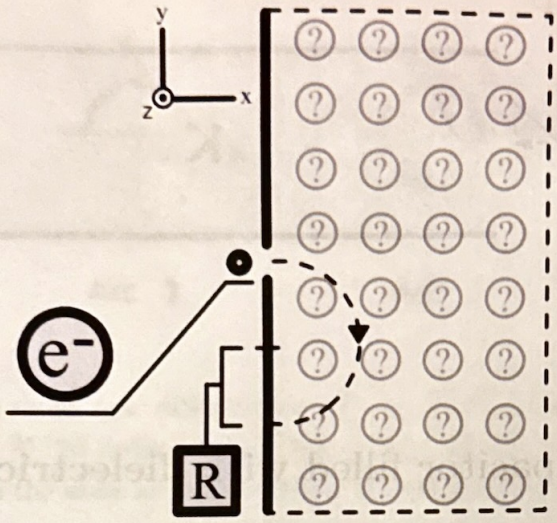


Problem 1: Trajectory [15 points]

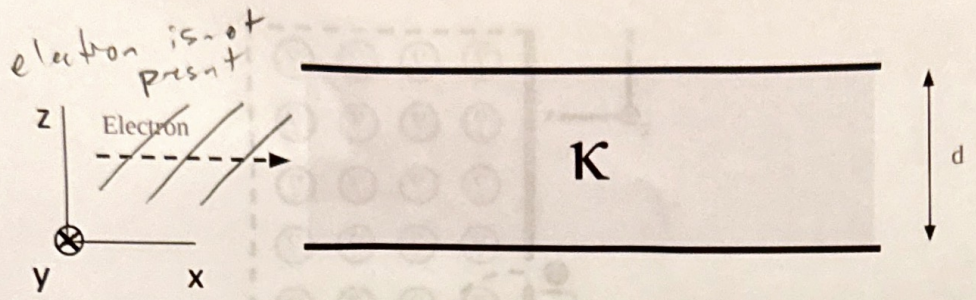
An electron is fired with an initial velocity v in the positive x-direction. Upon entering a region of uniform magnetic field B, the electron is observed to curve downward in the negative y-direction, following a circular arc at constant speed.

(a) [5 points] Based on the electron's observed motion, determine the direction of the magnetic field. Copy the provided sketch in your answer sheet and draw in the magnetic field as viewed from above, using appropriate symbols. Explain your reasoning both mathematically and in words.

(b) [2.5 points] Applying Newton's Second Law and the Lorentz Force, derive an expression for the radius R of the electron's circular trajectory in terms of m, v , q, and B.

(c) [5 points] Suppose a proton is fired with the same initial velocity v into the same magnetic field. In the figure you previously drew, qualitatively sketch the proton's trajectory.

(d) [2.5 points] Would the radius of the proton's trajectory be larger, smaller, or the same compared to that of the electron's? Justify your reasoning conceptually or mathematically knowing that the proton is approximately 2000 times heavier than the electron.



Problem 2: Capacitor filled with dielectric [20 points]

Consider a parallel plate capacitor with a separation d between the plates and an area A for each plate. The plates are parallel to the xy-plane with separation along the z-axis.

(a) [2.5 points] Calculate the electric field between the plates of the empty capacitor using Gauss Law. Write it as a function of σ_{free}, where σ_{free} is the free surface charge density on the top plate (assuming that the top plate has a positive charge and the bottom plate is negative). Feel free to assume that $A >> d^2$ and that the electric field is constant inside the capacitor.

(b) [2.5 points] Calculate the capacitance of the empty capacitor.

Now we fill the space between the plates with a linear and isotropic dielectric material with a dielectric constant $\kappa = 1 + \chi$.

(c) [5 points] Calculate the electric field (magnitude and direction) inside the capacitor when filled with the dielectric material.

(d) [5 points] Calculate the polarization vector $\vec{P}$ inside the dielectric material. Express your answer in terms of χ. Indicate the direction of the polarization vector.

(e) [5 points] Determine the surface bound charge density σ_b on the surfaces of the dielectric in contact with the capacitor plates. Specify the sign of the bound charge on each surface.

