# Final Report

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ME 407

Preliminary Design of Robotic Systems

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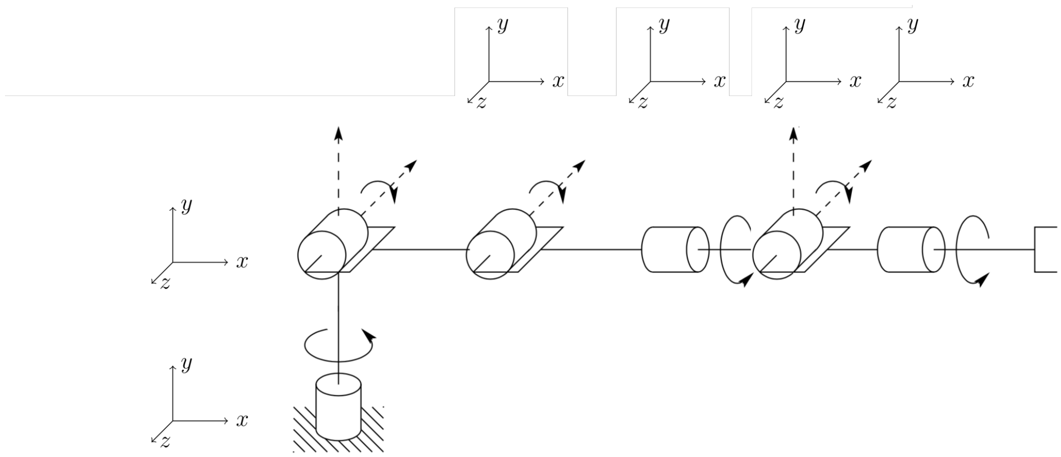



Figure 1: Coordinate Systems

$$ {}^{I}_{B}r_1 = \begin{bmatrix} 0 \\ 0 \\ \ell_b \end{bmatrix} \quad {}^{1}_{1}r_2 = \begin{bmatrix} 0 \\ 0 \\ \ell_1 \end{bmatrix} \quad {}^{2}_{2}r_3 = \begin{bmatrix} 0 \\ \ell_2 \\ 0 \end{bmatrix} \quad {}^{3}_{3}r_4 = \begin{bmatrix} 0 \\ \ell_3 \\ 0 \end{bmatrix} \quad {}^{4}_{4}r_5 = \begin{bmatrix} 0 \\ \ell_4 \\ 0 \end{bmatrix} \quad {}^{5}_{5}r_6 = \begin{bmatrix} 0 \\ \ell_5 \\ 0 \end{bmatrix} $$

$$ {}^{I}_{B}r_1 = \begin{bmatrix} 0 \\ 0 \\ \ell_b \end{bmatrix} \quad {}^{I}_{B}r_2 = \begin{bmatrix} 0 \\ 0 \\ \ell_b + \ell_1 \end{bmatrix} \quad {}^{I}_{B}r_3 = \begin{bmatrix} -\ell_2 c_{\theta_2} s_{\theta_1} \\ \ell_2 c_{\theta_{12}} \\ \ell_b + \ell_1 + \ell_2 s_{\theta_2} \end{bmatrix} \quad {}^{I}_{B}r_4 = \begin{bmatrix} -s_{\theta_1}(\ell_3 c_{\theta_{23}} + \ell_2 c_{\theta_2}) \\ c_{\theta_1}(\ell_3 c_{\theta_{23}} + \ell_2 c_{\theta_2}) \\ \ell_1 + \ell_b + \ell_3 s_{\theta_{23}} + \ell_2 s_{\theta_2} \end{bmatrix} $$

$$ {}^{I}_{B}r_5 = \begin{bmatrix} -s_{\theta_1}(\ell_3 c_{\theta_{23}} + \ell_4 c_{\theta_{23}} + \ell_2 c_{\theta_2}) \\ c_{\theta_1}(\ell_3 c_{\theta_{23}} + \ell_4 c_{\theta_{23}} + \ell_2 c_{\theta_2}) \\ \ell_1 + \ell_b + \ell_3 s_{\theta_{23}} + \ell_4 s_{\theta_{23}} + \ell_2 s_{\theta_2} \end{bmatrix} \quad {}^{I}_{B}r_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} $$

