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ME131 HW 3
Due: 2/20/19

Prob. 1) no deliverable

Prob. 2)

Use the eqns presented in prob. 1 to estimate β, C_d, R_x given an experimental dataset for a coast down test of a passenger vehicle.

2.1) straightlineTest.mat includes the measured:

V_x - longitudinal velocity

t - time

m - mass $= 1755\ kg$

ρ - air mass density $= 1.225\ kg/m^3$

- and $g = 9.81\ m/s^2$
- A_F is frontal area of the vehicle, we'll estimate it:

$$A_F = 1.6 + 0.00056(m - 756)\ [m^2]$$

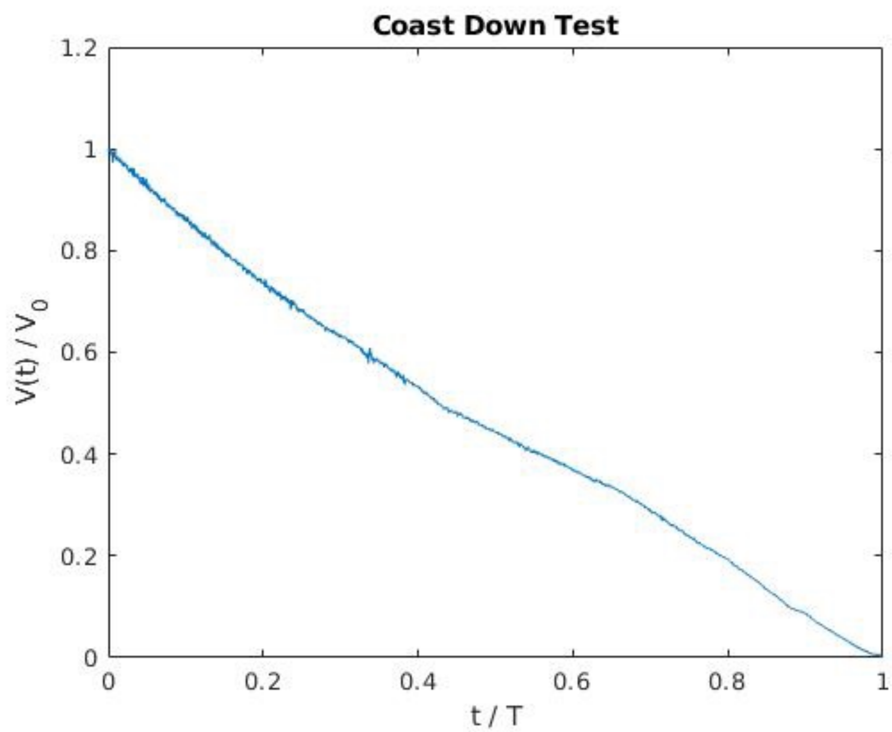
$$V_0 = V_x(0),\ \text{initial longitudinal velocity}$$

- define T as the time the vehicle stops, $V(T)=0$

→ submit a plot of $\frac{v(t)}{V_0}$ vs. $\frac{t}{t(\text{length}(t))}$

- all of the V_x vector is nonzero, so

$T = 52.93\,s$



2.2) Nonlinear curve fitting to find parameters β, C_d, and R_x.

$$\left\{ \frac{v(t)}{V_0} = \frac{1}{\beta}\tan\left[\left(1-\frac{t}{T}\right)\tan^{-1}\beta\right] \quad w/\ \beta = V_0\left(\frac{\rho A_F C_d}{2R_x}\right)^{1/2} \right\}$$

$$A_F = 1.6 + 0.00056(1755-765) = 2.15944$$

using MATLAB's lsqcurvefit, we get:

$$\boxed{\beta = 0.7649}$$

We can use that $V(T) = 0$ to arrive at eqns 4.9 in the book.

the C_d and R_x in terms of β:

$\rho = 1.225$

$\beta = 0.7649$

$T = 52.93$

$V_0 = 7.7257$

$A_F = 2.159$

$m = 1755$

$$C_d = \frac{2m\beta\tan^{-1}(\beta)}{V_0 \rho T A_F}$$

$$R_x = \frac{V_0 m \tan^{-1}(\beta)}{\beta T}$$

$$C_d = \frac{2(1755)(0.7649)\tan^{-1}(0.7649)}{(7.7257)(1.225)(52.93)(2.159)}$$

$$R_x = \frac{(7.7257)(1755)\tan^{-1}(0.7649)}{(0.7649)(52.93)}$$

$$\boxed{\begin{aligned} C_d &= 1.621 \\ R_x &= 218.68 \end{aligned}}$$

Prob. 3 | Method of Least Squares

n points (x_i, y_i) with $i = 1 \dots n$

$x_i \in \mathbb{R}^d$ independent variable

$y_i \in \mathbb{R}$ dependent scalar

We have a linear model with m parameters $w \in \mathbb{R}^d$ linking $x_i \to y_i$ so

$$\hat{y}_i = x_i^T w \in \mathbb{R}$$

predicted from model

Sum of square errors b/w prediction and measured is:

$$S = \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 = \sum_{i=1}^{n} \left(x_i^T w - y_i\right)^2$$

3.1)

define $\hat{Y} = Xw$

where $Y, \hat{Y} \in \mathbb{R}^n$ collecting all the $y_i, \hat{y}_i$

$X \in \mathbb{R}^{n \times d}$ collecting all the $x_i \in \mathbb{R}^d$

$$\Delta Y = \hat{Y} - Y = \begin{pmatrix} \hat{y}_1 - y_1 \\ \hat{y}_2 - y_2 \\ \vdots \\ \hat{y}_n - y_n \end{pmatrix} \; ; \quad S = \Delta Y^T \Delta Y$$

$$\Delta Y = Xw - Y \qquad \boxed{S = (Xw - Y)^T (Xw - Y)}$$

3.2) $w = \underset{w}{\operatorname{argmin}} \left(S(w) \right)$

which occurs when $Xw - Y = \varepsilon$ is minimized. ε will be minimized when Y is orthogonally projected onto the subspace spanned by the columns of X. Then the elements of w will be the coefficients of a vector decomposition of $\hat{Y}$, which is the result of Y being projected onto X.

The orthogonal projection:

$$(Y - Xw)^T X = 0 \qquad (*)$$

we have originally $Y = Xw$

$$\Rightarrow \quad w = X^{-1} Y = \boxed{(X^T X)^{-1} X^T Y}$$

Prob. 4 | Parameter Fitting for Diff Eqs

→ we didn't have zero incline in prob. 2

Longitudinal dynamics become:

$$m\dot{V} = -\frac{1}{2}\rho C_d A_F V^2 - R_x \underbrace{- mg\sin\theta}$$

(includes inclined road.)

We get the linear curve fitting model:

$$\dot{V} = w_1 V^2 + \underbrace{w_2 - w_3}$$

we need to differentiate these, we have two tests one up the incline $(+w_3)$ and one down $(-w_3)$

$$w_1 = -\frac{1}{2m}\rho C_d A_F$$

$$w_2 = -\frac{R_x}{m}$$

$$w_3 = g\sin\theta$$

$V_1 \to$ uphill $(+\omega_3)$

$V_2 \to$ downhill $(-\omega_3)$

$dt \to$ sampling rate

4.1) obtain acceleration data by approximating

$$\dot{V}_i = a_i \cong \frac{V(i+1) - V(i)}{dt} \quad \text{in MATLAB}$$

construct the Y mtx of prob. 3 using a_i.

$$Y \in \mathbb{R}^n \quad ; \quad X \in \mathbb{R}^{n \times d} \quad ; \quad \omega \in \mathbb{R}^d$$

(dependent) (independent) (parametes)

The concatination of the a_i will be the Y vector, $Y \in \mathbb{R}^{n-1}$ b/c we had to use differences of the elements of $V_1 \in \mathbb{R}^n$ to obtain the accelerations.

matlab code is at:

github.com/treyfortmuller/me131

4.2) construct the $X \in \mathbb{R}^{n-1 \times d}$

$\hat{Y} = X\omega$ is the model prediction

for V_1 (uphill) $\longrightarrow +\omega_3$

$$\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} V_1^2 & 1 & -1 \\ \vdots & 1 & -1 \\ V_n^2 & 1 & \vdots \\ & \vdots & \\ & 1 & -1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

4.3) perform least squares.

