Cycle 1

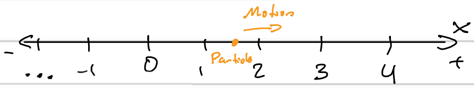
Introduction to Motion - Kinematics

Kinematics vs. Dynamics, the how vs. the why

↳ ask yourself what part a problem is in (dynamics or Kinematics)

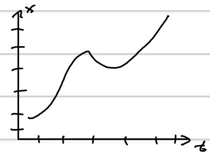
1-D Kinematics

Default units for this class is SI units



our coordinate x is dependent on time

$x(t)$ = some function

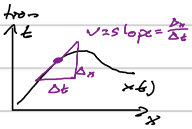


Example: $x(t) = Bt^2$

Must be a one-to-one function

Definition

velocity: $v \equiv \frac{dx}{dt}$, rate of change of position

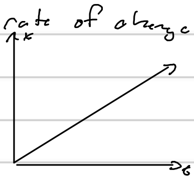


Given: $x(t) = K\cos(Bt)$

$v(t) = \frac{dx}{dt} = -KB\sin(Bt)$

Acceleration: $a \equiv \frac{dv}{dt} = \frac{d^2x}{dt^2}$, rate of change of velocity

Example: Tricycle with constant velocity



x = linear
v = constant
a = 0

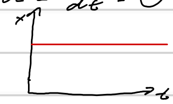
Will not use prime notation for derivatives in this class

Example: Book at rest

x = constant, C

$v = \frac{dx}{dt} = 0$

$a = \frac{d^2x}{dt^2} = 0$



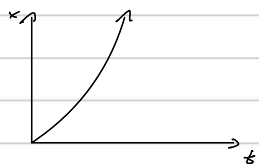
constant velocity means 0 acceleration

If going backwards, there is a neg. constant velocity

↳ still no acceleration

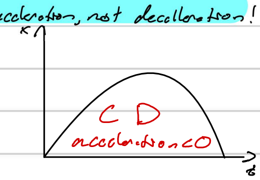
Speed: speed = $|v|$

Example: Tricycle w/ inc. velocity



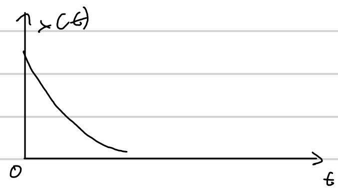
neg. acceleration, not decelleration!

Example: Tricycle going back and forth at inc/dec velocity + pos.



Velocity becoming more and more negative after peak

Example: Reading a graph



- velocity is becoming more positive, or less negative which translates to a positive acceleration.
- The graph's concavity is upward, so the acceleration is positive

"Forward" Derivatives

$x(t) \rightarrow v(t) \rightarrow a(t)$

"Backwards" Integrals

$a(t) \rightarrow v(t) \rightarrow x(t)$

> More complicated be you need initial conditions to solve for C

Integrating to solve, going "backwards"

- $x = \int v\, dt + C$ ← indefinite
- $x(t) = \int_{t'=0}^{t'=t} v(t')\,dt' + x_0$ ← we like definite integrals for solving in this class

t' is used as an indicator of respected variable

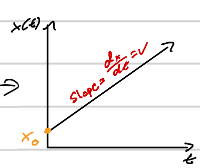
- $v(t) = \int_{t'=0}^{t'=t} a(t')\,dt' + v_0$ ← These are always true

Special Case: Constant Velocity

$a = 0$ — *The position changes

v = constant — linearly with time

$x = x_0 + \int_0^t v\,dt \rightarrow x = x_0 + vt$



These equations only apply if v = constant

Special Case: Constant acceleration

a = constant

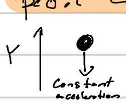
$v = v_0 + at$

$x = x_0 + v_0t + \frac{1}{2}at^2$

These apply only when a = constant

Special Case: Free Fall Kinematics

For this course, air resistance is neglected



$a_y = -g$ where $g = +9.81\ m/s^2$ — g is positive!

$v_y = v_{y_0} - gt$

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$

Subscript y means vertical direction

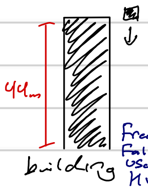
> These equations correspond to 1D Free-Fall Kinematics

Example: Building

Explain your work = identify the main concept

Box of mass 12 Kg → release box it falls!

① How long will the box fall?



Free Fall. Can use on HW

F.F. Kin: $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$0_m = 44_m + 0_{m/s} - \frac{1}{2}(9.81_{m/s^2}) t^2$

$\underbrace{44_m = t^2 (\tfrac{1}{2})(9.81_{m/s^2})}_{\text{This works, but don't do it!}}$

Don't do pure numerical algebra!

② How fast will it fall just before impact

$v_y = v_{y_0} - g t$

$v_y = 0 - g\left(\sqrt{\frac{2h}{g}}\right)$ ← Plug in symbol from ①

$v_y = -\sqrt{2hg} = -38_{m/s}$

How fast = speed: $v_y = \sqrt{2hg}$ or $38_{m/s}$

① Symbolic Algebra with given parameters

ⓘ Define Parameters: $h = 44_m$, $m = 12\,Kg$, $g = 9.81_{m/s^2}$

② F.F. Kin: $y = 0$ $v_{y_0} = 0$ ← Plug in no numbers until the end, excluding 0!

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$0 = h + 0 - \frac{1}{2} g t^2$

③ Solve for t: $\frac{1}{2} g t^2 = h$ ⟶ $t = \sqrt{2h/g}$ ← They will look for this as a solution on the

④ Solve: $t = \sqrt{\dfrac{2(44_m)}{9.81_{m/s^2}}} = 3.9\ sec$

- Put boxes around answers on exams
- Do the same on hw, but you can highlight

} Just indicate the final answer

Introduction to Motion - Dynamics

Newton's Laws

① → not useful for the scope of this course

② → Force ($\vec{F}_{Net}$) = Mass (m) · Acceleration ($\vec{a}$), $\vec{F}_{Net} = m\vec{a}$

Net Force: Vector Sum of actual forces on a body

Suppose... ∴ $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ → $F_{Net,x} = F_{1x} + F_{2x} + F_{3x}$ Can be split into components!

The Net Force is not a force itself, just a sum of forces!

Abbr. by N2L 1D

Newton's Second Law in 1-D

$F_{Net,x} = m a_x$ or $F_{Net,y} = m a_y$

Recipe For NL Problems

① Sketch, Given Parameters
② "Consider Acceleration"
③ Free Body Diagram
④ Apply N2L or N3L

Example 1: Apple Falling

Find all the forces

① M given

② accel? $a_y = -g$ ← falling down

③ Free Body Diagram ← Use a point for the body

④ $F_y = m a_y$ ← net force in y direction

$-W = m(-g)$

$\boxed{W = mg}$

Everything has a weight force, and its magnitude is always mg! $W = mg$

Force Menu

Use w to represent force of gravity bringing obj. down → $\vec{W}$ Weight, $W = mg$

only when in contact → $\vec{N}$ Normal, ⊥ surface contact, Push

Example 2: Book on Table

① M given

② $a = 0$ → $a_x = 0$ and $a_y = 0$

③ $\vec{N}$, $\vec{w}$, y↑

④ N2L, $F_y = m a_y$

$N - W = 0$ ← magnitudes here, not vectors!

$N - mg = 0$

Because $a = 0$, there must be another force, in this case its the normal force $\vec{N}$

$\boxed{N = mg}$ $\boxed{W = mg}$

Example 3: Apple on Table in Elevator

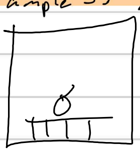
①  ↑v

Table Mass
Elevator Mass
Constant Velocity
Apple mass
} Given

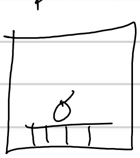
② $a_y = 0$ bc constant velocity

③ $\vec{N}$, $\vec{w}$, ↑y

④ Same answer as Ex 2 $\boxed{N = mg}$ and $\boxed{W = mg}$

As far as N2L is concerned, Constant velocity will do nothing to the diagram ∴ Same result as no speed

Example 4: Apple on Table in Elevator w/ accel

①  ↑A

A constant
m given
} given

② $a_y = A$

③ $\vec{N}$, $\vec{w}$, ↑y

④ $F_y = m a_y$

$N - W = mA$

$N - mg = mA$

$N = mA + mg$

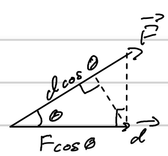
$\boxed{N = m(A + g)}$

Normal Force for this situation > N

Work

$$W_F \equiv \oint \vec{F}(\vec{r}) \cdot d\vec{r} \equiv \vec{F} \cdot \vec{d} = F \cdot \Delta x = Fd\cos\theta$$

Equivalency or a definition; a little too hard; simplification; dot product: $\vec{A} \cdot \vec{B} = AB\cos\theta$



Suppose $\vec{F}$, $\vec{d}$: For F and D in the same direction, $W_F = Fd\cos\theta = Fd\cos 0 = Fd$

Suppose $\vec{F}$, $\vec{d}$: For F and D in opposite directions, $W_F = Fd\cos\theta = Fd\cos 180° = -Fd$

Suppose $\vec{N}$, $\vec{d}$: Work done by the normal force $W_F = Fd\cos\theta = Fd\cos 90° = 0$

Any work done by the normal force is 0, $W_F = 0$!

Kinetic Energy

$v^2 = v_0^2 + 2a(x - x_0)$ $\quad K \equiv \frac{1}{2}mv^2$ $\quad \frac{m}{2}v^2 = \frac{m}{2}v_0^2 + F(x - x_0)$

$F_0 = ma_0$, $a_0 = \frac{F_0}{m}$ $\Big\rangle\ v^2 = v_0^2 + \frac{2F_0}{m}(x - x_0)$ $\quad \frac{m}{2}v^2 - \frac{m}{2}v_0^2 = F(x - x_0)$ $\quad$ let $v = v_f, v_0 = v_i, x = x_f, x_0 = x_i$

$\frac{m}{2}v_f^2 - \frac{m}{2}v_i^2 = F(x_f - x_i)$ $\quad x_f - x_i = \Delta x$

$\frac{m}{2}v_f^2 - \frac{m}{2}v_i^2 = F\Delta x \leftarrow K = \frac{1}{2}mv^2$

Work Energy Relation: $W_{Tot} = \Delta K$ $\quad$ Always True

$K_f - K_i = F\Delta x \rightarrow \Delta K = F\Delta x \rightarrow \Delta K = W_F$ $\quad$ $F\Delta x = $ Work

Potential Energy

$U_F \equiv -\oint \vec{F}(\vec{r})\cdot d\vec{r} + U_0 \longrightarrow U_F = -W_F + U_0 \rightarrow W_F = -\Delta U$

reference pt

$U(y) = mgy$

$W_w = U_{wy}' - U_{wy}$

$W_w = -mg\Delta y$

Work done by the weight force

Conservation Laws

Condition: If for each force on a body, either: force is conservative

or: force does 0 work

Energy cannot be created or destroyed $\therefore$ Before = After

$E_T = E_T'$

$W_T = W_c + W_{nc}$ $\quad$ conservative force $\quad$ non conservative

$\Delta K = -\Delta U$ $\quad$ U is used for potential energy

$K_f - K_i = U_i - U_f$

$K_f + U_f = K_i + U_i$

$U_{Tot} + K_{Tot} = U_{Tot}' + K_{Tot}'$

$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$

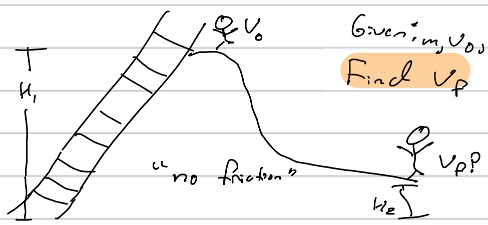
For a conservative force, you can actually find the potential energy through calculus

Forces like tension or friction are not

when to use...

Newtons	Conservative
time	usually
acc is const	speed, height
	varying force

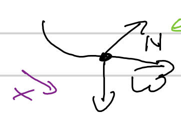
Example 1: Shutes and Ladders



Given: m, v_0, H_1, H_2

Find v_f

FBD



Perpendicular to Surface

Path not straight, can not use NL of Motion!

Forces: w yes conservative

N not conservative, but does no work

> Must check condition

Conditions Met $\quad$ Indicate this

Before = After

$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$

$\cancel{m}gH_1 + \frac{1}{2}\cancel{m}v_0^2 = \cancel{m}gH_2 + \frac{1}{2}\cancel{m}v_f^2$

m is common, can cancel out

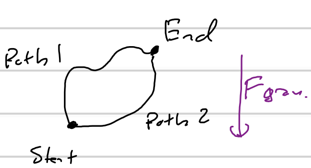
$gH_1 + \frac{1}{2}v_0^2 - gH_2 = \frac{1}{2}v_f^2$

$2gH_1 + v_0^2 - 2gH_2 = v_f^2$

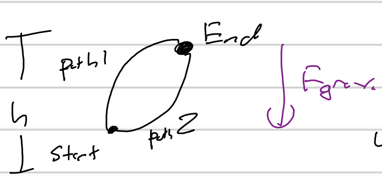
$\boxed{v_f = \sqrt{2g(H_1 - H_2) + v_0^2}}$

Conservative Force: -The work done by a conservative force is independent of the path $\quad$ > Calc 3

-The work done by a conservative force around a closed loop is 0



Path Independent, Only care about start/finish



$W_{1\to 2} = -mgh$

$W_{2\to 1} = mgh$

$W_{Tot} = mgh + (-mgh) = 0$

Newton's Laws

Second Law: $\vec{F}_{Net} = m\vec{a}$

↳ $F_x = ma_x$ and $F_y = ma_y$

"Recipe"

① Sketch parameters
② Consider acceleration
③ FBDs
④ Apply N2L

Force Menu

$\vec{w}$	weight	$w = mg$
$\vec{N}$	Normal	"Push" unknown
$\vec{T}$	Tension	"Pull"
$\vec{f}$	Friction	Parallel

Contact

Look @ # of contacts, that's how many nonweight forces on FBD

Notation

- $\vec{w}$ (one Body)
- $\vec{w}_A$ => weight force on body A
- $\vec{N}_B$ => Normal force on body B
- $\vec{N}_{BA}$ => Normal force on body B due to A "on-due"

N3L: Action reaction law

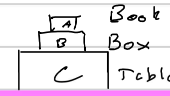
If Ⓐ ⟷ Ⓑ then $F_{AB} = F_{BA}$ for real force F

↳ for 2 different bodies Ⓐ and Ⓑ

$\vec{N}_{AB}$ points opposite $\vec{N}_{BA}$

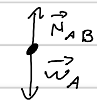
Same as HW1 #5

Example: Find Forces on Book and Box



m_A, m_B, m_c given

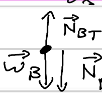
FBD Book A



y↑

Length is not important here

FBD Box B



Two contacts, Must be another force!

