T

p1) Two Capacitor Circuit

(a) (i) $i_2 = \frac{dQ_2}{dt}$ ✓

(ii) $\mathcal{E} - i_2 R_2 - \frac{Q_2}{C_2} = 0$ ✓

(b) Steady State $\Rightarrow \frac{dQ_2}{dt} = 0$

Now (a-i) $\Rightarrow \boxed{i_2 = 0}$

(a-ii) $\Rightarrow \boxed{Q_2 = C_2 \mathcal{E}}$

(c) (i) $i_1 = \frac{dQ_1}{dt}$ ✓

$i_2 = i_1 + \frac{dQ_2}{dt}$ ✓

(ii) $\mathcal{E} - i_2 R_2 - i_1 R_1 - \frac{Q_1}{C_1} = 0$ ✓

$\mathcal{E} - i_2 R_2 - \frac{Q_2}{C_2} = 0$ ✓

(c) (iii) $t = 0^+$ $\boxed{Q_1 = 0, \quad Q_2 = C_2 \mathcal{E}}$ (since capacitor charge doesn't jump) p2) Two Capacitor Circuit

(c) (iii) contd

use (c-ii) $\mathcal{E} - i_2 R_2 - \frac{q_2}{C_2} = 0$ sub $q_2 = C_2 \mathcal{E}$

$\Rightarrow -i_2 R_2 = 0 \Rightarrow \boxed{i_2 = 0}$

use (c-ii) $\mathcal{E} - i_2 R_2 - i_1 R_1 - \frac{q_1}{C_1} = 0$

sub $q_1 = 0$, $i_2 = 0$

$\Rightarrow \boxed{i_1 = \frac{\mathcal{E}}{R_1}}$

(iv) $t \to \infty$ $\frac{dq_1}{dt} = 0$, $\frac{dq_2}{dt} = 0$ (in steady state)

(c-i) $\Rightarrow \boxed{i_1 = 0, \; i_2 = 0}$

(c-ii) $\Rightarrow \mathcal{E} - i_2 R_2 - \frac{q_2}{C_2} = 0$ sub $i_2 = 0$

$\Rightarrow \boxed{q_2 = C_2 \mathcal{E}}$

(c-ii) $\Rightarrow \mathcal{E} - i_2 R_2 - i_1 R_1 - \frac{q_1}{C_1} = 0$ sub $i_1 = i_2 = 0$

$\Rightarrow \boxed{q_1 = C_1 \mathcal{E}}$

Practice 5 Problem 2

(2) Two loop LR circuit

(a) Initial analysis

(i) $\boxed{i = i_1}$

(ii) $\boxed{\mathcal{E}_1 - iR_1 - L\frac{di}{dt} = 0}$

(iii) Set $\frac{di}{dt} = 0$ (steady state)

$\Rightarrow \boxed{i = \frac{\mathcal{E}_1}{R_1}}$

(b) Kirchoff analysis

(i) $\boxed{i = i_2}$

(ii) $\boxed{\mathcal{E}_2 - iR_2 - L\frac{di}{dt} = 0}$

$t \to 0^+$ behavior

(c) (i) $\boxed{i = \frac{\mathcal{E}_1}{R_1}}$ (current in an inductor cannot change abruptly)

from (a-iii)

(ii) Set $i = \frac{\mathcal{E}_1}{R_1}$ in part (b-ii)

$$\Rightarrow \mathcal{E}_2 - \frac{\mathcal{E}_1 R_2}{R_1} - L\frac{di}{dt} = 0$$

$$\Rightarrow \boxed{\frac{di}{dt} = \left(\mathcal{E}_2 - \frac{\mathcal{E}_1 R_2}{R_1}\right)\frac{1}{L}}$$

Practice 5 Problem 2

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(2) Two loop LR

(d) $t \to \infty$ behavior

Set $\frac{di}{dt} = 0$ in part (b-ii),

$\Rightarrow \varepsilon_2 - i R_2 = 0$

$\Rightarrow \boxed{i = \frac{\varepsilon_2}{R_2}}$

Practice 5 Problem 3

p1) Physics 122 Series LCR Circuit

(a) (i) $\boxed{Z_1 = i\omega L}$ $\boxed{Z_2 = -\frac{i}{\omega C}}$ $\boxed{Z_3 = R}$

