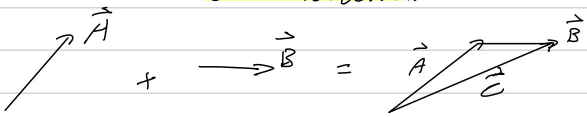
Vectors and 2D kinematics

Recall: Vectors have both magnitude and direction

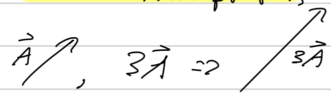
Length = "magnitude" $|\vec{A}|$

Vector Addition

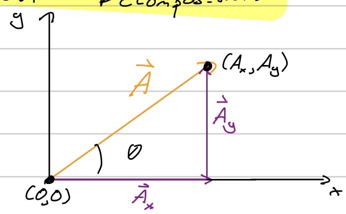


$\vec{C} = \vec{A} + \vec{B}$

Scalar Multiplication



Vector Decomposition



Vectors are defined by numbers!

$|\vec{A}| = $ magnitude $= A$ (no → symbol)

Direction $= \theta$ (some reference eg $\hat{x}$)

Or we can break into components!

$\vec{A} = \vec{A}_x + \vec{A}_y$

Components

$|\vec{A}_x| = A_x = A\cos\theta$

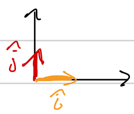
$|\vec{A}_y| = A_y = A\sin\theta$

You can find the components of a vector if given magnitude and direction

Reverse Direction: To find A if A_x, A_y given: $A = \sqrt{A_x^2 + A_y^2}$ Woah!

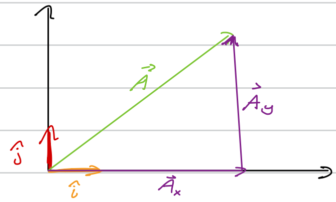
To find θ if A_x, A_y given: $\theta = \arctan\left(\frac{A_y}{A_x}\right)$ Crazy

Cartesian Basis Unit Vector



Don't use < > Notation in this class!

Unit Vector = Length of One!



Means...

- $\vec{A}_x = A_x\hat{\imath}$
- $\vec{A}_y = A_y\hat{\jmath}$

$\therefore$ $\boxed{A = A_x\hat{\imath} + A_y\hat{\jmath}}$

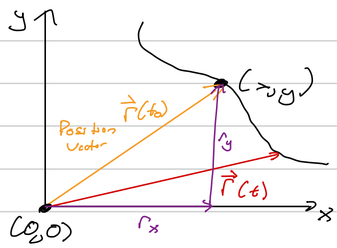
Vector Representation

Example: $\vec{C} = \vec{A} + \vec{B}$

$$\begin{cases} \vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} \\ \vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} \end{cases} \therefore \vec{C} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$

Path Integrals will Come Later

Kinematics w/ Vectors



$\vec{r} = \vec{r}_x + \vec{r}_y = r_x\hat{\imath} + r_y\hat{\jmath}$

Position Vectors – a vector whose components are its own coordinates

$\vec{r} \equiv x\hat{\imath} + y\hat{\jmath} \Rightarrow \vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$

$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$

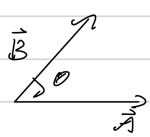
$\vec{a} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath}$

$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$

Example: $\vec{r}(t) = (Bt^2)\hat{\imath} + (Ce^{-t/\tau})\hat{\jmath}$ τ, B, C Given Constants. Find Velocity and accel

$\vec{v}(t) = (2Bt)\hat{\imath} - \left(\frac{C}{\tau}e^{-t/\tau}\right)\hat{\jmath}$ and $\vec{a}(t) = (2B)\hat{\imath} - \left(\frac{C}{\tau^2}e^{-t/\tau}\right)\hat{\jmath}$

Dot Products!



$C = \vec{A}\cdot\vec{B} = AB\cos\theta$

$\hat{\imath}\cdot\hat{\imath} = 1, \hat{\imath}\cdot\hat{\jmath} = 0$

Using Decomposition

$\therefore C = (A_x\hat{\imath} + A_y\hat{\jmath})\cdot(B_x\hat{\imath} + B_y\hat{\jmath})$

FOIL $= A_xB_x(1) + A_xB_y(0) + A_yB_x(0) + A_yB_y(1)$

$$\boxed{C = A_xB_x + A_yB_y}$$

Example: Related to HW4

Define: $\vec{r}(t) = \underbrace{R\cos(\omega t)}_{x(t)}\hat{\imath} + \underbrace{R\sin(\omega t)}_{y(t)}\hat{\jmath}$, R, ω given

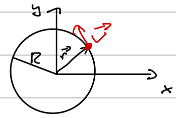
Take Derivatives of functions of time!