↑y

Normal force is always away from body

N2L Body A y-dir

$F_{Ay} = m_A a_{Ay}$

$N_{AB} - W_A = 0$

$N_{AB} - m_A g = 0$

$\boxed{N_{AB} = m_A g \quad W_A = m_A g}$

N2L Body B y-dir

$F_{By} = m_B a_{By}$

$N_{BT} - N_{BA} - W_B = 0$

$N_{BT} - N_{BA} - m_B g = 0$ two unknowns!

by N3L $N_{AB} = N_{BA}$

$N_{BA} = N_{AB} = m_A g$

$N_{BT} - m_A g - m_B g = 0$

$\boxed{N_{BT} = (m_A + m_B) g}$

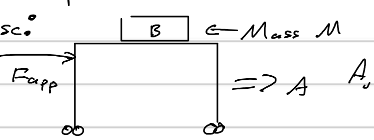
Box answers that are unknown in terms of known

"Friction opposes Motion" eNo! wrong direction

Friction

No slip and No slide: Assumption Case

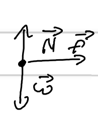
Suppose:



A, M Given

"static" friction: $f_s = \cancel{\mu_s \cdot N}$ Never use!

$a_T = a_{Box} = a_x = A$



Friction acts in whatever direction it needs to in order to satisfy N2L

Not the same

"non-static" Friction

$f_k = \mu_k N$ This is valid

Kinetic / sliding

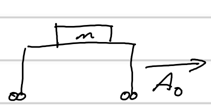
Forces must be on the same body: $f_{Ak} = \mu_k \cdot N_{AB}$

Still not "really" true

Cycle #1 Force Menu

Symbol	Name	Contact	Known?	What
$\vec{w}$	weight	No	$W = mg$	Down
$\vec{N}$	normal	Yes	No	Push into surface
$\vec{T}$	tension	Yes	No	Pull along wait/string
$\vec{f}$	Friction	Yes	No!	Force ∥ to surface

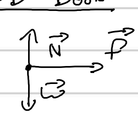
Example: Book on a Cart



$\begin{cases} A_0 = \text{given} = \text{const} \\ M = \text{given} \end{cases}$

No slip ⟵ Must be given

FBD Book



Friction here is in the direction of motion as the book is accelerating

Static Friction Revisited

$f_s = \mu_s \cdot N$ ⟵ Never use!

but

$f_s \leq \mu_s N$ ⟵ TRUE, but useless for calculating friction

If given, don't use!

N2L

$F_{Bx} = m_B a_{Bx}$

$f = M \cdot A_0$

$\boxed{f = M \cdot A_0}$

friction is the only force in the x-direction

Only use N2L to find friction

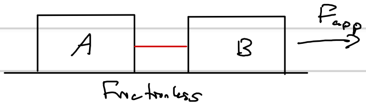
This is almost identical to HW2 #2, but there are two bodies

Look back to calc 360 to explain again and hints (P.)

Example: Ideal string

Remember a rope can only pull, not push!

You can only put $\vec{F}_{app}$ on FBD if it's given in that problem



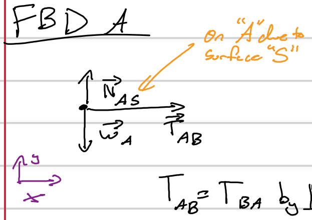
m_A, m_B, and F_{app} all given

Find: ① acceleration of Ⓐ

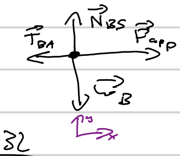
② Force of Tension on Ⓑ

a_{ax} not given
a_{bx} not given
$\Big\}$ but $a_{ax} = a_{bx} = a_x$ — Kinematic Constraint

FBD A



FBD B



$T_{AB} = T_{BA}$ by N3L

N2L A in x-direction

$F_{Ax} = m_A a_{Ax}$

$T_{AB} = m_A \cdot a_x$ Stop!

a_x and T_{AB} are unknown !!

B in x-direction

$F_{Bx} = m_B a_{Bx}$

$F_{app} - T_{BA} = m_B a_x$ Stop!

$T_{BA} = T_{AB}$ N3L

3 eq., 3 unknowns, the rest is algebra

Solution or guide to HW2 #1

Have everything in terms of given parameters

Let $T = T_{AB} = T_{BA}$ $\quad T = m_A \cdot a_x$

$+ \quad F_{app} - T = m_B \cdot a_x$

$F_{app} = m_A a_x + m_B a_x$

$F_{app} = (m_A + m_B) a_x$

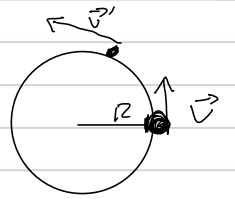
$$\boxed{a_x = \frac{F_{app}}{m_A + m_B}}$$

$$\boxed{T_{AB} = T_{BA} = m_A \left(\frac{F_{app}}{m_A + m_B} \right)}$$

Uniform Circular Motion, Systems, & CofME/CofLM

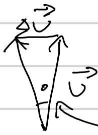
Circular Motion

UCM is Uniform Circular Motion



Birds eye view

$m, R, \vec{v}$ given



Δv points left

$\Delta\theta$

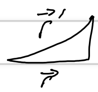
$\lim_{\theta \to 0} \sin\theta = \theta$

$$|\vec{a}| = \left|\frac{\Delta \vec{v}}{\Delta t}\right| = \frac{v \sin \Delta\theta}{\Delta t} \approx \frac{v\,\Delta\theta}{\Delta t} = v \cdot \omega$$

Centripetal "Towards Center"

Kinetics

Define $\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$ Omega

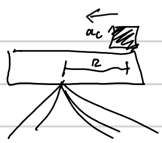


$$\frac{\Delta r}{\Delta t} = v = r\frac{d\theta}{dt} = r \cdot \omega$$

Uniform Circ Motion

$$\boxed{\begin{aligned} v &= \omega R \\ a_c &= \omega^2 R \\ a_c &= \frac{v^2}{R} \end{aligned}}$$

Example: Turntable



m, R, v given

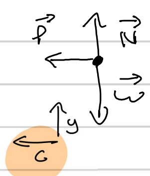
Find Forces

Accel: $a_y = 0$

UCM: $a_c = \frac{v^2}{R}$

a_c is given

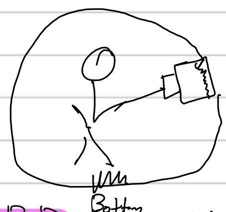
FBD Car



$F_c = m a_c$

$$\boxed{f = \frac{mv^2}{R}}$$

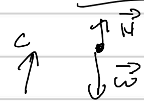
Example: water in bucket



m, v_0, R Find N on water due to bucket

$a_c = \frac{v^2}{R}$ UCM

FBD Bottom



N2L

$F_c = m a_c$

$N - w = \frac{mv^2}{R}$

$N = \frac{mv^2}{R} + w$

$$\boxed{N = m\left(\frac{v^2}{R} + g\right)}$$

FBD Top

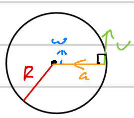
$F_c = m a_c$

$N + w = \frac{mv^2}{R}$

$$\boxed{N = m\left(\frac{v^2}{R} - g\right)}$$

Review

$a_c = \frac{v^2}{R} = \omega^2 R$



$\omega = \frac{d\theta}{dt}$

Centripetal Coordinate will point towards Center of circle

Conservation of Mechanical Energy

$E_{Tot} = U + K$, $U_{(grav)}$ is potential Energy

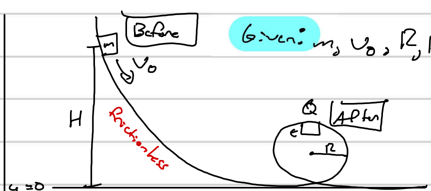
$U_{\vec{w}} = mgy$ $K = \frac{1}{2}mv^2$, v is speed $\therefore$ K is non-negative

"Before" "After"

$E_{Tot} = E'_{Tot}$ ' = Prime = after

$U + K = U' + K'$

$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$



Given: m, v_0, R, H

what is the normal force on block at Q

① Conservation of mechanical energy

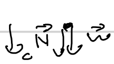
Conditions: all forces must be conservative or do 0 work

$y = H$, $v = v_0$, $y' = 2R$, $v' = v_Q = ?$

$\cancel{m}gH + \frac{1}{2}\cancel{m}v_0^2 = \cancel{m}g2R + \frac{1}{2}\cancel{m}v_Q^2$

$v_Q^2 = 2gH - 4gR + v_0^2$

FBD Q



N2L - c-coord

$F_c = m a_c$

$N + w = m\left(\frac{v_Q^2}{R}\right)$

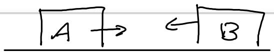
$N = m\left(\frac{v_Q^2}{R} - g\right)$ where $v_Q^2 = 2gH - 4gR + v_0^2$

Systems

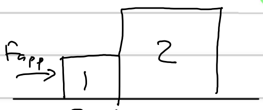
A system divides the universe into unambiguous, well defined, spaces.

2 blocks collide, given m_A, m_B, v_A, v_B

Find v_A' and v_B' = ??



3 Model Question



N2L $F_{sys} = m_{sys} a_{sys}$

$= m \frac{dv}{dt}$

$F_{sys} = \frac{d(mv)}{dt}$

$F_{sys,\,Net\,Ext} = \frac{dP}{dt}$, $p \equiv mv$ — Newton's 2nd Law

calculus → $\int F dt = \int dp$

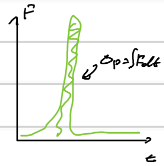
$F_{avg} \cdot \Delta t = \Delta p$

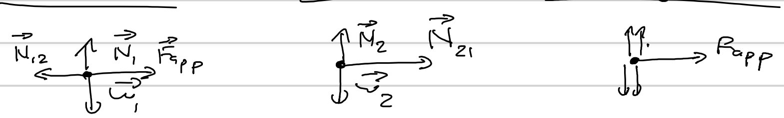
Work Energy Relation

$W_{Tot} = \Delta K = \vec{F} \cdot \vec{d}$

Impulse Momentum Relation

Impulse $\equiv \Delta p = F_{avg} \cdot \Delta t$





If $F_{Net,\,sys,\,ext} = 0 \Rightarrow \frac{dP}{dt} = 0$

$\therefore P =$ constant

$P_{tot} = P_{tot}'$

Conservation of Linear Momentum → Can only use if and only if $F_{Net} = 0$!

Conservation of Linear Momentum

Moment: "amount of motion" in a body

Def: $\boxed{\vec{p} = m\vec{v}}$

N1L: In an absence of net external force, ($F_{Net\,Ext} = 0$) velocity of a particle remains a constant

$\Rightarrow p =$ [constant]

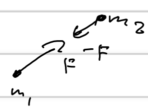
(Momentum is constant)

N2L: $F_{Net} = ma$

$= m\left(\frac{dv}{dt}\right)$

$= \frac{d}{dt}(mv)$

$\boxed{F_{Net\,Ext} = \frac{dP}{dt}}$ but $F_{Net\,Ext} = 0 \therefore \frac{dP}{dt} = 0$ and $P =$



N3L: $\frac{dP_1}{dt} = \frac{-dP_2}{dt}$

$\frac{dP_1}{dt} + \frac{dP_2}{dt} = 0$

$\frac{d}{dt}(P_1 + P_2) = 0 \therefore P_1 + P_2 =$ Constant

$F_{Net\,sys\,Ext} = 0$ so System is isolated

Impulse, Collisions, Rotational Motion, & Cycle 1 Review

Impulse

$$I = \int_0^{\Delta t} F \cdot dt = \int_0^{\Delta t} \frac{dp}{dt}\, dt$$

$$\boxed{I = \int dp = \Delta p}$$

Impulse Momentum Relation

Recall:

$U + K = \text{constant} = E_{Tot}$

Before = After

$U + K = U' + K'$

$P_1 + P_2 = \text{constant} = P_{Tot}$

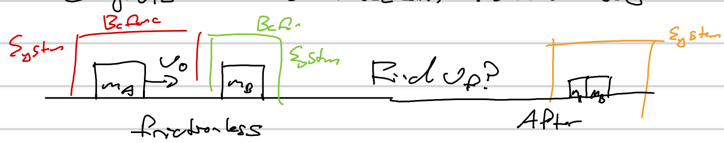
Before = After

$P_1 + P_2 = P_1' + P_2'$

$P_{Tot} = P'_{Tot}$

Inelastic Collisions

Objects stick and move with the same velocity



$P_A + P_B = P_A' + P_B'$

$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$m_A v_A + 0 = m_A v_A' + m_B v_B'$

$m_A v_0 = m_A v_A' + m_B v_A'$

$$\boxed{v_0 \left(\frac{m_A}{m_A + m_B} \right) = v_A' = v_f}$$

$m_A = m_B \qquad v_f = \frac{1}{2} v_0$

Every Review Sheet Outlines the Exam Content

Some Review of Major Conservation Concepts

Bridge Concept (Always True)	Condition IF	Conservation Law THEN "Before" = "After"
Work Energy $W_{Tot} = \Delta K$ $F \cdot \Delta x = \Delta K$	Forces are all either conservative or do zero work	$E_{Tot} = E_{Tot}'$ CofME $U + K = U' + K'$ $mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$
Impulse Momentum $F_{ave} \cdot \Delta t = \Delta p$	$F_{Net, sys, Ext} = 0$ "isolated" "Net force on system due to external forces only"	$P_{TOT} = P_{Tot}'$ CofLM $P_A + P_B = P_A' + P_B'$ $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

The only Conservative force we have so far is weight!

Totally Inelastic Collisions

2 bodies stick

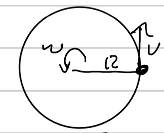
$v_A' = v_B'$ "catch"

Reverse = Explosion

$v_A = v_B$ = given

"throw"

Circular Motion



$a_c = v^2 / R$

$a_c = \omega^2 R$

$v = \omega R$

Rotational Motion

Analogy Table

"Quantity"	Translational "linear"	Rotational "angular"
Kinematics		
Position	x distance	θ angle
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
Accel	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Dynamics		
N2L	$F_{Net} = ma$	$\tau_{Net} = I\alpha$

Tau τ = torque

Not the torque Rule!

This is "N2L Rotational"

Constant This accel

$a = \text{const}$

$v = v_0 + at$

$x = x_0 + v_0 t + \frac{1}{2} a_0 t^2$

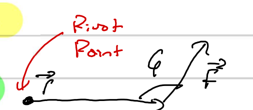
$\alpha = \text{const}$

$\omega = \omega_0 + \alpha t$

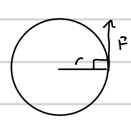
$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

Definition of Torque:

$$\tau_P = rF \sin\phi$$



ϕ angle btwn. $\vec{r}$ and $\vec{F}$

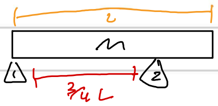


maximum at $90°$

as $\sin 90° = 1$

Rod on 2 Trestles

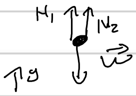
Trestles holding up rod, trying to find 2 to the left



Mass and lengths given

accel = 0

FBD Rod



N2L - y dir

$F_y = m a_y$

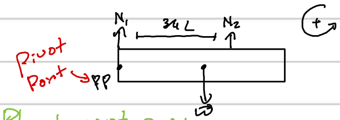
$N_1 + N_2 - mg = 0$

Stop!

