Physics 122 Fall 24

Practice Problems

Practice problems are not for credit and will not be graded.

They will be helpful in preparation for the first midterm exam on Friday, Sept 20, 2024

Checklist for First Midterm

The list is not meant to be comprehensive but if you can do all of the following you should do very well on the exam.

☐ 1. Use a symmetry argument to determine the direction of the electric field.

☐ 2. Determine the direction of the electric field by breaking up the source into pairs of symmetrically placed charges.

☐ 3. Use Coulomb's law to write a vector expression for the electric field due to a point charge. By vector expression I mean using unit vectors.

☐ 4. Use Coulomb's law to write a vector expression for the electric field due to an infinitesimal segment of charge.

☐ 5. Use superposition to add the electric field vectors due to several point charges.

☐ 6. Use superposition and integration to determine the electric field of linear distributions of charges such as lines, rings and circular arcs.

☐ 7. For simple charge distributions be able to guess the form of the electric field at large and small distances.

☐ 8. Make simple approximations such as these.

$$a + b \approx a, \quad a^2 + b^2 \approx a^2, \quad \sqrt{a^2 + b^2} \approx a \tag{1}$$

provided $a \gg b$. In case of more complicated approximations we will supply you with necessary formulae.

☐ 9. Be able to determine the charge enclosed by a Gaussian surface.

☐ 10. Be able to evaluate the flux through a Gaussian surface. This involves being able to determine the direction of the outward normal unit vector $\hat{\mathbf{n}}$ on the Gaussian surface;

knowing that if the electric field lies in the plane of a surface, then the flux through that surface is zero; and knowing that if $\hat{\mathbf{n}} \cdot \mathbf{E}$ is constant over a surface, then the flux is just the area of that surface times the constant value of $\hat{\mathbf{n}} \cdot \hat{\mathbf{E}}$.

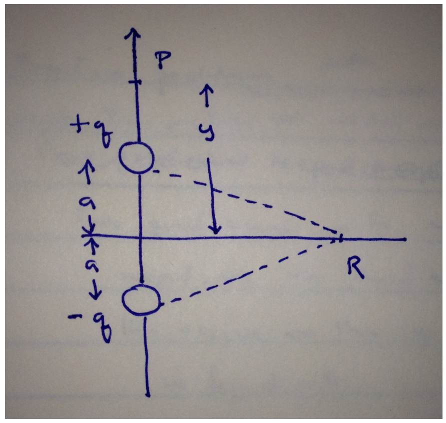


Figure 1: An electric dipole.

1. Electric dipole moment.[1] Two point charges with charge $+q$ and $-q$ respectively are placed symmetrically about the origin on the y-axis as shown in Fig 1.

(a) In what direction does the electric field at P point? If it is vertical, is it up or down? If horizontal, is it left or right?

(b) Determine the electric field at P. Give your answer in terms of q, ϵ_0, a, y and the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$. Here y is the y-coordinate of P.

(c) Simplify your result of part (a) for the circumstance that $y \gg a$ making use of the binomial approximation

$$\frac{1}{(y+a)^2} \approx \frac{1}{y^2} - \frac{2a}{y^3}. \tag{2}$$

(d) In what direction does the electric field at R point? If it is vertical, is it up or down? If horizontal, left or right?

(e) Determine the electric field at the point R that lies on the x-axis as shown in Fig 1. Give your answer in terms of $q, \epsilon_0, a, x =$ the x-coordinate of R, and the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

(f) Simplify your result of part (c) for the circumstance $x \gg a$.

[1] This problem was worked out in class. That is probably why it looks so familiar.

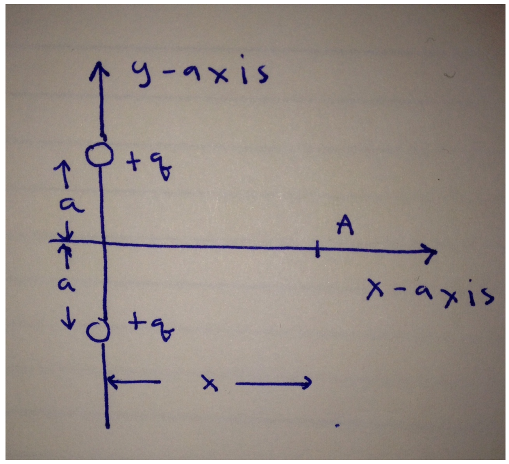


Figure 2: Two positive charges.

