Physics 122 Spring 2025

Electrostatics

Homework 2

Due: 11:59 PM Friday, Feb 2, 2025

Figure 1: Atmospheric electricity provides striking examples of electric phenomena that we will explore in lectures and homework.

Guidelines

1. Please write legibly. If necessary solve the problem first on scratch paper. Then write a cogent and legible solution based on your scratch work. Learning to make a clear and logical presentation of your work is an invaluable asset, well worth the trouble of having to write it twice.

2. Please begin each problem on a new page just as we have done with the questions.

3. Parts of problems marked with an asterisk (if any) are only to be read. No response is needed.

How to submit homework: Submit your homework in PDF format via Canvas. You will find a link to submit under Assignments. Please submit before the deadline. Deadlines are enforced and late homework may not be graded. If you have compelling reasons to miss the deadline please contact the instructor preferably before the deadline. In order to generate a PDF of your homework you should scan it. If you do not have access to a scanner there are a number of effective and free scanner apps available for phones. AdobeScan is one that many students have found worked well for them in previous courses. The other option is to take pictures of your homework and convert them to PDFs using your computer. Finally if you work with a tablet you can of course easily save your work as a PDF.

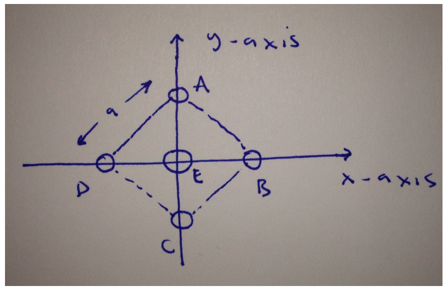

Figure 2: Four charges are placed at the vertices of a square of side a. A fifth charge is placed at the center. The central charge is Q; the charges at the corners are all equal to q.

1. Equilibrium of a square[1] Four charges are placed at the vertices of a square with side of length a and a fifth charge is placed at the center as shown in figure 2. The corner charges each have the same charge q; the central charge is Q.

(a) What is the total electrostatic force on the central charge?

(b) What is the total electrostatic force on the charge at the vertex A? Your answer may depend on $q, Q, 4\pi\epsilon_0, a$ and the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

(c) Write down the total force on each of the other three corner charges also. [*Hint:* Use your result of part (b) and symmetry to write down the answer. No calculation is needed].

(d) What value should the central charge Q have in order that the forces on all five charges are exactly zero?

[1] Adapted from E.M. Purcell, *Electricity and Magnetism*.

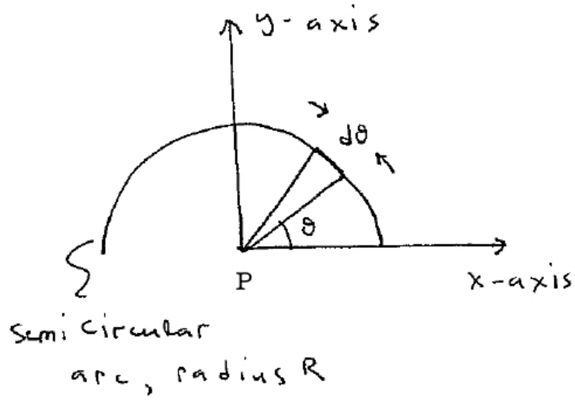

Figure 3: A uniformly charged semi-circular arc of radius R. λ is the charge per unit length.

2. Field of a semi-circular arc A semi-circular arc of radius R has a charge per unit length λ. We wish to compute the electric field $\vec{E}$ at P, the center of the arc.

(a) What is the total charge of the arc?

(b) In what direction does $\vec{E}$ point? Explain briefly.

(c) Consider the infinitesimal segment of arc located at an angle θ from the x-axis. $d\theta$ is the angle subtended by this segment at P as shown in figure 3.

(i) What is the charge of this segment?

(ii) What is the distance of this segment from P?

(iii) Write down the displacement vector $\vec{r}$ from the segment to P. Give your answer in terms of $R, \theta, \hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

(iv) Using Coulomb's law write down the electric field $d\vec{E}$ at P that is produced by the infinitesimal arc segment.

(d) By integration of the result of part (c) determine E_x, the x-component of the total electric field at P.

Helpful Integral:

$$\int_0^{\pi} d\theta \ \cos\theta = 0. \tag{1}$$

(e) By integration of the result of part (c) determine E_y, the y-component of the total electric field at P.