$\alpha = 0$

$\tau_{Net} = I\alpha = 0$

X FBD Rod



Place the pivot point @ an unknown force

$\tau = r F \sin\theta$

N2L Rot

$\tau_{Net} = I\alpha$

$\tau_{N_1} + \tau_{N_2} + \tau_w = I\alpha \quad \alpha = 0$

$0 + \frac{3L}{4} N_2 \sin 90° - \left(\frac{L}{2}\right) mg \sin 90 = 0$

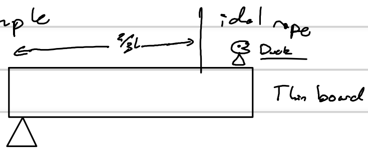
$\frac{3\cancel{L}}{4} N_2 - \frac{\cancel{L}}{2} mg = 0$

$\therefore N_2 = \frac{2}{3} mg$

$N_1 + \frac{2}{3}mg - mg = 0$

$\boxed{N_1 = \frac{1}{3}mg}$ $\boxed{W = mg}$ $\boxed{N_2 = \frac{2}{3}mg}$

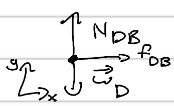
Example



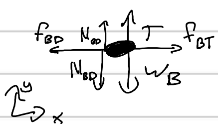
Given: $m_B, m_D, L, A_0, \mu s$

Find all forces

FBD Duck



FBD Board



N2L Duck

y: $F_y = m_D a_y$

$N_{BD} - W_D = 0$

$N_{BD} = m_D g$

x: $F_{Dx} = m_D a_{Dx}$

$f_{DB} = m_D A_0$

N2L Board

x: $F_{Bx} = m_B a_{Bx}$

$f_{BT} - f_{BD} = 0$

$f_{BT} - f_{DB} = 0$

$f_{BT} = f_{DB}$

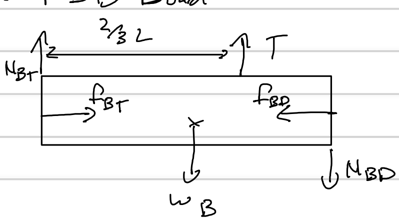
N3L $f_{BT} = m_D A_0$

y: $F_{By} = m_B a_{By}$

$T + N_{BT} - w_B - N_{BD} = 0$

$T + N_{BT} - m_B g - m_D g = 0$

X FBD Board



N2L Rot to solve

Exam Review

25% to 33% - Identifying concepts correctly
25% to 33% - Articulate correct concepts
33% to 50% - Applying the concepts

} 5% of total grade
3 questions, 50 minutes
25% of the points from

Formal answer in box in terms of given parameters
Simply writing N2L, for example

You are responsible for everything on the first 3 review sheets

- Homework solutions and textbook can help clarify material

Application of concepts must be methodical and coherent

No paragraphs just say the concept

Practice Exams!

Mechanics

How — Kinematics

Explanation

Def V: $v = \frac{dx}{dt}$

Def position, velocity, accel.

Const a, Free Fall

UCM

$a_c = \frac{v^2}{R} = \omega^2 R$

$v = \omega R$ "Rolling Constraint"

Why? — Dynamics

Force Menu (Pattern C.S.)

(X) FBDs!

Don't forget conditions!

Newton's Laws and Conservation Laws ⟷ Analogy

Work

$W_p = \int_{path} \vec{F} \cdot d\vec{r}$

$= \vec{F} \cdot \vec{d}$

$= Fd\cos\theta$

N3L

$N_{AB} = N_{BA}$

"on-due"

Consider acceleration and kinematic constraints

N2L

$F_{Net} = ma$

Mechanical

Energy

Cof ME

$U + K = U' + K'$

Linear

Momentum

Cof LM

$P_{Tot} = P'_{Tot}$

Conditions (!/?)

Rotations

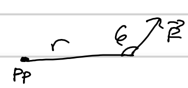
$x, v, a \rightarrow \theta, \omega, \alpha$

$F_{Net} = ma \rightarrow \tau_{Net} = I\alpha$

Have Mastery of all Topics Here!

Torque

$\tau_p \equiv rF\sin\theta$



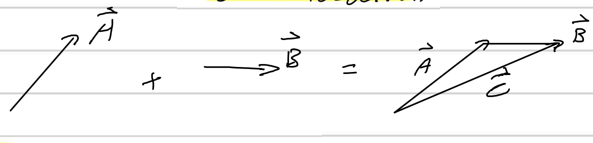
Cycle 2

Vectors and 2D kinematics

Recall: Vectors have both magnitude and direction

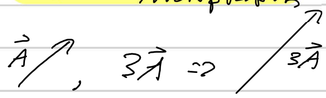
Length = "magnitude" $|\vec{A}|$

Vector Addition

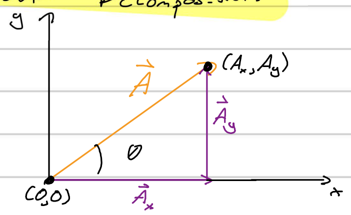


$\vec{C} = \vec{A} + \vec{B}$

Scalar Multiplication



Vector Decomposition



Vectors are defined by numbers!

$|\vec{A}| = $ magnitude $= A$ (no > symbol)

Direction $= \theta$ (some reference ex !)

Or we can break into components!

$\vec{A} = \vec{A}_x + \vec{A}_y$

Components

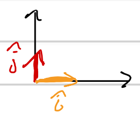
$|\vec{A}_x| = A_x = A\cos\theta$

$|\vec{A}_y| = A_y = A\sin\theta$

You can find the components of a vector if given magnitude and direction

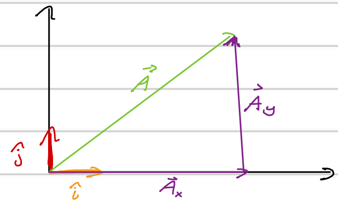
Reverse Direction:
- To find A if A_x, A_y given: $A = \sqrt{A_x^2 + A_y^2}$ Woah!
- To find θ if A_x, A_y given: $\theta = \arctan\left(\frac{A_y}{A_x}\right)$ Crazy

Cartesian Basis Unit Vector



Don't use < > Notation in this class!

Unit Vector = Length of One!



Means...
- $\vec{A}_x = A_x\hat{\imath}$
- $\vec{A}_y = A_y\hat{\jmath}$

$\therefore$ $\boxed{A = A_x\hat{\imath} + A_y\hat{\jmath}}$

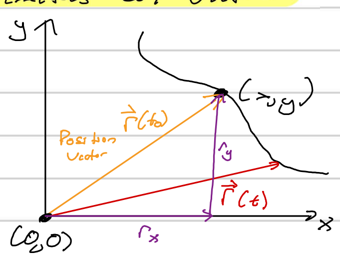
Vector Representation

Example: $\vec{C} = \vec{A} + \vec{B}$

$$\begin{cases} \vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} \\ \vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} \end{cases} \quad \therefore \quad \vec{C} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$

Path Integrals will Come Later

Kinematics w/ Vectors



$\vec{r} = \vec{r}_x + \vec{r}_y = r_x\hat{\imath} + r_y\hat{\jmath}$

Position Vectors — a vector whose components are its own coordinates

$$\vec{r} \equiv x\hat{\imath} + y\hat{\jmath} \Rightarrow \vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$$

$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$

$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$

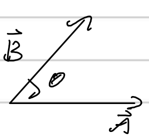
$\vec{a} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath}$

$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$

Example: $\vec{r}(t) = (Bt^2)\hat{\imath} + (Ce^{-t/\tau})\hat{\jmath}$ τ, B, C Given Constants   Find Velocity and accel

$\vec{v}(t) = (2Bt)\hat{\imath} - \left(\frac{C}{\tau}e^{-t/\tau}\right)\hat{\jmath}$ and $\vec{a}(t) = (2B)\hat{\imath} - \left(\frac{C}{\tau^2}e^{-t/\tau}\right)\hat{\jmath}$

Dot Products!



$C = \vec{A}\cdot\vec{B} = AB\cos\theta$

$\hat{\imath}\cdot\hat{\imath} = 1$, $\hat{\imath}\cdot\hat{\jmath} = 0$

Using Decomposition

$\therefore C = (A_x\hat{\imath} + A_y\hat{\jmath})\cdot(B_x\hat{\imath} + B_y\hat{\jmath})$

FOIL $= A_xB_x(1) + A_xB_y(0) + A_yB_x(0) + A_yB_y(1)$

$$\boxed{C = A_xB_x + A_yB_y}$$

Example: Related to HW4

Define: $\vec{r}(t) = \underbrace{R\cos(\omega t)}_{x(t)}\hat{i} + \underbrace{R\sin(\omega t)}_{y(t)}\hat{j}$, R, ω given

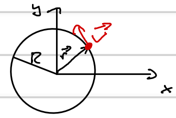
Take Derivatives of Functions of time!

$|\vec{r}| = \sqrt{x^2+y^2}$ → $|\vec{r}| = \sqrt{(R\cos(\omega t))^2 + (R\sin(\omega t))^2}$

Determine Magnitude

$= \sqrt{R^2(\cos^2(\omega t) + \sin^2(\omega t))} = \sqrt{R^2} = \boxed{R}$

Suppose



Speed $= |\vec{v}| = \sqrt{v_x^2+v_y^2}$

$\vec{v} = \frac{d\vec{r}}{dt} = -\omega R\sin(\omega t) + \omega R\cos(\omega t)$

$\therefore |\vec{v}| = \sqrt{R^2\omega^2(\sin^2(\omega t) + \cos^2(\omega t))} = \boxed{R\omega}$

Omg! its the really constant

$\vec{a} = -R\omega^2\cos(\omega t) - R\omega^2\sin(\omega t)$

$|\vec{a}| = \sqrt{R^2\omega^4(\cos^2(\omega t) + \sin^2(\omega t))}$

$= \sqrt{R^2\omega^4} = R\omega^2 = \frac{v^2}{R}$

$\boxed{a = \frac{v^2}{R} = a_c}$

woah! Its centripetal acceleration

Projectile Motion

x-direction:

no air resistance → velocity is constant

$$\boxed{\begin{aligned} a_x &= 0 \\ v_x &= \text{Const} = v_{x_0} \\ x &= x_0 + v_{x_0}t \end{aligned}}$$

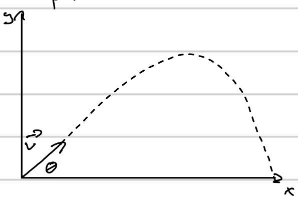
Two sets of equations to describe an objects motion

y-direction:

Free Fall

$$\boxed{\begin{aligned} a_y &= -g \\ v_y &= v_{y_0} - gt \\ y &= y_0 + v_{y_0}t - \tfrac{1}{2}gt^2 \end{aligned}}$$

Graph



we say: $v_{x_0} = v_0\cos\theta$ and $v_{y_0} = v_0\sin\theta$

$\therefore \vec{r} = x\hat{i} + y\hat{j}$

Motion of Projectile in vector form

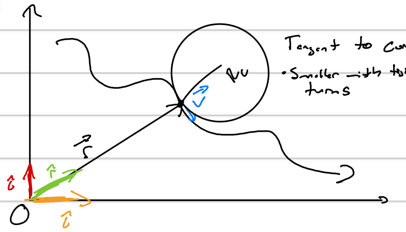
Identical to relations shown above

$\vec{r}(t) = (x_0 + v_{x_0}t)\hat{i} + (y_0 + v_{y_0}t - \frac{1}{2}gt^2)\hat{j}$

$\vec{v}(t) = (v_x)\hat{i} + (v_{y_0} - gt)\hat{j}$

$\vec{a}(t) = 0\hat{i} - g\hat{j}$

Example: A particle on a given path



Tangent to Curve

• Smaller with tighter turns

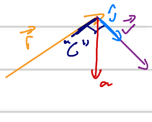
$\vec{r} = x\hat{i} + y\hat{j}$

Unit vector in the r direction

$$\boxed{\begin{aligned} \vec{r} &= r\hat{r} \\ \vec{v} &= v\hat{v} \end{aligned}}$$

where $r = |\vec{r}|$ and $|\hat{r}| = 1$ r is mag, $\hat{r}$ is dir

where $v = |\vec{v}|$ and $|\hat{v}| = 1$ v is mag, $\hat{v}$ is dir



$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{a} = a_v\hat{v} + a_c\hat{c}$

Tangential Component   Centripetal Component

$\hat{c}$ is $\perp$ to $\hat{v}$

magnitude

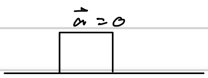
$a_v = \frac{d}{dt}(v)$ where v is the speed

$a_c = \frac{v^2}{R_c}$

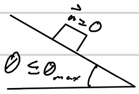
Both depend on speed

R_c is radius of Curvature

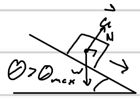
Forces at Angles



$F_{Net}=0$



$F_{Net}=0$



$F_{Net} > 0$

θ_{max} is the maximum angle required for the body to remain at rest.

Steps

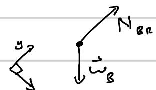
1. FBD
2. Coordinate System
3. Kinematic Constraint
4. N2L
   4.1 Decompose Forces

Only decompose forces Not aligned with the chosen coordinate System!

Decomposing Necessary forces into their necessary components is very important

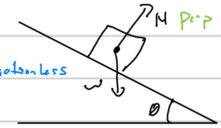
Given: m, θ; find (i) a and (ii) all forces

FBD Block



avoids having to break net force into components "tipped coordinate"

Kinematic Constraint
$a_x = a$
$a_y = 0$



frictionless

N2L $F_{Net} = ma$ Y-dir

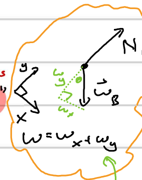
$F_y = ma_y$ ← Normal force and y component of the weight force

$N - w_y = ma_y$

$N - w\cos\theta = 0$

$N - mg\cos\theta = 0$

$\boxed{N = mg\cos\theta}$



$\cos\theta = \frac{w_y}{w}$
$w_y = w\cos\theta$
$w_x = w\sin\theta$

N2L $F_{Net} = ma$ x-dir

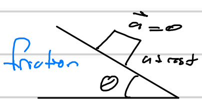
$w_x = ma_x$ ← only w_x in x dir

$w\sin\theta = ma$

$mg\sin\theta = ma \Rightarrow \boxed{a = g\sin\theta}$

Small θ → Small a
Large θ → a approaching g

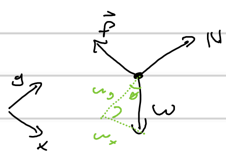
Suppose there is friction (Static)



Given m, θ; Find (i) all forces

$a = 0$ as at rest

FBD Block



$a_x = 0 \quad a_y = 0$

N2L

x: $F_x = ma_x$
$w_x - f = 0$
$w_x = w\sin\theta = mg\sin\theta$
$mg\sin\theta - f = 0$
$\boxed{f = mg\sin\theta}$

y: $F_y = ma_y$
$N - w_y = 0$
$w_y = w\cos\theta = mg\cos\theta$
$\boxed{N = mg\cos\theta}$

Suppose there is Kinetic Friction

Given μ_K, m, θ Find (i) a
(ii) all forces

N2L

y: $F_y = ma_y$
Same as the bit to the left
$\boxed{N = mg\cos\theta}$

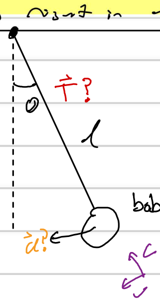
$f = \mu_K N$
$\boxed{f = \mu_K mg\cos\theta}$

x: $w_x - f = ma_x$
$mg\sin\theta - f = ma$
$mg\sin\theta - \mu_K N = ma$
$N = mg\cos\theta$
$mg\sin\theta - \mu_K mg\cos\theta = ma$
$\cancel{m}g(\sin\theta - \mu_K\cos\theta) = \cancel{m}a$
$\boxed{a = g(\sin\theta - \mu_K\cos\theta)}$

$f = \mu_K N$

Pendulums

Jumping right in with an example



Circular Motion

- a_c points radially inward
- v is tangential

Pendulum bob with mass m released at rest angle θ

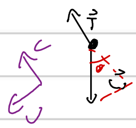
Q1 What is the tension on string?

