P121 Exam 3 Summary Session

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April 15th, 2024

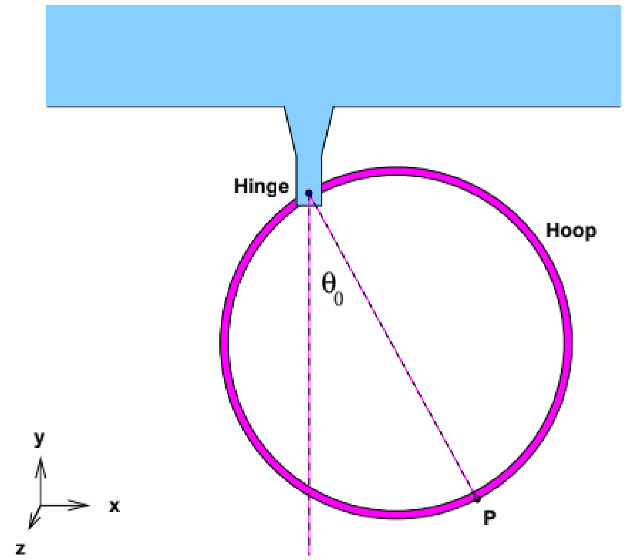
1 Learning Objectives

By the end of this session, students will be able to:

- Apply Rotational N2L to find all the forces
- Find tangential/rotational acceleration on different bodies
- Use CofME to find speed and kinetic energy
- Call Corbin Covault "Bestie" (again)

2 Hoops I Did it Again

Problem 3: A New Pivot Point (40 points)



A hula-hoop of radius R and mass M is attached to an ideal hinge as shown in the figure above. The hoop is positioned at an angle θ_0 and released. Note that we define a coordinate system, where y is a vertical coordinate and z is the horizontal coordinate the points "out of the page".

The moment of inertia of a hoop rotating about a point on the hoop's circumference is $I = 2MR^2$.

a Find the net torque on the hoop calculated about the hinge point immediately after the hoop is released. Give your answer as a vector (magnitude and direction).

All we need to do for this one is consider each of the forces on the hoop and calculate the sum of their torques. To do this, we use the **Definition of Torque**.

$$\tau = Fr \sin \phi_{Fr}$$

The only forces acting on the hoop are the weight force and the hinge force.

$$\tau_{net} = \tau_W + \tau_H$$

$$\tau_{net} = Wr \sin \phi + Hr \sin \phi$$

Since we're calculating the net torque about the hinge, the distance r from the pivot point to the hinge is 0. We also know that the distance between the hinge and the center of mass is R and the angle between the W vector and the r vector is equal to θ_0 So,

$$\tau_{net} = MgR \sin \theta_0 + 0$$

But the question asked for the magnitude and direction, so we use the Right Hand Rule and find that

$$\tau_{net} = -MgR\sin\theta_0\hat{k}$$

b What's the angular acceleration of the hoop immediately after it has been released?

Since we found the net torque in part a, we can easily use **Rotational N2L** to find the angular acceleration. For I, we can plug in the moment of inertia given to us above.

*Note: The moment of inertia comes from the **Parallel Axis Theorem** that you saw in class on Monday. You should check that you get the same moment of inertia for the hoop by using the parallel axis theorem.*

$$\tau_{net} = I\alpha$$

$$MgR\sin\theta_0 = 2MR^2\alpha$$

$$\frac{g\sin\theta_0}{2R} = \alpha$$

c At point P, there is a small bit of paint on the hoop. Find the maximum speed of the paint sometime after the hoop has been released.

We're looking for the maximum *speed*, so we should probably use a conservation law. We know that the weight force is conservative and the hinge force does no work about the pivot point, so we can use **CofME**.

$$'BEFORE' =' AFTER'$$

We say that "before" is the instant the hoop is released, and "after" is when the blob of paint reaches the max speed. We can infer that this happens when the point P reaches its lowest point.

$$E_{TOT} = E'_{TOT}$$

$$U + K = U' + K'$$

$$mgy + \frac{1}{2}I\omega^2 = mgy' + \frac{1}{2}I\omega'^2$$

It is best to evaluate the hoop as it rotates about the hinge, so we do not consider the translational kinetic energy of the hoop. Also, we need to pick a convenient spot for our "zero point" for gravitational potential energy. A good place to set $U = 0$ is the hinge. Remember that the y position refers to the position of the center of mass, not the position of p.

