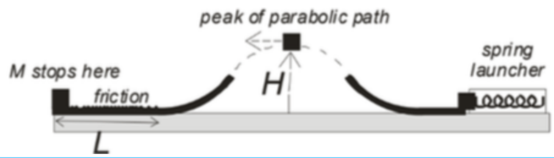
SI Summary Session 2

Problem 1)



Given: Spring Const k, Displacement x
Block Mass M, m at height H
Frictionless → Friction with μ_k

a) Normal Force points perp. to direction of motion. Friction Stops Block over dist L
So the Normal Force does 0 work for the whole time.

b) C of ME

Conditions: Frictionless so no work done by friction. Same with Normal Force. F_{sp} is also conservative and so is weight. So conditions met!

Before = After

$$E_{Tot} = E'_{Tot}$$

$$U_{sp} + U_w + K = U'_{sp} + U'_w + K'$$

$$\tfrac{1}{2}kx^2 + mgy + \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx'^2 + mgy' + \tfrac{1}{2}mv'^2$$

$y=0,\ v=0,\ x'=0,\ y'=H$

$$\tfrac{1}{2}kx^2 = mgH + \tfrac{1}{2}mv^2$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kx^2 - mgH$$

$$v^2 = \tfrac{k}{m}x^2 - 2gH$$

$$\boxed{v_p = \sqrt{\tfrac{k}{m}x^2 - 2gH}}$$

c) Work Energy Thm.

$W_{Tot} = \Delta K$

$\Delta K = K' - K$

$K' = \tfrac{1}{2}mv'^2 \qquad v' = 0 \leftarrow$ stopped

$K = \tfrac{1}{2}mv^2 \qquad v = 0 \leftarrow$ at rest

Work $= Fd\cos\theta$

$W_{Tot} = W_w + W_{sp} + W_f$

$W_w = -mg\,\Delta y$

$W_{sp} = F_{sp}(x' - x)\cos 180°$

$= -\tfrac{1}{2}k\,\Delta x$

Work done by Conservative force is equal to negative change in potential energy

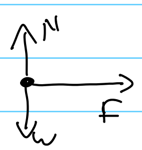
$W_f = F_f L \cos 180°$

$= F_f L(-1)$

$F_{sp} = kx$

N2L Block y-dir

$F_y = m\cancel{a_y}^{\,0}$



$N - W = m0$

$N - mg = 0$

$N = mg$

$f = \mu N \Rightarrow f = \mu_k mg$

$\therefore W_{Tot} = mgy - mgy' + \tfrac{1}{2}kx^2 - \tfrac{1}{2}kx'^2 - \mu_k mgL$

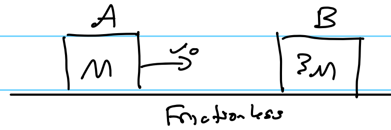
$W_{Tot} = 0,\ y = 0,\ y' = 0,\ x'^2 = 0$

$0 = \tfrac{1}{2}kx^2 - \mu_k mgL$

$\mu_k mgL = \tfrac{1}{2}kx^2$

$$\boxed{L = \frac{kx^2}{2\mu_k mg}}$$

Problem 2)



Given: $m_A = M$

$m_B = 3M$

A has $v_A = v_0$, B has $v_0 = 0$

a) Totally Inelastic

C of LM

Condition: Isolated systems, frictionless surface.

Before = After

$P_{Tot} = P'_{Tot}$

$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$m_A V_0 = v_f (m_A + m_B)$

Kinematic Constraint

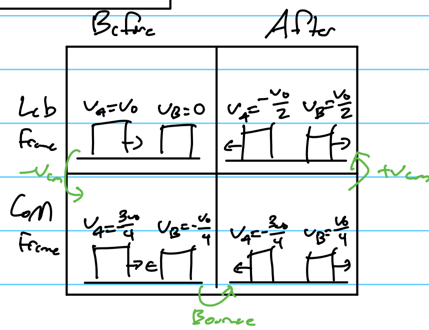
$v_A' = v_B' = v_f$ as totally inelastic "stick"

$$v_f = \frac{m_A v_0}{m_A + m_B} \qquad m_A = M \quad m_B = 3M$$

$$v_f = \frac{M}{4M} v_0 \implies \boxed{v_f = \frac{v_0}{4}}$$

b) Totally Inelastic

$$V_{cm} = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{v_0}{4}$$



$$\boxed{\begin{aligned} V_A' &= -\frac{v_0}{2} \\ V_B' &= \frac{v_0}{2} \end{aligned}}$$

c) C of LM

$v_A = v_0,\ v_B = 0,\ v_A' = 0,\ v_B' = ?$

Condition: Isolated systems, frictionless surface.

