PHYS 121: Nine Practice Exam Problems With Solutions

September 22, 2023

Please note: The First Hour Exam is Friday, February 16, 11:40 AM in Strosacker. See Document #10 for all details and rules related to the exam.

Because of the exam, there is *no additional homework assignment* due until *after* the exam. (The next Homework will posted on the weekend and will not be due until Monday, February 26.

Physics 121: Spring 2024: Nine Practice Problems for First Hour Exam

These are for Practice Only. They will not be collected nor graded.

Important: Do not look at solutions! Read First:

Here you will find 9 practice problems which are all actual problems used on past versions of the PHYS 121 First Hour Exam. The problems are organized into groups of three problems, each corresponding to a "Practice First Hour Exam". All of these problems come directly from past First Hour Exams. Practice problems can be useful for students. However, there are pitfalls to using practice problems and so these general guidelines might be helpful: students:

- First and foremost, all students are encouraged to attempt to work all problems "under exam conditions". In other words, ***work each problem without notes or books, and without looking at the solutions.*** Ideally, give yourself about 50 minutes (corresponding to the allowed exam time) and try to completely solve one of the groups of three problem. Doing the practice problems under exam conditions will tell you if you are "well-prepared" to do well on the exam as it will actually be given.

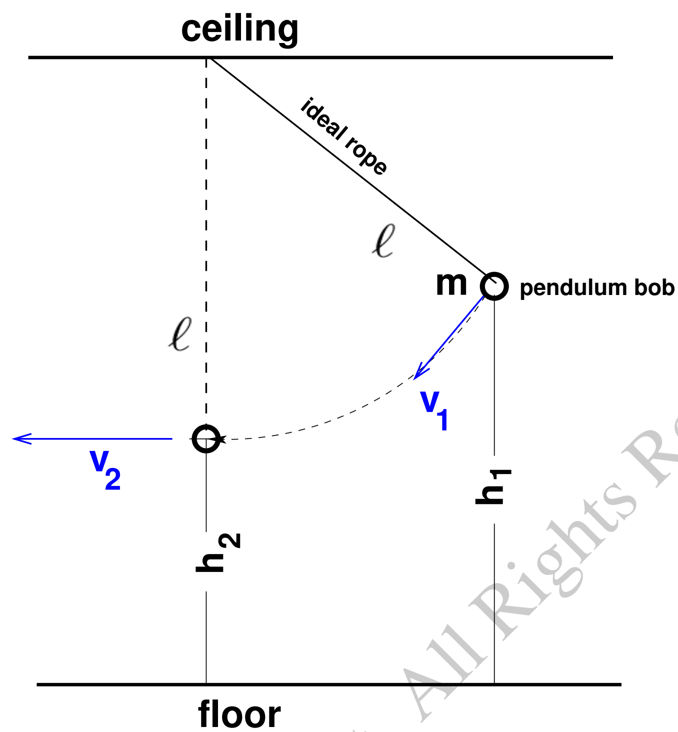
- Again, *resist any temptation to look at the solutions before you have made a decent effort to solve the problem on your own..* This is really important. Student who quickly look at the solutions before they actually work the problems risk fooling themselves into thinking they know how to solve the problem. **Physics is learned by doing, not observing.** Simply looking at solutions before you try the problems as preparation for an exam is like trying to learn how to ride a bicycle by watching someone else riding. You tell yourself "Oh I understand that. I can do that." But you may well be fooling yourself. The only way to know if you have mastered a problem is to demonstrate that you have actually done the problem on your own.

- So give yourself a fixed period of time to work the problems. Use the whole time to work the problem even if you are stuck. At the end of the time, look at the solutions. First, for each problem, make sure that you are using the correct ***Main Physics Concept*** for any given part of a problem. This is most important. If you pick the wrong Main Physics Concept you will get nowhere. Study the solution. Make sure you understand it. Make sure you understand where you went wrong. Then *put the problem away for at least 12 to 24 hours.* Now, say, the next day, try to solve that problem – again without looking at the solution. Maybe you will get further. Repeat until you have mastered the problem and you can solve it quickly without looking at the solution at all.

- If you are struggling with a Main Concept, better to make sure you get some help with that Concept instead of doing many practice problems. Every student has different learning preferences. Some students do best by studying the materials. Others do well with practice problems. You want to develop a study strategy that is best for you. Remember, struggle is a good thing.

- Remember the goal of the course is to have students *demonstrate that they understand the Main Physics Concept by successfully solving problems.* We are not looking for "the right answer". We are looking for you to show us that you understand the main concepts and can apply these correctly to solve a wide variety of problems. The problems on the exam will be *different* from the practice problems, guaranteed. Do not "memorize solutions" to problems. Instead, make sure you understand the problems so that you can tackle new problems using the same Physics Concepts – problems that may seem both familiar and unfamiliar. Mastery means being ready to apply the Main Physics Concepts to new problems that you have not yet encountered, working your way methodically towards the correct solution quickly and with confidence.

Problem X01: Pendulum fun – Straight from 2013 First Hour Exam:



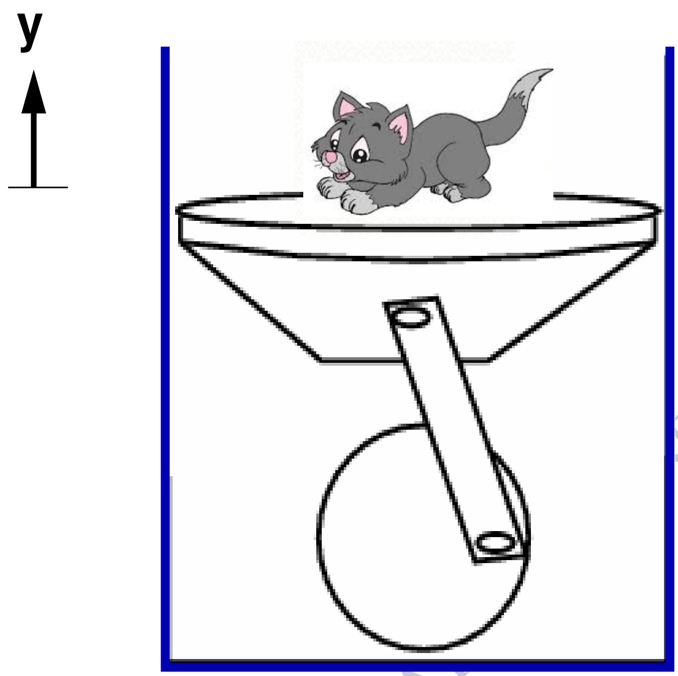
A pendulum is constructed from a bob of given mass m attached to an ideal rope of given length ℓ. The bob is given an initial speed v_1 at an initial height h_1 from the floor.

Part (a) – What is the work done by the force of Weight on the bob as it swings from it's starting position at height h_1 to the position at the lowest position correspond to a height h_2 above the floor? Express your answer in terms of the given parameters only. Explain how you know this.

Part (b) – What is the work done by the tension due to the rope on the bob as it swings from it's starting position to the lowers position? Express your answer in terms of the given parameters only. Explain your answer.

Part (c) – What is the speed of the bob v_2 when it reaches the lowest position? Express your answer in terms of the given parameters only. Explain how you know this. Show your work.

Problem X02: Kitten Ride



A kitten of given mass M goes on the Kitten Ride as follows: The kitten sits on a horizontal platform that is connected to a large piston so that the platform is driven vertically. The position of the surface of the piston as a function of time is given according to the following equation:

$$y(t) = K\sin(\Omega t)$$

Here Ω is a constant given angular speed, K is a given positive constant (with appropriate physical units) and y represents the vertical position, positive upward as indicated.

Part (a) (5 points) – What is the **velocity** of the kitten at time $t = 0$? Explain your work. Present your answer in terms of the given parameters. Hint: the correct answer is not zero.

Part (b) (5 points) – What is the magnitude of the **net force** on the kitten at time $t = \frac{\pi}{2\Omega}$? Explain your work. Present your answer in terms of the given parameters. Hint: the correct answer is not zero.

Problem X02 continues next page....

Problem X02 continues....

Part (c) (5 points) – What is the magnitude of the **Normal Force** on the kitten at time $t = \frac{\pi}{2\Omega}$? Explain your work. Present your answer in terms of the given parameters. Hint: the correct answer is not zero, and it is not Mg.

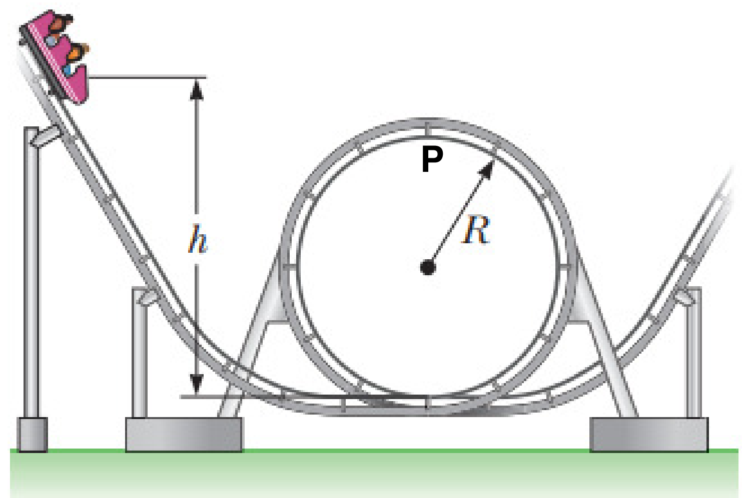
Some Possibly Useful Equations: Here b is a constant:

$$\frac{d}{dx}\sin(bx) = b\cos(bx) \qquad \frac{d}{dx}\cos(bx) = -b\sin(bx)$$

$$\int \sin(bx)\,dx = -\frac{\cos(bx)}{b} + C \qquad \int \cos(bx)\,dx = \frac{\sin(bx)}{b} + C$$

$$\sin(0) = 0 \qquad \cos(0) = 1 \qquad \sin(\pi/2) = 1 \qquad \cos(\pi/2) = 0$$

Problem X03: Toy Roller-coaster



An oddly simple toy roller-coaster is made as follows. A cart is released at rest on a track that is effectively frictionless with a given mass M as shown above. The cart then moves on the track and then does a "loop-the-loop". The height of the track h and the radius of the loop R are given parameters as shown in the figure.

