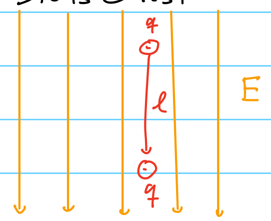
2. Dielectric break down of gas

a) Starts @ rest → $K_i = 0$



Work energy thm.

$W = \Delta K$

$= K_f - \cancel{K_i}^{\,0}$ @ rest

$W = K_f$

$\int_{i \to f} d\vec{l} \cdot \vec{F} = \Delta K$ $\quad \theta = 0$

$\int F \cdot dl = K_f$

$qE \int dl = K_f$

$\boxed{K_f = qEl}$

b) Given $\begin{cases} K_f = 10\,eV \\ l = 1.0 \times 10^{-6}\,m \end{cases}$ $\Rightarrow$ $10\,eV = (-1.6 \times 10^{-19}\,C)(E)(1.0 \times 10^{-6}\,m)$

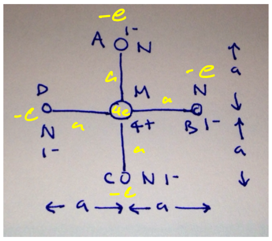
$$K_f = 10\,eV \times \frac{1.6 \times 10^{-19}\,J}{1\,eV} = 1.6 \times 10^{-18}\,J$$

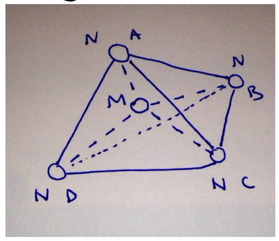
$$E = \frac{1.6 \times 10^{-18}\,J}{(-1.6 \times 10^{-19}\,C)(1.0 \times 10^{-6}\,m)} = -1.0 \times 10^{7}\,\frac{J}{C \cdot m}$$

Magnitude

$\boxed{E = 1.0 \times 10^{7}\,\frac{V}{m}}$

3. Electrostatic energy of complex ions





a) Total energy is the sum of energy of each pair

$$U=\frac{1}{4\pi\varepsilon_0}\left(\frac{MN_A}{a}+\frac{MN_B}{a}+\frac{MN_C}{a}+\frac{MN_D}{a}+\frac{N_AN_B}{a}+\frac{N_AN_C}{a}+\frac{N_AN_D}{a}+\frac{N_BN_C}{a}+\frac{N_BN_D}{a}+\frac{N_CN_D}{a}\right)$$

$$U=\frac{1}{4\pi\varepsilon_0 a}\left[M(N_a+N_B+N_C+N_D)+N_aN_B+N_aN_C+N_aN_D+N_BN_C+N_BN_D+N_CN_D\right]$$

$\rightarrow N_A=N_B=N_C=N_D=-e$

$$U=\frac{1}{4\pi\varepsilon_0 a}\left[4e(4\cdot -e)+\overset{N_A\,N_B}{(-e)(-e)}+\overset{N_a\,N_C}{(-e)(-e)}+\overset{N_a\,N_D}{(-e)(-e)}+\overset{N_B\,N_C}{(-e)(-e)}+\overset{N_B\,N_D}{(-e)(-e)}+\overset{N_C\,N_D}{(-e)(-e)}\right]$$

$$U=\frac{1}{4\pi\varepsilon_0 a}\left[-16e^2+6e^2\right]\rightarrow \quad U=-\frac{10e^2}{4\pi\varepsilon_0 a}$$

b) $A\rightarrow a\hat{k}$ $\qquad r_{M\text{-}A}=a$

$B\rightarrow -\frac{1}{3}a\hat{k}+\frac{\sqrt{8}}{3}a\hat{i}$ $\qquad r_{M\text{-}B}=\sqrt{(-\frac{1}{3}a)^2+(\frac{\sqrt{8}}{3}a)^2}=\sqrt{a^2(1)}=a$

$C\rightarrow -\frac{1}{3}a\hat{k}-\frac{\sqrt{2}}{3}a\hat{i}+\sqrt{\frac{2}{3}}a\hat{j}$ $\qquad r_{M\text{-}C}=\sqrt{(-\frac{1}{3}a)^2+(-\frac{\sqrt{2}}{3}a)^2+(\sqrt{\frac{2}{3}}a)^2}=\sqrt{a^2(1)}=a$