Q2 What is the acceleration of bob immediately after its released?

Given mass m, θ, and length of string l

a_c and a_v must be considered

FBD Bob



N2L : c-coord

$F_c = m a_c$

$F_c = m\left(\frac{v^2}{R}\right) = m\left(\frac{0^2}{l}\right) = 0$

$T - W\cos\theta = 0$

$T - mg\cos\theta = 0 \rightarrow$ ① $\boxed{T = mg\cos\theta}$

N2L : v-coord

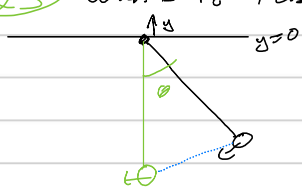
$F_v = m a_v$

$W\sin\theta = m a_v$

$mg\sin\theta = m a_v$

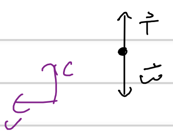
② $\boxed{a_v = g\sin\theta}$

Q3 What is the Tension on string when bob at lowest position?



Still using coordinates as path is still circular

FBD Bob



N2L c-coord

$F_c = m a_c$

$T - w = m\left(\frac{v^2}{l}\right)$

$T - mg = m\left(\frac{v^2}{l}\right)$ stop!

v is unknown here!

$T - mg = \frac{m v_e^2}{l}$ and $v' = \sqrt{2gl(1-\cos\theta)}$

Let $v' = v_e$

$T = mg + \frac{m(2gl(1-\cos\theta))}{l}$

$T = mg + 2mg(1-\cos\theta)$

$T = mg(1 + 2(1-\cos\theta))$

$T = mg(1 + 2 - 2\cos\theta) \rightarrow \boxed{T = mg(3 - 2\cos\theta)}$

Find v

CofME $\vec{w}$ conservative

$\vec{T}$ does no work as $\perp$ to path

Before = After

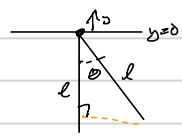
$E_{Tot} = E'_{Tot}$

$U + K = U' + K'$

"Before" = release

"After" = lowest position

Conditions met!



$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$

$y = -l\cos\theta$, $y' = -l$

$v = 0$, $v' = ?$

$mg(-l\cos\theta) + \frac{1}{2}m(0)^2 = mg(-l) + \frac{1}{2}mv'^2$

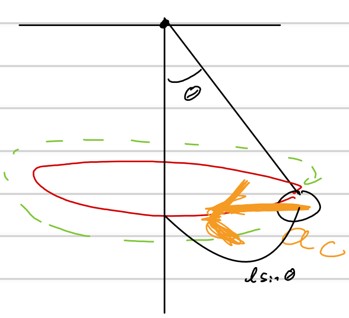
$-\cancel{m}gl\cos\theta = -\cancel{m}gl + \frac{1}{2}\cancel{m}v'^2$

$gl - gl\cos\theta = \frac{1}{2}v'^2$

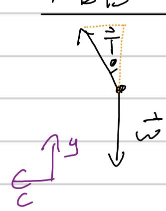
$v'^2 = 2gl(1-\cos\theta)$

$\therefore \boxed{v' = \sqrt{2gl(1-\cos\theta)}}$

Conical Pendulums



FBD Bob



N2L y-coord

$F_y = m a_y$ $a_y = 0$

$T\cos\theta - W = 0$

$\boxed{T = \frac{mg}{\cos\theta}}$

N2L c-coord

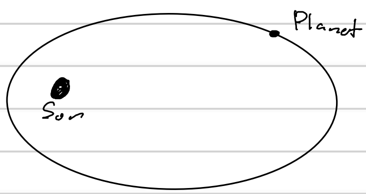
$F_c = m a_c$

$T\sin\theta = m\frac{v^2}{l\sin\theta}$

$\frac{mg\sin\theta}{\cos\theta} = \frac{mv^2}{l\sin\theta}$

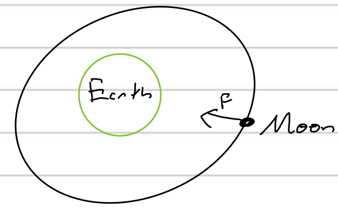
$mg\tan\theta = \frac{mv^2}{l\sin\theta}$, solve for v

Planets!



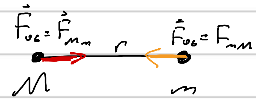
Kepler's 3rd Law (K3L)

$$\frac{P^2}{R^3} = \frac{(\text{Period})^2}{(\text{"radius"})^3} = \text{constant?}$$



Universal Gravity

$$F_{UG} = \frac{G \cdot M \cdot m}{r^2}$$

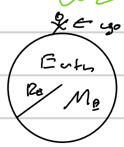


G = Gravitational Constant

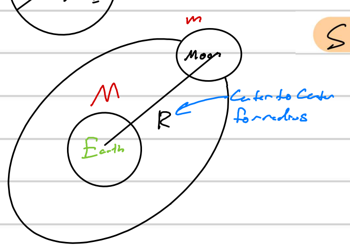
$= 6.67 \times 10^{-11}\ N \cdot m^2 \cdot kg^{-2}$ ← Super small

$F_{UG} = \frac{G(1)(1)}{1} = G$ ← barely anything at all

we only notice it bc of earth's size



$\Rightarrow F_{UG} = \frac{G M m}{R_E^2} = w = mg \rightarrow \boxed{g_E = \frac{G M_E}{R_E^2}}$ ← where "g" comes from!!



Situation $M \gg m$

Orbit ≈ Circular

(1) what are the forces on the moon

N2L Moon

$F_{moon} = m a_c$

$\frac{G M \cancel{m}}{R^2} = \cancel{m} \frac{v^2}{R}$

Solve for $\frac{P^2}{R^3}$

$\rightarrow$ Period $\frac{2\pi R}{P} = v$ ← $\frac{\text{distance}}{\text{time}}$

$\frac{GM}{R^2} = \frac{4\pi^2 R^2}{P^2 R}$

$GM = 4\pi^2 \left(\frac{R^3}{P^2}\right) \rightarrow \boxed{\frac{P^2}{R^3} = \frac{4\pi^2}{GM}}$ ← K3L back!

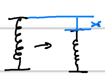
Force Menu

Symbol	what	Known?	Conservative?	Contact?
$\vec{w}$	weight	$w = mg$	Yes	No
$\vec{N}$	Normal Push	No	No	Yes
$\vec{T}$	Tension Pull	No	No	Yes
$\vec{f}$	Friction	Sometimes	Absolutely No!	Yes ← Known if kinetic and given μ_k
$\vec{F}_{UG}$	Universal Gravity	Yes	Yes	No
$\vec{F}_{sp}$				

Hooke and Springs

"Spring"

Compressed or Stretched



x is the change in length

$F_{sp} = K|x|$ ← x is displacement doesn't care about actual spring length

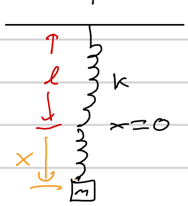
Force points in the direction opposite to x

$F_{sp} = K|x|$ Larger K equates to increased force

K = spring constant with equal displacement compared to small K

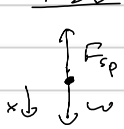
x = displacement relative to a relaxed spring

Example



what is x s.t. mass m in equilibrium? $\rightarrow a = 0$

FBD



N2L x

$F_x = m a \rightarrow 0$

$w - F_{sp} = 0$

$mg - Kx = 0$

$\boxed{x = \frac{mg}{K}}$

$w = mg$

$F_{sp} =$

You can only assume $a = 0$ when mass or object is in equilibrium

Definition of Potential Energy

$$U_{\vec F}(P) - U_{ref}(P_{ref}) = -\int_{P_{ref}}^{P} \vec F \cdot d\vec r$$

Path integral

$\vec F \cdot d\vec r = F dr \cos\theta$ — Change w/ time

W.K.: $W_{cons} = -\Delta U_F$ — work done by a conservative force

Computing the integral
Weight w/ coord y } for weight

Pot. Energy

$$U_{\vec w}(y) - U_{ref}(y_{ref}) = -\int \vec w \cdot d\vec r$$

$\rightarrow \vec w \cdot d\vec r = mg\, dy \cos(180°) = -mg\, dy$

mg of weight; Angle btwn. $\vec w$ and $d\vec r$ is 180°; $d\vec r$ mag is dy

$$U_{\vec w}(y) - U_{ref}(\cancel{y_{ref}=0}) = -\int_{y=0}^{y'=y} -(mg\, dy)$$

$$\boxed{U_{\vec w}(y) = mgy}$$

Just an easy integral

Set ref to 0 in PHYS 121, not always true in PHYS 122

Calculating for Spring Force

$$U_{sp}(x) - U_{ref}(x_{ref}) = -\int \vec F_{sp} \cdot d\vec r$$

Spring force against motion

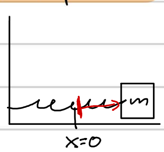
x_{ref}, $\vec F_{sp}$, $d\vec r$, x

$\vec F_{sp} \cdot d\vec r = Kx\, dx \cos(180°) = -Kx\, dx$ — $|\vec F_{sp}|$, $|d\vec r|$

$$U_{sp}(x) - \cancel{U_{ref}(x=0)} = -\int_{x=0}^{x'=x} -(Kx'dx')$$

$$\boxed{U_{sp}(x) = \frac{1}{2}Kx^2}$$

Example



Release at rest at $x = x_i$. Find max speed

Given: K, m, x_i

Consider C of ME when asked to find speed

All conservative or no work done

"Before" = "After"

$U_{sp} + K = U'_{sp} + K'$

$\frac{1}{2}Kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}Kx'^2 + \frac{1}{2}mv'^2$

$x = x_i,\ v = 0,\ x' = 0,\ v' = v_f ?$

$\frac{1}{2}Kx_i^2 + 0 = 0 + \frac{1}{2}mv_f^2$

$\therefore \boxed{v_f = x_i\sqrt{\frac{K}{m}}}$

Computing for Universal Gravity

$$U_{UG}(r) - U_{ref}(r_{ref}) = -\int \vec F_{UG} \cdot d\vec r$$

$d\vec r$, $\vec F_{UG}$, r_{ref}, r

$\vec F_{UG} \cdot d\vec r = \frac{GMm}{r^2} dr \cos 180° = -\frac{GMm}{r^2} dr$

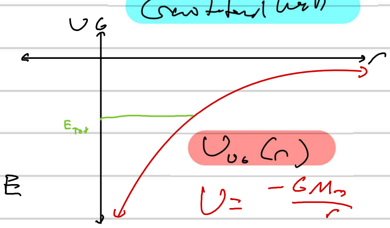
F_{UG} goes to 0 when $r = \infty$

can't be 0, can't be divided by 0

$$U_{UG}(r) - \cancel{U_{ref}(r_{ref} = \infty)} = -\int_{r=\infty}^{r'=r} -\left(\frac{GMm}{r^2}\right) dr$$

$$\boxed{U_{UG}(r) = -\frac{GMm}{r}}$$

"Gravitational well"



$U_{UG}(r)$

$U = -\frac{GMm}{r}$

Conservative Forces

$\vec w$ — Weight — $U_{\vec w} = mgy$

$\vec F_{sp}$ — Spring Force — $U_{sp} = \frac{1}{2}Kx^2$

$\vec F_{UG}$ — Universal Gravity — $U_{UG} = -\frac{GMm}{r}$

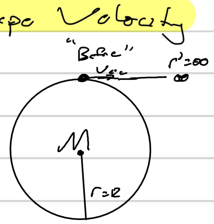
C of ME

$E_{Tot} = E'_{Tot}$

$U + K = U' + K'$ ← $U^{(')}$ can be a sum of different potential Energies

Escape Velocity

← Ignoring Air resistance, high K sends an object out of atmosphere as $\frac{1}{r^2}$ decreases



After: $v' = 0$

C of ME

$U + K = U' + K'$

$-\frac{GMm}{r} + \frac{1}{2}mv^2 = -\frac{GMm}{r'} + \frac{1}{2}mv'^2$

$-\frac{GMm}{R} + \frac{1}{2}mv_{esc}^2 = 0 + 0$

Algebra: $\boxed{v_{esc} = \sqrt{\frac{2GM}{r}}}$ $\approx 1.2 \times 10^4$ m/s or ~7 miles/sec

Thrown with some velocity v_{esc}. Initial radius = R. Mass of object not needed. Planet mass M. Thrown, knowing the following is 0 and $r' = \infty$. Find the escape velocity.

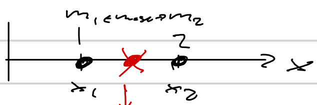
More on Systems

N2L for Systems

$$F_{\substack{Net\\Sys\\Ext}} = M_{sys}\, a_{sys}$$

works for systems of n objects and directions only with CoM

Center of Mass (CoM)



$$X_{Avg} = \frac{x_1 + x_2}{2}$$

CoM is not an actual thing, just a concept

Suppose $m_1 \neq m_2$, find weighted Avg

2pts: $\boxed{X_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$ ← weighted avg equation

N pts: $$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_n}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \dots + m_n y_n}{m_1 + m_2 + \dots + m_n}$$

2 coordinates with n bodies with given masses and positions

Time derivatives of CoM

$$\frac{dx_{cm}}{dt} = V_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

derivative of a sum is the sum of derivatives

$$\frac{dv_{cm}}{dt} = a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

Can be expanded to N points

N2L System

$$\vec{F}_{\substack{Net\\Sys\\Ext}} = M_{sys}\, \vec{a}_{cm}$$

vector b/c it applies to any number of dimensions

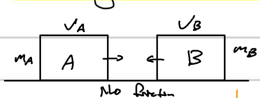
if $F_{\substack{Sys\\Net\\Ext}} = 0 \Rightarrow$ "Isolated" $\Rightarrow$ $a_{cm} = 0$

$V_{cm} = $ Constant ← "conserved"

$M_{sys} \cdot V_{cm} = P_{Tot}$ — CofLM Momentum!

Inertial reference frame is defined to be at rest or moving with constant velocity

Totally Elastic Collisions



$K = K'$ is what totally elastic means

$$\therefore \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 = \tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} m_B v_B'^2$$

CofK

Isolated ⇒ $P_{Tot} = P_{Tot}'$ — CofLM

"les 2 unknowns"

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

Annoying and a waste of time to solve for quadratically

Use this instead!

Bob Brown Master Eq

$$\boxed{V_A - V_B = V_B' - V_A'}$$

Only true in
- Totally Elastic Collisions
- 1-D Equations

Must be true!!!

