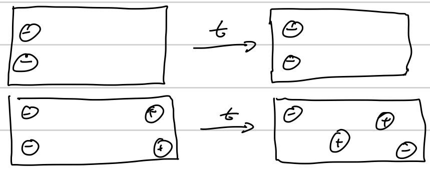
Midterm Exam 1

Need to write the Maxwell equations on the final exam!

Electric Charges & Fields, Coulomb's Law, Superposition

Electric Charge

- charge is a fundamental prop. of matter
  - ↳ two types: ⊕ & ⊖ — matter (atoms) are normally neutral
- **Triboelectricity:** Friction transfers charge
- Metal Rod in demo?
  - ∘ Plastic & crystal are insulators
    - → charge "stays where put"
  - ∘ Metals are conductors
    - → charge "flow freely"



t meaning "after some time"

Demo - Rods

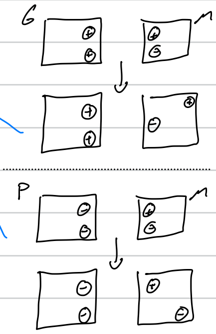
⊖ P P ⊖ ← →

⊕ G G ⊕ ← →

⊖ P G ⊕ → ←

Demo w/ Metal

Note: the metal is still neutral



- Charge is conserved
  - ∘ Rubbing transfers charges, but does not create nor destroy it
- Charge is quantized
  - ∘ Charge is always a multiple of $e = 1.6 \times 10^{-19}$ Coulombs (C) → electron $= -e$, proton $= e$
- why the rods are moving!
  - ∘ Attraction or Repulsion ⇒ There is a force between the charges
    - ↳ Force is known as coulomb's force

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{r^2} \qquad \text{where } \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \ \frac{N\cdot m^2}{C^2}$$

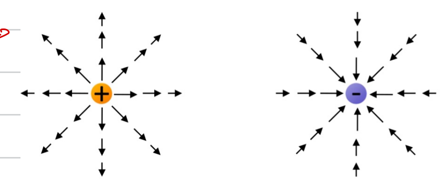
- ▫ when two charges q and Q are same signed → F is positive = Repulsive
- ▫ when they have opposite sign → F is negative = Attractive

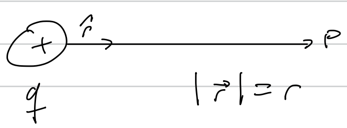
Electric Field and Coulomb's Law

Laws of Electrostatics

- Electrostatics ≡ charges @ rest
  - ∘ charges produce electric fields
  - ∘ electric fields push charges

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$





$\hat{r}$ is a unit vector that points away from charge q

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

Note: $r = |\vec{r}|$

or alternatively (use $\vec{v} = |\vec{v}| \hat{v}$) so $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ and

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^3} \vec{r}$$

Equivalent!

Superposition Principle

In Words:

Electric field produced by many point charges = Sum of the electric field produced by each point charge independently

In Math:

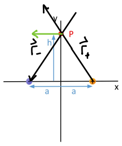
$$\vec{E}(q_1, q_2) = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1^3} \vec{r}_1 + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2^3} \vec{r}_2$$

Second Law of Electrostatics

Fields push/pull charges: $\vec{F} = Q\vec{E}$ ⇒ $\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2} \hat{r}$

$\{N\}$ $\{C\}$ $\{\frac{N}{C}\}$

Example: PollEv



Calculate the Electric field at point P.

Pos vector $\vec{r}$ from ⊕ to p: $-a\hat{i} + h\hat{j}$

Pos vector $\vec{r}$ from ⊖ to p: $a\hat{i} + h\hat{j}$ (not moving, position vector not dependent on charge!)

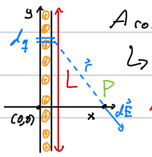
Here, we are assuming point-like charges but this is not realistic

Calculate the Electric Field @ P:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r} \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{(\sqrt{a^2+h^2})^3}\left[q(-a\hat{i}+h\hat{j}) + -q(a\hat{i}+h\hat{j})\right]\right) = \frac{2qa}{4\pi\varepsilon_0\sqrt{a^2+h^2}^3}\cdot(-\hat{i})$$

$$\left.\begin{array}{l} r_- = a\hat{i} + h\hat{j} \rightarrow r_- = \sqrt{a^2+h^2} \\ r_+ = -a\hat{i} + h\hat{j} \rightarrow r_+ = \sqrt{a^2+h^2} \end{array}\right\} \vec{E} = \vec{E}_+ + \vec{E}_-$$

Continuous Distribution of Charge



$dq = \lambda dy$

A rod of Length 'L' is charged with a positive amount of charge Q

↳ Calculate the Electric field @ P

Assumptions: Rod is very long ($L \to \infty$)

Rod is very thin (1-D)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r} \rightarrow d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}$$

$$\left.\begin{array}{l} \vec{r} = x\hat{i} - y\hat{j} \\ r = \sqrt{x^2+y^2} \end{array}\right\} d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+y^2}^3}\cdot(x\hat{i} - y\hat{j})$$

Definition: Charge per unit length, or linear charge density

$$\lambda = \frac{\text{Charge}}{\text{Length}} = \frac{Q}{L}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \sum_{i=1}^{N}\vec{E}_i = \int_a^b d\vec{E}$$

$$\text{so, } \vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\lambda}{\sqrt{x^2+y^2}^3}(x\hat{i} - y\hat{j})\,dy$$

Solve $\hat{i}$ and $\hat{j}$ components separately

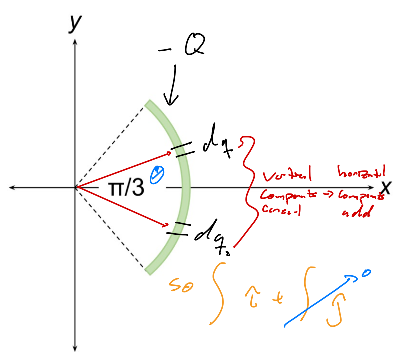
Example

A plastic rod with a uniformly distributed charge –Q is bent into a circular arc of radius *r* that subtends an angle of π/3 radians. We place coordinate axes such that the axis of symmetry of the rod lies along the *x*-axis and the origin is the center of curvature for the rod. What is the electric field (magnitude and direction) at the origin?

Hint: Imagine that the arc is made up of many infinitesimally small point charges *dq*. Each *dq* creates a differential electric field of magnitude $dE = k\,dq/r^2$.

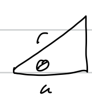
Sum up all the *dE*s from all the *dq*s to get the magnitude of the overall electric field.

Integration is a way to add many infinitely small elements.



$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}$$

$$|d\vec{E}| = \left|\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}\right|$$



$$\left.\begin{array}{l} \cos\theta = \frac{a}{r} \\ \sin\theta = \frac{b}{r} \end{array}\right\} \vec{r} = -r\cos\theta\,\hat{i} - r\sin\theta\,\hat{j}$$

x-component of $|d\vec{E}|$: $dE_x = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta$

we need to integrate over all θ, but dE_x is in terms of dq, not $d\theta$!

$\Rightarrow dq = \lambda\,ds$ where ds is segment of arc

↖ linear charge density

$\Rightarrow 2\pi r$ so $ds = r\,d\theta \rightarrow dq = r\lambda\,d\theta$

To integrate let's use polar! $(x, y) \rightarrow (r, \theta)$

$$\int d\vec{E}_x = \frac{1}{4\pi\varepsilon_0}\int_{-\pi/6}^{\pi/6}\frac{r\lambda\cos\theta\,d\theta}{r^2}$$

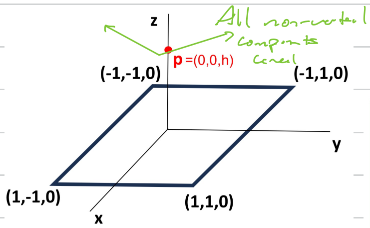
$$= \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin\theta\right]_{-\pi/6}^{\pi/6}$$

but what is lambda $= \frac{Q}{L} = \frac{Q}{r(\pi/3)}$ Just think length = angle · radius (like in $C = 2\pi r$)

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{3Q}{\pi r^2} \rightarrow \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{\pi r^2}\hat{i}}$$

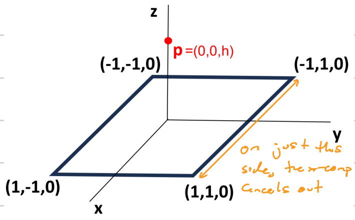
Example

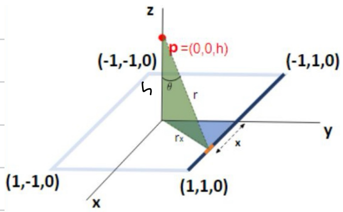
A plastic rod with a uniformly distributed charge Q is shaped forming a square whose side is 1m. We place coordinate axes such that the square is on the x-y plane and the origin is the center of the square. What is the electric field (magnitude and direction) at the point p=(0,0,h)?



