Physics 122 Spring 2025

Second Exam

Wednesday, March 5, 2025

You have 45 minutes to complete the exam.

There are two problems each worth 50 points

Guidelines

1. **Write your name clearly on this cover page.**

2. If you have done and understand the homework, you will be fine. While the exam starts, if you need to relax yourself, you can draw something here:

$\exists x \in f(x)$ s.t. $\forall y \in g(y)$,
$x - y \neq 0$ and $f(x) \cdot g(y) \neq 0$

3. Do not open the exam until you are instructed to.

4. Write your answer on the blank pages provided for each problem. If you need more space you may request additional paper and staple it to your exam when you finish.

5. The questions are on the last three pages of this booklet. Please separate those pages from the rest of the booklet. You do not need to submit the questions, only your answers.

6. After you complete your work (or time is up) you should scan it and upload it to Canvas. Look for an Assignment called "Second Exam". You are responsible of the quality of your submission. **A correct Canvas submission will give you 1 extra credit point.**

7. After scanning **submit the hard copy** of your work. The proctors will let you know where the submission bins are located.

8. The exam is closed book. You are only allowed the provided formula sheet.

9. Keep your work clear, neat and well-organized. Illegible answers might be graded as incorrect. Often it helps to put a box around your final answer.

10. Use pictures and words to explain what you are doing. State the central physics concept associated with each problem (e.g. Coulomb's law, Gauss's law, superposition). You will earn partial credit for knowing how to solve the problem even if you are not able to fully implement the solution.

Write your name here: Trevor Swan

Problem 1

a) (i) $(0 < r < r_a) \Rightarrow \boxed{E = 0}$ as the core is a conductor whose charges are distributed on the surface to result in an E of 0 inside

(ii) $(r_a < r < r_b)$ choose cylindrical gaussian surface w/ radius r, length L

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma_a A}{\varepsilon_0} = \frac{\sigma_a (2\pi r_a L)}{\varepsilon_0} = \frac{\frac{\lambda}{\cancel{2\pi r_a}} \cdot (\cancel{2\pi r_a} L)}{\varepsilon_0}$$

$$E_a = E(2\pi r \cancel{L}) = \frac{\lambda \cancel{L}}{\varepsilon_0} \qquad \boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$

(iii) $(r > r_b)$ choose gaussian surface w/ radius r, length L

$$Q_{enc} = \sigma_a A_a + \sigma_b A_b$$

$$= \frac{\lambda}{2\pi r_a}(2\pi r_a L) + \left(-\frac{\lambda}{2\pi r_b}\right)(2\pi r_b L) = L + -L = 0$$

$\therefore \boxed{E = 0}$ by $E_a = \frac{Q_{enc}}{\varepsilon_0} = 0$ (gauss' law)

b) $$\Delta \Phi = -\int_a^b \vec{E} \cdot d\vec{l} = -\int_{r_a}^{r_b} E\, dr\, \hat{r} \cdot \hat{r} = -\int_{r_a}^{r_b} E\, dr$$

$(\vec{E} = E\hat{r},\ d\vec{l} = \hat{r}\, dr)$

$$\Delta \Phi = -\int_{r_a}^{r_b} \frac{\lambda}{2\pi r \varepsilon_0} dr = -\frac{\lambda}{2\pi \varepsilon_0} \int_{r_a}^{r_b} \frac{1}{r} dr = \boxed{-\frac{\lambda}{2\pi \varepsilon_0} \ln\left(\frac{r_b}{r_a}\right)}$$

c) $C = \frac{\lambda}{V}$

to ensure $C > 0$, take $V = |V| = |\Delta\Phi|$

$$\Rightarrow \Delta\Phi = \left| -\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_B}{r_A}\right) \right| = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_B}{r_A}\right) > 0 \quad \text{as } r_B > r_A$$

$$C = \frac{\lambda}{\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_B}{r_A}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_B}{r_A}\right)}$$

$$\boxed{C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_B}{r_A}\right)}}$$

