \quad (F_{1x} = F_1 > 0, \\ F_{2x} = F_2 > 0)$$

$$F_1 - F_2 = ma \quad (F_{1x} = F_1 > 0, \\ F_{2x} = -F_2 < 0)$$

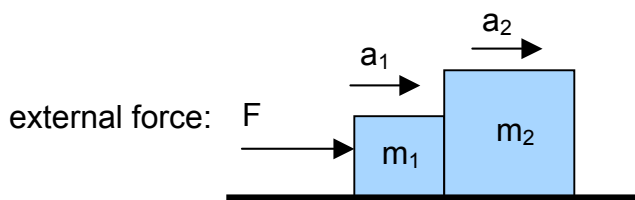
(to the right is the positive x-direction)

Free-Body Diagrams*



*FBD convention: Place tails of all force arrows at the point used to represent the block. While the **vertical forces** don't come into the horizontal force equations, we show them anyway. The support force on the block due to the table is **N** (perpendicular or “normal” to the table – the table or any surface cannot exert horizontal forces until we include friction). In this simple case, the weight **W=mg** of the block must balance **N** by the second law because the block is not even moving, let alone accelerating, vertically. Remember, we can **isolate** any part of a system and draw its FBD.

Two Blocks in Contact and Sliding on a Frictionless Table Due to a Single External Push



- A) A constraint: the blocks move together which means their accelerations are the same. We define this common acceleration to be simply a (we are using magnitudes for acceleration, too, so, for example, $a_{1x} = +a_1$ since block 1 is accelerating to the right, etc.)

$$a_1 = a_2 \equiv a$$

- B) The system's common acceleration a is determined with the second law applied to the two blocks considered together as the overall system. Since we know the net external force:

$$F_{\text{external}}^{\text{net}} = F = (\text{total mass}) a = (m_1 + m_2) a$$

$$\Rightarrow a = \frac{F}{m_1 + m_2}$$

Again, we use “right is positive” once again. Again, we think of the horizontal direction as the x-axis, with the positive x-axis pointing to the right. Again, $F_x = +F > 0$ where instead of components, we use plus or minus magnitudes if the direction is right or left, respectively.

- C) Thus we know the net forces on each individual block. Why? Since we know each block's common acceleration a (from above) along with its mass, then we know the net force on each from the second law:

$$\text{net force on } m_1: F_1^{\text{net}} \stackrel{\text{must}}{=} m_1 a_1 = m_1 a = \frac{m_1}{m_1 + m_2} F$$

$$\text{net force on } m_2: F_2^{\text{net}} \stackrel{\text{must}}{=} m_2 a_2 = m_2 a = \frac{m_2}{m_1 + m_2} F$$

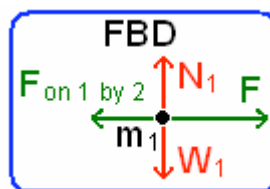
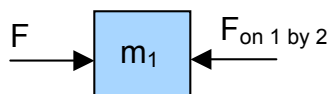
- D) Next we can dig inside and calculate the contact force between the two blocks because, as we will show on the next page, we now know both the net force on each individual block and their common acceleration.

Preliminary remarks:

- We will draw both plain vanilla diagrams and FBD's for each block.
- In all the drawings, we just **guess the arrow magnitudes (and sometimes even the directions!)**. The calculations tell us how big they are, and confirm their directions, afterwards.
- **We don't dare draw any contact force into any picture unless it really IS in contact with the body we're analyzing.**

Calculate the contact force on the first block due to the second block:

First Block (isolated)



The net force on block 1 is: $F_1^{\text{net}} = F - F_{\text{on 1 by 2}}$

From the previous page, we know: $F_1^{\text{net}} = \frac{m_1}{m_1 + m_2} F$

Putting the above together: $F_1^{\text{net}} = F - F_{\text{on 1 by 2}} = \frac{m_1}{m_1 + m_2} F$

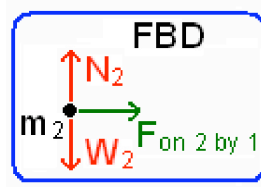
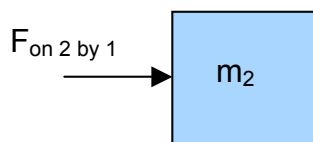
Solve for $F_{\text{on 1 by 2}}$: $F_{\text{on 1 by 2}} = F - \frac{m_1}{m_1 + m_2} F$

