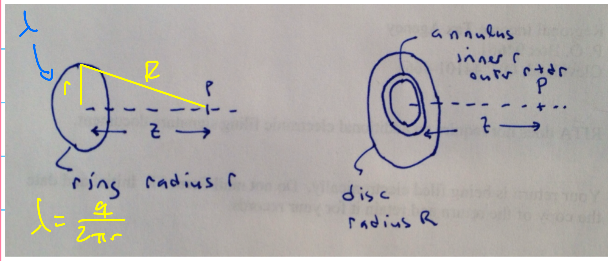
PHYS122 Homework 5 - Due 03/02/25 **Trevor Swan (tcs94)**

1. Potential of a Disk



a) $R = \sqrt{r^2+z^2}$

$$d\Phi = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{r^2+z^2}}$$

$$dq = \lambda r\, d\theta = \left(\frac{q}{2\pi r}\right) r\, d\theta = \frac{q\, d\theta}{2\pi}$$

$$\Phi = \int_0^{2\pi} \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{r^2+z^2}}\frac{d\theta}{2\pi} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{r^2+z^2}}\int_0^{2\pi}\frac{d\theta}{2\pi}$$

$$\Phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{r^2+z^2}}$$

b) $R = \sqrt{r^2+z^2}$

$$d\Phi = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{r^2+z^2}}$$

(i) $dq = \sigma\, dA$

dA = circumference × thickness = $2\pi r\, dr$

$$d\Phi = \frac{\sigma}{4\cancel{\pi}\varepsilon_0}\frac{2\cancel{\pi} r\, dr}{\sqrt{r^2+z^2}} \Rightarrow d\Phi = \frac{\sigma}{4\varepsilon_0}\frac{2r}{\sqrt{r^2+z^2}}dr$$

(ii)
$$\Phi = \frac{\sigma}{4\varepsilon_0}\int_0^R \frac{2r}{\sqrt{r^2+z^2}}dr = \frac{\sigma}{4\varepsilon_0}\int_{z^2}^{R^2+z^2} u^{-1/2}\, du$$

$u = r^2+z^2$

$\frac{du}{dr} = 2r$

$dr = \frac{du}{2r}$

$$= \frac{\sigma}{\cancel{4}2\varepsilon_0}\left(\cancel{2}u^{1/2}\Big|_{z^2}^{R^2+z^2}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - z\right)$$

$$\Phi = \frac{1}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - z\right)$$

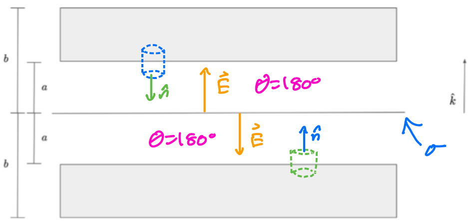
Doesn't match the helpful integral on HW questions, but I think I did my integral right?

(iii) For $z \gg R$, we should expect a potential close to that of a point like charge with total charge $Q = \sigma\pi R^2$. Expect $\Phi = \frac{Q}{4\pi\varepsilon_0 z}$

$$\lim_{z\gg R}\Phi = \frac{\sigma}{2\varepsilon_0}\left(z + \frac{R^2}{2z} - z\right) = \frac{\sigma R^2}{4z\varepsilon_0}$$

$$\Phi = \frac{\sigma\pi R^2}{4\pi\varepsilon_0 z}$$

2. Slabs



a) Assuming each conductor is perfect, then the charge inside both slabs must be 0, as the charges at the surfaces of the slabs will arrange to cancel out any enclosed field.

b) Field produced by an infinite plane: $E = \frac{\sigma}{2\varepsilon_0}$ E is vertical only, no field inside conductor

σ_u? $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$, $-E\cancel{A_{cap}} = \frac{\sigma_u \cdot \cancel{A_{cap}}}{\varepsilon_0} \Rightarrow \boxed{\sigma_u = -\frac{\sigma}{2}}$

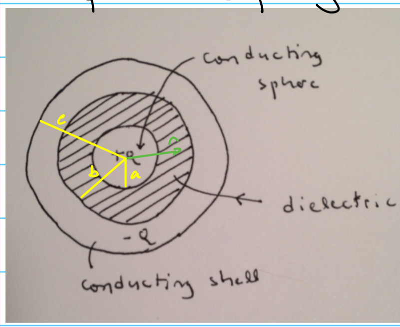
$\cos\theta = -1$

σ_l? $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$, $-E A_{cap} = \frac{\sigma_l \cdot A_{cap}}{\varepsilon_0} \Rightarrow \boxed{\sigma_l = -\frac{\sigma}{2}}$

$\cos\theta = -1$

c) Since each slab is given to be neutral, the outer surfaces must be equal and opposite to the inner surfaces. So, the upper surface of the upper slab is equal to the lower surface of the lower slab which is equal to $+\frac{\sigma}{2}$.

3. Spherical Capacity

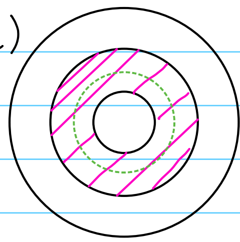


a) As the sphere is a conductor, then it must have an electric field of 0 for $r < a$.

b) As the sphere is a conductor, all charge is on its surface, so its charge on the surface is $+q$

σ? $\quad \sigma = \frac{\text{charge}}{\text{area}} = \frac{+q}{4\pi a^2}$ $\qquad \sigma = \frac{q}{4\pi a^2}$

c)



$a < r < b$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint E \, da \cos 0 = \frac{q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$E = \frac{q}{4\pi\varepsilon_0 r^2}$, and is radially outward

d) Similar to a, the field inside the conductor is 0.

e) Choose surface $b < r < c$, w/ field $E = 0$

$\oint E \, da = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow Q_{enc} = 0$ $\quad \therefore$ enclosed charge must be 0 including the center conductor and the inner surface of the conductor.

$\Rightarrow Q_{enc} = q + q_{inner\ surface} = 0$ $\quad$ so $\quad q_{inner\ surface} = -q$

Choose surface $r > c$

$\oint E \, da = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0} \rightarrow$ $\quad q_{outer\ surface} = 0$

Since the total charge of the shell is $-q$, its entire charge must be on its inner surface to neutralize the field inside the conductor.

f) $\Delta V = V_2 - V_1 = -\int_{r_1}^{r_2} \vec{E} \cdot d\vec{l}$ $\qquad \left.\begin{array}{l} \vec{E} = E\hat{r} \\ d\vec{l} = dr\,\hat{r} \end{array}\right\} \theta = 0 \rightarrow \cos\theta = 1$

$$V_b - V_a = -\int_a^b \frac{q}{4\pi\varepsilon_0 r^2} dr = -\frac{q}{4\pi\varepsilon_0} \int_a^b r^{-2} dr = -\frac{q}{4\pi\varepsilon_0} \left(-\frac{1}{r}\right)_a^b = -\frac{q}{4\pi\varepsilon_0} \left(-\left[\frac{1}{b} - \frac{1}{a}\right]\right)$$

$$\Rightarrow V_b - V_a = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{b} - \frac{1}{a}\right)$$

g) Charge: $+q$

Potential difference: $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)$

$$C = \frac{\cancel{q}}{\frac{\cancel{q}}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)} = \frac{4\pi\varepsilon_0}{\frac{1}{b}-\frac{1}{a}}$$

$$C = \frac{4\pi\varepsilon_0}{\frac{1}{b}-\frac{1}{a}}$$