

# EOM

## Charge to Mass Ratio of the Electron

revised August, 2021

*(You will do two experiments; this one and the Digital Storage Oscilloscope experiment. Sections will switch rooms and experiments half-way through the lab.)*

### Learning Objectives:

During this lab, you will

1. communicate scientific results in writing.
2. estimate the uncertainty in a quantity that is calculated from quantities that are uncertain.
3. compare a measurement of a physical quantity to a measurement of the same quantity from a different experiment.

### A. Introduction

In 1897 J. J. Thomson obtained the first definitive evidence that electricity is carried in gases by particles. Our version of the experiment is similar to one devised several years ago by K. T. Bainbridge of Harvard and R. Winch of Williams as a simple cousin to the original Thomson experiment. You will visually observe the path of an electron beam and the effect of a magnetic field on this path. While you may not yet have encountered magnetic fields in the course lectures, these lab notes contain all that is required to complete the experiment. By measuring the path of the electron using a simple ruler, you will in effect measure the mass of an electron!

The data that you take may appear to you to be prone to large errors, since the electron beam path will not be a perfect cir-

cle beam and it may take irreproducible jumps. However, you should find that with appropriate methods of data analysis you can still determine a reasonable value for the ratio of charge to mass for the electron. Furthermore, statistical analysis of the data acquired by the entire PHYS 122 class should provide an excellent estimate for this quantity.

**You must write a FULL paper worth 60 points for this lab.**

### B. Theory

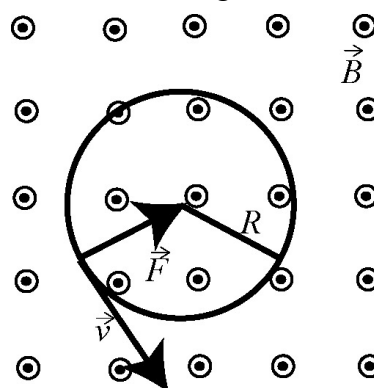
One can produce free electrons in a vacuum tube by ‘boiling’ them off of a hot filament. These electrons can then be accelerated across a potential difference  $V$  using an electric field. The electrons will gain kinetic energy according to the relation

$$\frac{1}{2}mv^2 = eV \quad (1)$$

where  $m$  is the mass of an electron,  $v$  is the speed of the electron, and  $e$  is the magnitude of the charge of an electron.

The electrons can then be focused and steered using either electrostatic and/or magnetic fields. Electron guns based on these principles form the basis of the picture tubes in most television sets and computer monitors.

An electron moving with velocity  $\vec{v}$  in



**Figure 1:** Path of an electron in a perpendicular magnetic field.

a magnetic field  $\vec{B}$  feels a transverse (*Lorentz*) force given by

$$\vec{F} = -e(\vec{v} \times \vec{B}) \quad (2)$$

The magnetic force acts in a direction perpendicular to both the velocity of the electron and the direction of the magnetic field, so that an electron moving with constant speed  $v$  perpendicular to a uniform field will follow a circular trajectory of radius  $R$  as illustrated in Figure 1.

The magnetic force has a magnitude  $evB$  and acts as the centripetal force for this motion. Applying Newton's Second Law, we obtain

$$evB = mv^2/R. \quad (3)$$

Eliminating  $v$  from Equations 1 and 3 gives us an expression for  $e/m$ :

$$\frac{e}{m} = \frac{2V}{(BR)^2} \quad (4)$$

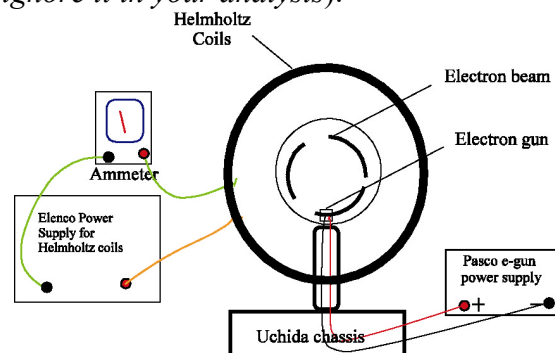
We expect that in this experiment we should measure a value close to the CODATA recommended value (see Halliday, Resnick and Walker page A-3) of

$$\frac{e}{m} = (1.75882017 \pm 0.00000007) \times 10^{11} \frac{\text{C}}{\text{kg}}$$

(Although one would expect *much* lower precision from an experiment lasting less than 1.5 hours.) Note that in this experiment, the measurements of  $R$ ,  $B$  and  $V$  give a value for  $e/m$ , but not for  $e$  or  $m$  separately. However, the value of  $e$  can be determined independently in the Millikan oil drop experiment. This means that in this experiment you will in effect be measuring the mass of the electron.

