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# AC Power Flow Importance Measures Considering Multi-element Failures

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**Abstract**:

Quantifying the criticality of individual components of power systems is essential for overall reliability and management. This paper proposes an AC-based power flow element importance measure, while considering multi-element failures. The measure relies on a proposed AC-based cascading failure model, which captures branch overflow, bus load shedding, and branch failures, via AC power flow and optimal power flow analyses. Taking the IEEE 30, 57 and 118-bus power systems as case studies, we find that *N*-3 analyses are sufficient to measure the importance of a bus or branch. It is observed that for a substation bus, its importance is statistically proportional to its power demand, but this trend is not observed for power plant buses. While comparing with other reliability, functionality, and topology-based importance measures popular today, we find that a DC power flow model, although better correlated with the benchmark AC model as a whole, still fails to locate some critical elements. This is due to the DC-based models focus on real power and ignoring reactive power. The proposed importance measure is aimed to inform decision makers about important components in complex systems, while improving cascading failure prevention, system backup siting, and overall resilience.

**Keywords**: AC power flow, *N*-*k* reliability, importance ranking, cascading failure, functional reliability

## 1. Introduction

Modern society depends on critical infrastructure systems, such as electricity, water, gas and oil, transportation, and telecommunication networks, among others. Electric power systems are particularly critical because the operation of most other infrastructure systems relies on stable and reliable supply of electricity [1]. Besides, disruptions in bulk power grids can result in diverse consequences including economic, social, physical, and psychological impacts. The purpose of power system functional reliability assessment and risk quantification studies is to probabilistically estimate various indices that measure the adequacy of power systems for supplying the total load with minimum cost. Hence, analyzing the importance or criticality of individual components based on their contribution to system-level reliability is of importance for a variety of purposes such as prioritizing resources based on rankings, identifying root causes of outages, improving maintenance scheduling, informing expansion planning, increasing reliability and resilience, preventing cascading failures, and optimizing restoration, among others [2–5].

Consistent with the available literature, this paper associates component importance quantification and ranking to a disruptive event, and quantifies importance as the performance drop of a power system under the failure event of the component(s) in question. Hence, to quantify component importance, adequate models to simulate the *dynamical* processes of power grids are necessary, with and without disruptive events, to quantify component importance based on system-level performance drops.

For simulating power system dynamics (and cascading failure processes), as well as to assess system reliability and quantify component importance, scholars and practitioners have adopted Direct Current (DC) power flow models, which are a tractable relaxation of the accurate Alternating Current (AC) model, to simulate the dynamics of power grids according to power balance equations and electrical engineering constraints. Because of its linear features, DC flow models are computationally efficient and as a result widely applied (and attractive for probabilistic studies). . However, DC power flow models sometimes fail to approximate the actual cascading process since a number of assumptions that govern the DC model could be violated. In particular, DC models ignore the reactive power balance equations, assume all voltage magnitudes identically equal to one per unit, ignores line losses, and ignores tap dependence in the transformer reactance. Hence, AC-based models are presented [6]. Nedic et al. [7] verified and examined criticality of a 1,000 bus network with an AC blackout model that represents many of the interactions in cascading failures. However, generation adjustment is not taken into consideration in this model. Hence, Mei et al. [8] developed a novel approach to study the cascading failures and blackouts in power systems based on AC-OPF and self-organized criticality theory, while both fast and slow dynamics processes are considered. As an alternative, and with the aim of focusing on cascading processes this paper adopts an AC power flow model along with AC optimal power flow considerations, and proposes an AC-based model to simulate the cascading failures of power systems so as to better inform resource prioritization. The AC-based cascading failure approach in this paper not only relies on more accurate power flow assessments, but also corrects important phenomena inherent to the DC flow analyses, such as the DC-based model tendency to overestimate the importance of high load nodes and underestimate low load nodes.

With power system dynamics computed, a method to measure the importance of each component based on the performance drop is desirable. Since the early work on importance measures in the 1960s [9], various importance measures (IMs) have been investigated to judge the relative strength of components in a system with respect to different criteria. Among others, topological importance measures, which mainly originate from network science, are among the most widely used ones, including degree centrality, closeness centrality [10], betweenness centrality [11], information centrality [12], PageRank [13], Hypertext Induced Topic Selection algorithm (HITS) [14], and so on. Besides the topological IMs, structural importance measures [15], lifetime importance measures [2], resilience importance measures [3,4], risk importance measures [16,17] and reliability importance measures [18,19] are also applied in different disciplines.

Taking the risk importance measures as examples, there are Risk reduction (RR) measures, Risk reduction worth (RRW) measures, Fussel-Vesely (FV) importance metrics [20], risk achievement (RA) measures, risk achievement worth (RAW) metrics, as well as Birnbaum importance (BI), and partial derivative (PD) measure [21], among others [22]. Some of these importance measures could easily be transformed into performance-based importance measures. For example, RA, RAW and FV are computed by $P(\text{base})-P(i=0)$, $P(\text{base})/P(i=0)$, and $[P(\text{base})-P(i=0)]/P(\text{base})$, where $P(\text{base})$ is the performance of the original system and $P(i=0)$ is the system performance when element $i$ is removed from the system. These measures work well for most scenarios, but not all. For example, power systems are typically managed to satisfy $N$-1 contingencies, which means the failure of any one component will not result in any performance decrease (except for the loss of the component itself), and as a result, it is not possible to distinguish component importance levels. In addition, incidents are usually caused by combinations of failures. This means the rankings of some components are not comparable or are the same. To avoid such situations, this paper proposes an AC-based power flow element importance measure that considers the failures of multiple elements in the system.

The proposed element importance measure not only minimizes importance ranking ties, but also applies an AC-based power flow cascading failure model to capture unique phenomena across power systems. With the AC-based power flow cascading model, power flows better reflect system performance and reliability, and, as a result, element importance could be assessed more accurately. Hence, the proposed importance measure can assist decision makers and utilities to explicitly locate key elements in the system, and more efficaciously execute reliability improvements.

The rest of this paper is structured as follows: in Section 2, this paper proposes an AC-based power flow cascading model to simulate the power flow and cascading process given that some components are removed from the system. Then, an element importance measure is presented considering *multi-element* failures. Section 3 gives a brief introduction to the cases studied in this paper. Section 4 discusses how loads (of substations and generators) and maximum supplies (of generators) impact the importance and rankings, and shows relationships between bus importance and branch importance. In Section 5, this paper compares several widely used reliability- and topology-based importance measures as well as DC-based power flow importance measures with the proposed AC-based power flow importance measure. Finally, Section 6 provides general conclusions and ideas for future research.



