Final Report

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ME 407

Preliminary Design of Robotic Systems

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1 Forward Kinematics

The forward kinematics of the manipulator can be found using the Denavit–Hartenberg convention.

Where $\ell_1 = 22.08$cm, $\ell_2 = 25$cm, $\ell_3 = 20$cm, $\ell_4 = 8$cm, $\ell_6 = 5.25$cm, and $d = 1$cm.

Table 1: DH Table for 6 DOF Manipulator

DH	d_i	θ_i	a_i	α_i
1	ℓ_1	θ_1	0	$\pi/2$
2	$-d$	θ_2	ℓ_2	0
3	0	$\theta_3 + \pi/2$	0	$\pi/2$
4	$\ell_3 + \ell_4$	θ_4	0	$-\pi/2$
5	0	θ_5	0	$\pi/2$
6	ℓ_6	θ_6	0	0

$$A = Rot_{z,\theta}\ Trans_{z,d}\ Trans_{x,a}\ Rot_{x,\alpha} \tag{1}$$

Given an arbitrary homogeneous matrix T_i^{i-1} (computed by matrix multiplication of A matrices $\rightarrow \Big[A_1 A_2 \cdots A_{i-1} A_i\Big]$), the orientation vector $\bar{z}_i$ (with respect to φ, θ and ψ) and the relative joint position (displacement) vector $\bar{o}_i$ (with respect to x, y, and z) can be obtained via the 3^{rd} and 4^{th} columns of the matrix respectively, as shown in Equation 2 (given β as an arbitrary rotation angle about the z-axis).

$$T_i^{i-1} = \begin{bmatrix} c_\beta & -s_\beta & z_i^\varphi & o_i^x \\ s_\beta & c_\beta & z_i^\theta & o_i^y \\ 0 & 0 & z_i^\psi & o_i^z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$A_1 = rotz(\theta_1)\ transz(\ell_1)\ rotx(\pi/2)$$

$$A_1 = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 22.08 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = rotz(\theta_2)\ transz(-d)\ transx(\ell_2)$$

$$A_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 25\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 25\sin(\theta_2) \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = rotz(\theta_3 + {}^{\pi}/_{2})\ rotx({}^{\pi}/_{2})$$

$$A_3 = \begin{bmatrix} \cos(\theta_3 + {}^{\pi}/_{2}) & 0 & \sin(\theta_3 + {}^{\pi}/_{2}) & 0 \\ \sin(\theta_3 + {}^{\pi}/_{2}) & 0 & -\cos(\theta_3 + {}^{\pi}/_{2}) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = rotz(\theta_4)\ transz(\ell_3 + \ell_4)\ rotx(-{}^{\pi}/_{2})$$

$$A_4 = \begin{bmatrix} \cos(\theta_4) & 0 & -\sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & \cos(\theta_4) & 0 \\ 0 & -1 & 0 & 28 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = rotz(\theta_5)\ rotx({}^{\pi}/_{2})$$

$$A_5 = \begin{bmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) & 0 \\ \sin(\theta_5) & 0 & -\cos(\theta_5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = rotz(\theta_6)\ transz(\ell_6)$$

$$A_6 = \begin{bmatrix} \cos(\theta_6) & 0 & -\sin(\theta_6) & 0 \\ \sin(\theta_6) & 0 & \cos(\theta_6) & 0 \\ 0 & 0 & 1 & 5.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 Motor Dynamics