Part (a) (5 points) – What is the Work done by the force of *Weight* on the cart as it moves from the start of the track to point P as shown? Express your answer in terms of the given parameters. Is this positive work or negative work? Explain your work.

Part (b) (5 points) – What is the ***speed*** of the cart at point P? Express your answer in terms of the given parameters. Explain your work.

Part (c) (5 points) – More Difficult: What is the minimum value of h required to ensure that the cart is pressing upward against the track at point P? Express your answer in terms of parameters M, R, and g, as applicable. Explain your work.

SOLUTION TO PROBLEM X01:

Part (a):

The force of Weight $\vec{W}$ on the bob points *downward* with magnitude: $W = mg$. The path is circular, but since the force is downward we only need to worry about the component of the path that moves downward:

$$\mathcal{W}_{\vec{W}} = \int_{path} \vec{W} \cdot d\vec{r}$$

$$\mathcal{W}_{\vec{W}} = mg(\Delta y)$$

$$\mathcal{W}_{\vec{W}} = -mg(h_2 - h_1)$$

$$\boxed{\mathcal{W}_{\vec{W}} = mg(h_1 - h_2)}$$

Alternatively: we can argue that since the force of Weight is *Conservative* then the work done by Weight is just the negative of the change in the Potential Energy:

$$\mathcal{W}_{\vec{W}} = -\Delta U$$

$$\mathcal{W}_{\vec{W}} = -(mgy' - mgy)$$

$$\mathcal{W}_{\vec{W}} = -(mgh_2 - mgh_1)$$

$$\boxed{\mathcal{W}_{\vec{W}} = mg(h_1 - h_2)}$$

Alternatively: we can argue that since the force of Weight is Conservative, the work done is *Path Independent* and we can invoke a path such as the sides of a right triangle where the length of vertical displacement vector is just the differences in the two heights:

$$\mathcal{W}_{\vec{W}} = \vec{W} \cdot \vec{d}$$

$$\boxed{\mathcal{W}_{\vec{W}} = mg(h_1 - h_2)}$$

Part (b):

The force of Tension is *not* a Conservative force. But in this problem Tension is always applied in a direction that is *perpendicular* to the curved path of the bob. Therefore $\vec{T} \cdot \vec{d} = 0$ on the whole curved path and so $\boxed{\textbf{The work done by the force of Tension is zero.}}$.

(Problem solution continues next page....)

Part (c):

Given that we are asked for a speed, we are inclined to use **Conservation of Energy**. However, *we need to check the conditions:* There are two forces on the bob. (1) Weight which is *conservative*, and (2) Tension, which (since it applied at a right angle to the motion of the ball) *does no work*. So we have **met the Conditions and we can use Conservation of Mechanical Energy** and we may now proceed to apply this to the problem at hand:

$$\text{“BEFORE”} = \text{“AFTER”}$$
$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

We plug in initial values and solve for $v_2 = v'$:

$$mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_2^2$$
$$\frac{1}{2}mv_2^2 = mgh_1 - mgh_2 + \frac{1}{2}mv_1^2$$
$$v_2^2 = 2gh_1 - 2gh_2 + v_1^2$$
$$v_2^2 = 2g(h_1 - h_2) + v_1^2$$
$$\boxed{v_2 = \sqrt{2g(h_1 - h_2) + v_1^2}}$$

NOTE TO GRADER: If the student correctly applies Conservation of Energy but fails to check and explain the Conditions above, this is a one-half point deduction.

Solution to Problem X02: (15 points)

Part (a) – (5 points):

We are asked for the ***velocity*** of the kitten. This is a **Kinematics** problem. The kitten is in contact with the piston platform and nothing else. The motion of the kitten is entirely vertical. The position as a function of time is specified:

$$y(t) = K\sin(\Omega t)$$

To get the velocity, we apply the **definition of velocity:**

$$\text{velocity} \equiv \frac{d}{dt}(\text{position})$$

$$v(t) = \frac{dy}{dt}\left[K\sin(\Omega t)\right]$$

This is calculus: the derivative of the sine is the cosine:

$$v(t) = \Omega K\cos(\Omega t)$$

And at $t = 0$ the cosine function is one, so:

$$v(t = 0) = (\Omega K)\cdot(1)$$

$$\boxed{v(t = 0) = \Omega K}$$

Part (b) – (5 points):

We are asked to calculate the **Net Force** on the Kitten at a particular time. This can be done simply if we consider the **Definition of Acceleration** and then apply **Newton's Second Law:** First, we take another derivative to get the acceleration:

$$a(t) \equiv \frac{dv}{dt}$$
$$a(t) = \frac{d}{dt}\left[\Omega K \cos(\Omega t)\right]$$
$$a(t) = -\Omega^2 K \sin(\Omega t)$$

Plugging in $t = \frac{\pi}{2\Omega}$ we get:

$$a\left(t = \frac{\pi}{2\Omega}\right) = -\Omega^2 K \sin\left[\Omega\left(\frac{\pi}{2\Omega}\right)\right]$$
$$= -\Omega^2 K \sin\left(\frac{\pi}{2}\right)$$

Trigonometry:

$$= (-\Omega^2 K) \cdot (1)$$
$$a = -\Omega^2 K$$

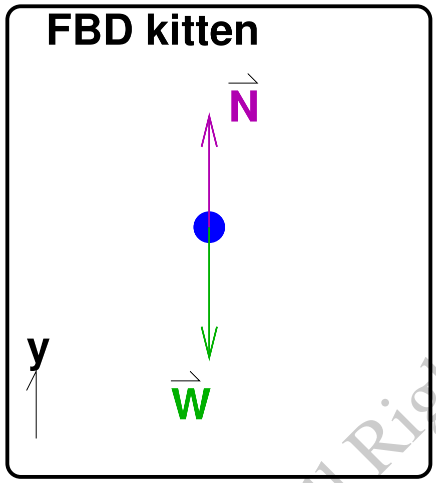
Finally then we just apply **Newton's Second law:**

$$F_{net} = ma$$
$$|F_{net}| = |ma|$$
$$\boxed{|F_{net}| = M\omega^2 K}$$

Graders take note: Although we asked for the ***magnitude*** of the Net Force here, any students who incorrectly leaves the negative sign on this answer should not be penalized.

Part (c) – (5 points):

Finally we need to consider the specific forces on the kitten. We note that there is no net friction on the kitten (if there were, the kitten would be accelerating in a horizontal direction, which is not happening here). So the only two forces on the kitten are Weight and the Normal force on the kitten due to the piston platform:



We define positive up in accordance with the coordinate system as given. [1] We use our result from **Part (b)** and apply **Newton's Second Law:** at time $t = \frac{\pi}{2\Omega}$:

$$F_y = Ma_y$$
$$N - W = M(-\Omega^2 K)$$
$$N - W = -M\Omega^2 K$$

Note here we have defined the direction up as positive, so since the acceleration relative to this direction is negative we need that negative sign on our acceleration.

Solving for the one unknown force in terms of other parameters:

$$N = W - M\Omega^2 K$$
$$N = Mg - M\Omega^2 K$$
$$\boxed{N = M(g - \Omega^2 K)}$$

Solution to Problem X03: (15 points)

Part (a) – (5 points):

Definition of Work: Work associated with a particular force in going from point to point Q is defined as follows:

[1] A perfectly acceptable alternate approach is to define the downward direction as positive and then the left side of the equation is $W - N$.

$$\mathcal{W}_{\vec{F}} \equiv \int_{\text{path}} \vec{F} \cdot d\vec{r}$$

In other words, the Work results from the integral of the component of the force aligned parallel to the direction of the path of the body.

The Work done by the force of Weight is easily calculated because the Weight force is **Conservative** and therefore path-independent. So we need only consider the vertical component of the motion:

$$\mathcal{W}_{\vec{W}} = \int_{y}^{y'} W dy$$

$$\mathcal{W}_{\vec{W}} = \int_{y}^{y'} (-mg) dy$$

$$\mathcal{W}_{\vec{W}} = -mg(y' - y)$$

In this problem the initial position is given by $y = h$ and the "final" position at the right side of the loop (point P) is given by $y' = 2R$. So we have:

$$\mathcal{W}_{\vec{W}} = -mg(2R - h)$$

$$\boxed{\mathcal{W}_{\vec{W}} = mg(h - 2R)}$$

Note also that the work done by a **Conservative Force** is equal to the negative of the change in **Potential Energy**, as expected. Note also that the Work is **positive** because the force of Weight (downward) is applied in the same direction as the vertical displacement (downward).

