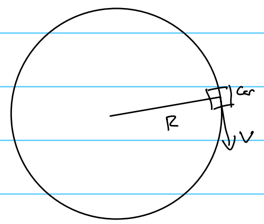
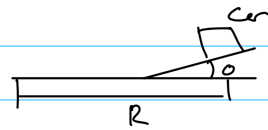
Practice Exam 1

1.





Given

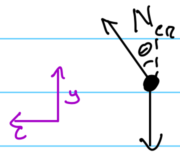
- Car Mass M
- Radius of Path R
- θ not given

a) UCM Circular Path with Radius R. Given Speed v $\boxed{a=\frac{v^2}{R}}$

b) N2L $F_{Net}=ma$

$a=\frac{v^2}{R}$ $m=M$ $\boxed{F_{Net}=M\frac{v^2}{R}}$

c) FBD Car



d) $N_c = N\sin\theta$

$N_y = N\cos\theta$

N2L c-dir

$F_c = m a_c$

$N\sin\theta = M\frac{v^2}{R}$

N2L y-dir

$F_y = m\cancel{a_y}^{0}$

$N\cos\theta - mg = 0$

$N\cos\theta = mg$

$N=\frac{mg}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta}\cancel{M}g = \cancel{M}\frac{v^2}{R}$

$\tan\theta = \frac{v^2}{gR}$

$\boxed{\theta=\arctan\left(\frac{v^2}{gR}\right)}$

2. $x(t) = R\omega t - C\sin(\omega t)$

$y(t) = R - C\cos(\omega t)$

a) $\vec{r}(t) = (R\omega t - C\sin(\omega t))\hat{i} + (R - C\cos(\omega t))\hat{j}$

$\vec{r}\left(\frac{\pi}{\omega}\right) = \left(R\omega\left(\frac{\pi}{\omega}\right) - C\sin(\pi)\right)\hat{i} + (R - C\cos(\pi))\hat{j}$

$= (R\pi)\hat{i} + (R+C)\hat{j}$

$$\boxed{\vec{r}(t) = (R\pi)\hat{i} + (R+C)\hat{j}}$$

b) $\vec{v}(t) \equiv \frac{d}{dt}\vec{r}(t) = (R\omega - C\omega\cos(\omega t))\hat{i} + (C\omega\sin(\omega t))\hat{j}$

Speed $= \sqrt{(R\omega - C\omega\cos(\omega t))^2 + (C\omega\sin(\omega t))^2}$

$= \sqrt{R^2\omega^2 - 2RC\omega^2\cos(\omega t) + C^2\omega^2\cos^2(\omega t) + C^2\omega^2\sin^2(\omega t)}$

$= \sqrt{R^2\omega^2 - 2RC\omega^2\cos(\omega t) + C^2\omega^2}$

Speed $= \omega\sqrt{R^2 - 2RC\cos(\omega t) + C^2}$

@ $t = \frac{\pi}{\omega} \therefore = \omega\sqrt{R^2 - 2RC\cos(\pi) + C^2}$

$= \omega\sqrt{R^2 + 2RC + C^2} = \omega\sqrt{(R+C)^2} = \omega(R+C)$

$$\boxed{\text{Speed} = \omega(R+C)}$$

c) Circular Path $\therefore$ UCM

$a = \frac{v^2}{R_c}$, R_c = Radius$_c$

$\therefore R_c = \frac{v^2}{a}$

$R_c = \frac{(\omega(R+C))^2}{C\omega^2}$

$$\boxed{R_c = \frac{(R+C)^2}{C}}$$

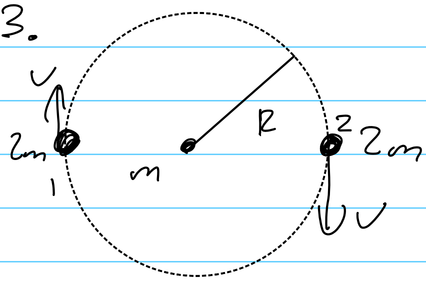
$\vec{a}(t) = C\omega^2\sin(\omega t)\hat{i} + C\omega^2\cos(\omega t)\hat{j}$

$|\vec{a}(t)| = \sqrt{(C\omega^2\sin(\omega t))^2 + (C\omega^2\cos(\omega t))^2}$

$= \sqrt{C^2\omega^4(\sin^2(\omega t) + \cos^2(\omega t))}$

$= C\omega^2$

3.



