p1) Problem 9 Breakthrough Starshot Final 2016

(a) $P_{rad} = \frac{2I}{c}$ (reflecting surface)

$\Rightarrow F_{rad} = \frac{2I}{c} \times \ell^2$ area of sail

$\Rightarrow a = \frac{F_{rad}}{m} = \frac{2I\ell^2}{mc}$ ✓

(b) $\frac{\ell}{d} = \frac{\lambda}{\ell} \Rightarrow \boxed{d = \frac{\ell^2}{\lambda}}$

Numerically $d = 9 \times 10^8 m = \boxed{900,000 \text{ km}}$

(c) $I = \frac{1}{4} \frac{\lambda m c v^2}{\ell^4} = \frac{1}{4} \frac{\lambda m c^3}{\ell^4} \left(\frac{v}{c}\right)^2$

$$\boxed{I = 3 \times 10^{11} \frac{J}{m^2 s}}$$

p7) Physics 122 Spring 2014 Practice 4

(5) Comet's Tail

(a) L = energy that passes sphere per second

$4\pi R^2$ = area of sphere

$$\Rightarrow \boxed{I = \frac{L}{4\pi R^2}}$$

(b) $P_{rad} = \frac{I}{c}$ (formula for radiation pressure given in class)

$$= \frac{L}{4\pi c R^2}$$

F_{rad} = force of radiation

$$= P_{rad}\,(\pi r^2)$$

cross section area of dust sphere

$$\Rightarrow F_{rad} = \frac{L}{4\pi c R^2}\,\pi r^2$$

$$\Rightarrow \boxed{F_{rad} = \frac{L r^2}{4 c R^2}}$$

Comet's Tail

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(5) Comet's tail

(c) $$F_{gravity} = \frac{GM\left(\frac{4}{3}\pi r^3 \rho\right)}{R^2}$$ ← mass of dust particle

Set $F_{gravity} = F_{rad}$

$$\Rightarrow \frac{\frac{4}{3}\pi\rho GM}{R^2} r^3 = \frac{L}{4cR^2} r^2$$

$$\Rightarrow \boxed{r = \frac{3}{16\pi}\frac{L}{GM\rho c}}$$

p1) Physics 122 Practice 4

(1) Porro prism

(a) (i) $n \sin\theta_1 = \sin\theta_2$ (Snell's law)

$$\Rightarrow \sin\theta_1 = \frac{\sin\theta_2}{n}$$

$$\Rightarrow \sin\theta_1 < \sin\theta_2$$

$$\Rightarrow \boxed{\theta_1 < \theta_2}$$ ✓

(ii) $\sin\theta_1 = \dfrac{\sin\theta_2}{n}$ from part (i)

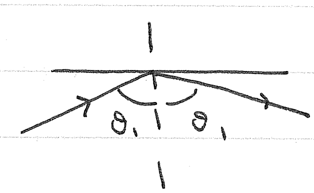
$$\Rightarrow \boxed{\theta_1 = \sin^{-1}\left(\frac{\sin\theta_2}{n}\right)}$$

(iii) Set $\theta_2 \to \frac{\pi}{2}$ above

$$\Rightarrow \boxed{\theta_1 = \sin^{-1}\left(\frac{1}{n}\right)}$$

(iv) $\boxed{\theta_1 = 41.8^\circ}$

(v)



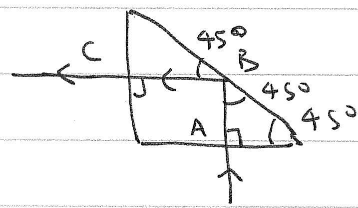
Ray is reflected with angle of reflection equal to angle of incidence

Porro Prism and Internal Reflection

p2) Physics 122 Spring 2014 Practice 4

(1) Porro prism

(b)



A: normally incident light is transmitted without deflection.