Part (b) There are only two forces on the block for this whole problem: Weight and Nornmal. Because the Weight Force is ***Conservative*** and the Normal Force does ***zero work***, the **Conditions are Satisfied** for applying **Conservation of Mechanical Energy**:

$$\text{“BEFORE”} = \text{“AFTER”}$$
$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

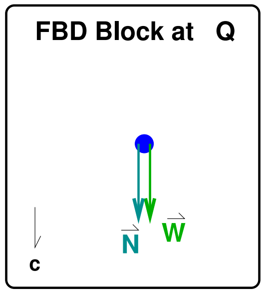
In this problem, the “Before” corresponds to the top of the ramp where $y = h$ and $v = 0$. The “After” correponds to the position at point P corresponding to $y' = R$. We plug these values in and solve for the unknown final velocity $V_P = v'$.

$$mgh + 0 = mg(2R) + \frac{1}{2}mV_P^2$$

Solving for V_P

$$mg2R + \frac{1}{2}mV_P^2 = mgh + \frac{1}{2}mV_0^2$$
$$\frac{1}{2}mV_P^2 = mgh + \frac{1}{2}mV_0^2 - 2mgR$$
$$V_P^2 = 2gh + 4gR$$
$$V_P^2 = 2g(h - 2R)$$
$$\boxed{V_P = \sqrt{2g(h - 2R)}}$$

Part c) – 5 points At Point P we have this **Free Body Diagram:**



For this part, we are interested in the vertical (down) coordinate corresponding to the direction of **centripetal acceleration** We apply Newton's Second Law in the "centripetal" direction:

$$F_c = ma_c$$

$$N + W = m\frac{v^2}{R}$$
$$N = m\frac{v^2}{R} - W$$
$$N = m\frac{V_P^2}{R} - mg$$

Here $v = V_P$ where we the result from Part (b).

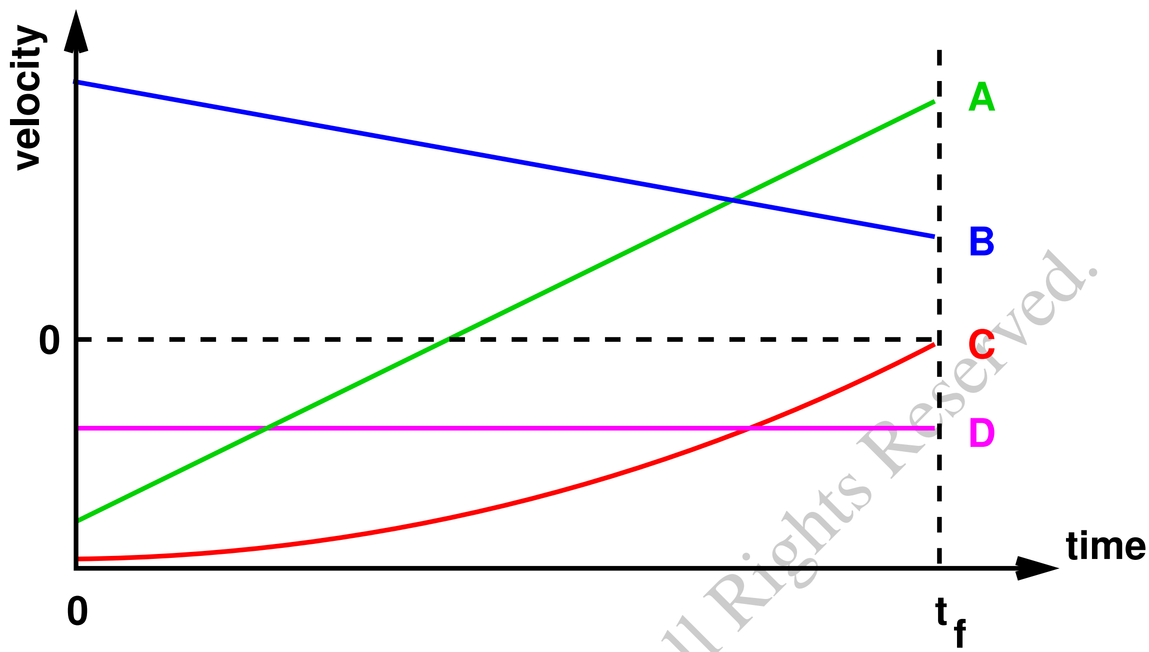
$$N = m\left[\frac{2g(h-2R)}{R}\right] - mg$$

Next we make the argument that in order for the cart to be "pressing up" into the track, there must be a Normal force pressing down onto the cart (by Newton's Third Law). This force must be greater than zero if the cart is in contact with the track.

$$N > 0$$

$$m\left[\frac{2g(h-2R)}{R}\right] - mg > 0$$
$$\left[\frac{2g(h-2R)}{R}\right] > g$$
$$\left[\frac{(h-2R)}{R}\right] > \frac{1}{2}$$
$$h - 2R > \frac{R}{2}$$
$$h > 2R + \frac{R}{2}$$
$$\boxed{h > \frac{5R}{2}}$$

Problem Y01: One-D Kinematics Concept (20 points)



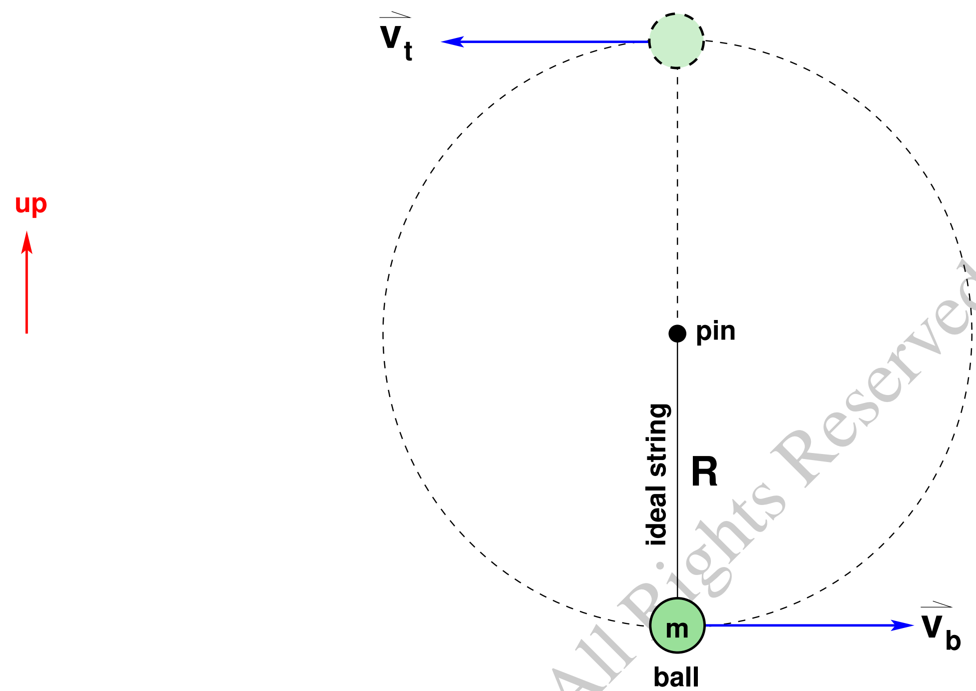
Four different particles move on a one dimensional path. The **velocity as a function of time** is shown each of the four particles on the graph above The four particles are labeled "A", "B", "C" and "D". Note that the dashed horizontal line represents zero velocity. Note that each of the following questions applies to the ***entire trip of each of the four particles***. In other words, each question below should be considered in the context of the entire time interval from time $t = 0$ to time $t = t_f$.

Part (a) – (5 points): Make a list of all of the particles (if any) that are moving with **constant acceleration** during the entire time interval from $t = 0$ to $t = t_f$. Important: Explain your work. Write one or two sentences to explain how and why you know the acceleration is constant in these cases?

Part (b) – (5 points): In terms of speed, make a list of all of the particles (if any), that are **always slowing down** during the entire time interval. Important: Explain your work. Write one or two sentences to explain.

Part (c) – (5 points): Which of these four particles has the **largest position displacement in the positive direction** as a result of the motion during this time interval? Here displacement is defined as the final positin relative to the intitial position. If you wanted to determine the value of this displacement, describe how this could be done using this kind of graph. If applicable, indicate on the graph (on your answer sheet) how this measurement could be done. Important: Explain your work. Write one or two sentences to explain.

Problem Y02: Ball on a String (30 points)



A ball of given mass m is tied to an ideal string of given length R which is tied to a fixed pin so that the ball is constrained to travel vertically within a circular path as shown above. The ball is given an initial horizontal speed v_b which is large enough to ensure that the ball will arrive at top of the circle. Ignore air resistance.

Part a) (5 points) – What is the Work done by the force of Weight on the ball as the ball moves from the bottom of the circle to the top of the circle? Give your answer in terms of given parameters. Explain your work.

Part b) (5 points) – What is the Work done by the force of Tension on the ball as the ball moves from the bottom of the circle to the top of the circle? Give your answer in terms of given parameters. Explain your work.

Part c) (10 points) – Determine the value of v_t, the speed of the ball when it reaches the top of the circle. Give your answer in terms of given parameters. Explain your work. Important: you must explain and justify any physics concepts that you are using here.

