ECE 453: Wireless Communication Systems

Laboratory Experiment No.5: Colpitts Crystal Oscillator

Author: Timothy Green

Partner: Marco Rojas

Date Performed: October 19th, 2021

Date Written: November 1st, 2021

1 Objective

This lab introduces Quartz Crystal Resonators along with RF oscillators. The lab first analyzes an existing Crystal Resonator (Xtal), then builds and tests an RF oscillator, the Colpitts Common Collector Oscillator, using the Xtal.

2 Prelab

Crystal Oscillators are used often because of their ability to behave as an inductor with a very high Q value. It is known that the equivalent circuit of a crystal oscillator takes the form shown in Figure 1. The device has a series RLC portion along with an additional parallel capacitance. The total impedance of the device can be calculated as

$$Z_{Xtal} = (R_s + j\omega L_s + \frac{1}{j\omega C_s})|||(\frac{1}{j\omega C_p}) = \frac{R_s + j\omega L_s + \frac{1}{j\omega C_s}}{j\omega C_p(R_s + j\omega L_s + \frac{1}{j\omega C_s}) + 1}. \tag{1}$$

The impedance is a function of frequency and can be plotted over a sweep of frequencies. An example of such a device with $L_s = 15mH$, $C_s = 16fF$, $R_s = 15\Omega$, and $C_p = 5pF$ is used to calculate the impedance as a function of frequency. The resulting plots for the real and imaginary components of the frequency are shown in Figure 2 and Figure 3.

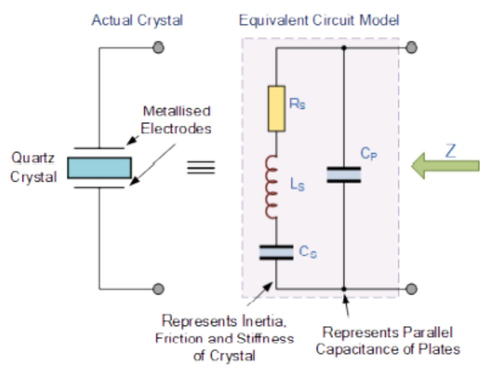


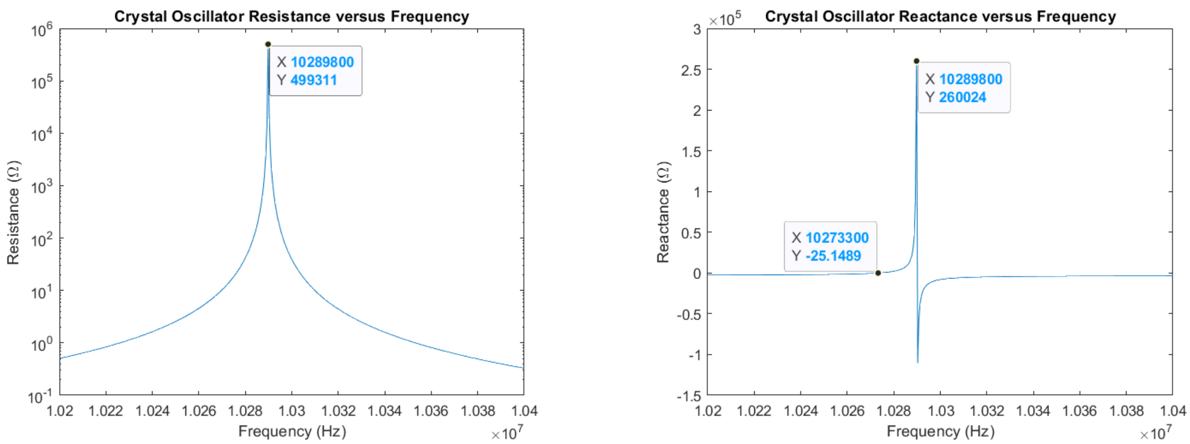
Figure 1: Crystal Oscillator Equivalent Circuit

The resonant frequencies can be best determined by examining the impedance magnitude. The series resonant frequency is characterized by an impedance magnitude minimum and the parallel resonant frequency is characterized by an impedance magnitude maximum. The series and parallel resonant frequencies can then be determined to be

$$f_s = 10.273MHz, \tag{2}$$

$$f_p = 10.289MHz. \tag{3}$$

The Xtal behaves as a high Q inductor when between frequencies f_s and f_p allowing a high Q resonator to be constructed by matching the inductance of the Xtal with some



(a) Resistance (b) Reactance

Figure 2: Crystal Oscillator Impedance versus Frequency Plots

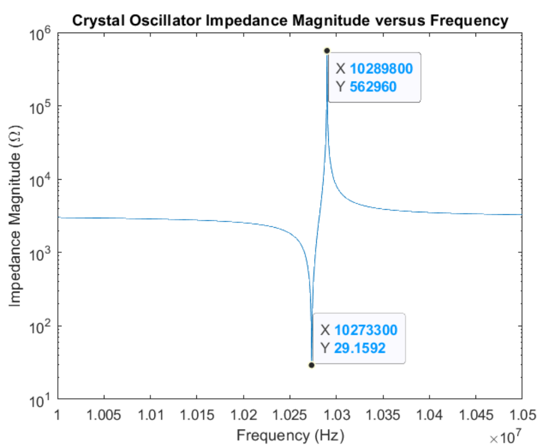


Figure 3: Crystal Oscillator Impedance Magnitude versus Frequency Plot

capacitance. The quality factor of the Xtal is equivalent to the quality factor of the series portion and can be determined to be

$$Q = \frac{\omega_s L_s}{R_s} = \frac{(2\pi \times 10.273 \times 10^6)(15 \times 10^{-3})}{15} = 64547. \quad (4)$$

