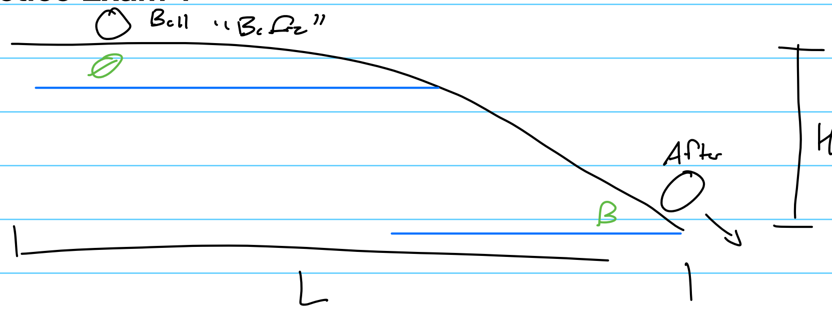
Practice Exam 1

1)



Hollow Ball:
$I = \frac{2}{3} m R^2$
m and R given
No slip, no slide

a) C of ME

Conditions: No slip no slide, weight is conservative, N does no work be by friction does not contribute @ contact.

$$\boxed{v = \sqrt{\frac{6}{5} g H}}$$

Before = After

$E_{Tot} = E'_{Tot}$

$U + K_T + K_A = U' + K'_T + K'_A$

$mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgy' + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$

$y = H,\ v = 0,\ \omega = 0,\ y' = 0,\ v' = ?,\ \omega = \frac{v'}{R}$

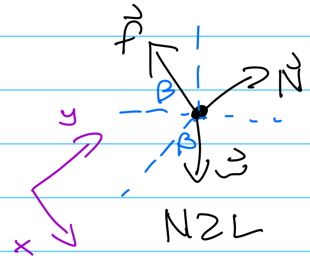
$mgH = 0 + \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$

$mgH = 0 + \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v}{R}\right)^2$

$\cancel{m}gH = \frac{1}{2}\cancel{m}v^2 + \frac{1}{3}\cancel{m}v^2$

$gH = \frac{5}{6}v^2$

b) FBD Ball After

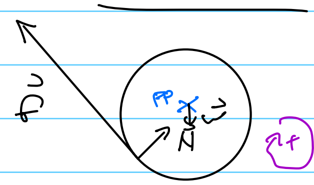


N2L

$F_{Net_x} = ma_x$

$W\sin\theta - f = ma_x$

xFBD



N2L Rot

$\tau_{Net} = I\alpha \qquad \alpha = \frac{a}{R}$

$\tau_H + \tau_w + \tau_f = \left(\frac{2}{3}mR^2\right)\left(\frac{a}{R}\right)$

$\cancel{R}f = \frac{2}{3}m\cancel{R}a$

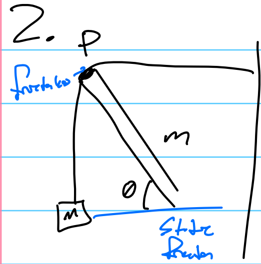
$f = \frac{2}{3}ma$

$\cancel{m}g\sin\beta - \frac{2}{3}\cancel{m}a_x = \cancel{m}a_x$

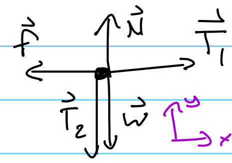
$g\sin\beta = \frac{5}{3}a_x$

$$\boxed{a_x = \frac{3}{5}g\sin\beta}$$

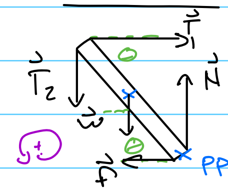
2.



