First Summary Session Answers

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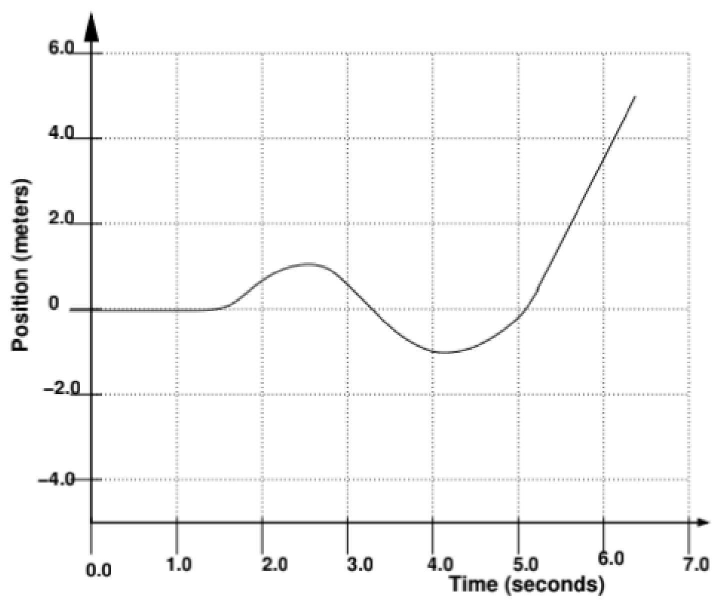
1 Question 1: Position vs Time Graph



The above plot shows the 1-D position of a particle as a function of time. Please answer these questions about the motion of the particle.

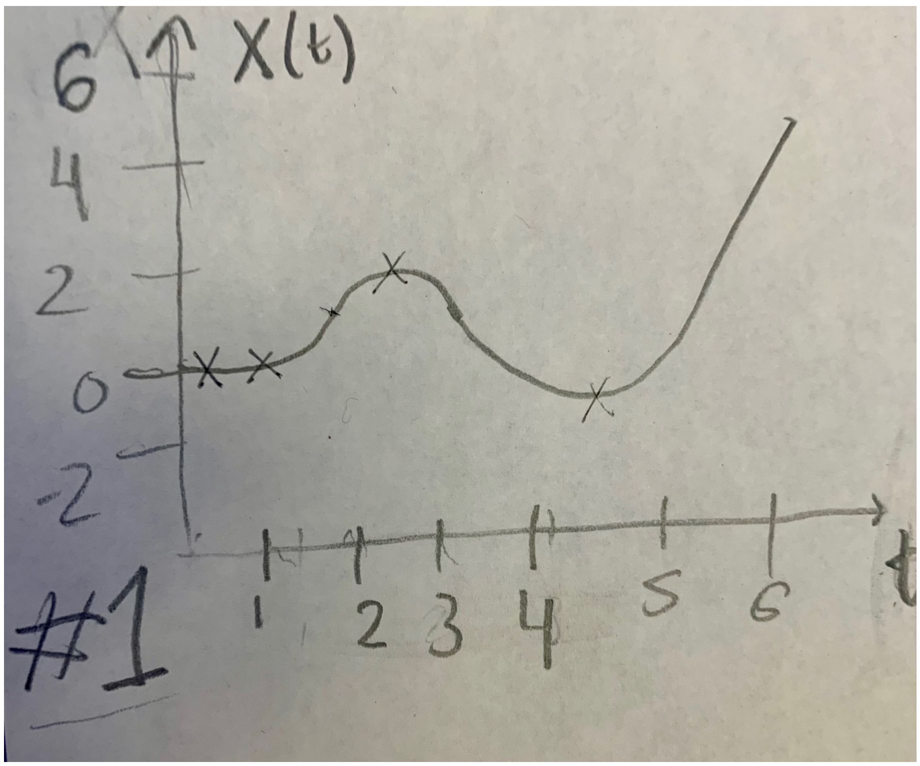
a) Are there any times during the motion when the particle is instantaneously at rest. Please clearly indicate these times by putting an "X" on them. Please explain how you determined this.

b) Are there any times during the motion when the velocity is constant. Please clearly indicate these times by putting an "O" on or around them. Please explain how you determined this.

c) What is the *maximum speed* the particle obtains during this motion? Please explain how you determined this.

