Physics 122 Spring 24

Practice Problems

Practice problems are not for credit and will not be graded.
They will be helpful in preparation for the second midterm exam on Friday, October 18.

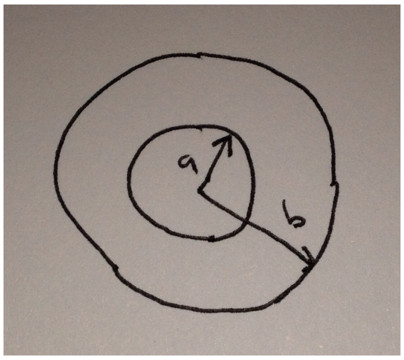


Figure 1: A cylindrical capacitor.

1. Cylindrical capacitor. A cylindrical capacitor consists of two thin cylindrical shells with a common axis. The radius of the inner shell is a and of the outer shell b. For simplicity both shells are taken to be infinite in length. The shells are made by rolling up conducting sheets. The inner shell has a charge per unit length λ; the outer has a charge per unit length $-\lambda$. [We use linear charge density so in a piece oc capacitor of a given length, there is the same amount of charge in both plates. You can relate λ and σ as $\lambda = 2\pi a\sigma_a$ and $\lambda = 2\pi b\sigma_b$.]

(a) What is the electric field for

(i) $0 < r < a$ (inside the inner shell)?

(ii) $a < r < b$ (between the shells)?

(iii) $b < r$ (outside the shells)?

Here r denotes the distance from the axis of the shell.

Useful info: The field of a single thin cylindrical shell of radius R with a charge per unit length λ is zero inside the shell; outside the shell,

$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{r}\hat{\mathbf{n}}, \tag{1}$$

where $\hat{\mathbf{n}}$ is a unit vector at the point of interest that points radially away from the axis of the shell.

(b) Calculate the potential difference $V_2 - V_1$ where V_2 is the potential of the outer shell and V_1 is the potential of the inner shell.

(c) Calculate $\mathcal{C}$, the capacitance per unit length of the cylindrical capacitor. $\mathcal{C}$ is defined as λ/V where $V = V_1 - V_2$ is the potential difference between the capacitor plates.

Helpful integral:

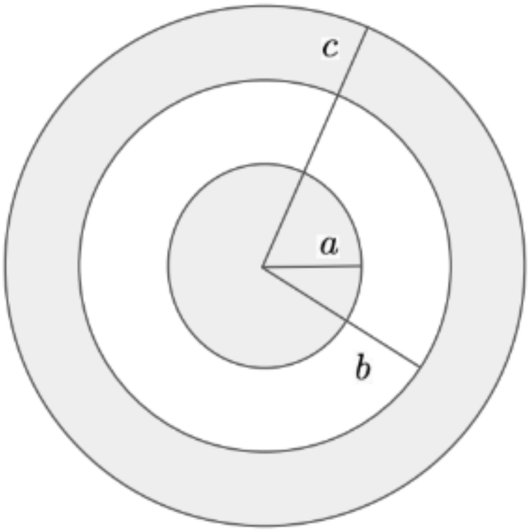
$$\int dr\ \frac{1}{r} = \ln r. \tag{2}$$

2. Capacitor filled with dielectric

A parallel plate capacitor has plates of area $A = 100\,\mathrm{cm}^2$ separated by a distance $d = 1.0\,\mathrm{mm}$. Half of the area of the plates is filled with a dielectric material of dielectric constant $\kappa = 4$ that spans the entire distance between the plates.

1. Calculate the capacitance of the capacitor when there is no dielectric present.
2. Calculate the capacitance of the region of the capacitor filled with the dielectric.
3. Determine the total effective capacitance of the capacitor.
4. If a potential difference of $V = 5.0\,\mathrm{V}$ is applied across the capacitor, determine the total stored energy.

3.Potential of the Earth A sphere the size of the earth has 1 C of charge distributed evenly over its surface. What is the electric field strength just outside the surface? What is the potential of the sphere, with zero potential at infinity?

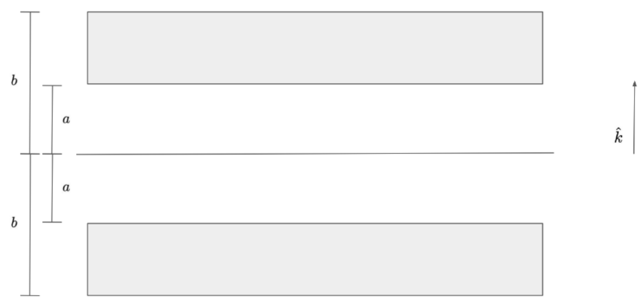


4. Spherical capacitor A conducting sphere (radius a) is surrounded by a spherical conducting shell (inner radius b, outer radius c). The inner sphere has a total charge $+q$; the outer shell has a charge $-q$. Throughout this problem r denotes distance from the center of the sphere.

