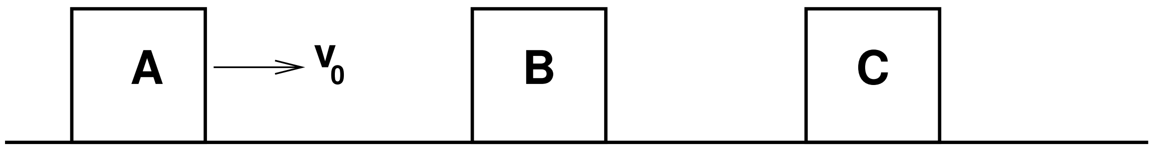
PHYS 121: Solutions to Written Homework #05
March 15, 2024

Note: this homework is worth **15 points** as follows:

___	Problem 1	0 points	Do not grade
___	Problem 2	5 points	5 pts total
___	Problem 3	5 Points	5 pts total
___	Problem 4	0 points	Do not grade
___	Problem 5	5 points	5 pts total
___	Problem 6	0	Do not grade

Problem 1: Two 1-D Collisions



Three blocks, are placed on a frictionless surface as shown above. Block A has a given mass of m, Block B has a given mass of $2m$ and Block C has a given mass of $5m$. Block A has initial velocity given as v_0 while the other two blocks are *initially* at rest. Then the following happens:

- First Collision: Block A collides with Block B and this collision is ***totally inelastic***.
- Second Collision: Block B collides with Block C and this collision is ***totally elastic***.

Part (a): Determine the final velocity of every block immediately after the First Collision.

Part (b): Determine the final velocity of every block immediately after the Second Collision.

Part (c): Suppose we define V_{CM} as the velocity of the Center-of-Mass of the *entire system* of three blocks, A, B, and C. What is V_{CM} just *before* the First collision. What is V_{CM} just *after* this First Collision? What is V_{CM} just *after* this Second Collision?

SOLUTION Problem 1:

Part (a)

Let's start with the first collision: Block A hits Block B *inelastically*. This means that they stick: $v_{AB} \equiv v'_A = v'_B$. We use **Conservation of Momentum**:

$$P_{tot} = P'_{tot}$$
$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$
$$m v_0 + 0 = m v_{AB} + 2m v_{AB}$$
$$m v_0 = 3m v_{AB}$$
$$\boxed{v_{AB} = \frac{v_0}{3}}$$

Also **Block C still at zero velocity.**

Part (b)

Now we consider the second collision which is *elastic* between the *combined* object of ***Blocks A and B together*** and Block C. This is a "standard" elastic collision, isolated system (no net external force on the two blocks) so we can use both **Conservation of Momentum** and **Conservation of Kinetic Energy**:

In a *totally elastic* collision between "two" bodies where one body is "AB" (combined) and other is C, by definition we know that the total kinetic energy in the bodies is conserved:

$$\tfrac{1}{2} m_{AB} v_{AB}^2 + \tfrac{1}{2} m_C v_C^2 = \tfrac{1}{2} m_{AB} v'^2_{AB} + \tfrac{1}{2} m_C v'^2_C$$

We can often combine this expression with the conservation of linear momentum (here in the 1-D case):

$$m_{AB} v_{AB} + m_C v_C = m_{AB} v'_{AB} + m_C v'_C$$

In general, we can combine these two equations to solve for the two unknowns: v'_{AB} and v'_C. This is algebra, not physics, and students should feel free to find whatever method works for them.

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This time let's use **Bob Brown's Master Equations** for 1-D Elastic Collisions:

$$m_{AB}v_{AB} + m_C v_C = m_{AB}v'_{AB} + m_C v'_C$$

$$v_{AB} - v_C = v'_C - v'_{AB}$$

Here $m_{AB} = 3m$, $m_C = 5m$, $v_{AB} = v_0/3$ and $v_c = 0$. Plugging these into the two Master Equations:

$$3m\frac{v_0}{3} + 0 = 3mv'_{AB} + 5mv'_C$$
$$\frac{v_0}{3} - 0 = v'_C - v'_{AB}$$

Second Equation, solving for v'_{AB}:

$$v'_{AB} = v'_C - \frac{v_0}{3}$$

Plug this back into First equation:

$$3m\frac{v_0}{3} = 3m(v'_C - \frac{v_0}{3}) + 5mv'_C$$
$$v_0 = 3v'_C - v_0 + 5v'_C$$
$$8v'_C = 2v_0$$
$$\boxed{v'_C = \frac{v_0}{4}}$$

Plug this back into Second equation:

$$v'_{AB} = \frac{v_0}{4} - \frac{v_0}{3}$$
$$v'_{AB} = \frac{3v_0}{12} - \frac{4v_0}{12}$$
$$\boxed{v'_{AB} = -\frac{v_0}{12}}$$

Part (c)

We consider the Center-of-Mass velocity for the whole system before the first collision:

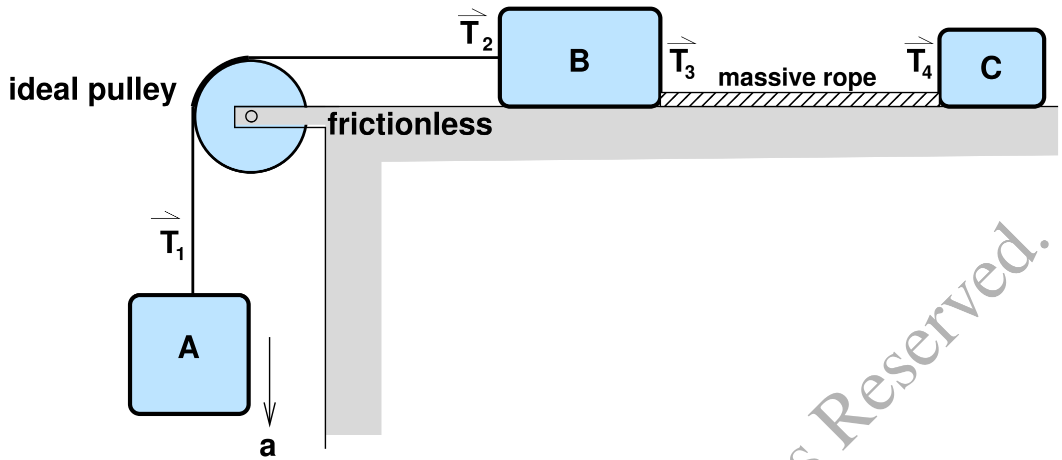
$$v_{CM} = \frac{m_A v_A + m_B v_B + m_C v_C}{m_A + m_B + m_C}$$

$$v_{CM} = \frac{m v_0 + 0 + 0}{m + 2m + 5m}$$

$$\boxed{v_{CM} = \frac{v_0}{8}}$$

Here, we have calculated the Center-of-Mass velocity for the time corresponding to before the first collision, but because the system is *completely isolated* we can say that this also corresponds to the velocity of the Center-of-Mass for the system at *any* time, including **before, during, and after any collisions**.