$$-MgR\cos\theta_0 + 0 = -MgR + MR^2\omega'^2$$

$$\omega'^2 = \frac{g(1-\cos\theta_0)}{R}$$

$$\omega = \sqrt{\frac{g(1-\cos\theta_0)}{R}}$$

To find the tangential speed, we use the **Rolling Constraint**. Remember that the fleck of paint is $2R$ away from the pivot point.

$$v' = \omega'(2R)$$

$$v' = 2\sqrt{gR(1-\cos\theta_0)}$$

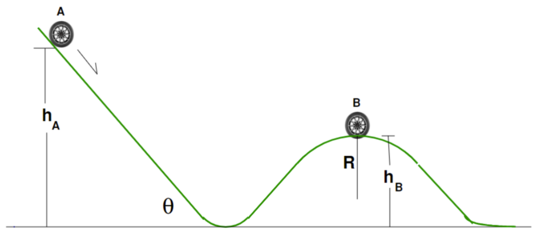
IMPORTANT: Since there was a lot of confusion on Monday night, let's highlight some important things.

1. The y in the gravitational potential energy formula is always the y position of the *center of mass*.

2. For "hinge problems" like this, the best spot to set $U = 0$ is the hinge.

3. For the rolling constraint, $v = \omega r$, the r refers to the distance between the pivot point and the point of interest. This isn't always equal to the radius! In this hoop problem, it wasn't, but for "rolling problems", like the problem below, it is.

3 You spin me right round, baby (right round)

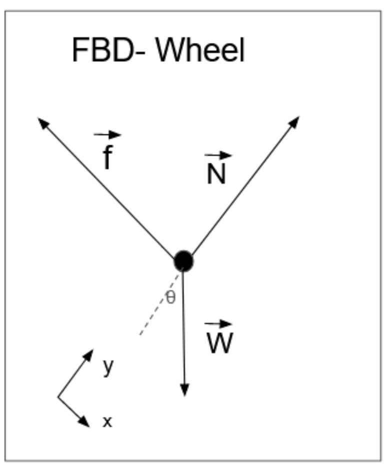


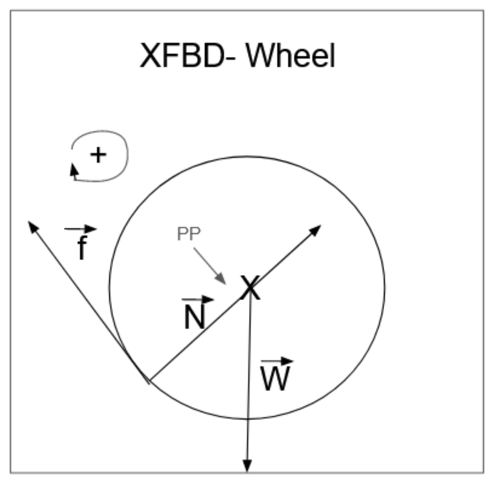
A wheel of mass m and radius r is released from rest on the side of a hill at point A as shown. The height of point A (relative to the lowest point of the hill) is given as h_A. After its release, the wheel rolls with no slipping or sliding.

The wheel has a rotational inertia of $I_{cm} - Cmr^2$. The side of the hill on which the wheel is released can be likened to a ramp at an angle θ relative to the horizontal. The second hill has a radius of curvature R.

Assume $m, r, C, h_A, h_B, \theta$, and R are all given parameters, where $r << R$.

a Draw a FBD and an XFBD for the wheel at position A (immediately after release).





b What is the magnitude of the **linear acceleration** of the wheel immediately after release?

Well, we're looking for linear (translational) acceleration, so we can just do **N2L**, right? The y direction isn't gonna tell us anything useful since

$a_y = 0$, so I'm gonna jump straight to the x coordinate.

$$F_{net,x} = ma_x$$

$$W_x - f = ma$$

$$mg\sin\theta - f = ma$$

Oh no! We have 1 equation and 2 unknowns!! We need another equation. Let's use **Rotational N2L**

$$\tau_{net} = I\alpha$$

We can use the **Rolling Constraint** to replace that α with $\frac{a}{r}$

$$\tau_N + \tau_W + \tau_f = Cmra$$

$$Nr\sin phi + Wr\sin phi + fr\sin phi = Cmra$$

We know that W and N apply no torque, but for different reasons. The weight force is being applied at the pivot point, so $r = 0$. $\tau_N = 0$ because the angle N makes with the radius vector is 180 degrees. So,

$$fr = Cmra$$

$$f = Cma$$

Plug this result into our other N2L equation

$$mg\sin\theta = am(1 + C)$$

$$a = \frac{g\sin\theta}{1 + C}$$

c What is the magnitude of the **Friction force** on the wheel immediately after release? (Hint: it is not zero.)