Combine terms: $F_{\text{on 1 by 2}} = F \left(\frac{m_1 + m_2}{m_1 + m_2} \right) - \frac{m_1}{m_1 + m_2} F = \frac{\cancel{m_1} + m_2 - \cancel{m_1}}{m_1 + m_2} F$

$$\therefore F_{\text{on 1 by 2}} = \frac{m_2}{m_1 + m_2} F$$

Calculate the contact force on the second block due to the first block:

Second Block (isolated)



**N.B. There is no F here.
It does not act directly on m_2**

The net force on block 2 is: $F_2^{\text{net}} = F_{\text{on 2 by 1}}$

From the previous page, we know: $F_2^{\text{net}} = \frac{m_2}{m_1 + m_2} F$

Putting the above together: $F_2^{\text{net}} = F_{\text{on 2 by 1}} = \frac{m_2}{m_1 + m_2} F$

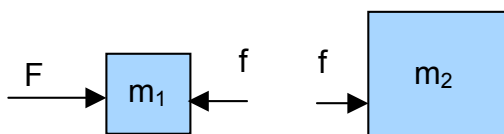
Shedding light on Newton's third law through this example: From the results on the previous page, we find the contact forces $F_{\text{on 1 by 2}}$ and $F_{\text{on 2 by 1}}$ have the same magnitudes:

$$F_{\text{on 1 by 2}} = \frac{m_2}{m_1 + m_2} F = F_{\text{on 2 by 1}}$$

While their magnitudes are the same, notice in the figures that the directions of the two forces are drawn in opposite directions. It is also important to note that the two contact forces act on **two different bodies**: one on block 1 and the other on block 2. **This is all consistent with the third law, which says that the contact force on block 1 due to block 2 must be equal and opposite to the contact force on block 2 due to block 1.**

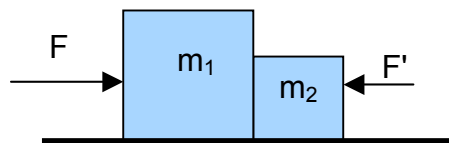
The third law comes out automatically because we assume the only contribution to $F_{\text{external}}^{\text{net}}$ is F , which carries the hidden assumption that internal forces cancel.

Shortcut notation: From now on, automatically put the contact forces between blocks equal and opposite, such as shown in the following plain vanilla diagram, and make your life easier:



As in the above example, F is NOT directly applied to the second block, so PLEASE PLEASE draw f but NOT F on m_2 in the above diagram. If F causes f and if the **whole system accelerates**, F is partly “eaten up” by m_1 on its way to INDIRECTLY accelerating m_2 .

Problem 5-1 Two blocks with masses as shown and on a frictionless horizontal surface are in contact with each other. If an external force (magnitude) F is applied as shown on the left side of block 1, and another external force (magnitude F') pushes back in the opposite direction on the right side of block 2,

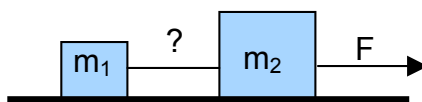


- Draw the FBD* with both horizontal and vertical forces shown (**always show all forces in FBD's**) for the whole system as a “point” and use its horizontal forces to determine the acceleration of the system in terms of F , F' , m_1 , and m_2
- Determine the net force on each individual block in terms of F , F' , m_1 , and m_2
- Draw the FBD for block 2, and, from its horizontal forces and (b), determine the magnitude of the force that block 1 exerts on block 2 in terms of F , F' , m_1 , and m_2
- Draw the FBD for block 1, and, from its horizontal forces and (b), determine the magnitude of the force that block 2 exerts on block 1 in terms of F , F' , m_1 , and m_2 . Of course, the answers (c) and (d) should agree!

* We will often be asking for FBD's because 1) “teaching research” has shown that when students draw them, they tend to do a better job on writing down and understanding the second law, and 2) experts need them as much as beginners! The FBD's help us blend our physical mental space better with our math mental space!