## 2. AC Power Flow importance measure considering multi-element failures

Our first step is to find a way to simulate power flow before measuring element importance. This section summarizes the fundamental assumptions and definitions used for such a purpose, and then proposes a cascading failure model based on AC power flow analysis. Afterwards, we introduce the element importance metric.

### *2.1 AC-based cascading failure model*

In this paper, an importance measure is proposed under the following assumptions (which hold true throughout unless stated otherwise):

(1) The status of a branch is binary: up (functional) or down (failed). Note that the supply and demand of a generator or substation bus could have a continuous value.

(2) Repair is not performed during cascading failures.

(3) Failure of a bus disconnects all the transmission lines that connect to it.

With the above assumptions, this paper uses MATPOWER, an open-source MATLAB power system simulation package [23], to simulate AC and DC power flow and optimal power flow (OPF).

Different from most of the current cascading failure models, this paper develops a model based on AC power flow and AC optimal power flow, so that the model could adequately approximate the actual cascading process. We call this model AC-based cascading failure (ACCF) model. In the model, one or a certain number of elements fail initially given a contingency, and this operation may disconnect the grids. As a result, the first step of the ACCF is to identify each subgraph (or island) of the grids to ensure feasibility of operation (flowchart shown in Fig. 1). For each island, the model does the following:

Check if there is at least one generator in the island (S1). If no, the demands in this island will not be supplied (S2). If yes, then check whether there is only one bus in the island (S3). If there is only one bus, then the real and reactive supplied demand will be min($P_i^{\text{max}}$, $D_i^{\text{real}}$) and min($Q_i^{\text{max}}$,$D_i^{\text{reactive}}$), respectively (S4). If there is more than one bus, it is necessary to assign a slack bus in the island if there is no such an assignment in the current island (S5). In this paper, we assign the slack bus as the generator with the smallest ID number, just as done in test networks, such as the IEEE 30 and 57-bus systems.

Then, the model runs an AC optimal power flow (ACOPF) and saves the results (S6). However, the ACOPF may not converge, and this may indicate that the system would suffer from a voltage collapse if the operator does not take action [24]. The model assumes that the operators have enough time and understanding to implement load shedding in an attempt to arrest this voltage collapse (S7). As a result, the model runs AC power flow (ACPF), and checks the overflowed branches (S8). Following Nedic et al. [7], the model sheds 5% load in the area with a mismatch in each time step (S9). This load-shedding is repeated until convergence of the optimal power flow is achieved (S10). If convergence has not been achieved after 20 load-shedding steps, which means all the load in the mismatch area is shed, the system is deemed to have collapsed, which means the overloaded branches will be removed (S11).

Once overloaded branches are removed, the model will repeat the above process that begins with identifying each island. Only if all the islands achieve a stable power flow, the simulation ends.

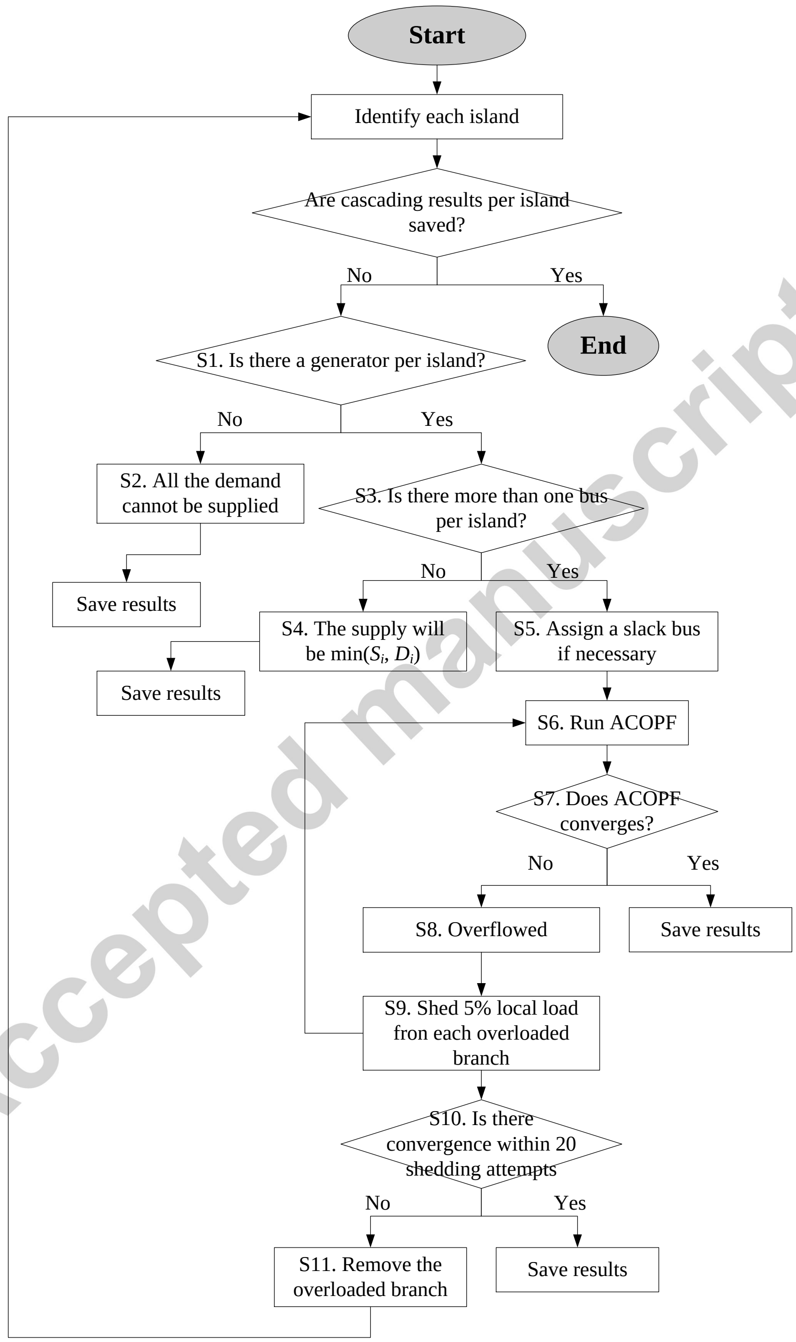



Fig. 1 AC-based cascading failure model (ACCF)

*2.2 Element importance measures considering multi-element failures*

Building upon the Fussell-Vesely importance ideas [25], this paper proposes an importance measure considering system performance decrease ratios, so as to quantify element importance while certain elements are removed from the networks.

