Cycle 1 Review Sheet, Part 2

30 September 2024

Conservation Laws:

Remember that we introduced the notion of conservation laws as an *alternative toolbox* to approach problems of motion where the forces are not constant or are otherwise complicated so that direct application of Newton's second law is difficult. We have shown that by using **Conservation of Mechanical Energy** and **Conservation of Linear Momentum**, we can describe the motion of bodies in systems without applying Newton's idea of force at all. This is important, in that the Conservation Laws allow us to deal with problems without worrying about the details of forces. Certain problems are much easier to solve using Conservation Laws. You should be able to recognize which problems call for dealing with Newton's laws, and which problems call for applying conservation laws.

Linear Momentum:

For one-dimensional motion, we define *linear momentum* as the quantity p of motion of a body:

$$p \equiv mv$$

where m is the mass of a body and v is the velocity of a body. We can then re-write Newton's Second Law as:

$$F_{net} = \frac{dp}{dt}$$

Definition of Impulse:

For one-dimensional motion, *impulse* is the change in momentum of a body that results from the application of a force over a short interval of time Δt:

$$\text{Impulse} \equiv \Delta p = F_{ave}\Delta t$$

where $\vec{F}_{ave}$ is the average force applied during the small interval of time Δt. When we deal with impulse we often are able to talk about the total change in the momentum even though we do not actually know how the force actually changes in time.

Condition for Using Conservation of Linear Momentum:

In order to consider using Conservation of Linear Momentum we have to meet this one Condition. We need to make sure that the system is *isolated*. This means that the net force on the entire system (as a whole) is zero.

We note that linear momentum is *additive* and signed (as a vector). So the total momentum is just the sum of the individual momenta: For example, for the one-dimensional linear momentum, p we say:

$$p_{total} = p_A + p_B + p_C + ...$$

When we say a quantity is conserved, what we mean is that the total quantity in the system is *constant* and does not change with time. For example, if we say that momentum is conserved, then we mean:

$$p_{total} = constant$$

or in other words:

$$\text{“BEFORE”} = \text{“AFTER”}$$

$$p_{tot} = p'_{tot}$$

Two colliding bodies:

We consider two bodies, A and B, colliding as a system (perhaps two billiard balls). Before they collide, each body has constant velocity. During the collision, the two bodies exert force upon each other so that the velocities change. After the collision, the velocities are constant again. How can we use the law of the conservation of linear momentum to constrain the motion of these two bodies? We say that the total momentum AFTER the collision is the same as the momentum BEFORE the collision. Assuming that the collision is one dimensional, we can write:

$$\text{BEFORE} = \text{AFTER}$$

$$p_{total} = p'_{total}$$

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

(Important: Do not "memorize" the above equation. Rather understand where it comes from and be ready to derive it for any given problem.)

Totally inelastic collisions (bodies stick together):

In the case where the two bodies collide and *stick* together so that they both have the same final velocity: ($v'_A = v'_B \equiv v'$).

In this case we can solve for the final velocity v' after the collision:

$$m_A v_A + m_B v_B = m_A v' + m_B v'$$

where here we has shown the case for a 1-D collision. Then we can re-arrange to get:

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B}$$

(Important: Again do not "memorize" the above equation. Rather understand where it comes from and be ready to derive it for any given problem.)

Note that the case where one body *catches* another body is equivalent to a totally inelastic collision.

Explosions (Reverse Collision: initial momentum is zero

If two bodies fly apart as a result of an internal force, conservation of linear momentum constrains their velocities:

$$\text{BEFORE} = \text{AFTER}$$
$$p_{total} = p'_{total}$$
$$0 = m_A v'_A + m_B v'_B$$

Therefore, if we are given the velocity of one of the bodies, v'_A, we can solve for the other velocity:

$$v'_B = -\frac{m_A}{m_B} v'_A$$

Note that when one body *throws* another body, this is the equivalent of an explosion.

Note that an explosion can be thought of as an inelastic collision in reverse.