Chapter 5+

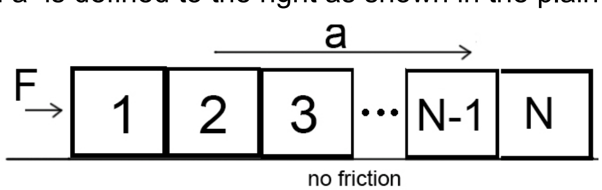
Revisiting Contact Forces: Massive Ropes

Revisit:	To-Do:
• Contact forces • Tension • Ideal ropes	• String of Blocks • Massive Rope (like a continuous string of blocks)

NOTE: We say massive to just mean it has mass; too bad there isn't a word that just means 'has mass,' since massive sounds so big; we could invent something like 'massious' but that sounds nauseous.

A String of Blocks: A prelude to massive ropes

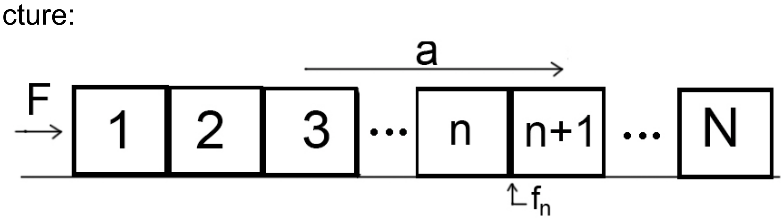
Following our earlier Ch. 5 discussion, let's add more blocks and create a "string of blocks." This will help us understand a massive rope, which we'll discuss later in the chapter. Consider a horizontal force F applied to the left end of a row of N identical blocks each with mass m resting on a frictionless table. With a nonzero external net force, the system accelerates and the common horizontal acceleration a is defined to the right as shown in the plain vanilla diagram:



We wish to find the contact forces at any point along the string of blocks, given F, N, and m. Recalling how we approached the two-block system,* we first find the common acceleration a, which tells us the net external force on each portion of the block system, and thus we can find the individual internal contact forces.

Newton's second law for the whole system yields: $a = \dfrac{F}{M_{total}} = \dfrac{F}{Nm}$

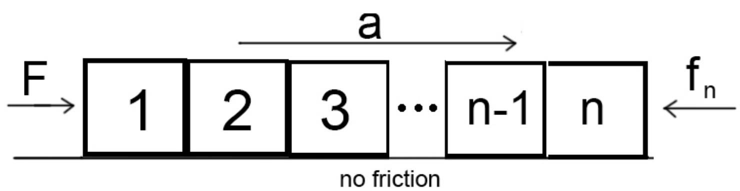
We want to find the horizontal contact force (defined as f_n) between the n^{th} and $n+1^{st}$ blocks, as located in the picture:



The contact force f_n is the action-reaction force on the wall separating the **first n blocks** from the **last N – n blocks**.

* **It is always helpful to use the way you approach a simple version of a complicated system to guide you in the more general case.**

To get an equation for f_n, isolate the **first n blocks** as your subsystem:



You know the net external force on the first n blocks must be their total mass times the common acceleration a by Newton's second law:

$$F - f_n = (\text{mass of the first n blocks})\, a = (nm)\, a$$

Now solve for the magnitude of the contact force f_n, and substitute for a from the previous page:

$$f_n = F - (nm)\, a = F - (nm)\frac{F}{N\,m} = F - n\frac{F}{N} = \left(1 - \frac{n}{N}\right) F$$

Thus $$\boxed{f_n = \left(1 - \frac{n}{N}\right) F}$$

We see that as we go down the line of blocks (from n = 1 to n = N-1) the contact force gets more and more eaten up by the blocks' masses, as we expect for an accelerating system.

Now, you might complain to us, why didn't you derive this more easily by considering the **last N – n block**s, and we say ... see the homework!
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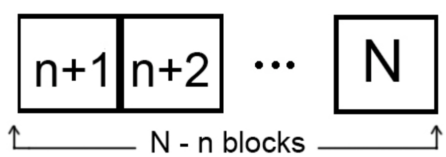
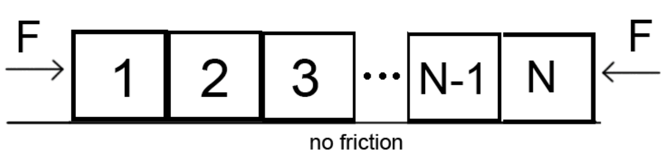
Problem 5-4

a) *Instead of analyzing the first n blocks, as in the above derivation, derive the magnitude f_n by considering Newton's second law for the **last N – n blocks** and check to see that your answer agrees with the above boxed answer.*



b) *What is f_n changed to, if, in addition to the original force* F*, another equal and opposite force is applied to the other end, as shown below?*



