Solutions

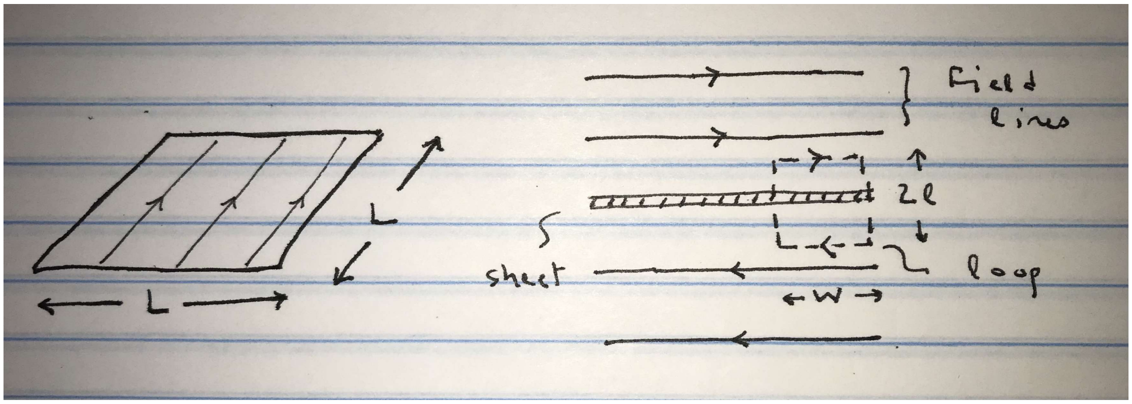


Figure 5: A flat sheet carrying electrical current.

1. Magnetic field of a sheet of current.

(a) Along the top segment

$$\oint d\ell \cdot \mathbf{B} = Bw \tag{1}$$

because $\mathbf{B}$ is constant and the total displacement along the segment is parallel to the magnetic field. For the same reason

$$\oint d\ell \cdot \mathbf{B} = Bw \tag{2}$$

along the bottom segment also. On the other hand for the two vertical segments

$$\oint d\ell \cdot \mathbf{B} = 0 \tag{3}$$

Since $\mathbf{B}$ and the displacements $d\ell$ are parallel to each other.

Thus, circulation of $\mathbf{B}$ around Ampere loop in figure is

$$\oint d\ell \cdot \mathbf{B} = 2Bw \tag{4}$$

(b) Since current per unit length is κ and loop encloses length w of sheet, current enclosed is κw.

(c) Ampere's law implies

$$B = \frac{1}{2}\mu_0\kappa \tag{5}$$

which leads to final answer

$$B = \frac{\mu_0}{\mu_0}kw.$$

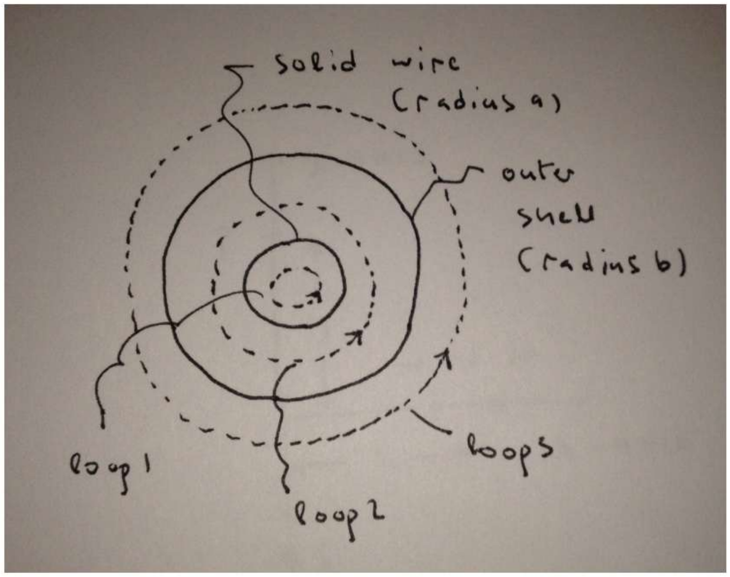


Figure 6: A coaxial cable.

2. Co-axial cable. (a) By symmetry we expect the magnetic field lines are circles oriented counterclockwise. The magnitude of the magnetic field depends only on the distance from the common axis of the cylinder and shell. Hence the circulation of the magnetic field

$$\oint \mathbf{B} \cdot d\ell = 2\pi r B \tag{6}$$

Here B is the magnitude of the magnetic field at the distance r.

The current enclosed by the Amperian loop 2 is i = current carried by the solid cylindrical wire.

