Stat 344 – HW 6

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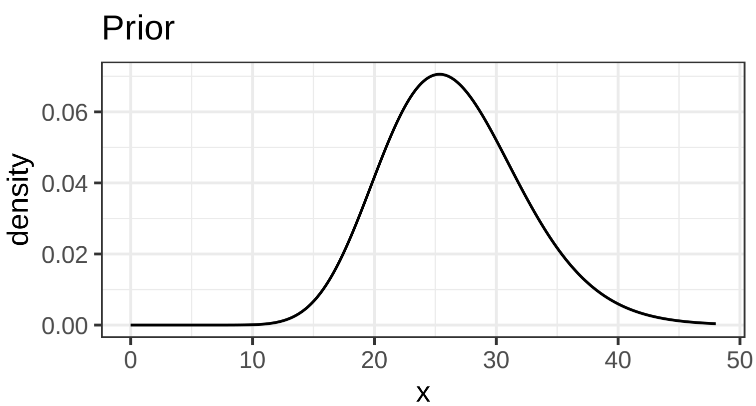
Problem 5.46 d.)

```
classes <- c(9, 39, 22, 35, 28)

lambda <- mean(classes)

lambda
```

```
## [1] 26.6
```

We want a gamma prior with a mean of 26.6

```
gf_dist(dist = 'gamma',
        params = list(shape = 21.28,
                      rate = .8)) |>
  gf_labs(title = 'Prior')
```




```
alpha <- 21.28
theta <- .8

sum(classes) + alpha
```

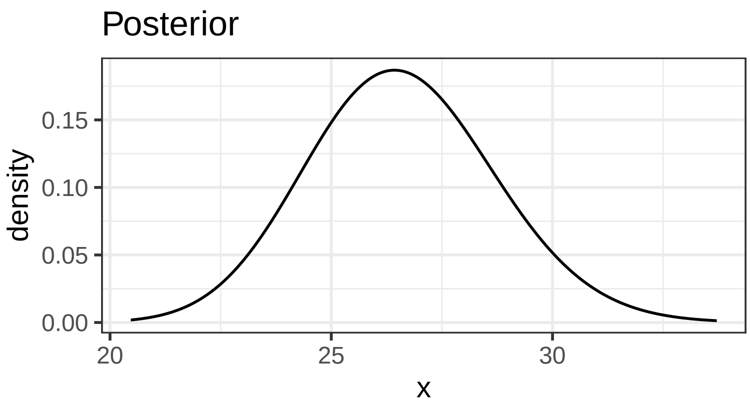
```
## [1] 154.28
```

```
length(classes) + theta
```

```
## [1] 5.8
```

$$g(\lambda) = \text{Gamma}(\alpha = 154.28, \theta = 5.8)$$

```r
gf_dist(dist = 'gamma',
        params = list(shape = 154.28,
                      rate = 5.8)) |>
  gf_labs(title = 'Posterior')
```




```r
set.seed(032601)
post_sample <- rgamma(n = 10000, shape = 154.28, rate = 5.8)
```

```r
cdata(~post_sample, p = 0.95)
```

```
##         lower   upper central.p
## 2.5% 22.54241 30.9835      0.95
```

A 95% credible interval for lambda for our data is (22.54, 30.98). This means that we are 95% confident that this interval contains the true value of lambda (also known as the mean) for a poisson distribution that our data came from.

Problem 5.47

a.)

```r
set.seed(032601)
Positions <- rgeo(25)
leaflet_map(position = Positions, mark = TRUE)
```

```
## PhantomJS not found. You can install it with webshot::install_phantomjs(). If it is installed, please
```

Observed: land = 5, water = 20.

Using uniform prior of Unif(0,1):

prior: $g(\pi) = 1$ likelihood: $f(x|\pi) = \pi^a(1-\pi)^b$

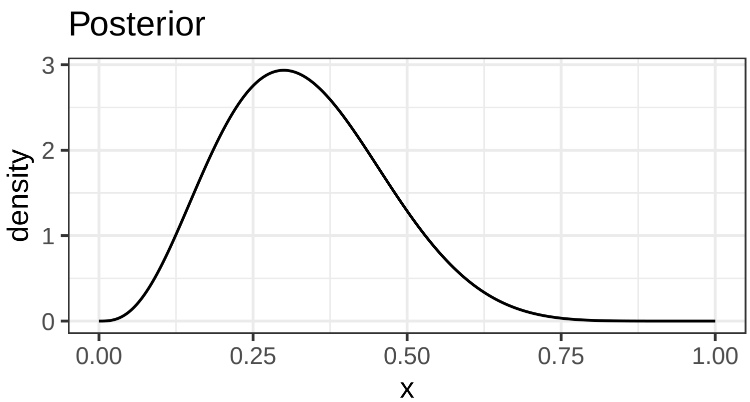
which means our posterior is: $h(/pi|x) = (1) * (\pi^a(1-\pi)^b)$, so $\pi \sim Beta(5+1, 20+1)$

```r
set.seed(032601)
post_unif <- rbeta(n = 10000, shape1 = 6, shape2 = 21)
cdata(~post_unif, p = 0.90)
```

```
##        lower     upper central.p
## 5% 0.1064814 0.3608645       0.9
```