$\Rightarrow$ This problem's solution will be $\vec{E} = E\hat{k}$

So we just need to calculate the vertical (z) component of one side and add the rest!





Blue triangle: $r_x^2 = x^2 + 1^2$

Green triangle: $r^2 = r_x^2 + h^2$

Hint: What is the square formed by? Is there any symmetry?

Imagine that the square is made up of many infinitesimally small point charges *dq*. Each *dq* creates a differential electric field of magnitude $dE = k\,dq/r^2$.

Sum up all the *dE*s from all the *dq*s to get the magnitude of the overall electric field.

Integration is a way to add many infinitely small elements.

$\vec{r}_h = h\hat{k}$

$\vec{r}_{xy} = -x\hat{i} - \hat{j} \Rightarrow r_x^2 = x^2 + 1^2$ $\Big\}$ $\vec{r} = -x\hat{i} - \hat{j} + h\hat{k}$

So, $r^2 = r_x^2 + h^2$

$\to r = \sqrt{x^2 + h^2 + 1}$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(x^2+h^2+1)^{3/2}}(-x\hat{i} - \hat{j} + h\hat{k})$$

x and y components cancel

$$\Rightarrow \vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{(x^2+h^2+1)^{3/2}}(\cancel{-x\hat{i}} - \cancel{\hat{j}} + h\hat{k})$$

$$\vec{E}_{Right} = \int d\vec{E}_{Right} = \int_{-1}^{1}\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(\sqrt{x^2+h^2+1})^3}(-x\hat{i} - \hat{j} + h\hat{k}) = +c\hat{j} + d\hat{k}$$

↑ In one side $\hat{j}$ component present, but disappears when using Left as well

$$\Rightarrow \vec{E}_{Left} = \int d\vec{E}_{Left} = \dots = -c\hat{j} + d\hat{k}$$

$$\vec{E}_{Top} = \int_{-1}^{1}\frac{1}{4\pi\varepsilon_0}\frac{dy}{(\sqrt{x^2+h^2+1})^2}(-\hat{i} - y\hat{j} + h\hat{k})$$

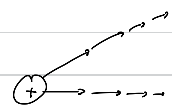
$$\vec{E}_{Bottom} = \int_{-1}^{1}\frac{1}{4\pi\varepsilon_0}\frac{dy}{(\sqrt{x^2+h^2+1})^2}(+\hat{i} - y\hat{j} + hk)$$

So $\boxed{\vec{E} = \vec{E}_{Right}\cdot 4\hat{k}}$

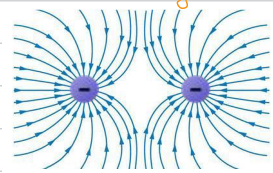
Gauss Law

Electric Field Lines

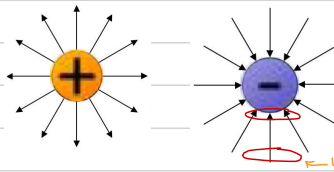
Single Charge



Two charges



(i) Lines are tangent to $\vec{E}$ at every point in space
(ii) The density of lines is proportional to $|\vec{E}|$
(iii) Lines never cross
(iv) No closed loops
(v) Lines start or stop on charges
  ↳ start should be a positive charge, end should be a negative charge
  ↳ This rule inherently enforces part (iv)

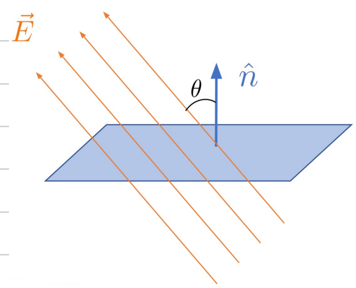


← weaker charge → fewer lines

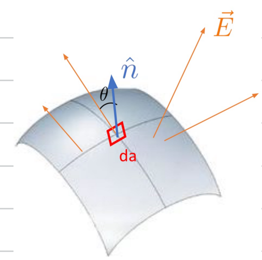
Recall Vector Multiplication
$$\begin{cases} \vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z \\ \quad = |\vec{A}||\vec{B}|\cos\theta \end{cases}$$
for θ the angle between $\vec{A}$ and $\vec{B}$

Electric Flux

- Flat surface and uniform $\vec{E}$
- Defined:
$$\Phi_E = A\hat{n}\cdot\vec{E} = A\overbrace{|\hat{n}|}^{1}|\vec{E}|\cos\theta = A|\vec{E}|\cos\theta$$
  - $\hat{n}$ is a unit vector perpendicular to the surface (normal vector)
  - θ is the angle between $\hat{n}$ and $\vec{E}$
  - A is the area of the surface



- Non-Flat surface and/or Non-uniform $\vec{E}$
$$\Phi_E = \int \hat{n}\cdot\vec{E}\,da = \int \overbrace{|\hat{n}|}^{1}|\vec{E}|\cos\theta\,da = \int|\vec{E}|\cos\theta\,da$$
  - $\hat{n}$ is a unit vector perpendicular to the surface da
  - θ is the angle between $\hat{n}$ and $\vec{E}$



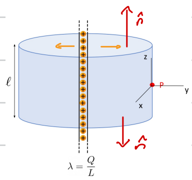
Gauss Law

- First of Maxwell's Equations
- In words: The electric flux through a closed surface $= \dfrac{\text{Amount of charge inside the surface}}{\varepsilon_0}$
- Equation:
$$\Phi_{E_{\text{closed}}} = \oint \hat{n}\cdot\vec{E}\,da = \frac{Q}{\varepsilon_0}$$

- $\hat{n}$ points outside the surface by convention
- $\oint \vec{E}\cdot d\vec{a} = \dfrac{Q}{\varepsilon_0}$ is also valid, where $d\vec{a} = da\cdot\hat{n}$

• **Example / Review:** Infinite Line

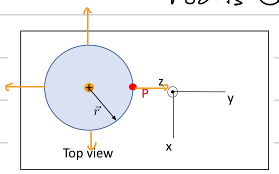
Assumptions: rod is very long ($L \to \infty$)
rod is very thin (1-D)



Length ℓ of rod inside cylinder

$\Rightarrow Q_{ENC} = \lambda \ell$

$$\phi_E = \frac{Q_{ENC}}{\varepsilon_0} = \frac{\lambda \ell}{\varepsilon_0}$$



$$= \int_{Cap\uparrow} \hat{n}\vec{E}\,da + \int_{Cap\downarrow} \hat{n}\vec{E}\,da + \int_{Side} \hat{n}\vec{E}\,da$$

Extra credit Question: What is the direction of the total electric field @ P? $\hat{s}$

CAPS — radial

$\vec{E} = E\hat{r}$, $\hat{n} = \pm\hat{k}$ } $\theta = 90°$

SIDE — radial

$\vec{E} = E\hat{r}$, $\hat{n} = \hat{r}$ } $\theta = 0$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}\hat{r}\ ,\quad E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

$$\underbrace{2\int_{Caps} |\hat{n}||\vec{E}|\cos\theta\,da}_{\theta=90°,\text{ so } 0} + \int_{Side} \hat{n}\vec{E}\,da$$

$$= 0 + \int |\hat{n}||E|\cos\theta\,da$$

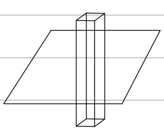
$$= \int E\,da = E\int da = EA$$

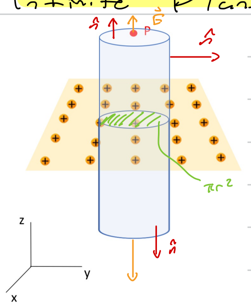
The magnitude is constant for all points on SIDE

$$\Phi_E = E \cdot 2\pi r \ell$$

$$E \cdot 2\pi r \cancel{\ell} = \frac{\lambda \cancel{\ell}}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Infinite Plane





Also works for this and smaller

Direction of Electric Field?

