Chapter 1++
Relative Uniform Motion with General Angles

Revisit:	To-Do:
• Relative motion • Wind as the medium	• Triangularization for general angles • Rectangular components for general angles

An Airplane at Arbitrary Angles to the Wind

We finally free ourselves to consider general "angles." Consider an airplane in the wind **with an arbitrary angle** anywhere, anyhow. It's not too big a deal, since our previous procedures will still work. We can continue to use our relative velocity relation:

$$\vec{v}_{\text{airplane/ground}} = \vec{v}_{\text{airplane/air}} + \vec{v}_{\text{air/ground}}$$

or, in a simpler notation: $\vec{v}_{\text{ap/g}} = \vec{v}_{\text{ap/air}} + \vec{v}_{\text{air/g}}$ with $\vec{v}_{\text{air/g}} \equiv \vec{w}$

Problem solving procedures (notice if we are adding only two vectors, this defines a plane (no pun intended) so this stuff remains 2D:

First procedure:

- **Draw the triangle corresponding to the above vector addition**

  We called this "triangularization." We use the information we have about the legs to draw the triangle representing graphical vector addition.

Second procedure (two steps):

- **Choose x and y axes that are convenient**

  Perhaps align one of them parallel to a particular velocity.

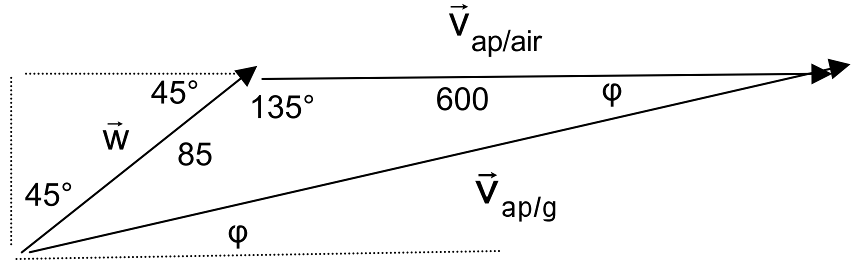
- **Consider the separate components of the vector equation**

  I.e., the components of $\vec{v}_{\text{ap/g}} = \vec{v}_{\text{ap/air}} + \vec{w}$

Example: An airplane has 600 km/h airspeed east and a wind is blowing from the southwest at 85 km/h. What is the airplane's ground velocity (magnitude and direction)?

Answer: Let's draw a triangle right away (see the top of the next page). We know the directions of the two legs $\vec{v}_{\text{ap/air}}$ (East) and $\vec{w}$ (NorthEast) that have to add up to the resultant ground velocity, so it is easy to draw the resultant. (Two different triangles can be drawn with the same resultant, corresponding to adding the two legs in different order. We can choose to start with $\vec{w}$ but either one is fine to use first. See Ch. 1+.)

Putting the tail of $\vec{v}_{ap/air}$ to the tip of $\vec{w}$, we get:



Two methods for analyzing:

1.) First way (analyze the triangle):

Law of Cosines: $v^2_{ap/g} = v^2_{ap/air} + w^2 - 2\,v_{ap/air}w\cos(135^\circ)$

$= (600)^2 + (85)^2 - 2(600)(85)\cos(135^\circ)$

$\therefore \boxed{v_{ap/g} = 663\ \text{km/h}}$

Law of Sines: $\dfrac{\sin\phi}{85} = \dfrac{\sin(135)^\circ}{663} \quad \therefore \boxed{\phi = 5^\circ}$

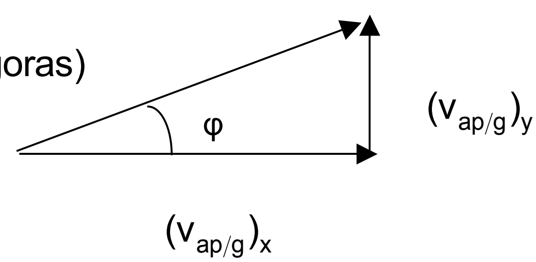
2.) Second way (analyze the rectangular coordinates):

$$(v_{ap/g})_x = w_x + (v_{ap/air})_x = 85\cos(45^\circ) + 600 = 660$$

$$(v_{ap/g})_y = w_y + 0 = 85\sin(45^\circ) = 60$$

$$\therefore\ v_{ap/g} = \sqrt{(660)^2 + (60)^2} = 663\ \text{km/h} \quad \text{(by Pythagoras)}$$

$$\tan\phi = \frac{60}{660} \Rightarrow \phi = 5^\circ \quad \text{(see the figure)}$$



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Problem 1- 5 An airplane is **supposed to fly in a straight path 30° N of E** (this means with respect to the **ground!**). The navigator finds that she must "head" 40° N of E (this is the direction the plane would go if there were no wind) in order to fly this path. The airplane **airspeed** is 400 km/h, but the pilot calculates (by land distances) that they are going 425 km/h with respect to the ground.

a) After triangularization, use the law of cosines to find the wind speed w in km/h. Here and below keep two significant figures.
b) Then use the law of sines to find the (acute) angle in degrees between the wind and the ground velocity.
c) What is the wind direction in degrees E of S?
d) Now start over, using the rectangular-coordinate approach, calling east the positive x-direction, and north the positive y-direction. Find the rectangular coordinates w_x and w_y of the wind.
e) Check your answers in (a) and (c) by using your answers from (d).

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