Problem 2: Newton's Second Law and a Massive Rope



Three blocks A, B, and C with a given mass m_A, m_B, and m_C respectively, are positioned as shown above. Blocks B and C are accelerated to the left along the frictionless surface . The pulley is ideal as is the string between Blocks A and B. A massive rope of mass m_r connects Blocks B and C. Assume that the rope sits flush on the frictionless surface so that there is no component of the tension in the vertical direction.

Part (a): Rank the four tension forces from largest to smallest. Explain how you determined this. Hint: this is a *conceptual* question. No algebra or calculations required. Important hint: any solution that includes either of the words "system" or "motion" is probably not correct. You want to invoke a Law of Physics here.

Part (b): Determine the magnitude of the acceleration a of Block A. Explain your work. Give your answer in terms of the given parameters. Hint: you will simplify your calculation if you consider these three items: Blocks B, Block C and the massive rope, altogether, as one "system".

Part (c): Assume that the massive rope has length L. Determine the tension *in the massive rope* as a function of horizontal position x along the rope measured relative to the *right* edge of Block B. Give your answer in terms of the given parameters. Explain your work.

SOLUTION TO PROBLEM 2:

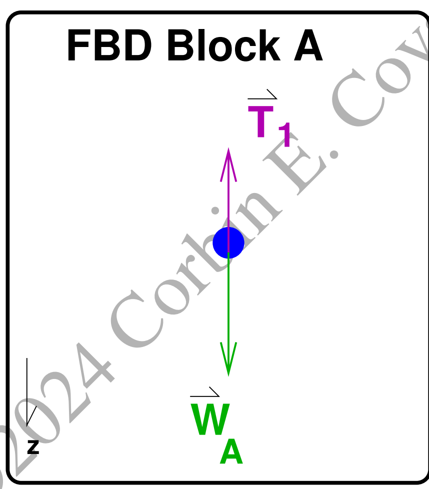
Part (a): We consider the relative strengths of tension forces starting with the left side:

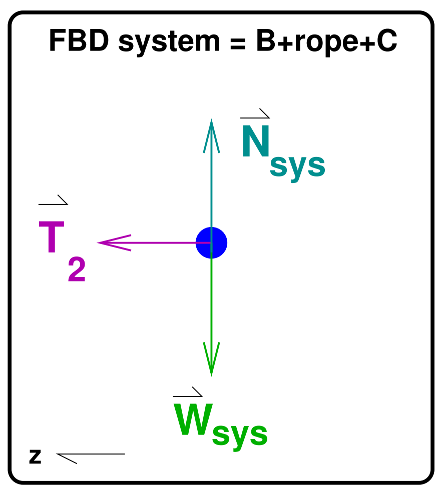
- We note that the rope segment over the pulley is an **ideal rope** which, *by definition,* has the same tension at both ends. So we know $T_1 = T_2$.
- We use **Newton's Second Law** and the fact that Block B is *accelerating to the left* to argue that the net force on Block B is to the left. This means that $T_2 > T_3$
- We use **Newton's Second Law** and the fact that the massive rope is *accelerating to the left* to argue that the net force on the rope is to the left. This means that $T_3 > T_4$.

Putting it all together:

$$\boxed{T_1 = T_2 > T_3 > T_4}$$

Part (b): We apply **Newton's Second Law** to get the acceleration. All of the block have the same magnitude of acceleration. We can simplify the work by considering the set of objects which are Block B plus the rope plus Block C as one "system" moving together in the horizontal direction: The Free Body Diagrams for Block A and "the system" look like this:





Here we have selected a coordinate system for each FBD which we call z corresponding to the direction of acceleration of the blocks. We can then apply **Newton's Second Law** to each FBD in the z-coordinate direction:

Newton's Second Law: Block A, z-direction (down):

$$F_{Az} = m_A a_z$$
$$W_A - T_1 = m_A a_z$$

$$m_A g - T_1 = m_A a_z \tag{1}$$

That's as far as we can go. We have one equation but two unknowns: T_1 and a_z.

Newton's Second Law: System (Block B, rope, Block C), z-direction (left):

$$F_{sys,z} = M_{sys} a_z$$

$$T_2 = M_{sys} a_z \tag{2}$$

Now we have one equation but two unknowns: T_2 and a_z.

Finally we add the **Newton's Third Law** result (ideal rope):

$$T_1 = T_2 \tag{3}$$

Now we have three equations, three unknowns: T_1, T_2 and a_z. Some pretty easy algebra combining Equations (1), (2) and (3) and solving for acceleration:

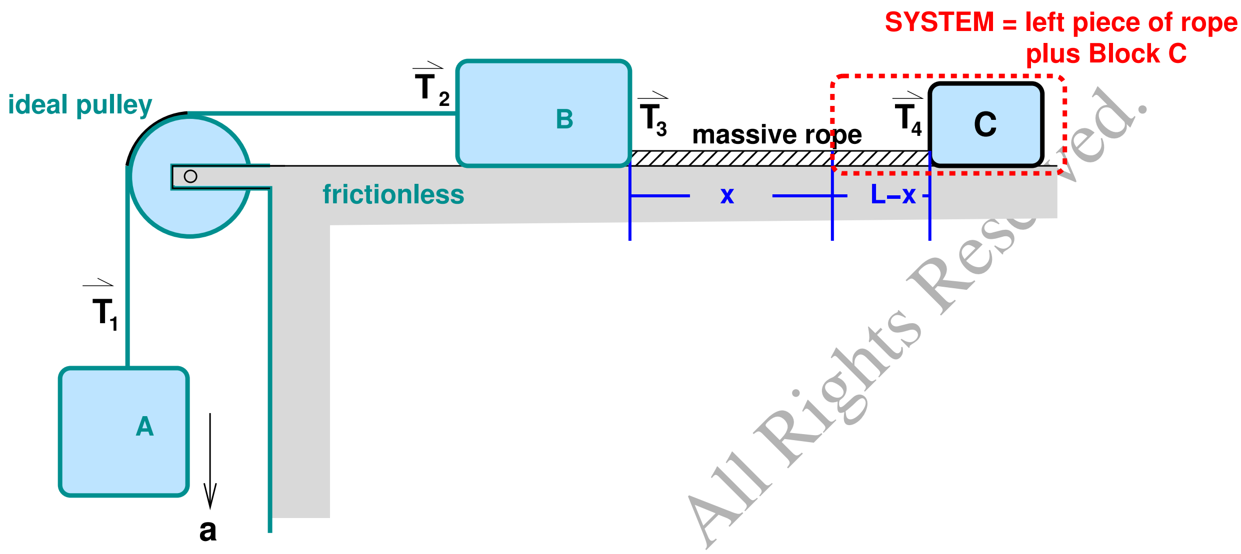
$$m_A g - T_2 = m_A a_z$$

$$m_A g - M_{sys} a_z = m_A a_z$$
$$M_{sys} a_z + m_A a_z = m_A g$$
$$(M_{sys} + m_A) a_z = m_A g$$
$$a_z = \frac{m_A g}{M_{sys} + m_A}$$
$$\boxed{a_z = g\left(\frac{m_A}{m_A + m_B + m_r + m_C}\right)}$$

Problem solution continues next page...

