IND

Magnetic Induction

revised August, 2021

(*You will do two experiments; this one (in Rock 403) and the Magnetic Fields experiment (in Rock 402). Sections will switch rooms and experiments half-way through the lab.*)

Learning Objectives:

During this lab, you will

1. explore the phenomena of magnetic induction.
2. test a physical law experimentally.

A. Introduction

In this lab, you will investigate the fundamental principles of electromagnetic induction. Most things that draw power from a wall outlet rely on induction. Electric generators use induction to convert mechanical energy to power transmitted to these outlets. Motors may rely on induction to convert electrical power back to mechanical motion. Almost every piece of electronics, including computers, televisions, and microwave ovens, has a power supply to convert 110 V_{AC} to other voltages. The first stage in such a power supply is a transformer whose operation is based on the principle of induction.

You must complete and turn in a worksheet worth 30 points for Lab #5B; submit it with your worksheet for Lab #5A. The worksheets are in Appendix XI.

You may notice that this worksheet does not include space for error bars. Many of the measurements that you must make for this lab require the use of *Logger Pro* functions that don't return error estimates. We are therefore relaxing the usual requirement that every number you report must include error estimates. For this lab, you may make "judgment calls" about whether any two numbers you are asked to compare are roughly equal. It's possible to do better either by using another software package or by taking many measurements of each quantity and then deriving error estimates from a statistical analysis, but this is not required because of the limited time available.

B. Theory

B.1. Faraday's Law

Faraday's Law of Induction succinctly describes the principles behind this effect.

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d\left(\vec{B}\cdot\vec{A}\right)}{dt} \qquad (1)$$

$\mathcal{E}$ or EMF is the *electromotive force* caused by a change in the magnetic flux through a circuit. This EMF is basically just a voltage that is created around a circuit, with one important difference from other voltage sources. If you have a closed loop of wire, there can't normally be a voltage drop as a charge travels around the loop back to its

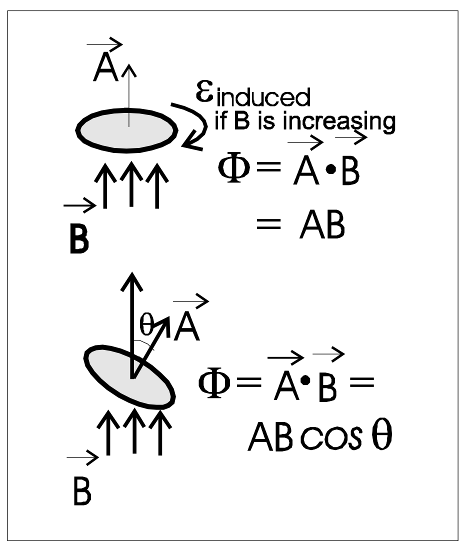

Figure 1: Magnetic flux though a loop of area A.

starting point. However, there **can** be an induced EMF. The EMF will cause a current to flow around this loop, with the magnitude of the current given by the EMF divided by the resistance of the loop.

Φ is the *flux* of magnetic field that flows through the circuit and is given by the dot product of the magnetic field and the area of the circuit. Figure 1 illustrates the concept of flux. Measuring the flux of a magnetic field is very much like measuring the flow of water through a pipe. In this pipe, flux is a measure of the amount of water flowing through the area at the end of the pipe, per unit time. For the case of a current carrying loop of cross-sectional area A in a uniform magnetic field $\vec{B}$ perpendicular to the loop, the flux is just the dot product $\vec{B}\cdot\vec{A}$. (*The vector* $\vec{A}$ *is defined as perpendicular to the surface it describes*.)

If you wanted the magnetic flux in a system to be zero, $\vec{B}$ and $\vec{A}$ would need to be perpendicular. This happens when the angle between $\vec{B}$ and $\vec{A}$ is 90º, so $\vec{B}\cdot\vec{A} = BA\cdot\cos(90°) = 0$. This could correspond to a system where the magnetic field does not "flow through" a current carrying loop, so we say there is no flux.

There are many ways in which the magnetic flux through a circuit can change and you will examine a few of these in this lab. Some examples are:

i. The amount of change in the overlap between the magnetic field and circuit loop. For example, imagine stretching the loop in Fig. 1 to increase its radius or, if the field is confined to some region of space, moving the loop into and out of this region.
ii. The magnetic field and the area of the loop are constant, but the relative angle between them changes so that the changing dot product leads to a change in flux.
iii. The magnitude of the magnetic field changes with time, either because the source of the field moves towards or away from the loop or, if the source is another current distribution, that other current changes with time.