It is naive to only consider the *N*-1 problem while measuring element importance as the computed importance might be partial or without discerning well across elements. As a result, a more general *N*-*k* problem (for small *k*) is considered in this paper. One could consider the failure of different types of elements, including buses and branches. The importance measures proposed in this paper work for both types.

Suppose $k$ elements are removed from the grids, then the ACCF model simulates power flow $m$ times, where $m = C(N,k) = N!/(k!\times(N-k)!)$. An $m\times k$ matrix $R$ is introduced to store the possible configurations of removed elements. The importance of the $i^{\text{th}}$ element is measured by

$$IM_i^{N-k} = \frac{PE_0 - PE_i^{N-k}}{PE_0}, \tag{1}$$

where $PE_0$ is the system performance under the default configuration, and $PE_i^{N\text{-}k}$ is the average system performance among all the scenarios in which element $i$ is removed (along with other $k$-1 elements). Here the system performance is the power demand that could be supplied in the system. $PE_i^{N\text{-}k}$ is computed via the following function,

$$PE_i^{N-k} = \frac{1}{C(N-1,k-1)} \sum_{j=1}^{C(N,k)} PE_j \cdot W_{i,j}\ , \tag{2}$$

where $PE_j$ is the system performance of the $j^{\text{th}}$ removal out of $C(N,k)$ possible removals ($N > 1$). $W_{i,j}$ captures whether element $i$ is removed in the $j^{\text{th}}$ removal, which is computed by

$$W_{i,j} = \begin{cases} 1, & i \in R(j,:) \\ 0, & i \notin R(j,:) \end{cases}. \tag{3}$$

With Eq. (1)-(3), the absolute importance under *N*-*k* of the $i^{\text{th}}$ element can be captured, and in the following we will first normalize the element importance, as follows:

$$IM_{i,norm}^{N-k} = \frac{IM_i^{N-k}}{\max(IM^{N-k})}, \tag{4}$$

where $IM^{N-k} = [IM_1^{N-k}, IM_2^{N-k}, \ldots, IM_i^{N-k}, \ldots, IM_N^{N-k}]$ and $IM_{i,norm}^{N-k}$ is the normalized element importance. The element importance considering *N*-1 to *N*-*k* can be captured by

$$IM_i^{N-k'} = IM_{i,norm}^{N-1} + IM_{i,norm}^{N-2} + \ldots + IM_{i,norm}^{N-k}. \tag{5}$$

With the above element importance measure, critical elements can be identified with reduced ambiguity, accounting for their role in a variety of contingencies, as well as in topology and functional power flow. This paper will apply this approach to practical grids, including the IEEE 30, 57 and 118-bus systems. A brief introduction to these power grids is presented next.

## 3. Case study

In this paper, the IEEE 30, 57 and 118-bus systems are used as case studies. The IEEE 30, 57 and 118-bus test cases represent a portion of the American Electric Power in the early 1960's. The grids' data, summarized in Table 1, is taken from MATPOWER v5.1 [23], and all the power flow analyses in this paper is based on this version too.

Among the grids, the IEEE 30-bus system contains 6 generators, 24 substations, 20 loads, and 41 lines; the IEEE 57-bus system contains 7 generators, 50 substations, 42 loads, and 80 lines; and the IEEE 118-bus system contains 54 generators, 64 substations, 99 loads, and 186 lines.

Table 1 IEEE 30, 57 and 118-bus test systems [23]

| Grids | Items | No. | Real Power (MW) | | | Reactive Power (MVAr) | | | Capacity (MVA) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Total | Min. | Max. | Total | Min. | Max. | Total | Min. | Max. |
| IEEE 30 | Generators | 6 | 335 | 30 | 80 | 405.9 | -20 | 150 | - | - | - |
| | Load | 20 | 189.2 | 2.2 | 30 | 107.2 | 0.7 | 30 | - | - | - |
| | Branches | 41 | - | - | - | - | - | - | 1954 | 16 | 130 |
| IEEE 57 | Generators | 7 | 1975.9 | 100 | 575.9 | 699 | -150 | 200 | - | - | - |
| | Load | 42 | 1250.8 | 1.6 | 377 | 336.4 | 0.5 | 88 | - | - | - |
| | Branches | 80 | - | - | - | - | - | - | 7702 | 16 | 500 |
| IEEE 118 | Generators | 54 | 9966.2 | 100 | 805.2 | 11777 | -1000 | 1000 | - | - | - |
| | Load | 99 | 4242 | 2 | 277 | 1438 | 0 | 113 | - | - | - |
| | Branches | 186 | - | - | - | - | - | - | 43148 | 16 | 4000 |

## 4. AC-based power flow importance rankings

Prior to conducting *N*-*k* importance analysis, it is necessary to find a suitable *k*, because a *k* that is too small (e.g. $k = 1$) may result in tied rankings, while a *k* that is too large renders the *N*-*k* problem computationally intractable [26]. We start with the IEEE 30-bus system as a reference to seek a suitable *k* for the *N*-*k* enumeration, while confirming with similar trends form the IEEE 118-node system.

According to the proposed AC-based cascading failure model (ACCF) in Section 2.1, this study manually removes *k* elements (buses and branches alike) from the grids, then simulates the power system cascading failure process, and monitors real and reactive power loss in the system (or power not supplied, PNS) as a result of the removal of $k$ elements. Then, the analyst computes the importance value of each element through Eq. (5), to finally rank the importance of each element in descending order (as most important, $2^{nd}$, …, $N^{th}$), consistent to the loss monitored. The bus importance (BuI) rankings and branch importance (BrI) rankings based on different $N$-$k$ measures ($1 \leq k \leq 5$) for the IEEE 30-bus system are shown in Fig. 2, where the range is based on practicality of computation. The ranking based on $N$-5 is set as the $X$-axis and the rankings based on the other $N$-$k$ levels are reflected in the $Y$-axis. It can be seen that for the $N$-1 analysis, many branches have the same importance value [as wrapped by the red oval in Fig. 2 (b)] and as a result the same importance ranking, which in turn highlights the necessity of $N$-$k$ based importance measures for $k > 1$. This is because the grids are designed to satisfy $N$-1 contingency, and the removal of *one* branch will not lead to major PNS to the system. It should be noted that the grid is not entirely $N$-1 contingency compliant as we still observe measurable PNS levels due to the removal certain branches, including {11, 15, 19, 22, 36, and 40}.