B: Total internal reflection since $45° > \theta_c$

C: Same as A

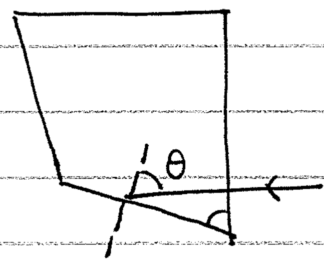
(c) The left fig is correct.

Light from air → glass will not undergo total internal reflection.

Total Internal Reflection

p1) Total Reflection

(a)



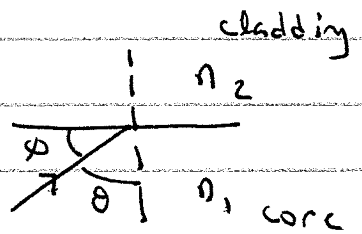
The first figure from left is wrong

$\theta = 67.5°$ by simple geometry

$> 41.8°$

$\Rightarrow$ ray should be reflected not transmitted

(b)



(i) expect total internal reflection since $n_1 > n_2$

Obvi $\theta = \frac{\pi}{2} - \phi$

In critical case Snell

$\Rightarrow n_1 \sin\left(\frac{\pi}{2} - \phi_c\right) = n_2 \sin 90°$ ← refracted ray skims the interface

$\Rightarrow \cos\phi_c = \frac{n_2}{n_1} \Rightarrow \boxed{\phi_c = \cos^{-1}\left(\frac{n_2}{n_1}\right)}$

(ii) $\boxed{\phi_c = 14°}$

Michelson Interferometer

PS) Physics 122 Spring 2014 Practice Probs 4

(4) Michelson Interferometer

Let l_1 = length of path that reflects from M_1

l_2 = length of path that reflects from M_2

Initially $l_2 - l_1 = N\lambda$, N = integer

since interference is constructive

i) Now if M_2 is moved a distance x_1

$l_2 \rightarrow l_2 + 2x_1$

$\Rightarrow (l_2 + 2x_1) - l_1 = N\lambda + \frac{\lambda}{2}$

[Since interference becomes destructive]

$\Rightarrow 2x_1 = \frac{\lambda}{2}$

$\Rightarrow \boxed{x_1 = \frac{\lambda}{4}}$

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(4) Michelson Interferometer

(ii) Assume mirror moved by x_2

Then $l_2 \to l_2 + 2x_2$

$\Rightarrow l_2 + 2x_2 - l_1 = N\lambda + \lambda$

[since interference is constructive again]

$\Rightarrow 2x_2 = + \lambda$

$\Rightarrow \boxed{x_2 = \frac{\lambda}{2}}$

(iii) Let M_2 be moved by x_3

Then $l_2 \to l_2 + 2x_3$

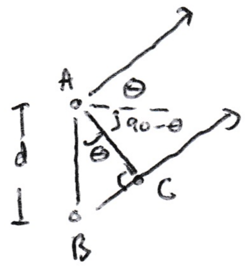
& $l_2 + 2x_3 - l_1 = N\lambda + 50\lambda$

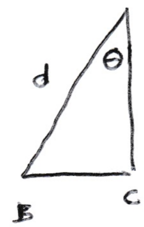
$\Rightarrow \boxed{x_3 = 25\lambda}$

2

(a)

(i)





$$\sin\theta = \frac{BC}{d}$$

$$\Rightarrow \boxed{BC = d\sin\theta}$$

(ii) $BC = n\lambda$ for constructive interference

$$\Rightarrow d\sin\theta = n\lambda \qquad \boxed{\sin\theta = \frac{n\lambda}{d}}$$

(iii) If $\lambda/d = \frac{1}{3}$, then $\theta = \sin^{-1}\left(n\frac{1}{3}\right)$

$\theta_{max} = 90° = \frac{\pi}{2} = \sin^{-1}(1)$. Suggests $n_{max} = 3$

$\Rightarrow$ Angles corresponding to $n = 0, 1, 2, 3$ (or $-1, -2, -3$) will lead to constructive interference.

