Notes on "Minimax Play at Wimbledon" by Mark Walker and John Wooders

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1 Research Question & Motivation

- **Main Question:** Using field data from championship professional tennis matches to see if players' behavior is consistent with the minimax hypothesis.
- **Context:** Previous experimental work on mixed strategy play has not shown that players adhere to the minimax hypothesis.

2 Theoretical Framework

- **Minimax Hypothesis:** states that a rational player will minimize their maximum possible loss. This principle is commonly applied in zero-sum games.

  Formally, if a player chooses a strategy s from a set of possible strategies S, and the opponent chooses a strategy t from a set T, leading to a payoff function $u(s,t)$, the minimax strategy is:

  $$\max_{s \in S} \min_{t \in T} u(s,t)$$

  which means the player selects the strategy s^* that maximizes their minimum possible payoff, assuming the opponent plays optimally to minimize their gains.

  The **Minimax Theorem** states that in a two-player zero-sum game:

  $$\max_{s \in S} \min_{t \in T} u(s,t) = \min_{t \in T} \max_{s \in S} u(s,t)$$

  This guarantees that the player's optimal strategy secures a certain payoff, regardless of the opponent's actions.

- **Assumptions:**
  - Every point in a tennis match is played as a 2 X 2 constant-sum normal-form game with a unique equilibrium in strictly mixed strategies.
  - There are four point games in a tennis match, distinguished by which player is serving for the point, and by whether the point is a deuce-court point or an ad-court point.
  - for the serial independence test continuity is required, so the statistic is modeled as a random draw from a uniform distribution instead of the discreteness of the actual decision of choosing the length of a run.
- **Mathematical Model:** Formulating it as a 2 x 2 game.

3 Data

Data Source: professional match data where only the first serve is analyzed because there are only a few second serves.

4 Findings & Results

- **Optimal Play Strategy:** Minimax strategy dictates that optimal play would be demonstrated through the right and left serves having the same win rate.
- **Empirical Validation:** Based upon the data it looks like the data is generated from equilibrium play if the hypothesis is looked at jointly.
- **Key Figures/Tables:**
  - **Table 1 and Figure 2:** There are only two matches where their p-value is significant: one is significant under the 10-percent level and the other is under the 5-percent level. The Kolmogorov Test (defined below) shows a p-value of 0.76, meaning that the distribution that the data is generated from is not significantly different from the theorized distribution under the null hypothesis.
  - **Kolmogorov-Smirnov Test:** The Kolmogorov-Smirnov (KS) test is a nonparametric test that measures the difference between the empirical cumulative distribution function (ECDF) of a sample and a given cumulative distribution function (CDF) or between two ECDFs. The test statistic is given by:

    $$D_n = \sup_x |F_n(x) - F(x)|$$

    where $F_n(x)$ is the empirical CDF of the sample, and $F(x)$ is the theoretical CDF (one-sample test) or another ECDF (two-sample test). The null hypothesis H_0 states that the sample follows the given distribution (one-sample test) or that the two samples come from the same distribution (two-sample test). A large D_n indicates a stronger deviation, leading to rejection of H_0.

- **Table 2 and Figure 3:** In the O'Neil table there are more significant p-values and the overall differences in the Kolmogorov Test is nearly significant at a value of 0.06. This is a striking difference from the data in the previous table. This may be indicative that the data used by the O'Neil paper may have just been a bad sample to choose.
- **Table 3 and Figure 5 on Runs Test:** The table shows quite a few significant values and the figure shows a large difference between the data and the theoretical distribution according to the Kolmogorov test (p-value of 0.001).
- **Table 4 and Figure 6 on Runs Test:** About every other value on the table is significant indicating a large departure from the null hypothesis of the choices being independent. The Kolmogorov Test corroborates this in figure 6 by illustrating a massive difference between the O'Neil data and the theoretical distribution (p-value of 0.000007).

5 Implications & Discussion

- **Main Takeaways:**
  - **Main Result**: Professional tennis players generally demonstrate behavior in line with the minimax hypothesis, effectively mixing their strategies to approximate equilibrium play. However, their choices across points appear to be dependent, likely due to a well-documented cognitive bias. When attempting to randomize their decisions, individuals tend to switch too often, creating shorter streaks than true independence would produce. This pattern suggests they are unconsciously mimicking what they perceive as randomness rather than following a genuinely random process.
  - **Other Experiments:** This paper's results likely differ from previous literature because the subjects in previous literature were not as 'strategically sophisticated' as the professional tennis players.