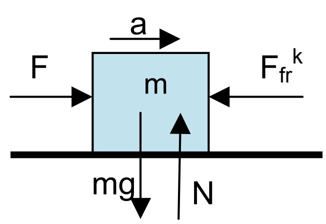
Chapter 7

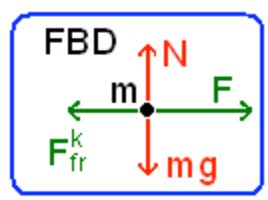
More Contact Forces: Static and Kinetic (sliding) Friction in 1D

- **Kinetic Friction Resists Sliding Motion**
- **Static Friction Tries to Stop Motion From Getting Started**
- **Add Friction to the Block-on-Table Problem: Need Normal Force**

Sliding Friction

Consider a block m sliding on a table, subject to a constant external force F (we again deal with magnitudes and we stick in minus signs as needed when we know things are pointing to the left for x-components and down for y-components), and now we suppose there is friction between the block and the table. This frictional force is parallel to the two surfaces and in a direction so that it opposes the relative motion between the block and the table. We define $\boxed{F_{friction}(\text{sliding}) \equiv F_{fr}^{k}}$ using the label k for kinetic (we can't use superscript s for sliding because we need that later to refer to static friction). The plain vanilla picture and the fancy-shmancy FBD **(you can freely guess the sizes of the arrows)** are:





Now the vertical second law is crucial, as we will see, and, with zero vertical acceleration, it is:

$$N - mg = 0 \quad \Rightarrow \quad N = mg$$

For acceleration a defined to the right, the horizontal second law is:

$$F - F_{fr}^{k} = ma$$

We have something wonderfully simple to use for the kinetic friction force! We find from experiment that

$$\boxed{F_{fr}^{k} = \mu_k N}$$

where

- N is the normal force on the block due to table (so we need to do the normal force analysis first before we can do the horizontal motion analysis)
- μ_k is the proportionality constant, the **coefficient of sliding or kinetic friction.**

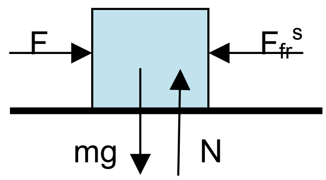
Therefore, with the vertical second law N = mg as input, the horizontal second law output is

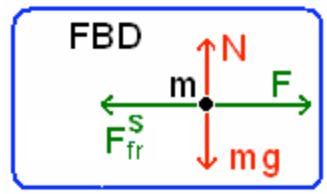
$$F - \mu_k mg = ma$$

for the block **sliding** on a table subject to external F and friction μ_k .

Static Friction

Now consider the block m resting on the table and not moving. For typical blocks on typical tables, if we apply a very small push or pull F , there still ain't gonna be no motion no how . This is because of what we call "static friction." If we define $\boxed{F_{fr}(\text{static}) \equiv F_{fr}^{s}}$ for the static friction force, the plain vanilla picture and FBD are:





While the vertical second law is the same as in the sliding case

$$N - mg = 0 \;\Rightarrow\; N = mg$$

the horizontal second law with zero acceleration just gives a horizontal force balance

$$F - F_{fr}^{s} = 0 \;\Rightarrow\; F = F_{fr}^{s}$$

We see that F_{fr}^{s} is equal and opposite to the applied force F, and must be growing when the applied force F is increased (if there's no motion). But when F is too large, you know from experience the block moves and math-wise this means F_{fr}^{s} can no longer match the increase in F. From experiments, we find the **maximum value** of the static friction force is proportional to the normal force, to excellent approximation:

$$\boxed{F_{fr}^{s}(\text{maximum}) = \mu_s N}$$

Therefore, the range for F_{fr}^{s} is restricted to: $\boxed{0 \le F_{fr}^{s} \le \mu_s N}$

where

- N is again the normal force on the block due to table (so again we need to do the normal force analysis first before we can do the horizontal motion analysis – N is NOT always mg!)
- μ_s is the **coefficient of static friction.**

Picture for Sliding and Static Friction

A microscopic picture of surface bumps sliding over each other and trying to stick to each other (momentarily bonding together) is shown below:

Let's use this picture to help us understand sliding and static friction better – see the comments on the next page.

Comments utilizing the microscopic picture:

1) If you press the two surfaces vertically together more strongly (i.e., N is larger), you deform the bigger bumps letting more of the smaller bumps come into contact, implying more bump-bump bonding between the two surfaces and thus more attraction and thus more friction! This is why the friction forces grow with N . By the way, by Newton's third law, we can equally well be talking about the block pushing into the table or the push-back by the table on the block.

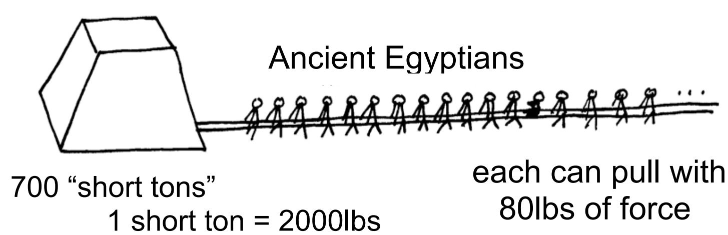
2) **The frictional forces depend only on the total weight regardless of the contact area.** (We'd like to say "irregardless," but they won't let us!) For example, a block with larger contact area has less friction per unit area (fewer bumps being bonded in 1 cm^2) than a block with the same weight but smaller contact area. But the total friction is the same (same number of bonded bumps).

3) The constant sliding friction is almost always less than the maximum static friction: $\mu_k < \mu_s$ We see this from the microscopic picture because during sliding there is less time for the bumps to stick together and form a strong bond.