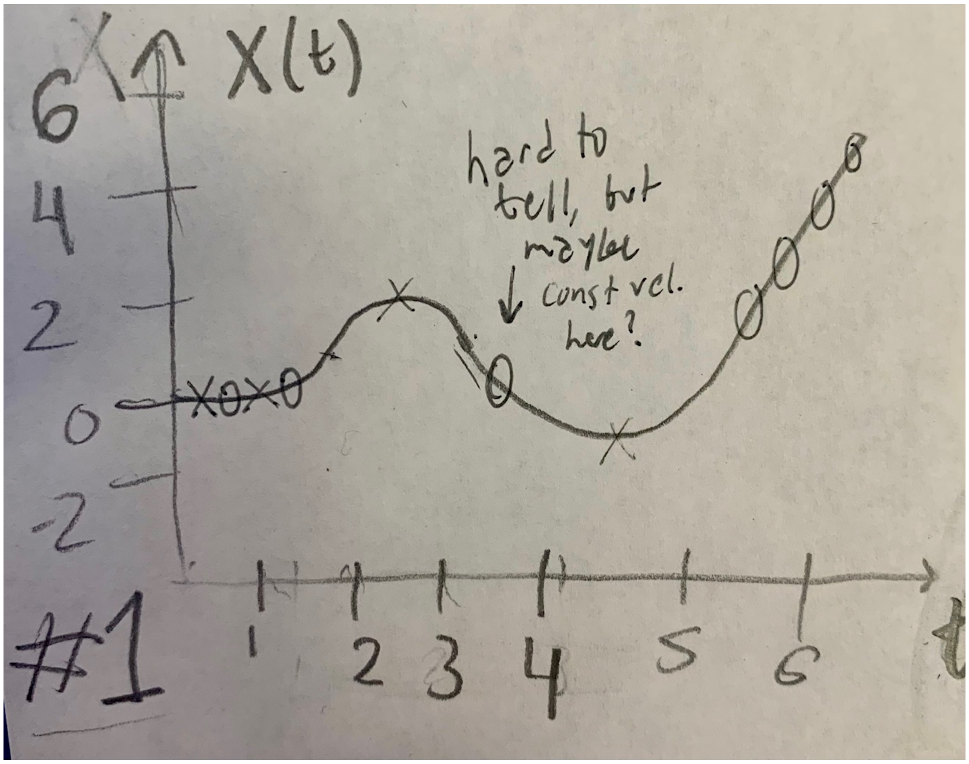
1.1 Instantaneously At Rest

When we say "instantaneously at rest", we're looking for any instant on this graph where the particle has a velocity of 0. By the Definition of Velocity, we know that on a position vs. time graph, velocity corresponds to the slope of the line. All we need to do is look for and mark the times when the slope equals zero. It should look something like this:



1.2 Constant Velocity

Once again, we want to use the Definition of Velocity here. $v = \frac{dx}{dt}$, so the particle will have constant velocity in any time frame where the slope does not change. This includes the first second or so, when the slope and the position are both zero. There are a few spots where it's harder to tell if the slope is constant, but your graph should look something like:



1.3 Maximum Speed

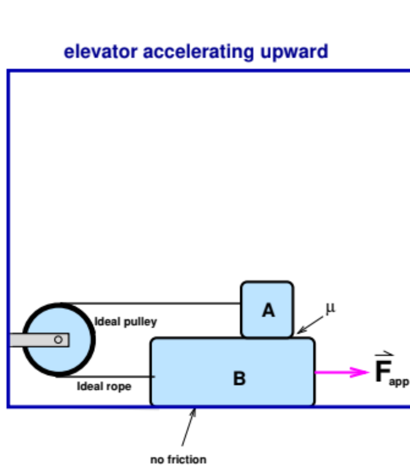
Since the velocity corresponds to the slope of the line, the maximum speed will occur when the slope is the steepest. If you line up your pencil with the slope, you'll see that the steepest slope happens at $t > 5.3$

