Physics 122 Spring 2025

Ampere's and Faraday's law

Homework 8

Due: April 8th, at 10 PM

Note special time

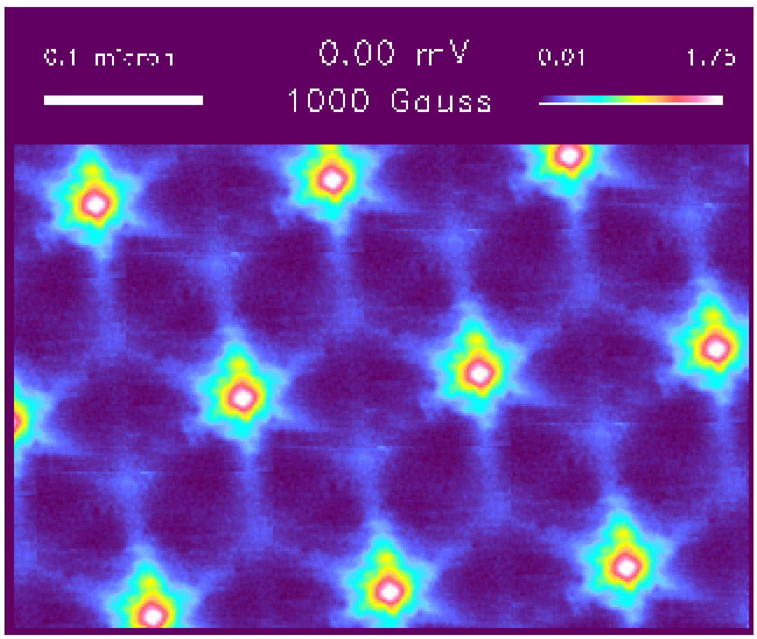


Figure 1: Vortex lattice in a type II superconductor. The bright yellow circles are the locations of vortices: whirlpools of circulating electrical current seen from above. Ampere's law dictates that there is a strong magnetic field perpendicular to the page in the core of the vortices and that there is no magnetic field outside the cores. Hence superconducting vortices are also called magnetic flux tubes. This image was made using scanning probe microscopy by Harald Hess at Bell Labs.

Guidelines

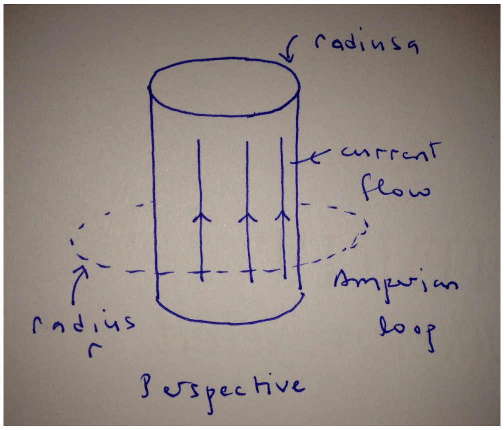
1. Please write legibly. If necessary solve the problem first on scratch paper. Then write a cogent and legible solution based on your scratch work. Learning to make a clear and logical presentation of your work is an invaluable asset, well worth the trouble of having to write it twice.

2. Please begin each problem on a new page just as we have done with the questions.

3. Parts of problems marked with an asterisk (if any) are only to be read. No response is needed.

How to submit homework: Submit your homework in PDF format via Canvas. You will find a link to submit under Assignments. Please submit before the deadline. Deadlines are enforced and late homework may not be graded. If you have compelling reasons to miss the deadline please contact the instructor preferably before the deadline. In order to generate a PDF of your homework you should scan it. If you do not have access to a scanner there are a number of effective and free scanner apps available for phones. AdobeScan is one that many students have found worked well for them in previous courses. The other option is to take pictures of your homework and convert them to PDFs using your computer. Finally if you work with a tablet you can of course easily save your work as a PDF.

1. **Axial current on a cylindrical surface.** A current i flows along the surface of a cylinder parallel to the axis of the cylinder. The radius of the cylinder is a and its length is presumed to be infinite.



