

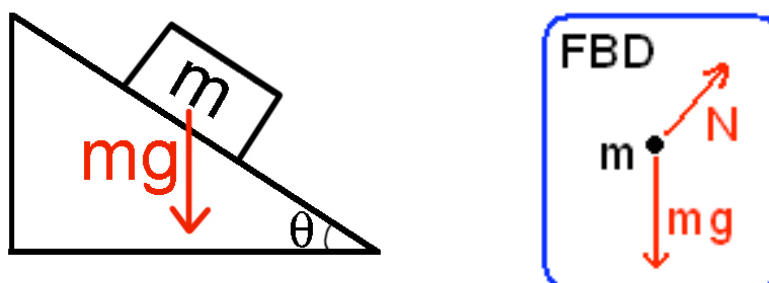
## Chapter 6+

### Revisit Force Components

Revisit: <ul style="list-style-type: none"> <li>Force components</li> </ul>	New: <ul style="list-style-type: none"> <li>Inclined planes without friction</li> <li>Forces at angles in circular motion</li> </ul>
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#### The Inclined Plane

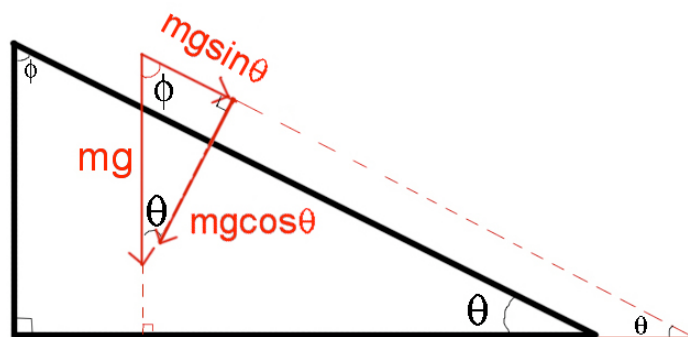
Consider a block  $m$  sliding down due to gravity at an angle without friction and its FBD:



What are the force components parallel and perpendicular to the hill? (You may like to think of the direction along the hill is our new “horizontal axis” or “x-axis.” The normal to the hill is our new vertical or y-axis.) Afterwards we will write down Newton’s second law.

1) There is no component of  $N$  along the hill.

2) There is a component of the weight  $mg$  along the hill. Draw the **GOLDEN** triangle with its hypotenuse in the direction of the force due to gravity, one leg parallel to the hill (the component we want), and thus the other leg perpendicular to the hill. We superimpose the right triangle on the hill to see how one of its angles is equal to the hill’s angle  $\theta$ . (First, notice that the hill triangle and the **GOLDEN** triangle have the same angle  $\phi$  because of the mutually parallel sides. Thus the other acute angles of these two triangles are equal to each other because they are complementary to the same angle.)

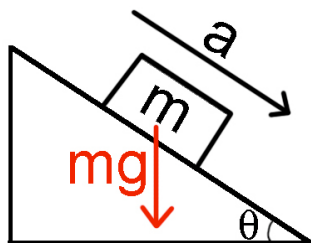


3) From the **GOLDEN** triangle and the SOHCAH without the TOA, the **magnitudes** of the two components are

$$(mg)_{\parallel} = mg \sin \theta \quad \text{and} \quad (mg)_{\perp} = mg \cos \theta$$

4) We will use these components over and over in inclined plane problems. Notice that you can remember which has the sine and which has the cosine by looking at the limit  $\theta = 0$ , where we know the limits  $(mg)_{\parallel} = 0$  and  $(mg)_{\perp} = mg$ .

**Newton's Second Law:** We're ready for " $\vec{f} = m \vec{a}$ " for the block  $m$  going down the hill.



Defining the acceleration  $a$  along and down the hill (so it is now the "x-component" and the only acceleration component), we can write down the second law for the parallel and perpendicular directions:

**Parallel:** (down the hill is positive)

$$mg \sin\theta = ma$$

**Perpendicular:** (going along the normal up from the hill is positive)

$$N - mg \cos\theta = 0$$

Follow-up on our answers:

- In the first equation, the mass  $m$  cancels out and we get

$$a = g \sin\theta \text{ (no friction)}$$

which is a good old constant acceleration situation that we know how to handle. It's like falling in gravity but with acceleration reduced by a factor of  $\sin\theta$ .

- From the second equation,

$$N = mg \cos\theta \quad (\text{Aha! A case where } N \neq mg !)$$

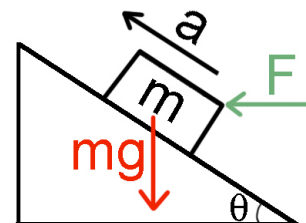
which will be important in our friction discussions in the next chapter.

**Digression:** Before we look at a problem, we digress for a story: A toboggan on snow hasn't a lot of friction and in fact we can recall taking our family down a hill where we had wondered why there were no previous tracks in the snow going down that side. Well we found out why. The hill turned into a  $\theta = \pi/2$  descent; there was actually zero friction then! The neighborhood kids all gathered around and applauded when we came to a really, really painful "thud" at the bottom. It was a miracle no one was hurt, so we could revel in the applause. You see, no one had ever gone down that side before ... or after.

