Machine Learning Homework Week 1

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1 Question 1

a. We have the marginal probability mass function here: b. We first note that

	x1	x2	x3	x4	x5	Row sum = P(Y = y)
y1	0.01	0.02	0.03	0.1	0.1	0.26
y2	0.05	0.1	0.05	0.07	0.2	0.47
y3	0.1	0.05	0.03	0.05	0.04	0.27
Column sum = P(X = x)	0.16	0.17	0.11	0.22	0.34	1

$$p_Y(Y = y1) = 0.26 \tag{1}$$

Besides, we have the fomula below:

$$p_{X|Y}(x|y_1) = P(X = x|Y = y_1) = \frac{P(X = x, Y = y_1)}{P(Y = y_1)} \tag{2}$$

Hence

$$p_{X|Y}(x_1|y_1) = P(X = x_1|Y = y_1) = \frac{P(X = x_1, Y = y_1)}{P(Y = y_1)} = \frac{0.01}{0.26} = 0.038 \tag{3}$$

$$p_{X|Y}(x_2|y_1) = P(X = x_2|Y = y_1) = \frac{P(X = x_2, Y = y_1)}{P(Y = y_1)} = \frac{0.02}{0.26} = 0.077 \tag{4}$$

$$p_{X|Y}(x_3|y_1) = P(X = x_3|Y = y_1) = \frac{P(X = x_3, Y = y_1)}{P(Y = y_1)} = \frac{0.03}{0.26} = 0.115 \tag{5}$$

$$p_{X|Y}(x_4|y_1) = P(X = x_4|Y = y_1) = \frac{P(X = x_4, Y = y_1)}{P(Y = y_1)} = \frac{0.1}{0.26} = 0.385 \tag{6}$$

$$p_{X|Y}(x_5|y_1) = P(X = x_5|Y = y_1) = \frac{P(X = x_5, Y = y_1)}{P(Y = y_1)} = \frac{0.1}{0.26} = 0.385 \tag{7}$$

We do similarly with the probability of X given that Y = $y_3, sowehave: P(X = x_1|Y = y_3) = \frac{0.1}{0.27} = 0.37(8)$

$$P(X = x_2|Y = y_3) = \frac{0.05}{0.27} = 0.185 \tag{9}$$

$$P(X = x_3|Y = y_3) = \frac{0.03}{0.27} = 0.11 \tag{10}$$

$$P(X = x_4|Y = y_3) = \frac{0.05}{0.27} = 0.185 \tag{11}$$

$$P(X = x_5|Y = y_3) = \frac{0.04}{0.27} = 0.148 \tag{12}$$

2 Question 2

We have 2 cases to prove this fomula:

- Case 1: Y is a discrete random variable, so we have:

$$E_X(X) = \sum_y E(X|Y = y) * P(Y = y) \tag{13}$$

$$E_X(X) = \sum_y \sum_x P(X = x|Y = y) * P(Y = y) \tag{14}$$

$$E_X(X) = \sum_y \sum_x x * \frac{P(X = x, Y = y)}{P(Y = y)} * P(Y = y) \tag{15}$$

$$E_X(X) = \sum_y \sum_x x * P(X = x, Y = y) \tag{16}$$

$$E_X(X) = \sum_x x \sum_y P(X = x, Y = y) \tag{17}$$

$$E_X(X) = \sum_x x * P(X = x) = E(X) \tag{18}$$

- Case 2: Y is a continuous random variable, so we have:

$$E(X) = \int_{-\infty}^{\infty} E(X|Y = y) f(y) dy \tag{19}$$

$$E(X) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \times f_{X|Y}(x|y) dx \times f(y) dy \tag{20}$$

$$E(X) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \times f_{X|Y}(x|y) \times f(y) dx dy \tag{21}$$

$$E(X) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \times f_{X,Y}(x,y) \times f(y) dx dy \tag{22}$$

$$E(X) = \int_{-\infty}^{\infty} x \times \int_{-\infty}^{\infty} f_{X,Y}(x,y) \times f(y) dy dx \tag{23}$$

$$E(X) = \int_{-\infty}^{\infty} x \times f(x) dx = E(X) \tag{24}$$

3 Question 3

We have P(X) = 0.0207 , P(Y) = 0.5 , P(X |Y) = 0.365

$a.P(X,Y) = P(XjY) \times P(Y) = 0.365 \times 0.5 = 0.1825$

$b.P(Y|\overline{X}) = \frac{P(\overline{X} \times Y)}{p(\overline{X})} = \frac{P(Y)-P(X \times Y)}{1-P(X)} = \frac{0.5-0.1825}{1-0.207} = 0.4$

4 Question 4

Firstly, we have the standard definition of the variance is:

$$Var(X) = E[(X - E(X)^2] \tag{25}$$

$$Var(X) = E(X^2 - 2XE(X) + E(X)^2) \tag{26}$$

$$Var(X) = E(X^2) - E(2XE(X) + E[E(X)^2] \tag{27}$$

$$Var(X) = E(X^2) - 2E(X)E(X) + E(X)^2 \tag{28}$$

$$Var(X) = E(X^2) - 2E(X)^2 + E(X)^2 \tag{29}$$

$$Var(X) = E(X^2) - E(X)^2 \tag{30}$$

5 Question 5

Event A: 'The car is in the door number 1.'
Event B: 'The car is in the door number 3.'
Event C: 'Monty will open the door number 2.'
So we have the probability that the car is in the door number 1 is $P(A) = \frac{1}{3}$
We have $P(C|A) = \frac{1}{2}$ is the probability that Monty will open the door number 2 (event C happened) when the car is in the door number 1 (event A happened) and he will only open the door number 2 or number 3. We have $P(C) = \frac{1}{2}$ is the probability that Monty will open the door number 2 since the door number 1 is opened.
Therefore, we have: $P(A|C) = \frac{P(C|A) \times P(A)}{P(C)} = \frac{1}{3}$
is the probability that the car is in the door number 1 (event A happened) given that Monty will open the door number 2. The probability that the car is in the door number 3 is: $P(B) = 1 - P(A) = \frac{2}{3}$
We can easily see that the probability that the car is in the door number 3 is bigger that that in the door number 1, therefore, we should change the decision.