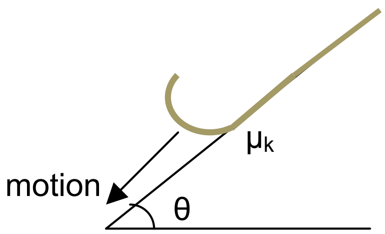
Chapter 7+

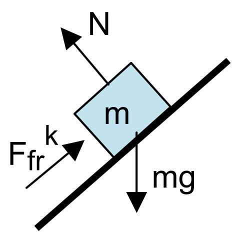
Revisit:	To-Do:
• Kinetic (sliding) friction • Static friction	• Add friction to the inclined plane • Friction on a moving surface – blocks sliding on blocks that can move

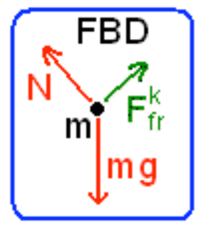
Inclined Plane: Now Add Friction To The Hill

Recall our frictionless inclined plane discussion from Ch. 6+. We will now remove the "less" from it. Consider a toboggan and riders with a total mass m going down a hill with sliding friction μ_k . (This scenario is what reminded us of our story on p. 6-7 !)

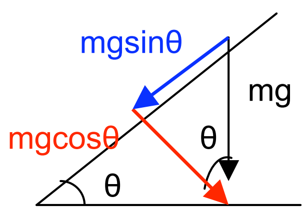
The toboggan, hill, force pictures (including sliding friction) and FBD are:







Recall the **GOLDEN** right triangle from Chapter 6+ for finding the weight components is



1) The second law equation for the direction perpendicular to the hill (the normal direction) has zero acceleration and is

$$N - mg\cos\theta = 0$$

or

$$\boxed{N = mg\cos\theta}$$

2) The second law equation for the direction parallel to the hill, including sliding friction and with the acceleration a defined downhill as positive, is

$$mg\sin\theta - F_{fr}^{k} = ma$$

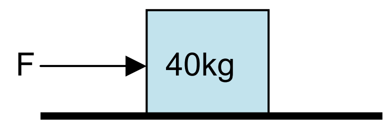
From the normal force analysis, we can substitute for N in $F_{fr}^{k} = \mu_k N$ such that

$$mg \sin\theta - \mu_k mg \cos\theta = ma$$

or, with the mass cancelling out, we've got a good old constant acceleration situation again:

$$\boxed{a = g \sin\theta - \mu_k g \cos\theta}$$

Using an inclined plane to measure friction parameters: We have said that we expect to find $\mu_k < \mu_s$ when we measure them for a given body sliding on or sticking to a given surface. But how can we measure them?



1) A crude way to determine μ_s is as follows: Measure the "critical value" of F, which is the maximum value we can push just before sliding. Suppose, for example, the critical value is measured to be 300 N for the 40 kg block shown. With N = mg, and F(critical) = μ_s N:

$$300 = \mu_s N = \mu_s mg = \mu_s (40)(9.8) \Rightarrow \mu_s = 0.76$$

But it is hard to find the <u>exact point</u> at which the block breaks free!

2) A better way is to consider **a crate of mass m resting on a loading ramp** inclined at a low angle. We are about to find a way to measure both μ_s and μ_k independent of the mass m and g! (And your lab experience will help us here.)

a) First, consider θ small enough that the crate does not move. From the FBD and the **GOLDEN** right triangle on the previous page (BUT with F_{fr}^{s} instead of F_{fr}^{k}), we have $mg \sin\theta - F_{fr}^{s} = 0$ for the second law applied along the direction of the incline:

$$F_{fr}^{s} = mg \sin\theta$$

b) Second, using the **GOLDEN** triangle again, we get the familiar force balance in the normal direction, $N - mg \cos\theta = 0$, or

$$N = mg \cos\theta$$

continued on the next page

c) We can eliminate mg by dividing these two equations:

$$\frac{F_{fr}^{s}}{N} = \frac{mg\ \sin\theta}{mg\ \cos\theta} = \tan\theta \qquad \text{static case}$$

But the maximum value of F_{fr}^{s} is $\mu_s N$, so that the maximum value of the left-hand-side of this equation is $\frac{\mu_s \cancel{N}}{\cancel{N}} = \mu_s$, so $\tan\theta$ cannot get bigger than μ_s .

$$\boxed{\tan\theta_{max}^{static} = \mu_s}$$

By measuring the angle θ_{max}^{static} at which the block breaks loose and starts to slide, we can calculate its tangent and obtain μ_s . We don't need m and g, or anything else.

