Physics 121 Exam #1

February 16, 2024

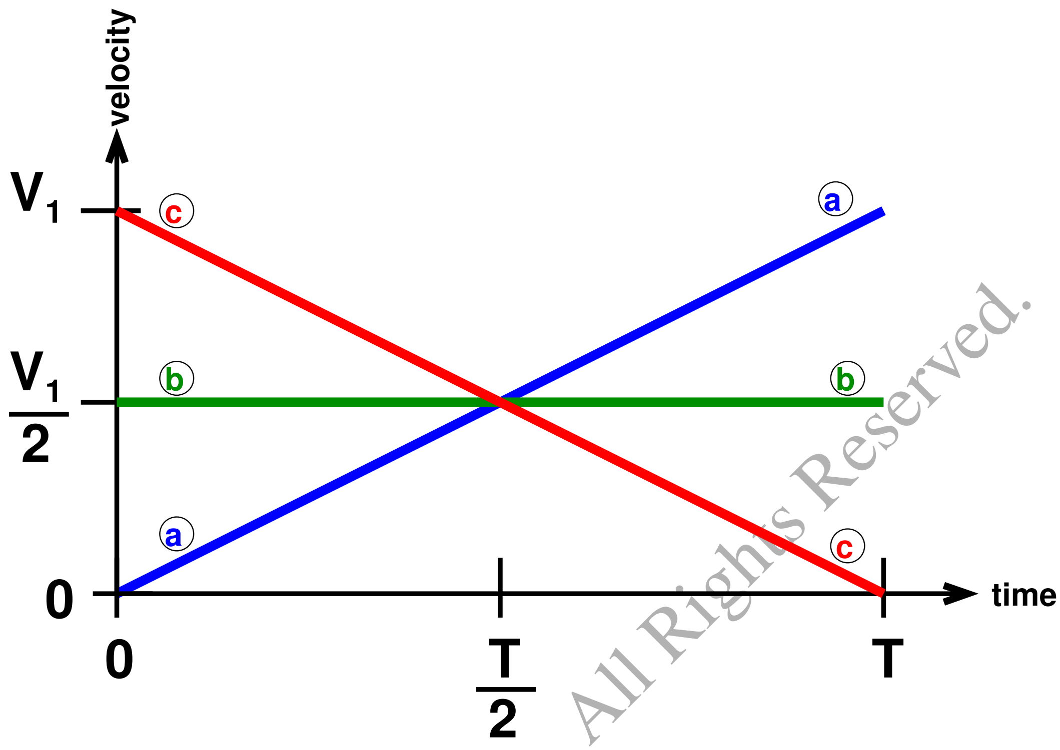
Important: Only these vector forces allowed on your Free-Body-Diagrams:

- $\vec{W}$ = Weight Force
- $\vec{N}$ = Normal Force
- $\vec{T}$ = Tension Force
- $\vec{f}$ = Friction Force
- $\vec{F}_{app}$ = Applied force

Example Notation for force subscripts:

- $\vec{T}$ = "The Tension Force."
- $\vec{W}_A$ = "The Weight Force *on* Body A."
- $\vec{N}_{BT}$ = "The Normal Force *on* Body B *due* to the Table."
- f_{AB} = "The magnitude of the Friction Force *on* Body A *due* to Body B."
- F_c = "The magnitude of the *net* force in the *centripetal* direction."
- F_{Ax} = "The magnitude of the *net* force on Body A in the x-direction."

Problem 1: Three particles moving along the x-axis (15 points)



Three particles are moving in 1-D. The plot above shows the **velocity** as a linear function of time for each of three particles labeled ⓐ, ⓑ, and ⓒ as shown. The time scale of the plot ranges from time $t = 0$ to time $t = T$ where T is a given parameter. The velocity scale ranges from velocity $v = 0$ to $v = V_1$ where V_1 is a given parameter. The masses of the three particles are specified as $m_a = 3M$, $m_b = M$, and $m_c = 2M$ where M is a given parameter.