B.2. Lenz's Law

Lenz's Law is useful for understanding the direction of induced EMFs or currents. Lenz's Law states that an induced current is always in such a direction as to oppose the motion or change causing it. This induced current will create a magnetic field, just as a normal current would. For example, if the magnetic field shown in Fig. 1 is upward and increasing with time, the induced current is in the direction shown in that figure; from the *right hand rule,* this will produce a magnetic field that points downward, thus opposing the change in field that created the induced current in the first place.

B.3. Changing the Area

Consider a uniform magnetic field that points in the $+z$ direction and is confined to the region of space between $x = \pm 10$ cm as shown in Figure 2. The field is zero outside this region. This field might, for instance, be produced by a flat magnet indicated as a shaded region in Fig. 2. If a rectangular coil of wire of width W and length L, oriented as shown in the figure, is moved back and forth in the $\pm x$-direction with speed v, you might make the following six predictions, based on Faraday's and Lenz's Laws.

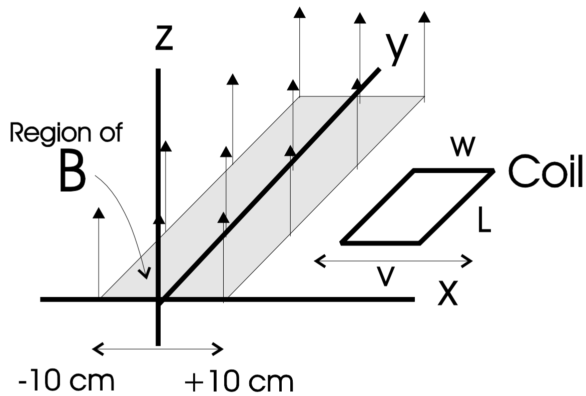

Figure 2: Coil moving through a magnetic field.

i. As long as the coil is kept outside the region of the magnetic field, the magnetic flux is constant (*zero*) and there is no EMF induced in the coil.

ii If the coil is kept **completely** within the region of nonzero field as it is moved, there is no change in flux (*it is a non-zero constant*) and therefore no EMF induced in the coil. (*This obviously requires $W < 20$ cm.*)

iii If the coil is wider than the width of the magnetic field ($W > 20$ *cm*), you could arrange to have the center of the coil within the field region but both sides of the coil outside the field as you move the coil around a bit. Here the amount of magnetic flux remains constant even though the field is passing though different regions of the coil as the coil moves. There will still be no EMF as long as the coil's sides never cross the line where the field begins.

iv As one side of the coil enters or leaves the field region, there will be an EMF. If W^* represents that part of the coil's width within the field region, the magnitude of the EMF will be given as

$$\mathcal{E} = -\frac{d\left(\vec{B}\cdot\vec{A}\right)}{dt} = -\frac{d\left(\vec{B}\cdot\vec{L}W^*\right)}{dt} = -BL\frac{dW^*}{dt} = -BLv \qquad (2)$$

where v is the velocity of the coil.

v. Lenz's law can be used to predict the direction of the induced current. For example, if the coil is positioned as shown in Fig. 2 and is moving to the left, the induced EMF will be clockwise when viewed from above. This is because the flux is increasing as the loop enters the field region, with $\vec{B}$ pointing up. To counteract this, the induced EMF must have a direction that produces a downward-pointing $\vec{B}$ field, a clockwise EMF (*using the right hand rule*).

vi. If we do a time integration of the current induced in the coil, we can determine the total charge that passes any given point. From Ohm=s Law, the induced current equals the induced EMF divided by the resistance of the coil, $I = \mathcal{E}/R$. So we can use Eq. 2 to write

$$\Delta Q = \int I dt = \int \frac{\mathcal{E}}{R} dt = \frac{BL}{R}\int v dt = \frac{BL\Delta x}{R} \qquad (3)$$

The charge passing through the coil depends on the constants B, L, R and the net displacement of the coil Δx. So if you move a coil into or out of a magnetic field, the total charge that flows through the coil is independent of the speed of the coil as well as the exact path taken, only the net displacement matters. Although Eq. 2 predicts that the faster you move the coil between two points in space, the larger the magnitude of the

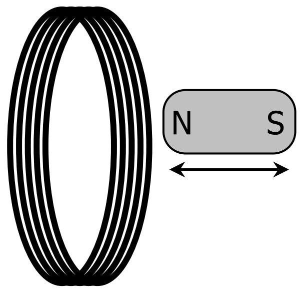

Figure 3: Magnet moving near a coil.

induced EMF, the total charge that moves is same, regardless of speed.