(b) $\mathcal{E}_0 - \tilde{I}(Z_1 + Z_2 + Z_3) = 0$ (Kirchoff loop rule)

$$\Rightarrow \tilde{I} = \frac{\mathcal{E}_0}{Z_1 + Z_2 + Z_3} = \boxed{\frac{\mathcal{E}_0}{R + i\left(\omega L - \frac{1}{\omega C}\right)}}$$

(c) $$\tilde{I} = \frac{\mathcal{E}_0}{R + i\left(\omega L - \frac{1}{\omega C}\right)} \frac{\left[R - i\left(\omega L - \frac{1}{\omega C}\right)\right]}{\left[R - i\left(\omega L - \frac{1}{\omega C}\right)\right]}$$

$$= \frac{\mathcal{E}_0}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\left[R - i\left(\omega L - \frac{1}{\omega C}\right)\right]$$

$$\Rightarrow \boxed{\mathrm{Re}\ \tilde{I} = \frac{\mathcal{E}_0 R}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$$

$$\boxed{\mathrm{Im}\ \tilde{I} = \frac{-\mathcal{E}_0\left(\omega L - \frac{1}{\omega C}\right)}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$$

Practice 5 Problem 3

p2) Physics 122 Series LCR Circuit

(d) $$|\tilde{I}| = \frac{|\mathcal{E}_0|}{\left|R + i\left(\omega L - \frac{1}{\omega C}\right)\right|} = \boxed{\frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}}}$$

[we have used the rule

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$ for z_1 & z_2 complex numbers

Also if $z = x + iy$ then $|z| = \sqrt{x^2+y^2}$]

$$\phi = \tan^{-1}\left(\frac{\mathrm{Im}\,\tilde{I}}{\mathrm{Re}\,\tilde{I}}\right)$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left[\frac{\left(\frac{1}{C\omega} - L\omega\right)}{R}\right]}$$

(e) $$|\tilde{I}|^2 = \frac{\mathcal{E}_0^2}{\left[R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2\right]}$$

$|\tilde{I}|^2$ is max when denominator is minimum

R^2 is fixed so denominator is min

when $\left(\frac{1}{C\omega} - L\omega\right)^2$ is minimum

Practice 5 Problem 3

p3) Physics 122 Series LCR Circuit

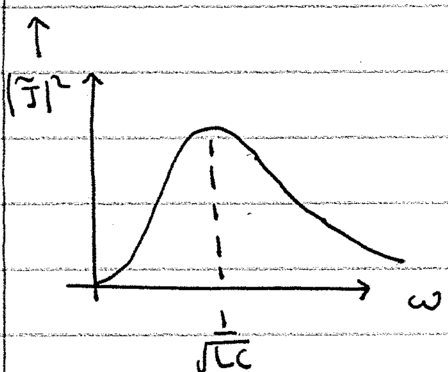
(e) contd

Since $\left(\frac{1}{C\omega} - L\omega\right)^2$ is not negative its minimum value is zero

$$\left(\frac{1}{C\omega} - L\omega\right)^2 = 0 \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \boxed{\omega = \frac{1}{\sqrt{LC}}}$$

Also $|\tilde{I}|^2 \to 0$ as $\omega \to 0$

& $|\tilde{I}|^2 \to 0$ as $\omega \to \infty$



(f)

$$I = \mathrm{Re}\left(\tilde{I} e^{i\omega t}\right)$$

$$= \mathrm{Re}\left(|\tilde{I}| e^{i\phi} e^{i\omega t}\right)$$

$$= \mathrm{Re}\left[|\tilde{I}| e^{i(\omega t + \phi)}\right]$$

$$\Rightarrow \boxed{I = |\tilde{I}| \cos(\omega t + \phi)}$$

using Euler's formula

$e^{i\theta} = \cos\theta + i\sin\theta$

Practice 5 Problem 4

p1) Two loop AC Circuit

(a) $\boxed{Z_1 = i\omega L}$, $\boxed{Z_2 = -\frac{i}{\omega C}}$, $\boxed{Z_3 = R}$