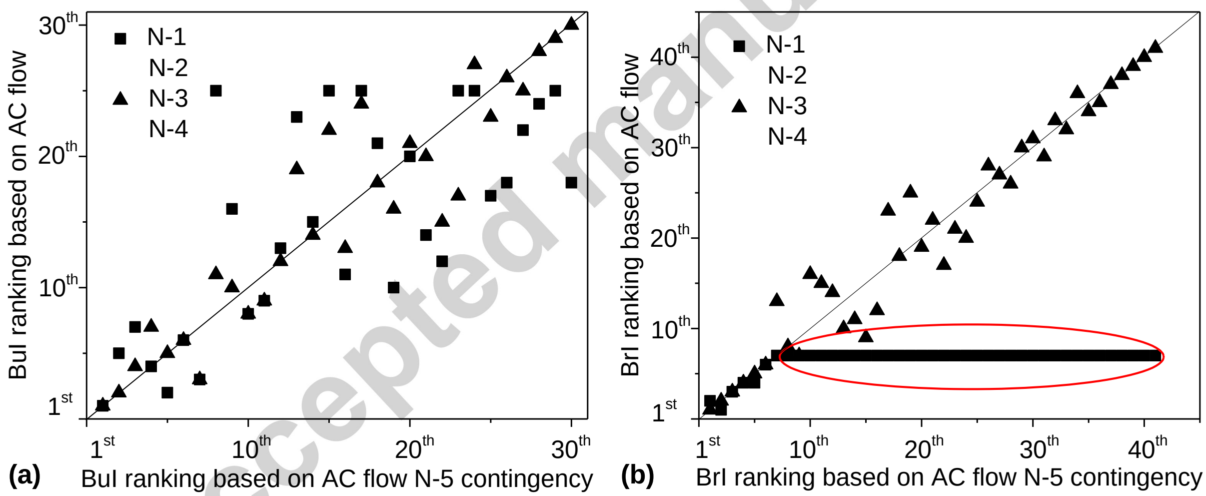



Fig. 2 Bus importance (BuI) rankings and Branch importance (BrI) rankings in terms of different $N$-$k$ for the IEEE 30-bus system: (a) BuI ranking; (b) BrI ranking



In Fig. 2, it is also observed that as $k$ increases from 1 to 4, the ranking tends to get closer to the ranking based on $N$-5. To quantify the correlation between rankings based on different $N$-$k$ levels, this paper uses the Pearson correlation coefficient. Tables 2 and 3 present the Pearson correlations between different $N$-$k$ removals for BuI rankings and BrI rankings, respectively. One can find that the rankings based on $N$-1 analysis always have low correlations with others, particularly with $N$-$k$ ($k$>2) analysis. The rankings tend to be high and stable with $N$-5 when $N$-3 is reached. The IEEE 118-bus system exhibits similar trends as shown in Tables 4 and 5, albeit for smaller $k$ levels, given that the system's state space is significantly larger.

Hence, this paper will apply *up to* $N$-3 analyses to explore element rankings on power grids, given the following reasons: (a) $N$-3 analysis will achieve practical rankings for feasible contingencies, (b) $N$-3 analysis is not computationally prohibitive even for large networks, and (c) the event of more than three components *initially failed* is less likely during operational contingencies, and if it happens it is highly correlated to $N$-3 analyses. Note that this reasoning excludes devastating natural disasters or coordinated terrorist attacks.

It is also observed that BrI rankings based on $N$-4 and $N$-5 are exactly the same, while BuI rankings are not. This is mainly because the importance of a branch (BrI) is related to the cascading failure size when the branch in question is removed (along with other $k$-1 branches if $k > 1$), while the importance of a bus (BuI) is related to not only the cascading failure size, but also to the power demand at the bus location. The power demand is fixed but not necessary of the same importance. Because of this, the correlation between $N$-$k$ and $N$-($k$+1) for BrI increases faster than for BuI as $k$ increases.

Table 2 Bus importance (BuI) ranking correlations among different $N$-$k$ removals for the IEEE 30-bus system

| Correlations | $N$-1 | $N$-2 | $N$-3 | $N$-4 | $N$-5 |
|---|---|---|---|---|---|
| $N$-1 | - | 0.93 | 0.83 | 0.75 | 0.70 |
| $N$-2 | 0.93 | - | 0.95 | 0.87 | 0.81 |
| $N$-3 | 0.83 | 0.95 | - | 0.96 | 0.93 |
| $N$-4 | 0.75 | 0.87 | 0.96 | - | 0.98 |
| $N$-5 | 0.70 | 0.81 | 0.93 | 0.98 | - |

Table 3 Branch importance (BrI) ranking correlations among different $N$-$k$ removals for the IEEE 30-bus system

| Correlations | $N$-1 | $N$-2 | $N$-3 | $N$-4 | $N$-5 |
|---|---|---|---|---|---|
| $N$-1 | - | 0.58 | 0.58 | 0.58 | 0.58 |
| $N$-2 | 0.58 | - | 0.94 | 0.85 | 0.85 |
| $N$-3 | 0.58 | 0.94 | - | 0.97 | 0.97 |
| $N$-4 | 0.58 | 0.85 | 0.97 | - | 1.00 |
| $N$-5 | 0.58 | 0.85 | 0.97 | 1.00 | - |

Table 4 Bus importance (BuI) ranking correlations among different *N*-*k* removals for the IEEE 118-bus system

