# PHYS 121: Cycle 2 Review Sheet, Part 4

**March 20, 2024**

## Rotational Motion Review: The Definition of Torque

Review: Torque is the quantity that when applied to a body results in angular acceleration. In other words, torque is applied to get something spinning. Think of torque as a **twist**.

You are free to define the pivot point at any place you want.[1] However, it is usually a good idea to choose a pivot point corresponding to an actually axis of rotation for a rigid body for most problems.

To calculate the torque, we note that the magnitude of the cross-product is **defined** so that we calculate the magnitude according to:

$$|\vec{\tau}_{\vec{F}}| \equiv rF\sin\phi$$

Torque results from a force, but torque and force are not the same thing. The torque is **defined** as being proportional to both the Force and the radius. Here "radius" means the distance from the pivot point (axis of rotation) to where the force is applied. Torque depends only on the component of force that is perpendicular to the radius line.

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[1] Actually calling it a"pivot axis" is more accurate than a pivot "point".

## Review: Newton's Second Law applied to Rotational Motion:

The rotational analog of $F_{net} = ma$ is:

$$\vec{\tau}_{net} = I\vec{\alpha}$$

where $I$ is the analogue to inertial mass, called the *rotational inertia*. Your textbook calls this quantity the *moment of inertia*. As a rule the moment of inertia is proportional to some factor times $MR^2$. The factor depends on the shape of the body.

Note the **vector nature** of angular acceleration. Use the **Curly-fingered Right-Hand-Rule** where the curled fingers point in in the direction of angular motion and the thumb represents the vector quantity.

## Review: The Rolling Constraint:

Another example that we have considered in class are "rolling objects" – that is objects such as wheels, hoops, or spheres that roll on a surface without slipping. For these objects there is a constraint between the velocity of the wheel and the rotational velocity of the wheel about the central axis of rotation:

$$v = r\omega$$

The constraint also applies to acceleration:

$$a = r\alpha$$

Note that this constraint is a constraint on the rotational and translational speeds; it is *not* a vector relationship. Generally speaking, $\vec{v}$ and $\vec{\omega}$ do not point in the same direction for a spinning body.

Note also that this constraint applies to two different scenarios:

- The relationship between the linear and rotational speeds of a wheel that rolls without slipping, and
- The relationship between the linear speed of a point-like object on or part of a spinning body and the rotational speed of that body.

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## More on The Rolling Constraint for Rigid Bodies:

A general and powerful **constraint** exists between the angular motion of any rigid body that is rotating about a fixed axis and the linear motion corresponding to some mass point embedded in that rigid body. This constraints is called the **Rolling Constraint:**

$$v = r\omega$$

$$a = r\alpha$$

It's worth emphasizing that $r$ is the distance from the chosen rotational axis to the point embedded in the rigid body. This is a very general relation between the rotation and angular motion of any rigid body that is rotating about a fixed axis.

For a wheel or a disk or a hoop that is rotating about a fixed axis, the rolling constraint immediately tells you the relationship between the angular motion of the object and the translational (tangential) motion of any point on the outer edge of the wheel.

This also turns out to be the relationship between the translational and angular motion of a wheel that rolls without slipping on surface. This is where the name comes from.

It's also worth emphasizing that this is a relationship between *scalars*, not vectors. Generally speaking, $\vec{v}$ and $\vec{\omega}$ do not point in the same direction for a spinning body.

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## Rotational Motion Examples and Issues:

### Massive Pulleys

Don't forget: when we deal with a massive pulley we usually treat the pulley as a disk with the appropriate value of $I = \frac{1}{2}mR^2$ and we write down the XFBD for that pulley and solve Newton's Second Law for Torques for the pulley. Note in particular that the rope on one side of a massive pulley does *not* have the same tension as both sides of the pulley. We treat each rope piece as separate tension ropes attached to the pulley at the point of contact.

### Hinge Force:

If we have a rigid body attached to a hinge, axle, axis, or pin, we can use a new force in our Force Menu: The **Hinge force** is generally unknown and can act with both vertical and horizontal components. It acts in such a way to ensure that this attached part of the rigid body stays put, transitionally. A key point is this. If we chose the hinge as the "pivot point' this ensures that the hinge force contributes zero torque.

### Choice of Rotational axis:

Note that for many problems in statics or with rotation, you have to *choose* the axis of rotation that you will use to calculate your torques. You are free to chose any axis you want. However, some choices will make the problem much easier to solve. Here are some hints as to how to choose a rotational axis:

- If there is only one object in the system and it is spinning and not static, then chose the axis of rotation for the spin described. For example, with a wheel chose the central axle of the wheel. For a swinging door, chose the axis of the hinge.
- Otherwise, most of the time the best axis to choose is the central axis through the center-of-mass of the system. This means that at least one force (weight) will not contribute to the calculated torque.
- If the object is static, you can write down equations that correspond to the torque about any point – since the object does not move the net torque has to be zero about any axis. To eliminate a force from the net torque, chose an axis of rotation that is centered on the point where that force is applied.
- If the object is experiencing a net torque then you must keep track of the rotational inertia ($I$) of the system. This depends both on the shape and the position of the center of rotation.