Solar Radiation Prediction from Other Climatic Variables

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ABSTRACT

A procedure to estimate daily solar radiation from other climatic variables which are easy to measure and are readily available has been developed. Two equations were developed using regressing techniques and measured climatic variables for a ten year period at Columbia, Missouri. When date and temperature were used, an R-square value of 76 percent was obtained. Adding the daily minimum relative humidity increased the R-square value to 85 percent.

INTRODUCTION

Daily solar radiation is often required to predict plant water use. Unfortunately only a few major weather stations record daily solar radiation. Many weather stations, airports, and radio and TV stations, however, record maximum and minimum air temperature and relative humidity data. In this paper an attempt was made to predict daily solar radiation as a function of day of year, daily maximum and minimum air temperature, and minimum relative humidity.

Previous attempts to predict daily solar radiation have centered around estimating duration of sunshine or the amount of cloud cover. Penman (1948) used clear day solar radiation and possible sunshine hours to estimate potential evaporation from an open water surface. Clover and McCulluch (1958) give the following equation for estimating daily solar radiation reaching the earth's surface:

$$R_s = I_o\ (0.29 \cos \phi + 0.52\ n/N) \quad [1]$$

where

- R_s = is the solar radiation reaching the earth's surface
- I_o = the total solar radiation at the top of the atmosphere
- ϕ = the latitude of the location
- n/N = the ratio of time of actual bright sunshine (n) over possible sunshine hours or day length (N).

The difficulty of estimating actual bright sunshine hours, however, limits the usefulness of this equation. Predicting solar radiation from other climatic variables which are easy to measure would avoid this problem.

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CONCEPTS AND DEVELOPMENT

The clear day solar radiation reaching the ground for a given day and location can be computed with equation [1] by setting n/N equal to 1.0. For a partly cloudy day the solar radiation will be less than the clear day solar radiation. It was assumed that for any day of a year, the solar radiation reaching the ground is a function of clear day solar radiation, daily maximum and minimum air temperature, and minimum relative humidity. Air temperature is strongly dependent on the incoming solar radiation. On a cloudy day, less solar radiation will reach the ground than on a clear day. The air temperature thus will increase more on a clear day than on a cloudy day. This suggests that the range of daily maximum and minimum air temperature should be an indicator of daily solar radiation. Usually minimum relative humidity occurs in the afternoon for any given day. For a cloudy day the vapor pressure of the air will be higher than for a clear day. The minimum relative humidity will be high for that day; thus, minimum relative humidity should be another indicator of solar radiation.

Maximum and minimum air temperature is easy to measure and is often available. The measurement of minimum relative humidity or dew point temperature is more difficult and may not be available for some locations. Thus, two different regression models were developed with ten years of measured data at Columbia, MO.

CLEAR DAY SOLAR RADIATION

Clear day solar radiation can be predicted from the solar radiation at the top of the atmosphere for any given location. At the top of the atmosphere, incoming solar radiation can be defined as (Smithsonian Meterological Tables 1951),

$$\frac{dI_o}{dt} = \frac{J_o}{r^2} \cos z \quad [2]$$

where

- I_o = the total solar radiation at the top of the atmosphere
- r = the radius vector of the earth
- t = the time in hours from sunrise to sunset
- J_o = the solar constant 1.94 ℓy/min.
- z = the sun's zenith distance given by

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h \quad [3]$$

where

- ϕ = the latitude of the location
- δ = the declination angle
- h = the sun's hour angle.

Substituting equation [3] into equation [2] and convert-

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ing the solar constant from ℓy/min to ℓy/hr gives

$$\frac{dI_o}{dt} = \frac{60\ J_o}{r^2}(\sin\phi\sin\delta + \cos\phi\cos\delta\cos h) \quad [4]$$

The sun's declination angle is dependent on the position of the earth, so it is dependent on the days of the year and can be defined as

$$\delta = 0.41\cos(2\pi(I - 172)/365) \quad [5]$$

where I is the Julian days of the year (Tscheschke and Gilley, 1978). The sun's hour angle is also given by Tscheschke and Gilley as

$$h = 0.2618\,(t - \frac{DL}{2}) \quad [6]$$

where t is the time and DL is the day length in hours. Differentiating equation [6] with respect to t and rearranging the terms and substituting into equation [4] for t gives an equation for total solar radiation at the top of the atmosphere

$$dI_o = \frac{60\ J_o}{0.2618\ r^2}\ (\sin\phi\sin\delta + \cos\phi\cos\delta\cos h)\ dh \quad [7]$$

For sunrise, the sun's hour angle h is $-\frac{\pi DL}{24}$

and for sunset h is $\frac{\pi DL}{24}$.

