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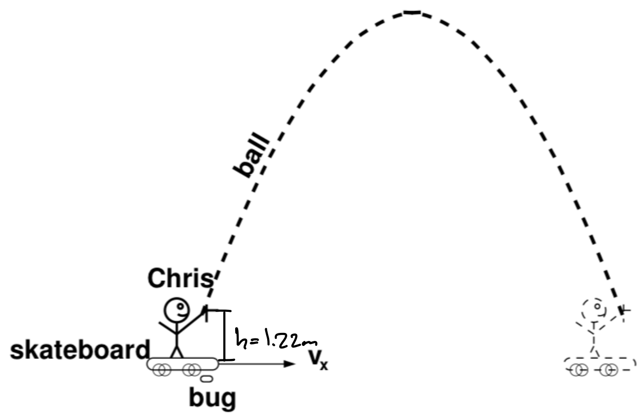
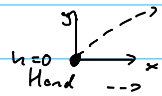
## PHYS121 - Homework 4

Problem 1)




Given
$V_x = 4.35\ m/s$
$h = 1.22\ m$
ground is $h=0$
time in air $= 2.85$ seconds $= T$

a)




Velocity upwards is only vertical component ← $V_{y_0}$ is not 0, calculate at catch

FF Velocity $\quad V_y = V_{y_0} - gt \quad V_y = 0$ @ catch $(t=T)$

$0 = V_{y_0} - gT \quad \therefore \quad \cancel{\boxed{V_{y_0} = gT}}$

Numerically: $V_{y_0} = \text{speed} = gT = (9.81\ m/s^2)(2.85\ s) = \boxed{27.96\ m/s}$ $*\frac{1}{2}$

$y = y_0 + v_0 t - \frac{1}{2}gt^2$
$0 = 0 + V_0 t - \frac{1}{2}gt^2$
$\rightarrow \boxed{V_{y_0} = \frac{1}{2}gt}$

b) FF Kinematics $\quad$ Let T be the total time, so maximum height at $\frac{T}{2}$ by symmetry

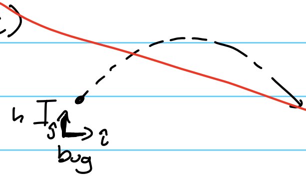
$y = y_0 + V_{y_0} t - \frac{1}{2}gt^2$
$y_0 = h$ ← given starting
$V_{y_0} = V = 27.96\ m/s$
$t = \frac{T}{2}$ ← Symmetry

$\boxed{y = h + v\left(\frac{T}{2}\right) - \frac{1}{2}g\left(\frac{T}{2}\right)^2}$

Numerically: $y = 1.22 + 27.96\left(\frac{2.85}{2}\right) - \frac{1}{2}(9.81)\left(\frac{2.85}{2}\right)^2$ ← off bc of (A)

$y = \boxed{31.10\ m/s}$

c)




Projectile Motion Kinematics

$\begin{cases} x = x_0 + V_{x_0} t \\ y = y_0 + V_{y_0} t - \frac{1}{2}gt^2 \end{cases}$ $\quad x_0 = 0,\ V_{x_0} = V_x$ ← bug aligned @ start
$\quad y_0 = h,\ V_{y_0} = V_y = V$ ← established above

$\boxed{\vec{r}(t) = (V_x t)\hat{i} + (h + V_y t - \frac{1}{2}gt^2)\hat{j}}$

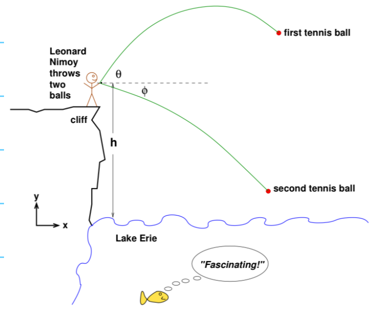
velocity $= \frac{d}{dt}(\vec{r}(t))$ $\quad$ acceleration $= \frac{d}{dt}v(t)$

$\boxed{\vec{v}(t) = (V_x)\hat{i} + (V_y - gt)\hat{j}}$ $\quad \vec{a}(t) = 0\hat{i} - g\hat{j} \Rightarrow \boxed{\vec{a}(t) = -g\hat{j}}$