Part (a) (5 points) – Suppose a_a, a_b and a_c correspond to the accelerations of each of the three particles. Determine the **accelerations $\mathbf{a_a}$, $\mathbf{a_b}$ and $\mathbf{a_c}$.** Express your answers in terms of the given parameters. Explain your work.

Part (b) (5 points) – Suppose each particle starts at the position $x = 0$ when $t = 0$. Suppose X_a, X_b and X_c correspond to the position of each particle at time $t = T$. Determine the **positions $\mathbf{X_a}$, $\mathbf{X_b}$ and $\mathbf{X_c}$.** Express your answers in terms of the given parameters. Explain your work.

Part (c) (5 points) – Determine the **net Force** applied to Particle ⓐ. ***Also***, use this result, together with your result from Part (b), to determine the **Work done by the net force** on Particle ⓐ. Express your answers in terms of the given parameters. Explain your work.

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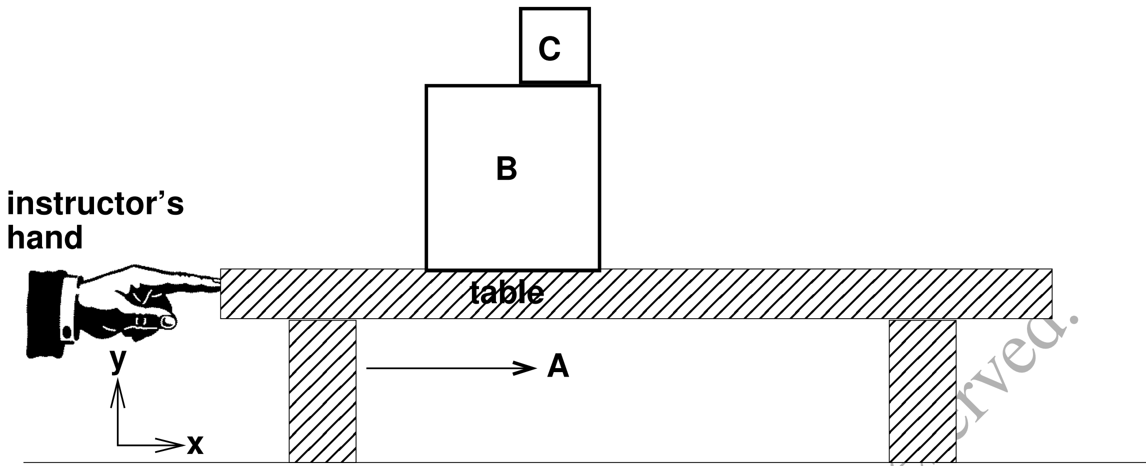
Problem 2: Stacked Blocks with Friction



Two blocks are placed together on a table as shown. Block C has a given mass m_C and Block B has a given mass m_B. There is *friction* between the two blocks. There is *friction* between the table and Block B.

An instructor applies an ***unknown horizontal force*** to the table which has an ***unknown mass*** with the result that the table accelerates with a given (known) constant horizontal acceleration A.

We observe these things as the table accelerates:

- Block B moves *with no slipping or sliding* relative to the table.
- Block C *slides* relative to Block B.

Assume that known values of both the coefficient of static friction μ_s between the table and Block B and the coefficient of kinetic friction μ_k between Block B and Block C are given.

Part (a) (5 points) – Draw two free body diagrams (FBDs), one for each block, showing all vertical and horizontal forces on each. Make sure that your diagrams clearly indicate the nature and direction of each force. Make sure that your forces are labeled with appropriate subscripts, as needed. Important: Do ***not*** draw an FBD for the table.

Part (b) (5 points) – Apply ***Newton's Laws:*** to calculate a_{Cx} which is the horizontal acceleration of Block C. Hint: $a_{Cx} \neq A$ here. Another hint: $a_{Cx} > 0$ in the given coordinate system.

Part (c) (10 points) – Apply ***Newton's Laws:*** to calculate the magnitude and direction of $\vec{f}_{BT}$ which is the force of *Friction* on Block B due to the Table.

