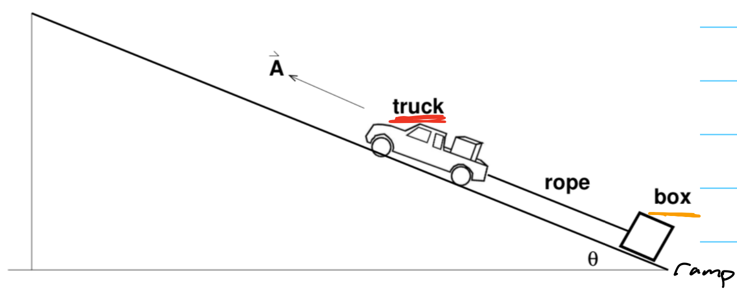
Trevor Swan
tcs94

PHYS121 - Homework 5

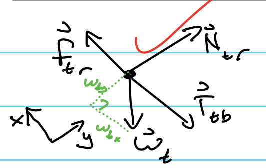
Problem 1)



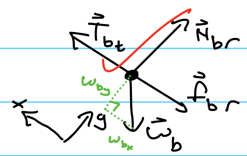
Given

- Truck constant Accel: A
- Ramp angle θ
- No slip no slide for truck
- μ kinetic friction for ramp + box
- mass of truck m_t
- mass of box m_b

a) FBD Truck



FBD Box



Weight Components and Forces

① $\boxed{w_t = m_t g}$ ① $\boxed{w_b = m_b g}$

$\vec{w}_t$: $w_{tx} = w_t \sin\theta$ $\vec{w}_b$: $w_{bx} = w_b \sin\theta$

$w_{ty} = w_t \cos\theta$ $w_{by} = w_b \cos\theta$

b) N2L Truck x-dir

$F_{tx} = m_t a_{tx}$ $a_x = A$

$f_{tr} - T_{tb} - w_{tx} = m_t A$

$f_{tr} - T_{tb} = m_t A + m_t g \sin\theta$ stop!

N2L Box x-dir

$F_{bx} = m_b a_{bx}$ $a_{tx} = a_{bx} = A$ Kinematic constraint

$T_{bt} - f_{br} - w_{bx} = m_b A$

$T_{bt} - f_{br} = m_b A + m_b g \sin\theta$ stop!

N2L box y-dir

$F_{by} = m_b a_{by}$, $a_{by} = 0$

$N_{br} - w_{by} = 0$

② $\boxed{N_{br} = m_b g \cos\theta}$

$f_{br} = |N_{br}|\mu$, $\boxed{f_{br} = \mu m_b g \cos\theta}$ ③ Kinetic Friction

$\therefore T_{bt} - N_{br}\mu = m_b A + m_b g \sin\theta$

$T_{bt} = m_b(A + g\sin\theta) + \mu m_b g \cos\theta$

$\boxed{T_{bt} = m_b(A + g(\sin\theta + \mu\cos\theta))}$ ④

N2L Truck y-dir

$F_{ty} = m_t a_{ty}$ $a_{ty} = 0$

$N_{tr} - w_{ty} = 0$

$\boxed{N_{tr} = m_t g \cos\theta}$ ②

N3L

$T_{bt} = T_{tb}$

$\boxed{T_{tb} = m_b(A + g(\sin\theta + \mu\cos\theta))}$ ③

$f_{tr} = m_t(A + g\sin\theta) + T_{tb}$

$\hookrightarrow f_{tr} = m_t(A + g\sin\theta) + m_b(A + g(\sin\theta + \mu\cos\theta))$

$= m_t(A + g\sin\theta) + m_b(A + g(\sin\theta + \mu\cos\theta))$

$= m_t(A + g\sin\theta) + m_b(A + g\sin\theta) + m_b g\mu\cos\theta$

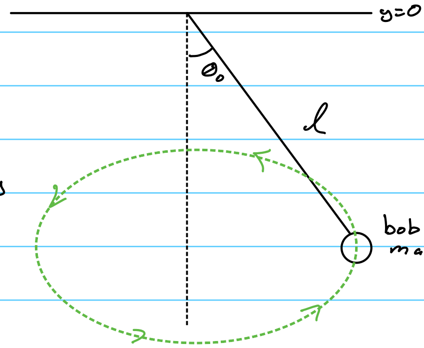
$= (m_t + m_b)(A + g\sin\theta) + m_b g \mu \cos\theta$

