Chapter 2+

1D Constant Acceleration: “Throw-up” and “Catch-up”

Revisit:	Learn:
• Constant acceleration in 1D	• 1D “Throw-up” problems: Knowing what variables to look for • “Catch-up:” Intersections of two 1D trajectories • Graphical Approach to Motion

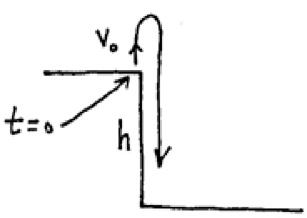
. Two General Classes of Problems

- **THROW-UP:** things thrown straight up or down with some initial velocity
- **CATCH-UP:** one “thing” accelerated to catch up to another “thing”

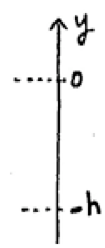
I) THROW-UP

Example: A stone thrown straight up from the edge of a cliff

We will see that we have to decide what we know (e.g., the position) and what we want to know (e.g., the velocity), and thus we look for the equation relating the two (e.g., the velocity as a function of position). We look back at p. 2-6.



$a_0 = -g$



v_0, h given

a) How high does it get? (i.e., what is y at v = 0? so we should look for an equation that only involves v and y, which we can solve for y!) Note that v is shorthand for the y-component of the velocity (the only component in this 1D problem). The equation we grab therefore (from p. 2-7) is

$$v^2 - v_0^2 = -2g(y - y_0)$$

and we insert the initial position $y_0 = 0$ and we solve for y at the top where v = 0, getting

$$\boxed{y = \frac{v_0^2}{2g}}$$

b) How long does it take to reach maximum height? (That is, what is t when v = 0 ?) So we should look for an equation involving v and t that we can solve for t. The equation we grab now is

$$v = v_0 - g\,t$$

and we solve for t at the top where v = 0, getting

$$\boxed{t = \frac{v_0}{g}}$$

c) When does the ball hit bottom? (That is, what is t at y = - h?) So we look for a y(t) equation that we can solve for t. The equation grabbed now is

$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

and we insert the initial position $y_0 = 0$ and the final position y = - h and we arrive at a quadratic equation for the time t we hit the bottom,

$$-\frac{1}{2} g t^2 + v_0 t + h = 0$$

Remembering the well-known quadratic solution formula,

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

we find

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4(-\frac{1}{2}g)h}}{2(-\frac{1}{2}g)}$$

and, since the square root is bigger than v_0 , we must choose the minus sign of the $\pm$ possibilities in order to get a positive time (remember the minus sign in the denominator). (By the way, the other solution gives a negative time, which corresponds to the fact that if we went backwards – and the cliff weren't in the way – we would hit y = -h again!). Simplifying and rearranging a little, we obtain

$$\boxed{t = \frac{\sqrt{v_0^2 + 2gh} + v_0}{g}}$$

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Problem 2-4: *An astronaut shipwrecked on a distant planet is on top of a sheer vertical cliff (let's call it point P), which he wishes somehow to climb down. He does not know the acceleration due to gravity on the planet, and he has only a good watch with which to make measurements.*

He wants to learn the height of the cliff (from P down to the bottom). To do this, he makes two measurements. 1) He drops a rock fall from rest off the cliff edge (from point P) and finds that the rock takes 4.15 s to reach the bottom. 2) He tosses another rock straight upward from P such that it rises a height of what he estimates to be 2 m (up from P) before it falls to the ground below. This time the rock takes 6.30 s from the original toss to hit the bottom.

What is the height of the cliff (point P down to the bottom below)?

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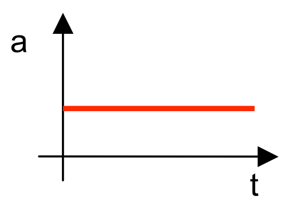
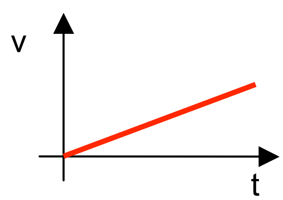
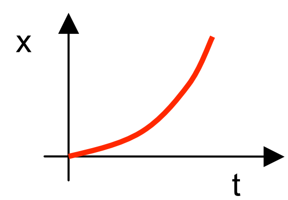
II) CATCH-UP ("things accelerated or decelerated to catch up or slow down to join other things) -- A better name might be "intersections."

