IP

Inclined Plane

revised August 2021

Learning Objectives:

During this lab, you will

1. be introduced to how to write a lab paper.
2. learn how to take data with *Logger Pro*.
3. estimate the uncertainty in a quantity that is calculated from quantities that are uncertain.
4. test a physical law experimentally.

A. Introduction

You will check the validity of Newton's Second Law by measuring the motion of a cart as it accelerates up an inclined plane under the action of gravitational and other forces. You will use an "encoded pulley" to convert the motion of the cart to an electronic signal that will be monitored by a computer. The computer will use the program *Logger Pro* to read and display this signal in terms of position, velocity and acceleration as a function of time.

You must complete a fill-in-the-blank paper for this lab experiment, worth 30 points. Appendix II has details of how to write papers. Appendix XI is a sample paper. The fill-in-the-blank paper is in Appendix IX. Additional guidance may be supplied to you in lab.

B. Apparatus

You will measure the motion of a low-friction PASCO® cart along its mating track. A ring stand holds the track at a fixed angle while a string, mass holder and masses supply an additional force to balance or accelerate the cart. An encoded pulley and a computer running the program *Logger Pro*® monitor the motion. An electronic scale and meter stick are used for measuring key characteristics of the system.

C. Theory

Newton's Second Law provides the basic theory for this experiment. The vector form of this law for the motion of a body of constant mass m is

$$\sum \vec{F} = m\vec{a} \qquad (1)$$

where Σ denotes a sum and $\sum \vec{F} =$ $F_1+F_2+F_3+\ldots$ therefore represents a vector sum of all external forces acting on the body. $\vec{a}$ is the acceleration of the body. For motion in two dimensions, this corresponds to two independent equations,

$$\Sigma F_x = ma_x \text{ and } \Sigma F_y = ma_y \qquad (1a)$$

that is, the vectors can be resolved into components along two axes. For an inclined plane a natural choice is for the x-direction to be parallel to the plane and the y-direction to be perpendicular to the plane. For example see Figure 1, in particular look at how the x-axis and y-axis are arranged.

The forces that act on the cart (which has mass m_2) in this experiment while the cart is moving up the inclined plane under the influence of a string and counterweight are illustrated in Figure 1. The gravitational force $m_2\vec{g}$ points directly down towards the center of the earth. The normal force $\vec{N}$ is

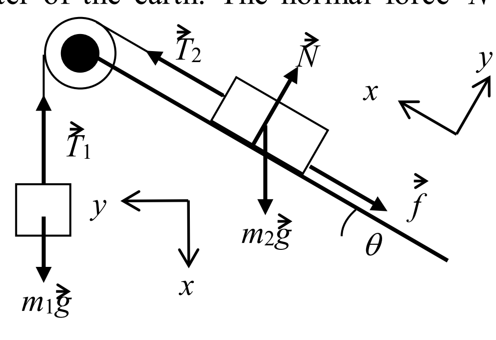

Figure 1: Schematic of Forces in Experiment

the force on the cart that prevents it from sinking into the metal track. This force points perpendicularly to the track. The force of friction $\vec{f}$ points along the track, opposite to the direction of motion of the cart. If the pulley is frictionless and massless (*which we will assume is true in this experiment*), the magnitude of the string tension is the same on both sides of the pulley $\left(\left|\vec{T}_1\right| = \left|\vec{T}_2\right| \equiv T\right)$ and acts to "pull" the objects to which it is connected. In this example, tension in the string is pulling on the cart and the hanging weight. If the cart and weight are motionless, $T = m_1 g$, where m_1 is the mass of the counterweight. However, if the cart and counterweight are accelerating, the tension is different from $m_1 g$. This can be seen by writing the equation of motion for the counterweight as

$$-T + m_1 g = m_1 a \qquad (2)$$

where the positive direction is downward. Solving for T, we find

$$T = m_1(g-a) \qquad (2a)$$

Along the direction of the track, the equation of motion for the cart is given by

$$T - f - m_2 g \sin\theta = m_2 a \qquad (3)$$

Using Eq. 2a to eliminate T from Eq. 3 and assuming that f is negligible leads to

$$m_1(g-a) - m_2 g \sin\theta = m_2 a \qquad (4)$$

which can be rearranged to give

$$a = g(m_1 - m_2 \sin\theta)/(m_1 + m_2) \quad (4a)$$

D. Procedure

The procedure for this experiment is divided into two main parts. **First, to get an estimate of the frictional force, you will study the case of *zero* acceleration (and zero velocity). Second, to test Newton's Second Law, you will add an additional counterweight so that the cart accelerates up the incline.**