## C. Apparatus

Figure 2 illustrates the setup. You will use a commercial apparatus based on a vacuum tube that contains an electron gun and a small amount of helium gas, roughly  $10^{-2}$  Torr (*about*  $10^{-5}$  atmospheres). This tube is designed specifically for measuring the ratio of the electron's charge to its mass,  $e/m$ . The tube is mounted on a chassis. The chassis also supports Helmholtz coils for controlling the magnetic field used to make this measurement. The current to operate the Helmholtz coils comes from an Elenco power supply (or a TE Quad Power supply) and is read by a DMM. (*Don't trust the analog current meter on the Elenco for this reading.*) A special Pasco power supply provides the potentials required to operate the electron gun. You may **assume that the meters on the Pasco supply have an accuracy of  $\pm 1\%$  (if this is significantly less than your other uncertainties, you can ignore it in your analysis).**



**Figure 2:** Experimental setup (simplified).

### C.1. Electron Source

A red hot metal wire (or *filament*) will boil off electrons in a process similar to evaporating molecules of  $\text{H}_2\text{O}$  by boiling water. The energy of these electrons can be estimated from thermodynamics using the *Equipartition Theorem*, which says that their energy will be approximately  $k_B T$ , where  $k_B = 8.6 \times 10^{-5}$  eV/K is the Boltzmann constant and  $T$  is the filament temperature in

kelvin. (*An eV is a unit of energy that is usually more convenient than joules when working with atomic energy levels or particle beams. You will encounter it in the lecture part of this course.*) The filament temperature will be between 1000-2000 K, so the electrons leave the filament with a thermal kinetic energy of only a few tenths of an electron volt.

The circular trajectories of such electrons would be too small for convenient laboratory measurement, even in magnetic fields as low as that of the earth (*about 2 gauss*). We can increase the energy, and therefore the velocity, of the electrons by accelerating them through a potential difference of a few hundred volts (Eq. 1). The increased velocity will result in trajectories of larger radii (Eq. 3).

The accelerating apparatus is one element of the electron gun inside the *e/m* tube. Thermal electrons boil off the filament and are accelerated towards a *plate* which is maintained at a positive potential  $V$  relative to the filament. An aperture in the plate allows some of the electrons to escape into the tube. The paths of these electrons can be observed as a faint glow resulting from collisions of some electrons with the He gas atoms in the tube.

### C.2. Helmholtz Coils

The electron gun and vacuum tube are in the middle of a Helmholtz coil that provides a uniform magnetic field perpendicular to the electron beam. Herman Helmholtz was the first to realize and use the fact that the magnetic field is very spatially uniform near the middle of a pair of identical circular coils separated by a distance equal to their own radius  $r$ . The field at the center of the Helmholtz coil is given by the equation

$$B = \frac{8\mu_0 N I_c}{5r\sqrt{5}} \quad (5)$$

where  $\mu_0 = 4\pi \times 10^{-7}$  T·m/A,  $I_c$  is the current in the coil, and  $N$  is the number of turns in each coil. For your apparatus, the number of turns in each coil is  $N = 130$  while their radii are  $r = (0.158 \pm 0.005)$  m.

With this arrangement we can control the radius of the electron trajectory by varying either the accelerating voltage  $V$  or the magnetic field  $B$ , with the latter determined by the current  $I$  in the Helmholtz coil.

### C.3. Equipment Checkout

Examine your equipment to identify its various components. Circuits are pre-wired for you; no wiring changes should be made without the assistance of an instructor.

Identify the functions of the various switches, controls and meters so that you understand how to operate the apparatus. ***Check that the high voltage plate control on the Pasco supply is set to its minimum value before turning on the apparatus.*** This control is the second knob from the left; the leftmost knob is not used in this experiment. **Don't touch** the filament setting, the knob on the right labeled "AC." This controls the heating of the filament and can be used to destroy your \$1,000 vacuum tube. This will also destroy your chance of an amicable relationship with the Lab Director.