Assuming the maximum solar radiation occurs at solar noon for a day, we can integrate equation [7] between sunrise and sunset to get total daily solar radiation at the top of the atmosphere.

$$I_o = \frac{120\ J_o}{0.2618\ r^2}(\frac{\pi DL}{24}\sin\phi\sin\delta + \cos\phi\cos\delta\sin(\frac{\pi DL}{24})) \quad [8]$$

The work of Clover and McCulluch (1958) can now be used to modify equation [8] to predict clear day solar radiation I_s at the earth's surface:

$$I_s = \frac{120\ J_o}{0.2618\ r^2}(\frac{\pi DL}{24}\ \sin\phi\sin\delta + \cos\phi\cos\delta\sin\ (\frac{\pi DL}{24})). \\ (0.29\cos\phi + 0.52) \quad [9]$$

where n/N in equation [1] was given the value of 1.0.

Seasonally, δ and r cannot be considered constant, thus equation [9] has only limited value in computing clear day solar radiation. To keep calculations simple, clear day solar radiation was assumed to follow a sine wave pattern of the form:

$$I_s = A + B\sin(\frac{2\pi(I + 10.5)}{365} - \frac{\pi}{2}) \quad [10]$$

where A and B are constants for a given location and I is day of year. The 10.5 in equation [10] is required to obtain the peak value on June 21. The form of this equation was suggested by Anderson in 1975. Anderson evaluated A and B by analyzing local data; however, equation [9] can be used to obtain general solutions for A and B. During the year, the maximum and minimum daily solar radiation occurs on June 21 and December 21, respectively, for the northern hemisphere. For June 21 r = 1.0163 and δ = 23.45 deg and for December 21 r = 0.98372 and δ = −23.45 deg (Smithsonian Meteorological Tables, 1951). The substitution of values of r and δ into equation [9] for maximum clear day solar radiation on June 21 and for minimum clear day solar radiation on December 21 gives

$$I_{smax} = 860.933\ (0.05209\ DL\sin\phi + \\ 0.9174\cos\phi\sin(\frac{\pi DL}{24}))\ (0.29\cos\phi + 0.52) \quad [11]$$

$$I_{smin} = 918.904\ (-0.05209\ DL\sin\phi + \\ 0.9174\cos\phi\sin(\frac{\pi DL}{24}))\ (0.29\cos\phi + 0.52) \quad [12]$$

For June 21, DL will equal the longest day ℓd and on December 21, DL will equal 24 − ℓd. Substituting maximum and minimum clear day solar radiation predicted by equation [10] and longest day of the year in equations [11] and [12], respectively, A and B can be determined in terms of the variables of latitude and longest day of the year.

$$A = [\sin\phi\ (46.355\ \ell d - 574.3885) + \\ 816.41\cos\phi\sin(\frac{\pi\ \ell d}{24})]\ (0.29\cos\phi + 0.52) \quad [13]$$

$$B = [\sin\phi\ (574.3885 - 1.509\ \ell d) - \\ 26.59\cos\phi\sin(\frac{\pi\ \ell d}{24})]\ (0.29\cos\phi + 0.52) \quad [14]$$

The longest day of the year can be obtained from Fig. 1 for any given latitude or from the equation (Tables of Sunrise, Sunset and Twilight, 1946),

$$\ell d = 0.267\sin^{-1}(\sqrt{0.5 + \frac{0.007895}{\cos\phi} + 0.2168875\tan\phi}) \quad [15]$$

Fig. 2 gives the magnitude of A and B for different latitudes for the northern hemisphere. A similar development can be made for the southern hemisphere.

A comparison of measured clear day solar radiations and predictions using equation [10] for Columbia, MO is given in Fig. 3.

ESTIMATION OF SOLAR RADIATION FOR COLUMBIA MISSOURI

The latitude at Columbia, MO, is about 39 deg. A for-

FIG. 1 Longest day of the year for different latitudes computed with equation [15].

