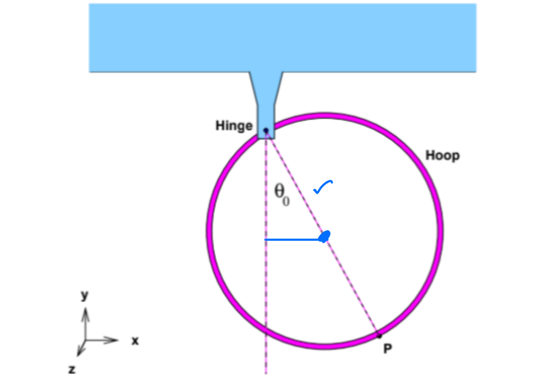
SI Summary Session 3

1)

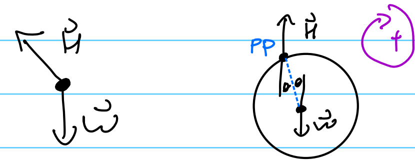
Problem 3: A New Pivot Point (40 points)



A hula-hoop of radius R and mass M is attached to an ideal hinge as shown in the figure above. The hoop is positioned at an angle θ_0 and released. Note that we define a coordinate system, where y is a vertical coordinate and z is the horizontal coordinate the points "out of the page".

$I_{Hoop} = 2MR^2$ for this problem

a)



$$\tau_{Net} = \tau_H + \tau_W$$
$$= 0 + RW\sin\theta$$
$$= \boxed{RMg\sin\theta_0 \ \hat{k}}$$

don't need to know RHR for exam 3

b) N2L Rot

$$\tau_{Net} = I\alpha$$
$$RMg\sin\theta_0 = 2MR^2\alpha$$
$$\boxed{\alpha = \frac{g\sin\theta_0}{2R}}$$

c) C of ME

Conditions: Hinge does no work relative to pp, CM starts at $-R\cos\theta_0 = y$ R.C $v=\omega r$ Weights conservative.

assume v_{max} @ $-R = y$ $\omega = \frac{v}{r}$ r is dist to pp

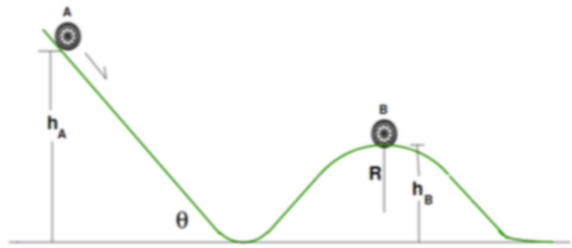
Before = After

$$E_{Tot} = E'_{Tot}$$
$$U + K_T + K_A = U' + K_T' + K_A'$$
$$mgy + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgy' + \tfrac{1}{2}mv'^2 + \tfrac{1}{2}I\omega'^2$$

If $\vec{H}$ is pp, then motion is purely rotational!

$$-MgR\cos\theta_0 + 0 + 0 = -MgR + 0 + \tfrac{1}{2}I\left(\frac{v}{2R}\right)^2$$
$$MgR(1-\cos\theta_0) = \tfrac{1}{2}(2MR^2)\left(\frac{v^2}{4R^2}\right)$$
$$\frac{v^2}{4} = gR(1-\cos\theta_0)$$
$$\boxed{v' = 2\sqrt{gR(1-\cos\theta_0)}}$$

2)

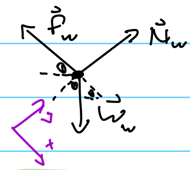


Given

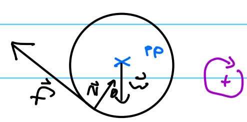
- wheel mass m, radius r, height h_A
- No slip no slide after release
- Slide at angle θ
- wheel inertia $I = Cmr^2$
- assume $r \ll R$ where R is radius of curvature for second hill.

- Everything is given

a) FBD A



xFBD A



b) N2L x-dir

$F_{Net,x} = m a_x$

$W\sin\theta - f_w = m a_x$

$mg\sin\theta - aCm = ma$

$\cancel{m}g\sin\theta = \cancel{m}a + aC\cancel{m}$

$\cancel{m}g\sin\theta \; a(1+C)$

$$\boxed{a = \frac{g\sin\theta}{1+C}}$$

N2L Rot

$\tau_{Net} = I\alpha \quad R.C\ \alpha = \frac{a}{r}$

$\tau_{\vec{f}} + \tau_{\vec{N}} + \tau_{\vec{w}} = I\left(\frac{a}{r}\right)$

$Rf\sin 90° + 0 + 0 = CmR^{\cancel{2}}\left(\frac{a}{\cancel{R}}\right)$

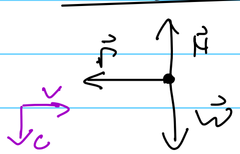
$Rf = aCmR$

$f = aCm$

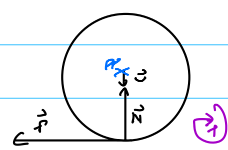
c) $f = aCm$ from (b)