Note that although it is the coil that is described as moving in this example, you would obtain exactly the same results if the coil was fixed and the source of magnetic field was moved instead.

B.4. Changing the Angle

Even if we hold the magnetic field and the area of the coil constant, we can still change the flux and induce an EMF by changing the angle between the two. This is illustrated in Fig. 1. If the coil is spun at a constant angular velocity ω, starting with an initial orientation $\theta_0 = 0$, the flux is given by

$$\begin{aligned}\mathcal{E} &= -\frac{d}{dt}\left(BA\cos\theta\right) = -\frac{d}{dt}\left[BA\cos\left(\omega t\right)\right] \\ &= BA\omega\sin\left(\omega t\right) \qquad (4)\end{aligned}$$

Thus if you spin a coil with constant angular velocity in the region of a magnetic field, you will create a sinusoidal EMF; in other words, you will have built an AC generator. If you increase the rate of rotation, ω, Eq. 4 predicts that the magnitude of the induced EMF should increase since there is an ω ***outside*** the $\sin(\omega t)$ term. If you consider the time integral of the EMF or the net charge that flows through the coil, you will again find that it depends only on the net change in orientation as in Eq. 3. The time integral of Eq. 4 over a 180º flip is just equal to $\pm 2BA$, which is also obviously the net change in flux as the coil is flipped. (*The final flux is -BA and the initial flux is +BA, or vice versa.*) You will use this effect to build a gaussmeter, an instrument to measure magnetic fields.

B.5. Changing the Field

The flux through a loop can also change if the magnetic field causing that flux is changing in time. There are a few ways for this to happen. The field source itself could be moving or, if the field source is another current carrying wire or circuit, the current through that circuit could be changing in time. The easiest way to observe the first effect is to move a permanent magnet towards or away from, into and out of a small coil. Fig. 3 illustrates this situation.

The second possibility can be understood with the help of Figure 4. This is a cross-sectional view of a solenoid (*a long uniform helical coil*), creating a magnetic field while the EMF induced in a second coil, held within the solenoid, is monitored. An ideal solenoid has a uniform field $B = \mu_0 ni$ in its interior. (*n is the number of turns per length of the solenoid and i is the current flowing through each turn.*) The direction of this field is parallel to its axis. The ideal solenoid has zero magnetic field outside its walls.

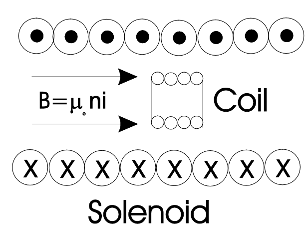

Figure 4: Coil inside a solenoid.

If the current in the solenoid changes as a function of time as $i(t)$, the field within the solenoid also changes as a function of time and the EMF measured across an N turn coil of area A at the center of the solenoid should be

$$\mathcal{E} = -NA(dB/dt) = -\mu_0 nNA(di/dt) \quad (5)$$

Since the magnetic field within the solenoid is spatially uniform, this EMF should remain constant as the coil is moved around inside the solenoid, as long as it does not approach the ends. The EMF should fall to zero if the coil is removed from the solenoid.

If the coil is placed inside the solenoid and current through the solenoid is made to vary with time as $i_0\cos\omega t$, then the EMF across the coil is given by

$$\mathcal{E} = -\mu_0 nNAi_0\omega\sin\omega t \quad (6)$$

Note that the factor of ω outside the sine function means that a higher frequency fed to the solenoid results in a larger signal across the coil.

C. Apparatus

You will take measurements with several different setups for this lab. Most of these setups will employ the LabPro™ electronics and *Logger Pro* software to record the induced EMF as a function of time. You should refer to the photographs posted on the lab web site or on Canvas for help in understanding each setup.

