Chapter 8+

Revisit Pulleys and Weights

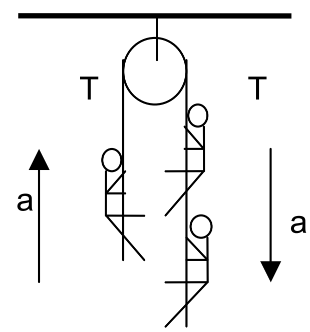
Revisit:	To-Do:
• Pulleys • Basic pulley-weight system • Ideal ropes and pulleys	• Several bodies draped over a pulley and going up and down. • Bootstraps – people pulling themselves up using pulleys

Throughout this discussion we'll stay with "ideal pulleys" and "ideal ropes." Recall that an ideal pulley has no mass (so no moment of inertia) and no friction (so the tension is not reduced if it is slipping around the pulley). So all along a given rope throughout the discussion in Ch.8+: **If the tension is T somewhere, it's T everywhere!**

Revisit The Basic Pulley System With People Instead Of Weights!

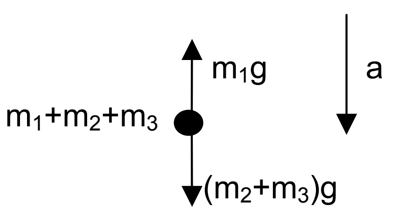
Huey, who weighs m_1g =150 lbs, grabs and hangs on tight to a vertical light rope which is hung from a light, low-friction pulley.

Grabbing and hanging on tight to the other end together are Dewey and Louie, each also weighing 150 lbs ($m_2g = m_3g$ = 150 lbs).



a) What is Huey's acceleration upwards (in ft/s^2)?

Answer: A fast way to get this acceleration a is to apply the second law to the whole system since everyone is going in lock-step with the same acceleration. If the "clockwise" direction around the pulley is called positive, we can imagine an "unfolded" overall FBD (be careful – this could be tricky and not always possible) and an overall second law equation follows:



Overall FBD

$$\Rightarrow \quad (m_2 + m_3)\,g - m_1\,g = (m_1 + m_2 + m_3)\,a$$

With $m_1 = m_2 = m_3 \equiv m$, we have

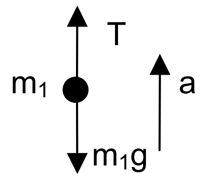
$$a = \frac{(m_2 + m_3)\,g - m_1\,g}{m_1 + m_2 + m_3} = \frac{(m + m)\,g - m\,g}{m + m + m} = \frac{mg}{3m} = \frac{1}{3}g$$

Thus $$a = \frac{1}{3}32 \text{ ft/s}^2 = \boxed{10.7 \text{ ft/s}^2}$$

(since the masses cancelled out, we didn't have to convert to slugs, which are gotten by dividing the weight by g=32 ft/s^2)

b) What is the tension (in lb.) in the rope?

Answer:
From Huey's FBD:

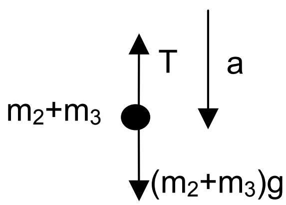


$\Rightarrow \quad T - m_1g = m_1a$

so, from our acceleration result on the previous page: $T = m_1g + m_1a = m_1(g+a) = m_1(g+1/3g)$

Therefore $T = 4/3\ m_1g = 4/3\ (150\text{ lb}) = \boxed{200\text{ lb}}$

Check this via the Dewey-Louie FBD and its second law (down is positive)



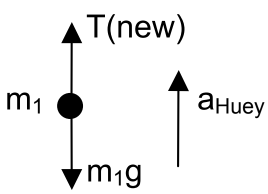
$\Rightarrow \quad (m_2 + m_3)g - T = (m_2 + m_3)\,a$

With $m_2 = m_3 \equiv m$, we verify that we get the same T:

$T = (m_2 + m_3)g - (m_2 + m_3)\,a = 2m(g - a) = 2m\,(2/3\ g) = 4/3\ mg$

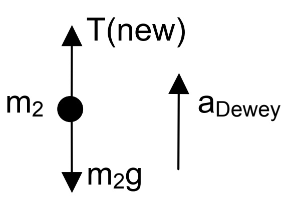
c) Louie jumps off, and briefly brakes the rope with his hand so that Huey and Dewey stop moving and balance each other. Then Huey starts to climb up the rope. What happens to Dewey who is still hanging on tight? Does he go up, down, or stay put? And why?