Algebraic Simplification

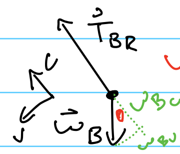
$\therefore$ $\boxed{f_{tr} = (m_t + m_b)(A + g\sin\theta) + m_b g \mu \cos\theta}$ ④

Problem 2) a) Sketch and FBD

Given
- m - mass of bob
- ℓ - length of string
- θ_0 - instance angle
- $\theta_0 < 90$



FBD Bob



$W_{Bv} = W\sin\theta$

T_{BR} Components

$W_{B_c} = W\cos\theta$

$W_{B_v} = W\sin\theta$

Centripital Accel inwards

Bob moves in a circular arc

b) N2L v-dir

$F_v = m a_v$

$W_{B_v} = m a_v$

$W_B \cancel{\cos}\theta = m a_v$

$\boxed{a_v = g\cancel{\cos}\theta}$ sin

c) Centripital Accel along String

N2L c-dir: $F_c = m a_c$, $a_c = \frac{v^2}{\ell}$ UCM

$T_{Br} - W_{B_c} = m\left(\frac{v^2}{\ell}\right)$

Released @ rest $v=0$ immediately

$T_{br} - mg\cos\theta = m\left(\frac{0}{\ell}\right)$

$\therefore \boxed{T_{br} = mg\cos\theta}$

d) Tension points perpendicular to the motion of the bob so it does $\boxed{\text{no work, } 0}$

e) Weight is conservative

$\therefore W_{\vec{w}} = -\Delta U$

$W_{\vec{w}} = -(mgy' - mgy)$

Let $\begin{cases} y' = -\ell \\ y = -\ell\cos\theta_0 \end{cases}$

$\therefore W_{\vec{w}} = -(mg\,{-\ell}\cos\theta_0 - mg\,{-\ell})$

$\boxed{W_{\vec{w}} = mg\ell(\cos\theta_0 - 1)}$

$1 - \cos\theta_0$

f) C of ME conditions: Tension does no work bc $\perp$ to path. No other forces to consider

Weight is conservative

"Before" = "After"

$U + K = U' + K'$

$\cancel{m}gy + \frac{1}{2}\cancel{m}v^2 = \cancel{m}gy' + \frac{1}{2}\cancel{m}v'^2$

Let $\begin{cases} y' = -\ell \\ y = -\ell\cos\theta_0 \\ v = 0 \\ v' = v_f\,? \end{cases}$

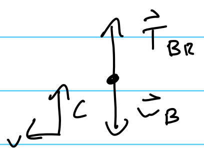
$-g\ell\cos\theta_0 + \frac{1}{2}(0)^2 = -g\ell + \frac{1}{2}v_f^2$

$\Rightarrow -g\ell\cos\theta_0 + g\ell = \frac{1}{2}v_f^2$

$v_f^2 = 2g\ell(1-\cos\theta_0)$

$\boxed{v_f = \sqrt{2g\ell(1-\cos\theta_0)}}$

g) FBD bottom



Recall: UCM

N2L c-dir

$F_c = m a_c$, $a_c = \frac{v^2}{R}$

$T_{Br} - W_B = m\frac{v^2}{R}$, $R = \ell$, $v = v_\ell$

$T_{Br} = m\left(\frac{v_\ell^2}{\ell} + g\right)$

$T_{Br} = m\left(\frac{v_\ell^2}{\ell} + g\right)$ $v_\ell = v_f$

$\therefore T_{Br} = m\left(\frac{\sqrt{2g\ell(1-\cos\theta_0)}^2}{\ell} + g\right)$

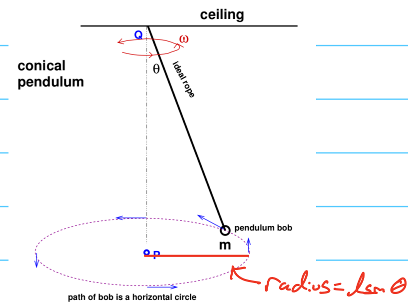
$T_{Br} = m\left(\frac{2g\cancel{\ell}(1-\cos\theta_0)}{\cancel{\ell}} + g\right)$

