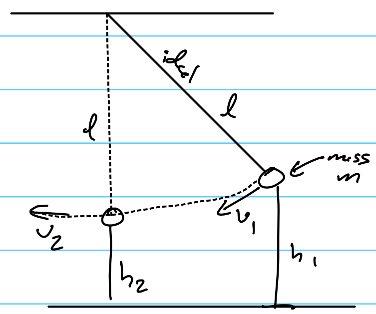
Practice Exam 1

1a)



Potential Energy Def. and Work Energy Relation

$W_{\vec{w}} = -\Delta U$

$= -(mgy' - mgy) = mgy - mgy'$

$= mg(h_1 - h_2)$

$$\boxed{W_{\vec{w}} = mg(h_1 - h_2)}$$

$W_{\vec{w}} = \int_{Path} \vec{w} \cdot d\vec{r}$

1b) Definition of work

work = $Fd\cos\theta$, here the angle the Rope makes with the bob is $90°$ and $\cos 90° = 0$ ∴ work done by Tension is 0. It is also perp. to motion direction, which, by definition, does 0 work.

1c) C of ME

Weight is a conservative force, assume no air resistance, Tension does 0 work.

Before = After

$E_{Tot} = E'_{Tot}$

$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$ $\quad v = v_1$ given

$\cancel{m}gh_1 + \frac{1}{2}\cancel{m}v_1^2 = \cancel{m}gh_2 + \frac{1}{2}\cancel{m}v_2^2$ $\quad v' = v_2$?

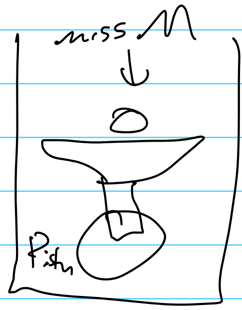
$gh_1 + \frac{1}{2}v_1^2 = gh_2 + \frac{1}{2}v_2^2$

$\frac{1}{2}v_2^2 = gh_1 + \frac{1}{2}v_1^2 - gh_2$

$v_2^2 = 2(g(h_1 - h_2) + \frac{1}{2}v_1^2)$

$$\boxed{v_2 = \sqrt{2(g(h_1 - h_2) + \frac{1}{2}v_1^2)}}$$

2



$y = K\sin(\Omega t)$

Ω is angular speed

y is position

a) Definition of Velocity

$v(t) = \frac{d}{dt}(y(t))$

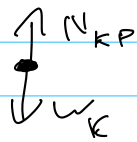
$v(t) = \Omega K\cos(\Omega t)$

$v(0) = \Omega K\cos(0)$

$= \Omega K$

$\boxed{\text{At } t=0 \quad v(t) = \Omega K}$

2b) FBD Kitten



Def of Accel

$a(t) = \frac{d}{dt}(v(t))$

$a(t) = \frac{d}{dt}(\Omega K\cos(\Omega t))$

$a(t) = -\Omega^2 K\sin(\Omega t)$

@ $t = \frac{\pi}{2\Omega}$ $\quad a(\frac{\pi}{2\Omega}) = -\Omega^2 K\sin\frac{\pi}{2}$

$a(\frac{\pi}{2\Omega}) = -\Omega^2 K$

N2L $\quad F_K = m_K a_K$

$a_K = -\Omega^2 K$ Kinematic constant

$\boxed{F_{Net_K} = -\Omega^2 K M}$

2c) $a(\frac{\pi}{2\Omega}) = -\Omega^2 K = a_K$ from part b

N2L $\quad F_{Net_K} = m_K a_K \quad$ Def of weight

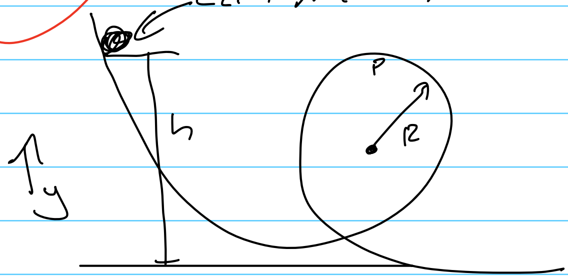
$N_{KP} - W_K = M(-\Omega^2 K) \quad W_K = Mg$

$N_{KP} = M(-\Omega^2 K) + Mg$

$\boxed{N_{KP} = M(-\Omega^2 K + g)}$

3.