a) FBD Beam



x FBD



b) N2L x-dir

$F_{Net} = m\cancel{a_x}^{\,0}$

$T_1 - f = 0$

N2L y-dir

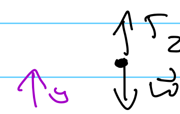
$F_{Net} = m\cancel{a_y}^{\,0}$

$N - W - T_2 = 0$

$N - mg - T_2 = 0$

$\boxed{W = mg}$

FBD block



N2L $F_{Net} = m\cancel{a}^{\,0}$

$T_2 - W = 0$

$T_2 = w = Mg$

N3L $T_1 = T_2 = Mg$

$T_1 - f = 0$

$Mg - f = 0$

$\boxed{f = Mg}$

$\boxed{\begin{matrix} T_1 = Mg \\ T_2 = Mg \end{matrix}}$

$N - mg - Mg = 0 \rightarrow N = mg + Mg$ $\boxed{N = g(m+M)}$

c) N2L Rot

Let rod be an arbitrary length L

$\tau_{Net} = I\cancel{\alpha}^{\,0}$

$\tau_N + \tau_{T_1} + \tau_{T_2} + \tau_w + \tau_f = 0$

$0 - \cancel{L}T_1 \sin\theta + \cancel{L}T_2\cos\theta + \frac{\cancel{L}}{2}W\cos\theta = 0$

$M\cancel{g}\cos\theta - M\cancel{g}\sin\theta + \frac{1}{2}m\cancel{g}\cos\theta = 0$

$M\cos\theta + \frac{1}{2}m\cos\theta - M\sin\theta = 0$

$\cos\theta(M + \frac{1}{2}m) - M\sin\theta = 0$

$M\sin\theta = \cos\theta(M + \frac{1}{2}m)$

$\tan\theta = \frac{M + \frac{1}{2}m}{M}$

$\boxed{\theta = \arctan\left(\frac{M + \frac{1}{2}m}{M}\right)}$

d) $f_s \leq \mu_s N$

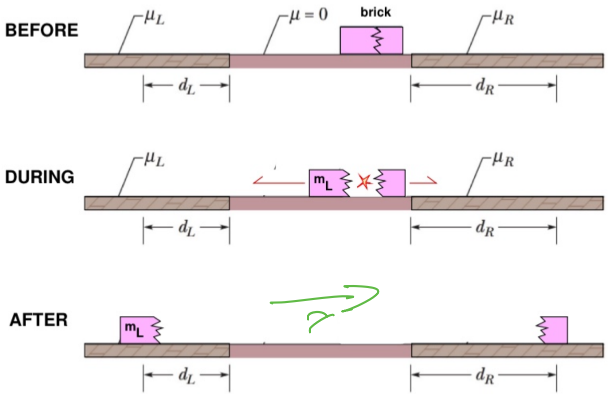
$Mg \leq \mu_s(g(M+m))$

$\mu_s \geq \frac{M\cancel{g}}{\cancel{g}(M+m)}$

$\boxed{\mu_s \geq \frac{M}{M+m}}$

Ecs, once I knew where to start

3. Mass of original block?



C of LM   Before Explosion part

Conditions: Isolated sys no
friction done

$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$

$v_a = 0, v_b = 0$

$0 = m_L v_L' + m_R v_R'$

$M = m_L + m_R$

Stop!

Work Energy Thm

$W = \Delta K$

$W = Fd\cos\theta \quad \theta = 180°$

Block L $-f_s d = \frac{1}{2} m_L v'^2 - \frac{1}{2} m_L v^2$

$f_s = \mu_L N_{AB} \leftarrow N_{AB} = W = m_L g$

$-d_L \mu_L \cancel{m_L} g = \frac{1}{2} \cancel{m_L} v'^2 - \frac{1}{2} \cancel{m_L} v^2$