Center of Mass Reference Frame

$$V_{cm} = \frac{m_A V_A + m_B v_B}{m_A + m_B} = \dots$$

Constant!

can be calculated if m_A, m_B, v_A, and v_B given such as that above!

Choose coordinate System that moves with Center of mass (CoM) of System!

Relative to CoM Reference Frame V_{cm} is 0 → $V_{cm} = 0$

Example: $m_A = m_B = m$

$V_A = V_0 \quad V_B = 0$

① $m v_0 = m v_A' + m v_B'$

② $v_0 = v_B' - v_A'$

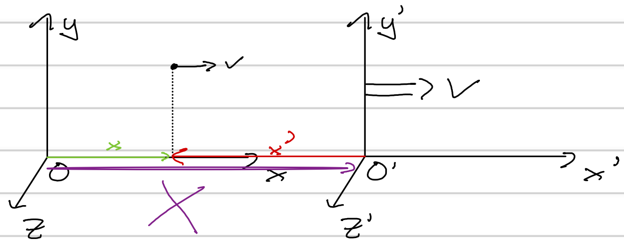
$v_B' = v_0 + v_A'$ — into ①

$m v_0 = m v_A' + m(v_0 + v_A')$

$2 v_A' = 0 \Rightarrow v_A' = 0$

② → $v_B' = v_0$

Reference Frames



$x' = x - X$

$x' = x - (X_0 + Vt)$

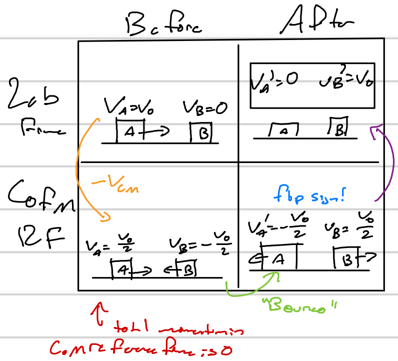
$\boxed{v' = v - V}$

- Lab = stdy still
- Center-of-mass Ref. Frame

$V_{CoM\,RF} = V_{Lab} - V_{cm}$

$\therefore X = X_0 + Vt$ constant velocity

Couault's 4-box method



$V_{cm} = \frac{m_A v_A + m_B v_B}{m_A + m_B}$

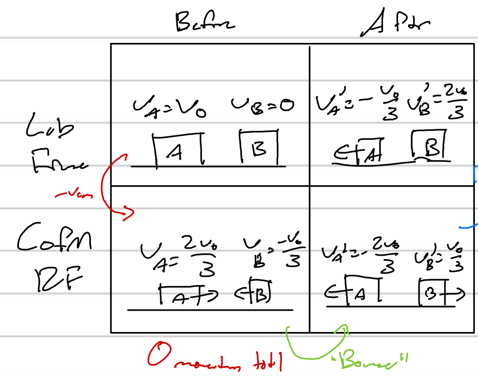
$= \frac{m_A V_0}{m_A + m_B}$ $m_A = m_B = m$

$V_{cm} = \frac{m v_0}{m+m} = \frac{v_0}{2}$

$\boxed{v_A' = 0 \quad v_B' = v_0}$

Example: $m_A = m$ $m_B = 2m$

$v_A = v_0$ $v_B = 0$

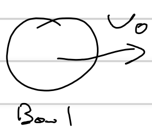


$V_{cm} = \frac{m v_0 + 0}{m + 2m}$

$V_{cm} = \frac{v_0}{3}$

$\boxed{v_A' = -\frac{v_0}{3} \quad v_B' = \frac{2v_0}{3}}$

Example: Totally elastic with no things given



Estimate V_{cm}!

Let Bowl mass $= m_B$

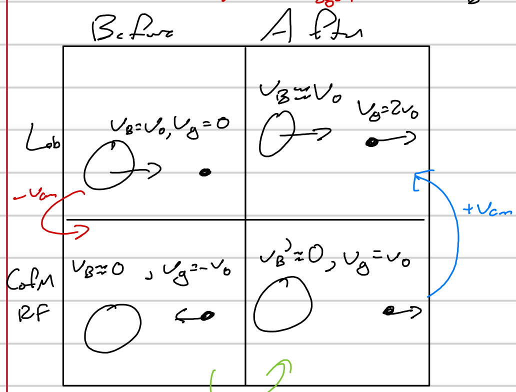
Let Golf mass $= m_G$

Much Bigger! $\rightarrow m_B >> m_G$

$v_{cm} = \frac{m_B v_0 + 0}{m_B + m_G}$ ← assume negligible terms

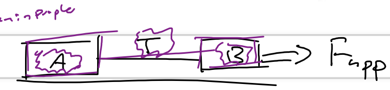
$\approx \frac{m_B v_0}{m_B + 0} = v_0$

If one mass much greater assume it dominates V_{cm}



Example: Given $m_A = m_B = m$ and F_{app} ← F_{app} always given in 121 — result of using rope

System in purple



Find Tension

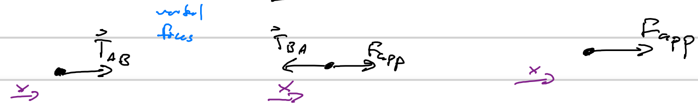
Kinematic Constant accel? $a_A = a_B = a$

Shortcut!

FBD A — Ignore vertical forces

FBD B

FBD System



N2L x-dir System (A+B)

$F_{x\,sys} = m_{sys} a_{sys}$

$F_{app} = 2m\, a$

$\therefore a = \frac{F_{app}}{2m}$

N2L A x-dir

$F_x = m_A a_A$ ← Calculated already

$T_A = m\left(\frac{F_{app}}{2m}\right)$

$\therefore \boxed{T = \frac{F_{app}}{m}}$

Example: n number of blocks

No friction

Given: m, N, F_{app}. Find T_i

$a_1 = a_2 = a = a_{sys} = ?$

FBD System ← all blocks

$\longrightarrow F_{app}$

N2L x-dir Sys

$F_{x_{sys}} = M_{sys} a_{sys}$

$F_{app} = (Nm)a$

$a = \frac{F_{app}}{Nm}$

N2L new System

$F_{sys} = M_{sys} a_{sys}$

$T_i = (im)\frac{F_{app}}{Nm}$

$\therefore \boxed{T_i = \frac{i}{N} F_{app}}$

To find T_i, define a system where the boundary of the system is at the point where that force is applied

Suppose $i = 3$

New system = leftmost i blocks

FBD new system

$\longrightarrow T_i$ ← only force on FBD is the one we want!

Example: Massive Rope

Massive Rope, length ℓ, mass m, F_{app} given

F_{app}

Friction less

T(x): Tension in rope a distance x from left end

FBD Rope

$\longrightarrow F_{app}$

N2L Rope System

$F_{x_{sys}} = m_{sys} a$

$F_{app} = m a$

$a = \frac{F_{app}}{m}$

New System T(x) applied @ left side up to x

FBD

$\longrightarrow T(x)$

N2L new system

$F_{x_{sys}} = m_{sys} a$

$T(x) = (\frac{x}{\ell} m) a$

$T(x) = (\frac{x}{\ell} m)(\frac{F_{app}}{m})$ ← ratio of rope times mass

$\therefore \boxed{T(x) = \left(\frac{x}{\ell}\right) F_{app}}$

Example: Massive rope pulls block

ℓ — Frictionless — M — F_{app}

Block M mass

Rope m mass

F_{app}, ℓ

Find T(x)

N2L whole

$F_{sys} = M_{sys} a$

$F_{app} = (M + m) a_{sys}$

$a_{sys} = \frac{F_{app}}{M + m}$

New System = Boundary @ x

FBD

$\longrightarrow T(x)$

N2L new system

$F_{new_{sys}} = M_{sys} a_{sys}$ ← $M + \frac{x}{\ell} m$

$T(x) = \left(\left(\frac{x}{\ell}\right) m + M\right)\left(\frac{F_{app}}{m + M}\right)$

$T(x) = \left(\frac{xm}{\ell} + M\right)\frac{F_{app}}{m + M}$

simplify

$\therefore \boxed{T(x) = \left(\frac{M + \frac{x}{\ell} m}{m + M}\right) F_{app}}$

Example: Fun

moves up @ constant velocity

m_s, m_p, m_b all given

What is Tension in rope?

System defined this way to include everything

4 contact points

FBD System

$4\vec{T}$

W_{sys}

N2L System

$F_{sys} = m_{sys} a_{sys}$ ← 0 constant

$4\vec{T} - M_{sys} g = 0$

$\vec{T} = \frac{M_{sys} g}{4}$

$\boxed{\vec{T} = \frac{(m_s + m_p + m_b) g}{4}}$

Rotations!

Translational Motion	Rotational Motion
Kinematics: x, v, a	Kinematics: θ, ω, α
Dynamics: $F_{Net} = ma$	Dynamics: $\tau_{Net} = I\alpha$
N2L	N2L Rot

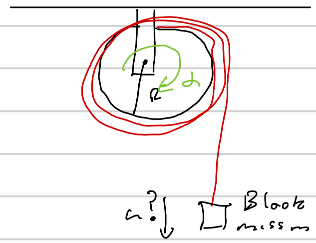
} Review of Motion Components

New Force

$\vec{H}$ = "Hinge Force"

axles, gears, pin

Example: Massive Pulley



Find accel of block

Given
- Radius of Pulley
- Block of Mass m
- Pulley Mass is 4m
  - ↳ Pulley Mass = m_p
- $I_p = \frac{1}{2} m_p R^2$
  - ↳ given for now!

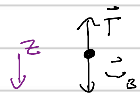
$a = ?$ $\alpha = ?$

Always assume string/rope is not slipping or sliding on the Pulley

Pulley has $a=0$ so a Force must be points upward to hold it in place

Hinges/Axels apply force in any direction needed to satisfy N2L

FBD Block



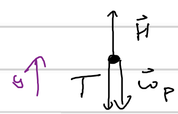
N2L Block

$F_{Bz} = m_B a_{Bz}$

$w_B - T = ma$

$\boxed{mg - T = ma}$ Stop!

FBD Pulley



N2L Pulley

$F_{Py} = m_P \cancel{a_{Py}}$ 0

$H - T - w_P = 0$

$\boxed{H - 4mg - T = 0}$ Stop!

Two equations w/ Too many unknowns

① $mg - T = ma$

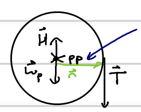
② $H - 4mg - T = 0$

Always with a to b

Solve problem w/o rotational motion

Extended FBD (XFBD) Pulley ← No longer a point-like object

- Weight Force always @ CoM
- Coordinate System is lined up w/ rotation



PP for pivot point

(+)

N2L Rot Pulley

$\tau_{Net} = I_p \alpha$

$\tau_w + \tau_H + \tau_T = (\frac{1}{2} m_p R^2)\alpha$

$0 + 0 + RT\sin 90 = \frac{1}{2} m_p R^2 \alpha$

0 distance from PP for w and H so $\tau = 0$

$\therefore RT = \frac{1}{2}(4m)R^2\alpha$

Pulley $\alpha = \frac{a}{R}$ Rolling Constraint

R.C is acceptable

$RT = 2mR^2(\frac{a}{R})$

$\therefore T = 2ma$

Recall: $mg - 2ma = ma$

$mg = 2ma + ma$

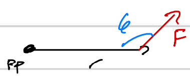
$g = a(2+1)$

$\boxed{a = \frac{1}{3}g}$

Very Helpful for HW6 #6

Definition of Torque

$\tau_F \equiv rF\sin\phi$



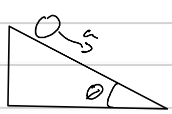
Important Equation!

Rolling Constant

$a = \alpha r$, $v = \omega r$

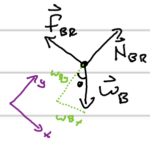
Tangential acceleration, rotational, radius from PP

Example: Ball on Ramp



Given
- $\theta, R, m, I = \frac{2}{5}mR^2$
- Rolls w/ no slip or slide

FBD ball

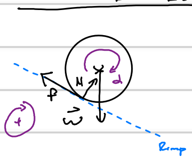


N2L x-coord

$F_{Bx} = m_B a_{Bx}$

$\boxed{mg\sin\theta - f = ma}$ Stop!

XFBD ball



N2L Rot ball

$\tau_{net} = I\alpha$

$\tau_w + \tau_N + \tau_f = (\frac{2}{5}mR^2)\alpha$

$0 + \cancel{RN(\sin 180°)} + Rf = \frac{2}{5}mR^2\alpha$

$\alpha = \frac{a}{R}$ R.C

$Rf = \frac{2}{5}mR^2\frac{a}{R}$

$f = \frac{2}{5}ma$

↳ $mg\sin\theta - f = ma$

$\cancel{m}g\sin\theta - \frac{2}{5}\cancel{m}a = \cancel{m}a$

$g\sin\theta = \frac{7}{5}a$

$\boxed{a = \frac{5}{7}g\sin\theta}$

Review Lecture

Topics

2D Kinematics

Vectors

- $\vec{A}$: $|\vec{A}|$ is magnitude $= A$
- Notation: $\vec{A} = A_x \hat{i} + A_y \hat{j}$
- Projectile Motion, UCM
- Centripetal vs. Tangential Acceleration
  - $\hookrightarrow \vec{a} = \vec{a}_x + \vec{a}_y$
  - $= \vec{a}_v + \vec{a}_c$

$a_v = \frac{d}{dt}(\text{speed})$

$a_c = \frac{v^2}{R_c}$ for R_c - radius of curvature

Conservative Forces

Know Formula

All Topics Outlined on the Cheat Sheets

Dynamics

Forces at Angles

- Tipped Coordinate System in direction of Acceleration

New Forces

Both Conservative

- Universal Grav. $F_{UG} = \frac{GMm}{R^2}$ → RWB's magic Equation
- Spring Forces $F_{sp} = k|x|$ → 4 - box Method
  - x is displacement

Systems of Multiple Bodies

- Massive Rope
- Massive Pulley ⟸ Rotational Motion

Systems

N2L

$F_{Net,\,sys,\,ext} = M_{sys} a_{cm}$

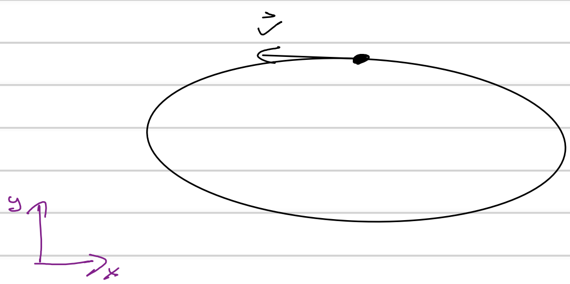
CoM apply and calculate

Elastic Collisions

$K_{Tot} = K'_{Tot}$

- CoM Frame vs. Lab Frame

First Exam Topics all Fair Game



$\vec{v} = -v\hat{i}$

speed → slows

$\frac{d(\text{speed})}{dt} = \text{negative}$

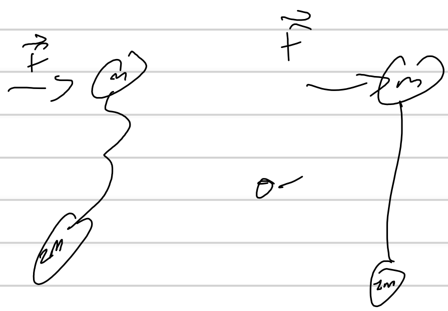
$\vec{a}_v = +P\hat{i}$

$\vec{a}_c = -Q\hat{j}$

$\boxed{P\hat{i} - Q\hat{j}}$

B

$\text{Impulse} \equiv \int F\,dt \equiv \Delta p = F_{avg} \cdot \Delta t$

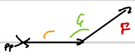


Cycle 3

More on Rotations

Summary of Important Collisions

Torque: $\tau_P = rF\sin\theta$



N2L Rot: $\tau_{Net} = I\alpha$

R.C: $v = \omega r$ and $a = \alpha r$

Values for Rotational Inertia (I)

