AMATH 482 Homework 1

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This report analyzes acoustic measurements from a submarine in the Puget Sound using Fourier analysis to extract meaningful insights from noisy data. First, we compute the averaged Fourier transform to determine the submarine's dominant frequency, verifying our findings through visualizations. Next, we design and implement a filter to isolate this center frequency, reducing noise and improving the clarity of the submarine's trajectory. We then visualize the denoised measurements and assess the effectiveness of the filtering process. Finally, we extract and plot the submarine's x, y coordinates over a 24-hour period, providing valuable insights for future sub-tracking efforts.

1 Introduction/Overview

In this project, we analyze submarine acoustic measurements from the Puget Sound to extract meaningful insights using Fourier analysis. We begin by averaging the Fourier Transform of the recorded signals to determine the dominant frequency, or center frequency, generated by the submarine. Using this information, we design and apply a Gaussian filter to remove noise and obtain a clearer representation of the submarine's movement. Finally, we extract and plot the submarine's x,y coordinates over a 24-hour period, providing valuable data for aircraft-based tracking systems.

2 Theoretical Background

Understanding and analyzing acoustic signals from a submarine requires advanced signal processing techniques, particularly Fourier analysis. The Fourier Transform is a fundamental mathematical tool that allows us to analyze signals in the frequency domain, helping us identify dominant frequency components and filter out unwanted noise.

2.1 The Fourier Transform and Its Importance in Signal Analysis

The Fourier Transform (FT) is a mathematical operation that converts a time-domain signal into its frequency components. Given a signal $s(t)$, its Fourier Transform is defined as:

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-2\pi i f t} \, dt.$$

This transformation decomposes a complex signal into a sum of sinusoidal components, each associated with a specific frequency. This is crucial in signal processing because many real-world signals, including acoustic waves from a submarine, contain a mixture of frequencies. By analyzing the Fourier spectrum, we can identify dominant frequencies that characterize the source of the signal. In this project, we leverage the Fourier Transform to extract the center frequency of the submarine's acoustic signal. The presence of noise and environmental disturbances can obscure this frequency, so we take the averaged Fourier Transform over time to enhance accuracy.

2.2 Averaging the Fourier Transform to Determine the Center Frequency

Noise in real-world signals often fluctuates unpredictably. If we were to analyze a single Fourier Transform snapshot, random noise might distort the true dominant frequency. To mitigate this, we compute multiple Fourier Transforms over time and average them, which suppresses high-variance noise and amplifies consistent signal components.

2.3 Constructing a Gaussian Filter for Denoising

Once we identify the submarine's center frequency, we design a Gaussian filter to isolate this frequency and remove surrounding noise. A Gaussian function is a smooth, bell-shaped curve defined as:

$$G(k_x, k_y, k_z) = e^{-\tau((k_x-k_x^*)^2+(k_y-k_y^*)^2+(k_z-k_z^*)^2)},$$

where $\tau = \frac{1}{2\sigma^2}$ and σ^2 denotes the variance. This Gaussian filter is applied to the Fourier-transformed data by multiplication, effectively suppressing frequencies that deviate significantly from the central frequency. The filtered signal is then reconstructed using the Inverse Fourier Transform, yielding a denoised version of the original data:

$$s_{filtered}(t) = \mathcal{F}^{-1}(S(f) \cdot G(k_x, k_y, k_z))$$

By applying this denoising process, we obtain a more robust submarine trajectory, minimizing the impact of random disturbances in the raw acoustic data. This improved path estimation allows for more effective tracking and predictive analysis, which is crucial for deploying surveillance aircraft or other tracking technologies.

3 Algorithm Implementation and Development

To analyze and visualize the submarine's acoustic signal, we utilize the NumPy package for data storage and mathematical operations, including Fourier analysis, and Plotly for three-dimensional visualization. This section details the implementation of the Fourier Transform, the Gaussian filtering process, and the 3D visualization of the submarine's trajectory.