Helpful Integral:

$$\int_0^\pi d\theta \ \sin\theta = 2. \tag{2}$$

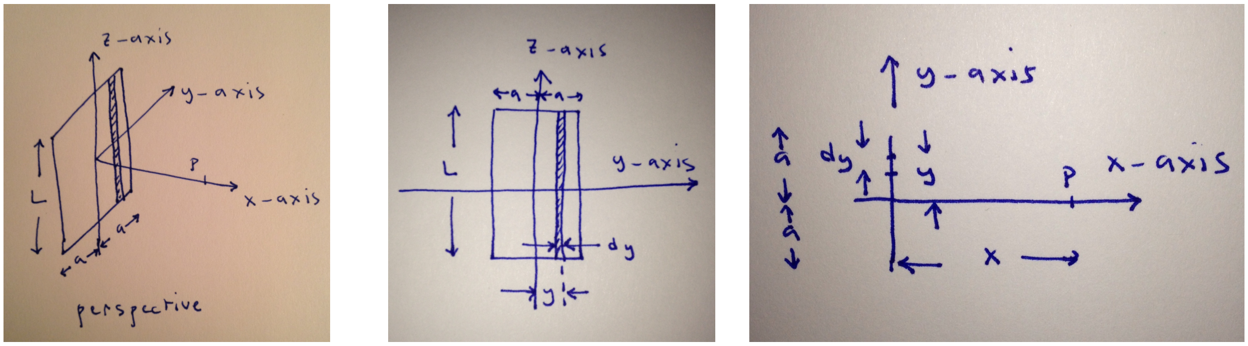

Figure 4: A uniformly charged strip of length L and width $2a$. σ is the charge per unit area. The strip lies in the y-z plane and is shown in perspective (left), viewed from above the y-z plane (center) and from above the x-y plane (right).

3. Field of a charged strip An infinite strip of width $2a$ lies in the y-z plane symmetrically about the z-axis. In figure 4 the strip is shown to have a finite length L for ease of drawing; however in the problem we shall take $L \to \infty$. The strip has a uniform surface charge density σ and we are interested in the electric field at the point P on the x-axis. To calculate the field we will mentally subdivide the strip into narrow strips of infinitesimal width dy, such as the one shown, and add up their contributions.

(a) In what direction do you expect the field at P to point? Briefly explain your reasoning.

(b) What is the charge per unit length of the narrow strip shown in the figure?

(c) What is the electric field at P due to the narrow strip? [*Hint:* The narrow strip may be regarded as a line of charge. Use the result for the field of a line of charge derived in class].

(d) Integrate your result of part (c) to obtain the total electric field at point P.
Helpful integrals:

$$\int_{-a}^{a} dy\, \frac{1}{x^2+y^2} = \frac{2}{x}\tan^{-1}\left(\frac{a}{x}\right), \qquad \int_{-a}^{a} dy\, \frac{y}{x^2+y^2} = 0. \tag{3}$$

(e) What electric field do you expect at P in the limit that $a \gg x$? Does your answer in part (d) reduce to the expected result? [*Hint:* $\tan^{-1}\xi \approx \pi/2$ for $\xi \gg 1$].

(f) What electric field do you expect at P in the limit that $a \ll x$? Does your answer in part (d) reduce to the expected result? [*Hint:* $\tan^{-1}\xi \approx \xi$ for $\xi \ll 1$].

Problem 4 is optional.

There is no extra credit for doing this problem.

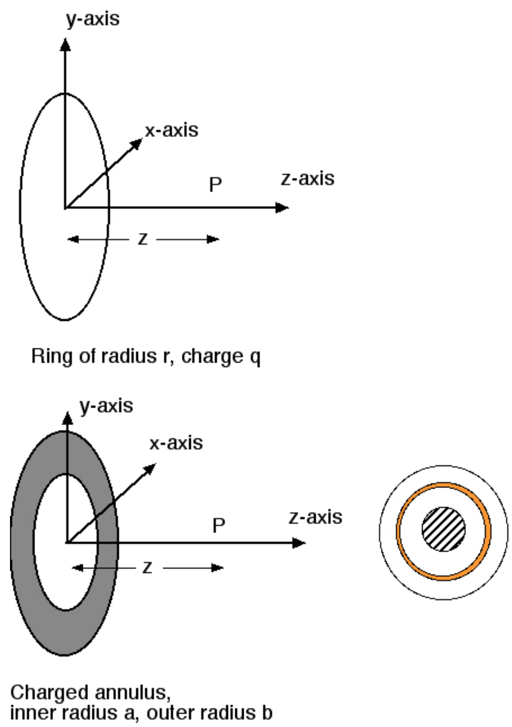

Figure 5: A charged ring (upper diagram) and a charged annulus show in perspective (lower left) and as a front view (lower right). The imaginary ring of radius r and width dr discussed in part (b) of problem 6 is shown in gold.