$v'^2 = 0$ ends @ rest

$-d_L \mu_L g = -\frac{1}{2} v^2$

$v^2 = 2\mu_L d_L g$

$v_L = \sqrt{2\mu_L d_L g}$

Block R   Independent of Mass,
Just change vars

$v_R = \sqrt{2\mu_R d_R g}$

$0 = m_L(-v_L) + m_R v_R$

$m_L v_L = m_R v_R$

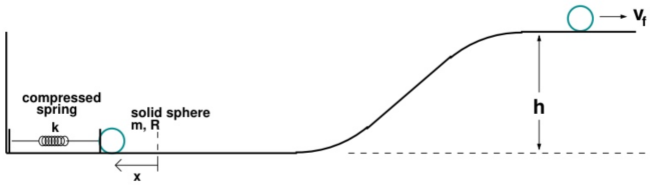
$m_L \sqrt{2\mu_L d_L g} = m_R \sqrt{2\mu_R d_R g} \qquad m_R = \sqrt{\frac{2\mu_L d_L g}{2\mu_R d_R g}}\, m_L$

$m_R = \sqrt{\frac{\mu_L d_L}{\mu_R d_R}}\, m_L$

$$\boxed{M = m_L\left(1 + \sqrt{\frac{\mu_L d_L}{\mu_R d_R}}\right)}$$

Practice Exam 2

1.



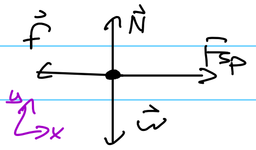
Given sphere Solid

$I = \frac{2}{5} m R^2$ m, R given

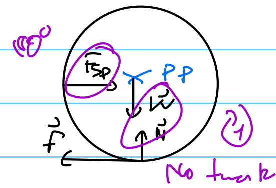
Spring displaced x, K given

Ball rolls to h

a) FBD



xFBD



N2L x-dir

$F_{net} = m a_x$

$F_{sp} - f = m a_x$

$Kx - f = m a_x$

$Kx - \frac{2}{5} Ma = m a_x$

$Kx = \frac{7}{5} Ma$

N2L Rot

$\tau_{Net} = I\alpha$ R.C

$\tau_f = \frac{2}{5} m R^2 \left(\frac{a}{R}\right)$

$\cancel{R} f = \frac{2}{5} M \cancel{R} a$

$f = \frac{2}{5} Ma$

$f = \frac{2}{5} Ma$

$$\boxed{a = \frac{5Kx}{7M}}$$

b) C of ME

Conditions: No slip no slide. Weight constant, Normal ⊥ to motion. Friction no work at contact

Before = After

$U_{sp} + U_w + K_T + K_R = U'_{sp} + K'_T + K'_R + U_w$

$\frac{1}{2}Kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Kx'^2 + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$

$y = 0, y' = H$ $v = 0, \omega = 0, x' = 0, v' = ?$ $\omega = \frac{v}{r}$

$\frac{1}{2}Kx^2 = mgH + \frac{1}{2}mv'^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$

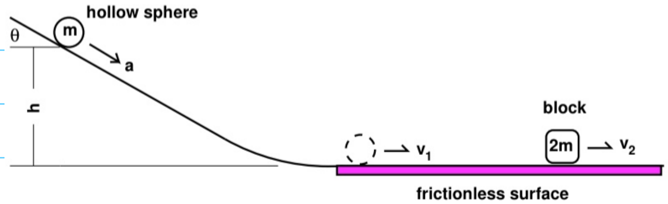
$\frac{1}{2}Kx^2 = mgH + \frac{1}{2}mv^2 + \frac{1}{5}mv^2$

$Kx^2 - 2mgH = v^2\left(m + \frac{2}{5}m\right)$

$v^2 = \frac{5}{7}\left(\frac{Kx^2}{m} - 2gH\right)$

$$\boxed{v = \sqrt{\frac{5}{7}\left(\frac{Kx^2}{m} - 2gH\right)}}$$

2.