3.1 Using NumPy for Data Storage and Manipulation

NumPy is a fundamental library for numerical computing in Python. It provides efficient data structures such as ndarrays, which allow for fast element-wise operations and matrix manipulations. In this project, we store the four-dimensional submarine acoustic data (with three spatial dimensions and one time dimension) in a NumPy array of shape (64, 64, 64, 49). This format enables efficient indexing and mathematical operations across time and space.
Key NumPy functions used in this project include:

- np.mean() – To compute the averaged Fourier Transform over time.
- np.fft.fftn() – To perform the Fourier Transform along selected axes.
- np.fft.fftshift() – To center the frequency components

3.2 Visualization Using Plotly

To analyze the submarine's movement, we extract and plot its denoised 3D path using Plotly. The plotly.graph_objects module allows for dynamic and interactive 3D visualizations.
Key Plotly functions used:

- Scatter3d – Plots individual 3D data points.
- Isosurface – Generates smooth 3D representations of scalar fields.
- Figure() – Combines multiple visual elements.

4 Computational Results

In the first task, we start by taking the Fourier transform of the signal over the three spatial dimensions, and then averaging the frequencies over the 49 different time measurements to reduce some of the random noise in the data. Using np.unravel_index and np.argmax from the NumPy package, we then found the indices in each of the x, y, z-axes where the maximum average frequency is achieved. Then, we extracted these largest frequencies from a meshgrid of frequency values using these indices. From this, we found the dominant frequencies to be

$$(k_x^*, k_y^*, k_z^*) = (5.3407, 2.1991, -6.9115).$$

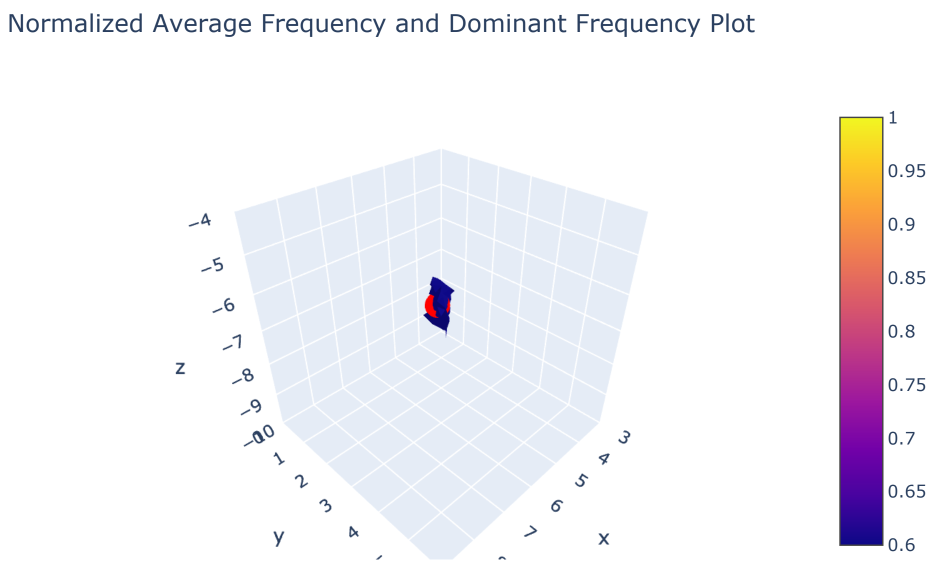


Figure 1: Average Frequencies and Maximum Average Frequency

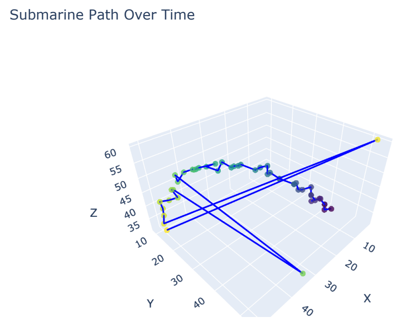
Figure 1 shows these average frequencies graphed in the frequency domain, with the large red dot representing the central frequency shown above. As we can see, (k_x^*, k_y^*, k_z^*) is indeed the central frequency we were attempting to extract.