The capacitance added to produce the high Q resonator is often referred to as the load capacitance. When a load capacitance of $C_L = 20pF$ is included, the calculation for the impedance of the crystal oscillator is the same as shown in equation 1 with $C_p = 5pF + C_L = 25pF$. This can be plotted against frequency to determine the new parallel resonance frequency. The resulting plot is shown in Figure 4.

The parallel resonant frequency or the overall resonator frequency can then be determined to be

$$f_p' = 10.277MHz. \quad (5)$$

With the load capacitance included in the circuit, the loaded Q of the circuit will be

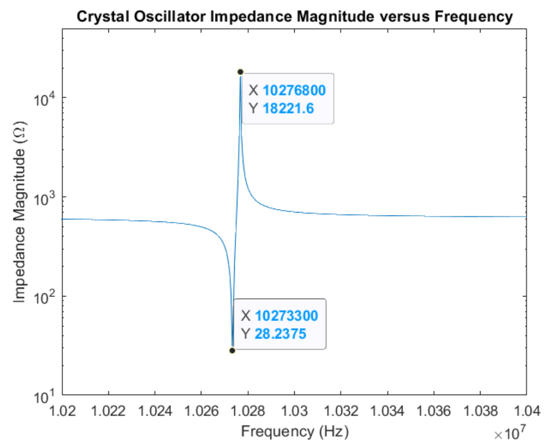


Figure 4: Crystal Oscillator Impedance Magnitude versus Frequency Plot with Load Capacitance

different from the Q value shown previously. Since the Xtal behaves as an inductor, the resistance and reactance of the crystal at the new resonant frequency can be used to determine the loaded Q value.

$$R = Re\{Z_{Xtal}\}|_{\omega=2\pi f_p'} = 24.57\Omega \tag{6}$$

$$X = Imag\{Z_{Xtal}\}|_{\omega=2\pi f_p'} = 866.3\Omega \tag{7}$$

The loaded Q can then be calculated as

$$Q_{loaded} = \frac{|X|}{R} = 35.26. \tag{8}$$

With an additional $R_L = 20\Omega$ in parallel with the load capacitance, the series R and X from the previous portion will need to be converted into a parallel equivalent form. The parallel resistance will be very large compared to R_L so the equivalent resistance will be R_L. The new loaded Q can then be found to be

$$Q'_{loaded} = \frac{R_L}{|X|} = 0.0231. \tag{9}$$

The crystal oscillator in combination with an amplifier circuit, which often uses some type of transistor, can create a full RF oscillator. Assuming the amplifier is a non-inverting amplifier ($sgn(A) = 1$), then the required phase shift and frequency range can be determined. The expression $sgn(A) = 1$ simply indicates that $\angle A = 0$. As shown in the Lab Manual [1], the loop gain must be

$$A_{lo} = 1\angle 0^\circ. \tag{10}$$

To reach a phase of 0°, the phase shift of β must be

$$\angle\beta = 0 - \angle A = -\angle A = 0^\circ. \tag{11}$$

An inverting amplifier has an output of $sgn(A) = -1$ and the phase shift from A is 180^circ. This means the phase shift due to the feedback must be

$$\angle\beta = -\angle A = 180^\circ. \tag{12}$$

Since the feedback circuitry is provided by the crystal oscillator in parallel with a capacitors, $\angle\beta$ will be able to reach 0° if Xtal is inductive and 180° if Xtal is capacitive. This means a non-inverting amplifier requires the Xtal to operate between f_s and f_p while the inverting amplifier requires operation outside this range.

The Common Collector (CC) amplifier circuit is shown in Figure 5 below and in the Lab Manual [1]. The DC operating point of the circuit affects the amplifier performance and thus the oscillations of the circuit.

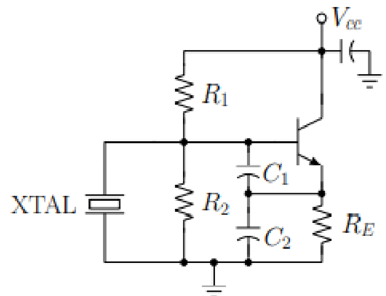


Figure 5: Common Collector Amplifier Colpitts Oscillator Schematic

The voltage at the base of the transistor is determined by the biasing resistors.

$$V_B = \frac{R_1}{R_1 + R_2} V_{cc} = \frac{V_{cc}}{R_1}(R_1||R_2) \tag{13}$$

Assuming the transistor is in active operation, the voltage at the emitter can be found as

$$V_E = V_B - V_{BE,on}. \tag{14}$$

The current at the emitter is then

$$I_{E,Q} = \frac{V_E}{R_E}. \tag{15}$$

The current at the collector is then

$$I_{C,Q} = \frac{\beta + 1}{\beta} I_{E,Q}. \tag{16}$$

Combining all of these leads to the following expression for the DC bias current.

$$I_{C,Q} = \frac{\beta + 1}{\beta R_E}\left(\frac{V_{cc}}{R_1}(R_1||R_2) - V_{BE,on}\right) \tag{17}$$

This current can be sensitive to each of the listed factors, but certain conditions can be met to reduce this sensitivity. For the current to be insensitive to β, $\beta >> 1$. For the current to be insensitive to $V_{BE,on}$, $V_B >> V_{BE,on}$. For the current to be insensitive to $R_1||R_2$, $R_1 >> (R_1||R_2)$.

