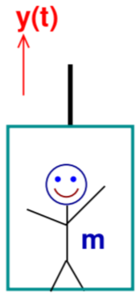
Practice Set

Problem 1)



Given

- Mass m of person
- $y(t) = A + Bt - Ct^2 + Dt^3$ for position
- Constants of $y(t)$ given
- $0 < t < T$ the interval where T is given

a) $v = \frac{d}{dt}(y(t))$ definition of velocity

$v(t) = \frac{d}{dt}(A + Bt - Ct^2 + Dt^3) = B - 2Ct + 3Dt^2$

$v(t=0) = B - 0 + 0 \quad \boxed{v(0) = 0}$

b) $a = \frac{d}{dt}(v(t))$ definition of acceleration

$a(t) = \frac{d}{dt}(B - 2Ct + 3Dt^2) = -2C + 6Dt$

$v(t=T) = \boxed{-2C + 6DT}$

c) N2L $\quad F_{Net} = ma$

Net force $= 0$, so $a = 0$

$0 = ma$

$a = 0$

$-2C + 6Dt = 0$

$6Dt = 2C$

$t = \frac{2C}{6D} \rightarrow \boxed{t = \frac{C}{3D}}$

d) Effective weight

$\vec{W}_{eff} = \vec{W}_{real} + \vec{W}_{fict}$

$= mg_{real} + mg_{fict}$

$g_{fict} = -A = 2C - 6Dt$

$\vec{W}_{eff} = -mg\hat{j} + m(2C - 6Dt)$

$= m(-g + 2C - 6Dt)$

$\boxed{\vec{W}_{eff} = -m(g - 2C + 6Dt)}$

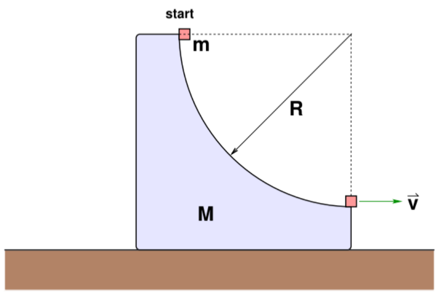
At $t=0$: $W_{eff} = -m(g - 2C) < -mg$

At $t=t$: $W_{eff} = -m(g - 2C + 6Dt) \downarrow$ increase

At $t=T$: $W_{eff} = -m(g - 2C + 6DT) \downarrow$

At $t=0$, you feel lighter than normal, but $0 < t < T$ will cause you to feel slightly heavier, and past time T it follows that you will become increasingly heavy.

Problem 2)



Given

- Radius R of cube cut
- frictionless path and surface
- Released at rest
- m of small block, M of cube

Let v_A = small block, v_B = big block

CofLM conditions: big and small block isolated, no friction

Before = After

$$P_{Tot} = P'_{Tot}$$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

Looking for v_A'

@ rest $$0 + 0 = m_A v_A' + m_B v_B'$$

$$v_B' = -\frac{m_A}{m_B} v_A'$$ stop!

CofME Conditions: frictionless, weight is conservative, normal does zero work ⊥ to motion.

Before = After

$$U_{Tot} + K_{Tot} = U'_{Tot} + K'_{Tot}$$

Not large block does not have potential energy here

$$m_A g y + \cancel{\tfrac{1}{2} m_A v_A^2}^{\,0} + \cancel{\tfrac{1}{2} m_B v_B^2}^{\,0} = m_A g y' + \tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} m_B v_B'^2$$

$y = R$, $y' = 0$, $v_A' = ?$, $v_B' = -\frac{m_A}{m_B} v_A'$

$$m_A g R = \tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} m_B \left(-\frac{m_A}{m_B} v_A'\right)^2$$

$$= \tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} \cancel{m_B} \left(\frac{m_A^2}{m_B^{\cancel{2}}} v_A'^2\right)$$

