Physics 122 Spring 24

Practice Problem Set 4

Practice problems are not for credit and will not be graded.

1. Dynamics of a Comet's Tail.[1] Cometary nuclei are mountain sized globs of dust and ice. As the nucleus approaches the sun, the surface ices vaporize and tiny particles of dust and ice are blown off to form the brilliant cometary tail (fig 1).

Figure 1: The Bayeux tapestry depicting the appearance of Halley's comet in 1066 before the Battle of Hastings.

(a) Let L denote the total power radiated by the sun. Calculate I, the intensity of sunlight, at a distance R from the sun. [*Hint:* L is the total energy per second radiated by the sun. The intensity is the energy per second per unit area. To calculate the intensity imagine that the sun is surrounded by a sphere of radius R. How much energy passes this sphere per second? How much energy passes the sphere per second per unit area?]

(b) Model the tail particles as dark spheres of radius r which completely absorb all incident solar radiation (together with the field momentum it carries). Calculate the magnitude of the force exerted by solar radiation on the tail particles. Give your answer in terms of L, R, r and $c =$ speed of light. [*Hint:* The force is the radiation pressure of sunlight multiplied by the cross section area of the tail particles.]

[1] In case this problem looks familiar that is because it appeared as problem 4 on HW 10.

(c) Calculate the radius of a dust particle for which radiation pressure forces precisely balance the sun's gravitational attraction. Give your answer in terms of $G =$ Newton's constant of gravitation, $M =$ mass of the sun, $\rho =$ density of silicate dust, c and L.

(d) Particles of radius smaller than the critical value calculated in part (c) will be blown outward by radiation pressure. Calculate the critical radius in μm. Take $\rho = 2.5 \times 10^3$ kg/m^3 and $L = 4 \times 10^{26}$ W. Also $M = 2 \times 10^{30}$ kg, $c = 3.0 \times 10^8$ m/s, and $G = 6.67 \times 10^{-11}$ S.I. units.

2. Breakthrough Starshot. The Breakthrough starshot initiative proposes to propel a tiny probe to the nearest star at one fifth the speed of light using a high intensity laser. For the purpose of this problem we model the probe as a reflecting square sail of side $\ell = 30$ m. The mass of the probe $m = 10^{-3}$ kg.

(a) [10 points] Show that the acceleration of the probe is given by

$$a = \frac{2I\ell^2}{mc} \tag{1}$$

where I is the intensity of the laser beam used to accelerate the probe.

(b) [5 points] The laser drives the probe out to a distance d from the earth. At that distance the angular diameter of the probe is ℓ/d. On the other hand due to the phenomenon of diffraction the laser beam fans out at an angle λ/ℓ. By equating these angles obtain the distance d out to which the laser can usefully drive the sail. Give your answer in terms of λ and ℓ. Assuming that $\lambda = 10^{-6}$m what is d? Give your answer in kilometers.

(c) [5 points] Recall that the final speed v of an object that starts from rest and is accelerated uniformly a distance d is given by $v^2 = 2ad$. Combining formula with the results of parts (a) and (b) reveals that the necessary intensity of the laser to drive the probe to a velocity v is given by

$$I = \frac{1}{4}\frac{\lambda mcv^2}{\ell^4}. \tag{2}$$

You do not need to prove this formula. Calculate the intensity of the laser in S.I. units. assuming $v/c = 1/5$. What is the S.I. unit of intensity? *Useful datum:* The speed of light is 3×10^8m/s.