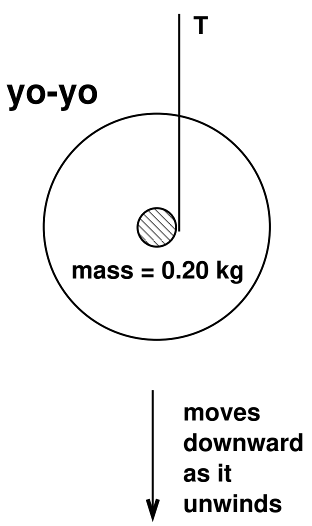
Part d) (10 points) – Determine the value of the *tension* in the string when the ball reaches the top of the circle. Give your answer in terms of given parameters. Explain your work. Important: you must explain and justify any physics concepts that you are using here.

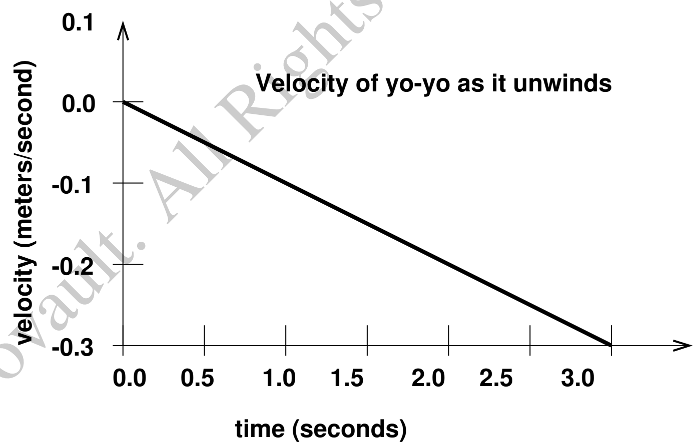
Problem Y03: Stopping a Spinning Space Station (30 points)

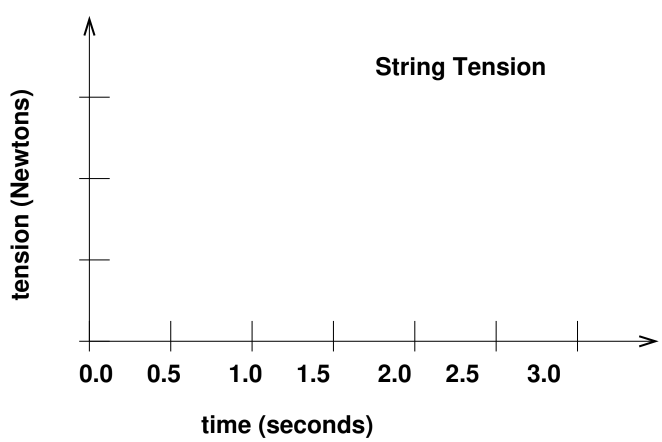
A yo-yo is an interesting toy for demonstrating physics principles. The string of a yo-yo ($m = 0.20kg$) is wound up on the central axle. Suppose the wound-up yo-yo is released from rest and begins to move downward towards the floor as the string unwinds. Suppose you are able to measure accurately the velocity of the yo-yo as it unwinds. A plot of the measured velocity of the yo-yo vs. time is shown below.

Use the plot shown below to determine the *string tension* as a function of time for this same time interval. Plot the tension as a function of time on the incomplete graph below. Be sure to label the y-axis scale. Explain your reasoning.

In this problem, use the correct value for the acceleration due to gravity g is equal to 9.81 m/s^2.







Solution to Problem Y01: (15 points)

Part (a): (5 points)

This is a problem in **1-D Kinematics**. In the case that the **acceleration is constant**, we expect the velocity to vary as a ***linear function of time***. In other words, we are looking for those particles where the plot of velocity versus time is a straight line. This correponds to **Particles A, B, and D** but not particle C, where the plot of velocity versus time is curved, not linear.

Part (b): (5 points)

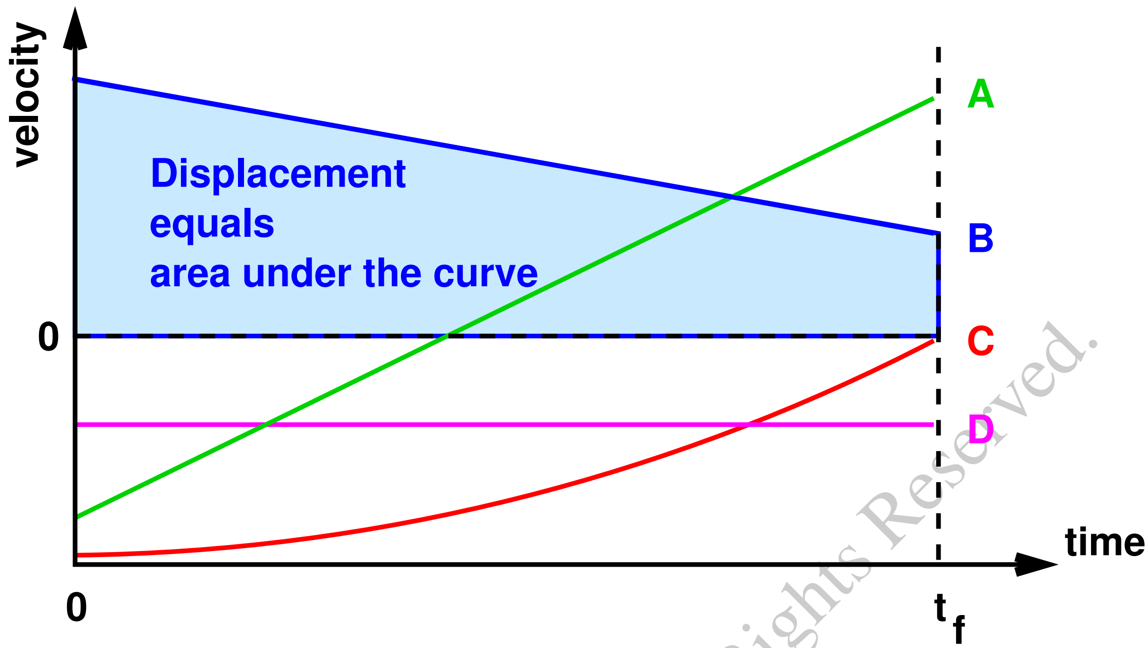
This is a problem in **1-D Kinematics**. we define **speed as the magnitude of the velocity.** Here, "slowing down" means the magnitude of the velocity is always getting closer to zero. We can rule out Particle D because the speed is constant. We can also rule out Particle A – even though the speed is getting closer to zero for the first part of the time interval, it is increasing for the second part. Particle B is indeed slowing down (decreasing positive velocity) during the whole time interval. Particle D is also slowing down – the (negative) velocity is always getting closer to zero with time. Therefore, **Particles C and D** (and not particles A and B), correspond to the particles that are slowing down in terms of speed.

Part (c): (5 points)

This is a problem in **1-D Kinematics**. Since velocity is defined as the time-derivative of the position, the displacement (change in position) can be infered from the **anti-derivative (with respect to time)** of the velocity:

$$x(t_f) = x_0 + \int_0^{t_f} v(t)\, dt$$

Graphically, the anti-derivative of a function can be represented as the **area under the curve**. Since we asked for the particle that has the largest ***positive*** displacement, we want the particle which has the largest positive area under the curve. By inspection, **Particle B** clearly has the positive area under the curve which can be demonstrated graphically this way:



Note that while a case can be made that the displacement for Particle C is larger than for Particle B, in the case of Particle C, the velocity is always negative and so the displacement is negative.

Solution to Problem Y02:

Part (a): $(5 points)$

We cannot "simply" use the expression $\mathcal{W}_{\vec{F}} = \vec{F} \cdot \vec{d}$ because the circular path here does not correspond to a straight path. However, the force of Weight is easily calculated because this force is **Conservative** and therefore the Work done is *path-independent*. So we need only consider a vertical path that moves us from the bottom position to the top position: The displacement vector $\vec{d}$ is of length $d = 2R$ and points *upward*. The force vector $\vec{F} = \vec{W}$ has magnitude $W = mg$ and points *downward*.

$$\mathcal{W}_{\vec{W}} = \vec{W} \cdot \vec{d}$$
$$\mathcal{W}_{\vec{W}} = Wd\ \cos\theta$$
$$\mathcal{W}_{\vec{W}} = (mg)(2R)\ \cos\theta$$

Since $\vec{d}$ and $\vec{W}$ point in opposite directions, $\cos\theta = -1$ and so the work that is done is negative:

$$\boxed{\mathcal{W}_{\vec{W}} = -2mgR}$$

Alternatively we know that for any **Conservative force**, the work done is the negative of the change in **Potential Energy**. Since $\Delta P = mg(\Delta y) = mg(2R)$ we get the same answer:

$$\boxed{\mathcal{W}_{\vec{W}} = -2mgR}$$

Guidelines for graders:

- **Perfect score with explanation = 5 points.**
- **Negative sign missing = 4 points.**
- **Applying the Mechanical Work-Energy Relation ($\mathcal{W}_{tot} = \Delta K$) and giving correct answer but in terms of non-given parameter v_t = 3 points.**
- **Incorrectly calculating the Work as the force of weight (mg) times the distance around the arc $(d = \pi R)$ = 2 points.**
- **Incorrectly arguing that ($\mathcal{W}_{\vec{W}} = 0$ because for force is perpendicular to the path at the top and/or bottom = 1 point.**

Part (b): $(5points)$

We consider the **definition of Work** associated with a particular force in going from point P to point Q is defined as follows:

$$\mathcal{W}_{\vec{F}} \equiv \int_P^Q \vec{F} \cdot d\vec{r}$$

In other words, the Work results from the integral of the component of the force aligned in the direction of the path of the body.

