Physics 122 Spring 24

Practice Problem Set 5

Practice problems are not for credit and will not be graded.

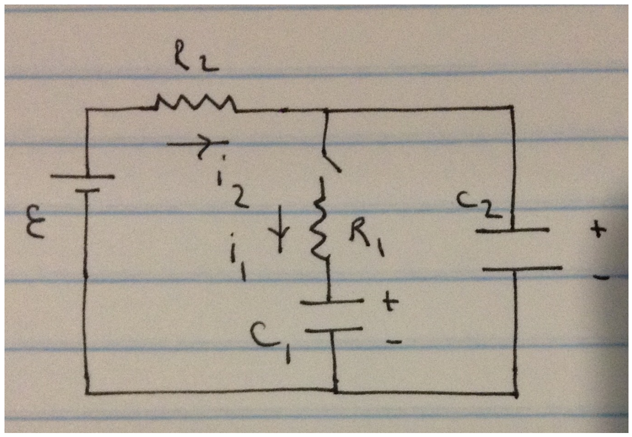

Figure 1: A circuit with two capacitors.

1. Two capacitor circuit. Consider the circuit shown in the figure. Initially the switch is open and the charge on the both capacitors is zero.

(a) *Kirchoff analysis.* (i) What is the relation between i_2 and Q_2 when the switch is open?

(ii) Apply Kirchoff's loop rule to the circuit to obtain a second relation involving i_2 and Q_2. (Your answer may also involve $\mathcal{E}, R_2$ and C_2).

(b) *Steady state analysis.* Assuming that a steady state has been attained determine (i) i_2 and (ii) Q_2.

(c) *Closed switch analysis.* Now assume that the switch is closed suddenly at time $t = 0$. Assume that before the switch is closed the charges on the capacitors are $Q_1 = 0$ and $Q_2 = C_2\mathcal{E}$.

(i) What is the relation between i_1 and dQ_1/dt? Between i_1, i_2 and dQ_2/dt?

(ii) *Kirchoff analysis.* Apply Kirchoff's loop rule to the loop that includes the battery, R_2, R_1 and C_1. Repeat for the loop that includes the battery, R_2 and C_2.

(iii) $t = 0^+$ *analysis.* What are Q_1 and Q_2 immediately after the switch is closed? Use these results and your results of part (ii) to determine i_1 and i_2 immediately after the switch is closed as well.

(iv) $t \to \infty$ *analysis.* What are i_1 and i_2 a long time after the switch is closed? Use this result and your result of part (ii) to also determine Q_1 and Q_2 a long time after the switch is closed.

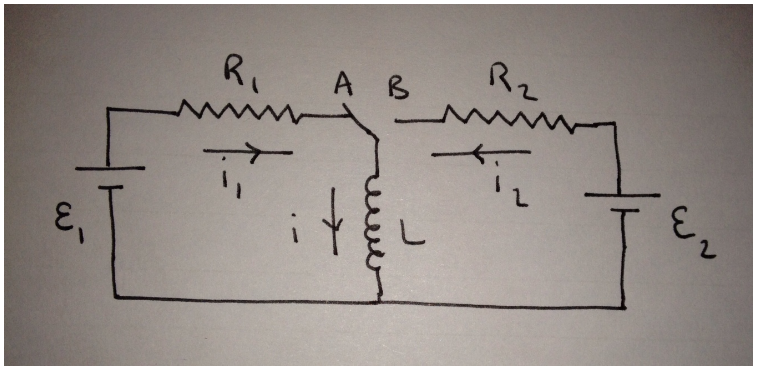

Figure 2: An LR circuit with a switch.

2. Two loop L-R circuit. Initially the switch is in position A and the circuit has reached a steady state wherein $di/dt = 0$.

(a) [6 points] *Initial state analysis.* (i) What is the relationship between i and i_1?

(ii) Apply Kirchoff's loop rule to the circuit.

(iii) Determine i. Give your answer in terms of $\mathcal{E}_1, L$ and R_1.

At $t = 0$ the switch is flipped from position A to position B.

(b) [4 points] *Kirchoff analysis.* (i) What is the relationship between i and i_2 after the switch is flipped.

(ii) Apply Kirchoff's loop rule to the circuit.

(c) [6 points] $t = 0^+$ *behavior.* At $t = 0^+$ determine (i) i and (ii) di/dt. In both cases give your answer in terms of $\mathcal{E}_1, \mathcal{E}_2, R_1, R_2$ and L.

(d) [4 points] $t \to \infty$ *behavior.* After a long time a steady state is attained wherein $di/dt = 0$. In this steady state determine i. Give your answer in terms of $\mathcal{E}_2, R_2$ and L.

