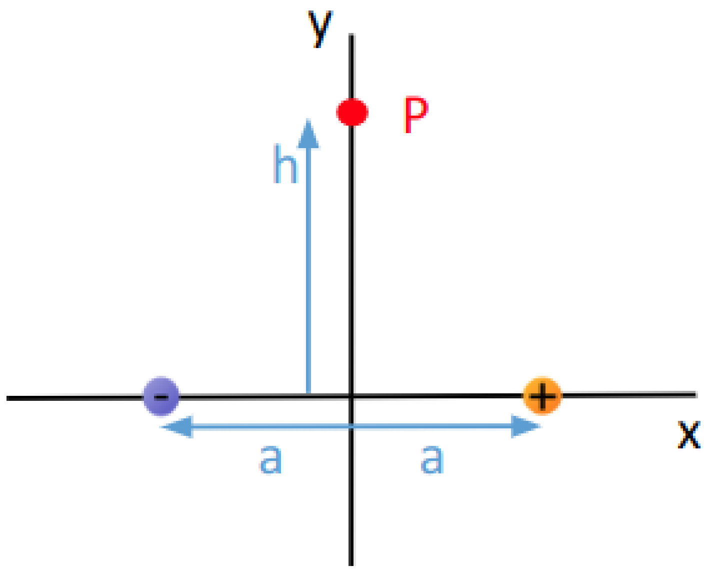


1. Point like charges. Two charges of the same magnitude q but different signs are located on the x-axis at a distance a of the origin of coordinates. The negative one is on the left and the positive one on the right. A point P is defined to be on the y-axis at a distance h from the origin of coordinates.

(a) [5 points] In what direction does the electric field point at the origin (0,0) based on symmetry?

(b) [13 points] Calculate the Electric Field at point P (give magnitude and direction using vector notation).

(d) [7 points] Calculate the magnitude and direction of the force that a negative particle (charge -Q) will feel if placed at point P.

Solution

(a) Left $(-\hat{\imath})$

(b)

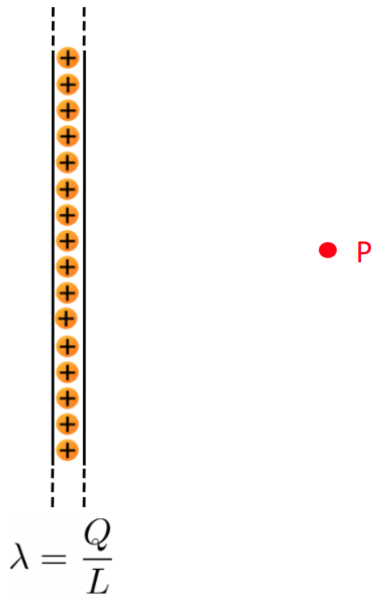
$$\vec{r}_{-} = a\hat{\imath} + h\hat{\jmath} \quad \text{and} \vec{r}_{+} = -a\hat{\imath} + h\hat{\jmath}$$

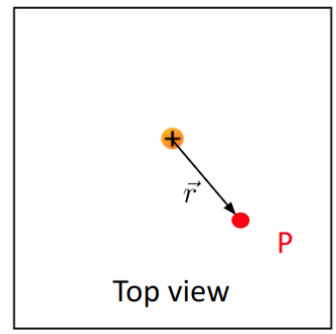
in both cases $r = \sqrt{a^2 + h^2}$. Plug it in Coulombs law:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{(a^2 + h^2)^{3/2}} \left[(-a\hat{\imath} + h\hat{\jmath}) - a\hat{\imath} + h\hat{\jmath}\right] = \frac{1}{4\pi\varepsilon_0} \frac{2aq}{(a^2 + h^2)^{3/2}} (-\hat{\imath})$$

(c)

$$\vec{F} = -Q\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2aqQ}{(a^2 + h^2)^{3/2}} \hat{\imath}$$





2. Infinite line of charge. An infinite line is charged uniformly with a linear charge density λ. For simplicity, assume $\lambda > 0$. You will need to solve this problem using Gauss' Law.

(a) [3 points] Tell us which Gaussian surface are you going to use and explain why that is the smartest choice. Give as many details as possible.

(b) [3 points] In what direction does the electric field point based on symmetry? Use vector notation or precise words and explain why. Hint: Use polar (aka cylindrical) coordinates.

(c) [3 points] What is the charge enclosed by the Gaussian surface?

(d) [6 points] What is the electric flux through the Gaussian surface?

(e) [10 points] Use Gauss's law to determine $\vec{E}$ (magnitude and direction).

Solution

(a) By symmetry, we want the E field and the normal vector to the surface to form an angle that is either 0 or 90º. In this case since the E field is radial, a cylinder is the best choice.
(b) $\vec{E} = E\hat{r}$ since all the vertical components will cancel due to the symmetry of an infinite line. Also, the line has cylindrical symmetry so it can only depend on r.
(c) They need to define a cylinder of a given length L, and then $Q_{enc} = \lambda L$.
(d)

$$\Phi_E = \oint \vec{E} \cdot \hat{n} da = \oint E \cos\theta da = \oint E da = E \oint da = EA = E 2\pi r L$$

(e)

$$E 2\pi r L = \frac{\lambda L}{\varepsilon_0}$$

so $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 \; r}$