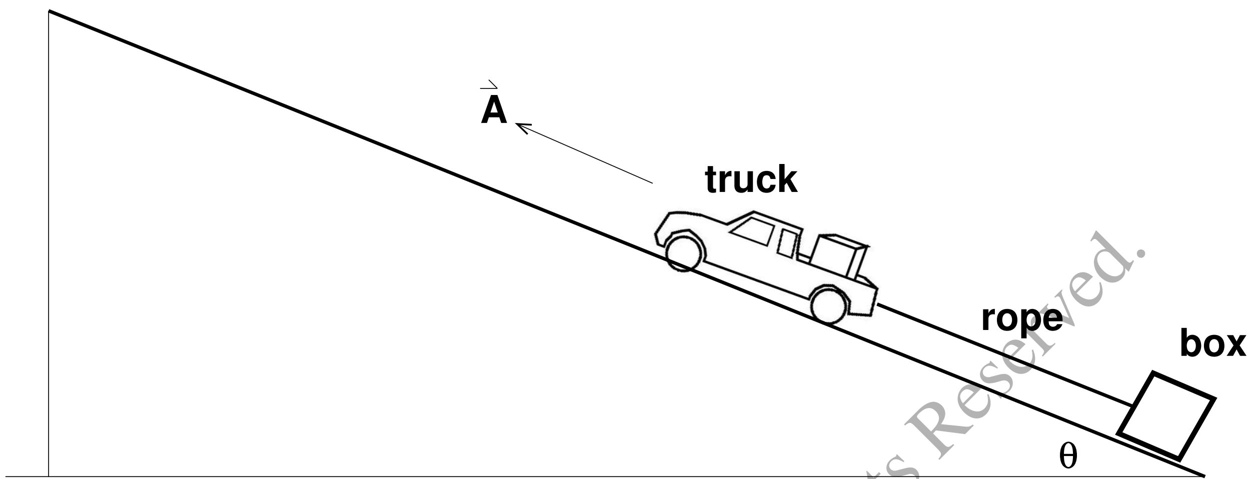
PHYS 121: Solutions to Written Homework #05
March 15, 2024

Note: this homework is worth **15 points** as follows:

___	Problem 1	0 points	Do not grade
___	Problem 2	5 points	5 pts total
___	Problem 3	5 Points	5 pts total
___	Problem 4	0 points	Do not grade
___	Problem 5	5 points	5 pts total

Problem 1:



A truck pulls a box up a ramp at a constant given constant acceleration A as shown above with an ideal rope. The angle of the ramp is given as θ. The wheels of the truck pull the truck up with no slipping and no sliding. The box is greased on the bottom surface so that there is a given coefficient of sliding friction between the box and the ramp of μ. The mass of the truck is given as m_t and the mass of the box is given as m_b.

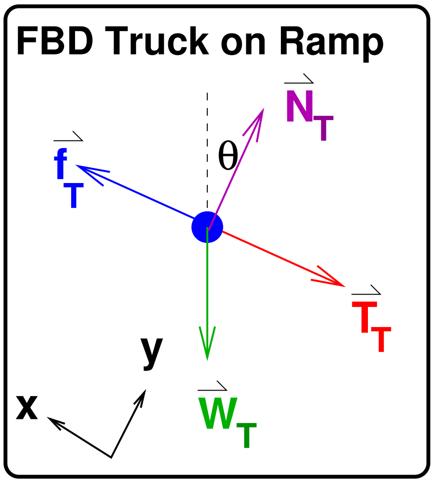
a) Draw a careful Free Body Diagram for both the truck and the box. Include a proper coordinate system and be sure to indicate every force on each body. Make very sure you have each force clearly and properly labeled. At this time we expect all students use the format and notation given in this class for every FBD.

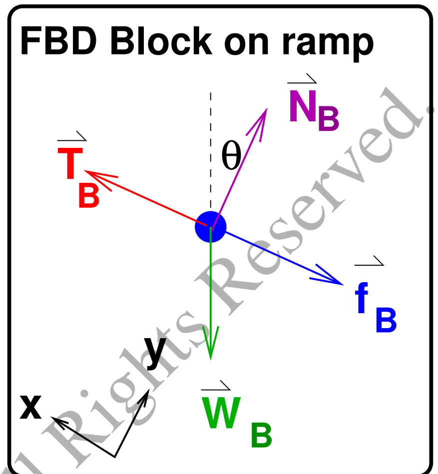
b) In terms of given parameters A, m_t, m_b, μ and θ, determine the magnitude of each and every force on both the truck and the box. Note: thee are four forces on each. Be sure you find all eight forces and express your answers in terms of the given parameters.

SOLUTION TO PROBLEM 1:

We are asked to draw Free-Body-Diagrams and calculate all of the forces on each body, so we obviously want to apply **Newton's Laws** here.

