PHYS122 Homework 1 - Due 1/26/25 **Trevor Swan (tcs94)**

1. Triboelectricity

a) Scotch Tape

(i) The two strips initially repel each other as they were both transferred the same charge from the roll, and since they were transferred charge from the same source, they are similarly charged and hence repel each other.

(ii) The friction created from ripping the pieces of tape apart causes charge from being transferred between the pieces. This results in one piece being positively charged and the other being negatively charged → attraction.

b) Laundry

(i) The clothes cling together after going through the dryer as the friction caused by the tumbling transfers charges between the fabrics. This creates oppositely charged regions on the clothes that attract each other.

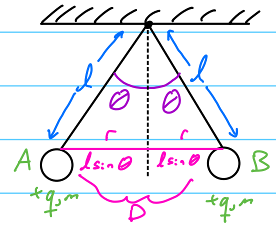
(ii) Clothes made up of different materials should cling together more because they have different tendencies to gain or lose electrons during the tumbling. This results in higher electrostatic attraction due to greater charge imbalances.

c) Nylon Threads & Metal Spheres

(i) The two spheres must have opposite charges at the region where they're being attracted. It is possible that one sphere is charged while the other is not. The charge imbalance is required for attraction to occur.

(ii) Since Metals are conductors, their charges can travel freely. Eventually, the charges of the spheres will redistribute until the charges reach an equilibrium. This means the spheres will have the same charge ⇒ no longer attract → repel slightly.

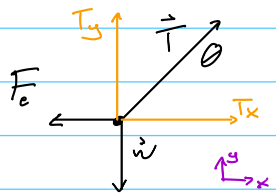
2. Electroscope



Given

- Plastic spheres A & B, both mass m
- Threads of length ℓ
- Charge $+q$ imparted to each sphere
- Imparted charges result in an angle θ to the vertical

FBD A



$$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cdot q}{D^2}$$

$$T_x = T\sin\theta$$

$$T_y = T\cos\theta$$

$$W = mg$$

where $D = 2\ell\sin\theta$

N2L x-dir $\Sigma\vec{F}_x = 0$

$$T\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2\ell\sin\theta)^2}$$

$$T\sin\theta = \frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{4\ell^2\sin^2\theta}$$

N2L y-dir $\Sigma\vec{F}_y = 0$

$$T\cos\theta = mg$$

$$T = \frac{mg}{\cos\theta}$$

$$\frac{mg}{\cos\theta}\sin\theta = \frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{4\ell^2\sin^2\theta}$$

$$\frac{(4\pi\varepsilon_0)(mg\sin\theta)}{\cos\theta} = \frac{q^2}{4\ell^2\sin^2\theta}$$

$$q^2 = \frac{(16\pi\varepsilon_0)(\ell^2 mg\sin^3\theta)}{\cos\theta}$$

$$\Rightarrow q = \sqrt{\frac{(16\pi\varepsilon_0)(\ell^2 mg\sin^3\theta)}{\cos\theta}}$$

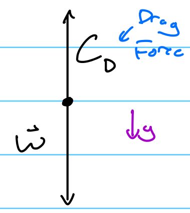
$$= \sqrt{(\cancel{16}\pi\varepsilon_0)(\cancel{\ell^2}mg\cancel{\sin^2\theta})(\tan\theta)}$$

$$\boxed{q = 4\ell\sin\theta\sqrt{(\pi\varepsilon_0)(mg\tan\theta)}}$$

3. Millikan & charge quantization

a) Terminal Velocity

(i)



N2L y-dir

$\Sigma F_y = m\cancel{a}^{\,0}$ const. velocity

$W - C_D = 0$

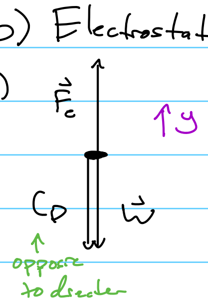
$W = C_D \rightarrow mg = Kv$

$$\boxed{v = \frac{mg}{K}}$$

(ii) $\boxed{K = \dfrac{mg}{v}}$

b) Electrostatic Forces

(i)



