Charge to Mass Ratio of the Electron

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1 Abstract

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2 Introduction and Theory

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3 Experimental Procedure

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4 Results and Analysis

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5 Conclusion

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5.1 Acknowledgments

I would like to thank Pratham Bhashyakarla, CWRU Department of Physics, for his help in obtaining the experimental data, preparing the figures, and checking my calculations.

5.2 References

1. Driscoll, D., General Physics II: E&M Lab Manual, “Charge to Mass Ratio of the Electron,” CWRU Bookstore, 2016.

A Appendix

A.1 Fixed Voltage Data and Figures

Amps (A)	Trevor's D (cm)	Pratham's D (cm)	Average Radius (m)
0.66	16.3 ± 0.1	14.5 ± 0.1	7.700E-4 ± 3.5E-6
0.98	13.4 ± 0.1	12.7 ± 0.1	6.525E-4 ± 3.5E-6
1.28	10.2 ± 0.1	10.2 ± 0.1	5.100E-4 ± 3.5E-6
1.59	8.3 ± 0.1	7.6 ± 0.1	3.975E-4 ± 3.5E-6
1.91	6.6 ± 0.1	6.4 ± 0.1	3.25E-4 ± 3.5E-6

Table 1: Fixed voltage at $V = 104 \pm 1V$, with steps of voltage from a minimum Amps of $0.66A$ and a maximum of $1.91A$. Trevor's and Pratham's D refers to their measured diameter values, respectively. Average radius is calculated by taking the average of the two measured values from me and Pratham, dividing the value by 2 (diameter $\rightarrow$ radius), and then converting that average radius value to meters. Uncertainty of these values is discussed in the following section.

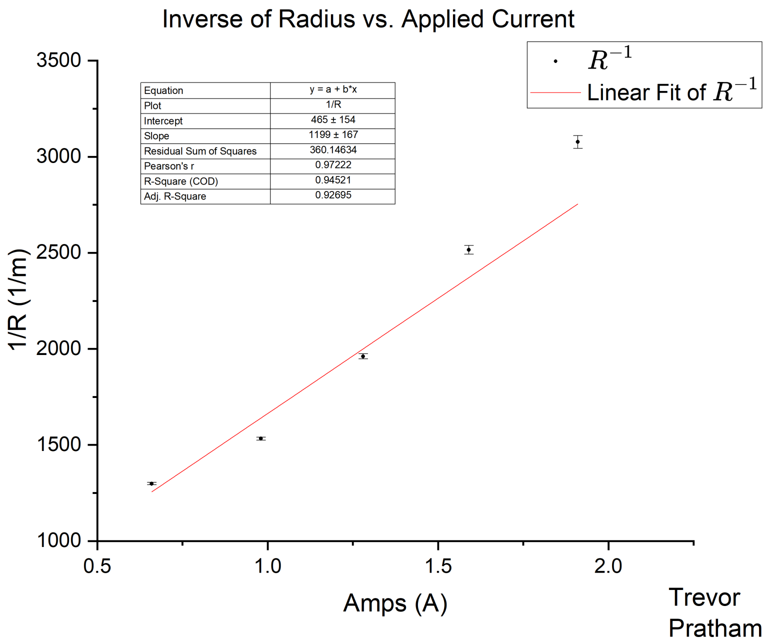

Figure 1: Inverse of radius $\frac{1}{R}$ vs. applied current A. 1/R is calculated as 1 over the Average Radius values reported in Table 1.

A.2 Fixed Amps Data and Figures

Voltage	Trevor's D (cm)	Pratham's D (cm)	Average Radius (m)
84	10.4 ± 0.1	10.3 ± 0.1	5.175E-4 ± 3.5E-6
113	11.8 ± 0.1	12.5 ± 0.1	6.075E-4 ± 3.5E-6
139	14.4 ± 0.1	13.7 ± 0.1	7.025E-4 ± 3.5E-6
168	15.2 ± 0.1	14.8 ± 0.1	7.500E-4 ± 3.5E-6
197	15.8 ± 0.1	15.5 ± 0.1	7.825E-4 ± 3.5E-6

Table 2: Fixed Amps at $A = 1.02 \pm 0.01A$, with steps of voltage from a minimum Voltage of $84V$ and a maximum of $197V$. Trevor's and Pratham's D refers to their measured diameter values, respectively. The Average Radius Values and Uncertainties here are identical to the values calculated Table 1. See the next section for uncertainty discussions.