Part (c): There are several ways to get the answer here, most of these involve a clever choice of systems. Here is perhaps the most elegant way:

If we choose a system that includes everything to the *right* of some point on the rope a distance x down the length from Block B (as shown below) then we can draw a single simple Free-Body Diagram for that system:



We see by inspection that if the piece of rope outside the system has length x then the piece of rope inside the system has length $L - x$. This allows us to fairly easily calculate the mass of this system:

$$M_{sys} = m_C + m_{\text{left-part-of-rope}}$$
$$M_{sys} = m_C + m_r \text{ (fraction of rope in system)}$$
$$M_{sys} = m_C + m_r \left(\frac{L - x}{L} \right)$$

Now we can apply **Newton's Second Law** to this system in the horizontal component:

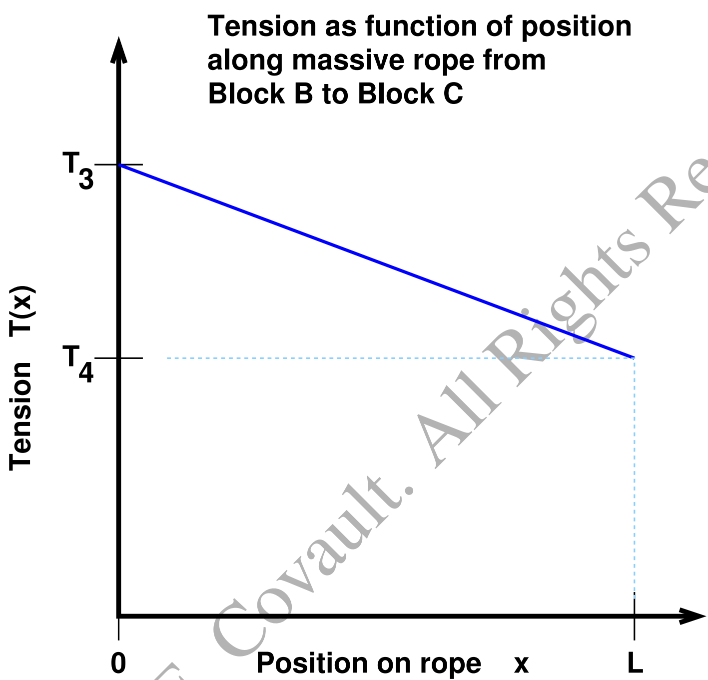
$$F_{sys,z} = M_{sys} a_z$$
$$T(x) = M_{sys} a_z$$
$$T(x) = \left[m_C + m_r \left(\frac{L - x}{L} \right) \right] a_z$$

$$\boxed{T(x) = \left[m_C + m_r \left(\frac{L - x}{L} \right) \right] \left[g \left(\frac{m_A}{m_A + m_B + m_r + m_C} \right) \right]}$$

This is "the answer" but we can learn something by multiplying through and very slightly re-arranging the terms:

$$\boxed{T(x) = \left(\frac{m_A m_C g}{m_A + m_B + m_r + m_C}\right) + \left(1 - \frac{x}{L}\right)\left(\frac{m_A m_r g}{m_A + m_B + m_r + m_C}\right)}$$

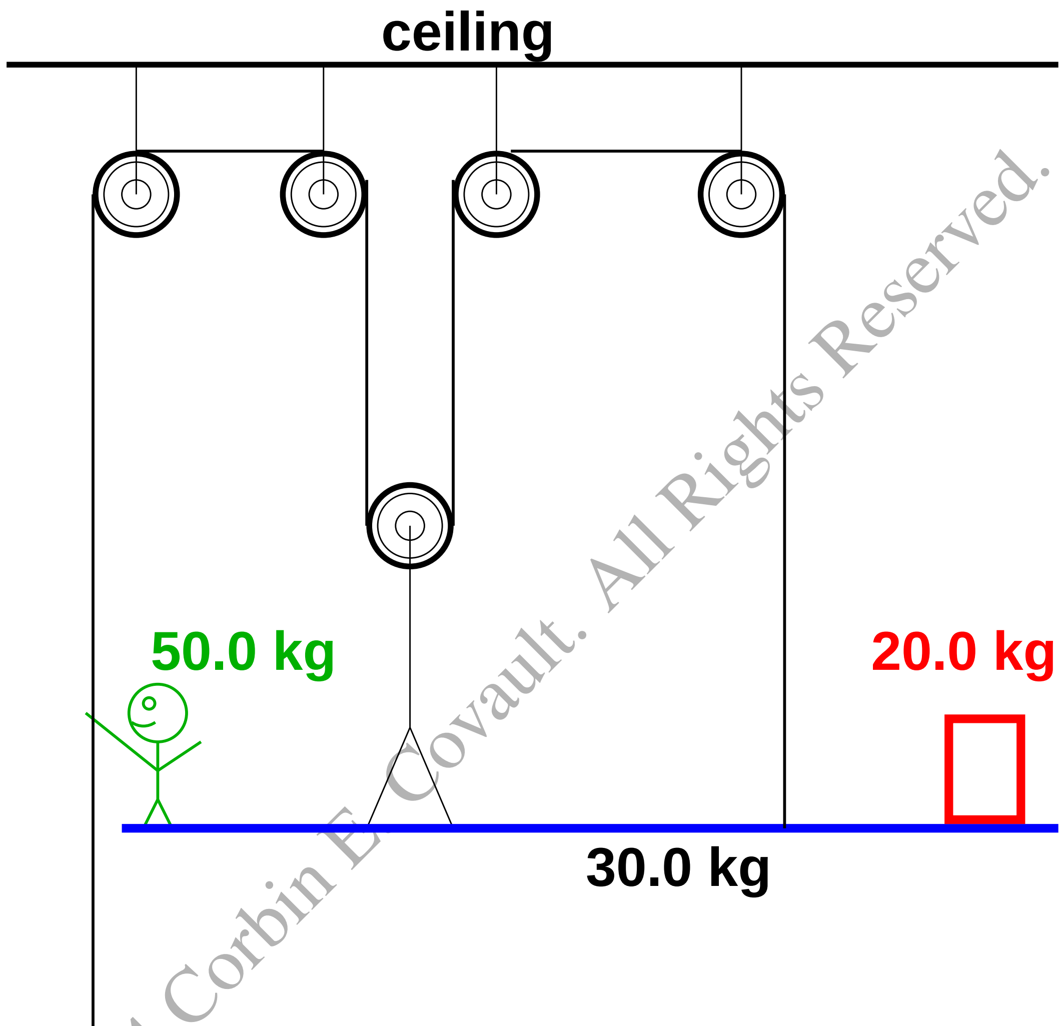
The first term is the correct (constant) expression for the tension T_3. The second term includes a factor that changes linearly with x times an expression which is $T_4 - T_3$. In other words, we expect the tension to vary **linearly** from the value T_3 to the value T_4. If we made a plot of tension vs. position along the rope, it would look like this:



Note: A perfectly acceptable "alternate solution" is to work out the tensions T_4 and T_3 using Newton's Second Law and then calculate the linear function of position between one end of the rope and the other. This is not quite as elegant but will give the same answer.

Problem 3. Pulleys and Systems:

Leonard Nimoy stands on a platform and pulls on an ideal rope, which is attached to a system of ideal pulleys as shown:



The mass of Leonard is given as $m_\ell \equiv$50.0 kg. The mass of the platform is given as $m_p \equiv$30.0 kg. The mass of a box that sits on the platform is given as $m_b \equiv$20.0 kg. Assume that Leonard pulls the rope so that the platform moves upward with given constant acceleration $A_0 \equiv$0.200 meters per second squared.

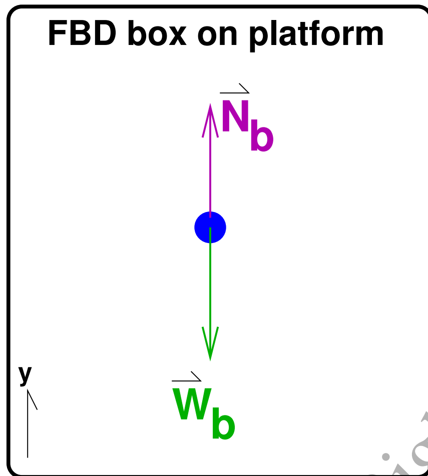
a) What are the magnitudes and directions of all forces on the box? Give your answer both in terms of given symbolic parameters and in terms of numeric values. Explain your work.

b) What is the tension in the rope? Give your answers in terms of given symbolic parameters and in terms of numeric values. Explain your work.

c) What are the magnitudes and directions of all forces on Leonard? Give your answers in terms of given symbolic parameters and in terms of numeric values. Explain your work.

SOLUTION TO PROBLEM 3:

Part (a): To answer this question, we consider only the forces on the box. We draw a **Free Body Diagram:** of the box on the platform:



Note in particular that since the rope is not in contact with the box, there is *no tension force* on the FBD for the box.

Since we need to report all forces, for starters, we know immediately the value and direction of the weight force:

$$\boxed{W_b = m_b g}$$

And plugging in numbers:

$$W_b = (20.0\text{ kg})(9.81\text{ m/s}^2)$$
$$\boxed{W_b = 196\text{ Newtons}}\text{ (downward)}$$

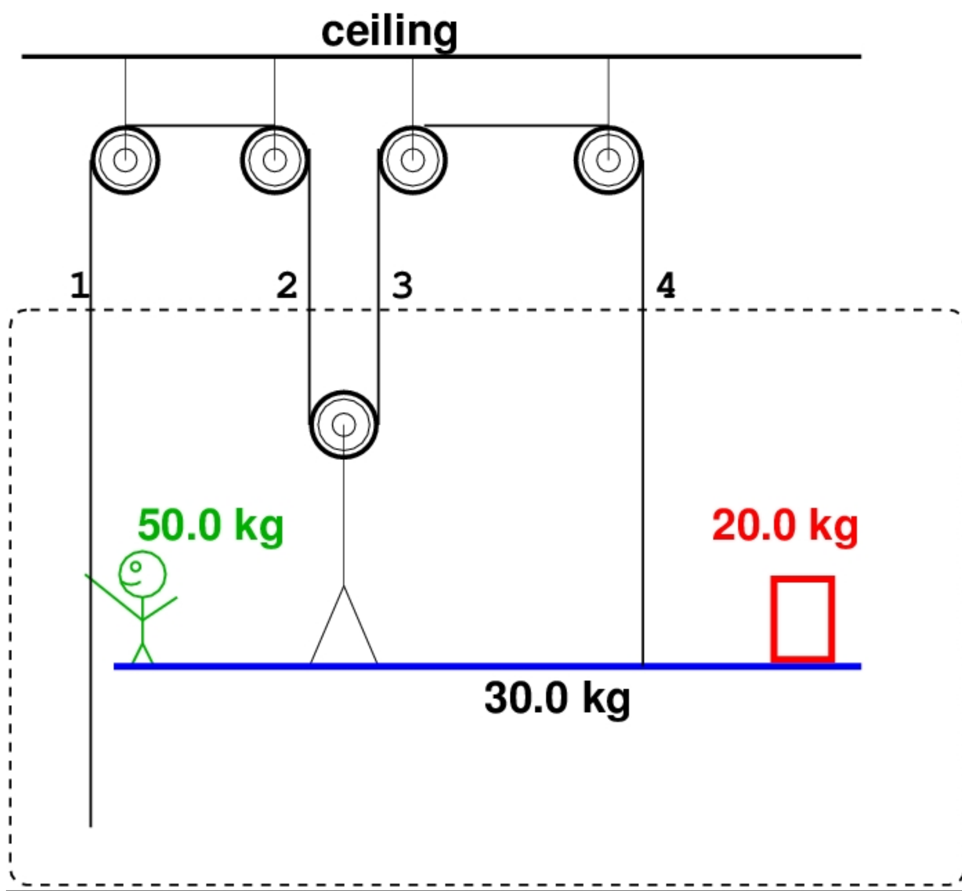
To get the Normal force we **must** use **Newton's Second Law** on the box in the vertical coordinate:

$$F_b = m_b a_y$$
$$N_b - W_b = m_b a_y$$
$$N_b = m_b a_y + W_b$$
$$N_b = m_b a_y + m_b g$$
$$\boxed{N_b = m_b(a_y + g)}$$