Cart mass = M



a) $W_w = \int_{path} W\,dy$ Def of work

$$W_w = \int_y^{y'} -mg\,dy$$

$$W_w = -(mgy' - mgy) = -mg(y'-y)$$

Substitute

$$W_{\vec{w}} = -mg(2R-h) \qquad \boxed{W_{\vec{w}} = mg(h-2R)}$$

b) Released = at rest

C of ME

Weight is conservative, frictionless, Normal Force ⊥ to motion → no work

Before = After

$$mgy + \tfrac{1}{2}mv^2 = mgy' + \tfrac{1}{2}mv'^2$$

$$v_0 = 0,\ y = h,\ y' = 2R \quad v' = ?$$

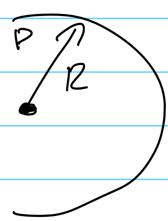
$$\cancel{m}gh + 0 = \cancel{m}g2R + \tfrac{1}{2}\cancel{m}v'^2$$

$$gh = g2R + \tfrac{1}{2}v'^2 \longrightarrow \tfrac{1}{2}v'^2 = gh - g2R$$

$$\longrightarrow v'^2 = 2g(h-2R)$$

$$\boxed{v' = \sqrt{2g(h-2R)}}$$

b)



UCM

$$a_c = \frac{v^2}{R}$$

FBD Cart

N2L

$$F_c = ma_c$$

$$N_{CT} + W_c = M\cdot\frac{v^2}{R}$$

$$N_{CT} = M\cdot\frac{v^2}{R} - W_c$$

$$N_{CT} = M\cdot\frac{2g(h-2R)}{R} - Mg$$

v' @ P = (b) answer

N3L

$N_{CT} = N_{TC}$, pressure

$$N_{TC} > 0$$

$$0 < \frac{M2g(h-2R)}{R} - Mg \qquad \text{Solve for } h$$

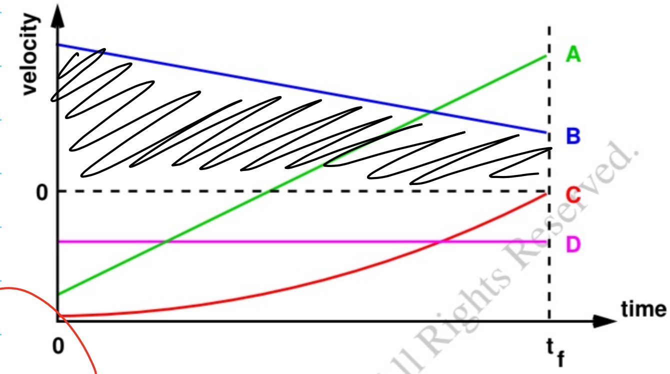
$$RMg < M2g(h-2R)$$

$$R < 2h - 4R$$

$$5R < 2h \longrightarrow \boxed{h > \frac{5R}{2}}$$

Practice Exam 2

1.



a) Constant Accel = Linear velocity

Definition of Acceleration

$a \equiv \frac{dv}{dt}$

A and B are linear so they have constant, non zero acceleration. D is linear but constant velocity. This means accel is 0, constant but zero nonetheless.

b) Always Slowing Down = negative acceleration

Def of Accel: $a \equiv \frac{dv}{dt}$ velocity slope must be negative to have negative accel.

