Chapter 8++

More Pulleys

Revisit:	To-Do:
• FBD'S and XFBD's • Newton's second law for usual translations and for rotations • Bootstraps	• More bootstraps: first with an ideal pulley and then later with a massive pulley

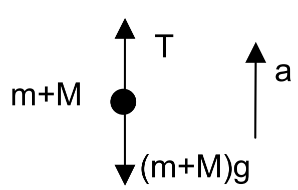
Act III, Scene 1: An Ideal Bootstrap

Let's consider one more example in practicing looking at internal forces, where we need to do a good job isolating the different parts of the problem through "free-body diagrams." We'll include ropes and pulleys **but here we'll go back to ignoring their mass.**

Example: A man with mass m is standing as shown on a platform with mass M. He is pulling mightily on the light fall rope as illustrated, such that both the platform and he are accelerating together upward with constant acceleration a. We ignore any friction as well as the mass of the pulley (and the mass of the rope), and we assume the rope motion rotates the pulley without any slippage.

a. What is the tension T in terms of the given parameters?
   Answer: We first draw an FBD along with an arrow representing the acceleration direction, noting that the only external forces come from the part of the rope going up to the ceiling. We then write the second law for the whole system:

total FBD

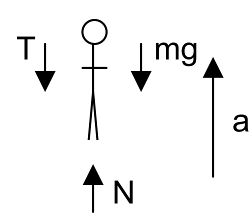


$$T - (m+M)g = (m+M)a$$

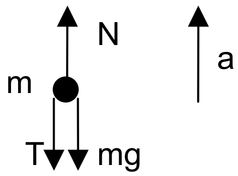
$$\boxed{T = (m+M)(a+g)}$$

b. What is the total normal support force N exerted on (both of) the man's feet in terms of the given parameters?

   Answer: We first sketch on the right a cartoon to help us see the forces and then on the next page draw an FBD for the man as the system and use it to help us write the second law for this system. Then we use T from (a). **Note the direction of T on the man!** (continued on the next page)



m FBD



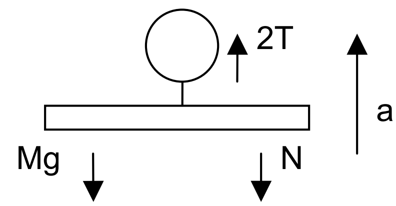
$$N - mg - T = ma$$

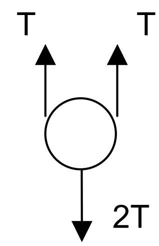
$$\therefore\ N = T + m(a+g) = (m+M+m)(a+g)$$

or

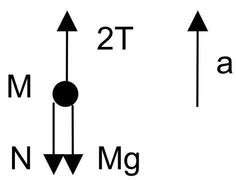
$$\boxed{N = (2m+M)(a+g)}$$

c. Let’s find N in a second way by looking at the platform as a system. We first sketch on the right a cartoon again to help us see the forces and then we draw the FBD and write its second law. Notice that the pulley support post must have tension 2T (as in tutti fruiti) by reaction to the two ends of the rope pulling up on it (see the little XFBD on the right, where the 2T is demanded by force balance).





M FBD



$$2T - Mg - N = Ma$$

$$N = 2T - M(a+g)$$

or

$$N = [2(m+M) - M](a+g) = (2m+M)(a+g)$$

d. Dear reader, why don’t you guess what happens to the tension T and the normal force N when, after he has risen for a bit, the man lets go of the rope. Close your eyes and read no further and guess. Now open them and read the answer below:

Well, we expect $\boxed{T = N = 0}$. Let's check it: If the man lets go, we have free fall $\Rightarrow$ $a = -g$

$\therefore T = (m+M)(-g+g) = 0$ and $N = (2m+M)(-g+g) = 0$ This checks!

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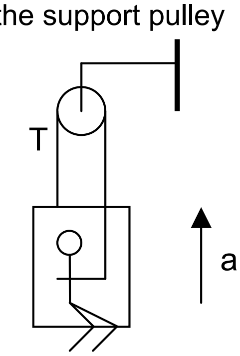
Problem 8-7 A painter with mass m is seated on a scaffolding chair (with mass M < m), hung on the side of a building as shown. She is pulling on the light fall rope as illustrated, such that she and the chair are accelerating upward with constant acceleration a. Also, the support pulley is ideal (frictionless and light in weight)



(a) From an overall FBD, find an expression for the tension T in the rope in terms of the given parameters.
(b) From an FBD for her or for the chair, find an expression for the force F exerted by the chair upwards on her seat, in terms of the given parameters.
(c) Guess what happens if her weight is in fact less than the weight of the chair (M > m), and check it by using your formula from (b).

