UNC
Uncertainty
revised August, 2021

Learning Objectives:
During this lab, you will learn how to
1. estimate the uncertainty in a directly measured quantity.
2. estimate the uncertainty in a quantity that is calculated from quantities that are uncertain.
3. use *Origin* to perform calculations, make graphs, and perform linear regression.
4. look at a graph with data and a model and determine if the data support the model.

(*You will upload to Canvas-Assignments two worksheets and two graphs for this lab. The worksheets (UNC Worksheet and Error Analysis and Propagation Exercise) are included in Appendix IX of the lab manual. Attach a cover sheet found on Canvas-Modules. The material for this lab is due one week from the date of your lab at 6:00 PM.*)

A. Introduction
The purpose of this laboratory is to accustom you to recording measurements. You will measure mass, time, length, and force. Besides learning some basic measurement techniques, you must realize that all measurements possess some **uncertainty** in their values. When you make a measurement (x), you must estimate the **absolute uncertainty** (δ_x) in your measurement. This quantity is based upon how well you think that you could make the measurement. The absolute uncertainty may be a function of the quality of the measuring instrument, the nature of the quantity being measured, the ability of the individual making the measurement, and the conditions under which the measurement is made. Too often we assume that the absolute uncertainty is based solely on the **resolution** of the measuring instrument. The resolution is the smallest graduation on a scale or the last decimal place in a digital readout. The absolute uncertainty often depends on several of the factors above. There is no prescribed formula of how to calculate the absolute uncertainty for a measured quantity. For this reason, it is necessary for you to discuss how you determine the absolute uncertainty of a measurement in the Procedure paragraph of a laboratory paper. Taking the measured value and the absolute uncertainty together, you report the **measurement interval** ($x \pm \delta_x$). This symbol $x \pm \delta_x$ represents the interval in which we have confidence the measured value lies. Remember to always report the absolute uncertainty (δ_x) together with your best estimate of the measured value (x).

There are **rounding rules** that guide us in reporting the number of digits in the answer. For example, we would not report a number such as 2.1789345 meters if we were really only sure of the reading to within 0.01 meters. Instead, we would report the value as 2.18 ± 0.01 meters, i.e., we estimate that the value lies somewhere between 2.17 and 2.19 meters. In Physics laboratories, we will adopt the simple rule that the absolute uncertainty of a measurement is reported to only one, or at most two, significant figure(s), and the measurement is then rounded to the same decimal place as its absolute uncertainty. For example, 0.01224 ± 0.00002 kg is properly rounded. The absolute uncertainty is reported to one significant digit, but that digit lies five decimal places to the right of the decimal mark. Therefore, the measured value is reported all the way out to five decimal places. Remember that the measured value plus or minus the absolute uncertainty, e.g., 0.01224 ± 0.00002 kg, represents the measurement interval. See Appendix V, Section D.

Relative uncertainty is a measure of the **precision** of the measurement. If a measurement has a high degree of precision, repeated measurements (using the same equipment and technique) will produce about the same results. We calculate the relative uncertainty by dividing the absolute uncertainty of the measurement by the value of the measured quantity, $\frac{\delta_x}{x}$. For example, the relative uncertainty of the value above is

$$\frac{\delta_x}{x} = \frac{0.00002\ \text{kg}}{0.01224\ \text{kg}} = 0.002 = 0.2\%.$$

Note that, as with absolute uncertainty, we will report the relative uncertainty to one or two significant figures. We will also normally report the relative uncertainty in percent form.

When you make a measurement, there is always some error present. Errors can be divided into two categories: **systematic** and **random**. Systematic errors can cause measured values to be consistently either too high or too low. Systematic errors affect the **accuracy** of a measurement. The accuracy of a measurement indicates how close it is to a true or given value. Random errors are positive or negative fluctuations that cause a measurement to be too high sometimes and too low at other times. If we take enough measurements, the effect of the random error normally averages out to nearly zero. See Appendix V, Section A.

B. Apparatus

Each group member will make measurements of length, diameter, mass, time, and force of a simple pendulum. **You will measure the length of the pendulum using a meter stick, the diameter of the ball with the vernier calipers, the mass of the ball with an electronic balance, the period of the pendulum (how long it takes to complete one swing) with a stopwatch, and the weight (force of gravity on) of the pendulum with a spring scale.** Make the measurements independently of other group members. Do not be influenced by their results.

C. Procedure

Set up a table in your notebook with a title of "Length" and columns for the following: trial number, person who did the measurement, and measurement value. Take one measurement and record the value in the table. <u>Estimate the uncertainty in the measurement and record it</u> along with your reasoning for that estimate in your notebook. Have each partner take a measurement and estimate of uncertainty without telling each other what the measurement was. **Alternate taking measurements until the group has taken a total of ten measurements. Add your partner's measurements to your table.**

Repeat this procedure for measurements of diameter, mass, period, and weight. Note that each lab room will only have one balance; you do not need to take the measurements in order.

