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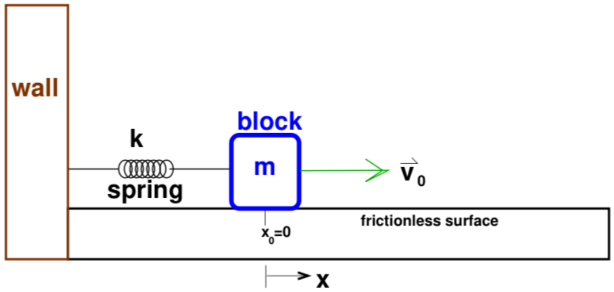
PHYS121 - Homework 10

Problem 1)



Given
- Block mass m
- Spring Constant k
- $x(0)=0$, $v(0)=v_0$
- Frictionless Surface

Equation of Motion

$$\frac{d^2x}{dt^2}+\frac{k}{m}x=0$$

General Solution to DE

$$x(t)=B\cos(\omega t)+C\sin(\omega t)$$

Kinematic Expression

$$v(t)=x'=\frac{dx}{dt}=\frac{d}{dt}(x(t))$$

$$\therefore x'(t)=-\omega B\sin(\omega t)+\omega C\sin(\omega t)$$

The frequency of an undamped harmonic oscillator is given by the roots of its characteristic polynomial: $s^2+\frac{k}{m}=0$

With roots: $s=\pm i\sqrt{\frac{k}{m}}$

Using principles of Differential Equations, we derive:

$$x(t)=B\cos\left(\sqrt{\tfrac{k}{m}}\,t\right)+C\sin\left(\sqrt{\tfrac{k}{m}}\,t\right)$$

$\therefore$ The frequency is $\omega=\sqrt{\frac{k}{m}}$

We are given initial conditions:

$$\begin{cases} x(0)=0 \\ v(0)=\frac{dx}{dt}(0)=v_0 \end{cases}$$

We can substitute these into the above $x(t)$ and $x'(t)$ above

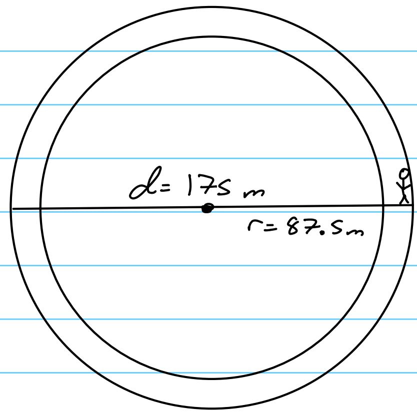
$$\begin{cases} 0=B\cos(0)+C\sin(0) \\ v_0=-\omega B\sin(0)+\omega C\cos(0) \end{cases}$$

$$\therefore \begin{cases} 0=B \\ v_0=\omega C \end{cases} \longrightarrow \begin{array}{l} B=0 \\ C=\frac{v_0}{\omega}=\frac{v_0}{\sqrt{\frac{k}{m}}}=v_0\sqrt{\frac{m}{k}} \end{array}$$

With these constants:

$$\boxed{x(t)=v_0\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}\,t\right) \quad \text{with} \begin{cases} \omega=\sqrt{\frac{k}{m}} \\ B=0 \\ C=v_0\sqrt{\frac{m}{k}} \end{cases}}$$

Problem 2)



Max distance from center is 87.5m

part 'a' is rotational rate question, 'b' is the question containing the transport cart.

a) Circular Rotation: $a_c = g$ to experience earth-like gravity.

Centripital Acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$

$a_c = g$, $r = \frac{D}{2}$, $\omega = ?$

$g = \omega^2 \left(\frac{D}{2}\right) \longrightarrow \boxed{\omega = \sqrt{\frac{2g}{D}}}$ $f = \frac{\omega}{2\pi}$ $= 0.053$ rot/sec $\left(60 \frac{s}{min}\right)$ $= 3.20 \frac{rot}{s}$

Numerically: $\omega = \sqrt{\frac{2(9.81 m/s^2)}{175 m}} = 0.3348 s^{-1}$

$\frac{0.3348}{s} \times \frac{60 s}{1 min} = \cancel{\boxed{20.09 \frac{rad}{min}}}$

b) If you drive the cart in the same direction that the ship is rotating in, then your experienced gravity given by $a_c = \omega^2 r$, will be greater as your velocity will be higher than you being stationary. This difference will result in you feeling more weight. The opposite is true if you drive against rotation. You will move with less velocity, and by the same reasoning as above, you will feel less weight.

$V_{rim} = \omega R$

$= \left(\sqrt{\frac{g}{R}}\right) R$

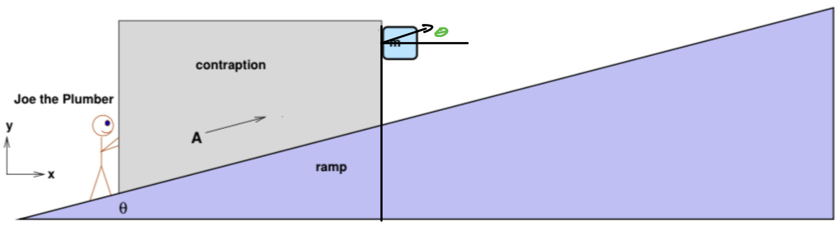
$= \left(\sqrt{\frac{2g}{D}}\right) \frac{D}{2}$

$= \sqrt{\frac{gD}{2}}$

$= 29.3 m/s$

$a_c = \frac{V_{net}^2}{2} = \frac{(V_{rim} + V_{cart})^2}{R} = \frac{2(V_{rim} + V_{cart})^2}{D}$

