Physics 122 Spring 2025

Final Exam

Tuesday, May 6, 2025

You have 3 hours to complete the exam.

There are 6 problems and 2 Short Questions (extra credit).

Guidelines

1. You have put a lot of effort into this course. You have done Homework, Partial Exams and Quizzes so you are very well prepared for this exam. Now is just a matter of showing us what you have learned during this course.

2. Write your name clearly on this cover page.

3. Do not open the exam until you are instructed to.

4. Write your answer on the blank pages provided for each problem. If you need more space you may request additional paper and staple it to your exam when you finish.

5. The questions are on a separate booklet together with the formula sheet.

6. After you complete your work (or time is up) you should scan it and upload it to Canvas. Look for an Assignment called "Final Exam".

7. After scanning submit the hard copy of your work. The proctors will let you know where the submission bins are located.

8. The exam is closed book. You are only allowed the provided formula sheet.

9. Keep your work clear, neat and well-organized. Illegible answers might be graded as incorrect. Often it helps to put a box around your final answer.

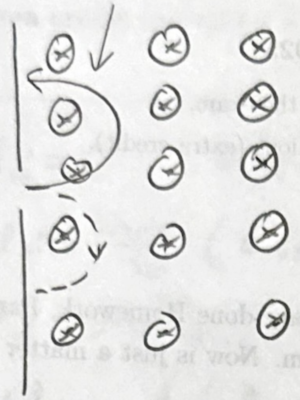
10. Use pictures and words to explain what you are doing. State the central physics concept associated with each problem (e.g. Gauss' Law, Ampere's law, Ohm's Law, superposition). You will earn partial credit for knowing how to solve the problem even if you are not able to fully implement the solution.

DISCLAIMER: You need to upload the exam to canvas correctly and on time for these extra credit questions to be graded.

Write your name here: Trevor Swan

Problem 1

a) proton trajectory



↗ negative particle

As the electron is taking a path downwards $(-\hat{j})$, the RHR states that the field must be into the page to have this effect on an electron moving in x-direction

negative $\hat{i} \times ? = -\hat{j} \rightarrow -\hat{k}$ $\boxed{\vec{B} = -B\hat{k}}$

$\vec{F} = q\vec{v} \times \vec{B}$, and forces $-\hat{j}$

b) $F = ma$ ← centripital

$F_B = q\vec{v} \times \vec{B}$

$q\cancel{v}B = m\dfrac{v^{\cancel{2}}}{R} \rightarrow \boxed{R = \dfrac{mv}{qB}}$

c) Drawn as a solid line in the figure from a. Follows opposite path to electron.

d) • q is the same, so it won't impact R apart from its sign

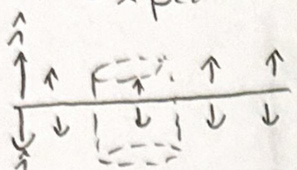
• $m_p = 2000 m_e \rightarrow R_p = 2000 \dfrac{m_e v}{qB}$

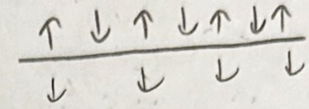
Therefore the radius will also be 2000 times larger for the proton compared to the electron

Problem 1

Problem 2

a) Expected: $E_0 = \dfrac{\sigma_{free}}{\epsilon_0}$ $\qquad E_{out} = 0$

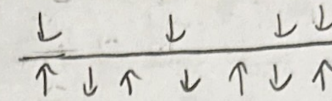




$\dfrac{\sigma_{free}}{2\epsilon_0}$

$\oint \vec{E}\cdot d\vec{a} = \dfrac{Q_{enc}}{\epsilon_0}$ ← Gauss Law

$E(\pi r^2 2)\cos 0 = \dfrac{\sigma_{free}(\pi r^2)}{\epsilon_0}$



$\dfrac{\sigma_{free}}{2\epsilon_0}(-1)$

$E = \dfrac{\sigma_{free}}{2\epsilon_0}$ $\qquad E_{out} = 0$

As per my drawing, $E_{in} = E_{top} - E_{bottom}$

$= \dfrac{\sigma_{free}}{2\epsilon_0} - \left(-\dfrac{\sigma_{free}}{2\epsilon_0}\right) = \dfrac{\sigma_{free}}{\epsilon_0}$

downwards

$$\boxed{\vec{E} = \frac{\sigma_{free}}{\epsilon_0}(-\hat{k})}$$

b) $\boxed{C = \dfrac{\epsilon_0 A}{d}}$ ϵ_0 is constant, A & d are assumed known. This is from formula sheet!