$A = \pi r^2$

$$\sigma = \frac{Q}{L^2} \qquad \oint \hat{n}\vec{E}\,da = \frac{Q_{ENC}}{\varepsilon_0} \qquad Q_{ENC} = \sigma\pi r^2$$

Surface Charge density

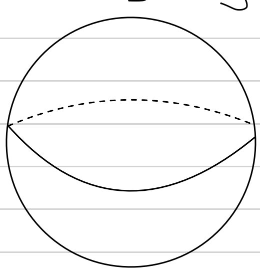
$$= \int_{Caps} \hat{n}\vec{E}\,da = 2\int E\,da = 2EA = 2E\pi r^2$$

SIDE $\hat{n} = \hat{r}$ $\theta = 90°$

CAPS $\hat{n} = \pm\hat{k}$ $\theta = 0$

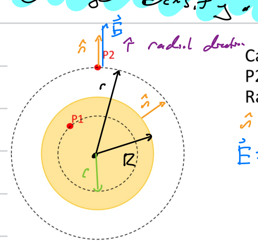
$$2E\cancel{\pi r^2} = \frac{\sigma\cancel{\pi r^2}}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

Spherical Symmetry



Charge Density: $\rho = \frac{Q}{V}$ $\left\{\frac{C}{m^3}\right\}$

$$\oint \hat{n}\cdot\vec{E}\,da = \frac{Q_{enc}}{\varepsilon_0}$$



Calculate the Electric field at points P1 and P2 **using Gauss Law** of the sphere of Radius R and charged with a density ρ

$\hat{n} = \hat{r}$

$\vec{E} = E\hat{r}$

$$\frac{Q_{enc}}{\varepsilon_0} = \frac{\rho\frac{4}{3}\pi R^3}{\varepsilon_0}$$

Flux through gaussian surface which's outer radius sphere → $\oint E\cos(0)\,da = Ea = E(4\pi r^2)$

$r > R$: $\frac{\rho\frac{4}{3}\pi R^3}{\varepsilon_0} = E(4\pi r^2) \Rightarrow \boxed{E = \frac{\rho R^3}{3\varepsilon_0 r^2}}$

outside the sphere

enclosed radius

$r < R$: $\hat{n} = \hat{r}$, $\vec{E} = E\hat{r}$

$$\frac{Q_{enc}}{\varepsilon_0} = \frac{V\rho}{\varepsilon_0} = \frac{\frac{4}{3}\pi r^3\rho}{\varepsilon_0}$$

Gaussian surface is r

$$\oint E\,da = EA = E(4\pi r^2)$$

$$\frac{\rho\frac{4}{3}\pi r^3}{\varepsilon_0} = E(4\pi r^2) \Rightarrow \boxed{E = \frac{\rho r}{3\varepsilon_0}}$$

inside the sphere

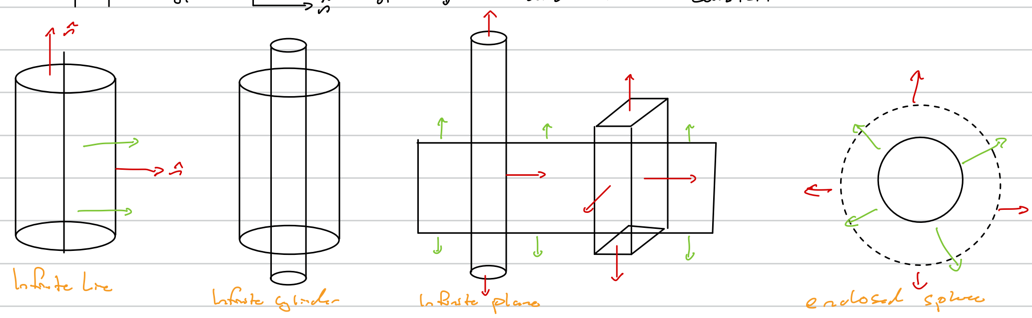
Any charge outside the gaussian surface does not affect the flux

Exam 1 Review Lecture

Gauss Law

$$\oint \vec{E}\,\hat{n}\,da = \oint \vec{E}\cdot d\vec{a}$$

$\vec{E} \uparrow \uparrow \hat{n}$ or $\vec{E} \perp \hat{n}$ or $\vec{E} \updownarrow \hat{n}$ and $\vec{E}$ is constant

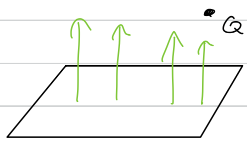


Coulombs Law

- $\vec{F} = Q\vec{E}$
- E by point q: $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

If you have only two charges: $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r}$

But if not 2 point-like:



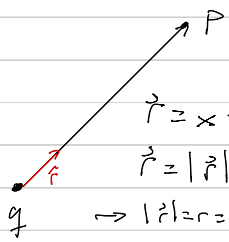
Q's $\vec{F}$?

$\vec{E} = \frac{\sigma}{2\varepsilon_0} \rightarrow \vec{F} = Q\frac{\sigma}{2\varepsilon_0}$

$\hat{r}$ vs. $\vec{r}$

charge densities

$\lambda = \frac{C}{m}$ linear, $\sigma = \frac{C}{m^2}$ surface, $\rho = \frac{C}{m^3}$ volume



$\vec{r} = x\hat{i} + y\hat{j}$

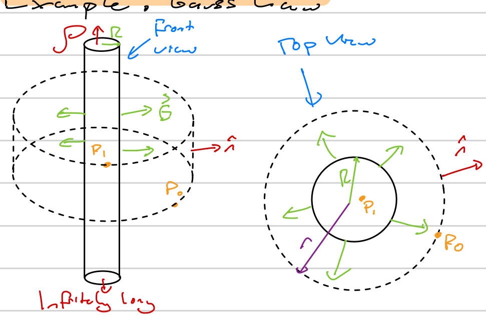
$\vec{r} = |\vec{r}|\cdot\hat{r}$

Direct Integration

$\rightarrow |\vec{r}| = r = \sqrt{x^2+y^2}$ magnitude

$\rightarrow \hat{r} = \frac{x}{\sqrt{x^2+y^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2}}\hat{j}$ direction

Example: Gauss Law



$$\oint \vec{E}\cdot\hat{n}\,da = \frac{Q_{enc}}{\varepsilon_0} = \frac{\pi R^2 l\cdot\rho}{\varepsilon_0}$$

$\vec{E} = E\hat{r}$

Sides: $\hat{n} = \hat{r}$

CAPS: $\hat{n} = \pm\hat{k}$

$$= 2\int_{CAPS} E\cos(90)\,da + \int_{SIDES} E\cos 0\,da$$

area of sides

$$= 0 + Ea = E(2\pi r l)$$

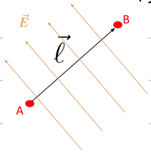
$$\Rightarrow \boxed{E = \frac{R^2\rho}{2r\varepsilon_0}}$$ outside the cylinder

← Cannot depend on parameter of Gaussian surface (l in this case)

Midterm Exam 2

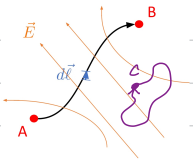
Circulation Law

Line Integrals



Straight line & uniform $\vec{E}$

- Line Integral $= \vec{\ell} \cdot \vec{E}$ for $\vec{\ell}$ the displacement vector
  - ↳ For this simple application (straight line & uniform $\vec{E}$)



General

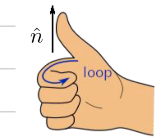
- Line integral $= \int_A^B \vec{E} \cdot d\vec{\ell}$

Closed Loops

- Circulation: $\oint \vec{E} \cdot d\vec{\ell}$

Right Hand Rule

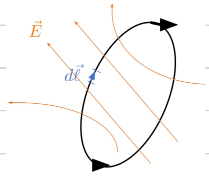
- Stokes convention
- Use direction of loop to determine $\hat{n}$ direction using:



Circulation

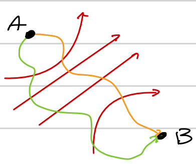
- The circulation of the electric field equals zero

$\rightarrow$ Circulation $= \oint d\vec{\ell} \cdot \vec{E} = 0$
(electrostatics)



- **Alternative**: The line integral between two points do not depend on path

$$\int_A^B \vec{E} \cdot d\vec{\ell} = -\int_B^A \vec{E} \cdot d\vec{\ell} \; \left\{ \int_A^B \vec{E} \cdot d\vec{\ell} \right\}$$

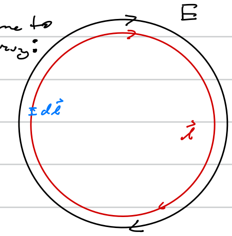


$$\Rightarrow \oint \vec{E} \cdot d\vec{\ell} = \int_A^B \vec{E} \cdot d\vec{\ell} + \int_B^A \vec{E} \cdot d\vec{\ell} = 0$$

Choose the path that is most easy to integrate

No closed Loops! (Proof by Contradiction)

Assume to the contrary: $\vec{E}$



So, $\oint \vec{E} \cdot d\vec{\ell} = \oint |\vec{E}||d\vec{\ell}|\cos\theta$

>0 strictly

$\theta = 0$ $\quad = \oint |\vec{E}||d\vec{\ell}|$

$= (+) > 0$

No closed loops! but $\oint \vec{E} \cdot d\vec{\ell} = 0$ by circ. law

Application

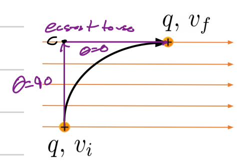
P121

- **Recall**: Work-Energy Theorem

$$\int_{i \to f} d\vec{\ell} \cdot \vec{F}_{Total} = \Delta K = \tfrac{1}{2}m(v_f^2 - v_i^2)$$

- But for electric force we don't need the path!

$$\rightarrow \int_{i \to f} d\vec{\ell} \cdot (q\vec{E}) = \Delta K = \tfrac{1}{2}m(v_f^2 - v_i^2) = \text{Same for all paths btwn } i \text{ and } f$$

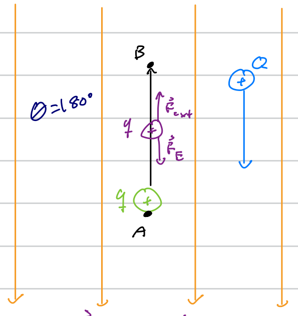


Example

$$\int_A^B \vec{E} d\vec{\ell} = \int_A^C \vec{E} d\vec{\ell} + \int_C^B \vec{E} d\vec{\ell}$$
$$= \int_A^C E d\ell \cos 90 + \int_C^B E d\ell \cos 0$$
$$= E \int_C^B d\ell = E \cdot L$$

Electrostatic Energy

Overview / Introduction



$\vec{E} = E(-\hat{y})$

$\vec{F} = Q\vec{E} = QE(-\hat{y})$

- Apply force to bring +q to B from A
- Counteracts $\vec{E}$
- Release q at B → moves down

$\vec{F}_{ext} = -\vec{F}_E$ by N2L

Assumption: We move the charge <u>very</u> slowly, so the initial, final, and intermediate velocity is zero and hence there is <u>No Kinetic Energy</u>

↳ Spend an infinite amount of time from A to B $\left(v = \frac{\Delta x}{\Delta t}\right)$

→ Energy Input = Work done by external agent $= \int_A^B \vec{F}_{ext} \cdot d\vec{l}$

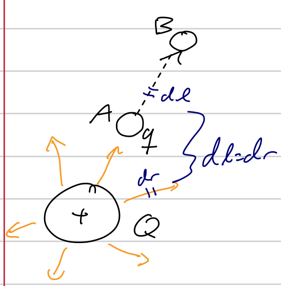
Using our assumption

- The external force matches the electric force: $U_B - U_A = \Delta U = \int_A^B d\vec{l} \cdot \vec{F}_{ext} = -\int_A^B d\vec{l} \cdot (\underbrace{q\vec{E}}_{\vec{F}_E})$, $\vec{F}_{ext} = -\vec{F}_E$

  ↳ In this simple (uniform $\vec{E}$) case: $U_B - U_A = \Delta U = qE(z_B - z_A)$ wow! GPE $(U_G = mgy)$

This is a particular formula, <u>not</u> general!

Point Charge



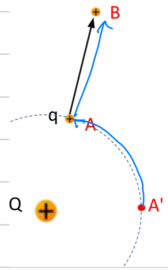
$$\Delta U = -q\int_A^B d\vec{l} \cdot \vec{E} = -q\frac{Q}{4\pi\varepsilon_0}\int_A^B \frac{\hat{r} \cdot \hat{r}}{r^2}\, dl = -\frac{qQ}{4\pi\varepsilon_0}\int_A^B \frac{dl}{r^2} = \frac{-qQ}{4\pi\varepsilon_0}\int_A^B \frac{dr}{r^2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$d\vec{l} = \hat{r}\, dl$$

$$\Rightarrow \Delta U = \frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

If $r_A \to \infty$ chosen as reference: $U = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}$



$\Delta U_{AB} = \Delta U_{A'B}$

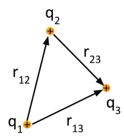
choose path in green

→ along dashed line

integral is 0 as you

move tangent

Arrangement of Charges



- The total energy of the system equals the sum of the energy of each pair of charges

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Electrostatic Potential

- Defined Φ, also seen as V
- Do not confuse w/ electrostatic energy (U)
- Another way to describe an electric field
- Electrostatic potential is a scalar (not a vector)
- To know $\vec{E}$ or Φ for any position space provides same amount of info (only electrostatics)
- If you know one you can figure out the other
- Voltage $V = \Delta\Phi$

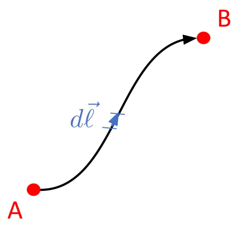
$\vec{E}(r) \longrightarrow \Phi(r)$ ⟵ not discussed here

$\Phi(r) \longrightarrow \vec{E}(r)$ not easily but derivable

Examples of ... Scalar Fields: • Elevation • Temperature

Vector Fields: • Gravity • Wind

Definition



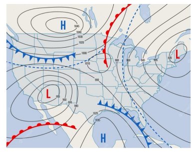
- $\Delta\Phi = \Phi(B) - \Phi(A) \equiv -\int_A^B d\vec{\ell}\cdot\vec{E}$
- If we set a reference point ($\Phi = 0$)

$$\Phi(\vec{r}) \equiv -\int_{ref\to\vec{r}} d\vec{\ell}\cdot\vec{E}$$

$\Phi(ref) = 0$

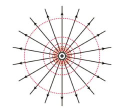
Scalar Quantity - Use Equipotential Lines

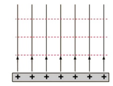
Equipotential lines



Same atmospheric preasure

Point charge

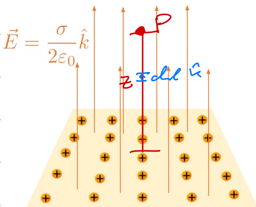




Infinite Plane

- Always perpendicular to electric lines

Example: Potential - Infinite Plane

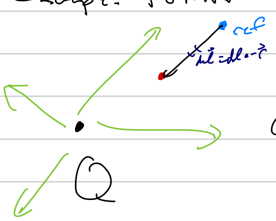


- Choose $z = 0$ so $\Phi(z=0) = 0$

$\vec{E} = \frac{\sigma}{2\varepsilon_0}(\pm\hat{k})$ (from gauss law)

$$\Delta\Phi = \cancel{\Phi_{ref}} - \Phi_P = -\left(-\int_{ref\to P} d\vec{\ell}\cdot\vec{E}\right) = \int E\,d\ell\cos\theta = E\int_{z_P}^{ref=0} dz = Ez = \boxed{-\frac{\sigma z}{2\varepsilon_0}}$$

