PHYS 121: Solutions to Written Homework #03
February 13, 2024

Note: this homework is worth **15 points** as follows:

Problem 1	0 points	Do Not Grade.
Problem 2	0 points	Do Not Grade.
Problem 3	4 points	4 Points total.
Problem 4	6 points	6 Points total.
Problem 5	4 points	4 Points total.

Problem 1: Science-Fiction Marshmallow Battle

A strange science-fiction battle takes on a tiny moon in deep space; here, the moon is so small that we can completely ignore gravity. A deranged invading space monster of given mass 28.2 kg kicks off a rocky structure to achieve a horizontal speed of 3.45 m/s directly toward a startled space cadet. The cadet is braced firmly on the surface of the moon and is armed with a rapid-fire automatic marshmallow blaster gun which fires multiple sticky marshmallows toward the monster, each of mass 11.5 grams and each with a speed of 53.4 m/s. The cadet attempts to stop the monster with the marshmallow gun, slowing the monster down by hitting the monster with marshmallow after marshmallow until the monster is finally brought to a stop. **How many marshmallows will the cadet have to fire at the monster to stop the monster's horizontal motion?** (Assume each fired marshmallow harmlessly ***sticks*** to the monster. Assume that the monster is flying horizontally, with no vertical motion and no friction or other forces on the monster except for those due to the collision with the marshmallows. Explain your work. Indicate what Physics Concept (law of physics) you used to solve this problem.

Also: is kinetic energy conserved here? Explain how you know this.

Hints: If the velocity of the monster is defined as positive, the velocities of the marshmallows must be negative. We are essentially asking for the number N corresponding to the the minimum number of marshmallows fired by the cadet to bring the monster from positive velocity to zero (or lower). Define a system that includes the monster and the N marshmallows, determine the linear momentum of the system before and after the N collisions, apply the appropriate Conservation Law, and then solve for N.

SOLUTION TO PROBLEM 1:

We assign M=28.2 kg as the mass of the monster and m=0.0115 kg (convert to SI units!) as the mass of each marshmallow. The initial velocity of the monster is V=3.45 m/s and the initial velocity of each marshmallow is v= -53.4 m/s. (Note velocity of one of either of the marshmallows or the monster needs to be negative since they are heading in opposite directions).

When the marshmallows collide and stick, this is a ***totally inelastic collision***. If we consider the collision between monster and marshmallows in mid-air and we ignore gravity (we are in space after all), this is an *isolated system* so we can use **Conservation of Linear Momentum**. Since time does not come into play here, we can treat the N collisions of one marshmallows each as equivalent to one single collision of N marshmallows:

$$P_{tot} = P'_{tot}$$

$$P_{monster} + NP_{marshmallows} = P'_{tot}$$

Since in the "after" case, the monster has been stopped, $P'_{tot} = 0$.

$$P_{monster} + NP_{marshmallows} = 0$$

$$MV + Nmv = 0$$

$$\boxed{N = -\frac{MV}{mv}}$$

Plugging in numbers:

$$N = -\frac{(28.2 \text{ kg})(3.45 \text{ m/s})}{(0.0115 \text{ kg})(-53.4 \text{ m/s})}$$

$$N = 158.4$$

This means that the cadet must fire at least $\boxed{\text{159 marshmallows}}$ if he is going to stop the monster. You need alot of marshmallows to stop a monster.

Problem 2. A medicine ball with given mass $M = 8.11$ kg drops vertically, eventually hitting the floor. Just before impact, the ball has a given downward *speed* of 5.11 m/s. It then rebounds straight back with a given upward speed of 4.42 m/s.

Part (a) – What is conserved with respect to the system that is defined as *the medicine ball* during the entire collision? Is Mechanical Energy Conserved? Is Linear Momentum Conserved? Explain how you know this?

Part (b) – What is the ***total impulse*** imparted to the ball as a result of contact with the floor?

Part (c) – If the ball is in contact for a time interval $\Delta t = 0.056$ seconds, what is the *average force* exerted on the floor during this collision?

SOLUTION TO PROBLEM 2:

Part (a): In order to consider carefully what we mean by *Conservation* we have to clearly define a "BEFORE" and an "AFTER" where these are before and after *something happens*. In this case, **the "something happens" is the medicine ball hitting the floor.** So in order to consider Conservation, we have to define "BEFORE" as *just-immediately before the ball comes in contact with the floor* and we need to define "AFTER" as *just-immediately after the ball loses contact with the floor*. This "before" and "after" then surrounds the "event" which is the ball colliding with the floor. So to determine whether or not Energy and/or Linear Momentum are conserved, all we need to do is calculate these quantities just "BEFORE" and just "AFTER" and see if they have changed. If they have changed, then then are not conserved.

