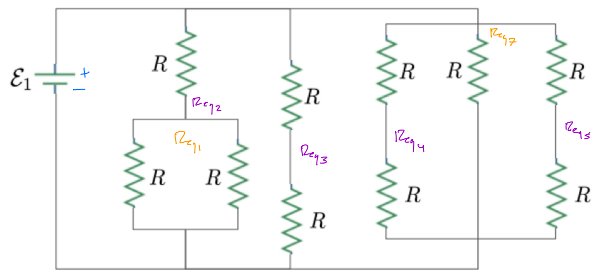
PHYS122 Homework 9 - Due 4/20/25 — Trevor Swan

1.




$$R_{eq_1} = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{R}{2}$$

$$R_{eq_2} = R + R_{eq_1} = R + \frac{R}{2} = \frac{3R}{2}$$

$$R_{eq_3} = R + R = 2R$$

$$R_{eq_4} = R + R = 2R$$

$$R_{eq_5} = R + R = 2R$$

$$R_{eq_7} = \left(\frac{1}{R} + \frac{1}{R_{eq_4}} + \frac{1}{R_{eq_5}}\right)^{-1}$$

$$= \left(\frac{1}{R} + \frac{1}{2R} + \frac{1}{2R}\right)^{-1}$$

$$= \left(\frac{1}{R}\left(1 + \frac{1}{2} + \frac{1}{2}\right)\right)^{-1} = \frac{R}{2}$$

$$R_{total} = \left(\frac{1}{R_{eq_2}} + \frac{1}{R_{eq_3}} + \frac{1}{R_{eq_7}}\right)^{-1}$$

$$= \left(\frac{2}{3R} + \frac{1}{2R} + \frac{2}{R}\right)^{-1}$$

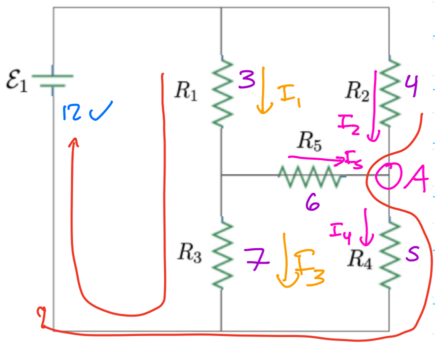
$$= \left(\frac{1}{R}\left(\frac{2}{3} + \frac{1}{2} + 2\right)\right)^{-1}$$

$$= \left(\frac{19}{6R}\right)^{-1}$$

$$\boxed{R_{total} = \frac{6}{19}R}$$

The total resistance of the circuit will be less than $R$, specifically $R_{total} = \frac{6}{19}R$

2.




$$\boxed{\begin{aligned} I_1 &= 0.477A \\ I_2 &= 1.907\,A \\ I_3 &= 1.510\,A \\ I_4 &= 0.874\,A \\ I_5 &= 1.033\,A \end{aligned}}$$

our assumption for $I_5$ was left to right but it should've been right to left, just flip the sign to match convention!

**System of Equations (Kirchhoff Rules)**

$\mathcal{E}_1 - I_1R_1 - I_3R_3 = 0$ (Kirchhoff Loop)

$\mathcal{E}_1 - I_2R_2 - I_4R_4 = 0$ (Loop)

$I_1R_1 - I_5R_5 - I_2R_2 = 0$ (Loop w/ Bridge)

$I_1 = I_3 + I_5$ (Kirchhoff Junction)

$I_4 = I_2 + I_5$ (Junction)

Solve for $I_i$'s: 5 equations, 5 unknowns

$I_3 = I_1 - I_5$

$\mathcal{E}_1 - I_1R_1 - (I_1 - I_5)R_3 = 0$

$12 - 3I_1 - (I_1 - I_5)7 = 0$

$12 - 3I_1 - 7I_1 + 7I_5 = 0$

$12 = 10I_1 - 7I_5$

$I_4 = I_2 + I_5$

$\mathcal{E}_1 - I_2R_2 - (I_2 + I_5)R_4 = 0$

$12 - 4I_2 - (I_2 + I_5)5 = 0$

$12 - 4I_2 - 5I_2 - 5I_5 = 0$

$12 = 9I_2 + 5I_5$

$3I_1 - 6I_5 - 4I_2 = 0$

$$\left.\begin{aligned} &4I_2 = 3I_1 - 6I_5 \\ &12 = 10I_1 - 7I_5 \\ &12 = 9I_2 + 5I_5 \end{aligned}\right\}\ \text{3 eq, 3 unknowns}$$

$I_1 = \dfrac{12 + 7I_5}{10}$

$3\left(\dfrac{12 + 7I_5}{10}\right) - 6I_5 = 4I_2$

$\dfrac{36 + 21I_5}{10} - 6I_5 = 4I_2$

$36 + 21I_5 - 60I_5 = 40I_2$

$36 - 39I_5 = 40I_2$

$I_2 = \dfrac{12 - 5I_5}{9}$

$36 - 39I_5 = 40\left(\dfrac{12 - 5I_5}{9}\right)$

$324 - 351I_5 = 480 - 200I_5$

$-151I_5 = 156$

$I_5 = -1.03\ A$

$I_1 = \dfrac{12 + 7(-1.03)}{10} = 0.477A$

$I_2 = \dfrac{12 - 5(-1.03)}{9} = 1.907$

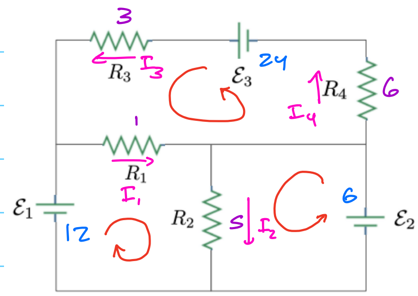
$$\text{rref}\begin{bmatrix} 3 & -4 & -6 & 0 \\ 10 & 0 & -7 & 12 \\ 0 & 9 & 5 & 12 \end{bmatrix}$$