D. Analysis

<u>Calculate the mean, standard deviation, and standard error for each of the five measurements</u>. See Appendix V, section B.1 for how to calculate these values. Compare the standard errors with your estimates of the uncertainties. <u>Were your estimates much larger, much smaller, or essentially the same as the standard errors? Can you explain any differences?</u>

E. Propagation of Uncertainties and Discrepancies

The theoretical relationship between the period T and length ℓ of a simple pendulum and the acceleration due to gravity g is

$$T = 2\pi\sqrt{\frac{\ell}{g}}\,. \qquad (1)$$

Thus, if we treat our pendulum as a simple pendulum, we can find g by

$$g = 4\pi^2\frac{\ell}{T^2} \qquad (2)$$

Apply Eq. 2 to your mean values of length and period above to find g. Since you are uncertain about both ℓ and T, you are uncertain about g as well. Treat the standard errors in your values of length and period as your uncertainties (δ_ℓ and δ_T) to find your uncertainty in g (δ_g). You should review Appendix V, Section C.

We can find δ_g by adding in quadrature our uncertainties in g due to ℓ ($\delta_{g\ell}$) and T (δ_{gT}):

$$\delta_g = \sqrt{\delta_{g\ell}^2 + \delta_{gT}^2} \qquad (3)$$

We have two methods of calculating $\delta_{g\ell}$ and δ_{gT}: The derivative method and the computational method.

E.1. Derivative Method

(PHYS115 students *may* skip to section E.2. at this point.)

We can apply Eq. 5a from Appendix V, Section C.1. to Eq. 2 above to determine $\delta_{g\ell}$ and δ_{gT}:

$$\delta_{g\ell} = \left|\frac{\partial g}{\partial \ell}\right|\delta_\ell = \left[\frac{\partial}{\partial \ell}\left(4\pi^2\frac{\ell}{T^2}\right)\right]\delta_\ell$$

$$\Rightarrow \delta_{g\ell} = \frac{4\pi^2}{T^2}\delta_\ell \qquad (4)$$

and

$$\delta_{gT} = \left|\frac{\partial g}{\partial T}\right|\delta_T = \left|\frac{\partial}{\partial T}\left(4\pi^2\frac{\ell}{T^2}\right)\right|\delta_T$$

$$\Rightarrow \delta_{gT} = \frac{8\pi^2\ell}{T^3}\delta_T\,. \qquad (5)$$

Apply Eqs. 3, 4, and 5 to your data to determine an estimate of δ_g.

E.2. Computational Method

Alternatively, we can directly calculate how much g changes when ℓ and T vary by their uncertainties by applying Eq. 9 from Appendix V, Section C.1. to Eq. 2 above:

$$\delta_{g\ell} = \left|g(\ell+\delta_\ell, T) - g(\ell, T)\right|$$

$$\Rightarrow \delta_{g\ell} = \left|4\pi^2\frac{(\ell+\delta_\ell)}{T^2} - 4\pi^2\frac{\ell}{T^2}\right|$$

$$\Rightarrow \delta_{g\ell} = \frac{4\pi^2}{T^2}\delta_\ell \qquad (6)$$

and

$$\delta_{gT} = \left|g(\ell, T+\delta_T) - g(\ell, T)\right|$$

$$\Rightarrow \delta_{gT} = \left|4\pi^2\frac{\ell}{(T+\delta_T)^2} - 4\pi^2\frac{\ell}{T^2}\right|$$

$$\Rightarrow \delta_{gT} = 4\pi^2\ell\left|\frac{1}{(T+\delta_T)^2} - \frac{1}{T^2}\right| \qquad (7)$$

Apply Eqs. 3, 6, and 7 to your data to determine an estimate of δ_g. Report your value of g as a measurement interval ($x \pm \delta_x$).

E.3. Discrepancies

The difference between your experimental value and another experimental or theoretical value (the "accepted" value) is the

discrepancy. The discrepancy should never be quoted as an "error." If a discrepancy is not much larger than a result's uncertainty (say, not more than two times as large), the result is considered to be consistent with expectations. **Calculate the discrepancy between your measured value and the "accepted" value.** For this course, we will use (9.81 ± 0.02) m/s^2 as the "accepted" value for *g*.

If possible, **offer some explanation for discrepancies that are larger than the expected uncertainties.** For example, suppose you perform an experiment to measure the speed of sound and find that your measured value is higher than the accepted value by more than two times your estimated error. You may justify this discrepancy by mentioning that the unusually high humidity and temperature may have affected your results, if this is in fact the case and if they might have had the appropriate effect. Ideally, your explanation should have some physical basis or describe a limitation in your technique with a clear line of reasoning as to how it could have caused the discrepancy, and the explanation should be quantitative.