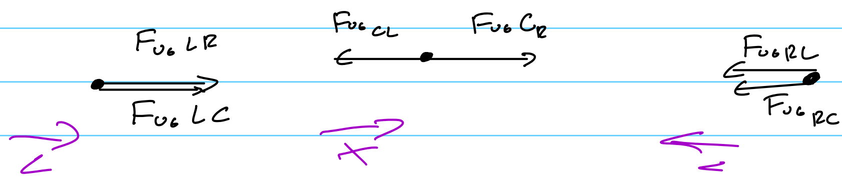
Given

$M_{outer} = 2m$ for both

Radius $= R$

Small Star Center $M_{center} = m$

a) FBD Outer L — FBD Central — FBD Outer R



b) By N3L $F_{UG\,LC} = F_{UG\,CL}$. By symmetry we can see this for the right outer star as well. The forces balance each other out results in zero net force on central.

c) Net Force is Vector Sum

$$F_{Net} = F_{LR} + F_{LC}$$

$$= \frac{G M_L m_R}{(2R)^2} + \frac{G M_L m_C}{(R)^2}$$

$$= \frac{G(2m)(2m)}{4R^2} + \frac{G(2m)(m)}{R^2}$$

$$= \frac{Gm^2}{R^2} + \frac{G2m^2}{R^2} = \frac{3 \cdot G \cdot m^2}{R^2}$$

Vector Sum Magnitude $= \boxed{\dfrac{3 \cdot G \cdot m^2}{R^2}}$

d) UCM Circular Path

$$a_c = \frac{v^2}{R}$$

$$F_{LC} = m_L a_c$$

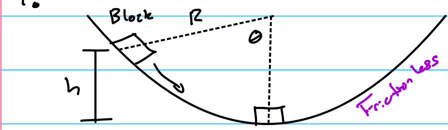
$$\frac{3 \cdot G \cdot m^2}{R^2} = m_L\left(\frac{v^2}{R}\right)$$

$$\frac{3 \cdot G \cdot m^2}{R^2} = 2m\frac{v^2}{R}$$

$$v^2 = \frac{3 \cdot G \cdot m}{2R} \qquad v = \sqrt{\frac{3 \cdot G \cdot m}{2R}}$$

Practice Exam 2

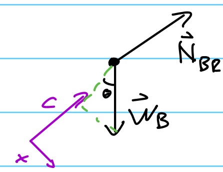
1.



• $h = R(1-\cos\theta)$, R and θ given

• Block of Mass m @ rest

a) FBD Block



b) N2L Block c-dir

$F_c = m_B \cancel{a_{Bc}}^{0}$ UCM $a_c = \frac{v^2}{R}$

$N_{BR} - W_{\cos\theta} = 0$ Released @ rest → $v=0$

$\boxed{N_{BR} = mg\cos\theta}$ ∴ $a_c = \frac{0}{R} = 0$

c) N2L Block x-dir

$F_x = m_B a_{Bx}$

$W_{\sin\theta} = m_B a_x$

$\boxed{a_x = g\sin\theta}$

d) $W_{cons} = -\Delta U_{cons}$

$U_{\vec{w}} = mgy$, $W_{\vec{w}} = -(mgy' - mgy)$

$= mgy - mgy'$

Let $y'=0$ and $y = h = R(1-\cos\theta)$

$\boxed{W_{cons} = mgR(1-\cos\theta)}$

e) C of ME

Conditions: No friction, Normal Force does no work, and weight is conservative so we chill!

e) Before = After

$E_{Tot} = E'_{Tot}$

$U + K = U' + K'$

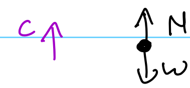
$mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2$

$y'=0$, $y = R(1-\cos\theta)$, $v=0$

$\cancel{m}g(R(1-\cos\theta)) = \frac{1}{2}\cancel{m}v'^2$

$v'^2 = 2gR(1-\cos\theta)$

FBD Block Bottom



N2L Block Bottom c-dir

$F_{Bc} = m_B a_{Bc}$ UCM $a_c = \frac{v^2}{R}$

$N - W = m\left(\frac{v'^2}{R}\right)$

$N = m\left(\frac{2g\cancel{R}(1-\cos\theta)}{\cancel{R}}\right) + mg$

$N = mg(2(1-\cos\theta)+1)$

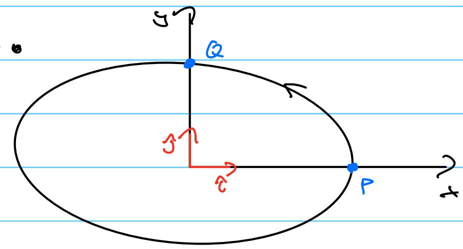
$N = mg(2-2\cos\theta+1)$

$N = mg(3-2\cos\theta)$

$N_{BR} = mg(3-2\cos\theta)$ so $N_{RB} = mg(3-2\cos\theta)$ by N3L

$\boxed{N_{RB} = mg(3-2\cos\theta) \text{ points downwards}}$

2.