In this problem we go from the *bottom* to the *top* on a circular path. During that time, the force of Tension is always perpendicular to the tangent line to the path. This means that the work done by Tension is zero because the Tension force is always applied perpendicular to the direction of motion.

Guidelines for graders:

- **Perfect score with explanation = 5 points.**
- **A incorrectly non-zero answer that invokes something to do with Work: = 2 points.**

Part (c): $(10 points)$

For this problem we are asked for a *speed* so it sounds like we want to pick up the toolbox called **Conservation Laws** and in particular we want to look at Energy. However, ***first we need to check that the Conditions Are Met:*** Specifically, there are two forces on the ball, Weight and Tension. We know:

- The *Weight force is conservative* (part a).
- and because the *Tension force does no work* (part b).

Therefore we can state that ***The Conditions for the application of Conservation of Energy are satisfied***.

So we apply **Conservation of Energy** where the “BEFORE” corresponds to the ball at the bottom and the “AFTER” corresponds to the ball at the top:

$$\text{“BEFORE”} = \text{“AFTER”}$$
$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

We need to define a “zero-point” One of several allowed options is to select the position $y = 0$ at the bottom. So $y = 0$, $v = v_b$, $y' = 2R$ and we want to solve for the unknown $v' = v_t$:

$$0 + \frac{1}{2}mv_b^2 = mg(2R) + \frac{1}{2}mv_t^2$$

Solving for v_t

$$\frac{1}{2}mv_t^2 = \frac{1}{2}mv_b^2 - 2mgR$$
$$v_t^2 = v_b^2 - 4gR$$
$$\boxed{v' = \sqrt{v_b^2 - 4gR}}$$

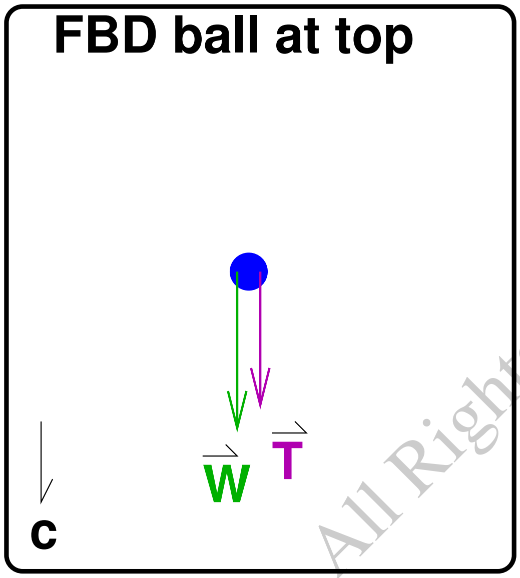
Guidelines for graders:

- **Perfect score with explanation = 10 points.**
- **Missing consideration of *Conditions for Conservation of Energy* = 8 points.**
- **Correct answer but not in terms of given parameters = 5 points.**
- **Attempt to apply Newton’s Second Law instead of Conservation of Energy = 2 points.**

Part (d): $(10 points)$

For this problem, we are asking for the force of tension, so we really want to pick up the toolbox we call **Newton's Laws:**

We consider the FBD of the ball at the top position:



Then we apply **Newton's Second Law** in the **centripetal** direction (downward) which corresponds to the direction of centripetal acceleration since the ball is moving on a *circular path*.

$$F_c = ma_c$$

We know the **centripetal acceleration** is given by: $a_c = \frac{v_t^2}{R}$ so:

$$T + W = m\frac{v_t^2}{R}$$

$$T + mg = m\frac{v_t^2}{R}$$

$$T = m\frac{v_t^2}{R} - mg$$

At this point we plug in our result from Part (b):

$$\boxed{T = m\frac{(v_b^2 - 4gR)}{R} - mg}$$

or

$$T = m\left(\frac{v_b^2}{R} - 4g \;- g\right)$$

$$\boxed{T = m\left(\frac{v_b^2}{R} - 5g\right)}$$

Grader guidelines on next page...

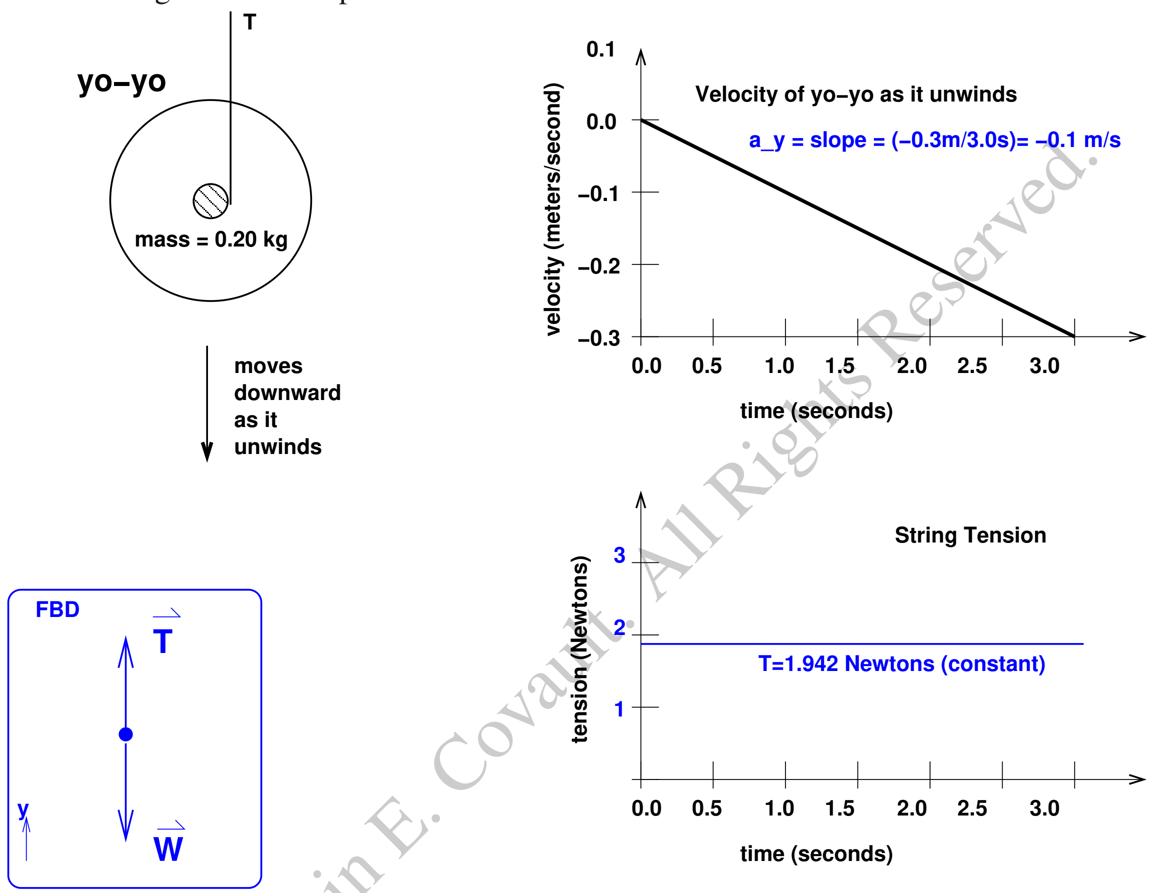
Guidelines for graders:

- **Perfect score with good FBD, explanation using Newton's Second Law, centripetal acceleration = 10 points.**
- **Correct answer in terms of v_t but not in terms of given parameters = 8 points.**
- **Attempt to apply some incorrect ideas about circular motion = 1 or 2 points.**

Solution to Problem Y03:

SOLUTION TO PROBLEM Y03:

Here's the figure marked up with the answer:



We want the tension, which is a *force* so we apply **Newton's Second Law**.