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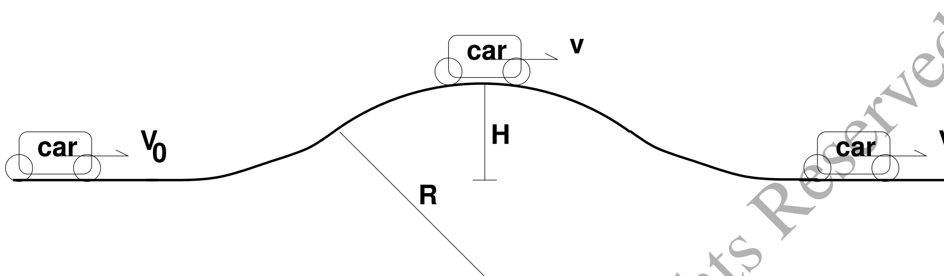
Problem 3: Driving over a Hill (15 points)

A driver is in a car with given mass m. The car is rolling on level ground with given initial speed V_0 and approaches a large rolling hill as shown. The hill causes the car to move on a circular path with given radius of curvature R. The hill rises given height H above the level ground as shown. Assume that the size of the car is quite small compared to both R and H.



As the car rolls up the hill, the driver is *coasting* so there are **negligible** forces of friction and air resistance on the car while it rolls up the hill. This remains true up to the point where the car is at the very top of the hill. However once the car starts to roll down the hill, the driver *then* applies the brakes to slow down the car. The car eventually comes to a final steady given speed of V_F on the level ground after the driver releases the brakes. Note that we *cannot* assume that the braking force (friction) itself is constant here. The breaking force is *not* constant and it is *not* given. There is no way to calculate the breaking force (friction) here.

Part (a) (5 points) – What is the **speed** of the car when it is right at the top of the hill as shown? Explain how you know this. Present your answer in terms of the given parameters.

Part (b) (5 points) – What is the magnitude and direction of the **Normal force** on the car when it is right at the top of the hill as shown? Explain how you know this. Present your answer in terms of the given parameters. Hint: The Normal force is not equal to mg here.

Part (c) (5 points) – What is the **Work done by the force of Friction** on the car as it rolls down the hill? Explain how you know this. Present your answer in terms of the given parameters.

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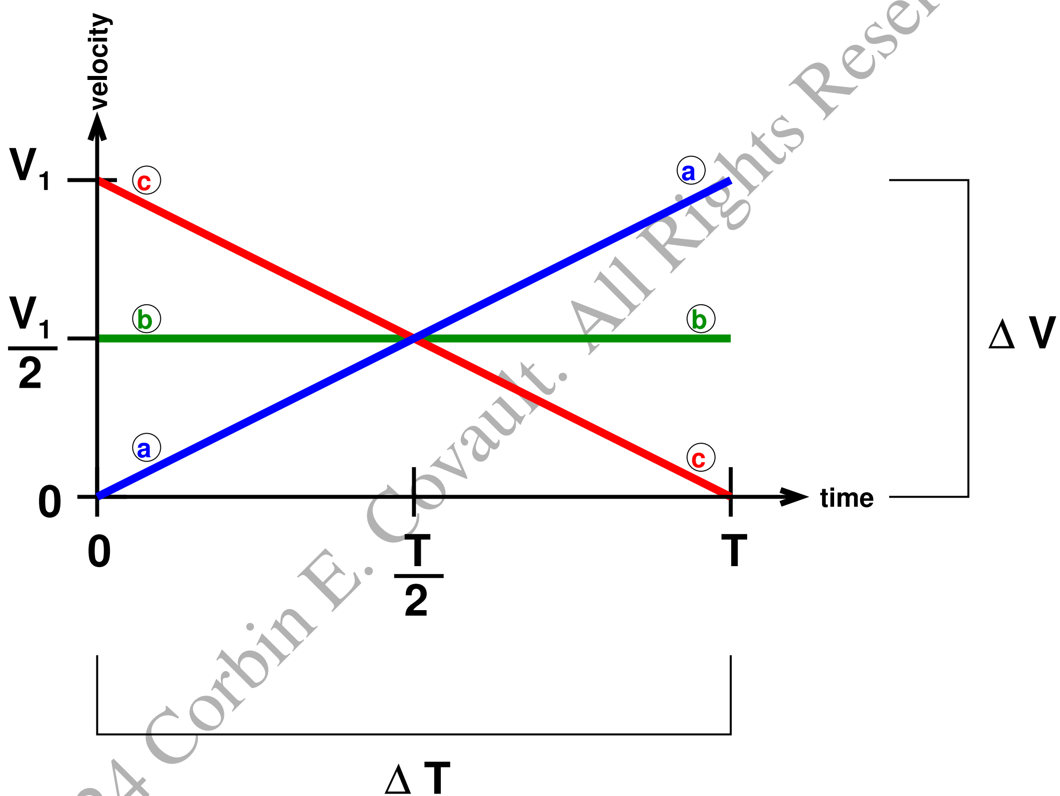
Solution to Problem 1 (15 points):