1. What is the electric field in all regions of space? *Hint: where is the charge located on a conductor? What is the electric field inside a conductor?*

2. What is the potential difference between the outer shell and the inner sphere?

3. A pair of conductors with equal and opposite charge constitute a capacitor. The capacitance of the capacitor is defined as $C = q/V$. Here q is the charge on the positively charged conductor and V is the potential difference between the positively charged conductor and the negatively charged conductor. Calculate C for the spherical capacitor analyzed in this problem.



length

5. Slabs A flat insulating sheet has surface charge σ per unit area. For simplicity we assume it has an infinite lateral area. In addition to the insulating sheet there are also two conducting slabs. These slabs are parallel to the sheet and are placed symmetrically on either side of it. For simplicity we assume that the slabs also have infinite lateral area. We also assume that each slab is neutral.

1. What is the electric field inside the upper conducting slab? Inside the lower? Explain your answer briefly (in one or two sentences).

2. What is the surface charge density on the lower surface of the upper slab and on the upper surface of the lower slab?[*Hint:* Use symmetry and apply Gauss's law to the Gaussian surface shown in the figure]

3. What is the surface charge density on the upper surface of the upper slab and on the lower surface of the lower slab? Briefly explain your answer (one or two sentences).

6. Dielectric cylinder with axial free charge. Free charge is deposited along the axis of an essentially infinite dielectric cylinder of radius a. λ_f is the amount of free charge per unit length along the axis of the cylinder. The dielectric is linear and has susceptibility χ. Assume that the electric field within the dielectric is radial and of the form $\mathbf{E} = E\hat{\mathbf{r}}$. Here $\hat{\mathbf{r}}$ is a unit vector that points radially outward from the axis of the cylinder.

(a) What is the polarization of the dielectric? Give your answer in terms of χ, ϵ_0, E and appropriate unit vectors.

(b) Consider a Gaussian surface that is a cylinder of radius r and length ℓ. The axis of the Gaussian surface coincides with the axis of the dielectric cylinder. For the moment assume $r < a$. How much bound charge does the Gaussian surface contain? Give your answer in terms of r, ℓ, E, χ and ϵ_0.

(c) How much free charge does the Gaussian surface contain? Give your answer in terms of r, ℓ and λ_f.

(d) Apply Gauss's law to the surface defined in part (b) to determine the electric field $\mathbf{E}$. Give your answer in terms of $\lambda_f, \epsilon_0, \chi$ and r.

(e) *External field:* Now suppose that the radius of the Gaussian surface $r > a$. (i) How much free charge does it contain? (ii) How much bound charge? (iii) Apply Gauss's law to determine the electric field $\mathbf{E}$ at points outside the dielectric cylinder.

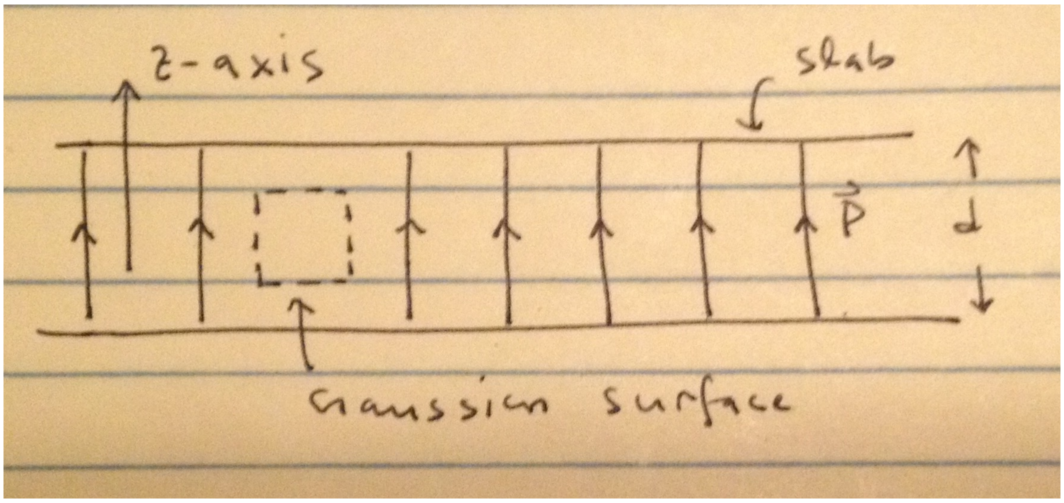


Figure 2: A uniformly polarized dielectric slab. A cubic Gaussian surface is depicted as a square in the figure.

4. Dielectric slab. A infinite dielectric slab of thickness d lies in the x-y plane. The slab is uniformly polarized along the z-axis. The polarization inside the slab is $\mathbf{P} = P\hat{\mathbf{k}}$.

(a) [4 points] Consider the cubic Gaussian surface shown in the figure as a dotted square. What is the bound charge enclosed by this surface?

(b) [4 points] Assume that the dielectric is linear and isotropic with susceptibility χ. What is the electric field inside the dielectric? Give your answer in terms of ϵ_0, P, χ and $\hat{\mathbf{k}}$.

(c) [4 points] What is the total charge inside the cubic Gaussian surface (shown in the figure as a dotted square)? How much free charge lies within the Gaussian surface?

(d) [4 points] What is the surface density of bound charge, σ_b, on the upper surface of the dielectric slab?

(e) [4 points] Assume that the electric field outside the slab is zero. What is the free charge density, σ_f, on the upper surface of the dielectric slab?