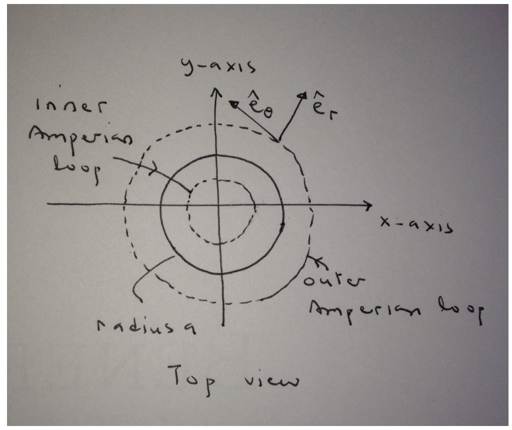


Figure 2: Cylinder with an axial surface current.

(a) *Current density:* What is the current per unit length, K? (Hint: think about the units that current per unit length must have.) Give your answer in terms of i and a.

(b)* *Field direction:* At any point there are three possible directions along which the magnetic field can point in principle. These directions are "radial" (perpendicular to the axis of the cylinder and outwards), "axial" (along the axis of the cylinder) or "azimuthal" (perpendicular to the other two directions). The unit vectors along these directions are denoted $\hat{\mathbf{e}}_r$, $\hat{\mathbf{e}}_z = \hat{\mathbf{k}}$ and $\hat{\mathbf{e}}_\theta$ respectively.

(c) *Gaussian analysis:* A professor assumes that the magnetic field due to the cylinder described above points in the radial direction. Explain why this assumption is untenable. [*Hint:* Consider a suitable Gaussian surface and apply Gauss's law for magnetism.]

(d) *Amperian analysis:* Now assume that the magnetic field points in the azimuthal direction. Consider the outer Amperian loop shown in the figure.

(i) What is the current enclosed by the loop?

(ii) What is the integral of $\mathbf{B}$ around the loop? Give your answer in terms of B, the magnitude of B at a distance r from the axis, a and r.

(iii) Apply Ampere's law to determine B in terms of μ_0, i, a and r.

(e) Repeat part (d) for the inner Amperian loop shown in the figure.

(f) Make a plot of B (the magnitude of the magnetic field) as a function of r (the distance from the axis).

2. **Circulating current around a cylindrical surface.** Current circulates around the circumference of a cylinder. The current per unit length is K. The radius of the cylinder is a; its length is assumed to be infinite.

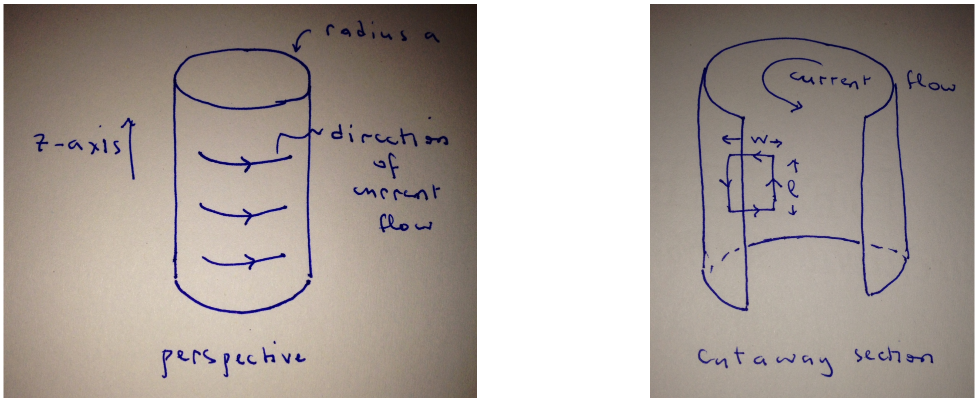


Figure 3: Cylinder with a circulating surface current.

(a)* *Direction of field:* As in the previous problem we can rule out a radial component for the magnetic field using Gauss's law for magnetism. We can also rule out that it has an azimuthal component by using Ampere's law as in the previous problem. Thus the magnetic field must point along the z-axis. We also assume that the magnetic field is zero outside the cylinder.