$T_{Br} = mg(2(1-\cos\theta_0)+1)$

$T_{Br} = mg(2 - 2\cos\theta_0 + 1)$

$\therefore \boxed{T_{Br} = mg(3 - 2\cos\theta_0)}$

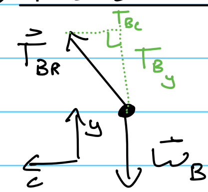
Problem 3)



Given

- bob mass = m
- moves in circular path
- constant angle θ
- String length = l

a) FBD bob



b) N2L y-dir   no vertical accel!

$F_y = m a_y$, $a_y = 0$

$T_{By} - W_B = 0$

$T_{By} = W_B$

$T_{BR}\cos\theta = mg$

$\boxed{T_{BR} = \frac{mg}{\cos\theta}}$

c) N2L c-dir   Let R = l sin θ

$F_c = m a_c$, $a_c = \frac{v^2}{R}$ UCM

$T_{Bc} = m\left(\frac{v^2}{l\sin\theta}\right)$

$T_{BR}\sin\theta = \frac{mv^2}{l\sin\theta}$ $T_{BR} = \frac{mg}{\cos\theta}$ from (b)

$\frac{mg\sin\theta}{\cos\theta} = \frac{mv^2}{l\sin\theta}$

$l\sin\theta\, mg\tan\theta = mv^2$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

∴ $v = \sqrt{l\sin\theta\, g\tan\theta}$

$v = \sqrt{gl\sin\theta\tan\theta}$

$v = \omega r$ Rolling constraint

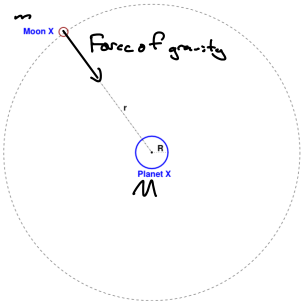
$r = l\sin\theta$

∴ $\omega = \frac{v}{l\sin\theta}$

$\omega = \frac{\sqrt{gl\sin\theta\tan\theta}}{l\sin\theta} = \sqrt{\frac{g\tan\theta}{l\sin\theta}}$ $\frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta}$

∴ $\boxed{\omega = \sqrt{\frac{g}{l\cos\theta}}}$

Problem 4) a)



Given

planetary radius → • X_{Planet}: $R = 5.20\times10^5\,m$

orbital radius → • X_{moon}: $r = 4.61\times10^6\,m$

• X_{moon} Period: $P = 14.32\,hrs$

• $M >> m$ for mass

The vector indicating the force of gravity on Moon X points radially inward along its orbit with Planet X because the gravitational force is an attractive force.

b) Universal Grav $F_{UG} = \dfrac{GMm}{r^2}$ Period (definition) $\dfrac{2\pi r}{P} = v$ $\dfrac{\text{distance}}{\text{time}} = \text{velocity}$ (circumference / period)

N2L Moon X

$F_{Moon} = m a_c$ Assume UCM $a_c = \dfrac{v^2}{R}$

$F_{UG} = m\dfrac{v^2}{R}$ Let $R = r$

$\dfrac{GM\cancel{m}}{r^2} = \cancel{m}\dfrac{v^2}{r}$

$v^2 = \dfrac{GM}{r}$

$\therefore \dfrac{4\pi^2 r^2}{P^2} = \dfrac{GM}{r}$

$$\boxed{M = \frac{4\pi^2 r^3}{P^2 G}}$$

$M = \dfrac{4\pi^2(4.61\times10^6\,m)^3}{(14.32\,hrs)^2 G}$

$G = 6.67\times10^{-11}\,\dfrac{N\,m^2}{kg^2} = 6.67\times10^{-11}\,\dfrac{kg\cdot m\cdot m^2}{s^2\cdot kg^2}$

Continuing Numerical Sln:

$14.32\text{ hours} = 51552\text{ seconds}$ $\therefore M = \dfrac{4\pi^2(4.61\times10^6\,\cancel{m})^3}{(51552\,\cancel{s})^2\left(6.67\times10^{-11}\,\frac{\cancel{m^3}}{\cancel{s^2}\,kg}\right)} = \boxed{2.18\times10^{22}\,kg}$

$\text{hours}\times\dfrac{3600\,s}{\text{hours}} = \text{seconds}$

c) Assume Air resistance is negligible

FBD phone   N2L Phone $g_x = \dfrac{GM}{R^2}$ and $M = \dfrac{4\pi^2 r^3}{P^2 G}$

$y\uparrow$ $\downarrow\vec{W}_p$

$F_y = m_p a_y$

falls $\ominus W_p = m_p a$ $a_g = -\dfrac{GM}{R^2}$

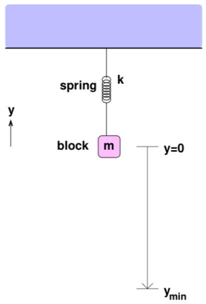
$-\cancel{m_p} g_x = \cancel{m_p} a_g$ $\therefore a_g = -\dfrac{\cancel{G}\,4\pi^2 r^3}{R^2 P^2 \cancel{G}}$

$a_g = -g_x$

Phone is falling! $\boxed{a_g = -\dfrac{4\pi^2 r^3}{R^2 P^2}}$ Actual Accel to gravity on phone $\boxed{a_g = \dfrac{4\pi^2 r^3}{R^2 P^2}}$

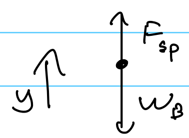
Numerically: $a_g = \dfrac{4\pi^2(4.61\times10^6\,m)^3}{(5.20\times10^5\,m)^2(51552\,sec)^2} = \boxed{5.38\,m/s^2}$

Problem 5)



Given
- Block mass = m
- At rest position: y=0
- Spring in eq. @ release
- Spring constant: K
- Block falls vertically

a) FBD Block



CofME

Conditions: Weight is conservative; F_{sp} is conservative → only 2 forces on block

"Before" = "After"

$$U_{Tot} + K_{Tot} = U'_{Tot} + K'_{Tot}$$

$$U_w + U_{sp} + \tfrac{1}{2}mv^2 = U'_w + U'_{sp} + \tfrac{1}{2}mv'^2$$

$$mgy + \tfrac{1}{2}Kx^2 + \tfrac{1}{2}mv^2 = mgy' + \tfrac{1}{2}Kx'^2 + \tfrac{1}{2}mv'^2$$

$$0 + 0 + 0 = mgy_{min} + \tfrac{1}{2}Ky_{min}^2 + 0$$

$$-\tfrac{1}{2}Ky_{min}^2 = mgy_{min} \longrightarrow \boxed{y_{min} = \frac{-2mg}{K}}$$

Assumptions:
- $y = x$, $y' = x'$
- $v' = 0$ @ bottom
- $y = 0$, $y' = x' = y_{min}$

b) $a \neq 0$ bc spring is no longer at equilibrium

N2L Block

$$F_B = m a_B$$

$$F_{sp} - W_B = m a_B$$

$$K|x| - mg = m a_B$$

$$K|x| - mg = m a_B, \quad x = y_{min} = \frac{-2mg}{K}$$

$$K\left|-\frac{2mg}{K}\right| - mg = m a_B$$

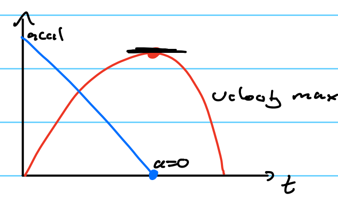
$$2mg - mg = m a_B$$

$$\therefore \boxed{a_B = g}$$

c) Position and speed are variables, CofME will ~~not~~ work

Assumption: Velocity is not linear and it is not 0!

↳ Acceleration is 0 at maximum velocity!



N2L Block

$$F_B = m a_B, \quad a_B = 0$$

$$F_{sp} - W_B = 0$$

$$F_{sp} = mg$$

$$K|x| = mg \quad \therefore \quad |x| = \frac{mg}{K}$$

Because all positions are below the zero point, I will choose the negative branch of absolute value.

$$\boxed{x = -\frac{mg}{K}}$$