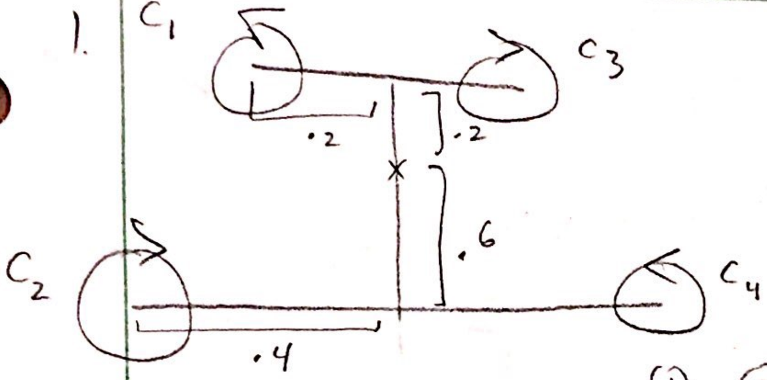
PS4 | ME136

1.



$$\begin{bmatrix} c_\Sigma \\ n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{array}{r} \\ \text{thrust} \\ \text{roll} \\ \text{pitch} \\ \text{yaw} \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ .2 & .4 & -.4 & -.2 \\ -.2 & .6 & .6 & -.2 \\ -.016 & .016 & -.016 & .016 \end{bmatrix} \end{array} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

inverting the mtx:

$$M = \frac{1}{4} \begin{bmatrix} 1 & .2^{-1} & -.2^{-1} & -.016^{-1} \\ 1 & .4^{-1} & .6^{-1} & .016^{-1} \\ 1 & -.4^{-1} & .6^{-1} & -.016^{-1} \\ 1 & -.2^{-1} & -.2^{-1} & .016^{-1} \end{bmatrix}$$

2. $c_\Sigma = mg$ =

Euler's Law

$$\left[\frac{d}{dt}\underline{\omega}\right]^B = \begin{bmatrix} 30 \\ 0 \\ 10 \end{bmatrix}$$

$$D^I \underline{\ell}_B^{BI} = \underline{n}_B$$

apply an Euler transformation and coordinate in B.

$$\left[\frac{d}{dt}\underline{\omega}^{BE}\right]^B = \left([\underline{I}_B^B]^B\right)^{-1}\left([\underline{n}_B]^B - [\underline{\Omega}^{BE}]^B[\underline{I}_B^B]^B[\underline{\omega}^{BE}]^B\right)$$

$$\underbrace{[\underline{I}_B^B]^B\left[\frac{d}{dt}\underline{\omega}^{BE}\right]^B}_{\text{we compute the desired moments}} = [\underline{n}_B]^B - [\underline{\Omega}^{BE}]^B[\underline{I}_B^B]^B\cancel{[\underline{\omega}^{BE}]^B}$$

this is 0 b/c the quad is hovering

$$\begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 29 \end{bmatrix} \times 10^{-6} \begin{bmatrix} 30 \\ 0 \\ 10 \end{bmatrix} = \begin{bmatrix} 4.8\times 10^{-4} \\ 0 \\ 2.9\times 10^{-4} \end{bmatrix}$$ desired moments

$$c_\Sigma = (30\times 10^{-3})(9.81) = \underline{.2943\ N}$$

now we plug and chug using the mixer on the cheat sheet.

$$\begin{bmatrix} c_{p_1} \\ c_{p_2} \\ c_{p_3} \\ c_{p_4} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & (33\times10^{-3})^{-1} & -(33\times10^{-3})^{-1} & (.01)^{-1} \\ 1 & -(33\times10^{-3})^{-1} & -(33\times10^{-3})^{-1} & -(.01)^{-1} \\ 1 & -(33\times10^{-3})^{-1} & (33\times10^{-3}) & (.01)^{-1} \\ 1 & (33\times10^{-3})^{-1} & (33\times10^{-3}) & -(.01)^{-1} \end{bmatrix}\begin{bmatrix} 0.2943 \\ 4.8\times10^{-4} \\ 0 \\ 2.9\times10^{-4} \end{bmatrix}$$

using matlab:

$$\begin{bmatrix} c_{p_1} \\ c_{p_2} \\ c_{p_3} \\ c_{p_4} \end{bmatrix} = \begin{bmatrix} 0.0845 \\ 0.0627 \\ 0.0772 \\ 0.0700 \end{bmatrix}$$

3.

$$c_\Sigma = mg = \sum_{i=1}^{4} c_i$$

to cause a maximum yaw moment (say to the left) we max out the prop speed of motors 1 & 3 which spin in the opposite direction as this yaw motion.

Notice that full throttle for motors 1 & 3 is not enough thrust to keep us from loosing altitude $c_{1,max} + c_{3,max} = .13 + .13\ N$

$= .26\ N < mg$

w/ $mg = .2943\ N$

we evenly distribute the remaining required thrust among motors 2 & 4 in order to prevent any roll or pitch moments.

$$.2943 - .26 = \frac{.0343}{2} = .01715\ N$$

now we have all the motor forces:

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} .13 \\ .01715 \\ .13 \\ .01715 \end{bmatrix} N$$

we plug these values into the the inverse of the mixer to get the yaw moment n_3 in order calculate the angular acceleration.

$$\begin{bmatrix} c_\Sigma \\ n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} .2943 \\ 0 \\ 0 \\ .0023 \end{bmatrix}$$

$$\Sigma \tau = I \alpha$$

$$\frac{.0023}{29 \times 10^{-6}} = \alpha = \boxed{79.31\ m/s^2}$$

4. This configuration is problematic b/c in trying to induce a yaw moment for example, to the right (spinning up motors 3 & 4 relative to 1 & 2) we produce an unintended pitch moment. We can see this using the mixer matrix