Now for the second class ("catch-up") of problems, the quadratic equations continue to be important (they arise, as we saw, from constant accelerations, which will again be considered). The **1D** position-time, velocity-time, and acceleration-time graphs for constant accelerations from the formulas on p. 2-6 are

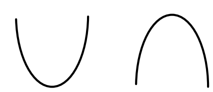




v(t) is some part of a straight line



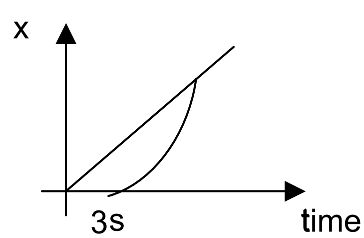
x(t) is some part of a parabola



and combinations of pieces of these are our graphical building blocks in what's coming. Also, see the last section in this chapter for more on the graphical relationships when the acceleration is not constant.

Example: A police car is stopped for a red light. A truck moving at a constant velocity of 15 m/s passes the police car, and illegally goes through a red light. **Three seconds later**, the police car begins accelerating with a constant acceleration of 2.28 m/s^2 in order to catch-up with the truck (which continues at the original constant speed). How many seconds after passing will it take for the police car to catch-up with the truck?

PICTURE:



"La Cucaracha" :)

CALCULATION: Start the clock so that t = 0 is now the time when the police car begins to accelerate (then add 3 seconds at the end of the calculation). Call x = 0 the place where the truck first passes the police car. The steps, starting with the position as a function of time (x^T for the truck and x^P for the police car) are:

Position $x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$ good for constant (and zero) acceleration

The police car: $x_0^P = 0\,\text{m}$, $v_0^P = 0\,\text{m/s}$, $a_0^P = 2.28\ \text{ms}^{-2}$

and the truck: $x_0^T = (15\text{m/s})(3\text{s}) = 45\,\text{m}$, $v_0^T = 15\,\text{m/s}$, $a_0^T = 0\,\text{ms}^{-2}$

They intersect when $x^P(t) = x^T(t) \quad \Rightarrow \quad t = t_{catch}$

i.e., $(0\text{ m}) + (0\tfrac{\text{m}}{\text{s}})t_{catch} + \tfrac{1}{2}(2.28\tfrac{\text{m}}{\text{s}^2})t_{catch}^2 = 45\text{m} + (15\tfrac{\text{m}}{\text{s}})t_{catch} + (0\tfrac{\text{m}}{\text{s}^2})t_{catch}^2$

And solve quadratic equation $\Rightarrow \; t_{catch} = \begin{cases} 15.7\text{s} \\ -2.5\text{s} \end{cases}$

The positive-time solution is the one we're looking for – the second is the time in the past where the police car and the truck would have crossed had we extrapolated the police car's trajectory backwards in time (if the police had been accelerating from the far past! Ha!).

$$\boxed{\text{The time after passing is } 15.7\text{s} + 3\text{s} = 18.7\text{ s}}$$

ALTERNATIVE CALCULATION: Start the clock so t' = 0 is now the time when the truck passes the police car (the time when they first have the same position which we still call x = 0).

Then the truck's position is $x^T(t') = v_0^T t'$ for $t' > 0$

and * the police car's position is $x^P(t') = \begin{pmatrix} 0 & t' < 3\text{s} \\ \frac{1}{2} a_0^P (t' - 3\text{s})^2 & t' > 3\text{s} \end{pmatrix}$

The truck and police intersect again for t' > 3 s when $v_0^T t' = \frac{1}{2} a_0^P (t' - 3\text{s})^2$

or, putting in the numbers, when $(15\frac{m}{s})\, t' = \frac{1}{2}(2.28\frac{m}{s^2})(t' - 3\text{s})^2$

and, if we define t = t' – 3s (so t' = t + 3s), this is the same equation as on the previous page: $(15\frac{m}{s})\, t + 45\text{m} = \frac{1}{2}(2.28\frac{m}{s^2})\, t^2$

