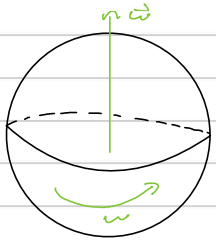
Vectors and Rotational Quantities

Vector Nature of Rotational Quantities



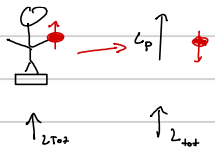
Right-Hand Rule

Curly Fingered RHR

1. Stroke out Right Hand
2. Curl Fingers in direction of rot
3. Thumb direction points in vector direction

Names $\vec{\omega}, \vec{\alpha}, \vec{L}, \vec{\tau}$

Example: Bike Wheel / Gyro



Recall dot Product

$\vec{A} \cdot \vec{B} = AB\cos\theta$

Formal Definition of cross

$\vec{\tau} = \vec{r} \times \vec{F} \rightarrow |\vec{\tau}| = rF\sin\theta$

$\vec{L}_{pp} = \vec{r} \times \vec{p} \rightarrow |L_{pp}| = rmv\sin\theta$

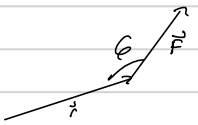
Cross Product

$\vec{C} = \vec{A} \times \vec{B}$

$|\vec{C}| = AB\sin\theta$

$\vec{C} \perp \vec{A}, \vec{C} \perp \vec{B}$

Vector Cross Product Right Hand Rule



1. Point lmp Right Hand in direction of vector 1
2. Rotate hand so palm is $\vec{F}$ dir
3. Curl Fingers in $\vec{F}$ direction
4. Thumb Points to cross dir

Simple Harmonic Motion (SHM)

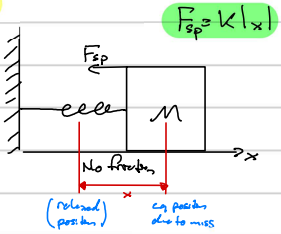
(Simple harmonic oscillator)

Hooke's Law

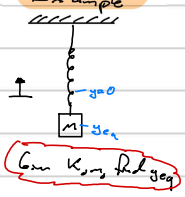
$F_{sp} = k|x|$

F_{sp}

- Known Force
- Contact Force
- Conservative

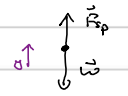


Example



Can know k and y_{eq}

FBD block



Very simple problem to solve with FBD/N2L

N2L y-dir

$F_{sp} - w = ma_y$

$a = 0$ @ equilibrium

$F_{sp} - w = 0$

$k|y_{eq}| - mg = 0$

$\rightarrow |y_{eq}| = \frac{mg}{k}$ below zero line

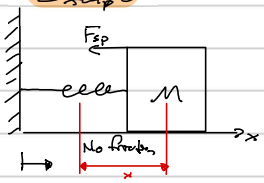
$y_{eq} = \frac{-mg}{k}$

Mechanics

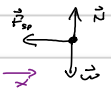
How? Kinematics — $y(t)$? $x(t)$?

Dynamics — why?

Example



FBD block



N2L x-dir

$-F_x = ma_x$

$-kx = ma$

$ma + kx = 0$

$a + \frac{k}{m}x = 0$

2nd order DE can solve below

$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$ — Equation of Motion

$s^2 + \frac{k}{m} = 0$

$s = \pm i\sqrt{\frac{k}{m}}$

Motion of $\frac{d^2y}{dt^2} + \frac{k}{m}x = 0$

$s^2 + \frac{k}{m} = 0$

$s = \pm i\sqrt{\frac{k}{m}}$

$x(t) = A\sin(\sqrt{\frac{k}{m}}t) + B\cos(\sqrt{\frac{k}{m}}t)$

Plug $x(t)$ and $x''(t)$ into equation to get 0 because ✓

$x'(t) = \sqrt{\frac{k}{m}}A\cos(\sqrt{\frac{k}{m}}t) - \sqrt{\frac{k}{m}}B\sin(\sqrt{\frac{k}{m}}t)$

$x''(t) = -\frac{k}{m}A\sin(\sqrt{\frac{k}{m}}t) - \frac{k}{m}B\cos(\sqrt{\frac{k}{m}}t)$

IVP: at $t = 0$ $x = x_0$ $v = 0$

$x(t) = A\sin(\sqrt{\frac{k}{m}}t) + B\cos(\sqrt{\frac{k}{m}}t)$

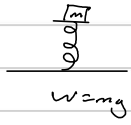
$x'(t) = \sqrt{\frac{k}{m}}A\cos(\sqrt{\frac{k}{m}}t) - \sqrt{\frac{k}{m}}B\sin(\sqrt{\frac{k}{m}}t)$

$\begin{cases} x_0 = B(1) & B = x_0 \\ 0 = \sqrt{\frac{k}{m}}A & A = 0 \end{cases}$

$x(t) = x_0\cos(\sqrt{\frac{k}{m}}t)$

Wait! Let's Revisit it!

