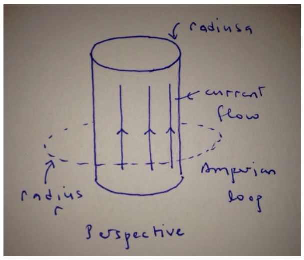
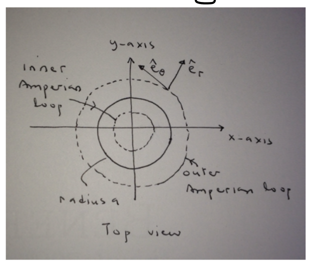
PHYS122 Homework 8 - Due 04/08/25 **Trevor Swan (tcs94)**

1. Axial Current on a cylindrical surface



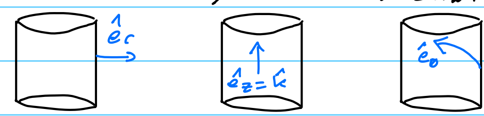


a) Per unit length: $K = \frac{I}{L}$ $\frac{\text{current}}{\text{length}}$

↳ Length $= 2\pi a$ for a cylinder (circumference)

$$K = \frac{i}{2\pi a}$$

b) Radial Axial Azimuthal



c) Gauss Law of Magnetism

$\Phi_B = \oint \vec{B} \cdot d\vec{a} = 0$ Assume $\vec{B} = B\hat{e}_r$, also $\hat{n} = \hat{e}_r$ for surface

$= \int B(2\pi r L) = 0$

$\Rightarrow B(2\pi r L) = 0$

This contradicts the professor as r & $L > 0$, and $|B|$ must be nonzero to have a direction. Thus, the assumption that $\vec{B} = B\hat{e}_r$ is contradicted. Radial lines prevent flux from ever being 0 when a magnetic field is present.

d) Assume $\vec{B} = B\hat{e}_\theta$ for amperian loop $r > a$

(i) $I_{enc} = K \times \underset{\text{circumference}}{L_{enc}} = \frac{i}{2\pi a} \cdot 2\pi a = i$; $I_{enc} = i$

(ii) Circulation $= \oint \vec{B} \cdot d\vec{l}$ $\theta = 0$

$= \oint \vec{B} \cdot (r\,d\theta)$ Circulation $= B(r)(2\pi r)$ $\{$function of $r\}$

$= B(r) r \int_0^{2\pi} d\theta$

(iii) $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$

$B(r)(2\pi r) = \mu_0 i$

$$B(r) = \frac{\mu_0 i}{2\pi r}$$

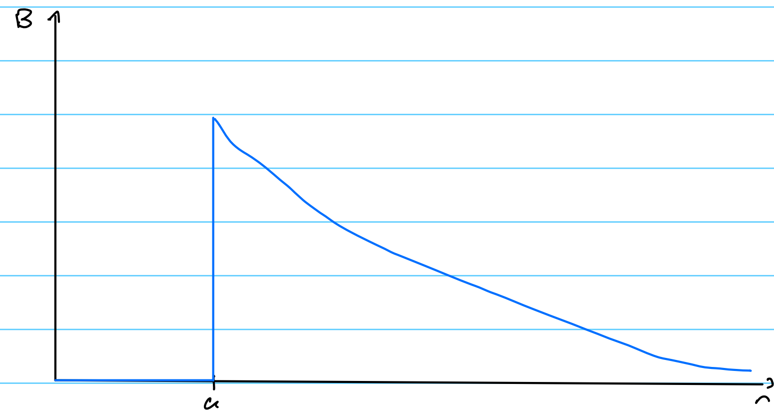
e) Assume $\vec{B} = B\hat{e}_\theta$ for $r < a$

(i) $I_{enc} = 0$, Current is along surface of cylinder only

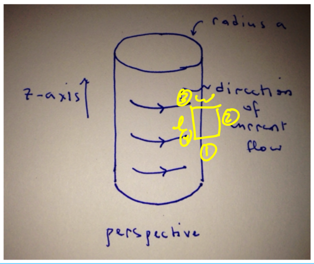
(ii) Circulation $= \oint \vec{B} \cdot d\vec{l} = B(r)(2\pi r)$

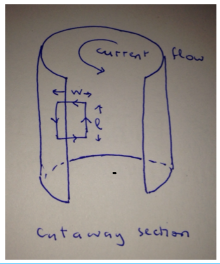
(iii) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow B(r) = 0$

f)



