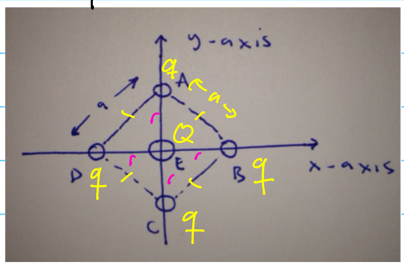
PHYS122 Homework 2 - Due 02/02/25 Trevor Swan (tcs94)

1. Equilibrium of a Square



a) All vertices of the square are equidistant and equal in magnitude, so they do not exert any force on the central charge. The charges opposite to each other cancel each other out.

$$F_{Net} = 0 \text{ N}$$

b) For vertex A, the x-components of B and D cancel out, so there should be no $\hat{i}$ contribution in our answer. $F_{tot} = F_A + F_B + F_C + F_D + F_E$

$\frac{a}{\sqrt{2}} = \frac{\sqrt{2}a}{2} \to 2\cdot\frac{a}{\sqrt{2}} = \sqrt{2}a$

$r, a : 2r^2 = a^2 \to r = \frac{a}{\sqrt{2}}$

$$\vec{E}_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^3}\left(-\frac{a}{\sqrt{2}}\hat{i} + \frac{a}{\sqrt{2}}\hat{j}\right)$$
$$\vec{E}_C = \frac{1}{4\pi\varepsilon_0}\frac{q}{(2\cdot\frac{a}{\sqrt{2}})^3}\left(0\hat{i} + 2\left(\frac{a}{\sqrt{2}}\right)\hat{j}\right)$$
$$\vec{E}_D = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^3}\left(\frac{a}{\sqrt{2}}\hat{i} + \frac{a}{\sqrt{2}}\hat{j}\right)$$
$$\vec{E}_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(\frac{a}{\sqrt{2}})^3}\left(0\hat{i} + \frac{a}{\sqrt{2}}\hat{j}\right)$$

$\vec{E}_{B_x} + \vec{E}_{D_x} = 0$, $\vec{E}_{B_y} = \vec{E}_{D_y}$

$$E_{tot} = \cancel{2}\left(\frac{1}{\cancel{4}\,2\pi\varepsilon_0}\cdot\frac{q}{a^{\cancel{3}\,2}}\cdot\frac{\cancel{a}}{\sqrt{2}}\hat{j}\right) + \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(\sqrt{2}a)^{\cancel{3}\,2}}\cdot\cancel{\sqrt{2}a}\,\hat{j}\right) + \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(\frac{a}{\sqrt{2}})^{\cancel{3}\,2}}\cdot\cancel{\frac{a}{\sqrt{2}}}\hat{j}\right)$$
$$= \frac{q}{2\sqrt{2}\pi\varepsilon_0 a^2}\hat{j} + \frac{q}{8\pi\varepsilon_0 a^2}\hat{j} + \frac{Q}{2\pi\varepsilon_0 a^2}\hat{j}$$

$F_{Net_A} = q\cdot E_{tot}$

$$F_{Net_A} = q\left(\frac{q}{2\sqrt{2}\pi\varepsilon_0 a^2}\hat{j} + \frac{q}{8\pi\varepsilon_0 a^2}\hat{j} + \frac{Q}{2\pi\varepsilon_0 a^2}\hat{j}\right)$$ Pos y

c) $$F_{Net_B} = q\left(\frac{q}{2\sqrt{2}\pi\varepsilon_0 a^2}\hat{i} + \frac{q}{8\pi\varepsilon_0 a^2}\hat{i} + \frac{Q}{2\pi\varepsilon_0 a^2}\hat{i}\right)$$ Pos x

$$F_{Net_C} = -q\left(\frac{q}{2\sqrt{2}\pi\varepsilon_0 a^2}\hat{j} + \frac{q}{8\pi\varepsilon_0 a^2}\hat{j} + \frac{Q}{2\pi\varepsilon_0 a^2}\hat{j}\right)$$ Neg y

$$F_{Net,} = -q\left(\frac{q}{2\sqrt{2}\pi\varepsilon_0 a^2}\hat{i} + \frac{q}{8\pi\varepsilon_0 a^2}\hat{i} + \frac{Q}{2\pi\varepsilon_0 a^2}\hat{i}\right)$$ Neg x

d) The charge of Q doesn't affect its force, but should cancel out each individual charge on the vertices. $F_Q = F_{Net}$, find Q

$$F_Q = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(\frac{a}{\sqrt{2}})^2} = \frac{2qQk}{a^2}$$