2. Electrostatics of molecular hydrogen. A pair of charges, each with positive charge $+q$, are placed on the y-axis, symmetrically about the origin, separated by a distance $2a$ from each other, as shown in fig 2.

(a) [15 points] (i) What is the direction of the electric field at the point marked A in fig 2?

(ii) Determine the electric field at A. Give your answer in terms of $q, \epsilon_0, x, a, \hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

(iii) What do you expect the electric field to be at large distances (for $x \gg a$)? Explain briefly (one or two sentences). Does your answer of part (ii) reduce to the expected result?

Useful approximation:

$$\frac{1}{(x^2 + a^2)^{3/2}} \approx \frac{1}{x^3}. \tag{3}$$

(b) [5 points] Suppose the two particles are protons in a hydrogen molecule. Calculate the force between the protons in Newtons. *Useful data:* Charge of proton, $q = 1.6 \times 10^{-19}$ C. The bond length of molecular hydrogen is 0.74 Å corresponding to the parameter $a = 3.7 \times 10^{-11}$ m. $1/4\pi\epsilon_0 = 9.0 \times 10^9$ N m^2/C^2.

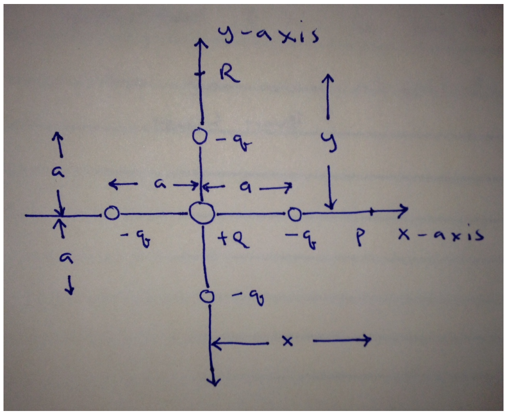


Figure 3: A complex ion.

3. Complex ion. A complex ion consists of a central cation of charge $+Q$ surrounded by four "ligands" of charge $-q$ as shown in the figure 3.

(a) In what direction do you expect the electric field to point at the points P and R shown in the figure?

(b) Let S be a point on the z-axis an elevation z above the central cation. In what direction do you expect the electric field to point at S?

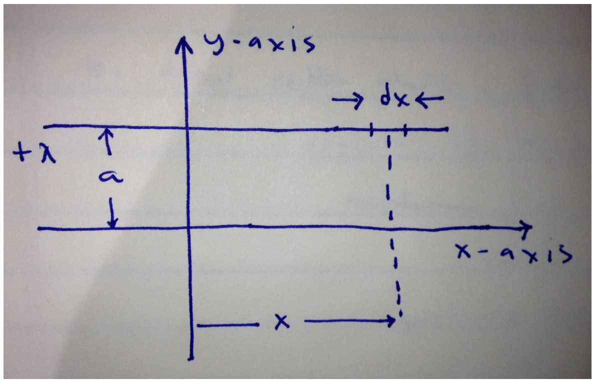
(c) Determine the electric field at P. Give your answer in terms of Q, q, ϵ_0, a, x and the unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$.

(d) What electric field do you expect at P if $x \gg a$? Does your result of part (c) simplify to the expected answer?

(e) Determine the electric field at S. Give your answer in terms of Q, q, ϵ_0, a, z and the unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$.

(f) What do you expect the electric field to be if S is far away from the complex ion, (*i.e.*, $z \gg a$)? Does your result of part (e) reduce to the expected result?

4. Parallel lines.



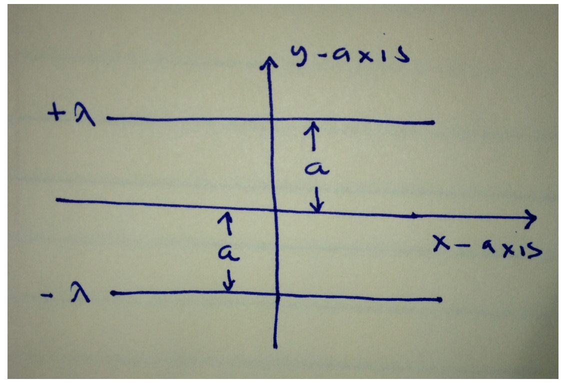


Figure 4: (*Left*) A uniformly charged infinite line parallel to the x-axis. (*Right*) Two infinite parallel lines, one with charge per unit length $+\lambda$, the other with $-\lambda$.

