Chapter 1

ZERO FORCE: Constant Velocity in One Dimension (1D)

- **Newton's First Law and Constant Velocities**
- **Intersecting Motion for Constant Velocities**
- **Relative Motion and Constant Velocities**
- **Appendix on Inertial Reference Frames**

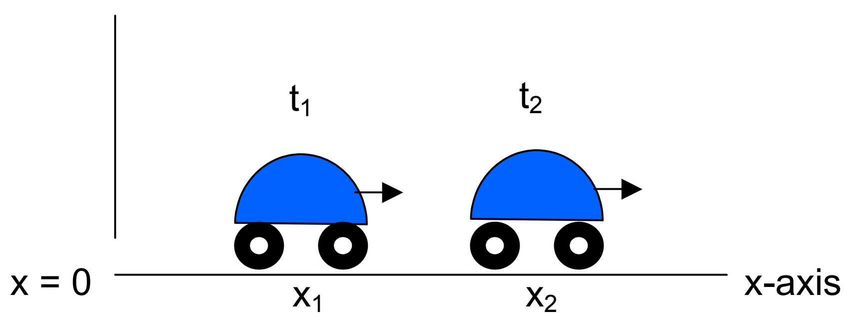
Recall Newton's First Law from Chapter 0, p. 0-3:

If there is no acceleration, there is no net (total) force, and conversely, if there is no net force, there is no acceleration:

$$\vec{F}_{net} = \sum \vec{F}_{individual} = 0 \iff \vec{a} = 0 \qquad (1.1)$$

Also, remember that the symbol $\Leftrightarrow$ means "implies" both ways.

Example: When there is zero net force acting on a moving object, the first law says the object moves with zero acceleration. Zero acceleration means constant velocity, which means we will always get constant ratios for the change in position in any given time interval. Consider, respectively, the positions x_1 , x_2 at two different times t_1 , t_2 of a little VW bug feeling zero force and thus moving at constant velocity. The bug is moving along a straight line, which we have called the x-axis.



The rate of change of x is defined to be the component of the velocity along that axis $\frac{\Delta x}{\Delta t} = v_x$, which is the reason we use the subscript x. For this bug we always get the constant ratio

$$\frac{x_2 - x_1}{t_2 - t_1} = \text{constant} \equiv v_x^c \qquad (1.2)$$

Since the rate of change is constant for any time interval, we have added the superscript c, and we end up with v_x^c as notation for the one-dimensional constant vector.

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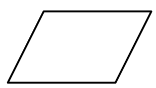
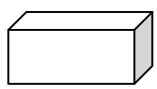
Digression on dimensions, time, position, velocity, and speed: (You can usually follow the non-digressive stuff without pausing for these digressions, but they should be both helpful and easy to read.)

Dimensions - The phrase "one dimension" means that we need only one number to label where we are; this could be referring to a position along a sidewalk, or a road, or on a ladder, etc. Contrast the three standard systems:

LINE – this has one dimension (we need one number to locate a point on it)



AREA – two dimensions (need two numbers to locate a point on it)



VOLUME – three dimensions (need three numbers to locate a point on it)

Time - How about the “fourth dimension” of time? For the motion of something relative to another thing, then "time t" naturally arises as a useful label. We compare our motion to a clock, and use that clock to label our different positions.

Position - By the way, when we talk about position of a body (in particular, our "one-dimensional" position), well, for now, we ignore "size." So we are talking about a particular point on a car, an antelope, a ball, etc.

Velocity - We see that velocity in one dimension has just one component, but that component still includes information about “direction.” It remains a vector (see Chapter 0), though a simple one. Why does it include information about direction? In our bug example, for instance, v_x can be either positive or negative. $v_x > 0$ means the bug is going to the right and $v_x < 0$ means it is going to the left.

Speed - "Speed" is the magnitude of the velocity vector, so, even in one dimension, the velocity is not its speed. Speed is just a positive number indicating how fast something is traveling.

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Now if we know the velocity is always constant for certain bodies of interest, we can make predictions like those on the next page.

