Chapter 1+

ZERO FORCE: Constant Velocity in Two Dimensions (2D)

- **Revisit 1D Relative Uniform Motion**
- **Relative Motion in 2D – Right Angle Examples**
- **Find How Fast Snow Is Falling With Our Car's Speedometer!**

By the way, you know uniform motion means zero net force, but for what we are doing we don't care – we just assume uniform straight line motion and look at it in different reference frames.

Revisit 1D Relative Velocity:

Recall the general formula $\boxed{V_{X/Y} = V_{X/Z} + V_{Z/Y}}$ for any body Z serving as an intermediate reference frame (which "cancels" out) between the two bodies X and Y. In the following example, X is the airplane, Y is the ground, and Z is the air. X/Y refers to the airplane relative to the ground, etc. We can immediately find $V_{\text{airplane/ground}}$ since we are given $V_{\text{airplane/air}}$ (also called the airspeed) and $V_{\text{air/ground}}$ (also called the windspeed). Remember that the V's here are really 1D vectors (i.e., x-components that can be positive or negative).

Quickie Example: An airplane with 200 MPH airspeed (this means it would go 200 MPH with respect to the air), is flying downwind where the wind is blowing 50 MPH with respect to the ground. What is the airplane groundspeed? What is it when the airplane is flying upwind? You know in your heart that it will be faster going with the wind and slower going against it. Let's check it. Calling downwind the positive x-direction, we find:

$$\text{downwind: } V_{\text{airplane/ground}} = V_{\text{airplane/air}} + V_{\text{air/ground}}$$
$$= \quad 200 \quad + \quad 50 \quad = \quad 250 \text{ MPH}$$
$$\text{upwind: } V_{\text{airplane/ground}} = V_{\text{airplane/air}} + V_{\text{airplane/ground}}$$
$$= \quad -200 \quad + \quad 50 \quad = \quad -150 \text{ MPH}$$

where the change in sign for the second case arises from the airplane going in the negative x-direction. (The wind is always positive.) It is verified that the airplane has the higher speed with the wind.

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Problem 1-3 *The speed of a boat relative to water (its "waterspeed") is* v*. The boat is to make a round trip in a river whose current travels at speed* u *relative to the shore. Derive a formula for the time needed to make a round trip of total distance* D ***relative to the shore*** *if the boat makes the round trip by moving the distance* D/2 *(relative to the shore) upstream and back downstream* D/2 *(relative to the shore). (We must assume that* u < v*. Why?) If you felt like it, you could put the two terms over a common denominator to make a nice compact answer.*

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Consider 2D Relative Velocity: Example At Right Angles

What happens when the airplane flies at an angle with the wind? We'll consider right angles in this chapter and more general angles the final time through this stuff in 1++.

Suppose an airplane has a 600 km/h airspeed east. This airspeed and its direction gives us an "airvelocity," which means it would go east at 600km/h if the air was not moving. Another neat way of saying it is also important in our relative velocity discussion, namely that the airvelocity is the velocity of the airplane relative to the air!

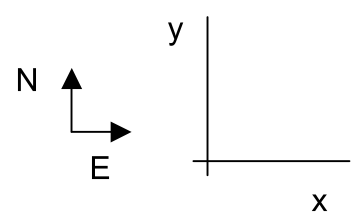
Now suppose a strong wind is blowing from the south at 100 km/h. What are the airplane's speed and direction with respect to the ground?

To analyze this, we use the 2D (vector) version of our relative velocity formula

$$\vec{v}_{\text{airplane/ground}} = \vec{v}_{\text{airplane/air}} + \vec{v}_{\text{air/ground}}$$

Let's solve for $\vec{v}_{\text{airplane/ground}}$ in two ways:

A) First method is to look at the **Cartesian components**. We simply look at the x (East) and y (North) components of the relative velocity formula:



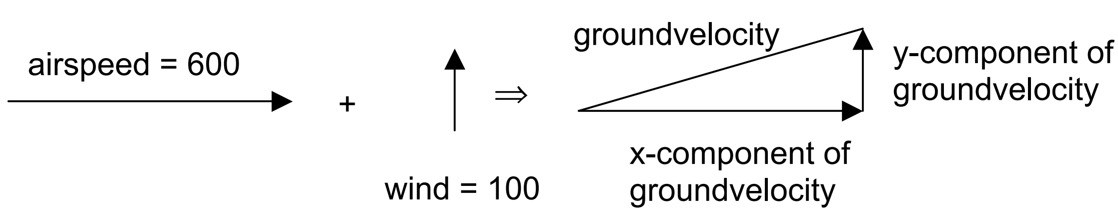
east (x) component:

$$v_{(\text{airplane/ground}),\,x} = v_{(\text{airplane/air}),\,x} + v_{(\text{air/ground}),\,x} = 600 + 0 = 600$$

north (y) component:

$$v_{(\text{airplane/ground}),\,y} = v_{(\text{airplane/air}),\,y} + v_{(\text{air/ground}),\,y} = 0 + 100 = 100$$

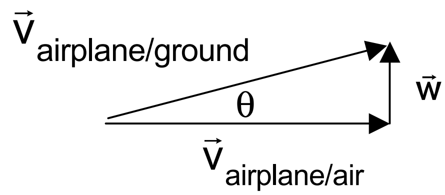
We can arrange these components as the sides of a right triangle in which the groundvelocity is its hypotenuse (whose magnitude and direction we could easily get from Pythagoras and trigonometry, respectively):



B) The second method is to solve for $\vec{v}_{\text{airplane/ground}}$ from the triangle corresponding to the **vector addition** indicated in

$$\vec{v}_{\text{airplane/ground}} = \vec{v}_{\text{airplane/air}} + \vec{v}_{\text{air/ground}}$$
$$\text{with } \vec{v}_{\text{air/ground}} \equiv \vec{w} \text{, the wind velocity}$$

That is, adding tails to tips (recall the little vector primer in the appendix of Ch. 6), we draw the triangle:



It is obvious we get the (same) angle θ (North of East) and magnitude $v_{\text{airplane/ground}}$ of the airplane's ultimate ground velocity from both methods:

$$\tan\theta = \frac{100}{600} \Rightarrow \theta = 9.5°$$
$$v_{\text{airplane/ground}} = \sqrt{(600)^2 + (100)^2} = 608 \text{ km/h}$$

Comments:

- When you think in terms of a **graphical vector addition,** it will help you draw the correct triangle! Here, we were given the two legs and we knew that their addition had to be the vector $\vec{v}_{\text{airplane/ground}}$, the object of our desire! Sometime you may have the resultant and one leg and, by remembering that the other leg must add to the first to give the resultant, you can figure out how to draw the complete triangle.

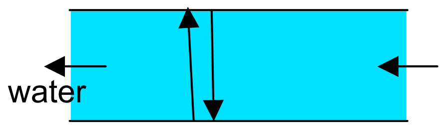
- In both cases, you use trigonometry with SOHCAHTOA and Pythagoras to find angles and magnitudes.

- But the two methods will have their own advantages and disadvantages when we return in the third cycle to arbitrary angles for the wind.

Another Example At Right Angles:

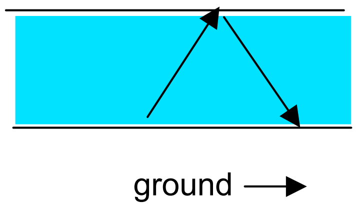
Consider a famous relative velocity problem: a boat going across a river. We are interested in the relationship between the boat's velocity relative to the shore (ground) and its velocity relative to the water: $\vec{v}_{boat/ground} = \vec{v}_{boat/water} + \vec{v}_{water/ground}$. If we knew the two velocities on the right-hand side, we could just plunge right and use the previous methods. But let's get more insight.

Suppose the boat is to go straight across (i.e., perpendicularly across) from one side to the other side and then back again. This tells us the direction of the velocity of the boat relative to the ground, $\vec{v}_{boat/ground}$, and it is what we see from the shore:



$\Rightarrow$ $\vec{v}_{boat/ground}$

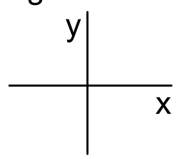
If we were floating along on a log in the river, and, if the river flows to the left, we would see the boat do the following (and we also see the ground "flow" to the right):



$\Rightarrow$ $\vec{v}_{boat/water}$

Notice in the bottom picture the boat has to move at some angle upstream relative to the river so that the component of its velocity relative to the water CANCELS the river velocity completely. Thus we are left only with a component for the boat perpendicular to the river in the top picture and it goes straight across. Keep the above insight in mind as we use the two different methods to analyze this.

