Chapter 14++

More Non-Zero Torque Situations

Revisit:	To-Do:
• Moment of Inertia formulas • Parallel axis theorem • Rotational second law	• Rolling, falling, and cello-playing yo-yos

Review Moments of Inertia

We revisit moments of inertia to remind ourselves what goes into the single-axis Newton rotational second law, $\tau = I\,\alpha$ as well as the rotational kinetic energy $\frac{1}{2}\, I\,\omega^2$ and the rotational angular momentum $I\,\omega$. Recall (in descending order of fraction coefficient) for total mass M and radii R

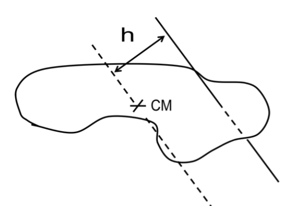
I (uniform **hollow cylinder** with axis along its symmetry axis) = $\mathbf{MR^2}$
I (uniform **hollow sphere** with axis through its CM) = $\mathbf{2/3\ MR^2}$
I (uniform solid cylinder with axis along its symmetry axis) = $\mathbf{1/2\ MR^2}$
I (uniform **solid sphere** with axis through its CM) = $\mathbf{2/5\ MR^2}$

and for sticks of length L (or thin doors on edge) – **Don't confuse L with R!!!**

I (uniform **rod** with axis perpendicular through its **end**) = $\mathbf{1/3\ ML^2}$
I (uniform **rod** with axis perpendicular through its **CM**) = $\mathbf{1/12\ ML^2}$

Recall the parallel-axis theorem that saves us much work:

$$I = I_{CM} + M h^2$$

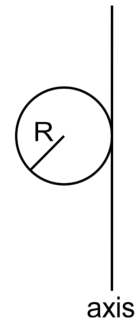


where the moment of inertia about any axis is equal to the moment of inertia about the axis through the CM which is parallel to the given axis plus the total mass M times h^2 with h the (shortest) distance between the two parallel axes.

More examples using the parallel-axis theorem on the next page.

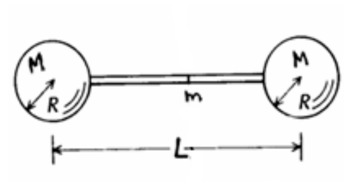
Parallel-axis theorem examples

a) Find the moment of inertia of a uniform sphere (radius R, mass M) about an axis tangent to the sphere's surface as shown.



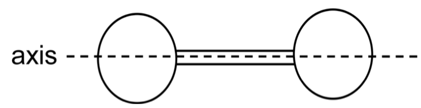
$$I = I_{CM} + M h^2 = \frac{2}{5} MR^2 + MR^2 = \frac{7}{5} MR^2$$

b) A dumbbell consists of two uniform spheres of mass M and radius R joined by a thin rod of mass m and length L (see figure). L is the distance between sphere centers.



What is the moment of inertia of the above device about an axis along the rod (this does not need the parallel-axis theorem but see the problem)?

Draw the axis (along the symmetry axis):



$$I_{axis} = 2 I_{ball}^{CM} + I_{rod}^{axis} = \frac{4}{5} MR^2 + 0$$

(two eyeballs, of course ☺)

where $I_{ball}^{CM} = \frac{2}{5} MR^2$ and, because it's so thin, $I_{rod}^{axis} \cong 0$ (the axis is parallel to the rod and along its center symmetry axis).

****

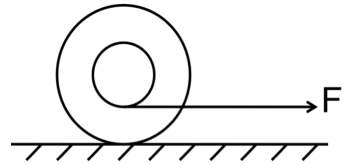
Problem 14-7

A quickie! What is the moment of inertia of the above dumbbell about an axis through the center of the rod perpendicular to the rod?

****

Rolling, Falling, and Cello-playing Yo-yos

• Example: A yo-yo on the table - a nonzero torque situation!



A yo-yo is initially at rest on a table as shown.