The **Free Body Diagrams** for the truck and the box look like this:





Note in particular that with the exception of the Tension force, there are no interacting forces applied between the truck and the box. We are *told* that **the acceleration is constant** and given as A pointing up the ramp for both the block and the truck. For this reason, we chose a ***tipped coordinate system*** that is aligned with the acceleration of the two bodies.

First we apply Newton's Second Law to the Block in the "y" direction:

$$F_{By} = m_B a$$

The acceleration is zero:

$$F_{By} = 0$$

Solution continues next page...

Next, we use the FBD to unpack the net force, noting that the "y" component of Weight correspond to a "negative cosine":

$$N_B - W_B \cos\theta = 0$$
$$N_B = W_B \cos\theta$$

We know that $\boxed{W_B = m_B g}$ and so:

$$\boxed{N_B = m_B g \cos\theta}$$

Next we apply Newton's Second Law to the Block in the "x" direction:

$$F_{Bx} = m_B a$$

The acceleration in the x-direction is given as A:

$$F_{Bx} = m_B A$$

We use the FBD to get the forces, noting that the "x" component of Weight correspond to a "negative sine":

$$T - f_B - W_B \sin\theta = m_B A$$

We use the rule for **sliding friction** that says $f = \mu N$ so in this case $f_B = \mu N_B$ where we use our result above:

$$T - \mu m_B g \cos\theta - m_B g \sin\theta = m_B A$$
$$\boxed{T = \mu m_B g \cos\theta + m_B g \sin\theta + m_B A}$$

or

$$\boxed{T = m_B \left(\mu g \cos\theta + g \sin\theta + A\right)}$$

Next we apply Newton's Second Law to the Truck in the "y" direction:

$$F_{Ty} = m_T a$$

The acceleration is zero:

$$F_{Ty} = 0$$

We use the FBD to get the forces, as above:

$$N_T - W_T \cos\theta = 0$$
$$N_T = W_T \cos\theta$$

We know that $\boxed{W_T = m_T g}$ and so:

$$\boxed{N_T = m_T g \cos\theta}$$

Finally we apply Newton's Second Law to the Truck in the "y" direction:

$$F_{Tx} = m_T a$$

The acceleration is zero:

$$F_{Tx} = m_T A$$

We use the FBD to get the forces. We note that **the friction on the truck is not sliding, and therefore is unknown.**

$$f_T - T - W_T \sin\theta = m_T A$$
$$f_T = T + W_T \sin\theta + m_T A$$
$$f_T = T + m_T g \sin\theta + m_T A$$

Finally we use our results from the Block and (implicitly) **Newton's Third Law** which tells us that the two tensions T are the same on each end of the rope:

$$f_T = (\mu m_B g \cos\theta + m_B g \sin\theta + m_B A) + m_T g \sin\theta + m_T A$$

$$\boxed{f_T = \mu m_B g \cos\theta + m_B g \sin\theta + m_B A + m_T g \sin\theta + m_T A}$$

There are lots of ways to combine like-terms. For example, you can gather terms by mass:

$$\boxed{f_T = m_B (\mu g \cos\theta + g \sin\theta + A) + m_T (g \sin\theta + A)}$$

Or you can combine terms by $theta$-dependence:

$$\boxed{f_T = \mu m_B g \cos\theta + (m_B g + m_T g) \sin\theta + (m_B + m_T) A}$$

All of these are algebraically equivalent.

Problem 2. The Simple Pendulum represents a rich system for checking our understanding of forces, work, and energy. This from a previous final exam:

A pendulum is made from a bob of mass m and an ideal string of length ℓ. Suppose the bob is pulled back and released from rest at a position with the string at an angle of θ_0 relative to the vertical where θ_0 has some particular but arbitrary value less than 90 degrees. For all of these questions give your answer in terms of the given parameters m, ℓ and θ_0.

(part a) Draw a careful sketch of what is going on here.

(part b) What is the magnitude of the acceleration of the bob immediately after the bob is released? Hint: no it is not zero. Another hint. If you choose the right coordinate system this problem is relatively simple.

(part c) What is the tension on the string immediately after the bob is released? No, it is not zero. No, it is not mg. Hint: What kind of acceleration applies in the direction of the string?

(part d) What is the Work done by the force of Tension as the bob falls from its initial position to the position corresponding to the bottom of the arc?

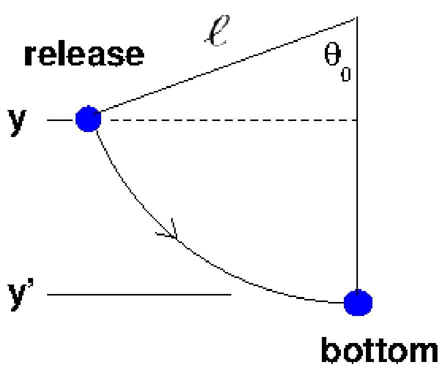
(part e) What is the Work done by the Weight force as the bob falls from its initial position to the position corresponding to the bottom of the arc?