$$F_y = ma_y$$

Let's start with the acceleration term. We look at the plot which is velocity vs. time. It's *linear* so this corresponds to **constant acceleration** where a_y given by the slope:

$$a_y = \frac{dv_y}{dt} = \frac{\Delta v_y}{\Delta t}$$

We can read this from the plot directly:

$$a_y = \frac{-0.30m/s}{3.0s} = -0.10m/s^2$$

Now to get the tension, we work the force side of Newton's Second Law with a **Free Body Diagram**. There are only two forces on the yo-yo: Weight and Tension, as indicated in the Free Body Diagram above. We apply the Second Law:

$$F_y = ma_y$$

$$T - W = ma_y$$

$$T = W + ma_y$$

$$T = mg + ma_y$$

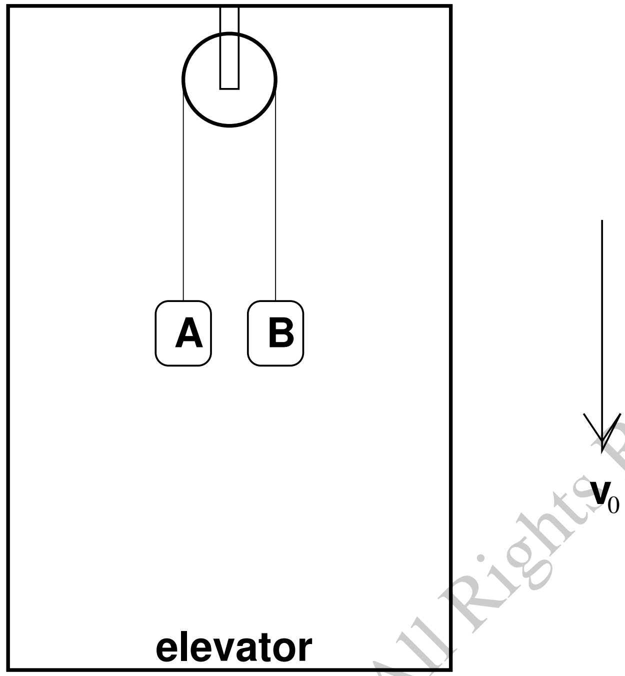
$$\boxed{T = m(g + a_y)}$$

Note that this is constant with time and does not depend on position. Plugging in the numbers:

$$T = (0.2kg)(9.81m/s^2 - 0.10m/s^2)$$

$$\boxed{T = 1.942 \ \text{Newtons}}$$

Problem Z01: Pulley and Blocks in an Elevator



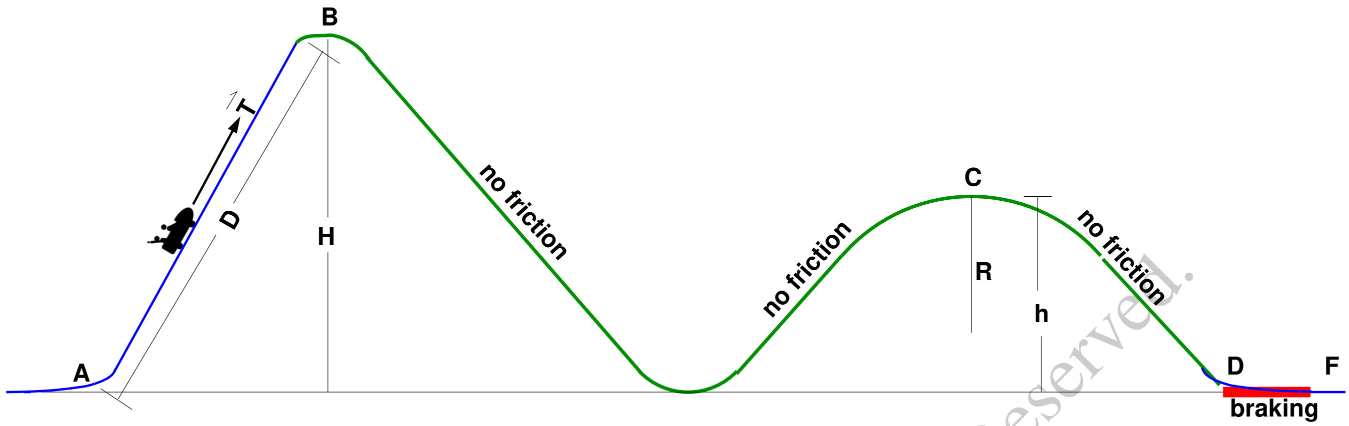
An elevator is moving downward with constant velocity v_0. Inside the elevator two blocks A and B are suspended on an ideal pulley as shown. Block A has mass $m_A = M$ and Block B has mass $m_B = 4M$ where M is a given parameter.

Draw two clear and properly labeled Free-Body Diagrams. What is the ***magnitude*** of the force of Tension on the string? Also: what is the acceleration of Block A?

Important: Explain your work. Show your work. Be sure to follow the general steps for solving a problem with Newton's Laws. Be sure your final answers are presented in terms of given parameters only.

Important: please do not try to solve this problem by applying the concept of a "system" to the two blocks together. For this homework you are obligated to apply Newton's Second Law to each of the blocks separately.

Problem Z02: A Roller-Coaster Cart: (40 points)



A small roller-coaster cart starts at position A. The cart has a given mass m. It is pulled up a the first hill a given displacement D to a given height H by a chain with a given constant force of tension T. At the top of the first hill, corresponding to position B the cart is released with a given initial speed V_B. It then travels to position C corresponding to the top of the second hill with a given height h and a given radius of curvature R. The cart travels down the second hill to position D and then encounters a horizontal region of track where an *unknown* braking force is applied to slow the cart to a final given speed of V_F at position F.

Important: You should assume that the track has ***zero friction*** between position B and position D only.

Part (a) – (5 points) – What is the **Work done by the force of Tension** on the cart as it travels from position A to position B? Give your answer in terms of given parameters only. Explain your work.

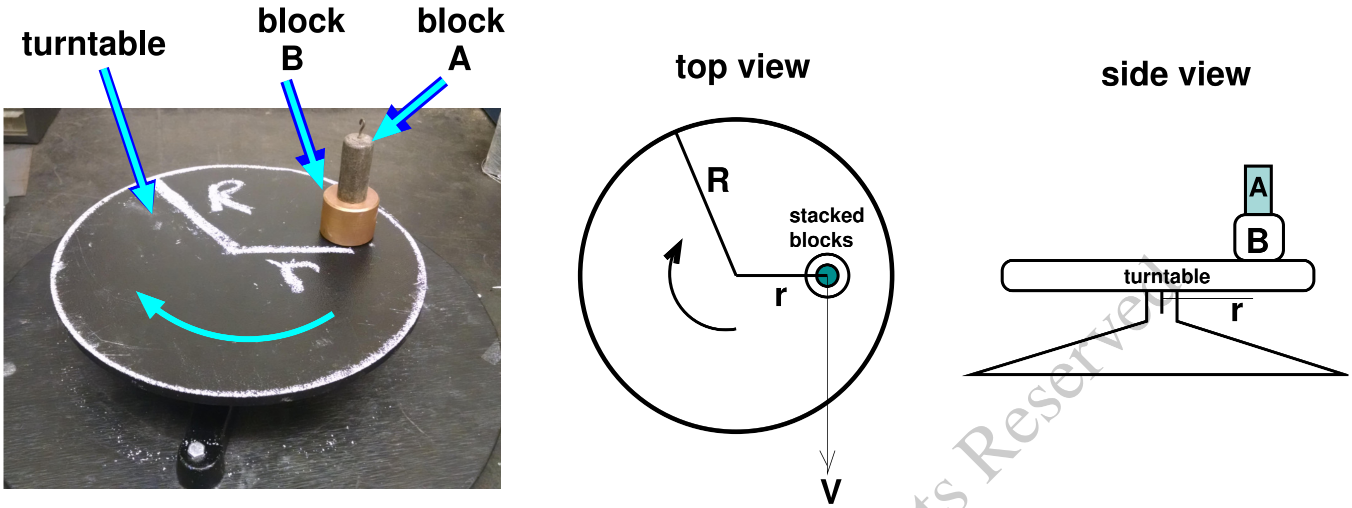
Part (b) – (5 points) – What is the **Work done by the force of Weight** on the cart as it travels from position A to position B? Give your answer in terms of given parameters only. Explain your work.

Part (c) – (10 points) – What is the **speed** of the cart at Position C? Give your answer in terms of given parameters only. Explain your work.

Part (d) – (10 points) – What is the magnitude of the **Normal Force** on the cart due to the track at Position C? Give your answer in terms of given parameters only. Explain your work.

Part (e) – (10 points) – ***More Difficult:*** What is the **Work done by the *unknown braking force*** applied after the bottom of the second hill? Give your answer in terms of given parameters only. Explain your work.

Problem Z03: Turntable Friction – (30 points)



A turntable of given radius R is found to be rotating at a constant angular speed. A snapshot of the arrangement is shown. Two cylindrical blocks, labeled A and B with given masses m_A and m_B are vertically stacked on the turntable as shown at a given distance r from the center. There is ***static friction*** between Block A and Block B and there is also ***static friction*** between Block B and the spinning turntable. The constant speed of Block B is given as V. The coefficient of static friction between Block A and Block B is given as μ_1 and the coefficient of static friction between Block B and the turntable is given as μ_2.

Part (a) – (5 points) – What is the magnitude of **net force** on Block B? Explain your work. Present your answer in terms of the given parameters.

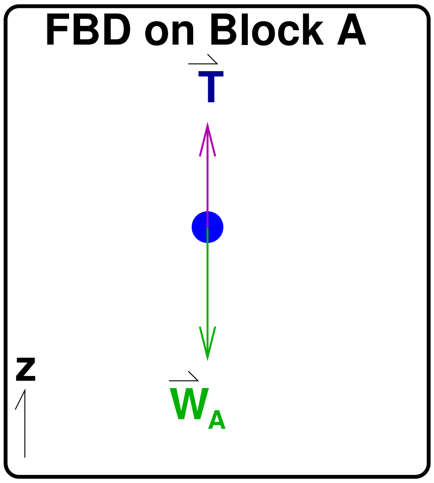
Part (b) – (5 points) – Draw two **Free Body Diagrams** showing ***all*** of the forces on both cylindrical blocks. Be sure to label each force appropriately and be sure to indicate a proper coordinate system that is lined up with the direction of acceleration.