# 6 Motor Dynamics

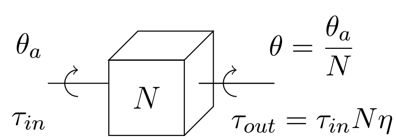
Given robot dynamics described by $H(\gamma)\ddot{\gamma} + n(\gamma, \dot{\gamma}) = \tau$, the torque, $\tau$, provided by the servo motors is necessary to solve the closed loop dynamics of the system. Assuming the servo is driven by a D.C. motor with proportional derivative control,

$$\tau_a = Ki_a = J_a\ddot{\theta}_a + b_a\dot{\theta}_a + \tau_L \tag{1}$$

Where $\tau_a$ is the actuator torque, $K$ is the back-EMF constant, $i_a$ is the motor current, $J_a$ is the armature inertia, $\theta_a$, $\dot{\theta}_a, \ddot{\theta}_a$ is the motor position and it's first and second time derivatives, respectively, $b_a$ is the viscous friction coefficient, and $\tau_L$ is the torque available for the actuator to do work. The basic equation for a motor is known to be:

$$V_a = i_aR_a + K\dot{\theta}_a \tag{2}$$

Where $V_a$ is the voltage applied to the actuator and $R_a$ is the armature resistance. Given a gearbox with $^{\text{in}}/_{\text{out}}$ ratio $N$ and efficiency $\eta$,




The motor equation (1) can be expressed in the output coordinates:

$$Ki_a = J_aN\ddot{\theta} + b_aN\dot{\theta} + \frac{\tau}{N\eta}$$

Substituing into equation (2) and solving for $i_a$:

$$i_a = \frac{J_aN}{K}\ddot{\theta} + b_aN\dot{\theta} + \frac{\tau}{N\eta}$$

$$V_a = \frac{R_aJ_aN}{K}\ddot{\theta} + \frac{R_ab_aN}{K}\dot{\theta} + \frac{R_a}{KN\eta}\tau + KN\dot{\theta} \tag{3}$$

Assuming P.D. control, $V_a = K_p(\theta - \theta_d) + K_d\dot{\theta}$, where $\theta_d$ is the desired orientation of the actuator, the following solution is found by setting the P.D. solution equal to (3). After collecting like terms:

$$\frac{R_aJ_aN}{K}\ddot{\theta} + \left(\frac{R_aJ_aN}{K} - K_d + KN\right)\dot{\theta} - K_p\theta = -K_p\theta_d - \frac{R_a}{KN\eta}\tau \tag{4}$$

The following parameters of the system can be obtained by applying a step input to the system with $\tau = 0$ and measuring the characteristics of it's response. Denoting $\zeta$ as the damping ratio and $\omega_n$ as the natural frequency of the system,

$$\%\text{ Overshoot} = \left(\frac{\theta_{max} - \theta_{ss}}{\theta_{ss}}\right) \times 100 \ , \quad \zeta = \frac{-\ln({}^{\%\mathrm{OS}}/_{100})}{\sqrt{\pi^2 + \ln^2({}^{\%\mathrm{OS}}/_{100})}} \ , \quad \omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$$

Given $\theta_{max}$, $\theta_{ss}$, and $T_p$ as measured parameters of the system's max output, steady state, and time to peak, respectively.

Refactoring equation (4) and equating with the general solution for a second order system given by $\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = \omega_n^2\theta_d$,

$$2\zeta\omega_n = \frac{b_a}{J_a} - \frac{KK_d}{R_aJ_aN} + \frac{K^2}{R_aJ_a} \ , \quad \omega_n^2 = \frac{-KK_p}{R_aJ_aN} \tag{5}$$

Performing a similar experiment as previously described, except with a known inertial load $\tau = J_m\ddot{\theta}$, the following parameters can be found:

$$\alpha_m \equiv 2\zeta\omega_n = \frac{R_ab_aN^2\eta - KK_dN\eta + K^2N^2\eta}{R_aJ_aN^2\eta + R_aJ_m} \ , \quad \beta_m \equiv \omega_n = -\frac{KK_pN\eta}{R_aJ_aN^2\eta + R_aJ_m} \tag{6}$$

$$\begin{bmatrix} 1 & -(\alpha_1J_1 + \beta_1J_1) \\ 1 & -(\alpha_2J_2 + \beta_2J_2) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \dfrac{R_ab_aN^2\eta - KK_dN\eta + K^2N^2\eta - KK_pN\eta}{R_aJ_aN^2\eta} \\ \\ \dfrac{1}{J_aN^2\eta} \end{bmatrix} = \begin{bmatrix} \alpha_1 + \beta_1 \\ \alpha_2 + \beta_2 \\ \vdots \end{bmatrix} \tag{7}$$

Finally, with multiple datasets (varying inertial loads, $J_m$) the coefficients of the second order system equation can be found using the least squares solution of (7).

# Acknowledgements & Attributions

We would like to acknowledge the following people for their contibutions to creating this report?

— Dr. Isenberg

# I Appendix

## i Drawings

## ii Salient Code