To calculate the slope, then, estimate two points on that stretch of the line. You should get, roughly, $3.5 \leq v \leq 4.5$

MG's Calculations:

$$v = \frac{3.5m - 1.75m}{6.0s - 5.5s} = \frac{1.75m}{0.5s} \approx 3.5m/s$$

2 Question 2: stop putting things in elevators istg



Two blocks A and B with a given masses m_A, and m_B respectively, are positioned as shown above. Block A is observed to be *sliding* on Block B. The coefficient of kinetic (sliding) friction between Block A and Block B is given as μ. The pulley is an ideal massless pulley. The ropes are ideal. A given force $\vec{F}_{app}$ is applied to the right on Block B. The entire arrangement sits inside an elevator that is accelerating upward with a constant given acceleration, a_0.

Part (a) – Draw two clear, clean, and properly labeled "Free-Body Diagrams" (FBDs), one for each block, indicating *all* of the forces on each body.

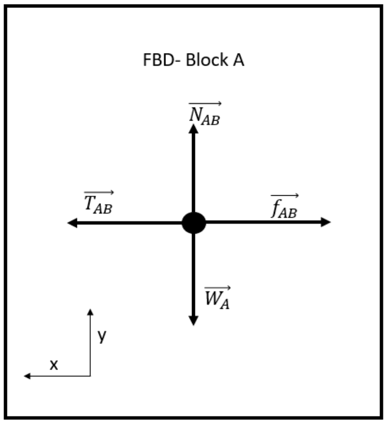
Part (b) – Using your FBD and Newton's Second Law, determine N_{AB}, the magnitude of the normal force on Block A due to Block B. Express your answer in terms of the given parameters only. Explain your work.

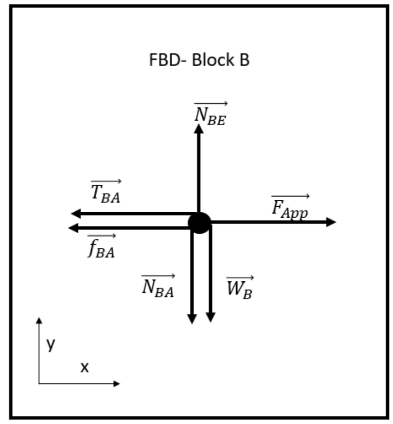
Part (c) – Using your FBD and Newton's Second Law, determine N_{BE}, the magnitude of the normal force on Block B due to being in contact with the elevator. Express your answer in terms of the given parameters only. Explain your work.

Part (d) – Rather More Difficult: Using your FBD and Newton's Second Law, determine the force of Tension in the rope. Express your answer in terms of the given parameters only. Explain your work.

2.1 Free Body Diagrams!! :D

Make sure your FBDs include all your forces, a coordinate system, and "on-due" notation!





**NOTE: Here, I've defined x to be a coordinate that points in the direction of the pulley's motion. This is allowed and recommended for pulleys (as long as you state that that's what you're doing!). However, you can totally do this problem with x pointing to the right for both blocks too. It just means you have to deal with negative signs.

Before we continue, we should note that, with the way we've set our coordinate system,

$$a_{yA} = a_{yB} = a_0$$

and

$$a_{xA} = a_{xB} = ax$$

Where a_x is unknown. Note that if you have x pointing to the right in both FBDs,

2.2 Find N_{AB}

Now that we've got some good FBDs set up, this should be easy! Use your FBD and <u>Newton's Second Law</u> to solve.