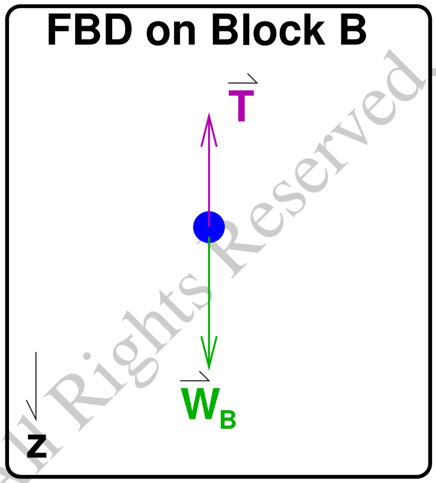
Part (c) – (5 points) – Determine both the magnitude and the direction of $\vec{f}_{BA}$ corresponding to **Force of Friction on the Block B due to Block A.** Explain your work. Present your answer in terms of the given parameters.

Part (d) – (10 points) – Determine both the magnitude and the direction of $\vec{f}_{BT}$ corresponding to the **Force of Friction on the Block B due to the turntable**. Explain your work. Present your answer in terms of the given parameters.

Solution to Problem Z01a:

First we consider the forces on the two blocks: Here's the **Free Body Diagrams** Here (for now) we note that the two blocks accelerate at the same magnitude but (because of the pulley) in opposite directions. So we define a single "coordinate" correspond to the z-direction where z points up for Block A and down for Block B. In this case, the z-direction corresponds to the direction of acceleration of both of the blocks.





We apply **Newton's Second Law:** Block A:

$$F_A = m_A a_z$$
$$T - W_A = m_A a_z$$

$$T - m_A g = m_A a_z \tag{1}$$

We apply **Newton's Second Law:** Block B: (note z positive is down)

$$F_B = m_B a_z$$
$$W_B - T = m_B a_z$$

$$m_B g - T = m_B a_z \tag{2}$$

We have **Two equations and two unknowns:** T and a_z. *We are done with physics, the rest is algebra.* There are several different ways to work this. For example we can add equations (1) and (2):

$$-m_A g + m_B g = m_A a_z + m_B a_z$$
$$(m_A + m_B) a_z = m_B g - m_A g$$
$$\boxed{a_z = g\left(\frac{m_B - m_A}{m_A + m_B}\right)}$$

To get the tension we just plug back into (1) for example:

$$T = m_A g + m_A a_z$$

$$T = m_A g + m_A g\left(\frac{m_B - m_A}{m_A + m_B}\right)$$

$$\boxed{T = m_A g\left[1 + \left(\frac{m_B - m_A}{m_A + m_B}\right)\right]}$$

Note that this gives exactly what we expect at the limits of different masses. If $m_A = m_B$ then the two masses are "balanced" and there is no acceleration. If $m_B >> m_A$ then Block B is simply in free-fall with $a_z = g$.

Finally we plug in the particular values for the two blocks:

$$a_z = g\left(\frac{4M - M}{4M + M}\right)$$

$$a_z = g\left(\frac{3M}{5M}\right)$$

$$a_z = g\left(\frac{3}{5}\right)$$

$$\boxed{a_z = \frac{3g}{5}}$$

$$T = m_A g\left[1 + \left(\frac{m_B - m_A}{m_A + m_B}\right)\right]$$

$$T = Mg\left[1 + \left(\frac{4M - M}{M + 4M}\right)\right]$$

$$T = Mg\left(1 + \frac{3}{5}\right)$$

$$\boxed{T = \frac{8Mg}{5}}$$

Note that the constant velocity v_0 has no bearing on this problem whatsoever.

Solution to Problem Z02: (40 points)

Part (a) – (5 points):

We ask for the Work done by the force of Tension. Here we apply the **Definition of Work** as relevant for a constant force (here $\vec{T}$ over a straight-line path and a displacement of distance D.) Since $\vec{T}$ and $\vec{D}$ are parallel, the angle between them is zero.

$$\begin{aligned} \mathcal{W}_{\vec{F}} &\equiv \vec{F} \cdot \vec{d} = F d \cos\theta \\ &= T D \cos(0) \\ &\boxed{\mathcal{W}_{\vec{T}} = TD} \end{aligned}$$

Part (b) – (5 points):

The force of Weight is easily calculated because this force is **Conservative** and therefore the Work done is *path-independent*. So we need only consider a vertical path that moves us from the bottom position to the top position: The displacement vector $\vec{d}$ is of length H and points *upward*. The force vector $\vec{F} = \vec{W}$ has magnitude $W = mg$ and points *downward*.

$$\begin{aligned} \mathcal{W}_{\vec{W}} &= \vec{W} \cdot \vec{d} \\ \mathcal{W}_{\vec{W}} &= W d \ \cos\theta \\ \mathcal{W}_{\vec{W}} &= (mg)(H) \ \cos\theta \end{aligned}$$

Since $\vec{d}$ and $\vec{W}$ point in opposite directions, $\cos\theta = -1$ and so the work that is done is negative:

$$\boxed{\mathcal{W}_{\vec{W}} = -mgH}$$

Alternatively we know that for any **Conservative force**, the work done is the negative of the change in **Potential Energy**. Since $\Delta U = mg(\Delta y) = mg(2R)$ we get the same answer:

$$\boxed{\mathcal{W}_{\vec{W}} = -mgH}$$

Guidelines for graders for Parts (a) and (b) (each worth 5 points):

- Perfect score with explanation = 5 points for each part.
- Sign error on Work = minus one point
- Correct idea and "correct" answer given, but in terms of non-given parameters, such as an angle "theta" = at most 4 points.
- For Part (b) Incorrectly calculating the Work as the force of Weight (mg) times the distance D = 2 points.
- Incorrectly applying Work-Energy Relation ($\mathcal{W}_{tot} = \Delta K$) at most 2 points.
- Incorrectly arguing that $\mathcal{W}_{\vec{W}} = 0$ because force is perpendicular to the path = at most 2 points.

Part (c) – (10 points):

For this problem we are asked for a *speed* so it sounds like we want to pick up the toolbox called **Conservation Laws** and in particular we want to look at Energy. However, ***first we need to check that the Conditions Are Met:*** Specifically, there are two forces on the ball, Weight and Tension. We know:

- The *Weight force is conservative* (part a).
- and because the *Normal force does no work* on the path from B to C.

There are no other forces in play here (no friction). Therefore we can state that ***The Conditions for the application of Conservation of Energy are satisfied***.

So we apply **Conservation of Energy** where the "BEFORE" corresponds to the ball at the bottom and the "AFTER" corresponds to the ball at the top:

$$\text{"BEFORE"} = \text{"AFTER"}$$
$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

We need to define a "zero-point" One of several allowed options is to select the position $y = 0$ at position A. So $y = H$, $v = V_B$, $y' = h$ and we want to solve for the unknown $v' = V_C$:

$$mgH + \frac{1}{2}mV_B^2 = mgh + \frac{1}{2}mV_C^2$$

Solving for V_C

$$\frac{1}{2}mV_C^2 = \frac{1}{2}mV_B^2 + mgH - mgh$$
$$V_C^2 = V_B^2 + 2g(H - h)$$
$$\boxed{V_C^2 = \sqrt{V_B^2 + 2g(H - h)}}$$

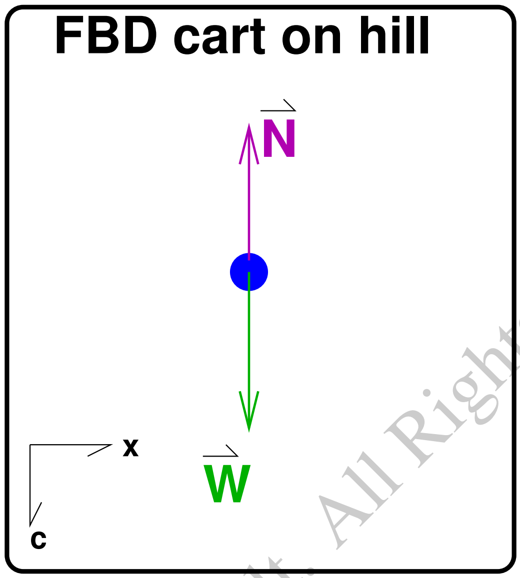
Guidelines for graders:

- Perfect score with explanation = 10 points.
- Missing consideration of *Conditions for Conservation of Energy* = 8 points.
- Correct answer but not in terms of given parameters = at most 7 points.
- Failure to consider initial speed and/or final position = at most 6 points
- Attempt to apply Newton's Second Law instead of Conservation of Energy = 2 points.
- Sign errors or algebra mistakes = 1 or 2 points off for each mistake.

Part (d) – (10 points):

For this problem, we are asking for the Normal force, so we really want to pick up the toolbox we call **Newton's Laws:**

We consider the FDB of the cart at the top position:



Then we apply **Newton's Second Law** in the **centripetal** (downward) direction which corresponds to the direction of centripetal acceleration since the ball is moving on a *circular path*.

$$F_c = ma_c$$

We know the **centripetal acceleration** is given by: $a_c = \frac{v_t^2}{R}$ so:

$$W - N = m\frac{v_C^2}{R}$$

$$N - W = -m\frac{v_C^2}{R}$$

$$N = mg - m\frac{v_C^2}{R}$$

At this point we plug in our result from Part (c):

$$\boxed{N = mg - m\left[\frac{V_B^2 + 2g(H-h)}{R}\right]}$$

Grader guidelines on next page...

