ST-WAVE
Standing Waves on a String
revised August 2021

Learning Objectives:
During this lab, you will
1. estimate the uncertainty in a quantity that is calculated from quantities that are uncertain.
2. test a physical law experimentally.

A. Introduction
We are continually bombarded by waves - radio, television, infrared heat, microwave communication, visible light and sound to name a few. Although these waves appear to be very different from each other, they have many features in common. In this experiment we shall investigate one particular type of mechanical wave, waves that travel on a string.

A string can support two different types of mechanical waves; longitudinal waves and transverse waves. Sound waves are longitudinal; however, these waves are difficult to observe and measure since the waves depend upon the compression and expansion of elements of the string along its length. Transverse waves are easy to see; the string oscillates perpendicular to its length. You will examine transverse waves to learn about wave propagation.

You will not write a paper for this lab. Rather, you should fill out a worksheet for this experiment and for the accompanying lab on the velocity of sound and submit both to Canvas the day you perform the lab. Each lab experiment is set up at a different station; you will switch between the stations halfway through the lab period.

B. Apparatus
You will use a function generator, an audio amplifier with a separate power supply, a wave driver based on an audio speaker, string, a pulley, hanger, weight, and meter stick.

C. Theory
We can represent a sinusoidal wave propagating in the x-direction as

$$\psi(x,t) = \psi_m \sin(kx \pm \omega t + \phi) \qquad (1)$$

(ψ is the Greek letter "psi") where $\psi(x,t)$ is the amplitude of the wave, ψ_m is the maximum value of the amplitude, k is the *wave number*, ω is the *angular frequency*, and ϕ is the *phase angle* of the wave. The wave number k is related to the wavelength λ by $k = 2\pi/\lambda$, while the angular frequency ω is related to the period T of the oscillation by $\omega = 2\pi/T$. The phase angle ϕ provides the 'initial' condition of the string when $t = 0$ and $x = 0$. The $\pm$ sign in the argument is negative for a wave traveling in the positive x-direction and positive for a wave traveling in the negative x-direction.

The speed v with which a wave propagates on a string is related to the linear density μ (*mass per unit length*) of the string and the tension τ in the string by the equation

$$v = \sqrt{\tau/\mu} \qquad (2)$$

The speed v with which a wave travels in any medium may also be expressed in terms of the wavelength and period by

$$v = \lambda f = \lambda/T = \omega/k. \qquad (3)$$

Thus, for a given frequency, the wavelength is a function of the linear density and tension.

For a transverse wave propagating in the x-direction on a string, $\psi(x,t)$ is the displacement of the string perpendicular to x as a function of position and time. If the wave travels along the string and is reflected from a fixed end, the combination of two waves of the same frequency traveling in opposite directions may result in a standing wave which can be represented by the equation

$$\psi = \psi_0 \sin(kx - \omega t) + \psi_0 \sin(kx + \omega t). \qquad (4)$$

This reduces to

$$\psi = 2\psi_0 \sin(kx)\cos(\omega t), \qquad (5)$$

Standing Waves on a String

the equation of a standing wave on the string.

The frequencies at which standing waves are formed are referred to as *resonant* frequencies. For a string fixed at its ends, the lowest resonant frequency (*the fundamental frequency*) corresponds to a single loop with a wavelength λ given by $L = \lambda/2$, where L is the length of the string. Higher frequencies (*overtones*) correspond to additional loops, two loops for $L = \lambda$, three loops for $L = 3\lambda/2$, and in general n loops for $L = n\lambda/2$. It is the pattern of overtones that is in part responsible for the rich sound of certain musical instruments.

Figure 1 shows the extremes of the motion for a standing wave on a string of length 1 (*arbitrary units*). In this figure, we have two loops, so the wavelength is equal to the length of the string $\lambda = L$. During this motion, the string will at times form a straight line while it oscillates between a "positive sine" and a "negative sine." The two endpoints of this string are fixed and, if the frequency is properly adjusted, the middle of the string will also remain stationary even though nothing apparently holds it in place. The points on a standing wave that remain fixed are called *nodes* while the points of maximum excursion are called *antinodes*.