Plugging in numbers:

$$N_b = (20.0\text{ kg})(0.200\text{ m/s}^2 + 9.81\text{ m/s}^2)$$
$$\boxed{N_b = 200.2\text{ Newtons}}\text{ (upward)}$$

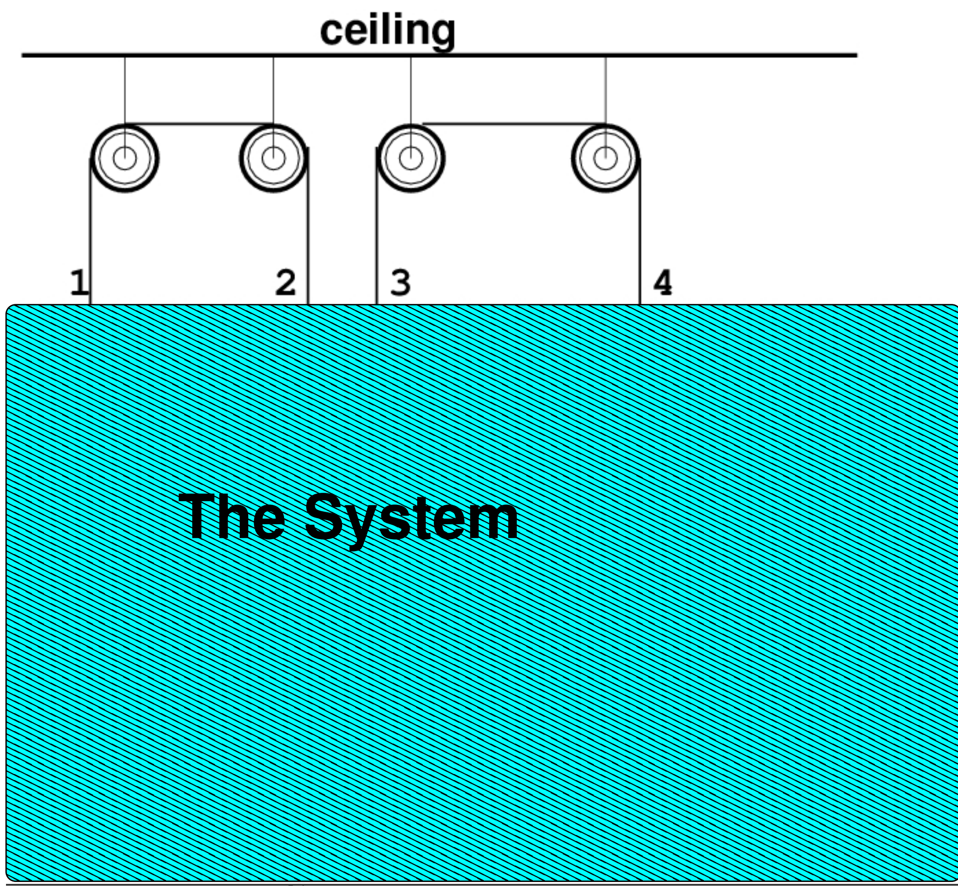
Part (b): The only way we can get the tension in the rope is to devise a **system** that allows us to consider effects of tension without other unknown forces (such as the forces due to and/or on the upside-down-y-shaped cable, for example). Here is our only good selection for such a system:



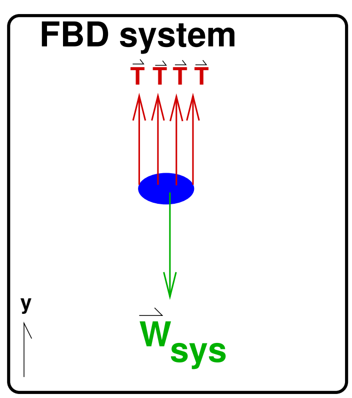
The dashed line represents an imaginary surface that completely encloses our system. It's like a plastic bag: everything inside the boundary is part of the system. Everything outside is not part of the system.

Problem solution continues next page....

We can take this idea even further, graphically "blacking out" everything that is inside "The System" like this:



The reason that this is a good system is that the only thing in physical contact with this system is the rope. Indeed, the rope is in contact with the system at four locations (labeled 1, 2, 3, and 4). We treat everything inside the boundary as one "object" with a total mass $M_{sys} = m_b + m_p + m_\ell$, the masses of the box, platform, and Leonard, respectively (the pulleys and cables have no mass). The **Free Body Diagram** for the system as defined is quite simple:



Note in particular that an ideal rope always pulls with the same tension at every location on the rope, and note that this is true regardless of the fact that the rope is moving in any particular direction.

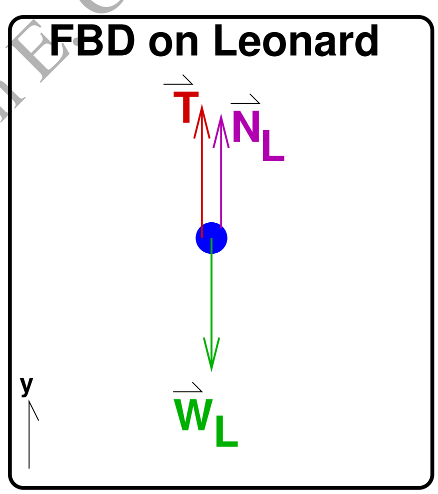
Now we solve **Newtons Second Law:** noting that we have four times the tension upward:

$$F_{sys} = m_sysa_y$$
$$4T - W_{sys} = m_sysa_y$$
$$4T = m_{sys}a_y + W_{sys}$$
$$4T = m_{sys}a_y + m_{sys}g$$
$$T = \frac{m_{sys}(a_y + g)}{4}$$
$$\boxed{T = \frac{(m_b + m_p + m_\ell)(a_y + g)}{4}}$$

Plugging in numbers:

$$T = \frac{(100.0 \text{ kg})(10.01 \text{ m/s}^2)}{4}$$
$$\boxed{T = 250.3 \text{ Newtons}}$$

Part (c): Finally, we consider the forces on Leonard alone: Leonard is pulling the rope firmly (he must be, otherwise the platform would fall) and so there is a force of tension on Leonard due to the rope.