Hence by Ampere's law

$$2\pi r B = \mu_0 i \tag{7}$$

Solving for B we obtain

$$B = \frac{\mu_0 i}{2\pi r} \tag{8}$$

(b) The circulation of the magnetic field is given again by eq (7) for exactly the same reasons. The current enclosed by Amperian loop 1 is the total current in the cylindrical wire i multiplied by the ratio of area of Amperian loop 1 to area of solid cylinder, namely,

$$i\frac{\pi r^2}{\pi a^2} = i\frac{r^2}{a^2}$$

Hence by Ampere's law

$$2\pi r B = \mu_0 i \frac{r^2}{a^2}$$

Solving for magnetic field we obtain

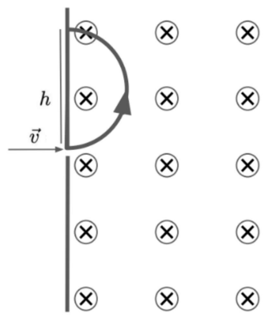
$$B = \mu_0 i \frac{r}{2\pi a^2} \tag{9}$$

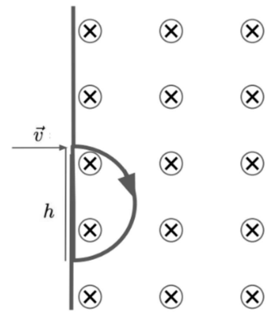
(c) The circulation of magnetic filed about loop is again given eq (7) for same reason.

The current enclosed by loop 3 is $i + (-i) = 0$. The first term in that sum is current due to cylinder, second term due to shell.

Hence Ampere's law

$$2\pi r B = 0 => B = 0.$$





3. Trajectory

1. When first entering the screen, the particle will fee a force up by applying the right hand rule, so the left figure shows the correct path.

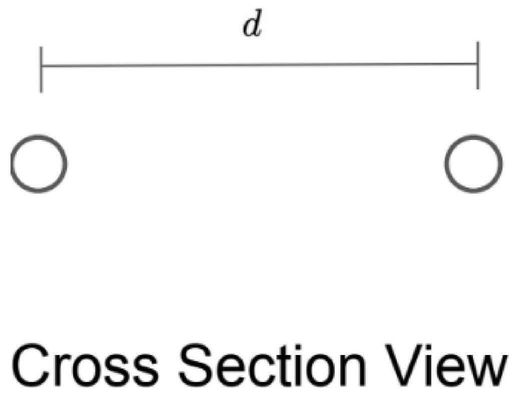
2. From N2L we can get the cyclotron radius:

$$R = \frac{mv}{qB}$$

h will be twice this radius:

$$h = \frac{2mv}{qB}$$

3. The next hit will occur another distance h farther up, or $2h$ from the initial entrance. This is because after the direction is reversed, we are back to the initial conditions. If this process goes on, the path will be repeated semicircular arcs continuing to the upwards direction.



4. Parallel Currents

1. The force from one wire on the other can be found by first finding the magnetic field due to one of the wires. We know this force will be attractive based on Ampere's initial experiments. For that, we can use Ampere's Law:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

Picking a circular loop a distance d from one of the wires gives:

$$B 2\pi d = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi d}$$

If we consider this field originating from the wire on the left, and the current coming out of the page, B will be up according to the figure. The force on a wire in a magnetic field is:

$$\vec{F} = I\vec{\ell}\vec{B}$$

The force per unit length is then:

$$d\vec{F} = Id\vec{\ell}\vec{B}$$

Since $\vec{I}$ and $\vec{B}$ are orthogonal, this cross product simplifies to:

$$|\vec{F}| = IdlB$$

So the force per unit length is:

$$\frac{|\vec{F}|}{dl} = \frac{\mu_0 I^2}{2\pi d}$$

Here we plugged in B from earlier. The direction by the right hand rule is a force to the left on the wire on the right, attractive.

2. To balance this force, we need a repulsive force from the electric force. The electric field from a charged wire is:

$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 d}$$

We can get the force per unit length by multiplying by the charge per unit length that the other wire has added:

$$\frac{|\vec{F}|}{dl} = \frac{\lambda^2}{2\pi\varepsilon_0 d}$$

We need this to equal the attractive force:

$$\frac{\lambda^2}{2\pi\varepsilon_0 d} = \frac{\mu_0 I^2}{2\pi d}$$

$$\lambda^2 = \mu_0\varepsilon_0 I^2$$

$$\lambda = I\sqrt{\mu_0\varepsilon_0}$$

This is the final answer, but one interesting note is that $\sqrt{\mu_0\varepsilon_0}$ is a special expression in electromagnetism, it is equal to $1/c$, the inverse of the speed of light in a vacuum. So we can actually simplify this answer a bit to find:

$$\lambda = \frac{I}{c}$$

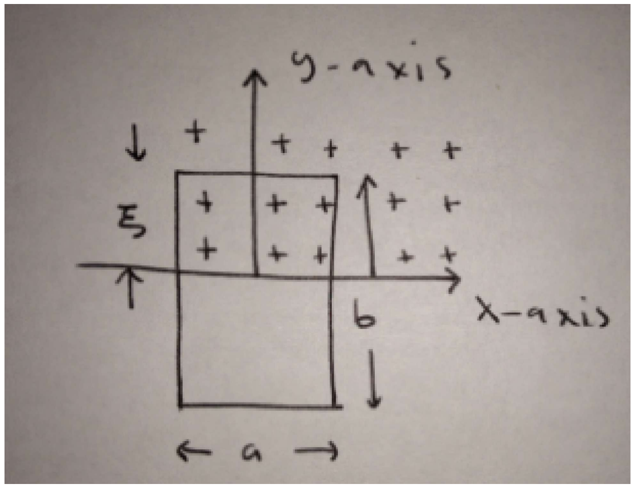


Figure 7: Rectangular Loop in a Magnetic Field

Problem 5

(a)

$$\begin{aligned}\phi &= 0 \quad \text{for } \xi < 0 \\ &= Ba\xi \quad \text{for } 0 < \xi < b \\ &= Bab \quad \text{for } \xi > b\end{aligned}$$

(b) (i)

$$\begin{aligned}\epsilon &= -\frac{d\phi}{dt} = 0 \quad \text{for } \xi < 0 \\ &= Ba\frac{d\xi}{dt} = Bav \qquad \text{for } 0 < \xi < b \\ &= 0 \qquad \text{for } \xi > b\end{aligned}$$

(ii) Magnetic forces

* $\vec{B}$ is fixed, so no induced electric fields.

(c) (i) $i = \frac{\epsilon}{R} = \frac{Bav}{R}$ (Ohm's law)

(ii) Clockwise (Then induced field will point into the page and oppose the increase in external flux brought by the moving loop)

(iii) $P_{Joule} = I^2 R = \frac{B^2 a^2 v^2}{R}$

(d) (i) Use $\vec{F} = i\vec{\ell} \times \vec{B}$

For top segment:

$$\vec{\ell} = a\hat{i}$$
$$\vec{B} = B\hat{k}$$
$$I = \frac{Bav}{R}$$

$$\Rightarrow \vec{F} = -\frac{B^2a^2v}{R}\hat{j} \quad \text{since} \quad \hat{i} \times \hat{k} = -\hat{j}$$

(ii)

$$P_{ext} = \vec{F}_{ext} \cdot \vec{v} = \left(\frac{B^2a^2v}{R}\hat{j}\right) \cdot (v\hat{j})$$

$$= \boxed{\frac{B^2a^2v^2}{R}}$$

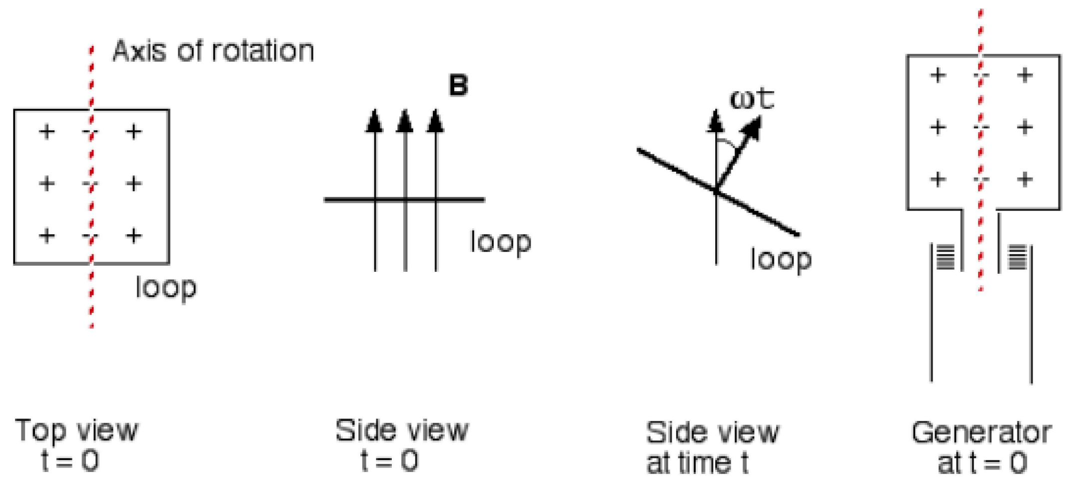


Figure 8: A square loop of side a rotates in a magnetic field.

Problem 6

(a)

$$\begin{aligned} \phi &= (\hat{n} \cdot \vec{B})A \\ &= BA\cos(\omega t) \quad \text{(angle between } \vec{B} \text{ and } \hat{n}) \end{aligned}$$

where: * B: magnitude of $\vec{B}$ * $A = a^2$: area of the loop
Thus,

$$\phi = Ba^2 \cos(\omega t)$$

(b)

$$\epsilon = -\frac{d\phi}{dt} = a^2 B\omega \sin(\omega t)$$

by Faraday's flux rule.