4. Field of an annulus In class we considered a ring of radius r and net charge q (see fig 5). At a point on the axis of the ring at a distance z from the center of the ring, the electric field is

$$E_z = \frac{q}{4\pi\epsilon_0} \frac{z}{(z^2 + r^2)^{3/2}}. \tag{4}$$

Now consider a charged annulus of inner radius a and outer radius b with a uniform charge density per unit area of σ (see fig 5).

(a) What is the total charge Q of the annulus? Give your answer in terms of σ, a and b.

(b) Now let us determine the electric field due to the annulus at a point P that lies on the axis at a distance z from the center of the annulus. To this end let us imagine that the annulus is composed of a set of nested concentric rings. (i) What is the charge of the ring of radius r and with dr shown in the figure? Give your answer in terms of σ, r and dr. (ii) What is the electric field due to this ring at P? (iii) Integrate the contributions of all such rings to obtain the desired field net field due to an annulus.

Useful integral:

$$\int dr \, \frac{r}{(r^2 + z^2)^{3/2}} = -\frac{1}{(r^2 + z^2)^{1/2}}. \tag{5}$$

(c) What do you expect the electric field to be if we take $a = 0$ and $b \gg z$? Explain why briefly. Does your result of part (b) reduce to the result you expected? [Other interesting special cases to think about: (i) $z \gg a, b$. (ii) $z = 0$. (iii) $a = b$. You do not need to turn in your work on these cases].

5. Electric field lines.

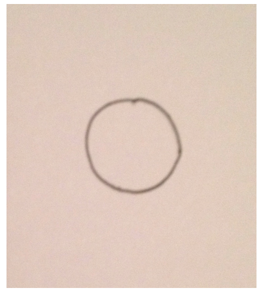

Figure 6: A cylinder viewed from above.

(a) *Charged cylinder:* Fig 6 shows a uniformly charged infinite cylinder viewed from above. Draw the electric field lines assuming (i) the cylinder is positively charged and (ii) negatively charged. Remember to indicate the direction of the field lines.

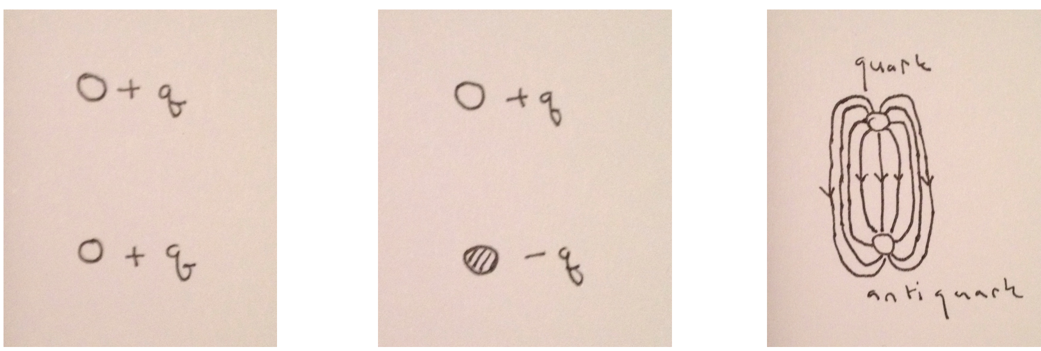

Figure 7: (a) A pair of positively charged particles. (b) A pair of negatively charged particles. (c) A quark-antiquark pair.

(b) *Charged pair:* Fig 7 (a) shows a pair of positively charged particles; fig 7 (b) shows an oppositely charged pair of particles. In each case draw field lines to help visualize the corresponding electric field. Remember to indicate the direction of each field line. Also remember that each field line either starts from the positive charge or ends in the negative charge (or both).

(c) *The nutty professor:* A professor draws the pictures shown in fig 8 that he claims are the field lines for an electrostatic field. Explain what is wrong with each drawing.