Let us discuss what happens when we pull on the yo yo string by way of a problem

****

Problem 14-8

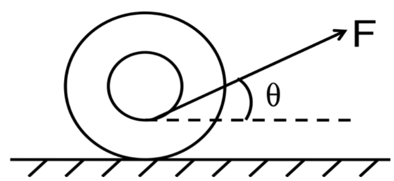
Answer all of the questions (a)-(d) found below. You will get help in your lecture demonstrations and discussions

a) **Suppose there is no friction** between the yo-yo and the table. If you pull the string horizontally to the right as shown, what happens and why?

b) **Suppose now there IS friction** between the yo-yo and the table. If you pull the string gently horizontally, which way will it move, and why?

c) **In the case that there is friction and you yank on** the string, what happens and why?

d) **Suppose now you pull gently at an angle** as shown on the right, what happens and why?



****

• Example: A falling yo-yo – another nonzero torque situation.

In terms of the angular momentum change, the rotational form of Newton's second law for a simple rotation about a fixed axis P is (where the vectors $\vec{\tau}$, $\vec{\alpha}$, $\vec{\omega}$, $\vec{L}$ have only one component): $\tau_P = I_P\alpha = \dfrac{dL_P}{dt}$ where $L_P = I_P\omega$

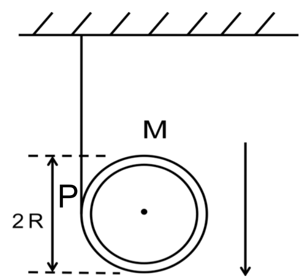
We have emphasized the subscript P to remind us that torque, moment of inertia, and angular momentum all have to be defined with respect to a point (axis) and, in fact, the same point in a given rotational second-law formula. By the way, the torque here is the net torque due to all the external forces.

(continued)

What if the object is moving as well as rotating? The rotational form above still holds if we consider the axis through the CM (and moving with it):

$$\tau_{CM} = I_{CM}\alpha = \frac{dL_{CM}}{dt} \text{ where } L_{CM} = I_{CM}\omega$$

The torque here is the net torque due to all the external forces but now around the CM.



Example: Like a spool of thread, consider a light rope wrapped around a hoop and the loose end tied to the ceiling. The hoop falls as shown and unwinds with no slipping. Newton's translational second law for the CM is easy to write down:

$$Mg - T = Ma_{CM}$$

Newton's rotational second law about CM:

$$\tau_{net,CM} = I_{CM}\alpha \Rightarrow TR = MR^2\alpha \Rightarrow T = MR\alpha = Ma_{CM}$$

where 1) we chose CW as +, 2) the torque due to T is just +TR, and the torque around the CM due to the weight is zero, 3) the moment of inertia is that of a hoop, 4) we have $a_{CM} = \alpha R$, the rolling constraint! (Ch. 12+).

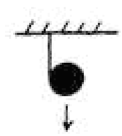
$$\text{Substitute } T = Ma_{CM} \text{ into } Mg\text{-}T\text{=}Ma_{CM} \Rightarrow \boxed{a_{CM} = \frac{1}{2}g \text{ and } T = \frac{1}{2}mg}$$

As an alternative derivation, we can consider the torque about rope point P, which is instantaneously at rest:

$$\tau_{net,\,P} = I_P\alpha = (I_{CM} + MR^2)\alpha \Rightarrow MgR = 2MR^2\alpha \Rightarrow \alpha = \frac{1}{2}\frac{g}{R}$$

where 1) CW is + again, 2) now the torque due to T is zero, and the torque due to the weight is +MgR, 3) the moment of inertia is calculated with the parallel axis theorem with h = R, 4) and, since $a_{CM} = \alpha R$, this agrees with the previous calculation.

****



Problem 14-9 Another quickie! What is the CM acceleration downward of an unwinding uniform solid cylinder? Use either torque approach (it would be great to verify that either the torque around the CM point or the point P, as discussed above, would agree with each other).

****

- **Example: Yo-Yo Ma – never mind!**