$|\vec{r}| = \sqrt{x^2 + y^2}$ $\rightarrow$ $|\vec{r}| = \sqrt{(R\cos(\omega t))^2 + (R\sin(\omega t))^2}$

Determine Magnitude

$= \sqrt{R^2(\cos^2(\omega t) + \sin^2(\omega t))} = \sqrt{R^2} = \boxed{R}$

Suppose



Speed $= |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

$\vec{v} = \frac{d\vec{r}}{dt} = -\omega R\sin(\omega t) + \omega R\cos(\omega t)$

$\therefore |\vec{v}| = \sqrt{R^2\omega^2(\sin^2(\omega t) + \cos^2(\omega t))} = \boxed{R\omega}$

Omg! its the rolling constant

$\vec{a} = -R\omega^2\cos(\omega t) - R\omega^2\sin(\omega t)$

$|\vec{a}| = \sqrt{R^2\omega^4(\cos^2(\omega t) + \sin^2(\omega t))}$

$= \sqrt{R^2\omega^4} = R\omega^2 = \frac{v^2}{R}$

$\boxed{a = \frac{v^2}{R} = a_c}$

woah! Its centripetal acceleration

Projectile Motion

x-direction:

no air resistance $\rightarrow$ velocity is constant

$$\boxed{\begin{aligned} a_x &= 0 \\ V_x &= \text{Const} = V_{x_0} \\ x &= x_0 + V_{x_0}t \end{aligned}}$$

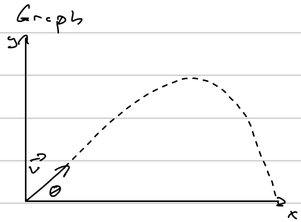
Two sets of equations to describe an objects motion

y-direction:

Free Fall

$$\boxed{\begin{aligned} a_y &= -g \\ V_y &= V_{y_0} - gt \\ y &= y_0 + V_{y_0}t - \tfrac{1}{2}gt^2 \end{aligned}}$$

Graph



we say: $V_{x_0} = V_0\cos\theta$ and $V_{y_0} = V_0\sin\theta$

$\therefore \vec{r} = x\hat{\imath} + y\hat{\jmath}$

Motion of Projectile in vector form

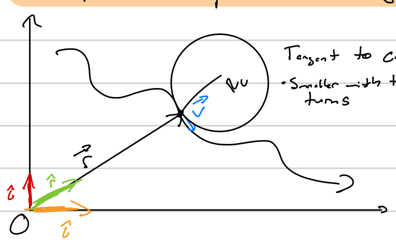
Identical to relations shown above

$\vec{r}(t) = (x_0 + V_{x_0}t)\hat{\imath} + (y_0 + V_{y_0}t - \frac{1}{2}gt^2)\hat{\jmath}$

$\vec{v}(t) = (V_x)\hat{\imath} + (V_{y_0} - gt)\hat{\jmath}$

$\vec{a}(t) = 0\hat{\imath} - g\hat{\jmath}$

Example: A particle on a given path



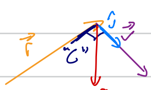
Tangent to Curve
- Smaller with tighter turns

$\vec{r} = x\hat{\imath} + y\hat{\jmath}$

Unit vector in the r direction

$\vec{r} = r\hat{r}$ where $r = |\vec{r}|$ and $|\hat{r}| = 1$ r is mag, $\hat{r}$ is dir

$\vec{v} = v\hat{v}$ where $v = |\vec{v}|$ and $|\hat{v}| = 1$ v is mag, $\hat{v}$ is dir



$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$

$\vec{a} = \underbrace{a_v\hat{v}}_{\text{Tangential Component}} + \underbrace{a_c\hat{c}}_{\text{Centripetal Component}}$

R_c is radius of Curvature

$\hat{c}$ is $\perp$ to $\hat{v}$

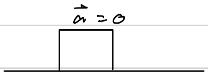
magnitude

$a_v = \frac{d}{dt}(v)$ where v is the speed

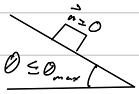
$a_c = \frac{v^2}{R_c}$

Both depend on speed

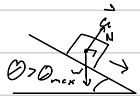
Forces at Angles



$F_{Net}=0$



$F_{Net}=0$



$F_{Net} > 0$

θ_{max} is the maximum angle required for the body to remain at rest.

Steps

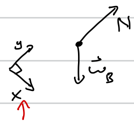
1. FBD
2. Coordinate System
3. Kinematic Constraint
4. N2L
   - 4.1 Decompose Forces

Only decompose forces Not aligned with the chosen coordinate System!

Decomposing Necessary forces into their necessary components is very important

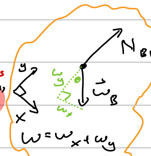
Given: m, θ; find (i) a and (ii) all forces

FBD Block

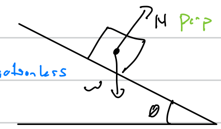


avoids having to break net force into components
"tipped coordinate"

