MCQ Paper - test

Generated Question Paper

MCQ Examination (ANSWERS)

Name: ______	**Instructions:**
Class: ______ Section: ______	• Fill OMR sheet with blue/black pen.
Roll no.: ______	• Fill circles completely.
	• No stray marks.
	• Enter Name, Class, Section.

SET A

Questions: **9**
Duration: **120min**

Calculus: Differentiation and Integration

This section covers fundamental and advanced topics in differential and integral calculus.

1. *Evaluate the definite integral:* $\int_0^{(\pi/2)} \sin^2(x)\,dx$

A. 1 **B.** π/2
C. **π/4 *** **D.** 0

Explanation:
To solve, use the power-reduction identity $\sin^2(x) = (1 - \cos(2x))/2$. The integral becomes $\int_0^{(\pi/2)} (1/2 - \cos(2x)/2)\,dx$. Integrating yields $[x/2 - \sin(2x)/4]$ from 0 to π/2. Evaluating at the limits gives $(\pi/4 - \sin(\pi)/4) - (0 - \sin(0)/4) = \pi/4$.

2. *Consider the function of two variables provided below:*

Let f(x, y) be a differentiable function defined as $f(x, y) = e^{(xy)} * \cos(x^2 + y^2)$.

What is the value of the partial derivative ∂f/∂x at the point (0, 0)?

A. e **B.** -1
C. 1 **D.** **0 ***

Explanation:
Using the product rule for differentiation, $\partial f/\partial x = (\partial/\partial x\, [e^{(xy)}]) * \cos(x^2 + y^2) + e^{(xy)} * (\partial/\partial x\, [\cos(x^2 + y^2)]) = y*e^{(xy)}*\cos(x^2+y^2) - 2x*e^{(xy)}*\sin(x^2+y^2)$. Evaluating at (0,0) gives $0*e^0*\cos(0) - 0*e^0*\sin(0) = 0$.

3. *The Fundamental Theorem of Calculus connects the concepts of differentiation and integration.*

Statement I:

If F'(x) = f(x) for all x in [a, b], then ∫ f(x) dx = F(b) - F(a).

Statement II:

If f is continuous on [a, b], then the function G(x) = ∫ˣ f(t) dt is differentiable on (a, b) and G'(x) = f(x).

Which of the statements above are considered parts of the Fundamental Theorem of Calculus?

A. Statement II only
B. Neither statement is correct
C. Statement I only
D. **Both Statement I and Statement II ***

Explanation:
The Fundamental Theorem of Calculus is commonly presented in two parts. Statement I is the second part, which provides a method for evaluating definite integrals. Statement II is the first part, which establishes the relationship between the derivative and the integral.

Linear Algebra: Matrices and Vector Spaces

This section tests knowledge of matrix properties, linear transformations, and vector spaces.

4. *Match each type of matrix in Column A with its defining property in Column B.*

Column A: Matrix Type	-	Column B: Property
1. Orthogonal	-	a. A = A
2. Symmetric	-	b. A² = A
3. Skew-Symmetric	-	c. A = -A
4. Idempotent	-	d. A⁻¹ = A

Which option represents the correct matching?

A. 1-b, 2-a, 3-d, 4-c
B. 1-a, 2-d, 3-b, 4-c
C. 1-d, 2-c, 3-a, 4-b
D. **1-d, 2-a, 3-c, 4-b ***

Explanation:
The definitions are: Orthogonal matrix (A⁻¹ = A), Symmetric matrix (A = A), Skew-Symmetric matrix (A = -A), and Idempotent matrix (A² = A). The choice correctly aligns these definitions.

5. *For any two n × n matrices A and B, which of the following properties is NOT always true?*

i. det(A) = det(A)
ii. det(AB) = det(A)det(B)
iii. det(A + B) = det(A) + det(B)
iv. If A is invertible, $\det(A^{-1}) = 1/\det(A)$

A. Property iv
B. **Property iii ***
C. Property ii
D. Property i

Explanation:
The determinant of a sum of matrices is generally not equal to the sum of their determinants. This is a common misconception. All other listed properties are fundamental theorems of matrix determinants.