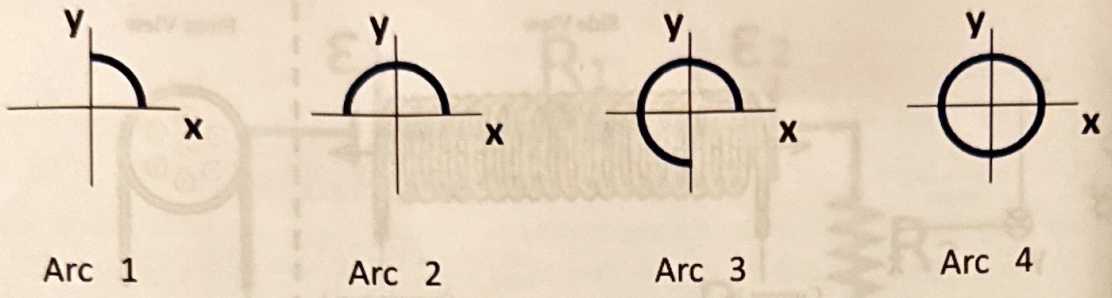


Figure 1: The picture shows four different arcs of wire, each of them subtending an angle (left to right) of $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π radians respectively. The radius is R for all of them. All of them are charged with the same amount of charge Q uniformly deposited.

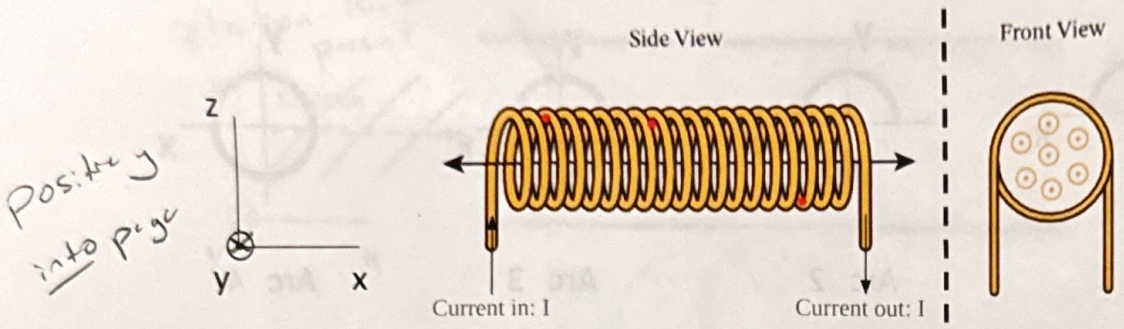
Problem 3: Biot-Savart [15 points]

Four arcs of different length have been charged with a Q, uniformly deposited, along the whole length. For simplicity assume $Q > 0$.

(a) [5 points] Calculate the linear charge density λ_i for each arc.

(b) [5 points] In what direction does the electric field point at the origin (0,0) based on symmetry in each case? Use vector notation.

(c) [5 points] Calculate the magnitude of the Electric field at the origin (0,0) for each of the four configurations. Order them from bigger to smaller magnitude of the electric field. Your answer may depend on some (but not necessarily all) of the following: Q, λ_i and R.



Problem 4: Ampère's and Faraday's Law [15 points]

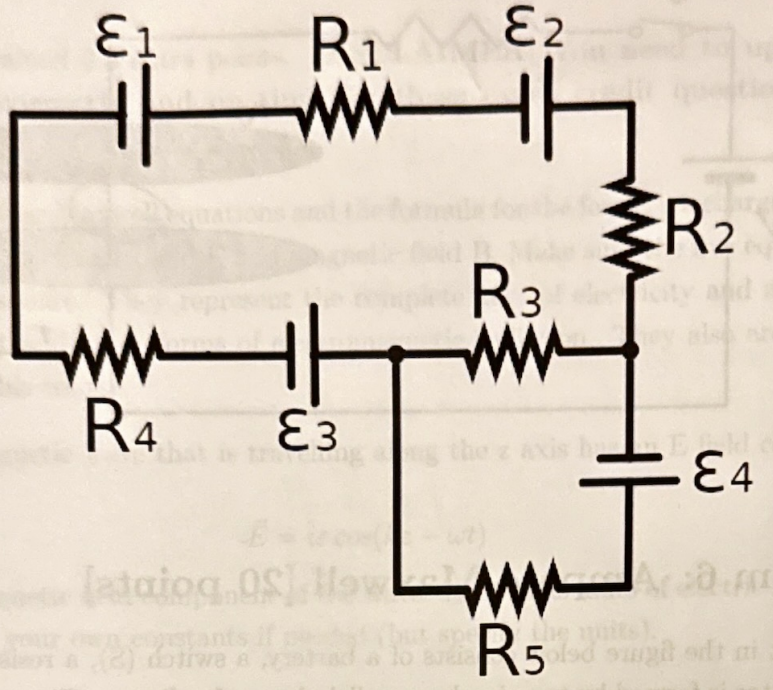
Consider an infinite solenoid with radius R and n turns per unit length carrying a time-varying current $I(t)$ which is a **strictly increasing function of time**.