As you work, record your measurements, derivations and calculations in your notebook. For any measured

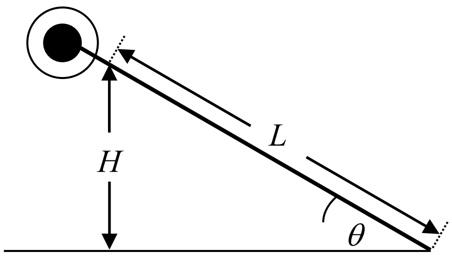

Figure 2: Calculating the angle

quantities, also record an estimate of the uncertainty.

D.1 Zero Acceleration and the Angle of Incline

The first task is to **estimate the mass of the counterweight required to balance the cart**; to do this you need to know the mass of the cart and the angle of the track. **Determine the mass m_2 of the cart** using one of the electronic balances in the lab. Assume that the balance is accurate to within 1 unit of its most significant digit, *i.e.* a reading of 512.4 g has an estimated error of ± 0.1 g.

Next **measure the angle θ to which the plane has been set. To get this angle, carefully measure the height H and distance L shown in Figure 2.** These measurements may not be as easy as they sound. **Discuss the difficulties you encounter and what you do to overcome them. Be certain to estimate the uncertainty of your measurements and discuss how you made those estimates.**

Calculate $\sin\theta$ and the estimated error in $\sin\theta$. Note that $\sin\theta$ = opposite side divided by the hypotenuse of a 90° triangle. The latter is determined from the estimated errors in L and H. Also **calculate θ and estimate the error in your calculation of θ**; you may calculate the error using the computational method described in Appendix V. Simply see how much θ changes when you plug in the value for L or H, including the estimated error in each, and then use the quadrature rule to add these two contributions.

Next, **calculate the mass m_b of the counterweight** needed to balance the force of gravity acting on the cart. Assume (*for the moment)* that the friction is negligible, so that zero acceleration occurs when the component of the cart's weight along the incline is equal to the counterweight. Use Eq. 4a with a = 0 to determine the counterweight mass m_1 in terms of m_2 and θ; we will call this mass $m_b = m_1$.

Place the cart near the middle of the track. If it has not already been done for you, **loop a string over the pulley and attach one end to the cart and the other to the mass hanger. Add masses to the hanger until the total mass is m_b.** (*Note that the mass hanger itself has some mass and should be measured using a scale. This should be labeled on the mass hanger; it is most likely 50 g.*) Can you balance the system so the cart is not moving? Does the cart roll up or down?

You may discover that it is difficult to balance the cart this way; the problem can be traced to measuring $\sin\theta$ with sufficient precision to balance the system *a priori.* To get a better determination of the angle, you will find the range of counterweight masses that balance the cart and then work backwards to find θ.

By adding and removing mass to the weight hanger, **find the minimum and maximum masses that lead to zero acceleration**, $m_{\min}$ and $m_{\max}$. Within this range, you should be able to position the cart anywhere along the track and have it not move. Because of *static* (stationary) *friction* you may find that there is a range of masses for which the cart will remain stationary. Alternatively, you may find the range of masses for which a moving cart continues to move with a constant velocity (remember, constant velocity still implies zero acceleration). Because *kinetic* friction (friction while an object is moving) is smaller than static friction, this latter range would be smaller.

Calculate the mass in the middle of the range $m_b = (m_{\max} + m_{\min})/2$ and its uncertainty $\delta m_b = (m_{\max} - m_{\min})/2$. Use Eq. 4a with m_1 set equal to m_b to **calculate a new determination of $\sin\theta$ from m_b and m_2.** How does this new value compare to the values found from direct measurement of L and H? You will use this new value of $\sin\theta$ to make predictions in the next part of the experiment. Errors due to friction are now contained within δm_b. Why do you think this is the case? Any errors due to the masses being off from their stamped values are negligible in comparison; however you should still measure the masses!

