PHYS 121: Homework #09

April 10, 2024

Physics 121: Spring 2024: Week 12 Reading assignment:

$\boxed{\textbf{REQUIRED:}}$ ***Physics 121 Online Class Notes* Cycle 3, Chapters 10++ through 14++, posted on the Course Website, as follows:**

- **Skim Chapter 10++**: We will return to the rocket equation in Cycle 4. For now, just skim this material so you have a sense of where we will be going with this.
- **Read Carefully Chapter 11++**: This is a pretty important chapter. The example of the board tipped at an angle is more complicated than the one I will show in class where the board is horizontal. RWB uses the term "radial" which is sort of the opposite of "centripetal".
- **Read Chapter 12++**: We look in even more detail that you ever want to know about falling rod and boards. This is probably more detail than we need.
- **Read Chapter 13++**: An even deeper look at statics problems, doors and hinges, leaning ladders.
- **Read Chapter 14++**: Yo-yo's and spools. This is good stuff. Make sure you can set up these problems on your own.

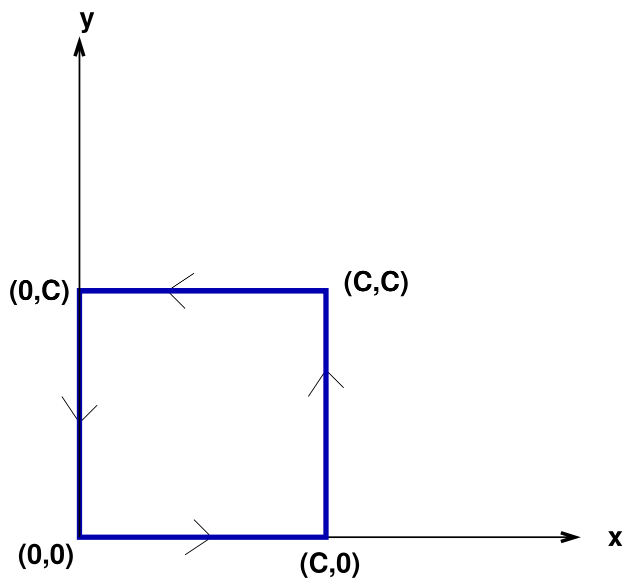
Homework continues next page....

Problem 1: The Path Integral

Consider a **new force** which is defined as a function of two-dimensional coordinates this way:

$$\vec{F} \equiv Ay\,\hat{\boldsymbol{\imath}} + Bx^4\,\hat{\boldsymbol{\jmath}}$$

Here x and y are standard Cartesian coordinates and A, and B are given real positive constants with appropriate units.



Consider the loop that goes around a square path as show in the figure. Each side of the square has a given length C.

Part a) What is the **Work** done by this specific force $\vec{F}$ for the closed path that goes around the loop in the *counter-clockwise* direction, starting from the origin $(0,0)$? Important: You must *explicitly calculate* the path integral here. Explain your work. Give your answer in terms of the given parameters.

Part b) Based on your calculation from Part (a), answer this question: Is this force $\vec{F}$ Conservative or Not Conservative? Cite specific evidence and/or examples to support your claim either way. Explain your work completely here to get full credit.

Important: Problem 1 continues next page.....

Problem 1 continues .....

Possibly helpful hints for Problem 1: You **cannot** assume that just because the path is closed the integral is zero. You need to **calculate** the path integral explicitly here. Don't forget the Definition of Work on a path that goes from point P to point Q is given by:

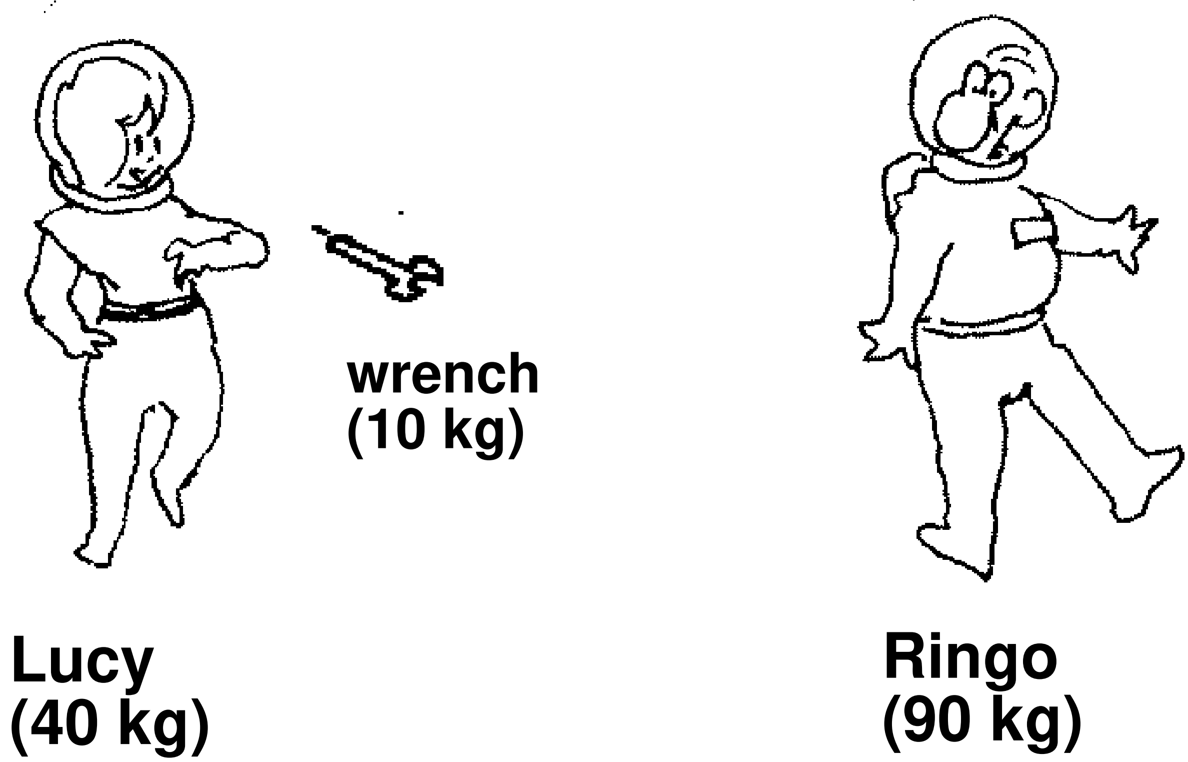
$$\mathcal{W}_{\vec{F}}(P \to Q) \equiv \int_P^Q \vec{F} \cdot d\vec{r} = \int_{x_P}^{x_Q} F_x dx + \int_{y_P}^{y_Q} F_y dy$$

