Final Report

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ME 407

Preliminary Design of Robotic Systems

Embry-Riddle Aeronautical University

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1 Introduction

2 Requirements

— The system shall cost the end-user no more than $1000.

— The system shall be fully dexterous without being kinematically redundant.

— The system end-effector shall maintain a positional accuracy magnitude of ± 1 mm and an orientation accuracy of $\pm 5°$ eigen angle from the base frame.

— The system end-effector shall maintain a pose repeatability magnitude between 0.1—1.5 mm for the position and $\pm 4°$ eigen angle from the base frame for the orientation.

— The system's reachable workspace shall be a hemisphere with a radius of 300-700 mm.

— The system's dexterous workspace shall contain a hemispherical shell within the reachable workspace with a thickness of 280 mm.

— The system shall have a removable end-effector capable of picking and placing a low-odor chisel tip Expo dry erase marker.

— The system shall be able to write with a low-odor chisel tip Expo dry erase marker.

— The system shall be open source.

— The system shall be capable of operating given only desired end-effector cartesian coordinates specified with respect to the base frame.

3 Conceptual Design

4 Specifications

— The cost for the MEIOSIS team to develop the manipulator shall be no more than \$800.

— The system shall consist of six rotational joints connected by four links. The last three joints will create a spherical wrist.

— The system shall have no link offsets.

— The system shall accommodate a process in which the end user can calibrate the end-effector's position and orientation to within 0.5 mm and 1 degree of the manipulator's precision.

— Joints one and two of the system shall possess an angular error of no more than .025 degrees.

— Joint three of the system shall possess an angle error of no more than .03 degrees.

— Joints four, five, and six shall possess an angle error of no more than .29 degrees.

— The length of link one, two, three, four, and the wrist shall be 220.8 mm, 250 mm, 200 mm, 80 mm, and 52.5 mm respectively.

— The rotational limit of joint one, two, three, four, five, and six shall be $\pm180°$, $-9.7°$ to $177.5°$, $-150.6°$ to $-19.3°$, $\pm180°$, $-180°$ to $-1.6°$, and $\pm180°$ respectively.

— The system shall use a gripper that can close to 18mm.

— The end-effector shall attach to the manipulator using screws configured in a pattern that can accommodate a Dynamixel AX-12A servo.

— The end-effector shall be able to support 0.004 Newton meter moments about the axes normal to its gripping surfaces.

— The software shall be hosted publicly on an online repository and maintain an MIT license for distribution.

— The system shall have a user interface capable of accepting the end-effector's desired cartesian position and Euler angle orientation as a six element row vector.

— The system shall be capable of performing floating point arithmetic.

5 Preliminary Design

5.1 CAD

5.2 Forward Kinematics

5.3 Velocity Kinematics

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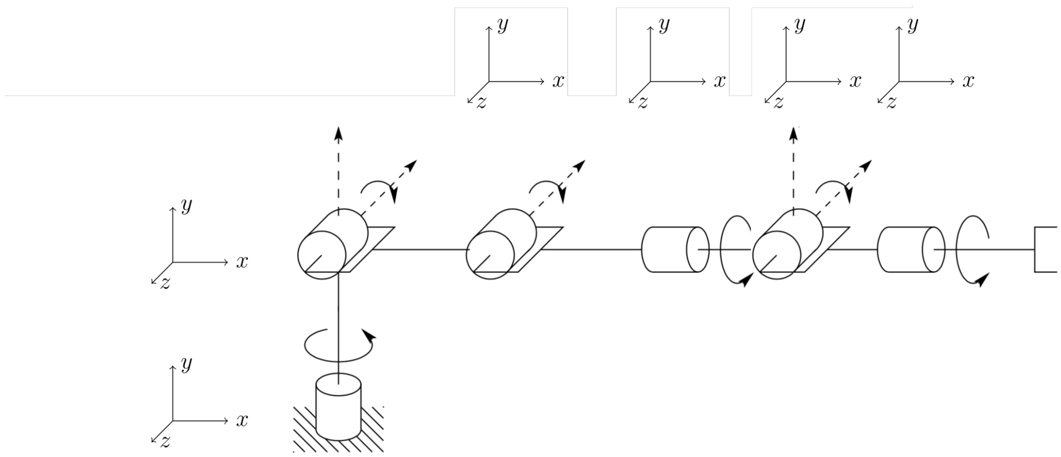