Predictions:

- Formula and its graph for predicting the future for constant velocity in one dimension (1D):

If we are moving at constant velocity (say, speed v_x^c in the positive direction), and we know where we are at some time t_1, we know from equation (1.2) where we are at any later time t_2:

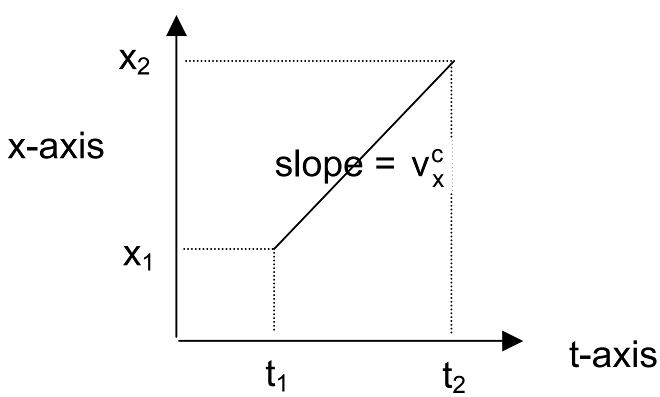
$$x_2 - x_1 = v_x^c(t_2 - t_1) \Rightarrow x_2 = x_1 + v_x^c(t_2 - t_1)$$

or, for the convenient notation of $x_2 \equiv x$, $t_2 \equiv t$, and $t_1 \equiv 0$ (by the way, recall when we write " $\equiv$ ", we mean "defined as"), we have as an equivalent form for (1.2)

$$\boxed{x = v_x^c t + C} \tag{1.3}$$

with $x_1 \equiv C = x(0) \equiv x_0$. The constant C is determined by the "initial condition." We always get such linear relations (like between x and t here) for zero acceleration.

To plot the linear relation, $x = v_x^c t + C$, let's use the vertical axis now as the x-axis and the horizontal axis for time (the t-axis):



For zero acceleration, we always get a straight line for the 1D velocity over time intervals. The slope of the straight line is the constant velocity v_x^c.

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Digression on a calculus approach to equation (1.3)

As an alternate derivation of (1.2), or its equivalent, (1.3), we use the fact that the velocity at any time is given by the calculus derivative of the position (the instantaneous velocity – see Chapter 2). We also use the fact that for zero force this velocity is constant, leading to an easy calculus analysis. If $\frac{dy}{dz} = A$ = constant, then $y = Az + C$, where C is another constant. Thus we get the form in equation (1.3)

$$\frac{dx}{dt} = v_x^c \Rightarrow x = v_x^c\, t + C$$

Equivalently, we can rewrite the derivative in differential form and integrate both sides of the equation, using the pairs of points from p. 1-1. We get the form in (1.2)

$$dx = v_x^c dt \Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x^c\, dt \Rightarrow x_2 - x_1 = v_x^c(t_2 - t_1)$$

This is a solution of a simple "differential equation" (a differential equation is simply an equation with derivatives in it!)

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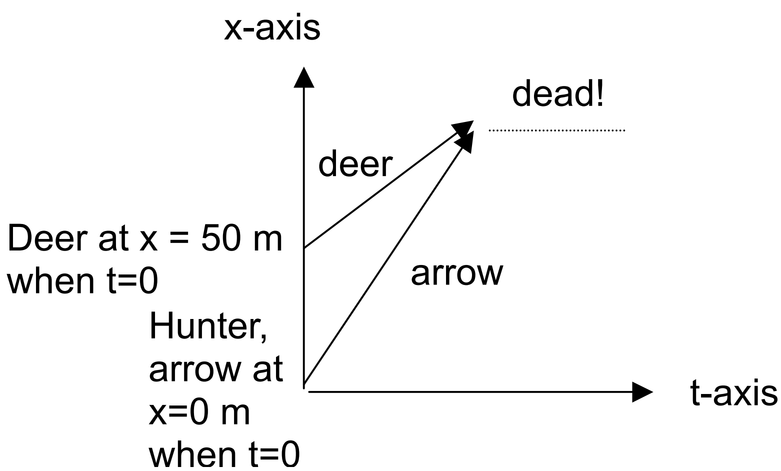
Intersection Example:

Knowing how to write formulas and draw graphs for constant velocities in 1D, we can consider two bodies, each moving at constant velocity, and compare their motions:

A hunter standing still shoots an arrow straight at a deer running directly away from her at a constant speed of 10 m/s. (Thus the deer has constant velocity of $v_x^c(\text{deer}) = +10\text{ m/s}$ where we have chosen the positive x-axis to be along the direction of the deer's motion.) When the arrow first leaves the bow, the deer is at a distance of 50 m from the hunter. We assume the arrow has an approximately constant 1D velocity of $v_x^c(\text{arrow}) = +60\text{ m/s}$ (at least before impact). When and where did the arrow hit the deer?

We answer this question in two ways on the next page – graphically and analytically (i.e., using formulas).