F. Origin Exercise

Astronaut Dr. C. C. Taylor was selected to go on the NASA mission to Europa. One of the experiments Dr. Taylor performed while on Europa was to drop a steel ball bearing from several different heights and measure the time it took to reach the ground. Dr. Taylor sent his data set to Glenn Research Center where you are assigned the task of analyzing this data set to determine the acceleration due to gravity on Europa's surface.

Your friend Aristotle tells you that you should use the equation

$$h = a_A t \qquad (8)$$

to analyze the data where h is the height of release, t is the time of fall, and a_A is the acceleration due to gravity.

Your friend Newton tells you that you should use the equation

$$h = \tfrac{1}{2} a_N t^2 \qquad (9)$$

to analyze the data where a_N is the acceleration due to gravity.

Dr. Taylor's data is

Trial	Height h (± 0.01 m)	Time t (± 0.01 s)
1	1.45	1.51
2	1.30	1.42
3	1.20	1.34
4	1.00	1.23
5	0.90	1.16
6	0.75	1.07
7	0.60	0.97
8	0.45	0.83
9	0.30	0.69
10	0.15	0.48

F.1. Direct Calculation

One way to analyze this data is as a set of 10 measurements of the acceleration. **Open up *Origin***. Go to the columns menu, choose "Add New Columns," and add 4 new columns. Right-click on the top of first column and choose "Properties." Type in "Height (m)" in the long name area. Change the plot designation to "Y" and click the "Next" button. Choose "yes" to the question "Do you wish to automatically display the Column Label?" Change the next long name to "Uncert. in h," and change the plot designation to "Y Error." Click on the "Next" button. Make the next plot designation "X," and long name "Time (s)." Click on the "Next" button again and make the next plot designation "X Error" and long name "Uncert. in t." Make the next long name "Aristotle's acceleration (m/s)" and the last long name "Newton's acceleration (m/s^2)." Click "OK." **Enter Dr. Taylor's values for *h* and *t*** into the appropriate columns.

To put the uncertainties into *Origin*, select the column dh, go to COLUMN\SET COLUMN VALUES. In the box that pops up, change "Row(i):From Auto to Auto" to "for rows 1 to 10" and change the default equation in the box below "col(B)=" to 0.01 and click OK. Use the same procedure to set column D to 0.01.

Now we're ready to start calculating a_A and a_N. From Eq. 8, we can see that

$$a_A = h/t \tag{10}$$

and from Eq. 9,

$$a_N = 2h/t^2. \tag{11}$$

Rather than doing 20 calculations on a calculator, we can have *Origin* do the calculations for us. Select the column E and bring up the set column values box. Set the range of rows to 1 to 10. At this point, you want to make Origin calculate the whole new column for Aristotle's version of acceleration for you (aA = h/t). To have this done on whole column by origin, type "col(A)/col(C)" to the equation box then click "OK".
Select the column F and set its first 10 rows values to 2*col(A)/col(C)^2.

Now we are ready to calculate the mean, standard deviation, and standard error in our experiment. Right-click on column E and select "Quantities", check "Mean" "standard deviation" and "SE of Mean", and hit "ok". **Record the mean (Mean(Y)), standard deviation (sd(yEr±)), and standard error (se(yEr±)). Report a_A as a measurement interval.** Go through the same procedure on column E **to find the mean, standard deviation, and standard error of a_N and report a_N as a measurement interval.** (*Do not print out the tables produced by STATISTICS ON COLUMNS; just record the appropriate numbers in your laboratory notebook.*)

What conclusions can you make as to which model (Aristotle's or Newton's) better describe the data? What is the reasoning behind these conclusion or reasons why you can't make a conclusion?

F.2. Fitting a Model to Data

Origin is a powerful graphing and fitting program. We will now use that power to more precisely test the validity of our models.

F.2.a. Aristotle's Model

Eq. 8 implies that if we plot h on the y-axis and t on the x-axis, the slope of the best fit line will be a_A. Furthermore, if Aristotle's model is valid, the data should randomly scatter about that best fit line.

In *Origin*, make sure no columns are selected. **Go to PLOT / SCATTER. Select columns A, B, C, and D as your Y, yEr, X, and xEr columns respectively.** (If the "xEr" check-boxes don't appear in the "Plot Setup" window, right-click on the window and select "X Error Bars.") **Click ADD and then OK**. We now have a fairly ugly looking graph; we need to "pretty it up" to make it informative to others. **Double-click on the "Y Axis Title" and change it to "*h* (± 0.01 m)**." (To insert the "±" symbol, hit Ctrl-m to bring up the symbol map.) **Change the "X Axis Title" to "*t* (± 0.01 s)." Double click on a data point**. A "Plot Details" window will appear. Make sure the first set under "Layer 1" is selected (Data 1: t(X), h(Y).) **Change the size of the data points to 3 points and click OK.** (With 8-point data markers, the data markers are bigger than the error bars!) Use the text tool to **give the graph the title "Acceleration Due to Gravity on Europa-Aristotle's Model."** Use the text tool to **add your name and your partner's name to the graph**. Make sure the legend doesn't overlap the title.

