Physics 122 Spring 2025

Electrostatics

Homework 1

Due: 11.59 PM Sunday, Jan 26, 2025

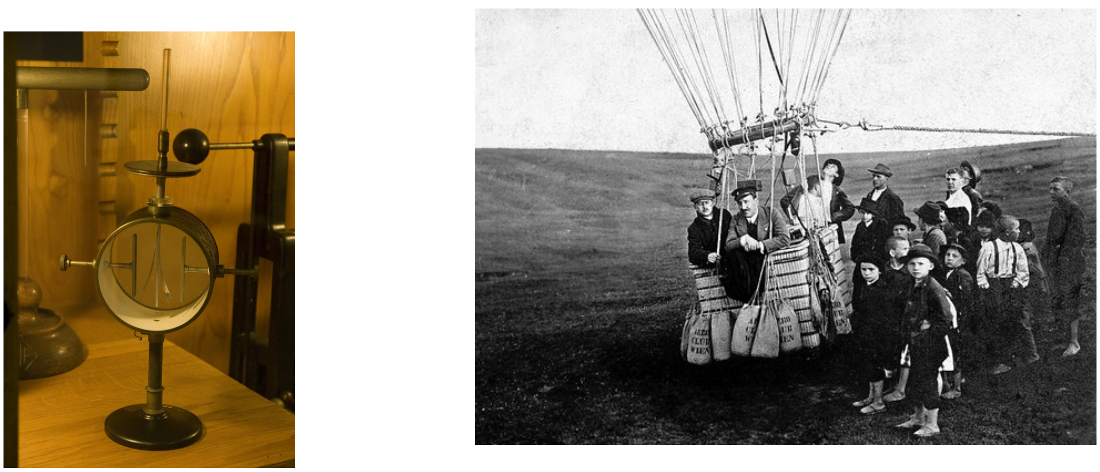

Figure 1: *Left panel:* A nineteenth century gold leaf electroscope at the University of Rome that works on the principle analyzed in Problem 2. *Right panel:* Balloonist Victor Hess near Vienna in 1911. Using an electroscope on balloon flights Hess discovered cosmic rays: a mysterious stream of energetic subatomic particles that come to earth from all directions. For this discovery Hess was awarded the Nobel Prize in physics in 1936.

Guidelines

1. Please write legibly. If necessary solve the problem first on scratch paper. Then write a cogent and legible solution based on your scratch work. Learning to make a clear and logical presentation of your work is an invaluable asset, well worth the trouble of having to write it twice.

2. Please begin each problem on a new page just as we have done with the questions.

3. Parts of problems marked with an asterisk (if any) are only to be read. No response is needed. Problems marked optional are optional and for zero credit. If you choose to do them you don't need to submit your work as it will not be graded.

How to submit homework: Submit your homework in PDF format via Canvas. You will find a link to submit under Assignments. Please submit before the deadline. Deadlines are enforced and late homework will have only partial credit (15% less per day). If you have compelling reasons to miss the deadline please contact the instructor preferably before the deadline. In order to generate a PDF of your homework you should scan it. If you do not have access to a scanner there are a number of effective and free scanner apps available for phones. AdobeScan is one that many students have found worked well for them in previous courses. The other option is to take pictures of your homework and convert them to PDFs using your computer. Finally if you work with a tablet you can of course easily save your work as a PDF.

1. Triboelectricity

(a) *Fun with scotch tape.* (i) You peel two strips of scotch tape off the same roll. The strips repel one another. (ii) A long time goes by. Then you stick the sticky side of one strip to the smooth side of the other and rip them apart. The strips now attract one another. Explain why in one or two sentences.

(b) *Physics of Laundry.* (i) Your clothing tends to cling together after going through the dryer. Why? (ii) In one load the clothing is all of the same material, cotton; in the other load there is clothing of various different kinds of materials. In which case do you expect the clothes to cling together more?

(c) *Hanging by a thread.* Two metal spheres hang from nylon threads and attract each other when brought close together. (i) What can you say about the charges on the two spheres? (ii) The spheres are now brought into contact. Will they continue to attract each other?