1) We first sketch a solution graphically, showing the intersection of the two straight paths (notice that the deer and the arrow are at rest after the intersection – sob!). The slope (rise/run) of the deer trajectory is less than the slope for the arrow, leading to an (unfortunate) intersection:



2) We next solve it algebraically, using two different constant-velocity formulas (1.3) for the deer and the arrow. Between t = 0 and the intersection:

$$x_{deer} = 10 m/s\ t + 50m$$

$$x_{arrow} = 60 m/s\ t + 0m$$

They are equal, $x_{deer} = x_{arrow}$, at the intersection time t_{hit} :

$$10 m/s\ t_{hit} + 50m = 60 m/s\ t_{hit} + 0m \quad \text{or} \quad 50m = 50 m/s\ t_{hit}$$

Thus $\boxed{t_{hit} = 1.0\ s}$ and a substitution into either x_{deer} or x_{arrow} gives $\boxed{x_{hit} = 60\ m}$

Significant digits:

Ah, yes, the first thing we ask is, how many significant figures should we keep in doing our work (especially our homework)? In the above problem, we see that it looks like one significant figure is given for the numbers stated (10, 50, 60). While our simple example had no round-off needed, a good guide is to keep lots of digits during the calculation, and then round off only at the end. This avoids accumulated errors that arise when we round off at every step. Frankly, don't be frightened if you quote one more significant figure than you are officially allowed. It is more important to avoid accumulated round-off errors.

Comments:

- The assumption of constant 1D velocity for the arrow obviously means we are neglecting the small slowing down due to air friction and the modest drop in height due to the force of gravity.

- If there were several forces acting on an object observed to be going at a constant velocity, the forces must be canceling each other, according to Newton's first law. For example, a feather falling in air due to gravity reaches a terminal (constant) velocity quickly. Here, the drag force (acting upwards) is now cancelling the force of gravity (which acts downwards). Two equal forces act in the opposite direction and add up to zero.

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Problem 1-1 *A reference we got from Googling "cheetah" tells us it can run up to 110 km/h (kilometers per hour), Suppose it takes a cheetah 20 s to catch a Thomson gazelle, which began 200 m ahead of the cheetah, from the time they both start running, with the gazelle trying to run directly away from the cheetah. You can make the approximation that their maximum speeds are achieved almost instantaneously.*

(a) Convert 110 km/h to m/s and thus find a factor that you can use from now on whenever you need to go back and forth between km/h and m/s.

(b) Find the speed of the gazelle (the algebraic approach is easiest here).

(c) Make a rough sketch of this cheetah-catching-gazelle saga like the arrow-deer saga in the example above.

(d) Google to verify that your answer for the gazelle's speed is reasonable.

(e) Google to see how long the cheetah actually can run at top speed, to see if it really can catch the gazelle starting from this distance.

(f) A cheetah quickly gets up to a maximum speed while chasing a gazelle. (as do the fleeing gazelle and, in our earlier example, the hunted deer). But while it is at this constant speed, the cheetah seems to be constantly exerting a force through its feet. Doesn't this violate the first law?

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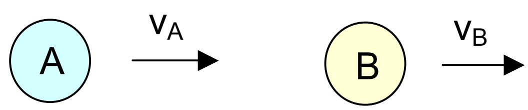
RELATIVE VELOCITY AND DIFFERENT REFERENCE FRAMES IN 1D

Consider the cheetah (call it body A or reference frame A) and the gazelle (body B or reference frame B) from the previous homework problem. How fast are they moving **with respect to each other?** Well, the velocities discussed in the problem and shown below are with respect to the ground as a reference frame (call it reference frame C):*



$$v_A \equiv v_{A/C} \equiv \text{ the velocity of A with respect to the ground C}$$
$$v_B \equiv v_{B/C} \equiv \text{ the velocity of B with respect to the ground C}$$

where we use the subscript notation "**object/frame.**" That is, A/C refers to A relative to C.

Below we first guess the answers, develop a procedure and a formula for calculating what velocity (speed and direction) each animal has relative to the other, and then check our guesses.

Borrowing from the last homework problem, $v_{A/C} = 110\text{ km/h}$ and $v_{B/C} = 74\text{ km/h}$ (Hah! We just gave away an answer to that problem!), we can guess that the antelope sees the cheetah moving forward (horrors!) at 36 km/h ($v_{A/B} = +36\text{ km/h}$) and the cheetah sees the antelope moving backward (yum yum!) at 36 km/h ($v_{B/A} = -36\text{ km/h}$).

We can immediately write down and understand a handy formula, for one-dimensional motion, to verify the above guesses. The formula is simply

$$v_{B\text{ with respect to ground C}} = v_{B\text{ with respect to A}} + v_{A\text{ with respect to ground C}}$$

That is, take the velocity of B relative to A and add to it the velocity of A relative to the ground and – hooray – we get the velocity of B relative to the ground.