Part (a) – : (5 points)

We are asked to determine the **acceleration** of each of the three particles. Acceleration is **defined as the time-derivative of the velocity as a function of time:**

$$a \equiv \frac{dv}{dt}$$

For this problem we are given a graphical representation showing the velocity of each particle as a function of time. Graphically, the acceleration represents the ***slope of the tangent line.*** Since the velocity for these students is **linear as a function of time** we just need to look at the slopes for each particle:



Looking at the plot we see:

$$a_a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{V_1 - 0}{T - 0} = \frac{V_1}{T}$$

Likewise

$$a_b = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{V_1}{2} - \frac{V_1}{2} = 0$$

$$a_c = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{0 - V_1}{T - 0} = -\frac{V_1}{T}$$

Solution to Problem 1, part (a) continues next page....

Solution to Problem 1, part (a) continues....

In summary:

$$\boxed{a_a = \frac{V_1}{T}}$$

$$\boxed{a_b = 0}$$

$$\boxed{a_c = -\frac{V_1}{T}}$$

Part (b) – : (5 points)

We are asked to determine the **position** of each of the three particles. Since the **acceleration is constant** we can use the equation for position:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

For each of the three particles, we are told $x_0 =$. We can infer the starting velocities from the plot: $v_0 a = 0$, $v_0 b = \frac{V_1}{2}$, and $v_0 c = V_1$. Using these with our results from Part (a) we get:

$$x_a = 0 + 0 + \frac{1}{2} a_a T^2 = \frac{1}{2}\left(\frac{V_1}{T})\right) T^2 = \frac{V_1 T}{2}$$

$$x_b = 0 + \frac{V_1}{2} T + \frac{1}{2} a_b T^2 = \frac{V_1 T}{2} + \frac{1}{2}(0) T^2 = \frac{V_1 T}{2}$$

