Physics 122 Fall 24

Practice Problem Set 3

Practice problems are not for credit and will not be graded.

They will be helpful in preparation for the third midterm exam.

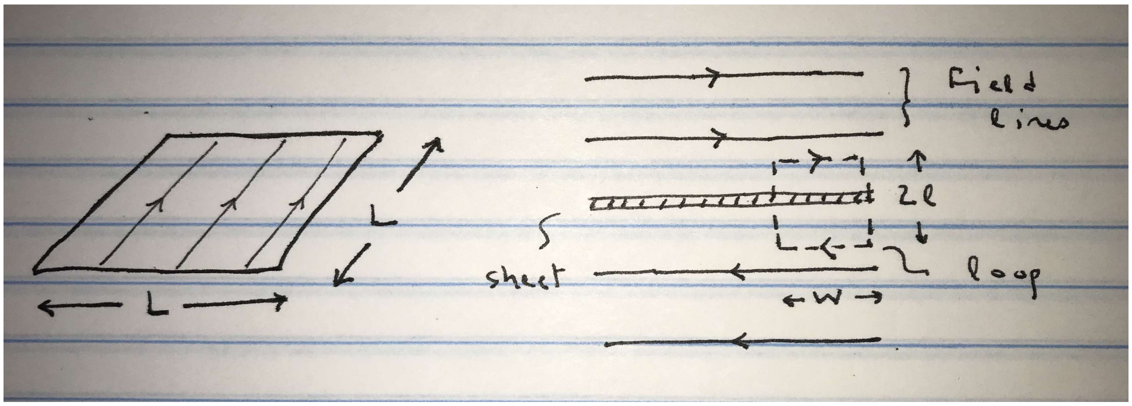


Figure 1: A flat sheet carrying electrical current.

Problem 1. Magnetic field of a sheet of current.

Fig 5 (a) shows a flat $L \times L$ sheet that carries current on its surface as indicated by the arrows. In this problem we assume that the sheet is actually infinite; in other words $L \to \infty$. The current flowing on the sheet corresponds to a current per unit length of κ.

Fig 5 (b) shows a front view of the same flat infinite sheet. In this drawing for clarity the sheet is shown to have a finite thickness and its cross section has been shaded. Also shown in the figure are the magnetic field lines caused by the sheet of current. The form of the field lines can be understood by imagining that the sheet consists of parallel infinite wires stacked side by side.

Also shown in Fig 5 is an Amperian loop (dotted rectangle of height 2ℓ and width w that is assumed to be placed symmetrically so that half of it is below the sheet and the other half is above. The arrows indicate the direction in which the loop is traversed in computing the line integral of **B** around the loop.

(a) What is the circulation of the magnetic field around the Amperian loop shown in the figure? Give your answer in terms of B, the magnitude of the magnetic field along the top and bottom segments of the loop, and w and ℓ.

(b) How much current flows through the Amperian loop? Give your answer in terms of κ and w and ℓ.

(c) Use Ampere's law to determine the magnitude of the magnetic field B. Give your answer in terms of κ and μ_0.

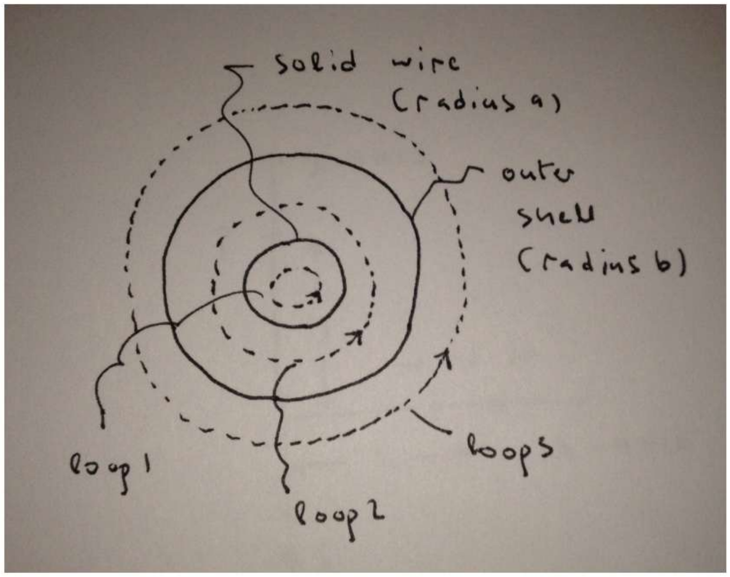


Figure 2: A coaxial cable.