$$a_1 = \omega_1 V^2 + \omega_2 - \omega_3$$

$$\omega = (X^T X)^{-1} X^T Y$$

$$\omega = \begin{pmatrix} -0.0009 \\ 0.0084 \\ 0.1157 \end{pmatrix}$$

$$\begin{cases} \omega_1 = -\frac{1}{2m}\rho C_d A_F = -0.0009 \\ \omega_2 = -\frac{R_x}{m} = 0.0084 \\ \omega_3 = g\sin\theta = 0.1157 \end{cases}$$

$$\frac{-2m}{\rho A_F}(0.0009) = C_d = \frac{2(1755)}{(1.225)(2.159)}(0.0009)$$

$$\boxed{C_d = 0.19}$$

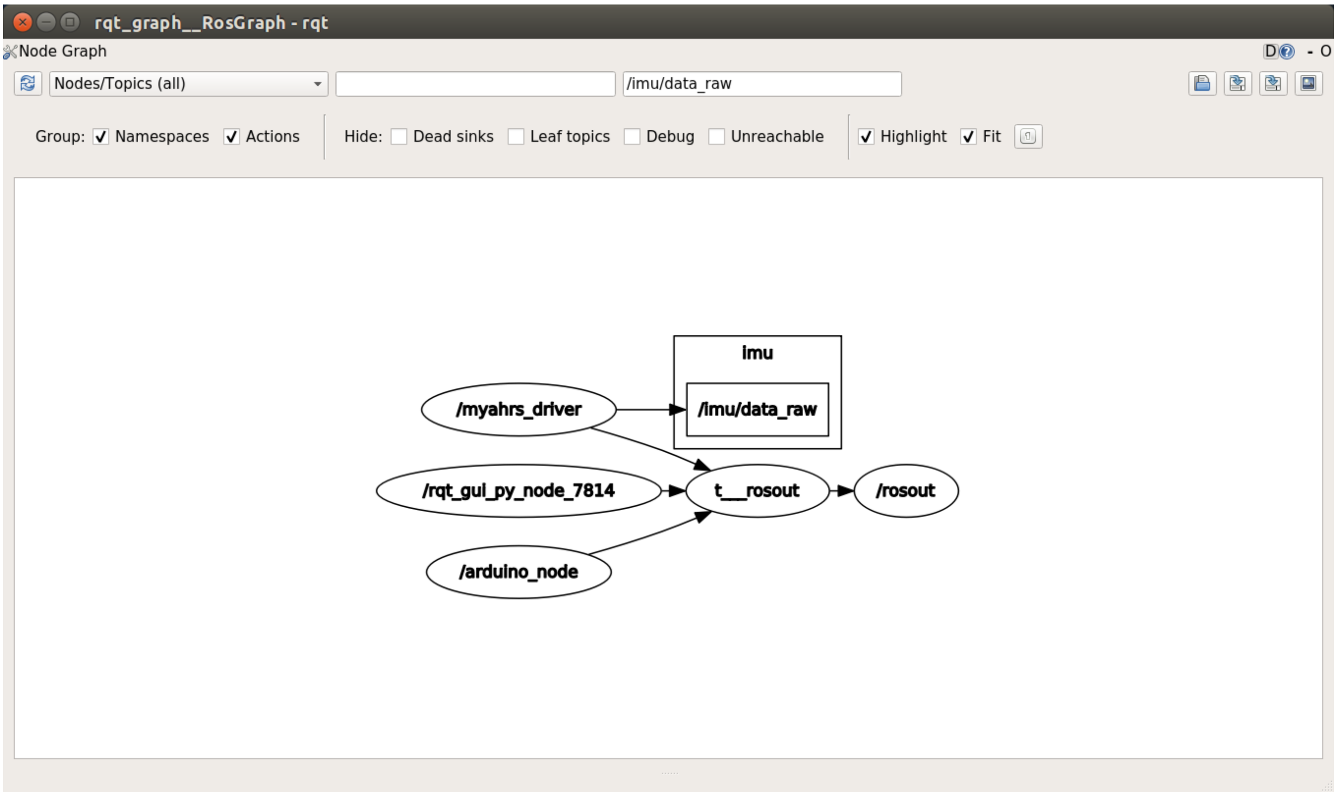
$$R_x = (0.0084)(1755) = \boxed{14.742}$$

$$\theta = \sin^{-1}\left(\frac{0.1157}{9.81}\right) = 0.012 \text{ rad}$$

$$\theta \approx \boxed{0.687^\circ}$$

ME131 Lab3 Deliverables

- Task 6.7 (ROS Architecture)
    - `rqt_graph` of BARC IMU and actuators topics and nodes.



