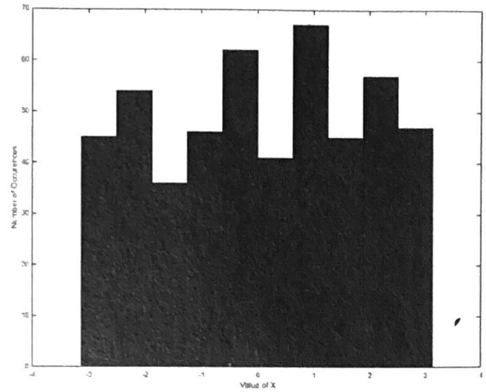
Trevor Shaw HW5

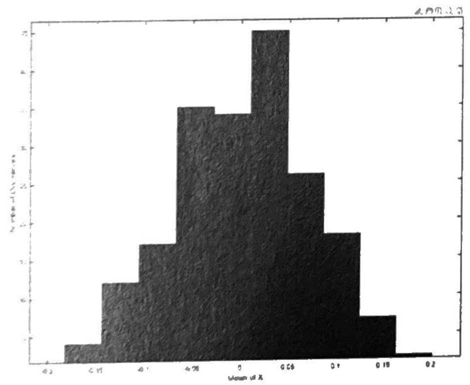
i. To generate 500 samples of X~U(-pi, pi), we can use the command:
X=-pi+2*pi*rand(500,1);

ii. To sketch the PDF of X, we can use the command: hist(X)
The sketch shows a flat distribution / Uniform distribution
mean(X) = 0.0790 which is close to zero as expected
var(X) = 3.1244



iii. To generate Y, we can use the command
Y=sum(X)/500 which returns 0.0790 which is the same as mean(X)

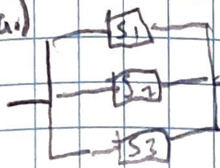
iv. To generate and plot 200 samples of Y, we can use the following code:
X=-pi+2*pi*rand(500,200);
Y=(sum(X))/500;
Hist(Y)
The distribution is Gaussian because the histogram resembles a bell curve.



The mean and variance are closer to zero as expected mean(Y) = 0.0037 and var(Y)=0.0055.

Trevor Shaw HW5

2.) a.)



$P_1 = P_2 = P_3 = P_i$

P(Receiving an input signal at output) $= P($at least 1 of 3 works$)$

$= P(\text{sure event}) - P(\text{no switches work})$

$= 1 - P(\bar S_1 \cap \bar S_2 \cap \bar S_3) = 1 - P(\bar S_1) \cdot P(\bar S_2) \cdot P(\bar S_3)$

$= 1 - (1-p)(1-p)(1-p)$

$= \boxed{3p - 3p^2 + p^3}$

b.) $P(\bar S_1 \mid (S_1 \cup S_2 \cup S_3)) = \dfrac{P(\bar S_1 \cap (S_1 \cup S_2 \cup S_3))}{P(S_1 \cup S_2 \cup S_3)} =$

$= \dfrac{P(\bar S_1 \cap (S_2 \cup S_3))}{1-(1-p)(1-p)(1-p)} = \dfrac{P(\bar S_1) \cdot P(S_2 \cup S_3)}{3p - 3p^2 + p^3} =$

$(1-p)(2p-p^2) =$
$= 2 - 3p + p^2$

$= \dfrac{(1-p)[1 - P(\bar S_2 \cap \bar S_3)]}{3p - 3p^2 + p^3} = \dfrac{(1-p)[1-(1-p)^2]}{3p - 3p^2 + p^3} = \boxed{\dfrac{(1-p)(2-p)}{3 - 3p + p^2}}$

3.) a.) $g(x)\delta(x - x_0) = g(x_0)\delta(x - x_0)$

For $\frac{x}{x+1}\delta(x-2)$ $\quad g(x) = \frac{x}{x+1}$ $\quad \delta(x - x_0) = \delta(x-2)$ $\quad x_0 = 2$

$\therefore \frac{x}{x+1}\delta(x-2) = \left(\frac{2}{2+1}\right)\delta(x-2) = \boxed{\frac{2}{3}\delta(x-2)}$

b.) $g(x)\delta(x) = g(0)\delta(x)$

for $\frac{x-1}{x+1}\delta(x)$ $\quad g(x) = \frac{x-1}{x+1}$, $\quad \delta(x) = \delta(x)$

$\therefore \frac{x-1}{x+1}\delta(x) = \frac{0-1}{0+1}\delta(x) = (-1)\delta(x) = \boxed{-\delta(x)}$

c.) $\int \frac{x}{x+1}\delta(x-2)\,dx = g(x_0)\int \cancel{\delta(x - x_0)\,dx} = g(x_0)$