Given

- Particle mass m
- Oval Path
- $\vec{r}(t) = 2A\cos(\omega t)\hat{\imath} + A\sin(\omega t)\hat{\jmath}$
  - ↳ A and ω given and positive

a) Position Vector's Components = coordinates

$\vec{r}(0) = 2A\cos(0)\hat{\imath} + A\sin(0)\hat{\jmath}$

$\vec{r}(0) = 2A\hat{\imath} + 0\hat{\jmath}$

$$\boxed{\begin{array}{l} t=0:\ (2A, 0) \\ \text{Speed} = A\omega \end{array}}$$

$\vec{v}(t) = \frac{d}{dt}\vec{r}(t)$

$\vec{v}(t) = -2A\omega\sin(\omega t)\hat{\imath} + A\omega\cos(\omega t)\hat{\jmath}$

$\vec{v}(0) = -2A\omega\sin(0)\hat{\imath} + A\omega\cos(0)\hat{\jmath}$

$= A\omega$

b) N2L $\quad F_{Net} = m\vec{a}$

$\vec{a}(t) = -2A\omega^2\cos(\omega t)\hat{\imath} - A\omega^2\sin(\omega t)\hat{\jmath}$

$\vec{a}(0) = -2A\omega^2\cos(0)\hat{\imath} - A\omega^2\sin(0)\hat{\jmath}$

$= -2A\omega^2\hat{\imath}$

$$\boxed{F_{Net} = -2A\omega^2 m\hat{\imath}}$$

c) $t = \frac{\pi}{2\omega}$: $\vec{a}\left(\frac{\pi}{2\omega}\right) = -2A\omega^2\cos\left(\frac{\pi}{2}\right)\hat{\imath} - A\omega^2\sin\left(\frac{\pi}{2}\right)\hat{\jmath}$

$= 0\hat{\imath} - A\omega^2\hat{\jmath}$

$\vec{v}\left(\frac{\pi}{2\omega}\right) = -2A\omega\sin\left(\frac{\pi}{2}\right)\hat{\imath} + A\omega\cos\left(\frac{\pi}{2}\right)\hat{\jmath}$

$= -2A\omega\hat{\imath} + 0\hat{\jmath}$

$a_c = \frac{v^2}{R_c}$ $\quad$ UCM Assumed

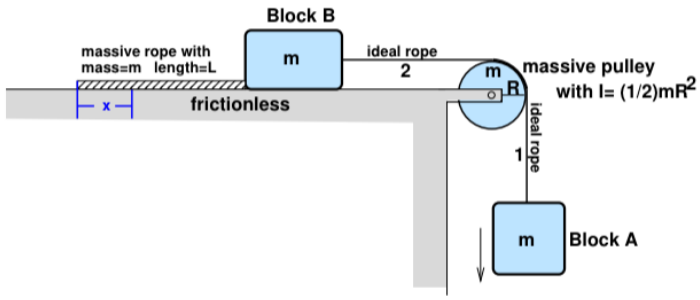
$R_c = \frac{v^2}{a_c}$

$v = \sqrt{(-2A\omega)^2} = 2A\omega$

$a = \sqrt{(-A\omega^2)^2} = A\omega^2$

$R_c = \frac{(2A\omega)^2}{A\omega^2} = \frac{4A^2\omega^2}{A\omega^2} = 4A \quad \therefore \quad \boxed{R_c = 4A}$

3.