Well,

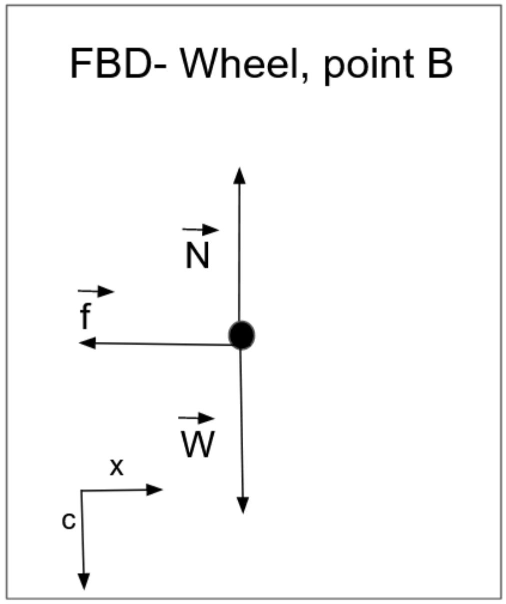
$$f = Cma$$

So we can just plug in our answer from part a.

$$f = \frac{Cmg\sin\theta}{1 + C}$$

d After it is released, the wheel rolls down the first hill and up the second hill to position B, as indicated. What is the magnitude of the **Normal** force on the wheel at position B? (Hint: the velocity of the wheel at position B is not zero.)

Okay, so we want a Normal Force. That means we should probably draw another FBD:



Note that we use the c coordinate since the wheel is on a curved hill.

Great, let's try **N2L** in the c coordinate:

$$F_c = ma_c$$

$$W - N = m\frac{V^2}{R}$$

$$N = m\left(g - \frac{V^2}{R}\right)$$

But we don't know what V is. How can we find it? We can use a conservation law. Let's think about our forces:

W- conservative

N- does no work (perpendicular to path of motion)

f- "does no work". (Swindle! Since the wheel is not moving at the point of contact, technically there is no displacement and therefore no work)

So we can use **CofME**.

$$"BEFORE" = "AFTER"$$

$$U + K_T + K_R = U' + K_T' + K_R'$$

Here we have rotational and translational K, so we must include both. However, the wheel starts at rest, so the initial K equals 0

$$mgy = mgy' + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2$$

We can use the rolling constraint, $v = \omega r$ to get rid of the omega. We can also go ahead and set $U = 0$ at the floor.

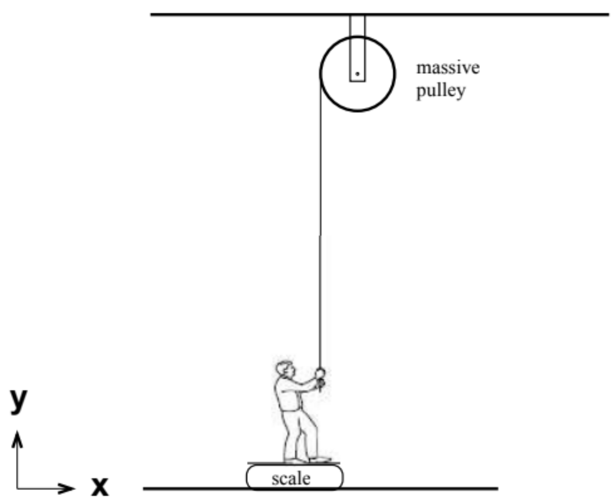
$$mgh_A = mgh_B + \frac{1}{2}mV'^2 + \frac{1}{2}CmV'^2$$

$$V'^2 = \sqrt{\frac{2g(h_A - h_B)}{(1+C)}}$$

Plug this back into our N2L...

$$N = mg\left(1 - \frac{2(h_A - h_B)}{R(1+C)}\right)$$

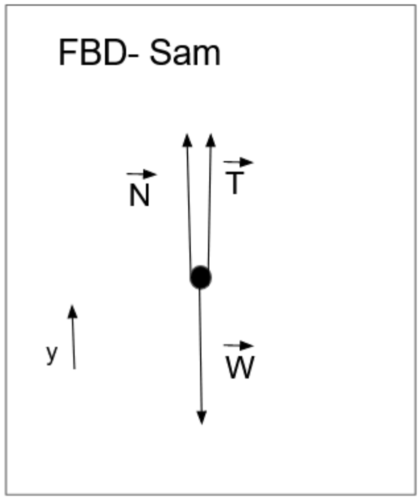
4 Yet Another Pulley Problem (They're fun, I promise.)