Problem 2. Co-axial cable.

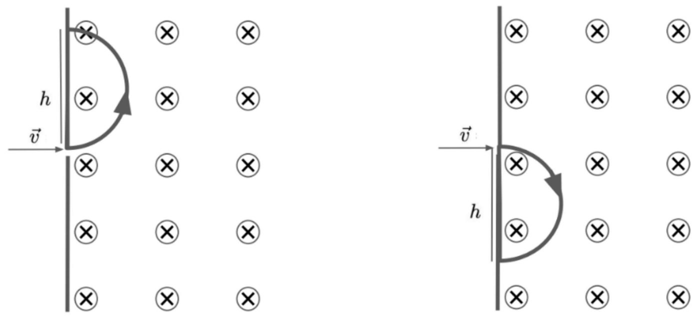
A co-axial cable consists of a solid cylindrical wire surrounded by a thin cylindrical shell. The solid wire has radius a; the shell has a radius b. The wire and shell have a common axis. A current i flows out of the page through the solid cylindrical wire. The same current flows through the shell but into the page. The current through the solid wire is uniformly distributed over its cross section.

(a) Apply Ampere's law to loop 2 show in the figure in order to determine the magnetic field between the solid wire and the shell (i.e. for $a < r < b$ where r is the distance from the common axis).

(b) Repeat part (a) for loop 1 to determine the magnetic field inside the solid wire (i.e. for $0 < r < a$).

(c) Repeat part (a) for loop 3 to determine the magnetic field outside the co-axial cable (i.e. for $r > b$).

In each part explain your reasoning which in context of Ampere's law means writing an expression for the circulation of $\mathbf{B}$ and the current enclosed by the loop.



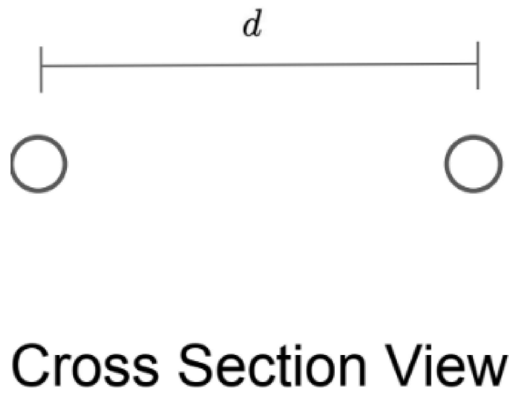
Problem 3. Trajectory

To the right of the screen shown in the figure, a uniform magnetic field of magnitude B points directly into the page. A particle of mass m and charge q moving to the right at a speed v enters this region through a hole in the screen.

1. The figure shows two possible orbits for the particle. Which one does it follow? Explain briefly. You should assume the particle is positively charged.

2. Use Newton's second law to determine h (the distance between the hole and the point at which the particle collides with the screen). Give your answer in terms of q, B, m and v.

3. Assume that the collision with the screen reverses the velocity of the particle. Where will the particle next hit the screen? Briefly explain your reasoning. *Hint: This is called a skipping orbit*



4. Parallel Currents Consider two current carrying wires a distance d apart each with current I in the same direction. Both are electrically neutral.

1. What is the force per unit length on wire one wire due to the magnetic field of the other? Is it attractive or repulsive?

