Chapter 5++

Tension Along a Tug-of-War

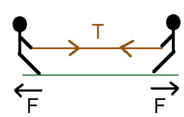
Revisit:	Integrate:
• Tension • String of Masses	• Tug-of War

Tugs-of-War

Ideal Ropes Again: We put our ropes on a diet and go back to situations where the ropes are "lightweights" compared with other bodies in the system, so ideal ropes are again called for (another call for Mr. Ideal Rope, calling Mr. Ideal Rope, …)

How does the tension change along a tug-of-war? The difference between the string of blocks and the tug-of-war is that each "block" (i.e., kid) now has an additional external force applied to it: **the kids are pulling on the rope by digging their feet into the ground** (the friction of their shoes against the ground is needed).

EXAMPLE 1: One kid against one kid, each can pull with a force of 130 N
Each kid pushes against the ground, so the ground pushes back on the kid. Thus in the figure we show the external force F = 130 N on each kid due to the ground as well as the tension in the rope between them:



With a zero net external force (the F's cancel out), the common acceleration a of the two-kid system is zero (well, the kids are in a stalemate here not only with zero acceleration but with zero velocity, too, since they're certainly not sliding!):

$$F^{net}_{\text{two kids}} = 0 \Rightarrow a = 0 \Rightarrow \text{stalemate}$$

Isolate the right kid to find T_1 : Second law (usual convention: right arrows positive)

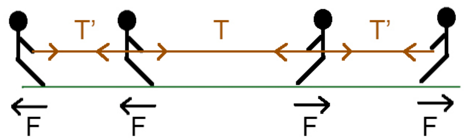
$$\Rightarrow F - T = 0 \Rightarrow \boxed{T = F = 130\text{ N}}$$

Comments:

1) It's easy to see that the second law for the kid on the left gives the same answer for T as it must (recall that the third law is automatically satisfied when we assume the internal forces don't contribute to the net external force)

2) Notice $T \neq 2F$!!!!! The double arrows drawn on the rope help us to avoid this old psychological pitfall !

EXAMPLE 2: Two kids against two kids, each again can pull with a force of 130 N
By symmetry, the two kids at the ends feel the same tension; we call this common tension T' :



With a zero net external force (the F's again all cancel out), the common acceleration a of the four-kid system is zero (and, again, they're not sliding):

$$F^{net}_{\text{four kids}} = 0 \quad \Rightarrow \quad a = 0 \quad \Rightarrow \quad \text{stalemate}$$

Isolate the left-most kid: Second law $\Rightarrow$ $T' - F = 0$

$$\boxed{T' = F = 130 \text{ N}}$$

Isolate the two kids on the left: Second law $\Rightarrow$ $T - 2F = 0$

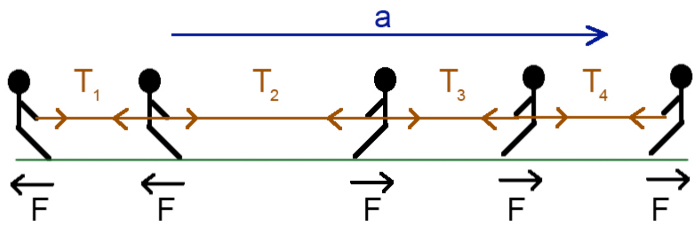
$$\boxed{T = 2F = 260 \text{ N}}$$

Comments:

1) You can easily verify that any other system of kids (the two on the right, etc.) give no new information.

2) We see the tension is greatest in the middle and decreases as we go away from the center in either direction. This is a general trend in tugs-of-war.

EXAMPLE 3: Two kids against three kids, each again can pull with a force of 130 N
Now the net external force is not zero; the whole system must be accelerating, and it must be accelerating in the direction of the three kids:



With a nonzero acceleration, we finally need to know the masses of the kids. (Mathematically, we need the masses for the "ma" part of the second law; physically, accelerated masses eat up the forces.) Assume they all have the same mass m.

Find the acceleration most quickly by considering the whole system:

$$F^{net}_{\substack{\text{whole}\\\text{system}}} = 3F - 2F = M_{total}\, a = (5m)\, a$$

$$\Rightarrow \quad \boxed{ma = \tfrac{1}{5} F} \qquad \text{Use this to substitute for "ma" later.}$$

Isolate the left-most kid to find T_1 :

$$T_1 - F = m a \;\Rightarrow\; T_1 = ma + F = \frac{1}{5} F + F$$

$$\Rightarrow \boxed{T_1 = \frac{6}{5} F}$$

Since $T_1 = \frac{6}{5} F > F$, the kid does move to the right. $\surd$

Isolate the two left-most kids to find T_2 :

$$T_2 - 2F = 2\,ma \;\Rightarrow\; T_2 = 2ma + 2F = \frac{2}{5}F + 2F$$

$$\Rightarrow \boxed{T_2 = \frac{12}{5} F}$$

Since $T_2 = \frac{12}{5} F > 2\,F$, the two-kid system does move right. $\surd$

Isolate the three left-most kids (not all on the same team!) to find T_3 :

$T_3 + F - 2F = 3\,ma$, etc.

$\Rightarrow \boxed{T_3 = \frac{8}{5} F}$ and $T_3 = \frac{8}{5} F > F$ $\surd$

Isolate the right-most kid to find T_4 :

$F - T_4 = ma$, etc.

$\Rightarrow \boxed{T_4 = \frac{4}{5} F}$ and $F > T_4 = \frac{4}{5} F$ $\surd$

Comments:

1) Again, you can easily verify that any other system of kids give the same answers

2) We again see the tension is greatest in the rope between the two teams. The tension decreases as we go left or right from that point.

Problem on the next page.

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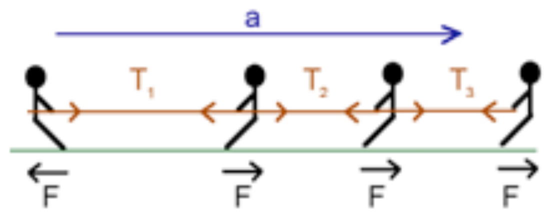
Problem 5-6

a) Quite an unfair tug-of-war is taking place, with one kid against three, as shown. If each kid (all have the same mass 50.0 kg) can dig into the ground such that she can pull with a force of 100.0 N, what is the common acceleration all the kids have and what are the tensions T_1, T_2, and T_3 ?



b) Another ghastly problem! In 1978, in an accident at a school in Harrisburg, Pennsylvania, several children lost parts of their fingers when a nylon rope suddenly snapped during a giant tug-of-war. Suppose each child exerted a pull of 120 N, and the rope was known to have a breaking tension of 60,000 N. What must have been the minimum number of children involved in this tug-of-war if they were equally divided on both sides?

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