PHYS122 Homework 10 - Due 4/27/25 **Trevor Swan**

2. Electromagnetic Waves

$\vec{E} = \mathcal{E}\cos(kx-\omega t)\hat{j}$ & $\vec{B} = \frac{\mathcal{E}}{c}\cos(kx-\omega t)\hat{k}$

google: $u = \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2$

$\frac{\mathcal{E}}{c} = \frac{\mathcal{E}}{\frac{1}{\sqrt{\mu_0\varepsilon_0}}} = \mathcal{E}\sqrt{\mu_0\varepsilon_0}$

a) $u = \frac{\varepsilon_0}{2}\left(\mathcal{E}\cos(kx-\omega t)\hat{j}\right)^2 + \frac{1}{2\mu_0}\left(\frac{\mathcal{E}}{c}\cos(kx-\omega t)\hat{k}\right)^2$

$= \frac{\varepsilon_0}{2}\left[\mathcal{E}^2\cos^2(kx-\omega t)\right] + \frac{1}{2\cancel{\mu_0}}\left[\mathcal{E}^2(\cancel{\mu_0}\varepsilon_0)\cos^2(kx-\omega t)\right]$

$= \frac{\varepsilon_0\mathcal{E}^2}{2}\left[2\cos^2(kx-\omega t)\right] = \varepsilon_0\mathcal{E}^2\cos^2(kx-\omega t)$

constants!

$\langle u\rangle = \frac{1}{T}\int_0^T \varepsilon_0\mathcal{E}^2\cos^2(kx-\omega t)\,dt \longrightarrow \langle u\rangle = \varepsilon_0\mathcal{E}^2\langle\cos^2(kx-\omega t)\rangle$

$\Rightarrow$ $\langle u\rangle = \frac{1}{2}\varepsilon_0\mathcal{E}^2$ and $u = \varepsilon_0\mathcal{E}^2\cos^2(kx-\omega t)$

b) $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

$\vec{E}\times\vec{B} = \langle 0, \mathcal{E}\cos(kx-\omega t), 0\rangle \times \langle 0, 0, \frac{\mathcal{E}}{c}\cos(kx-\omega t)\rangle$

$= \langle \mathcal{E}\cos(kx-\omega t)\cdot\frac{\mathcal{E}}{c}\cos(kx-\omega t) - 0, 0, 0\rangle$

$= \frac{\mathcal{E}^2}{c}\cos^2(kx-\omega t)\hat{i}$

$\vec{S} = \frac{\mathcal{E}^2}{\mu_0 c}\cos^2(kx-\omega t)\hat{i} \rightarrow \vec{S} = \varepsilon_0 c\mathcal{E}^2\cos^2(kx-\omega t)\hat{i}$

$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ $\mu_0 c = \frac{1}{\varepsilon_0 c^2}\cdot c = \frac{1}{\varepsilon_0 c}$ $\langle S\rangle = \varepsilon_0 c\mathcal{E}^2\hat{i}\langle\cos^2(kx-\omega t)\rangle$

$\sqrt{\mu_0\varepsilon_0} = \frac{1}{c}$ $\frac{1}{\mu_0 c} = \varepsilon_0 c$ $= \varepsilon_0 c\mathcal{E}^2\frac{1}{2}\hat{i}$

$\mu_0 = \frac{1}{\varepsilon_0 c^2}$

$\vec{S} = \varepsilon_0 c\mathcal{E}^2\cos^2(kx-\omega t)\hat{i}$ and $\langle S\rangle = \frac{1}{2}\varepsilon_0 c\mathcal{E}^2\hat{i}$

c) google: $\Pi = \frac{\vec{S}}{c^2} = \frac{\varepsilon_0}{c}\mathcal{E}^2\cos^2(kx-\omega t)\hat{i}$

$\langle\pi\rangle = \frac{\varepsilon_0}{c}\mathcal{E}^2\langle\cos^2(kx-\omega t)\rangle\hat{i} = \frac{\varepsilon_0}{2c}\mathcal{E}^2\hat{i}$

$\pi = \frac{\varepsilon_0}{c}\mathcal{E}^2\cos^2(kx-\omega t)\hat{i}$ and $\langle\pi\rangle = \frac{\varepsilon_0}{2c}\mathcal{E}^2\hat{i}$

d) $P = \frac{I}{c} = \frac{\langle S\rangle}{c}$ (absorber) $P = 2\cdot\frac{\langle S\rangle}{c}$ (reflector) $P = \frac{F}{A} \rightarrow F = PA$

$\frac{\langle S\rangle}{c} = \frac{\frac{1}{2}\varepsilon_0 c\mathcal{E}^2\hat{i}}{c} = \frac{1}{2}\varepsilon_0\mathcal{E}^2\cancel{\hat{i}}$ F perp to A

(i) $F = A\left(\frac{1}{2}\varepsilon_0\mathcal{E}^2\right)$

(ii) $F = 2A\left(\frac{1}{2}\varepsilon_0\mathcal{E}^2\right)$

e)

(i) $I = \langle S \rangle$ is energy per second per square meter $\frac{J/s}{m^2} = \frac{W}{m^2}$

