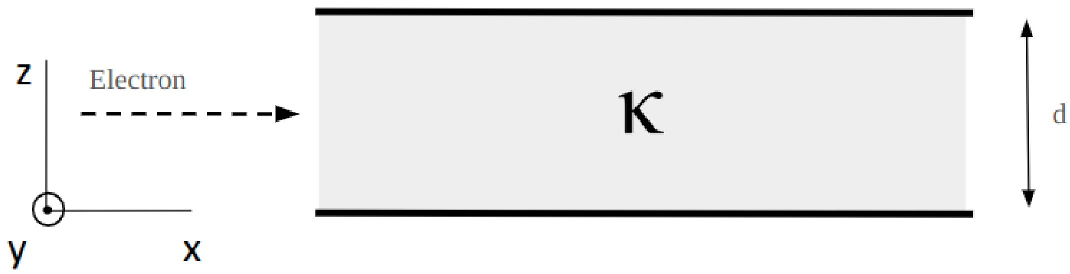

Figure 1: Note that Y axis should be into the page

Problem 1: Capacitor filled with dielectric

Consider a parallel plate capacitor with a separation d between the plates and an area A for each plate. When the capacitor is empty, the magnitude of the electric field between the plates is given by $E_0 = \frac{\sigma_{free}}{\epsilon_0}$, where σ_{free} is the free surface charge density on the top plate (assuming the top plate has positive charge and the bottom plate negative, and that the plates are parallel to the xy-plane with separation along the z-axis). The space between the plates is completely filled with a linear and isotropic dielectric material with a dielectric constant $\kappa = 1 + \chi$.

1. [10 points] Calculate the electric field (magnitude and direction) inside the capacitor when filled with the dielectric material.

   The magnitude of the electric field E inside the capacitor is given by:

   $$E = \frac{E_0}{\kappa} = \frac{\sigma_{free}}{\epsilon_0 \kappa}$$

   Since $\kappa = 1 + \chi$, we can write:

   $$\vec{E} = \frac{\sigma_{free}}{\epsilon_0(1+\chi)}(-\hat{k})$$

   The direction of the electric field remains the same as when the capacitor is empty, which is from the positive plate to the negative plate (along the z-axis).

2. [10 points] Calculate the polarization vector $\vec{P}$ inside the dielectric material. Express your answer in terms of χ and E_0. Indicate the direction of the polarization vector.

   The polarization vector $\vec{P}$ inside the dielectric material is given by:

   $$\vec{P} = \epsilon_0 \chi \vec{E}$$

Substituting the expression for $\vec{E}$:

$$\vec{P} = \epsilon_0 \chi \left(\frac{\sigma_{free}}{\epsilon_0(1+\chi)} \right) \hat{k} = \frac{\chi \sigma_{free}}{1+\chi} \hat{k}$$

The direction of the polarization vector $\vec{P}$ is the same as the direction of the electric field, which is along the z-axis.

3. [15 points] Determine the surface bound charge density σ_b on the surfaces of the dielectric in contact with the capacitor plates. Specify the sign of the bound charge on each surface.

   The surface bound charge density σ_b on the surfaces of the dielectric in contact with the capacitor plates is given by:

   $$\sigma_b = \vec{P} \cdot \hat{\vec{n}}$$

   For the top plate (positive charge), $\hat{n} = \hat{k}$, the bound charge density is:

   $$\sigma_b = -\frac{\chi \sigma_{free}}{1+\chi}$$

   For the bottom plate (negative charge), $\hat{n} = -\hat{k}$, the bound charge density is:

   $$\sigma_b = \frac{\chi \sigma_{free}}{1+\chi}$$

4. [15 points] Assume that an electron is injected with a horizontal velocity $\vec{v} = v_0 \hat{\vec{i}}$ into the capacitor, at a height midway between the plates. Determine the magnitude and direction of the magnetic field $\vec{B}$ that must be applied for the electron to pass through the capacitor horizontally without being deflected by the electric force. Neglect gravitational effects and assume that the magnetic field is uniform within the capacitor.

   To ensure the electron passes through the capacitor horizontally without being deflected by the electric force, the magnetic force must counteract the electric force. The electric force on the electron is given by:

   $$\vec{F}_E = -e\vec{E}$$

   The magnetic force is given by:

   $$\vec{F}_B = -e(\vec{v} \times \vec{B})$$

   For the electron to pass through horizontally, $\vec{F}_E + \vec{F}_B = 0$. Therefore:

   $$-e\vec{E} = -e(\vec{v} \times \vec{B})$$

Since $\vec{v} = v_0\hat{\vec{i}}$ and $\vec{E} = E\hat{k}$, we have:

$$E\hat{k} = v_0\hat{\vec{i}} \times \vec{B}$$

The cross product $\hat{\vec{i}} \times \vec{B}$ must result in a vector along the z-axis. Therefore, $\vec{B}$ must be along the y-axis:

$$\vec{B} = B\hat{\vec{j}}$$

The magnitude of $\vec{B}$ is given by:

$$E = v_0 B$$

Thus:

$$\vec{B} = \frac{E}{v_0} = \frac{\sigma_{free}}{\epsilon_0(1+\chi)v_0}\hat{j}$$

The direction of $\vec{B}$ is along the y-axis.