← also works to get $I_1, I_2, I_5$

$I_3 = 0.477A - (-1.033) = 1.510\ A$

$I_4 = 1.907A + (-1.033) = 0.874\ A$

3.




| |
|---|
| $I_1 = 6A$ |
| $I_2 = 1.2A$ |
| $I_3 = 3.6A$ |
| $I_4 = 4.8A$ |

← Was negative direction is opposite to that assumed

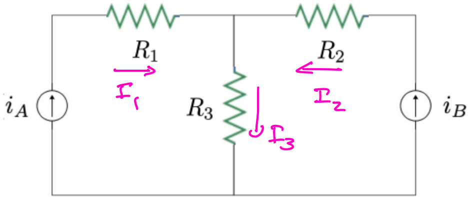
$$\varepsilon_1 - I_1R_1 - I_2R_2 = 0$$
$$\varepsilon_2 - I_2R_2 = 0$$
$$\varepsilon_3 - I_3R_3 - I_1R_1 - I_4R_4 = 0$$
$$I_4 = I_1 - I_2$$

4 equations 4 unknowns

$$I_2R_2 = \varepsilon_2$$
$$I_2 = \frac{\varepsilon_2}{R_2} = \frac{6}{5} = 1.2A$$
$$I_1 = \frac{\varepsilon_1 - I_2R_2}{R_1} = \frac{12 - 1.2(5)}{1} = 6A$$
$$I_4 = 6A - 1.2A = 4.8A$$

$$I_3 = \frac{\varepsilon_3 - I_1R_1 - I_4R_4}{R_3}$$
$$= \frac{24 - 6(1) - 4.8(6)}{3}$$
$$= -3.6A$$

4.




a) $I_1 = i_A$

$I_2 = i_B$

$I_1 + I_2 = I_3$ (Junction)

$I_3 = i_A + i_B$

$$\boxed{\begin{aligned} I_1 &= i_A \\ I_2 &= i_B \\ I_3 &= i_A + i_B \end{aligned}}$$

b) $V_A - R_1 I_1 - R_3 I_3 = 0$

$V_A - R_1 i_A - R_3 (i_A + i_B) = 0$

$V_A = R_1 i_A + R_3 (i_A + i_B)$

$V_B - R_2 I_2 - R_3 I_3 = 0$

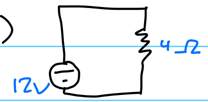
$V_B - R_2 i_B - R_3 (i_A + i_B) = 0$

$V_B = R_2 i_B + R_3 (i_A + i_B)$

$$\boxed{\begin{aligned} V_A &= R_1 i_A + R_3 (i_A + i_B) \\ V_B &= R_2 i_B + R_3 (i_A + i_B) \end{aligned}}$$

## 5. Power Supply

a)



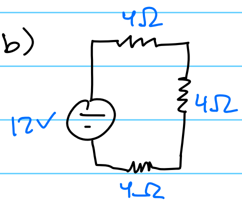

$$\boxed{P_1 = 36W}$$

$$\varepsilon_1 - I_1 R_1 = 0$$

$$I_1 = \frac{\varepsilon_1}{R_1} = \frac{12V}{4\Omega} = 3A$$

$$P_1 = I_1^2 R_1 = 3^2 A \cdot 4\Omega = 36W$$

b)




$$R_{eq} = R_1 + R_2 + R_3 = 12\Omega$$

$$P_{tot} = I^2 R = \underset{\text{Ohm's}}{\frac{V^2}{R}} = \frac{12V^2}{12\Omega} = 12W$$

$$\varepsilon_1 - I_1 R_1 - I_2 R_2 - I_3 R_3 = 0$$

$$\varepsilon_1 - 4 I_{Resistor} R_{Resistor} = 0$$

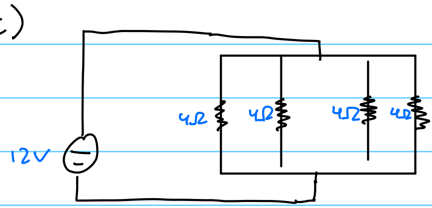
$$12 - 3 I_{Resistor}(4) = 0$$

$$I_{Resistor} = 1A$$

$$P_{resistor} = I^2 R = 1^2 4 = 4W$$

$$\boxed{\begin{array}{l} P_{tot} = 12W \\ P_{resistor} = 4W \end{array}}$$

c)




$$R_{eq} = \left(\frac{1}{4} \cdot 4\right)^{-1} = 1\Omega$$

$$P_{tot} = \frac{V^2}{R_{eq}} = \frac{12V^2}{1\Omega} = 144W$$

$$P_{resistor} = \frac{V^2}{R} = \frac{12V^2}{4\Omega} = 36W$$

$$\boxed{\begin{array}{l} P_{tot} = 144W \\ P_{resistor} = 36W \end{array}}$$