* **We leave the x subscript understood here!** We will imagine a horizontal x-axis where to the right is positive and to the left is negative. We will drop the x subscript but remember all velocities v here really are the x-components, which are strictly labeled as v_x . So $v > 0$ is moving to the right and $v < 0$ to the left.

In our simpler notation:

$$\boxed{v_{B/C} = v_{B/A} + v_{A/C}} \tag{1.4}$$

We can solve this formula for the velocity of B with respect to A (the velocity A "sees" that B has):

$$\boxed{v_{B/A} = v_{B/C} - v_{A/C}} \tag{1.5}$$

and, just from switching A and B, we see:

$$\boxed{v_{A/B} = - v_{B/A}} \tag{1.6}$$

Comments:

- We verify, for our example, that $v_{B/A} = 74 - 110 = -36$ km/h (the negative sign verifies that the antelope goes backwards relative to the cheetah). And then we also verify $v_{A/B} = -v_{B/A} = +36$ km/h , for the cheetah relative to the antelope.

- These formulas are good for any three "reference frames" A, B, C.

- Notice how A "cancels out" in $v_{B/C} = v_{B/A} + v_{A/C}$

- A neat way to derive (1.4) is to realize, first, that the position of B relative to C can be written in terms of B's position relative to A and add that to A's position relative to C: $x_{B/C} = x_{B/A} + x_{A/C}$. Now take a time derivative of this relation:

$$\frac{dx_{B/C}}{dt} = \frac{dx_{B/A}}{dt} + \frac{dx_{A/C}}{dt} \quad \Rightarrow \quad v_{B/C} = v_{B/A} + v_{A/C}$$

- Talking about relative velocities and reference frames motivates us to add an appendix following the problem to comment on Newton's first law as a way of figuring out special reference frames: inertial reference frames.

- In the famous problem coming up, you can put formulas like $v_{B/C} = v_{B/A} + v_{A/C}$ to good use!

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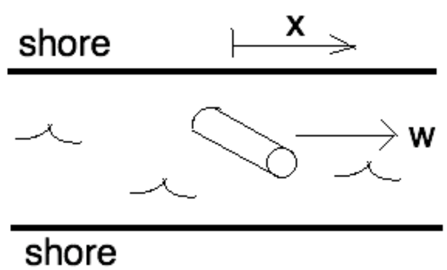
Problem 1-2 *A famous problem!*

*You are floating on a log down a river. The river is flowing at the uniform speed w relative to the shore (i.e., the ground). You can swim at a speed v relative to the water with v > w. For distances **relative to the shore**, use an x-coordinate with the positive x direction the same as the river flow direction.*



*a) You jump off the log and swim for a time **ΔT upstream** (against the river). Let x = 0 be the initial location of the log relative to the shore at the instant you jump off, what is your new coordinate position x(ΔT) relative to the shore in terms of v, w, ΔT (it will be negative for v > w and, as a check, should vanish if v = w). Hint: You can use* $v_{B/C} = v_{B/A} + v_{A/C}$*, where B is you, C is the shore, A is the water.*

*b) Now you turn around instantly and swim for a time **Δt downstream** (with the river). What is your new coordinate position x(ΔT+Δt) in terms of v, w, ΔT and Δt (again, remember that x = 0 is the initial location of the log when you originally jumped off).*

c) What is the coordinate position of the log, X(ΔT+Δt), relative to the shore after the total time ΔT+Δt has passed?

d) For a given ΔT, solve for the time Δt that it takes for you to get back to the log.

e) Please explain the simplicity of your answer in (d). That is, you did not need to do the previous long calculation to get this answer!

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Appendix on Newton's First Law and Inertial Frames!

Newton's first law helps define an "inertial frame," and some people say this is the most important thing about the first law (otherwise it is a trivial version of the second law). The importance of the first law is that Newton's second law applies only to inertial frames, and we can use the first law to define them.

To define the inertial frames, first recall what we mean by "reference frames." Positions, velocities, accelerations are all defined with respect to some reference system. For motion with respect to the ground, the ground is our reference frame. Alternatively, we could use a moving car as our reference frame. To say it again, since motion is always relative to something, what we use for that "something" is our "reference frame."

Now what do we mean by "inertial frames?" We might just quickly say that an inertial frame is a reference frame that is not accelerating. (The car mentioned above is one if it is moving at a constant speed in a straight line.) But then some smarty-pants will say, "not accelerating with respect to what?" To answer this, we say, imagine a body that has **no interaction** with the rest of the world (feels no external forces). Then, if this body is not accelerating with respect to some reference frame, then that reference frame is a good inertial frame.