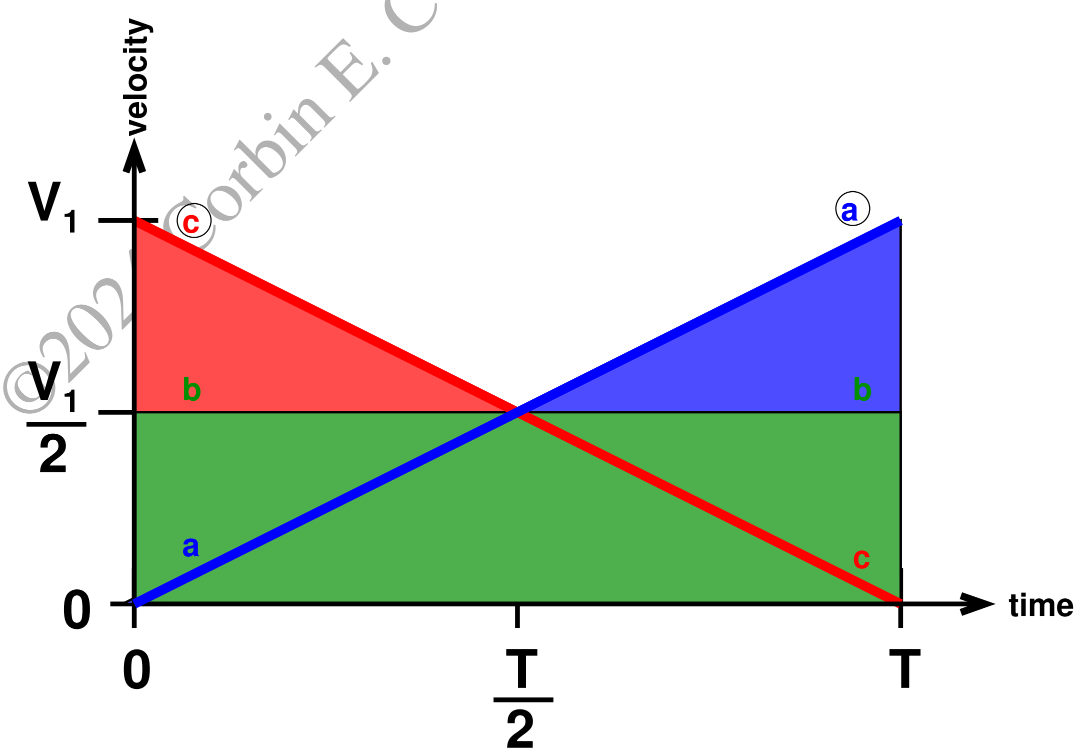
$$x_c = 0 + V_1 T + \frac{1}{2} a_c T^2 = V_1 T - \frac{1}{2}\left(\frac{V_1}{T})\right) T^2 = \frac{V_1 T}{2}$$

In summary:

$$\boxed{x_a = x_b = x_c = \frac{V_1 T}{2}}$$

Note that the fact that the three distances must be the same can be immediately inferred from the fact that a casual inspection of the plots show that the area under each curve is the same, where

$$\boxed{x = \text{Area under each curve} = \frac{V_1 T}{2}}$$



Part (c) – : (5 points)

First we are asked to find the ***net force*** on particle ⓐ. To do this we simply applied **Newton's Second Law:** and our result from Part (a):

$$F_{a,net} = m_a a_a$$

$$F_{a,net} = (3M)\frac{V_1}{T}$$

$$\boxed{F_{a,net} = \frac{3MV_1}{T}}$$

Next we are asked to calculate the work done by the net force. The **Definition of Work** for a constant force applied over a linear displacement is given by the dot product between the force and the displacement vector.

$$\mathcal{W}_F \equiv \vec{F} \cdot \vec{d}$$

Here the force is positive in the direction of the displacement. We already calculated the displacement in Part (b):