→ $\langle u \rangle$ is energy per unit volume $\frac{J}{m^3}$

→ c is speed: $\frac{m}{s}$

$\langle u \rangle \cdot c \; [=] \; \frac{J}{m^3} \cdot \frac{m}{s} = \frac{J}{m^2 s} = \frac{W}{m^2}$ ✓ so $I = \langle u \rangle c$ is correct

$\langle u \rangle \cdot \frac{1}{c} \; [=] \; \frac{J}{m^3} \cdot \frac{s}{m} = \frac{J \cdot s}{m^4} \neq$

(ii) $\langle \Pi \rangle$ is $\frac{\vec{S}}{c^2}$, so it has units of $\frac{kg}{s^3} \cdot \frac{s^2}{m^2} = \frac{kg}{s \cdot m^2}$ in base units

→ $\langle u \rangle$ is energy per unit volume $\frac{J}{m^3} = \frac{kg \cdot m^2}{s^2} / m^3 = \frac{kg}{s^2 \cdot m}$

→ c is speed: $\frac{m}{s}$

$\langle u \rangle \cdot c \; [=] \; \frac{J}{m^2 s} = \frac{kg \cdot m^2}{s^2} \cdot \frac{1}{m^2 \cdot s} = \frac{kg}{s^3}$

$\langle u \rangle \cdot \frac{1}{c} \; [=] \; \frac{kg}{s^2 \cdot m} \cdot \frac{s}{m} = \frac{kg}{s \cdot m^2}$ ✓ so $\langle \Pi \rangle = \frac{\langle u \rangle}{c}$

f) Sunlight: $I = 1.36 \frac{kW}{m^2} = \langle S \rangle = 1360 \frac{W}{m^2}$ Note: $F = \frac{J}{m}$

(i) $I = \langle u \rangle c = \frac{1}{2} \varepsilon_0 \mathcal{E}^2 \cdot c$

$1360 \frac{J}{s \cdot m^2} = \frac{1}{2}\left(8.854 \times 10^{-12} \frac{J}{mV^2}\right)(\mathcal{E}^2)\left(299{,}792{,}458 \frac{m}{s}\right)$

$\mathcal{E} = 1012.29 \frac{V}{m}$

$\frac{\mathcal{E}}{c} = \frac{1012.29 \frac{V}{m}}{299792458 \frac{m}{s}} = 3.38 \times 10^{-6} \frac{V \cdot s}{m^2}$ $\qquad \frac{\mathcal{E}}{c} = 3.38 \times 10^{-6}\, T$

(ii) $\langle u \rangle = \frac{1}{2} \varepsilon_0 \mathcal{E}^2$

$= \frac{1}{2}\left(8.854 \times 10^{-12} \frac{J}{mV^2}\right)\left(1012.29 \frac{V}{m}\right)^2$

$\langle u \rangle = 4.54 \times 10^{-6} \frac{J}{m^3}$

(iii) $\langle \pi \rangle = \frac{\langle u \rangle}{c} = \frac{4.54 \times 10^{-6} \frac{J}{m^3}}{299792458 \frac{m}{s}} \Rightarrow \langle \pi \rangle = 1.51 \times 10^{-14} \frac{kg}{m^2 s}$

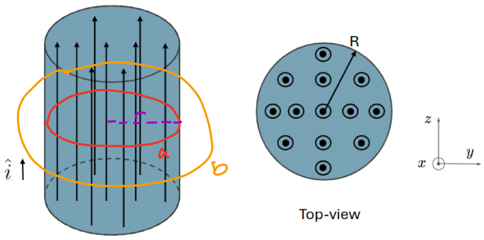
(iv) $A = 1\, mile^2 = 2.59 \times 10^6\, m^2$ perfectly reflecting

$F = 2A\left(\frac{1}{2} \varepsilon_0 \mathcal{E}^2\right) = 2A \langle u \rangle$

$F = 2(2.59 \times 10^6\, m^2)\left(4.54 \times 10^{-6} \frac{J}{m^3}\right) = 23.5\, N$

$F_{radiation} = 23.5\, N$

5.



$$\vec{E} = E_0 \sin(\omega t)\hat{i}$$

$$\oint d\vec{l}\cdot\vec{B} = \mu_0\left(\cancel{I}^{\,0} + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$

$$B(2\pi r) = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

a) Distance R from cylinder (amperian loop a)

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\left[\int \vec{E}\cdot d\vec{a}\right] = \frac{d}{dt} EA = \pi R^2 \frac{d}{dt}\left(E_0 \sin(\omega t)\right) = E_0 \pi R^2 \omega \cos(\omega t)$$

$$\oint \vec{B}\cdot d\vec{l} = B(2\pi \overset{r=R}{R}) = \mu_0 \varepsilon_0 E_0 \pi R^2 \omega \cos(\omega t)$$

$$B = \frac{\mu_0 \varepsilon_0 E_0 \omega R}{2}\cos(\omega t)$$ Counterclockwise from above

b) Distance r = 2R from cylinder (amperian loop b)

$$\frac{d\Phi_E}{dt} = E_0 \pi R^2 \omega \cos(\omega t)$$ Same as field is enclosed by cylinder

$$\oint \vec{B}\cdot d\vec{l} = B(4\pi R) = \mu_0 \varepsilon_0 E_0 \pi R^2 \omega \cos(\omega t)$$

$$B = \frac{\mu_0 \varepsilon_0 E_0 \omega R}{4}\cos(\omega t)$$ counter-clockwise