Textbook/Labs may call it moment of Inertia

$I = C m R^2$, for some const C

Common Values for I:

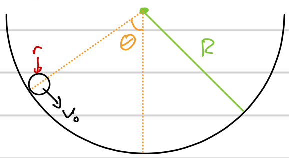
$I_{Hoop} = mR^2$ $I_{Hollow\ Sphere} = \frac{2}{3} mR^2$

$I_{Disk} = \frac{1}{2} mR^2$ $I_{Rod\ CM} = \frac{1}{12} m\ell^2$

$I_{Solid\ Sphere} = \frac{2}{5} mR^2$ $I_{Rod\ End} = \frac{1}{3} m\ell^2$

Good to have on Cheatsheet to reference easily

Example: Hollow ball rolling in bowl



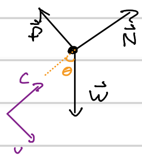
Given

- m, r, v_0, θ of ball, no slip/slide
- R of Bowl
- $I = \frac{2}{3} m r^2$

Find Normal Force, Friction force, a

Assume $R \gg r$

FBD Ball



N2L c-coord centripetal

$F_c = m a_c$

$N - W\cos\theta = m\frac{v_0^2}{R}$ (radius of bowl as motion defined by the bowl)

$N = mg\cos\theta + \frac{m v_0^2}{R}$

$\boxed{N = m\left(g\cos\theta + \frac{v_0^2}{R}\right)}$

N2L v-coord tangential

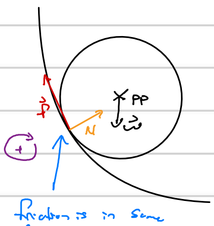
$F_v = m a_v$

$W\sin\theta - f = m a_v$

Stop!

Now Contd with Rotational

xFBD Ball



friction is in same direction as coordinate sys so torque is positive

N2L Rot

$\tau_{Net} = I\alpha$

$\tau_W + \tau_N + \tau_P = I\alpha$

$0 + 0 + rf\sin(90°) = I\alpha$

$\cancel{r} f = \frac{2}{3} m r^{\cancel{2}}\alpha$ $\alpha = \frac{a_v}{r}$ R.C

$f = \frac{2}{3} m r\left(\frac{a_v}{r}\right)$

Stop! $f = \frac{2}{3} m a_v$

① $\tau_W: 0W\sin\theta = 0$

② $\tau_N: rN\sin(180°) = 0$

Checking G in torque

force is positive for torque if it is aligned with coordinate system.

Two Equations, Two Unknowns

① $mg\sin\theta - f = m a_v$

② $f = \frac{2}{3} m a_v$

$\cancel{m} g\sin\theta - \frac{2}{3}\cancel{m} a_v = \cancel{m} a_v$

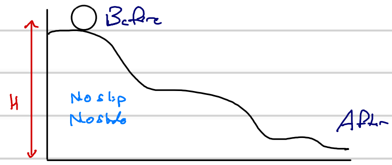
$g\sin\theta = \frac{5}{3} a_v$

$\boxed{a_v = \frac{3}{5} g\sin\theta}$

$f = \frac{2}{3} m\left(\frac{3}{5} g\sin\theta\right)$

$\boxed{f = \frac{2}{5} mg\sin\theta}$

Example: Ball rolling on slide



Given: Release @ rest

$m, R, I = \frac{2}{5} mR^2$ of ball

H of ball initially

Translational Kinetic: $K_T = \frac{1}{2} m v^2$

Rotational Kinetic: $K_R = \frac{1}{2} I\omega^2$

$\therefore K_{Tot} = K_T + K_R = \frac{1}{2} m v^2 + \frac{1}{2} I\omega^2$

Friction does "no work" @ point of contact

$E_{Tot} = E'_{Tot}$

$U + K_T + K_R = U' + K'_T + K'_R$

$mgy + \frac{1}{2} m v^2 + \frac{1}{2} I\omega^2 = mgy' + \frac{1}{2} v'^2 + \frac{1}{2} I\omega'^2$

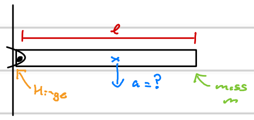
$y = H,\ y' = 0,\ v = 0,\ \omega = 0,\ \omega = \frac{v}{r}$ R.C

$mgH + 0 + 0 = 0 + \frac{1}{2} m v_P^2 + \frac{1}{2}\left(\frac{2}{5} m \cancel{R^2}\right)\left(\frac{v_T}{\cancel{r}}\right)^2$

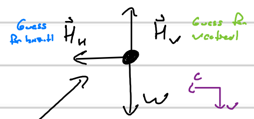
$\cancel{m} gH = \frac{1}{2}\cancel{m} v_P^2 + \frac{1}{5}\cancel{m} v_T^2$ $v_T = v_P$

$gH = \frac{7}{10} v_P^2 \rightarrow \boxed{v_P = \sqrt{\frac{10}{7} gH}}$

Rod on Hinge



FBD Rod



Hinge Force is completely unknown! Just assign two arbitrary forces to represent it. One is vertical one is horizontal

① Find Acceleration of CoM @ release. Rod starts @ rest!

Not worth spending a lot of time determining direction of H components

If you misguess direction, answer is negated. Just make note of it and move on to get points back.

CoM is moving on circular path so use v/c-coordinates.

N2L Horizontal c-coord

$F_c = m a_c$

$H_H = m \frac{v^2}{"R"}$ @ rest so $v = 0$

$\therefore H_H = 0$ woah!

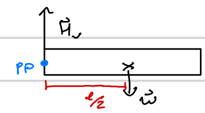
N2L Vertical v-coord

$F_v = m a_v$

$W - H_v = m a_v$

$mg - H_v = m a_v$ Stop!

x FBD Rod



(+) Neglect H_H bc it is 0 from N2L

N2L Rot

$\tau_{Net} = I\alpha$ Known for Rod

$\tau_{H_v} + \tau_W = \frac{1}{3} m \ell^2 \alpha$

$0 + \frac{\ell}{2} mg(\sin 90°) = \frac{1}{3} m \ell^2 \alpha$

$\frac{1}{2} \cancel{\ell} \cancel{m} g = \frac{1}{3} \cancel{m} \ell^{\cancel{2}} \left(\frac{a_v}{\cancel{\ell}/2}\right)$

$\frac{1}{2} g = \frac{1}{3} 2 a_v$

$\boxed{a_v = \frac{3}{4} g}$

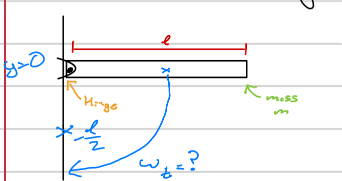
R.C. $a = \alpha r$

$\alpha = \frac{a}{r}$

r is radial distance from pp to CoM here!

$r = \frac{\ell}{2}$

Another Rod on Hinge



C of ME Choose End of Rod as Reference, no translational K!

Conditions: W nonconservative and Hinge does no work @ point of contact!

Before = After

$E_{Tot} = E'_{Tot}$

$U + K_T + K_R = U' + K'_T + K'_R$

$U + 0 + K_{Rod} = U' + 0 + K'_{Rod}$

$mgy + 0 + \frac{1}{2} I \omega^2 = mgy' + 0 + \frac{1}{2} I \omega'^2$

$0 + 0 + 0 = mg\left(-\frac{\ell}{2}\right) + \frac{1}{2}\left(\frac{1}{3} m \ell^2\right) \omega_f^2$

Algebra!

$\frac{1}{2} \cancel{m} g \cancel{\ell} = \frac{1}{6} \cancel{m} \ell^{\cancel{2}} \omega_f^2$

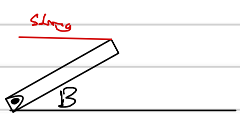
$\boxed{\omega_f = \sqrt{\frac{3g}{\ell}}}$

If you choose a different Reference Point, It's different!

Hinge Force does no work bc its @ point of contact. No Force applied over a distance

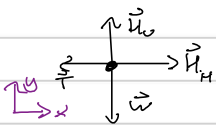
You will still get the right answer if you use a different reference point!

Hinge with a String



$a = 0, \alpha = 0$

FBD Rod



At rest! so H_x and the must hold rod in place

N2L y-dir

$F_y = m \cancel{a_y}$ 0

$H_v - W = 0$

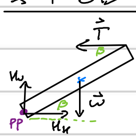
$\boxed{H_v = mg}$

N2L x-dir

$F_x = m \cancel{a_x}$ 0

$H_H - T = 0$ Stop!

x FBD Rod



$\tau_H = 0$

$\tau_T = \ell T \sin\beta$

$\tau_W = \left(\frac{\ell}{2}\right) mg \sin(90 - \beta)$

$\tau_W = \frac{\ell}{2} mg \cos\beta$ Trig!

Statics

Static Friction

$f_s \le \mu_s N$

Constrained

Kinetic Friction

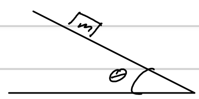
$f_k = \mu_k N$

Equality

For Statics, $a = 0$ and $\alpha = 0$

Use N2L to find N → N2L to find f_s → Apply constraint

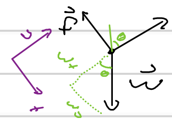
Example



Given: m, θ, μ_s

Find θ_{max}

FBD Block



N2L y-dir

$F_y = m a_y^{\nearrow 0}$

$N - W\cos\theta = 0$

$N = mg\cos\theta$

N2L x-dir

$F_x = m a_x^{\nearrow 0}$

$W\sin\theta - f_s = 0$

$f_s = mg\sin\theta$

Example shows $f_s \le \mu_s N$ is not useless! Good for determining maximums!

Apply Constraint: $f_s \le \mu_s N$

No an Equal Sign!

$mg\sin\theta \le \mu_s mg\cos\theta$

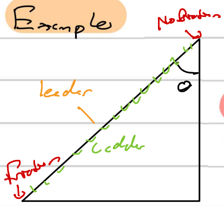
$\frac{\sin\theta}{\cos\theta} \le \mu_s$

$\tan\theta \le \mu_s$

$\theta \le \arctan\mu_s$

$\therefore \boxed{\theta_{max} = \arctan(\mu_s)}$

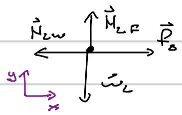
Example



Given: m, θ, L

Find all Forces

FBD Ladder



N2L y-dir

$F_y = m a_y^{\nearrow 0}$

$N_{LF} - W = 0$

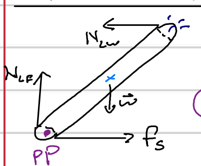
$\boxed{N_{LF} = mg \quad W = mg}$ (1) (2)

N2L x-dir

$F_x = m a_x^{\nearrow 0}$

$f_s - N_{LW} = 0$ Stop!

FBD Ladder



Place PP where it kills most forces

N2L Rot

$\tau_{Net} = I\alpha^{\nearrow 0}$

$\tau_{N_{LF}} + \tau_{f_s} + \tau_w + \tau_{N_{LW}} = 0$

$0 + 0 - \frac{L}{2} W\sin\theta + L N_{LW}\sin(90-\theta) = 0$

$-\frac{1}{2}\cancel{L} mg\sin\theta + \cancel{L} N_{LW}\cos\theta = 0$

$N_{LW}\cos\theta = \frac{1}{2} mg\sin\theta$

$\boxed{N_{LW} = \frac{1}{2} mg\tan\theta}$ (3)

Recall: $\sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

$f_s - N_{LW} = 0$

$f_s = N_{LW}$

$\boxed{f_s = \frac{1}{2} mg\tan\theta}$ (4)

Center of Mass and Matter Distribution

CoM:

$\vec{r} \equiv \frac{m_1 r_1 + m_2 r_2 + \dots m_n r_n}{M}$ $\quad x_{cm} \equiv \frac{m_1 x_1 + m_2 x_2 + \dots m_n x_n}{M}$ $\quad a_{cm} \equiv \frac{d}{dt}(\vec{v}_{cm}) = \frac{m_1 a_1 + m_2 a_2 + \dots m_n a_n}{M}$

$M \equiv m_1 + m_2 + \dots m_n$ $\quad v_{cm} \equiv \frac{d}{dt}(\vec{x}_{cm}) = \frac{m_1 v_1 + m_2 v_2 \dots m_n v_n}{M}$

Good definitions to have in your back pocket

Continuous Distribution of Matter

$$\boxed{\begin{aligned}\vec{r}_{cm} &\equiv \frac{1}{M}\sum_{i=1}^{n} m_i r_i \\ \vec{r}_{cm} &\equiv \frac{1}{M}\int_{volume} r\, dm\end{aligned}}$$

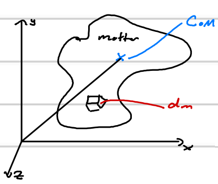
where $M = \sum_{i=1}^{n} m_i$

Uniform Density

$\rho = \frac{m}{V}$

Not Uniform Density

$\rho = \frac{dm}{dV} \rightarrow dm = \rho\, dV$



$x_{cm} = \frac{1}{M}\int_{volume} x\, dm$

$= \frac{1}{M}\int x\rho\, dV$

$= \frac{1}{M}\iiint x\rho\, dx\, dy\, dz$

Triple Integral DN

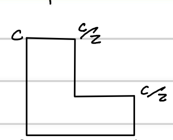
Calculus 3 CoM

$\vec{x}_{cm} \equiv \frac{1}{M}\iiint x\rho\, dx\, dy\, dz$

$\vec{y}_{cm} \equiv \frac{1}{M}\iiint y\rho\, dx\, dy\, dz$

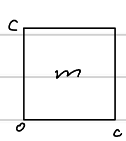
$\vec{z}_{cm} \equiv \frac{1}{M}\iiint z\rho\, dx\, dy\, dz$

Example: A cut-out Square from a Square



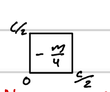
Find x_{cm}?

=



Original Shape

+



Negative Area of the square

$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m(\frac{c}{2}) + (-\frac{m}{4})(\frac{c}{4})}{m + (-\frac{m}{4})}$

$= \frac{\frac{mc}{2} - \frac{mc}{16}}{\frac{3m}{4}} = \frac{2\cancel{m}c - \frac{\cancel{m}c}{4}}{3\cancel{m}} = \frac{\frac{7}{4}c}{3} = \boxed{\frac{7c}{12}}$

Recall: $\vec{F}_{\substack{Net\\Sys\\Ext}} = m\vec{a}_{cm}$

Special Cases

① If $\vec{F}_{\substack{Net\\Sys\\Ext}} = 0$ means $\vec{a}_{cm} = 0$, $\vec{v}_{cm} = $ constant

Individual forces don't need to be 0, just need to cancel

then $\vec{P}_{Tot} = m\vec{v}_{cm} = $ constant CofLM

This means system is isolated

② If $\vec{v}_{cm} = 0$ means $P_{Tot} = 0$

$V_{cm} = 0 \rightarrow$ CoM is not changing!