Sam, a man of given mass m stands on a scale at rest. At time $t = 0$ Sam uses his arms to pulls on an ideal rope with some unspecified constant force of tension T. The rope is wound around and attached to a massive pulley mounted to the ceiling as shown. The pulley has a mass $m_p = 2m$ and the rotational inertia of the pulley is given as $I = \frac{1}{2}m_pR^2$ where R is the given radius of the pulley.

a Sam looks at the scale while pulling on the rope and sees that the scale reads exactly 3/4ths of his mass. What is the tension of the rope?

Let's start by drawing an FBD of Sam.



And let's go ahead and do **N2L**:

$$F_{net} = ma$$

$$N + T - W = 0$$

$$T = W - N$$

So this "3/4ths of his mass" bit is kinda confusing. What does it mean? You can say that a scale measures the normal force applied to it. When you push down on a scale, it will read a heavier weight than expected. If the scale reports 3/4ths of Sam's mass, then that means that Sam is not pressing as hard against the scale. In other words, we can say that the Normal force that Sam and the scale apply to each other is equal to $\frac{3mg}{4}$.

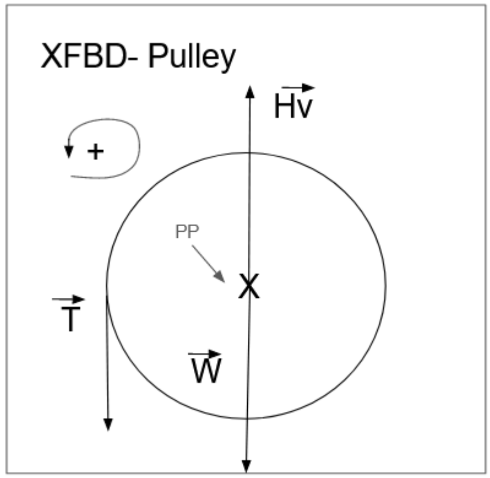
Let's plug that in to our N2L:

$$T = mg - \frac{3mg}{4}$$

$$T = \frac{mg}{4}$$

b What is the net torque applied to the pulley about the center of rotation? Express it as a vector.

Since we're asked about a torque, we should probably make an XFBD:



We are asked about the *net torque*. We do not need to use **N2L** here because we don't know the angular acceleration of the pulley (yet). Rather, let's find the net force by adding all the torques up.

$$\tau_{net} = \tau_W + \tau_H + \tau_T$$

Plug in the **Definition of Torque**

$$\tau_{net} = Wr\sin\phi + Hr\sin\phi + Tr\sin\phi$$

We can see from our XFBD that the weight and the the hinge force are applied at the pivot point, which means that $r = 0$ for both. We also can see that the r for Tension is equal to the radius of the pulley and that it is pulling at a 90 degree angle. So,

$$\tau_{net} = TR$$

We know T from part a, and we can use the **RHR** to find that the torque is pointing out of the page, or "positive z"

$$\tau_{net} = \frac{mgR}{4}\hat{k}$$

c In terms of given parameters, find the total kinetic energy of the pulley as a function of time.

This one sounds like a lot, but you know how to do this. Let's consider what "Total Kinetic Energy" means.

$$K_{TOT} = K_T + K_R$$

Since the pulley is just rotating in place, we can definitely say that the only Kinetic Energy is Rotational Kinetic Energy.

$$K_{TOT} = \frac{1}{2}I\omega^2$$

We know from N2L that since there is a constant torque on the pulley (part b), there must be a constant angular acceleration, which means that *the angular speed ω must be changing over time.* This means that the Kinetic Energy is also changing as a function of time.

To find ω we can use **Constant Acceleration Rotational Kinematics**.

$$\omega(t) = \omega_0 + \alpha t$$

We know that ω_0 is 0 because the pulley starts at rest. To find α we use **Rotational N2L** and the torque we already found. (I'm leaving it in terms of I for now to make calculations easier.)

$$\tau_{net} = I\alpha$$

$$\alpha = \frac{mgR}{4I}$$

$$\omega(t) = \frac{mgRt}{4I}$$

Now we can plug this into our kinetic energy expression:

$$K_{TOT}(t) = \frac{1}{2}I\left(\frac{mgRt}{4I}\right)^2$$

$$K_{TOT}(t) = \frac{(mgRt)^2}{32I}$$

$$K_{TOT}(t) = \frac{m(gt)^2}{32}$$

5 Reminders

1. MG will be holding "office hours" on Thursday from 5:30 to 7:00p.m. in Bingham 140 in case you have any questions. If you can't make it and have questions, feel free to shoot us an email.

2. Good luck everybody! You can do this!

3. Here are some cat pics to help you feel less stressed :)