* Here are the general formulas for the initial position at $t = t_o$ instead of $t = 0$:

$$\boxed{\begin{array}{l} a = a_0, \quad v = v_0 + a_0(t - t_0), \quad x = x_0 + v_0(t - t_0) + \frac{1}{2} a_0 (t - t_0)^2 \\ \text{Note that } v^2 - v_0^2 = 2a_0(x - x_0) \text{ is unchanged since time doesn't enter into it.} \end{array}}$$

To officially derive this, either just integrate step by step with the constants of integration fixed by knowing the initial values at $t = t_o$, or, even easier, just redefine your time variable in the old formulas as $t = t' - t_o$ ... and then call t' t again!

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Problem 2-5: *An unmarked police car traveling a constant 95 km/h is passed by a speeder traveling 160 km/h. Precisely 1.00 s after the speeder passes, the policeman steps on the accelerator and with constant acceleration catches up to the speeder who does not see the policeman and just keeps going at constant speed.*

a) *Please draw a rough* x *versus* t *picture of this "intersection."*
b) *If the police car's acceleration is constant at 2.30 m/s^2, how much time is needed after the speeder passes, before the police car catches the speeder?*

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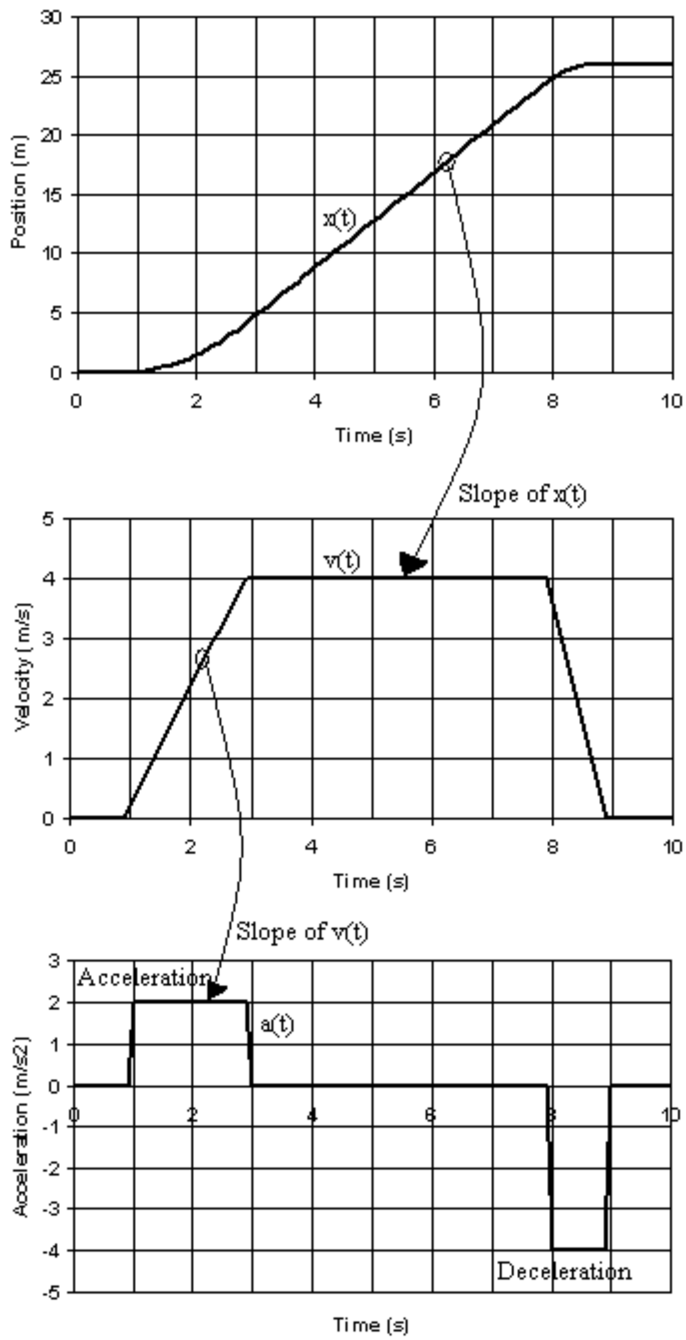
Graphical Approach