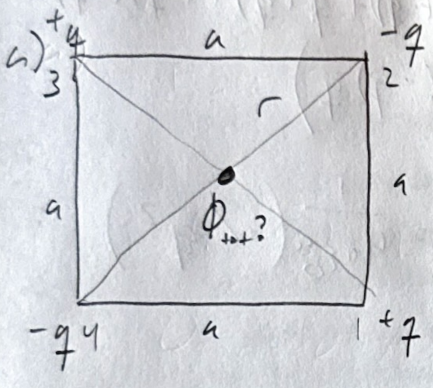
Helpful

$$U_{\substack{pairs\\tot}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{13}} + \dots\right)$$

Derived from superposition property

$$U_{tot} = U_{12} + U_{23} + \dots = U_{pairs,\,tot}$$

a)



$$r = \frac{1}{2}\left(\sqrt{a^2 + a^2}\right)$$

$$= \frac{1}{2}\sqrt{2a^2} = \frac{\sqrt{2}}{2}a$$

$$\boxed{\phi_{tot} = 0}$$

$$\phi_{tot} = \phi_1 + \phi_2 + \phi_3 + \phi_4$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{\frac{\sqrt{2}}{2}a}\left(\cancel{q + -q + q + -q}\right)\right) = 0$$

b) Use (*)

$$U_{tot} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_4}{r_{34}} + \frac{q_4 q_1}{r_{41}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_4}{r_{24}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q(-q)}{a} + \frac{-q(q)}{a} + \frac{q(-q)}{a} + \frac{-q(q)}{a} + \frac{q(q)}{\sqrt{2}a} + \frac{-q(-q)}{\sqrt{2}a}\right)$$

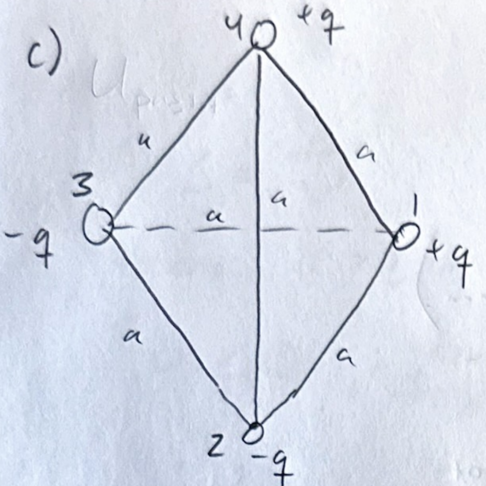
$$= \frac{1}{4\pi\varepsilon_0 a}\left(-q^2 - q^2 - q^2 - q^2 + \frac{q^2}{\sqrt{2}} + \frac{q^2}{\sqrt{2}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0 a}\left(-4q^2 + \frac{2q^2}{\sqrt{2}}\right)$$

$$\boxed{U_{tot} = \frac{q^2}{4\pi\varepsilon_0 a}\left(\frac{2}{\sqrt{2}} - 4\right)}$$

Problem 2

c)



$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{a} + \frac{q_1 q_3}{a} + \frac{q_1 q_4}{a} + \frac{q_2 q_3}{a} + \frac{q_2 q_4}{a} + \frac{q_3 q_4}{a}\right)$$

$$U = \frac{1}{4\pi\varepsilon_0 a}\Big(q(-q) + q(-q) + q(q) + (-q)(-q) + (-q)q + (-q)q\Big)$$

$$\Rightarrow U = \frac{1}{4\pi\varepsilon_0 a}\left(-q^2 - q^2 + \cancel{q^2} + \cancel{q^2} - \cancel{q^2} - \cancel{q^2}\right) = \frac{1}{4\pi\varepsilon_0 a}\left(-2q^2\right).$$

$$\boxed{U = \frac{q^2}{4\pi\varepsilon_0 a}(-2)}$$

d) $U_{square} = \frac{q}{4\pi\varepsilon_0 a}\left(\frac{2}{\sqrt{2}} - 4\right)$ vs. $U_{tet} = \frac{q^2}{4\pi\varepsilon_0 a}(-2)$

$$\frac{2}{\sqrt{2}} - 4 = 2\left(\frac{1}{\sqrt{2}} - 2\right) = \left(\frac{\sqrt{2}}{2} - 2\right)2 = \sqrt{2} - 4 = 1.7 - 4 = -2.3$$

$-2.3 < -2$, so $U_{tet} < U_{square}$. The square has a lower potential energy, and is more stable. This makes sense as the square has two further apart pairs, while the tetrahedron is fully evenly spread.

$\boxed{U_{square}}$, Square is more stable