Notes

Trey Plante

December 1, 2024

Contents

1 The type theory

The type theory is a two-level type theory, meaning it contains two notions of equality.

2 Algorithmic Definitions

We present an algorithmic procedure for deciding the judgments of the type theory.

Definition 2.1. Head reduction is defined as follows:

$$\begin{array}{ll}
\Gamma \triangleright (\lambda x.e)f \to e[f/x] & \\
\Gamma \triangleright ef \to e'f & \text{if } \Gamma \triangleright e \to e' \\
\Gamma \triangleright [\alpha \to e, \beta \to f] \to e & \text{if } \Gamma \vdash \alpha \\
\Gamma \triangleright [\alpha \to e, \beta \to e] \to f & \text{if } \Gamma \vdash \beta \\
\Gamma \triangleright \mathrm{out}_\psi(a) \to t & \text{if } \Gamma \triangleright a \uparrow A[\psi \to t] \text{ and } \Gamma \vdash \psi \\
\Gamma \triangleright \mathrm{out}_\psi(in(a)) \to a & \\
\Gamma \triangleright \mathrm{out}_\psi(a) \to a' & \text{if } \Gamma \triangleright a \to a' \\
\Gamma \triangleright \mathrm{out}_{[\alpha]}(in(a)) \to a & \\
\Gamma \triangleright \mathrm{out}_{[\alpha]}(a) \to a' & \text{if } \Gamma \triangleright a \to a'
\end{array}$$

Definition 2.2. Term equivalence is defined as follows:

$$\begin{array}{ll}
\Gamma \triangleright M \Leftrightarrow N : A[\psi \to t] & \text{if } \Gamma \triangleright \mathrm{out}_\psi(M) \Leftrightarrow \mathrm{out}_\psi(N) : A \\
\Gamma \triangleright M \Leftrightarrow N : [\alpha] \to A & \text{if } \Gamma, \alpha \triangleright \mathrm{out}_{[\alpha]}(M) \Leftrightarrow \mathrm{out}_{[\alpha]}(N) : A \\
\Gamma \triangleright M \Leftrightarrow N : (x : \mathbb{I}) \to A & \text{if } \Gamma, x : \mathbb{I} \triangleright Mx \Leftrightarrow Nx : A(x) \\
\Gamma \triangleright M \Leftrightarrow N : (x : \mathbf{cof}) \to A & \text{if } \Gamma, x : \mathbf{cof} \triangleright Mx \Leftrightarrow Nx : A(x)
\end{array}$$

Definition 2.3. Neutral equivalence is defined as follows:

$$
\begin{array}{ll}
\Gamma \triangleright [\alpha \to p_1, \beta \to p_2] \leftrightarrow q \uparrow A & \text{if } \Gamma, \alpha \triangleright p_1 \Leftrightarrow q : A \text{ and } \Gamma, \beta \triangleright p_2 \Leftrightarrow q : A \\
\Gamma \triangleright p \leftrightarrow [\alpha \to q_1, \beta \to q_2] \uparrow A & \text{if } \Gamma, \alpha \triangleright p \Leftrightarrow q_1 : A \text{ and } \Gamma, \beta \triangleright p \Leftrightarrow q_2 : A \\
\Gamma \triangleright \mathrm{out}_\psi(p) \Leftrightarrow \mathrm{out}_\psi(q) \uparrow A & \text{if } \Gamma \triangleright p \leftrightarrow q \uparrow A[\psi \to t] \\
\Gamma \triangleright \mathrm{out}_{[\alpha]}(p) \Leftrightarrow \mathrm{out}_\psi(q) \uparrow A & \text{if } \Gamma \triangleright p \leftrightarrow q \uparrow [\alpha] \to A
\end{array}
$$