One setup is based on a 4 cm × 15 cm rectangular coil with 400 turns that you can slide across a 9 cm × 15 cm permanent magnet. The magnet is embedded in an oak board and covered with a sheet of Plexiglas. The field just above the surface of this magnet is approximately 300 gauss but it isn't uniform and may have variations of ± 100 gauss.

You will also use a small 1600 turn coil of radius 1.2 cm mounted in a thick gray plastic rod and a long , thin bar magnet (*13 cm long, 5 mm diameter*) that can be passed completely through the center of this coil.

A 25 cm long solenoid of radius 2.5 cm and n = 1708 turns per meter wound on a blue plastic form (or an aluminum cylinder) will be used in the setup of Fig. 4. Three small coils will also be used with this solenoid; one coil with 1600 turns, one with 160 turns and one with 16 turns. All three coils are mounted on rods that let you more easily use the coils as probes of the fields inside the solenoid. A power amplifier connected to the LabPro will function as a signal generator to drive the solenoid. (*The amplifier is necessary because the solenoid will draw more current than the computer/Logger Pro can safely deliver.*) A DMM will be used to read the voltage supplied to the solenoid and the computer will read out the EMF across the small coil.

D. Measurements

To gain some familiarity with induction, everyone will do one simple experiment, described in Section D.1 below.

D.1. Changing Magnetic Field

You should spend no more than 10 minutes on this section! TIME YOURSELF! This part is not on the worksheet, so don't waste time on it.

Locate your 5 mm diameter long cylindrical magnet. **Attach the two ends of the 1600 turn circular coil,** mounted in the gray plastic rod, **to the interface input cable**. (*There is another 1600 turn coil that is part of a set of three coils mounted on* ***black*** *rods used in section D.4.* ***Don't Use This Coil Now****!*)

Open up the file 'Programs on WERTSrv'(P:)\LoggerPro3 \ _E and M Labs \IND (if the default folder hasn't been changed, you should be in the "_E and M Labs" folder when you go to the Open command.)

Start a scan (*click the "Collect" button*) and **start to pass one end of the magnet through the center of the coil.** You should see an induced voltage on the screen. As you pass the magnet all the way through the coil, you should see the following.

1. A positive (or negative) voltage corresponding to the EMF induced by the increasing magnetic flux as the magnet enters the coil.
2. A region of nearly no voltage. This corresponds to the middle region of the magnet passing through the coil. Even though the magnet is moving, the magnetic flux through the coils remains unchanged.
3. A negative (or positive) voltage corresponding to the decreasing flux as the magnet is removed from the coil.

Play with this setup for a couple of minutes. Each time you try something, notice what happens to the magnitude and sign (positive or negative) of the induced EMF. Think about the explanation of each result based on the idea that the EMF is only induced when the magnetic flux is changing. Great care is not necessary, nothing you do in this section will be included in your worksheet but a few things worth trying are:

1. Put the magnet in halfway and pull it back out. The faster you do this the bigger the signal should be. See which lab partner can produce the tallest peak.
2. Switch the magnet end for end and try it again.
3. Flip the coil over and repeat.
4. Move the magnet at different speeds
5. Reverse the direction from which you insert the magnet into the coil.

D.2. Translational Motion

The magnet that you will be using for this section is the one mounted in the oak board. Use the 4 cm × 15 cm flat, rectangular coil.

D.2.1. Voltage Offset

For parts D.2 and D.3 there is one particular problem with the apparatus that you need to worry about. The interface electronics introduce a small voltage offset which must be calibrated out. This offset may be different for each of the two coils that you connect to the interface and should be accounted for before you take readings with a new coil.

Connect the rectangular coil to the interface and set the coil down on the oak board far from the magnet, near the 40 cm mark. Go to EXPERIMENT/ ZERO. This procedure will essentially subtract out the unwanted voltage offset from the next set of experiments.

D.2.2. Measurements

Take a 20 second scan while you perform each of the following procedures. The signals will vary widely, so you may prefer to **set the software to autoscale** (*double-click on the y-axis and change scaling to AUTOSCALE.*) When you use autoscale, it is sometimes helpful to run through a procedure very quickly, aborting it when you complete the maneuvers, and let the software reset the scale before you repeat the process more carefully. If a sweep requires that you make several moves one after the other, be sure to pause for long enough, perhaps 3 seconds, between each move so that you can separately analyze each part of the motion.