$g(x_0) = \frac{2}{2+1} = \boxed{\frac{2}{3}}$

d.) $\int \frac{x-1}{x+1}\delta(x)\,dx = g(0)\int \cancel{\delta(x)\,dx} = g(0)$

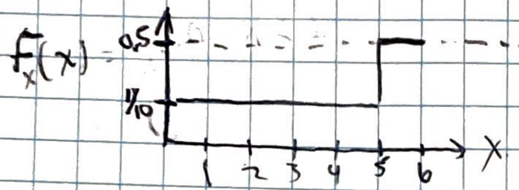
$g(0) = \frac{0-1}{0+1} = \boxed{-1}$

4.) a.) $P_i = \frac{1}{10}$ for $i = 1:5$

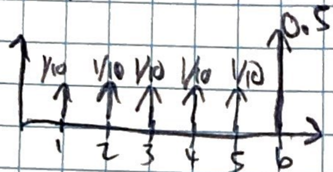
$$P_6 = P_S - P_{1:5} = 1 - P_{1:5} = 1-(P_1+P_2+P_3+P_4+P_5)$$

$$P_6 = 1 - \left(\tfrac{1}{10}+\tfrac{1}{10}+\tfrac{1}{10}+\tfrac{1}{10}+\tfrac{1}{10}\right) = 1-\left(\tfrac{5}{10}\right) = \boxed{0.5 = P_6}$$

b.) CDF:



PDF: $f_x(x)$



5.) a.) $f_X(x) = \begin{cases} Kx & \text{if } 0 \le x \le 2 \\ 0 & \text{else} \end{cases}$ $\boxed{K = 1/2}$

$$\int f_X(x)\,dx = 1 = \int_0^2 Kx\,dx \rightarrow K(2) - K(0) = 1 \qquad 2K = 1$$

b.) $P(x_1 < x < x_2) = \int_{x_1}^{x_2} f(x)\,dx = F(x_2) - F(x_1)$

$P(x<3) = \int_{-\infty}^{3} 1/2\, x = F(3) - F(-\infty) = \boxed{1 = P(x<3)}$

$P(x<1) = F(1) - F(-\infty) = 1/2(1) - 0 = \boxed{1/2 = P(x<1)}$

$P(x>2) = F(\infty) - F(2) = 1 - 1/2(2) = \boxed{0 = P(x>2)}$

c.) $F(x) = \begin{cases} 0 & x<0 \\ 1/2\, x & 0 \le x \le 2 \\ 1 & x > 0 \end{cases}$

d.) $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^2 1/2\, x^2 = 1/2 \int_0^2 x^2 = \frac{1}{2}\left(\frac{1}{3}[x^3]_0^2\right)$

$= \left(\frac{1}{2}\left(\frac{1}{3}[8-0]\right)\right) = \boxed{\frac{4}{3} = E(x)}$

$Var(x) = E(x^2) - E(x)$ $\qquad E(x^2) = \frac{1}{2}\int_0^2 x^3\,dx = \frac{1}{2}\left(\frac{1}{4}[x^4]_0^2\right)$

$E(x^2) = \left(\frac{1}{2}\left(\frac{1}{4}[16-0]\right)\right) = 2$ $\qquad \boxed{Var(x) = 2 - \frac{4}{3} = \frac{2}{3}}$

6.) $\boxed{P(X<2)} = 1 - P(x>2) = 1 - Q\left(\frac{2-1}{\sqrt{3}}\right) = \boxed{1 - Q\left(\frac{2}{\sqrt{3}}\right)}$

$\boxed{P(y>-1)} = Q\left(\frac{-1}{1}\right) = \boxed{Q(-1)}$

$P(1<x<3) = P(x<3) - P(x<1) = (1-P(x>3)) - (1-P(x\geq 1))$

$= \left(1 - Q\left(\frac{3-1}{\sqrt{3}}\right)\right) - \left(1 - Q\left(\frac{1-1}{\sqrt{3}}\right)\right) = \left(1 - Q\left(\frac{2}{\sqrt{3}}\right)\right) - (1 - 0.5)$

$= \left(1 - Q\left(\frac{2}{\sqrt{3}}\right)\right) - 0.5 = \boxed{0.5 - Q\left(\frac{2}{\sqrt{3}}\right) = P(1<x<3)}$

$P(1<x<3)$ and $P(y>-1) = \underline{\cancel{P(A|B)}} \times P(B) = P(A\cap B)$
$P(A)$

$= \boxed{P(A) \times P(B) = \left(0.5 - Q\left(\frac{2}{\sqrt{3}}\right)\right)(Q(-1))}$

$P(A|B) = P(A)$
because they are independent events

7.) $U \sim U[0,1]$

$F_U(x) = x$ for $0 \leq x \leq 1$

Let $U' = 1-U$

$\therefore F_{U'}(x) = P(U' \leq x) = P(1-U \leq x)$

$= P(U \geq 1-x) = 1 - P(U \leq (1-x))$

$= 1 - (1-x) = x \quad \therefore U' \sim U[0,1]$