(b) Consider the Amperian loop shown in the figure.

(i) What is the integral of the magnetic field around the loop? Give your answer in terms of ℓ, w and B. Here $\mathbf{B} = B\hat{\mathbf{k}}$ is the magnetic field along the inner vertical edge of the Amperian loop.

(ii) What is the current that passes through the loop? Give your answer in terms of K, ℓ and w.

(iii) Apply Ampere's law to determine B in terms of μ_0 and K.

(c)* *Realizations:* One way to realize a cylindrical sheath with circulating current is to stack rings. If N rings form a stack of height L and each ring has a circulating current i, then the current per unit length $K = Ni/L$. More common than a stack of rings is to coil a wire into a helix (a corkscrew shape). Such coils are called solenoids and are important components of circuits, transformers, generators and machines. If the solenoid with current i has n turns per unit length it can be modeled as a cylindrical sheath of current with current density $K = ni$. When a strong magnetic field is applied to a type II super-conductor vortices (such as those depicted in fig 1) form. A vortex is a whirlpool of current with a magnetic field along its axis. The geometry analyzed here is a crude caricature of a vortex.

Optional (Zero credit) but highly recommended

3. **Cylindrical wire and wire with a cylindrical bore.** A cylindrical wire of radius a carries a current i out of the page. The current is uniformly distributed over the cross section of the wire.

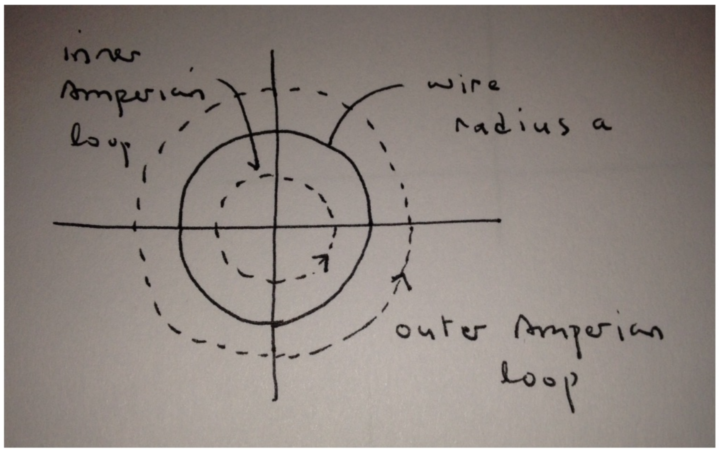


Figure 4: A solid cylindrical wire.

(a) Calculate j, the current density, defined as the current per unit area of cross section.

(b) Apply Ampere's law to the outer loop shown in the figure in order to calculate the magnetic field $\mathbf{B}$ outside the wire (for $r > a$ where r is the distance from the axis of the wire).

(c) Apply Ampere's law to the inner loop shown in the figure in order to calculate the magnetic field $\mathbf{B}$ inside the wire (for $0 < r < a$).

Now consider a cylindrical wire in which a cylindrical hole has been excavated. The wire has a radius a; the cylindrical bore has a radius $a/2$. As shown in the figure the axis of the bore is parallel to the axis of the wire but offset from it. Assume that the wire carries a current i out of the page and that the current is uniformly distributed over the crescent shaped cross section of the wire (shown shaded in the figure).

(d) Repeat part (a) for this wire.

(e) Determine the magnetic field along the y-axis for

(i) $y > a$.

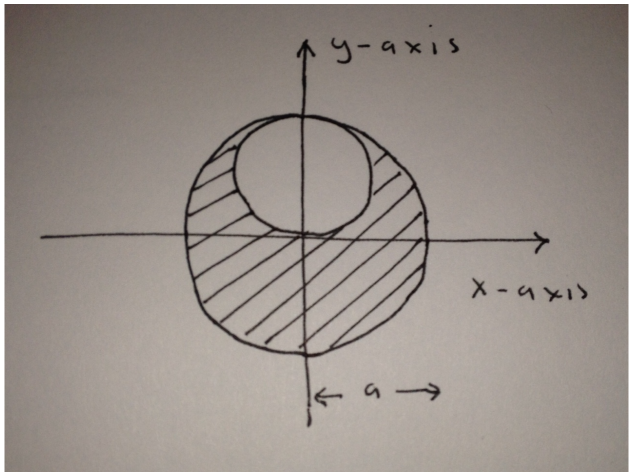


Figure 5: A cylindrical wire with a cylindrical hole bored through it.