(a) [5 points] Using Ampère's law, determine the magnitude and direction of the magnetic field $\vec{B}$ inside the solenoid ($r < R$). Clearly justify the choice of your Amperian loop and the obtained direction (if no justification you will lose points).

(b) To study the electric field induced by the changing current, consider a circular loop of radius r concentric with the axis of the solenoid. You can assume that the magnetic field is pointing out of the page in the front view.

(i) [5 points] Apply Faraday's law to calculate the induced electric field $\vec{E}$ inside the solenoid ($r < R$). Indicate the direction of the electric field.

(ii) [5 points] Apply Faraday's law to calculate the induced electric field $\vec{E}$ outside the solenoid ($r > R$). Indicate the direction of the electric field.



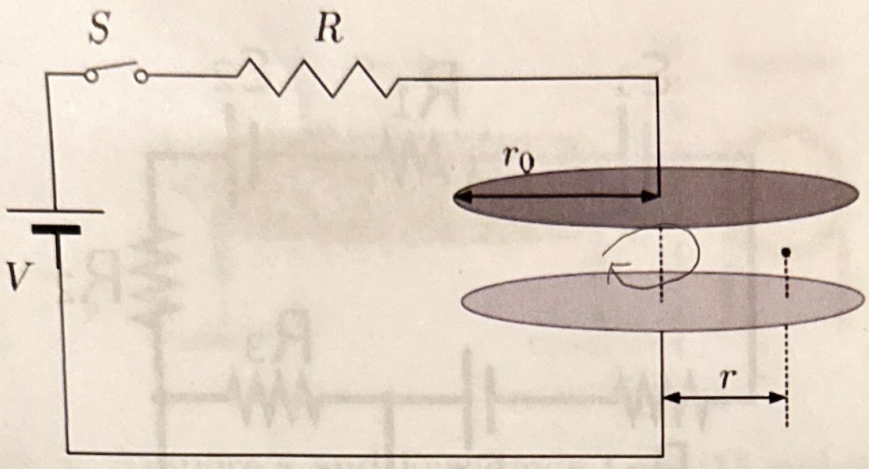
Problem 5: DC Circuit [15 points]

Consider the following circuit with five resistors and four batteries:

(a) [2.5 points] How many pairs of parallel resistors are there? If any tell us which are connected in parallel and why. Calculate the equivalent resistance of them.

(b) [2.5 points] How many pairs of series resistors are there? If any tell us which are connected in ~~parallel~~ series and why. Calculate the equivalent resistance of them.

(c) [10 points] What is the current flowing through each resistor? (Do not solve this part with equivalent resistors but with the circuit as given.)



Problem 6: Ampere-Maxwell [20 points]

The circuit in the figure below consists of a battery, a switch (S), a resistor and capacitor. The capacitor is formed by two circular parallel plates of radius r_0. The switch in the circuit is closed at t = 0. This creates a potential drop in the capacitor that reads:

$$V_C(t) = V\left(1 - e^{-t/RC}\right)$$

Where V is the voltage produced by the battery, R is the resistivity of the resistor, and C is the capacitance of the circular plates capacitor.

(a) [5 points] What is the magnetic field along the dashed line between the plates (center of the plates).

(b) [5 points] Calculate the electric field at any point inside the capacitor as a function of V_C (substitute with the function provided). Assume that that the area is much larger than the distance between the plates (you can use the formulas for the parallel plate capacitor).

(c) [5 points] Calculate the magnetic field at a point that is at a distance $r < r_0$ from the center of the plates.

(d) [5 points] Calculate the magnetic field at a point that is at a distance $r > r_0$.

Short Questions

Each of them are valued 2.5 extra points. **DISCLAIMER: You need to upload the exam to canvas correctly and on time for these extra credit questions to be graded.**

1. Write out the four Maxwell equations and the formula for the force on a charged particle of charge q in an electric field E and magnetic field B. Make sure the five equations fit on the given square. They represent the complete laws of electricity and magnetism including light and other forms of electromagnetic radiation. They also are the best summary of this course.

2. An electromagnetic wave that is travelling along the z axis has an E field component given by:

$$\vec{E} = \hat{i}\varepsilon \cos(kz - \omega t)$$

Write the magnetic field component of the wave. Here ε has units of electric field. Feel free to define your own constants if needed (but specify the units).