Kinematic Constraint
$a_x=a$
$a_y=0$



$\cos\theta = \frac{w_y}{w}$
$w_y = w\cos\theta$
$w_x = w\sin\theta$



N perp to surface

frictionless

N2L $F_{Net}=ma$ Y-dir

$F_y = ma_y$ ← Normal force and y component of the weight force

$N - w_y = ma_y$

$N - w\cos\theta = 0$

$N - mg\cos\theta = 0$

$\boxed{N = mg\cos\theta}$

N2L $F_{Net} = ma$ x-dir

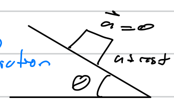
$w_x = ma_x$ ← only w_x in x-dir

$w\sin\theta = ma$

$mg\sin\theta = ma \Rightarrow \boxed{a = g\sin\theta}$

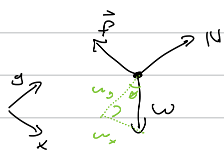
Small θ → Small a
Incr. θ → a approaching g

Suppose there is friction (S,f,ree)



Given m, θ; Find (i) all forces
$a=0$ as at rest

FBD Block



$a_x = 0 \quad a_y = 0$

N2L

x: $F_x = ma_x$

$w_x - f = 0$

$w_x = w\sin\theta = mg\sin\theta$

$mg\sin\theta - f = 0$

$\boxed{f = mg\sin\theta}$

y: $F_y = ma_y$

$N - w_y = 0$

$w_y = w\cos\theta = mg\cos\theta$

$\boxed{N = mg\cos\theta}$

Suppose there is Kinetic Friction

Given μ_K, m, θ Find (i) a (ii) all forces

N2L

y: $F_y = ma_y$

Same as the bit to the left

$\boxed{N = mg\cos\theta}$

$f = \mu_K N$

$\boxed{f = \mu_K mg\cos\theta}$

x: $w_x - f = ma_x$

$mg\sin\theta - f = ma$

$f = \mu_K N$

$mg\sin\theta - \mu_K N = ma$

$N = mg\cos\theta$

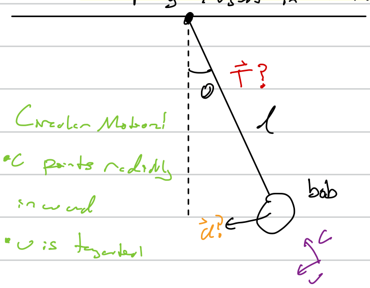
$mg\sin\theta - \mu_K mg\cos\theta = ma$

$\cancel{m}g(\sin\theta - \mu_K\cos\theta) = \cancel{m}a$

$\boxed{a = g(\sin\theta - \mu_K\cos\theta)}$

Pendulums

Jumping right in with an example



Circular Motion!
- c points radially inward
- v is tangential

Pendulum bob with mass m released at rest angle θ

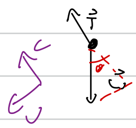
Q1 What is the tension on string?

Q2 What is the acceleration of bob immediately after its released?

Given mass m, θ, and length of string l

a_c and a_v must be considered

FBD Bob



N2L: c-coord

$F_c = m a_c$

$F_c = m\left(\frac{v^2}{R}\right) = m\left(\frac{0^2}{l}\right) = 0$

$T - W\cos\theta = 0$

$T - mg\cos\theta = 0 \rightarrow$ ① $\boxed{T = mg\cos\theta}$

N2L: v-coord

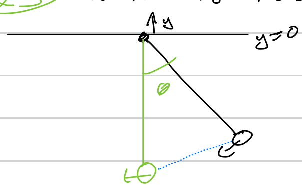
$F_v = m a_v$

$W\sin\theta = m a_v$

$mg\sin\theta = m a_v$

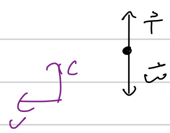
② $\boxed{a_v = g\sin\theta}$

Q3 What is the Tension on string when bob at lowest position?



Still align coordinates as path is still circular

FBD Bob



N2L c-coord

$F_c = m a_c$

$T - w = m\left(\frac{v^2}{l}\right)$

$T - mg = m\left(\frac{v^2}{l}\right)$ stop!

v is unknown here!

$T - mg = \frac{m v_e^2}{l}$ and $v' = \sqrt{2gl(1-\cos\theta)}$

Let $v' = v_e$

$T = mg + \frac{m(2gl(1-\cos\theta))}{l}$

$T = mg + 2mg(1-\cos\theta)$

$T = mg(1 + 2(1-\cos\theta))$

$T = mg(1 + 2 - 2\cos\theta) \rightarrow \boxed{T = mg(3-2\cos\theta)}$

Find v

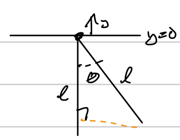
CofME $\vec{w}$ conservative

$\vec{T}$ does no work as $\perp$ to path

Before = After ("Before" = release, "After" = lowest position)

$E_{Tot} = E'_{Tot}$ Conditions met!

