Inclined Plane

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Abstract:

I have tested the theory of Newton's Second Law of Motion with a system of a cart on an inclined plane connected to a counterweight by a string over a pulley. After releasing the system from rest, I measured the velocity as a function of time. According to Newton's theory, the velocity should vary *linearly* with time. The data that I have collected does not support a linear dependence between velocity and time to within the uncertainties on the data points. I have also measured the average acceleration of the system as a_{meas} = ______ ± _______ ________. Newton's Second Law predicts that the acceleration of the system should be a_{pred} = ______ ± ______ _______. I find that the measured acceleration is not consistent with the predicted acceleration. ____

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(*If your measured velocity supports a linear model and/or your accelerations are consistent with the predicted acceleration, cross out the "not's" in the above abstract paragraph. Give a one-sentence conclusion about the lab.*)

Theory and Background:

One can determine the acceleration of the system depicted in Figure 1 by using Newton's Second Law to analyze the motion. Assuming that the frictional force $\vec{f}$ is negligible and that the pulley is massless and frictionless, the acceleration a of the system is[1]

__________________________________. (1)

where ____

____

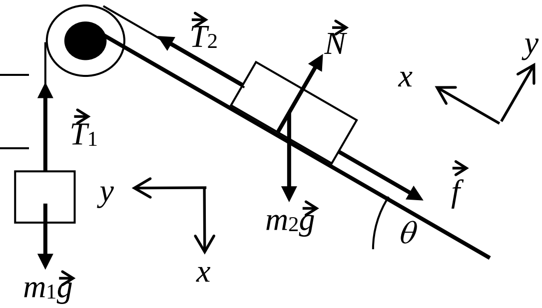

Figure 1: Schematic of Forces in Experiment. Courtesy Driscoll, (year).

(*Write down the appropriate equation; define all variables that haven't been defined yet.*)

One can find the sine of the angle of the incline using Eq. 1. If we adjust m_1 and m_2 so that the acceleration is zero and call this hanging mass the balancing mass m_b, then

________________________________ (2)

$\Rightarrow$________________________________ (3)

$\Rightarrow$________________________________. (4)

(*Eq. 2 should be Eq. 1 with a = 0; Eq. 3 should be an intermediate algebra step; Eq. 4 should be* $\sin\theta$ *in terms of* m_b *and* m_2.)

Equation 1 also implies that the acceleration of the system will be constant, so the velocity as a function of time will be

________________________________ (5)

where ____.

(*For equation 5 write down the expression for velocity in terms of time and other variables. Explain any new variables you introduce.*) From Eq. 5, we can see that if we fit a straight line to a plot of v vs. t, the slope of the line will be the acceleration and the intercept will be the velocity at time zero.

Procedure:

To get an estimate of the angle of the incline I estimated the length L and height H of the incline as in Figure 2. I used ________________________ to measure H = ______ __ and L = ______ ___. Getting accurate and precise measurements of H and L was difficult because ____. I compensated for these difficulties by ____ ____.

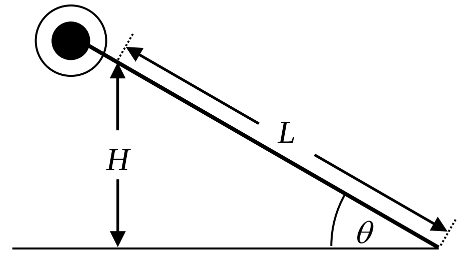

Figure 2: Estimating the angle. Courtesy Driscoll, (year).

Because of these issues, I estimate that my uncertainty in H is δ_H = ____ ___ and my uncertainty in L is δ_L = ____ ___.

My first estimate of $\sin\theta$ is then

$$\sin\theta = \frac{\qquad}{\qquad} \tag{6}$$

$$\Rightarrow \sin\theta = \frac{\qquad}{\qquad} = \underline{\qquad\quad}$$

(*Put the appropriate variables in the first line; put your actual measurements and final value for* $\sin\theta$ *in the second line.*)

The uncertainty in $\sin\theta$ is

$$\delta_{\sin\theta} = \sqrt{\delta^2_{\sin\theta,H} + \delta^2_{\sin\theta,L}} \tag{7}$$

where $\delta_{\sin\theta,H}$ is the uncertainty in $\sin\theta$ due to δ_H and $\delta_{\sin\theta,L}$ is the uncertainty in $\sin\theta$ due to δ_L.