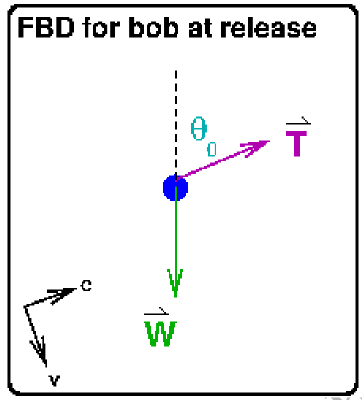
(part f) What is the maximum speed V of the bob during the swing of the pendulum?

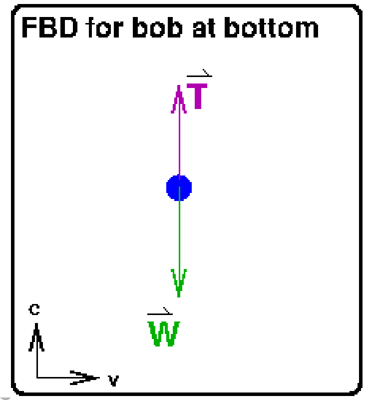
(part g) What is the magnitude of the force of Tension on the string when the bob is at the position corresponding to the bottom of the arc? Hint: No, $T = mg$ is not the correct answer.

SOLUTION TO PROBLEM 2:

Part (a) The "given parameters" in this problem are m, ℓ and θ_0 corresponding to the release angle of the pendulum.







This is a problem where it is important to consider the best possible coordinate system. Given the circular path of the bob, it is natural to consider a coordinate system that is aligned with both the *centripetal* and the *tangential* components of the acceleration:

$$\vec{a}_{tot} = \vec{a}_c + \vec{a}_v$$

Here "c" is the coordinate aligned with the centripetal direction and "v" is the coordinate aligned with the tangential direction, also called the "velocity" direction.[1]

As we can see on the figure above, there are two interesting points in the motion: First there is the *release* of the bob at some angle θ_0. Second there is the point when the bob reaches the *bottom* of the swing.

At the *release* point, the speed is zero (released "at rest"). The bob moves on a circular path. But since the speed is zero, the centripetal acceleration is zero at this point. However, the tangential acceleration is not zero.

At the *bottom* point the speed is at the *maximum*. The bob definitely experiences a non-zero centripetal acceleration. However, at the point the bob is neither speeding up nor slowing down, so the tangential acceleration is zero.

With this we consider the parts one-by-one:

Part (b) – We use the FBD for the "release" point. We apply **Newton's Second Law** in in the tangential (velocity) coordinate:

$$F_v = ma_v$$

[1] Students are free to use completely different names for these coordinates. For example, you can call them x and y if you want, just so long as the tipped orientation of the coordinate system is clearly indicated in the Free Body Diagram.

$$W \sin\theta_0 = ma_v$$

$$mg \sin\theta_0 = ma_v$$

$$g \sin\theta_0 = a_v$$

$$\boxed{a_v = g \sin\theta_0}$$

Note that this is very similar to the result for a box on a frictionless inclined plane.

Part (c) – We use the FBD for the "release" point. We apply **Newton's Second Law** in in the centripetal coordinate: We note that since the speed at release is zero, the centripetal acceleration must also be zero, so the components of force in the "c direction" must contribute zero net force in this direction:

$$F_c = ma_c$$

$$T - W \cos\theta_0 = ma_c$$

$$T - W \cos\theta_0 = 0$$

$$T = W \cos\theta_0$$

$$\boxed{T = mg \cos\theta_0}$$

Note that this makes sense. If we released the bob at an angle of zero degrees, the tension and the weight would be equal.

Part (d) We use the **Definition of Work** which tells us that work is a path integral:

$$\mathcal{W}_{\vec{F}} \equiv \int_{path} \vec{F} \cdot d\vec{r}$$

Here, the tension force do is **always** applied perpendicular to the circular path of the bob. Therefore

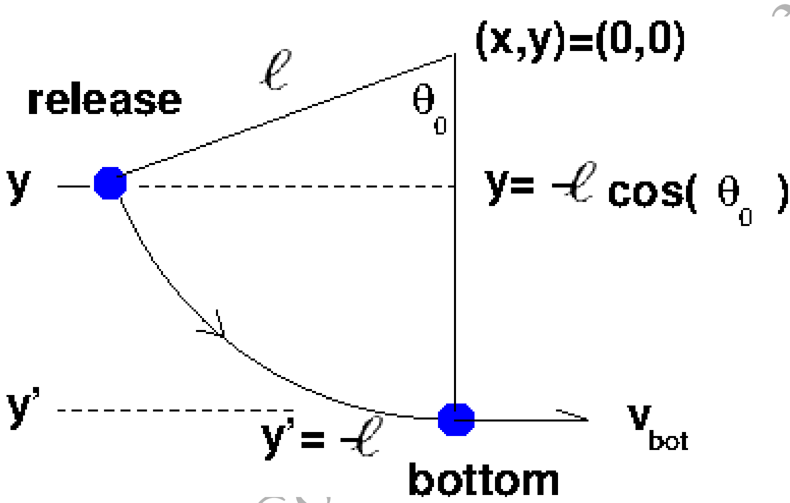
$$\boxed{\textbf{The Normal force does zero work.}}$$

Solution continues next page....