c) $\boxed{\vec{E} = \dfrac{\sigma_{free}}{\epsilon_0 \kappa}(-\hat{k}) = \dfrac{\sigma_{free}}{\epsilon_0(1+\chi)}(-\hat{k})}$ Both correct!

d) $\vec{P} = \chi\epsilon_0\vec{E} = \chi\cancel{\epsilon_0}\dfrac{\sigma_{free}}{\cancel{\epsilon_0}(1+\chi)}(-\hat{k})$

$$\boxed{\vec{P} = \frac{\chi\sigma_{free}}{1+\chi}(-\hat{k})}$$

Problem 2

e) $\sigma_{Bound} = \hat{n} \cdot \vec{P}$ (no field outside)

Top: $\hat{n} = \hat{k}$ on surface

$$\sigma_{bound} = \hat{k} \cdot \frac{\chi \sigma_{free}}{1+\chi}(-\hat{k})$$

$$\boxed{\sigma_{bound} = \frac{\chi \sigma_{free}}{1+\chi}(-1)}$$

Bottom: $\hat{n} = -\hat{k}$ on surface

Same as top, just negated

$$\boxed{\sigma_{bound} = \frac{\chi \sigma_{free}}{1+\chi}}$$

Problem 3

$$\cancel{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}}$$ Biot-Savart?

$\lambda_i = \frac{Q}{L}$

Length of circle:
L = angle × radius

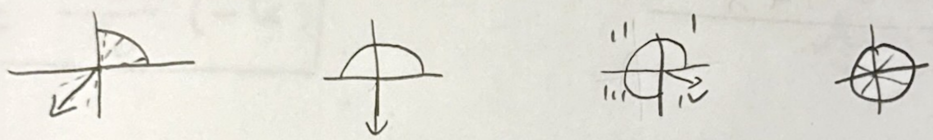
a) $\lambda_1 = \frac{Q}{\frac{\pi}{2}r} = \frac{2Q}{\pi r}$

$\lambda_2 = \frac{Q}{\pi r}$

$\lambda_3 = \frac{Q}{\frac{3\pi}{2}r} = \frac{2Q}{3\pi r}$

$\lambda_4 = \frac{Q}{2\pi r}$

b)



(Arc 1) $(-\hat{\imath}-\hat{\jmath})$

(Arc 2) All horizontal components cancel out
$-\hat{\jmath}$

(Arc 3) Components in Quadrants I & III cancel out
$(\hat{\imath}-\hat{\jmath})$

(Arc 4) All components $(\hat{\imath},\hat{\jmath})$ cancel out, no electric field
no direction

Problem 3

c) Biggest to smallest:

$$E_2 = E_1 = E_3 > E_4 = 0$$

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$ I want $d\theta$!

length = angle × radius

$dq = \lambda dr$

$dr = r d\theta$

$dq = \lambda r\, d\theta$

(General) $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r\, d\theta}{r^2}$

$\rightarrow dE_i = \frac{1}{4\pi\varepsilon_0}\frac{\lambda_i d\theta}{r}$

(Arc 1) $E_1 = \int_0^{\frac{\pi}{2}} \frac{1}{4\pi\varepsilon_0 r}\frac{2Q}{\pi r} d\theta = \frac{1}{4\pi\varepsilon_0 r}\frac{2Q}{\pi r}\left(\frac{\pi}{2}\right)$

$$\boxed{E_1 = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{1}{4\pi\varepsilon_0 r}\left(\frac{\pi}{2}\right)\lambda_1}$$

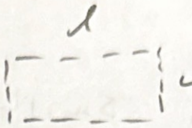
(Arc 2) $E_2 = \int_0^{\pi} \frac{1}{4\pi\varepsilon_0 r}\frac{Q}{\pi r} d\theta = \frac{1}{4\pi\varepsilon_0 r}\frac{Q}{\pi r}(\pi)$ $\boxed{E_2 = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{1}{4\pi\varepsilon_0 r}\pi\lambda_2}$