Example: Potential Point Like Charge



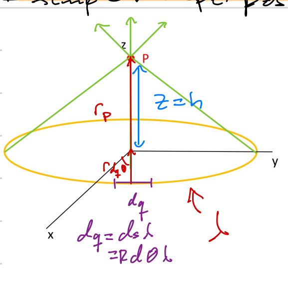
$\Phi(r\to\infty) = 0$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

$\Phi(r) = -\int d\vec{\ell}\cdot\vec{E} = -\int \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\cdot\hat{r}\,dr$

$$\boxed{\Phi(r) = \frac{Q}{4\pi\varepsilon_0 r}}$$

Example: Superposition Principle ($\vec{E}$ vs. Φ)



a) Develop the field @ P

Should point up

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{z^2+R^2}^{\,3}}\left(z\hat{k} - R(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})\right)$$

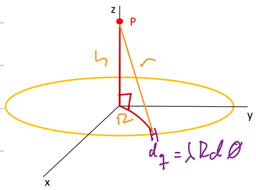
$\left.\begin{array}{l}\vec{r}_P = z\hat{k} \\ \vec{r}_{dq} = R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath}\end{array}\right\}$ $\vec{r} + \vec{r}_{dq} = \vec{r}_P \rightarrow \vec{r} = z\hat{k} - R(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$

$\hookrightarrow r = \sqrt{z^2 + R^2(1)}$

$$\Rightarrow \vec{E} = \frac{R\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi}\left(z\hat{k} - \cancel{R\cos\theta\hat{\imath}} - \cancel{R\cos\theta\hat{\jmath}}\right)d\theta \rightarrow \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\pi R\lambda z}{(z^2+R^2)^{3/2}}\hat{k}}$$

$\cos\theta$ and $\sin\theta$ are 0 to $2\pi = 0$

b) Develop potential without using $\vec{E}$



$\Phi(P) = ?$

$$d\Phi = \frac{1}{4\pi\varepsilon_0}\,\frac{dq}{r}$$

Its easier, in general, to find Φ

$$\Phi(h) = \int_0^{2\pi} \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda R\, d\theta}{\sqrt{R^2+z^2}} \Rightarrow \boxed{\Phi(h) = \frac{\lambda R}{2\varepsilon_0}\,\frac{1}{\sqrt{R^2+h^2}}}$$

$r = \sqrt{R^2+h^2}$

Converting between Φ and $\vec{E}$ ← outside of scope of course

$$\begin{tikzcd} \vec{E} \arrow[r, bend left, "\Phi = -\int d\vec{l}\cdot\vec{E}"] & \Phi \arrow[l, bend left, "\vec{E} = -\nabla\Phi"] \end{tikzcd}$$

The gradient Omg calc 3!

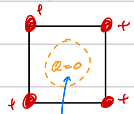
$$\vec{E} = -\nabla\Phi = -\left(\frac{\partial\Phi}{\partial x}, \frac{\partial\Phi}{\partial y}, \frac{\partial\Phi}{\partial z}\right)$$

Conductors

Perfect Conductor Ideal, doesn't exist in nature

- Electric Field inside is zero
  - If $\vec{E} \neq 0$, charges rearrange themselves so $\vec{E} = 0$ — instantaneous for perfect conductors
  - After some time, real conductors behave this way — time is a factor for real conductors (low amt)
- In the surface (outside), the Electric field is perpendicular to the surface of the conductor

→ All net charges reside in the surface



$Q_{enc} = \varepsilon_0 \oint d\vec{a} \cdot \vec{E} = 0$

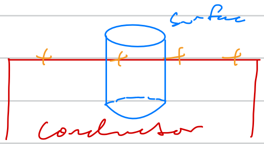
Enclosed charge is 0, and all charge is in surface

$Q = Q_+ - Q_- = 0$ (0 charge, but particles exist inside)

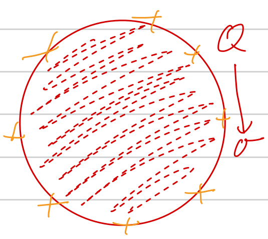
Charge Density

$$\oint d\vec{a} \cdot \vec{E} = \int_{\uparrow} E\, da + \int_{\downarrow} \overset{\text{inside conductor}}{0} + \int_{\text{side}} (0) = E A_{cap}$$

inside conductor; $\hat{n}$ $\vec{E}$ is vertical; $\hat{n}$ horizontal $\Big\}$ $\theta = 90°$

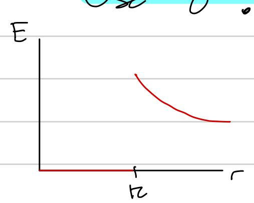


$$\longrightarrow E \cancel{A_{cap}} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma \cdot \cancel{A_{cap}}}{\varepsilon_0} \quad \text{so} \quad \boxed{E = \frac{\sigma}{\varepsilon_0}, \quad \sigma = E\varepsilon_0}$$

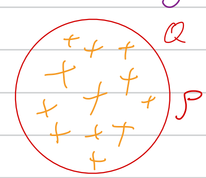


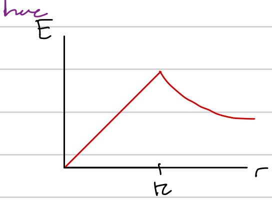
If the sphere is a perfect conductor, all charges are on the surface!

Use σ!

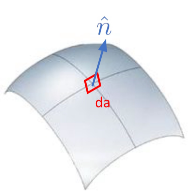


Different from insulators worked w/ previously as they had charges evenly distributed throughout volume; we use ρ here





Force on the Conductor



Force: $\vec{F} = q\vec{E}$ (known)

$\rightarrow d\vec{F} = \frac{1}{2}\sigma\, da\, E\hat{n}$

$\vec{E} = E\hat{n}$ ← outside conductor

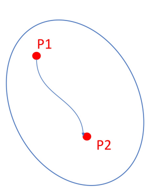
$\frac{1}{2}E$ ← on surface? average

$\vec{E} = 0$ ← inside conductor

$$\vec{F} = \frac{1}{2}\int \sigma E \hat{n}\, da \quad \left(\text{use } \sigma = E\varepsilon_0, \text{ or } E = \frac{\sigma}{\varepsilon_0}\right)$$

$$= \frac{1}{2}\varepsilon_0 \int E^2 \hat{n}\, da = \frac{1}{2\varepsilon_0}\int \sigma^2 \hat{n}\, da$$ ← always positive (outwards)

Potential inside conductor



$$\Delta\Phi = \int d\vec{l} \cdot \vec{E} = -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}$$

→ but $E = 0$ inside conductor $\therefore \Delta\Phi = 0$

So, conductors are equipotential surfaces

Watch out!

$\Delta\Phi = \Phi_B - \Phi_A = 0$

$\rightarrow \Phi_A = \Phi_B \neq 0$

↑ just the difference

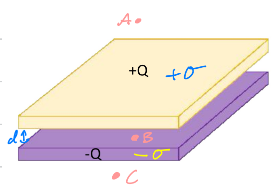
Had to use this on final last year

Capacitance

• Charge Q is proportional to potential difference (often called voltage) between two electrodes by a quantity called Capacitance

$Q = CV$ where $Q \equiv$ Charge (coulombs, C)
$C \equiv$ Capacitance (Farads, F)
$V \equiv$ Voltage (Volts, V) ⟵ Convention: Use volts instead of potential difference (equiv to $\Delta\Phi$)

Parallel plates



Assume Area $A \gg d^2$

$C = \frac{Q}{V} = \frac{\sigma A}{}$

$V = \Delta\Phi = -\int_1^2 d\vec{l}\cdot\vec{E}$

Q: What's $\vec{E}$ at all 3 Points?

above (+) → A: $\uparrow\frac{\sigma}{2\varepsilon_0}$ $\downarrow -\frac{\sigma}{2\varepsilon_0}$ $+\sigma$ ⟶ $E = 0$

btwn → B: d $\downarrow 2\times\frac{\sigma}{2\varepsilon_0}\downarrow$ $-\sigma$ ⟶ $E = \frac{\sigma}{\varepsilon_0}$

below (−) → C: $\downarrow -\frac{\sigma}{2\varepsilon_0}$ $\uparrow\frac{\sigma}{2\varepsilon_0}$ ⟶ $E = 0$

Derive from Gauss Law (Previous lectures)

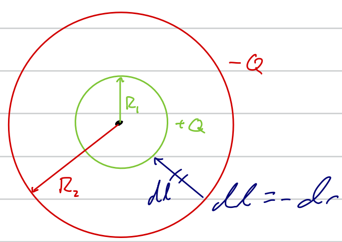
Use $\vec{E}$ @ all 3 points to get C of parallel plate capacitors?

