Appendix XI. Sample Paper

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A sample lab paper written by "Jane Doe" who had a lab partner "John Jones" is on the following pages.

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Free Fall Acceleration

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Abstract:

By using a brass object and commercially available tape timer, I have been able to measure the magnitude of the acceleration due to gravity near the Earth's surface. I have measured the magnitude of the acceleration to be (9.54 ± 0.04) m/s^2. The discrepancy between my results and the accepted value of (9.81 ± 0.02) m/s^2 is probably a result of the small amount of friction between the paper tape and the tape timer. Our data also indicates that the acceleration was constant and that the equations of motion are valid for describing the position and velocity of the brass weight as a function of time.

Introduction and Theory:

Newton's law of gravitation allows one to calculate the gravitational force between two massive bodies separated by a distance r. We can combine Newton's law of gravitation with his second law to determine that the magnitude of the acceleration g of a particle a distance r_e from the Earth is

$$g = \frac{GM_e}{r_e^2}. \tag{1}$$

where G is the Universal Gravitational Constant and M_e is the mass of the Earth. See References 1 and 2.

For objects moving near the Earth's surface, we can make the approximation that the distance from the object to the center of the Earth is the radius of the Earth and remains constant, *i.e.*, $r \approx r_e$. With this approximation, the gravitational force on an object, and thus the acceleration, remain constant.

If we take up as the positive y direction, the equations of motion reduce to a relatively simple form. The expression for the position as a function of time is

$$y(t) = y_0 + v_0 t - \tfrac{1}{2}gt^2 \tag{2}$$

and the expression for the velocity as a function of time is

$$v(t) = v_0 - gt \tag{3}$$

where y_0 and v_0 are the position and velocity at time $t = 0$.

From Eq. 2, we can see that if we fit a quadratic equation to a plot of t vs. y, the second-order slope will be $-\tfrac{1}{2}g$, or g will be negative twice the second-order slope. From Eq. 3, we can see that if we fit a straight line to a plot of t vs. v, the slope of the line will be $-g$.

Experimental Procedure:

We used a 2-g binder clip to attach the end of approximately 2 meters of paper tape to a brass object of mass 523 grams. We threaded the other end of the paper tape through a Pasco Scientific Tape Timer (model 9283). The timer was attached to a tall ring stand and was positioned above a catch bucket filled with sand. See Figure 1.

To reduce the amount of friction between the paper tape and the tape timer, we held the paper tape loosely above the timer and not draped over the back or the front of the timer. Prior to turning on the tape timer we allowed any swinging motion of the object to dampen away. The timer was turned on so it was marking the paper tape with small dots at a frequency of 40 Hz. The brass object was then released and the force of gravity accelerated it downward into the catch bucket.

Figure 1: Apparatus used for experiment

We then taped the paper tape to the table top (with care taken to ensure that the tape was both straight and as stretched as it had been under the force of the brass object's weight), and measured and recorded the distance between the first dot and every subsequent dot with a two-meter stick. These positions are listed in Table 1. We plotted the position and velocity of the brass object as a function of time and compared the results to the theoretical values of position and velocity.

Due to the size of the dots and parallax effects, I estimate that the uncertainty for the position measurements, δ_y, is $\pm$ 0.002 m. At the suggestion of my Teaching Assistant, Joe Smith, I will adopt the manufacturer's claimed accuracy in the time between successive dots of 0.001 s as my uncertainty $\delta_{\Delta t}$. These are the values for the uncertainties I used for each data point in Table 1.

Results and Analysis:

The measurements of the position versus time are in Table 1. Also in Table 1 are the average velocities of the brass object as a function of time. These velocities were calculated by using the relation

$$v = \Delta y / \Delta t. \tag{4}$$

where Δy is the distance between adjacent dots and Δt is the time interval between dots, 25 ms, which corresponds to 40 Hz.

Because the tape timer was turned on prior to releasing the brass object, I omitted the first few points on the paper tape and did not include them in the tables or graphs. They were omitted because they correspond either to dots printed prior to releasing the brass object or to those which were printed during the initial release period where friction or forces due to the releasing mechanism (my fingers) may have affected the results. I also noted that the brass object hit the bucket just before the tape came out of the tape timer. This circumstance is apparent in the data; the last
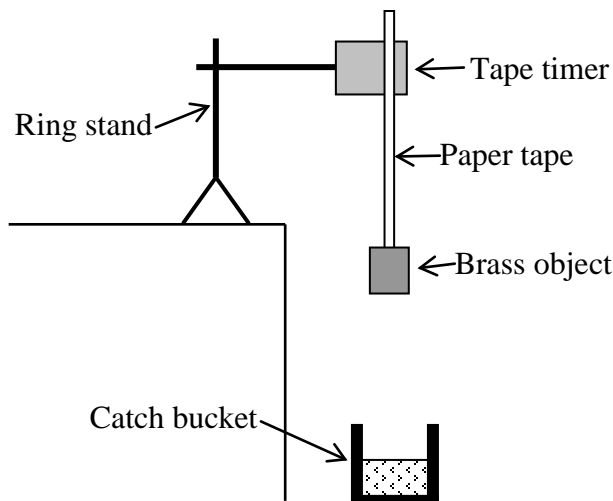
data point has a *lower* speed than the previous point. I therefore excluded the last data point from our analysis.