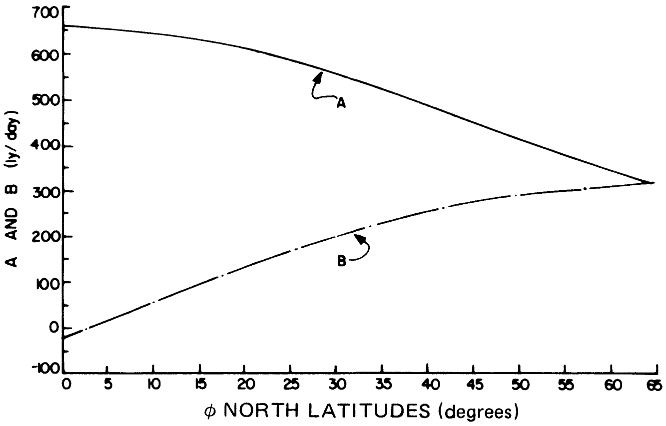

FIG. 2 Intercept (A) and amplitude (B) for the clear day solar radiation curve for northern latitudes.

mula to predict the potential solar radiation for each Julian day at Columbia can be obtained either using equations [10], [13], and [14], or Fig. 2. The result is

$$I_S = 495 + 245 \sin \left[\frac{2\pi\,(I + 10.5)}{365} - \frac{\pi}{2}\right] \qquad [16]$$

Measured solar radiation values were obtained from a U.S. Weather Service magnetic tape which had hourly readings for each day. Weather data for 1953 through 1964 was used. Daily measured solar radiation, R_s, was obtained by adding up the hourly values each day and converting the units to Langly's per day.

Dry bulb minimum and maximum were simply the lowest and highest hourly values for each day. Dry bulb range, DBR, was the maximum minus the minimum temperature (°C).

Minimum relative humidity, RHMIN, was calculated using Brooker's (1967) formulas, which require maximum dry bulb and the associated dew point temperatures. In the event that several time periods had the same maximum dry bulb temperature, the minimum dew point temperature was used.

Regression models will now be discussed. The standard error of the estimate of regression, s (the square root of the mean square regression) is used as the measure of precision. The percent of variation accounted for by the fitted regression line, R^2, is also reported. Terms in models are only reported when it could be shown that the population coefficient for the term was significantly different from zero ($\alpha = 0.05$).

Using only daily potential solar radiation computed from equation [16], one obtains

$$\hat{R}_S = -51.6 + 0.85\, I_S \qquad [17]$$

with s = 131.83 and $R^2 = 55.4$ percent. By using the daily range in dry bulb temperature, an easily available value, the following model was derived:

$$\hat{R}_S = 49.03 + 0.10\, I_S - 7.26\, DBR + 0.06\, (I_S\, DBR) \qquad [18]$$

with s = 96.45 and $R^2 = 76.2$ percent.

Next, models were examined which also include the minimum relative humidity. All of the possible interaction terms were statistically significant giving

$$\begin{aligned}\hat{R}_S = &-96.34 + 1.39\, I_S + 8.39\, DBR + 2.52\, RHMIN \\ &- 0.024\, (I_S\, DBR) - 0.020\, (I_S\, RHMIN) \\ &- 0.30\, (DBR\, RHMIN) + 0.0013\, (I_S\, DBR\, RHMIN) \qquad [19]\end{aligned}$$

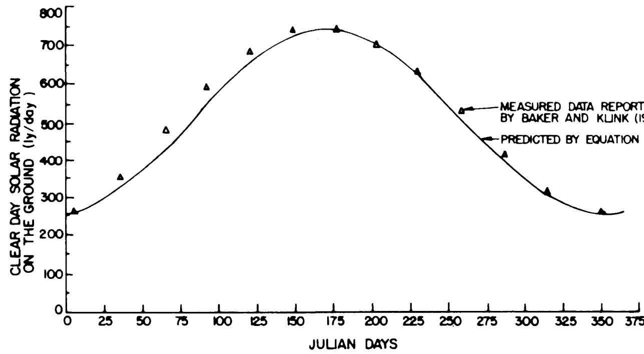

FIG. 3 Comparison of measured clear day solar radiation with predicted values from Columbia, MO.

with s = 77.62 and $R^2 = 84.6$ percent.