From a plot of position versus time, x(t), you can learn a lot about the motion and the forces that are causing this motion. After all, the time derivative is the velocity v(t) = dx/dt, and the second time derivative is the acceleration a(t) = dv/dt (which is proportional to the force). There's a nice graph from Halliday, Resnick, and Walker that we have reproduced here. It is a plot of the vertical position of an elevator that is initially at rest, then moves upward as measured by positive x(t), and then stops. Acceleration is varying here.



First, we find the velocity v(t) just by looking at the slope of x(t). We see that the velocity is zero at first, then increasing up to a constant value, then staying at that constant value for a while, and then decreasing back down to zero. (*To be certain that the velocity graph has a constant slope during the time segments where the velocity is increasing and decreasing, we have to estimate the slope fairly accurately at a couple of points, but we often just want a rough idea of what the velocity is doing.)* To find the acceleration graph a(t) is easy, using the slopes from v(t). (If we wanted to go backwards, given the acceleration, we just look at the area under its curve, as we are taught to do in calculus. Actually, just thinking "anti-derivative" can get you most of the way there! We'll return to this integration step in the third cycle where we'll note the need to have some initial value – the constant of integration – when we go backwards.)

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Problem 2-6

*Draw three nice plots, each with the same time axis, for the entire test described in Problem 2-3, **assuming that the manufacturer's claims are all true.** Be sure to put labels and units on your plot axes.*

That is, accepting the manufacturer's numbers, plot:

1. *The car's acceleration,*
2. *The car's velocity, and*
3. *The car's position,*

as functions of time for the prescribed test.

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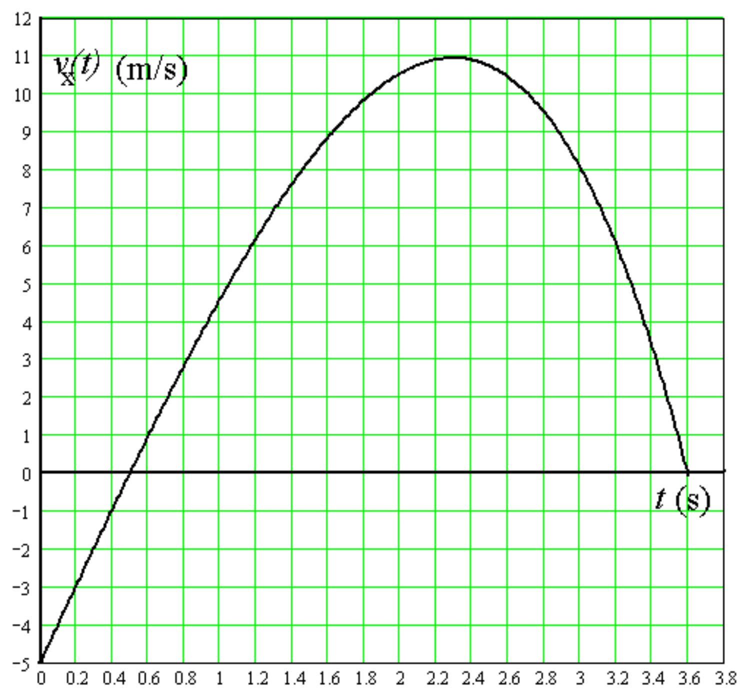
Problem 2-7

Suppose a particle moves in one-dimension (say, along the x *axis). The velocity of this particle is graphed in the figure below as a function of time.*

(a) Is the acceleration of the particle constant? Explain your answer.

(b) Estimate the acceleration of the particle at the instant 1.0 s.

(c) At what instants of time (if any) is the particle at rest? If such instants exist, is the acceleration of the particle also zero at these instants? Explain your answer.



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