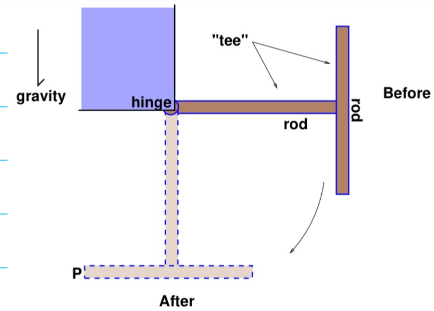
$$m_A g R = \tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} \frac{m_A}{m_B} \left(m_A v_A'^2\right)$$

$$\cancel{m_A} g R = \tfrac{1}{2} \cancel{m_A} v_A'^2 \left(1 + \frac{m_A}{m_B}\right)$$

$$g R = \tfrac{1}{2} \left(1 + \frac{m_A}{m_B}\right) v_A'^2$$

$$\boxed{v_A' = \sqrt{\frac{2gR}{1 + \frac{m_A}{m_B}}}}$$

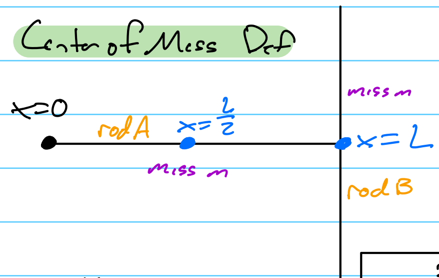
Problem 3)



Given

- Rigid Body
- Each rod is length L mass m

a) Center of Mass Def



$$\boxed{x_{cm} = \frac{3L}{4}}$$

$$x_{cm} = \frac{m_A x_A + m_B x_B}{m_A + m_B}$$

$$= \frac{m\left(\frac{L}{2}\right) + mL}{2m} = \frac{\frac{3}{2}\cancel{m}L}{2\cancel{m}} = \frac{3L}{4}$$

b) Total Inertia: $I_T = I_A + I_B$

I_A: Rod about end

$$I_A = \frac{1}{3} mL^2$$

I_B: Rod about CoM, Parallel Axis Thm

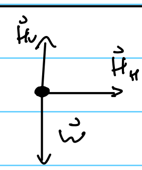
$$I_B = I_{cm} + mD^2$$

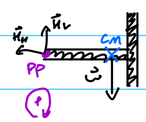
$$= \frac{1}{12} mL^2 + mL^2$$

$$I_T = \frac{1}{3} mL^2 + \frac{13}{12} mL^2 \quad \therefore \quad \boxed{I_T = \frac{17}{12} mL^2}$$

$$= \frac{4}{12} mL^2 + \frac{13}{12} mL^2$$

c) FBD 'tee'   x FBD





Torque: $\tau = rF\sin\phi$

$\rightarrow \tau_H = 0$

$\rightarrow \tau_w = \frac{3}{4} L(2mg)$

N2L Rot $\quad \tau_{Net} = I\alpha$

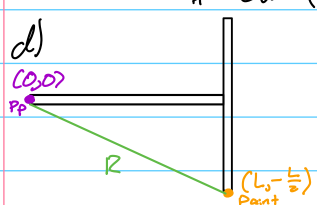
$$\tau_H + \tau_w = \left(\frac{17}{12} mL^2\right)\alpha$$

$$\frac{3}{2}\cancel{m}g\cancel{L} = \frac{17}{12}\cancel{m}L^{\cancel{2}}\alpha$$

$$\frac{3g}{2L} = \frac{17}{12}\alpha$$

$$\alpha = \frac{12}{17} \cdot \frac{3}{2} \cdot \frac{g}{L} \qquad \boxed{\alpha = \frac{18\,g}{17L}}$$

d)



Treat Two boards and point as a system with mass $2m$.

