Physics 122 Spring 2025

Dielectrics; DC Circuits

Homework 5

Due: Sunday March 2nd, 11.59PM

Figure 1: (Left) Sketch from the lab notebook of Jack Kilby dated July 24, 1958. Here he describes his "monolithic idea" that circuit elements such as resistors, capacitors, distributed capacitors and transistors—if all made from the same material—could be included in a single chip. (Right) Kilby's first realization of his idea, the world's first integrated circuit. The images are from Kilby's Physics Nobel Prize lecture in 2000.

Guidelines

1. Please write legibly. If necessary solve the problem first on scratch paper. Then write a cogent and legible solution based on your scratch work. Learning to make a clear and logical presentation of your work is an invaluable asset, well worth the trouble of having to write it twice.

2. Please begin each problem on a new page just as we have done with the questions.

3. Parts of problems marked with an asterisk (if any) are only to be read. No response is needed.

How to submit homework: Submit your homework in PDF format via Canvas. You will find a link to submit under Assignments. Please submit before the deadline. Deadlines are enforced and late homework may not be graded. If you have compelling reasons to miss the deadline please contact the instructor preferably before the deadline. In order to generate a PDF of your homework you should scan it. If you do not have access to a scanner there are a number of effective and free scanner apps available for phones. AdobeScan is one that many students have found worked well for them in previous courses. The other option is to take pictures of your homework and convert them to PDFs using your computer. Finally if you work with a tablet you can of course easily save your work as a PDF.

1. Potential of a disc

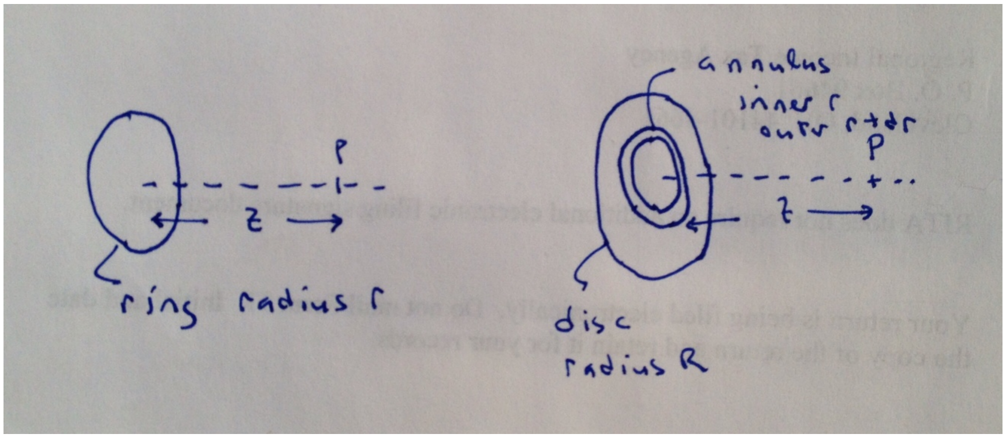


Figure 2: A charged ring (left) and disc (right). The disc may be subdivided into concentric annuli in order calculate its potential.

(a) *The Ring:* A ring of radius r has a charge q. The charge is distributed uniformly around the circumference of the ring. Determine the electrostatic potential ϕ at the point P that lies on the axis of the ring at a distance z from the center of the ring.

(b) *The Disc:* Now consider a uniformly charged disc with a radius R and a surface charge per unit area σ. We wish to determine the potential at a point that lies on the axis of the disc at a distance z from the center of the disc.

(i) To this end first consider the infinitesimal annulus of the inner radius r and outer radius $r + dr$. Determine the charge of this annulus. Determine the potential at P due to this infinitesimal annulus. [*Hint:* Use your result of part (a)].

(ii) Now integrate the result of part (i) to obtain the potential at P due to the disc.