In the second task, we take the dominant frequency and constructed a Gaussian filter of the form

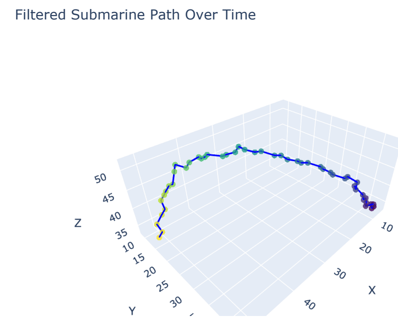
$$G(k_x, k_y, k_z) = e^{-\tau((k_x-k_x^*)^2+(k_y-k_y^*)^2+(k_z-k_z^*)^2)},$$

in order to reduce signal noise around this central frequency. After some experimentation with different σ^2 values, a σ^2 value of 100 was found worked well to denoise the measurements without losing too much information from the data.

After applying the Gaussian filter to the noisy Fourier-transformed signal, we take the inverse Fourier transform to extract a less noisy path of the submarine. The two figures below show the different "paths", where each point represents the largest acoustic measurement at each point in time.



(a) Unfiltered Submarine Path

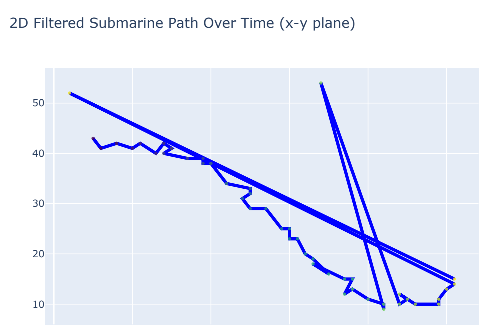


(b) Submarine Path after Gaussian filtering

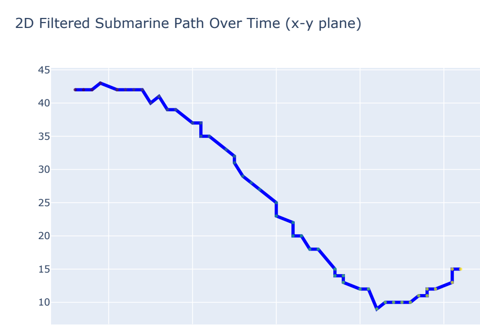
Figure 2: Unfiltered and Filtered Submarine Paths

Figure 2 shows the effectiveness of the Gaussian filtering in determining a more robust path for the submarine. In Figure 2a, we can see that the noiseness in the data creates large deviations in the observed path of the sub, while Figure 2b shows how filtering can eliminate this noise to more accurately visualize the sub's path.

In the third task, we take the 3D path and project it down to the x, y-coordinates, in order to get a two-dimensional path that could be tracked by an aircraft or some other device. Using a similar plotting method as in task 2 above, we can plot each x, y-coordinate of the denoised submarine path at each point in time.



(a) Unfiltered 2D Submarine Path



(b) 2D Submarine Path after Gaussian filtering

Figure 3: Unfiltered and Filtered 2D Submarine Paths

Again, we plot the unfiltered and filtered signals side-by-side for comparison. As we can see, the filtering is important and effective in determining a more accurate path of the submarine.

5 Summary and Conclusions

In this report, we analyzed acoustic measurements from a submarine operating in the Puget Sound by applying Fourier analysis and signal processing techniques to extract valuable insights. By averaging the

Fourier transform, we successfully identified the dominant frequency, or center frequency, of the submarine's acoustic signal. This frequency was crucial in constructing a Gaussian filter, which was used to denoise the data, effectively isolating the submarine's path from noise.

After applying the Gaussian filter, we were able to track the submarine's path over time with greater accuracy. Visualizations of the denoised data showed a clearer and more robust path, providing better insight into the submarine's movement through the sound. The path, specifically the x and y coordinates, was plotted over a 24-hour period to demonstrate the submarine's trajectory and improve the reliability of future tracking efforts.

The key findings from this analysis include:

- The dominant frequency of the submarine's acoustic signal was found to be $(k_x^*, k_y^*, k_z^*) = (5.3407, 2.1991, -6.9115)$.
- The application of the Gaussian filter improved the clarity of the submarine's path by removing unwanted noise.
- The denoised x, y coordinates over the 24-hour period were successfully plotted, which can be utilized for future tracking purposes. This work contributes to the development of more accurate tracking techniques for submarines, with potential applications in military and civilian maritime operations.

5.1 Acknowledgements

I would like to thank Rohin Gilman for providing me with valuable insights and tips when constructing my code for this project, as well as helping me understand some of the theoretical concepts behind what we were doing.

5.2 References

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