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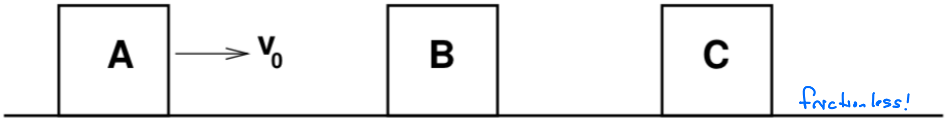
PHYS121 - Homework 6

Problem 1)



Given
- A mass = m, initial velocity v_0
- $B_{mass} = 2m$, $C_{mass} = 5m$
- B and C @ rest

a) Collision One, A and B collision is totally inelastic

Block C is not involved in the first collision, so $v_c' = 0$

C of LM Conditions: System of block A and B is isolated so C of LM is valid!

"Before" = "After"

$P_{Tot} = P'_{Tot}$

$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$ $v_A = v_0,\ v_B = 0$

Totally Inelastic

$m_A v_0 = m_A v_f + m_B v_f$ Kinematic Constraint: $v_A' = v_B' = v_f$

$m v_0 = (m + 2m) v_f$ $m_A = m,\ m_B = 2m$

$\therefore v_f = \frac{m v_0}{3m} = \frac{v_0}{3}$

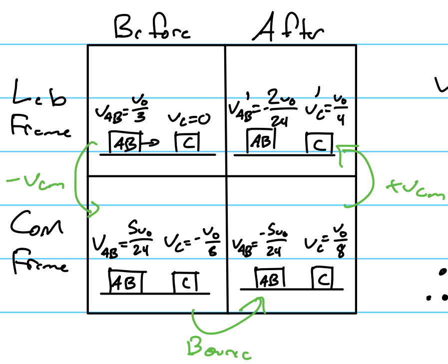
$$\boxed{\begin{aligned} v_A' &= \frac{v_0}{3} \\ v_B' &= \frac{v_0}{3} \\ v_c' &= 0 \end{aligned}}$$

b) Collision Two, B and C is totally elastic

Define a System as block AB and block C. AB is a result of an inelastic collision

- $AB_{mass} = m_A + m_B = 3m$
- $AB_{initial\ vel} = \frac{v_0}{3}$ from (a)

A and B move together $\Rightarrow v_A' = v_B' = v_{AB}'$



$$V_{cm} = \frac{m_{AB} V_{AB} + m_c V_c}{m_{AB} + m_c} \quad V_c = 0$$

$$= \frac{3m\left(\frac{v_0}{3}\right) + 0}{8m} = \frac{v_0}{8}$$

$\therefore v_{AB}' = -\frac{v_0}{12},\ v_c' = \frac{v_0}{4}$

$$\boxed{\begin{aligned} v_A' &= -\frac{v_0}{12} \\ v_B' &= -\frac{v_0}{12} \\ v_c' &= \frac{v_0}{4} \end{aligned}}$$

c) Define the system as all 3 blocks, A, B, and C

Before COL1: $v_{cm} = \frac{m_A v_A + m_B v_B + m_c v_c}{m_A + m_B + m_c}$ $v_A = v_0,\ v_B = 0,\ v_c = 0$ ← given

$$= \frac{m v_0 + 0 + 0}{m + 2m + 5m} = \frac{v_0}{8} \quad \therefore \boxed{V_{cm} = \frac{v_0}{8}}$$

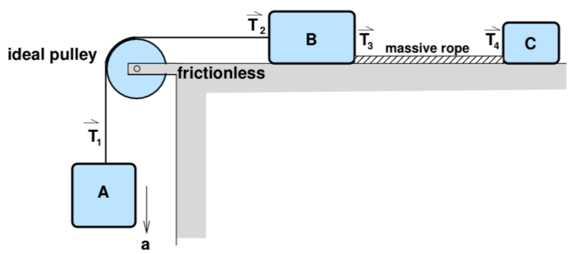
After COL1: $V_{cm} = \frac{m_A v_A + m_B v_B + m_c v_c}{m_A + m_B + m_c}$ $v_A = \frac{v_0}{3},\ v_B = \frac{v_0}{3},\ v_c = 0$ ← part (a)

$$= \frac{m\left(\frac{v_0}{3}\right) + 2m\left(\frac{v_0}{3}\right) + 0}{m + 2m + 5m} = \frac{m v_0}{8m} = \frac{v_0}{8} \quad \therefore \boxed{V_{cm} = \frac{v_0}{8}}$$