$$\mathcal{W}_F = (F_{a,net})(x_a)$$

$$\mathcal{W}_F = \left(\frac{3MV_1}{T}\right)(\left(\frac{V_1 T}{2}\right)$$

$$\boxed{\mathcal{W}_F = \frac{3MV_1^2}{2}}$$

An alternate approach is simply to use the **Classical Work-Energy Relation:**

$$\mathcal{W}_F = \Delta K$$

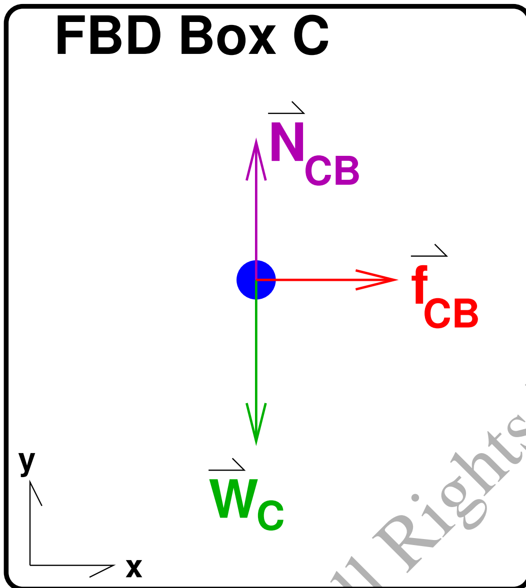
$$\mathcal{W}_F = K_f - K_i$$

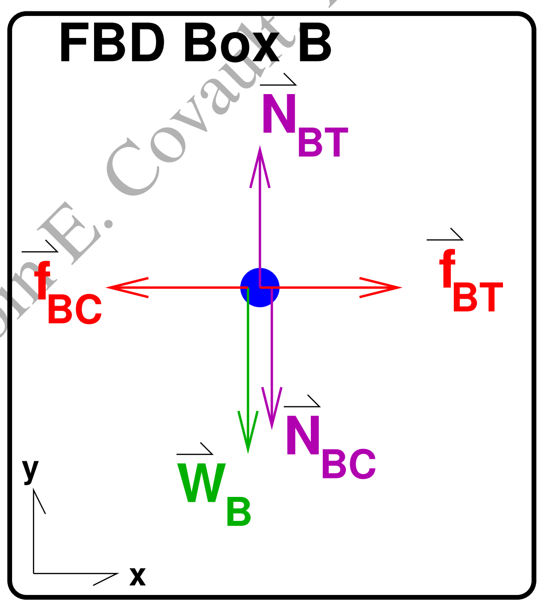
$$\mathcal{W}_F = \frac{1}{2} m_a v_f^2 - 0$$

$$\boxed{\mathcal{W}_F = \frac{3MV_1^2}{2}}$$

Solution to Problem 2 (20 points):

Part (a) – : (5 points)





To get full credit, all **eight forces** need to be listed with appropriate subscripts, and the coordinate systems have to be included. Note that there are three Normal forces and three Friction forces, so *each* of these must be unambiguously labeled. Full credit also requires a clear coordinate system.

Part (b) – : (5 points)

We apply **Newton's Second Law**, to Block C, first in the *vertical* component:

$$F_{Cy} = m_C a_{Cy}$$

There is zero acceleration in the y-direction, therefore:

$$N_{CB} - W_C = 0$$

$$N_{CB} = W_C$$

$$N_{CB} = m_C g$$

Next, we apply **Newton's Second Law**, Block C in the *horizontal* component:

$$F_{Cx} = m_C a_{Cx}$$

$$f_{CB} = m_C a_{Cx}$$

For **sliding (kinetic) friction**: $f_k = \mu_k N$. In this case $f_{CB} = \mu_k N_{CB}$

$$\mu_k N_{CB} = m_C a_{Cx}$$

$$\mu_k m_C g = m_C a_{Cx}$$

Solving for the acceleration:

$$\mu_k g = a_{Cx}$$

$$\boxed{a_{Cx} = \mu_k g}$$

Note that by our coordinate system, positive acceleration means that the acceleration points in the $\boxed{\text{positive-x (rightward) direction}}$.

Part (c) – : (10 points)

Graders Please Note: This part is worth 10 points!

Next, we apply **Newton's Second Law**, Block B in the *Horizontal* component:

$$F_{Bx} = m_B a_B$$

We know the acceleration of Block B:

$$F_{Bx} = m_B A$$

We use the FBD of Block B which only has friction forces:

$$f_{BT} - f_{BA} = m_B A$$

We know from **Newton's Third Law:** that $f_{BA} = f_{AB}$ which we already calculated:

$$f_{BT} - f_{BA} = m_B A$$

$$f_{BT} - \mu_k m_C g = m_B A$$

Solving for the unknown friction, then:

$$\boxed{f_{BT} = \mu_k m_C g + m_B A}$$

Solution to Problem 3:

Part (a): (5 points)

For this problem we are asked for a *speed* so it sounds like we want to pick up the toolbox called **Conservation Laws** and in particular we want to look at Energy. However, ***first we need to check that the Conditions Are Met:*** We are told that there is no friction on the car any time before it gets beyond the top of the hill. This means that there are only two forces on the car, Weight and Normal. So we consider these two forces.

- The *Weight force is conservative* (part a).
- and because it is applied perpendicular to the direction of displacement (the direction of motion) everywhere on the path, the *Normal force does no work*.

Therefore we can state that ***The Conditions for the application of Conservation of Energy are satisfied***.