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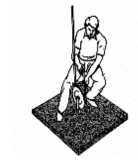
Act III, Scene 2: A Bootstrap with a Massive Pulley

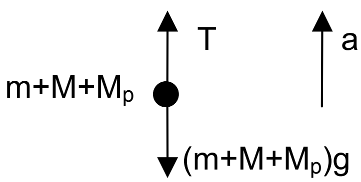


Previous Example: Please return with us to the previous pages where the man is pulling himself up on that platform. Suppose we now take into account the mass M_p of the pulley (but we still forget about the rope's mass and, as always, we assume the rope pulls the pulley around without slipping). Let the pulley be a nice uniform disk with radius R.

(a) What is the change in the overall system's FBD? What T does the second law and this FBD imply now?

total FBD

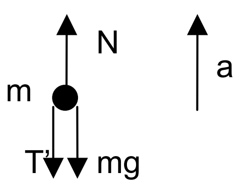
Answer: The FBD is the same as for (a) on p. 8-13, except that we have one more mass and one more weight (M_pg) pulling down on it. So we now get



$$\Rightarrow T-(M+m+M_p)g = (M+m+M_p)a \Rightarrow \boxed{T=(M+m+M_p)(a+g)}$$

(b) What is the change in the FBD and the second law for the man? On what does the normal support force N depend now?

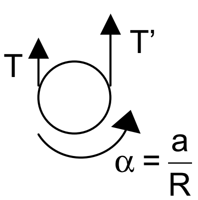
m FBD



Answer: Now the tension pulling him down is not T but a bigger tension T', since the pulley eats up part of the force through its moment of inertia. From the the FBD which involves T' and the second law we get

$$\Rightarrow N-T'-mg=ma \Rightarrow N=T'+m(a+g)$$

(c) What is the additional information you need to solve for N completely?



Answer: We now need the rotational second law for the pulley to relate T and T'. and for that we need the XFBD on the left. The rotational second law is thus $T'R - TR = I\alpha$ from the usual simple 90^o torques for pulleys. The cylindrical moment of inertia $I = \frac{1}{2} MR^2$ and the rolling constraint $\alpha = a/R$ lead to

$$T'R - TR = \frac{1}{2} M_pR^2(a/R) \Rightarrow T'=T+\frac{1}{2}M_pa$$

(d) What are the final answers for N and T'?

Answer: Substitute T from (a) into (c) to get

$$T'=(M+m+M_p)(a+g)+\frac{1}{2}M_pa \Rightarrow \boxed{T'=(M+m)(a+g)+M_p(\frac{3}{2}a+g)}$$

and substitute the T' expression into (b) to get

$$N = (M+m)(a+g)+M_p(\frac{3}{2}a+g)+m(a+g) \Rightarrow \boxed{N = (M+2m)(a+g)+M_p(\frac{3}{2}a+g)}$$

Comments:

- A massive pulley increases the outside upward pull T simply because it's heavier. It increases T' both because of the weight and the additional pulley inertia against its rotation. And thus N gets increased in two ways through T'.

- It is interesting that, if the rope slid around the pulley without turning it and without friction, we would only need to consider the pulley's weight in the translational second law and we could forget the moment of inertia part and the rotational second law!

To practice with both a massive pulley and an inclined plane:

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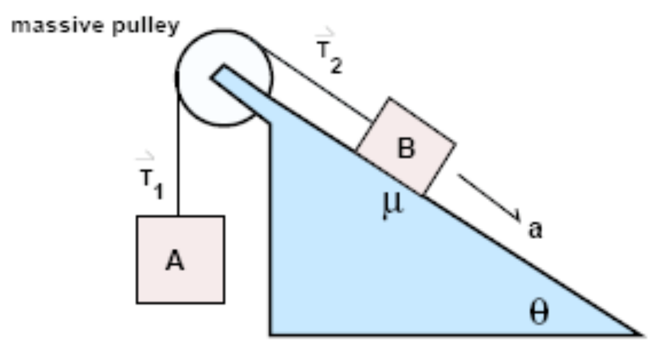
Problem 8-8

Two blocks A, and B with masses m_A, and m_B, respectively, are positioned as shown above. Block B is accelerated down the incline (angle θ) with acceleration a as shown. The coefficient of sliding friction between Block B and the incline is given as μ. The pulley has a mass m_p and radius R. The rope connecting A and B is considered to be massless.



a) *Without doing any calculations*, answer this question: Is the tension T_1 greater than, less than, or equal to the tension T_2? Explain.

b) Draw accurate Free-Body Diagrams (FBDs) to indicate the forces on Block A and on Block B. Also draw an Extended Free Body Diagram (XFBD) for the pulley, which can be modeled as a **disk**.

c) Find three equations with which you can determine the acceleration a, as defined on the figure, and the two tensions.

d) For good algebra practice, solve for the acceleration a by "eliminating" the two tensions via two of your equations so that the third equation involves only the acceleration. To save you time, don't bother solving for the tensions.

Notice in your answer in (d) that the acceleration would be negative (i.e., a < 0) if, for example, m_A is sufficiently large, as you would expect.

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