Figure 1: Coordinate Systems

Given the lengths of each of the manipulator links,

$$ {}^{I}_{B}r_1 = \begin{bmatrix} 0 \\ 0 \\ \ell_b \end{bmatrix} \quad {}^{1}_{1}r_2 = \begin{bmatrix} 0 \\ 0 \\ \ell_1 \end{bmatrix} \quad {}^{2}_{2}r_3 = \begin{bmatrix} 0 \\ \ell_2 \\ 0 \end{bmatrix} \quad {}^{3}_{3}r_4 = \begin{bmatrix} 0 \\ \ell_3 \\ 0 \end{bmatrix} \quad {}^{4}_{4}r_5 = \begin{bmatrix} 0 \\ \ell_4 \\ 0 \end{bmatrix} \quad {}^{5}_{5}r_6 = \begin{bmatrix} 0 \\ \ell_5 \\ 0 \end{bmatrix} $$

The position of each link relative to the inertial frame is given as:

$$ \begin{aligned} &{}^{I}_{B}r_1 = {}_{B}r_1 && {}^{I}_{B}r_2 = r_1 + {}^{I}T_1{}^{1}_{1}r_2 && {}^{I}_{B}r_3 = r_2 + {}^{I}T_2{}^{2}_{2}r_3 \\ &{}^{I}_{B}r_4 = r_3 + {}^{I}T_3{}^{3}_{3}r_4 && {}^{I}_{B}r_5 = r_4 + {}^{I}T_4{}^{4}_{4}r_5 && {}^{I}_{B}r_6 = r_5 + {}^{I}T_5{}^{5}_{5}r_6 \end{aligned} $$

$$ {}^{I}_{B}r_1 = \begin{bmatrix} 0 \\ 0 \\ \ell_b \end{bmatrix} \qquad {}^{I}_{B}r_2 = \begin{bmatrix} 0 \\ 0 \\ \ell_b + \ell_1 \end{bmatrix} \quad {}^{I}_{B}r_3 = \begin{bmatrix} -\ell_2 c_{\theta_2} s_{\theta_1} \\ \ell_2 c_{\theta_{12}} \\ \ell_b + \ell_1 + \ell_2 s_{\theta_2} \end{bmatrix} \qquad {}^{I}_{B}r_4 = \begin{bmatrix} -s_{\theta_1}(\ell_3 c_{\theta_{23}} + \ell_2 c_{\theta_2}) \\ c_{\theta_1}(\ell_3 c_{\theta_{23}} + \ell_2 c_{\theta_2}) \\ \ell_1 + \ell_b + \ell_3 s_{\theta_{23}} + \ell_2 s_{\theta_2} \end{bmatrix} $$

$$ {}^{I}_{B}r_5 = \begin{bmatrix} -s_{\theta_1}(\ell_3 c_{\theta_{23}} + \ell_4 c_{\theta_{23}} + \ell_2 c_{\theta_2}) \\ c_{\theta_1}(\ell_3 c_{\theta_{23}} + \ell_4 c_{\theta_{23}} + \ell_2 c_{\theta_2}) \\ \ell_1 + \ell_b + \ell_3 s_{\theta_{23}} + \ell_4 s_{\theta_{23}} + \ell_2 s_{\theta_2} \end{bmatrix} $$

$$ {}^{I}_{B}r_6 = \begin{bmatrix} \ell_5 c_{\theta_1} s_{\theta_4} s_{\theta_5} - \ell_4 c_{\theta_{23}} s_{\theta_1} - \ell_2 c_{\theta_2} s_{\theta_1} - \ell_5 c_{\theta_{23}} c_{\theta_5} s_{\theta_1} - \ell_3 c_{\theta_{23}} s_{\theta_1} \\ \quad + \ell_5 c_{\theta_2} c_{\theta_4} s_{\theta_1} s_{\theta_3} s_{\theta_5} + \ell_5 c_{\theta_3} c_{\theta_4} s_{\theta_1} s_{\theta_2} s_{\theta_5} \\ \ell_5(s_{\theta_5}(s_{\theta_1} s_{\theta_4} - c_{\theta_4}(c_{\theta_1} c_{\theta_2} s_{\theta_3} + c_{\theta_1} c_{\theta_3} s_{\theta_2})) - c_{\theta_5}(c_{\theta_1} s_{\theta_2} s_{\theta_3} - c_{\theta_1} c_{\theta_2} c_{\theta_3})) \\ \quad - \ell_3(c_{\theta_1} s_{\theta_2} s_{\theta_3} - c_{\theta_1} c_{\theta_2} c_{\theta_3}) - \ell_4(c_{\theta_1} s_{\theta_2} s_{\theta_3} - c_{\theta_1} c_{\theta_2} c_{\theta_3}) + \ell_2 c_{\theta_1} c_{\theta_2} \\ \ell_1 + \ell_b + \ell_3 s_{\theta_{23}} + \ell_4 s_{\theta_{23}} + \ell_2 s_{\theta_2} + \dfrac{\ell_5 c_{\theta_{23}} s_{\theta_{45}}}{2} + \ell_5 s_{\theta_{23}} c_{\theta_5} - \dfrac{\ell_5 s_{\theta_4 - \theta_5} c_{\theta_{23}}}{2} \end{bmatrix} $$