CofME:

Conditions: System is obviously isolated, forces conservative

Before = After

$$E_{Tot} = E'_{Tot}$$

$$\cancel{mgy} + \cancel{\tfrac{1}{2} I_T \omega_T^2} = mgy' + \tfrac{1}{2} I_T \omega_T'^2$$

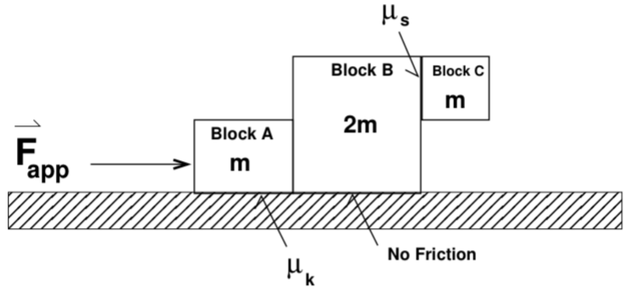
$$0 = 2\cancel{m}g\left(-\frac{3L}{4}\right) + \frac{1}{2}\left(\frac{17}{12}\cancel{m}L^2\right)\omega_T^2$$

$$\frac{3gL}{2} = \frac{17}{24} L^2 \omega_T^2$$

$$\frac{36gL}{17} = L^2 \omega_T^2$$

$$\omega_T^2 = \frac{36\,g\cancel{L}}{17\,L^{\cancel{2}}} \qquad \boxed{\omega_T = \sqrt{\frac{36g}{17L}}}$$

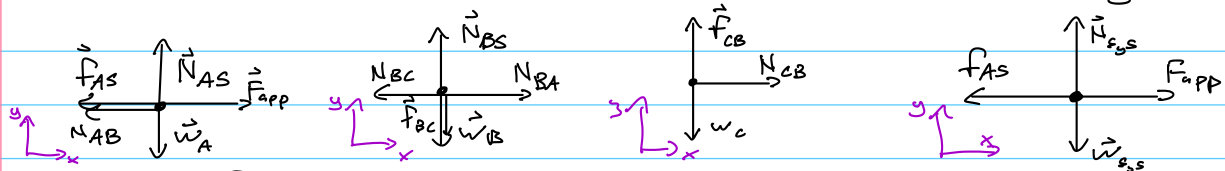
Problem 4)



Given

- F_{app} to block A
- A mass = m, B mass = 2m, C mass = m
- C held in place by static friction
- B and C static friction μ_s
- A and surface μ_k, B frictionless

a) FBD A FBD B FBD C FBD System



b) Kinetic Friction

$f_{AS} = \mu_k |N_{AS}|$

N2L block A y-dir

$F_{Net} = m\cancel{a_y}^{\,0}$ No vertical!

$N_{AS} - W_A = 0 \quad N_{AS} = m_A g, \; m_A = m$

$\boxed{f_{AS} = \mu_k m g}$ friction is leftward!

c) System, $a_A = a_B = a_C = a_x$

N2L System x-dir

$F_{Net} = m_{sys} a_{sys}$

$m_{sys} = m + 2m + m = 4m$

$a_{sys} = a_x = ?$

Kinematic Constraint

$F_{app} - f_{AS} = 4m a_x$

$\frac{F_{app} - \mu_k m g}{4m} = a_x$

$$\boxed{a_x = \frac{F_{app} - \mu_k m g}{4m}}$$

d) N2L A x-dir

$F_{Net} = m a_x$

$F_{app} - f_{AS} - N_{AB} = \cancel{m}\left(\frac{F_{app} - \mu_k m g}{4\cancel{m}}\right)$

$N_{AB} = F_{app} - \mu_k m g - \frac{1}{4}(F_{app} - \mu_k m g)$

$N_{AB} = \frac{3}{4}(F_{app} - \mu_k m g)$

N3L $N_{AB} = N_{BA}$

$\boxed{N_{BA} = \frac{3}{4}(F_{app} - \mu_k m g)}$ rightward!