Now we're ready to fit a line to our data. **Go to ANALYSIS / LINEAR FIT**. A red line will appear on your graph (the best fit line) and some numbers will appear in the results window (usually at the bottom right-hand corner of the *Origin* window). **Copy the linear regression data and paste it into the graph.** You will probably need to make the

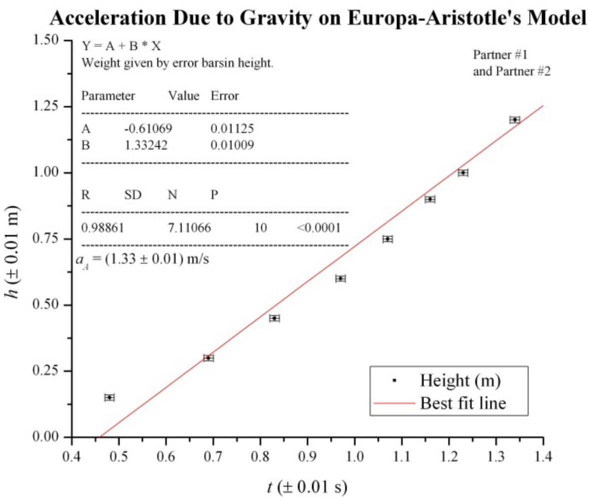

Figure 1: Sample graph of Aristotle's Model.

font size smaller. Note that *Origin* does **not** handle significant figures properly; you will have to edit these values manually so that the number of significant figures is appropriate for the errors that *Origin* calculates for each value. Your graph should look something like Figure 1. **Save your *Origin* "project" to a folder you produce on the lab server and <u>print out a copy of the graph for each member of the partnership to include with the worksheets.</u> <u>Report a_A from your best fit as a measurement interval</u>.**

F.2.b Newton's Model

Eq. 9 implies that if we plot h on the y-axis and t^2 on the x-axis, the slope of the best fit line will be $\frac{1}{2}a_N$; i.e., a_N will be twice the slope of the best fit line. Furthermore, if Newton's model is valid, the data should randomly scatter about that best fit line.

In order to create this plot, we need to **create two new columns: one for t^2 and one for δ_{t^2}.** Create two more columns in your *Origin* project and make the first of these new columns the t^2 column and set as a plot designation of X. Use the "Set Column Values" command to set the values of these cells to t^2.

The uncertainty in t^2 (δ_{t^2}) is **not** the square of the uncertainty in t (δ_t^2). We can apply Eq. 5a from Appendix V to find δ_{t^2}:

$$\delta_{t^2} = \left|\frac{d}{dt}t^2\right|\delta_t$$
$$\Rightarrow \delta_{t^2} = 2t\delta_t \qquad (12)$$

So each of our values of t^2 will have a different uncertainty associated with it.

(Alternatively, we could use the computational method to find

$$\delta_{t^2} = \left|(t+\delta_t)^2 - t^2\right|$$
$$\Rightarrow \delta_{t^2} = t^2 + 2t\delta_t + \delta_t^2 - t^2$$
$$\Rightarrow \delta_{t^2} = 2t\delta_t + \delta_t^2$$
$$\Rightarrow \delta_{t^2} \approx 2t\delta_t\,. \qquad (13)$$

As you can see, both methods yield the same results when $\delta_t << t$.)

Use the second new column as your δ_{t^2} column. Use the "Set Column Values" command to set the values of these cells to δ_{t^2}. **Then create a new plot with h as your Y variable and t^2 as your X variable. *Don't forget to include error bars!* Make your plot "pretty" and find the best fit line and the best fit line parameters as above**. Save your *Origin* "project" to a folder you produce on the lab server and **<u>print out a copy of the graph for each member of the partnership to include with the worksheets.</u> Calculate a_N and δ_{aN} from the slope of the best fit line and <u>report a_N from your best fit line as a measurement interval</u>.**

F.2.c. Conclusions

Study your two graphs. **<u>Which model more closely fits the data, Aristotle's or Newton's? What is your evidence that one is a better fit than the other (or they are equally good)? What value of the acceleration due to gravity at Europa's surface would you report to your</u>**

<u>supervisor at Glenn Research Center based on Dr. Taylor's data? Justify this choice.</u> <u>Your continued employment depends upon it!</u>

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