Pulling With Ropes: The Force is Called the Tension T

Consider two blocks tied together by a rope. Also, consider an external force pulling one of the blocks (perhaps through another rope). Because of this external force, the first mass pulls on the in-between rope and thus the in-between rope pulls on the second mass. The in-between rope has become “tense” which leads to a natural name, **tension**, for the force it exerts on both blocks:

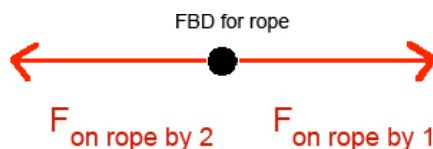


In this chapter, we use light ropes, which means that we are using **massless** ropes, which we refer to as “**ideal**” ropes.

A nice connection to the previous page: Consider a rope whose mass is not negligible compared with the other masses in the system. If we pull on one end of this rope and there are no other external forces, the rope accelerates such that the rope’s mass “eats up” the force, and the tension force decreases along the length of the rope as we go farther away from our hands. By the way, if the net external force is zero on a massive rope, the tension is constant along the rope – see below and Ch. 5+.

Back to ideal ropes and their feelings: We go back now to ideal ropes where the tension is the same all along an ideal rope, whether the rope accelerates or not! Let’s prove this.

First, what forces does the in-between rope in the above picture feel? The only external forces acting directly on this rope are the pulls from the two blocks (but nothing directly from F, remember!). Thus, draw the following FBD and note the opposite directions (arrows) for these forces.



Newton’s 2nd law applied to this FBD in terms of the force magnitudes and with a minus sign for the arrow pointing to the left:

$$F_{\text{net}}^{\text{on rope}} = F_{\text{by } m_1}^{\text{on rope}} - F_{\text{by } m_2}^{\text{on rope}} = m_{\text{rope}} a = 0 \quad \text{IF the rope is massless (OR not accelerated)}$$

Hence, for a massless rope, $F_{\text{by } m_1}^{\text{on rope}} = F_{\text{by } m_2}^{\text{on rope}}$, and by Newton’s third law, $F_{\text{by rope}}^{\text{on } m_1} = F_{\text{by rope}}^{\text{on } m_2}$

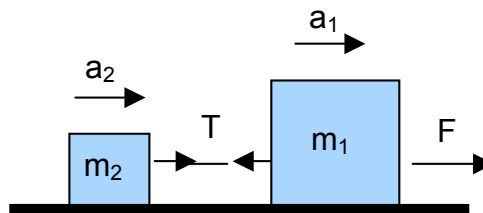
Therefore, the in-between rope in the picture above pulls forward on block 2 with the same magnitude of force that it pulls backward on block 1: $F_{\text{by rope}}^{\text{on } m_1} = F_{\text{by rope}}^{\text{on } m_2}$

Second, using the same argument on any little piece of the rope, the net force must be zero on that piece because it’s massless. Thus that piece is pulled left and right by forces with identical magnitudes, and thus that little piece pulls back on its two “pullers” with identical forces by Newton’s third law! **So every lousy little piece of the rope feels the same force pulling on it left and right.** This is what we mean by constant tension along a rope.

Usual notation: We will always call the force along the rope, the tension T. It might be less confusing if we called it “ F_T ” but why start now in trying to be clear!

Analysis of the two blocks with a rope between them and an external pull: Like the contact force f in the “push contact” case earlier, we are interested in the tension T that arises in the two-block system, given F and the masses m_1 and m_2 .

Putting double arrows on the tense rope in figures like this one reminds us that the rope pulls left and right with equal force.

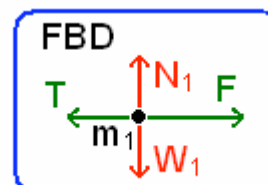
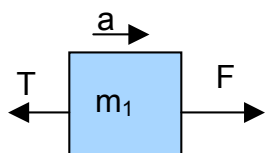


We start again with the constraint that the blocks move together so that they have a common acceleration, $a_1 = a_2 \equiv a$. Now, we automatically satisfy Newton’s third law by taking the same tension force T to be pulling on both masses. Thus, if we wrote down three second-law equations (one for the whole system, one for the first block and one for the second block), one would be redundant.