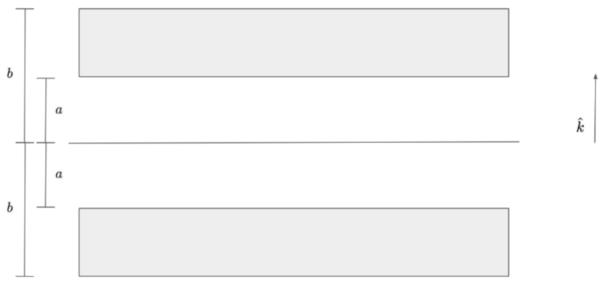
Helpful integral:

$$\int dr \frac{2r}{\sqrt{r^2 + z^2}} = 2\sqrt{r^2 + z^2}. \tag{1}$$

(iii) What form do you expect the potential to have for $z \gg R$. Explain briefly. Does your result of part (ii) reduced to the expected result?

Helpful approximation: For $z \gg R$

$$\sqrt{z^2 + r^2} \approx z + \frac{R^2}{2z}. \tag{2}$$



2. Slabs A flat insulating sheet has surface charge σ per unit area. For simplicity we assume it has an infinite lateral area. In addition to the insulating sheet there are also two conducting slabs. These slabs are parallel to the sheet and are placed symmetrically on either side of it. For simplicity we assume that the slabs also have infinite lateral area. We also assume that each slab is neutral.

1. What is the electric field inside the upper conducting slab? Inside the lower? Explain your answer briefly (in one or two sentences).
2. What is the surface charge density on the lower surface of the upper slab and on the upper surface of the lower slab?[*Hint:* Use symmetry and apply Gauss's law to the Gaussian surface shown in the figure]
3. What is the surface charge density on the upper surface of the upper slab and on the lower surface of the lower slab? Briefly explain your answer (one or two sentences).

3. Spherical capacitor. A conducting sphere (radius a) is surrounded b a spherical conducting shell (inner radius b, outer radius c). The inner sphere has a charge $+q$; the outer shell has a charge $-q$. Throughout this problem r denotes distance from the center of the sphere.

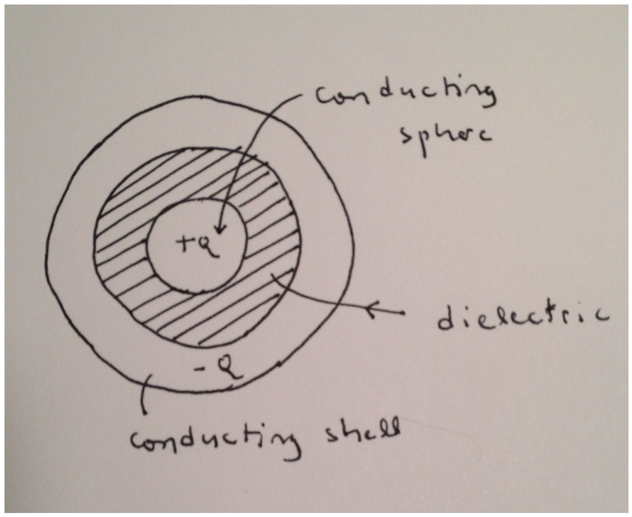


Figure 3: A spherical capacitor and two Gaussian surfaces.

(a) What is the electric field inside the conducting sphere (for $r < a$)?

(b) Where is the charge located on the conducting sphere: in the interior or on the surface? What is the surface charge density?

(c) Determine the electric field in the gap between the sphere and the shell (for $a < r < b$). [*Hint:* Apply Gauss's law to the inner Gaussian surface depicted in the figure.]

(d) What is the electric field within the conducting shell (for $b < r < c$)?

(e) Where is the charge located in the conducting shell? In the interior? On the inner surface? On the outer surface? Or divided in some proportion between the inner surface and the outer surface? Explain briefly. [*Hint:* Apply Gauss's law to the outer Gaussian surface depicted in the figure.]

(f) What is the potential difference between the outer shell and the inner sphere? [*Hint:* Recall that the potential difference between two points $\mathbf{r}_1$ and $\mathbf{r}_2$ is given by

$$V_2 - V_1 = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot d\mathbf{l} \tag{3}$$

where the integral is evaluated along any path that runs from point 1 to point 2.]

(g) A pair of conductors with equal and opposite charge constitute a capacitor. The capacitance of the capacitor is defined as $C = q/V$. Here q is the charge on the positively charged conductor and V is the potential difference between the positively charged conductor and the negatively charged conductor. Calculate C for the spherical capacitor analyzed in this problem.