e) N2L B x-dir

$F_{Net} = m_B a_x$

$N_{BA} - N_{BC} = 2m a_x$

$\frac{3}{4}(F_{app} - \mu_k m g) - N_{BC} = \cancel{2m}\left(\frac{F_{app} - \mu_k m g}{\cancel{2}4\cancel{m}}\right)$

$\frac{3}{4}(F_{app} - \mu_k m g) - N_{BC} = \frac{1}{2}(F_{app} - \mu_k m g)$

$\boxed{N_{BC} = \frac{1}{4}(F_{app} - \mu_k m g)}$ leftward!

f) N2L C y-dir

$F_{Net} = m\cancel{a_y}^{\,0}$ no vertical movement!

$f_{CB} - W_C = 0$

$\boxed{f_{CB} = mg}$ upward

g) Static Friction Constraint

$f_s \le \mu_s N$ N3L $N_{CB} = N_{BC} = \frac{1}{4}(F_{app} - \mu_k m g)$

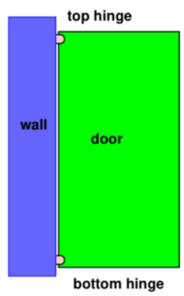
$f_{CB} \le \mu_s N_{CB}$

$mg \le \mu_s \cdot \frac{1}{4}(F_{app} - \mu_k m g)$

$\frac{4mg}{\mu_s} \le F_{app} - \mu_k m g$

$$\boxed{F_{app} \ge \frac{4mg}{\mu_s} + \mu_k m g}$$

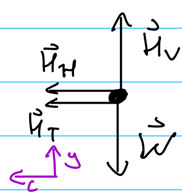
Problem 5)



Given

- mass m, height h, width w of door
- Bottom hinge applies not-given $\vec{H}_H$ and $\vec{H}_v$
- Top hinge " " $\vec{H}_t$, zero vertical force
- Door swings about hinge @ Ω rot speed
- Door inertia about hinge: $I = \frac{1}{3} m w^2$

FBD Door



N2L Door y-dir

$H_v - W = m a_y$ ($a_y = 0$, only horizontal)

$H_v = W$

$$\boxed{H_v = mg}$$

N2L c-dir

$H_H + H_t = m a_c \qquad a_c = \frac{v^2}{r}$

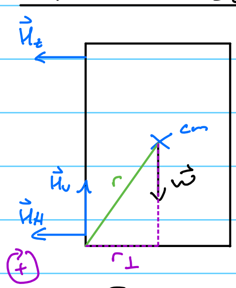
$H_H + H_t = m \frac{\Omega^2 r^2}{r} \qquad$ R.C $v = \Omega r$

$H_H + H_t = m \Omega^2 r \qquad r = \frac{w}{2}$ from left

$H_H + H_t = m \Omega^2 \left(\frac{w}{2}\right)$ Stop!

Door moving in path of $\frac{w}{2}$ circularly

xFBD Door



$\tau_F = r F \sin\varphi = r_\perp F$

N2L Rot

$\tau_{Net} = I \alpha$ ($\alpha = 0$, const speed, 0 accel)

$\cancel{\tau_{H_H}} + \cancel{\tau_{H_v}} + \tau_{H_t} + \tau_{H_w} = 0$

$h \cdot H_t + r_\perp W = 0$

$h \cdot H_t + \frac{w}{2} mg = 0$

$$\boxed{H_t = \frac{wmg}{2h}}$$

$\frac{wmg}{2h} + H_H = m\Omega^2 \left(\frac{w}{2}\right)$

$H_H = m\Omega^2\left(\frac{w}{2}\right) - \frac{wmg}{2h}$

$H_H = \frac{1}{2} m w \left(\Omega^2 - \frac{g}{h}\right)$

$$\boxed{H_H = \frac{1}{2} m w \left(\Omega^2 - \frac{g}{h}\right)}$$

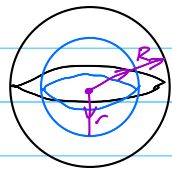
Problem 6)