Then $\vec{r}_{cm} = $ constant in CoMRF

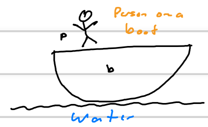
Recall: Elastic $\rightarrow K = K'$

Moving to CoMRF (center of mass reference frame)

$v' = v - v_{cm}$

velocity in CoMRF; CoMRF in "Lab" frame; CoM in "Lab" frame

Used in Ubox Method!



$x_{cm} = \dfrac{m_p x_p + m_b x_b}{m_p + m_b}$

x_{cm} is constant, so if you move one way, the boat will move the other way to keep x_{cm} constant

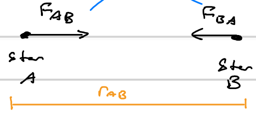
Universal Gravity Revisited

$F_{UG} = \dfrac{GMm}{r^2}$

"attractive"

distance of separation

Example

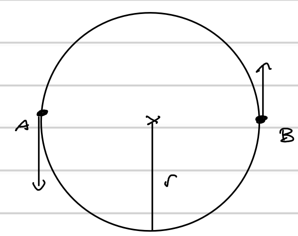


$F_{AB} = F_{BA} = F_{UG} = \dfrac{GM_AM_B}{r_{AB}^2}$

$a_A = a_B = a$ is true!

$a = \dfrac{v^2}{r}$

Find v_A, v_B



N2L

Let $m_A = m_B = m$

$F_{Net} = ma$

$\dfrac{GMm}{r^2} = m\dfrac{v_a^2}{r}$ ← radius; r^2 ← distance of separation

$\dfrac{GM^2}{(2r)^2} = m\dfrac{v^2}{r}$

$\dfrac{GM^2}{4r^2} = \dfrac{mv^2}{r}$

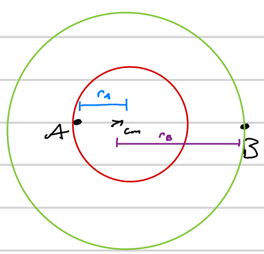
$\dfrac{GM}{4r} = v^2$

$\boxed{v = \sqrt{\dfrac{GM}{4r}}}$

Both stars move w/ the same velocity

$v_A = v_B = v$

Example: $m_A > m_B$



Find $v_A, v_B, \omega_A, \omega_B$

$x_{cm} = \dfrac{m_Ax_A + m_Bx_B}{m_A + m_B} = 0$

$= \dfrac{m_A(r_A) + m_B(r_B)}{m_A + m_B} = 0$

$m_Ar_A + m_Br_B = 0$

$\dfrac{r_A}{r_B} = \dfrac{m_B}{m_A}$

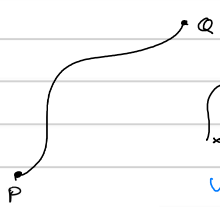
N2L $F_{Net} = ma$

$\dfrac{GM_AM_B}{(r_A + r_B)^2} = m_A\dfrac{v_A^2}{r_A}$ (radius; Distance between; Replace A with B for star B)

$\dfrac{GM_Br_A}{(r_A+r_B)^2} = v_A^2 \rightarrow \boxed{v_A = \sqrt{\dfrac{GM_Br_A}{(r_A+r_B)^2}}}$

R.C $v = \omega r \rightarrow \omega = \dfrac{v}{r}$

Path Integrals and Conservative Forces



Work Definition

$W_P = \oint \vec{F}\cdot d\vec{r}$

$\int_{x_P}^{x_Q} F_x dx + \int_{y_P}^{y_Q} F_y dy + \int_{z_P}^{z_Q} F_z dz$

Work is Conservative Force if not

Conservative Forces

① Force is conservative if work done by that force from P to Q is independent of path taken

② The work done by that force around a closed loop is zero

Example

Given $F = Axy\,\hat{i} + Bz\,\hat{j}$, is conservative?

$\int_a^b Axy\,dx + \int_a^b Bz\,dy$ $W_f \neq 0$ so not conservative

$\frac{1}{2}Ax^2y\Big|_a^b + Bzy\Big|_a^b$

Example

$F = \int Ax\,\hat{i} + \int By^3\,\hat{i} + \int Ce^{-bz}\,\hat{k}$

$\dfrac{x^2}{2}\Big|$ $\dfrac{y^4}{4}\Big|$ $-be^{-bz}\Big|$

Bridge Concepts

Newton's Laws: 1st, N2L, N3L

Conservative Laws: CofME, CofLM, CofAM

Classical Work Energy Theorem

$W_T = W_{F_1} + W_{F_2} + W_3 \ldots = \Delta K$

$W_T = \int F_{Net}\,dr = \Delta K$

Momentum Impulse Relation

$F_{Net} = \dfrac{dP}{dt}$

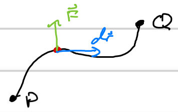
$\int F_{Net}\,dt = \Delta p$

Spring Force

$F_{sp} = Kx$ $\int Kx\,dx$

$\rightarrow \frac{1}{2}Kx^2$

$W_{\vec{F}}(P \to Q) = -\int_{path} \vec{F} \cdot d\vec{r}$



$\vec{F} = F_x \hat{i} + F_y \hat{j}$

$d\vec{r} = (dx)\hat{i} + (dy)\hat{j}$

Add z components if needed

$W_{\vec{F}}(P \to Q) = -\int_{x_P}^{x_Q} F_x dx + -\int_{y_P}^{y_Q} F_y dy$

Recall Rotational Inertia

$I_{hoop} = mR^2$

$I_{Disk} = \frac{1}{2} mR^2$

$I_{Solid\ Sphere} = \frac{2}{5} mR^2$

$I_{Hollow\ Sphere} = \frac{2}{3} mR^2$

$I_{Rod@\ Cent} = \frac{1}{12} ml^2$

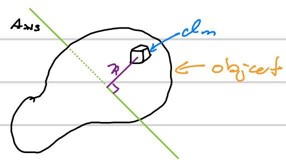
$I_{Rod@\ End} = \frac{1}{3} ml^2$

Definition of Rotational Inertia

$I \equiv \iiint R^2 dm$

R - distance from mass element to axis of rotation (Perp. to dist!)

dm - mass element



$\rightarrow \quad I = \int_x \int_y \int_z R^2 \rho \, dz\, dy\, dx$

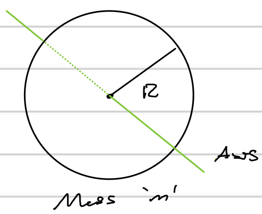
Contd with dm

dm $\therefore dm = \rho dV$

mass density: $\rho = \frac{M}{V}$ ← mass, ← volume

e.g. $dV = dx\,dy\,dz$

Example: A hoop



$I = \iiint R^2 dm$

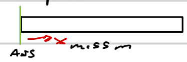
Here, $R = R$

$\therefore I_{hoop} = R^2 \underbrace{\iiint dm}$

$\rightarrow \boxed{I_{hoop} = mR^2}$

mass integrated over all space

Example: Rod about end



$I_{end\ rod} = \iiint R^2 dm$

Here $R = x$

$dm = \rho dV$

$V_{rod} = lA$ $\quad A =$ Cross Sectional area of rod

$dV = A dx$

$\rho = \frac{m}{lA}$ def of density

$dm = \left(\frac{m}{lA}\right)(A dx) = \frac{m}{l} dx$

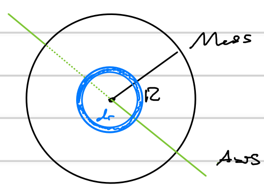
$I_{end\ rod} = \int_{x=0}^{x=l} x^2 \frac{m}{l} dx$

$= \frac{m}{l} \int_0^l x^2 dx$

$= \frac{1}{3}\left(\frac{m}{l}\right)(x^3)\Big|_0^l$

$\boxed{I_{end\ Rod} = \frac{1}{3} ml^2}$

Example: A disk



Here $R = r$

$dm = \rho dV$

$V_{disk} = A_{disk} T = \pi r^2 T$ $\quad$ Thickness T

$dV = dA_{annulus} T = 2\pi r (dr) T$

$\rho = \frac{m}{V} = \frac{m}{\pi r^2 T}$

$\boxed{I_{disk} = \frac{1}{2} mR^2}$

Use integration and Inertia def to solve

Angular Momentum

Angular Momentum

$L \equiv I\omega$

Conservation of Angular Momentum

Condition: "Isolated" "CoAM"

$\rightarrow \tau_{Net\ Sys\ Ext} = 0$

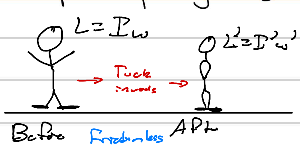
Check Torques! Not Forces

Individual Forces can be $\neq 0$ but must Sum to 0

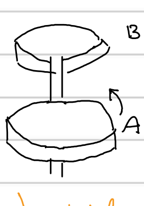
Then: Before = After

$L_{Tot} = L'_{Tot}$

Example: Spinning Figure Skater



Example: Disk Dropped on a turn table



Isolated

Given

m_A, m_B, R_A, R_B

$\omega_A = \omega_0 \quad \omega_B = 0$

Friction must be there so CoAMB no!

Find $\omega'_A = \omega_A = \omega_B$ after disk gets dropped

Before = After

$L_{Tot} = L'_{Tot}$

$L_A + L_B = L'_A + L'_B$

$I_A \omega_A + I_B \omega_B = I_A \omega'_A + I_B \omega'_B$

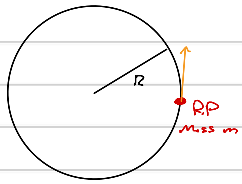
$I_A \omega_A + 0 = I_A \omega_f + I_B \omega_f$

$\omega_f = \frac{I_A \omega_A}{I_A + I_B} = \frac{\frac{1}{2} m_B R_B^2}{\frac{1}{2} m_B R_B^2 + \frac{1}{2} m_A R_A^2} \omega_A$

$\boxed{\omega_f = \frac{\omega_A m_B R_B^2}{m_B R_B^2 + m_A R_A^2}}$

Angular Momentum of a point Particle

P.P. UCM



Recall: $I = \int_{\text{All space}} R^2 dm$

$= mR^2$ for P.P. UCM

Particle in Circular Motion

$I_{PP} = mR^2$

$L_{UCM} = I\omega = mR^2\omega$

$= mvR$

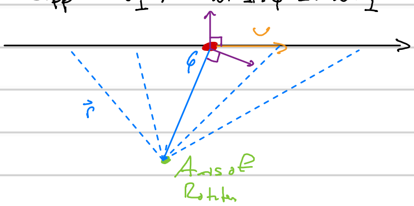
R.C $v = \omega r$

Particle in Circular Motion

$L_{UCM} = mvR$

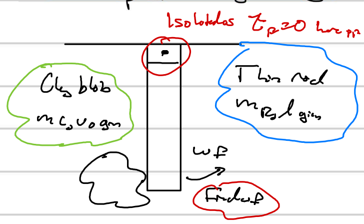
Example: P.P. moving on straight line

$L_{PP} = mv_\perp r = mvr\sin\phi = mvr_\perp$



$r_\perp = b$ is "impact parameter"

Example: Clay Blob



Before = After

$L_{Tot} = L'_{Tot}$

$L_c + L_R = L'_c + L'_R$ L_{UCM}

$m_c v_o l \sin(90)° + 0 = m_c l^2 \omega_f + \frac{1}{3} m_R l^2 \omega_f$

$m_c v_o l = \omega_f (m_c l^2 + \frac{1}{3} m_R l^2)$

$$\boxed{\omega_f = \frac{m_c v_o l}{m_c l^2 + \frac{1}{3} m_R l^2}}$$

Angular Momentum for a Rigid body

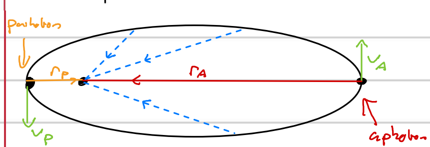
$L_{RB} = I\omega$

Recall: Point Particle

UCM: $L_{PP,\,UCM} = mvR = mR^2\omega$

Arbitrary Path: $L_{PP} = mvr\sin\phi = mv_\perp r = mvr_\perp$

Example: A Comet



$\tau_{UG} = rF_{UG}\sin\phi$ radial lines 180° to Force direction

$\therefore \tau_{UG} = 0$

Assume r_P, r_A, and v_P given

$$\boxed{v_A = \left(\frac{r_P}{r_A}\right) v_P}$$

C of AM

$\vec{r} \parallel \vec{F}_{UG} \rightarrow \tau = 0$

$\rightarrow L$ is conserved

$\therefore$ Before = After

$L_{Tot} = L'_{Tot}$

$L_P = L_A$

$mv_P r_P \sin 90° = mv_A r_A \sin 90°$

$mv_P r_P = mv_A r_A$

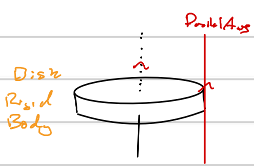
$v_P r_P = v_A r_A$

Parallel Axis Theorem

$I_{PB} = I_{cm} + mD^2$

where D = distance from CM to axis of rot.

Disk edge where $D = R$



$I_{\text{Disk Edge}} = I_{cm} + mD^2$

$= \frac{1}{2}mR^2 + mR^2$

$$\boxed{I_{\text{Disk Edge}} = \frac{3}{2}mR^2}$$

Review Lecture

Topics — Review sheets on the content as well as HW 7, 8, 9

Systems

NII: $\vec{F}_{Net,\ Sys,\ Ext} = M_{sys}\vec{a}_{cm}$

If $F_{Net,\ Sys,\ Ext} = 0$ → "isolated"

Then $a_{cm} = 0$ → v_{cm} = constant

Also C of LM $P_{Tot} = P'_{Tot}$

Work ⇒ Path integral

$W_F = \int_{Path} \vec{F}\cdot d\vec{r}$

$\vec{F} = F_x\hat{i} + F_y\hat{j}$

$\int \vec{F}\cdot d\vec{r} = \int \vec{F}(\vec{r}(t))\cdot \vec{r}\,'(t)\,dt$

Conservative Forces:

IFF $\oint_{closed\ loop} \vec{F}\cdot d\vec{r} = 0$

Pot. Energy of cons. force $U_{F_c} = -\text{Work} = -\int_{path} \vec{F}\cdot d\vec{r}$

Potential Energy

$U_w(y) = mgy$

$U_{sp}(x) = \frac{1}{2}kx^2$

$U_G(r) = \frac{-GmM}{r}$

Work–Energy Relation

- W–E relation
- $W_{Tot} = \Delta K$

Rotations

$\sin(90-\beta) = \cos\beta$

$\tau_o = rF\sin\varphi$

NII Rot $\tau_{Net} = I\alpha$

} FBD w/ PP and coord system

R.C.: $v = \omega r$

$a_{Tan} = \alpha r$

Can apply if rolling no slip/slide

Everything else is important but these are the most important!