$\vec{r}(t) = x\hat{i} + y\hat{j}$

$\vec{v}(t) = V_x\hat{i} + V_y\hat{j}$

## Problem 2)




**Given**

height = h above lake

given initial speed $= v_0$ for both balls

Ball 1 is $\theta$ above horizontal

Ball 2 is $\Phi$ below horizontal

Assume $\theta > \Phi$

a) y-direction of F.F

$\sin\theta = \frac{V_{y_0}}{V_0}$

$V_{y_0} = V_0 \sin\theta$

$y = y_0 + V_{y_0} t - \frac{1}{2} g t^2$

$V_y = V_{y_0} - g t$

$V_{y_0} = V_0 \sin\theta$ ← given

$V_y = 0$ @ max height

$0 = V_0 \sin\theta - g t$

$t = \frac{-V_0 \sin\theta}{-g}$

$\therefore \boxed{t = \frac{V_0 \sin\theta}{g}}$

time at max height

Kinematics Velocity definition

$y = y_0 + V_{y_0} t - \frac{1}{2} g t^2$ ← vertical proj. motion

$y_0 = h$, $V_{y_0} = V_0 \sin\theta$, $t = \frac{V_0 \sin\theta}{g}$

$y = h + V_0 \sin\theta \left(\frac{V_0 \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{V_0 \sin\theta}{g}\right)^2$

$y = h + \frac{(V_0 \sin\theta)^2}{g} - \frac{(V_0 \sin\theta)^2}{2g}$

$\boxed{y = h + \frac{(V_0 \sin\theta)^2}{2g}}$

Decomposing velocity into its components reveals $V_{y_0} = V_0 \sin\theta$. Thus allows the $y_{max}$ to be determined by finding the time it takes to be at 0 velocity, or the max y.

b) $t = \frac{V_0 \sin\theta}{g}$ @ max height from part (a)

$x_0 = 0$, $y_0 = h$, $\theta_{given}$, $y = h + \frac{(V_0 \sin\theta)^2}{2g}$

$\Rightarrow$ $\sin\theta = \frac{V_{y_0}}{V_0}$, $\cos\theta = \frac{V_{x_0}}{V_0}$ $\Big\}$ $V_{y_0} = V_0 \sin\theta$, $V_{x_0} = V_0 \cos\theta$

Position Parametrization

$\vec{r}(t) = (x_0 + V_{x_0} t)\hat{\imath} + (\underbrace{y_0 + V_{y_0} t - \tfrac{1}{2} g t^2}_{y = y_0 + V_{y_0} t - \frac{1}{2} g t^2})\hat{\jmath}$

$\vec{r}(t) = \left(0 + V_0 \cos\theta \left(\frac{V_0 \sin\theta}{g}\right)\right)\hat{\imath} + \left(h + \frac{(V_0 \sin\theta)^2}{2g}\right)\hat{\jmath}$

$\boxed{\vec{r} = \left(\frac{V_0^2 \cos\theta \sin\theta}{g}\right)\hat{\imath} + \left(h + \frac{(V_0 \sin\theta)^2}{2g}\right)\hat{\jmath}}$

Velocity parametrization

$\vec{v}(t) = (V_{x_0})\hat{\imath} + (V_{y_0} - g t)\hat{\jmath}$

$\vec{v}(t) = (V_0 \cos\theta)\hat{\imath} + \left(V_0 \sin\theta - g\left(\frac{V_0 \sin\theta}{g}\right)\right)\hat{\jmath}$

$\therefore \boxed{\vec{v} = (V_0 \cos\theta)\hat{\imath}}$

acceleration Parametrization

$\boxed{\vec{a}(t) = -g\hat{\jmath}}$

Using the t parameter found in (a) in conjunction with given parametrizations, the 3 vector expressions can be derived as shown.

c) Acceleration Components

$\vec{a}(t) = a_v \hat{v} + a_c \hat{c}$

$a_v = 0$ @ max height

$\vec{a} = 0\hat{v} + \frac{v^2}{R_c}\hat{c}$ Centripetal Accel

$-g\hat{\jmath} = \frac{V_0^2}{R}\hat{c}$ for R = radius of curve

$\hat{c} = -\hat{\jmath}$ as $\hat{\jmath}$ points ↑ and $\hat{c}$ points ↓

$\therefore -g\hat{\jmath} = \frac{V_0^2}{R}(-\hat{\jmath}) \Rightarrow g = \frac{V_0^2}{R}$

$\Rightarrow \boxed{R = \frac{V_0^2}{g}}$

Knowing $\hat{c} = -\hat{\jmath}$, a simplification can be done allowing centripetal accel. to be the solution.

