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PHYS121 - HW3

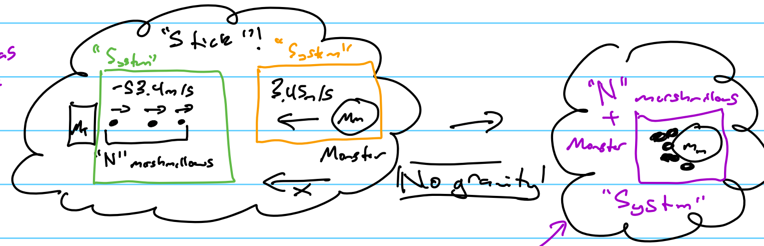
Problem 1)

Monster mass = 28.2 kg = M_m

→ Horizontal Speed = 3.45 m/s = V_m defined as positive

Marshmallow mass = 11.5 g = M_T

→ Speed = 53.4 m/s = V_T negative



CofLM Before = After

$P_{Tot} = P'_{Tot}$

Monster and Marshmallow move together:

$V_A' = V_B' = V_f$

V_T is a negative quantity

$m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$

$M_m V_m - N \cdot M_T V_T = V_f (M_m + N \cdot M_T)$

$\therefore V_f = \dfrac{M_m V_m - N M_T V_T}{M_m + N M_T}$ for isolated system.

Condition: System of interest is isolated because there is no external forces such as gravity or friction affecting them! We can use the CofLM.

$V_f = 0$ to make monster stop ⟹ $0 = M_m V_m - N M_T V_T$

$N M_T V_T = M_m V_m$

The cadet must fire $N = \dfrac{M_m V_m}{M_T V_T}$

$\boxed{N = \dfrac{M_m V_m}{M_T V_T} \text{ marshmallows, rounded up}}$

Numerical Solution:

$N = \dfrac{(28.2\text{ kg})(3.45\text{ m/s})}{(0.0115\text{ kg})(53.4\text{ m/s})} = 158.43$

$\boxed{\text{The cadet must fire 159 marshmallows}}$

The marshmallows being fired onto the monster are doing work on the system. Because the forces on the monster are not doing zero work, we can say that Kinetic Energy is not conserved either.

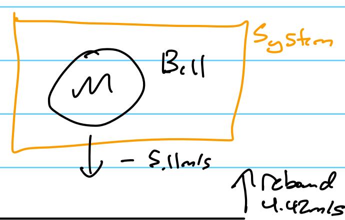
Problem 2)

Medicine Ball Dropped

mass M = 8.11 kg

Downward speed (−) = V_0 = 5.11 m/s

Rebound speed (+) = V_f = 4.42 m/s



a) CofME and CofLM Neither the energy or momentum is conserved with respect to the system of the ball. This is because the outside force of the floor acts on the ball and the velocity before and after impact changes, indicating a loss of energy.

b) Impulse

$I = \Delta p$

$= m_A V_A' - m_A V_A$ — V_0 is negative

$\therefore M \cdot V_f + M \cdot V_0$

$\boxed{I = M(V_f + V_0)}$

$I = 8.11\text{ kg}(4.42\text{ m/s} + 5.11\text{ m/s})$

$= \boxed{77.3 \dfrac{\text{kg}\cdot\text{m}}{\text{s}}}$

Ball rebounds so net force should be in positive direction

c) Impulse, $\Delta t = 0.056$ sec ← given

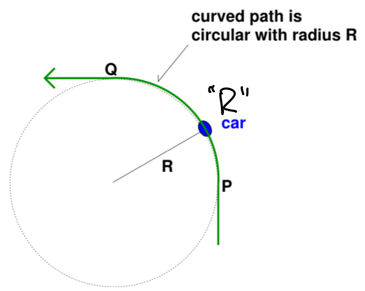
$F_{Avg} \cdot \Delta t = \Delta p$

$F_{Avg} = \dfrac{\Delta p}{\Delta t} = \dfrac{I}{\Delta t}$

$\boxed{F_{Avg} = \dfrac{M(V_f + V_0)}{\Delta t}}$ ⟹ $F_{Avg} = \dfrac{77.2883\text{ N}\cdot\text{s}}{0.056\text{ s}}$ — From previous question

$= \boxed{1380\text{ N}}$

Problem 3)



Given

Constant speed: V

driver mass: m

Car mass: M

Radius: R

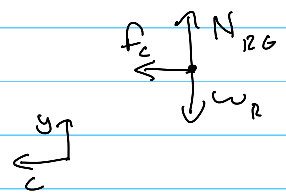
a) UCM and N2L

$F_{Net} = m a_c$, $a_c = \frac{V^2}{R}$ as the track is circular and we move at constant speed.

$$\boxed{F_{Net} = m \cdot \frac{V^2}{R}}$$

b) The student is incorrect. The direction of the centripetal acceleration always points along the radial line towards the circles center in UCM. This means the net force must be radial inwards, or to the drivers left!

c) FBD Point R



Because the car is not moving off its path and its motion is circular, there must be a frictional force present. It is in the centripetal direction, as shown to the left.