C of ME w/ Rotations

→ No slip, No slide

$E_{Tot} = E'_{Tot}$

$U + K_{Rot} = U' + K'_{Rot}$

For $K_{Tot} = K_{Trans} + K_{Rot}$

$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

→ Statics: $a = 0$ $\alpha = 0$

→ Massive Pulley

→ Rolling Ball/Hoops

→ Rods on Hinges

Angular Momentum

$L_{RB} = I\omega$

Rigid Body

$L_{PP} = mvr = m\omega R^2$

UCM

$L_{PP} = mvr\sin\varphi$

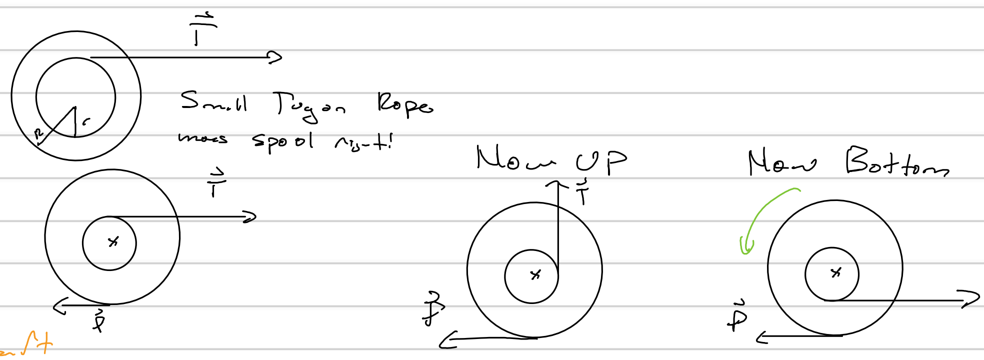
Gen

rot UCM

Clicker topics

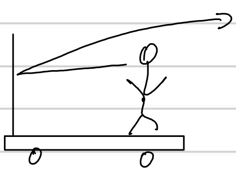
Hoop and disk race down

Solid Disk wins bc $I = \frac{1}{2}mR^2$



Small Tug on Rope — mass spool — Now it!

Now UP — Now Bottom



Define Cart Person ball wall as System

Throw ball at boarder

Frictionless Cart.

Cart moves left to conserve LM

Balls

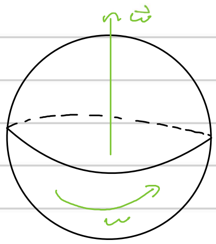
$P_{BB} = P_{PB}$

$m_{BB}v_{BB} = m_{PB}v_{PB}$

Cycle 4

Vectors and Rotational Quantities

Vector Nature of Rotational Quantities



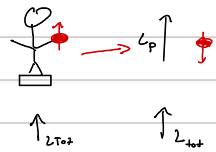
Right-Hand-Rule

Curly Fingered RHR

1. Stick out Right Hand
2. Curl Fingers in direction of rod
3. Thumb direction points in vector direction

Works $\vec{\omega}, \vec{\alpha}, \vec{L}, \vec{\tau}$

Example: Ross w/ Gyro



Recall dot Product

$\vec{A} \cdot \vec{B} = AB\cos\theta$

Formal Definition w/ cross

$\vec{\tau} = \vec{r} \times \vec{F} \rightarrow |\vec{\tau}| = rF\sin\theta$

$\vec{L}_{pp} = \vec{r} \times \vec{p} \rightarrow |L_{pp}| = rmv\sin\theta$

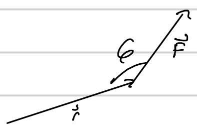
Cross Product

$\vec{C} = \vec{A} \times \vec{B}$

$|\vec{C}| = AB\sin\theta$

$\vec{C} \perp \vec{A}, \vec{C} \perp \vec{B}$

Vector Cross Product Right Hand Rule



1. Point limp Right Hand direction of vector 1
2. Rotate hand so palm is $\vec{F}$ dir
3. Curl Fingers in $\vec{F}$ direction
4. Thumb Points to cross dir

Simple Harmonic Motion (SHM) (simple harmonic oscillator)

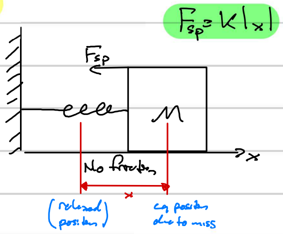
Hooke's Law

F_{sp}

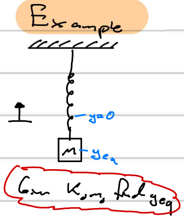
- Known Force
- Contact Force
- Conservative

wall

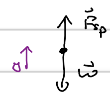
$F_{sp} = k|x|$



Example



FBD block



N2L y-dir

$F_y = ma_y$

$F_{sp} - w = ma_y$ → a=0 @ equilibrium

$F_{sp} - w = 0$

$k|y_{eq}| - mg = 0$

$\rightarrow |y_{eq}| = \frac{mg}{k}$

below zero — here

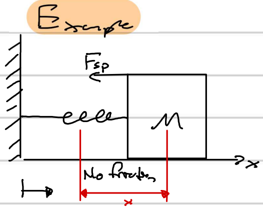
$$\boxed{y_{eq} = \frac{-mg}{k}}$$

very simple problem to solve with FBD/N2L

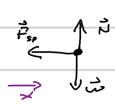
Mechanics

- how? Kinematics: $y(t)$? $x(t)$?
- Dynamics: why?

Example



FBD block



N2L x-dir

$-F_x = ma_x$

$-kx = ma$

$ma + kx = 0$

$a + \frac{k}{m}x = 0$

2nd order DE can solve

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Equation of Motion

$s^2 + \frac{k}{m} = 0$

$s = \pm i\sqrt{\frac{k}{m}}$

Motion of $\frac{d^2y}{dt^2} + \frac{k}{m}x = 0$

$s^2 + \frac{k}{m} = 0$

$s = \pm i\sqrt{\frac{k}{m}}$

Plug $x(t)$ and $x''(t)$ into equation to get 0 because ✓

$x(t) = A\sin\left(\sqrt{\frac{k}{m}}t\right) + B\cos\left(\sqrt{\frac{k}{m}}t\right)$

$x'(t) = \sqrt{\frac{k}{m}}A\cos\left(\sqrt{\frac{k}{m}}t\right) - \sqrt{\frac{k}{m}}B\sin\left(\sqrt{\frac{k}{m}}t\right)$

$x''(t) = -\frac{k}{m}A\sin\left(\sqrt{\frac{k}{m}}t\right) - \frac{k}{m}B\cos\left(\sqrt{\frac{k}{m}}t\right)$

IVP: at $t=0$ $x=x_0$ $v=0$

$x(t) = A\sin\left(\sqrt{\frac{k}{m}}t\right) + B\cos\left(\sqrt{\frac{k}{m}}t\right)$

$x'(t) = \sqrt{\frac{k}{m}}A\cos\left(\sqrt{\frac{k}{m}}t\right) - \sqrt{\frac{k}{m}}B\sin\left(\sqrt{\frac{k}{m}}t\right)$

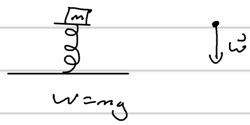
$x_0 = B(1)$ $\quad B = x_0$

$0 = \sqrt{\frac{k}{m}}A$ $\quad A = 0$

$$\boxed{x(t) = x_0\cos\left(\sqrt{\frac{k}{m}}t\right)}$$

Wait! Let's Revisit it!

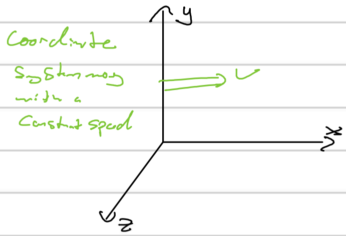
Suppose



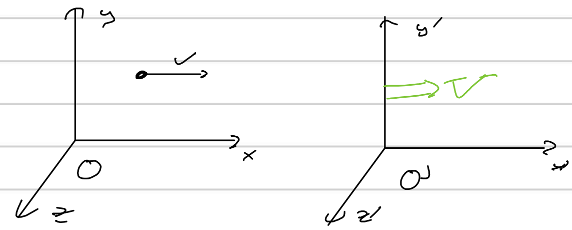
Newton's First Law!

If $F_{Net} = 0$

Then $v = \text{constant}$



Coordinate System with constant velocity



$v' = v - V$

velocity of particle from O as seen by O' frame

$$\frac{dv'}{dt} = \frac{dv}{dt} - \frac{dV}{dt}$$

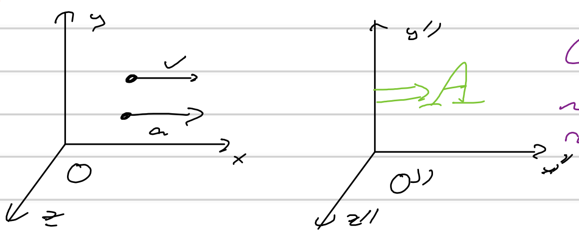
In a inertial frame, accelerations are the same

$\rightarrow a' = a - 0 \quad \therefore a' = a$

By Expression $\begin{cases} F = ma \\ F' = ma' \end{cases} \rightarrow F' = F$ N2L still works!

This is what N1L means

What if we have an accelerating frame



Calculate non-inertial reference frame

$a'' = a - A$

acceleration of particle in O as seen from O''

Then $F'' = F - mA$ Forces in non-inertial is not equal to other one

$\rightarrow \vec{F}_{\text{Non-iner}} = \vec{F}_{\text{Real}} + \vec{F}_{\text{Fict}}$

non-inertial = real + fictitious Force vectors

Fict Force

$F_{Fict} = -m\vec{A}$

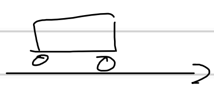
$\Rightarrow W_{Fict} = m\vec{g}_{Fict}$ where $\vec{g}_{Fict} = -\vec{A}$

effective weight

$\vec{W}_{eff} = \vec{W}_{real} + \vec{W}_{Fict}$ O in inertial reference frames

$m\vec{g}_{eff} = m\vec{g}_{real} + m\vec{g}_{Fict}$
$= m g\hat{j} + m(-\vec{A})$

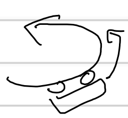
Think of Being in a car



stop quickly

$a = \leftarrow$ $-a$

but fict = $\rightarrow$



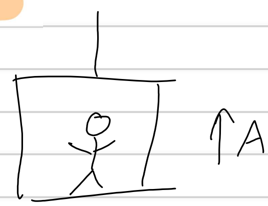
Sharp Turn left

$a = a_c$

but Fict $= -a_c$

Example

elevator

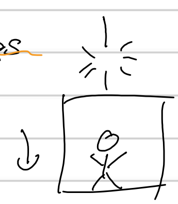


Feel heavier in an elevator moving up

w_{real} is $\downarrow$ $w_{Fict}\downarrow$

opposite of $A\downarrow$

Suppose Cable Snaps



Now in FF

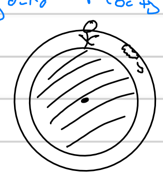
FF: $\vec{A} = \vec{g}_{real} = -g\hat{j}$

$\therefore W_{eff} = \vec{W}_{real} + \vec{W}_{Fict}$
$= -mg\hat{j} + m(-(-g\hat{j}))$

$\boxed{W_{eff} = 0}$

You feel like you're "floating" while falling here

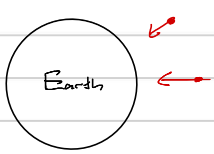
"weightless" Just like astronaut in space



Newton's Worries About Gravity

① Extended Objects Solution: Yes, invented calculus
↳ Can you treat them as point masses at CoM?

② Spooky action at a distance, what mechanism?



Solution: "Invent" the gravitational field

③ The equivalence of Gravitational and Inertial mass.

Einstein and the Universe

N2L $F = m a$

actually m_{iner} or inertial mass

UG: $F_{UG} = \frac{GMm}{r^2}$ ← m_{grav}

Why is it that $m_{iner} = m_{grav}$?

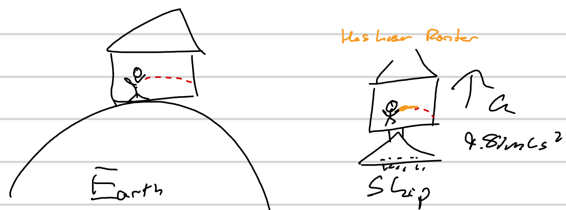
Albert Einstein!!!

1905 → Special relativity

↳ $E = mc^2$ ⟶ Inertial Ref Frame

Principle of Equivalence

"Gravity" indistinguishable from box in an accelerating reference frame



Stand. lobe the on earth → Laser would 'bend down' since its nonuniform...

General Relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$$

"gravity" "curvature" · "constant" · "stuff"

Einstein's Gravity

Gravity ⟺ Curved Space Time

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$ — cosmological constant

→ Gravity Bends Light

$$V_{esc} = \sqrt{\frac{2GM}{R}} = c \longrightarrow R_s = \frac{2GM}{c^2}$$

Escape velocity. At some radius, not even light can escape

Physics → Universe as a whole

"Cosmology"

Copernican Principle

→ Universe is homogeneous and is isotropic in space.

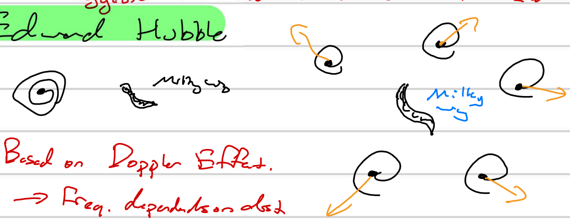
Laws of nature work the same everywhere

↳ There is no special concentration of stuff

→ Universe is " " in time.

"Suggests no real start nor a real end"

Edward Hubble

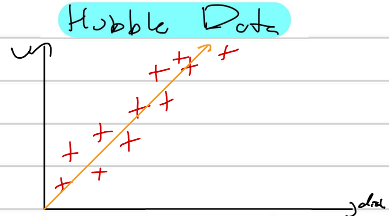


Based on Doppler Effect.

→ Freq. depends on dist.

Everything seems to move away from us, and moves faster at greater distances

Hubble Data



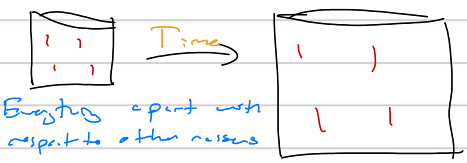
Leads to $v = HD$ Hubble Law

$v = HD$

Interpreted as space between expanding!

Not really away from us specifically, it all moves apart based on reference.

Plum Pudding



Everything apart with respect to other raisins

Then where did it come from

Past universe is denser and hotter

Keep going back → Started infinitely hot and dense.

Hot Big Bang

$t = 0$ 13.6 B years ago

Future

Expansion → Bigger

BUT!

Gravity → Smaller

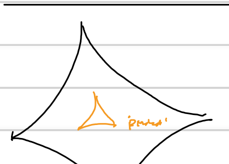
Microwave Rays in Universe

Proves Big Bang Happened, radiation from big bang decays over time and distance

Fate of the Universe

← "closed universe"

→ Fate of universe is that expansion will reverse



← "open universe"

→ Fate of universe is infinite expansion