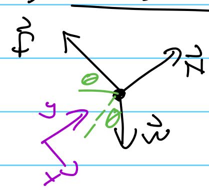
Hollow sphere

$I=\frac{2}{3}mR^2$, m, R given

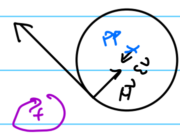
No slip no slide down

Block 2m Totally elastic

a) FBD



xPBD



N2L x-dir

$F_{Net}=ma_x$

$w\sin\theta - f = ma_x$

$mg\sin\theta - f = ma_x$

$\cancel{m}g\sin\theta - \frac{2}{3}\cancel{m}a = \cancel{m}a$

$g\sin\theta = \frac{5}{3}a$

N2L Rot

R.C $\frac{a}{R}$

$\tau_{Net} = I\alpha$

$\cancel{R}f = \frac{2}{3}mR^{\cancel{2}}\left(\frac{a}{\cancel{R}}\right)$

$f = \frac{2}{3}ma$

$$\boxed{a = \frac{3}{5}g\sin\theta}$$

b) C of M E   conditions met!

$B_{efore} = A_{fter}$

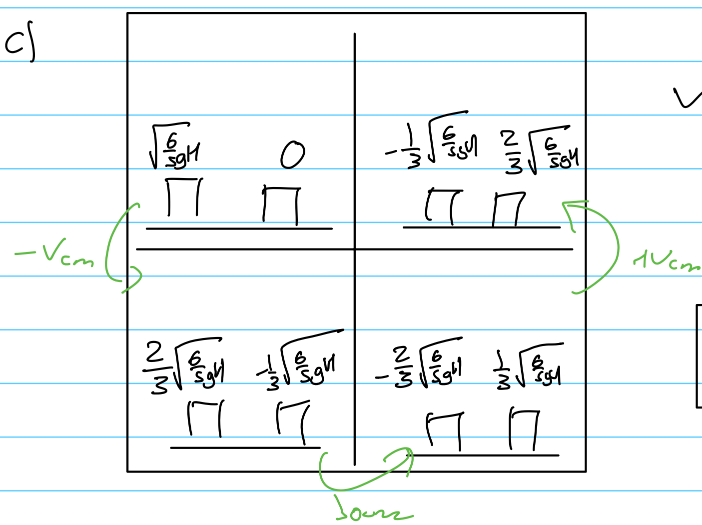
$mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgy' + \frac{1}{2}mv'^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$

$\cancel{m}gH = \frac{1}{2}\cancel{m}v^2 + \frac{1}{3}\cancel{M}v^2$

$gH = \frac{5}{6}v^2 \rightarrow v^2 = \frac{6}{5}gH$

$$\boxed{v = \sqrt{\frac{6}{5}gH}}$$

c)

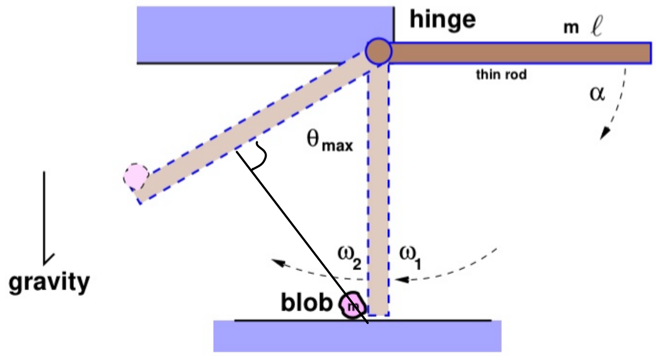


$V_{cm} = \frac{m\sqrt{\frac{6}{5}gH} + 2m(0)}{m+2m}$

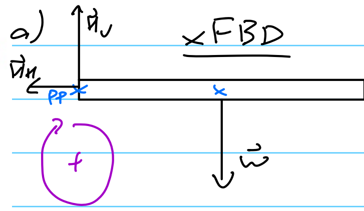
$= \frac{\sqrt{\frac{6}{5}gH}}{3} = \frac{1}{3}\sqrt{\frac{6}{5}gH}$

$$\boxed{V_2 = \frac{2}{3}\sqrt{\frac{6}{5}gH}}$$

3.