We start by considering the potential energy due to the weight of the medicine ball,

$$U_{\vec{W}} = mgy$$

In this problem the ball the position of the ball remains (approximately) constant just barely (not) in contact with the floor. This means that y before is very nearly equal to y' before so that there is no significant change to the potential energy during the collision. However, the speed of the ball changes as a result of the collision so the kinetic energy, and therefore the total energy has changed:

$$\text{“}BEFORE''$$

$$E_{tot} = K + U$$

$$\text{“}AFTER''$$

$$E'_{tot} = K' + U'$$

$U \approx U'$ but $K \neq K'$ therefore

$\boxed{E_{tot} \neq E'_{tot} \text{ therefore energy is } \textbf{not} \text{ conserved.}}$

Likewise, during the collision, the direction of the ball *reverses* so the momentum of the ball has clearly changed sign, going from negative to positive. Since linear momentum is a signed (vector) quantity, the momentum before cannot equal the momentum after. Therefore:

$\boxed{\vec{P}_{tot} \neq \vec{P}'_{tot} \text{ therefore linear momentum is } \textbf{not} \text{ conserved.}}$

This is not a surprising result. Indeed it is what we expect because we know that we can only apply Conservation Laws if we have met the conditions for applying these laws. In this case:

- The condition for the application of Conservation of Energy is that all of the forces in the problem be *either* **Conservative** *or* **do zero work**. In this problem the forces on the medicine ball are Weight and (during the collision) a Normal force (push). Weight is conservative, but Normal is not. And in this case since the motion of the ball is not perpendicular to the direction of the normal force, we cannot argue that the normal force does no work. So the condition **fails** and we **cannot apply Conservation of Mechanical Energy** to this system.
- The condition for the application of Conservation of Linear Momentum is that the system as defined be **isolated**, that is to say that there be zero net force on the system in the relevant component coordinate. In this problem there are vertical forces of weight and normal that do not cancel out and so the system is **not isolated**. So the condition **fails** and we **cannot apply Conservation of Linear Momentum** to this system.

Part (b): Impulse is defined as the change in momentum. Note that direction matters, so if we define positive up, then the initial momentum of the medicine ball as it falls is negative:

$$\text{impulse} \equiv \Delta p = p' - p = mV' - mV$$
$$\boxed{= m(V' - V)}$$

Plugging in numbers:

$$= (8.11 \text{ kg})[(4.42 \text{ m/s}) - (-5.11 \text{ m/s})]$$
$$\boxed{= 77.3 \text{ kg} \cdot \text{m/s}}$$

Important: A common "mistake" here is to subtract speeds and not velocities.

Part (c): The average force is just calculated by using the **impulse-momentum relation:**

$$\Delta p = \int F dt$$
$$\Delta p = F_{ave} \Delta t$$
$$\boxed{F_{ave} = \frac{\Delta p}{\Delta t}}$$

Plugging in numbers:

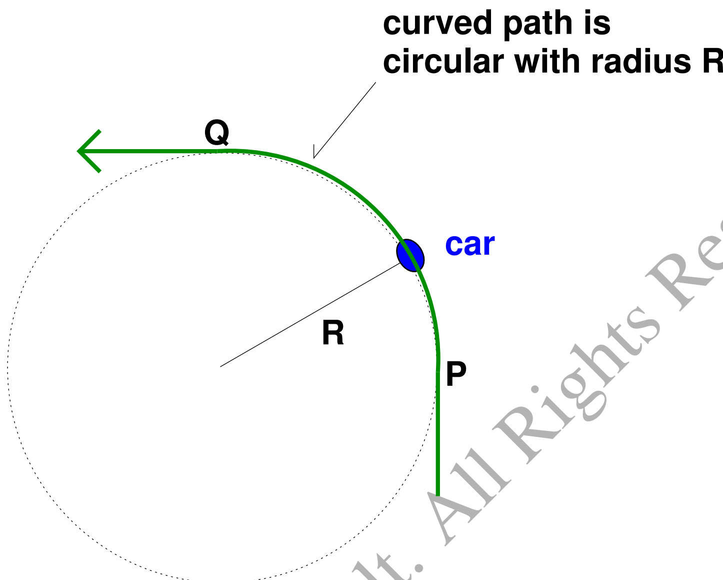
$$F_{ave} = \frac{77.3 \text{ kg} \cdot \text{m/s}}{0.056 \text{ s}}$$
$$\boxed{F_{ave} = 1,380 \text{ Newtons}}$$

One last detail: We actually asked for the average force on the Floor and not the Ball. The calculation shown is for the Ball but by Newton's Third Law this must also be the force on the Floor.