This is consistent with the fact that there are only two unknowns— a and T —so we only need two independent equations. This is why the additional equation is redundant.

Our procedure: First, we will draw the FBD’s, guessing that $T < F$, which will be confirmed by the calculation. (You won’t get thrown in jail if you misguess the length of your arrows.) Second, for each FBD, write the two corresponding second-law equations for the horizontal forces. Third, solve them for T and a .

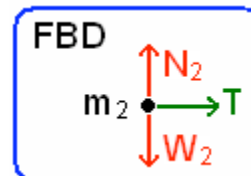
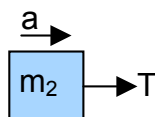
Our details: Plain vanilla and FBD for m_1 as the system:



The second law for the horizontal motion of m_1 :

$$F - T = m_1 a$$

Plain vanilla and FBD for m_2 as the system:



The second law for the horizontal motion of m_2 :

$$T = m_2 a$$

The two boxed equations are a **system of equations** to be solved for the two unknowns, a and T , on the next page.

Solution by substitution (just like on Sesame Street):

Plug $T = m_2 a$ into $F - T = m_1 a$: $F - m_2 a = m_1 a$

$$F = m_2 a + m_1 a = (m_2 + m_1) a$$

Solve for a:

$$a = \frac{F}{m_1 + m_2}$$

Solve for T by substituting a back in :

$$T = m_2 a = m_2 \frac{F}{m_1 + m_2}$$

We see indeed that a nonzero F gives rise to a nonzero tension T , but $T < F$ because m_1 “eats up” part of the force for this accelerated system.

Solution by combination: By combination we mean that we will add or subtract the two equations. The idea is to try to cancel out – get rid of – one of the unknowns. Add:

$$\begin{array}{rcl} \cancel{T} & = & m_2 a \\ + & F - \cancel{T} & = m_1 a \\ \hline F & = & (m_1 + m_2) a \end{array}$$

Once we have $F = (m_1 + m_2) a$, solve for a and substitute this into $T = m_2 a$ to get the same answers as above: $a = F/(m_1 + m_2)$ and $T = m_2 F/(m_1 + m_2)$.

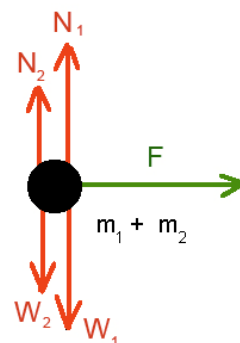
Notice how T canceled in the above addition. This math corresponds to the cancellation of internal forces, according to Newton’s third law. We already built Newton’s third law into our system, when we used equal and opposite tension forces in the two individual block FBD’s.

Comments:

- Verify the FBD for the whole system gives nothing new: The second law for the horizontal motion of $m_1 + m_2$: $F = (m_1 + m_2) a$

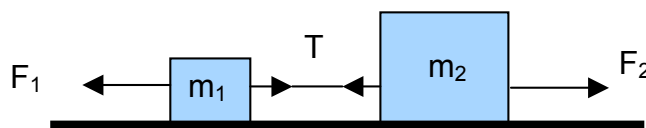
Clearly, this is the same as what we had. It is a short-cut you could use in place of either FBD of the individual blocks.

Notice there’s no T here, consistent with the third law.



- In the latest example, the forces have been referred to as forces of “**tension**.” In the example at the beginning of the chapter, we pressed one block into another. The forces there could thus be called forces of “**compression**.”

Problem 5-2 Two blocks with masses as shown and on a frictionless horizontal surface are connected by a light rope. Assume the force magnitudes F_1 , F_2 and masses m_1 , m_2 are known, along with the directions of the forces as shown.



- a) Before you start, what would you guess the tension T would be if $F_1 = F_2$? Don't calculate, just guess. You will get full credit for any honest guess. You can check this below when you're done calculating.

Now consider any two of the following three systems: the whole system, the isolated system m_1 , or the isolated system m_2 .

- b) Draw FBD's for the two systems you consider. Remember, each FBD involves ONLY those forces DIRECTLY on the considered body.

We take this opportunity to remind you that you don't have to know which force arrows are longer. We haven't told you yet who is bigger – F_1 or F_2 .