$U + K = U' + K'$



$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$

$y = -l\cos\theta$, $y' = -l$

$v = 0$, $v' = ?$

$mg(-l\cos\theta) + \frac{1}{2}m(0)^2 = mg(-l) + \frac{1}{2}mv'^2$

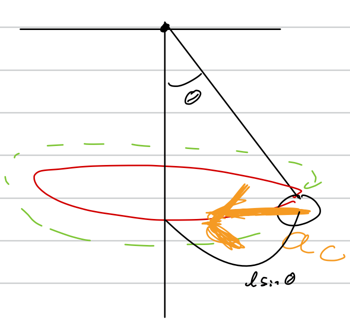
$-\cancel{m}gl\cos\theta = -\cancel{m}gl + \frac{1}{2}\cancel{m}v'^2$

$gl - gl\cos\theta = \frac{1}{2}v'^2$

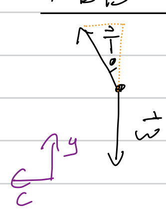
$v'^2 = 2gl(1-\cos\theta)$

$\therefore \boxed{v' = \sqrt{2gl(1-\cos\theta)}}$

Conical Pendulums



FBD Bob



N2L y-coord

$F_y = m a_y$ $a_y = 0$

$T\cos\theta - W = 0$

$\boxed{T = \frac{mg}{\cos\theta}}$

N2L c-coord

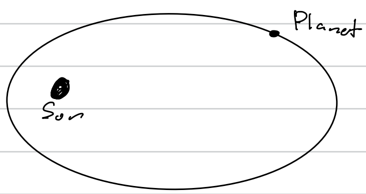
$F_c = m a_c$

$T\sin\theta = m\frac{v^2}{l\sin\theta}$

$\frac{mg\sin\theta}{\cos\theta} = \frac{mv^2}{l\sin\theta}$

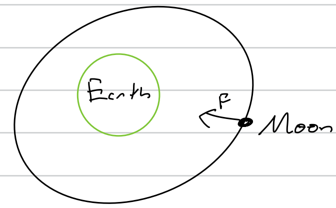
$mg\tan\theta = \frac{mv^2}{l\sin\theta}$, solve for v

Planets!



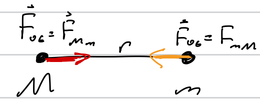
Kepler's 3rd Law (K3L)

$$\frac{P^2}{R^3} = \frac{(\text{Period})^2}{(\text{"radius"})^3} = \text{constant?}$$



Universal Gravity

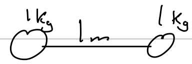
$$F_{UG} = \frac{G \cdot M \cdot m}{r^2}$$



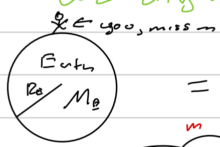
G = Gravitational Constant

$= 6.67 \times 10^{-11}\ N \cdot m^2 \cdot kg^{-2}$ ← Super small

we only notice it bc of earth's size

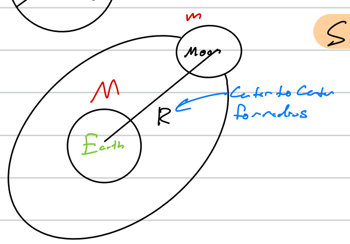


$F_{UG} = \frac{G(1)(1)}{1} = G$ ← barely anything at all



$$\Rightarrow F_{UG} = \frac{G M m}{R_E^2} = w = mg \rightarrow \boxed{g_E = \frac{G M_E}{R_E^2}}$$

← where "g" comes from!



Situation $M \gg m$

Orbit ≈ Circular

(1) what are the forces on the moon

N2L Moon

$F_{moon} = m a_c$

$$\frac{G M \cancel{m}}{R^2} = \cancel{m} \frac{v^2}{R}$$

Solve for $\frac{P^2}{R^3}$

$\rightarrow \frac{\text{Period}}{} \frac{2\pi R}{P} = v$ $\frac{\text{distance}}{\text{time}}$

$$\frac{GM}{R^2} = \frac{4\pi^2 R^2}{P^2 R}$$

$$GM = 4\pi^2 \left(\frac{R^3}{P^2}\right) \rightarrow \boxed{\frac{P^2}{R^3} = \frac{4\pi^2}{GM}}$$

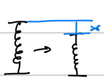
K3L back!

Force Menu

Symbol	what	Known?	Conservative?	Contact?
$\vec{w}$	weight	$w = mg$	Yes	No
$\vec{N}$	Normal Push	No	No	Yes
$\vec{T}$	Tension Pull	No	No	Yes
$\vec{f}$	Friction	Sometimes	Absolutely No!	Yes ← Known if kinetic and given μ_k
$\vec{F}_{UG}$	Universal Gravity	Yes	Yes	No
$\vec{F}_{sp}$				

Hooke and Springs

"Spring"



Compressed or stretched

x is the change in length

$F_{sp} = K|x|$ ← x is displacement doesn't care about actual spring length

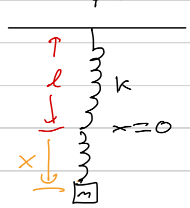
Force points in the direction opposite to x

$F_{sp} = K|x|$ Larger K equates to increased force

K = spring constant with equal displacement compared to small K

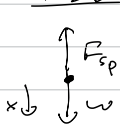
x = displacement relative to a relaxed spring