6. Let T: $\mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by the rule T(x, y, z) = (x + 2y - z, 3x + y + 4z).

Based on the transformation T, evaluate the following statements:

Statement I:

The kernel (null space) of T has a dimension greater than zero.

Statement II:

The transformation T is surjective (onto).

Which assessment of the statements is correct?

A. Both statements are false
B. Statement I is true, Statement II is false
C. **Both statements are true ***
D. Statement I is false, Statement II is true

Explanation:
The matrix for T is A = [[1, 2, -1], [3, 1, 4]]. The rank of A is 2, as the rows are linearly independent. By the Rank-Nullity Theorem, rank(T) + nullity(T) = dim(domain) = 3. So, 2 + nullity(T) = 3, which implies nullity(T) = 1. Since the nullity is > 0, the kernel is non-trivial (Statement I is true). Since the rank(T) = 2, which equals the dimension of the codomain $\mathbb{R}^2$, the transformation is surjective (Statement II is true).

Complex Analysis and Series

Questions on complex numbers, Euler's formula, and the convergence of infinite series.

7. *For a complex number $z = a + ib$, where $i^2 = -1$, consider the following statements:*

Assertion (A):

The modulus $|z|$ is given by the formula $\sqrt{(a^2 + b^2)}$.

Reason (R):

$|z|$ geometrically represents the distance of the point (a, b) from the origin in the Argand (complex) plane.

Which of the following is true regarding statements (A) and (R)?

- A. **Both (A) and (R) are true, and (R) is the correct explanation of (A). ***
- B. (A) is false but (R) is true.
- C. (A) is true but (R) is false.
- D. Both (A) and (R) are true, but (R) is not the correct explanation of (A).

Explanation:

The assertion (A) correctly states the formula for the modulus of a complex number. The reason (R) provides the correct geometric interpretation. The formula in (A) is derived from the Pythagorean theorem applied to the distance described in (R).

8. Consider the infinite series $S = \Sigma_{n=1 \text{ to } \infty} ((-1)^n * n) / (n^2 + 1)$.

Determine the nature of the convergence of the series S.

- A. Absolutely convergent
- B. The series does not converge or diverge
- C. Divergent
- D. **Conditionally convergent ***

Explanation:

The series is an alternating series. We apply the Alternating Series Test with $b = n/(n^2+1)$. Since b is positive, decreasing for $n \geq 1$, and $\lim(n\to\infty)\ b = 0$, the series converges. To test for absolute convergence, we examine $\Sigma\ n/(n^2+1)$. Using the Limit Comparison Test with the divergent harmonic series $\Sigma\ 1/n$, we find that $\Sigma\ n/(n^2+1)$ also diverges. Therefore, the series is conditionally convergent.

9. Euler's formula states that e^(iθ) = cos(θ) + i sin(θ), linking complex exponentials with trigonometric functions.

Using this formula, which step in the following simplification of e^(iπ) + 1 contains an error?

Step 1: Substitute θ = π into the formula, yielding e^(iπ) = cos(π) + i sin(π).

Step 2: Evaluate the trigonometric functions: cos(π) = -1 and sin(π) = 0.

Step 3: Conclude that e^(iπ) = -1 + i(0) = -1.

Step 4: Perform the final addition: e^(iπ) + 1 = 1 + 1 = 2.

A. Step 3
B. Step 1
C. **Step 4 ***
D. Step 2

Explanation:
Steps 1, 2, and 3 correctly derive that e^(iπ) = -1. This is a famous result known as Euler's Identity. Step 4 makes an error in the final calculation. It should be e^(iπ) + 1 = -1 + 1 = 0, not 1 + 1 = 2.

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