A slight increase in precision was gained by determining that the measured solar radiation does not continue to increase for dry bulb ranges greater than 12 °C or for minimum relative humidities less than 37 percent. A model which takes this into account is as follows (including significant interaction terms)

$$\begin{aligned}\hat{R}_S = &\ 135.13 + 0.38\, I_S - 10.07\, DBR_{12} - 1.07\, RHMIN_{37} \\ &+ 0.078\, (I_S\, DBR_{12}) - 0.0052\, (I_S\, RHMIN_{37}) \\ &- 0.00032\, (I_S\, DBR_{12}\, RHMIN_{37}) \qquad [20]\end{aligned}$$

with s = 76.05 and $R^2 = 85.2$ percent. DBR_{12} is set equal to 12 if DBR is greater than 12. $RHMIN_{37}$ is set equal to 37 if RHMIN is less than 37.

It was thought that if predictions for solar radiation are only needed for June through September, models could be based only on that time period. However, only a slight gain in precision was obtained.

Including the interaction terms, the following formula was obtained

$$\begin{aligned}\hat{R}_S = &-316.70 + 1.89\, I_S + 30.74\, DBR + 7.80\, RHMIN \\ &- 0.071\, (I_S\, DBR) - 0.031\, (I_S\, RHMIN) \\ &- 0.91\, (DBR\, RHMIN) + 0.0024\, (I_S\, DBR\, RHMIN) \qquad [21]\end{aligned}$$

with s = 77.12 and $R^2 = 74.3$ percent.

Again, a slight increase in precision was gained by determining that the measured solar radiation does not continue to increase for dry bulb ranges greater than 12 or for minimum relative humidities less than 47. A model which takes this into account is as follows (including significant interaction terms):

$$\begin{aligned}\hat{R}_S = &-551.41 + 1.25\, I_S + 66.03\, DBR_{12} + 7.54\, RHMIN_{47} \\ &- 0.016\, (I_S\, RHMIN_{47}) - 1.10\, (DBR_{12}\, RHMIN_{47}) \\ &+ 0.00068\, (I_S\, DBR_{12}\, RHMIN_{47}) \qquad [22]\end{aligned}$$

with s = 75.89 and $R^2 = 75.1$ percent. DBR_{12} is set equal to 12 if DBR is greater than 12 °C. $RHMIN_{47}$ is set equal to 47 if RHMIN is less than 47 percent.

SUMMARY AND CONCLUSIONS

To summarize, a need exists for a procedure to estimate solar radiation data when measured data is not available. A procedure was presented to estimate the clear day solar radiation for northern and southern latitudes. Data specific to Columbia, MO, was then used to develop equations to predict daily solar radiation with inputs of clear day solar radiation, dry bulb range and minimum relative humidity.

When only the minimum and maximum temperature are available, Equation 18 is the most useful equation explaining 76 percent of the variation in the data. If the minimum relative humidity can also be determined, then equations [19] and [20] are the most useful explaining approximately 85 percent of the variations in the data. These equations are all thought to be a reasonable substitute for missing radiation data and should thus be useful in a variety of hydrologic models.

List of Symbols

Symbol	Units	Description
R_s	ℓy/day	Solar radiation reaching earth's surface
I_o	ℓy/day	Solar radiation at the top of the atmosphere
ϕ	degrees	Latitude
n/N	—	Ratio of sunshine over possible sunshine
r	—	Radius vector of the earth (the ratio of the earth-sun distance and its mean)
t	hours	Time from sunrise to sunset
J_o	1.94 ℓy/min	Solar constant
z	degrees	Sun's zenith distance
δ	degrees	Declination angle
h	radians	Sun's hour angle
I	—	Day of year from Jan. 1
DL	hours	Day length
I_s	ℓy/day	Clear day solar radiation
A	ℓy/day	Constant for given location
B	ℓy/day	Constant for given location
I_{smax}	ℓy/day	Maximum clear day solar radiation for given location
I_{smin}	ℓy/day	Minimum clear day solar radiation for given location
ℓd	hours	Longest day of the year for given location
DBR	°C	Dry bulb temperature range
RHMIN	%	Minimum daily relative humidity

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