The equivalent small-signal model of the Colpitts Oscillator is shown in Figure 6. The schematic on the left shows the oscillator with each individual part shown while the schematic on the right shows the simplified version of it.

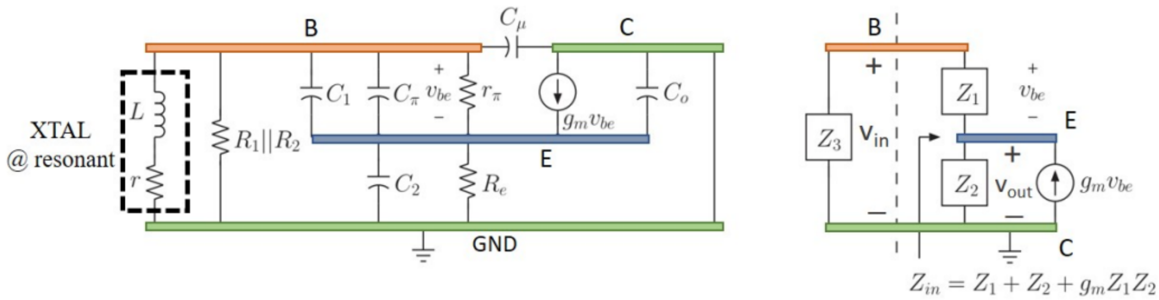


Figure 6: Colpitts Oscillator Small Signal Model

Using these schematics, the expressions for Z_1, Z_2, and Z_3 can be determined to be as follows.

$$Z_1 = (\frac{1}{j\omega(C_1 + C_\pi)}||r_\pi) \tag{18}$$

$$Z_2 = (\frac{1}{j\omega(C_0 + C_2)}||R_E) \tag{19}$$

$$Z_3 = (\frac{1}{j\omega C_\mu}||(j\omega L + r)||\frac{R_1 R_2}{R_1 + R_2}) \tag{20}$$

The loop gain of the circuit can be determined by cutting the feedback portion above Z_1 and terminating with Z_3. The circuit to calculate the loop gain is shown in Figure 7.

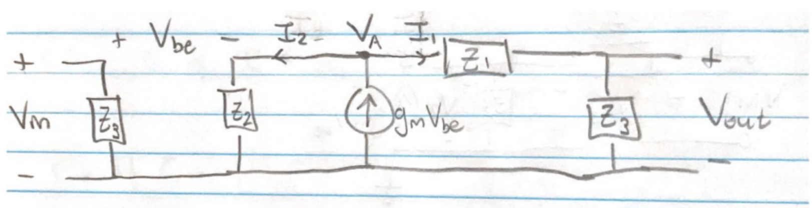


Figure 7: Colpitts Oscillator Small Signal Loop Gain Model

From this circuit we know the following.

$$V_{in} = V_{be} + V_A \tag{21}$$

$$g_m V_{be} = g_m(V_{in} - V_A) = I_1 + I_2 = \frac{V_A}{Z_2} + \frac{V_A}{Z_1 + Z_2} = \frac{Z_1 + Z_2 + Z_3}{Z_2(Z_1 + Z_3)} V_A \tag{22}$$

$$V_A = \frac{g_m}{\frac{Z_1+Z_2+Z_3}{Z_2(Z_1+Z_3)} + g_m} V_{in} \tag{23}$$

$$V_{out} = Z_3 I_1 = \frac{Z_3}{Z_1 + Z_3} V_A = \frac{Z_3}{Z_1 + Z_3} \frac{g_m}{\frac{Z_1+Z_2+Z_3}{Z_2(Z_1+Z_3)} + g_m} V_{in} \tag{24}$$

$$A = \frac{V_{out}}{V_{in}} = \frac{-g_m Z_1 Z_2}{Z_1 + Z_2 + Z_3} \tag{25}$$

3 Procedure

This lab requires use of the Keysight ADS software for the virtual portion. A soldering iron is then needed along with a prototyping board and several THT components used for the construction of the oscillator. Finally, the in-person portion of the lab requires use of the following lab equipment:

- Spentrum Analyzer
- DC Power Source
- Digital Multimeter

3.1 Quartz Crystal Characterization

The virtual lab portion begins by analyzing the Crystal Oscillator's equivalent circuit similar to the procedure from Lab 1. The S-parameters for the 10.2X2 Xtal is provided. The component within the Lab Kit is the 10.2X7 Xtal which has the same properties and was only manufactured at a different time. The equivalent circuit is of the Xtal is shown in Figure 1. The values of each component within this circuit can be determined by anayzing the resistance and reactance measurement at different frequencies. From here, the parameters are tuned to best match the measured impedance of the Xtal. This equivalent model can then be used to determine the required load capacitance for oscillation at $10.245MHz$ which may be different from the load specified in the datasheet for the Xtal.