So we apply **Conservation of Energy** where the "BEFORE" corresponds to the car coasting as it approaches the hill. The "AFTER" corresponds to the car at the top of the hill:

$$\text{“BEFORE”} = \text{“AFTER”}$$
$$E_{tot} = E'_{tot}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

We need to define a "zero-point" One of several allowed options is to select the position $y = 0$ at the bottom. So $y = 0$, $v = V_0$, $y' = H$ and we want to solve for the unknown $v' = v_t$:

$$0 + \frac{1}{2}mV_0^2 = mg(H) + \frac{1}{2}mv_t^2$$

Solving for v_t

$$\frac{1}{2}mv_t^2 = \frac{1}{2}mV_0^2 - mgH$$
$$v_t^2 = V_0^2 - 2gH$$
$$\boxed{v_t = \sqrt{V_0^2 - 2gH}}$$

Solution to Problem 3, part (a) continues next page....

Problem 3, Part (a) Guidelines for graders:

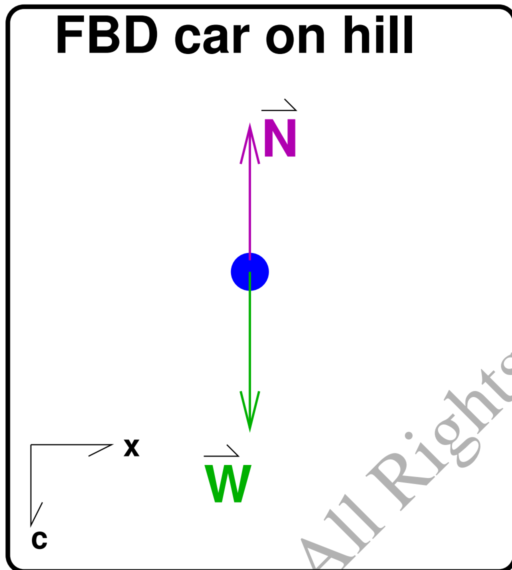
- **Perfect score with explanation including Conditions = 5 points.**
- **Missing consideration of *Conditions for Conservation of Energy* = 4 points.**
- **Mostly correct answer but not in terms of given parameters = 2 or maybe 3 points.**
- **Attempt to incorrectly apply Centripetal Acceleration instead of Conservation of Energy = 1 point.**
- **Attempt to incorrectly assert that $v' = 0$ "because we are coasting" = 1 or 2 points at most, depending on explanation.**
- **Attempt to apply Newton's Second Law or some other incorrect physics instead of Conservation of Energy at most = 1 or 2 points.**
- **Alternate solution from first principles using the Classical Work-Energy relation (instead of Conservation of Energy): Full credit (5 points) if done correctly.**

Solution to Problem 3 continues next page....

Part (b): $(5 points)$

For this part of the problem, we are asking for the force Normal, so we really want to pick up the toolbox we call **Newton's Laws:**

We consider the FBD of the ball at the top position:



Then we apply **Newton's Second Law** in the **centripetal** direction (downward) which corresponds to the direction of centripetal acceleration since the ball is moving on a *circular path*.

$$F_c = ma_c$$

We know the **centripetal acceleration** is given by: $a_c = \frac{v_t^2}{R}$ so:

$$W - N = m\left(\frac{v_t^2}{R}\right)$$

$$mg - N = m\left(\frac{v_t^2}{R}\right)$$

$$N = mg - m\left(\frac{v_t^2}{R}\right)$$

At this point we plug in our result from Part (a):

$$\boxed{N = mg - m\frac{(V_0^2 - 2gH)}{R}}$$

Problem 3, Part (b) Guidelines for graders:

- **Perfect answer with good FBD, coordinate system pointing down in direction of centripetal acceleration, explanation explicitly using Newton's Second Law, starting with $F_c = ma_c$, correct handling of centripetal acceleration =5 points.**
- **Correct answer in terms of v_t but not in terms of given parameters = not more than 3 points.**
- **Student works problem perfectly except fails to contend with fact that centripetal acceleration is downward (usually the coordinate downward is missing from the FBD) = 4 points.**
- **Attempt to apply some incorrect ideas about circular motion = 1 or maybe 2 points.**

Solution to Problem 3 continues next page....