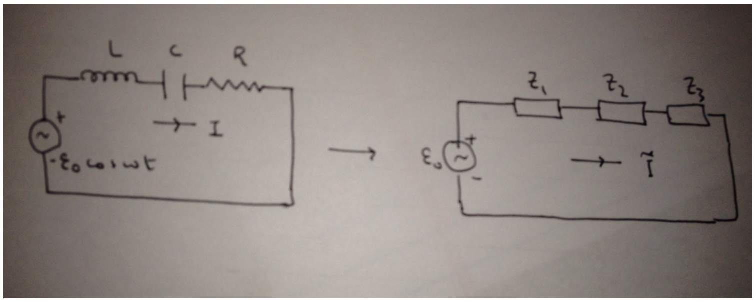

Figure 3: A series LCR circuit.

3. A series LCR circuit. We wish to analyze the circuit shown to the left. To this end we analyze the complex circuit shown to the right.

(a) What is (i) Z_1, (ii) Z_2 and (iii) Z_3 in terms of L, C, R and ω?

(b) Apply Kirchoff's loop rule to the circuit to determine $\tilde{I}$ the complex current amplitude. Give your answer in terms of $\mathcal{E}_0, L, C, R$ and ω.

(c) What are the real and imaginary parts of $\tilde{I}$?

(d) One can write $\tilde{I} = |\tilde{I}|e^{i\phi}$ where $|\tilde{I}|$ is the magnitude of $\tilde{I}$ and ϕ is its phase. Determine $|\tilde{I}|$ and ϕ in terms of $\mathcal{E}_0, L, C, R$ and ω.

(e) Sketch $|\tilde{I}|^2$ as a function of ω. For what value of ω is $|\tilde{I}|^2$ maximum? Give your answer in terms of L, C and R.

(f) The actual current in the circuit is given by $I = \text{Re}\,(\tilde{I}e^{-i\omega t})$. Determine I. It is sufficient to give your answer in terms of $|\tilde{I}|$, ϕ, ω and t.

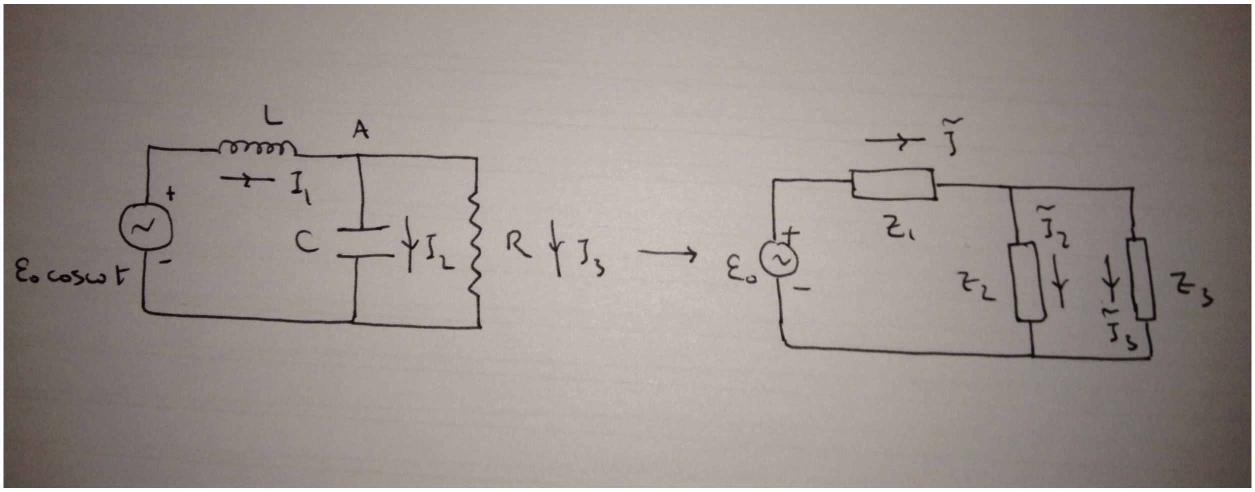

Figure 4: A two loop circuit with an a.c. source.

4. A two loop AC circuit. We wish to analyze the circuit shown to the left. To this end analyze the complex circuit shown to the right.

(a) *Impedances.* What is (i) Z_1? (ii) Z_2? (iii) Z_3? Give your answer in terms of L, C, R and ω.

(b) *Kirchoff analysis.* (i) Apply Kirchoff's loop rule to the circuit involving $\mathcal{E}_0, Z_1$ and Z_2.

(ii) Apply Kirchoff's loop rule to the circuit involving Z_1 and Z_2.

(iii) Apply Kirchoff's junction rule to the junction marked A in the figure.

(c) Determine $\tilde{I}_1$. Give your answer in terms of R, L, C and ω.

