Chapter 13++

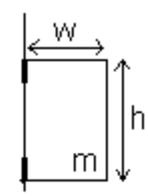
More Zero Motion Situations

Revisit:	To-Do:
• Torque formulas • Common errors in torque calculations • Statics – zero net force, zero net torque	• Doors and forces on their hinges • Ladders with friction on the floor

Revisit Statics

There's no motion at all and thus both the net external force is zero and the net torque is zero (around any axis whatsoever).

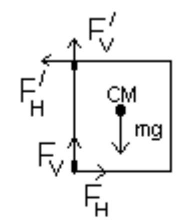
Example: **Door Torque Problem***:* The door shown is made of a uniform piece of wood and measures 3 ft by 7 ft and weighs 40 lb. The door is entirely supported by two hinges, one at the bottom corner and one at the top corner. We want to find the force (magnitude and direction) that each hinge exerts **on** the door.



Answer: To start, let's draw the appropriate XFBD with some convenient choices for the hinge forces (if we guess wrong, no problem! – the sign will just come out negative).

(1) Remember that the torque due to uniform gravity is that torque due to the weight applied at the center of mass – this guides us in drawing the weight force in our XFBD.

(2) Assume that each hinge is sharing equally the door weight such that the *vertical* force by each hinge is the same. Otherwise, we would need more information about how the door is "hung."



First, we use the fact that the vertical and horizontal forces must separately add up to zero. Assuming equal vertical hinge forces (by symmetry), we have

$$F_V' = F_V$$

We must have $\sum \vec{F}_i = 0$, force balance on the door in all directions. This yields

$$\text{vertical force balance:} \quad F_V + F_V' = mg = 40\text{ lb} \Rightarrow \boxed{F_V = F_V' = 20\text{ lb}}$$

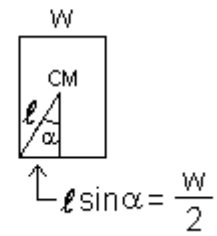
$$\text{horizontal force balance:} \quad F_H - F_H' = 0 \quad \Rightarrow \quad F_H = F_H'$$

Now to find F_H, we can get an equation for it from the fact that the sum of torques around any point must vanish:

$$\sum \vec{\tau}_i = 0 \quad \text{on the door about any given axis}$$

We make a convenient choice for this point (axis) on the next page.

It is convenient to calculate the **torque sum about the bottom hinge (the bottom-left corner)** because there the torques due to F_H, F_V are zero!. In writing this sum of torques, the most interesting term is the torque due to gravity. We have a force mg applied at the center of the door (see the figure on the right) giving a torque due to mg equal to $-mg\,\ell\sin\alpha$ (since R = ℓ, $\theta = \alpha$ in the FRsinθ formula and it's CW). Since $\ell\sin\alpha = w/2 = R_\perp$, we could have written $-mg\,\ell\sin\alpha$ down immediately from the equivalent torque magnitude formula $FR_\perp$. Some ways are faster than others depending on the problem.



Whichever way you derive it, the torque due to mg is simply $-mg\,w/2$, and the complete sum of torque terms about the bottom hinge (together with an explanation for each term in parenthesis) must vanish:

$$\tau_{F_H} + \tau_{F_V} + \tau_{F'_V} + \tau_{F'_H} + \tau_{mg} = 0$$

where

$\tau_{F_H} = 0$ (lever arm is zero), $\tau_{F_V} = 0$ (lever arm is zero), $\tau_{F'_V} = 0$ (F'_V angle is zero),

$\tau_{F'_H} = +F'_H h$ (CCW and F'_H angle is 90°), $\tau_{mg} = -\ mg\dfrac{w}{2}$ (see previous discussion)

Thus

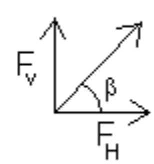
$$\therefore 0 + 0 + 0 + F'_H h - mg\frac{w}{2} = 0$$

or

$$\therefore F'_H = F_H = \frac{1}{2}\,mg\,\frac{w}{h} \quad \text{so} \quad \boxed{F'_H = F_H = 8.6\ \text{lb}}$$

The magnitude and direction of the bottom hinge force can be calculated from

$$F = \sqrt{F_H^2 + F_V^2},\ \tan\beta = \frac{F_V}{F_H}$$



The top hinge force has the same magnitude, and the same angle except that it points outward

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Problem 13-7 We can highlight some important issues by the following questions pertaining to the previous door example:

a) Can you find things wrong with every term in the following torque equation involving the net torque around the **bottom-right corner** of the door using our usual conventions (CCW =+)? Yours truly was watching a Seinfeld rerun while he was writing the equation and not paying attention!