Example



what is x s.t. mass m in equilibrium? → a=0

FBD



N2L x

$F_x = m a$ → 0

$w - F_{sp} = 0$

$mg - Kx = 0$

$$\boxed{x = \frac{mg}{K}}$$

$w = mg$

$F_{sp} =$

You can only assume a=0 when mass or object is in equilibrium

Definition of Potential Energy

$$U_{\vec F}(P) - U_{ref}(P_{ref}) = -\oint_{P_{ref}}^{P} \vec F \cdot d\vec r$$ Path integral

$\vec F \cdot d\vec r = F\,dr\cos\theta$ Change w/ time

W.r.t. $W_{cons} = -\Delta U_F$ work done by a Conservative force

Computing the integral for weight

Weight w/ coord $\hat{y}$

Pot. Energy

$$U_{\vec w}(y) - U_{ref}(y_{ref}) = -\int \vec w \cdot d\vec r$$

$\vec w \cdot d\vec r = mg\,dy\,\cos(180°)$

$$U_{\vec w}(y) - \cancel{U_{ref}(y_{ref}=0)}^{\,0} = -\int_{y=0}^{y'=y} -(mg\,dy) \qquad = -mg\,dy$$

Angle btwn. $\vec w$ and $d\vec r$ is $180°$

$d\vec r$ only is dy

$$\boxed{U_{\vec w}(y) = mgy}$$

Just an easy integral

Set ref to 0 in PHYS121, not always true in PHYS122

Calculating for Spring Force

$$U_{sp}(x) - U_{ref}(x_{ref}) = -\int \vec F_{sp} \cdot d\vec r$$

$$U_{sp}(x) - \cancel{U_{ref}(x=0)}^{\,0} = -\int_{x=0}^{x=x'} -(Kx'dx')$$

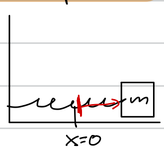
$$\boxed{U_{sp}(x) = \tfrac{1}{2}Kx^2}$$

Spring force against motion

$\vec F_{sp} \cdot d\vec r = Kx\,dx\cos(180°)$

$|\vec F_{sp}|$ $= -Kx\,dx$ $|d\vec r|$

Example



Release at rest at $x = x_i$. Find max speed

Given: K, m, x_i

Consider C of ME when asked to find speed

All conservative or no work done

"Before" = "After"

$U_{sp} + K = U'_{sp} + K'$

$\tfrac{1}{2}Kx^2 + \tfrac{1}{2}mv^2 = \tfrac{1}{2}Kx'^2 + \tfrac{1}{2}mv'^2$

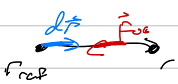
$x = x_i,\ v = 0,\ x' = 0 \quad v' = v_f\,?$

$\tfrac{1}{2}Kx_i^2 + 0 = 0 + \tfrac{1}{2}mv_f^2$

$\therefore\ \boxed{v_f = x_i\sqrt{\tfrac{K}{m}}}$

Computing for Universal Gravity

$$U_{UG}(r) - U_{ref}(r_{ref}) = -\int \vec F_{UG} \cdot d\vec r$$



$$\vec F_{UG} \cdot d\vec r = \frac{GMm}{r^2}\,dr\cos 180°$$
$$= -\frac{GMm}{r^2}\,dr$$

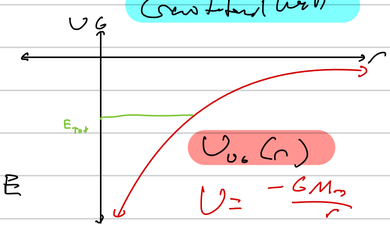
F_{UG} goes to 0 when $r = \infty$

$$U_{UG}(r) - \cancel{U_{ref}(r_{ref}=\infty)}^{\,0} = -\int_{r=\infty}^{r'=r} -\left(\frac{GMm}{r^2}\right)dr$$

can't be 0 b/c divide by 0

$$\boxed{U_{UG}(r) = -\frac{GMm}{r}}$$

"Gravitational well"



$U = -\dfrac{GMm}{r}$

Conservative Forces

$\vec w$ Weight $U_{\vec w} = mgy$

$\vec F_{sp}$ Spring Force $U_{sp} = \tfrac{1}{2}Kx^2$

$\vec F_{UG}$ Universal Gravity $U_{UG} = -\dfrac{GMm}{r}$

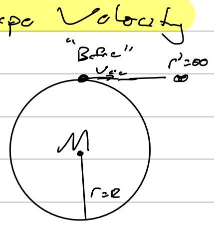
C of ME

$E_{Tot} = E'_{Tot}$

$U + K = U' + K'$ ← $U^{(')}$ can be a sum of different potential Energies

Escape Velocity

← Ignoring Air resistance, high K sends an object out of atmosphere as $\frac{1}{r^2}$ decreases



Thrown with some velocity v_{esc}. Initial radius = R. Masses of object not needed, Planet mass M. thrown towards the final velocity is 0 and $r' = \infty$. Find the escape velocity ☺