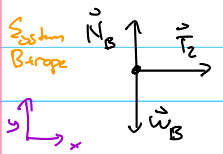


Given

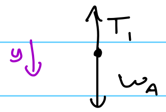
- A and B mass m
- Ideal Rope around Massive
- Pulley: mass m, R radius, $I = \frac{1}{2}mR^2$
- B on frictionless
- Massive Rope mass m Length L

a) FBD B

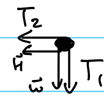
System B+rope



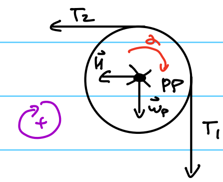
FBD A



FBD Pulley



*FBD Pulley



N2L B x-dir

$F_{B_x} = m_B a_{B_x}$

$T_2 = 2m a_B$

N2L A y-dir

$F_{A_y} = m_A a$

$W_A - T_1 = m_A a_A$

$mg - T_1 = m a_B$

N2L Rot Pulley

$\tau_{Net} = I\alpha$

$\tau_N + \tau_w + \tau_{T_1} + \tau_{T_2} = \frac{1}{2}mR^2\alpha$

$0 + 0 + \cancel{R}T_1\sin(90°) + \cancel{R}T_2\sin(90°) = \frac{1}{2}mR^{\cancel{2}}\alpha$

$T_1 - T_2 = \frac{1}{2}mR\alpha$

Kinematic Constraint

$a_B = a_A$ move together

R.C. $a = R\alpha$, $\alpha = \frac{a_B}{R}$ } $T_1 - T_2 = \frac{1}{2}mR\left(\frac{a_B}{R}\right)$

$T_1 - T_2 = \frac{1}{2}m a_B$

3 equations: $T_2 = 2m a_B$

$T_1 = m(g - a_B)$

$T_1 - T_2 = \frac{1}{2}m a_B$

$mg - ma_B - 2ma_B = \frac{1}{2}ma_B$

$mg - 3ma_B = \frac{1}{2}ma_B$

$\cancel{m}g = \frac{7}{2}\cancel{m}a_B$

$\boxed{a_B = \frac{2}{7}g}$

$T_2 = 2m a_B$

$T_2 = 2m\left(\frac{2}{7}g\right)$

$\boxed{T_2 = \frac{4}{7}mg}$

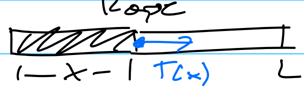
$T_1 = m(g - a_B)$

$T_1 = m\left(g - \frac{2}{7}g\right)$

$T_1 = mg\left(1 - \frac{2}{7}\right)$

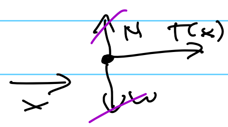
$\boxed{T_1 = \frac{5}{7}mg}$

b) System Rope



$\text{mass} = \left(\frac{x}{L}\right)m$

FBD System



N2L System x-dir

$F_{sys_x} = m_{sys} a_{sys}$

$a_{sys} = \frac{2}{7}g$ from (a)

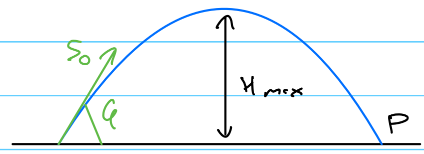
$m_{sys} = \left(\frac{x}{L}\right)m$

$T(x) = \left(\frac{x}{L}\right)m\left(\frac{2}{7}\right)g$

$\therefore \boxed{T(x) = \frac{2mgx}{7L}}$

Practice Exam 3

1.



S_0 given, θ given, Constant Speed

a) $\vec{r}(t) = (x_0 + v_{x_0} t)\hat{i} + (y_0 + v_{y_0} t - \frac{1}{2} g t^2)\hat{j}$

$x_0 = 0, \quad v_{x_0} = S_0 \cos\theta$

$y_0 = 0, \quad v_{y_0} = S_0 \sin\theta$

$\vec{v}(t) = v_{x_0}\hat{i} + (v_{y_0} - gt)\hat{j}$

$\vec{v}(t) = S_0 \cos\theta\,\hat{i} + (S_0 \sin\theta - gt)\hat{j}$

$v_x = S_0 \cos\theta \qquad v_y = 0$ @ H_{max}

$v_y = S_0 \sin\theta - gt \qquad 0 = S_0 \sin\theta - gt \rightarrow t = \frac{S_0 \sin\theta}{g}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$, $y_0 = 0$, $v_{y_0} = S_0 \sin\theta$, $t = \frac{S_0 \sin\theta}{g}$

$H = S_0 \sin\theta \left(\frac{S_0 \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{S_0 \sin\theta}{g}\right)^2$

$H = \frac{(S_0 \sin\theta)^2}{g} - \frac{(S_0 \sin\theta)^2}{2g} \rightarrow \boxed{H = \frac{(S_0 \sin\theta)^2}{2g}}$

b) $a_c = \frac{v^2}{R_c}$ Centripetal component of acceleration