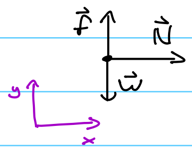
Problem 3)



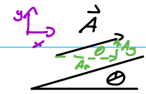
Given

- Coordinate system
- Contraption pushed at θ and $\vec{A}$
- Small box of mass m
- Static friction btwn box and contraption

a) FBD Box



Acceleration



$A_x = A\cos\theta$

$A_y = A\sin\theta$

N2L x-dir

$F_{Net_x} = m a_x$

$\vec{N} = m A_x$

$$\boxed{\begin{aligned} N &= mA\cos\theta \\ W &= mg \end{aligned}}$$

N2L y-dir

$F_{Net_y} = m a_y$

$\vec{f} - \vec{W} = m A_y$

$f - mg = mA\sin\theta$

$$\boxed{f = m(A\sin\theta + g)}$$

Although not asked, friction is constrained by $f \leq \mu N$, so $f \leq \mu m A\cos\theta$.

b) System is assumed to be in equilibrium so don't consider the constraint on friction.

$\vec{g}_{eff} = \vec{g}_{real} + \vec{g}_{fict}$

$\vec{g}_{real}$: $\vec{g}_{real} = 0\hat{i} - g\hat{j}$ gravity pulling down, so vector is negative

$\vec{g}_{fict}$: $\vec{g}_{fict} = -\vec{A}$

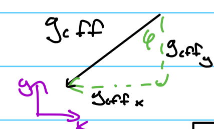
$= -(A_x\hat{i} + A_y\hat{j})$

$g_{fict} = -A$

$= -A\cos\theta\hat{i} - A\sin\theta\hat{j}$

$\therefore g_{eff} = -g\hat{j} - A\cos\theta\hat{i} - A\sin\theta\hat{j}$

$g_{eff} = -A\cos\theta\hat{i} - (A\sin\theta + g)\hat{j}$

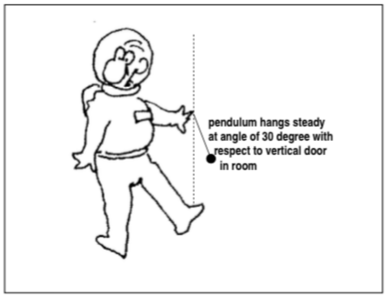


$\tan\varphi = \dfrac{g_{eff,x}}{g_{eff,y}}$

$= \dfrac{-A\cos\theta}{-(A\sin\theta + g)}$

$$\boxed{\varphi = \tan^{-1}\left(\frac{A\cos\theta}{A\sin\theta + g}\right)}$$

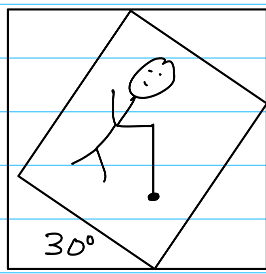
Problem 4)



Given

- Pendulum @ $30°$ relative to level
- Cannot hear any outside factors to indicate if room is tilted, or an illusion is occurring regarding the room moving

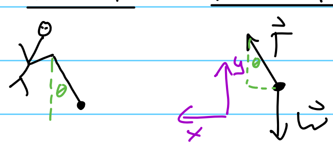
a)



b) No. The tilt of the pendulum suggests an effective gravity in a non vertical direction. This is given by $\vec{g}_{eff} = \vec{g}_{real} + \vec{g}_{fict}$, where $\vec{g}_{fict} = -A$. Here, $A = 0$ as the ship is moving with constant, purely vertical velocity, so $\vec{g}_{eff} = \vec{g}_{real} = -g\hat{j}$, which should not be tilted at all.

c) Yes, using the same logic as b, constant acceleration horizontally would explain the tilt.

Sketch | FBD Pend | N2L x-dir | N2L y-dir | Combine Expressions



$F_{Net_x} = ma_x$

$T_x = ma_x$

$T\sin\theta = ma_x$

$F_{Net_y} = m\cancel{a_y}^{0}$

$T_y - W = 0$

$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$

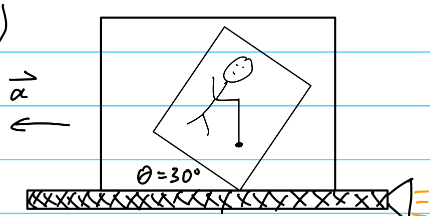
$\begin{cases} a_x = \frac{T\sin\theta}{m} \\ T = \frac{mg}{\cos\theta} \end{cases}$

$a_x = \frac{\frac{\cancel{m}g}{\cos\theta}\sin\theta}{\cancel{m}} = g\tan\theta$

$\boxed{a_x = g\tan\theta}$

let pendulum have arbitrary mass m

d)



Numerically: $a_x = 9.81\,m/s^2(\tan 30°) = \boxed{5.66\ m/s^2}$

The ship must move with constant horizontal acceleration leftwards. This would cause the room to tilt right by an angle of $30°$, here. The acceleration's magnitude is the same as c:

$$\boxed{\begin{aligned} a_x &= g\tan\theta \\ a_x &= 5.66\,m/s^2 \end{aligned}}$$

e) Einstein's principle of equivalence states that experiments performed in a constantly accelerating reference frame give indistinguishable results to those done in a gravitational field. This means that without seeing the outside of the ship, Ringo cannot verify if he is in deep space or on earth.

Problem 5) A bear travels 100 miles due south (precisely). Then he travels 100 miles due east. Then he travels 100 miles due north, to arrive at the exact same location from where he started. ***What color is the bear?***

Because the Earth is curved, the triangle traced out by the bear's path is curved in such a way that the bear ends up at the same place, which is the north pole. To my knowledge, polar bears live on the north pole, so the bear is white.