The orientation of each link with respect to the inertial frame is given as:

$$
\begin{aligned}
{}^{I}T_1 &= rotz(\theta_1) \\
{}^{I}T_2 &= rotz(\theta_1)\ rotx(\theta_2) \\
{}^{I}T_3 &= rotz(\theta_1)\ rotx(\theta_2)\ rotx(\theta_3) \\
{}^{I}T_4 &= rotz(\theta_1)\ rotx(\theta_2)\ rotx(\theta_3)\ roty(\theta_4) \\
{}^{I}T_5 &= rotz(\theta_1)\ rotx(\theta_2)\ rotx(\theta_3)\ roty(\theta_4)\ rotx(\theta_5) \\
{}^{I}T_6 &= rotz(\theta_1)\ rotx(\theta_2)\ rotx(\theta_3)\ roty(\theta_4)\ rotx(\theta_5)\ roty(\theta_6)
\end{aligned}
$$

Given the direction cosine matrices,

$$
rotx(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \quad , \quad roty(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}
$$

$$
rotz(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \end{bmatrix}
$$

$$
{}^{I}T_1 = \begin{bmatrix} c_{\theta_1} & -s_{\theta_1} & 0 \\ s_{\theta_1} & c_{\theta_1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad {}^{I}T_2 = \begin{bmatrix} c_{\theta_1} & -c_{\theta_2}s_{\theta_1} & s_{\theta_1}s_{\theta_2} \\ s_{\theta_1} & c_{\theta_1}c_{\theta_2} & -c_{\theta_1}s_{\theta_2} \\ 0 & s_{\theta_2} & c_{\theta_2} \end{bmatrix} \quad {}^{I}T_3 = \begin{bmatrix} c_{\theta_1} & -c_{\theta_{23}}s_{\theta_1} & s_{\theta_{23}}s_{\theta_1} \\ s_{\theta_1} & c_{\theta_{23}}c_{\theta_1} & -s_{\theta_{23}}c_{\theta_1} \\ 0 & s_{\theta_{23}} & c_{\theta_{23}} \end{bmatrix}
$$

$$
{}^{I}T_4 = \begin{bmatrix} c_{\theta_1}c_{\theta_4} - s_{\theta_{23}}s_{\theta_1}s_{\theta_4} & -c_{\theta_{23}}s_{\theta_1} & c_{\theta_1}s_{\theta_4} + s_{\theta_{23}}c_{\theta_4}s_{\theta_1} \\ c_{\theta_4}s_{\theta_1} + s_{\theta_{23}}c_{\theta_1}s_{\theta_4} & c_{\theta_{23}}c_{\theta_1} & s_{\theta_1}s_{\theta_4} - s_{\theta_{23}}c_{\theta_1}c_{\theta_4} \\ -c_{\theta_{23}}s_{\theta_4} & s_{\theta_{23}} & c_{\theta_{23}}c_{\theta_4} \end{bmatrix}
$$