Part (e) We note that the work done by Weight is easily calculated in the one dimension (y-coordinate) and since Weight is conservative this is the same as the negative of the potential energy:

$$\mathcal{W}_{\vec{W}} = \int_y^{y'} mg\,dy = -\Delta U_{\vec{W}}$$
$$\mathcal{W}_{\vec{W}} = -(mgy' - mgy)$$
$$\mathcal{W}_{\vec{W}} = -mg(y' - y)$$
$$\mathcal{W}_{\vec{W}} = mg(y - y')$$

At this point we need to sort out the value of y, and y' We consider the geometry of the situation here:



For this part we are constrained to choose a coordinate system that is lined up with y corresponding to the vertical direction. But we are free to choose any location corresponding to $y = 0$. Whenever we have a situation where there is motion in a circular path about a central point, it is often helpful to choose that central point as the point of origin. In other words, we choose the pivot point where the string connect to the ceiling as the point of origin corresponding to $(x, y) = (0, 0)$. We can now determine the coordinates of the bob using standard trigonometry. We can see from above that the release point corresponds to a y-position that is the cosine of our angle times the length of the string in the negative direction. Likewise the bottom point corresponds to a y-position that is the negative of the radius (length of the string). In other words:

$$y = -\ell \cos(\theta_0)$$

and

$$y' = -\ell$$

Plugging these in:

$$\mathcal{W}_{\vec{W}} = mg[-\ell\cos(\theta_0) - (-\ell)]$$
$$\mathcal{W}_{\vec{W}} = mg[\ell - \cos(\theta_0)]$$
$$\boxed{\mathcal{W}_{\vec{W}} = mg\ell\,(1 - \cos\theta_0)}$$

Part (f) We note from the FBD that the only forces on the bob are *Weight* which is **a conservative force**, and *Tension* which, while not conservative, **does zero work** (because it applies perpendicular to the motion always). Therefore we have **Met the Conditions** and we can use **Conservation of Mechanical Energy.**

$$\text{“BEFORE”} = \text{“AFTER”}$$
$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

We know the value of the speed at the point of release $v = 0$ and we also have the values of y and y' from Part (e).[2] Plugging these values into our equation:

$$mg[-\ell\cos(\theta_0)] + 0 = mg(-\ell) + \frac{1}{2}mv'^2$$

Solving for v':

$$v'^2 = 2g\ell - 2g\ell\cos(\theta_0)$$
$$v'^2 = 2g\ell\,(1 - \cos\theta_0)$$
$$\boxed{V = v_{bot} = v' = \sqrt{2g\ell\,(1 - \cos\theta_0)}}$$

Part (g) At the bottom, the acceleration is *purely centripetal* with the bob neither speeding up nor slowing down. This corresponds to the maximum speed of the bob. We use **Newton's Second Law:**

[2] A completely acceptable alternate approach is to set $y' = 0$ (the lowest part of the arc) and then we can define the "height" as the initial position $y = h$. In this case the algebra is quite similar:

$$mgh + 0 = 0 + \frac{1}{2}mv'^2$$

Solving for v':

$$\frac{1}{2}mv'^2 = mgh$$
$$v'^2 = 2gh$$
$$v' = \sqrt{2gh}$$

All that remains is determining h in terms of *given parameters*. We see from the diagram above that we can infer this from trigonometry. The length of the string is ℓ and we can see that the height h corresponds to the difference between the length of the string and the cosine of the angle times this length:

$$h = \ell - \ell\cos\theta_0$$
$$h = \ell\,(1 - \cos\theta_0)$$

Therefore:

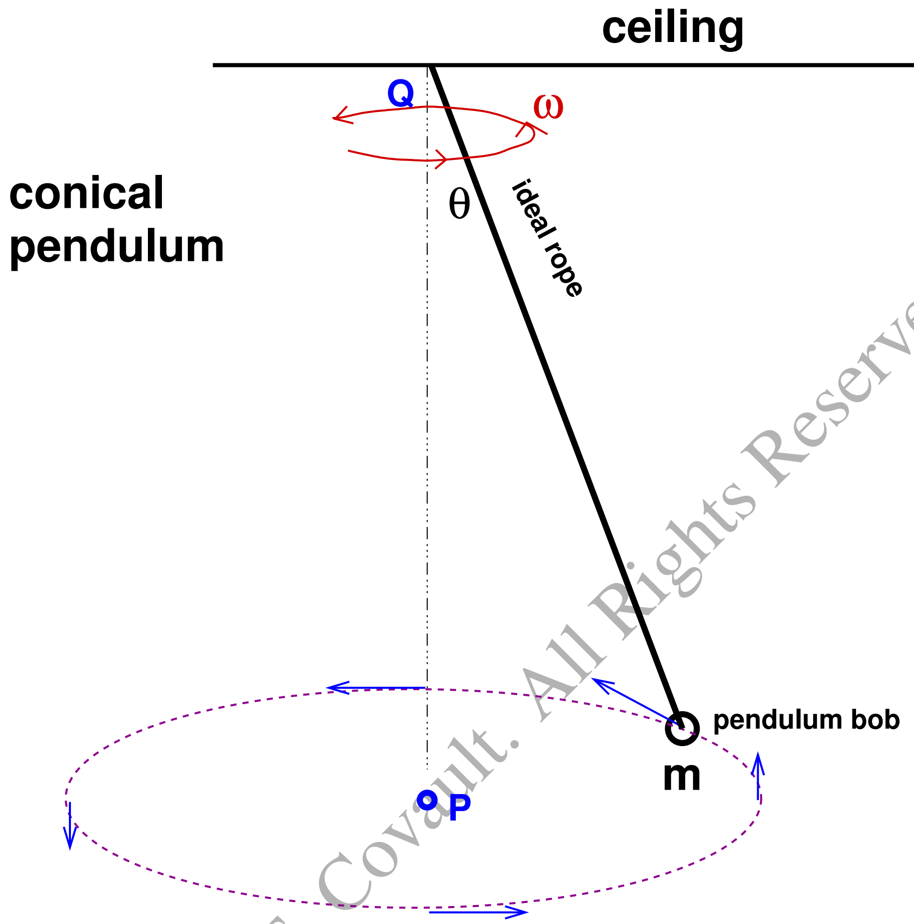
$$\boxed{v' = \sqrt{2g\ell\,(1 - \cos\theta_0)}}$$

which is the same answer we determined above.

$$F_c = ma_c$$
$$T - W = ma_c$$
$$T = W + m\frac{v'^2}{\ell}$$
$$T = mg + m\frac{v'^2}{\ell}$$
$$T = m\left(g + \frac{v'^2}{\ell}\right)$$
$$T = m\left[g + \frac{2g\ell\left(1 - \cos\theta_0\right)}{\ell}\right]$$
$$T = mg\left[1 + 2\left(1 - \cos\theta_0\right)\right]$$
$$\boxed{T = mg\left(3 - 2\cos\theta_0\right)}$$

Note that the tension does not depend on the length of the string.

Problem 3: A Conical Pendulum



A "Conical Pendulum" is shown in the figure above. The bob of given mass m travels in a *horizontal circular path*. The point P indicate the point at the center of the circular path of the bob as shown. The angle of the ideal string with respect to the vertical is **constant** and given as θ. The length of the string ℓ is also given.

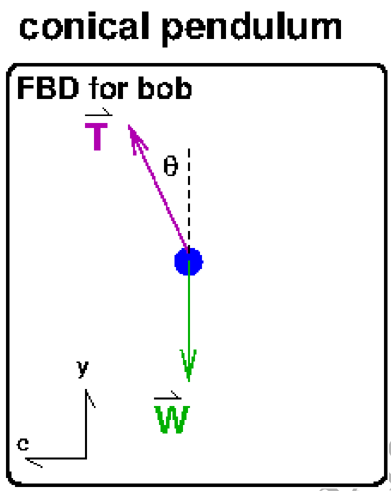
Part (a) – : Draw a careful and complete **Free-Body-Diagram** showing all of the forces on the bob. Be sure to include an appropriately labeled coordinate system.

Part (b) – : Calculate the magnitude of the force of Tension on the string. Explain your work. Give your answer in terms of the given parameters. Hint: Apply Newton's Second Law in the vertical Component.

Part (c) – : Calculate the **angular speed** ω of the conical pendulum. Explain your work. Give your answer in terms of the given parameters. Hint: Apply Newton's Second Law in the centripetal component. Use the Rolling Constraint: $v = \omega r$ that relates the (linear) speed of an object with Uniform Circular Motion to the angular speed of rotation about the axis.

Solution to Problem 3:

Part (a)– :



There are only two forces on the bob: **Weight** downward and **Tension** which is applied at an angle. The coordinate system is important: the horizontal coordinate is *centripetal*. We do ***not*** tip the coordinate system in this problem because the direction of acceleration of the bob is horizontal (centripetal).