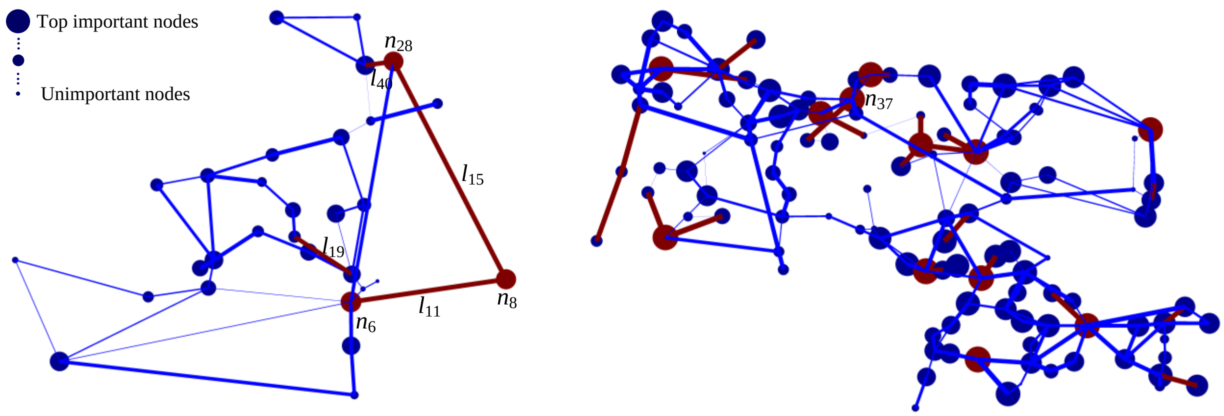
| Correlations | *N*-1 | *N*-2 | *N*-3 |
|---|---|---|---|
| *N*-1 | - | 0.968 | 0.937 |
| *N*-2 | 0.968 | - | 0.973 |
| *N*-3 | 0.937 | 0.973 | - |

Table 5 Branch importance (BrI) ranking correlations among different *N*-*k* removals for the IEEE 118-bus system

| Correlations | *N*-1 | *N*-2 | *N*-3 |
|---|---|---|---|
| *N*-1 | - | 0.422 | 0.422 |
| *N*-2 | 0.422 | - | 0.993 |
| *N*-3 | 0.422 | 0.993 | - |

With $k$ = 3 for practical *N*-*k* analyses, as well as the data in MATPOWER and the proposed AC and *N*-*k* based element importance proposed measures, one can compute the element relative importance for all the buses and branches with Eq. (5). Fig. 3 visualizes the importance of each bus and branch. The relative coordinates of each bus are approximations of the IEEE data provided by the University of Washington [27]. The IEEE 57-bus system coordinates are not available.

Taking the IEEE 30-bus grid as example, the authors find that buses $n_6$, $n_8$, and $n_{28}$ are the top 10% important nodes, and branches $l_{11}$, $l_{15}$, $l_{19}$ and $l_{40}$ are the top 10% links. Both the 10% buses and branches are labeled in Fig. 3 (a). Bus $n_8$ has the largest demand in the grids (30MW of real power and 30MVAr of reactive power), which makes it easy to justify its relevance. This also results in branches $l_{11}$ and $l_{15}$ having high importance because their capacity needs to route flow to bus 8. In addition, the failure of buses $n_6$ and $n_{28}$ will disconnected branches $l_{11}$ and $l_{15}$, thus ranking among the most important buses. These results show that the important buses tend to connect to important branches, or many branches, or both. The same reasoning and observations apply to the IEEE 118-bus system, as seen in Fig. 3 (b).




**(a) IEEE 30-bus system** **(b) IEEE 118-bus system**

Fig. 3 Bus and branch importance in the IEEE 30 and 118-bus systems: (a) IEEE 30-bus system, (b) IEEE 118-bus system. The size of a node denotes its relative importance, and the top 10% important buses are in red color. The thickness of a line denotes line importance, and the top 10% important branches are in red color.

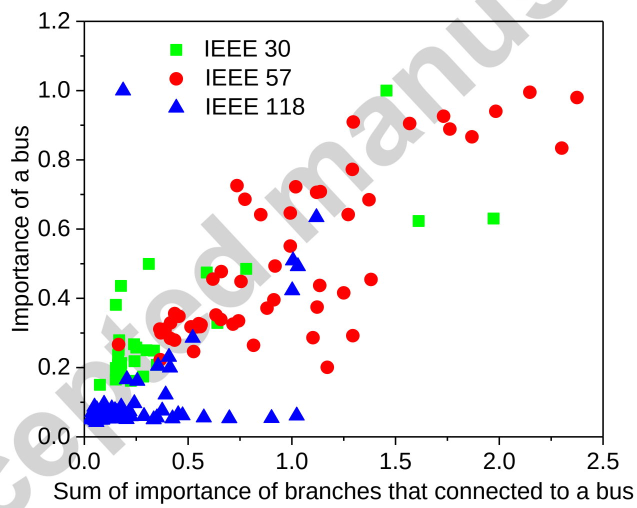
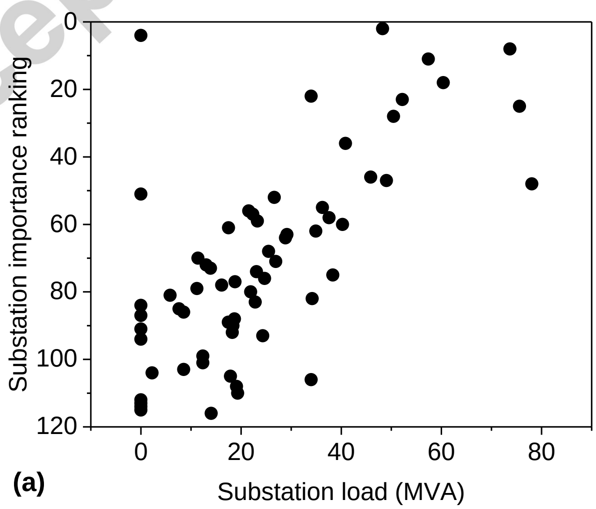



Fig. 4 Relationship between the importance of a bus and the sum of the importance of the branches that connect to the bus

To further confirm the bus-link correlation, this paper investigates the relationship between the importance of a bus and the sum of the importance of the branches that connected to the bus, as shown in Fig. 4. The observed trends relate to the removal of a bus which equates to the removal of all the connected branches of that bus along with the bus's supply and demand. Hence, an important bus is usually connected with many branches (whether they are important or unimportant) or to a few important branches.

We also detect how the functional bus load and power plant supply affect their rankings. There are only a few power plants in the IEEE 30 and 57-bus systems, making it hard to see the trend and distribution between the load (supply) and rankings. Hence, for demand and supply effects analyses, this paper focuses on the IEEE 118-bus system, as shown in Fig. 5. Naturally, a bus with large demand or supply is more important than a bus with small demand or supply. This is almost the case for substation nodes, as shown in Fig. 5 (a). But for power plant nodes, the ranking shows almost no correlation with the maximum supply [Fig. 5 (b)] or the supply based on OPF [Fig. 5 (c)].

