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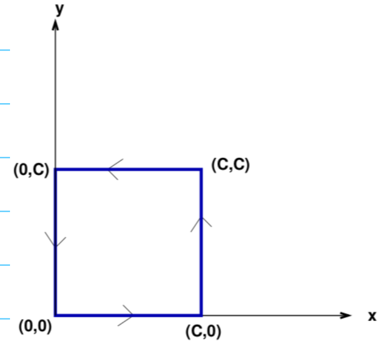
PHYS121 - Homework 9

Problem 1)



Given

- $\vec{F} \equiv Ay\hat{i} + Bx^4\hat{j}$ with $0 < A \le B \in \mathbb{R}$
- Counterclockwise direction around a square with side length C

$$\oint \vec{F}\cdot d\vec{r} = \int_a^b \left(\vec{F}(\vec{r}(t))\cdot \vec{r}\,'(t)\right)dt$$

a) $W_{\vec{F}} = \oint \vec{F}\cdot d\vec{r} = \overset{1}{\int_{P_1}^{Q_1} \vec{F}\cdot d\vec{r}} + \overset{2}{\int_{P_2}^{Q_2} \vec{F}\cdot d\vec{r}} + \overset{3}{\int_{P_3}^{Q_3} \vec{F}\cdot d\vec{r}} + \overset{4}{\int_{P_4}^{Q_4} \vec{F}\cdot d\vec{r}}$

1 $\int_{P_1}^{Q_1} \vec{F}\cdot d\vec{r} = \int_{t=0}^{t=1} \langle 0, B(Ct)^4\rangle\cdot\langle C,0\rangle dt = 0$

$r(t) = \langle Ct, 0\rangle$ $\quad \vec{F}(\langle Ct,0\rangle) = A(0)\hat{i} + B(Ct)^4\hat{j}$

$r'(t) = \langle C, 0\rangle$

2 $\int_{P_2}^{Q_2} \vec{F}\cdot d\vec{r} = \int_0^1 \langle ACt, BC^4\rangle\cdot\langle 0,C\rangle dt = \int_0^1 BC^5 dt = BC^5$

$r(t) = \langle C, Ct\rangle$ $\quad \vec{F}(\langle C,Ct\rangle) = A(Ct)\hat{i} + B(C)^4\hat{j}$

$r'(t) = \langle 0, C\rangle$

3 $\int_{P_3}^{Q_3} \vec{F}\cdot d\vec{r} = \int_0^1 \langle AC, BC^4t^4\rangle\cdot\langle -C,0\rangle dt = \int_0^1 -AC^2 dt = -AC^2$

$r(t) = \langle -Ct, C\rangle$ $\quad \vec{F}(\langle -Ct,C\rangle) = AC\hat{i} + B(-Ct)^4\hat{j}$

$r'(t) = \langle -C, 0\rangle$

4 $\int_{P_4}^{Q_4} \vec{F}\cdot d\vec{r} = \int_0^1 \langle -ACt, 0\rangle\cdot\langle 0,-C\rangle dt = \int_0^1 0\, dt = 0$

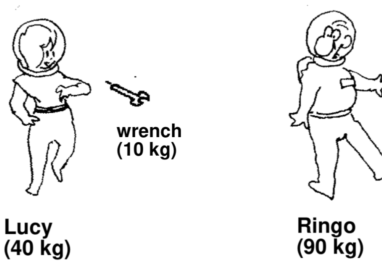
$\vec{r}(t) = \langle 0, -Ct\rangle$ $\quad \vec{F}(\langle 0,-Ct\rangle) = A(-Ct)\hat{i} + B(0)^4\hat{j}$

$r'(t) = \langle 0, -C\rangle$

$\oint \vec{F}\cdot d\vec{r} = 0 + BC^5 - AC^2 + 0 = C^2(BC^3 - A)$ $\quad\therefore\quad \boxed{W_{\vec{F}} = C^2(BC^3 - A)}$

b) The work done along the closed path is zero, so by definition $\vec{F}$ is not conservative. Unless $(BC^3 - A) = 0$, then the force does non-zero work. Even then, the force is not guaranteed to be conservative along any path, as conservative forces don't care about path. This is just like the normal force. The force $\vec{F}$ is not conservative!

Problem 2)



Given

- Zero velocity for both
- Lucy mass $m_L = 40\,Kg$
- wrench mass $m_w = 10\,Kg$
- Ringo mass $m_R = 90\,Kg$
- Lucy throws wrench to Ringo (throw → catch)

① Translational Speed: $v_w = 5\,m/s$

② Angular Velocity: $\omega_w = 16\,rad/s$

- $I_{wrench} = 0.2\,Kg\cdot m^2$

① $I_{Lucy} = I_{wrench} * 80 = 16\,Kg\cdot m^2$

② $I_{Ringo} = I_{wrench} * 125 = 25\,Kg\cdot m^2$

Define a system as Lucy, wrench, and Ringo

a) C of LM

Conditions: Is system isolated?

No external forces as it is given that they are floating free in space.

Only interested in Lucy and wrench for this part!

Before = After

$P_{Tot} = P'_{Tot}$

$m_L v_L + m_w v_w = m_L v_L' + m_w v_w'$

$v_L = 0,\ v_w = 0,\ v_L' = ?,\ v_w' = v_w$

$0 = m_L v_L + m_w v_w$

$-m_w v_w = m_L v_L$

$$\boxed{v_L = \frac{-m_w v_w}{m_L}}$$

Numerically: $v_L = \dfrac{-(10Kg)(5\,m/s)}{40Kg} = -1.25\ m/s$

$$\boxed{v_L = -1.25\,m/s}$$

b) C of LM

Conditions: Is system isolated?

