Practice Problems 1 - Solutions

February 28, 2024

1 Problem 1: Dipole Field

part a

It points up based on the figure below.

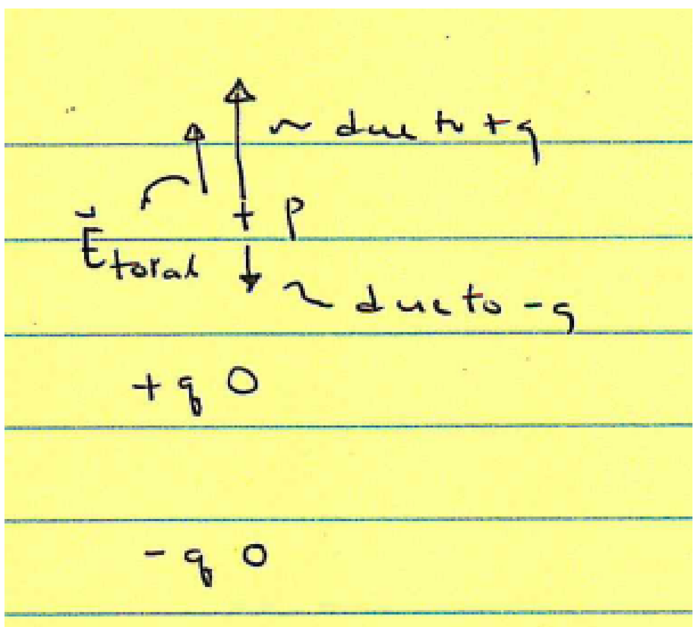

Figure 1: The overall electric field points upward based on the placement of the positive and negative charges

part b

Based on Coloumb's Law: $\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{1}{(y-a)^2}\hat{j} + \frac{-q}{4\pi\epsilon_0}\frac{1}{(y+a)^2}\hat{j}$

$$= \frac{q}{4\pi\epsilon_0}[\frac{1}{(y-a)^2} - \frac{1}{(y+a)^2}]\hat{j}$$

part c

$$\frac{1}{(y+a)^2} \approx \frac{1}{y^2} - \frac{2a}{y^3}$$
$$\frac{1}{(y-a)^2} \approx \frac{1}{y^2} + \frac{2a}{y^3}$$
$$\implies \frac{1}{(y-a)^2} - \frac{1}{(y+a)^2} \approx \frac{4a}{y^3}$$

Thus, $\vec{E} \approx \frac{4aq}{4\pi\epsilon_0}\frac{1}{y^3}\hat{j}$ for y much larger than a

part d

It points down based on the figure below.

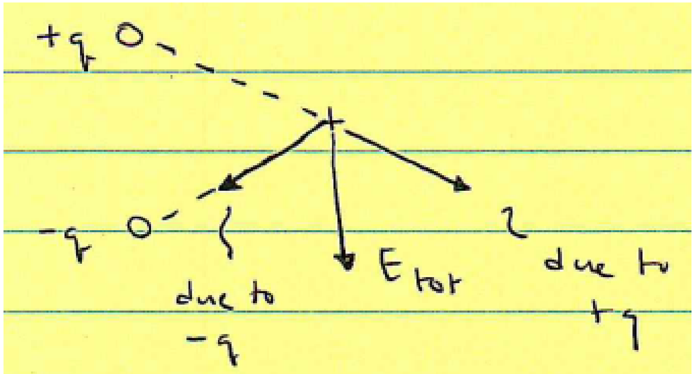

Figure 2: The overall electric field points down based on the placement of the positive and negative charges because the horizontal components cancel

part e

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{x\hat{i} - a\hat{j}}{(x^2+a^2)^{3/2}} + \frac{-q}{4\pi\epsilon_0}\frac{x\hat{i} + a\hat{j}}{(x^2+a^2)^{3/2}}$$
$$\implies \vec{E} = \frac{-2aq}{4\pi\epsilon_0}\frac{1}{(x^2+a^2)^{3/2}}\hat{j}$$

part f

$(x^2+a^2)^{3/2} \approx x^3$ when x much larger then a

$$\implies \vec{E} \approx \frac{-2aq}{4\pi\epsilon_0}\frac{1}{x^3}\hat{j}$$

2 Problem 2: Electrostatics of Molecular Hydrogen

a.i

It points right because the vertical components cancel

a.ii

Using Coulomb's Law and Superposition: $\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{x\hat{i}+a\hat{j}}{(x^2+a^2)^{3/2}} + \frac{q}{4\pi\epsilon_0}\frac{x\hat{i}-a\hat{j}}{(x^2+a^2)^{3/2}}$

$$\implies \vec{E} = \frac{2q}{4\pi\epsilon_0}\frac{x}{(x^2+a^2)^{3/2}}$$

a.iii

It looks like a point charge of 2q at the origin.

$$\implies \vec{E} = \frac{2q}{4\pi\epsilon_0}\frac{1}{x^2}\hat{i}$$

and, we know $\frac{1}{(x^2+a^2)^{3/2}} \approx \frac{1}{x^3}$ from part ii

$\implies \vec{E} = \frac{2q}{4\pi\epsilon_0}\frac{1}{x^2}\hat{i}$ as expected

part b

Using Coloumb's force law $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$

and we know the following:

$$q = 1.6 \times 10^{-19}\text{C}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{Nm}^2 \backslash \text{C}^{\ 2}$$

$$r = 7.4 \times 10^{-11}\text{m}$$

$$\implies \vec{F} = 4.2 \times 10^{-8}\text{N}$$

3 Complex Ion

part a

The electric field will be along the x-axis at point P and along the y-axis at point R.

part b

The electric field will be along the z-axis at point S.