Answer: As Huey starts to climb, the second law tells us he increases the tension to T(new) from 150 lb (they had been balancing each other, remember) corresponding to an FBD like before but now with T(new) in the second law (up positive) and a new acceleration a_{Huey} :



$\Rightarrow \quad T(new) - m_1 g = m_1\, a_{Huey}$

But Dewey experiences exactly that same new, larger tension and his FBD and second law (up positive) are:

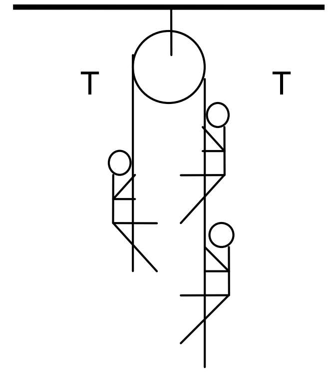


$\Rightarrow \quad T(new) - m_2\, g = m_2\, a_{Dewey}$

Since Dewey has the same mass $m_1 = m_2$, the equations are identical, and we must have

$$\boxed{a_{Huey} = a_{Dewey}}$$

Dewey goes up at exactly the same acceleration as Huey and thus has the same speed and moves the same distance at any time!!



d) Now Louie jumps back on Dewey's side, grabbing the rope and hanging on tight. Huey climbs up the rope so vigorously that both are lifted! What must be the minimum acceleration (in ft/s^2), relative to the ground, with which Huey must climb, in order that he could <u>just barely</u> begin to lift both Louie and Dewey?

Answer: Huey must be producing a tension T = 2mg, since on the "verge" of lifting Louie and Dewey, T – 2mg = 0 for the Louie-Dewey system. But for Huey, he is accelerating and

Huey's second law: T – mg = ma

$$\text{Thus} \quad a = \frac{T\text{-}mg}{m} = \frac{2mg\text{-}mg}{m} = \frac{mg}{m} = g = \boxed{32 \text{ ft/s}^2}$$

This turns out to be g just by coincidence since the difference between the combined weight of Louie-Dewey and Huey happens to equal Huey's weight.

Again, note that you can either (1) wait until the very end of the algebra, as we have done above, before putting in numbers (which often lets you avoid talking about slugs), or (2) you could put in numbers at the beginning by converting lbs to mass (units of slugs) by dividing them by 32. A neat shortcut in the coming problem in using approach (1) is to write ma = m(g/g) a = mg (a/g) and you have thus expressed ma in terms of weight =mg and you can now put in lb and ft/s^2 without an intermediate step of converting weight to slugs, etc. Either way is OK.

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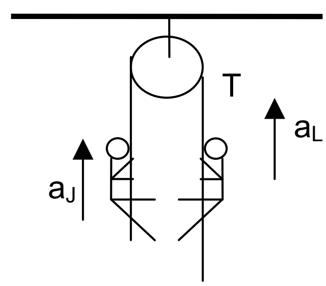

Problem 8-5 *Jennifer, who weighs 120 lbs, wants to climb up a vertical massless rope which is hung down from a fixed massless, frictionless pulley. Lisa is holding on for dear life to the other end of the rope which is hanging down from the other side of the pulley. Although Lisa is only 110 lbs, Jennifer can still climb up if she goes hand-over-hand vigorously enough. Suppose Jennifer is managing to climb up at a constant acceleration* a_J = 4.0 ft/s^2 *relative to the ground (probably breaking some kind of world record):*

a) From an FBD for Jennifer, write a second law for the vertical motion with which you can find the tension T *in lbs for the rope.*

b) From an FBD for Lisa, write a second law for her vertical motion with which you can find her acceleration a_L *(defined upwards as shown, so if she goes down,* a_L *will be negative) in* ft/s^2 .

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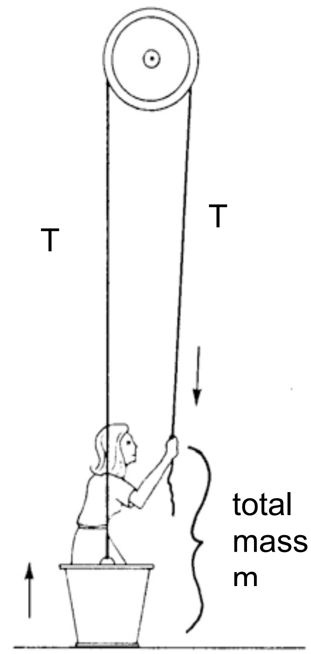
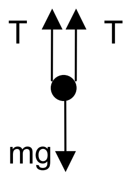
Bootstrap Example

A window washer pulls herself upward using the bucket-pulley apparatus shown. The pulley is attached to a ceiling and the **total** mass of the woman and bucket is m.

(a) How hard must she pull downward to raise herself slowly at constant speed?