d) Projectile Velocity Components

I do not care about the x-direction here

Ball 1: $V_{y_1} = V_0 \sin\theta - g t$

Ball 2: $V_{y_2} = V_0 \sin\Phi - g t$

$\Big\}$ Speeds identical to velocity y comp. as x-dir ignored

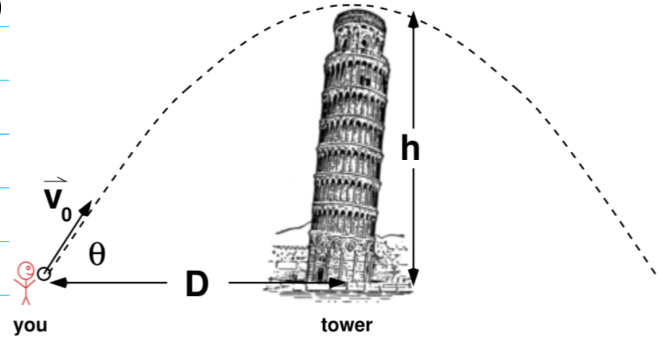
$V_0 \sin\theta - g t \; ? \; V_0 \sin\Phi - g t$

$\Rightarrow \sin\theta > \sin\Phi$ as $\theta > \Phi$

Equal Speed

$\boxed{\text{Ball 1 will have a } \cancel{\text{greater speed}}}$ as its instance angle is greater meaning the sin of the angle is greater.

Problem 3)




Given

Distance D to base

Height h for tower

Negligible

- Air resistance
- Your Height
- Diameter/Deflection of Tower

$\theta = ?$

$V_0 = ?$

s.t. ball barely clears tower's height "h"

Air resistance neglected

## FF Parametrization y-dir

$v_y = v_{y0} - gt$, $v_y = 0$ @ max as diameter ignored

$0 = V_{y0} - gt \Rightarrow t = \frac{V_{y0}}{g}$

$y = y_0 + V_{y0} t - \frac{1}{2} g t^2$, plug in $t = \frac{V_{y0}}{g}$

$y = h$ @ max height $y_0 = 0$

$h = 0 + V_{y0}\left(\frac{V_{y0}}{g}\right) - \frac{1}{2} g \left(\frac{V_{y0}}{g}\right)^2$

$h = \frac{V_{y0}^2}{g} - \frac{V_{y0}^2}{2g} = \frac{V_{y0}^2}{2g}$

$h = \frac{V_{y0}^2}{2g} \Rightarrow V_{y0} = \sqrt{2gh}$

## FF Parametrization x-dir

$x = x_0 + V_{x0} t$, $x_0 = 0$

max height @ Distance D

$\Rightarrow t = \frac{V_{y0}}{g} = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}}$

$D = 0 + V_{x0}\left(\sqrt{\frac{2h}{g}}\right)$

$V_{x0} = D\sqrt{\frac{g}{2h}}$

## Speed Def:

Speed $= |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

$|\vec{v}| = \sqrt{\sqrt{2gh}^2 + \left(D\sqrt{\frac{g}{2h}}\right)^2}$

$$\boxed{V_0 = \sqrt{2gh + \frac{D^2 g}{2h}}}$$

## Components of Velocity

$V_{x0} = V_0 \cos\theta$

$V_{y0} = V_0 \sin\theta$

choose one and solve for $\theta$

arcfn to solve

$V_{y0} = V_0 \sin\theta$

$\sqrt{2gh} = \sqrt{2gh + \frac{D^2 g}{2h}} \cos\theta$

$\cos\theta = \frac{\sqrt{2gh}}{\sqrt{2gh + \frac{D^2 g}{2h}}}$

$$\therefore \theta = \cos^{-1}\left(\sqrt{\frac{2gh}{2gh + \frac{D^2 g}{2h}}}\right) \Rightarrow \boxed{\cancel{\theta = \cos^{-1}\left(\sqrt{\frac{2h}{2h + \frac{D^2}{2h}}}\right)}}$$

Problem 4) Lacking (a) (b) (c) parts, I designated these to the 3 "proofs" below

$\vec{r}(t) = (R\cos(\omega t))\hat{i} + (R\sin(\omega t))\hat{j}$

a) Prove the particle is always "R" distance from the origin

Displacement = Distance Mag = $|\vec{r}| = \sqrt{r_x^2 + r_y^2}$

$r_x = R\cos(\omega t)$, $r_y = R\sin(\omega t)$ $\Big\}$ $|\vec{r}| = \sqrt{(R\cos(\omega t))^2 + (R\sin(\omega t))^2}$

$= \sqrt{R^2\cos^2(\omega t) + R^2\sin^2(\omega t)}$

Trig Identity $= \sqrt{R^2(\cos^2(\omega t) + \sin^2(\omega t))} = \boxed{R}$ $\therefore$ Displacement = R !