Part (b)– :

We apply **Newton's Second Law** in the vertical direction:

$$F_y = ma_y$$
$$T\cos\theta - W = 0$$
$$T\cos\theta - mg = 0$$
$$T\cos\theta = mg$$
$$\boxed{T = \frac{mg}{\cos\theta}}$$

Solution continues next page....

Part (c)– :

Since the system is moving with **Uniform Circular Motion** we know the acceleration in the centripetal direction:

$$a_c = \omega^2 R$$

where R is the radius of the circle. Now we just apply **Newton's Second Law:**

$$F_{net} = ma_c$$
$$T \sin\theta = m\omega^2 R$$

Now we plug in our result from Part (a):

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = m\omega^2 R$$

We also use simple trig to note that $R = \ell \sin\theta$:

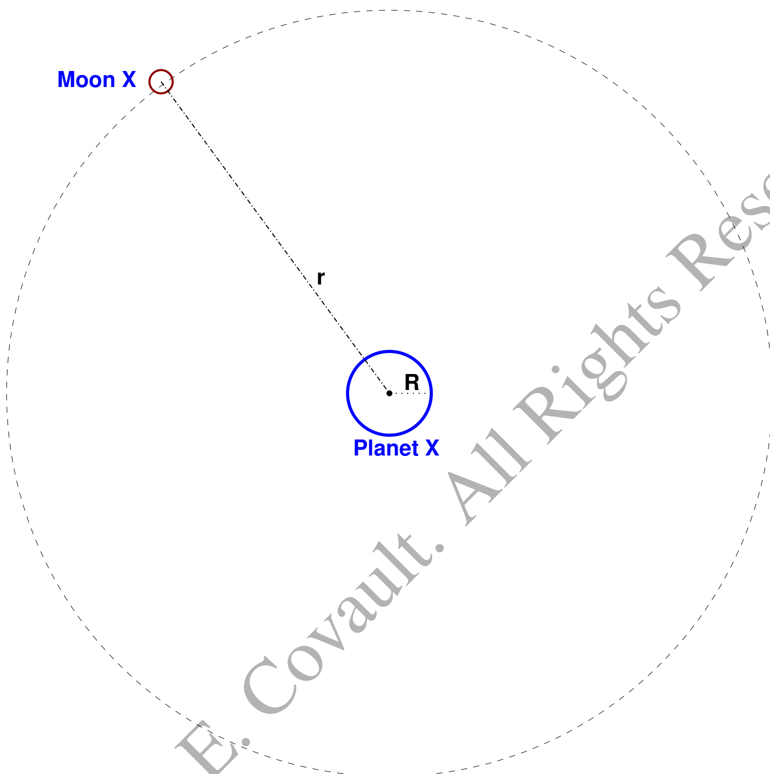
$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = m\omega^2 \ell \sin\theta$$
$$\left(\frac{mg}{\cos\theta}\right) = m\omega^2 \ell$$
$$\left(\frac{g}{\cos\theta}\right) = \omega^2 \ell$$

Solving for the angular speed:

$$\frac{g}{\cos\theta} = \omega^2 \ell$$
$$\omega^2 = \frac{g}{\ell\cos\theta}$$
$$\boxed{\omega = \sqrt{\frac{g}{\ell\cos\theta}}}$$

Problem 4: Planet X and it's Moon

Suppose a new planet is discovered at a great distance from the sun in deep space. The new planet is called *Planet X*. Suppose further that *Planet X* has *moon*, called *Moon X*, that orbits the planet in a circular orbit. Assume that the mass of the planet is quite large compared to the mass of the moon.



Suppose astronomers measure the radius of *Planet X* to be $R = 5.20 \times 10^5$ meters, and the orbital radius of *Moon X* to be $r = 4.61 \times 10^6$ meters, as shown. Suppose also that the orbital period of *Moon X* P is measured to be 14.32 hours. Assume that the mass of *Planet X* is known to be much larger than the mass of *Moon X*. In other words $M >> m$.

a) On the figure, draw a vector to indicate the *direction* of the gravitational force on *Moon X*. Explain how you know this.

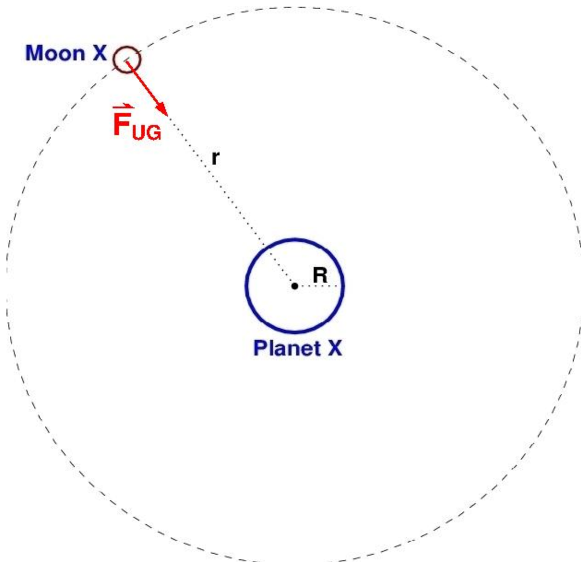
b) In terms of the given parameters, can you calculate the mass M of *Planet X*? Here, start from First Principles assuming Uniform Circular Motion and Newton's Law of Universal Gravity. Give your answers both in terms of symbolic constants and numerically. Explain how you calculated this. Show your work.

c) Suppose you are standing on *Planet X* and you drop your mobile phone. What is the observed acceleration due to the gravity of *Planet X* on that phone? Explain your work.