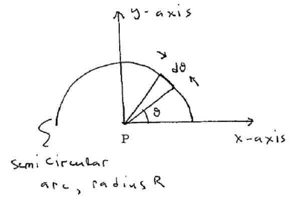
Adjacent $$F_a = \frac{kq^2}{(a)^2} = \frac{kq^2}{a^2} \Rightarrow F_{net,adjacent} = \sqrt{2F_a^2} = \sqrt{2}F_a = \sqrt{2}\frac{kq^2}{a^2}$$

Across $$F_D = \frac{kq^2}{(2\frac{a}{\sqrt{2}})^2} = \frac{kq^2}{2a^2}$$

$$F_{Net} = \frac{kq^2}{2a^2} + \frac{\sqrt{2}kq^2}{a^2} = \left(\sqrt{2}+\frac{1}{2}\right)\left(\frac{kq^2}{a^2}\right)$$

$$\Rightarrow \frac{2\cancel{q}Q\cancel{k}}{\cancel{a^2}} = \underbrace{\left(\sqrt{2}+\frac{1}{2}\right)}_{\left(\frac{2\sqrt{2}}{2}+\frac{1}{2}\right)}\left(\frac{\cancel{k}q^{\cancel{2}}}{\cancel{a^2}}\right) \Rightarrow Q = -\left(\frac{2\sqrt{2}+1}{4}\right)q$$

2. Field of a Semi-circular arc



a) $\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{\pi R} \Rightarrow \boxed{Q = \pi \lambda R}$

given/known

b) Because the charge is positive and uniform around P, the x-components cancel out, and the vector sum is just in the positive y-direction. $\vec{E}$ is in the $\hat{j}$ direction

c) Arc Analysis

(i) Find infinitesimal segment charge dq

ds is arc length

$$dq = \lambda\, ds \rightarrow ds = R\,d\theta \longrightarrow dq = \lambda R\, d\theta \quad \text{or} \quad dq = \frac{Q}{\pi} d\theta$$

(ii) $\cos\theta = \frac{a}{R}$ $\quad \sin\theta = \frac{b}{R}$ $\quad \text{dist} = \sqrt{R^2\cos^2\theta + R^2\sin^2\theta} = R$

(iii) $\vec{r} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$

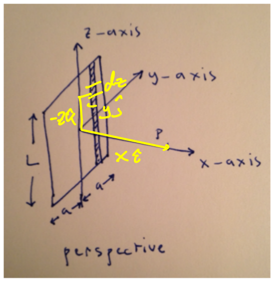
(iv) $dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{R^{\cancel{3}\,2}}(\cancel{R}\cos\theta\,\hat{i} + \cancel{R}\sin\theta\,\hat{j}) = \frac{1}{4\pi\varepsilon_0} \frac{\lambda \cancel{R}\, d\theta}{R^{\cancel{2}}}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

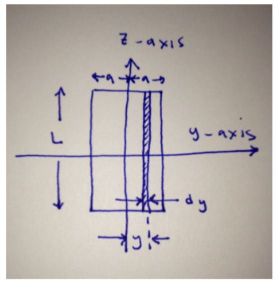
$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, d\theta}{R}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

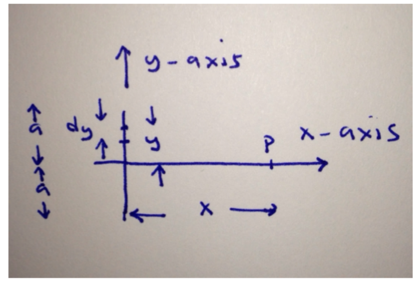
d) $E_x = \frac{\lambda \hat{i}}{4\pi\varepsilon_0 R}\int_0^\pi \cos\theta\, d\theta = 0 \rightarrow \vec{E}_x = 0$

e) $E_y = \frac{\lambda \hat{j}}{4\pi\varepsilon_0 R}\int_0^\pi \sin\theta\, d\theta = \frac{2\lambda\hat{j}}{4\pi\varepsilon_0 R} \rightarrow \vec{E}_y = \frac{\lambda\hat{j}}{2\pi\varepsilon_0 R}$