$$
{}^{I}T_5 = \begin{bmatrix} c_{\theta_1}c_{\theta_4} - s_{\theta_{23}}s_{\theta_1}s_{\theta_4} & s_{\theta_5}(c_{\theta_1}s_{\theta_4} + s_{\theta_{23}}c_{\theta_4}s_{\theta_1}) - c_{\theta_{23}}c_{\theta_5}s_{\theta_1} & c_{\theta_5}(c_{\theta_1}s_{\theta_4} + s_{\theta_{23}}c_{\theta_4}s_{\theta_1}) + c_{\theta_{23}}s_{\theta_1}s_{\theta_5} \\ c_{\theta_4}s_{\theta_1} + s_{\theta_{23}}c_{\theta_1}s_{\theta_4} & s_{\theta_5}(s_{\theta_1}s_{\theta_4} - s_{\theta_{23}}c_{\theta_1}c_{\theta_4}) + c_{\theta_{23}}c_{\theta_1}c_{\theta_5} & c_{\theta_5}(s_{\theta_1}s_{\theta_4} - s_{\theta_{23}}c_{\theta_1}c_{\theta_4}) - c_{\theta_{23}}c_{\theta_1}s_{\theta_5} \\ -c_{\theta_{23}}s_{\theta_4} & s_{\theta_{23}}c_{\theta_5} + c_{\theta_{23}}c_{\theta_4}s_{\theta_5} & c_{\theta_{23}}c_{\theta_4}c_{\theta_5} - s_{\theta_{23}}s_{\theta_5} \end{bmatrix}
$$

$$
{}^{I}T_6 = \begin{bmatrix} {}^{I}T_{6(1,1)} & {}^{I}T_{6(1,2)} & {}^{I}T_{6(1,3)} \\ {}^{I}T_{6(2,1)} & {}^{I}T_{6(2,2)} & {}^{I}T_{6(2,3)} \\ {}^{I}T_{6(3,1)} & {}^{I}T_{6(3,2)} & {}^{I}T_{6(3,3)} \end{bmatrix}
$$

$$
\begin{aligned}
{}^{I}T_{6(1,1)} &= c_{\theta_6}(c_{\theta_1}c_{\theta_4} - s_{\theta_{23}}s_{\theta_1}s_{\theta_4}) - s_{\theta_6}(c_{\theta_5}(c_{\theta_1}s_{\theta_4} + s_{\theta_{23}}c_{\theta_4}s_{\theta_1}) + c_{\theta_{23}}s_{\theta_1}s_{\theta_5}) \\
{}^{I}T_{6(1,2)} &= s_{\theta_5}(c_{\theta_1}s_{\theta_4} + s_{\theta_{23}}c_{\theta_4}s_{\theta_1}) - c_{\theta_{23}}c_{\theta_5}s_{\theta_1} \\
{}^{I}T_{6(1,3)} &= c_{\theta_6}(c_{\theta_5}(c_{\theta_1}s_{\theta_4} + s_{\theta_{23}}c_{\theta_4}s_{\theta_1}) + c_{\theta_{23}}s_{\theta_1}s_{\theta_5}) + s_{\theta_6}(c_{\theta_1}c_{\theta_4} - s_{\theta_{23}}s_{\theta_1}s_{\theta_4}) \\
{}^{I}T_{6(2,1)} &= c_{\theta_6}(c_{\theta_4}s_{\theta_1} + s_{\theta_{23}}c_{\theta_1}s_{\theta_4}) - s_{\theta_6}(c_{\theta_5}(s_{\theta_1}s_{\theta_4} - s_{\theta_{23}}c_{\theta_1}c_{\theta_4}) - c_{\theta_{23}}c_{\theta_1}s_{\theta_5}) \\
{}^{I}T_{6(2,2)} &= s_{\theta_5}(s_{\theta_1}s_{\theta_4} - s_{\theta_{23}}c_{\theta_1}c_{\theta_4}) + c_{\theta_{23}}c_{\theta_1}c_{\theta_5} \\
{}^{I}T_{6(2,3)} &= s_{\theta_6}(c_{\theta_4}s_{\theta_1} + s_{\theta_{23}}c_{\theta_1}s_{\theta_4}) + c_{\theta_6}(c_{\theta_5}(s_{\theta_1}s_{\theta_4} - s_{\theta_{23}}c_{\theta_1}c_{\theta_4}) - c_{\theta_{23}}c_{\theta_1}s_{\theta_5}) \\
{}^{I}T_{6(3,1)} &= s_{\theta_6}(s_{\theta_{23}}s_{\theta_5} - c_{\theta_{23}}c_{\theta_4}c_{\theta_5}) - c_{\theta_{23}}c_{\theta_6}s_{\theta_4} \\
{}^{I}T_{6(3,2)} &= s_{\theta_{23}}c_{\theta_5} + c_{\theta_{23}}c_{\theta_4}s_{\theta_5} \\
{}^{I}T_{6(3,3)} &= c_{\theta_6}(s_{\theta_{23}}s_{\theta_5} - c_{\theta_{23}}c_{\theta_4}c_{\theta_5}) - c_{\theta_{23}}s_{\theta_4}s_{\theta_6}
\end{aligned}
$$