2. Circulating Current around a cylindrical surface





Amperean loop: $\begin{cases} L = \text{height/length} \\ w = \text{width} \end{cases}$

Magnetic field: $\begin{cases} \vec{B} = B\,\hat{k} & \text{inside} \\ \vec{B} = 0 & \text{outside} \end{cases}$

Cylinder: Current/unit length K, radius a

b)

(i) Circulation $= \oint \vec{B}\cdot d\vec{s} = \cancel{\left(\oint \vec{B}\cdot d\vec{s}\right)_1}^{0} + \cancel{\left(\oint \vec{B}\cdot d\vec{s}\right)_2}^{0} + \cancel{\left(\oint \vec{B}\cdot d\vec{s}\right)_3}^{0} + \left(\oint \vec{B}\cdot d\vec{s}\right)_4 = B\cdot l$

$\theta = 90°$ $\quad B = 0$ outside $\quad \theta = 90°$ $\quad \theta = 0°$ inside

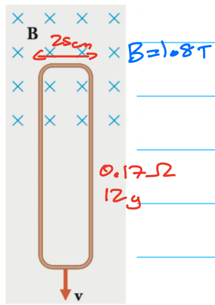
(ii) I_{enc} = density · height $= K\cdot l$

↳ accounts for current crossing loop many times by using density

(iii) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$

$B\cdot l = \mu_0 K\cdot l \implies \vec{B} = \mu_0 K\cdot\hat{k}$

4.



$F_g = mg$

Faradays law: $\frac{d\Phi_B}{dt} = Bwv = \varepsilon$

$I = \frac{\varepsilon}{R} = \frac{wvB}{R}$

$F_B = I\int |d\vec{l}||\vec{B}|\sin\theta$

RHR: $d\vec{l} = dl(-\hat{i})$ $\theta = 90$

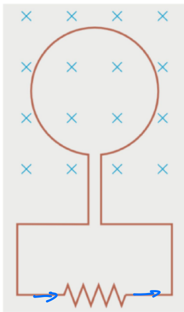
$\Rightarrow F_B = IB\int dl = IBw = \frac{wvB}{R}(Bw) = \frac{w^2 v B^2}{R}$

$F_g = F_B$ for (terminal velocity)

$mg = \frac{w^2 v_{term} B^2}{R}$

$v_{term} = \frac{Rmg}{w^2 \cdot B^2} = \frac{0.17\,\Omega\,(0.012\,kg)(9.81\,\frac{m}{s^2})}{(0.25\,m)^2(1.8\,T)^2} = 0.0988\,\frac{m}{s}$

5.



15Ω

200 to 100 cm^2 loop area in 0.020s

- Flux ↓ as there is less area
- Faraday will produce current to cause flux ↑

∴ Current will be counter-clockwise around loop

⇒ $+\hat{i}$ direction for resistor PoV

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\Delta\Phi_B}{\Delta t} = \frac{\Delta\Phi_B}{0.020s}$$

$$\Phi_B = B \cdot A \Rightarrow \Phi_B = B \cdot \Delta A$$

$$= 0.30T \cdot (0.01\,m^2 - 0.02m^2)$$

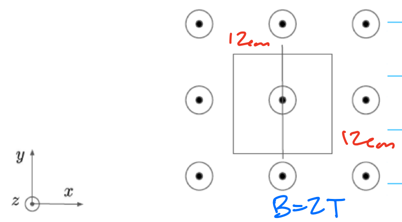
$$= -0.003\ T \cdot m^2$$

$$\mathcal{E} = -\left(\frac{-0.003\ T \cdot m^2}{0.020\ s}\right) = 0.15V$$

$$\vec{I} = 0.01\hat{i}\ A$$

$$V = IR \Rightarrow I = \frac{V}{R} = \frac{0.15V}{15\Omega} = 0.01\ A$$

6.



$$\mathcal{E}(t) = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = B \cdot A \cdot \cos\theta$$

$$\Rightarrow d\Phi_B = BA\cos(\theta t)$$

$$\mathcal{E}(t) = -\frac{d}{dt}\left(BA\cos(\omega t)\right) = -\left(-BA\sin(\omega t)\right)$$

$$= BA\sin(\omega t) \qquad \omega = 1\,rad/s$$

$$\mathcal{E}(t) = (2T)(0.12m)^2 \sin(t)$$

$$\mathcal{E}(t) = 0.0288 \sin t$$

Plot using python