Error Analysis:

As mentioned previously, my position measurements had an associated uncertainty of $\delta_y = 0.002$ m. The velocity measurements can be determined by Eq. 4, where Δy is the distance between two adjacent points. Since each distance measurement, y, has its own uncertainty equal to 0.002 m, I must calculate the uncertainty in Δy, the difference between any two data points,

$$\Delta y = y_i - y_j \tag{5}$$

where y_i and y_j represent the positions of the two points i and j. The uncertainty of Δy resulting from the uncertainty in y_i can be determined by the derivative method and is

$$\delta_{\Delta y, i} = \delta_{y, i} \,. \tag{6}$$

Likewise, the uncertainty in Δy as a result of the uncertainty in y_j is

$$\delta_{\Delta y, j} = \delta_{y, j} \,. \tag{7}$$

Adding these two uncertainties in quadrature gives the total uncertainty in Δy,

$$\delta_{\Delta y} = \sqrt{\left(\left(\delta_{y, i} \right)^2 + \left(\delta_{y, j} \right)^2 \right)} \tag{8}$$

Since this calculation only depends on the uncertainty of the measurement, δy, and not the measurement of y itself, we find that $\delta_{\Delta y}$ is a constant for all points. Using the values of 0.002 m for both $\delta_{y, i}$ and $\delta_{y, j}$, we find that $\delta_{\Delta y} = 0.002828427$ m ≈ 0.003 m.

I will now use the computational method to determine the uncertainty in the velocity measurement. I can calculate the average velocity of the brass object by applying Eq. 4. To determine the uncertainty of that velocity, we calculated the uncertainty of the velocity resulting from the uncertainty in the position and time measurements. The uncertainty of the velocity, $\delta_{v, y}$ due to the uncertainty in the position is

$$\delta_{v, y} = \frac{\Delta y + \delta_{\Delta y}}{\Delta t} - \frac{\Delta y}{\Delta t} \,. \tag{9}$$

Equation 9 reduces to

$$\delta_{v, y} = \frac{\delta_{\Delta y}}{\Delta t} \,. \tag{10}$$

Here again I find that this quantity is a constant for all data points.

The uncertainty of the velocity measurement which results from the uncertainty in the time measurement is

$$\delta_{v,t} = \frac{\Delta y}{\Delta t} - \frac{\Delta y}{\Delta t + \delta_{\Delta t}},$$

(11)

which reduces to

$$\delta_{v,t} = \frac{\Delta y \cdot \delta_{\Delta t}}{\Delta t (\Delta t + \delta_{\Delta t})}.$$

(12)

Adding Eqs. 10 and 12 in quadrature allows me to calculate the total uncertainty in the velocity calculation:

$$\delta_{\Delta v} = \sqrt{\left((\delta_{v,y})^2 + (\delta_{v,t})^2 \right)}$$

(13)

Because of the Δy dependence in Eq. 12, one would expect that the velocity calculation for each point will have a different uncertainty associated with it.

I used the *Origin* software to calculate the uncertainty in the velocity and position of the brass object in Table 1; they are also included in the position and velocity graphs, Figures 2 and 3.

Figures 2 and 3 are graphical representations of the measurements and calculations obtained for this experiment. Figure 2 is a plot of the computed velocity versus time along with a linear fit to the data. The linear fit corresponds to the equation of motion, Eq. 3, with a constant added to compensate for any offset in the experimental data resulting from neglecting the first few data points, as mentioned in the Procedure section. The slope of the linear fit is equal to the acceleration of the brass object and is $a = (9.45 \pm 0.07)$ m/s^2.

Figure 3 shows the position as a function of time. Note that the error bars for the uncertainty in the position are smaller than the data points. The solid curve is a polynomial fit to the experimental data. The polynomial fit used includes the zero-th and first order terms, A and B_1*X, to allow me to compensate for any offset in time and position resulting from neglecting the first few data points as mentioned above. If we compare the equation of motion given in Eq. 2 to the polynomial fit of our data, we see that the coefficient B_2 (4.77 $\pm$ 0.02) represents one-half of the experimental acceleration. Using this value gives a magnitude of the acceleration due to gravity of $g = (9.54 \pm 0.04)$ m/s^2. The dashed curve in Figure 3 represents the theoretical position of the brass object as a function of time using the accepted value of the magnitude of the acceleration, $g = (9.81 \pm 0.02)$ m/s^2. The theoretical curve has been shifted in time by 0.100 seconds to compensate for neglecting some initial data points.