3.1 part c

The electric field due to the following charges will be:
The center charge: $= \frac{Q}{4\pi\epsilon_0}\frac{1}{x^2}\hat{i}$
The right charge: $= \frac{-q}{4\pi\epsilon_0}\frac{1}{(x-a)^2}\hat{i}$
The left charge: $= \frac{-q}{4\pi\epsilon_0}\frac{1}{(x+a)^2}\hat{i}$
The top charge: $= \frac{-q}{4\pi\epsilon_0}\frac{(x\hat{i}-a\hat{j})}{(x^2+a^2)^{3/2}}$
The bottom charge: $= \frac{-q}{4\pi\epsilon_0}\frac{(x\hat{i}+a\hat{j})}{(x^2+a^2)^{3/2}}$

$$\implies \vec{E} = \frac{1}{4\pi\epsilon_0}\hat{i}[\frac{Q}{x^2} - \frac{q}{(x-a)^2} - \frac{q}{(x+a)^2} - \frac{2xq}{(x^2+a^2)^{3/2}}]$$

part d

If x much larger than a, then the following are true:

$$\frac{1}{(x-a)^2} \approx \frac{1}{x^2}$$

$$\frac{1}{(x+a)^2} \approx \frac{1}{x^2}$$

$$\frac{1}{(x^2+a^2)^{3/2}} \approx \frac{1}{x^3}$$

and if we substitute these into the electric field we got in part c,

$$\implies \vec{E} \approx \frac{Q-4q}{4\pi\epsilon_0}\frac{\hat{i}}{x^2}$$

part e

The electric field due to the following charges will be:
The center charge: $= \frac{Q}{4\pi\epsilon_0}\frac{1}{z^2}\hat{k}$
The right charge: $= \frac{-q}{4\pi\epsilon_0}\frac{z\hat{k}-a\hat{i}}{(z^2+a^2)^{3/2}}$
The left charge: $= \frac{-q}{4\pi\epsilon_0}\frac{z\hat{k}+a\hat{i}}{(z^2+a^2)^{3/2}}$
The top charge: $= \frac{-q}{4\pi\epsilon_0}\frac{z\hat{k}+a\hat{j}}{(z^2+a^2)^{3/2}}$
The bottom charge:$= \frac{-q}{4\pi\epsilon_0}\frac{z\hat{k}-a\hat{j}}{(z^2+a^2)^{3/2}}$

$$\implies \vec{E} = \frac{\hat{k}}{4\pi\epsilon_0}[\frac{Q}{z^2} - \frac{4qz}{(z^2+a^2)^{3/2}}]$$

part f

We know that when z is much larger than a, that $\frac{1}{(z^2+a^2)^{3/2}} \approx \frac{1}{z^3}$ and plugging that into the equation we got in part e, we get:

$$\vec{E} \approx \frac{Q-4q}{4\pi\epsilon_0}\frac{\hat{k}}{z^2}$$

4 Parallel Lines

a.i

It points downward, along the y-axis.

a.ii

$$d\vec{E} = \frac{\lambda dx}{4\pi\epsilon_0}\frac{-x\hat{i}-a\hat{j}}{(x^2+a^2)^{3/2}}$$

a.iii

$$\begin{gathered}
E_x = \int_{-\infty}^{\infty}\frac{\lambda x dx}{4\pi\epsilon_0(x^2+a^2)^{3/2}} \\
= \frac{-\lambda}{4\pi\epsilon_0}\int_{-\infty}^{\infty} dx\frac{x}{(x^2+a^2)^{3/2}} \\
= 0 \\
E_y = \int_{-\infty}^{\infty}\frac{\lambda a dx}{4\pi\epsilon_0(x^2+a^2)^{3/2}} \\
= \frac{-\lambda a}{4\pi\epsilon_0}\int_{-\infty}^{\infty} dx\frac{1}{(x^2+a^2)^{3/2}} \\
= \frac{-\lambda}{2\pi\epsilon_0}\frac{1}{a} \\
\implies \vec{E} = \frac{-\lambda}{2\pi\epsilon_0}\frac{1}{a}\hat{j}
\end{gathered}$$

b.i

It points downward along the y-axis

b.ii

$$E = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{a}$$

b.iii

Both lines produce the same field so,

$$\vec{E}_{\text{total}} = \frac{-2\lambda}{2\pi\epsilon_0}\frac{1}{a}\hat{j}$$

5 Charged Arc

part a

By symmetry, it points to the left

part b

$$d\vec{E} = \frac{\lambda R d\theta}{4\pi\epsilon_0}\frac{(-Rcos\theta\hat{i} - Rsin\theta\hat{j}}{R^3}$$