Part (c): $(10 points)$

For this part, we **cannot** use Conservation of Energy because the application of the brake implies friction between the car and the road. Further, we are told that we **cannot** assume that the force of friction is constant, so this means we cannot calculate the work done due to friction using the Definition of Work. We do not have a specification of the force as as a function of position on the path. We cannot calculate a path integral. We cannot apply $\mathcal{W}_{\vec{f}} = \vec{f} \cdot \vec{d}$. We do not have any way to calculate any of these things.

However, we do have the **Classical Work-Energy Relation:** which tells us about the *total work*

$$\mathcal{W}_{tot} = \Delta K$$

First we consider the right-hand side of this equation. We are given the final speed V_f and we know the "initial" speed V_0. So we can calculate ΔK as

$$\Delta K \equiv K_f - K_0$$

$$\Delta K = \frac{1}{2} m V_F^2 - \frac{1}{2} m V_0^2$$

$$\Delta K = \frac{1}{2} m \left(V_F^2 - V_0^2 \right)$$

As for the Total Work, this is the sum of the Work done by all three forces:

$$\mathcal{W}_{tot} \equiv \mathcal{W}_{\vec{W}} + \mathcal{W}_{\vec{N}} + \mathcal{W}_{\vec{f}}$$

Whenever we apply the Work-Energy Theorem we need to make it clear what we mean by initial vs. final. In this case the best choice of initial position is **before** the car starts rolling up the hill ($v = V_0$) and the best choice of final position is **after** the car gets to the bottom of the hill ($v = V_F$). However, many students chose the initial position as the top of the hill. This is acceptable but leads to an algebraically less elegant path to the solution. See the **alternative solution** below:

We know that the $\mathcal{W}_{\vec{N}}$, the Work done by the Normal force is **zero**, as we argued in Part (a).

As for Weight, this is a **Conservative Force**, which means the Work done is **path-independent**. For our choice of "BEFORE" and "AFTER", the final vertical position of the car is the same as the initial vertical position of the car (both corresponding to the flat part of the road on either side of hill) so $\mathcal{W}_{\vec{W}}$, the Work done by the Weight force is also **zero**:

$$\mathcal{W}_{tot} = 0 + 0 + \mathcal{W}_{\vec{f}}$$

$$\mathcal{W}_{tot} = \mathcal{W}_{\vec{f}}$$

So the total work is the same as the work done by the braking force, which is the force of friction. Setting this work equal to the change in kinetic energy in accordance with the Work-Energy Theorem:

$$\boxed{\mathcal{W}_{\vec{f}} = \frac{1}{2} m \left(V_F^2 - V_0^2 \right)}$$

Note that an acceptable **alternate solution** is to start with the initial position from the top of the hill, using the result from Part (a) to calculate the change in kinetic energy. In this case the Work done by the Weight force is no longer zero but will be $\mathcal{W}_{\vec{W}} = mgH$ now.[1] When you plug this in to our expression for the Total Work, along with the result from Part (a) where $v' = \sqrt{V_0^2 - 2gH}$ to calculate the change in kinetic energy, you will find that there are two terms, each with an H in them. that exactly cancel each other (assuming you finish the algebra, which few students will do), leaving the exact final answer as given above:

Problem 3, Part (c) Guidelines for graders:

- **Perfect answer with good explanation, clear logic = 5 points.**
- **Attempt to apply of Work-Energy but failure to correctly demonstrate that the relationship between the Total Work and the Work due to Friction = 3 points**
- **Incorrect attempt to apply Definition of Work directly to the friction force = at most 1 or 2 points.**
- **Correct answer with initial position at top of the hill but failure to complete algebra to show that terms that depend on the height of the hill cancel = subtract 1 or 2 points.**
- **Incorrect attempt to apply some other incorrect ideas at most 1 point.**

[1] This can be explained either by the definition of work or the fact that the Work done by a conservative force is the negative of the change in potential energy.