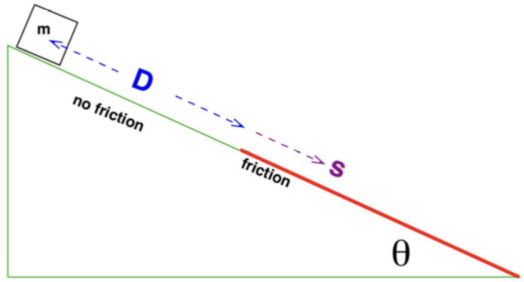
Final SI Summary Session

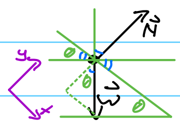
Problem 1)



Given

- θ with horizontal
- Block Released @ rest, mass m
- Frictionless section length D
- Moves to friction part with $\mu_K \rightarrow$ stop

a) FBD Block



$\boxed{a_x = g\sin\theta}$

N2L x-dir

$F_{Net} = ma_x$

$W\sin\theta = ma_x$

$mg\sin\theta = ma_x$

b) C of ME

Conditions: weight conservative and normal force ⊥ so no work done. Friction not present yet

b) Before = After

$E_{Tot} = E_{Tot}'$

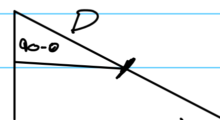
$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$

$y=0,\ v=0,\ y' = -D\sin\theta,\ v' = ?$

$0 = -\cancel{m}gD\sin\theta + \frac{1}{2}\cancel{m}v'^2$

$gD\sin\theta = \frac{1}{2}v'^2$

$v'^2 = 2gD\sin\theta$



$\cos(90-\theta) = \frac{y'}{D}$

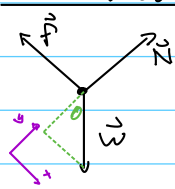
$y' = D\cos(90-\theta)$

$y' = D\sin\theta$

$\boxed{v' = \sqrt{2gD\sin\theta}}$

Simple Problem don't overthink it you bozo

c) FBD Block



N2L x-dir

$F_{Net} = ma_x$

$W\sin\theta - f = ma_x$

$mg\sin\theta - \mu_K N = ma_x$

$\cancel{m}g\sin\theta - \mu_K \cancel{m}g\cos\theta = \cancel{m}a_x \rightarrow a_x = g(\sin\theta - \mu_K\cos\theta)$

N2L y-dir

$F_{Net} = m\cancel{a_y}^{0}$

$N - W\cos\theta = 0$

$N = mg\cos\theta$

$x = x_0 + v_0 t + \frac{1}{2}a_0 t^2$ Let $x_0 = 0$ be @ friction start

$x = 0 + \sqrt{2gD\sin\theta}\, t + \frac{1}{2}(g(\sin\theta - \mu_K\cos\theta))t^2$

$t = -\frac{\sqrt{2gD\sin\theta}}{g(\sin\theta - \mu_K\cos\theta)}$

$V = V_0 + at$

$0 = \sqrt{2gD\sin\theta} + g(\sin\theta - \mu_K\cos\theta)t$

$s = \sqrt{2gD\sin\theta}\left(-\frac{\sqrt{2gD\sin\theta}}{g(\sin\theta - \mu_K\cos\theta)}\right) + \frac{1}{2}g(\sin\theta - \mu_K\cos\theta)\left(-\frac{\sqrt{2gD\sin\theta}}{g(\sin\theta - \mu_K\cos\theta)}\right)^2$

$s = \frac{-2gD\sin\theta}{g(\sin\theta - \mu_K\cos\theta)} + \frac{1}{2}\left(\frac{2gD\sin\theta}{g(\sin\theta - \mu_K\cos\theta)}\right)$

$s = \frac{-gD\sin\theta}{g(\sin\theta - \mu_K\cos\theta)}$

$\boxed{s = \frac{D\sin\theta}{\mu_K\cos\theta - \sin\theta}}$

d) $\vec{W}_{eff} = \vec{W}_{real} - m\vec{A}$

$\vec{A} = g(\sin\theta - \mu_K\cos\theta)\hat{i}$

$\vec{W}_{real} = W_x + W_y$

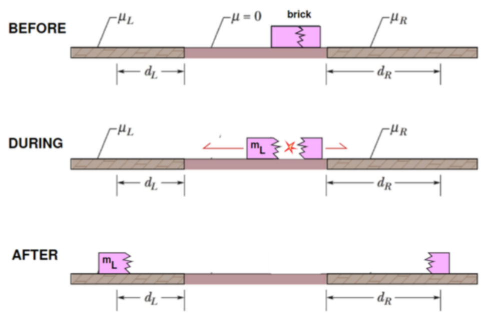
$= mg\sin\theta\hat{i} - mg\cos\theta\hat{j}$

$W_{eff} = M_D g\sin\theta\hat{i} - M_D g\cos\theta\hat{j} - M_D g(\sin\theta - \mu_K\cos\theta)\hat{i}$

$= M_D g(\sin\theta - \sin\theta + \mu_K\cos\theta)\hat{i} - M_D g\cos\theta\hat{j}$

$\boxed{\vec{W}_{eff} = M_D g\mu_K\cos\theta\hat{i} - M_D g\cos\theta\hat{j}}$

Problem 2)