Now we know the force of Weight on Leonard:

$$\boxed{W_\ell = m_\ell g}$$

And plugging in numbers:

$$W_\ell = (50.0 \text{ kg})(9.81 \text{ m/s}^2)$$

$$\boxed{W_\ell = 490.5 \text{ Newtons}} \text{ (downward)}$$

And we already calculated the **tension force** in Part (b), which pulls **upward** on Leonard. So all that remains is to calculate the normal force on Leonard due to the platform using **Newton's Second Law:**

$$F_L = m_\ell a_y$$
$$N_\ell + T - W_\ell = m_\ell a_y$$
$$N_\ell = m_\ell a_y + W_\ell - T$$
$$N_\ell = m_\ell a_y + m_\ell g - T$$
$$N_\ell = m_\ell(a_y + g) - \frac{(m_b + m_p + m_\ell)(a_y + g)}{4}$$
$$\boxed{N_\ell = \left(m_\ell - \frac{m_b + m_p + m_\ell}{4}\right)(a_y + g)}$$

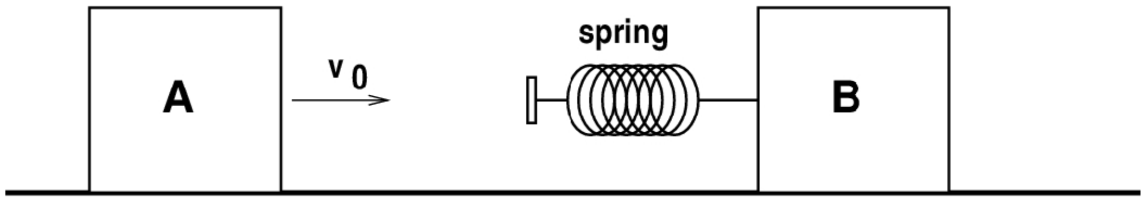
Plugging in numbers:

$$N_\ell = \left(50.0 \text{ kg} - \frac{100.0 \text{ kg}}{4}\right)(0.200 \text{ m/s}^2 + 9.81 \text{ m/s}^2)$$

$$\boxed{N_\ell = 250.3 \text{ Newtons}} \text{ (upward)}$$

By the way, the fact that the Normal force and the Tension force have the same numerical value is a *coincidence* that arises from the particular selection of mass values for each object.

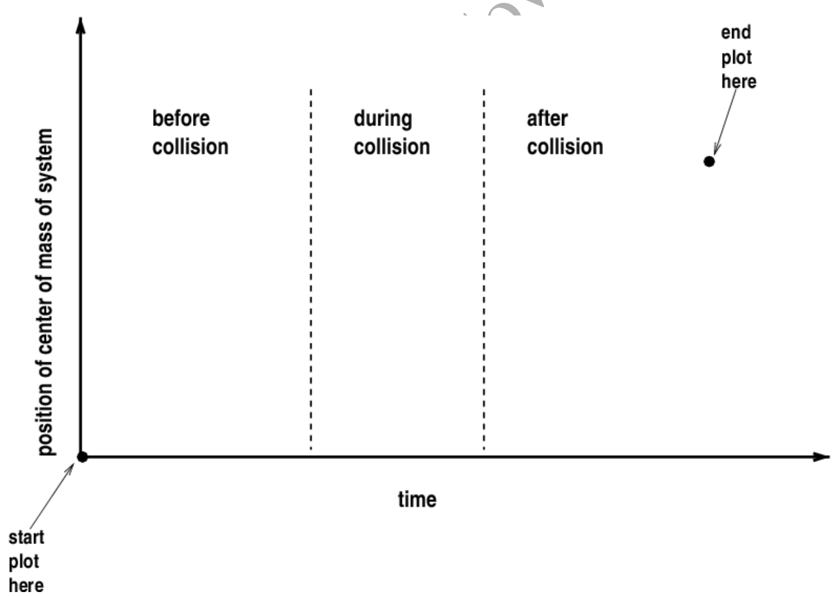
Problem 4: A Springy Collision that Is Way Simpler Than It Looks



Two blocks A and B each with mass m= 2.0 kg sit on a frictionless surface. Block A has a velocity v_0 = 5.0 m/s towards block B which is at rest. There is a massless spring with spring constant k =1.00 N/m attached to Block B as shown. Block A collides with the spring that is attached Block B. The spring is compressed, and then after a while it is uncompressed and the two blocks separate from each other. Note that because the spring force is Conservative, the collision is ***totally elastic***.

a) What is the final velocity of block A? What is the final velocity of block B? Be sure to show your work.

b) Consider the system which is made of block A, block B, and the spring. On the following figure make a *qualitative plot* of the position of the center of mass x_{cm} for this entire system as qa function of time. Start and end your plot at the points indicated. What is the slope of your plot during the intervals of time corresponding to each of these three periods: (1) before, (2) during, and (3) after the collision? Be sure to explain your work. Use physics!



SOLUTION to Problem 4.

Part (a):

This is a “standard” elastic collision, isolate system (no net external force on the two blocks) so we can use both **Conservation of Momentum** and **Conservation of Kinetic Energy**:

In a *totally elastic* collision between two bodies A and B, by definition we know that the total kinetic energy in the bodies is conserved:

$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2$$

We can often combine this expression with the conservation of linear momentum (here in the 1-D case):

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

In general, we can combine these two equations to solve for the two unknowns: v_A' and v_B'. This is algebra, not physics, and students should feel free to find whatever method works for them. Here we outline three acceptable methods:

Let’s use ***Bob Brown’s Master Equations*** for 1-D Elastic Collisions which can be found on page 10-11 (Bob Brown’s Online Notes Chapter 10+):

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$v_A - v_B = v_B' - v_A'$$

In this problem we know the following: $m_A = m_B = m$, $v_A = v_0$, and $v_b = 0$. We plus these values in:

$$mv_0 = mv_A' + mv_B'$$
$$v_0 = v_B' - v_A'$$

Solve the second equation for v_B' and plug this into the first equation, then solve for v_A':

$$v_B' = v_0 + v_A'$$
$$mv_0 = mv_A' + m(v_0 + v_A')$$
$$v_0 = v_A' + v_0 + v_A')$$
$$2v_A' = 0$$
$$\boxed{v_A' = 0}$$

This back into the second equation:

$$v_0 = v_B' - 0$$

$$\boxed{v_B' = v_0}$$

Important: In this one particular example where the two blocks have the same mass, all of the momentum and all of the kinetic energy transfers from one block to the other during the collision. But this only happens in this one special case where the masses are precisely equal. As a rule, only some of the energy will transfer during an elastic collision.