The current through the Helmholtz coils is controlled by knobs on both the Elenco (TE Quad Power) power supply and the Helmholtz coils chassis. The Helmholtz coil input is rated at 6.3 V, 3.5 A. You may exceed this slightly but at no time let the voltage from the Elenco (TE Quad Power) supply exceed 12 V or its current exceed 2.0 A, as this may burn out the coils.

**Switch on the Pasco High Voltage Power Supply and Elenco (TE Quad Power) supplies. Set the accelerating voltage  $V$  to about 100 V (using the "500 V Adjust" knob on the High Voltage Power Supply). Set the coil current at about**

**1.00 A, using the controls on the Elenco (TE Quad Power) supply.** The electron beam should be visible as a greenish glow forming a circle within the tube. This is easier to see with the room lights turned off; the instructor will turn them off when the first group is ready for this measurement. Each lab station has a small desk lamp to let you work without the room lights.

#### *C.3.1. Tube/Coil Alignment*

The vertical Helmholtz coils produce a horizontal field much stronger than the horizontal component of the Earth's field, so the latter can be ignored. However, you have to **align the tube with the field of the coils so that the electron trajectories are circles and not spirals. To do this, grasp the tube lightly by its neck and apply only very gentle forces.** (*It should rotate easily – don't force it!*) Stand up while doing this since you can't easily see the spirals if your eyes are level with the tube. This alignment will guarantee that the electrons are emitted in a plane which is perpendicular to the field of the coils.

Note that as the beam completes a circular path, it strikes the back of the electron gun. Because of the potentials on the gun, the beam may scatter from the gun at some odd angle. You can ignore the scattered beam.

## **D. Measurements**

### **D.1 Measurement Techniques**

To eliminate parallax (the error perceived when viewing two points at an angle) when measuring the beam radius, you must **visually align each side of the orbit with its reflection.** Close one eye and look at either the left or the right side of the beam. Move your head from side to side so that you can see the reflection of the beam in the mirror scale move. When its reflection is *obscured* by the beam itself,

**you are looking perpendicular to the mirror.** Taking a reading of the beam in this fashion nearly eliminates parallax errors.

You should **independently measure both the right and the left side of the beam this way and take the radius as half the diameter.** This eliminates artifacts from the beam being off-center with respect to the mirror scale. Note that as the circles get too small or too large, the diameter can be measured more easily if you adjust the position of the mirror.

Each lab partner should independently measure the radii. Don't compare your readings until *after* you've each written them in your lab notebooks, but do use *both* sets of data in your analysis. You can combine your data in a variety of ways; the choice is left to you (*unless your TA chooses to specify a method*). For example, you can average each radius measurement or you can wait until you each find the slopes  $\alpha$  and  $\beta$  described below and then average those. You must discuss how you combined your data and your reasons in your paper.

### **D.2. Varying Coil Current**

With the **beam voltage at 100 V, find the upper and lower limits of coil current for which you can clearly see and measure the circular path of the beam, but don't go higher than 2.0 A** since this could damage the Helmholtz coils. Then **take measurements of the beam diameter** (as per section D.1) **at these two extremes and at three, widely spaced, intermediate values of the coil current.**

### **D.3. Varying Beam Voltage**

Set the beam voltage at 100 V and adjust the Helmholtz coil current so that the beam almost fills the vacuum tube. Measure the diameter of the beam. Keeping the coil current fixed, find the minimum beam voltage at which you can reliably measure the beam diameter. Take

this reading and about five additional widely spaced readings between the minimum and maximum of  $V$ .

## E. Analysis

### E.1. Helmholtz Coil Current

You can use Eq. 5 to eliminate  $B$  from Eq. 4 and obtain

$$\frac{e}{m} = \frac{2V}{(8\mu_0 N / 5r\sqrt{5})^2 I_c^2 R^2} \quad (6)$$

which can be rewritten to show the dependence of the beam radius,  $R$ , on either the Helmholtz coil current,  $I_c$  or the beam voltage,  $V$ .