3.2 Colpitts Oscillator

The second part of the virtual lab involves preparing and building the Colpitts Oscillator. The circuit schematic for this oscillator is shown in Figure 5 along with the small-signal equivalent circuit in Figure 6. The component values R_1, R_2, C_1, C_2, and C_L are chosen to build an oscillator with the goal of oscillation at $10.245MHz$. The resistors are chosen to create a large parallel resistance which will keep the quality factor of the oscillator high. The capacitance in series with the load is added to make the equivalent parallel output resistance large. Without the series capacitance, the output resistance will be the 50Ω resistance of the measurement device. A larger equivalent parallel resistance keeps the emitter resistance at an acceptable value. The capacitors are chosen so the parallel capacitance (equal to the load capacitance) is about equal to that specified by the datasheet for the Xtal creating the desired resonant frequency and allowing the oscillation to occur at $10.245MHz$. With these choices made, the circuit is then built on the prototyping board using actual components.

3.3 Measuring the Crystal Oscillator

The in-person portion of the lab tests the performance of the oscillator. The device is connected to a 12V power source and the output is measured using the spectrum analyzer. This output can be used to determine the quality of the device.

4 Results and Observations

4.1 Quartz Crystal Characterization

The measured S-parameter values for the Xtal are plotted and compared with the model S-parameters values in Figure 8.

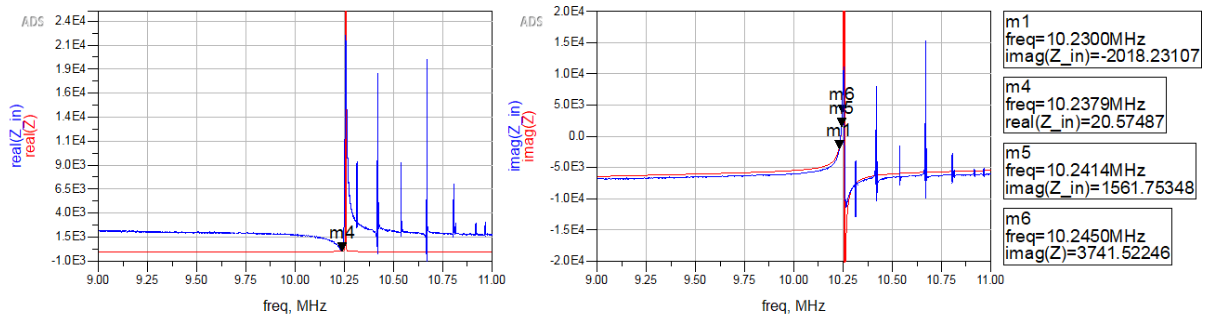


Figure 8: Crystal Oscillator Model Narrow Band Impedance

The parallel capacitance can be determined by examining the measured reactance at the lowest measured frequency.

$$X|_{f=7MHz} = -8424 \approx X_p = -\frac{1}{\omega C_p} \tag{26}$$

$$C_p \approx 2.7pF \tag{27}$$

The resonant frequencies of the component can be determined by examining the impedance. The parallel resonant frequency occurs at an impedance maximum while the series resonant frequency occurs at an impedance minimum.

$$f_s \approx 10.239MHz \tag{28}$$

$$f_p \approx 10.257MHz \tag{29}$$

The series resistance can be determined by examining the resistance at the series resonant frequency. The reactance at this frequency should be near zero and the resistance should be due mainly to the series resistance.

$$R_s \approx 17\Omega \tag{30}$$

The series capacitance and the series inductance can be determined using the resonant frequencies mentioned above and the calculations in the Lab Manual [1]. Using these equations, the series capacitance can be found to be

$$C_s = C_p((\frac{f_p}{f_s})^2 - 1) \approx 9.501fF. \tag{31}$$

The series inductance can then be determined using the series resonant frequency.

$$L_s = \frac{1}{C_s(2\pi f_s)^2} \approx 25.4mH \tag{32}$$

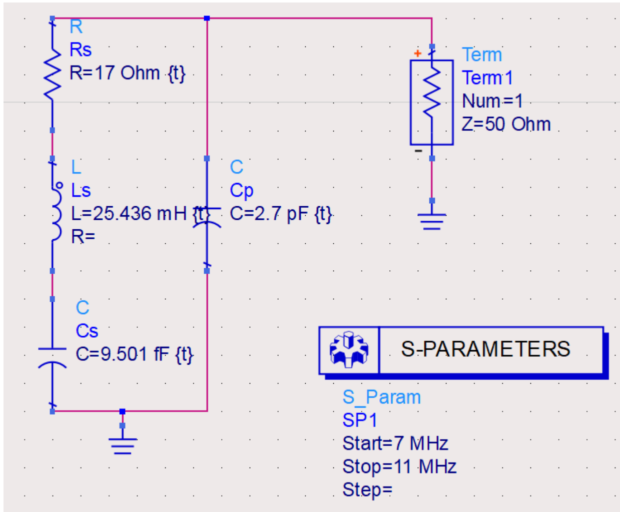


Figure 9: Crystal Oscillator Equivalent Circuit Model

These parameters are then tuned to best match the resistance and reactance of the measured component. The final schematic is shown in Figure 9.

