SI P121 Final Exam Summary Session S24-Answers

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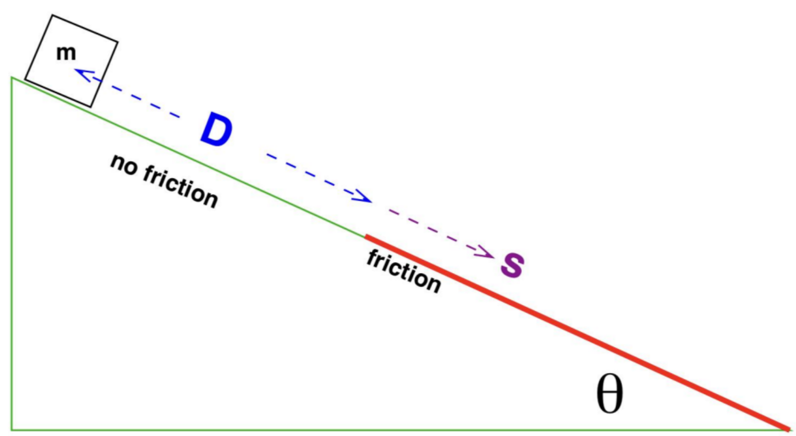
Learning Objectives

By the end of this session, students will be able to:

- Draw accurate and complete FBDs and XFBDs and apply N2L
- Recall the conditions for CofME, CofLM, and CofAM and apply them
- Find position, velocity, and acceleration using (translational/rotational) Kinematics
- Physics :)

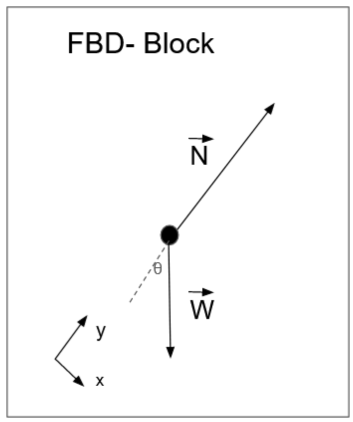
Question 1



A block of given mass m is placed on a fixed ramp as shown above. The angle of the ramp relative to the horizontal is given as θ. The block is released from rest at the top of the ramp and then is allowed to slide down the ramp. The top section of the ramp (with given distance D) is frictionless. After the frictionless section there is a section of the ramp with a known coefficient of kinetic friction μ. The block continues to slide down the ramp until it comes to a stop.

a. Immediately after the block is released at the top of the ramp, what is the magnitude of acceleration, a?

   This is a problem we've seen a few times before. It's a simple application of **N2L** and **The Force Recipe**. Our first step is to make an FBD:



   Note the tipped coordinate system. Next, let's consider the acceleration in both directions. Since the block is constrained to move on the ramp, we can say that $a_y = 0$. This means that the magnitude of the acceleration is equal to a_x, which we can find using N2L:

   N2L- BLOCK, X-COORDINATE:

$$F_{net,x} = ma_x$$

$$W_x = ma_x$$

$$mg\sin\theta = ma_x$$

$$a = \boxed{a_x = g\sin\theta}$$

b. Calculate the speed of the block after it has travelled a distance D. This will also be the speed of the block the instant it hits the non-frictionless section of the ramp.

   There's a couple ways to do this part. Let's start first by looking at the forces on the block. Weight is conservative, and Normal does no work because it is perpendicular to the path of motion. Therefore, we are allowed to use **CofME**.

$$'BEFORE' =' AFTER'$$

$$E_{TOT} = E'_{TOT}$$
$$U + K = U' + K'$$
$$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$$

For potential energy, we need to set a point where $y = 0$. The easiest way to do this is to make it so that the final y position of the box in this part is zero. This makes the initial position $y = D\sin\theta$

$$mgD\sin\theta = \frac{1}{2}mv'^2$$

$$\boxed{v' = \sqrt{2gD\sin\theta}}$$

c. Calculate the distance s that the block slides past the boundary before coming to a stop.
There are a couple of ways you can go about this problem. It's very possible to do with the Work-Energy Theorem, but since we're going to need acceleration for part (d) anyways, let's go ahead and use 1-D Kinematics. First we need to know whether the acceleration is constant or variable over the given interval (past the boundary). If we draw a FBD of our block with a tilted coordinate axis (aligned with the ramp), we get the following Newton's 2nd Law equations:

N2L X

$$\Sigma F_x = ma_x$$
$$\Sigma F_x = W_x - f_k$$
$$ma_x = mg\sin\theta - \mu N$$

N2L Y

$$\Sigma F_y = ma_y$$
$$\Sigma F_y = 0$$
$$\Sigma F_y = N - W_y$$
$$N = mg\cos\theta$$

Let's circle back around and plug in this value for N:

$$ma_x = mg\sin\theta - \mu mg\cos\theta$$
$$a_x = g(\sin\theta - \mu\cos\theta)$$

Once the block has slid onto the rough portion of the ramp, the coefficient of friction μ is not changing. Since none all of the components of the block's acceleration we have just calculated are constant, we can state that the the block is undergoing Constant Acceleration.

Constant Acceleration

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

We solved for the speed of the block as it crosses the border in part (b), we know the block comes to rest at the position we're trying to solve for, and just found its acceleration above, so let's go ahead and plug those in.

$$a = g(\sin\theta - \mu\cos\theta)$$
$$0 = \sqrt{2gD\sin\theta} + g(\sin\theta - \mu\cos\theta)$$
$$s = 0 + \sqrt{2gD\sin\theta}t + \frac{1}{2}g(\sin\theta - \mu\cos\theta)t^2$$

Solving for t, we get:

$$t = -\frac{\sqrt{2gD\sin\theta}}{g(\sin\theta - \mu\cos\theta)}$$

With a little algebra, we find that

$$s = -\frac{D\sin\theta}{\sin\theta - \mu\cos\theta}$$

$$\boxed{s = \frac{D\sin\theta}{\mu\cos\theta - \sin\theta}}$$

d. Suppose that the block is not a block at all but is in fact a box containing Darth Vader. The combined mass of the box and Darth Vader is equal to m, and the mass of Darth Vader is equal to M_D. Find the effective weight of Darth Vader right before the box comes to a stop. Express it as a vector with the correct unit vector notation.

When Darth Vader slides down the ramp, he is in a **non-inertial reference frame.** This means that he will experience a fictitious force that points in the opposite direction to the acceleration of his reference frame.

This fictitious force acts a weight force and causes Darth Vader to feel heavier or lighter, depending on the direction of acceleration.

The equation to determine Darth Vader's effective weight is

$$\vec{W}_{eff} = \vec{W}_{real} - M_D\vec{A}$$

We're going to use the coordinate system that we used in part a- this means that x, or $\hat{\imath}$, points along the ramp. This makes calculating $\vec{A}$ easy, but it means we have to break up $\vec{W}_{real}$ into components like so.

$$\vec{W}_{real} = M_D g \sin\theta\hat{\imath} - M_D g \cos\theta\hat{\jmath}$$

So we can go ahead and write the effective weight as:

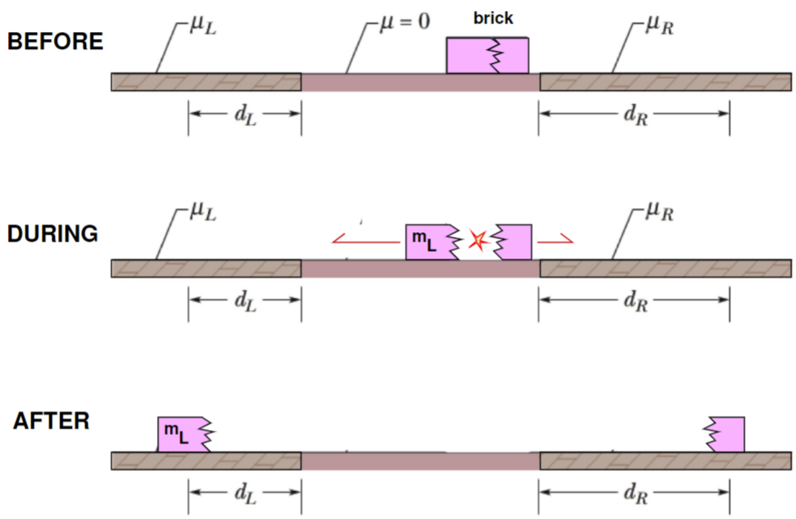
$$\vec{W}_{eff} = M_D g \sin\theta\hat{\imath} - M_D g \cos\theta\hat{\jmath} - M_D g(\sin\theta - \mu\cos\theta)\hat{\imath}$$