a)



N2L Rot

$\tau_{Net} = I\alpha$

$\frac{\cancel{\ell}}{2} w = \frac{1}{3} m \ell^{\cancel{2}} \alpha$

$\frac{3}{2} \cancel{m} g = \cancel{m} \ell \alpha$

$$\boxed{\alpha = \frac{3g}{2\ell}}$$

b) C of ME   Wont Cos. H dosnt work

$mg(0) + \frac{1}{2} I \omega^2 = mg\left(-\frac{\ell}{2}\right) + \frac{1}{2}\left(\frac{1}{3} m \ell^2\right)\omega^2$

$0 = -mg\frac{\ell}{2} + \frac{1}{6} m \ell^2 \omega^2$

$\frac{1}{2} \cancel{m} g \cancel{\ell} = \frac{1}{6} \cancel{m} \ell^{\cancel{2}} \omega^2$

$3g = \omega \ell$

$$\boxed{\omega = \sqrt{\frac{3g}{\ell}}}$$

c) C of AM

$L_{pp} + L_{Rod} = L_{pp}' + L_{Rod}'$

$\frac{1}{3} \cancel{m} \cancel{\ell^2} (\omega_1) = \cancel{m} \cancel{\ell^2} \omega' + \frac{1}{3} \cancel{m} \cancel{\ell^2} \omega$

$\frac{1}{3}\sqrt{\frac{3g}{\ell}} = \omega_2 + \frac{1}{3}\omega_2$

$\frac{1}{3}\sqrt{\frac{3g}{\ell}} = \frac{4}{3}\omega_2$

$$\boxed{\omega_2 = \frac{1}{4}\sqrt{\frac{3g}{\ell}}}$$

d) C of ME

Blob and Rod are System "stuck"

$y_{cm} = \frac{m\left(\frac{-\ell}{2}\right) + m(-\ell)}{m+m} = \frac{-\frac{3}{2}\cancel{m}\ell}{2\cancel{m}} = -\frac{3}{4}\ell$

$y_{sys_{max}} = -\frac{3}{4}\ell\cos\theta \qquad I_{sys} = \frac{1}{3}m\ell^2 + m\ell^2 = \frac{4}{3}m\ell^2$

$U_{sys} + K_{sys} = U_{sys}' + K_{sys}'$ max $\omega = 0$

$mgy_{sys} + \frac{1}{2} I_{sys}\omega_2^2 = mgy_{sys}' + \frac{1}{2} I_{sys}(0)$

$2mg\left(-\frac{3}{4}\ell\right) + \frac{1}{2}\left(\frac{4}{3}m\ell^2\right)\omega_2^2 = 2mg\left(-\frac{3}{4}\ell\cos\theta\right)$

$-\frac{3}{2}\cancel{m}g\ell + \frac{2}{3}\cancel{m}\ell^2\left(\frac{1}{4}\sqrt{\frac{3g}{\ell}}\right)^2 = -\frac{3}{2}\cancel{m}g\ell\cos\theta$

$-\frac{3}{2}\cancel{g}\cancel{\ell} + \frac{2}{3}\ell^{\cancel{2}}\frac{3\cancel{g}}{16\ell} = -\frac{3}{2}\cancel{g}\cancel{\ell}\cos\theta$

$-\frac{3}{2} + \frac{2}{3}\left(\frac{3}{16}\right) = -\frac{3}{2}\cos\theta$

$-\frac{3}{2} + \frac{1}{8} = -\frac{3}{2}\cos\theta$

$-\frac{2}{3}\left(-\frac{3}{2} + \frac{1}{8}\right) = \cos\theta$

$1 - \frac{1}{12} = \cos\theta$

$$\boxed{\theta = \arccos\left(1 - \frac{1}{12}\right)}$$