<u>N2L- Block A,y-coordinate:</u> $F_{net,A,y} = m_A a_y$

$$N_{AB} - W_A = m_A a_0$$

$$N_{AB} = m_A(a_0 + g)$$

2.3 Find N_{NE}

Exactly the same thing as the previous part: N2L

N2L- Block B,y-coordinate: $F_{net,B,y} = m_B a_y$

$$N_{BE} - N_{BA} - W_B = m_B a_{-0}$$

We also know by Newton's Third Law that $N_{BA} = N_{AB} = m_A(a_0 + g)$ So,

$$N_{BE} - m_A(a_0 + g) - m_B g = m_B a_0$$

$$N_{BE} = (m_B + m_A)(a_0 + g)$$

2.4 "Rather More Difficult" Tension in the Rope

Okay, we're definitely gonna have to do something with N2L in the x direction. Let's start with A since it looks a little nicer.
**NOTE: Use your FBDs for this! It's super helpful to look at your FBD while you're doing N2L. ALso, since we know that the tension in an ideal rope is the same throughout, I've denoted the tension as T

N2L- Block A,x-coordinate: $F_{net,A,x} = m_A a_x$

$$T - f_{AB} = m_A a_x$$

This is sliding (kinetic) friction. That means that we're allowed to use the equation:

$$f_k = \mu_k N$$

So let's do it (notice how the subscripts on the friction and normal force match?)

$$f_{AB} = \mu N_{AB} = \mu m_A(a_0 + g)$$

$$T - \mu m_A(a_0 + g) = m_A a_x$$

STOP!! We have 1 equation and 2 unknowns. Let's leave this one as-is and try to do something with our other N2L equation:

N2L- Block B,x-coordinate: $F_{net,B,x} = m_B a_x$

$$F_{app} - T - f_{AB} = m_B a_x$$

$$Fapp - T - \mu m_A(a_0 + g) = m_B a_x$$

STOP!! Another equation with 2 unknowns. That makes two equations and two unknowns. **The rest is algebra.**

So, let's do the algebra. Divide the first of our two equations. by m_A to get a value for our acceleration:

$$a_x = \frac{1}{m_A}(T - \mu m_A(a_0 + g))$$

Now, plug that into the other equation and rearrange. You should get something along these lines:

$$F_{app} - T - \mu m_A(a_0 + g) = \frac{m_B}{m_A}(T - \mu m_A(a_0 + g))$$

$$-T - \frac{m_B}{m_A}(T - \mu m_A(a_0 + g))$$

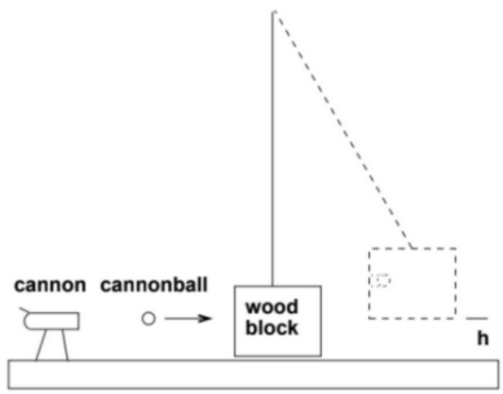
$$T + \frac{m_B}{m_A}T - \mu m_B(a_0 + g) = F - \mu m_A(a_0 + g)$$

$$T(1 + \frac{m_B}{m_A}) = F + \mu(m_B - m_A)(a_0 + g)$$

$$T = \frac{F + \mu(m_B - m_A)(a_0 + g)}{(1 + \frac{m_B}{m_A})}$$

That was messy, but that's okay. If you get lost in the algebra, try your best and move on to the next problem:

3 Question 3: The next problem (AKA the rest of the exam topics)



A cannon is fixed to a solid platform and fires a cannonball with mass m_c at a velocity V_0. The cannonball collides with a large wooden block (mass m_b) and becomes completely embedded inside the block. The block is suspended on an ideal rope and swings upward to a height h above its resting point.

a) Find the speed of the wood block after the cannonball is completely embedded in the block.
b) Find the maximum height h that the block will reach in terms of the given parameters.
c) What is the total work done on the cannonball by the cannon?
d) What is the work done on the block by the tension force as it rises to the maximum height?
e) What is the work done on the cannonball by the force of gravity as it rises to the maximum height?
f) Is the total mechanical energy conserved during the collision between the cannonball and the block?