That's a large force. Much larger than, say, the weight of the ball which is only about 80 Newtons. Collisions tend to produce large forces for short periods of time.

Problem 3: Conceptual Question – Straight from 2011 First Hour Exam:

Important: This is a series of *conceptual* questions. Each part requires essentially no calculations. If you find yourself working algebra here you are going down the wrong path.



You are driving a car that is moves with constant given speed V. Your mass is given as m and the mass of the car (not including yourself) is given as M. The road is completely flat and level. The car enters a curve in the road that can be described as a circular arc with given radius R as shown above in a "bird's-eye" view of the path of the car. Assume you are properly secured into your seat as you drive.

Part (a) – What is the magnitude of the **net force** on you, the driver, as you go around the curve? Explain how you know this.

Part (b) – A student says:

> *"As the car comes around the curve, there must be a net force on the driver to the driver's* right*. In other words, the net force points* away *from the center of the circle."*

Is the student correct or incorrect? Explain how you know this.

Part (c) – What is the magnitude of the force of ***friction*** on the car due to the road as you go around the curve? Hint: it is not zero. Explain how you know this?

Part (d) – What is the **Total Work** done by all forces on the car as it moves on the curve from point P (corresponding to where the car enters the curve) to the point Q (corresponding to where the car leaves the curve)? Explain how you know this? Important: you must use one or more Physics Concept(s) here

SOLUTION TO PROBLEM 3:

Part (a):

As the driver, you are moving with the car as it go around the circular curve at constant speed. This means you move with **Uniform Circular Motion**, and therefore your acceleration is **centripetal**.

To calculate the net force on you, just apply **Newton's Second Law:**

$$\vec{F}_{net} = m\vec{a}$$

$$F_c = ma_c$$

$$\boxed{F_c = \frac{mV^2}{R}}$$

Part (b):

The student is $\boxed{\textbf{completely incorrect}}$. The direction of the net force on the car is in the same direction as the direction of acceleration. Since the acceleration is **centripetal**, the net force is pointed **toward the center of the circle**, a direction that is to the **left** of the driver, not to the right.

NOTE TO GRADER: If the student argues that this statement is correct based on some notion of centrifugal force or some other concept that is explicitly or implicitly invoking a non-inertial reference frame, you can award at most 0.5 points for partial credit depending on the strength of the argument. Students were instructed explicitly not to invoke non-inertial reference frames to solve problems on this exam.

Part (c):

In this problem the *only* force applied from the road to the car parallel to the surface of the road is friction. This follow from the **definition of friction**. Since friction is the **only** force on the car-plus-driver system, it must also be equal to the net force. And so again, we apply **Newton's Second Law:**

$$F_{sys_{net}} = M_{sys}a$$

$$F_c = M_{sys}a_c$$

$$f = M_{sys}a_c$$

$$\boxed{f = \frac{(m+M)V^2}{R}}$$

NOTE TO GRADER: If the student uses M instead of $(m+M)$ for the mass of the car-system, this is a relatively minor error and you should note it, but not deduct points.

Part (d):

Here we apply the **Classical Work-Energy Relation** that tells us the total work is equal to the change in kinetic energy:

$$\mathcal{W}_{tot} = \Delta K$$

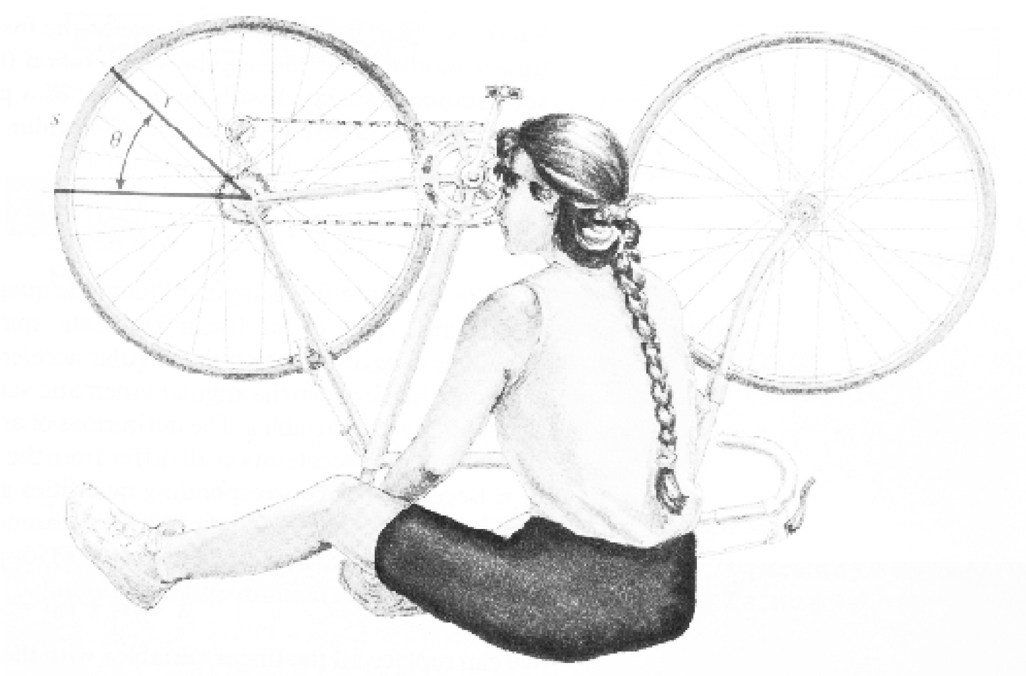
Since the speed of the car is *constant* the kinetic energy $K = \frac{1}{2}MV^2$ also remains constant and therefore unchanged. Therefore $\Delta K = 0$ and so:

$$\boxed{\mathcal{W}_{tot} = 0}$$

Alternative Acceptable Solution: As is demonstrated in Part (c), all three forces on the car (Weight, Normal, and Friction) are applied perpendicular to the path of the car. Therefore each of these forces does zero Work:

$$\boxed{\mathcal{W}_{tot} = \mathcal{W}_{\vec{W}} + \mathcal{W}_{\vec{N}} + \mathcal{W}_{\vec{f}} = 0 + 0 + 0 = 0}$$

Problem 4:



A *constant* given torque τ of 0.073 Newton-meters is applied to a free-spinning bicycle wheel of mass m = 4.3 kg, radius R of 0.27 meters as shown. The rotational inertia of the wheel is **given** by the expression: $I = mR^2$.

a) What is the angular acceleration of the wheel? (Hint: Use the rotational analog of Newton's 2nd Law).

b) Assuming the wheel starts at rest, how much time will it take for the wheel to be spun to a final rotational velocity of ω = 22.2 radians/second? (Hint: This is analogous to translational motion with constant acceleration).

c) A small piece of mud is stuck to the outside edge of of the wheel. What is the *centripetal acceleration* on the piece of mud after time T = 5.0 seconds of torque have been applied? (Hint: find the angular velocity, and then consider the piece of mud as instantaneously moving with Uniform Circular Motion).

SOLUTION TO PROBLEM 4:

Part (a):

Since we are given a torque and we want an acceleration, we use **Newton's Second Law in Rotational Form**:

$$\tau_{net} = I\alpha$$

Here we are given the net torque, τ_{app} = 0.073 Newton-meters, and the rotational inertia is given by $I = mR^2$ where m=4.3 kg and R=0.27 meters. Solving symbolically for the angular acceleration in terms of known parameters:

$$\alpha = \frac{\tau_{net}}{I}$$

$$\boxed{\alpha = \frac{\tau_{app}}{mR^2}}$$

Plugging in numbers:

$$\alpha = \frac{(0.073 \text{ N} \cdot \text{m})}{(4.3 \text{ kg})(0.27 \text{ m})^2}$$

$$\boxed{\alpha = 0.23 \text{ radians/s}^2}$$

Part (b):