The inverse kinematics can be calculated given desired position and orientation vectors, o and R, respecitvely.

$$\begin{bmatrix} x_c & y_c & z_c \end{bmatrix} = o \ , \quad R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Inverse Position :

$$\theta_1 = \text{atan2}\left(x_c,\ y_c\right) - \text{atan2}\left(d,\ \sqrt{x_c^2 + y_c^2 - d^2}\right) - \pi/2$$

$$\theta_2 = \text{atan2}\left(z_c - \ell_1,\ \sqrt{x_c^2 + y_c^2 - d^2}\right) - \text{atan2}\left(\ell_3 s_3,\ \ell_2 + \ell_3 c_3\right)$$

$$\theta_3 = \text{atan2}(-\sqrt{1 - D^2},\ D)$$

$$\text{where } D \equiv \frac{x_c^2 + y_c^2 - d^2 + \left(z_c - \ell_1\right)^2 - \ell_2^2 - \ell_3^2}{2\ell_2\ell_3}$$

Inverse Orientation :

$$^{I}T_3 = rotz(\theta_1)\ rotx(\theta_2)\ rotx(\theta_3)$$

$$^{3}T_6 = {}^{I}T_3^T R$$

$$\theta_4 = \text{atan2}\left({}^{3}T_{6(1,2)},\ {}^{3}T_{6(3,2)}\right)$$

$$\theta_5 = \text{atan2}\left({}^{3}T_{6(3,2)}/c_4,\ {}^{3}T_{6(2,2)}\right)$$

$$\theta_6 = \text{atan2}\left({}^{3}T_{6(2,1)},\ -{}^{3}T_{6(2,3)}\right)$$

$$H(\gamma)\ddot{\gamma} + d(\gamma, \dot{\gamma}) + G(\gamma) + B(\dot{\gamma}) + C\mathrm{sgn}(\dot{\gamma}) = F \tag{1}$$

$$J_B = \begin{bmatrix} {}^I T_B(:,3)^T \cdot \frac{\partial}{\partial\gamma}\, {}^I T_B(:,2) \\ {}^I T_B(:,1)^T \cdot \frac{\partial}{\partial\gamma}\, {}^I T_B(:,3) \\ {}^I T_B(:,2)^T \cdot \frac{\partial}{\partial\gamma}\, {}^I T_B(:,1) \end{bmatrix} = \begin{bmatrix} {}^B_B\omega_I \\ {}^I_B\dot{r}_B \end{bmatrix}$$

$${}^B_B\Gamma = {}^B_B\, r_{cm} m_b \ , \quad \mathring{S}(\omega) r = (\omega \times r)$$

$$G(\gamma) \equiv \left(\frac{\partial U({}^I r(\gamma))}{\partial \gamma} \right)^T$$

$$U_B = \begin{bmatrix} 0 & 0 & g \end{bmatrix} \left({}^I_B r_B m_B + {}^I T_B {}^B_B\Gamma \right)$$

$$H(\gamma) = \sum_B^N J_B(\gamma)^T \begin{bmatrix} {}^B_B J & \mathring{S}({}^B_B\Gamma)\, {}^I T_B^T \\ {}^I T_B \mathring{S}({}^B_B\Gamma)^T & m_B I \end{bmatrix} J_B(\gamma) \tag{2}$$

$$d(\gamma, \dot{\gamma}) = \sum_B^N J_B(\gamma)^T \begin{bmatrix} {}^B_B J & \mathring{S}({}^B_B\Gamma)\, {}^I T_B^T \\ {}^I T_B \mathring{S}({}^B_B\Gamma)^T & m_B I \end{bmatrix} \dot{J}_B(\gamma, \dot{\gamma})\dot{\gamma} + J_B(\gamma)^T \begin{bmatrix} {}^B_B\omega_I \times {}^B_B J {}^B_B\omega_I \\ {}^I T_B \left({}^B_B\omega_I \times ({}^B_B\omega_I \times {}^B_B \Gamma) \right) \end{bmatrix} \tag{3}$$