3. Geometric Optics: Camera Lucida and Optical Fiber.

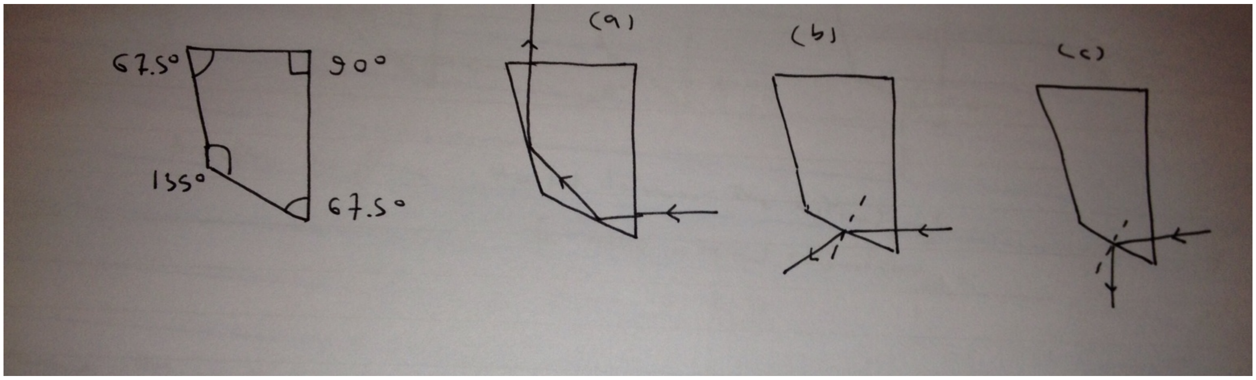


Figure 2: The glass prism used in a camera lucida (left). (a), (b) and (c) show conjectured trajectories for the ray of light incident from the right. Only one is correct. Which one?

(a) *Camera Lucida.* A key element of the camera lucida is a glass prism shown in fig 2. A ray of light is incident on the prism from the right. Which of the three figures shows the correct path of the ray of light through the prism? Briefly explain. *Useful datum:* The critical angle of incidence for total internal reflection in glass is 41.8°.

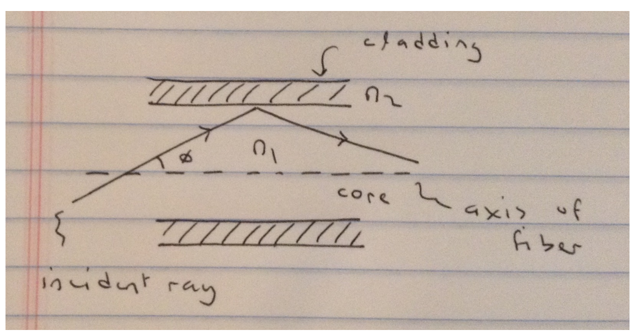


Figure 3: An optical fiber.

(b) *Optical fiber.* The core of an optical fiber is a glass cylinder with refractive index $n_1 = 1.63$. The cladding of the fiber is a cylindrical sheath the covers the core which is made of glass with refractive index $n_2 = 1.58$. A ray of light enters the fiber at an angle ϕ to the axis of the fiber as shown in figure 3.

(i) Show that the ray will be reflected and will remain inside the fiber provided that $\phi < \phi_c$ where $\phi_c = \cos^{-1}(n_2/n_1)$.

(ii) Calculate the angle ϕ_c in degrees.

4. Geometric Optics: Porro Prism and Total Internal Reflection.

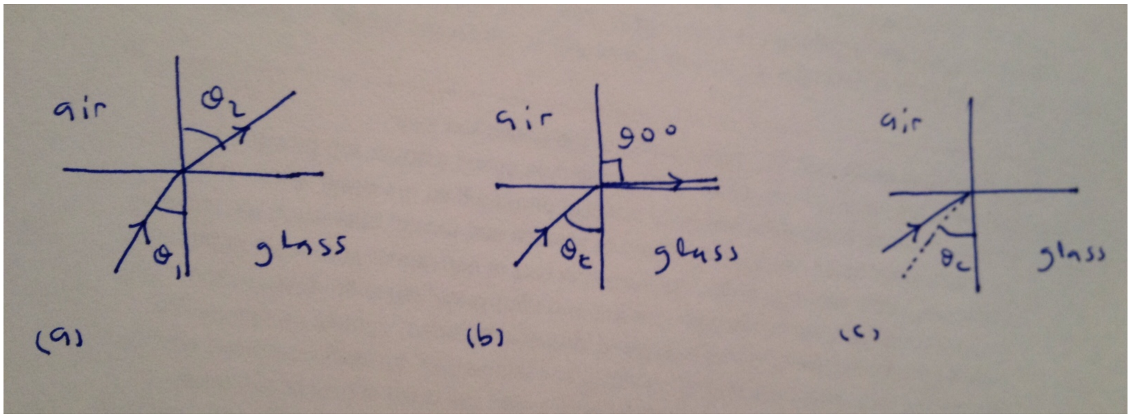


Figure 4: Light incident on air-glass interface from the glass side. (a) At a small angle of incidence. (b) At the critical angle of incidence. (c) At more than the critical angle of incidence.