(d)* *Quark confinement and String theory:* Protons and neutrons are made up of particles called quarks and anti-quarks. These particles are bound together by the strong nuclear

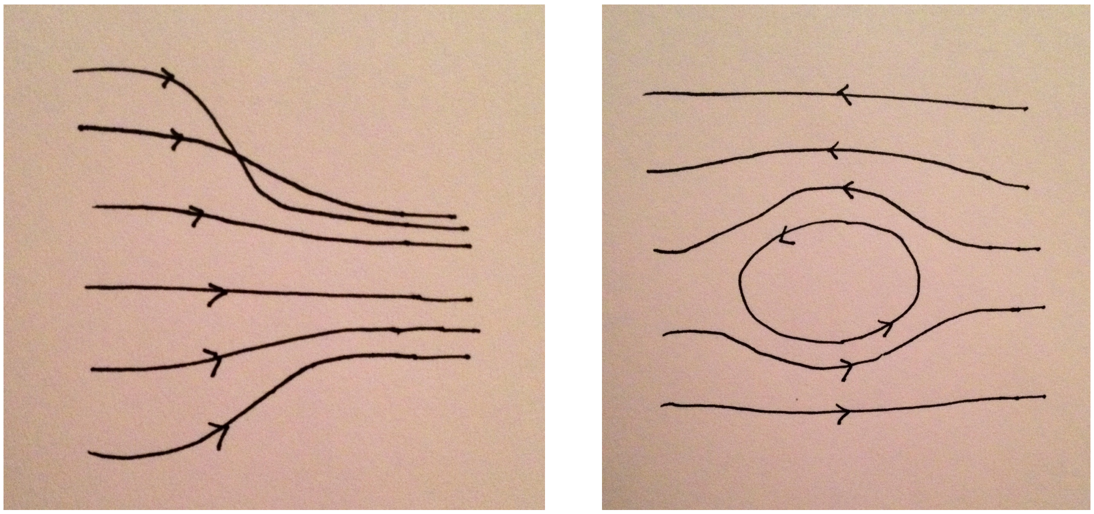

Figure 8: Field lines drawn by professor.

force. Chromodynamics, the theory of the strong nuclear force, is very similar in many ways to electromagnetism, but with some curious twists. The electrostatic forces between a pair of charges falls off with distance as $1/r^2$. The corresponding chromodynamic force between quarks, the "color electric force", however, is constant, independent of distance. One way to understand this is to assume that all the lines of color force that emanate from the quark end at the anti-quark and that these lines of force form a bundle called a flux tube. There isn't a complete understanding of why the color electric lines would behave in this way but in the 1970s, motivated by flux tubes, physicists began to explore the physics of string like objects that obey the laws of quantum mechanics and relativity. Thus was born string theory which in the 1980s became a still more ambitious attempt to understand all forces of nature in a single unified framework. Time will tell which, if any, of these hopes string theory will fulfill.

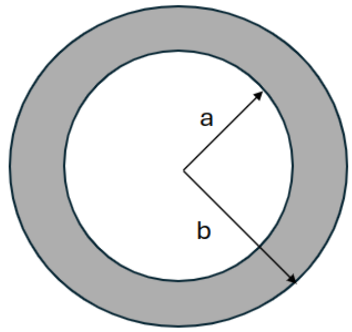

Figure 9: A uniformly charged spherical shell of inner radius a and outer radius b. ρ is the charge per unit volume.

6. Spherical shell. A spherical thick shell of inner radius a and outer radius b is uniformly charged with a volume charge density ρ.

(a) What is the total charge of the shell, Q_{tot}? Give your answer in terms of a, b and ρ.

(b) Based on symmetry in what direction do you expect the electric field to point outside the shell?

(c) Use Gauss's law to determine the magnitude of the electric field outside the shell at a distance r from the center of the shell (*i.e.* for $r > b$). Give your answer in terms of Q_{tot}, r and ϵ_0.

(d) Use Gauss's law to determine the magnitude of the electric field inside the shell at a distance r from the center of the shell (*i.e.* for $r < a$). Give your answer in terms of ρ, a, b, r and ϵ_0.

(e) Use Gauss's law to determine the magnitude of the electric field within the shell at a distance r from the center of the shell (*i.e.* for $a < r < b$). Give your answer in terms of ρ, a, b, r and ϵ_0.