For example, if we find that the crate just begins to slide at an angle of 30° , then we obtain

$$\mu_s = 1/\sqrt{3} = 0.58$$

d) We'll let you have the fun of finding μ_k in a related manner in the following problem.

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Problem 7-4

(a) After the crate begins to slide at the angle of 30°, the crate accelerates starting from rest at time t = 0 *down the ramp, because* $\mu_k < \mu_s$ *(the friction force is now less at the angle the sliding begins, but the component of weight down the ramp is still the same at that angle so the **net** force component down the ramp is now positive and no longer zero). Find the velocity* v *as a function of time* t *and in terms of* g *and* μ_k *.*

(b) While the crate is sliding, the ramp angle is slowly reduced (so the component of weight along the ramp is reduced while the friction grows, as seen by the formulas above) and just when we get to 20°, the crate no longer accelerates down the ramp but slides at a constant speed (the weight component along the ramp and the friction force exactly cancel each other). With this information, find the coefficient of kinetic friction μ_k *between the crate and the ramp. Notice how this answer is independent of* g *as well as* m*.*

So you could use this method for any mass and you could do it on any planet!

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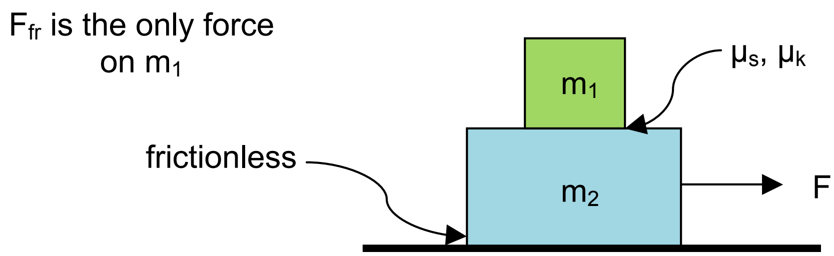
Friction on a Moving Surface

Blocks On Blocks With Friction Between Them

Consider a small block riding on top of a bigger one with friction between them. The bigger one can slide on a smooth table. We pull on the bigger one with a horizontal force F as shown, but not so large as to cause the small block to slide off the big one:



- **What is the largest F for which there is still no slipping <u>between</u> the blocks?**

Comments to help us think about this:

"Static" friction between them can allow the top block to stick to the bottom block, as long as the bottom block does not move too violently. In this case, static friction stops the small block from sliding off, so that the **friction force on m_1** points **in the direction of the motion** of the bottom block! (This is consistent with the idea that friction tries to stop the two surfaces from sliding against each other.)

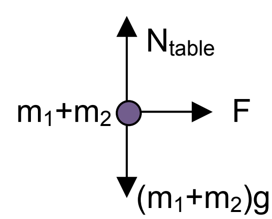
But an even better and easier way to figure out its direction, the friction force on m_1 must point IN THE DIRECTION OF ITS ACCELERATION which is to the right!

Constraint if they move together:

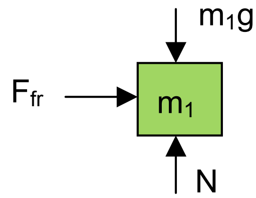
$$a_1 = a_2 \equiv a$$

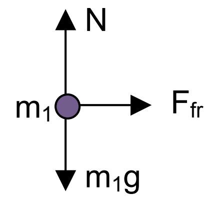
Overall horizontal force equation with overall FBD:

$$\boxed{F = (m_1 + m_2)\,a}$$



Plain vanilla diagram and FBD for small block:





Repeating our earlier remark, remember that you can figure out the direction of F_{fr} either by asking that it resists the tendency of m_1 to slide OR asking that it must be in the direction of the acceleration of m_1 . Either one gives the right answer.