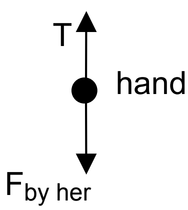


FBD for woman-bucket system plus the second law with zero acceleration (down positive):



$$\Rightarrow \; mg - 2T = 0 \Rightarrow \; T = \frac{1}{2}mg$$

Since her hand has negligible mass the force by her arm on her hand is simply equal to the tension T by the second law applied to her hand as a system. To see this*:



$$\Rightarrow \; F_{by\ her} - T + \cancel{m_{hand}g} = 0$$

(we have "X-ed" out the small term that can be neglected)

$$\Rightarrow \; F_{by\ her} = T \quad \text{(as if the hand were just part of the “massless” rope)}$$

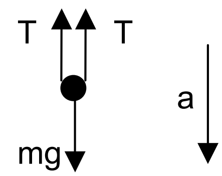
So we now can use the result for T found above:

$$\boxed{F_{by\ her} = \frac{1}{2}mg}$$

* Generally, in these kinds of hand-pulling situations, we can simply imagine the arm as part of the rope with the same tension traveling down it. So when we say "how hard is she pulling?" then we are simply inquiring as to the tension in the rope she is holding, and forget about the (light) hand.

(b) If she increases this force by 10 percent, what will her acceleration be?

Now the forces are not balanced, implying that the acceleration is not zero. The previous FBD still applies for the woman-bucket system:



so the second law now reads (we have defined a as positive if it's accelerating down – with this definition, we expect to find a < 0 !):

$$mg - 2T = ma \;\Rightarrow\; a = g - 2T/m$$

Again, since her hand has negligible mass, the force by her arm on her hand is simply equal to the tension T by the second law:

$$F_{\text{by her}} = T$$

Perhaps we can be pardoned if we revisit the equation with the hand included to see once more how to prove this:

$$F_{\text{by her}} - T + \cancel{m_{\text{hand}} g} = \cancel{m_{\text{hand}} a} \;\Rightarrow\; F_{\text{by her}} = T \qquad \text{(checks!)}$$

where we have again "X-ed" out the small terms that can be neglected)

$$\text{But } F_{\text{by her}} = \text{previous} + 10\%\text{ of previous} = \frac{1}{2}mg + 0.1\left(\frac{1}{2}mg\right) = 1.1\left(\frac{1}{2}mg\right)$$

$$\Rightarrow\; T = F_{\text{by her}} = (1.1)\,\frac{1}{2}mg$$

$$\Rightarrow\; a = g - \frac{2}{m}\,(1.1)\,\frac{1}{2}mg = g - 1.1g$$

$$\boxed{a = -0.1\,g}$$ Of course! She accelerates up!

Just as another reminder of our short-cuts taken. This a is the x-component (the only component, since it's 1D) of the acceleration vector with the x-axis defined going down. But we hate writing a_x all the time here and many places in the text.

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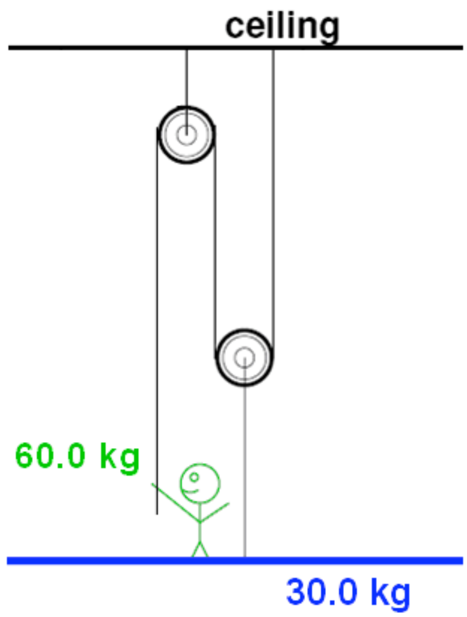
Problem 8-6: *Leonard Nimoy stands on a platform and pulls on an ideal rope, which is attached to a system of two ideal pulleys as shown below.*

The mass of Leonard is 60.0 kg. The mass of the platform is 30.0 kg. Assume that Leonard pulls the rope sufficiently hard such that the platform moves upward with constant acceleration $a_y = 0.20\ m/s^2$.



a) *Using an FBD for the whole system consisting of the lower pulley, Leonard, and the platform, find a second law equation for the vertical motion of the whole system and use it to determine the tension in the rope in N. Note that the only forces that show up in the overall FBD are the external ones involving "three rope pulls" and two weights.*

b) *Using an FBD for Leonard alone, find a second law equation and use it to determine the magnitude and direction of each and every force on Leonard (again in N).*

c) *Use an FBD for the platform to get a second law equation for its vertical motion and, to check your answers from (a) and (b), plug in your tension from (a) and your contact force from (b) and see if this new equation is satisfied.*

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