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**Problem 6-3**

A block of mass  $m$  is on a **frictionless** hill at an angle  $\theta$  and subject to gravity and a constant horizontal force  $F$  as shown in the figure to the right. Refer to the acceleration up the hill as  $a$  (so our axis along the hill here is positive going up the hill)



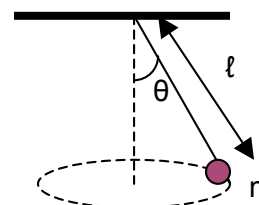
- (a) Draw an FBD for this block.
- (b) Find and solve two second-law equations for the acceleration  $a$ , and for the support force  $N$  normal to the hill, in terms of the given parameters. Define upward along the hill and upward normal to the hill as the respective positive directions.
- (c) Find  $a$  in  $\text{m/s}^2$  and  $N$  in  $\text{N}$  ( $N$  is the force,  $N$  is Newtons!) if  $m = 1.0 \text{ kg}$ ,  $F = 5 \text{ N}$ ,  $\theta = 30^\circ$ .

(As always: If we find that  $a < 0$ , it means the mass is accelerating down the hill.)

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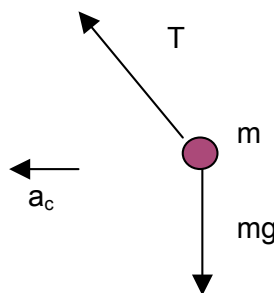
### Circular Motion and Forces at Angles

**Example:** A ball of mass  $m$  is attached to the ceiling by a string of length  $\ell$  moves at constant speed  $v$  in a horizontal circle. The string makes an angle  $\theta$  with the vertical, under the influence of gravity. Newton's second law allows us to connect the different parameters.

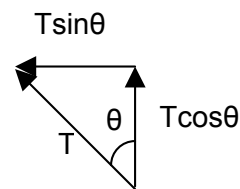


**Question:** So we can ask, what is the tension  $T$  in the string and the speed  $v$  of the ball in terms of  $\theta$ ,  $g$ ,  $m$ , and  $\ell$ .

**Solution:** First, let's draw an FBD:



We learn everything from our second law applied to both the vertical and the horizontal direction. First, note that we have zero vertical acceleration and nonzero horizontal centripetal acceleration (radially inward toward the center of the circular motion). Second, to get the force components, the golden triangles should have their legs vertical and horizontal. The weight  $mg$  is purely vertical and the tension  $T$  has upward vertical component  $T\cos\theta$  and leftward horizontal component  $T\sin\theta$ .



The vertical second law with zero acceleration implies the net vertical force is zero which in turn implies force balance with the upward vertical force equal to the downward vertical force:

$$T \cos \theta = mg \Rightarrow T = \frac{mg}{\cos \theta}$$

The horizontal second law with acceleration  $v^2/R$  radially inward (calling leftward in the previous picture the positive direction):

$$T \sin \theta = ma_c = m \left( \frac{v^2}{R} \right)$$

$$\Rightarrow v^2 = \frac{R T \sin \theta}{m}$$

With  $R = \ell \sin \theta$ , we then have

$$v^2 = \frac{\ell T \sin^2 \theta}{m}$$

Substitute T into this (from the vertical force balance result above) and cancel the mass as shown:

$$v^2 = \frac{\cancel{\ell} \cancel{m} g \sin^2 \theta}{\cancel{m} \cos \theta} = \frac{\ell g \sin^2 \theta}{\cos \theta}$$

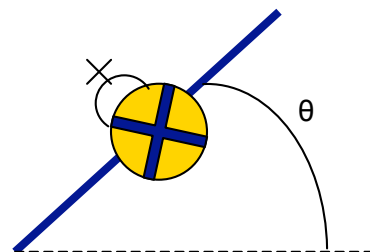
or

$$v^2 = g \ell \tan \theta \sin \theta$$

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#### Problem 6-4

Assume that the lift force on an airplane is **perpendicular** to the wing surface, so it is analogous to the normal force on a car due to the surface of a banked road. The normal force on a car is normal to the road surface. The normal lift force on an airplane is normal to the wing surface. But the airplane problem is simpler than a banked road problem because you can ignore any sideways friction force!



a) Why do we need the airplane to fly with its wings at an angle in order to go in a horizontal circle?

b) For the airplane shown, is the center of the circle to the left or right of this picture?

c) If the airplane in the picture is flying in a horizontal circle, consider both the vertical component and the horizontal component of the **net force** on the airplane (the net force is the total force\* due to the two forces: gravity and the normal lift). One of these two net components, vertical or horizontal, must be zero and one must NOT be zero. Which is which?

\* We ignore the possibility that the airplane may be accelerating or slowing down, the discussion of which would involve the propeller force and the air drag force, forces that would be perpendicular to the picture (the face of this page).

d) Suppose an airplane is flying in a large horizontal circle at a speed of 400 km/hr. At what angle  $\theta$  in degrees to the horizontal must the wings of the plane be tilted if the radius of the circle is 2.0 km?

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