ME131 HW 3

Due: 2/20/19

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Prob. 1 no deliverable

Prob. 2

Use the eqns presented in prob. 1 to estimate β, C_d, R_x given an experimental dataset for a coast down test of a passenger vehicle.

2.1) straightlineTest.mat includes the measured:

V_x - longitudinal velocity

t - time

m - mass = 1755 kg

ρ - air mass density = 1.225 kg/m³

- and $g = 9.81 \ m/s^2$

- A_F is frontal area of the vehicle, we'll estimate it:

$$A_F = 1.6 + 0.00056(m - 256) \ [m^2]$$

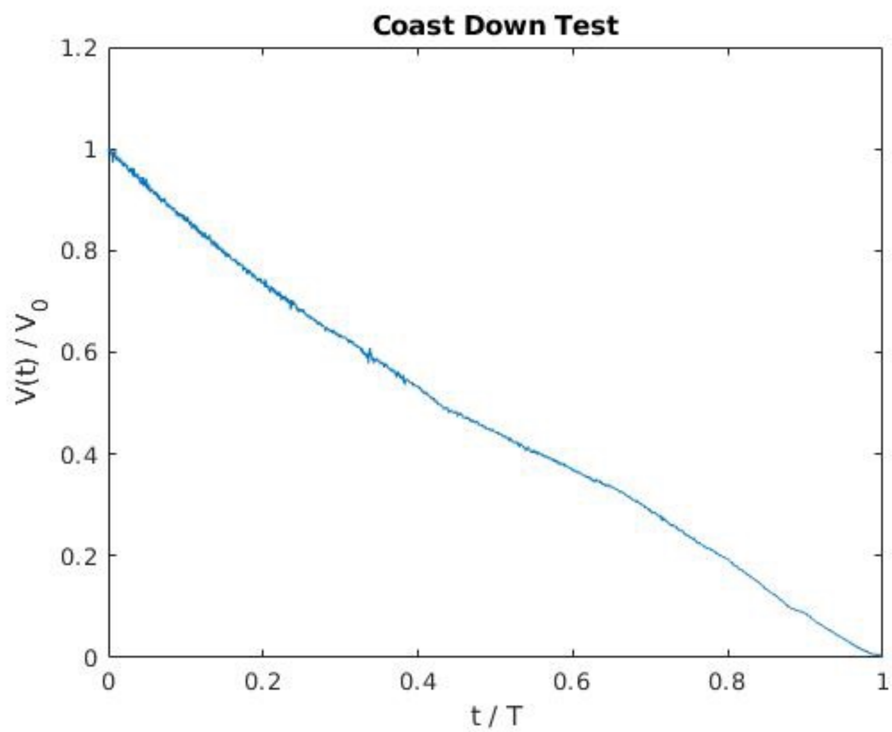
$V_0 = V_x(0)$, initial longitudinal velocity

- define T as the time the vehicle stops, $V(T)=0$

→ submit a plot of $\frac{v(t)}{V_0}$ vs. $\frac{t}{t(\mathrm{length}(t))}$

- all of the V_x vector is nonzero, so

$T = 52.93\,s$



2.2) Nonlinear curve fitting to find parameters β, C_d, and R_x.

$$\left\{ \frac{v(t)}{V_0} = \frac{1}{\beta}\tan\left[\left(1-\frac{t}{T}\right)\tan^{-1}\beta\right] \quad \text{w/ } \beta = V_0\left(\frac{\rho A_F C_d}{2R_x}\right)^{1/2} \right\}$$

$$A_F = 1.6 + 0.00056(1755-765) = 2.15944$$

using MATLAB's lsqcurvefit, we get:

$$\boxed{\beta = 0.7649}$$

We can use that $V(T)=0$ to arrive at eqns 4.9 in the book.

the C_d and R_x in terms of β:

$\rho = 1.225$

$\beta = 0.7649$

$T = 52.93$

$V_0 = 7.7257$

$A_F = 2.159$

$m = 1755$

$$C_d = \frac{2m\beta\tan^{-1}(\beta)}{V_0\rho T A_F}$$

$$R_x = \frac{V_0 m\tan^{-1}(\beta)}{\beta T}$$

$$C_d = \frac{2(1755)(0.7649)\tan^{-1}(0.7649)}{(7.7257)(1.225)(52.93)(2.159)}$$

$$R_x = \frac{(7.7257)(1755)\tan^{-1}(0.7649)}{(0.7649)(52.93)}$$

$$\boxed{\begin{aligned} C_d &= 1.621 \\ R_x &= 218.68 \end{aligned}}$$

Prob. 3 | Method of Least Squares

n points (x_i, y_i) with $i = 1 \ldots n$

$x_i \in \mathbb{R}^d$ independent variable

$y_i \in \mathbb{R}$ dependent scalar

We have a linear model with m parameters $w \in \mathbb{R}^d$ linking $x_i \to y_i$ so

$$\hat{y}_i = x_i^T w \in \mathbb{R}$$

predicted from model

Sum of square errors b/w prediction and measured is:

$$S = \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 = \sum_{i=1}^{n} (x_i^T w - y_i)^2$$