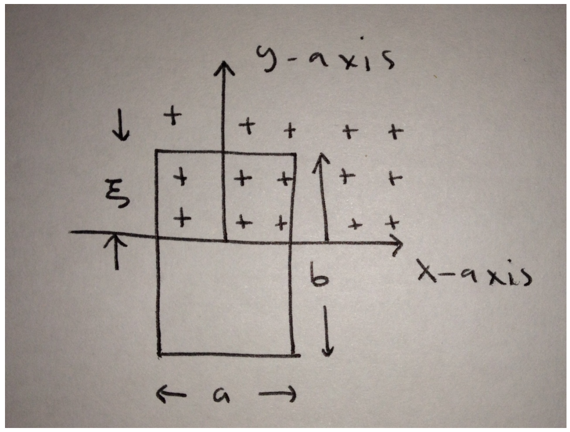

Figure 5: A rectangular loop is pulled into a magnetic field. The plus signs indicate that the magnetic field is out of the page.

5. A rectangular loop of dimensions $a \times b$ is pulled into a region with a uniform magnetic field from a region with zero magnetic field as shown in the figure. Thus $\mathbf{B} = B\hat{\mathbf{k}}$ for $y \geq 0$ and $\mathbf{B} = 0$ for $y < 0$. Assume the loop moves at a uniform velocity $\mathbf{v} = v\hat{\mathbf{j}}$. Let ξ represent the y-coordinate of the top of the loop.

(a) *Flux.* What is the flux ϕ through the loop? Give your answer in terms of B, ξ, a and b. Be careful to distinguish the three cases $\xi < 0, 0 < \xi < b$ and $\xi > b$.

(b) *E.m.f.* (i) What is the emf around the loop? Give your answer in terms of B, ξ, a, b and v.

(ii) Is the emf due to an induced electric field or due to magnetic forces? Explain (one or two sentences at most).

(c) *Current and power.* Assume the loop has resistance R.

(i) How much current flows through the loop?

(ii) In what direction does the current flow? Justify your answer using Lenz's law.

(iii) How much power is dissipated as Joule heat?

Give your answer to parts (i) and (iii) in terms of B, v, a, b and R.

(d) *Force and power.* (i) What is the net magnetic force on the loop?

(ii) An external agent must apply a force $\mathbf{F}_{\text{ext}}$ equal and opposite to the magnetic force in order to keep the loop moving at a uniform velocity $\mathbf{v} = v\hat{\mathbf{j}}$.What is the rate at which this external force does work on the loop?

Give your answer in terms of B, v, a, b, R and appropriate unit vectors.

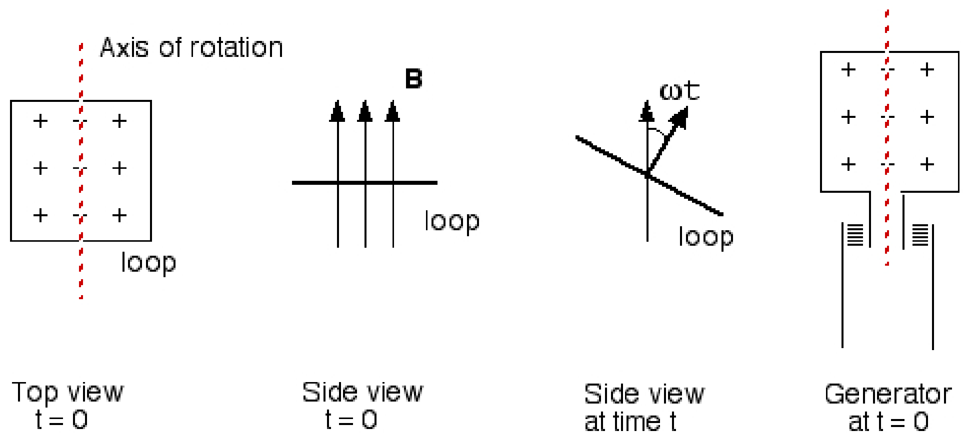

Figure 6: A square loop in a vertical magnetic field. Initially the loop is horizontal as shown in the top view (left) and side view (second from left). The loop rotates about the axis shown in the top view (left) at a frequency ω. As a result at time t the normal to the surface of the loop makes an angle ωt with the magnetic field (third diagram from left). An open loop rotating in a magnetic field and connected to an external circuit with a split ring and brush connectors can serve as an electric generator.

6. Generator. A square loop of side a rotates in a magnetic field as shown in fig 6.

(a) What is the magnetic flux through the loop at time t?

(b) What is the e.m.f. around the loop according to Faraday's flux rule?

By connecting an open loop to an external circuit via a split ring and brush connectors one can deliver the e.m.f. calculated in part (b) to an external circuit as shown in figure 6.