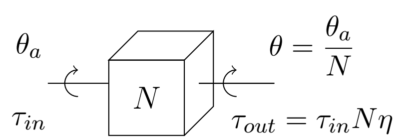
Given robot dynamics described by $H(\gamma)\ddot{\gamma} + n(\gamma, \dot{\gamma}) = \tau$, the torque, τ, provided by the servo motors is necessary to solve the closed loop dynamics of the system. Assuming the servo is driven by a D.C. motor with proportional derivative control,

$$\tau_a = Ki_a = J_a\ddot{\theta}_a + b_a\dot{\theta}_a + \tau_L \tag{3}$$

Where τ_a is the actuator torque, K is the back-EMF constant, i_a is the motor current, J_a is the armature inertia, $\theta_a,\ \dot{\theta}_a, \ddot{\theta}_a$ is the motor position and it's first and second time derivatives, respectively, b_a is the viscous friction coefficient, and τ_L is the torque available for the actuator to do work. The basic equation for a motor is known to be:

$$V_a = i_a R_a + K\dot{\theta}_a \tag{4}$$

Where V_a is the voltage applied to the actuator and R_a is the armature resistance. Given a gearbox with in/out ratio N and efficiency η,



The motor equation (3) can be expressed in the output coordinates:

$$Ki_a = J_a N\ddot{\theta} + b_a N\dot{\theta} + \frac{\tau}{N\eta}$$

Substituing into equation (4) and solving for i_a:

$$i_a = \frac{J_a N}{K}\ddot{\theta} + b_a N\dot{\theta} + \frac{\tau}{N\eta}$$

$$V_a = \frac{R_a J_a N}{K}\ddot{\theta} + \frac{R_a b_a N}{K}\dot{\theta} + \frac{R_a}{KN\eta}\tau + KN\dot{\theta} \tag{5}$$

Assuming P.D. control, $V_a = K_p(\theta - \theta_d) + K_d\dot{\theta}$, where θ_d is the desired orientation of the actuator, the following solution is found by setting the P.D. solution equal to (5). After collecting like terms:

$$\frac{R_a J_a N}{K}\ddot{\theta} + \left(\frac{R_a J_a N}{K} - K_d + KN\right)\dot{\theta} - K_p\theta = -K_p\theta_d - \frac{R_a}{KN\eta}\tau \tag{6}$$

The following parameters of the system can be obtained by applying a step input to the system with $\tau = 0$ and measuring the characteristics of it's response. Denoting ζ as the damping ratio and ω_n as the natural frequency of the system,

$$\%\ \text{Overshoot} = \left(\frac{\theta_{max} - \theta_{ss}}{\theta_{ss}}\right) \times 100\ , \quad \zeta = \frac{-\ln(\%\text{OS}/100)}{\sqrt{\pi^2 + \ln^2(\%\text{OS}/100)}}\ , \quad \omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$$

Given θ_{max}, θ_{ss}, and T_p as measured parameters of the system's max output, steady state, and time to peak, respectively.

Refactoring equation (6) and equating with the general solution for a second order system given by $\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = \omega_n^2\theta_d$,

$$2\zeta\omega_n = \frac{b_a}{J_a} - \frac{KK_d}{R_aJ_aN} + \frac{K^2}{R_aJ_a}\ , \quad \omega_n^2 = \frac{-KK_p}{R_aJ_aN} \tag{7}$$

Performing a similar experiment as previously described, except with a known inertial load $\tau = J_m\ddot{\theta}$, the following parameters can be found:

$$\alpha_m \equiv 2\zeta\omega_n = \frac{R_ab_aN^2\eta - KK_dN\eta + K^2N^2\eta}{R_aJ_aN^2\eta + R_aJ_m}\ , \quad \beta_m \equiv \omega_n = -\frac{KK_pN\eta}{R_aJ_aN^2\eta + R_aJ_m} \tag{8}$$

$$\begin{bmatrix} 1 & -(\alpha_1J_1 + \beta_1J_1) \\ 1 & -(\alpha_2J_2 + \beta_2J_2) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \dfrac{R_ab_aN^2\eta - KK_dN\eta + K^2N^2\eta - KK_pN\eta}{R_aJ_aN^2\eta} \\ \\ \dfrac{1}{J_aN^2\eta} \end{bmatrix} = \begin{bmatrix} \alpha_1 + \beta_1 \\ \alpha_2 + \beta_2 \\ \vdots \end{bmatrix} \tag{9}$$

Finally, with multiple datasets (varying inertial loads, J_m) the coefficients of the second order system equation can be found using the least squares solution of (9).