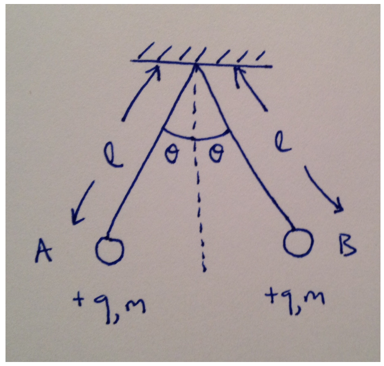

Figure 2: Two charged spheres, each hanging by a thread.

2. Electroscope Two plastic spheres A and B, both of mass m, hang by threads of length ℓ from a common point of support (see figure 2). An unknown charge $+q$ is imparted to each sphere. As a result the spheres hang with each thread making an angle θ to the vertical. Determine q. Give your answer in terms of m, θ, ℓ and g = acceleration due to gravity. *Hint:* First draw a free body diagram for sphere A.

Figure 3: The apparatus used by Millikan to determine the fundamental unit of charge e. Micron sized oil drops were created using the atomizer spray at left. The charge on the drops was varied by illuminating them with ultraviolet light or x-rays or by bombarding the chamber with α, β and γ radiation from radium. The motion of the drops was observed with a microscope. The mass of the drops could be determined by measuring their radii and the density of oil separately. In part for this experiment Millikan was awarded the 1923 Nobel Prize in Physics.

3. Millikan and the quantization of charge

(a) *Terminal velocity:* A small oil drop of mass m falls under the influence of gravity and air drag at a uniform speed v called the terminal velocity. The magnitude of the air drag force is Kv; the force points in the direction opposite to the motion of the oil drop. K is a constant that depends on the size of the oil drop and the viscosity of air.

(i) Calculate the speed v. Give your answer in terms of m, g and K. [*Hint:* Apply Newton's second law].

(ii) Invert your answer to part (i) to give K in terms of m, g and v.

(b) *Electrostatic forces:* A strong uniform electric field E that points vertically upward is now applied and a charge q is imparted to the oil drop. The oil drop is now observed to float upward at a uniform speed u.

(i) Determine u. Give your answer in terms of m, g, q, E and K.

(ii) Suppose that the charge of the droplet is changed from q to $q + e$ where e is the fundamental unit of charge (*i.e.* the charge of the electron is $-e$ in our notation). What is Δu, the change in the speed of the oil drop? Give your answer in terms of e, E and K.

(iii) Use the expression for K in part (a-ii) to determine Δu in terms of e, E, m, g and v.

(c) In his famous oil drop experiment Millikan observed a drop that drifted at a terminal velocity $v = 1.1 \times 10^{-4}$ m/s. He then applied an electric field of $E = 6 \times 10^5$ N/C and varied the charge of the drop by various means. He found that the drop drifted upward and that the speed of the drop changed in steps of size $\Delta u = 3.0 \times 10^{-4}$ m/s as charge was added or subtracted. Assuming, as Millikan did, that the jumps in the velocity correspond to the addition or subtraction of a fundamental unit of charge, determine e in Coulombs. Take the mass of the drop to be $m = 3.5 \times 10^{-15}$ kg.

Optional problem. Zero credit.

4. Cathode ray tube: In the cathode ray tube shown in figure 4, a beam of electrons is injected horizontally into the exact center of the parallel-plate region. Here $L = 3.0$ cm and $d = 0.20$ cm. In the region between the plates there is a uniform electric field $\mathbf{E}$ that points vertically downward. The magnitude of this electric field is $E = 1.7 \times 10^4$ N/C. What minimum speed v_0 should the electrons have in order to ensure that they do not strike the upper plate? (Source: Adapted from Ohanian and Markert, *Physics for Engineers and Scientists*).