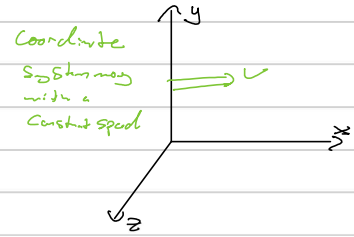
Suppose



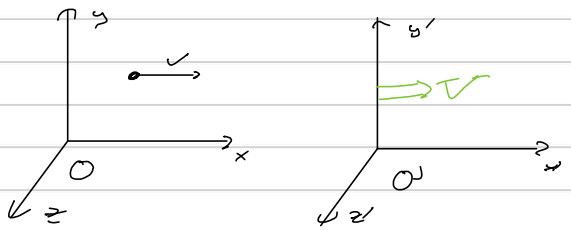
Newton's First Law:

If $F_{Net} = 0$

Then $v = \text{constant}$



Coordinate System with constant velocity



$v' = v - V$

velocity of particle from O as seen by O' frame

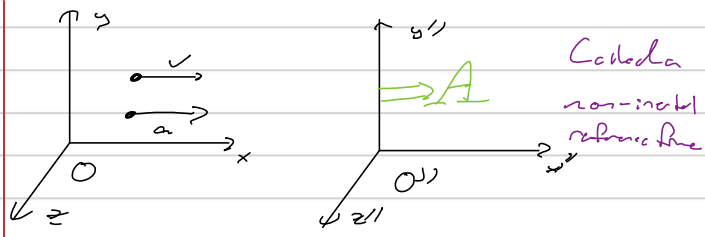
$\frac{dv'}{dt} = \frac{dv}{dt} - \frac{dV}{dt}$ In a inertial frame accelerations are the same

$\rightarrow a' = a - 0 \quad \therefore a' = a$

By Eqn: $\begin{cases} F = ma \\ F' = ma' \end{cases} \rightarrow F' = F$ N2L still works!

This is not N1L ans

What if we have an accelerating frame



Called a non-inertial reference frame

$a'' = a - A$

acceleration of particle in O as seen from O''

Then $F'' = F - mA$

Force in non-inertial is not equal to other one

$\rightarrow \vec{F}_{Non-inertial} = \vec{F}_{Real} + \vec{F}_{Fict}$

non-inertial = real + fictitious Force vectors

Fict Force

$\vec{F}_{Fict} = -m\vec{A}$

$\Rightarrow W_{Fict} = mg_{Fict}$ where $g_{Fict} = \vec{A}$

effective weight

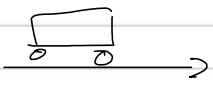
$\vec{W}_{eff} = \vec{W}_{real} + \vec{W}_{Fict}$

O in inertial reference frame

$mg_{eff} = mg_{real} + mg_{Fict}$

$= mg\hat{j} + m(-\vec{A})$

Think of Boy in a car



Stop quickly

$a = \Leftarrow$

$b_{at}\,Fict = \rightarrow$

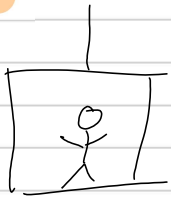
Stop Turn left

$a = a_c$

$b_{at}\,Fict = -a_c$

Example

elevator



$\uparrow A$

Feel heavier in an elevator moving up

W_{real} is $\downarrow$ W_{Fict} $\downarrow$

opposite of $A\downarrow$

Suppose Cable Snaps

Now in FF

FF: $\vec{A} = \vec{g}_{real} = -g\hat{j}$

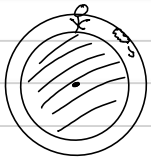
$\therefore W_{eff} = \vec{W}_{real} + \vec{W}_{Fict}$

$= -mg\hat{j} + m(-(-g\hat{j}))$

$\boxed{W_{eff} = 0}$

You feel like you're "floaty" like fully free

"Weightless" Just like astronaut in space

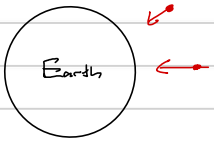


Newton's Worries About Gravity

① Extended Objects — Solution: yes, integral calculus!

↳ Can you treat them as point masses at CoM?

② Spooky action at a distance, what mechanism?



Solution: "Invent" the gravitational field

③ The equivalence of Gravitational and Inertial mass.

N2L $F = m a$

actually $m_{inertial}$ or inertial mass

UG: $F_{UG} = \frac{GMm}{r^2}$ ← m_{grav}

Why is it true $m_{inertial} = m_{grav}$?

Albert Einstein!!!

1905 → special relativity

↳ $E = mc^2$ → Inertial Ref Frame

Principle of Equivalence

"Gravity" is indistinguishable from being in an accelerating reference frame

General Relativity

$G_{\mu\nu}$ "gravity" "curvature" + $\Lambda g_{\mu\nu}$ "constant" $= 8\pi T_{\mu\nu}$ "stuff"