Final Report

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ME 407

Preliminary Design of Robotic Systems

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Meiosis

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1 Forward Kinematics

Table 1: DH Table for 6 DOF Manipulator

DH	d_i	θ_i	a_i	α_i
1	ℓ_1	θ_1	0	$\pi/2$
2	$-d$	θ_2	ℓ_2	0
3	0	$\theta_3 + \pi/2$	0	$\pi/2$
4	$\ell_3 + \ell_4$	θ_4	0	$-\pi/2$
5	0	θ_5	0	$\pi/2$
6	ℓ_6	θ_6	0	0

Where $\ell_1 = 22.08$cm, $\ell_2 = 25$cm, $\ell_3 = 20$cm, $\ell_4 = 8$cm, $\ell_6 = 5.25$cm, and $d = 1$cm.

$$A = Rot_{z,\theta}\ Trans_{z,d}\ Trans_{x,a}\ Rot_{x,\alpha} \tag{1}$$

Given an arbitrary homogeneous matrix T_i^{i-1} (computed by matrix multiplication of A matrices $\rightarrow \left[A_1 A_2 \cdots A_{i-1} A_i\right]$), the orientation vector $\bar{z}_i$ (with respect to φ, θ and ψ) and the relative joint position (displacement) vector $\bar{o}_i$ (with respect to x, y, and z) can be obtained via the 3^{rd} and 4^{th} columns of the matrix respectively, as shown in Equation 2 (given β as an arbitrary rotation angle about the z-axis).

$$T_i^{i-1} = \begin{bmatrix} c_\beta & -s_\beta & z_i^\varphi & o_i^x \\ s_\beta & c_\beta & z_i^\theta & o_i^y \\ 0 & 0 & z_i^\psi & o_i^z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$A_1 = rotz(\theta_1)\ transz(\ell_1)\ rotx(\pi/2)$$

$$A_1 = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 22.08 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = rotz(\theta_2)\ transz(-d)\ transx(\ell_2)$$

$$A_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 25\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 25\sin(\theta_2) \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = rotz(\theta_3 + \pi/2)\ rotx(\pi/2)$$

$$A_3 = \begin{bmatrix} \cos(\theta_3 + \pi/2) & 0 & \sin(\theta_3 + \pi/2) & 0 \\ \sin(\theta_3 + \pi/2) & 0 & -\cos(\theta_3 + \pi/2) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = rotz(\theta_4)\ transz(\ell_3 + \ell_4)\ rotx(-\pi/2)$$

$$A_4 = \begin{bmatrix} \cos(\theta_4) & 0 & -\sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & \cos(\theta_4) & 0 \\ 0 & -1 & 0 & 28 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = rotz(\theta_5)\ rotx(\pi/2)$$

$$A_5 = \begin{bmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) & 0 \\ \sin(\theta_5) & 0 & -\cos(\theta_5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = rotz(\theta_6)\ transz(\ell_6)$$

$$A_6 = \begin{bmatrix} \cos(\theta_6) & 0 & -\sin(\theta_6) & 0 \\ \sin(\theta_6) & 0 & \cos(\theta_6) & 0 \\ 0 & 0 & 1 & 5.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

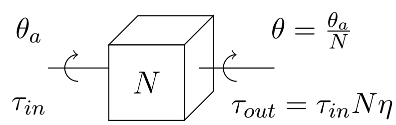
2 Motor Dynamics

$$H(\gamma)\ddot{\gamma} + n(\gamma, \dot{\gamma}) = \tau \tag{3}$$

$$\tau_a = Ki_a = J_a\ddot{\theta}_a + b_a\ddot{\theta}_a + \tau_L \tag{4}$$

$$V_a = i_aR_a + K\dot{\theta}_a \tag{5}$$

Given a gearbox with in/out ratio N and efficiency η,



The motor equations can be expressed in the output coordinates:

$$Ki_a = J_aN\ddot{\theta} + b_aN\dot{\theta} + \frac{\tau}{N\eta}$$

Substituing into equation (5) and solving for i_a:

$$i_a = \frac{J_aN}{K}\ddot{\theta} + b_aN\dot{\theta} + \frac{\tau}{N\eta}$$

$$V_a = \frac{R_aJ_aN}{K}\ddot{\theta} + \frac{R_ab_aN}{K}\dot{\theta} + \frac{R_a}{KN\eta}\tau + KN\dot{\theta} \tag{6}$$

Assuming P.D. control: $V_a = K_p(\theta - \theta_d) + K_d\dot{\theta}$, setting equal to 6, then collecting like terms:

$$\frac{R_aJ_aN}{K}\ddot{\theta} + \left(\frac{R_aJ_aN}{K} - K_d + KN\right)\dot{\theta} - K_p\theta = -K_p\theta_d - \frac{R_a}{KN\eta}\tau \tag{7}$$

$$2\zeta\omega_n = \frac{b_a}{J_a} - \frac{KK_d}{R_aJ_aN} + \frac{K^2}{R_aJ_a} \tag{8}$$

$$\omega_n^2 = \frac{-KK_p}{R_aJ_aN} \tag{9}$$

$$\%\ \text{Overshoot} = \left(\frac{\theta_{max} - \theta_{ss}}{\theta_{ss}}\right) \cdot 100 \tag{10}$$

$$\zeta = \frac{-\ln(\%\mathrm{OS}/100)}{\sqrt{\pi^2 + \ln^2(\%\mathrm{OS}/100)}} \tag{11}$$

$$\omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}} \tag{12}$$

$$2\zeta\omega_n = \frac{R_a b_a N^2\eta - K K_d N\eta + K^2 N^2 \eta}{R_a J_a N^2 \eta + R_a J_m} \tag{13}$$

$$\omega_n = -\frac{K K_p N\eta}{R_a J_a N^2 \eta + R_a J_m} \tag{14}$$