(a) An infinite line with a charge per unit length $+\lambda$ lies in the x-y plane. It is aligned with the x-axis and lies a distance a above the origin.

(i) In what direction does the electric field at the origin point?

(ii) Consider the infinitesimal segment of length dx located at the co-ordinates (x, a). What is the electric field produced by this segment at the origin? Give your answer in terms of $\lambda, \epsilon_0, a, x$ and the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

(iii) By integration determine the total electric field at the origin. Give your answer in terms of λ, ϵ_0, a, $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

(b) Now suppose that a second infinite line has charge per unit length $-\lambda$. This line is parallel to the first and lies a distance $-a$ below the origin.

(i) What is the direction of the field at the origin due to the second line?

(ii) What is the magnitude of the electric field at the origin due to the second line? Give your answer in terms of λ, ϵ_0 and a.

(iii) What is the total electric field at the origin? Give your answer in terms of $\lambda, \epsilon_0, a, \hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

Helpful integrals:

$$\int_{-\infty}^{\infty} dx \, \frac{x}{(x^2+a^2)^{3/2}} = 0; \qquad \int_{-\infty}^{\infty} dx \, \frac{1}{(x^2+a^2)^{3/2}} = \frac{2}{a^2}. \tag{4}$$

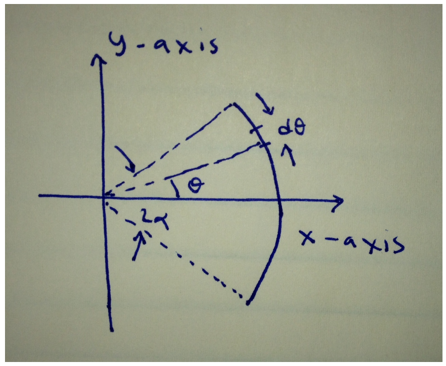


Figure 5: A charged circular arc.

5. Charged Arc. A circular arc of radius R subtends an angle 2α at its center. The arc is uniformly charged with a charge per unit length λ. As shown in the figure the arc is placed in the x-y plane symmetrically about the x-axis; its center lies at the origin.

(a) In what direction will the electric field at the origin point?

(b) Consider the infinitesimal segment of the arc of angular width $d\theta$ and located at an angle θ with respect to the x-axis. What is the electric field produced by this segment at the origin? Give your answer in terms of $\lambda, R, d\theta, \epsilon_0, \theta \hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

(c) What is the total electric field at the origin due to the complete arc? Give your answer in terms of $\lambda, R, \epsilon_0, \alpha, \hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$. *Useful integrals:*

$$\int d\theta \ \sin\theta = -\cos\theta, \qquad \int d\theta \ \cos\theta = \sin\theta. \tag{5}$$

(d) What electric field do you expect at the origin if $\alpha = 2\pi$? Explain your reasoning briefly (one or two sentences). Does your answer of part (c) reduce to the expected answer?

(e) What electric field do you expect at the origin if α is very small? Briefly explain your reasoning (one or two sentences). Does your answer of part (c) reduce to the expected answer?