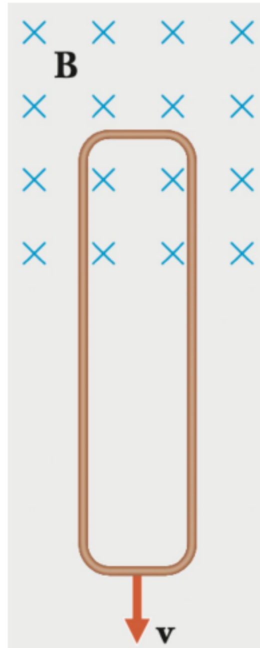
(ii) $a > y > a/2$.

(iii) $a/2 > y > 0$.

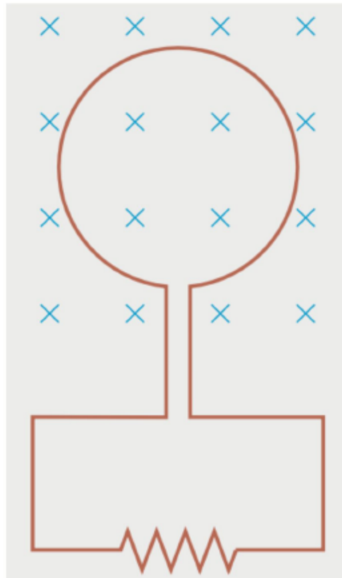
(iv) $0 > y - a$.

(v) $-a > y$.

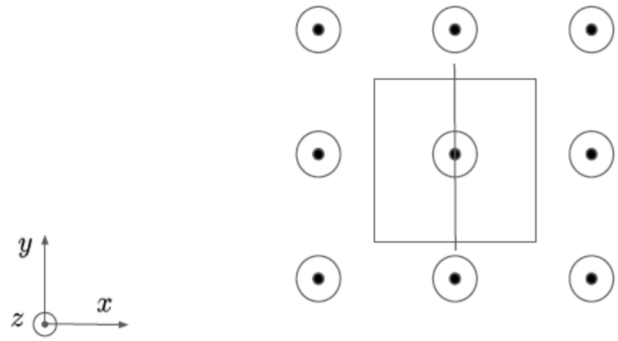
Hint: Use superposition.



4. A long rectangular conducting loop of width 25 cm is partially in a region of a horizontal magnetic field of 1.8T perpendicular to the loop as shown in the figure below. The mass of the loop is 12 g, and its resistance is 0.17Ω. If the loop is released, what is its terminal velocity? Assume that the top of the loop stays in the magnetic field. (Hint: The terminal velocity occurs when the magnetic force on the induced current is equal in magnitude to the gravitational force.)



5. A circular loop of wire is placed in a magnetic field of 0.30 T while the free ends of the wire are attached to a 15Ω resistor as shown in the figure below. When you squeeze the loop, the area of the loop is reduced at a constant rate from 200 to 100 cm^2 in 0.020 s. What are the magnitude and direction of the current in the resistor?



6. A square loop of wire with side length 12cm is placed in a uniform magnetic field of strength 2T. Initially $\vec{B}$ and $\vec{A}$ are parallel. At $t = 0$ the loop begins to spin about its axis with angular velocity $\omega = 1 rad/s$ (see below). Plot the EMF through this loop from $t = 0$ to $t = 7$ seconds.

Optional (Zero credit) but highly recommended

7. A circular wire loop of radius r and total resistance R is placed inside a region of uniform magnetic field that has magnitude given by $B = B_0 \sin(\omega t)$. $\vec{B}$ and $\vec{A}$ are parallel. Find the induced magnetic field (the magnetic field only from the wire loop) produced at the center of the wire loop at any given time t. Describe the the phase offset between the two fields.