A) Method of rectangular components (see the x-y axes on the right)



First, the river velocity $\vec{v}_{water/ground}$ has no y-component so, if its magnitude (speed) is defined to be u, its (only) x-component is $-u$ (it goes in the negative direction). Second, the boat velocity $\vec{v}_{boat/ground}$ has no x-component (it goes straight across), so if its magnitude is defined to be V, its (only) y-component is $+V$.

- **x component:**

$$v_{boat/ground,\, x} = v_{boat/water,\, x} + v_{water/ground,\, x} = v_{boat/water,\, x} - u$$

But

$$v_{boat/ground,\, x} = 0$$

$$\therefore \boxed{v_{boat/water,\, x} = u} \quad \text{(Just as we said above.)}$$

- **y component**:

$$v_{boat/ground,\,y} = v_{boat/water,\,y} + v_{water/ground,\,y} = v_{boat/water,\,y} + 0$$

But

$$v_{boat/ground,\,y} = +V$$

$$\therefore \boxed{v_{boat/water,\,y} = V}$$

The result is that we now know the magnitude of $\vec{v}_{boat/water}$ from Pythagoras:

$$v_{boat/water} = \sqrt{(v_{boat/water,\,x})^2 + (v_{boat/water,\,y})^2}$$
$$= \sqrt{u^2 + V^2}$$

Besides the river speed, we often know the waterspeed, by which we mean the magnitude $v_{boat/water}$, Suppose it is defined as $v_{boat/water} \equiv v$. Then

$$v = \sqrt{u^2 + V^2}$$

Therefore, we can solve for the groundspeed of the boat: the magnitude $V \equiv |\vec{v}_{boat/ground}|$, if we are given u, v:

$$\boxed{V = \sqrt{v^2 - u^2} = \text{the boat speed relative to the ground}}$$

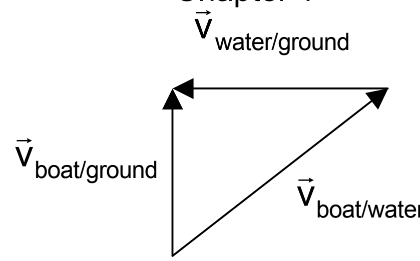
The time to go across the river is therefore $t_{across} = \frac{D/2}{V} = \frac{D/2}{\sqrt{v^2 - u^2}}$, if the river is a distance D/2 wide. By symmetry the total time is twice this:

$$\boxed{t_{across\ and\ back} = \frac{D}{V} = \frac{D}{\sqrt{v^2 - u^2}}}$$

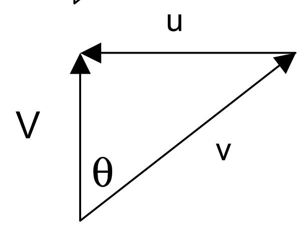
The total distance gone D matches the earlier homework Problem 1-3 so that the times could be compared.

B) Method of Triangularization (for this problem, this is neater and faster)

Recall the vector relation $\vec{v}_{boat/ground} = \vec{v}_{boat/water} + \vec{v}_{water/ground}$ corresponds to a triangle where the resultant $\vec{v}_{boat/ground}$ is the sum of the legs $\vec{v}_{boat/water}$ and $\vec{v}_{water/ground}$. The general tactic is to know enough about the magnitudes and/or the angles of part of the triangle to figure out the rest of the triangle. In the boat problem, we know the directions of the resultant $\vec{v}_{boat/ground}$ and one leg $\vec{v}_{water/ground}$, and that helps us draw the shape of the triangle. See the next page.

