ME136 PS2 Trey Fortmuller, Joey Kroeger, David D. Hooper

1) known

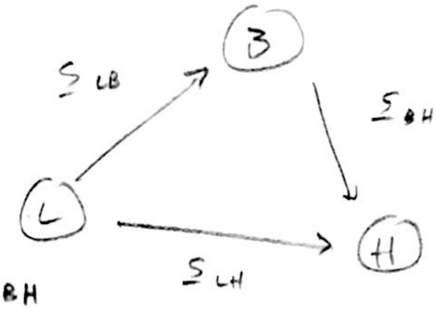
$$[\underline{s}_{LB}]^{B},\ [\underline{s}_{LH}]^{E},\ [T]^{BE}$$

find

$$[\underline{s}_{LH}]^{E}$$

we know $[\underline{v}]^{B} = [T]^{BA}[\underline{v}]^{A}$

and $\underline{s}_{AB} + \underline{s}_{BC} = \underline{s}_{AC}$



$$\underline{s}_{BL} + \underline{s}_{LH} = \underline{s}_{BH} \Rightarrow -\underline{s}_{LB} + \underline{s}_{LH} = \underline{s}_{BH}$$

coordinate in E

$$[-\underline{s}_{LB}]^{E} + [\underline{s}_{LH}]^{E} = [\underline{s}_{BH}]^{E}$$

$$[-\underline{s}_{LB}]^{E} = \underbrace{[T]^{EB}}_{\left([T]^{BE}\right)^{-1}}[-\underline{s}_{LB}]^{B}$$

so

$$\boxed{[\underline{s}_{BH}]^{E} = -\left([T]^{BE}\right)^{-1}[\underline{s}_{LB}]^{B} + [\underline{s}_{LH}]^{E}}$$

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2) $\|[\underline{x}]^A\| = \sqrt{\overline{[\underline{x}]}^A[\underline{x}]^A} = \sqrt{(x_1^A)^2 + (x_2^A)^2 + (x_3)^2}$

$$[\underline{x}]^B = [T]^{BA}[\underline{x}]^A \implies \overline{(AB)} = \overline{B}\,\overline{A}$$

$$\|[\underline{x}]^B\| = \sqrt{\overline{[\underline{x}]}^B[\underline{x}]^B} = \sqrt{\overline{[\underline{x}]}^A\,\overline{[T]}^{BA}[T]^{BA}[\underline{x}]^A}$$

using the orthogonality property of $[T]^{XY}$ $\longrightarrow$

$$= \sqrt{\overline{[\underline{x}]}^A \cancel{\left([T]^{BA}\right)^{-1}[T]^{BA}}[\underline{x}]^A}$$

$$\|[\underline{x}]^B\| = \sqrt{\overline{[\underline{x}]}^A[\underline{x}]^A} = \|[\underline{x}]^A\|$$

so $\|[\underline{x}]^B\| = \|[\underline{x}]^A\|$

the magnitude of the 1OT does not change as a function of coordinates used to express the tensor.

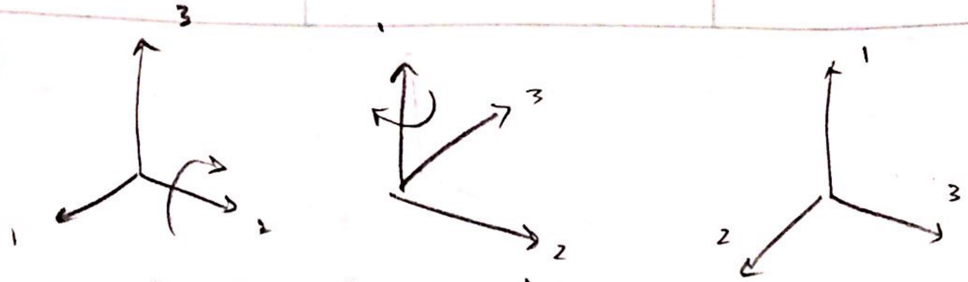
3) $[\alpha]^B = [T]^{BI}[\alpha]^I$

there is a -45° rotation about the z-axis from B to I.

$$= \begin{bmatrix} \cos(-45^\circ) & -\sin(45^\circ) & 0 \\ \sin(-45^\circ) & \cos(-45^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix} = \boxed{\begin{bmatrix} 5\sqrt{2}/2 \\ -5\sqrt{2}/2 \\ 10 \end{bmatrix}}$$

4)



$$R^{y}(-90)\,R^{x}(-90)\,R^{z}(\theta)$$

$$= \begin{bmatrix} \cos(-90) & 0 & \sin(-90) \\ 0 & 1 & 0 \\ -\sin(-90) & 0 & \cos(-90) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) \\ 0 & \sin(-90) & \cos(-90) \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\begin{bmatrix} (\cos\theta+0+0) & (0+0+0) & (0+0+0) \\ (0+0+0) & (0+0+0) & (0+0+1) \\ (0-\sin\theta+0) & (0-\cos\theta+0) & (0+0+0) \end{bmatrix}}$$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 1 \\ -\sin\theta & -\cos\theta & 0 \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}$$

$$[T]^{xy} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}$$

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