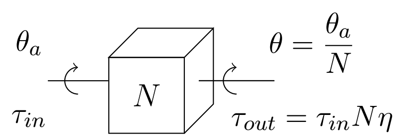
Given robot dynamics described by $H(\gamma)\ddot{\gamma} + n(\gamma, \dot{\gamma}) = \tau$, the torque, τ, provided by the servo motors is necessary to solve the closed loop dynamics of the system. Assuming the servo is driven by a D.C. motor with proportional derivative control,

$$\tau_a = Ki_a = J_a\ddot{\theta}_a + b_a\dot{\theta}_a + \tau_L \tag{4}$$

Where τ_a is the actuator torque, K is the back-EMF constant, i_a is the motor current, J_a is the armature inertia, θ_a, $\dot{\theta}_a, \ddot{\theta}_a$ is the motor position and it's first and second time derivatives, respectively, b_a is the viscous friction coefficient, and τ_L is the torque available for the actuator to do work. The basic equation for a motor is known to be:

$$V_a = i_aR_a + K\dot{\theta}_a \tag{5}$$

Where V_a is the voltage applied to the actuator and R_a is the armature resistance. Given a gearbox with in/out ratio N and efficiency η,



The motor equation (4) can be expressed in the output coordinates:

$$Ki_a = J_aN\ddot{\theta} + b_aN\dot{\theta} + \frac{\tau}{N\eta}$$

Substituing into equation (5) and solving for i_a:

$$i_a = \frac{J_aN}{K}\ddot{\theta} + b_aN\dot{\theta} + \frac{\tau}{N\eta}$$

$$V_a = \frac{R_aJ_aN}{K}\ddot{\theta} + \frac{R_ab_aN}{K}\dot{\theta} + \frac{R_a}{KN\eta}\tau + KN\dot{\theta} \tag{6}$$

Assuming P.D. control, $V_a = K_p(\theta - \theta_d) + K_d\dot{\theta}$, where θ_d is the desired orientation of the actuator, the following solution is found by setting the P.D. solution equal to (6). After collecting like terms:

$$\frac{R_aJ_aN}{K}\ddot{\theta} + \left(\frac{R_aJ_aN}{K} - K_d + KN\right)\dot{\theta} - K_p\theta = -K_p\theta_d - \frac{R_a}{KN\eta}\tau \tag{7}$$

The following parameters of the system can be obtained by applying a step input to the system with $\tau = 0$ and measuring the characteristics of it's response. Denoting ζ as the damping ratio and ω_n as the natural frequency of the system,

$$\%\text{ Overshoot} = \left(\frac{\theta_{max} - \theta_{ss}}{\theta_{ss}}\right) \times 100 \ , \quad \zeta = \frac{-\ln(\%\text{OS}/100)}{\sqrt{\pi^2 + \ln^2(\%\text{OS}/100)}} \ , \quad \omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$$

Given θ_{max}, θ_{ss}, and T_p as measured parameters of the system's max output, steady state, and time to peak, respectively.

Refactoring equation (7) and equating with the general solution for a second order system given by $\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = \omega_n^2\theta_d$,

$$2\zeta\omega_n = \frac{b_a}{J_a} - \frac{KK_d}{R_aJ_aN} + \frac{K^2}{R_aJ_a} \ , \quad \omega_n^2 = \frac{-KK_p}{R_aJ_aN} \tag{8}$$

Performing a similar experiment as previously described, except with a known inertial load $\tau = J_m\ddot{\theta}$, the following parameters can be found:

$$\alpha_m \equiv 2\zeta\omega_n = \frac{R_ab_aN^2\eta - KK_dN\eta + K^2N^2\eta}{R_aJ_aN^2\eta + R_aJ_m} \ , \quad \beta_m \equiv \omega_n = -\frac{KK_pN\eta}{R_aJ_aN^2\eta + R_aJ_m} \tag{9}$$

$$\begin{bmatrix} 1 & -(\alpha_1J_1 + \beta_1J_1) \\ 1 & -(\alpha_2J_2 + \beta_2J_2) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \dfrac{R_ab_aN^2\eta - KK_dN\eta + K^2N^2\eta - KK_pN\eta}{R_aJ_aN^2\eta} \\ \\ \dfrac{1}{J_aN^2\eta} \end{bmatrix} = \begin{bmatrix} \alpha_1 + \beta_1 \\ \alpha_2 + \beta_2 \\ \vdots \end{bmatrix} \tag{10}$$

Finally, with multiple datasets (varying inertial loads, J_m) the coefficients of the second order system equation can be found using the least squares solution of (10).

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We would like to acknowledge the following people for their contibutions to creating this report?

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- Dr. Schipper

I Appendix

i Drawings

ii Salient Code