(a) *Total internal reflection.* Light is incident from glass (refractive index n) to air (refractive index ≈ 1) as shown in fig 4 (a).

(i) Which one is greater, the angle of incidence (θ_1) or the angle of refraction (θ_2)?

(ii) Derive an expression for θ_1 in terms of n and θ_2.

(iii) For a critical angle of incidence $\theta_1 = \theta_c$ the refracted ray grades the interface; $\theta_2 = \pi/2$. Derive an expression for θ_c in terms of n.

(iv) For glass $n = 1.5$. Calculate θ_c. Give your answer in degrees not radians.

(v) What happens to the incident ray when the angle of incidence $\theta_1 > \theta_c$?

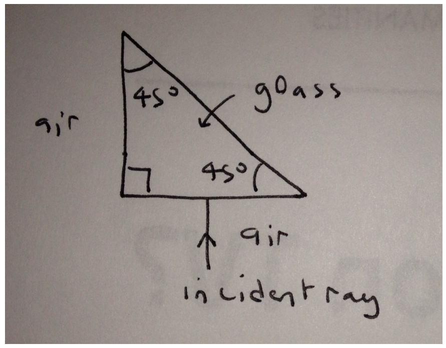


Figure 5: A Porro prism.

(b) *Porro prism:* A ray of light is incident on a Porro prism as shown in figure 5. Trace its subsequent trajectory.

(c) *More Porro prism:* Now consider a ray incident from the right as shown in fig 6. Which of the two diagrams depicts its subsequent trajectory correctly?

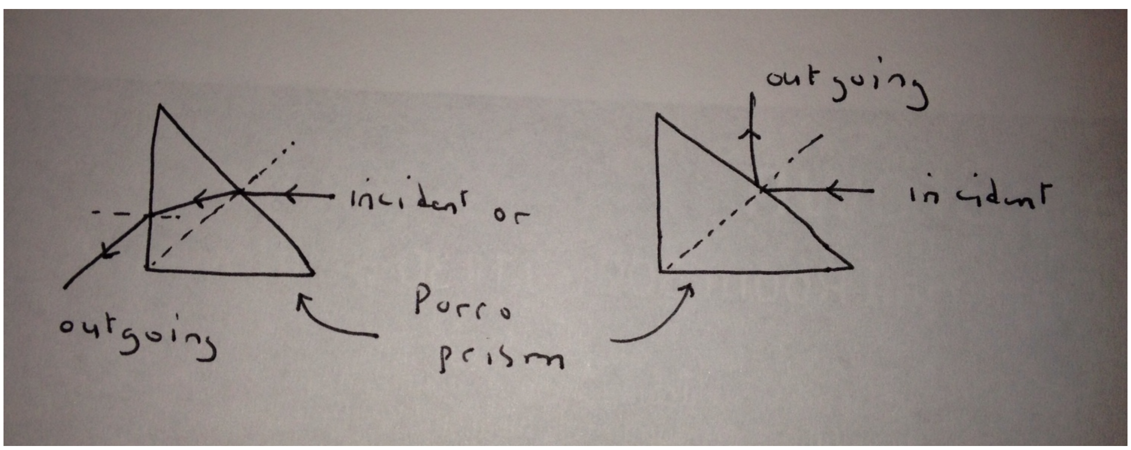


Figure 6: A Porro prism.

