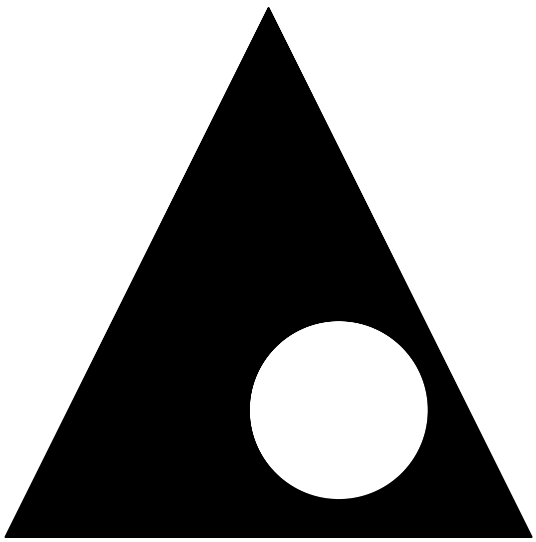
UNC LAB: Error Analysis and Propagation Exercise

Revised April 06, 2006

Your Name: ____



This exercise is designed to help you understand error analysis and error propagation. You need to determine the area of the shaded region in the figure above; that is, the area of a triangle minus the area of a circle. If the triangle has a height, *h*, and width, *w*, and the circle has diameter *d*, then the shaded area is given by the formula $A = hw/2 - \pi d^2/4$.

Every measurement has an associated uncertainty. The uncertainties can be labeled with the symbol, δ, which indicates a small change in the associated quantity. The uncertainties of *h*, *w*, and *d* are given by δ_h , δ_w , and δ_d respectively.

Use a metric ruler to measure *h, w,* and *d*, estimate the uncertainties δ_h, δ_w, and δ_d in your measurements of each quantity and enter these values below, in cm. For your convenience, copy these values onto the other side of this page.

________	±	________ cm	________	±	________	cm	______	±	______	cm
h	±	δ_h	w	±	δ_w		d	±	δ_d	

Now calculate $A = hw/2 - \pi d^2/4 =$ ______________________ cm^2

To estimate the *uncertainty* in *A*, δ_A, we need to *propagate* each individual contribution to the uncertainty (δ_h, δ_w, and δ_d) through the equation for *A* to find out how much each contributes to the uncertainty in *A* (*these terms are labeled as* δ_{Ah}, δ_{Aw}, *and* δ_{Ad}) and then add these contributions in quadrature $\delta_A = (\delta_{Ah}{}^2 + \delta_{Aw}{}^2 + \delta_{Ad}{}^2)^{1/2}$.

The first step is to determine δ_{Ah}, δ_{Aw}, and δ_{Ad}. This may be done by one of two methods. In the computational method, you calculate the change in *A* caused by substituting for each term, such as *h*, the value plus its estimated uncertainty, such as $h + \delta_h$ (or $h - \delta_h$). The derivative method has you calculate terms such as δ_{Ah} using the idea that any small change in *A* due to a small change in *h* is given by the derivative of *A* with respect to *h*, treating all the other terms such as *w* and *d* as constants. This is properly called a *partial derivative* and uses the symbol ∂ as in $\frac{\partial A}{\partial h}$ rather than $\frac{dA}{dh}$. Once you know how *A* changes as a function of *h*, you can simply multiply this by the estimated uncertainty in *h*, δ_h, to find $\delta_{Ah} = |\partial A/\partial h|\delta_h$.

Now, for some practice in error propagation, fill in each of the blanks on the other side of this page.

COMPUTATIONAL METHOD

δ_{Ah} = $|(hw/2 - \pi d^2/4) - ((h+\delta_h)w/2 - \pi d^2/4)|$ = { *this simplifies to* $\delta_h w/2$} = ________ _____
(units)

δ_{Aw} = $|(hw/2 - \pi d^2/4) -$ __________________________________$|$ = ________ _____

δ_{Ad} = $|(hw/2 - \pi d^2/4) -$ __________________________________$|$ = _________ _____

$\delta_A = (\delta_{Ah}{}^2 + \delta_{Aw}{}^2 + \delta_{Ad}{}^2)^{1/2}$ = ____________ _____

You should quote your value for A in the form $A \pm \delta_A$ (units): _______ ± ______ ____
(δ_A *is normally given with one or at most two significant figures while the most significant figure in the value of* A *should be determined from* δ_A, *as in* A = *3.65 ± 0.03 cm*^{2}. *See Appendix V, Section D.*)

DERIVATIVE METHOD

(*Optional for P115 students*)

$$\delta_{Ah} = \left|\frac{\partial A}{\partial h}\right| \delta_h = \left|\frac{\partial}{\partial h}\left(\frac{hw}{2} - \frac{\pi d^2}{4}\right)\right| \delta_h = \frac{\delta_h w}{2} = \text{\_\_\_\_\_\_\_\_ \_\_\_\_\_}$$

(units)

$$\delta_{Aw} = \left|\frac{\partial A}{\partial w}\right| \delta_w = \left|\frac{\partial}{\partial w}\left(\frac{hw}{2} - \frac{\pi d^2}{4}\right)\right| \delta_w = \text{\_\_\_\_\_\_\_\_} = \text{\_\_\_\_\_\_\_\_ \_\_\_\_\_}$$

$$\delta_{Ad} = \left|\frac{\partial A}{\partial \_}\right| \delta_{\_\_} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} = \text{\_\_\_\_\_\_\_\_\_\_\_\_} = \text{\_\_\_\_\_\_\_\_\_\_ \_\_\_\_}$$

$$\delta_A = \sqrt{\delta_{Ah}^2 + \delta_{Aw}^2 + \delta_{Ad}^2} = \text{\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_}$$

A = _______ ± _______ ____

You should find that the computational and derivative methods give similar results.

GRADE:_______
(out of 10 points)

GRADED BY ________
(TA's initials)