3.1)

define $\hat{Y} = Xw$

where $Y, \hat{Y} \in \mathbb{R}^n$ collecting all the $y_i, \hat{y}_i$

$X \in \mathbb{R}^{n \times d}$ collecting all the $X_i \in \mathbb{R}^d$

$$\Delta Y = \hat{Y} - Y = \begin{pmatrix} \hat{y}_1 - y_1 \\ \hat{y}_2 - y_2 \\ \vdots \\ \hat{y}_n - y_n \end{pmatrix} \quad ; \quad S = \Delta Y^T \Delta Y$$

$$\Delta Y = Xw - Y \qquad \boxed{S = (Xw - Y)^T (Xw - Y)}$$

3.2) $w = \underset{w}{\operatorname{argmin}} \left(S(w) \right)$

which occurs when $Xw - Y = \varepsilon$ is minimized. ε will be minimized when Y is orthogonally projected onto the subspace spanned by the columns of X. Then the elements of w will be the coefficients of a vector decomposition of $\hat{Y}$, which is the result of Y being projected onto X.

The orthogonal projection:

$$(Y - Xw)^T X = 0 \qquad (*)$$

we have originally $Y = Xw$

$$\Rightarrow \quad w = X^{-1} Y = \boxed{(X^T X)^{-1} X^T Y}$$

Prob. 4 | Parameter Fitting for Diff Eqs

→ we didn't have zero inclide in prob. 2

Longitudinal dynamics become:

$$m\dot{V} = -\frac{1}{2}\rho C_d A_F V^2 - R_x \underbrace{- mg\sin\theta}_{\text{(includes inclined road.)}}$$

We get the linear curve fitting model:

$$\dot{v} = w_1 V^2 + \underbrace{w_2 - w_3}$$

$$w_1 = -\frac{1}{2m}\rho C_d A_F$$

$$w_2 = -\frac{R_x}{m}$$

$$w_3 = g\sin\theta$$

we need to differentiate these, we have two tests one up the incline ($+w_3$) and one down ($-w_3$)

$V_1 \to$ uphill $(+\omega_3)$

$V_2 \to$ downhill $(-\omega_3)$

$dt \to$ sampling rate

4.1) obtain acceleration data by approximating

$$\dot{V}_i = a_i \cong \frac{V(i+1) - V(i)}{dt} \quad \text{in MATLAB}$$

construct the Y mtx of prob. 3 using a_i.

$$Y \in \mathbb{R}^n \quad ; \quad X \in \mathbb{R}^{n \times d} \quad ; \quad \omega \in \mathbb{R}^d$$

(dependent) (independent) (parametes)

The concatination of the a_i will be the Y vector, $Y \in \mathbb{R}^{n-1}$ b/c we had to use differences of the elements of $V_1 \in \mathbb{R}^n$ to obtain the accelerations.

matlab code is at:

github.com/treyfortmuller/me131

4.2) construct the $X \in \mathbb{R}^{n-1 \times d}$

$\hat{Y} = Xw$ is the model prediction

for V_1 (uphill) $\rightarrow +\omega_3$

$$\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} V_1^2 & 1 & -1 \\ \vdots & 1 & -1 \\ V_n^2 & \vdots & \vdots \\ & 1 & -1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

4.3) perform least squares.

$$a_1 = \omega_1 V^2 + \omega_2 - \omega_3$$

$$w = (X^T X)^{-1} X^T Y$$

$$w = \begin{pmatrix} -0.0009 \\ 0.0084 \\ 0.1157 \end{pmatrix}$$

$$\begin{cases} \omega_1 = -\frac{1}{2m} \rho C_d A_F = -0.0009 \\ \omega_2 = -\frac{R_x}{m} = 0.0084 \\ \omega_3 = g\sin\theta = 0.1157 \end{cases}$$

$$\frac{-2m}{\rho A_F}(0.0009) = C_d = \frac{2(1755)}{(1.225)(2.159)}(0.0009)$$

$$\boxed{C_d = 0.19}$$

$$R_x = (0.0084)(1755) = \boxed{14.742}$$

$$\theta = \sin^{-1}\left(\frac{0.1157}{9.81}\right) = 0.012 \text{ rad}$$

$$\theta \approx \boxed{0.687^\circ}$$