Given

- Left fragment mass m_L
- Friction coefficients
- Distances Traveled

CofLM isolated

$P_{Tot} = P_{Tot}'$

$m_L v_L + m_R v_R = m_L v_L' + m_R v_R'$

$0 = m_L v_L' + m_R v_R'$

$-m_L v_L' = m_R v_R'$

$M = m_L + m_R$

Work Energy Thm

$W = \Delta K$

$W = Fd\cos\theta$

$= f_k d\cos\theta$

$= f_k d$

$= \mu mgd$

N2L y-dir

$N - w = 0$

$N = w$

$N = mg$

Block L

$\mu_L \cancel{m_L} g(-d_L) = \frac{1}{2}\cancel{m_L} v_L'^2 - \frac{1}{2}\cancel{m_L} v_L^2$

$-\mu_L g d_L = \frac{1}{2}v_L'^2 - \frac{1}{2}v_L^2$

$v_L' = 0 \rightarrow$ ends at rest

$-\mu_L g d_L = -\frac{1}{2}v_L^2$

$v_L = \sqrt{2\mu_L g d_L}$

Masses cancel out so

sub in R for L and Negate

$v_R = -\sqrt{2\mu_R g d_R}$

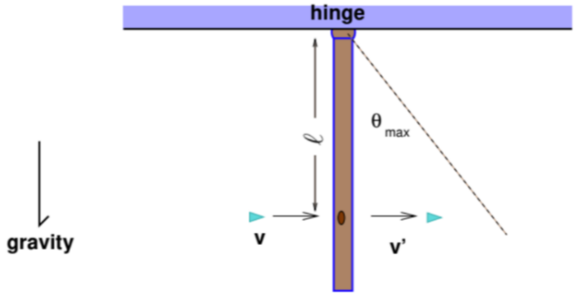
$-m_L v_L = m_R v_R$

$-m_L\sqrt{2\mu_L g d_L} = m_R\left(-\sqrt{2\mu_R g d_R}\right)$

$m_R = m_L\sqrt{\frac{\cancel{2}\mu_L \cancel{g} d_L}{\cancel{2}\mu_R \cancel{g} d_R}}$

$M = m_L + m_L\sqrt{\frac{\mu_L d_L}{\mu_R d_R}} \rightarrow \boxed{M = m_L\left(1 + \sqrt{\frac{\mu_L d_L}{\mu_R d_R}}\right)}$

Problem 3)



Given
- Mass M of Rod
- Total Rod Length L
- Bullet shot distance ℓ
- Ideal hinge connection
- v and v' given of bullet

Rod I about Hinge: $I = \frac{1}{3} m\ell^2$

Bullet as PP: $I = MR^2$ Treat PP's ω_{cm} with respect to hinge

C of AM: Bullet Rod Isolated

$$L_{Tot} = L'_{Tot}$$

$$I_B\omega_B + I_R\omega_R = I_B\omega_B' + I_R\omega_R'$$

$$\omega_B = \frac{v}{\ell},\quad \omega_R = 0,\quad \omega_B' = \frac{v'}{\ell},\quad \omega_R' = ?$$

$$m\ell^{\cancel{2}}\frac{v}{\cancel{\ell}} = m\ell^{\cancel{2}}\frac{v'}{\cancel{\ell}} + \frac{1}{3}ML^2\omega_R$$

$$m\ell v = m\ell v' + \frac{1}{3}ML^2\omega_R$$

$$m\ell(v-v') = \frac{1}{3}ML^2\omega_R$$

$$\omega_R = \frac{3m\ell(v-v')}{ML^2}$$

Stop! useful later!!

C of ME: Thin rod after hit so isolated system and no work done by $\vec{H}$, $\vec{\omega}$ is conservative

$$E_{Tot} = E'_{Tot}$$

$$mgy + \frac{1}{2}I_R\omega_R^2 = mgy' + \frac{1}{2}I_R\omega_R'^2$$

$$mg\left(-\frac{L}{2}\right) + \frac{1}{2}\left(\frac{1}{3}mL^2\right)(\omega_R)^2 = \frac{1}{2}(I)0^2 + mg\left(-\frac{L\cos\theta}{2}\right)$$

$$-\cancel{\frac{1}{2}}\cancel{m}g\cancel{L} + \frac{1}{6}\cancel{m}L^{\cancel{2}}\omega_R^2 = -\cancel{\frac{1}{2}}\cancel{m}g\cancel{L}\cos\theta$$

$$g - \frac{1}{3}L\omega_R^2 = g\cos\theta$$

$$1 - \frac{L}{3g}\omega_R^2 = \cos\theta$$

$$1 - \frac{L}{3g}\left(\frac{3m\ell(v-v')}{ML^2}\right)^2 = \cos\theta$$

$$1 - \frac{\cancel{L}}{\cancel{3}g}\left(\frac{9m^2\ell^2(v-v')^2}{M^2L^4}\right) = \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(1 - \frac{3m^2\ell^2}{M^2L^3g}(v-v')^2\right)}$$

Answer on S1

$$\cos^{-1}\left(1 - \frac{3m^2\ell^2}{M^2L^3}(v-v')\right)$$

where is $(v-v')^2$ and where did 'g' go?