Using the "computational method" to determine $\delta_{\sin\theta,H}$ and $\delta_{\sin\theta,L}$, I obtain

$$\delta_{\sin\theta,H} = \frac{\qquad}{\qquad} - \frac{\qquad}{\qquad} = \frac{\qquad}{\qquad} \tag{8}$$

$$\Rightarrow \delta_{\sin\theta,H} = \frac{\qquad}{\qquad} = \underline{\qquad\quad}$$

and

$$\delta_{\sin\theta,L} = \underline{\hspace{3cm}} - \underline{\hspace{3cm}} \tag{9}$$

$$\delta_{\sin\theta,L} = \underline{\hspace{3cm}} - \underline{\hspace{3cm}} = \underline{\hspace{2cm}}\,,$$

yielding $\delta_{\sin\theta} = \sqrt{(\underline{\hspace{1.5cm}})^2 + (\underline{\hspace{1.5cm}})^2} = \underline{\hspace{3cm}}$, or $\sin\theta$ = _____ ± _____. Since θ = $\sin^{-1}(\sin\theta)$, the uncertainty in θ is

$$\delta_\theta = \underline{\hspace{4cm}} - \underline{\hspace{3cm}} \tag{10}$$

$$\Rightarrow \delta_\theta = \underline{\hspace{4cm}} - \underline{\hspace{3cm}} = \underline{\hspace{3cm}}\ \underline{\hspace{1cm}},$$

or θ = ________ ± ______ __.

I measured m_2 with an electronic balance and determined that ___ = _______ ± _______ _____.
I estimated the uncertainty δ_{m2} as ____ ____ because ______________________________________
____.

I now used the first estimate of θ to determine an estimate of the mass m_b required to balance the system by taking Eq. 4 and solving for m_b:

$$m_b = \underline{\hspace{5cm}} \tag{11}$$

$$\Rightarrow m_b = \underline{\hspace{5cm}} = \underline{\hspace{5cm}}\ \underline{\hspace{1cm}}.$$

(*Solve Eq. 4 for m_b, substitute in the appropriate numbers, and solve.*)

I then set the mass of m_1 to ______ by adding masses to the hanger. After releasing the cart, the system was not in balance; the system ____.

(*If the system was balanced, cross out "not." Describe the system's motion in the blank.*)

I then found the minimum and maximum masses that lead to zero acceleration (m_{min} and m_{max}) by adding and removing mass from m_1 in order to obtain a better estimate of the angle of the incline and to account for the small amount of friction in the system. ____________________

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(*State if you tested zero acceleration by a stationary cart or cart moving with constant velocity. If you used the constant velocity test, also state how you determined the cart was moving at constant speed. Write a sentence about your reasons for choosing your methods, i.e., the advantages and disadvantages of your methods over other choices.*)

Using the procedure above, I determined that m_{min} = _______ __ and m_{max} = _______ __, each with negligible uncertainty. I then set the average of m_{min} and m_{max} to be the "balancing mass" m_b and half the difference between m_{min} and m_{max} as the uncertainty δ_{mb}, so m_b = _______ ± ______ __. Substituting m_b into Eq. 4, we see that sine of the angle of the incline is

$$\sin\theta = \frac{\qquad}{\qquad} = \underline{\qquad\quad} .$$

The uncertainty in $\sin\theta$ is

$$\delta_{\sin\theta} = \sqrt{\delta^2_{\sin\theta,m_b} + \delta^2_{\sin\theta,m_2}} \tag{12}$$

where $\delta_{\sin\theta,m_b}$ is the uncertainty in $\sin\theta$ due to δ_{mb} and $\delta_{\sin\theta,m_2}$ is the uncertainty in $\sin\theta$ due to δ_{m2}. Using the "computational method" to determine $\delta_{\sin\theta,m_b}$ and $\delta_{\sin\theta,m_2}$, I obtain

$$\delta_{\sin\theta,m_b} = \frac{\qquad}{\qquad} - \frac{\qquad}{\qquad} = \frac{\qquad}{\qquad} \tag{13}$$

$$\Rightarrow \delta_{\sin\theta,m_b} = \frac{\qquad}{\qquad} = \underline{\qquad\qquad}$$

and

$$\delta_{\sin\theta,m_2} = \frac{\qquad}{\qquad} - \frac{\qquad}{\qquad} \tag{14}$$

$$\delta_{\sin\theta,m_2} = \frac{\qquad}{\qquad} - \frac{\qquad}{\qquad} = \underline{\qquad},$$

yielding $\delta_{\sin\theta} = \sqrt{(\underline{\qquad})^2 + (\underline{\qquad})^2} = \underline{\qquad\qquad}$, or sin $\theta =$ _____ ± _____. ________________

____.

(*Compare this value of sinθ with the value from direct measurement of L and H.*)

I will adopt this value for sinθ.