N2L and UCM

$F_c = m a_c$, $a_c = \frac{V^2}{R}$

$f = M \cdot \frac{V^2}{R}$

Frictional Force on car only: $\boxed{f = M \cdot \frac{V^2}{R}}$

On car/driver system: $\boxed{f = (m+M)\frac{V^2}{R}}$

d) • The work done by the weight force is 0 because weight is a conservative force.

• The work done by the normal force is 0 because it points perpendicular to direction of motion.

Work Energy Relation

$W_{Tot} = \Delta K$

$\Delta K = \frac{1}{2} m v'^2 - \frac{1}{2} m v^2$

Because speed is constant,

$v' = v = V_{car}$

$\Delta K = \frac{1}{2} m (V_{car} - V_{car}) = 0$

$\Delta K = 0 \therefore W_{Tot} = 0$

$\Downarrow$

$\Rightarrow$ The total work done on the car by all forces is equal to zero.

Problem 4)

Given

Constant Torque: $\tau = 0.073\ N \cdot m$

Free-spinning wheel: mass $= m = 4.3\ kg$

Radius of wheel: $R = 0.27\ m$

Rotational Inertia: $I = mR^2$

a) N2L Rot $\quad \tau_{Net} = I\alpha$

$$\frac{\tau_{Net}}{I} = \alpha$$

$$\boxed{\alpha = \frac{\tau_{Net}}{mR^2}} = \frac{0.073\ N \cdot m}{4.3\ kg(.27\ m)^2}$$

$$= \frac{0.073\ kg \cdot m \cdot m \cdot \frac{1}{s^2}}{4.3\ kg(.27^2\ m^2)} = 0.23\ s^{-2}$$

$$\boxed{\alpha = 0.23\ s^{-2}}$$

b) Constant Translational Motion w/ Const. α

$\alpha = $ const. $\quad \omega_0 = 0 \rightarrow$ as wheel starts at rest

$\omega = \omega_0 + \alpha t$

Find t when $\omega = 22.2$ $\quad \alpha = 0.23\ s^{-2}$

$\omega = 0 + \alpha t$

$$\boxed{t = \frac{\omega}{\alpha}} \Rightarrow t = \frac{22.2\ s^{-1}}{0.23\ s^{-2}} = \boxed{96.5\ sec}$$

c) Constant Accel $\quad$ Rolling Constraint $\quad$ UCM

$\omega = \omega_0 + \alpha t \quad T = 5.0\ sec \quad v = \omega R,\ \omega = \alpha t \quad a_c = \frac{v^2}{R},\ v = \alpha t R$

$\omega = 0 + \alpha t \quad \omega_0 = 0 \quad \therefore v = \alpha t R \quad a_c = \frac{(\alpha t R)^2}{R}$

$\omega = \alpha t$

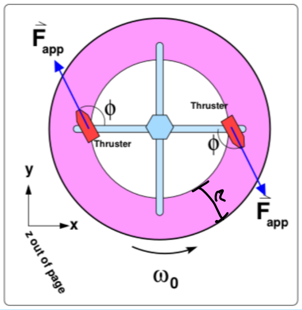
$$\boxed{a_c = (\alpha t)^2 R} \Rightarrow a_c = (0.23\ s^{-2}(5.0\ sec))^2 \cdot 0.27\ m$$

$$= \boxed{0.36\ m/s^2}$$

Instantaneous Accel

α not given, put in terms of parameters!