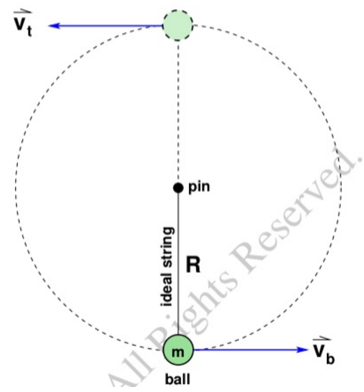
B only. Speed becomes closer to zero

c) Def of velocity and Position

Position: $x \equiv \int v \, dt$. The maximum displacement is determined by the signed area under the curve. Maximum area is B.

Trapezoid Area with known values to calculate

2.



a) Def of work

$$W_{\vec{w}} = \int_y^{y'} w \, dy$$

$$W_{\vec{w}} = -mgy\Big|_y^{y'}$$

$$= -mgy' + mgy$$

$$= -mg(0 + 2R)$$

$y' = 2R \quad y = 0$

$$\boxed{W_{\vec{w}} = -mg2R}$$

Algebra!

b) $W = Fd\cos\theta$ Def of work Rope is $\perp$ to the ball, so it will have an angle of $90°$. $\cos\theta$ when $\theta = 90°$ is 0. Tension does 0 work!

c) C of ME

Tension and weight on ball. Weight is conservative, so no work. Tension does not do work as established in b. Conditions met

Before = After $\quad y = 0$

$E_{Tot} = E'_{Tot} \quad y' = 2R$

$\cancel{m}gy + \frac{1}{2}\cancel{m}v^2 = \cancel{m}gy' + \frac{1}{2}\cancel{m}v'^2 \quad v = v_b$

$g0 + \frac{1}{2}v_b^2 = g2R + \frac{1}{2}v'^2$

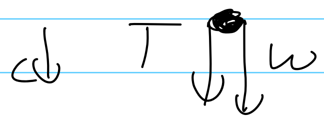
$\frac{1}{2}v'^2 = \frac{1}{2}v_b^2 - g2R$

$v'^2 = v_b^2 - 4gR$

$$\boxed{v' = \sqrt{v_b^2 - 4gR}}$$

d) UCM

FBD Top



$a_c = \frac{v^2}{R} \quad v' = \sqrt{v_b^2 - 4gR}$

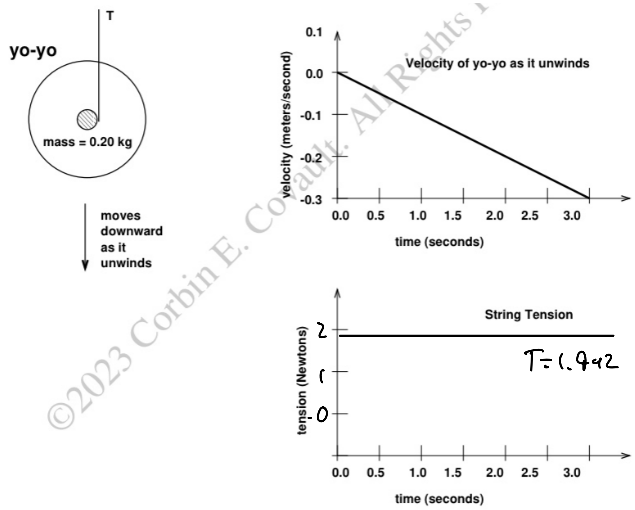
N2L $F_c = ma_c$

$T + w = m\frac{v^2}{R}$

$T = m\frac{v^2}{R} - mg \qquad T = m\left(\frac{v^2}{R} - g\right)$

Substitute: $\boxed{T = m\left(\frac{v_b^2 - 4gR}{R} - g\right)}$

3.



Def of velocity

$a_y = \frac{dv}{dt}$

$\text{slope} = \frac{(-.3-0)}{3-0}$

$= \frac{-.3}{3} = -\frac{3}{10} \cdot \frac{1}{3}$

$a_y = -\frac{1}{10} = -.1$

$a_y = -.1$

FBD Yoyo

y T W

N2L $F_y = m a_y$

$T - W = m a_y$

$T = m a_y + m g$

$T = m(a_y + g)$

$T = 0.2(-.1 + 9.81)$

$= 1.942$

Tension is independent bc velocity is linear.