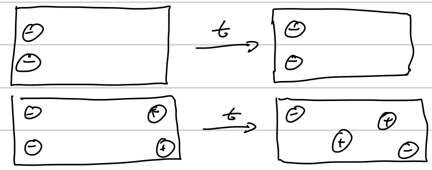
Need to write the Maxwell equations on the final exam!

Electric Charges & Fields, Coulomb's Law, Superposition

Electric Charge

- charge is a fundamental prop. of matter
  - ↳ two types: ⊕ & ⊖ matter (atoms) are normally neutral
- **Triboelectricity:** Friction transfers charge
- Metal Rod in demo?
  - ∘ Plastic & crystal are insulators
    - → charge "stays where put"
  - ∘ Metals are conductors
    - → charge "flow freely"

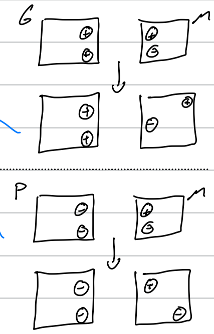


Demo - Rods

⊖P P⊖ ← →
⊕G G⊕ ← →
⊖P G⊕ → ←

Demo w/ Metal

Note: the metal is still neutral



- Charge is conserved
  - ∘ Rubbing transfers charges, but does not create nor destroy it
- Charge is quantized
  - ∘ Charge is always a multiple of $e = 1.6 \times 10^{-19}$ Coulombs (C) → electron $= -e$, proton $= e$
- why the rods are moving!
  - ∘ Attraction or Repulsion ⇒ There is a force between the charges
    - ↳ Force is known as coulomb's force

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{r^2} \qquad \text{where } \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \ \frac{N\cdot m^2}{C^2}$$

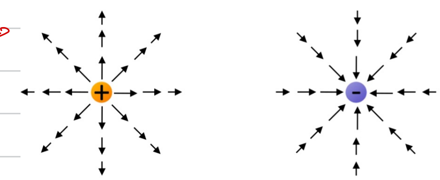
- ▫ When two charges q and Q are same signed → F is positive = Repulsive
- ▫ when they have opposite sign → F is negative = Attractive

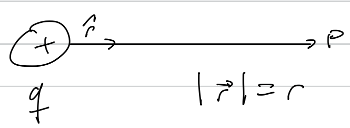
Electric Field and Coulomb's Law

Laws of Electrostatics

- Electrostatics ≡ charges @ rest
  - ∘ charges produce electric fields
  - ∘ electric fields push charges

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$





$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

$\hat{r}$ is a unit vector that points away from charge q

Note: $r = |\vec{r}|$

or alternatively (use $\hat{v} = |\vec{v}|\,\hat{v}$) so $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ and $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^3} \vec{r}$

Equivalent!

Superposition Principle

In Words:

Electric field produced by many point charges = Sum of the electric field produced by each point charge independently

In Math:

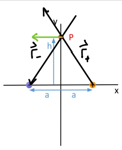
$$\vec{E}(q_1, q_2) = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1^3} \vec{r}_1 + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2^3} \vec{r}_2$$

Second Law of Electrostatics

Fields push/pull charges: $\vec{F} = Q\vec{E} \Rightarrow \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2} \hat{r}$

$\{N\} \quad \{C\} \{\frac{N}{C}\}$

Example: PollEv



Calculate the Electric field at point P.

Pos vector $\vec{r}$ from ⊕ to P: $-a\hat{\imath} + h\hat{\jmath}$

Pos vector $\vec{r}$ from ⊖ to P: $a\hat{\imath} + h\hat{\jmath}$ (not moves, position vector not dependent on charge!)

Here, we are assuming point-like charges but this is not realistic

Calculate the Electric Field @ P:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r} \rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{(\sqrt{a^2+h^2})^3}\left[q(-a\hat{\imath}+h\hat{\jmath}) + -q(a\hat{\imath}+h\hat{\jmath})\right]\right) = \frac{2qa}{4\pi\varepsilon_0\sqrt{a^2+h^2}^3}\cdot(-\hat{\imath})$$

$r_- = a\hat{\imath} + h\hat{\jmath} \rightarrow r_- = \sqrt{a^2+h^2}$

$r_+ = -a\hat{\imath} + h\hat{\jmath} \rightarrow r_+ = \sqrt{a^2+h^2}$

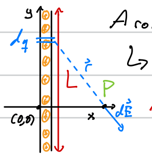
$\vec{E} = \vec{E}_+ + \vec{E}_-$

Continuous Distribution of Charge

Definition: Charge per unit length or linear charge density

$$\lambda = \frac{\text{Charge}}{\text{Length}} = \frac{Q}{L}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \sum_{i=1}^{N}\vec{E}_i = \int_a^b d\vec{E}$$



$dq = \lambda\, dy$

A rod of length 'L' is charged with a positive amount of charge Q

↳ Calculate the Electric field @ P

Assumptions: Rod is very long ($L \rightarrow \infty$)

Rod is very thin (1-D)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r} \rightarrow d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}$$

$\vec{r} = x\hat{\imath} - y\hat{\jmath}$

$r = \sqrt{x^2+y^2}$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+y^2}^3}\cdot(x\hat{\imath} - y\hat{\jmath})$$

So, $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\lambda}{\sqrt{x^2+y^2}^3}(x\hat{\imath} - y\hat{\jmath})\,dy$$

Solve $\hat{\imath}$ and $\hat{\jmath}$ components separately