The quality factor of the device is about equal to the Q value obtained from a series RLC circuit. The Q value is thus

$$Q = \frac{2\pi f_s L_s}{R_s} \approx 125700. \tag{33}$$

The impedance at the desired oscillation frequency can be used to determine the needed load capacitance to allow for the Xtal to resonate at that frequency. The reactance at $10.245MHz$ can be measured to be $X = 4084.6\Omega$. At this frequency the Xtal can be seen as an inductor and resistor in parallel. An additional capacitance in series acts as the load capacitance. The resonant frequency then occurs when the reactance due to the capacitance is the opposite of that due to the inductance.

$$X_C = -X_L = -4084.6\Omega \tag{34}$$

$$C = \frac{1}{\omega X_L} \approx 3.903pF \tag{35}$$

This value varies significantly from the load capacitance specified by the datasheet of $10pF$. This is due to the rapid change in impedance with small changes in frequency. A capacitance of $10pF$ corresponds to a reactance of $X_C = -1553.5\Omega$ which is matched by the Xtal at about $f = 10.2414MHz$. This is a characteristic of high Q components allowing for a narrow range of frequencies where resonance may occur. The narrowband impedance of the model is shown in Figure 8. The wideband impedance is shown in Figure 10.

4.2 Colpitts Oscillator

The C_L or the capacitance in series with the output is chosen as $0.1\mu F$ as this is one of the few options available within the lab kit. The DC bias of the oscillator has a desirable collector current around $1mA$. This leads to a transconductance of $g_m \approx 0.038$ which is

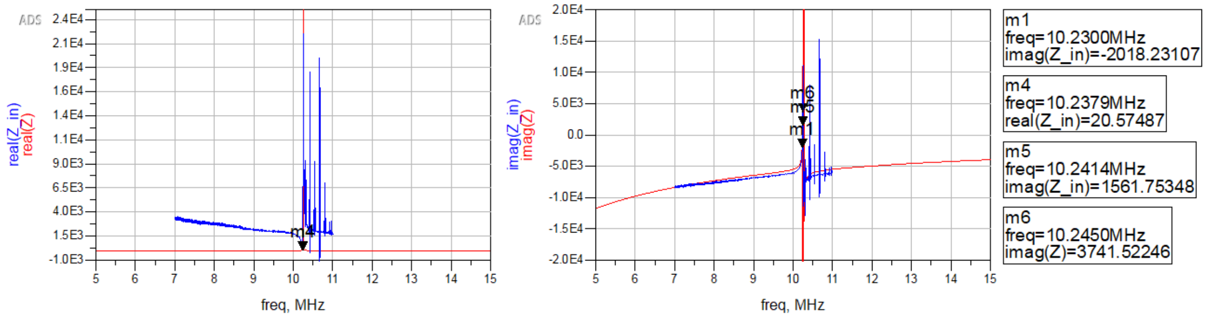


Figure 10: Crystal Oscillator Model Wideband Impedance

within the range to lead to the desired loop gain. This can be achieved with a base voltage of about 6V, an emitter resistance of $R_E \approx 5k\Omega$, and a $\beta >> 1$. Since the power source for this circuit is 12V, R_1 and R_2 must be about equal. A large parallel combination of the resistances leads to the best output for the small-signal analysis of the model. The best choices available within the kit are then $R_1 = 82k\Omega$ and $R_2 = 100k\Omega$.

The capacitors are then chosen to have a parallel combination to match the load capacitance specified in the datasheet of the Xtal. The various options for the combinations of C_1 and C_2 available within the components in the kit are measured through the gain values shown in Table 1 The program used to calculate the gain is a Python script included in Appendix A.

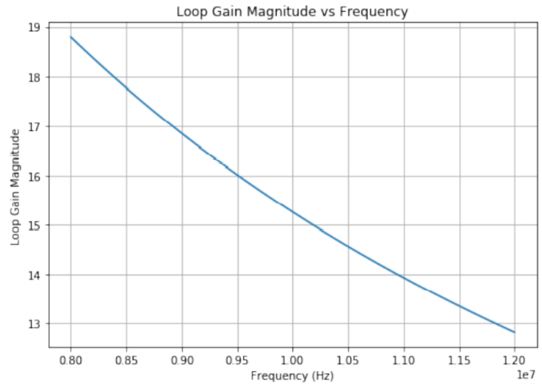
Table 1: Gain Calculation for Different Capacitor Combinations

C_1 (pF)	C_2 (pF)	$\lvert A_{lo}\rvert$	$\angle A_{lo}$ $(°)$
36	18	11.719	100.22
24	18	14.914	103.05
24	22	13.675	101.95

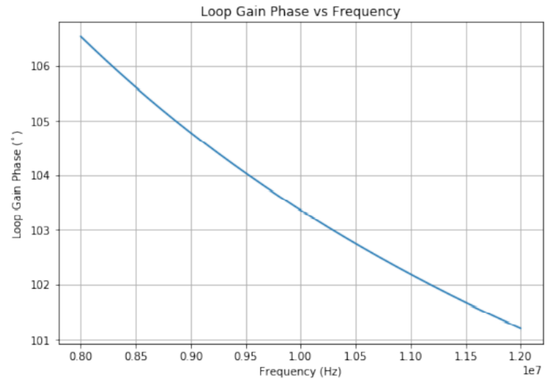
There is some error in the calculation of these gain values as the gain should be nearing $A_{lo} = 1\angle 0°$ with the desired gain calculation. Regardless, the confirmed optimal combination is $C_1 = 24pF$ and $C_2 = 18pF$. The corresponding loop gain magnitude and phase values for a range of frequencies are shown in Figure 11. With these component values determined, the circuit can be built on the prototyping board. The assembled circuit is shown below in Figure 12.