We use the standard expression for **Constant Acceleration (1-D Rotational):**

$$\omega = \omega_0 + \alpha t$$

$$\omega = 0 + \alpha t$$

$$\boxed{t = \frac{\omega}{\alpha} = \frac{\omega m R^2}{\tau_{app}}}$$

Plugging in numbers:

$$t = \frac{22.2 \text{ radians/s}}{0.23 \text{ radians/s}^2}$$

$$\boxed{t = 95 \text{ seconds}}$$

Part (c): We use our expression for **Centripetal Acceleration**, and we then use our previous solutions to express the acceleration in terms of *given parameters*:

$$a_c = \omega^2 r$$

$$a_c = (\alpha t)^2 R$$

$$\boxed{a_c = \alpha^2 t^2 R = \left(\frac{\tau_{app}}{mR^2}\right)^2 t^2 R}$$

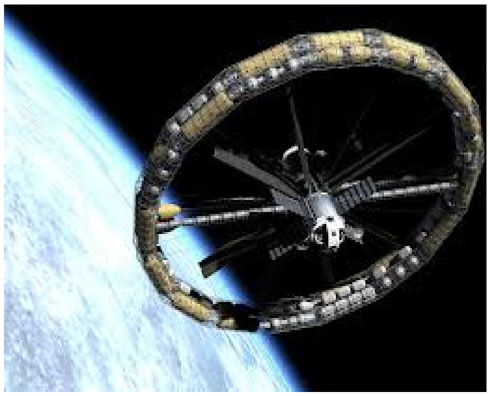
Plugging in numbers:

$$a_c = (0.23 \text{ radians/s}^2)^2 (5.0 \text{ s})^2 (0.27 \text{ m})$$

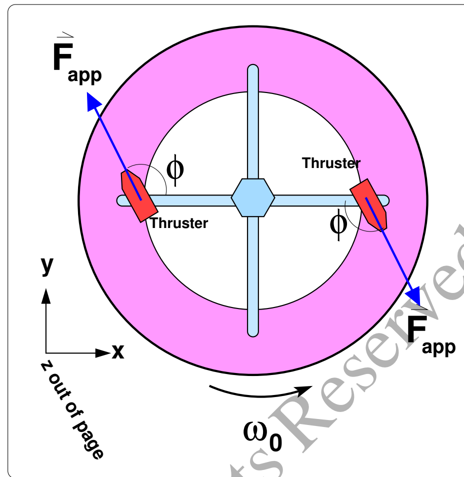
$$\boxed{a_c = 0.37 \text{ m/s}^2}$$

Note that we have only asked for the *centripetal* acceleration which depends only on the speed and the radius and not on whether or not the speed itself is changing. Note also that the acceleration is a *kinematic* quantity, and it's value does not depend on the mass of the mud blob in any way.

Problem 5: Stopping a Spinning Space Station



Spinning space station orbits Earth



Consider a hypothetical future space station that consists of a torus attached by spokes to a central hub as show above. Suppose the space station has an given initial angular speed of ω_0 and a rotational inertia $I = CMR^2$ where M is the given mass of the space station, R is the given outer radius of the torus, and C is a given numerical constant.

In this situation, the space engineers need to repair the station and in order to do this, they need to stop the rotation and bring the station to rest. Therefore, the engineers affix two large *thrusters* to the space station as shown. Each thruster is attached to a fixed position on the space station corresponding to a given distance r from the center and at a given angle ϕ relative to the radial direction. When activated, each thruster will result in a force of given constant magnitude F_{app} applied to the station as shown. Note that a coordinate system is given here, with x-direction corresponding to the right, y-direction corresponding to the top of the page, and z-direction corresponding to out of the page.

Part a) – What is the magnitude of the total *torque* τ applied to the space station that results from the action of both of the thrusters? Give your answer in terms of given parameters only. Explain your work.

Part b) – What is the magnitude of the *angular acceleration* α of the space station that results from the action of both of the thrusters? Give your answer in terms of given parameters only. Explain your work.

Part c) – What is the *linear acceleration* $\vec{a}$ of the entire space station as a single body that results from the action of both of the thrusters? Indicate both the magnitude and the direction. Give your answer in terms of given parameters only. Explain your work. Hint: The Rolling Constraint does *not* apply here.

Part d) – Assume that at time $t = 0$ both thrusters are activated. How many total rotations of the space station will occur before the space station is brought completely to rest? Give your answer in terms of given parameters only. Explain your work. Hint: one rotation equals 2π radians of angle.

Solution to Problem 5:

Part (a):

Since we are given the forces, we want to use the **Definition of Torque**:

$$\tau_{\vec{F}} \equiv rF \sin\phi$$

Here r is the length of a radial vector that points from the (central) pivot point to the point of application of the thruster force and ϕ is the (given) angle between the thruster force vectors and that same radial vector.

In this case, we have two forces $\vec{F}_{app}$ and each of them contributes a torque. The both forces are located at a given position r from the central point of rotation. And the forces are applied at a given angle ϕ relative to the radial vector. So the total torque applied is just twice the torque from each thruster: [1]

$$\tau_{tot}\ 2\tau_{\vec{F_{app}}}$$

$$\boxed{\tau_{tot} = 2rF_{app}\sin\phi}$$

Note that the correct radius to use is the distance to the thrusters r and *not* the radius of the space station R.