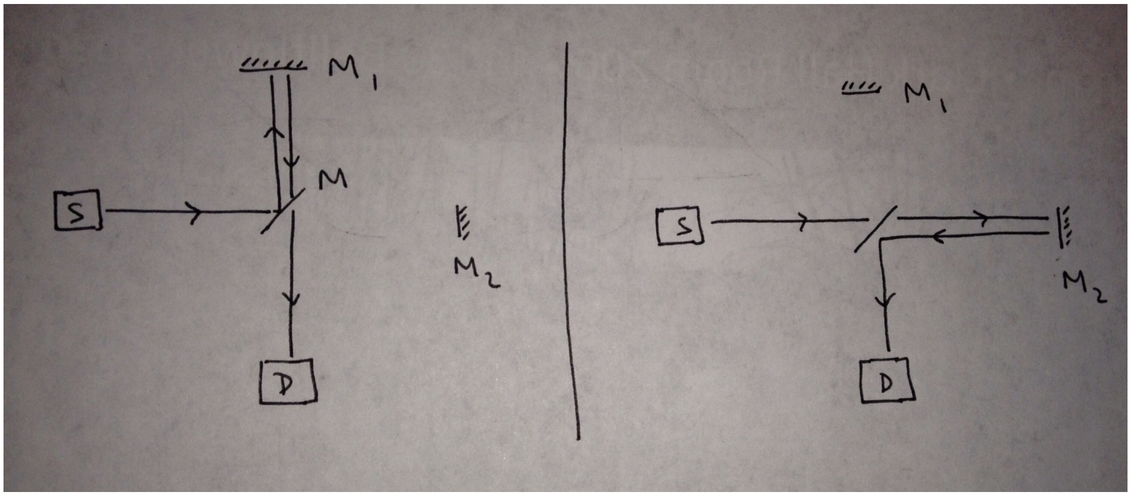


Figure 7: Light can travel from source to detector along one of two paths as shown in the figure.

5. Michelson Interferometer. A Michelson interferometer consists of a source of light S and a detector D, two perfect mirrors M_1 and M_2 and a partially transmitting mirror M arranged as shown in the figure. There are two paths light rays can follow from source to detector as shown in the figure.

The source produces light of fixed intensity and wavelength $\lambda = 633$ nm. Initially the detector sees bright light but as the mirror M_2 is moved slowly to the right the light seen by the detector becomes dark.

(i) By what distance has the mirror been moved? [*Hint:* Assume the interference between the two paths has gone from constructive to destructive.]

(ii) If the mirror is moved an additional small distance the light becomes bright again. By what distance has the mirror been moved now?

(iii) Suppose that the light observed by the detector undergoes the transition from bright to dark to bright again fifty times. How far has the mirror M_2 been moved?

*(iv) Among other applications the Michelson interferometer is a sensitive detector of small movements on a length scale comparable to the wavelength of light. For developing the interferometer, and for carrying out experiments at CWRU in 1887 that are the basis of Einstein's theory of relativity, Michelson became the first American to win the Nobel Prize in Science. This spring a Michelson interferometer was used to measure gravitational waves from the merger of black holes in a galaxy a billion light years away.

Figure 8: Albert A Michelson (left), first Professor of Physics at Case School of Applied Science, with Einstein (center) and Millikan (right) in 1931.

6. Interference and Diffraction.

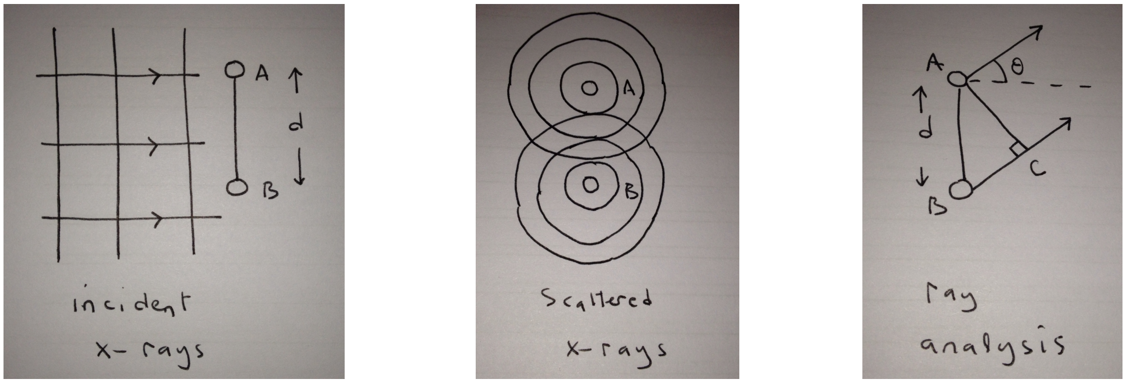


Figure 9:

(a) A plane wave of x-rays is incident on a diatomic molecule as shown in the figure. λ denotes the wavelength of x-rays and d the distance between the atoms. The atoms are excited by the x-rays and each atom radiates spherical waves outward as shown in fig (b). This outgoing radiation constitutes the scattered x-rays. The scattered radiation from each atom is of uniform intensity in all directions. However due to interference between the two atoms the total intensity depends strongly on the direction. To determine the total intensity in the direction θ we need to compare outgoing rays from the two atoms. As shown in fig (c), radiation from atom B has to go an extra distance BC.