$$+F_V w + F_H w + F_H' - F_V'\,w/h + m\,w/2 = 0$$

(continued)

b) Write down the **correct torque equation for that bottom-right corner** and check that it is satisfied by the solution $F'_H = F_H = \frac{1}{2} mg \frac{w}{h}$ and $F'_V = F_V = \frac{1}{2} mg$ found in the text on the previous two pages.

c) Now yours truly watches Scrubs reruns, too, so here comes another doozey. Find four errors in the following net torque equation for the **net torque around the door CM** that this poor soul has written down:

$$- F_V w/2 + F_H h + F_H{}' w - F_V{}' h + mg\, w/2 = 0$$

d) Write down the **correct torque equation for the CM point** and check that it is satisfied by the solution $F'_H = F_H = \frac{1}{2} mg \frac{w}{h}$ and $F'_V = F_V = \frac{1}{2} mg$ given above.

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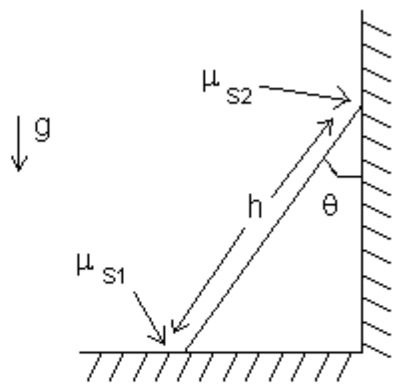
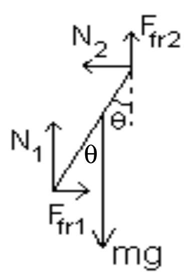
Example:

Leaning Card or Leaning Ladder Problem

A single playing card with mass m and height h stands on a table leaning against a wall. The frictional coefficient between the bottom of the card and the table is μ_{s1} and between the top of the card and the wall is μ_{s2}. The card is shown on the right with the forces indicated, too. The official XFBD is also shown below.





What equations do you have to work with if the card is not slipping? First, there is the zero net force $\sum \vec{F}_i = 0$ on the card

$$\Rightarrow \quad F_{fr1} = N_2 \text{ (horizontal) and } F_{fr2} + N_1 = mg \text{ (vertical)}$$

Now the above two equations for the balance of forces and a torque equation will give us three of the five unknowns (F_{fr1} , F_{fr2} , N_1 , N_2 , θ).

A torque equation is found on the next page.

To get that torque equation, we use $\sum \vec{\tau}_i = 0$ about any given axis. **Again, remember that once you have chosen an axis then EVERY torque term has to be calculated with respect to THAT SAME axis! You can't change axes in midstream. Also, remember that the torque due to weight is as if the weight were all at the CM.**

An example where we have chosen the bottom end as the point (the point where the axis goes through perpendicular to the page) is:

$$\tau_{F_{fr1}} + \tau_{N_1} + \tau_{mg} + \tau_{F_{fr2}} + \tau_{N_2} = 0$$

where we can substitute

$$\tau_{F_{fr1}} = 0 \text{ (lever arm is zero)}, \quad \tau_{N_1} = 0 \text{ (lever arm is zero)},$$

$$\tau_{mg} = -mg\frac{h}{2}\sin\theta \text{ (CW, angle is } \theta), \quad \tau_{F_{fr2}} = +F_{fr2} h \sin\theta \text{ (CCW, } F_{fr2} \text{ angle is } \theta),$$

$$\tau_{N_2} = +N_2 h\cos\theta \text{ (CCW, } R_\perp = h\cos\theta \text{ or use } \sin(\pi/2-\theta) = \cos\theta \text{)}$$

We thus have three equations, but we still need two more conditions.

In the following problem, we assume 1) the wall is frictionless (so $F_{fr2} = 0$) and 2) the friction on the floor is as big as possible (we're on the verge of slipping). The latter gives us the equation $F_{fr_1} = F_{fr_1}(\max) = \mu_{s_1} N_1$. We thus have two additional conditions we need to be able to calculate everything!

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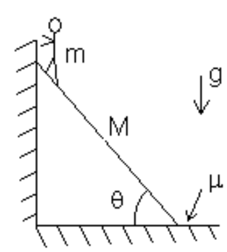
Problem 13-8 A ladder of length L (which ultimately cancels out of the answer) and of uniformly distributed mass M rests against a **frictionless wall** and makes an angle θ with the floor where the static coefficient is μ. A woman with mass m has climbed to the top of the ladder as shown.



a) Draw an XFBD for the ladder-woman system in which the friction force due to the floor is labeled F_{fr} .

b) What is the smallest angle θ allowed where the woman (mass m) can climb all the way to the top? HINT: The smallest angle means we are just on the verge of slipping. This in turn means the static friction force from the floor is as large as it can possibly be. With (1) no friction due to the wall and (2) maximum friction from the floor we now have the two additional equations we need to completely solve the problem!

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