(Arc 3) $E_3 = \int_0^{\frac{3\pi}{2}} \frac{1}{4\pi\varepsilon_0 r}\frac{2Q}{3\pi r} d\theta = \frac{1}{4\pi\varepsilon_0 r}\frac{2Q}{3\pi r}\left(\frac{3\pi}{2}\right)$ $\boxed{E_2 = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{1}{4\pi\varepsilon_0 r}\frac{3\pi}{2}\lambda_2}$

(Arc 4) $\boxed{E_4 = 0 \text{ @ the origin}}$

by symmetry

Problem 4

a) I will use a loop like this 

. This ensures that I only have to account for the circulation inside the solenoid as the sides $\hat{n}$ are $\perp$ to the field and the top bit is outside of the solenoid ($B=0$ outside).

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$\vec{B}$ is in the positive $\hat{z}$ direction by RHR

$$I_{enc} = n l I(t)$$

$$\underbrace{\cancel{\oint_{left} \vec{B} \cdot d\vec{l}}^{\,0} + \cancel{\oint_{right} \vec{B} \cdot d\vec{l}}^{\,0}}_{(\pm)\hat{z} \perp \hat{n},\ \text{so } \cos(90)=0} + \oint_{Bot} \vec{B} \cdot d\vec{l} + \underbrace{\cancel{\oint_{top} \vec{B} \cdot d\vec{l}}^{\,0}}_{B=0 \text{ outside}} = \mu_0 l n I(t)$$

$$\rightarrow \quad B l = \mu_0 l n I(t)$$

$$\boxed{B(t) = \mu_0 n I(t)(\hat{z})}$$

b) $$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint \vec{B} \cdot d\vec{a} \qquad \oint \vec{E} \cdot d\vec{l} = E(2\pi r)$$

$$\Phi_B = \oint \vec{B} \cdot d\vec{a} = \mu_0 n I(t) \cdot \pi r^2$$

$$\frac{d\Phi_B}{dt} = \mu_0 n \pi r^2 \frac{dI}{dt}$$

*Note: For both parts, the electric field will be clockwise as I is strictly increasing

(i) $$E(2\cancel{\pi} \cancel{r}) = \mu_0 n \cancel{\pi} r^{\cancel{2}} \frac{dI}{dt}(-1)$$

$$\boxed{E = -\frac{1}{2} \mu_0 n r \frac{dI}{dt}}$$

Clockwise

See *Note for direction explanation

(ii) $$E(2\cancel{\pi} r) = \mu_0 n \cancel{\pi} R^2 \frac{dI}{dt}(-1)$$

$$\boxed{E = -\frac{1}{2} \mu_0 n \frac{R^2}{r} \frac{dI}{dt}}$$

$B = 0$ outside, only flux for $r \le R$

Both Clockwise

Problem 4

Problem 5

a) R_3 & R_5 in parallel. The voltage drop, from ε_3, across them is the same, so they must be in parallel.

$$R_{eq} = \left(\frac{1}{R_3} + \frac{1}{R_5}\right)^{-1}$$

b) R_2 & R_3 in series. In the upper loop, either direction taken for Kirchhoff analysis indicates that the voltage drop across them is the same. They are connected tip to tail.

$$\underline{R_{eq} = R_2 + R_3}$$

Problem 5

c) $\varepsilon_4 - I_5 R_5 - I_3 R_3 = 0$

$\varepsilon_1 - I_1 R_1 + \varepsilon_2 - I_2 R_2 - I_3 R_3 - \varepsilon_3 - I_4 R_4 = 0$

$\varepsilon_1 - I_1 R_1 + \varepsilon_2 - I_2 R_2 + \varepsilon_4 - I_5 R_5 - \varepsilon_3 - I_4 R_4 = 0$

Currents: $I_1 = I_4$ $\quad I_2 + I_3 = I_5$

$I_1 = I_2$ $\quad I_4 = I_3 + I_5$ $\Big\}$ $I_4 = 2I_3 + I_2$

$I_4 = 2I_3 + I_2$, but $I_1 = I_2$ and $I_1 = I_4$

$I_1 = 2I_3 + I_1$ $\qquad \boxed{I_3 = 0}$

$\varepsilon_4 - I_5 R_5 - 0R_3 = 0$ $\qquad I_4 = 0 + \frac{\varepsilon_4}{R_5}$