The first two measurements below would ideally give you a reading of zero for the induced EMF. However they will in fact give you something other than zero, since there is noise in the electronics and small variations in the local magnetic field. So that you can better appreciate what you will see, start a sweep and randomly move the coil onto and off the rectangular magnet mounted in the oak board, lifting it, sliding it, etc. as you see fit. Note the magnitude of the signals you see.

As discussed in D.2.1, in order to get rid of any offset in voltage, we need to calibrate the coil. To do this, **move the coil 40 cm**

away from the magnet, then go to EXPERIMENT/ ZERO.

i. **Set the rectangular coil about 40 cm away from the magnet.** The coil should be oriented as shown in Fig. 2, with its long side parallel to the long side of the magnet. **Start a scan but leave the coil alone for about 5 seconds so that you get an idea of the amount of noise in the system. Then slide the coil right and left a few cm. Record on the worksheet the maximum magnitude of any induced emf you observe.** (*You may use the software's ANALYSIS / STATISTICS feature to read this value.*) Did you see any significant induced voltage (*significant compared to the EMF when you moved the coil onto or off the magnet*)? Probably not. This is because the coil remains in a region where there is little (*but NOT zero*) magnetic field; hence the magnetic flux never changed much and there was relatively little induced EMF.

ii. **Rotate the coil 90º** so that its long side is parallel to the short side of the magnet and center the coil over the magnet. (*The coil should still be flat on the table.*) **Move the coil back and forth** (*right and left*) **without letting the sides of the coil cross the edges of the magnet. Record the maximum magnitude of any induced EMF.** Did you induce any EMF in the coil? In this case, you may again have seen a small induced EMF because of edge effects in the magnetic field. However, if we ignore these edge effects, the answer should be **no**.

iii. **Place the coil back to roughly 40 cm away from the magnet. Slide the coil towards the magnet until it is centered over the magnet. Pause for about 5 seconds and then slide the coil back to its original position 40 cm away. Record the maximum magnitude of the EMF when moving both towards and from the magnet.** Explain why the sign of the induced EMF changed between the two moves. Integrate both of these movements separately. (*You will need to select an appropriate ROI using the mouse. The integrate function is under ANALYSIS/INTEGRAL.*) If a feature includes both positive and negative-going peaks, integrate over the entire feature. The two integrals should be equal but opposite. Explain why. If your two integrals are very different, check that your voltage offset hasn't changed (*and zero it again if needed*). **Attach a copy of this scan, complete with integrals, to your worksheet. Repeat this at a faster speed and record the maximum magnitude of the EMF and integral of the EMF.**

iv. **Repeat part iii at two different speeds, one faster and one slower than your original motion. Record the maximum magnitude of the induced EMF for all four signals** (*motion towards and away from the magnet at both speeds*). Explain why the different speeds result in different magnitudes for the induced EMF. **Use the software to integrate the portion of each curve corresponding to motion onto the magnet.** Are the integrals for two different speeds the same? Should they be? Explain why or why not.

v. The last item we want to investigate is whether the integrated EMF depends on the path the coil takes. Use the interface to record the following in one scan. **Put the coil back at a distance of 40 cm away from the magnet. Slide it to center it on the magnet. Wait 3**

seconds. Raise the coil straight up roughly 40 cm above the table, move it to the right 40 cm and place it back down on the table at its original position. Integrate over the two portions of the curve corresponding to the two motions, the first move onto the magnet and the second up and back to the starting point. Are they equal (*but opposite*)? Is this behavior expected? Explain why or why not.

D.3. Rotating a Coil

Reconnect the $N = 1600$ turn coil, mounted on the gray bar, to the interface. Hold the coil parallel to and just above the center of the rectangular magnet (*the axis of the coil should be vertical*), **no more than 1 cm away from the plastic sheet. Set up Logger Pro to take data for about 25 seconds. Start the scan, wait about 5 seconds, flip the coil 90º in about 2 seconds, wait again about 5 seconds and flip it back to its original position. Try this again but flip it much faster.** You should observe that the faster flip gives a larger signal and that the sign of the effect reverses when you reverse the direction of rotation. Use the software to **integrate each of the peaks on your plot** - four separate integrals.

Save this plot to the L: drive and attach it to your worksheet. You should be able to show that the magnitude (*ignore the sign*) of the integrated areas are roughly the same, even though the magnitude of the peak is higher for a faster flip.