Guidelines for graders:

- Perfect score with good FBD, explanation using Newton's Second Law, centripetal acceleration = 10 points.
- Correct answer in terms of v_t but not in terms of given parameters = 8 points.
- Argument made based on incorrect statement that the Normal Force = mg because the vertical acceleration is zero = at most 5 points.
- Attempt to apply some incorrect ideas about circular motion = 1 or 2 points.

Part (e) – (10 points):

The **Work-Energy Relation** relationship applies well here. We can define Position D as the initial of the motion and the Position F the final position. If we call $\vec{F}_B$ the unknown (and variable) braking force, then this is the only force that does work between D and F:

$$\mathcal{W}_{tot} = \Delta K$$
$$\mathcal{W}_{\vec{F_B}} = K_f - K_i$$

Now we know the final speed, that's given as V_F. What we need yet is V_D the speed at Position D. However, this is quite easily calculated using the same method as for Part (c) only now the $y' = 0$ instead of the height of the second hill. Applying **Conservation of Mechanical Energy:** as in Part (c):

$$\text{“BEFORE”} = \text{“AFTER”}$$
$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$
$$mgH + \frac{1}{2}mV_B^2 = 0 + \frac{1}{2}mV_D^2$$

Solving for V_D

$$\frac{1}{2}mV_D^2 = \frac{1}{2}mV_B^2 + mgH$$
$$V_D^2 = V_B^2 + 2gH$$

Plugging this in to our expression for Work:

$$\mathcal{W}_{\vec{F_B}} = K_F - K_D$$
$$\mathcal{W}_{\vec{F_B}} = \frac{1}{2}mV_F^2 - \frac{1}{2}mV_D^2$$
$$\mathcal{W}_{\vec{F_B}} = \frac{1}{2}m(V_F^2 - V_D^2)$$
$$\boxed{\mathcal{W}_{\vec{F_B}} = \frac{1}{2}m[V_F^2 - (V_B^2 + 2gH)]}$$

Grader guidelines on next page...

Guidelines for graders:

- Perfect score with explanation including Work-Energy Relation = 10 points.
- Incorrectly attempting to apply Definition of Work "Force times distance" = at most 4 points.
- Attempt to apply Newton's Second Law instead of Work-Energy or Conservation of Energy = 2 points.
- Sign errors or algebra mistakes = 1 or 2 points off for each mistake.

Solution to Problem Z03:

Part (a) – (5 points):

- We are **told** that the turntable spins with a constant angular speed. We are *told* that Block B sits on the turntable without slipping or sliding. So Block B is moving with **Uniform Circular Motion** and the speed and radius for Block B are given. So we know $a_B = a_c = \frac{V^2}{r}$.
- We also know **Newton's Second Law** which tells us that the net force on Block B is given by $F_{Bx} = m_B a_{Bx}$
- Therefore:

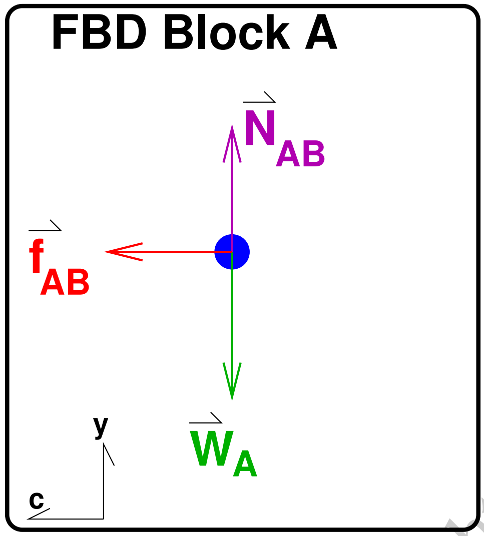
$$\boxed{F_{net} = F_{Bc} = \frac{m_B V^2}{r}}$$

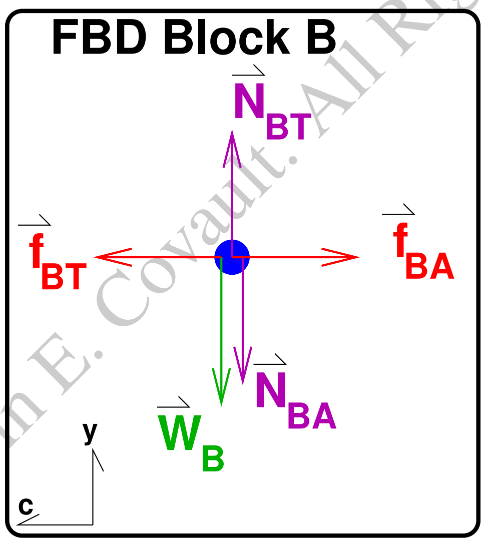
and the net force points in the $\boxed{\text{inward (lefttward) direction}}$.

Guidelines for graders:

- Correct answer with explanation that the acceleration of the Block B is known/given as centripetal due to Uniform Circular Motion *and* the invocation of Newton's Second Law = 5 points.
- Correct answer but no explanation as to why $a_c = \frac{V^2}{r}$ = at most 3 points.
- Correct answer with $F = ma$ shown but Missing Explanation: E.g.; Missing words such as Newton's Second Law (or "NSL" or "N2L") = at most 4 points.
- Net force expressed and/or calculated as a sum of individual forces (as opposed to applying Newton's Second Law) = at most 2 points.
- Some other wrong physics concept = at most 1 point.

Part (b) – (5 points):





To get full credit, all eight forces need to be listed with appropriate subscripts, and the coordinate systems have to be included. Note that there are three Normal forces and three Friction forces, so *each* of these must be unambiguously labeled.

Guidelines for graders:

- Two perfect FBDs = 5 points.
- All of the forces present and labeled but improper or ambiguous subscripts (for example, two different "friction" forces with the same labels) or missing coordinate system = 3 or 4 points depending on impact.
- Missing and/or incorrectly directed force(s): at most 3 points.
- Missing coordinate system on FBDs but otherwise perfect = 4 points.

Part (c) – (5 points):

Even though we want the force on Block B, we start with applying **Newton's Second Law**, to Block A. We know that the acceleration of Block A is the same as Block B: $a_c = \frac{V^2}{r}$. Since there is no sliding friction, we need consider only the horizontal (centripetal) forces:

$$F_{Ac} = m_A a_{Ac}$$
$$f_{AB} = m_A \left(\frac{V^2}{r}\right)$$

Now we simply apply **Newton's Third Law** to assert that the force on Block B f_{AB} has the same magnitude:

$$f_{BA} = f_{AB}$$
$$\boxed{f_{BA} = \frac{m_A V^2}{r}}$$

and from our FBD we see that:

$$\boxed{\vec{f}_{BA} \text{ points in the anti-centripetal (rightward) direction.}}$$

Guidelines for graders for Part (c)....

Guidelines for graders:

- Parts for credit totaling 5 points as following:
  - 2 pts = Writing down Newton's Second Law in the correct form for the vertical forces and solving the Normal force
  - 1 pts = Writing down Newton's Second Law in the correct form for the horizontal forces:
  - 2 pts = Invoking Newton's Third Law correctly.
- Correctly asserting $f_{BA} = \frac{m_A V^2}{r}$ but with no explanation and/or invocation of Newton's Laws to justify this: at most 2 points point.
- Correct work shown mathematically but no explanation or invocation of Newton's Laws, no words written: at most 3 points.
- Incorrectly asserting that the magnitude of the force is negative (as a result of introducing a minus sign for Newton's Third Law:) at most 4 points.
- Sign errors or other algebra mistakes: 1 point per mistake.

Part (d) – (10 points):

To find the force on Block B due to the turntable we carry on with Newton's Second Law:

$$F_{Bc} = m_B a_{Bc}$$
$$f_{BT} - f_{BT} = m_B \left(\frac{V^2}{r}\right)$$
$$f_{BT} - \frac{m_A V^2}{r} = \frac{m_B V^2}{r}$$
$$f_{BT} = \frac{m_A V^2}{r} + \frac{m_B V^2}{r}$$
$$\boxed{f_{BT} = \frac{(m_A + m_B)V^2}{r}}$$

and from our FBD we see that:

$$\boxed{\vec{f}_{BA} \text{ points in the centripetal (leftward) direction.}}$$

Alternate Completely Acceptable Solution: Consider Blocks A and B as a ***system*** and then apply the steps shown in Part (c) to the system.

Guidelines for graders for Part (d)....

Guidelines for graders:

- Parts for credit totaling 10 points as following:
  - 3 pts = Writing down Newton's Second Law in the correct form for the vertical forces and solving the Normal force
  - 3 pts = Writing down Newton's Second Law in the correct form for the horizontal forces:
  - 2 pts = Properly substituting in our result from Part (c).
  - 2 pts = Properly solving algebra in terms of given parameters for acceleration.
- Correctly asserting $f_{BT} = \frac{(m_A+M_B)V^2}{r}$ but with no explanation and/or invocation of Newton's Laws to justify this: at most 5 points point.
- Correct work shown mathematically but no explanation or invocation of Newton's Laws, no words written: at most 7 points.
- Sign errors or other algebra mistakes: 1 or 2 points per mistake.