4.3 Measuring the Crystal Oscillator

The built circuit was then connected to the power source and spectrum analyzer in the in-person portion of the lab. This output does not show the expected oscillation effect. All that is shown is the reference peak at $f = 0Hz$ and noise surrounding it. This indicates some error in the assembly of the circuit with either poor solder connections or incorrect component interconnections. The expected output of the device has been provided by the course staff for analysis and can be seen in Figure 13.

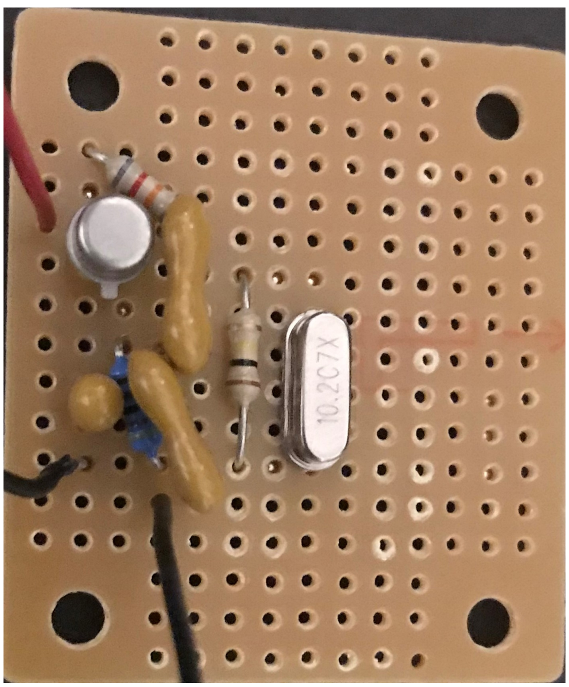


(a) Magnitude of Loop Gain

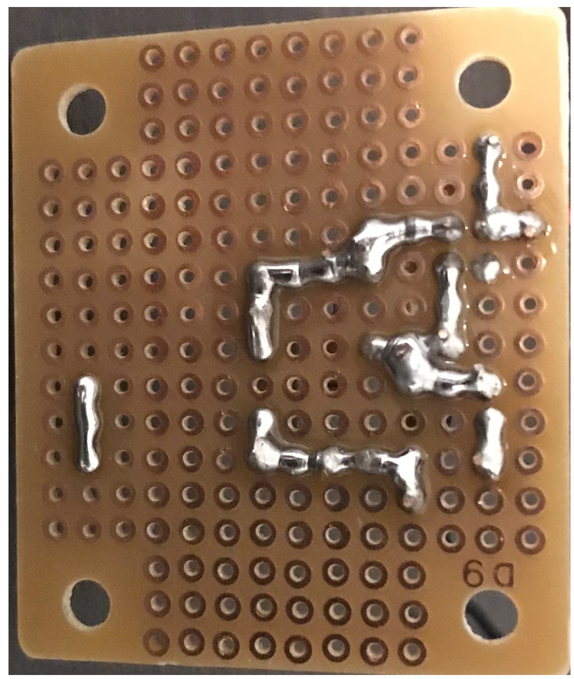


(b) Phase of Loop Gain

Figure 11: Loop Gain vs Frequency



(a) Component Side



(b) Soldered Side

Figure 12: Assembled Oscillator Circuit

The main peak can be seen at $f = 10.2MHz$. A higher resolution would allow for a more detailed view of the exact oscillation frequency. The power of this and the additional side harmonics can be seen in the peak table within the SA view in Figure 13. The total harmonic distortion of this output can be calculated by summing the power of the peaks with larger frequency then the fundamental frequency, then dividing by the total power of the output. The power of each peak must be converted to mW as two power measurements in dBm cannot be added to get the sum of their power. Additionally, there are many listed peak values around a single peak as small variations in measured power may lead to slight local maxima. Therefore, only the largest power value near the frequency of that peak will