D.   Procedure

In order to predict the wave velocity, you need to determine the value of the string's linear density (*mass per length*), μ, when it is under the tension used for the rest of the experiment. The
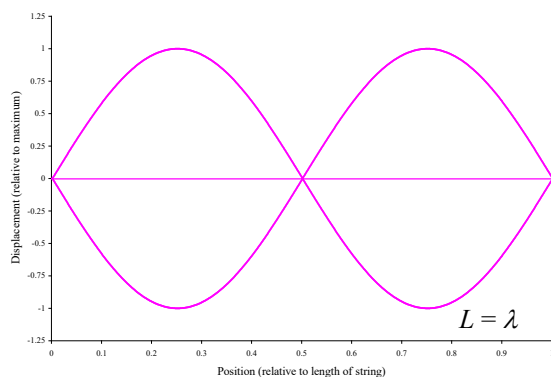

Figure 1: Representation of a two-loop standing wave

string is fairly elastic and can stretch significantly under such tension. Remove the string from the apparatus and weigh the string. Then determine and measure the *appropriate* length of the string, as well as the estimated uncertainty in this measurement. Consider carefully how to determine this *appropriate* length for finding μ. Your worksheet should include a discussion of your reasoning and measurement techniques.

If you untied the knots, retie a slip knot to each end of the string. Fasten one end of the string to the oscillator (*the knot slips over a clamp and the string slides into the slot on the driver using masking tape to keep it in place - see the photos posted on the web site*). Attach the other end of the string to a mass (*m*), hanging the string over the pulley as shown in Figure 3. You may assume that there is negligible uncertainty in *m*.

Turn on the power for the function generator, amplifier, and amplifier power supply.

Figure 2: Function generator

On the function generator, to the right of the display press the CH1/2 button to select the channel to which output the BNC cable is plugged into. The lower portion of the display will read "CH 1 Waveform" if that is
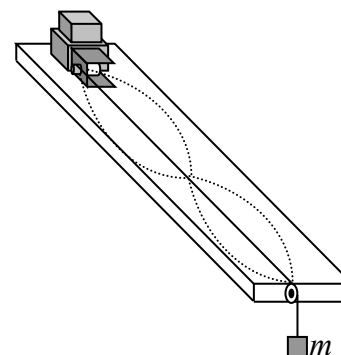

Figure 3: Experimental Setup

the channel you are using. Press the button below the CH1/2 button to select "Frequency," not "Period," which is displayed to the left of the button. Use the dial on the top right or the key pad to enter 10 and select Hz for the units. Then press the button below the frequency/period button to select "Amplitude," and use the dial or the keypad to enter 1.000 Vpp (Volts peak-to-peak). If the Offset and Phase are not 0, press the following buttons below the previous ones and set the offset and phase to zero. Once you have all these settings, press the "Output" button above the BNC cable.

Make sure that you have the frequency setting selected and starting with the frequency at 10 Hz, carefully adjust the frequency of the wave driver up or down using the dial, until a stable standing wave with a single loop forms on the string. You may be tempted to turn the amplitude higher to make it easier to see the motion of the string but if you hear a knocking sound, you have the amplitude too high (*and you are about to blow the speaker—this will displease the lab personnel*).

When the string is driven at a resonant frequency, you should notice that the motion is maximized, compared to nearby frequencies, and the nodes are stationary, moving neither along the string nor perpendicular to it. Note that the point where the string is attached to the driver cannot be a node, even though this point sometimes appears to be relatively stationary, since it is forced to move by the driver. The true node at this end of the string may be anywhere from a few mm to several cm away from the driver; try to produce it as close to the driver as you can. By contrast, the pulley provides a node for any wave motion since it in principle holds the string fixed, although it can be difficult to determine exactly where along the pulley the motion ceases. Any uncertainty of this type should be included in your error estimates.

Increase the frequency using the dial on the function generator to form 4, 5, 6... loops,

continuing on to form as many stable loops as possible. The current record is 38 (*Manuel Bernardo Arrillaga Tamez and William Neal Driscoll, Spring 2013*). Starting either with your maximum number of loops or your single loop, record the frequency of each standing wave from the display on the function generator and measure the wavelength with a meter stick. This latter measurement is improved if you measure the total, combined length of all your waves (*measure from the node nearest the driver to the node at the pulley*) and then divide by the number of waves. Calculate the period for each standing wave. You may assume that the frequency reading is exact, limited only by the number of significant figures it provides.

When you are done with all the readings, please turn all the equipment off and place the hanging mass on the wooden board, so that it doesn't continue providing tension to the axle of the wave driver.

E. Analysis

Use *Origin* to plot a graph of wavelength versus period, including error bars. Fit a straight line to the data. From the slope, calculate the *measured* wave velocity v_m. Staple a copy of this plot to your worksheet when you hand it in.

Calculate the wave velocity v_p predicted from the density of the string and the tension produced by the hanging weight.

Compare your measured and predicted values of the wave velocity. Are they consistent with each other? Justify your conclusions.

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