(b) (i) $\mathcal{E}_0 - \tilde{I}_1 Z_1 - \tilde{I}_2 Z_2 = 0$

(ii) $\tilde{I}_2 Z_2 - \tilde{I}_3 Z_3 = 0$

(iii) $\tilde{I}_1 = \tilde{I}_2 + \tilde{I}_3$

(c) (b-ii) $\Rightarrow$ $\tilde{I}_3 = \tilde{I}_2 \frac{Z_2}{Z_3}$

sub in (b-iii) $\Rightarrow$ $\tilde{I}_1 = \tilde{I}_2 \left(1 + \frac{Z_2}{Z_3}\right)$

$\Rightarrow$ $\tilde{I}_2 = \left(\frac{Z_3}{Z_2 + Z_3}\right) \tilde{I}_1$

Sub in (b-i) $\Rightarrow$ $\mathcal{E}_0 - \tilde{I}_1 Z_1 - \frac{Z_2 Z_3}{Z_2 + Z_3} \tilde{I}_1 = 0$

$\Rightarrow$ $\tilde{I}_1 = \frac{\mathcal{E}_0}{\left(Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}\right)}$

Note $\frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{1}{\frac{1}{Z_2} + \frac{1}{Z_3}}$

$= \frac{1}{i\omega C + \frac{1}{R}}$

$\Rightarrow$ $\boxed{\tilde{I}_1 = \frac{\mathcal{E}_0}{i\omega L + \frac{1}{\left(\frac{1}{R} + i\omega C\right)}}}$

Practice 5 Problem 5

p1) Loop in field

(a) $\phi = 0 \quad \text{for} \quad \xi < 0$

$= Ba\xi \quad \text{for} \quad 0 < \xi < b$

$= Bab \quad \text{for} \quad \xi > b$

(b) (i) $\mathcal{E} = -\frac{d\phi}{dt} = 0 \quad \text{for} \quad \xi < 0$

$= Ba\frac{d\xi}{dt} = Bav \quad \text{for} \quad 0 < \xi < b$

$= 0 \quad \text{for} \quad \xi > b$

(ii) Magnetic forces

$\vec{B}$ is fixed so no induced electric fields

(c) (i) $i = \frac{|\mathcal{E}|}{R} = \boxed{\frac{Bav}{R}}$ (Ohm's law)

(ii) Clockwise (Then induced field will point into the page and thereby oppose the increase in external flux through the moving loop)

(iii) $P_{Joule} = i^2 R = \boxed{\frac{B^2a^2v^2}{R}}$

Practice 5 Problem 5

p2) loop in field

(d) (i) Use $\vec{F} = i\vec{l} \times \vec{B}$

For top segment $\vec{l} = a\hat{i}$

$\vec{B} = \hat{k} B$

$i = \frac{Bav}{R}$

$\Rightarrow \boxed{\vec{F} = -\frac{B^2 a^2 v}{R} \hat{j}}$ since $\hat{i} \times \hat{k} = -\hat{j}$

(ii) $P_{ext} = \vec{F}_{ext} \cdot \vec{v}$

velocity of loop

$= \underbrace{\left(\frac{B^2 a^2 v}{R} \hat{j} \right)}_{\vec{F}_{ext}} \cdot (v\hat{j})$

$= \boxed{\frac{B^2 a^2 v^2}{R}}$

Practice 5 Problem 6

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(5) Generator (a) $\phi = (\hat{n}.\vec{B})A$

$\hat{n}.\vec{B} = B\cos(\omega t)$

magnitude of $\vec{B}$

angle between $\vec{B}$ & $\hat{n}$

magnitude of $\hat{n} = 1$

$A = a^2$

Thus $\boxed{\phi = a^2 B \cos(\omega t)}$

(b) $\mathcal{E} = -\dfrac{d\phi}{dt} = \boxed{a^2 B\omega \sin\omega t}$

By Faraday's flux rule.