Given

- Neglect air resistance
- Gravitational force on small body in large sphere is 0
- M and R are mass and radius of earth
- Person falling has mass m (assume earth uniformly dense)

a) $F_{UG}(r) = \frac{GMm}{r^2}$ M is earth fraction

m is your mass



Volume of inner sphere:
$\frac{4}{3}\pi r^3$

Volume of outer sphere:
$\frac{4}{3}\pi R^3$

Volume Fraction of Inner

$$V = \frac{V_i}{V_o} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{r^3}{R^3}$$

Mass of Inner Sphere

$$M = \frac{r^3}{R^3} \cdot M$$

$$\therefore F_{UG}(r) = \frac{G(\frac{r^3}{R^3}M)m}{r^2} = \frac{GrM}{R^3}m$$

$$\boxed{F_{UG}(r) = \frac{GMm}{R^3} \cdot r}$$

b) N2L $F_r = ma_r$

attractive force $-F_{UG} = m\frac{d^2r}{dt^2}$

$$\frac{d^2r}{dt^2}m + F_{UG} = 0$$

$$\frac{d^2r}{dt^2}\cancel{m} + \frac{GM\cancel{m}}{R^3}r = 0$$

$$\boxed{\frac{d^2r}{dt^2} + \frac{GM}{R^3}r = 0}$$

c) 2nd order DE

Characteristic polynomial! $s^2 + \frac{GMm}{R^3} = 0$

$$s = \pm i\sqrt{\frac{GM}{R^3}}$$

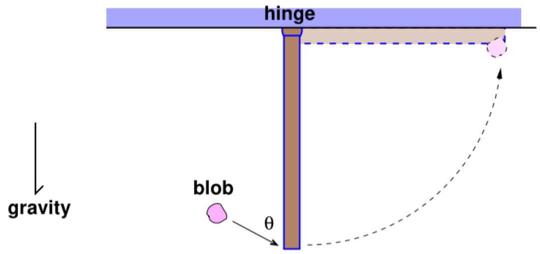
Allow $\begin{cases} r(0) = R \\ r'(0) = v(0) = 0 \end{cases}$

$$\begin{cases} \vec{r}(t) = K_1\cos\left(\sqrt{\frac{GM}{R^3}}t\right) + K_2\sin\left(\sqrt{\frac{GM}{R^3}}t\right) \\ \vec{r}'(t) = -\sqrt{\frac{GM}{R^3}}K_1\sin\left(\sqrt{\frac{GM}{R^3}}t\right) + \sqrt{\frac{GM}{R^3}}K_2\cos\left(\sqrt{\frac{GM}{R^3}}t\right) \end{cases}$$

$$\begin{cases} R = K_1 + 0 \\ 0 = 0 + \sqrt{\frac{GM}{R^3}}K_2 \end{cases} \quad \begin{matrix} K_1 = R \\ K_2 = 0 \end{matrix} \quad \Big\rangle \quad \boxed{\vec{r}(t) = R\cos\left(\sqrt{\frac{GM}{R^3}}t\right)}$$

d) According to Einstein's principle of equivalence you cannot perform any experiment to determine your location relative to the earth. Your reference frame accelerates at the same rate as gravity pulls you, so effective gravity is 0, which leads to a weightless feeling

Problem 7)



Given

- Thin rod mass m, length L
- Ideal hinge
- Blob sticks to rod @ θ to vertical

C of ME

stick → swing

Conditions: Ideal Hinge does no work, [illegible] conserve.