$$\boxed{f = \frac{g\sin\theta}{1+C}(Cm)}$$

d) FBD B



xFBD



$$\boxed{N = mg\left(1 - \frac{2(h_A - h_B)}{R(1+C)}\right)}$$

N2L

$F_{Net_c} = m a_c = \frac{v^2}{r}$ UCM

$W - N = m\left(\frac{v^2}{R}\right)$ v from a+B

$N = m\left(g - \frac{v^2}{R}\right)$

C of ME

Conditions: w is conservative, N does no work b/c ⊥, friction does no work @ point of contact

Before = After

$U + K_T + K_A = U' + K_T' + K_A'$

$mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgy' + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$

$y = h_A,\ v = 0,\ \omega = 0,\ y' = h_B\ \ v' = ?\ \ \omega' = \frac{v}{r}$

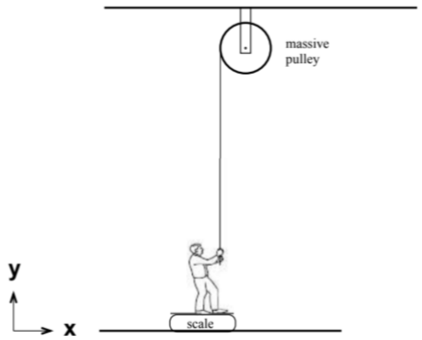
$mgh_A = mgh_B + \frac{1}{2}mv^2 + \frac{1}{2}(Cmr^2)\left(\frac{v}{r}\right)^2$

$\cancel{m}g(h_A - h_B) = v^2\left(\frac{1}{2}\cancel{m} + \frac{1}{2}C\cancel{m}\right)$

$$v^2 = \frac{2g(h_A - h_B)}{1+C}$$

$$v = \sqrt{\frac{2g(h_A - h_B)}{1+C}}$$

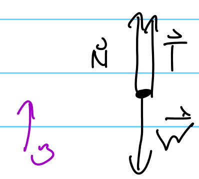
3)



Given

- Sam mass m
- Pulls on rope @ $t=0$ with tension T, not given
- Massive pulley $m_p = 2m$, rot inertia $I = \frac{1}{2} m_p R^2$
- Radius of Pulley R is given

a) Sam FBD



N2L

$F_{Net_y} = m a_y$

$T + N - W = m a_y$ $a_y = 0$

$T = mg - N$

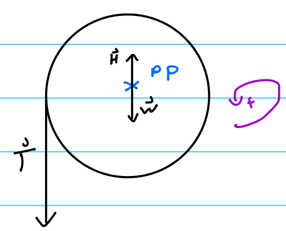
Scale reads 3/4 of his mass

Sam pushes down w/ 3/4 of his mass → Normal force = 3/4 mg

$T = mg - \frac{3}{4} mg$

$$\boxed{T = \frac{mg}{4}}$$

b) τ FBD Pulley



$\tau_{Net} = \tau_H + \tau_W + \tau_T$

$= 0 + 0 + rT\sin 90°$

$= rT$

$$\boxed{\tau_{Net} = \frac{rmg}{4}\ \hat{k}}$$

c) $K_{Tot} = K_T + K_A$

$= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$v = 0$

$= \frac{1}{2} \left(\frac{1}{2} m_p R^2\right) \omega^2$

$\omega(t) = \omega_0 + \alpha t$

$\omega_0 = 0$ @ rest

$\omega(t) = \frac{rmg}{4I} t$

$\tau_{Net} = I\alpha$

$\frac{rmg}{4} = I\alpha$

$\alpha = \frac{rmg}{4I}$

$K_{Tot} = \frac{1}{2}\left(\frac{1}{2} m_p R^2\right)\left(\frac{rmg}{4I} t\right)^2$

$= \frac{1}{2} * \frac{(rmgt)^2}{16I}$

$= \frac{(mgRt)^2}{32I} = \frac{(mg\cancel{R}t)^2}{32\left(\frac{1}{2}\cancel{m_p R^2}\right)}$

$$\boxed{K_{Tot}(t) = \frac{m(gt)^2}{16}}$$