After COL2: $V_{cm} = \frac{m_A v_A + m_B v_B + m_c v_c}{m_A + m_B + m_c}$ $v_A = -\frac{v_0}{12},\ v_B = -\frac{v_0}{12}\ \ v_c = \frac{v_0}{4}$ ← part (b)

$$= \frac{m\left(-\frac{v_0}{12}\right) + 2m\left(-\frac{v_0}{12}\right) + 5m\left(\frac{v_0}{4}\right)}{m + 2m + 5m} = \frac{-\frac{3m}{12} v_0 + \frac{15m}{12} v_0}{8m} = \frac{m v_0}{8m} \quad \therefore \boxed{V_{cm} = \frac{v_0}{8}}$$

Problem 2)



Given

- A, B, C masses m_A, m_B, and m_C
- B and C move left — frictionless!
- Pulley/String btwn A and B are ideal
- Massive Rope btwn B and C mass m_r

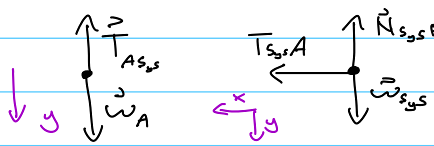
a) Tension is the only forces on the blocks! So N2L describes the forces exactly. More Mass is equal to greater Net Force as $F = ma$. a is constant. T_1 and T_2 pull the same mass ∴ $T_1 = T_2$. T_3 pulls the massive rope and block C, and T_4 pulls only Block C. T_1 and T_2 pull all blocks (minus A) and massive rope.

$$\boxed{\therefore\ T_1 = T_2 > T_3 > T_4}$$

b) Define a system as blocks B, C, and the massive rope

FBD A

FBD System



N2L y-dir A

$F_A = m_A a_A$

$W_A - T_{A,sys} = m_A a$

$m_A g - T_{A,sys} = m_A a$ Stop!

Kinematic Constraint

$a_A = a_{sys} = a$

N2L x-dir Sys

$F_{sys} = m_{sys} a_{sys}$

$T_{sys,A} = m_{sys} a$

$m_{sys} = m_A + m_B + m_r$

N3L $T_{A,sys} = T_{sys,A}$

$T_{sys,A} = a(m_A + m_B + m_r)$

Solve for 'a' with equations

$m_A g - a(m_B + m_C + m_r) = m_A a$

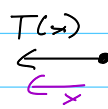
$m_A a - a(m_B + m_C + m_r) = m_A g$

$a(m_B + m_C + m_r + m_A) = m_A g$

$$\therefore\ \boxed{a = \frac{m_A g}{m_B + m_C + m_r + m_A}}$$

c) Define a system as everything from left end of massive rope to a point x

FBD System



from (b)

$$a = \frac{m_A g}{m_B + m_C + m_r + m_A}$$

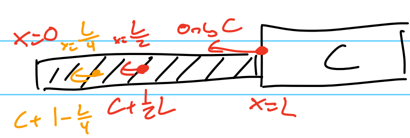
N2L System

$F_{sys} = m_{sys} a$

$T(x) = m_{sys} a$

$m_{sys} = \left(m_C + m_r\left(1 - \frac{x}{L}\right)\right)$

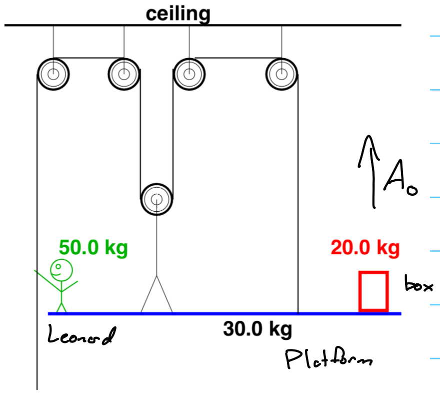
Rope given length L



Plug in a and m_{sys}:

$$\boxed{T(x) = \left(\frac{m_A g}{m_B + m_C + m_r + m_A}\right)\left(m_C + m_r\left(1 - \frac{x}{L}\right)\right)}$$

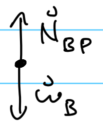
Problem 3)