Conclusions:

The data and linear fit in Figure 2 indicate that the magnitude of the acceleration of the brass object was (9.45 $\pm$ 0.07) m/s^2, which is 3.7% slower than the accepted value of (9.81 $\pm$ 0.02) m/s^2. This is a reasonable result because of the presence of a small amount of friction between the paper tape and the tape timer, which would be a force opposing the direction of motion, thus decreasing

the acceleration. Another possible source of the discrepancy is air resistance, although the speeds of the brass object are small enough so that this effect should be negligible. The linear fit to the velocity versus time data is extremely good. This would imply that the equation of motion Eq. 3 is sufficient in modeling the data we have obtained.

The polynomial fit of the data in Figure 3 gives a magnitude of the acceleration due to gravity of (9.54 ± 0.04) m/s^2. This is a reasonable result, given the expected presence of friction as described above. It is also agrees with the acceleration coefficient from the linear fit of the velocity versus time data, Figure 2 (the results in Figs. 2 and 3 differ by only 0.09 ± 0.08 m/s^2). One can see that the polynomial fit used in Figure 3 is sufficiently good to support the claim that the equation of motion, Eq. 2, is sufficient to describe the motion of the brass object for this experiment. Because the polynomial fit takes into account that the object has a non-zero speed at the first data point, I will adopt the value of $g = (9.54 \pm 0.04)$ m/s^2 as my experimental value.

This experiment gave a magnitude of the gravitational acceleration near the surface of the Earth to be $g = (9.54 \pm 0.04)$ m/s^2. This value is significantly (7 standard deviations) smaller than the accepted value of (9.81 ± 0.02) m/s^2. The discrepancy between the experimental and accepted values is probably due to the neglected friction between the paper tape and the tape timer. I have also shown that the constant-acceleration kinematic equations of motion are valid for this experiment.

Acknowledgements:

References:

1. Halliday, Resnick and Walker, *Fundamentals of Physics, 6th edition*, John Wiley and Sons, Inc, 2001, pg. 295.
2. Schultz, D., *General Physics I: Mechanics Lab Manual*, CWRU Bookstore, Spring 2004.

Table 1: Experimental data

Time (s) t	Position (m) y	Uncertainty in y (m) δ_y	change in position (m) Δy	Uncertainty in Δy (m) $\delta_{\Delta y}$	velocity (m/s) v	Uncertainty in velocity (m/s) δ_v
0.000	0	0.002				
0.025	-0.0135	0.002	-0.0135	0.003	-0.54	0.11
0.050	-0.033	0.002	-0.0195	0.003	-0.78	0.12
0.075	-0.059	0.002	-0.026	0.003	-1.04	0.12
0.100	-0.091	0.002	-0.032	0.003	-1.28	0.12
0.125	-0.128	0.002	-0.037	0.003	-1.48	0.12
0.150	-0.172	0.002	-0.044	0.003	-1.76	0.13
0.175	-0.219	0.002	-0.047	0.003	-1.88	0.13
0.200	-0.273	0.002	-0.054	0.003	-2.16	0.14
0.225	-0.332	0.002	-0.059	0.003	-2.36	0.14
0.250	-0.397	0.002	-0.065	0.003	-2.60	0.15
0.275	-0.469	0.002	-0.072	0.003	-2.88	0.16
0.300	-0.546	0.002	-0.077	0.003	-3.08	0.16
0.325	-0.63	0.002	-0.084	0.003	-3.36	0.17
0.350	-0.721	0.002	-0.091	0.003	-3.64	0.18
0.375	-0.816	0.002	-0.095	0.003	-3.80	0.18
0.400	-0.918	0.002	-0.102	0.003	-4.08	0.19
0.425	-1.026	0.002	-0.108	0.003	-4.32	0.2
0.450	-1.142	0.002	-0.116	0.003	-4.64	0.2
0.475	-1.262	0.002	-0.120	0.003	-4.80	0.2
0.500	-1.389	0.002	-0.127	0.003	-5.08	0.2
0.525	-1.522	0.002	-0.133	0.003	-5.32	0.2
0.550	-1.654	0.002	-0.132	0.003	-5.28	0.2

Figure 2: Velocity of a Falling Object

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Linear Regression for Data_v:
Y = A + B * X
Weight given by Data_dv error bars.

Parameter	Value	Error
A	-0.297	0.017
B	-9.45	0.07

velocity (m/s)

time (+/- 0.001 s)

v
Linear fit

Figure 3: Motion of a Falling Object

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y
- - - Theoretical calculations
——— Polynomial fit

Polynomial Regression for DATA_Y:
Y = A + B1*X + B2*X^2
Weight given by Data_dy error bars.

Parameter	Value	Error
A	-0.0029	0.0012
B1	-0.385	0.010
B2	-4.770	0.019

Position (+/- 0.02 m)

Time (+/- 0.001 s)