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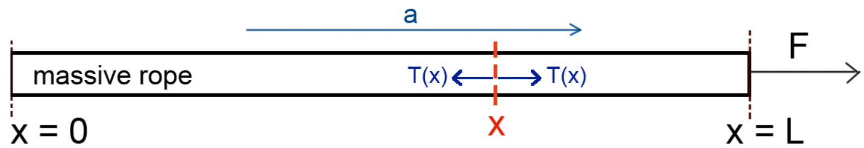
Tension Along a Massive Rope

We have neglected the mass of the rope up until now and that's fine, since it is usually much smaller than the loads involved. But armed with the understanding of the string of blocks, we can now consider a rope with mass. So let's do it!

Take a uniform rope of length L and total mass M (uniform officially means any two equal pieces of it have the same mass and the cross sectional area profile is the same throughout). Lay it straight along the x-axis, from x = 0 to x = L, on a frictionless table with nothing attached to the rope. Suppose we apply an external force F, as shown.

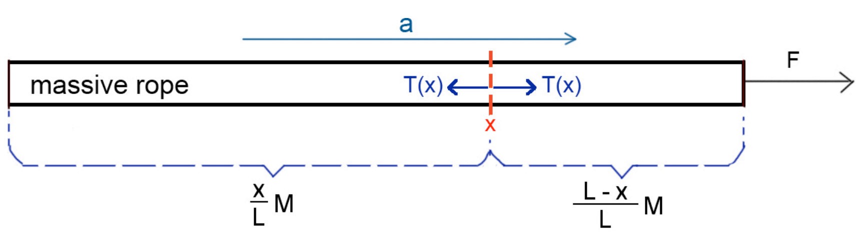


How does the tension, T(x), change with x along the rope, for a given F?

(Remember that the tension in any infinitesimal slice of the rope is the force pulling on that slice from the left **or** the right. The two pulls from the left and right must be equal and opposite according to the third law.)

Well, from our block experience above, we know that an accelerating rope with mass will **eat up the force as we go down the line**, so we expect the tension to depend on the distance from F and to **get smaller as we go to the left.**

Calculation of T(x): Notice the tiny red slice divides the rope into two parts, with masses $\frac{x}{L}M$ and $\frac{L-x}{L}M$:



Choosing to isolate the piece on the left, we find from the second law:

$$F^{net}_{left} = T(x) = m_{left} a = \frac{x}{L} M a$$

in terms of the acceleration a common to every point on the rope. The acceleration a is found from the second law applied to the whole rope:

$$F^{net}_{whole\ rope} = F = Ma \Rightarrow a = \frac{F}{M}$$

The tension at any point x is therefore given by $T(x) = \frac{x}{L} \cancel{M} \frac{F}{\cancel{M}}$ or

$$\boxed{T(x) = \frac{x}{L} F}$$

Comments on the result , $T(x) = \frac{x}{L} F$, for the above system:

1) Alternatively, we could have chosen to analyze the right side of the rope, which has mass $\frac{L - x}{L} M$. The second law derivation with this subsystem is a little longer, but yields the same result.

2) At $x = L$, $T(L) = F$ as expected since there the rope feels the full force F.

3) At $x = 0$, $T(0) = 0$ as expected since there's no more mass on the left to pull on .

4) The tension is growing linearly from the left end to the right end because the mass is accelerating and is increasing linearly from the left to the right. Equivalently, the "accelerated mass eats up the force" as we go from right to left.

5) If the rope is not accelerating, the net force on any portion of the rope is zero and the "accelerated mass eats up the force" story doesn't apply – see the Problem below.

6) By the way, sometimes people like to give a mass density λ (mass per unit length) for the rope instead of the total mass M. Then we have $M = \lambda L$ but λ cancels out, just as M did, and we get the same answer $T(x) = (x / L) M$

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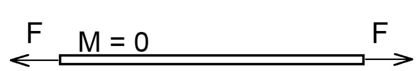
Problem 5-5 *Let's look at the same rope, with mass* M *and length* L*, laid out horizontally on a frictionless table as before, but now with some variations:*

a) Derive the tension T(x) *along the rope if there is an additional equal and opposite force* F *pulling on the left end, as shown*

b) Derive the tension T(x) *along the rope if we go back to just the single external force* F *pulling on the right end, but now there is a block with mass* m *attached to its left end.*

c) Can we talk about a tension T(x) *if* M = 0 *in the original rope system with just one external force?*

d) Can we talk about a tension T(x) *if* M = 0 *in the system of part (a)?*



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