Given

- Leonard mass: $m_\ell = 50.0\,kg$
- Platform mass: $m_p = 30.0\,kg$
- Box mass: $m_b = 20.0\,kg$
- Platform moves up at $A_o = 0.200\,m/s^2$

a) FBD Box



N2L y-dir Box

$F_B = m_B a_B$

$N_{BP} - W_B = m_B A_o$

$N_{BP} = m_B A_o + m_B g$

$$\boxed{\begin{aligned} W_B &= m_B g \\ N_{BP} &= m_B(A_o + g) \end{aligned}}$$

Kinematic Constraint: box moves w/ Platform

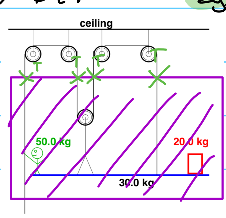
$a_B = A_o$

Numerically: $W_B = 20.0\,kg(9.81\,m/s^2) = 196.2\,N$

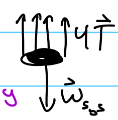
$N_{BP} = 20.0\,kg(9.81\,m/s^2 + .200\,m/s^2) = 200.2$

$\therefore$ $$\boxed{\begin{aligned} W_B &= 196.2\,N \\ N_{BP} &= 200.2\,N \end{aligned}}$$

b) Define a System as everything from the platform to just below upper pulleys



FBD System



N2L System

$F_{sys} = m_{sys} a_{sys}$

$4T - W_{sys} = m_{sys} A_o$

$T = \dfrac{m_{sys} A_o + m_{sys} g}{4}$, $m_{sys} = m_\ell + m_B + m_p$

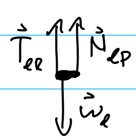
Kinematic Constraint: all move w/ Platform

$a_{sys} = A_o$

$\therefore$ $\boxed{T = \dfrac{(m_\ell + m_B + m_p)(A_o + g)}{4}}$

Numerically: $T = \dfrac{(50.0 + 30.0 + 20.0)(.200 + 9.81)}{4} = 250.25$ $\boxed{T = 250.25\,N}$

c) FBD Leonard



N2L Leonard

$F_\ell = m_\ell a_\ell$

$T_{\ell R} + N_{\ell R} - W_{\ell R} = m_\ell A_o$

$N_{\ell R} = m_\ell A_o + m_\ell g - T$

Kinematic Constraint: leonard move w/ Platform

$a_\ell = A_o$

$T_{\ell R} = T$ as rope pulls whole system @ T (from (b))

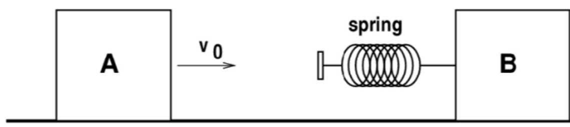
$$\boxed{\begin{aligned} W_{\ell R} &= m_\ell g \\ T_{\ell R} &= \frac{(m_\ell + m_B + m_p)(A_o + g)}{4} \\ N_{\ell R} &= m_\ell(A_o + g) - \frac{(m_\ell + m_B + m_p)(A_o + g)}{4} \end{aligned}}$$

Numerically: $N_{\ell R} = 50.0(9.81 + .200) - 250.25$
$= 250.25\,N$

$W_{\ell R} = 50.0\,(9.81) = 490.5\,N$

$$\boxed{\begin{aligned} W_{\ell R} &= 490.5\,N \\ N_{\ell R} &= 250.25\,N \\ T_{\ell R} &= 250.25\,N \end{aligned}}$$

Problem 4)



Frictionless

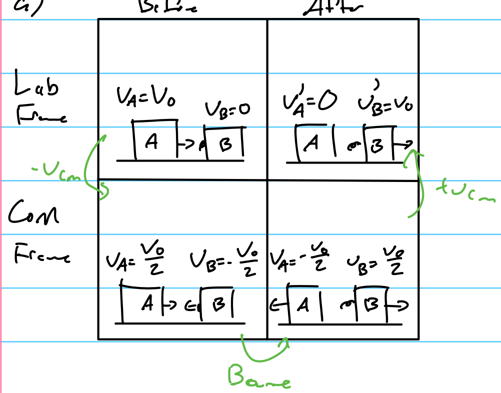
Given

- $m_A = m_B = m = 2.0\,kg$
- A velocity of $v_0 = 5.0\,m/s$

Collision is totally elastic

- Spring Constant: $k = 1.00\,N/m$

a)



$$V_{cm} = \frac{m_A V_A + m_B V_B}{m_A + m_B} = \frac{m v_0 + 0}{2m} = \frac{V_0}{2}$$

