

FINAL REPORT

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ME 407

Preliminary Design of Robotic Systems

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Meiosis

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1 Forward Kinematics

Table 1: DH Table for 6 DOF Manipulator

DH	d_i	θ_i	a_i	α_i
1	ℓ_1	θ_1	0	$\pi/2$
2	$-d$	θ_2	ℓ_2	0
3	0	$\theta_3 + \pi/2$	0	$\pi/2$
4	$\ell_3 + \ell_4$	θ_4	0	$-\pi/2$
5	0	θ_5	0	$\pi/2$
6	ℓ_6	θ_6	0	0

Where $\ell_1 = 22.08\text{cm}$, $\ell_2 = 25\text{cm}$, $\ell_3 = 20\text{cm}$, $\ell_4 = 8\text{cm}$, $\ell_6 = 5.25\text{cm}$, and $d = 1\text{cm}$.

$$A = Rot_{z,\theta} Trans_{z,d} Trans_{x,a} Rot_{x,\alpha} \quad (1)$$

Given an arbitrary homogeneous matrix T_i^{i-1} (computed by matrix multiplication of A matrices $\rightarrow [A_1 A_2 \cdots A_{i-1} A_i]$), the orientation vector \bar{z}_i (with respect to φ , θ and ψ) and the relative joint position (displacement) vector \bar{o}_i (with respect to x , y , and z) can be obtained via the 3rd and 4th columns of the matrix respectively, as shown in Equation 2 (given β as an arbitrary rotation angle about the z -axis).

$$T_i^{i-1} = \begin{bmatrix} c_\beta & -s_\beta & z_i^\varphi & o_i^x \\ s_\beta & c_\beta & z_i^\theta & o_i^y \\ 0 & 0 & z_i^\psi & o_i^z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$A_1 = rotz(\theta_1) transz(\ell_1) rotx(\pi/2)$$

$$A_1 = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 22.08 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = rotz(\theta_2) transz(-d) transx(\ell_2)$$

$$A_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 25 \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 25 \sin(\theta_2) \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = rotz(\theta_3 + \pi/2) rotx(\pi/2)$$

$$A_3 = \begin{bmatrix} \cos(\theta_3 + \pi/2) & 0 & \sin(\theta_3 + \pi/2) & 0 \\ \sin(\theta_3 + \pi/2) & 0 & -\cos(\theta_3 + \pi/2) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \text{rotz}(\theta_4) \text{transz}(\ell_3 + \ell_4) \text{rotx}(-\pi/2)$$

$$A_4 = \begin{bmatrix} \cos(\theta_4) & 0 & -\sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & \cos(\theta_4) & 0 \\ 0 & -1 & 0 & 28 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \text{rotz}(\theta_5) \text{rotx}(\pi/2)$$

$$A_5 = \begin{bmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) & 0 \\ \sin(\theta_5) & 0 & -\cos(\theta_5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \text{rotz}(\theta_6) \text{transz}(\ell_6)$$

$$A_6 = \begin{bmatrix} \cos(\theta_6) & 0 & -\sin(\theta_6) & 0 \\ \sin(\theta_6) & 0 & \cos(\theta_6) & 0 \\ 0 & 0 & 1 & 5.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 Motor Dynamics

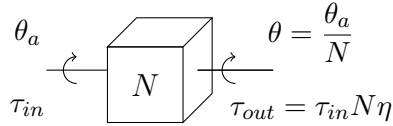
Given robot dynamics described by $H(\gamma)\ddot{\gamma} + n(\gamma, \dot{\gamma}) = \tau$, the torque, τ , provided by the servo motor is necessary to solve the closed loop dynamics of the system. Assuming the servo is driven by a D.C. motor with proportional derivative control,

$$\tau_a = Ki_a = J_a\ddot{\theta}_a + b_a\dot{\theta}_a + \tau_L \quad (3)$$

Where τ_a is the actuator torque, K is the back-EMF constant, i_a is the motor current, J_a is the armature inertia, θ_a , $\dot{\theta}_a$, $\ddot{\theta}_a$ is the motor position and it's first and second time derivatives, respectively, b_a is the viscous friction coefficient, and τ_L is the leftover torque available for the actuator to do work. The basic equation for a motor is known to be:

$$V_a = i_a R_a + K\dot{\theta}_a \quad (4)$$

Where V_a is the voltage applied to the actuator and R_a is the armature resistance. Given a gearbox with in/out ratio N and efficiency η ,



The motor equations can be expressed in the output coordinates:

$$Ki_a = J_a N \ddot{\theta} + b_a N \dot{\theta} + \frac{\tau}{N\eta}$$

Substituting into equation (4) and solving for i_a :

$$i_a = \frac{J_a N}{K} \ddot{\theta} + b_a N \dot{\theta} + \frac{\tau}{N\eta}$$

$$V_a = \frac{R_a J_a N}{K} \ddot{\theta} + \frac{R_a b_a N}{K} \dot{\theta} + \frac{R_a}{K N \eta} \tau + K N \dot{\theta} \quad (5)$$

Assuming P.D. control, $V_a = K_p(\theta - \theta_d) + K_d\dot{\theta}$, setting equal to (5), then collecting like terms:

$$\frac{R_a J_a N}{K} \ddot{\theta} + \left(\frac{R_a J_a N}{K} - K_d + K N \right) \dot{\theta} - K_p \theta = -K_p \theta_d - \frac{R_a}{K N \eta} \tau \quad (6)$$

The following parameters of the system can be obtained by applying a step input to the system with $\tau = 0$ and measuring the characteristics of its response. Denoting ζ as the damping ratio and ω_n as the natural frequency of the system,

$$\% \text{ Overshoot} = \left(\frac{\theta_{max} - \theta_{ss}}{\theta_{ss}} \right) \times 100, \quad \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}, \quad \omega_n = \frac{\pi}{T_p \sqrt{1 - \zeta^2}}$$

Given θ_{max} , θ_{ss} , and T_p as measured parameters of the system's max output, steady state, and time to peak, respectively.

Refactoring equation (6) and equating with the general solution for a second order system given by $\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = \omega_n^2\theta_d$,

$$2\zeta\omega_n = \frac{b_a}{J_a} - \frac{KK_d}{R_aJ_aN} + \frac{K^2}{R_aJ_a}, \quad \omega_n^2 = \frac{-KK_p}{R_aJ_aN} \quad (7)$$

Performing a similar experiment as previously described, except with a known inertial load $\tau = J_m\ddot{\theta}$, the following parameters can be found:

$$\alpha_m \equiv 2\zeta\omega_n = \frac{R_ab_aN^2\eta - KK_dN\eta + K^2N^2\eta}{R_aJ_aN^2\eta + R_aJ_m}, \quad \beta_m \equiv \omega_n = -\frac{KK_pN\eta}{R_aJ_aN^2\eta + R_aJ_m} \quad (8)$$

$$\begin{bmatrix} 1 & -(\alpha_1J_1 + \beta_1J_1) \\ 1 & -(\alpha_2J_2 + \beta_2J_2) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \frac{R_ab_aN^2\eta - KK_dN\eta + K^2N^2\eta - KK_pN\eta}{R_aJ_aN^2\eta} \\ 1 \\ \frac{1}{J_aN^2\eta} \end{bmatrix} = \begin{bmatrix} \alpha_1 + \beta_1 \\ \alpha_2 + \beta_2 \\ \vdots \end{bmatrix} \quad (9)$$

Finally, with multiple datasets (varying inertial loads, J_m) the coefficients of the second order system equation can be found using the least squares solution of (9)