SOLUTION TO PROBLEM 4:

Part (a):

We know that the force of **Universal Gravity** is *attractive*. That is to say that there is a force between any two bodies, and the force always acts on each body in the direction of the other body. In this case, the force on the Moon due to the Planet acts in the direction from the Moon to the Planet:



Part (b):

Yes, it is possible to calculate the mass of Planet X by applying **Newton's Second Law** to the circular motion of the Moon:

$$F_c = ma_c$$

$$F_{UG} = m\frac{v^2}{r}$$

Here, since we have Uniform Circular Motion, we can write the velocity down in terms of the *orbital period*:

$$v = \frac{2\pi r}{P}$$

therefore:

$$F_{UG} = m\frac{\left(\frac{2\pi r}{P}\right)^2}{r}$$

$$F_{UG} = m\frac{4\pi^2 r}{P^2}$$

We now plug in the expression for *Universal Gravity*:

$$\frac{GMm}{r^2} = m\frac{4\pi^2 r}{P^2}$$

Solving for M:

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{P^2}$$

$$\boxed{M = \left(\frac{4\pi^2}{G}\right)\left(\frac{r^3}{P^2}\right)}$$

By the way, this expression has a name, it's called **Kepler's Third Law** which says that the ratio of the radius cubed over the period squared is a constant for any planet in the solar system. Newton *expanded* Kepler's Third Law so that the constant can be calculated for any system dominated by a central mass.

Plugging in numbers we get:

$$M = \left(\frac{4\pi^2}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}}\right)\left[\frac{(4.61 \times 10^6\ \mathrm{m})^3}{(14.32\ \mathrm{hours} \cdot 3600\ \mathrm{s/hour})^2}\right]$$

$$\boxed{M = 2.18 \times 10^{22}\ \mathrm{kg}}$$

Does this number make sense? It's about 1/200th of the mass of the Earth, so pretty darn small for a "planet" but maybe not outrageously so, at least for a small, hard-to-find planet.

Part (c):

Here we make use the underlying fact that Weight and Universal Gravity are the same force, just written down different ways. We write down **Newton's Second Law** with Universal Gravity at the surface of Planet X: To match our expression for Weight, we will call the "y-direction" up (radially away from the center of the Planet). In this case, gravity points in the *negative*-y direction:

$$F_y = ma_y$$

$$-\frac{GMm}{R^2} = ma_y$$

$$\boxed{a_y = -\frac{GM}{R^2}}$$

or in terms of given parameters:

$$a_y = -\frac{G\left[\left(\frac{4\pi^2}{G}\right)\left(\frac{r^3}{P^2}\right)\right]}{R^2}$$

$$\boxed{a_y = -\frac{4\pi^2 r^3}{P^2 R^2}}$$

Again plugging in numbers and using our result from Part (a):

$$a_y = -\frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(2.18 \times 10^{22}\ \text{kg})}{(5.20 \times 10^5\ \text{m})^2}$$

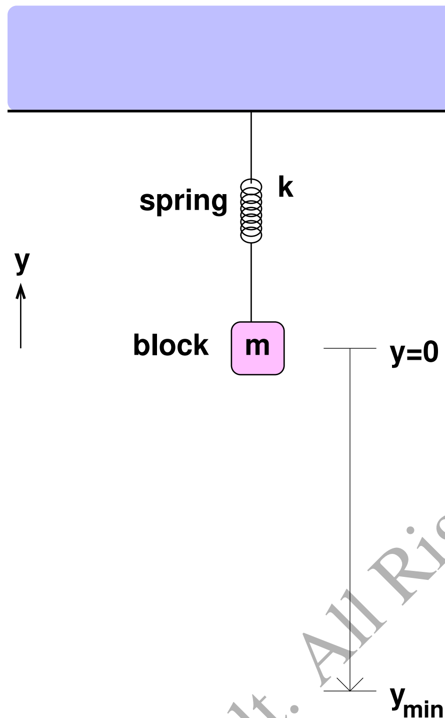
or alternately:

$$a_y = -\frac{(4\pi)^2(6.00 \times 10^6\ \text{m})^3}{(12.0\ \text{hours} \cdot 3600\ \text{s/hour})^2(5.2 \times 10^5\ \text{m})^2}$$

which is:

$$\boxed{a_y = -5.37\ \text{m/s}^2}$$

Problem 5: 2013 Second Hour Exam: Problem 2: Spring Block Drop



A block of given mass m is released at rest at a position $y = 0$ where it is connected to an ideal spring with a given spring constant k. At the release, the spring is at its equilibrium position (neither compressed nor extended). Once released, the block falls vertically and the spring stretches.