A 90% credible interval for the proportion using a uniform prior is (0.106, 0.361).

```r
gf_dist(dist = 'beta',
        params = list(shape1 = 4,
                      shape2 = 8)) |>
  gf_labs(title = 'Posterior')
```



Using Beta prior of Beta(4,8):

prior: $g(\pi) = \pi^{\alpha-1}(1-\pi)^{\beta-1}$ likelihood: $f(x|\pi) = \pi^a(1-\pi)^b$

which means our posterior is: $h(\pi|x) = \pi^{\alpha-1+a}(1-\pi)^{\beta-1+b}$, so $\pi \sim Beta(\alpha + a, \beta + b)$

$\pi \sim Beta(9, 28)$

```r
set.seed(032601)
post_beta <- rbeta(n = 10000, shape1 = 9, shape2 = 28)
cdata(~post_beta, p = 0.90)
```

```
##        lower     upper central.p
## 5% 0.1377843 0.3639031       0.9
```

A 90% credible interval for the proportion using a beta prior is (0.138, 0.364).

b.)

```r
set.seed(032641)
Positions <- rgeo(25)
leaflet_map(position = Positions, mark = TRUE)
```

Observed land=8, water=17.

Using Beta prior of Beta(9,28) (from last posterior):

Our new prior is $g(\pi) = h(\pi|x) = \pi^{\alpha-1+a}(1-\pi)^{\beta-1+b}$. likelihood: $f(x|\pi) = \pi^a(1-\pi)^b$

New posterior is: $h(\pi|x) = \pi^{\alpha-1+2a}(1-\pi)^{\beta-1+2b}$.

so $\pi \sim Beta(\alpha + 2a, \beta + 2b)$

$\pi \sim Beta(9 + 2(8), 28 + 2(17))$

```r
set.seed(032601)
post_beta2 <- rbeta(n = 10000, shape1 = 25, shape2 = 62)
cdata(~post_beta2, p = 0.90)
```

```
##        lower     upper central.p
## 5% 0.2117821 0.3695912       0.9
```

Final using original posterior with 50 observations:

Using Beta prior of Beta(9,28):

posterior is the same: $h(\pi|x) = \pi^{\alpha-1+2a}(1-\pi)^{\beta-1+2b}$.

so $\pi \sim Beta(\alpha + 2a, \beta + 2b)$

$\pi \sim Beta(9 + 2(13), 28 + 2(37))$

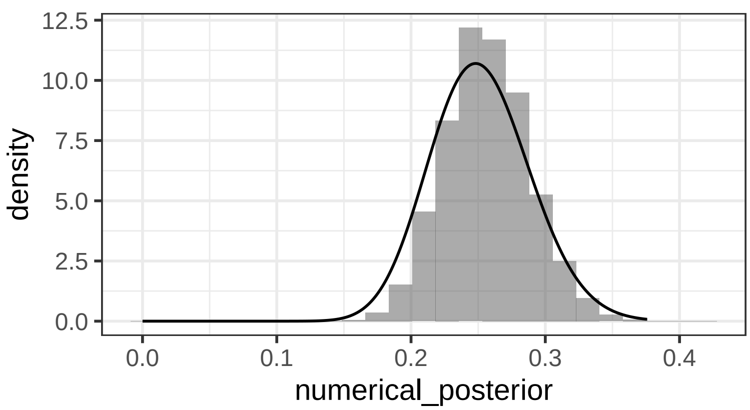
```
set.seed(032641)
post_beta2 <- rbeta(n = 10000, shape1 = 35, shape2 = 102)
cdata(~post_beta2, p = 0.90)
```

```
##        lower     upper central.p
## 5% 0.1961743 0.3172588       0.9
```

c.)

```
grid <- expand.grid(pi = seq(from = 0, by = 0.001, to = 1))
```

```
a <- 13
b <- 37
alpha <- 9
beta <- 28

grid <- grid |>
  mutate(prior = pi^(alpha - 1 + 2*a)*(1-pi)^(beta - 1 + 2*b), likelihood = dbinom(x = a, size = a + b,

numerical_posterior <- with(grid, sample(x = pi, size = 1e6, prob = posterior, replace = TRUE))

gf_dhistogram(~numerical_posterior) %>%
  gf_dist('beta', shape1 = 34, shape2 = 101)
```



The Numerically obtained posterior sample distribution of the proportion seems to agree with the analytically obtained posterior distribution.