$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$M v_0 = 3M v_B'$

$$\boxed{v_B' = \frac{v_0}{3}}$$

d)

Fapp causes external force ∴ N2L not C of LM or anything

N2L System

$F_{sys} = M_{sys} a_{sys}$ (Net Ext)

$F_{app} = M_{sys} a_{sys}$

$M_{sys} = M + 3M = 4M$

$a_{sys} = a_{cm}$

$F_{app} = 4Ma$

$a = \frac{F_{app}}{4M}$

$v_{cm} = \int a = V t + v_0$

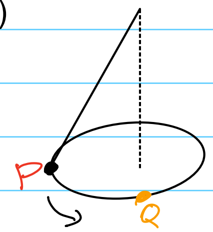
$V_{cm} = \frac{F_{app}}{4M} t + V_0$

$V_0 = \frac{v_0}{4}$ per part b

Velocity right before Fapp at $t = 0$. Original v_{cm}

$$\boxed{V_{cm} = \frac{F_{app}}{4M} t + \frac{v_0}{4}}$$

Problem 3)



$\vec{r}(t) = 3A\cos(\omega t)\hat{i} + 2A\sin(\omega t)\hat{j}$

Given: A and $\omega > 0$

Mass of Bob m

a) P is $t=0$

$\vec{v}(t) = -3A\omega\sin(\omega t)\hat{i} + 2A\omega\cos(\omega t)\hat{j}$

$\text{speed} = \sqrt{(-3A\omega\sin(\omega t))^2 + (2A\omega\cos(\omega t))^2}$

$= \sqrt{9A^2\omega^2\sin(\omega t)^2 + 4A^2\omega^2\cos(\omega t)^2}$

$\boxed{\text{Speed} = \sqrt{A^2\omega^2(9\sin^2(\omega t) + 4\cos^2(\omega t))}}$ @ $t=0$ $\text{Speed} = \sqrt{A^2\omega^2(4)}$

$\boxed{\text{Speed} = 2A\omega}$

b) N2L $F = ma$ $\qquad \vec{a} = \frac{d\vec{v}}{dt}$

$a = ?$

$v(t) = -3A\omega\sin(\omega t)\hat{i} + 2A\omega\cos(\omega t)\hat{j}$

$a(t) = -3A\omega^2\cos(\omega t)\hat{i} - 2A\omega^2\sin(\omega t)\hat{j}$

$t=0$: $a(0) = -3A\omega^2\hat{i} - 0 = -3A\omega^2\hat{i}$

$\boxed{F_{Net} = -3Am\omega^2\hat{i}}$

c) Bob! @ $t = \frac{\pi}{2\omega}$

Radius of Curvature?

UCM $a_c = \frac{v^2}{R_c}$

$R_c = \frac{v^2}{a_c}$

$v(t) = -3A\omega\sin(\omega t)\hat{i} + 2A\omega\cos(\omega t)\hat{j}$

$a(t) = -3A\omega^2\cos(\omega t)\hat{i} - 2A\omega^2\sin(\omega t)\hat{j}$

$v\left(\frac{\pi}{2\omega}\right) = -3A\omega\sin\left(\frac{\pi}{2}\right)\hat{i} + 2A\omega\cos\left(\frac{\pi}{2}\right)\hat{j}$

$= -3A\omega\hat{i}$

$a\left(\frac{\pi}{2\omega}\right) = -3A\omega^2\cos\left(\frac{\pi}{2}\right)\hat{i} - 2A\omega^2\sin\left(\frac{\pi}{2}\right)\hat{j}$

$= -2A\omega^2\hat{j}$

$\left|v\left(\frac{\pi}{2\omega}\right)\right| = 3A\omega$

$\left|a\left(\frac{\pi}{2\omega}\right)\right| = 2A\omega^2$

$R_c = \frac{(3A\omega)^2}{2A\omega^2} = \frac{9}{4}A \rightarrow \boxed{R_c = \frac{9}{4}A}$