Part (a) – What is y_{min} corresponding to the minimum *position* of the block after release? Express your answer in terms of the given parameters. Explain your work. Hint: Note that y_{min} is negative.

Part (b) – What is the magnitude and direction of the acceleration of the block when the block is at the position y_{min} as calculated in Part (a)? Express your answer in terms of the given parameters. Explain your work. Hint: no, the acceleration here is not zero.

Part (c) – What is y_{max}, corresponding to the position of the block when it has maximum *speed*? Express your answer in terms of the given parameters. Explain your work. Hint: You do not need to find the actual value of the maximum speed. Also Conservation of Energy is not what you want. Instead, think about the net force on the block as a function of position.

Solution to Problem 5:

Part (a):

For this part you want to use **Conservation of Energy**. You can do this because there only two forces on the bock: Weight and Spring Force, both of which are *conservative* so you have ***met the condition for the application of Conservation of Energy***. The "before" corresponds to release at rest where by definition $y = 0$ and $v == 0$. The "after" corresponds to the minimum position where $y' = y_{min}$ and again $v' = 0$: We have potential energy due to both the spring and the weight forces. $U_{\vec{W}} = mgy$ and $U_{\vec{F}_{sp}} = \frac{1}{2}kx^2$ where in this case $x = -y$ corresponds to the displacement in the downward direction of the spring.

$$E_{tot} = E'_{tot}$$

$$U_{tot} + K = U'_{tot} + K'$$

$$U_{\vec{W}} + U_{\vec{F}_{sp}} + K = U'_{tot} + U'_{\vec{F}_{sp}} + K'$$

$$mgy + \frac{1}{2}k{-}y^2 + 0 = mgy' + \frac{1}{2}k(-y')^2 + 0$$

$$0 + 0 + 0 = mgy_{min} + \frac{1}{2}ky_{min}^2 + 0$$

$$\frac{1}{2}ky_{min}^2 = -mgy_{min}$$

$$\frac{1}{2}ky_{min} = -mg$$

$$\boxed{y_{min} = -\frac{2mg}{k}}$$

Part (b):

To get the acceleration at y_{min} we just consider the application of **Newton's Second Law** In this case there are only two forces on the block:

- Weight downward: $W = mg$
- Spring force upward in accordance with Hooke's Law: $F_{sp} = k|x|$

In this case $|x| = -y_{min}$. So we write out Newton's Law:

$$F_y = ma_y$$

$$k(-y_{min}) - mg = ma_y$$

$$ma_y = k\left[-\left(-\frac{2mg}{k}\right)\right] - mg$$

$$ma_y = 2mg - mg$$

$$ma_y = mg$$

$$\boxed{a_y = g \quad \text{(upward)}}$$

Part (c):

For Part (c) we want to determine the position when the block has maximal velocity. There are two acceptable methods to determine this:

Method 1: Use Newton's Laws

We note that we have this expression for the net force as a function of position from the start of Part (b):

$$F_y = k(-y) - mg$$

We note that when $-y$ is small that the weight term is less than the spring force term. So the block has a net force is negative (pointing downward) and so the block is *speeding up*. Likewise, when the $-y$ becomes larger than the weight, the net force is positive (pointing upward) and therefore the block is *slowing down*. The transition between speeding up and slowing down corresponds to the maximum speed: this where the force due to the spring is equal to the opposing force of weight:

$$F_y = k(-y_{max}) - mg = 0$$

(Note: you get this same answer if you say that there must me a maximum of the velocity at the position where the acceleration (slope of velocity) is equal to zero.)

We solve this equation to find y_{max}

$$k(-y_{max}) - mg = 0$$

$$\boxed{y_{max} = -\frac{mg}{k}}$$

Method 2: Invoke Simple Harmonic Motion and Symmetry

We note that this system corresponds to a **Simple Harmonic Oscillator**. The solution to the equation of motion must be a position as a function of time in the form of some sinusoidal function. We know that the position oscillates between $y = 0$ and $y = y_{min}$ Sinusoidal functions are *symmetric*, oscillating about some mean value, half-way between the extremes. This half-way point is also where we expect maximum velocity (maximum value of the slope on the sinusoidal function):

So **by symmetry of harmonic oscillators:**

$$\boxed{y_{max} = \frac{1}{2}y_{min} = -\frac{mg}{k}}$$