$C = \frac{Q}{|V|} > 0$ ← $\Delta\phi$

$\Delta\phi = -\int_{P_1\to P_2} d\vec{l}\cdot\vec{E} = -\int_{P_1\to P_2} dl\,E\cos 180 = E\int_0^d dl = d\left(\frac{\sigma}{\varepsilon_0}\right)$, $Q = \sigma A$

So, $\boxed{C = \frac{\sigma A}{\frac{d\sigma}{\varepsilon_0}} = \frac{\varepsilon_0 A}{d}}$

Example



$C = \frac{Q}{V}$, $V = \Delta\phi = -\int_{-\to+} \vec{E}\cdot d\vec{l}$

① Determine $\vec{E}$ in 3 points (only really need to one btwn ⊖ and ⊕, just doing this for practice)

$\vec{E}_{inside} = 0$ $Q_{enc} = 0$

$\vec{E}_{btwn}$: $Q_{enc} = Q$

$\frac{Q_{enc}}{\varepsilon_0} = E_a$ (Area of Gaussian surface) ⟶ $E = \frac{Q}{4\pi r^2\varepsilon_0}$

• spherical & hollow shells
• Conductive materials
• Determine Capacitance (C)

$\vec{E}_{outside} = 0$ $Q_{enc} = Q - Q = 0$

② Determine $\Delta\phi$

$\Delta\Phi = -\int_{-\to+} E\,dl\cos 180 = \int \frac{Q}{4\pi r^2\varepsilon_0}dl = -\int_{R_2}^{R_1} \frac{Q}{4\pi r^2\varepsilon_0}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \boxed{C = \frac{Q}{V} = 4\pi\varepsilon_0\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1}}$

Exam 2 Review Lecture

- Dielectrics will <u>not</u> be on the homework

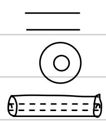
3 Key Topics

- Potential Energy
- Electric Potential
- Capacitance

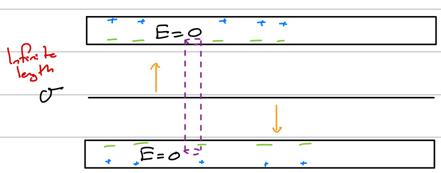
} Including all from Exam 1

Capacitance

$$C = \frac{Q}{V} \begin{cases} \varepsilon_0 \frac{A}{d} \\ \\ \end{cases}$$



Parallel plate



$E = \frac{\sigma}{2\varepsilon_0}$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \frac{A(\sigma + \sigma_u + \sigma_l)}{\varepsilon_0} = \frac{A(\sigma + 2\sigma_u)}{\varepsilon_0}$$

$\sigma_u = \sigma_l$ by symmetry

$$2\int_{caps} \cancel{\vec{E}\,d\vec{a}} + \int_{side} \overset{\hat{n}\perp\vec{E}}{0} = 0 = \frac{A(\sigma + 2\sigma_u)}{\varepsilon_0}$$

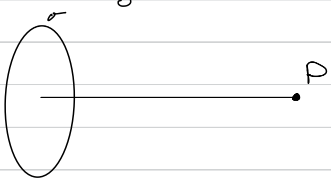
$$\sigma + 2\sigma_u = 0 \qquad \boxed{\sigma_u = \sigma_l = -\frac{\sigma}{2}}$$

Question 2 on hw 5, I got it right!

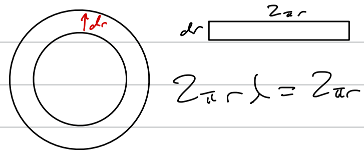
Outside edges of slabs should be $+\frac{\sigma}{2}$ to keep conductors neutral

Potential by a Disk

HW5 Q1



$$\phi_{ring} = \frac{1}{4\pi\varepsilon_0} \frac{\overbrace{\lambda(2\pi r)}^{q}}{\sqrt{r^2+z^2}} \Rightarrow \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{2\pi r\sigma\, dr}{\sqrt{r^2+z^2}}$$



$$2\pi r\lambda = 2\pi r\, dr\, \sigma$$

Don't expect something like this on the exam

Potential & Energy

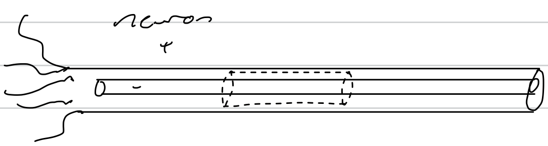
$$\Delta\phi = -\int \vec{E}\cdot d\vec{l}$$

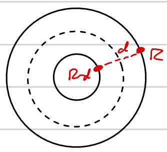
$$\Delta U = -Q\int \vec{E}\cdot d\vec{l}$$

$$\Delta U = Q\,\Delta\phi$$

Example Regarding Capacitance

neuron





$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2\pi r L) = \frac{Q_{enc}}{\varepsilon_0}$$

$$E = \frac{Q}{2\pi r L\varepsilon_0}$$

$$V = \Delta\phi = \int_{R-d}^{R} \frac{Q}{2\pi\varepsilon_0 r L}\, dr \propto \log\left(\frac{R}{R-d}\right)$$

Midterm Exam 3

Dielectrics

- Dielectrics = Insulators

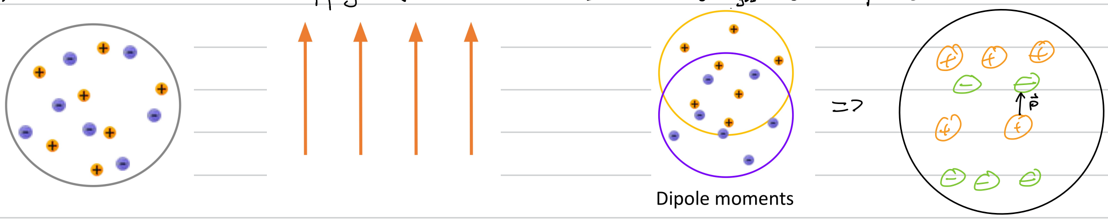
  examples: Glass, Plastic, Water, Diamond

- charges stay put

Internal charge aka. Bound Charge vs. External charge aka. Free Charge or Fields

Response of Internal Charges to Fields

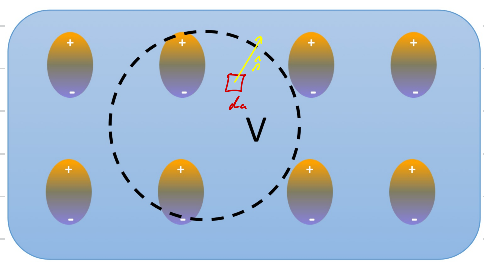
Atom of Dielectric → Apply electric Field → Bound charges are displaced



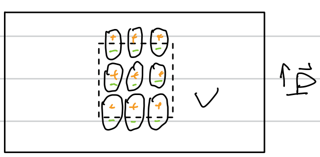
Displaced densities in the atom

Dipole

- $\vec{p} = q\,\vec{d}$
- Polarized medium
  → Polarization vector $\vec{P}$
  - Dipole Moment Density: $\vec{P} = \frac{\Sigma \vec{p}}{V}$ ← of all ← volume



How much bound charge inside V?