$$\text{Before} = \text{After}$$

$$E_{Tot} = E'_{Tot}$$

ends @ rest and $y=0$

$$U_B + E_B + U_R + E_R = U_B' + E_B' + U_R' + E_R'$$

$$mgy_B + \tfrac{1}{2}mv_B^2 + mgy_R + \tfrac{1}{2}I\omega^2 = 0$$

$$-mgL + \tfrac{1}{2}mv_B^2 + mg\left(-\tfrac{L}{2}\right) + \tfrac{1}{2}\left(\tfrac{1}{3}mL^2\right)\omega^2 = 0$$

$$-2mgL + mv_B^2 - mgL + \tfrac{1}{3}mL^2\omega^2 = 0$$

R.C $\quad v = \omega r \quad v_B = \omega L$ 'sticks'

$$-3mgL + m(\omega L)^2 + \tfrac{1}{3}mL^2\omega^2 = 0$$

$$\omega^2\left(\cancel{m}L^{\cancel{2}} + \tfrac{1}{3}\cancel{m}L^{\cancel{2}}\right) = 3\cancel{m}g\cancel{L}$$

$$\omega^2\left(\tfrac{4}{3}L\right) = 3g$$

$$\omega^2 = \frac{9}{4}\frac{g}{L} \rightarrow \omega = \frac{3}{2}\sqrt{\frac{g}{L}}$$

C of AM

Conditions: blob-rod system isolated

$$\text{Before} = \text{After}$$

$$L_{Tot} = L'_{Tot}$$

$$L_B + L_R = L'_B + L'_R$$

R.C

$$mv_iL\sin\theta + 0 = mv_fL + I\omega \qquad v = \omega r$$

$$\cancel{m}v_i\cancel{L}\sin\theta = \cancel{m}\omega L^{\cancel{2}} + \left(\tfrac{1}{3}\cancel{m}L^{\cancel{2}}\right)\omega$$

$$v_i\sin\theta = \omega L + \tfrac{1}{3}L\omega$$

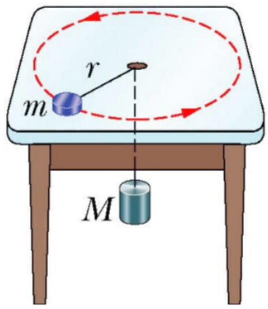
$$v_i\sin\theta = \tfrac{4}{3}L\omega$$

$$v_i = \frac{4L\omega}{3\sin\theta}$$

$$v_i = \frac{\overset{2}{\cancel{4}}L}{\cancel{3}\sin\theta}\cdot\frac{\cancel{3}}{\cancel{2}}\sqrt{\frac{g}{L}} = \frac{2L}{\sin\theta}\sqrt{\frac{g}{L}}$$

$$\boxed{v_i = \frac{2}{\sin\theta}\sqrt{gL}}$$

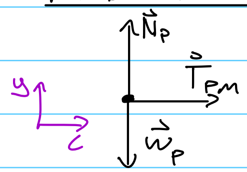
Problem 8)



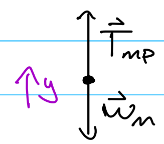
Given

- Small puck mass m on frictionless surface
- Massless string holding unknown mass M
- Puck moves in circle with constant radius r, speed v

a) FBD Puck



FBD Mass



N2L Puck c-dir

$$F_c = m a_c, \quad a_c = \frac{v^2}{r}$$

$$T_{pm} = m\frac{v^2}{r}$$

N2L Mass y-dir

$$F_y = m \cancel{a_y}^{\,0}$$

$$T_{mp} - W_m = 0 \rightarrow T_{mp} = Mg$$

N3L $T_{pm} = T_{mp}$

$$m\frac{v^2}{r} = Mg$$

$$\boxed{M = \frac{mv^2}{gr}}$$

b) CofAM conditions: isolated as surface is frictionless

$$\text{Before} = \text{After}$$

$$L_{Tot} = L'_{Tot}$$

Treat as point particle in UCM

$$L_p = L'_p$$

$$\cancel{m} v_i r = \cancel{m} v_f r' \qquad \boxed{V_f = \frac{v_i r}{r'}}$$

c) Work Energy theorem

$$W_{Tot} = \Delta K$$

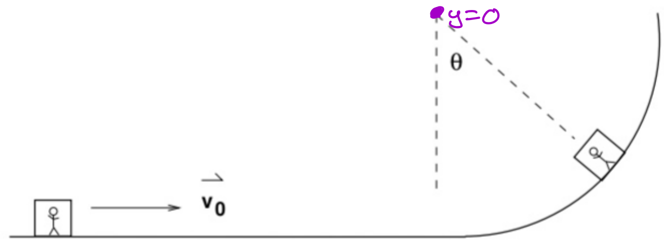
$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}m\left(\frac{vr}{r'}\right)^2 - \frac{1}{2}mv^2$$