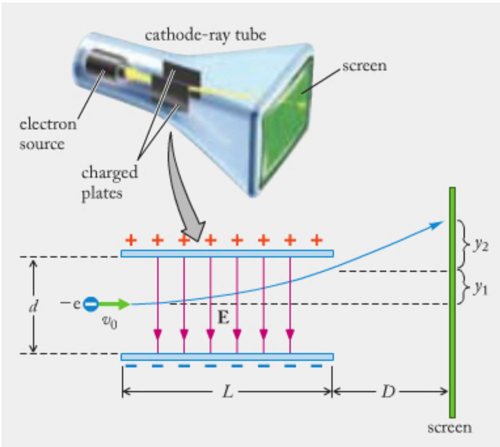

Figure 4: Schematic of a cathode-ray tube.

5. Hierarchy problem Two protons in an atomic nucleus are separated by a distance of 1.0×10^{-15} m. Each proton has a charge $e = 1.6 \times 10^{-19}$ C and a mass $m = 1.67 \times 10^{-27}$ kg. [*Useful data:* $1/4\pi\epsilon_0 = 9.0 \times 10^9$ kg m^3/C^2 s^2 and $G = 6.67 \times 10^{-11}$ m^3/kg s^2].

(a) What is the electric force between the protons, F_e? Is it attractive or repulsive?

(b) What is the gravitational force between the protons, F_g? Is it attractive or repulsive?

(c) What is the ratio of the magnitude of the two forces, F_e/F_g?

(d)* Inside a nucleus there is an additional attractive force between the protons called the strong nuclear force. This force is stronger than the electrostatic repulsion between the protons and is the force that holds together atomic nuclei. To a good approximation the strong force between a pair of protons, or a pair of neutrons, or a proton and a neutron are all the same. In contrast the neutron, being electrically neutral, does not experience the electrostatic force calculated in part (a). The relative weakness of gravity compared to other forces on the subatomic scale is a great mystery.

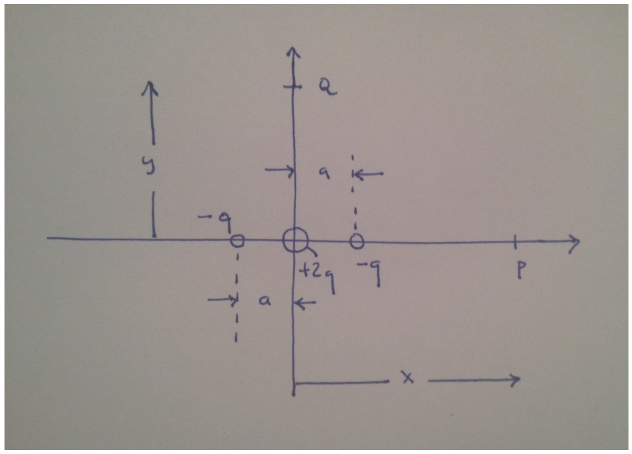

Figure 5: A point charge $+2q$ is placed at the origin. Two additional point charges $-q$ are placed symmetrically about the origin on the x axis each at a distance a from the origin. This configuration of charges is called an electric quadrupole.

6. Electric quadrupole moment

The arrangement of charges shown in figure 5 constitutes an electric quadrupole moment.

(a) (i) In which direction does the total electric field point at the location P?

(ii) Determine the electric field at P in terms of q, a, ϵ_0 and $x =$ the x-coordinate of P.

(iii) Simplify your expression in the limit that a is small compared to x. *Hint:* You may use the small a/x expansion

$$\frac{1}{(x+a)^2} = \frac{1}{x^2} - \frac{2a}{x^3} + \frac{3a^2}{x^4} + \ldots \tag{1}$$

(b) (i) At the location Q in which direction does the total electric field point?

(ii) Determine the electric field at Q in terms of q, a, ϵ_0 and $y =$ the y-coordinate of Q.

(iii) Simplify your expression in the limit that a is small compared to y. *Hint:* You may use the small a/y expansion

$$\frac{1}{(y^2+a^2)^{3/2}} = \frac{1}{y^3} - \frac{3}{2}\frac{a^2}{y^5} + \ldots \tag{2}$$