$$\boxed{\theta = 0,\ \pm 0.11\pi,\ \pm 0.23\pi,\ \pm\frac{\pi}{2}}$$

$n=0$, 1, 2, 3

(iv)

$$BC = \left(n+\tfrac{1}{2}\right)\lambda \Rightarrow \boxed{\sin\theta = \frac{\left(n+\frac{1}{2}\right)\lambda}{d}}$$

(v) Similarly as for part (iii), if $\lambda/d = 1/3$

$$\theta_{max} = \frac{\pi}{2} = \sin^{-1}(1)$$

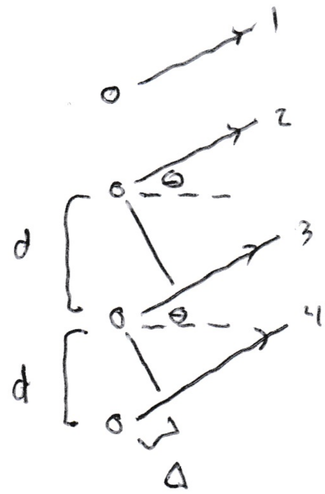
$\theta_n = \sin^{-1}\left(\left(n+\frac{1}{2}\right)\frac{1}{3}\right)$. Argument is < 1 for $n = 0, 1, 2$

$$\boxed{\theta = \pm 0.053\pi,\ \pm 0.17\pi,\ \pm 0.31\pi}$$

0, 1, 2

(b)

(i)



The difference in path length between any two neighboring rays (Δ) is still $d \sin\theta$, same for the double slit.

If the difference between ray 1 + 2 is 1λ, then the difference between ray 1 + 3 is 2λ. Thus, all the rays will constructively interfere.

So if Δ is any integer multiple of λ, you will get constructive interference.

$$\Delta = n\lambda = d\sin\theta \Rightarrow \boxed{\sin\theta = \frac{n\lambda}{d}}$$ same as for a double slit

(ii) For our DNA molecule, $d = 34$ Å + $\lambda = 1.54$ Å

$$\frac{\lambda}{d} = \frac{1}{22.1} \Rightarrow \sin\theta = \frac{n}{22.1}$$

No diffraction angle θ exists if $\frac{n}{22.1} > 1$, since $\sin\theta$ is always ≤ 1. The max n for which $\frac{n}{22.1} < 1$ is $\underline{n = 22}$

There are 22 orders on one side, ($\theta > 0$), 22 on the other side ($\theta < 0$), plus one at the center.

$22 + 22 + 1 = \boxed{45}$ angles, or orders

part (c)

↳

(c)

(i) $\mathcal{E}_x^{TOT} = \mathcal{E}_x^A + \mathcal{E}_x^B$

$$= \mathcal{E}\cos(kz-\omega t) + \mathcal{E}\cos(kz-\omega t+\phi)$$

using

$$\cos(\theta) + \cos(\theta+\phi) = 2\cos\left(\frac{\phi}{2}\right)\cos\left(\theta+\frac{\phi}{2}\right)$$

Gives

$$\boxed{\mathcal{E}_x^{TOT} = 2\mathcal{E}\cos\left(\frac{\phi}{2}\right)\cos\left(kz-\omega t+\phi/2\right)}$$

Also,

$$\boxed{\mathcal{E}_y^{TOT} = \mathcal{E}_z^{TOT} = 0}$$ since $\mathcal{E}_y^A, \mathcal{E}_y^B, \mathcal{E}_z^A, \mathcal{E}_z^B = 0$

(ii) This eqn. implies the amplitude of the total plane wave is

$$A^{TOT} = \boxed{2\mathcal{E}\cos(\phi/2)}$$

(iii) For $\phi = 0$, $A^{TOT} = \underline{2\mathcal{E}}$ (in-phase)

$\phi = \pi/2$, $A^{TOT} = 2\mathcal{E}\cos(\pi/4) = 2\mathcal{E}\sqrt{2}/2 = \underline{\sqrt{2}\,\mathcal{E}}$ $(1.4\,\mathcal{E})$

$\phi = \pi$, $A^{TOT} = 2\mathcal{E}\cos(\pi/2) = \underline{0}$ (out-of-phase)

(iv)