Constant displacement means a circle with radius R.

b) Prove a constant speed of $\omega R$ for the particle

Definition of Velocity

$\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$

$\vec{v}(t) = (\omega R\cos(\omega t))\hat{i} + (\omega R\sin(\omega t))\hat{j}$

Magnitude of Velocity $= |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

$|\vec{v}| = \sqrt{(\omega R\cos(\omega t))^2 + (\omega R\sin(\omega t))^2}$

$= \sqrt{\omega^2R^2\cos^2(\omega t) + \omega^2R^2\sin^2(\omega t)}$

$= \sqrt{\omega^2R^2(\cos^2(\omega t) + \sin^2(\omega t))}$

Trig Identity $= \sqrt{\omega^2R^2} = \omega R$

$v = \omega R$ is independent of time, as seen to the constant $\longrightarrow$ $\therefore \boxed{v = \omega R}$ $\leftarrow$ The rolling constraint

c) Prove a fixed acceleration of $\frac{v^2}{R}$

Definition of Acceleration

$\vec{a}(t) = \frac{d\vec{v}(t)}{dt}$

$\vec{a}(t) = (\omega^2R\cos(\omega t))\hat{i} + (\omega^2R\sin(\omega t))\hat{j}$

R is the distance from the origin so it is always positive and constant

Also, a is $\perp$ to velocity, so it is always pointed radially inward

Magnitude of Accel. $= |\vec{a}| = \sqrt{a_x^2 + a_y^2}$

$|\vec{a}| = \sqrt{(\omega^2R\cos(\omega t))^2 + (\omega^2R\sin(\omega t))^2}$

Similar algebra to (a), (b) $= \sqrt{\omega^4R^2(\cos^2(\omega t) + \sin^2(\omega t))}$

Trig $= \sqrt{\omega^4R^2} = \omega^2 R$

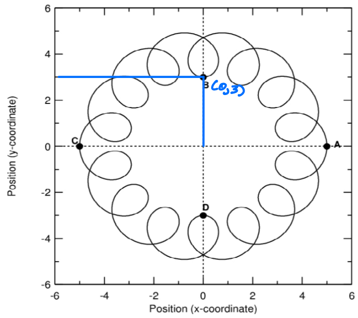
from (b) $v = \omega R$

$\therefore \omega = \frac{v}{R}$

Plugging into $|\vec{a}|$: $a = \frac{v^2}{R^2}(R)$

$|\vec{a}| = \boxed{a = \frac{v^2}{R}}$

Problem 5)




$\vec{r} = [P\cos(\Omega t) + Q\cos(\omega t)]\hat{i} + [P\sin(\Omega t) + Q\sin(\omega t)]\hat{j}$

Given

$P = 4m$ Point A corresponds to $t=0$ and $t=1$

$Q = 1m$ Full spirograph pattern

$\Omega = 2\pi \ sec^{-1}$ follows path over $0 < t < 1$

$\omega = 30\pi \ sec^{-1}$

a) Suppose Point B corresponds to $t = 0.25$, as this a quarter around the path

If $t = 0.25$ @ B, then $\vec{r}(0.25) = 0\hat{i} + 3\hat{j} = (0,3)$ ← estimate from Graph above

$\vec{r}(0.25) = [P\cos(0.25\Omega) + Q\cos(0.25\omega)]\hat{i} + [P\sin(0.25\Omega) + Q\sin(\Omega\omega)]\hat{j}$

$= [4\cos(\frac{\pi}{2}) + 1\cos(\frac{15\pi}{2})]\hat{i} + [4\sin(\frac{\pi}{2}) + 1\sin(\frac{15\pi}{2})]\hat{j}$

$= [4(0) + 1(0)]\hat{i} + [4(1) + 1(-1)]\hat{j}$

$= 0\hat{i} + 3\hat{j}$

Position vector's components are its coordinates $\therefore$ $\vec{r}(0.25) = (0,3)$

I conclude that $t = 0.25s$ correlates to point B

b) Definition of velocity: $\vec{v} = \frac{d}{dt}\vec{r}(t)$

$\boxed{\vec{v}(t) = -[\Omega P\sin(\Omega t) + \omega Q\sin(\omega t)]\hat{i} + [\Omega P\cos(\Omega t) + \omega Q\cos(\omega t)]\hat{j}}$ ← Symbolic

$\vec{v}(0.25) = -[2\pi(4)\sin(2\pi(\frac{1}{4})) + 30\pi\sin(30\pi(\frac{1}{4}))]\hat{i} + [2\pi(4)\cos(2\pi(\frac{1}{4})) + 30\pi\cos(2\pi(\frac{1}{4}))]\hat{j}$

$= -[8\pi\sin(\frac{\pi}{2}) + 30\pi\sin(\frac{15\pi}{2})]\hat{i} + [8\pi\cos(\frac{\pi}{2}) + 30\pi\cos(\frac{15\pi}{2})]\hat{j}$

$= -[8\pi - 30\pi]\hat{i} + [0+0]\hat{j} = 22\pi\hat{i} + 0\hat{j}$

Definition of speed

$Speed = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$ , I am not doing this symbolically, the expression would be far to long, and it is not explictly stated here.