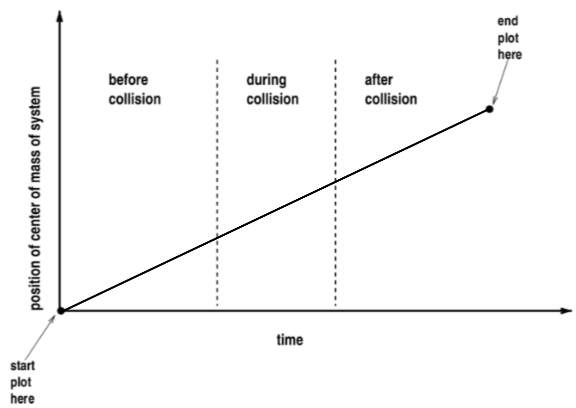
Totally Elastic Collision as F_{sp} is Conservative!

$$\boxed{\begin{aligned} V_A' &= 0 \\ V_B' &= V_0 \end{aligned}}$$

Numerically:

$$\boxed{\begin{aligned} V_A' &= 0\,m/s \\ V_B' &= 5\,m/s \end{aligned}}$$

b)



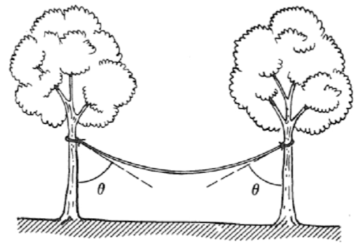
The system is not accelerating, so velocity is constant. Since velocity is constant, position is linear $x_{cm} = \int V_{cm}\,dt$

(from a)

$V_{cm} = \frac{V_0}{2}$. This is the slope of the position of the center of mass

$v_0 = 5\,m/s$ $\therefore$ $\boxed{V_{cm} = \text{slope} = \frac{5}{2}\,m/s}$

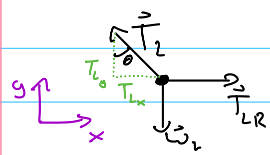
Problem 5)



Given
- Uniform rope of mass m
- Rope length L
- Rope angle θ with both trees

a) Both ends of the rope apply the same force by N3L
Identify a system from left end of rope to the center of the rope

FBD Left



- $T_{L_x} = T_L \sin\theta$
- $T_{L_y} = T_L \cos\theta$

N2L Left y-dir

$F_{L_y} = m_L \cancel{a_{L_y}} \to 0$ Rope fixed

$T_{L_y} - W_L = 0$

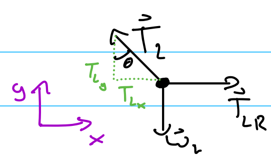
$T_L \cos\theta - m_L g = 0$, $m_L = \frac{m}{2}$ half of full rope!

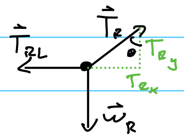
$T_L \cos\theta - \frac{m}{2} g = 0$

$T_L \cos\theta = \frac{mg}{2}$ $\therefore$ $T_L = \frac{mg}{2\cos\theta}$

$T_L = T_R$ by Newtons 3rd Law as the tree pulls on the rope with the same magnitude as the rope pulls on the tree on either side of the rope. $\therefore$ $\boxed{T_L = T_R = \frac{mg}{2\cos\theta}}$

b) FBD Left FBD Right





Although the tension in the middle of the rope contends with both ends of the rope, $T_{LR} = T_{RL}$ by N3L.
Therefore solving for T_{LR} solves the problem.