Collecting our terms...

$$\boxed{\vec{W}_{eff} = M_D g\mu \cos\theta\hat{\imath} - M_D g \cos\theta\hat{\jmath}}$$

We've shown that Darth Vader will feel a force with two components that depend on the angle of the ramp. We can check that this value matches our expectation by changing the angle θ. What weight does Darth Vader feel when the box is decelerating on a flat surface?

Question 2



An explosive charge is placed inside a brick which starts at rest on a table. The brick explodes horizontally into just two fragments. The left fragment has a given mass m_l and moves left and eventually encounters a surface region with a given coefficient of kinetic friction μ_l. The left fragment comes to rest in this after sliding a given distance d_l. The right fragment likewise encounters a region with a given coefficient of kinetic friction μ_r and eventually comes to rest after sliding a given distance d_r.

What was the **total mass** M of the original brick? Give your answer in terms of the given parameters of the problem. **Important**: Note that the mass of the **right** fragment is **not** a given parameter in this problem. Explain your work.

We're going to need two different strategies to solve this problem. Let's first talk about the 'before' and 'during' parts of this problem. During this period, if we define the system as the *brick*, we see that it is isolated. *(Right?)* Therefore, we're going to want to use the Conservation of Linear Momentum.

Conservation of Linear Momentum

$$P_{tot} = P'_{tot}$$

$$m_a v_a + m_b v_b = m_a v'_a + m_b v'_b$$

If we set $m_a = m_L$ and $m_b = m_R$, we get the following:

$$0 = m_L v_L + m_R v_R$$

$$m_L v_L = -m_R v_R$$

We're trying to find the mass of the entire brick, but we can easily say that $M = m_L + m_R$. Since m_L is a given parameter, we know we need to solve for m_R in the above equation. We're now going to use the Work-Energy Theorem to find the velocity of each piece of the brick directly after the explosion (before they reach the rough parts of the table).

Since the explosion happens on the frictionless surface, the initial velocity of each piece as it crosses the border is the same as just after the explosion. Knowing this, we can set up the Work-Energy Theorem:

Work-Energy Theorem (During→After)

$$W_{tot} = \Delta K$$

$$W_{tot} = W_f$$

Definition of Work:

$$W = \vec{F}\vec{d}$$

$$W = Fd\cos\theta$$

$$W_{fL} = (\mu_L N_L)(d_L)\cos(180)$$

$$W_{fR} = (\mu_R N_R)(d_R)\cos(180)$$

A quick FBD and invocation of Newton's 2nd Law finds that

$$N_L = m_L g$$

$$N_R = m_R g$$

Plugging these in, we can now find:

$$W_{fL} = -\mu_L m_L d_L g$$

$$W_{fR} = -\mu_R m_R d_R g$$

We know $\Delta K = K_f - K_i$. Since both pieces end at rest, this easily simplifies as follows.

$$-\frac{1}{2}m_L v_L^2 = -\mu_L m_L d_L g$$

$$-\frac{1}{2}m_R v_R^2 = -\mu_R m_R d_R g$$

$$v_L = \sqrt{2g\mu_L d_L}$$

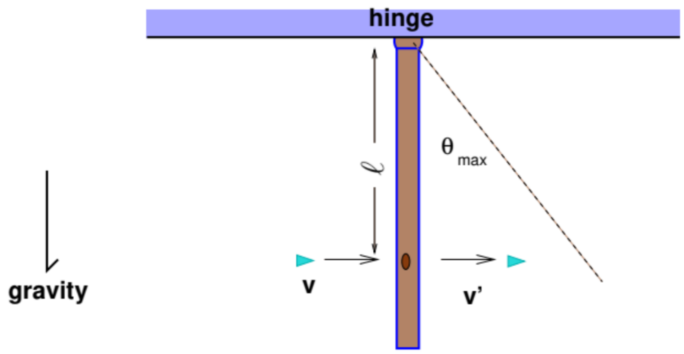
$$v_R = \sqrt{2g\mu_R d_R}$$

Now we get to wrap all the way back around to Conservation of Linear Momentum. Recalling the earlier equation we left off on, we can plug in our values for speed. With a little algebra, we come up with our final answer.

