Chapter 11++

More on CM Motion

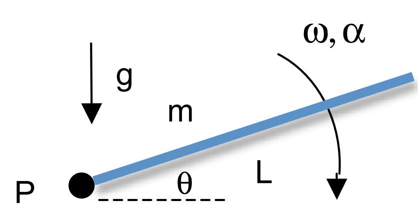
Revisit:	To-Do:
• Motion of CM determined by the net external force • Centripetal (radial) and tangential acceleration	• In the second law, the radial and tangential acceleration of a body, with size, going in a circle, refer to the acceleration of its CM - the rotating rod as an example

Another Blurb on CM Motion

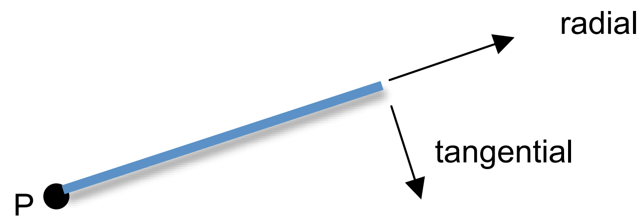
We have learned that **the motion of the CM point** of a system moves **like a single point particle** whose **mass is the total mass M** of the system and whose **acceleration is determined by the external net force through Newton's 2nd law.** In this chapter, we look at an example of the acceleration of the ***CM going in a circle for bodies with size.***

AN EXAMPLE OF THE CM ROLE IN THE SECOND LAW (circular motion)

A uniform rod or stick of mass m and length L has one end pivoted to rotate in a vertical plane under the influence of gravity around a frictionless hinge P. In Ch. 9++, we looked at the energy of such a rotating rod, which involved the angular velocity ω. In this chapter, we look at the external forces on the rod, which are related to both the angular velocity and the angular acceleration. Consider the rod at some instant of time to be at an angle θ (defined this angle above the horizontal) and to have angular velocity ω and angular acceleration $\alpha = d\omega/dt$, as shown below, at that instant. (As defined in this drawing, we actually would have $\omega = -d\theta/dt$ for these angular variables; θ is decreasing for increasing ω. So ω and α are both defined to be positive if CW !!)



The external forces on the rod are due to the pivot and gravity, which we need to think about if we want to write second-law equations. Let us set aside the rotational second law for now (the $\tau = I\alpha$ stuff), and concentrate on the two equations coming from the two components of the translational second law (the f = ma stuff). The lesson of the day is $\vec{a} \Rightarrow \vec{a}_{CM}$: the acceleration that enters into the translational second law is the CM acceleration (and the forces are the sum of the external forces on the rod due to the pivot and gravity). The fact that the rod is rotating tells us that radial and tangential directions first discussed in Ch. 12 are convenient here. That is, we should use radial and tangential components of the acceleration for the 2D motion here. See the figure below. The CM acceleration decomposition that's helpful therefore comes from Ch. 12.



With radial components, the second law in the radial direction can be written

$$F_{\text{external, radial}} = ma_{\text{CM, radial}}$$

where

$$a_{\text{CM, radial}} = -\frac{v_{CM}^2}{r_{CM}}$$

The minus sign is needed because we define radially outward from P to be the positive radial direction. The CM of the rod is, of course, at its midpoint. So the CM quantities are $r_{CM} = L/2$ and $v_{CM} = \omega r_{CM} = \omega L/2$. We find

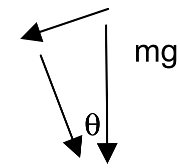
$$F_{\text{external, radial}} = -m\,\omega^2 L/2$$

Finally, if the external force on the rod due to the pivot and its components are defined to be

$$\vec{F}_P = (F_{\text{P, radial}}, F_{\text{P, tangential}})$$

and, by now, a golden triangle calculation of the weight is something we have done a million times, with the radial/tangential components as legs (we defined the positive radial direction as outward; let's also define the positive tangential direction to be clockwise). We get

$$m\vec{g} = (-mg\sin\theta, +mg\cos\theta)$$



To check ourselves, we can verify these reduce to the right limits for $\theta = 0, 90^O$. For example, $\theta = 0$ implies $m\vec{g} = (0, +mg)$, or the force of gravity is in the positive tangential direction when the rod is horizontal.

Thus the second law for the radial direction reduces to

$$\boxed{F_{P,\,radial} - mg\sin\theta = -m\,\omega^2 L/2}$$

If we knew ω and θ at some time, we could find what radial force the pivot must be exerting on the rod at that time.

In the following problem, we work out the second law for the tangential direction. The full story of a falling rod will be seen by combining the energy information from Ch.9++, the translational second-law equations for the CM from this chapter, and the rotational second law in the next chapter.

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Problem 11-8 Finish the above story. That is,

a) What is the second law for the rod along the tangential direction in terms of $F_{P,\,tangential}$, L, m, g, θ, and α? Hint: It will look like the radial second law with changes in some signs, a different trig function appearing, and the term involving ω will be replaced by a different term involving α. Also, remember α is defined to be positive if it is in the CW direction.

b) Look at your answer in (a) for the two different cases, θ = 0, 90^O, and explain in words why your limits are right (we hope).

c) Take a guess as to how we might find ω and α, which would give the complete force on the rod due to the pivot at a given θ. (Hint: think about the result in Ch. 9++ and the rotational second law, respectively.) Here, just describe in words what you might expect we'll have to do to predict ω and α.
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Digression: As a final word, it is kind of neat to take this chance to show how either a unit vector notation or the component notation can be used to describe the acceleration vector of interest:

$$\vec{a}_{CM,\,total} = a_{CM,\,radial}\hat{a}_{CM,\,radial} + a_{CM,\,tangential}\hat{a}_{CM,\,tangential} = (a_{CM,\,radial},\ a_{CM,\,tangential})$$

analogous to the two different notations, $\vec{r} = x\hat{x} + y\hat{y} = (x,\ y)$. Notice that the radial and tangential unit vectors change direction as we go through circular motion, in contrast to the rectangular unit vectors. We could write a similar decomposition for the total external force: $\vec{F}_{external} = (F_{external,\,radial},\ F_{external,\,tangential})$.