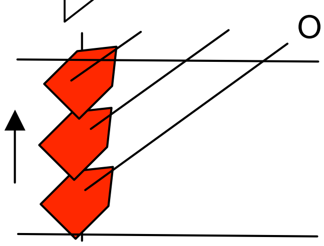


With what we know about their directions AND which ones are adding up to the resultant, we have the top triangle on the right. The second triangle shows the magnitudes (speeds) , and Pythagoras leads to $v^2 = u^2 + V^2$ and we immediately follow with the result we derived earlier:



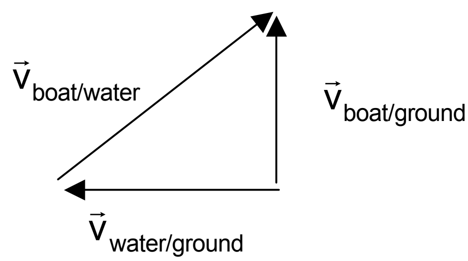
$$\boxed{V = \sqrt{v^2 - u^2} = \text{the boat speed relative to the ground}}$$

Here, the triangularization really sped up the analysis. But sometimes the component approach is more convenient.



Comment: By the way, how should you aim the boat as you traverse the river, in order to end up going straight across? You should keep the angle of the bow fixed as shown in the figure on the right – you do not keep aiming at some fixed point O on the other side. The fixed angle is θ, where $\sin\theta = u/v$ – see the second triangle for the definition of θ.

Another comment: We see that we must have $u < v$, as in Problem 1-4. If the river were going too fast, we couldn't go straight across. We can't cancel the river's velocity even if we were going opposite to it, and we'll always end up downstream.



Another comment: You could just as well add the vectors in the opposite order where, equivalently, you write $\vec{v}_{boat/ground} = \vec{v}_{water/ground} + \vec{v}_{boat/water}$ and you end up with the triangle shown on the right. It gives the same answers.

A final comment: In the above example, we were given the hypotenuse and one leg, and we were solving for the other leg so, in drawing the vector addition, we used $\vec{v}_{boat/ground} = \vec{v}_{boat/water} + \vec{v}_{water/ground}$. If, instead, you needed to solve for $\vec{v}_{boat/water}$ it may help you to write $\vec{v}_{boat/water} = \vec{v}_{boat/ground} + \vec{v}_{ground/water}$. Remembering that $\vec{v}_{ground/water} = -\vec{v}_{water/ground}$, you'll get the same equation and triangle as before!

Problem for practice on the next page -

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Problem 1-4

A) Relative to the ground, snow is falling vertically at a constant speed of 7.8 m/s. a car is traveling on a straight road with a constant speed of 55 km/h relative to the ground. (You might want to immediately convert this to m/s. How?) We are interested in what direction the driver of the car sees the snow is falling relative to her, and on the way to doing that, let's answer a series of questions:

1) Which of the following vector addition equations is correct? (Put true or false after writing down each letter.)

a) $\vec{v}_{\text{snow/ground}} = \vec{v}_{\text{car/ground}} + \vec{v}_{\text{car/snow}}$

b) $\vec{v}_{\text{snow/car}} = \vec{v}_{\text{snow/ground}} + \vec{v}_{\text{ground/car}}$

2) We can draw the arrows for the vectors: $\vec{v}_{\text{snow/ground}} = \downarrow$ *and* $\vec{v}_{\text{car/ground}} = \longrightarrow$ *if we choose the direction of the car as to the right. Use them to help you draw an appropriate triangle representing any of those vector addition equations in (1)* ***that was correct****. Remember that the negative of a vector is just the arrow reversed in its direction and that* $\vec{v}_{A/B} = -\vec{v}_{B/A}$.

3) For each triangle you drew in (2), draw another one that is just as good for finding any unknown information you want but differs from (2) by adding the vectors in a different order. That is, $\vec{v}_a + \vec{v}_b$ *can be drawn by putting the tip of* $\vec{v}_a$ *at the tail of* $\vec{v}_b$*, or by putting the tip of* $\vec{v}_b$ *at the tail of* $\vec{v}_a$ *. You will see that the hypotenuse will be the same for both of your right triangles.*

4) Using any one of your triangles, find the angle from the vertical that the snowflakes appear to be falling as viewed by the driver in the car. Does the driver see the snow falling at an angle toward her or away from her?

B) Suppose snowflakes are falling vertically at an unknown speed. Driving your car through the falling snow, you realize you can figure out how fast the snowflakes are falling down. How would you do it? Hint: If your (horizontal) car speed were the same as the snowflakes' (vertical) speed, at what angle would you see the snowflakes fall?

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