D.2 Uniform Acceleration

In this part of the experiment you will record the motion of the cart as it accelerates up the incline under the influence of a counterweight mass m_1 equal to m_b plus 25 grams. We will call this mass m_e. Examine the encoded pulley, light source and light detector at the top of the incline. The *Logger Pro* program records time intervals between successive blockages of the light path from the photo-diode (*source*) to the photocell (*detector*) caused by the spokes of the rotating pulley. Each time interval corresponds to a displacement of Δs = 0.015 m. Under your command, *Logger Pro* will measure the time intervals and automatically calculate position, velocity and acceleration as a function of time.

Place the cart at the bottom of the track and make sure that the encoded pulley turns freely (*try lifting the string and giving it a **light spin***). The string will probably rub against the pulley; remember however, that you have accounted for this error all ready. With the cart held near the bottom of the track, **add masses to the hanger so that the total mass is 25 grams more than m_b** (i.e. so that the total mass is now m_e) the mean balancing mass you found earlier. Make sure that the counter-weight doesn't swing. Be ready to catch the cart before it crashes into the stop at the top. If the counterweight will hit the floor before the cart hits the stop, note the spot along the track where this will happen and try to prevent it — or shorten the string.

Start the *Logger Pro* program by clicking on its desktop icon. A special file has been created to set up the program for use with the encoded pulley. **Load this program** using the FILE / OPEN command on the title bar and locating the file P:*Logger Pro 3__Mech Labs\IP.mbl* that you should be able to find on your local hard disk or on the P: drive of the lab server, *Wertsrv*. The P: drive on this server may show up as the PROGRAMS drive. When the screen prompting you to connect sensors pops up, send it away by **pressing the "OK" button**. If you accidentally press the "Cancel" button, re-open the file and try again.

To start recording data with Logger Pro, you can use the mouse to **click on the COLLECT button**. This button starts and, when you click it again, stops data collection. You may also start and stop data collection with the F11 key on the computer keyboard if you have previously highlighted the proper window.

Experiment by holding the cart and moving it along the track while recording the motion with the *Logger Pro* program. The computer will display a table of data consisting of time, time interval, position, velocity and acceleration. Does the data make sense based on how you moved the cart? **Do some practice runs, releasing the cart from near the base of the track and catching the cart before it hits the stop or the counterweight hits the floor**. If there are magnets installed on your track, release the cart a few cm from the base of the track; otherwise the magnets on the cart and track might influence your results.

Practice until you can smoothly operate the equipment and the program. Then **acquire a good set of data** to be used for analysis and your paper. When you have a good data set, **use the FILE / SAVE AS command to save your data.** Place your file in your group's subfolder under the appropriate course folder on the server's *L:* drive.

E. Analysis

Partners are encouraged to cooperate on the analysis, but each partner must report his or her own data set separately. The basic idea of the analysis is two-fold.

First, you need to **find the measured acceleration using *Logger Pro* and *Origin*.**

Second, you will analyze the motion based on the theory provided at the beginning of this lab and **predict the expected acceleration.** This prediction is itself based on measured quantities (*angles and masses*) therefore you *will* need to **propagate the uncertainties in your primary measurements** (i.e. masses and any angles) to arrive at the uncertainty in your prediction.

Your conclusions will be based on a comparison of the prediction and the measurement of the acceleration.

E.1. Data Analysis

Your first objective is to **determine the acceleration directly from the data acquired using *Logger Pro*.** You will do this with three different techniques. Hopefully they will give answers that agree with each other to within estimated errors.

The first two techniques rely on calculations performed by the *Logger Pro* program. To perform these calculations, you need to define a *Region of Interest* or ROI. This is the region of the *Logger Pro* plot that will be used in each calculation. This step is necessary because a graph made by *Logger Pro* typically includes regions that should be excluded from analysis. In your case, the beginning and end of the graph might be corrupted by the starting and stopping of the cart. To define a region of interest, **highlight data in the STime column.** This will also highlight data points on the plots. **Select a region of data that looks linear in the SAccel and SVel plots.**

Select the SAccel plot. *Logger Pro* can now **calculate the average acceleration within this ROI**. Use the mouse to select from the title bar ANALYZE / STATISTICS. A box should pop up that gives you the mean value and standard deviation (σ) of the measured acceleration. <u>**Record these numbers and calculate the uncertainty in the measured acceleration.**</u> This uncertainty δ_a (also called the *standard error* or the *uncertainty in the mean*) may be calculated from the standard deviation using the formula in Appendix V.B.1.3.