$$m_R = m_L\sqrt{\frac{\mu_L d_L}{\mu_r d_R}}$$

$$\boxed{M = m_L\left(1+\sqrt{\frac{\mu_L d_L}{\mu_r d_R})}\right)}$$

Question 3



A thin rod of given mass M and given length L hangs vertically from the ceiling attached to an ideal hinge. A unit of blaster fire which miraculously behaves exactly like a bullet of given mass m strikes the rod horizontally at a given point l from the hinge. The ~~bullet~~ unit of blaster fire pokes a small hole right through the rod. The speed of the *blaster fire* before the impact is given as v and the speed after the impact is given as v'. What is the angle θ corresponding to the maximum swing angle of the rod after impact? Ignore any effect of gravity on the bullet.
This is again going to be a two-part problem. First let's start by establishing the two different "before and after" periods we're going to use. If we consider just the time *directly before* to *directly after* the collision, we can see that the *net Torque on the system* of the "bullet" and the rod is equal to zero. Obviously the system is not isolated because of the hinge force and the weight force, but the hinge doesn't enact any torque on the system as it's the pivot point (r=0), and we've been told to neglect the effect of gravity on the "bullet". This means we're clear to use Conservation of Angular Momentum! We'll talk about the other "before and after" period later.

Conservation of Angular Momentum

$$I_R\omega_R + I_B\omega_B = I_R\omega'_R + I_B\omega'_B$$

Since we're dealing with a very small period of time before and after the collision, we can say that the "bullet" is *basically* a point particle moving in Uniform Circular Motion. (Yes, this *is* a swindle - but it's also a very good approximation which yields accurate results.) We can use the Rolling Constraint to convert between linear and angular speed. We also know that for a rod rotating about its endpoint, $I = \frac{1}{3}MR^2$. Now we can plug in our values for I and $\omega = 0$:

$$0 + ml^2\frac{v}{l} = \frac{1}{3}ML^2\omega_R + ml^2\frac{v'}{l}$$

$$mvl = \frac{1}{3}ML^2\omega_R + mlv'$$

$$\omega_R = \frac{3ml(v - v')}{ML^2}$$

Now we've calculated the angular velocity of the rod directly after the collision with the "bullet", but we still need to find the maximum angle the rod will reach with respect to the vertical. We've done problems like this before using Conservation of Mechanical Energy. To check whether or not this is valid, we first define our second interval to be *just after* the collision to the instant where the rod reaches its maximum height (and associated angle). Over this interval, our system is just the rod itself. The only forces acting on the rod are the Weight and Hinge forces. As we've proven before, the Weight force is conservative and the Hinge force does zero work, so we are free to use Conservation of Mechanical Energy.

Conservation of Mechanical Energy

$$E_{tot} = E'_{tot}$$

$$K_{tot} + U_{tot} = K'_{tot} + U'_{tot}$$

$$K_R + U_W = K'_R + U'_W$$

We'll define y=0 at the pivot point to simplify the trig we'll be using to find θ.

$$y_f = -\frac{L\cos\theta}{2}$$

Remember that our reference point for the y-position of the rod is the center-of-mass, which is why we're dividing L by 2 here.

$$\frac{1}{2}I\omega^2 + mg(-L/2) = \frac{1}{2}I(0)^2 + mg\left(-\frac{L\cos\theta}{2}\right)$$

From here, we plug in our previous value for ω and use the I of a rod just like before, and the rest is algebra.

$$\boxed{\theta = \cos^{-1}\left(1 - \frac{3m^2l^2}{M^2L^3}(v - v')\right)}$$

Good luck on Tuesday! If you have any additional questions, feel free to reach out via email. Thanks for the great semester everyone!