- Task 7.1 (rostopic pub)
    - We check the rosmsg fields with

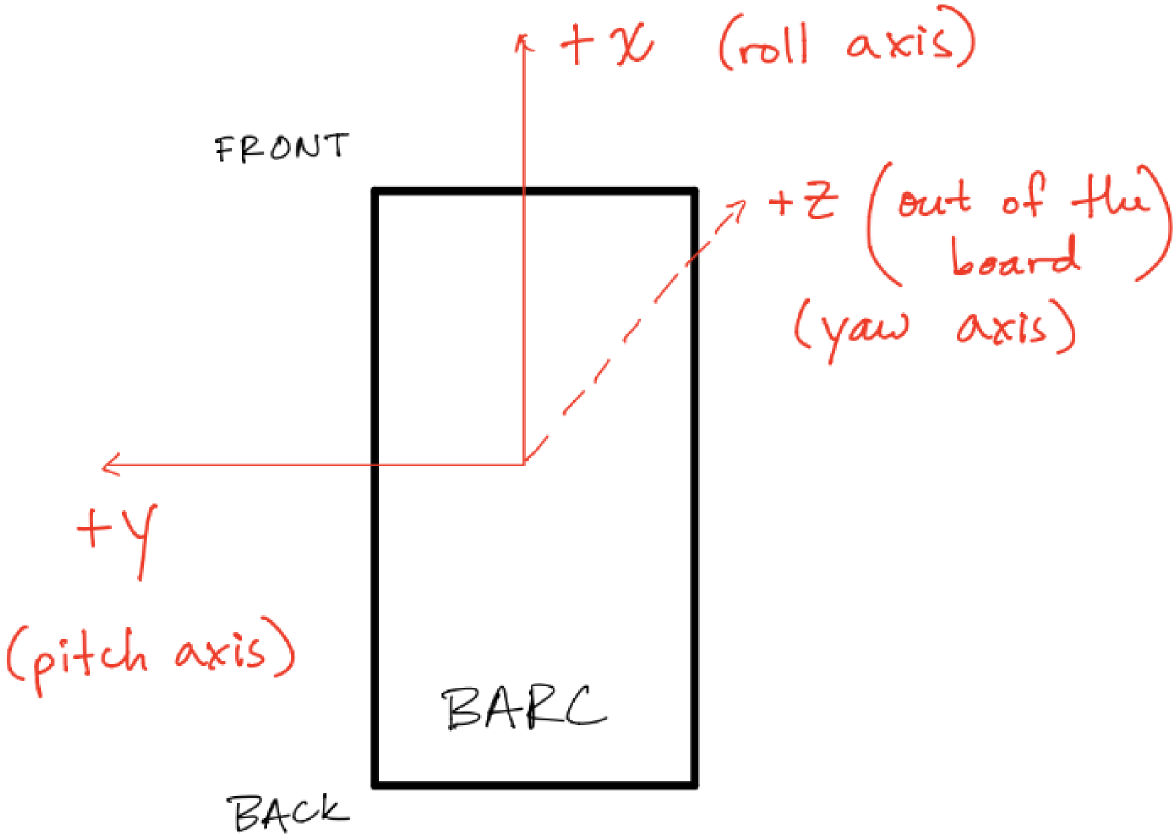
```
$ rosmsg info barc/ECU
float32 motor
float32 servo
```

    - The motor and servo commands are PWM microsecond commands in the range [1000, 2000]. 1500 corresponds to zero torque for the motor and 1600 gives a comfortable forward velocity. We are given that the steering servo is limited to +/-45 degree range of motion which maps to [1200, 1800] PWM. Linearly interpolating, we find a command of 1300 would steer to the left by 30 degrees. The command we used was:

      `rostopic pub --once /ecu_pwm barc/ECU "motor: 1600 servo: 1300"`

      rostopic pub publishes messages to a topic, the --once flag indicates it should send that command once and not repeat it, then we specify the topic name, message type, and finally fill the fields with our command.

- Task 8.1 (rqt_plot of IMU for determining body axis)
    - BARC car axes



- Values from the accelerometer while the vehicle is resting still on a table should be

```
a_x = 0, a_y = 0, a_z = 9.81
```

- The large offset in the z direction exists because the accelerometer (as with most MEMS accelerometers) measures "proper acceleration", so the effects of gravity are represented in the data. The accelerometer would read all zeros if the vehicle was in a free fall.

- Task 8.2 (Angular Velocity Measurements)
  - (See the above diagram for labels) these axes do respect the right hand rule.