$\varepsilon_4 = I_5 R_5$ $\qquad \boxed{I_4 = \frac{\varepsilon_4}{R_5}}$

$\boxed{I_5 = \frac{\varepsilon_4}{R_5}}$

$I_1 = I_2 = I_4$

$\boxed{I_1 = I_2 = \frac{\varepsilon_4}{R_5}}$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

Problem 6

a) Current is flowing down, so B is clockwise

$\oint \vec{B} \cdot d\vec{l}$ is circulation

$\left.\begin{array}{l}\vec{B} \text{ is clockwise} \\ d\vec{l} \text{ is vertical}\end{array}\right\} \theta = 90°$ $\rightarrow$ $\boxed{\text{Circulation is } 0, \text{ so } B = 0}$

$Bl \cos 90 = 0$

b) $C = \varepsilon_0 \frac{A}{d}$, $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$, $C = QV$

$C = \varepsilon_0 \frac{A}{d} \rightarrow \varepsilon_0 A = Cd$ or $\varepsilon_0 = \frac{dC}{A}$ so $E = \frac{\sigma}{\frac{dC}{A}}$

$\rightarrow E = \frac{Q}{Cd} \underset{\uparrow}{=} \frac{\cancel{Q}}{\cancel{Q}Vd} = \frac{1}{Vd}$ $\quad E = \frac{\sigma A}{dC} = \frac{\frac{Q}{A}A}{dQV} = \frac{1}{dV}$

$C = QV$

$\Rightarrow \boxed{E = \frac{1}{d}\left(V(1-e^{-t/RC})\right)^{-1}} (-\hat{z})$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}^{\nearrow 0} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ $\quad \frac{d\Phi_E}{dt} = \frac{d}{dt}(E\pi r^2)$

inside cap

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$

$0°$

$\Phi_E = \oint \vec{E} \cdot d\vec{a} = EA = E\pi r^2$ $\quad \Rightarrow B(2\cancel{\pi} r) = \cancel{\pi} r^2 \frac{dE}{dt} \mu_0 \varepsilon_0$

$0°$

$B(2r) = r^2 \frac{dE}{dt} \mu_0 \varepsilon_0$

Problem 6

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{d}(V)^{-1}\left(1-e^{-t/RC}\right)^{-1}\right)$$

$$\frac{d}{dt}\left(1-e^{-t/RC}\right) = \frac{1}{RC}e^{-t/RC}$$

$$= \frac{1}{dV}\cdot\frac{d}{dt}\left(\frac{1}{1-e^{-t/RC}}\right)$$

$$= \frac{1}{dV}\left(\frac{(1-e^{-t/RC})\cdot 0-\left(\frac{1}{RC}e^{-t/RC}\right)}{(1-e^{-t/RC})^2}\right)$$

$$\frac{dE}{dt} = \frac{1}{dV}\left(-\frac{\frac{1}{RC}e^{-t/RC}}{(1-e^{-t/RC})^2}\right)$$

radius of loop, enclosed radius

$$B(2r) = \mu_0 \varepsilon_0 r^2 \frac{dE}{dt}$$

c) $$B = \frac{1}{2}\mu_0\varepsilon_0 r\left(-\frac{1}{dVRC}\cdot\frac{e^{-t/RC}}{(1-e^{-t/RC})^2}\right)$$

d) $$B = \frac{1}{2}\mu_0\varepsilon_0\frac{r_0^2}{r}\left(-\frac{1}{dVRC}\cdot\frac{e^{-t/RC}}{(1-e^{-t/RC})^2}\right)$$

DISCLAIMER: You need to upload the exam to canvas correctly and on time for these extra credit questions to be graded.

Question 1

$\oint \vec{B} \cdot d\vec{a} = 0$ (Gauss Law for B)

$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$ (Gauss Law)

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$ (Amperes)

$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint \vec{B} \cdot d\vec{a}$ (Faraday)

$F = qE + q\vec{v} \times \vec{B}$ (force on charged particle q in fields E & B @ velocity v)

Question 2

$\vec{B} = \frac{\varepsilon}{c} \cos(kz - \omega t)\hat{k}$ (perpendicular to $\vec{E}$)

where c is the speed of light in m/s