The dependence on the coil current can be written as

$$\frac{1}{R} = \alpha I_c \quad (7)$$

where the proportionality constant  $\alpha$  is

$$\alpha = \frac{8\mu_0 N}{5r} \sqrt{\frac{e/m}{10(V)}} \quad (7a)$$

If we add an intercept term  $A$  to Eq. 7 to allow for the effects of any residual fields parallel to the  $B$ -field of the coils, we have

$$\frac{1}{R} = \alpha I_c + A \quad (7b)$$

Equation 7b describes a straight line with  $I_c$  as the independent variable,  $1/R$  as the dependent variable, slope  $\alpha$  and intercept  $A$ .

**Plot  $1/R$  versus  $I_c$  for the data of section D.2** where you kept the beam voltage fixed at 100 volts and varied the current through the Helmholtz coil. Remember that the equations given here use

the MKS system of units, so you will need  $R$  in meters, not centimeters. Be sure to **include the uncertainties in  $1/R$** . These should appear as error bars in your plot. (You would normally also include error bars for the current as well, but they are probably so much smaller than the error bars in  $1/R$  that you can ignore them.) Note that these uncertainties may not be the same for all radii. **To obtain these uncertainties from your estimated errors for  $R$ , use the derivative rule**, which tells you that the estimated error in the quantity  $1/R$  [call this error  $\delta_{1/R}$ ] can be found from the estimated error in  $R$  (call this  $\delta_R$ ) using the formula  $\delta_{1/R} = \delta_R/R^2$ . This tells you that, if you have the same error  $\delta_R$  in your measurement of all the radii, the resulting error in  $1/R$  will be larger for smaller radii. **Fit a straight line to your graph and obtain the slope and intercept with uncertainties. Use this slope,  $\alpha$ , and Equation 7a to calculate  $e/m$ .** (The propagation of uncertainty calculation is probably long enough to justify including it in an Appendix of your paper rather than in the text.) **Does a straight line fit the data? Compare your intercept  $A$  to the theoretical value.**

### E.2. Beam Voltage

The dependence on beam voltage can be written as

$$R = \beta V^{1/2} \quad (8)$$

where the proportionality constant  $\beta$  is

$$\beta = \frac{5r}{8\mu_0 N I_c} \sqrt{\frac{10}{e/m}}. \quad (8a)$$

If we now add an intercept term  $B$  to Equation 8, we have

$$R = \beta V^{1/2} + B. \quad (8b)$$

**Plot  $R$  as a function of the square root of the beam voltage for the data where you kept the Helmholtz coil current fixed. Make a linear fit to this data and use the slope  $\beta$  to calculate another value for  $e/m$ , using Equation 8a. Does a straight line fit the data? Compare your intercept  $B$  to the theoretical value.**

Compare your two values of  $e/m$ , the one for fixed beam voltage and the other for fixed coil current. Also compare the estimated errors in each of your values. Your paper should include a single best value for  $e/m$ . You must decide whether you should combine the two values obtained with different techniques to report a single, better value or justify using just one of them. *(Don't forget that lab partners are also supposed to combine their individual readings in some fashion.)*

## **F. Report Hints**

Unlike with the previous lab report, you must write up the full paper this time. That includes the Introduction/Theory, Apparatus diagrams, and Analysis/Error Analysis sections.

### **F.1. Introduction/Theory**

The Introduction/Theory section is perhaps the easiest section to write. You must show your starting equations and how you get to the final ones that you use for your analysis. You need not show all of the math, but you must at least explain your method. This is also the section to show your experimental procedures and Apparatus. Data collected can be put into another section. Make sure you properly reference your graphs, where appropriate.

### **F.2. Analysis**

In the Analysis section, you must show all of your work necessary to analyze your data to reach a conclusion. All of the

equations, derivations, manipulation of equations have to be done here. Make sure you show all of your Error Analysis in this section, or reference a separate page where it gets written out. You should also use the end of this section to put in a small amount of your conclusions based on values you just found.

### **F.3. Additional Information**

**You must submit the entire written report to Canvas.** Make sure that you properly acknowledge all sources that you used to reach your conclusion, including the lab manual. Partners may share graphs and data, but must write their own analysis and conclusions. See Appendix II for more hints and samples.