(i) Determine the distance BC. Give your answer in terms of d and θ.

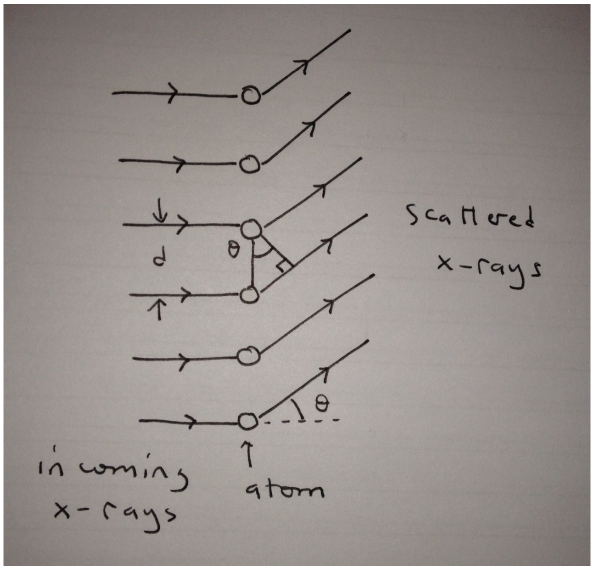
(ii) *Constructive interference:* Now if the distance $BC = n\lambda$ where n is an integer the interference is constructive and the scattered radiation will have a high intensity. Use this observation and the result of part (i) to derive a condition that must be obeyed by $\sin\theta$ for constructive interference.

(iii) Determine the angle for which constructive interference is obtained for $\lambda/d = 1/3$. Remember the range of θ is limited to $-\pi \leq \theta \leq \pi$.

(iv) *Destructive interference:* If the distance $BC = (n + \frac{1}{2})\lambda$ where n is an integer the interference is destructive. The intensity is zero for angles θ for which destructive intereference is obtained. Derive a condition on $\sin\theta$ for destructive interference.

(v) Determine the angles for which destructive interference is obtained given that $\lambda/d = 1/3$. Remember $-\pi \leq \theta \leq \pi$.

(b) *Diffraction:* As a greatly simplified model of DNA consider a linear chain of identical



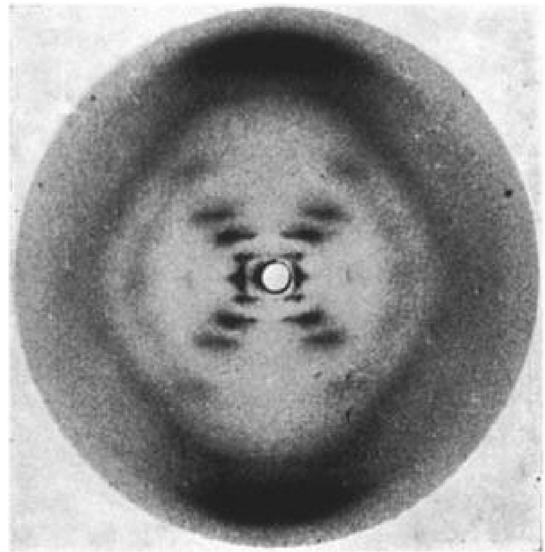


Figure 10: A simplified model of DNA (left). Photo 51 the famous x-ray diffraction image by Rosalind Franklin was instrumental in deciphering the structure of DNA, arguably the most important discovery of 20$^{\text{th}}$ century biology.

atoms separated by a distance $d = 34$ Å (see figure; 1Å $= 1 \times 10^{-10}$ m). X-rays of wavelength $\lambda = 1.54$ Å (corresponding to the K-α line of Cu) are incident normal to the axis of the molecule. Due to constructive interference, x-rays will be strongly scattered by the molecule at specific angles θ.

(i) Derive an expression to determine the angles at which there is strong scattering due to constructive interference.

(ii) Show that for the DNA molecule there is a total of 45 angles at which there is constructive interference.

Our simplified model suggests that the scattered radiation is cylindrically symmetric about the axis of the molecules. This cylindrical symmetry is broken by the double helical character of the DNA molecule. Measurements of the azimuthal dependence of scattering led to the discovery of the double helical structure of DNA in the 1950s.