Useful approximation: For small α,

$$\sin\alpha \approx \alpha. \tag{6}$$

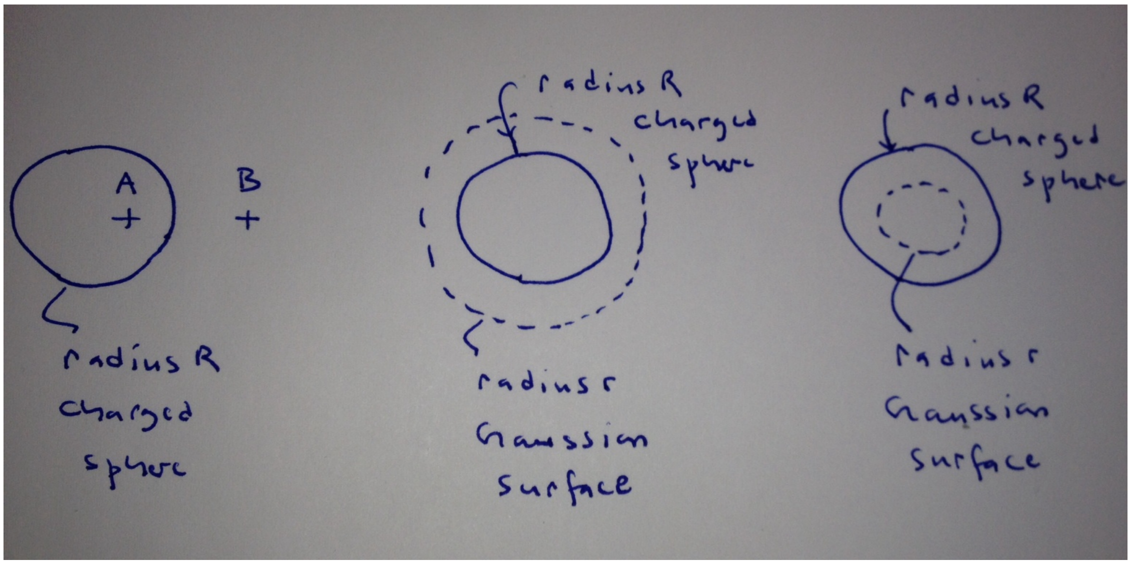


Figure 6: (a) A uniformly charged sphere. (b) Gaussian surface to determine external field. (c) Gaussian surface to determine internal field.

6. Uniformly charged sphere.[2] A sphere of radius R is uniformly charged with charge density ρ.

(a) What is the total charge of the sphere? Give your answer in terms of ρ and R.

(b) In what direction does the electric field point at the points marked A and B in the figure?

(c) Now let us determine determine E, the magnitude of the electric field at a distance r from the center of the sphere. In this part we focus on points outside the sphere for which $r > R$. To this end consider the Gaussian surface shown in fig 6 (b).

(i) What is the charge enclosed by the Gaussian surface? Give your answer in terms of r, R and ρ.

(ii) What is the flux through the Gaussian surface? Give your answer in terms of E, r and R.

(iii) Use Gauss's law to determine E. Give your answer in terms of ρ, R, r and ϵ_0.

(d) Now let us determine the electric field inside the sphere (*i.e.* assume that $r < R$). To this end consider the Gaussian surface shown in fig 6(c).

[2] This problem was worked out in class. That is probably why it looks so familiar.

(i) What is the charge enclosed by the Gaussian surface? Give your answer in terms of r, R and ρ.

(ii) What is the electric flux through the Gaussian surface? Give your answer in terms of E, r and R.

(iii) Use Gauss's law to determine E. Give your answer in terms of ρ, R, r and ϵ_0.

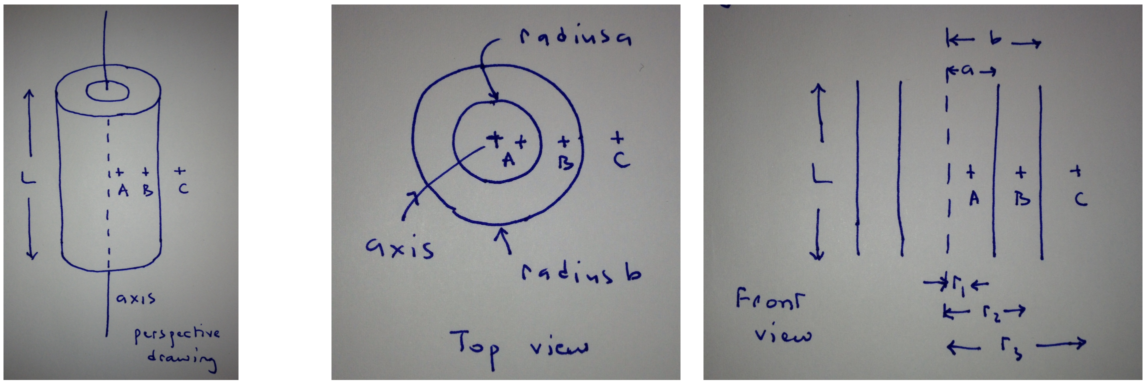


Figure 7:

7. Cylindrical shell. Consider a cylindrical shell of inner radius a and outer radius b with length L and a uniform charge density ρ. For simplicity we will assume that $L \to \infty$. We wish to determine the electric field at the points marked A, B and C which are located at a distance r_1, r_2 and r_3 respectively from the axis of the cylinder. Obviously $r_1 < a$, $a < r_2 < b$ and $r_3 > b$.

(a) (i) In what direction does the electric field point at C? You may assume $\rho > 0$.

(ii) Draw a Gaussian surface suitable for determining the electric field at C.

(iii) What is the charge enclosed by the Gaussian surface?

(iv) What is the flux through the Gaussian surface?

(v) What is the electric field at C?

(b) Repeat problem (a) for the point B.

(c) Repeat problem (a) for the point A.