After: $v' = 0$

C of ME

$U + K = U' + K'$

$-\dfrac{GMm}{r} + \tfrac{1}{2}mv^2 = -\dfrac{GMm}{r'} + \tfrac{1}{2}mv'^2$

$-\dfrac{GMm}{R} + \tfrac{1}{2}mv_{esc}^2 = 0 + 0$

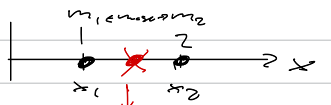
Algebra $\boxed{v_{esc} = \sqrt{\dfrac{2GM}{r}}}$ $\approx 1.2\times10^4\ m/s$ or ~7 miles/sec

More on Systems

NZL for Systems

$F_{\substack{Net\\Sys\\Ext}} = M_{sys}\, a_{sys}$ ← works for systems of n objects and directions only with CoM

Center of Mass (CoM)



$X_{AV} = \frac{x_1 + x_2}{2}$ CoM is not an actual thing, just a concept

Suppose $m_1 \neq m_2$, find weighted Avg

2pts: $\boxed{X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$ ← weighted avg equation

N pts: $x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_n x_n}{m_1 + m_2 + \ldots + m_n}$

$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \ldots + m_n y_n}{m_1 + m_2 + \ldots + m_n}$

2 coordinates with n bodies with given masses and positions

Time derivatives of CoM

$\frac{d x_{cm}}{dt} = V_{cm} = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$ derivative of a sum is the sum of derivatives

$\frac{d v_{cm}}{dt} = a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$ Can be expanded to N points

NZL System

$\vec{F}_{\substack{Net\\Sys\\Ext}} = M_{sys}\, \vec{a}_{cm}$ ← vector because it applies to any number of dimensions

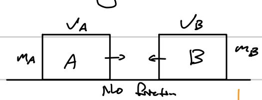
If $F_{\substack{Sys\\Net\\Ext}} = 0 \Rightarrow$ "Isolated" $\Rightarrow a_{cm} = 0$

$V_{cm} = \text{Constant}$ "conserved"

$M_{sys} \cdot V_{cm} = P_{Tot}$ CofLM Momentum!

Inertial reference frame is defined to be at rest or moving with constant velocity

Totally Elastic Collisions



← $K = K'$ is what totally elastic means

$\therefore \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$

Isolated ⇒ $P_{Tot} = P_{Tot}'$ "Isolated" (2 unknowns)

$m_A V_A + m_B v_B = m_A V_A' + m_B v_B'$ CofLM

CofK

Annoying and a waste of time to solve this quadratically

Use this method!

Bob Brown Master Eq

$$\boxed{V_A - V_B = V_B' - V_A'}$$

Only true in
- Totally Elastic Collisions
- 1-D Situations

Must be true!!!

Center of Mass Reference Frame

$V_{cm} = \frac{m_A V_A + m_B v_B}{m_A + m_B} = \ldots$ ← can be calculated if m_A, m_B, V_A, and v_B given such as that above!

Constant!

Choose coordinate System that moves with center of mass (CoM) of System!

Relative to CoM Reference Frame v_{cm} is 0 → $v_{cm} = 0$

Example: $m_A = m_B = m$

$V_A = V_0 \quad V_B = 0$

① $m v_0 = m V_A' + m v_B'$

② $v_0 = v_B' - v_A'$

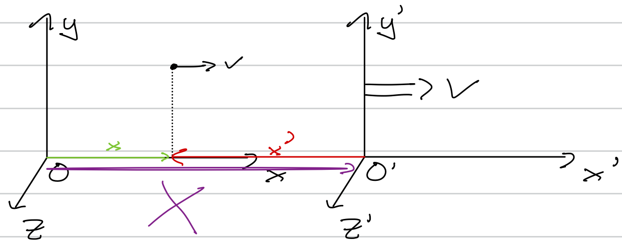
$v_B' = v_0 + v_A'$ into ①

$m v_0 = m v_A' + m(v_0 + v_A')$

$2 v_A' = 0 \Rightarrow v_A' = 0$

② → $v_B' = v_0$

Reference Frames



$x' = x - X$

$x' = x - (X_0 + Vt)$

$$\boxed{v' = v - V}$$

- Lab = standing still
- Center-of-mass Ref. Frame

$V_{CoM\,RF} = V_{Lab} - V_{cm}$

$\therefore X = X_0 + Vt$ constant velocity

Coucault's 4-box method

	Before	After
Lab Frame	$V_A = V_0$ $\quad V_B = 0$	$V_A' = 0$ $\quad v_B' = V_0$
C of M RF	$V_A = \frac{V_0}{2}$ $\quad V_B = -\frac{V_0}{2}$	$V_A' = -\frac{V_0}{2}$ $\quad V_B' = \frac{V_0}{2}$