I then set the counterweight m_1 to a value of ______ g to allow the cart to accelerate up the plane. I will refer to this value of m_1 as the experiment value m_e. I recorded the motion of the cart using an encoded pulley and *Logger Pro* software.[2] I subsequently exported the data from *Logger Pro* to *Origin* for a more complete analysis. Specifically, I plotted velocity vs. time to determine if the velocity has a linear dependence and to measure the slope. Previous experimenters[3] using this equipment have determined that the uncertainty in v has a value of 0.008 m/s; we adopted this value in our analysis.

Results:

The average acceleration recorded directly by *Logger Pro* statistics software was a_{meas1} = ______ ± ______ _____. Figure 3 shows a plot of velocity vs. time and a best linear fit using the *Origin* software. (*Attach a copy of your v vs. t graph labeled "Figure 3" to the end of the report.*) For this plot, vertical error bars are assigned based on an estimated uncertainty of the velocity measurements of ± ______ ______ for each point, where this value was determined by previous measurements done by the laboratory staff. As can be seen in the plot, the since not every data point lies within about one error bar of the best linear fit we conclude that these data are not consistent with Newton's model. (*If the data points fit the line to within about one error bar then delete the words "not" above. If the data are not consistent be sure to address this in your conclusion. Is Newton wrong? Or might there be systematic error in your data?*)

The slope of the graph as determined by *Origin's* fitting software is a_{meas2} = ______ ± ______ _____. In comparing this value to the value obtained directly from *Logger Pro* I note that these two values _____________ (*agree/do not agree to within their uncertainties/are exactly the same*). We expect that $a_{\text{meas\_}}$ should be more accurate because ___________________________ ____________________________________ and I will adopt it as the measured value, a_{meas}.

By substituting in known values into Eq. 1, one can determine a theoretical value for the acceleration of the system, a_{pred}. Since I did not measure θ directly, I will substitute Eq. 4 into Eq. 1 to obtain:

$$a_{pred} = \underline{\hspace{6cm}} \tag{15}$$

$$\Rightarrow a_{pred} = \underline{\hspace{6cm}} = \underline{\hspace{1.5cm}}\ \underline{\hspace{1cm}}\,.$$

Error Analysis:

To find the uncertainty in a_{pred}, $\delta_{a_{pred}}$, I must find the contribution to $\delta_{a_{pred}}$ for each of the quantities in Eq. 15 and add them in quadrature. The uncertainties in _________________ are negligible compared to the other quantities because ______________________________________ ____.

(*Identify any quantities that you will treat as having negligible uncertainty and justify your treatment. Then show your work in estimating the uncertainty in a_{pred} on the next page.*)

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So a_{pred} = __________ ± ___________ _________.

Conclusions:

The predicted value for the acceleration was a_{pred} = _______ ± ________ ______ and the measured value for the acceleration of the system was a_{meas} = _______ ± ________ ______. ________

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(*State whether or not your values agree within their uncertainties. If they do not agree, suggest at least one source of systematic error that were not adequately accounted for and suggest a way to reduce the effect of this error. If the two values do agree, suggest at least one source of random error and suggest a way to reduce the effect of this error. Make a quantitative statement about the effect friction should have had on this experiment. Make a conclusion about Newton's Second Law, especially regarding whether or not the data points support a linear model. If the model does not show a linear dependence give at least one reason why this might not be.)*

Acknowledgements:

I would like to thank ________________________, Case Department of Physics, for ___

help in obtaining the experimental data and preparing the figures. _________________________

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(*Thank your lab partner(s). **If they or anyone else gave you additional assistance, say who they were and specifically what their assistance was***.)

References:

(*If you have any additional references, list them below. Make sure to indicate with an endnote where in the report you referred to the reference.*)

1. Driscoll, D., *General Physics I: Mechanics Lab Manual*, “Inclined Plane,” CWRU Bookstore, 2014.

2.

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End Notes:

[1] Driscoll, D., p. 2.
[2] Driscoll, D., p. 3, describes the encoded pulley.
[3] Driscoll, D., p. 5