N2L y-dir $\quad \Sigma F_y = m\cancel{a}^{\,0}$ uniform speed

$\vec{F}_e - W - C_D = 0$

$\vec{F}_e = W + C_D$

$qE = mg + Ku$

$$\boxed{u = \frac{qE - mg}{K}}$$

(ii) $u_i = \dfrac{qE - mg}{K}$, $u_f = \dfrac{(q+e)E - mg}{K}$ $\qquad$ (iii) $K = \dfrac{mg}{v}$

$$\Delta u = \frac{(q+e)E - mg}{K} - \frac{qE - mg}{K} \Rightarrow \boxed{\Delta u = \frac{eEv}{mg}}$$

$$= \frac{\cancel{qE} + eE - \cancel{mg} - \cancel{qE} + \cancel{mg}}{K} \qquad \boxed{\Delta u = \frac{eE}{K}}$$

c) Millikan's Experiment

Solve for e in Coulombs (C): $e = \dfrac{mg}{Ev} \cdot \Delta u$

$m = 3.5 \times 10^{-15}$ kg

$g = 9.81$ m/s²

$\Delta u = 3.0 \times 10^{-4}$ m/s

$E = 6 \times 10^{5}$ N/C

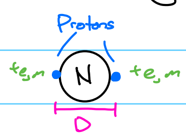
$v = 1.1 \times 10^{-4}$ m/s

$$e = \frac{(3.5 \times 10^{-15}\ \text{kg})(9.81\ \tfrac{m}{s^{\cancel{2}}})}{(6 \times 10^{5}\ \text{N/C})(1.1 \times 10^{-4}\ \cancel{\tfrac{m}{s}})} \cdot (3.0 \times 10^{-4}\ \text{m/s})$$

$$= 1.56 \times 10^{-19}\ \frac{\text{kg} \cdot \text{C} \cdot \text{m}}{\text{N} \cdot \text{s}^2} \qquad \left\{ \frac{\text{kg} \cdot \text{C} \cdot \text{m}}{\text{N} \cdot \text{s}^2} = \frac{\text{kg} \cdot \text{C} \cdot \text{m}}{\text{kg}\ \frac{\text{m}}{\text{s}^2} \cdot \text{s}^2} \right\}$$

$$\boxed{e = 1.56 \times 10^{-19}\ \text{C}}$$

5. Hierarchy Problem



Given: $D = 1.0\times10^{-15}\ m$

$e = 1.6\times10^{-9}\ C$

$m = 1.67\times10^{-27}\ kg$

Other useful data: $\frac{1}{4\pi\varepsilon_0} = 9.0\times10^{9}\ \frac{kg\cdot m^3}{C^2\cdot s^2}\ \left\{\frac{N\cdot m^2}{C^2}\right\}$

$G = 6.67\times10^{-11}\ \frac{m^3}{kg\cdot s^2}$

a) $$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cdot Q}{D^2} = \left(9.0\times10^{9}\ \frac{kg\cdot m^{\cancel{3}}}{\cancel{C^2}\cdot s^2}\right)\left(\frac{(1.6\times10^{-19}\ \cancel{C})^2}{(1.0\times10^{-15}\ \cancel{m})^2}\right) = \boxed{230\ N}$$

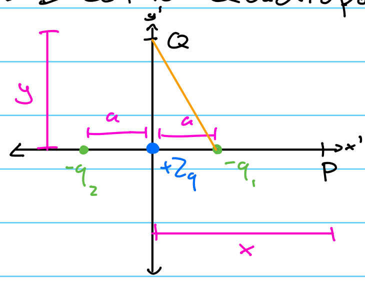
F_e is positive as q and Q are both positive, so the charges repel.