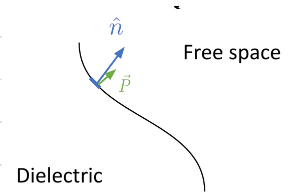
- $\vec{P} \equiv$ uniform, $Q_B(V) = 0$
- Answer depends on $\vec{P}$ at the surface $\qquad Q_B \propto \vec{P}_{surface}\, V$
- If $\vec{P}$ is parallel to the surface, no net charge

$$Q_{bound} = Q_B(V) = -\oint da\,\hat{n}\cdot\vec{P} = -\oint |\vec{P}|\,|d\vec{a}|\cos\theta$$

- Only for Dielectrics
- Not Gauss' Law, this is: $Q_{bound} + Q_{free} = \varepsilon_0 \oint da\,\hat{n}\cdot\vec{E}$ $\qquad$ Lone Charge

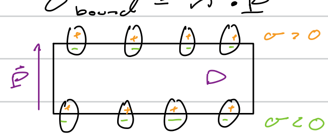
$$Q_{free} = \oint da\,\hat{n}\cdot\underbrace{(\varepsilon_0\vec{E} + \vec{P})}_{\vec{D}}$$

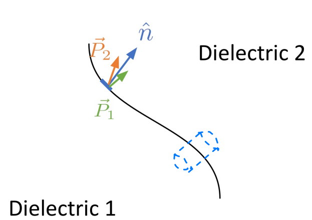
Surface Charge Density



One dielectric

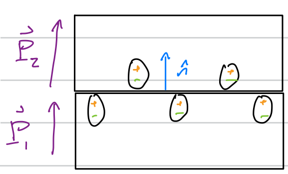
$$\sigma_{bound} = \hat{n}\cdot\vec{P}$$



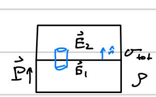


Two dielectrics

$$\sigma_{bound} = \hat{n}\cdot\vec{P}_1 - \hat{n}\cdot\vec{P}_2$$



Gauss Law to get σ_{total}



$\vec{E}_i = 2\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

$\sigma = \varepsilon_0 E_i$

$\sigma_{total} = -\varepsilon_0\,\hat{n}\cdot(\vec{E}_1 - \vec{E}_2)$

$\sigma_{free} = \sigma_{total} - \sigma_{Bound}$

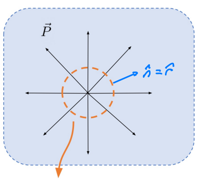
$$\sigma_{free} = \hat{n}\cdot\underbrace{(\varepsilon_0\vec{E}_2 + \vec{P}_2)}_{\vec{D}_2} - \hat{n}\cdot\underbrace{(\varepsilon_0\vec{E}_1 + \vec{P}_1)}_{\vec{D}_1}$$

Linear Isotropic Dielectrics

$\vec{P} \propto \vec{E}^{1}$ ← simplest case — proportionality depends on the material, temperature and pressure $\Rightarrow$ $\vec{P} = \chi \varepsilon_0 \vec{E}$ → Electric Susceptibility

Generally, $\vec{P} \propto \vec{E}^{x}$ (to a given power)

Example



• Assume $\vec{P} = \frac{A}{r^2}\hat{r}$ ← constant

• What is the bound charge inside a sphere with radius r?

$$Q_{bound} = \cancel{Q_B(V)} - \oint da\, \hat{n} \cdot \vec{P}$$

uniform $\vec{P}$

$$= 4\pi r^2 P \cos(\,)$$

$$Q_{bound} = -4\pi r^2 \left(\frac{A}{r^2}\right) = -4\pi A \qquad \boxed{Q_{bound} = -4\pi A}$$

Note: Q_{bound} independent of r, in fact its a constant value

↳ Why? All the charges at the center

• What is the electric field?

$$\vec{E} = \frac{\vec{P}}{\chi\varepsilon_0} = \frac{\frac{A}{r^2}\hat{r}}{\chi\varepsilon_0} = \frac{1}{\chi\varepsilon_0}\frac{A}{r^2}\hat{r} \qquad \boxed{\vec{E} = \frac{1}{\chi\varepsilon_0}\frac{A}{r^2}\hat{r}}$$

• What is the total charge inside a sphere with radius r

$$\frac{Q_{enc}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{a} \Rightarrow Q_{enc} = \varepsilon_0 E \oint da = \varepsilon_0 E 4\pi r^2 = \cancel{\varepsilon_0}\frac{1}{\chi\cancel{\varepsilon_0}}\frac{A}{\cancel{r^2}}4\pi \cancel{r^2} = \frac{4\pi A}{\chi} \qquad \boxed{Q_{total} = \frac{4\pi A}{\chi}}$$

• What is the external (free) charge inside a sphere with radius r?

$$Q_{free} = Q_{total} - Q_{bound} = 4\pi A \frac{1+\chi}{\chi} \qquad \boxed{Q_{free} = 4\pi A \frac{1+\chi}{\chi}}$$

You might see $1+\chi$ as ε_r (relative permittivity) or the dielectric constant Kappa (κ)

• What is the electric field as a function of free charge?

$$\vec{E}\left(Q_F = 4\pi A \frac{1+\chi}{\chi}\right)? \quad A = \frac{\chi Q_F}{4\pi(1+\chi)} \Rightarrow \vec{E} = \frac{1}{\cancel{\chi}\varepsilon_0 r^2}\left(\frac{\cancel{\chi} Q_F}{4\pi(1+\chi)}\right)\hat{r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0(1+\chi)}\frac{Q_F}{r^2}\hat{r}}$$

Basically coulombs law, but with $(1+\chi)$ factor.

We will define $\kappa = 1+\chi$ (Kappa)

Parallel Plate Capacitor

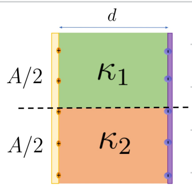
• Capacitance is affected: $C = \kappa C_{air} = (\kappa\varepsilon_0)\frac{A}{d}$

Material	Dielectric constant, κ
Air	1(0.85-1.21 at 20ºC)*
Water	80 at 20ºC (79.50-80.24)*
Ice	3 at -5ºC*
Basalt	12
Granite	7-9
Sandstone	9-11
Dry loam	3.5
Dry sand	2.5 (2.5-3.5 at 20ºC)*
Ethanol liquid	15.21 at 20ºC*
Ethanol Mixing Liquid	39.82 at 20ºC*
Acetone	21.20 at 20ºC*

*measured dielectric constant in this study

What is the new Capacitance with two dielectrics?

In Parallel



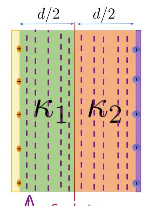
$C_1 = \frac{Q_1}{V}$, $C_2 = \frac{Q_2}{V}$ — for constant V $(V = Ed)$

Gauss Law: $E\frac{A}{2} = \frac{Q_i}{\kappa_i\varepsilon_0} \Rightarrow Q_i = (\kappa_i\varepsilon_0)E\frac{A}{2}$

$$\therefore C_i = \frac{\kappa_i\varepsilon_0}{2}\frac{A}{d}$$

$$\Rightarrow C = \frac{Q_1 + Q_2}{V} = \frac{\kappa_1 + \kappa_2}{2}\varepsilon_0\frac{A}{d} = C_1 + C_2$$

In Series



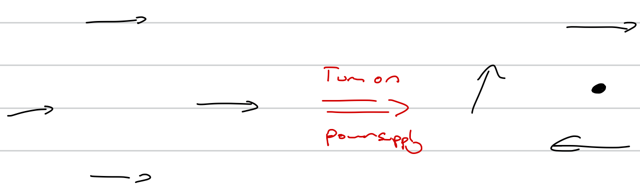
Equipotential Lines

$$\Delta\phi = \int \frac{\sigma}{\kappa\varepsilon_0} dl = \frac{\sigma}{\kappa\varepsilon_0}\int dl = \frac{\sigma}{\kappa\varepsilon_0} y$$

$$C = \frac{Q}{V_1 + V_2} = \left(\boxed{\frac{V_1 + V_2}{Q}}\right)^{-1} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

Magnetic Fields

Demo w/ Compasses



Electric Current produces a magnetic field which is circular as seen by the direction of the compass arrows

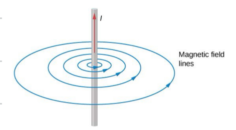
Magnetic Fields

- Moving electric charges produce magnetic fields
  - ↳ there are no "magnetic charges"
- Magnetic fields exert forces on moving electric charges

Note: Moving charges still produce electric fields!

