Timur Reziapov (reziapo1)

CS 438 Spring 2014 Homework 1

- 1. (a) Tradeoffs between copper and optical: Optical calbing is more expensive than copper, but allows to transfer data singnificantly faster (higher bandwidth), over longer distances, and is immune to Electromagnetic Interference and Radio Frequency Interference due to close proximitity to other electronic devices. Fibre optical cable is also smaller in diameter and weighs less, which is desirable for cabling solutions. Because a lot of technology was built on copper before fibre optics, it may be easier to build copper infrastructures and it is also cheaper, as no additional and expensive electronics are usually necessary.
 - (b) (Extra-credit) Different protocols:
 - (c) (Extra-credit) Delayed release and early release:
- 2. (a) Output Signal-to-Noise Ratio: Output power level P_2 is unknown

$$-20 \text{ (dB)} = 10 \times \log_{10} \left(\frac{P_2}{0.5 \text{ (W)}} \right) \text{ (Negative because it's a loss)}$$

$$\Rightarrow P_2 = 0.005 \text{ (W)}$$

$$SNR_{dB} = 10 \log_{10} \frac{0.005 \text{ (W)}}{8 \times 10^{-6} \text{ (W)}} = 10 \log_{10} 625 = 27.9588 \text{ (dB)}$$

(b) Capacity with a frequency range of 100-1000 Hz:

$$(Bandwidth) B = 1000 - 100 = 900 \text{ (Hz)}$$

$$\frac{S}{N} = 625$$

$$(Channel Capactiy) C = B \times \log_2 \left(1 + \frac{S}{N}\right)$$

$$= 900 \text{ (Hz)} \times \log_2 626$$

$$= 8361.02 \text{ (bps)} = 8.36102 \text{ (kbps)}$$

(c) Length of phone line before requiring a repeater:

$$SNR = 10 \log_{10} \left(\frac{0.00025 \text{ (W)}}{0.5 \text{ (W)}} \right) = -33.01 \text{ (dB)}$$

$$Max \ Phone \ Line \ Length = \frac{33.01}{2} = 16.505 \text{ (km)}$$

- 3. 4B/5B encoding and NRZI signals (See next page for NRZI signals)
 - (a) 1110 0101 0000 0011

4B/5B Encoding: 11100 01011 11110 10101

 6. (a) **Probability of a collision in the** *i***th round,** q_i : For a collision to happen in the *i*th round, both nodes must choose the same slot out of the 2^{i-1} slots for that round. Each slot is equally likely, so the probability for A and B to pick any specific slot is $1/2^{i-1}$. Therefore:

$$q_i = \sum_{n=0}^{2^{i-1}-1} \frac{1}{2^{i-1}} \times \frac{1}{2^{i-1}}$$
$$= \frac{1}{2^{i-1}}$$

(b) **Probability** p_i **that exactly** i **rounds are needed for success:** To reach ith round, there must have been i-1 coflicts, one at every round before. In order to finish on the ith round, we must not have a conflict at the last round. Therefore:

$$p_i = \left(\prod_{n=1}^{i-1} q_n\right) \times (1 - q_i)$$

$$\begin{split} p_1 &= 1 \\ p_2 &= 1 - \frac{1}{2} = \frac{1}{2} \\ p_3 &= \frac{1}{2} \times \left(1 - \frac{1}{4}\right) = \frac{3}{8} \\ p_4 &= \frac{1}{2} \times \frac{1}{4} \times \left(1 - \frac{1}{8}\right) = \frac{7}{64} \end{split}$$

(c) Probabiliy A wins again =

$$1-2 \times \frac{1}{2} \times \frac{1}{4} = \frac{3}{4}$$

(d) **Probability that** *A* **captures the network for the next 5 frames, given** *A* **won the first round:** At every round *A* must pick a time slot with value strictly smaller than *B*'s. Assuming after *A* wins round 1, its counter is 1 and *B*'s counter is 2:

$$P(win_2|win_1) = \frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{4} = \frac{5}{8}$$

$$P(win_3|win_2, win_1) = \frac{1}{2} \times \frac{7}{8} + \frac{1}{2} \times \frac{6}{8} = \frac{13}{16}$$

$$P(win_4|win_3, win_2, win_1) = \frac{1}{2} \times \frac{15}{16} + \frac{1}{2} \times \frac{14}{16} = \frac{29}{32}$$

$$P(win_5|win_4, win_3, win_2, win_1) = \frac{1}{2} \times \frac{31}{32} + \frac{1}{2} \times \frac{30}{32} = \frac{61}{64}$$

$$P(win_2 \wedge win_3 \wedge win_4 \wedge win_5 \mid win_1) = \frac{5}{8} \times \frac{13}{16} \times \frac{29}{32} \times \frac{61}{64} = 0.4386$$