To help explaining the previous results, let's revisit the model that quantifies the importance of a bus in the grids. It quantifies the importance of an element by removing the element, and monitoring the power not supplied (PNS). The PNS is composed of two parts: the demand of the bus that cannot be supplied, and the extra PNS due to cascading failures. For a bus, its removal will at least result in a PNS that equals the demand of the bus, and more PNS if a cascading failure happens. For a power plant, the cascading failure dominates the importance of the node, while the cascading size is not directly determined by the maximum supply or current supply of the generator. Therefore, we see no significant trend on the importance ranking of generators and their maximum or current supply. However, for most of the substations, their demand dominates their importance, and as a result the importance of a substation is mainly in direct proportion to their demands. Of course, there are still some low demand substations that rank high, because the removal of those buses will trigger larger scale cascading failures, [e.g., bus No. 37, even though its demand (and supply) is 0, it has an important topological role (degree $d$ = 6, betweenness $b$ = 0.1147).]




(a)

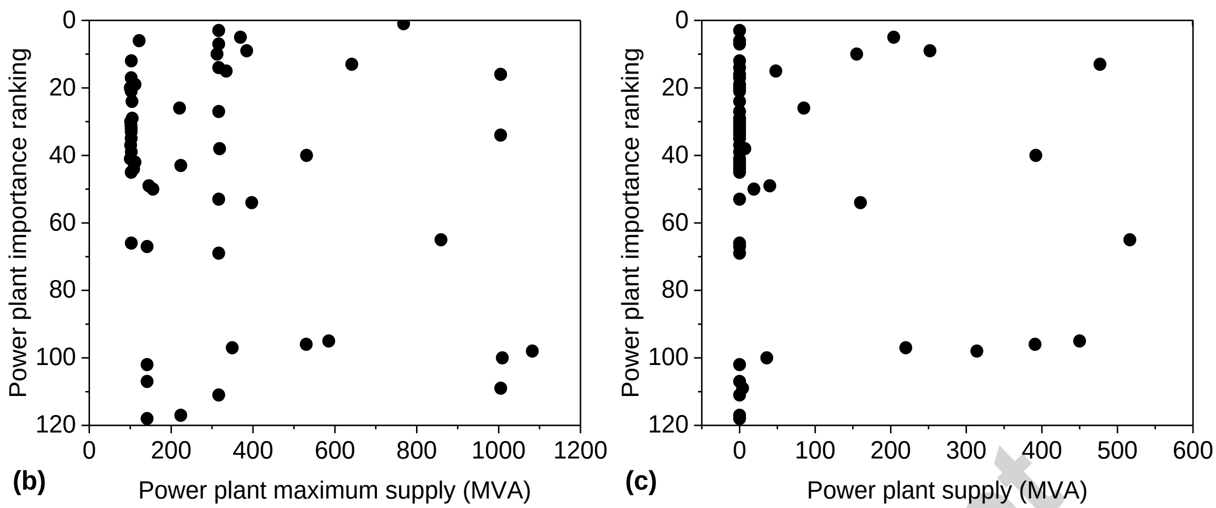



Fig. 5 Relationship between functional bus importance rankings and bus demands or supply: (a) substation importance rankings and substation loads; (b) power plant importance rankings and maximum supply; (c) power plant importance rankings and supply based on OPF.

In general, the proposed AC-based power flow element importance measure is practical as it only needs to be informed by up to *N*-3 contingencies, as showcased with three typical IEEE bus systems. The proposed measure can quantify the importance of buses and branches alike, and account for their operational constraints. The insights form this metric could inform reliability improvement, prioritization activities, maintenance scheduling, and expansion planning for the power utilities.

Despite the findings that the importance of an element is highly impacted by its physical attributes and functional flows (e.g. demand and supply), it is also found that the importance of an element has a relationship with its topology. However, how high the topology-based importance measures correlate with the physical AC-based power flow analyses is not explicit up to now. Hence, it is necessary to determine how approximate functional analyses and rankings, as from network topology as well as from DC-based power flow, match with the AC analyses.

## 5. Comparison with other ranking measures

This section compares different element importance measurement methods with the AC-based measures proposed in this paper. Regarding the DC-flow for functionality assessment, it is used instead of AC power flow in the cascading failure model, and it only monitors the real power rather than both real and reactive power as in the AC-based model.

As for the topology-based importance measures, this paper employs five practical element importance measures that build upon network structure, including connectivity reliability (CR) [28], PageRank [29], Normalized Wide network Ranking algorithm (NWRank) [30], degree centrality [31], and betweenness centrality [11]. The CR-based importance measure monitors the system's CR loss when an element is removed from the network, and quantifies the element importance based on loss levels. Li et al. [28,32] give a method to quantify element importance based on CR while contrasting with controllability principles—note that the other topological metrics are intrinsic characteristics of the grid layouts and do not rely on element removals. PageRank, which is an algorithm used by Google to rank websites in their search engine results, works by counting the number and quality of links to a page to determine how important the website is [29], while it is now used in applications far beyond its information science origins [33]. NWRank combines the mutual reinforcement feature of Hypertext Induced Topic Selection (HITS) and the weight normalization feature of PageRank. In NWRank, relative weights are assigned to links based on the degree of the adjacent neighbors and their Betweenness Centrality instead of assigning the same weight to every link as assumed in PageRank [30]. Simpler metrics, such as the degree centrality, give the highest score of importance to the node with the largest number of neighbors [31,34]. The betweenness centrality is an indicator of a node's (or link's) centrality in a network from an approximate flow perspective. It is equal to the number of shortest paths from all vertices to all others that pass through a node (link) in question [29].

This paper divides all the bus importance measures into two categories: the function-based measures and topology-based measures. The former contains the AC- and DC-based flow measures, and the later contains the rest.