$Speed = \sqrt{22\pi^2 + 0^2}$

$= 22\pi$ → $\boxed{|\vec{v}| = 22\pi \ @ \ B}$

c) Definition of acceleration: $\vec{a} = a_{Tot} = \frac{d}{dt}\vec{v}(t)$

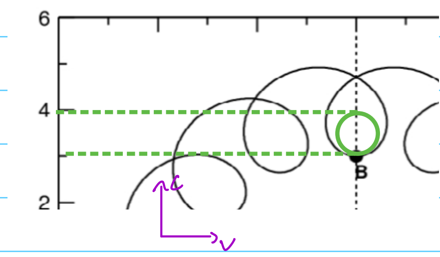
$\boxed{\vec{a}(t) = -[\Omega^2 P\cos(\Omega t) + \omega^2 Q\cos(\omega t)]\hat{i} - [\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)]\hat{j}}$

$\vec{a}(0.25) = -[(2\pi)^2(4)\cos(2\pi(\frac{1}{4})) + (30\pi)^2\cos(30\pi(\frac{1}{4}))]\hat{i} - [(2\pi)^2(4)\sin(2\pi(\frac{1}{4})) + (30\pi)^2\sin(30\pi(\frac{1}{4}))]\hat{j}$

$= -[16\pi^2\cos(\frac{\pi}{2}) + 900\pi^2\cos(\frac{15\pi}{2})]\hat{i} - [16\pi^2\sin(\frac{\pi}{2}) + 900\pi^2\sin(\frac{15\pi}{2})]\hat{j}$

$= -[0+0]\hat{i} - [16\pi^2 - 900\pi^2]\hat{j}$ $\therefore$ $\boxed{\vec{a}(0.25) = 0\hat{i} + 884\pi^2\hat{j}}$

d)




**Estimate**

Green circle to the left, I made the circle have an approximate radius of 0.5 because of the sharper curve/turn the path takes at the point. As you can see the circle in green very tightly touches the point and fails to interfere with the rest of the path.

Def of acceleration: $a_{Tot} = \vec{a}_c + \vec{a}_v$

↳ Since speed is constant and path is curved, therefore $\vec{a}_c = \frac{v^2}{R}$ and $\vec{a}_v = 0$

Part (c) $\rightarrow a_{Tot} = -[\Omega^2 P\cos(\Omega t) + \omega^2 Q\cos(\omega t)]\hat{i} - [\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)]\hat{j}$

As seen in purple from the v-c coordinates above we can setup the relationship here:

$$-[\Omega^2 P\cos(\Omega t) + \omega^2 Q\cos(\omega t)]\hat{i} - [\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)]\hat{j} = a_v\hat{v} + a_c\hat{c}$$

$$\Rightarrow -\underbrace{[\Omega^2 P\cos(\Omega t) + \omega^2 Q\cos(\omega t)]}_{\text{Goes to 0 from (c)}}\hat{i} - [\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)]\hat{j} = 0\hat{v} + \frac{v^2}{R}\hat{c}$$

$$\Rightarrow -[\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)]\hat{j} = \frac{v^2}{R}\hat{c}, \quad \hat{c} = \hat{j}$$

from (b)

$$\Rightarrow -[\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)] = \frac{v^2}{R}$$

$$R = \frac{v^2}{-[\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)]} \qquad v = \text{speed} = -[\Omega P\sin(\Omega t) + \omega Q\sin(\omega t)]$$

$$\therefore R = \frac{(-[\Omega P\sin(\Omega t) + \omega Q\sin(\omega t)])^2}{-[\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)]} \Rightarrow \boxed{R = \frac{[\Omega P\sin(\Omega t) + \omega Q\sin(\omega t)]^2}{-[\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)]}}$$

Numerically:

$$\begin{cases} [\Omega P\sin(\Omega t) + \omega Q\sin(\omega t)] = 22\pi \\ -[\Omega^2 P\sin(\Omega t) + \omega^2 Q\sin(\omega t)] = 884\pi^2 \end{cases}$$

from (b) and (c)

$$R = \frac{(22\pi)^2}{884\pi^2} = \frac{484\pi^2}{884\pi^2} = \frac{121}{221} = \boxed{0.5475}$$

This estimate of 0.5475 is very close to the radius of the circle explained above