To get a second estimate of the acceleration, **go to the plot of velocity versus time, and choose ANALYZE / LINEAR FIT.** From the box that pops up, <u>**record the slope and its uncertainty.**</u> To see the error in the slope, you may have to double-click on the text box that pops up after performing the linear fit and check the box that instructs *Logger Pro* to display the *STANDARD DEVIATION OF THE SLOPE*. This slope should also be a measure of the acceleration. <u>**Do these two values agree to within their uncertainties? Are they exactly the same? Why, why not? Which result should be more accurate?**</u>

Logger Pro has various limitations on its calculation and plotting abilities. You should therefore **transfer the data to *Origin*** for one last round of analysis. In *Logger Pro*, use the FILE / EXPORT AS / TEXT command to create in your group's directory an ASCII version of your data. ASCII files are plain text files and generally end with the extension *.txt*. **Start the *Origin* program and use its FILE / IMPORT / SIMPLE SINGLE ASCII command to load the ASCII file into *Origin.*** Clean up the file by deleting the Time and GateState columns and any rows at the top and bottom that you know to contain bad data. You can probably do this by inspecting the velocities or accelerations and looking for anomalies or only keeping data from your ROI in *Logger Pro*.

Plot the velocity versus time (with the SVel and STime columns) using the PLOT / SCATTER command. Use the Column Add function to add vertical error bars for the uncertainty in the velocity measurements δ_v. **Assume that the velocity is known to ± 0.008 m/s** for each data point. This number is based on measurements made previously by the staff using this equipment. When you make your scatter plot, be sure that these vertical error bars are included. Do not include any horizontal error bars. If you still see glitches at the beginning or end of your plot, you should delete this data and replot it. **Double-click on the axes to label them properly (and include units).**

Fit the data using ANALYSIS / LINEAR Fit. A new text window will appear with the results of the fit to the equation $y=a+b*x$. **The slope of this velocity-versus-time graph is the measured acceleration, $a_{meas.}$** The fit also provides you with the uncertainty, or error, in the slope. Copy the fit results into a text box on the graph. Adjust the font to something reasonable, add your name and date, save your project file, and print the graph. **Include the graph with your paper.**

E.2. Predicting the Acceleration

Calculate the predicted acceleration $a_{pred} = a$ based on Equation 4a. Also calculate the uncertainty on the predicted acceleration. Since your best estimate of $\sin\theta$ comes from your measurements of m_2 and m_b, first you should use the relationship between $\sin\theta$, m_2, and m_b to **rewrite Eq. 4a in terms of g, m_e, m_2, and m_b.** As always, you may **ignore the contributions to the uncertainty from variables whose uncertainty is negligible, but you should explain your thinking.**

F. Conclusions

Quote your values of *a* (***measured and predicted***) **and their uncertainties to the correct number of significant figures** (*the correct number of significant figures is determined by the size of the uncertainty; see appendix V for detailed instructions.*)

Write down as logical a completion to the "story" as you can. How well does your measurement agree with your prediction? Do they agree within their uncertainties? If not, can you suggest what additional sources of error or systematic effects were not adequately accounted for? What effect should friction have (be quantitative)? What can you conclude about Newton's Second Law?

G. Fill-in-the-Blank Lab Paper

You need to **complete a fill-in-the-blank lab paper for this experiment**, available in Section IX just behind the Error Analysis and Propagation Exercise you completed last time. You should read through Appendix II on paper writing. This appendix at the back of the lab manual describes the format for your paper. Also, take time to examine Appendix XI, which contains a sample paper. You are encouraged to work together with your partner or other classmates on completing derivations, calculations, and even discussing your approach to paper writing. However, your lab papers must be completed independently. Copying of paragraphs, sentences or even phrases will be taken as evidence of cheating and dealt with severely.

Cover sheets are available on Canvas in the Modules section. Attach one at the beginning of the Fill-in-the-Blank paper. The fill-in-the-blank papers are due one week from your laboratory meeting.