With the IEEE 30, 57 and 118-bus systems, this paper computes and ranks the importance of all the nodes with the seven mentioned measures. Then Pearson correlation coefficient is computed among the rankings based on different importance measures. Fig. 6 presents the correlation between different node (bus) importance ranking measures for all the 205 buses of the IEEE 30, 57 and 118-bus systems. Taking the AC-based power flow importance measure as a benchmark, the DC-based flow measure shows a relatively high correlation (rectangle with dotted lines), while topology-based measures have lower correlations (rectangle with solid line). Note that the topology based ranking measures have relatively high correlation among themselves (shown in the rectangle with long dash lines), but have lower correlation with function-based ranking measures (but still insightful at a much lower computational cost). NWRank also shows promise as it encapsulates multiple topological aspects at once. As for the gap in between function and topology, it is unavoidable as AC/DC flow involves not only the grids' topologies but also considers the physical flow and their unique physical attributes (e.g. demand and supply) of each bus and branch (including voltage and phase constraints).

| | | Function-based measures | | Topology-based measures | | | | |
|---|---|---|---|---|---|---|---|---|
| | | AC | DC | CR | PR | NWR | D | B |
| Function-based measures | AC | | 0.74 | 0.40 | 0.41 | 0.45 | 0.46 | 0.36 |
| | DC | 0.74 | | 0.30 | 0.42 | 0.42 | 0.45 | 0.30 |
| Topology-based measures | CR | 0.40 | 0.30 | | 0.72 | 0.64 | 0.49 | 0.69 |
| | PR | 0.41 | 0.42 | 0.72 | | 0.82 | 0.76 | 0.79 |
| | NWR | 0.45 | 0.42 | 0.64 | 0.82 | | 0.87 | 0.92 |
| | D | 0.46 | 0.45 | 0.49 | 0.76 | 0.87 | | 0.69 |
| | B | 0.36 | 0.30 | 0.69 | 0.79 | 0.92 | 0.69 | |

[0.8,1] [0.6,0.8) [0.4,0.6) [0.2,0.4) [-1,0.2)

Fig. 6 Correlation between different node (bus) importance ranking measures. AC denotes AC-based power flow node importance measure (considering multi-element failures); DC stands for DC-based power flow node measures; CR for connectivity reliability; PR for PageRank; NWR for NWRank; D for degree centrality; and B for betweenness centrality.

However, it should be noted that even for AC- and DC-based measures, there are certain differences. The reason is that DC-based measures only involve part of the physical attributes and constraints (e.g. they ignore reactive power and line losses), and as a result DC only captures part of the importance attributes. This paper takes the IEEE 30-bus system as an example to seek key differences between AC and DC based ranking measures, as shown in Fig. 7. One can see that for most of the buses the DC-based importance ranking is close to AC-based ranking. However, in terms of buses No. 6, 13 and 28 (with red circle in the figure), the ranking based on DC is much lower than that based on AC power flow. That is to say, these buses are underestimated when DC is applied. We track the cascading process when the above buses are removed from the grids, and find that some branches are overflowed (e.g. branches No. 11 and 28, when bus No. 13 is removed) in AC cascading but not overflowed in DC based cascading. This is because only real power is considered in the DC model, where the capacity of the branch is overestimated given that reactive power is not present, thus harboring inconsistencies with the physical processes under unstable operation.

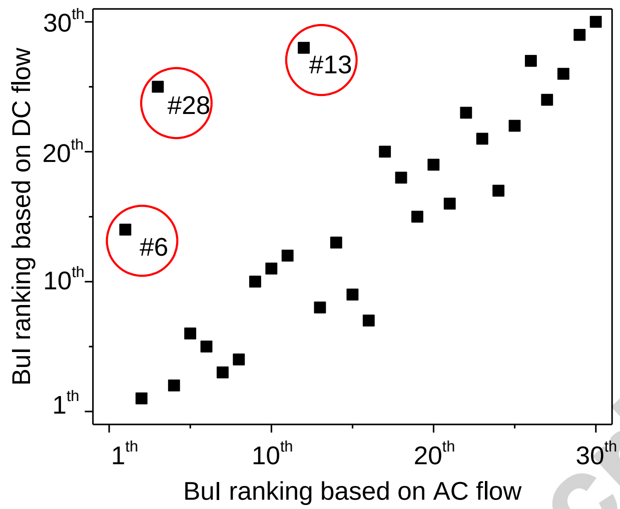



Fig. 7 Difference between AC- and DC-based importance ranking measures for the IEEE 30-bus system

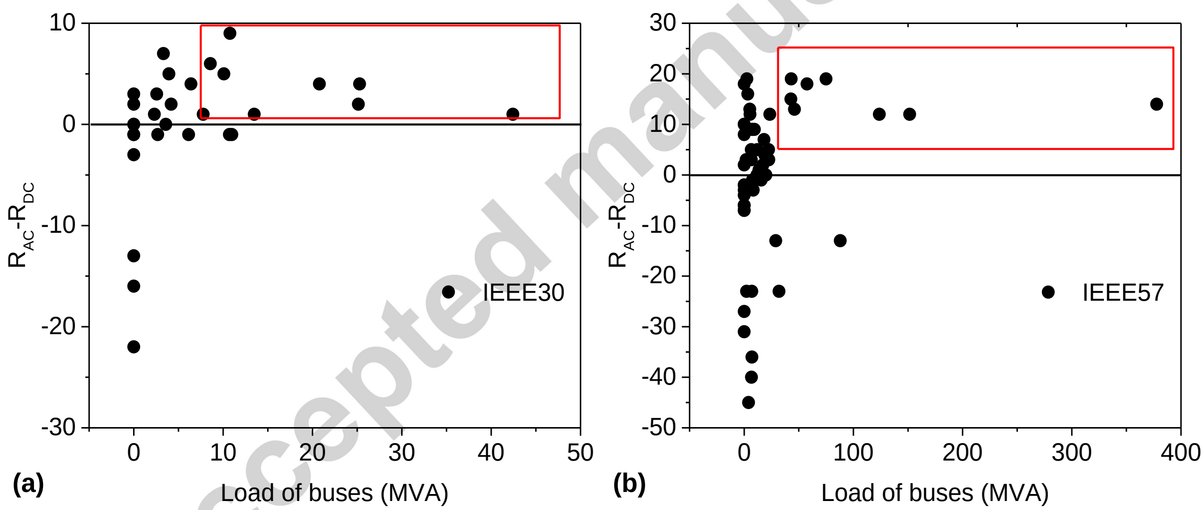



Fig. 8 Rankings difference between AC- and DC-based power flow models: (a) IEEE 30-bus system; (b) IEEE 57-bus system.