3.1 Speed after cannonball is embedded

So we have a cannonball that is embedded into a block, and the two move together. This looks like a textbook example of an inelastic collision. BUT! Let's check our conditions for CofLM first!

Our condition for CofLM is that the net Force external to the system must equal zero before, during, and after the collision. We can go ahead and ignore the forces in the vertical direction because they cancel out before, during, and right after the collision. If we define our system as the block and the cannonball, we can see that there are no net external forces on our system. So we can use Conservation of Linear Momentum

$$BEFORE = AFTER$$

$$P_{TOT} = P'_{TOT}$$

$$m_b v_b + m_c v_c = m_b v'_b + m_c v'_c$$

Since it's an inelastic collision, we know that the final velocity of the cannonball and the block are the same velocity, V'. We also know that the initial velocity of the block is 0, and the initial velocity of the cannonball is V_0.

$$m_c V_0 = (m_b + m_c)V'$$

$$V' = \frac{m_c V_0}{m_b + m_c}$$

(For the rest of this problem, I'll call this velocity V_1, for clarity.

3.2 Max height that the block reaches

So, after the collision, the block and the cannonball move together. We need an equation/law that involves a speed and a height. We could mayyybe use Kinematics for this, but that sounds hard. Let's look at the forces acting on our system (the block and the cannonball):

Weight Force- Conservative

Tension Force- Does no work. This is because the force of tension points perpendicular to the path of motion in this problem.

So we can use Conservation of Mechanical Energy to solve the problem.

CofME

$$BEFORE = AFTER$$

$$E_{TOT} = E'_{TOT}$$

$$U + K == U' + K'$$

Remember that this is the energy of the block and the cannonball, so we need to add the masses together!

$$(m_b + m_c)gy + \frac{1}{2}(m_b + m_c)v^2 = (m_b + m_c)gy' + \frac{1}{2}(m_b + m_c)v'^2$$

To make this as easy as possible, let's say that $y = 0$. Also, we know that $y' = h$ (this is the thing we're solving for). Since we're looking for the maximum height the block reaches, we'll want to set the final velocity equal to 0.
That leaves us with

$$\frac{1}{2}(m_b + m_c)V_1^2 = (m_b + m_c)gh$$

Divide the masses out and rearrange:

$$h = \frac{V_1^2}{2g}$$

Finally, sub in V_1

$$h = \frac{(m_c V_0)^2}{2g(m_b + m_c)^2}$$

3.3 Work done by... Cannon??

This one looks a little intimidating, but we can do it. Let's first think about what's happening to the cannonball. First, it's in the cannon at rest. Then, it is launched out of the cannon at a velocity V_0. As the cannon launches the cannonball, there are no forces outside of the forces from the cannon doing work on the cannonball. That means that the work done by the cannon on the cannonball is equal to the total work on the cannonball when it is launched.
Thankfully, we have an equation with Total Work in it- the Work-Energy Theorem

$$W_{TOT} = \Delta K$$

Our "Final Kinetic Energy" is the Kinetic Energy after the cannonball has been fired, and the initial is when the cannonball is sitting in the cannon ($V = 0$). So...

$$W_{TOT} = \frac{1}{2}m_c V_0^2$$

3.4 Work done on block by Tension force

As we said in part b- the Tension force does no work as the block rises to the max height. This is because of the Definition of Work, $W = Fd\cos\theta$. We know that the angle between the Tension and the path of motion of the block is 90 degrees for the entire path, so the Work done by Tension is 0.

3.5 Work done by Weight

The work done by weight is equal to the negative change in potential energy.

$$W_w = -\Delta U$$

$$W_W = -(U_f - Ui)$$

But we've already said that U_i=0, so...

$$W_W = -U_f = -mgh$$

$$= \frac{-m_c(m_c V_0)^2}{2(m_b+m_c)^2}$$

3.6 Energy Conserved in Collision?

No! Since this is an **Inelastic** Collision, kinetic energy is not conserved. Energy is actually lost to heat, the deformation of the block, etc.

4 That's all, folks! Good luck on Friday!