$$= \frac{1}{2}mv^2\left(\frac{r^2}{r'^2} - 1\right)$$

$$\boxed{W_{Tot} = \frac{1}{2}mv^2\left(\left(\frac{r}{r'}\right)^2 - 1\right)}$$

Problem 9)



Given
- Constant speed v_0
- Bills blind
- Ramp radius of R
- frictionless surface everywhere

a) C of ME

Conditions: normal force no work b/c const constante, frictionless surface.

$$E_{Tot} = E'_{Tot}$$

$$mgy + \tfrac{1}{2}mv^2 = mgy' + \tfrac{1}{2}mv'^2$$

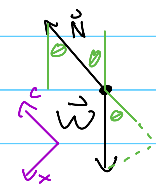
$$\cancel{m}g(-R) + \tfrac{1}{2}\cancel{m}v_0^2 = \cancel{m}g\,R\cos\theta + 0$$

$$\tfrac{1}{2}v_0^2 - gR = -gR\cos\theta$$

$$1 - \frac{v_0^2}{2gR} = \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(1 - \frac{v_0^2}{2gR}\right)}$$

b) FBD block



N2L x-dir

$$F_{Net} = m a_x$$

$$W\sin\theta = m a_x$$

$$a_x = g\sin\theta$$

$$\boxed{a_x = g\sin\left(\cos^{-1}\left(1 - \frac{v_0^2}{2gR}\right)\right)}$$

c) Total Acceleration

$$\vec{a}_{Tot} = \vec{a}_c + \vec{a}_x \qquad a_c = \frac{v^2}{R}$$

$$a_x = g\sin\theta\,\hat{\imath}$$

$$a_c = \frac{v_0^2 - gR(1-\cos\theta)}{R}\,\hat{c}$$

$$\boxed{\vec{a}_{Tot} = \frac{v_0^2 - gR(1-\cos\theta)}{R}\,\hat{c} + g\sin\theta\,\hat{u}}$$

C of ME

$$E_{Tot} = E'_{Tot}$$

$$mgy + \tfrac{1}{2}mv^2 = mgy' + \tfrac{1}{2}mv'^2$$

$$\cancel{m}g(-R) + \tfrac{1}{2}\cancel{m}v_0^2 = \cancel{m}g(-R\cos\theta) + \tfrac{1}{2}\cancel{m}v'^2$$