3. Field of a charged Strip







a) I expect the field at P to point in the positive x-direction ($+\hat{i}$) because the y-components are symmetric about the z axis and will cancel out. Similarly, since L is infinitely long centered at the y-axis, each +z-level has a −z-level to cancel it out, so the total field will just be the sum of $+\hat{i}$ components.

b) $\left.\begin{array}{l} \sigma = \dfrac{Q}{L^2} \\ \lambda = \dfrac{Q}{2} \end{array}\right\}$ For small squares $dq = \sigma\, dy\, dz \Rightarrow \lambda = \dfrac{dq}{dz} = \dfrac{\sigma\, dy\, dz}{dz} = \sigma\, dy$ $\quad \lambda = \sigma\, dy$

$(dq = \lambda\, dy)$

c) dz components will cancel out, so I'll ignore their contribution

$$\left.\begin{array}{l} \vec{r} = x\hat{i} - y\hat{j} \\ r = \sqrt{x^2+y^2} \end{array}\right\} d\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\, dy}{\sqrt{x^2+y^2}^{\,3}}\cdot(x\hat{i} - y\hat{j})$$

d)
$$\vec{E}_x = \frac{x\lambda\hat{i}}{4\pi\varepsilon_0}\int_{-a}^{a}\frac{dy}{\sqrt{x^2+y^2}^{\,3}} = \frac{\cancel{x}\lambda\hat{i}}{\cancel{4}_{2}\pi\varepsilon_0}\left(\frac{\cancel{2}}{\cancel{x}}\tan^{-1}\left(\frac{a}{x}\right)\right)$$
$$\vec{E}_y = \frac{-\lambda\hat{j}}{4\pi\varepsilon_0}\int_{-a}^{a}\frac{y\, dy}{\sqrt{x^2+y^2}^{\,3}} = \frac{-\lambda\hat{j}}{4\pi\varepsilon_0}(0) = 0$$
$$\vec{E}_{tot} = \frac{\lambda}{\pi\varepsilon_0}\tan^{-1}\left(\frac{a}{x}\right)\hat{i}$$

e) If $a \gg x$, then $a/x \to \infty$ and the electric field should be at its maximum in the $\hat{i}$ direction.

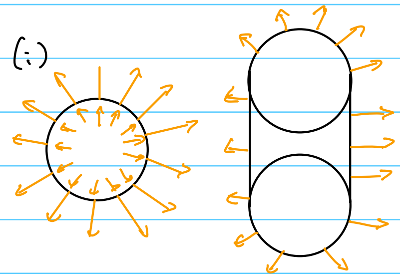
$$\lim_{\frac{a}{x}\to\infty}\frac{\lambda}{\pi\varepsilon_0}\tan^{-1}\left(\frac{a}{x}\right)\hat{i} = \frac{\lambda}{\pi\varepsilon_0}\cdot\frac{\pi}{2}\hat{i} = \frac{\lambda}{2\varepsilon_0}\hat{i}$$ Supported!

f) If $a \ll x$, then $\frac{a}{x}\to 0$ and the field should be at its minimum. This is supported by the fact that there are less strips to contribute to the field. Its direction will still be in the positive x ($\hat{i}$) direction.

$$\lim_{\frac{a}{x}\to 0}\frac{\lambda}{\pi\varepsilon_0}\tan^{-1}\left(\frac{a}{x}\right)\hat{i} = \lim_{\frac{a}{x}\to 0}\frac{\lambda}{\pi\varepsilon_0}\,0 = 0\hat{i}$$ Supported kind of; it will approach 0!

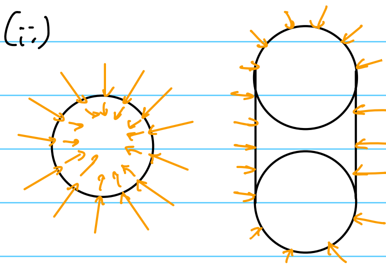
5. Electric Field Lines

a) (i)



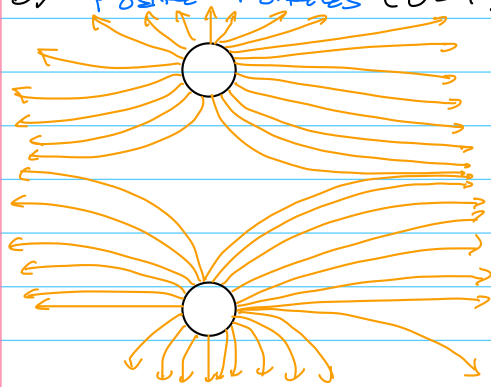
Top View Front View

(ii)