Law of Magnetostatics

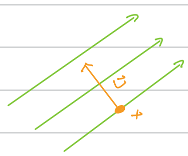
- Electric currents (or just currents) produce magnetic fields $\vec{B}$
- $\vec{B}$ exert forces on moving charges



Definition of Magnetostatics: Assume that there is no accumulation of charges, but the charges can move

Lorentz Force

- The force must depend on the charge, velocity, and field

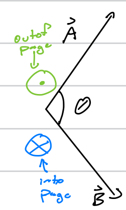


$$\vec{F} = \vec{F}(q, \vec{B}, \vec{v})$$

- The force is the vector product of $\vec{v}$ and $\vec{B}$

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Recall - Cross Product



- The cross product of two vectors is a vector

$$\vec{A} \times \vec{B} = \text{vector}$$

- Its magnitude:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin(\theta)$$

- Direction:

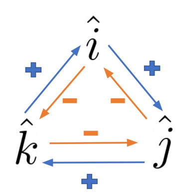
$\vec{A} \times \vec{B}$ is perpendicular to $\vec{A}$ and $\vec{B}$

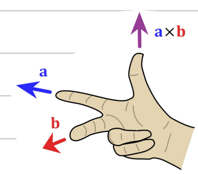
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$\vec{A} = 3\hat{i} + 4\hat{j}$

$\vec{B} = 6\hat{i} + 7\hat{k}$

$\vec{A} \times \vec{B} = 28\hat{i} - 21\hat{j} - 24\hat{k}$





Currents

- Which charges move?
  - → Metals ⟹ Negative charges move
  - → Semiconductors ⟹ Positive charges move

we will assume that positive charges move!

$I \equiv$ Current flowing through surface in direction

$$I \equiv \frac{\Delta Q}{\Delta t}$$

ΔQ → charge that cross surface

Δt → during a certain time

Units $\frac{\text{Coulombs}}{\text{Second}} = \text{Ampere}$

Currents: Line Wire

- Assume an ∞ line with infinitesimal thickness
  - ↳ current I means
    - → I coulombs going up
    - → $-I$ coulombs going down
- All charges (+) in the interval $v\Delta t$ will cross P in a time Δt
- The current: $I = \frac{\Delta Q}{\Delta t} = \frac{\lambda v \Delta t}{\Delta t} = \lambda v$

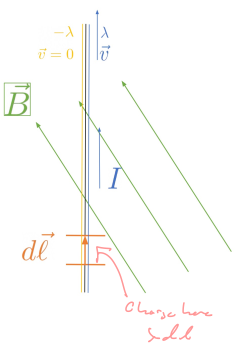
Coulomb vs. Lorentz

→ $F_E = q\vec{E}$

→ $F_B = q\,\vec{v} \times \vec{B}$

↳ Do cross product, then deal with q's sign

Lorentz Force on a Current



- Force on a piece of wire $d\ell$

$$d\vec{F} = (\lambda\, d\ell)\,\vec{v}\times\vec{B} \qquad d\vec{F} = (\lambda\, d\vec{\ell}\, v)\times\vec{B}$$

$\uparrow q = \lambda \cdot d\ell$

Note: $d\vec{\ell}\, v = d\ell\, \vec{v}$

- In terms of current

$$\boxed{d\vec{F} = I\, d\vec{\ell}\times\vec{B}}$$

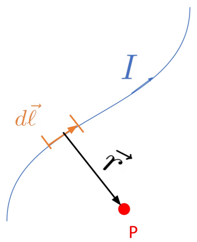
Note: $I = \lambda v$ (see previous!)

- Integrate to solve for $\vec{F}$

$$\boxed{\vec{F} = I\int d\vec{\ell}\times\vec{B} = I\int |d\vec{\ell}|\,|\vec{B}|\sin\theta}$$

for θ btwn $d\vec{\ell}$ and $\vec{B}$

Biot-Savart: Field Created by a current



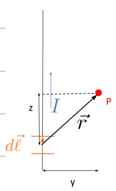
- Magnetic field at Point P due to $d\ell$

New constant

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell}\times\vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi} I \frac{d\vec{\ell}\times\hat{r}}{r^2}$$

Example: Infinite wire



$\vec{r} = y\hat{j} + z\hat{k} \to r = \sqrt{y^2+z^2}$

$d\vec{\ell} = d\ell\,\hat{k}$

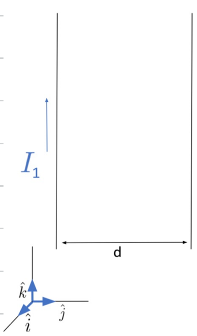
$d\vec{\ell}\times\vec{r} = -y\, d\ell\, \hat{i}$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{-y\, d\ell\, \hat{i}}{(z^2+y^2)^{3/2}}$$

$$\vec{B} = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi} I \frac{-y\, d\ell\, \hat{i}}{(z^2+y^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{y}(-\hat{i})$$

Force btwn 2 infinite wires

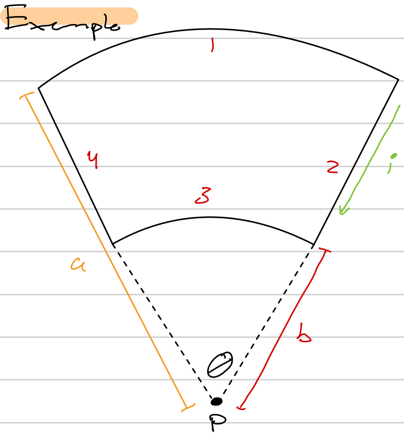


- What is the force that I_1 exert on I_2 ?

$$d\vec{F}_2 = \frac{\mu_0 I_1 I_2}{2\pi d} d\ell(-\hat{j}) = -d\vec{F}_1$$

- If the intensities are parallel the force is attractive,
- If the intensities are anti-parallel the force is repulsive

Example



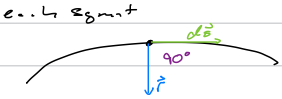
Find Magnetic Field @ P

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{r}}{r^3}$$

(i) Determine direction of $d\vec{s}$ for each segment

a) Angles btwn $d\vec{s}$ and $\vec{r}$ for each segment

→ Segment 1

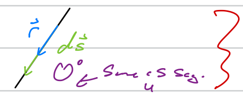


Angle btwn $d\vec{s}$ and $d\vec{r}$?

$|d\vec{s} \times \vec{r}| = |d\vec{s}||\vec{r}|\sin\theta$ $\quad \theta = 90°$ $\quad$ by symmetry, this is the same for seg 3

→ Segment 2

Angle btwn $d\vec{s}$ and $\vec{r}$ in segment 2



$dB_2 = dB_4 = 0$

b) Direction of Segments w/ RHR

① $\vec{B} = B \cdot \otimes$

② $\vec{B} = B \cdot \odot$

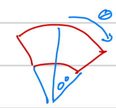
b) Integration

→ How is ds related to $d\theta$

$ds = r\,d\theta$

→ What is the position vector $\vec{r}$

$\vec{r} = -r\sin\theta\,\hat{i} - r\cos\theta\,\hat{j}$



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{s} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} I\, r \frac{d\theta\, \vec{r}}{r^3}$$

$$\int_{-\theta/2}^{\theta/2} d\vec{B}_1 = \int \frac{\mu_0}{4\pi} I \frac{d\theta}{a}\left(-\sin\theta\,\hat{i} - \cos\theta\,\hat{j}\right) = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \cdot \int_{-\theta/2}^{\theta/2} -\sin\theta\,\hat{i} - \cos\theta\,\hat{j}\, d\theta$$

$$\int_{-\theta/2}^{\theta/2} d\vec{B}_2 = \int \frac{\mu_0}{4\pi} I \frac{d\theta}{b}\left(-\sin\theta\,\hat{i} - \cos\theta\,\hat{j}\right) = \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \cdot \int_{-\theta/2}^{\theta/2} -\sin\theta\,\hat{i} - \cos\theta\,\hat{j}\, d\theta$$

This can be tricky to work out, many pitfalls

$$\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{I}{a} \theta \quad \otimes$$

much simpler

$$\vec{B}_2 = \frac{\mu_0}{4\pi} \frac{I}{b} \theta \quad \odot$$