$$\tfrac{1}{2}v_0^2 - gR = -gR\cos\theta + \tfrac{1}{2}v'^2$$

$$\tfrac{1}{2}v_0^2 - gR + gR\cos\theta = \tfrac{1}{2}v'^2$$

$$v_0^2 - gR(1-\cos\theta) = v'^2$$

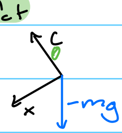
$$v'^2 = v_0^2 - gR(1-\cos\theta)$$

d) $W_{eff} = W_{real} + W_{fict}$

Look into solutions for concepts

$$W_{real} = -mg\,\hat{\jmath}$$

$$= -mg\cos\theta\,\hat{c} + mg\sin\theta\,\hat{\imath}$$

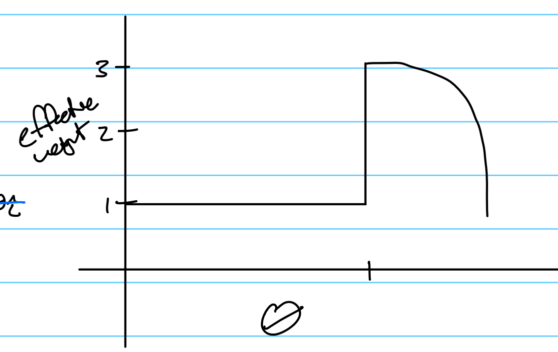


$$W_{fict} = -ma_{tot}$$

$$W_{eff} = -mg\cos\theta\,\hat{c} + \cancel{mg\sin\theta\,\hat{\imath}} - m\frac{v_0^2 - gR(1-\cos\theta)}{R}\,\hat{c} - \cancel{mg\sin\theta\,\hat{\imath}}$$

$$= -mg\cos\theta\,\hat{c} - \frac{m(v_0^2 - gR(1-\cos\theta))}{R}\,\hat{c}$$

$$\boxed{W_{eff} = m\left(-g\cos\theta - \frac{v_0^2 - gR(1-\cos\theta)}{R}\right)\hat{c}}$$

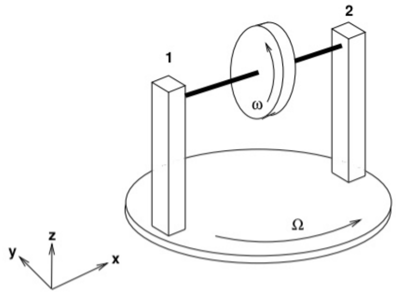


No $\hat{\imath}$ component so everything is centripetal.

Conceptually Challenging Problem Review!

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

Problem 10)



Given

- Given spin ω, mass M, radius R of uniform disk
- Length of axle is 2ℓ, disk exactly halfway
- Fixed on firm turntable @ Ω angular speed
- Ideal bearings and $\omega >> \Omega$
- Ω is not free precession frequency, it's a given rate

a) Curly RHR

$\vec{L}$ points negative x, $-\hat{i}$

$$L_{RB} = I\omega = \frac{1}{2}MR^2\omega$$

$$\boxed{L = -\frac{1}{2}MR^2\omega\hat{i}}$$

b) $\vec{\tau} = \frac{d\vec{L}}{dt}$

$$\tau = L\frac{d\theta}{dt} = L\Omega$$

$$\boxed{\tau = \frac{1}{2}MR^2\omega\Omega}$$

c) $\tau_{Tot} = \tau_1 + \tau_2$

Determine rotating Force!

$$= -F_1\ell + F_2\ell$$

$$= -\left(\frac{1}{2}mg - F\right)\ell + \left(\frac{1}{2}mg + F\right)\ell$$

$$= -\frac{1}{2}mg\ell + F\ell + \frac{1}{2}mg\ell + F\ell$$

$$\tau_{Tot} = 2F\ell \qquad F = \frac{\tau_{Tot}}{2\ell}$$

$$F = \frac{MR^2\omega\Omega}{4\ell}$$

$$\boxed{F_1 = \frac{1}{2}mg - \frac{MR^2\omega\Omega}{4\ell} \qquad F_2 = \frac{1}{2}mg + \frac{MR^2\omega\Omega}{4\ell}}$$

Pylon 2 net force clearly exceeds that of Pylon 1

d) $W_{Tot} = \Delta K$ and with velocities being held constant results in no work done by angle movement. Nothing moves vertically, so vertical forces does no work.

$\boxed{0 \text{ power}}$

$$\tau = \frac{dL}{dt}$$

$$\tau = I\alpha$$

$$L = I\omega \rightarrow I = \frac{L}{\omega}$$

$$\tau = \frac{L}{\omega}\alpha = \frac{L}{\frac{d\theta}{dt}} \cdot \frac{d^2\theta}{dt^2} = L\frac{d\theta}{dt}$$

$$\tau = L\frac{d\theta}{dt}$$