Problem 5)



Given

Angular Speed: ω_0

Mass of station: M

Radius of Torus: R

Given Constant: C

Rotational Inertia: $I = CMR^2$

Thrusters r distance from center

↳ ϕ angle from radial line

↳ F_{app} - constant magnitude

Clockwise is positive

↳ Thrusters apply neg. torque

a) Definition of Torque

$\tau_F = rF\sin\phi$

Total torque must be Double this, Should be negated but this is magnitude!

$$\boxed{\tau = 2rF_{app}\sin\phi}$$

b) N2L Rot

$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I}$

$\left.\begin{aligned} \tau &= -2rF_{app}\sin\phi \\ I &= CMR^2 \end{aligned}\right\}$ All given

$$\therefore \boxed{\alpha = \frac{2rF_{app}\sin\phi}{CMR^2}}$$

magnitude, so negate is dropped

c) N3L $F_{app_{T_1}} = F_{app_{T_2}}$

The thruster's applied force is equal and opposite, so there is no translational movement. The thrusters only slow spin, so the linear acceleration of the whole body is 0.

d) α doesn't depend on time, so it is constant. Constant accel

$\omega = \omega_0 + \alpha t$; at rest, $\omega = 0$

$0 = \omega_0 - \alpha t$; α is negative

$t = \frac{\omega_0}{\alpha}$

$\theta = \theta_0 + \omega_0 t - \frac{1}{2}\alpha t^2$ $\quad \theta_0 = 0$

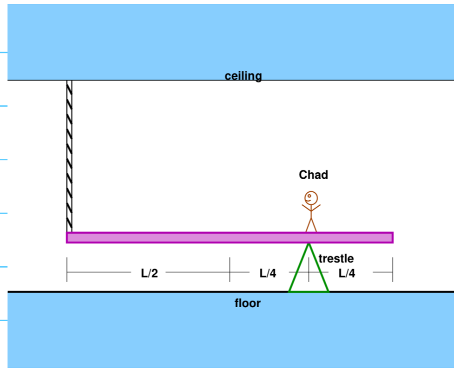
$\theta = \frac{\omega_0^2}{\alpha} - \frac{1}{2}\left(\frac{\omega_0}{\alpha}\right)^2(\alpha)$ $\quad$ (radians) $\times \frac{1 \text{ rotation}}{2\pi \text{ radians}}$

$\theta = \frac{\omega_0^2}{\alpha} - \frac{1}{2}\left(\frac{\omega_0^2}{\alpha}\right) = \frac{\omega_0^2}{\alpha}$

$\theta = \frac{1}{2}\left(\frac{CMR^2\omega_0^2}{2rF_{app}\sin\phi}\right)$

$$\boxed{\text{Rotations} = \frac{1}{2\pi}\left(\frac{CMR^2\omega_0^2}{4rF_{app}\sin\phi}\right)}$$

Problem 6)



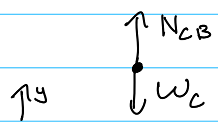
Given

Chad's mass: m

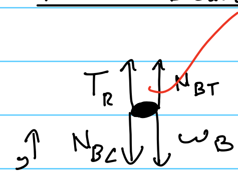
Board Mass: M

Total length: L

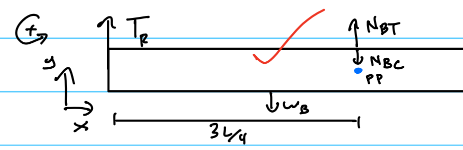
a) FBD Chad



FBD Board



XFBD Board



b) N2L Board

$F_B = m_B a \qquad a = 0$

$F_B = 0$

$T_r + N_{BT} - N_{BC} - W_B = 0$

$T_r + N_{BT} - N_{BC} - m_B g = 0$

Stop!

N2L Rot

$\tau_{Net} = I\alpha, \; \alpha = 0$

$\tau_{T_R} + \tau_{N_{BT}} - \tau_{N_{BC}} - \tau_{w_B} = 0$ ← all have sin 90° which = 1

$\frac{3}{4}L(T_r) + 0 - 0 - \frac{1}{4}L(w_B) = 0$

$\frac{3}{4}T_r - \frac{1}{4}m_B g = 0$

$\frac{3}{4}T_r = \frac{1}{4}m_B g \Rightarrow \boxed{T_r = \frac{1}{3}m_B g}$

N2L Chad

$F_c = m_c a, \; a = 0$

$N_{CB} - W_c = 0$

$N_{CB} = m_c g$

$N_{CB} = N_{BC}$ N3L

Recall: $T_r + N_{BT} - N_{BC} - m_B g = 0$

$\frac{1}{3}m_B g + N_{BT} - m_c g - m_B g = 0$

$N_{BT} = m_B g + m_c g - \frac{1}{3}m_B g$

$N_{BT} = g\left(m_B\left(1 - \frac{1}{3}\right) + m_c\right) \Rightarrow \boxed{N_{BT} = g\left(\frac{2}{3}m_B + m_c\right)}$