$-V_{cm}$ Add V_{cm} flip sign!   "Bounce"

total momentum Com reference frame is 0

$V_{cm} = \dfrac{m_A v_A + m_B v_B}{m_A + m_B}$

$= \dfrac{m_A V_0}{m_A + m_B}$ $\quad m_A = m_B = m$

$V_{cm} = \dfrac{m v_0}{m + m} = \dfrac{v_0}{2}$

$$\boxed{\begin{aligned} v_A' &= 0 \\ v_B' &= v_0 \end{aligned}}$$

Example: $m_A = m$ $\quad m_B = 2m$

$v_A = V_0 \quad v_B = 0$

	Before	After
Lab Frame	$v_A = V_0$ $\quad v_B = 0$	$V_A' = -\frac{V_0}{3}$ $\quad V_B' = \frac{2v_0}{3}$
C of M RF	$V_A = \frac{2v_0}{3}$ $\quad V_B = -\frac{V_0}{3}$	$V_A' = -\frac{2v_0}{3}$ $\quad V_B' = \frac{V_0}{3}$

$-V_{cm}$ add V_{cm} "Bounce"

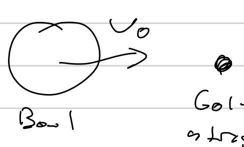
0 momentum total

$V_{cm} = \dfrac{m v_0 + 0}{m + 2m}$

$V_{cm} = \dfrac{v_0}{3}$

$$\boxed{\begin{aligned} v_A' &= -\frac{V_0}{3} \\ v_B' &= \frac{2v_0}{3} \end{aligned}}$$

Example: Totally elastic with rotating gives



Estimate V_{cm}!

Let Bowl mass $= m_B$

Let Golf mass $= m_G$

Much Bigger! $\rightarrow m_B \gg m_G$

$V_{cm} = \dfrac{m_B V_0 + 0}{m_B + m_G}$ ← assume negligible term

$\approx \dfrac{m_B v_0}{m_B + 0} = V_0$

If one mass much greater assume it dominates V_{cm}

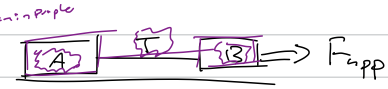
	Before	After
Lab	$V_B = V_0, V_g = 0$	$V_B \approx V_0$ $\quad V_g = 2v_0$
C of M RF	$V_B \approx 0$ $\quad, V_g = -V_0$	$v_B' \approx 0, V_g = v_0$

$-V_{cm}$ $+V_{cm}$

Example: Given $m_A = m_B = m$ and F_{app}

← F_{app} always given in 121; result of using a rope

System in purple



Frictionless

Find Tension

Kinematic Constant

accel? $\quad a_A = a_B = a$

Shortcut!

FBD A   Ignore vertical forces   FBD B   FBD System

$\vec{T}_{AB}$ x $\vec{T}_{BA}$ F_{app} x F_{app} x

N2L x-dir System (A+B)

$F_{x\,sys} = m_{sys}\, a_{sys}$

$F_{app} = 2m\, a$

$\therefore a = \dfrac{F_{app}}{2m}$

N2L A x-dir

$F_x = m_A a_A$

$T_A = m\left(\dfrac{F_{app}}{2m}\right)$ ← Calculated already

$$\therefore \boxed{T = \frac{F_{app}}{m}}$$

Example n number of blocks

No Friction

Given: m, N, F_{app}. Find T_i

$a_1 = a_2 = a = a_{sys} = ?$

FBD System ← all blocks

N2L x-dir Sys

$F_{x_{sys}} = M_{sys} a_{sys}$

$F_{app} = (Nm)a$

$a = \frac{F_{app}}{Nm}$

To find T_i, define a system where the boundary of the system is at the point where that force is applied

Suppose i=3

New system = leftmost i blocks

FBD new system

only force on FBD system we want! ☺

N2L new system

$F_{sys} = M_{sys} a_{sys}$

$T_i = (im)\frac{F_{app}}{Nm}$

$\therefore \boxed{T_i = \frac{i}{N} F_{app}}$

Example: Massive Rope

Massive Rope { length l, mass m }, F_{app} given

Friction less

$T(x)$: Tension in rope a distance x from left end

FBD Rope

N2L Rope System

$F_{x_{sys}} = m_{sys} a$

$F_{app} = m a$

$a = \frac{F_{app}}{m}$

New System $T(x)$ applied @ left side up to x

FBD

N2L new system

$F_{x_{sys}} = m_{sys} a$

$T(x) = (\frac{x}{l} m) a$

$T(x) = (\frac{x}{l} m)(\frac{F_{app}}{m})$

ratio of rope times tension

$\therefore \boxed{T(x) = (\frac{x}{l}) F_{app}}$

Example: Massive rope pulls block

Frictionless

Block M mass

Rope m mass

F_{app}, l

Find $T(x)$

N2L whole

$F_{sys} = M_{sys} a$

$F_{app} = (M+m) a_{sys}$

$a_{sys} = \frac{F_{app}}{M+m}$

New System = Boundary @ x

FBD

N2L new system

$M + \frac{x}{l} m$

$F_{new_{sys}} = M_{sys} a_{sys}$

$T(x) = ((\frac{x}{l})m) + M)(\frac{F_{app}}{M+m})$

$T(x) = (\frac{xm}{l} + M)\frac{F_{app}}{M+m}$

simplify

$\therefore \boxed{T(x) = \left(\frac{M + \frac{x}{l} m}{M+m}\right) F_{app}}$

Example: Fun

System

moves up @ Constant velocity

m_s, m_p, m_b all given

what is Tension in rope?