4) The sliding friction formula $F_{fr}^k = \mu_k N$ is independent of speed, but we can understand that when the blocks get going too fast (hundreds of MPH) the bumps don't have time to form bonds at all. At high speeds, friction is not independent of speed, and our simple formula is no longer a good approximation.

5) We understand that friction does depend on what kinds of material are rubbing against each other (wood or glass or metal). The bump bonding depends on the kinds of molecules and their abilities to bond.

Example: Suppose a huge team of ancient Egyptians is pulling a gigantic stone in the building of some monument. Assume $\mu_s = 0.5$ and $\mu_k = 0.3$ for the stone sled with respect to the sand surface. Other data are given in the figure shown.



How many people are needed to get the stone moving and how many are needed to keep it moving at **constant speed**?

Answer steps:

1) First, we know N = mg = (700) (2000) = 1,400,000 lbs.
2) Total force: F (to get it moving) must $\geq F_{fr}^s(\text{maximum}) = \mu_s N = 0.5\ N = 700{,}000$ lbs.
3) After it is moving, we need F (to keep it moving) must $= \mu_k N = 0.3\ N = 420{,}000$ lbs.
4) With 80 lbs per person, we have 700,000/80 = 6750 and 420,000/80 = 5250
   (a) We need about 6751 ancient Egyptians to get it moving.
   (b) We need about 5251 ancient Egyptians to keep it moving at constant speed.

   [We had to add one person to (a) to overcome the maximum static friction and get it going. We also added one more person to both (a) and (b) – notice the slacker!]

Also, we snuck in U.S. units, but only needed the product mg which is a force so it has units of pounds = lbs and we didn't have to deal with "slughs! And we do mean "slughs" and not slugs!

Constant Friction Forces and Good Old Constant Acceleration Equations

The sliding friction force is constant if N is constant, such as when N = mg. Thus, **if the other forces are constant, too,** then the acceleration will be constant. We can trot out all those formulas from Chapter 2 and predict the motion straight away! For example, see the next problem

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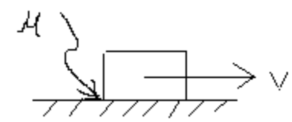
Problem 7-1



A package with mass m *is thrown horizontally onto a table where the coefficient of sliding friction is* μ *between the package and the table. The package has an initial horizontal speed of* v. *Gravity is present and the package comes to a stop.*

(a) *Guess what happens if we increase* m. *That is, does a heavier package or a lighter package travel for a longer time and distance before it stops due to sliding friction?*
(b) *Draw the fancy-shmancy FBD for the sliding package* ***after*** *the initial "throw."*
(c) *Find the time in terms of the given parameters* (v, μ, m, g) *that the package slides on the table. (While we say "in terms of the given parameters," know ye not all may be needed.)*
(d) *Calculate the distance in terms of the given parameters* (v, etc.) *that the package slides on the table (you could use a short-cut based on one of the formulas in Problem 2-2).*
(e) *How did your guess check out?*

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And to clarify the concepts here, consider this problem:

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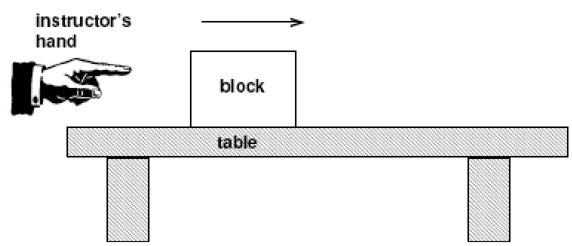
Problem 7-2 *Truth or Friction*

To demonstrate sliding friction, an instructor takes a rectangular block of wood and gives it a sharp push so that the block slides across a smooth table, eventually coming to rest (like the above problem!) The following statements pertain to the motion of the block after it has been released and before it comes to rest:



(a) *True or False: Immediately after the block leaves the instructor's hand, the net force on the block is zero.* ***Here and below, please explain your answer and don't just say T or F. Thank you***
(b) *True or False: At the instant the block leaves the instructor's hand, the velocity of the block remains constant for a while and then gradually begins to decrease.*
(c) *True or False: The weight of the sliding block does not depend on the block's velocity.*
(d) *True or False: The force of friction on the sliding block does not depend on the block's velocity.*
(e) *True or False: The normal force and the weight are equal* ***according to Newton's third law****. Hint: Newton's third law must always involve two forces acting on different bodies.*
(f) *True or False: There is a constant force of friction on the table in the direction of the motion of the block.*
(g) *True or False: While the block is sliding across the middle of the table, there is a constant force of friction on the block with magnitude* $F_{fr} = \mu_k N$.
(h) *True or False: At the moment the block finally comes to rest, the net horizontal force on the block abruptly goes to zero.*
(i) *True or False: At the moment the block finally comes to rest, the net vertical force on the block abruptly goes to zero.*

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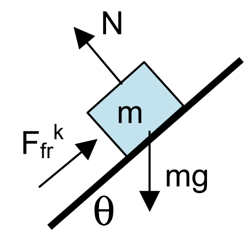
Problem 7-3 Quickies!

a) Give a simple explanation why the static or the sliding friction will be the same for two boxes of identical weight and material BUT with different area for the bottom side in contact with the floor. So for a given block of any material, we could slide it along any of its sides, even if they have different area, and still get the same friction resisting the motion along the floor. We just need the same area texture on the different sides, since friction depends on the makeup of the contact area.

b) Let's follow up on the question 6-2(b) from the last chapter. We again will try to anticipate something that is found in the second cycle. Return to the block of mass m sliding down a hill at an angle θ. *If there is friction included, there are now three forces acting on the block and shown in the figure. Guess what* F_{fr}^{k} *is in terms of* m, g, μ_k, *and* θ. *Does your answer agree with what you expect it to be if* θ=0? *Or if* $\theta=90^o$?



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