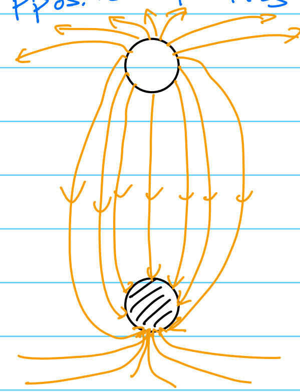


Top View Front View

b) Positive Particles (O=+)

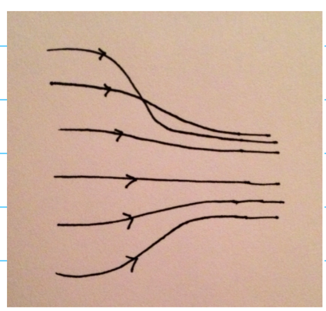


Opposite Particles (O=+, ⊘=-)



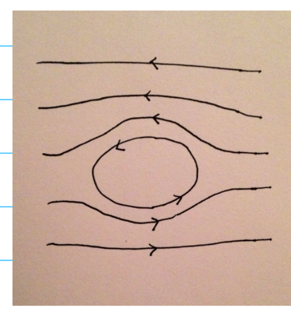
c)

a)



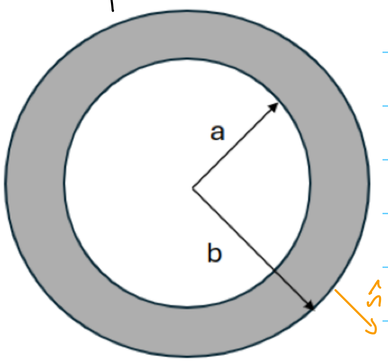
Lines can never cross; this is because crossing lines indicate that like charges play nice when they should repel and not directly interact

b)



Starts of charges should be at a positive terminal and there should be no closed loops. Closed loops fundamentally violate energy conservation laws.

6. Spherical Shell



$\frac{\text{charge}}{\text{unit volume}} = \rho \left\{ \frac{C}{\text{units}^3} \right\}$

a) Volume of sphere $= \frac{4}{3}\pi r^3$

Outer $= \frac{4}{3}\pi b^3$, Inner $= \frac{4}{3}\pi a^3$

Total Volume $(V) = \frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 = \frac{4}{3}\pi (b^3 - a^3)$

$\rho = \frac{Q_{tot}}{V} \Rightarrow Q_{tot} = \frac{4}{3}\pi \rho (b^3 - a^3)$

b) The sphere is uniformly charged and inherently symmetric, so the electric field will either point radially inward or outward depending on the sign of Q_{tot}

c) $\Phi_E = \oint \hat{n} \cdot \vec{E}\, da = \int |\vec{E}| \cos\theta\, da = |\vec{E}| a$ $\quad \theta = 0 \quad$ $a = 4\pi r^2$

$|\vec{E}| 4\pi r^2 = \frac{Q_{tot}}{\varepsilon_0} \Rightarrow |\vec{E}| = \frac{Q_{tot}}{4\pi r^2 \varepsilon_0}$

d) $\Phi_E = |\vec{E}| 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} \rightarrow |\vec{E}| = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0}$

$Q_{enc} = \rho V_{enc} = \rho \frac{4}{3}\pi r^3 \rightarrow |\vec{E}| = \frac{\rho \cancel{\frac{4}{3}}\cancel{\pi} r^{\cancel{3}}}{\cancel{4}\cancel{\pi r^2} \varepsilon_0} = \frac{\rho r}{3\varepsilon_0}$ $\quad |\vec{E}| = \frac{\rho r}{3\varepsilon_0}$

e) $\Phi_E = |\vec{E}| 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} \rightarrow |\vec{E}| = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0}$

$Q_{enc} = \rho V_{enc} = \rho \frac{4}{3}\pi (r^3 - a^3) \rightarrow |\vec{E}| = \frac{\cancel{\frac{4}{3}}\cancel{\pi}\rho (r^3 - a^3)}{\cancel{4}\cancel{\pi} r^2 \varepsilon_0}$ $\quad |\vec{E}| = \frac{\rho (r^3 - a^3)}{3 r^2 \varepsilon_0}$