System defined this way to include everything

FBD System

$4\vec{T}$

W_{sys}

4 contact points

N2L System

$F_{sys} = m_{sys} a_{sys}$, $a_{sys} = 0$ constant v

$4\vec{T} - M_{sys} g = 0$

$\vec{T} = \frac{M_{sys} g}{4}$

$\boxed{\vec{T} = \frac{(m_s + m_p + m_b) g}{4}}$

Rotations!

Translational Motion	Rotational Motion
Kinematics: x, v, a	Kinematics: θ, ω, α
Dynamics: $F_{Net} = ma$	Dynamics: $\tau_{Net} = I\alpha$
N2L	N2L Rot

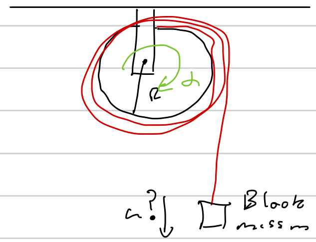
Review of Motion Components

New Force

$\vec{H}$ = "Hinge Force"

axles, gears, pin

Example: Massive Pulley



Always assume string/rope is not slipping or sliding on the Pulley

Find accel of block

$a = ?$ $\alpha = ?$

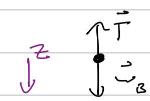
Given
- Radius of Pulley
- Block of Mass m
- Pulley Mass is $4m$
  - ↳ Pulley Mass = m_p
- $I_p = \frac{1}{2} m_p R^2$

↳ given for now!

Pulley has $a = 0$ so a Force must be points upward to hold the pulley

Hinges/Axels apply force in any direction needed to satisfy N2L

FBD Block



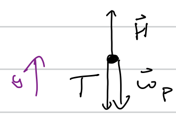
N2L Block

$F_{Bz} = m_B a_{Bz}$

$w_B - T = ma$

$mg - T = ma$ Stop!

FBD Pulley



N2L Pulley

$F_{Py} = m_P a_{Py}$ ← 0

$H - T - w_p = 0$

$H - 4mg - T = 0$ Stop!

Two equations w/ Too many unknowns

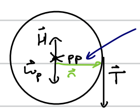
① $mg - T = ma$

② $H - 4mg - T = 0$

Always worth a try to solve problem w/o rotational motion

Extended FBD (xFBD) Pulley ← No longer a point-like object

- Weight Force always @ CoM
- Coordinate System is lined up w/ rotation



pp for pivot point

(+)

N2L Rot Pulley

$\tau_{Net} = I_p\alpha$

$\tau_w + \tau_H + \tau_T = (\frac{1}{2} m_p R^2)\alpha$

$0 + 0 + RT\sin 90 = \frac{1}{2} m_p R^2 \alpha$

0 distance from pp for w and H so $\tau = 0$

$\therefore RT = \frac{1}{2}(4m)R^2\alpha$

Pulley $\alpha = \frac{a}{R}$ Rolling Constraint

R.C is acceptable

$RT = 2mR^2(\frac{a}{R})$

$\therefore T = 2ma$

Recall: $mg - 2ma = ma$

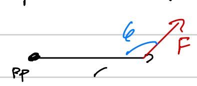
$mg = 2ma + ma$

$g = a(2+1)$

$\boxed{a = \frac{1}{3}g}$

Definition of Torque

$\tau_F \equiv rF\sin\theta$



Important Equation!

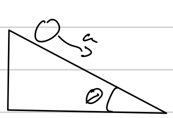
Rolling Constant

$a = \alpha r$, $v = \omega r$

Tangential acceleration ↑ rotational ↑ radius from pp ↑

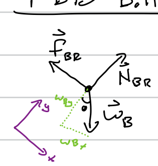
Very Helpful for HW6 #6

Example: Ball on Ramp



Given
- $\theta, R, m, I = \frac{2}{5}mR^2$
- Rolls w/ no slip or slide

FBD ball



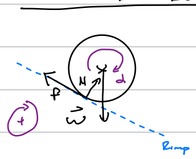
N2L x-coord

$F_{Bx} = m_B a_{Bx}$

$mg\sin\theta - f = ma$

Stop!

xFBD ball



N2L Rot ball

$\tau_{net} = I\alpha$

$\tau_w + \tau_N + \tau_f = (\frac{2}{5}mR^2)\alpha$

$0 + \cancel{RN(\sin 180°)} + Rf = \frac{2}{5}mR^2\alpha$

$\alpha = \frac{a}{R}$ R.C

$Rf = \frac{2}{5}mR^2\frac{a}{R}$

$f = \frac{2}{5}ma$

↳ $mg\sin\theta - f = ma$

$\cancel{m}g\sin\theta - \frac{2}{5}\cancel{m}a = \cancel{m}a$

$g\sin\theta = \frac{7}{5}a$

$\boxed{a = \frac{5}{7}g\sin\theta}$