- c) Derive two equations from a consideration of the horizontal forces in your FBD's using the convention that the common acceleration a is defined to the right.* If the system turns out to be accelerating to the left, a will be negative.
- d) Solve your two equations for a and T in terms of F_1 , F_2 , m_1 , and m_2 . Check your formula for a : a must be positive if $F_1 < F_2$ (the system would accelerate to the right) and a must be negative if $F_1 > F_2$ (acceleration is to the left).
- e) Now assume $F_1 = F_2$ in your answer in (d) and see if your guess in (a) was right. If it was wrong, try hard to analyze why, so your physics intuition can be strengthened.

Guessing is always a good self-disciplinary step. We have our own pre-conceptions and, if they fail to match up with the world around us, we need to work really hard to reshape them. (Of course, if your own private world is really much nicer than the one described in these notes, frankly, you might like to keep it just as it is.)

- f) Finally, independent of whether $F_1 = F_2$, what is the **net** force on any part of the in-between rope with tension T in it? This answer should also help you in building an intuition about the meaning of tension and questions like (a)!

* Here we define a as the x-component of the acceleration vector with the convention of a positive x-axis pointing to the right: $a_x \equiv a$. Thus if we find $a < 0$, it's accelerated left.

Shedding More Light on Newton's Third Law

In both examples in this chapter, good old Newton's third law tells us that the force block 1 exerts on block 2 must be equal and opposite to the force block 2 exerts on block 1.

Let's beat it to death by paraphrasing what we said in Chapter 0: **If the force on body 1 is due to an interaction with body 2, then body 2 feels an equal and opposite force due to body 1.**

$$\vec{F}_{\text{on 1 by 2}} = - \vec{F}_{\text{on 2 by 1}}$$

Comments:

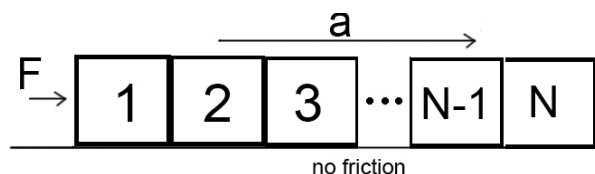
- This means the two forces have the same magnitude and the opposite direction.
- Thus, a "single isolated force" is impossible! We need two or more bodies for interactions/forces! A "force" means an interaction!! If there's an action, there must be a reaction. We must have "action-reaction." Stop us before we try to say it yet another way!
- Here is a neat thing. An action-reaction pair of forces always involves two forces that are each acting on **two different bodies!!!!!!** They may be equal and opposite, but they act on **different bodies** – namely, the two bodies interacting with each other.
- Newton's third law says that, for an FBD of a whole system made up of a bunch of bodies, we can ignore internal forces (like f and T in the two examples). So the **net force on the whole system is just the sum of all external forces**. See later chapters for lots of examples.

Problem 5-3 *Relatively quick and easy, but ... important!*

- If you push against a tree with some force of magnitude F , what is the force magnitude and relative direction with which the tree pushes back on you?*
- Suppose a person replaces the tree and you again push against her with the same force of magnitude F as in (a). If the two of you are at a stalemate, what is the force magnitude and direction with which this person pushes back on you?*
- You are talking about an action-reaction force pair in each of the cases (a) and (b). **Notice how each force in a given pair acts on a different body!** Explain why the support force on your shoes is NOT an action-reaction pair with your weight. What is the other member of the action-reaction pair involving your weight? What is the other member of the action reaction pair involving the support force on your shoes?*

continued

- d) Based on what we have learned, we can quickly figure out answers to the next two questions before we get to them in the second cycle. Are the contact forces between the blocks in the following string of identical blocks (mass m) the same, larger, or smaller as we go from left to right? Please think about this by drawing an FBD for the last block N isolated, from which the second law for the horizontal force tells us that the contact force on N is simply equal to ma . Now draw an FBD for the two blocks, $N-1$ and N , together. What must the contact force due to $N-2$ on the two-block system equal now? Do you see a pattern emerging? In agreement with F being “eaten up” as we go along the row of blocks?



- e) What about this string? That is, are the contact forces between the blocks in the following string of blocks the same, larger, or smaller as we go from left to right? Please use FBD's and the second law again to be sure of your answer.