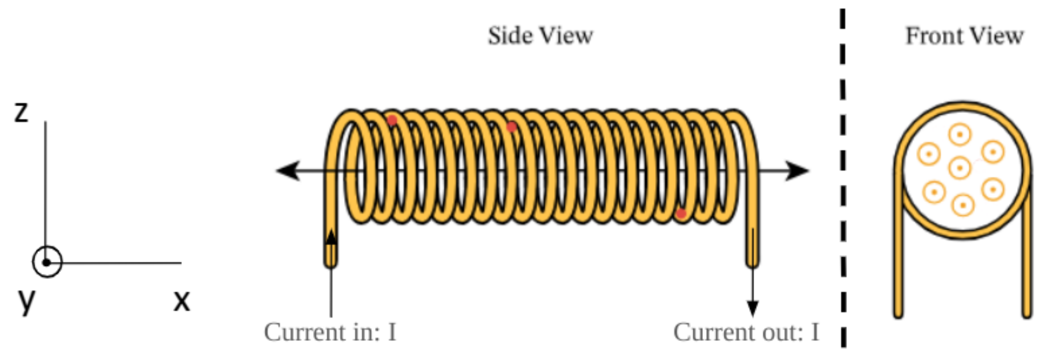

Figure 2: Note that Y axis should be into the page

Problem 2: Solenoid: Ampère's and Faraday's Law

Consider an infinite solenoid with radius R and n turns per unit length carrying a time-varying current $I(t)$.

1. [25 points] Using Ampère's law, determine the magnitude and direction of the magnetic field $\vec{B}$ inside the solenoid ($r < R$). Clearly justify the choice of your Amperian loop and the obtained direction (if no justification you will lose points).

   Using Ampère's law, we choose a rectangular Amperian loop with width w and length l, where one side of the loop is inside the solenoid and the other side is outside. Ampère's law states:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

   For an infinite solenoid with n turns per unit length carrying a current $I(t)$, the enclosed current I_{enc} is given by the number of turns per unit length n times the current $I(t)$ times the length l of the solenoid:

$$I_{\text{enc}} = nI(t)l$$

   The path integral of $\vec{B}$ around the rectangular Amperian loop is:

$$\oint \vec{B} \cdot d\vec{l} = B \cdot l$$

   Using Ampère's law:

$$B \cdot l = \mu_0 nI(t)l$$

   Solving for B:

$$\vec{B} = \mu_0 nI(t)\hat{i}$$

Using Lenz's rule we can get the direction of the magnetic field $\vec{B}$ is along the axis of the solenoid to the right, as seen in the side view, which corresponds to $\hat{\imath}$.

2. [25 points] To study the electric field induced by the changing current, consider a circular loop of radius r concentric with the axis of the solenoid. You can assume that the magnetic field is pointing out of the page in the front view.

   (a) Apply Faraday's law to calculate the induced electric field $\vec{E}$ inside the solenoid ($r < R$). Indicate the direction of the electric field.

   Inside the solenoid, the magnetic flux Φ_B through a circular loop of radius r is:

   $$\Phi_B = B \cdot \pi r^2$$

   Using Faraday's law:

   $$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}(B \cdot \pi r^2)$$

   Since $B = \mu_0 n I(t)$, we have:

   $$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt} = \pi r^2 \mu_0 n \frac{dI(t)}{dt}$$

   Therefore:

   $$\oint \vec{E} \cdot d\vec{l} = -\pi r^2 \mu_0 n \frac{dI(t)}{dt}$$

   The path integral of $\vec{E}$ around the circular loop is:

   $$\oint \vec{E} \cdot d\vec{l} = E \cdot 2\pi r$$

   Solving for E:

   $$E = -\frac{\mu_0 n r}{2} \frac{dI(t)}{dt}$$

   The direction of the induced electric field $\vec{E}$ is tangential to the circular loop and follows the right-hand rule, which means it circulates around the axis of the solenoid. If it is clockwise or counterclockwise will depend if I(t) is increasing or decreasing. If I is increasing, E will be clockwise, if it is decreasing it will be counterclockwise.

   (b) Apply Faraday's law to calculate the induced electric field $\vec{E}$ outside the solenoid ($r > R$). Indicate the direction of the electric field.

   Outside the solenoid, the magnetic flux Φ_B through a circular loop of radius r is:

   $$\Phi_B = B \cdot \pi R^2$$

Using Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}(B \cdot \pi R^2)$$

Since $B = \mu_0 n I(t)$, we have:

$$\frac{d\Phi_B}{dt} = \pi R^2 \frac{dB}{dt} = \pi R^2 \mu_0 n \frac{dI(t)}{dt}$$

Therefore:

$$\oint \vec{E} \cdot d\vec{l} = -\pi R^2 \mu_0 n \frac{dI(t)}{dt}$$

The path integral of $\vec{E}$ around the circular loop is:

$$\oint \vec{E} \cdot d\vec{l} = E \cdot 2\pi r$$

Solving for E:

$$E = -\frac{\mu_0 n R^2}{2r} \frac{dI(t)}{dt}$$

The direction of the induced electric field $\vec{E}$ is tangential to the circular loop and follows the right-hand rule, which means it circulates around the axis of the solenoid. Same as before.