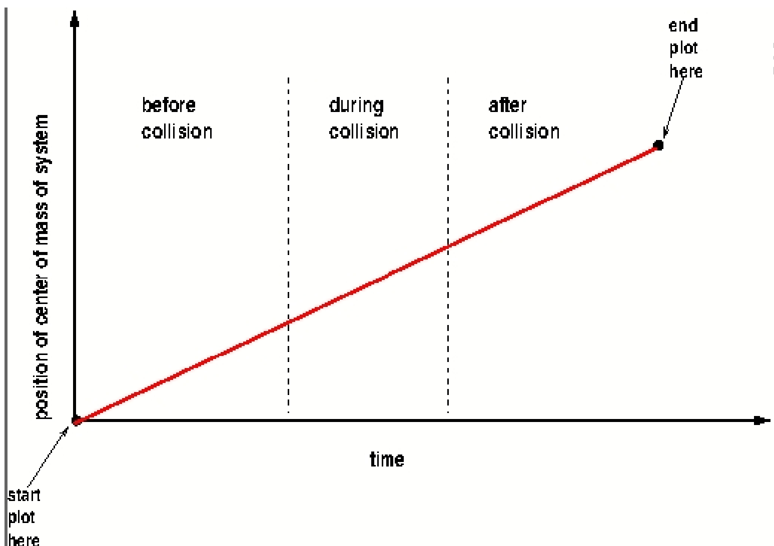
Part (b):

Since the **system is isolated** we can say that **Momentum is Conserved** and therefore **the velocity of the center of mass remains constant.** This remains true before, during, and after the collision.

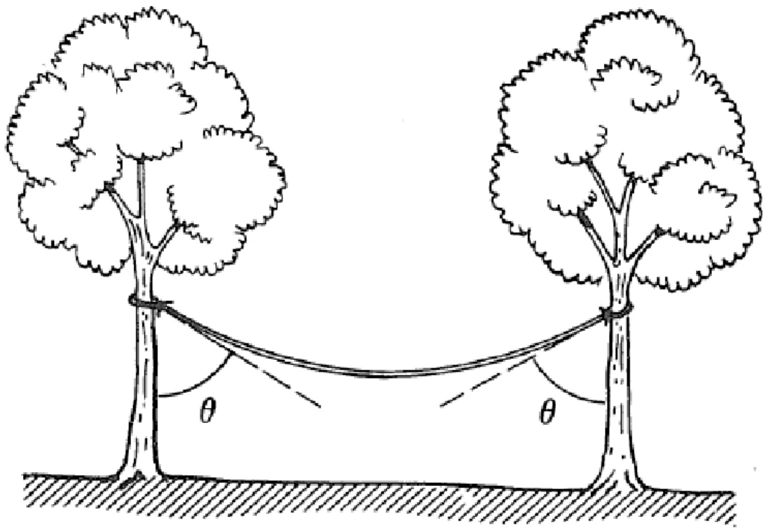
$$v_{cm} = \text{constant}$$

$$x_{cm} = x_0 + v_{cm}t$$

In other words **the position x_{cm} varies linearly with time:**



Problem 5: Massive Rope Hung Between Two Trees



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A massive uniform rope of total mass m and length L is attached to two trees as shown in the figure above. The connection point for the rope is the same height on each tree, each rope makes an given angle θ with each tree as shown.

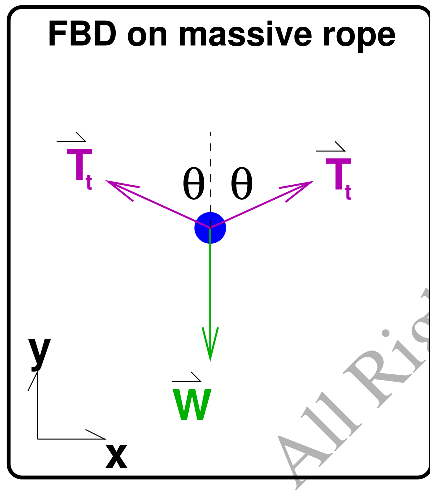
Part (a): Find the tension at either end of the rope.

Part (b): Find the tension at the center of the rope.

Hint: Perhaps this looks like a problem where detailed calculus is required. It's not. If you are doing the problem correctly, each part require just a few lines of simple mathematical work.

SOLUTION TO PROBLEM 5:

Part (a): Given that we are asking for tension related to the rope, we anticipate that this is a problem using **Newton's Second Law**. Although it might seem like we need to treat the rope as a sum of differential segments, in fact, the answers to the questions we ask can be obtained quickly by identifying and applying the Second Law to the **proper system**. For Part (a), we consider the *entire rope as the system*. In this case the **Free Body Diagram** is pretty simple:



Here, the force of tension on each end of the rope due to a tree is labeled T_t. We note that there is an axis of *symmetry* that tells us that these two forces should be mirror images of each other. We write down **Newton's Second Law** for the rope in y-component:

$$F_y = ma_y$$

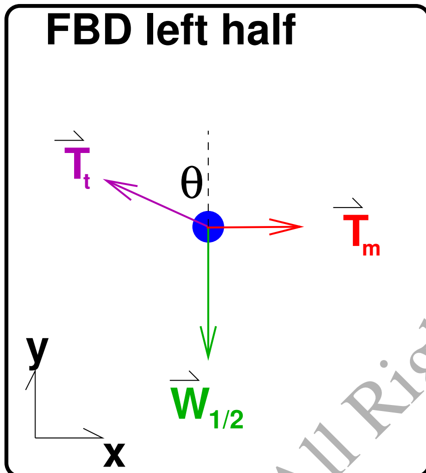
$$2T_t \cos\theta - W = 0$$

$$2T_t \cos\theta = mg$$

$$\boxed{T_t = \frac{mg}{2\cos\theta}}$$

Problem solution continues next page...

Part (b): Now, in order to get the tension at the *center* of the rope we must **redefine our system**. We must do this because the free body diagram we just wrote for the total rope does not include the tension at the center of the rope since this force is an *internal force* which does not contribute to Newton's Second Law. Therefore, we define a *new system* that is precisely **half of the rope**. By symmetry it does not matter which half: let's pick the left half. The Tension in the middle of the rope pulls the left half of the rope to the right:

