Chapter 4++

More on the Orbits and Total Energy for Newton's Gravity

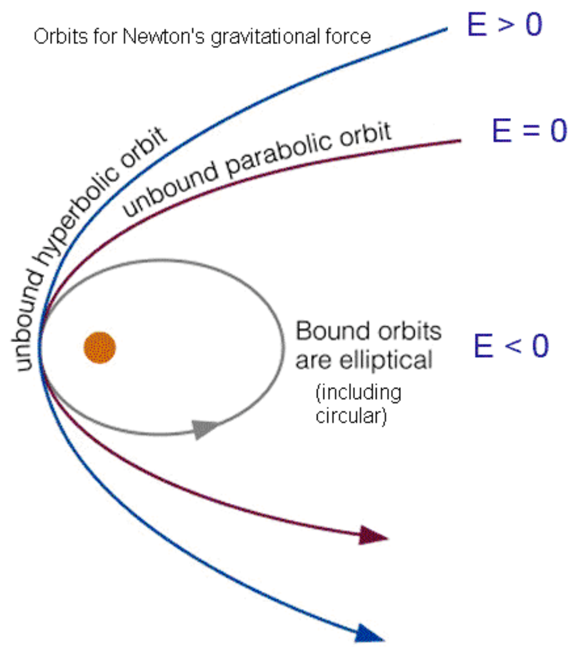
Revisit:	Learn:
• Total Energies Associated with the Orbits Generated by Newton's Gravitational Force	• More about Orbits and Their Total Energies • Escape velocity

Total Energy Associated with Newton's Gravitational Force

Recall what we learned in Chapter 4+: With no other forces: **the total mechanical energy is conserved for a body of mass m when it is subjected to the gravity of a fixed (or big!) mass M :**

$$E = \frac{1}{2}mv^2 + U(r) = \frac{1}{2}mv^2 - \frac{GMm}{r} \quad \text{is } \textbf{constant!}$$

Recall the three classes of orbits (see the figure):



Before we get to the **three classes**, keep in mind that, for E = KE + PE, we always have

KE is never negative: $KE = \frac{1}{2}mv^2 \geq 0$

PE is never positive: $PE = -\frac{GMm}{r} \leq 0$

1) BOUND: When m is **"bound"** to M , we must have E < 0. That is, PE dominates KE in the total energy, in the sense that |PE| > KE. This means that m can never escape to infinity ($r \to \infty$), because there PE vanishes, yet KE cannot be negative. We do not have enough KE to overcome the PE and we can't escape to infinity. Recall in circular orbits we were able to relate KE to PE – see Ch.4+ - and we indeed found E < 0. By the way, we need to know both the energy E and the angular momentum L in order to know the shape of the orbit (i.e., how eccentric the ellipse is). See your next mechanics course.

2) UNBOUND: When m is **"unbound,"** we must have $E \geq 0$ .That is, KE dominates PE in the total energy, in the sense that $KE \geq |PE|$. This means that m can and will always escape to infinity ($r \to \infty$), where the vanishing of PE does not contradict $KE \geq |PE|$. We do have enough KE now to overcome the PE and escape to infinity.* The so-called **"critical" unbound** case of E = 0 corresponds to when m just barely escapes to infinity. E = 0 means that, at infinity where PE vanishes, KE = 0, too. We have used up all of our KE to cancel the PE and escape. We consider the well-known escape velocity problem for a rocket to leave the earth on the next page.

* Dear reader, in your next mechanics course, you'll analyze the force and see how unbound orbits are hyperbolas (or parabolas for E = 0) and hence how the satellite will always eventually fly away forever!

Escape Velocity Example:

What is the minimum KE needed for a rocket to shoot off the ground and escape from the Earth's gravitational field?

Well, to barely escape we need E = 0 (E > 0 will give us some KE left over when we get to "infinity"). Although the rocket has some KE due to the Earth's spinning and its orbit around the sun, etc., it is not enough to overcome its PE due to the Earth. The amount of KE we need is such that E = KE + PE = 0 or KE = - PE, where PE is evaluated at the ground level.

To find the KE (and hence the velocity) needed to escape:

$$E_{escape} = KE_{escape} + PE = 0 \qquad \Rightarrow \qquad \tfrac{1}{2}mv_{escape}^2 - \frac{GMm}{R_{earth}} = 0$$

Notice that m cancels out (so this is going to be a universal escape velocity, true for any m). Solving for the escape velocity, we get

$$v_{escape} = \sqrt{\frac{2\,GM}{R_{earth}}} = \sqrt{\frac{2\,(6.67 \times 10^{-11})m^3/kg\cdot s^2(5.98 \times 10^{24}kg)}{6.37 \text{ x } 10^6 m}} = 11.2 \times 10^3 \text{ m/s}$$

where we inserted the Earth's mass in kg and its average radius in m . This result, 11.2 km/s translates into about 25,000 MPH!

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Problem 4-6 A small meteor of mass 1 kg is sailing into our solar system from outer space and at one instant in time is 5 x 10^9 km away from the sun.

a) What velocity in m/s must the meteor have in order that it could eventually escape from the sun's gravity?

b) Suppose instead that the meteor is moving such that it goes in a circular orbit around the sun. What velocity must the meteor have for this to happen?

c) Find the changes in your answers to (a) and (b) if the meteor has a mass of 1000 kg instead.

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A note of interest: We can't leave this orbit stuff without addressing something that you just might have puzzled over. We have described orbits of balls or rocks close to the surface of the Earth as parabolas. But we have also been saying that the orbits of balls around something like the Earth should be elliptical, in general. What gives? Well, the ellipse is the exact answer. But what's happening is that we are looking at a tiny part of it at or near its apogee (farthest point from the center of the Earth or perigee, the farthest point). That small part of the total orbit is approximately a parabola! (Mathematically, a "Taylor's Series" expansion gives a quadratic term for y as a function of x. So you get y = (constant) x^2 + another constant, giving a parabola!