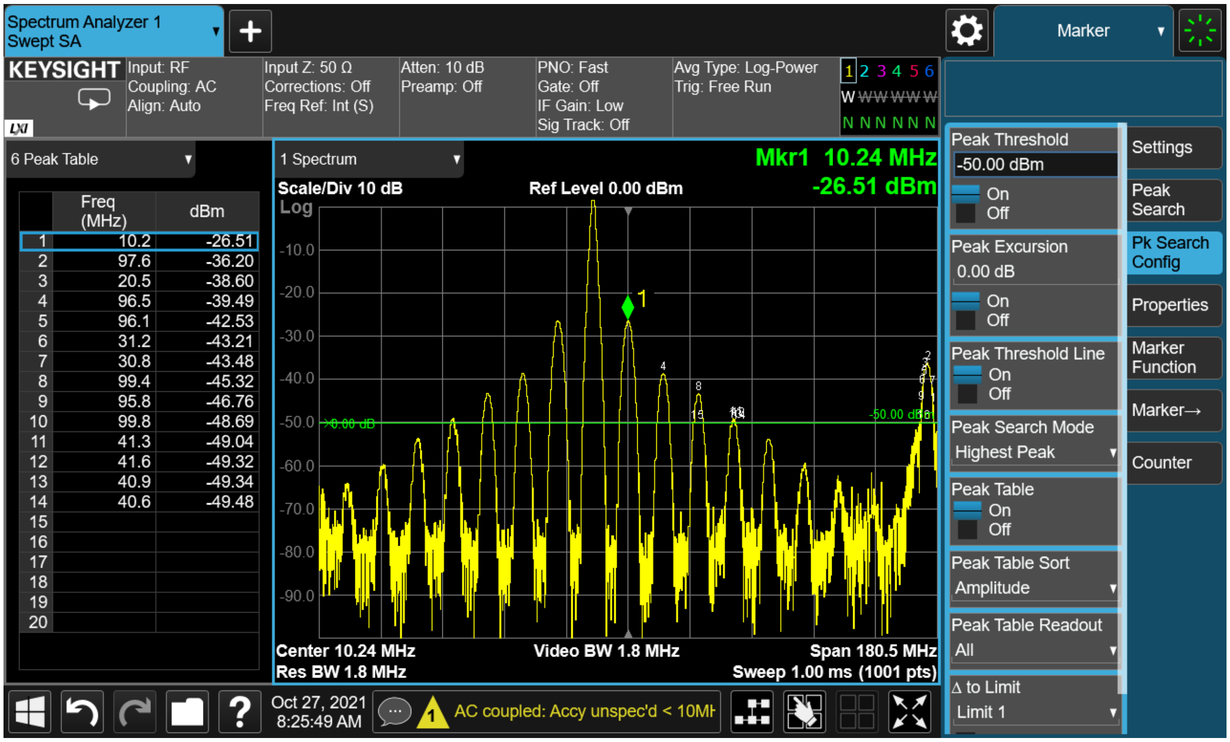


Figure 13: Expected Spectrum Analyzer Output

be included.

$$THD = \frac{P(97.6MHz) + P(20.5MHz) + P(31.2MHz) + P(41.3MHz)}{P(10.2MHz) + P(97.6MHz) + P(20.5MHz) + P(31.2MHz) + P(41.3MHz)} \tag{36}$$

$$THD = \frac{10^{\frac{-36.20}{10}} + 10^{\frac{-38.60}{10}} + 10^{\frac{-43.21}{10}} + 10^{\frac{-49.04}{10}}}{10^{\frac{-26.51}{10}} + 10^{\frac{-36.20}{10}} + 10^{\frac{-38.60}{10}} + 10^{\frac{-43.21}{10}} + 10^{\frac{-49.04}{10}}} \tag{37}$$

$$THD = 0.1640 = 16.40\% \tag{38}$$

5 Conclusion

The work in this lab had several errors or unexpected outputs. The model built within ADS matches the general shape of the measured Xtal, but varies at frequencies away from the fundemental frequency and varies when viewed very close to the fundemental frequency. A more accurate model may be built with more components included, but for the sake of simplicity and theory, this model functions well. The calculation of the gain of the oscillator as shown in Appendix A has some flaws. The gain should near a magnitude of 1 and a phase of $0°$ at the fundemental frequency as specified by the Lab Manual [1]. The built prototype circuit also did not function properly requiring further debugging or rebuilding as the oscillator will be important for future work.

The reasoning used behind the resistors was explained within the results portion of the report. The resistor choices match the equations and theory as a larger parallel resistor combination value will lead to a larger Q of the oscillator. The resistor values should also be about equal as the desired base voltage is about half the source voltage. The capacitor choices do not match the recorded values of loop gain. Other combinations of capacitors lead to a loop gain closer to the desired value of $1\angle 0°$, but this is likely due to the inaccuracy of the code used to calculate the gain. The capacitors used within this lab were chosen mainly due to suggestions by the course staff.

The calculated values vary significantly than the expected values. Since no oscillation frequency was able to be determined, that value cannot be compared. Regardless, the load capacitance varied significantly and the calculated gain values varied significantly. This is likely in part due to error, but also due to the inconsistency due to tolerance of components. The high Q oscillator allows for these variations without changing the oscillation frequency significantly though.

One potential limitation of the Xtal for RF applications would be the narrow range of frequencies that could be reached through tuning or adjustments. As has been explored in this lab, the Q of the Xtal Oscillator is very large meaning large changes in the load capacitance may not have a large effect on the oscillation frequency. This leads to part tolerance not being significant but makes it very difficult to tune the oscillation frequency to a different value even with variable capacitors involved.

References

[1] ECE Course Staff, *ECE 453 Wireless Communication Systems: Experiment 05 - Colpitts Crystal Oscillator*, University of Illinois, Urbana, IL, 2021.