- $T_{L_x} = T_L \sin\theta$
- $T_{L_y} = T_L \cos\theta$

N2L Left x-dir

$F_{L_x} = m_L \cancel{a_{L_x}} \to 0$ Rope is Fixed

$T_{LR} - T_{L_x} = 0$

$T_{LR} - T_L \sin\theta = 0$

$T_{LR} = T_L \sin\theta$, $T_L = \frac{mg}{2\cos\theta}$ (from a)

$T_{LR} = \frac{mg}{2\cos\theta} \sin\theta$

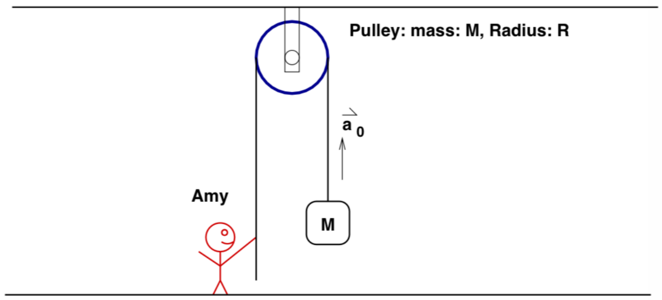
$T_{LR} = \frac{mg\tan\theta}{2}$

By N3L, $T_{LR} = T_{RL}$, so let

$T_{LR} = T_{RL} = T_c$ for the rope's center

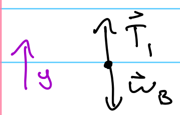
$\therefore$ $\boxed{T_c = \frac{mg\tan\theta}{2}}$

Problem 6)



Given
- Block and Pulley Mass = M
- Amy Mass = $3M$
- Block constant acceleration = $\vec{a}_0$
- Pulley Inertia: $I = \frac{1}{2} m R^2$
- Pulley Radius = R

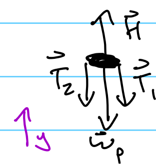
FBD Block



N2L Block

$$F_{By} = m_B a_{By} \quad a = a_0,\ m_B = M$$
$$T_1 - W_B = M a_0$$
$$T_1 = M a_0 + M g \rightarrow T_1 = M(a_0 + g)$$

FBD Pulley

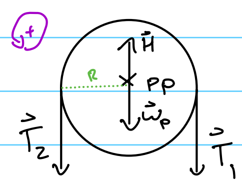


N2L Pulley

Pulley Fixed

$$F_{Py} = m_p \cancel{a_{Py}}^{0}$$
$$H - T_1 - T_2 - w_p = 0$$
$$H - T_2 = M(a_0 + g) + Mg$$
$$H - T_2 = M(a_0 + 2g)$$

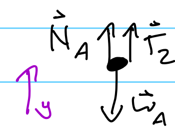
X FBD Pulley



N2L Rot Pulley

$$\tau_{Net} = I_{Net}\, \alpha$$
$$\tau_{\vec{w}} + \tau_{\vec{H}} + \tau_{T_1} + \tau_{T_2} = \frac{1}{2} m_p R^2 \alpha,\ \alpha = \frac{a_0}{R} \text{ R.C}$$
$$0 + 0 - RT_1 + RT_2 = \frac{1}{2} M R^2 \left(\frac{a_0}{R}\right)$$
$$R(T_2 - T_1) = \frac{1}{2} M R a_0$$
$$T_2 - T_1 = \frac{1}{2} M a_0$$
$$\therefore T_2 = \frac{1}{2} M a_0 + T_1$$
(from block) $$T_2 = \frac{1}{2} M a_0 + M(a_0 + g)$$
$$T_2 = M\left(\frac{1}{2} a_0 + a_0 + g\right)$$
$$\therefore T_2 = M\left(\frac{3}{2} a_0 + g\right)$$

FBD Amy



N2L Amy

Amy does not move

$$F_{Ay} = m_A \cancel{a_{Ay}}^{0}$$
$$N_A + T_2 - W_A = 0,\ W_A = 3Mg$$
$$N_A + T_2 = 3Mg$$
$$N_A = 3Mg - T_2,\ T_2 = M\left(\frac{3}{2} a_0 + g\right)$$
$$= 3Mg - M\left(\frac{3}{2} a_0 + g\right)$$
$$= M\left(3g - \frac{3}{2} a_0 - g\right)$$
$$N_A = M\left(2g - \frac{3}{2} a_0\right)$$

Forces on Amy: $\vec{W}_A$, T_2, and $\vec{N}_A$

$$\boxed{\begin{aligned} W_A &= 3Mg \\ T_2 &= M\left(\tfrac{3}{2} a_0 + g\right) \\ N_A &= M\left(2g - \tfrac{3}{2} a_0\right) \end{aligned}}$$