Another interesting phenomenon is that buses No. 6, 13 and 28 are all non-demand buses. This inspires us to explore loads' impacts on rankings based on different measures. Fig. 8 gives the ranking difference between AC- and DC-based power flow models in terms of different loads. $R_{AC}$ denotes the rankings based on AC power flow, and $R_{DC}$ for the rankings based on DC power flow. The buses with relatively high load tend to have $R_{AC}$ - $R_{DC}$ > 0, as shown in the red rectangle in the figure, which means the ranking based on AC measure is lower (high importance) than that based on DC, confirming the effect of the DC-based models ignoring reactive power.

This shortage in DC models also inspires scholars to develop DC power flow method with reactive power considerations [35].

Besides the bus (node) ranking difference, this paper also discusses branch (edge) importance among different measures. Since PageRank and degree centrality are not originally designed for ranking links, these two metrics are removed from the branch importance analysis. Fig. 9 shows the correlation between different branch (edge) importance ranking measures for all the 307 branches of the IEEE 30, 57 and 118-bus systems. The correlation between function-based measures and topology-based measures is low. It can also be seen that the correlation among the topology-based measures themselves is low, except for that between NWRank and betweenness centrality. A comprehensive topology based edge importance measure is imperative in the future; even though topology-based measures cannot capture all the features of physical systems, especially when processes on links are the target, they still offer insight at the node ranking level, whose two-element subsets form the ends of links—note that CR, based on link removals and path existence, could still be enhanced with flow-based reliability principles in the future to improve its AC/DC correlation.

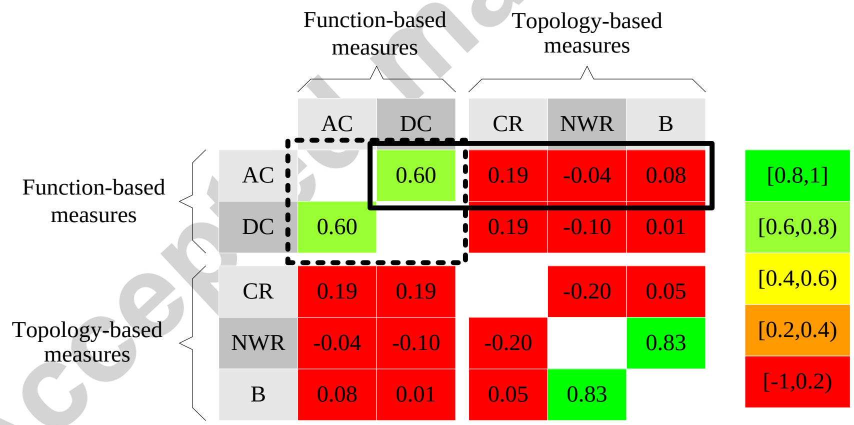



Fig. 9 Correlation between different branch (link) importance ranking measures. AC denotes the AC-based power flow node importance measure (considering multi-element failures); DC stands for DC-based power flow node importance; CR for connectivity reliability; NWR for NWRank; and B for betweenness centrality.

Balancing computational demands and insight relative to AC-based flow, DC-based metric provides a better estimation than topology-based measures. This and its relatively affordable computational efficiency, make DC models applicable to engineering system analysis, such as power planning, congestion management, economic dispatch, and probabilistic security check. But DC-based measures sometimes fail to identify the importance of particular elements, because DC power flow only involves part of the physical flow of power. Also, for high load buses, the DC-based measure tends to overestimate their importance since some of the cascading failures are not captured in DC model and demand dominates ranking. Still, DC-metrics can narrow down the contingency search space, and inform where AC flows which capture cascading failures correctly warrant usage. This is particularly important in maintenance scheduling, *N*-1 contingency checks, and cascading prevention.

## 6. Conclusions

This paper proposes an Alternating Current (AC)-based power flow element importance measure by considering multi-element failures. A cascading failure model is presented based on AC power flow. The importance ranking measure proposed in this paper takes into consideration the failure of multiple elements, as using the failure of one element as more commonly done today, may result in tied rankings and fail to capture the contribution of cascading failures on importance quantification. To illustrate the proposed measure, the IEEE 30, 57 and 118-bus systems are employed as functional case studies.

Using the proposed ranking approach, this study also investigates the relationship between bus importance and branch importance. We find that the importance of a functional bus is statistically proportional to the sum of importance levels of incident branches. It is also clear that the demand of a substation is a key indicator to the importance of the substation: a high demand load substation usually ranks high. But for power plants, the ranking shows almost no correlation as they have a wide range of operation before reaching maximum supply levels or even when dispatching under optimal power flow (OPF). Cascading failures dominate the importance of a power plant, but these effects are global, while the local demand dominates the importance of a substation.

This paper also conducts a comparison between the AC-based importance measure, and several classical and widely used functional and topological measures. Because of the lack of physical power flow considerations, the topology-based measures show low correlation with AC-based flow measures for bus and branch importance, although their low computational cost still offers valuable insights. Moving forward, this finding highlights the need for system dynamic analyses (power flow analysis for power infrastructure) when tracking critical elements in the system. Hence, the DC-based flow measure, emerges as a compromise, as it shows a good correlation with the AC model and it is computationally efficient. However, for certain high demand buses, the DC-based measure tends to overestimate their importance, or may underestimate the importance of some non-demand buses. Through tracking the cascading process, it is found that this mismatch is because the DC model fails to capture reactive power and their role in cascading failures.

Compared with some of the current importance measure, the proposed importance measure in this paper could assist decision makers and utilities to better locate important elements in the system when they are part of multiple contingencies. This approach is helpful for cascading failure prevention, system backup setting, maintenance scheduling, and reliability improvements. In addition, $N$-3 analyses are applied, which offer practical rankings for feasible contingencies, and are not computationally prohibitive. However, the optimal $N$-$k$ level for computing element importance remains unknown. While the $N$-3 insights may not guarantee the importance ranking each element (unless all $k$ are tried, which is infeasible), evidence is offered to support that this level is sufficient for practical functional applications. Finding the best $k$ is of great significance, and it remains a topic for the future investigations using modern algorithmic approaches for system reliability.

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# Highlights

- We propose a novel importance measure based on joint failures and AC power flow.
- A cascading failure model considers both AC power flow and optimal power flow.
- We find that *N*-3 analyses are sufficient to measure the importance of an element.
- Power demand impacts the importance of substations but less so that of generators.
- DC models fail to identify some key elements, despite correlating with AC models.