b) $$F_g = \frac{G\cdot m_1\cdot m_2}{r^2} = \frac{(6.67\times10^{-11}\ \frac{\cancel{m^3}}{\cancel{kg}\cdot s^2})(1.67\times10^{-27}\ kg)^2}{(1.0\times10^{-15}\ \cancel{m})^2} = \boxed{1.86\times10^{-34}\ N}$$

The gravitational force F_g is always attractive irregardless of charge.

c) $$R = \frac{F_e}{F_g} = \frac{230\ N}{1.86\times10^{-34}\ N} = \boxed{1.23\times10^{36}}$$

6. Electric Quadrupole moment



a) The x-direction

(i) Depending on the value of a, the field will point along the positive x direction if the +2q charge dominates in magnitude compared to the two −q charges. The field may point to the left (negative x) but more than likely is in the positive x-direction.

(ii) $E_{+2q} = k\cdot\frac{2q}{x^2}\hat{i}$, $E_{-q_1} = k\cdot\frac{-q}{(x-a)^2}\hat{i}$, $E_{-q_2} = k\cdot\frac{-q}{(x+a)^2}\hat{i}$

$$\vec{E} = k\left(\frac{2q}{x^2}+\frac{-q}{(x-a)^2}+\frac{-q}{(x+a)^2}\right)\hat{i} = \boxed{\frac{q}{4\pi\varepsilon_0}\left(\frac{2}{x^2}-\frac{1}{(x-a)^2}-\frac{1}{(x+a)^2}\right)\hat{i}}$$

(iii) Assuming a/x is relatively small

$$\frac{1}{(x+a)^2} = \frac{1}{x^2}-\frac{2a}{x^3}$$
$$\frac{1}{(x-a)^2} = \frac{1}{x^2}+\frac{2a}{x^3}$$

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{2}{x^2}-\left(\frac{1}{x^2}+\cancel{\frac{2a}{x^3}}\right)-\left(\frac{1}{x^2}-\cancel{\frac{2a}{x^3}}\right)\right)$$
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2}{x^2}-\frac{1}{x^2}-\frac{1}{x^2}\right) = 0 \qquad \boxed{E_{total} = 0}$$

b) (i) The electric field should point in the positive y-direction depending on the strength q and the distance a. Usually, the +2q charge will outweigh the two −q charges, but in rare cases, the opposite may be true → negative y-dir.

(ii) $E_{+2q} = k\cdot\frac{2q}{y^3}\vec{r} = k\frac{2q}{y^3}(0\hat{i}+y\hat{j})$

$E_{-q_1} = k\cdot\frac{-q}{\sqrt{y^2+a^2}^{\,3}}(-a\hat{i}+y\hat{j})$

$E_{-q_2} = k\cdot\frac{-q}{\sqrt{y^2+a^2}^{\,3}}(a\hat{i}+y\hat{j})$

$$\boxed{\vec{E}_{Total} = \frac{2qy}{4\pi\varepsilon_0}\left(\frac{1}{y^3}-\frac{1}{(y^2+a^2)^{3/2}}\right)\hat{j}}$$

$\hat{i}$ components go to 0

(iii) Assuming a/y is relatively small

$$\frac{1}{(y^2+a^2)^{3/2}} = \frac{1}{y^3}-\frac{3}{2}\cdot\frac{a^2}{y^5}$$

$$\vec{E}_{tot} = \frac{2qy}{4\pi\varepsilon_0}\left(\frac{1}{y^3}-\left(\frac{1}{y^3}-\frac{3}{2}\cdot\frac{a^2}{y^5}\right)\right) = \frac{\cancel{2}q\cancel{y}}{4\pi\varepsilon_0}\left(\frac{3}{\cancel{2}}\cdot\frac{a^2}{y^{\cancel{5}4}}\right) \qquad \boxed{\vec{E}_{tot} = \frac{3qa^2}{4\pi\varepsilon_0 y^4}}$$