Start another scan and give the coil four 180º flips. Give it a forward flip followed by a reverse flip and then repeat, using a different speed for each flip and pausing long enough between flips to clearly separate the peaks from each other. **Find the magnitude (*absolute values*) of their time integrals separately for each of the flips and average them together.** From this data and the fact that the value of the integral should be $\pm 2BA$ for a 180º turn for one loop (from B.4), **determine the field strength of the magnet** (*don't forget to divide by the number of turns in the coil*). You may assume that the area of the coil is 0.77 cm^2.

D.4. Coupled Circuits

This circuit is wired for you, but do double check that the Power Amplifier is connected to Channel 4 of LabPro and that the Instrumentation Amplifier is connected to Channel 1. You will **connect in turn each of three coils** (*the ones mounted on the rods*) to Logger Pro via the Instrumentation Amplifier. You should find the large solenoid wound on a metal form already connected to the Power Amplifier which is in turn connected to Channel 4 of LabPro which, in connection with the computer, will function as a signal generator. That way, the large coil will have current flowing through it (the coil is the "primary") and the three small coils will induce (secondary) currents which will be amplified and displayed in Logger Pro.

First, **turn on the power to the power amplifier.** You should leave the power on throughout the experiment! To **make sure the instruments are configured**, load LoggerPro from the desktop (there is no specific file for this on the *P:* drive), go to *EXPERIMENT / SET UP SENSORS / SHOW ALL INTERFACES*. A dialog showing available channels on the LabPro interface should open. On *CH 1,* select the associated *DROPDOWN BOX / CHOOSE SENSOR / INSTRUMENTATION AMPLIFIER,* if it is not already selected. An image representing the Instrumentation Amplifier should appear within the dropdown box. If this does not happen, please ask your TA for assistance. On *CH 4*, select the *DROPDOWN / ANALOG OUT*. Under *WAVEFORM*, select *SINE*. Once selected, enter a frequency of 20 Hz and

4 V amplitude. Select OK when you are done. A popup may appear asking for a Raw Voltage – if it does, select yes. These settings have given best results in the past, so even if you change the waveform, make sure to keep the same frequency and amplitude. Now go back to the *CHANNEL 4 DROPDOWN / CHOOSE SENSOR / VOLTAGE / RAW VOLTAGE (+/- 10V)*. Now **close the LabPro dialog**.

Go back to *EXPERIMENT / DATA COLLECTION*. Make sure the collection tab is open. The Mode should be set to **Time Based**, the **Length should be 0.5 seconds**, and the sampling rate should be **1000 samples per second.** Select Done. You should now be ready to perform this experiment.

Connect the 1600 turn coil to the amplifier. Start with coil on the tip of the probe centered along the length of the solenoid. Hit Collect and wait for the data to appear. Go to *ANALYZE / STATISTICS*, and **measure the amplitude of the wave**: the amplitude should simply be (max-min)/2. <u>**Record the amplitude of the induced EMF**</u>**. Collect data at several points within the solenoid** to prove to yourself that the induced EMF is constant throughout the solenoid as long as you avoid its ends. As you approach the ends of the solenoid with the coil, the amplitude of the induced EMF should fall roughly by a factor of 2 (*because the solenoid is now only "infinitely long" in one direction*) and should continue towards 0 as you probe the space outside the solenoid, including the space near the center of the solenoid but just *outside* its walls.

You will now observe what happens to the signal from the 1600 turn coil if you **change the waveform**. To change it, **go back to the LabPro dialog, choose Analog Output for channel 4, and adjust the settings**. You should **first change to a triangle waveform and then to a square waveform.** <u>**Save the plot for the current induced due to a square wave input to the L: drive.**</u> **Return the channel 4 setting to a sine wave output. Connect the 160 turn coil in place of the 1600 turn coil and position the tip of it in the center of the solenoid. Measure the amplitude of the induced EMF for a sine wave. Repeat this with the 16 turn coil**. <u>By what factor does the EMF change as you make each substitution (*1600 turns to 160 turns and then 160 turns to 16 turns*)? Compare to the theoretical prediction for the dependence on the number of turns.</u>

You have just examined some of the operating principles of transformers used in electronic equipment that connects to 110 V_{rms} wall outlets, where this voltage must be changed for proper operation of the electronics.

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