Second-law equations for the small block from the FBD and the inequality constraint from static friction:

$$\text{Vertical: } N = m_1 g \text{ , Horizontal: } F_{fr} = m_1 a \text{ , Static Friction: } F_{fr} \le \mu_s N$$

Substitute the two second-law results into the inequality:

$$\therefore \cancel{m_1} a \le \mu_s \cancel{m_1} g \Rightarrow \boxed{a \le \mu_s g}$$

From the overall force equation we thus have

$$\therefore \boxed{F \le \mu_s (m_1 + m_2)\, g}$$

- **What happens if F is bigger than the above?**

If F gets bigger than this, m_1 slides! Friction force alone cannot then supply the acceleration of m_1 needed to keep up.

Second-law equations for the small block from the same FBD as before but now using the sliding friction force model:

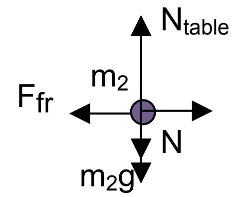
$$\text{Vertical: } N = m_1 g \text{ , Horizontal: } F_{fr} = m_1 a_1 \text{ , Sliding Friction: } F_{fr} = \mu_k N$$

Substitute the two second-law results into the sliding friction equation:

$$\therefore \cancel{m_1} a_1 = \mu_k \cancel{m_1} g \Rightarrow \boxed{a_1 = \mu_k g}$$

We see that m_1's acceleration is now constant independent of F !

Second-law equations for the big block using its FBD and the third law (telling us that the contact forces between the blocks are equal and opposite):



Vertical: $N_{table} = m_2 g + N$

(but don't really need N_{table} because there's no table friction)

Horizontal: $F - F_{fr} = m_2 a_2$

(notice F_{fr} produces a drag on m_2)

From previous page: $F_{fr} = \mu_k N = \mu_k m_1 g$

Substitute the sliding friction into the horizontal second-law equation:

$$\therefore \; F - \mu_k m_1 g = m_2 a_2 \Rightarrow \boxed{a_2 = \frac{F - \mu_k m_1 g}{m_2}}$$

We see that m_2's acceleration is reduced if m_1 is increased.

We now ask you to look at the opposite case, but as a problem:

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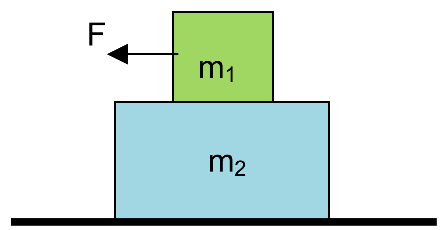
Problem 7-5) *A mass* m_1 *is on top of another mass* m_2*, which in turn is supported by a frictionless horizontal surface. A light string is attached to* m_1 *and pulled on the other end by force* F *as shown.*



a) Suppose the surface between the two masses is frictionless. Draw FBD's for each mass and find the horizontal acceleration of each mass (in terms of F*, etc.)*

b) Suppose instead the surface between the two bodies is rough and F *is small enough that the mass* m_1 *does not slide on* m_2*. Draw an FBD for the whole system and thus find the acceleration of each mass now. Notice that only the external forces F, weights, and the normal surface support enter into the overall FBD.*

c) If the coefficient of static friction between the two bodies is μ_s*, we want to know how big* F *can be before the top mass begins to slide on the bottom mass. Now draw an FBD for* m_1 *alone to help you write down the second law for both vertical and horizontal directions, which can be combined with the maximum static friction to answer this question.*

d) If the coefficient of kinetic friction between the two bodies is μ_k *and* F *is big enough that the top mass is now sliding on the bottom, draw FBD's for each mass. Be careful to draw the correct friction force direction for each. Now use these FBD's to help you write down the second law in both vertical and horizontal directions for each mass and thus find the acceleration of each mass.*

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