$$\implies d\vec{E} = \frac{-\lambda d\theta}{4\pi\epsilon_0}\frac{1}{R}(cos\theta\hat{i} - sin\theta\hat{j})$$

part c

$$E_x = \frac{\lambda}{4\pi\epsilon_0 R}\int_{-\infty}^{\infty} cos\theta d\theta$$

$$\int_{-\infty}^{\infty} d\theta cos\theta = 2sin\alpha$$

$$E_y = \frac{\lambda}{4\pi\epsilon_0 R}\int_{-\infty}^{\infty} sin\theta d\theta$$

$$\int_{-\infty}^{\infty} d\theta sin\theta = 0$$

$$\implies \vec{E} = \frac{-\lambda}{2\pi\epsilon_0 R}sin(\alpha)\hat{i}$$

6 Uniformly Charged Sphere

part a

$Q_{\text{total}} = \frac{4}{3}\pi R^3\rho$

part b

Radially outwards

part c.i

$Q_{\text{tot}} = \frac{4}{3}\pi R^3 \rho$

part c.ii

$4\pi r^2 E$

part c.iii

Gauss $\implies 4\pi r^2 E = \frac{4}{3}\pi R^3 \rho \frac{1}{\epsilon_0}$
$\implies E = \frac{\rho R^3}{3\epsilon_0}\frac{1}{r^2}$

part d.i

$Q_{\text{enclosed}} = \frac{4}{3}\pi r^3 \rho$

part d.ii

$4\pi r^2 E$

part d.iii

Gauss $\implies 4\pi r^2 E = \frac{4}{3}\pi r^3 \rho \frac{1}{\epsilon_0} \implies E = \frac{\rho r}{3\epsilon_0}$

7 Cylindrical Shell

part a.i

Radially outward.

a.ii

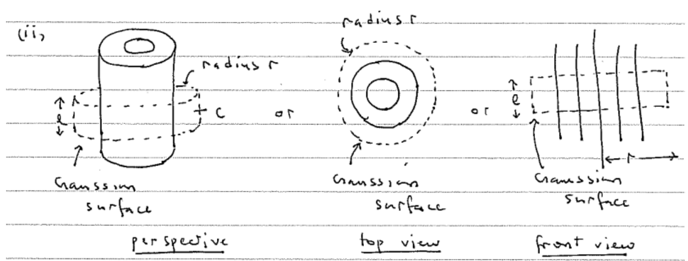

Figure 3: Shows perspective, top view, and front view

part a.iii

$\rho\pi(b^2 - a^2)l$

part a.iv

$2\pi rlE$

part a.v

Gauss law $\implies 2\pi rlE = l\rho\frac{\pi}{\epsilon_0}(b^2 - a^2)$
$\implies E = \frac{\rho}{2\epsilon_0}(b^2 - a^2)\frac{1}{r}$

part b.i

Radially outward

b.ii

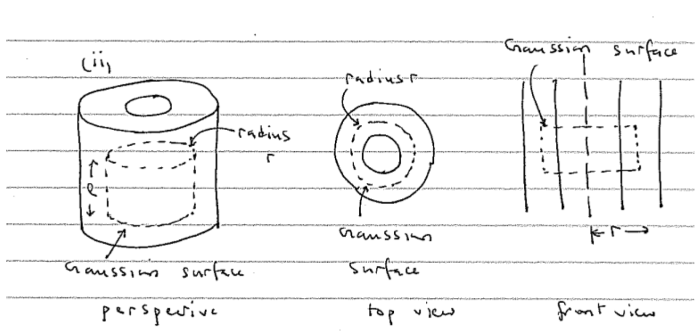

Figure 4: Shows perspective, top view, and front view

part b.iii

$\rho\pi(r^2 - a^2)l$

part b.iv

$2\pi rlE$

part a.v

Gauss law $\implies 2\pi rlE = \rho\pi(r^2 - a^2)l$
$\implies E = \frac{\rho}{2\epsilon_0}(r - \frac{a^2}{r})$

part c.i

Radial (assume outward to be definite)

c.ii

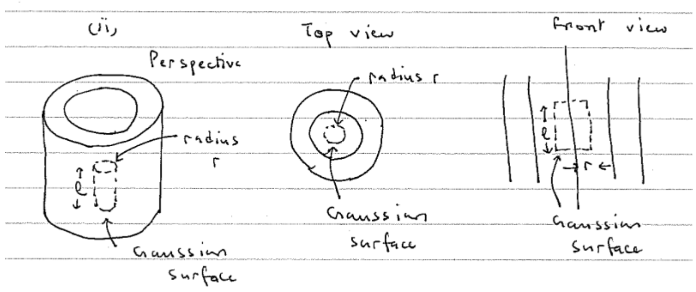

Figure 5: Shows perspective, top view, and front view

part c.iii

0 (charge enclosed)

part c.iv

$2\pi rlE$

part c.v

Gauss' Law $\implies 2\pi rlE = 0 \implies E = 0$