[1] An aside: to get the direction we use the **Right-Hand-Rule** for the cross-product which shows the that the forces generate a torque that results in a **clock-wise** twist corresponding to a torque vector that points into the page corresponding to the negative-z-direction. Note that for this exam, we only need the magnitude.

Part (b):

Here we use **Newton's Second Law** for rotational motion:

$$\tau_{net} = I\alpha$$

$$\alpha = \frac{\tau_{net}}{I}$$

We calculated the net torque in Part (a) and we are given the rotational inertial, so:

$$\boxed{|\alpha| = \frac{2rF_{app}\sin\phi}{CMR^2}}$$

That's it.

Part (c):

Here we use **Newton's Second Law** for linear (translational) motion:

$$\vec{F}_{net} = M\vec{a}$$

In this problem there are only two forces, each of magnitude F_{app} and each by construction directed in opposite directions. So therefore ***the net force on the station as a whole due to the thrusters is zero.*** Since the net force is zero, then by **Newton's Second Law** it must be true that **the linear acceleration of the station is zero.**.

That's it.

Part (d):

For this part we apply **Rotational Kinematics with Constant Angular Acceleration:**

First, we have to watch our signs here. If we define the direction of the angular speed (counter-clockwise) as positive, then our angular acceleration will be negative. From Part (b) then we get:

$$\alpha = -\frac{2rF_{app}\sin\phi}{CMR^2}$$

So we write down the three equations for Constant Acceleration:

$$\alpha = constant$$
$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

So when the station is brought to rest, this corresponds to the final angular speed being set to zero: $\omega = 0$. So we use the second equation above to get the time:

$$0 = \omega_0 + \alpha t$$
$$t = -\frac{\omega_0}{\alpha}$$

Now we plug this in to get our total rotational angle (defining the initial heading $\theta_0 = 0$):

$$\theta = 0 + \omega_0\left(-\frac{\omega_0}{\alpha}\right) + \frac{1}{2}\alpha\left(-\frac{\omega_0}{\alpha}\right)^2$$

Some algebra:

$$\theta = -\left(\frac{\omega_0^2}{\alpha}\right) + \frac{1}{2}\left(\frac{\omega_0^2}{\alpha}\right)$$
$$\theta = -\frac{1}{2}\left(\frac{\omega_0^2}{\alpha}\right)$$

