Chapter 6++

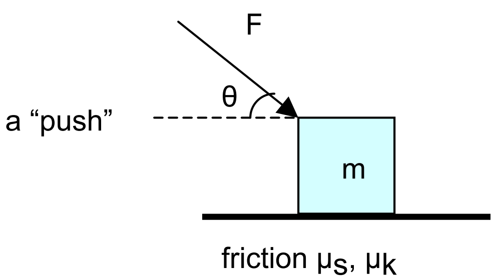
Force Components Reviewed

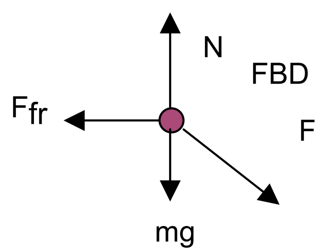
Revisit:	To-Do:
• Force components • Including friction	• An example combining both static and kinetic friction, components, and normal forces not equal to the weight

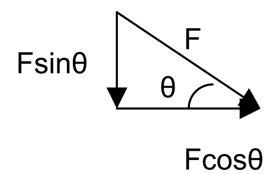
Friction and Forces at Angles

In the first cycle of this chapter, we considered an external force applied at an angle on a block. We saw examples where the normal force N depends on the applied force as well as the weight, so N was not simply mg. Let's return to that situation, for practice one more time on finding out when we can or cannot slide.

In the example below, the normal force on the block is larger than the weight because, besides the weight, it must also balance the downward component of the applied force.







With the FBD and the golden triangle shown, we're ready to write down the second laws for the vertical and horizontal directions:

- **vertical force equation -** $N - F\sin\theta - mg = 0$ or

$$\boxed{N = F\sin\theta + mg}$$

- **horizontal force equation -** $F\cos\theta - F_{fr} = ma$

1) If no slipping: $a = 0$ and $F_{fr} \le \mu_s N$ Thus

$$F\cos\theta = F_{fr} \le \mu_s N = \mu_s(F\sin\theta + mg)$$

or

$$F\cos\theta - \mu_s F\sin\theta \le \mu_s mg$$

continued on the next page

So we get no motion if F is too small

$$\boxed{F \le \frac{\mu_s mg}{\cos\theta - \mu_s \sin\theta}}$$

Notice: If $\cos\theta - \mu_s\sin\theta < 0$ (i.e., at larger angles and/or larger friction coefficients), then the original inequality is always satisfied for all F and the block never starts to slide: This just means the horizontal component of the applied force is then too small. That is, if $\cos\theta - \mu_s\sin\theta < 0$, then $F\cos\theta < \mu_s F\sin\theta$, so certainly $F\cos\theta < \mu_s F\sin\theta + \mu_s mg$, which means no slipping as we saw.

2) If there is slipping: $F_{fr} = \mu_k N = \mu_k(F\sin\theta + mg)$

so the second law reads

$$F\cos\theta - \mu_k(F\sin\theta + mg) = ma$$

or

$$\boxed{a = (F(\cos\theta - \mu_k \sin\theta) - \mu_k mg)/m}$$

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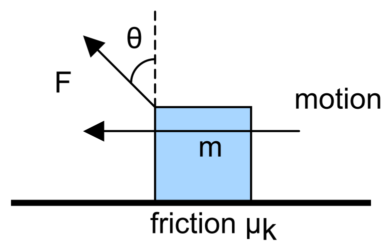
Problem 6-5:

A worker pulls a packing crate with mass m across a rough horizontal floor by exerting a force F on it at an angle of θ with respect to the vertical. This force is not sufficient to lift the crate.



a) After drawing an FBD and thinking through the components of F (a golden triangle), write out the vertical and horizontal force equations in terms of the general quantities m, g, F, the angle θ, the normal support force N of the floor on the crate, the coefficient of sliding (kinetic) friction μ_k, and a **leftward** acceleration a, daring not to copy the ones in the notes, since there are crucial sign and angle differences!

b) Now solve for N from the vertical equation and substitute it into the horizontal equation. Then solve for a in terms of the remaining given quantities.

c) If the crate moves at constant velocity, find a formula for the coefficient of sliding friction μ_k between the crate and the floor in terms of F, mg, and θ, and then calculate its numerical value if mg = 200 N, F = 55 N, and θ = 35°.

d) With what bigger force (but at the same angle) must the worker pull, to give the crate a leftward acceleration of a =1.3 m/s^2 along the floor? Give your answer as a formula involving a and the other general quantities and then evaluate it numerically (in terms of Newtons) using the μ_k value you found in (b).

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