No external forces as it is given that they are floating free in space.

Only interested in Ringo and wrench post throw for this problem!

Before = After

$P_{Tot} = P'_{Tot}$

$m_R v_R + m_w v_w = m_R v_R' + m_w v_w'$

$v_R = 0,\ v_w = 5\,m/s,\ v_R' = v_w' = v_f$ Kinematic Constraint

$0 + m_w v_w = v_f(m_R + m_w)$

$$\boxed{v_f = \frac{m_w v_w}{m_R + m_w}}$$

Numerically: $v_f = \dfrac{(10Kg)(5\,m/s)}{90Kg + 10Kg} = 0.5\ m/s$

$$\boxed{v_f = 0.5\,m/s}$$

c) C of AM

Conditions: Is system isolated w/ respect to external Torque? Yes! No EXT forces acting on Lucy or wrench!

Before = After

$L_{Tot} = L'_{Tot}$

$I_L \omega_L + I_w \omega_w = I_L \omega_L' + I_w \omega_w'$

$\omega_L = \omega_w = 0,\ I_L = 80 I_{wrench},\ \omega_L' = ?,\ \omega_w' = \omega_w$

$0 = 80 I_{wrench}\omega_L + I_{wrench}\omega_w$

$I_{wrench}\omega_w = -80 I_{wrench}\omega_L$

$\omega_L = \dfrac{\cancel{I_{wrench}}\,\omega_w}{-80\cancel{I_{wrench}}} = \dfrac{\omega_w}{-80}$

$$\boxed{\omega_L = -\frac{\omega_w}{80}}$$

Numerically: $\omega_L = -\dfrac{16\,rad/s}{80} = -0.2$

$$\boxed{\omega_L = -0.2\,rad/s}$$

d) C of AM

Conditions: Is system isolated w/ respect to external Torque? Yes! No EXT forces acting on Rigo or wrench when treated as a system!

Numericals: $\omega_f = \frac{16 \text{ rad/s}}{126} = \frac{8}{63} \text{ rad/s}$

$$\boxed{\omega_f = \frac{8}{63} \text{ rad/s} = 0.1269 \text{ rad/s}}$$

Before = After

$L_{Tot} = L'_{Tot}$

$I_R \omega_R + I_w \omega_w = I_R \omega_R' + I_w \omega_w'$

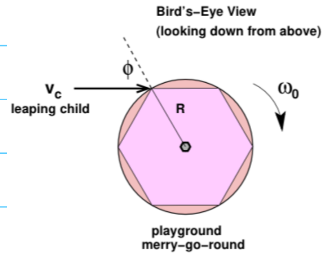
$\omega_R = 0,\ I_w = I_{wrench},\ I_R = 125 I_{wrench},\ \omega_w' = \omega_R' = \omega_f$

$0 + I_{wrench}\omega_w = \omega_f (125 I_{wrench} + I_{wrench})$

$\omega_f = \frac{\cancel{I_{wrench}}\,\omega_w}{126 \cancel{I_{wrench}}} = \frac{\omega_w}{126}$

$$\boxed{\omega_f = \frac{\omega_w}{126}}$$

Problem 3)



Given

- Ideal Bearing, Rotational Inertia I_m of Merry-go-round (independent!)
- Small Child mass m leaps onto with horizontal velocity v_c
- Child lands at R dist from center
- Child impacts at given angle ϕ as shown (relative to radius)
- Merry-go-round has ω_0 initial angular speed clockwise (will abbreviate as MgR or m)

Treat child as point-like object
↳ ignore all vertical motion

a) Combined Isolated System of Child and Merry-go-round. Child not touching MgR yet

$L_{Tot} = L_c + L_{MgR}$ (Not UCM Yet)

$= m_c v_c r \sin\phi + I_m \omega_m$

$m_c = m,\ v_c = v_c,\ r = R,\ \omega_0 = \omega_0$

$$\boxed{L_{Tot} = m v_c R \sin\phi + I_m \omega_0}$$

b) C of AM

Condition: Child and MgR are isolated! No torque done by bearing because it is 0 units from pp that is center of MgR.

Before = After

$L_{Tot} = L'_{Tot}$ (UCM motion @ land)

$\underline{L_c + L_m} = L'_c + L'_m$ (answer from a)

$m v_c R \sin\phi + I_m \omega_0 = m r_c^2 \omega'_c + I_m \omega_1$

$r_c = R,\ \omega'_c = \omega_1$ 'Rotational' Constraint

$m v_c R \sin\phi + I_m \omega_0 = (mR^2 + I_m)\omega_1$

$$\boxed{\omega_1 = \frac{mRv_0 \sin\phi + I_m \omega_0}{mR^2 + I_m}}$$

c) C of AM

Condition: Child and MgR are isolated! No torque on child post-jump as child lands w/ no velocity.

Before = After

$L_{Tot} = L'_{Tot}$ (No longer UCM)

$L_c + L_m = L'_c + L'_m$

$m r_c^2 \omega_c + I_m \omega_1 = m v r + I_m \omega_2$

$r_c = R,\ \omega_c = \omega_1,\ v = 0,\ \omega_2 = ?$

$(mR^2 + I_m)\omega_1 = 0 + I_m \omega_2$

Plug in ω_1: $\cancel{(mR^2 + I_m)}\left(\frac{mRv_0 \sin\phi + I_m \omega_0}{\cancel{mR^2 + I_m}}\right) = I_m \omega_2$

$$\boxed{\omega_2 = \frac{mRv_0 \sin\phi}{I_m} + \omega_0}$$

$\therefore\ \omega_2 = \frac{mRv_0 \sin\phi + I_m \omega_0}{I_m}$