Completed by: Timothy Green **On:** October 15th, 2021

6 Appendix A

```
import numpy as np
import matplotlib.pyplot as plt
import cmath

%matplotlib inline

# Defined values
R_s = 31  #Ohms
L_s = 25.436e-3 #H
C_s = 9.501e-15 #F
C_p = 2.7e-12 #F

V_cc = 12 #V
R_1 = 82e3 #Ohms
R_2 = 100e3 #Ohms
R_E = 4.7e3 #Ohms
C_1 = 24e-12 #F
C_2 = 22e-12 #F

beta = 100
C_mu = 1e-3 #F
V_BE = 1 #V
V_T = 26e-3 #V

f_start = 8e6 #Hz
f_0 = 10.245e6 #Hz
f_stop = 12e6 #Hz
steps = 1000
f = np.linspace(f_start, f_stop, 1000)

C_L = (C_1*C_2)/(C_1+C_2)
f_s = 1/(2*cmath.pi*cmath.sqrt(C_s*L_s));
f_p = f_s*cmath.sqrt(1+(C_s/(C_p+C_L)));

# Find the BJT DC Bias
V_BQ =
((R_2*R_E*(beta+1)*V_cc)+(R_1*R_2*V_BE))/((R_1*R_E*(beta+1))+(R_2*R_E*(beta+1))+(R_1
*R_2));
I_BQ = (V_BQ-V_BE)/(R_E*(beta+1));
I_CQ = beta*I_BQ;
g_m = I_CQ/V_T;
r_pi = beta/g_m;

w = np.zeros(steps)
Z_xtal = np.zeros(steps, dtype=complex)
L_xtal = np.zeros(steps)
Z_1 = np.zeros(steps, dtype=complex)
Z_2 = np.zeros(steps, dtype=complex)
Z_3 = np.zeros(steps, dtype=complex)
A = np.zeros(steps, dtype=complex)
```

```
A_mag = np.zeros(steps)
A_ang = np.zeros(steps)

for i in range(0, steps):
    w[i] = 2*cmath.pi*f[i]; #rad/s

    # Calculate the impedance and frequencies of the Xtal
    Z_xtal[i] =
((R_s+(1j*w[i]*L_s)+(1/(1j*w[i]*C_s)))*(1/(1j*w[i]*C_p)))/(R_s+(1j*w[i]*L_s)+(1/(1j*
w[i]*C_s))+(1/(1j*w[i]*C_p)))
    L_xtal[i] = Z_xtal[i].imag/w[i];

    # Find the three small signal impedances
    Z_1[i] = ((1/(1j*w[i]*C_1))*r_pi)/((1/(1j*w[i]*C_1))+r_pi);
    Z_2[i] = ((1/(1j*w[i]*C_2))*R_E)/((1/(1j*w[i]*C_2))+R_E);
    res = (R_1*R_2)/(R_1+R_2);
    imp = (res*Z_xtal[i])/(res+Z_xtal[i]);
    Z_3[i] = imp/(1+(1j*w[i]*C_mu*imp));

    # Find loop gain
    A[i] = (-1*g_m*Z_1[i]*Z_2[i])/(Z_1[i]+Z_2[i]+Z_3[i]);
    A_mag[i] = abs(A[i])
    A_ang[i] = cmath.phase(A[i])*180/cmath.pi

#calculate for the desired frequency now
w_0 = 2*cmath.pi*f_0; #rad/s

# Calculate the impedance and frequencies of the Xtal
Z_xtal_0 =
((R_s+(1j*w_0*L_s)+(1/(1j*w_0*C_s)))*(1/(1j*w_0*C_p)))/(R_s+(1j*w_0*L_s)+(1/(1j*w_0*
C_s))+(1/(1j*w_0*C_p)))
L_xtal_0 = Z_xtal_0.imag/w_0;

# Find the three small signal impedances
Z_1_0 = ((1/(1j*w_0*C_1))*r_pi)/((1/(1j*w_0*C_1))+r_pi);
Z_2_0 = ((1/(1j*w_0*C_2))*R_E)/((1/(1j*w_0*C_2))+R_E);
res = (R_1*R_2)/(R_1+R_2);
imp = (res*Z_xtal_0)/(res+Z_xtal_0);
Z_3_0 = imp/(1+(1j*w_0*C_mu*imp));

# Find loop gain
A_0 = (-1*g_m*Z_1_0*Z_2_0)/(Z_1_0+Z_2_0+Z_3_0);
A_mag_0 = abs(A_0)
A_ang_0 = cmath.phase(A_0)*180/cmath.pi

print(A_mag_0)
print(A_ang_0)

# Make some plots
plt.figure(figsize=(8,12))
```

```
plt.subplot(2,1,1)
plt.title("Loop Gain Magnitude vs Frequency")
plt.xlabel("Frequency (Hz)")
plt.ylabel("Loop Gain Magnitude")
plt.grid()
plt.plot(f, A_mag)
plt.subplot(2,1,2)
plt.title("Loop Gain Phase vs Frequency")
plt.xlabel("Frequency (Hz)")
plt.ylabel("Loop Gain Phase ($^\circ$)")
plt.grid()
plt.plot(f, A_ang)
```