2. What additional electric charge per unit length λ would need to be applied to each to have the total force be zero?

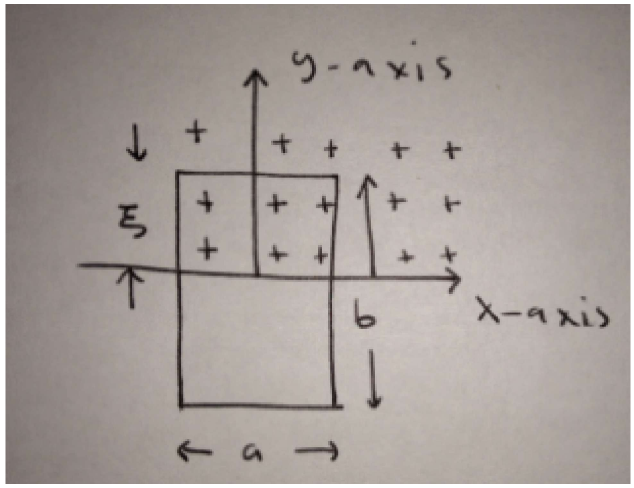


Figure 3: Rectangular Loop in a Magnetic Field

Problem 5: Rectangular Loop in a Magnetic Field

A rectangular loop of dimensions $a \times b$ is pulled into a region with a uniform magnetic field from a region with zero magnetic field as shown in the figure. Thus $\vec{B} = B\hat{k}$ for $y \geq 0$ and $\vec{B} = 0$ for $y < 0$. Assume the loop moves at a uniform velocity $\vec{v} = v\hat{j}$. Let ξ represent the y-coordinate of the top of the loop.

(a) **Flux.** What is the flux ϕ through the loop? Give your answer in terms of B, ξ, a and b. Be careful to distinguish the three cases $\xi < 0$, $0 < \xi < b$ and $\xi > b$.

(b) **E.m.f.**

(i) What is the emf around the loop? Give your answer in terms of B, ξ, a, b and v.

(ii) Is the emf due to an induced electric field or due to magnetic forces? Explain (one or two sentences at most).

(c) **Current and power.** Assume the loop has resistance R.

(i) How much current flows through the loop?

(ii) In what direction does the current flow? Justify your answer using Lenz's law.

(iii) How much power is dissipated as Joule heat? Give your answer to parts (i) and (iii) in terms of B, v, a, b and R.

(d) **Force and power.**

(i) What is the net magnetic force on the loop?

(ii) An external agent must apply a force $\vec{F}_{\text{ext}}$ equal and opposite to the magnetic force in order to keep the loop moving at a uniform velocity $\vec{v} = v\hat{\jmath}$. What is the rate at which this external force does work on the loop? Give your answer in terms of B, v, a, b, R and appropriate unit vectors.

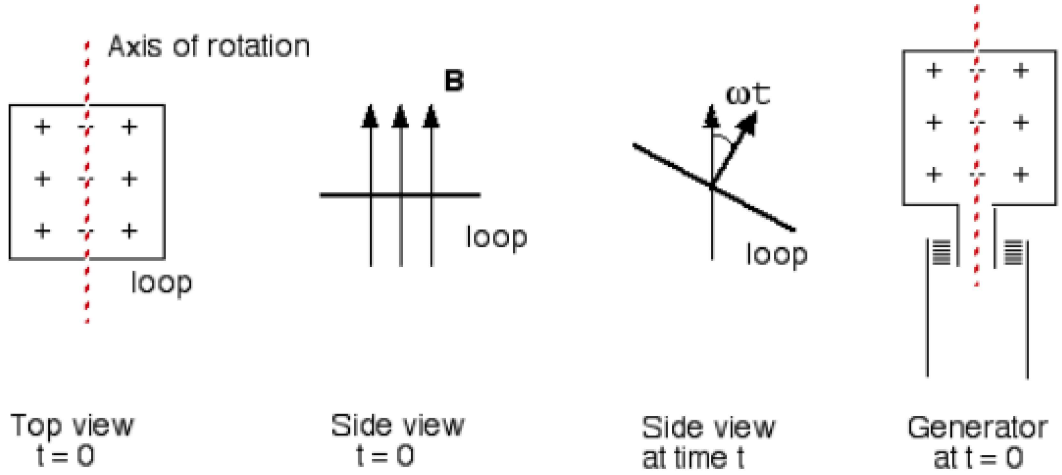


Figure 4: A square loop of side a rotates in a magnetic field.

Problem 6: Generator

A square loop of side a rotates in a magnetic field as shown in fig 6.

(a) What is the magnetic flux through the loop at time t?

(b) What is the e.m.f. around the loop according to Faraday's flux rule?

By connecting an open loop to an external circuit via a split ring and brush connectors one can deliver the e.m.f. calculated in part (b) to an external circuit as shown in figure 6.