$D\rightarrow -\frac{1}{3}a\hat{k}-\frac{\sqrt{2}}{3}a\hat{i}-\sqrt{\frac{2}{3}}a\hat{j}$ $\qquad r_{M\text{-}D}=\sqrt{(-\frac{1}{3}a)^2+(-\frac{\sqrt{2}}{3}a)^2+(-\sqrt{\frac{2}{3}}a)^2}=\sqrt{a^2(1)}=a$

$$r_{N_D\text{-}N_C}=\sqrt{\cancelto{0}{\left(-\frac{1}{3}a-\left(-\frac{1}{3}a\right)\right)^2}+\cancelto{0}{\left(-\frac{\sqrt{2}}{3}a-\left(-\frac{\sqrt{2}}{3}a\right)\right)^2}+\left(\sqrt{\frac{2}{3}}a-\left(-\sqrt{\frac{2}{3}}a\right)\right)^2}=\sqrt{\left(2a\sqrt{\frac{2}{3}}\right)^2}=2a\sqrt{\frac{2}{3}}$$

$$U_{M\text{-}N_{A\text{-}D}}=4\left[\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{4e\cdot -e}{a}\right)\right]=\frac{-16e^2}{4\pi\varepsilon_0 a}=\frac{-4e^2}{\pi\varepsilon_0 a}$$

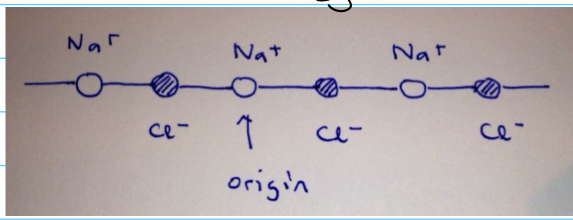
4 adjacent $\quad U_{adj}=4\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{-e\cdot -e}{2a\sqrt{\frac{2}{3}}}\right)\right]=\frac{e^2}{2\pi\varepsilon_0 a\sqrt{\frac{2}{3}}}$

2 opposing $\quad U_{opp}=2\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{-e\cdot -e}{2a}\right)\right]=\frac{e^2}{4\pi\varepsilon_0 a}$

$$U_{Tot}=-\frac{4e^2}{\pi\varepsilon_0 a}+\frac{e^2}{2\pi\varepsilon_0 a\sqrt{\frac{2}{3}}}+\frac{e^2}{4\pi\varepsilon_0 a}=\frac{e^2}{\pi\varepsilon_0 a}\left(-4+\frac{1}{2\sqrt{\frac{2}{3}}}+\frac{1}{4}\right)$$

c) For (a), $E=\frac{e^2}{\pi\varepsilon_0 a}(-2.5)$ and for (b), $E=\frac{e^2}{\pi\varepsilon_0 a}(-3.138)$, so the tetrahedral structure has lower energy. This makes sense, maximizing distance between atoms.

4. Electrostatic energy of a salt crystal



a) Cl^- @: $\pm a, \pm 3a, \pm 5a, \ldots$

Na^+ @: $\pm 2a, \pm 4a, \pm 6a, \ldots$

Cl^- @ $\pm a$: $\frac{U_1}{2} = \frac{e \cdot -e}{4\pi\varepsilon_0 a} = -\frac{e^2}{4\pi\varepsilon_0 a}$ Cl^- @ $\pm 3a$: $\frac{U_3}{2} = -\frac{e^2}{4\pi\varepsilon_0 (3a)}$

Na^+ @ $\pm 2a$: $\frac{U_2}{2} = \frac{e \cdot e}{4\pi\varepsilon_0 (2a)} = +\frac{e^2}{4\pi\varepsilon_0 (2a)}$ Na^+ @ $\pm 4a$: $\frac{U_4}{2} = \frac{e^2}{4\pi\varepsilon_0 (4a)}$

so $\frac{U_{Tot}}{2} = U_1 + U_2 + \ldots$ $U = \frac{e^2}{2\pi\varepsilon_0 a} \sum_1^{\infty} \frac{(-1)^n}{n}$

b) $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \ldots$

$-\ln 2 = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \ldots$

$$U = \frac{e^2}{4\pi\varepsilon_0 a}\left(-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \ldots\right) = \frac{e^2}{2\pi\varepsilon_0 a}(-\ln 2)$$

$$\Rightarrow U = \frac{-e^2}{2\pi\varepsilon_0 a}\ln 2$$

Matches given expression