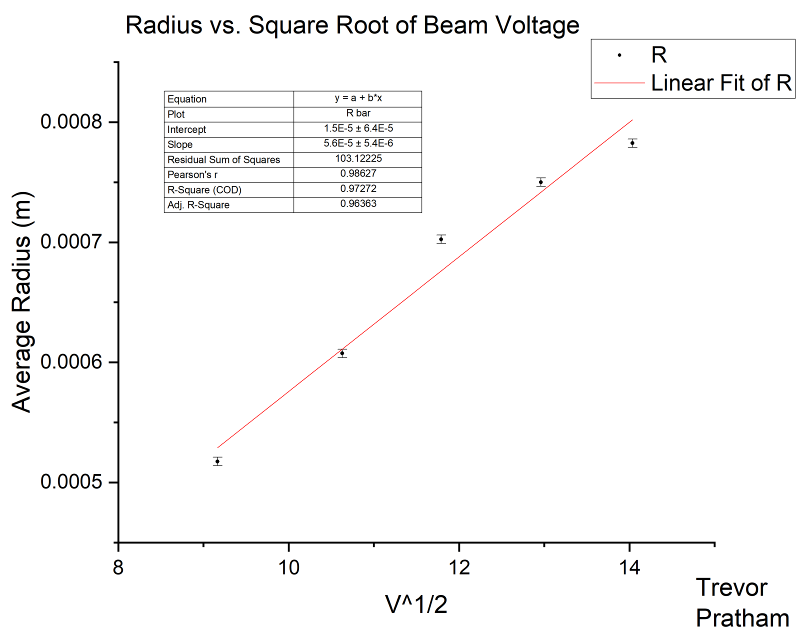

Figure 2: Radius R vs. applied beam voltage V

B Other Calculations

B.1 Average Radius Error Propagation

$$\begin{aligned}
\bar{R} =& \frac{1}{2} \cdot \frac{D_T + D_P}{2} \cdot \frac{1m}{100cm} = \frac{1}{400} \cdot (D_T + D_P)\ m && \text{for diameter measurements } D_T \text{ and } D_P \\
\delta_{\bar{R}} =& \sqrt{\delta^2_{\bar{R}_{D_T}} + \delta^2_{\bar{R}_{D_P}}} && \text{errors present only in } D_T \text{ and } D_P \\
\delta_{\bar{R}_{D_T}} =& \left(\frac{\partial \bar{R}}{\partial D_T}\right) \cdot \delta_{D_T} = \frac{1}{400} \cdot \delta_{D_T} && \text{use } \delta_{D_T} = 0.001 \text{ m – from measurements} \\
=& \frac{1}{400} \cdot 0.001 = 2.5 \times 10^{-6} m \\
\delta_{\bar{R}_{D_T}} =& \left(\frac{\partial \bar{R}}{\partial D_P}\right) \cdot \delta_{D_P} = \frac{1}{400} \cdot \delta_{D_P} && \text{use } \delta_{D_P} = 0.001 \text{ m – from measurements} \\
=& \frac{1}{400} \cdot 0.001 = 2.5 \times 10^{-6} m \\
\therefore \delta_{\bar{R}} =& \sqrt{(2.5 \times 10^{-6} m)^2 + (2.5 \times 10^{-6} m)^2} = 3.5 \times 10^{-6} m
\end{aligned}$$

$$\delta_{\bar{R}} = 3.5 \times 10^{-6} m \tag{1}$$

B.2 Error Propagation in 1/R

$$\begin{aligned}
\frac{1}{\bar{R}} =& \frac{1}{R} && \text{Used for determining } \alpha \\
\delta_{\frac{1}{\bar{R}}} =& \delta_{\frac{1}{\bar{R}}\bar{R}} && \text{No need for adding in quadrature, only once source of error} \\
\delta_{\frac{1}{\bar{R}}\bar{R}} =& \left| \left(\frac{\partial \frac{1}{\bar{R}}}{\partial \bar{R}}\right) \cdot \delta_{\bar{R}} \right| && \text{Derivative method for error propagation} \\
=& \frac{\delta_{\bar{R}}}{\bar{R}^2} && \text{Simple single-variable derivative}
\end{aligned}$$

This expression was used in determining the errors presented in Table 1, but calculations are omitted here to prevent redundancy. To calculate, use $\delta_{\bar{R}} = 3.5 \times 10^{-6} m$, as calculated in the above subsection.

$$\delta_{\frac{1}{\bar{R}}} = \frac{\delta_{\bar{R}}}{\bar{R}^2} \tag{2}$$

B.3 Hemholtz Coil Current Error Propagation - Fixed Voltage

$$\alpha = \frac{8\mu_0 N}{5r}\sqrt{\frac{e/m}{10V}} \quad \Rightarrow \quad \frac{e}{m} = \left(\frac{5r\alpha}{8\mu_0 N}\right)^2 \cdot (10V) \quad \text{Errors propagated in } \alpha, V, r \tag{3}$$

$$\delta_{\frac{e}{m}} = \sqrt{\left(\delta_{\frac{e}{m}\alpha}\right)^2 + \left(\delta_{\frac{e}{m}V}\right)^2 + \left(\delta_{\frac{e}{m}r}\right)^2} \qquad \text{General Error Equation}$$

$$\begin{aligned}
\delta_{\frac{e}{m}\alpha} = \left(\frac{\partial \frac{e}{m}}{\partial \alpha}\right) \cdot \delta_\alpha &= \left(20\left(\frac{5}{8\mu_0 N}\right)^2 (r\alpha^2)V\right) \cdot \delta_\alpha && \text{Error due to } \alpha \\
&= \left(20\left(\frac{5}{8 \cdot 4\pi \times 10^{-7} \cdot 130}\right)^2 (0.158 \cdot 1199^2) \cdot 104\right) \cdot 167 \\
&= 1.15 \times 10^{18}\frac{C}{Kg}
\end{aligned}$$