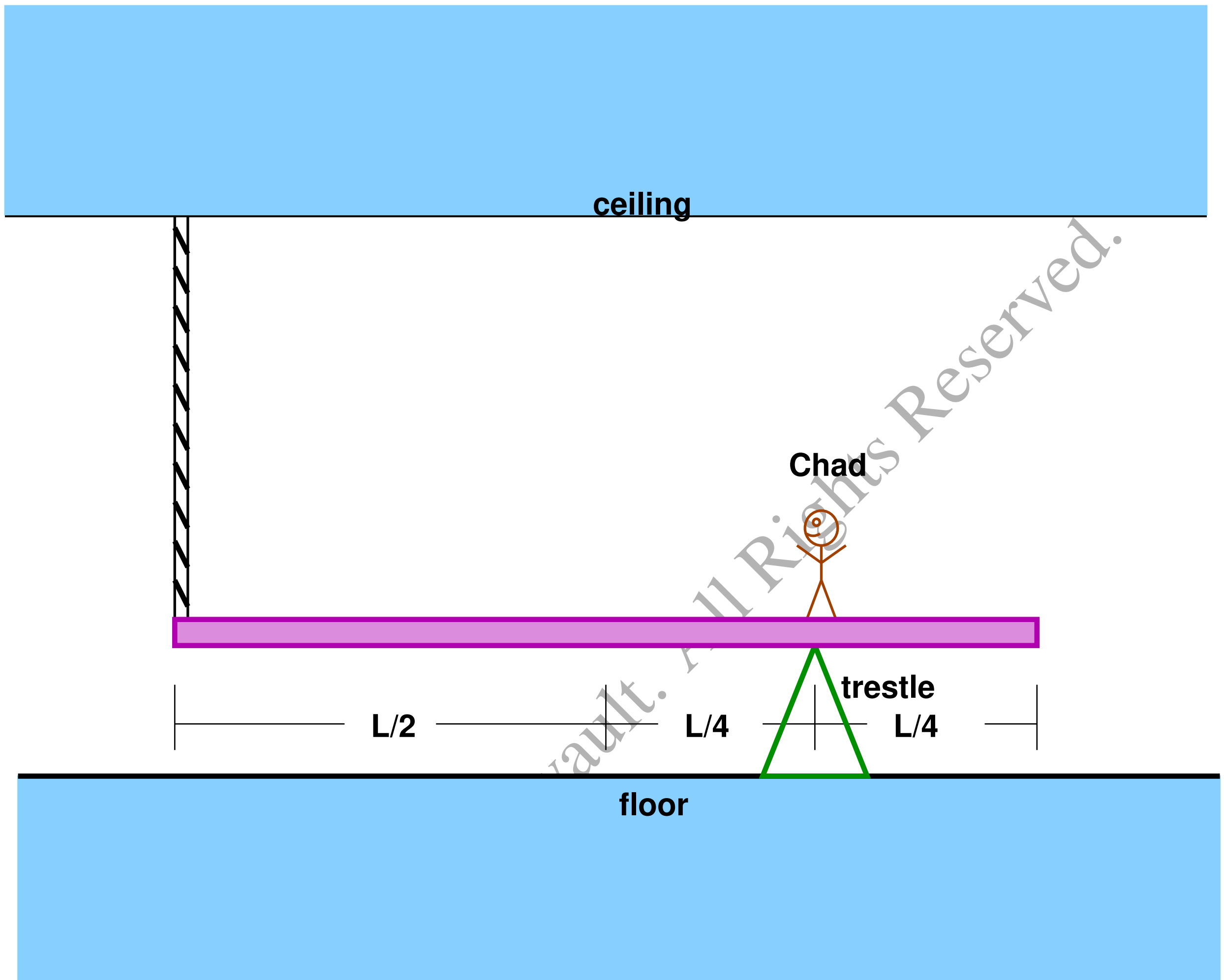
Then we plug in $\alpha = -\frac{2rF_{app}\sin\phi}{CMR^2}$ to get:

$$\theta = \left(\frac{1}{2}\omega_0^2\right)\left(\frac{CMR^2}{2rF_{app}\sin\phi}\right)$$
$$\theta = \frac{\omega_0^2 CMR^2}{4rF_{app}\sin\phi}$$

Finally we note that we get one rotation per 2π radians so we divide by 2π to get rotations:

$$\boxed{\text{rotations} = \frac{\theta}{2\pi} = \frac{\omega_0^2 CMR^2}{8\pi r F_{app}\sin\phi}}$$

Problem 6: Hanging Chad



Chad stands on a a large heavy uniform board. Chad has a mass m. The board has a mass M and a total length L. The board is suspended by a rope on the left side and a trestle which pushes the board upward at a position that is precisely three-fourths the length of the board from the left side as shown in the figure above. Chad is positioned directly above the trestle.

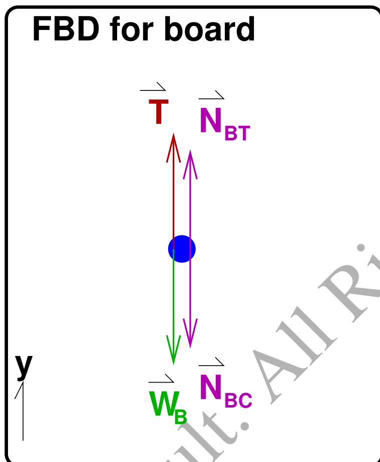
Draw complete and carefully labeled Free Body Diagrams (FBDs) for both Chad and the board. Also draw a complete and carefully labeled Extended Free Body Diagram (XFBD) for the board.

Determine T, the magnitude of the Tension on the board due to the rope. Also determine N_{BT}, the magnitude of the Normal force on the board due to the trestle. Express your answer in terms of the given parameters. Be sure to explain your work.

SOLUTION TO PROBLEM 2:

First, we note that as Chad is at rest and standing on the board, the *net force on Chad is zero*, and therefore by **Newton's Second Law**, the force Normal on Chad (up) must exactly cancel the force of Weight on Chad (down). Furthermore, by **Newton's Third Law** this force has the same magnitude as the force Normal of Chad *on the board*. In other words the force Normal on the Board due to Chad is given by $N_{BC} = mg$.