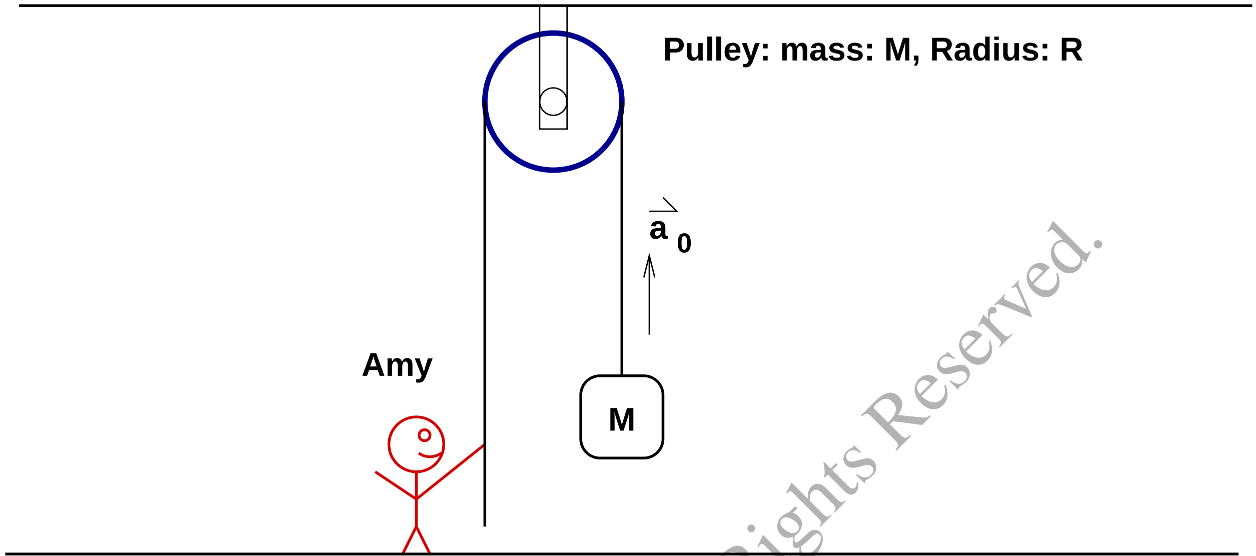


Here we are only interested in the application of of **Newton's 2nd Law** in the *horizontal* direction:

$$F_x = ma_x$$
$$T_m - T_t \sin\theta = 0$$
$$T_m = T_t \sin\theta$$
$$T_m = \left(\frac{mg}{2\cos\theta}\right)\sin\theta$$
$$T_m = \frac{mg\sin\theta}{2\cos\theta}$$
$$\boxed{T_m = \frac{mg\tan\theta}{2}}$$

This is interesting because it tells us that as the angle grows toward $\theta \to \pi/2$ the tension increases without bound.

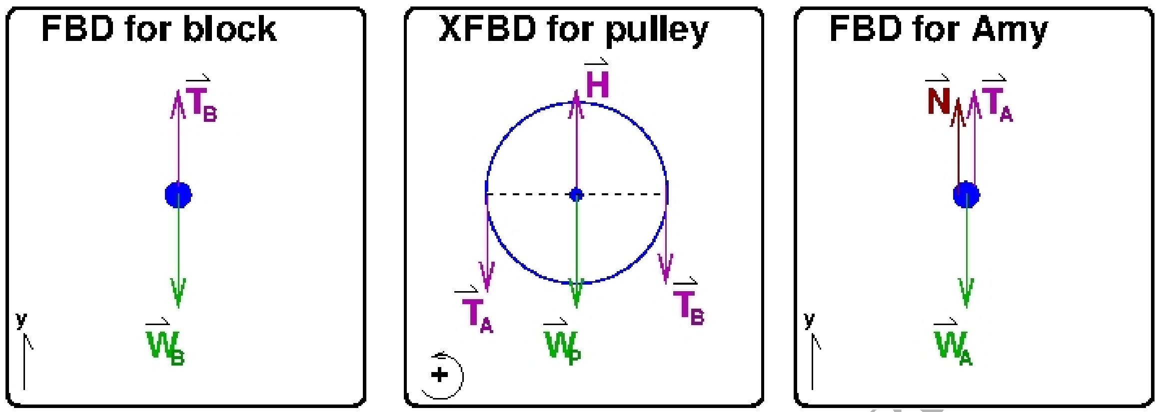
Problem 6: A massive pulley problem



Amy stands on the floor and pulls hard on a rope that is attached to a massive pulley and a block as shown above. The mass of the block is given as M. The mass of the pulley has the same given mass M. Amy's mass is given as $3M$. Amy pulls so that the block accelerates upward with a given constant acceleration $\vec{a}_0$. Assume that the pulley wheel can be represented as a solid disk with rotational inertia given by $I = \frac{1}{2}mR^2$.

What are all of the forces on Amy? Determine the type, magnitude, and direction of each force. Explain your work. If you are using Newton's Laws here, be sure to include clearly labeled Free-Body-Diagrams and/or Extended-Free-Body-Diagrams as needed. Your work for this problem needs to be complete, neat, coherent, and completely explained to earn full credit.

SOLUTION TO PROBLEM 6:



Since we are looking for forces, we will certainly want to apply **Newton's Second Law**. We start with the Free Body Diagrams. Above I have written the FBDs for the block and Amy and an XFBD for the massive pulley.

We apply Newton's Second Law to the block in the vertical component:

$$F_{B,net} = Ma_0$$
$$T_B - W_B = Ma_0$$
$$T_B - W_B = Ma_0$$

Solve for the unknown, T_B:

$$T_B = W_B + Ma_0$$
$$T_B = Mg + Ma_0$$

Next we apply the Second Law for Rotations to the pulley:

$$\tau_{net} = I\alpha$$

We note that the Weight of the pulley and the force due to the pulley hinge do not result in a torque. But the rope tensions do:

$$\tau_{T_A} - \tau_{T_B} = I\alpha$$

The forces are perpendicular to the radius, so the torque is simple:

$$RT_A - RT_B = I\alpha$$

Now we deal with I and α:

$$RT_A - RT_B = \left(\frac{1}{2}MR^2\right)\left(\frac{a_0}{R}\right)$$

And finally plug in result for T_B and solve for the unknown T_A

$$RT_A = RT_B + \left(\frac{1}{2}MR^2\right)\left(\frac{a_0}{R}\right)$$

$$RT_A = RMg + RMa_0 + \frac{1}{2}MRa_0$$

$$T_A = Mg + Ma_0 + \frac{1}{2}Ma_0$$

$$\boxed{T_A = Mg + \frac{3}{2}Ma_0}$$

Finally we consider the forces on Amy. We know the Weight: $\boxed{W_A = m_A g = 3Mg}$ and we just solved for the the tension on Amy. So we just need to use Newton's Second Law to get the Normal force, noting that Amy is *not* accelerating:

$$F_{A,net} = m_A a_A$$

$$F_{A,net} = 0$$

$$N + T_A - W_A = 0$$

$$N = W_A - T_A$$

$$N = 3Mg - Mg + \frac{3}{2}Ma_0$$

$$N = 2Mg - \frac{3}{2}Ma_0$$

$$N = M\left(2g - \frac{3}{2}a_0\right)$$

$$\boxed{N = M\left(\frac{4g - 3a_0}{2}\right)}$$

Note that if Amy pulls too hard, she will pull herself off the floor, corresponding to N having and non-positive number.