$$\begin{aligned}
\delta_{\frac{e}{m}V} = \left(\frac{\partial \frac{e}{m}}{\partial V}\right) \cdot \delta_V &= \left(10\left(\frac{5}{8\mu_0 N}\right)^2 (r^2\alpha^2)\right) \cdot \delta_V && \text{Error due to } V \\
&= \left(10\left(\frac{5}{8 \cdot 4\pi \times 10^{-7} \cdot 130}\right)^2 (0.158^2 \cdot 1199^2)\right) \cdot 1 \\
&= 5.25 \times 10^{12}\frac{C}{Kg}
\end{aligned}$$

$$\begin{aligned}
\delta_{\frac{e}{m}r} = \left(\frac{\partial \frac{e}{m}}{\partial r}\right) \cdot \delta_r &= \left(20\left(\frac{5}{8\mu_0 N}\right)^2 (r^2\alpha)V\right) \cdot \delta_r && \text{Error due to } r \\
&= \left(20\left(\frac{5}{8 \cdot 4\pi \times 10^{-7} \cdot 130}\right)^2 (0.158^2 \cdot 1199) \cdot 104\right) \cdot 0.005 \\
&= 4.38 \times 10^{7}\frac{C}{Kg}
\end{aligned}$$

$$\begin{aligned}
\delta_{\frac{e}{m}} &= \sqrt{(1.15 \times 10^{18})^2 + (5.25 \times 10^{12})^2 + (4.38 \times 10^{7})^2} && \text{Substitute Values} \\
&= 1.15 \times 10^{18}\frac{C}{Kg}
\end{aligned}$$

B.4 Beam Voltage Error Propagation - Fixed Amps

$$\beta = \frac{5r}{8\mu_0 N I_c}\sqrt{\frac{10}{e/m}} \quad \Rightarrow \quad \frac{e}{m} = 10\left(\frac{8\mu_0 N I_C \beta}{5r}\right)^{-2} \quad \text{Errors propagated in } \beta, I_c, r \tag{4}$$

$$\delta_{\frac{e}{m}} = \sqrt{\left(\delta_{\frac{e}{m}\beta}\right)^2 + \left(\delta_{\frac{e}{m}I_c}\right)^2 + \left(\delta_{\frac{e}{m}r}\right)^2}$$

$$\begin{aligned}
\delta_{\frac{e}{m}\beta} = \left|\left(\frac{\partial \frac{e}{m}}{\partial \beta}\right)\cdot \delta_\beta\right| &= \left(\left(\frac{8\mu_0 N}{5}\right)^{-2} \cdot \left(\frac{20r^2}{\beta^3 I_c^2}\right)\right)\cdot \delta_\beta \\
&= \left(\left(\frac{8\cdot 4\pi\times 10^{-7} * 130}{5}\right)^{-2} \cdot \left(\frac{20(0.158)^2}{(5.6\times 10^{-5})^3(1.02)^2}\right)\right)\cdot (5.4\times 10^{-6}) \\
&= 2.16\times 10^{14}\frac{C}{Kg}
\end{aligned}$$

$$\begin{aligned}
\delta_{\frac{e}{m}I_c} = \left|\left(\frac{\partial \frac{e}{m}}{\partial I_c}\right)\cdot \delta_{I_c}\right| &= \left(\left(\frac{8\mu_0 N}{5}\right)^{-2} \cdot \left(\frac{20r^2}{\beta^2 I_c^3}\right)\right)\cdot \delta_{I_c} \\
&= \left(\left(\frac{8\cdot 4\pi\times 10^{-7} * 130}{5}\right)^{-2} \cdot \left(\frac{20(0.158)^2}{(5.6\times 10^{-5})^2(1.02)^3}\right)\right)\cdot 0.01 \\
&= 2.20\times 10^{13}\frac{C}{Kg}
\end{aligned}$$

$$\begin{aligned}
\delta_{\frac{e}{m}r} = \left|\left(\frac{\partial \frac{e}{m}}{\partial r}\right)\cdot \delta_r\right| &= \left(\left(\frac{8\mu_0 N}{5}\right)^{-2} \cdot \left(\frac{20r}{\beta^2 I_c^2}\right)\right)\cdot \delta_r \\
&= \left(\left(\frac{8\cdot 4\pi\times 10^{-7} * 130}{5}\right)^{-2} \cdot \left(\frac{20(0.158)}{(5.6\times 10^{-5})^2(1.02)^2}\right)\right)\cdot 0.005 \\
&= 1.42\times 10^{14}\frac{C}{Kg}
\end{aligned}$$

$$\begin{aligned}
\delta_{\frac{e}{m}} &= \sqrt{(2.16\times 10^{14})^2 + (2.20\times 10^{13})^2 + (1.42\times 10^{14})^2} \\
&= 2.59\times 10^{14}\frac{C}{Kg}
\end{aligned}$$