where F_x is the x-component of the force and F_y is the y-component of the force.

In order to calculate the Work done by the force around the square loop, you will need to calculate the Work done on *each* of the four "straight-line" pieces of the path. In other words, you need to break the closed-path integral into four regular 1-D integrals. For each "straight-line" piece you are holding one coordinate *fixed* at a *known value* and integrating with respect to the other coordinate.

Problem 2: Throw and Catch Revisited

Two astronauts, Lucy, and Ringo, are floating free in deep space, as shown. Each astronaut is defined to have zero velocity. Lucy has a mass m_L of 40 kg. She holds a wrench of mass m_w of 10 kg. Ringo has a mass m_R of 90 kg. Lucy throws a the wrench at Ringo. She throws the wrench so that it moves towards Ringo at a translational speed v_w of 5 meters/second. She also puts a rather good spin on the wrench, so that it rotates with an angular velocity ω_w of 16 radians per second. Ringo then catches the wrench. Assume that the wrench has a rotational inertia given as $I_{wrench} = 0.2$ kg·m^2 and assume that Lucy has a rotational inertia that is 80 times that of the wrench. Assume Ringo has a rotational inertia that is 125 times that of the wrench.



Important: Solve this problem **symbolically** first, then plug in numbers at the end.

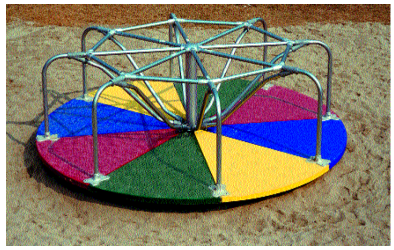
a) What is Lucy's translational velocity after she throws the wrench?

b) What is Ringo's translational velocity after he catches the wrench?

c) What is Lucy's rotational velocity after she throws the wrench?

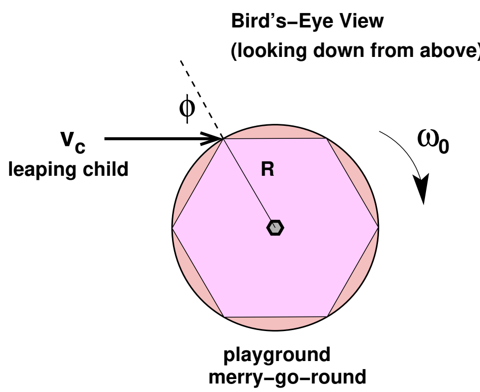
d) What is Ringo's rotational velocity after he catches the wrench?

Problem 3 – Merry-Go-Round ride From a Previous Exam



A playground merry-go-round.

A common recreational fixture in children's outdoor playgrounds is the "merry-go-round" which is generally a large circular platform mounted on a central bearing so that the platform can spin freely.



A "bird's-eye-view" of a merry-go-round is shown above. Assume that the bearing is ideal and that the total rotational inertia of the merry-go-round alone is given as I_m. Assume a small child with given mass m leaps from the ground with horizontal velocity given as v_c and then lands and sticks on the merry-go-round at a given distance R from the center. Assume that the child impacts the merry-go-round at a given angle ϕ relative to the radial direction as shown. Treat the child as a point-like-object. Ignore all vertical motion of the child. Assume the merry-go-round is initially moving with an angular speed of ω_0 in the clock-wise direction as shown.

Part (a) – What is the magnitude of the total angular momentum L_{tot} for the combined child-plus-merry-go-round system in the instant just before the child comes in contact with the merry-go-round as calculated for the pivot point corresponding to the central axle of the merry-go-round? Express your answer in terms of the given parameters. Explain your work.

Part (b) – Assume the merry-go-round is initially moving with a given angular speed of ω_0 but then comes up to angular speed ω_1 just after the child leaps on. Calculate the value of ω_1. Express your answer in terms of the given parameters. Explain your work.

Part (c) – Now assume that the child subsequently pushes straight off the merry-go-round so as to land just off the outside edge on the ground with precisely zero horizontal velocity. What is the angular speed ω_2 of the merry-go-round after the child performs this maneuver? Express your answer in terms of the given parameters. Explain your work.