Now let's write down a **Free Body Diagram** for the *board:*



There are four forces in the FBD: (1) $\vec{W}_B$ = the Weight of the board, (2) $\vec{N}_{BC}$ = the force Normal on the board due to Chad, (3) $\vec{N}_{BT}$ = The force Normal on the board due to the trestle, and (4) $\vec{T}$ = the force of Tension due to rope.

We apply **Newton's Second Law** using the Free Body Diagram:

$$F_y = Ma_y$$

From the FBD we see:

$$T + N_{BT} - N_{BC} - W_B = Ma_y$$

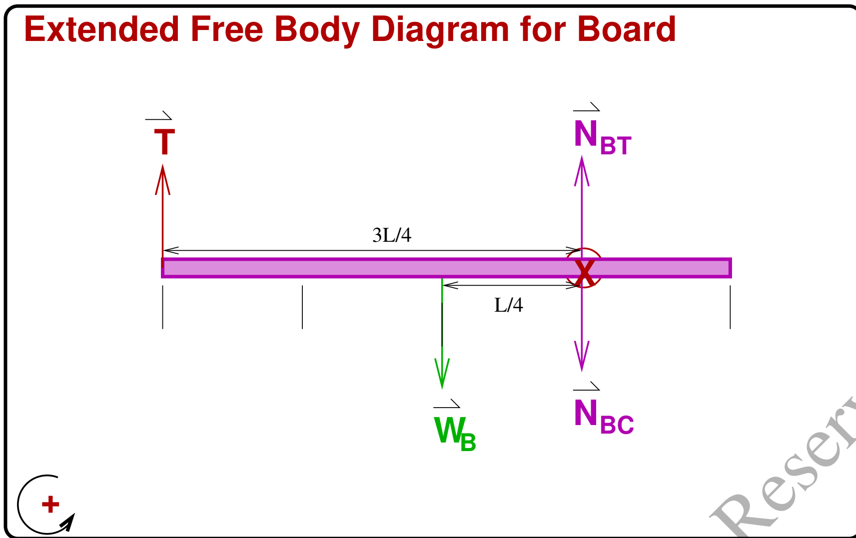
And we know the acceleration is zero because the board is static:

$$T + N_{BT} - N_{BC} - W_B = 0$$

$$T + N_{BT} - mg - Mg = 0$$

This is as far as we can go with a regular Free Body Diagram. *Two unknowns, T and N_{BT} and one equation.*

Next we consider the **torques** due to each force by writing down an **Extended Free Body Diagram** which includes the same four forces that we had in the regular Free Body Diagram.



We apply **Newton's Second Law for Rotations** using the Extended Free Body Diagram:

$$\tau_{net} = I\alpha$$

Here α, the angular acceleration, is *zero* because of this is a static situation and nothing is moving.

$$\tau_{net} = 0$$

Since there are four *forces*, there are also four *torques* to consider in sum:

$$\tau_T + \tau_{N_{BT}} + \tau_{N_{BC}} + \tau_{W_B} = 0$$

We need to chose a **pivot point**. *Any choice of a pivot point will do.* Note that while we are free to choose any we want, but we must define one to calculate torques, fr this problem it is very convenient to choose the position of the trestle as the pivot point. This is indicated by the "X" on the XFBD above.

Now can calculate the torques associate with each force according to $\tau_F = \pm F r \sin\theta$, where F is the magnitude of the force, and r is the radius from the pivot point to the point of application of the force, and θ is the angle between the radius vector and the force vector. In this problem, because the forces are all at right angles to the board, the term $\sin\theta = \pm 1$ always. We can read off the radius values from the XFBD: $r_T = \frac{3L}{4}$, $r_W = \frac{L}{4}$, and the radius for the two normal forces is zero. Note that the **weight of the board is applied at the center of mass of the board.**

So we add up the torques using the *convention that positive torque corresponds to a counter-clockwise twist*:

$$-T\left(\frac{3L}{4}\right) + W_B\left(\frac{L}{4}\right) + 0 + 0 = 0$$

We solve for T:

$$\frac{3TL}{4} = \frac{W_B L}{4}$$

$$\frac{3T}{4} = \frac{W_B}{4}$$

$$\boxed{T = \frac{Mg}{3}}$$

Plugging this back into our result from the regular FBD to N_{BT}:

$$T + N_{BT} - mg - Mg = 0$$

$$\frac{Mg}{3} + N_{BT} - mg - Mg = 0$$

$$N_{BT} = mg + Mg - \frac{Mg}{3}$$

$$\boxed{N_{BT} = mg + \frac{2Mg}{3}}$$