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BSHS

IEEJA

Number Theory (Quantitative Aptitude)

SET

B

Questions: 19

Duration: 120min

Question Text Array Format

This section demonstrates basic questions on Number Theory using the question_text array for line breaks.

1. What is a natural number greater than 1 that has no positive divisors other than 1 and itself?

A. Integer

B. Composite number

C. Prime number

D. Perfect number

2. The Fundamental Theorem of Arithmetic states that every integer greater than 1 can be uniquely represented as a product of prime numbers. Which of the following is the correct prime factorization of 84?

A. 2 × 6 × 7

B. 2 × 3 × 14

C. 4 × 3 × 7

D. 2² × 3 × 7

3. A perfect number is a positive integer that is equal to the sum of its proper positive divisors. For example, the proper divisors of 6 are 1, 2, and 3, and their sum is 1 + 2 + 3 = 6. Which of the following numbers is the next perfect number after 6?

A. 30

B. 28

C. 24

D. 12

LIST Placeholder Format

This section demonstrates questions on Number Theory using the LIST placeholder with a list_items array.

4. The Euclidean algorithm is a method for finding the greatest common divisor (GCD) of two integers. The steps for finding the GCD of 48 and 18 are shown below:

Step 1: 48 = 2 × 18 + 12

Step 2: 18 = 1 × 12 + 6

Step 3: 12 = 2 × 6 + 0

What is the GCD of 48 and 18?

A. 48

B. 12

C. 18

D. 6

5. Consider the set of integers below:

2, 9, 15, 23, 51

Which of the numbers in the list are prime?

A. All are prime

B. 2, 23, and 51

C. 9 and 15

D. 2 and 23

6. Consider the properties of the number 36:

i. It is a perfect square.

ii. It is a prime number.

iii. It is an abundant number.

iv. Its prime factorization is 2² × 3².

Which of the properties listed above are true for the number 36?

A. i and iv only

B. i, iii, and iv only

C. ii and iii only

D. All of the above

STATEMENT/STATEMENTS Placeholder Format

This section demonstrates questions on Number Theory using STATEMENT and STATEMENTS placeholders.

7. Consider the following assertion and reasoning:

Assertion (A):

The numbers 15 and 28 are coprime.

Reasoning (R):

Two integers are coprime (or relatively prime) if their greatest common divisor (GCD) is 1.

Evaluate the correctness of the assertion and reasoning.

A. R is correct, but A is incorrect.

B. Both A and R are incorrect.

C. Both A and R are correct, and R is the correct explanation for A.

D. A is correct, but R is incorrect.

8. Consider the following statement from number theory:

Statement:

The Twin Prime Conjecture states that there are infinitely many pairs of prime numbers that differ by 2.

Is this statement a proven theorem or an unproven conjecture?

A. It has been disproven.

B. It is an unproven conjecture.

C. It is a proven theorem.

D. It is not a valid mathematical statement.

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9. Review the following statements about divisibility rules:

- Statement I:
A number is divisible by 3 if the sum of its digits is divisible by 3.
- Statement II:
A number is divisible by 9 if the sum of its digits is divisible by 9.

Which of these statements are correct?

- A. Statement II only
B. Both Statement I and Statement II
C. Neither statement is correct
D. Statement I only correct

PARAGRAPH Placeholder Format

This section demonstrates questions on Number Theory using the PARAGRAPH placeholder.

10. Read the passage about the Fibonacci sequence:

The Fibonacci sequence is a series of numbers where each number is the sum of the two preceding ones, usually starting with 0 and 1. The sequence begins: 0, 1, 1, 2, 3, 5, 8, 13, ...

What is the next number in this sequence?

- A. 23
B. 21
C. 20
D. 18

11. The following paragraph describes modular arithmetic:

In modular arithmetic, we are concerned with the remainder when an integer is divided by another integer, called the modulus. The expression ' $a \equiv b \pmod{n}$ ' means that a and b have the same remainder when divided by n. The value of ' $a \pmod{n}$ ' is the remainder of the division $a \div n$.

Based on this definition, what is the value of $17 \pmod{5}$?

- A. 2
B. 4
C. 1
D. 3

12. Read the following description of a famous problem in number theory:

Goldbach's Conjecture is one of the oldest and best-known unsolved problems in number theory. It states that every even integer greater than 2 is the sum of two prime numbers.

Which of the following expresses the number 20 as a sum of two prime numbers, consistent with the conjecture?

- A. $10 + 10$
B. $5 + 15$
C. $1 + 19$
D. $7 + 13$

MTF (Match The Following) Placeholder Format

This section demonstrates match-the-following questions on Number Theory using the MTF_DATA placeholder.

13. Match the mathematical notation with its meaning in number theory:

Notation	-	Meaning
1. $a \mid b$	-	a. The greatest common divisor of a and b.
2. $\gcd(a, b)$	-	b. a and b have the same remainder when divided by n.
3. $a \equiv b \pmod{n}$	-	c. a divides b (b is a multiple of a).

Which option correctly matches the notations to their meanings?

- A. 1-c, 2-b, 3-a
B. 1-a, 2-b, 3-c
C. 1-b, 2-c, 3-a
D. 1-c, 2-a, 3-b

14. Match each number with its unique prime factorization:

Number	-	Prime Factorization
A. 56	-	1. $3^2 \times 11$
B. 99	-	2. $2^3 \times 3 \times 5$
C. 120	-	3. $2^3 \times 7$

Which pairing is correct?

- A. A-1, B-2, C-3
B. A-3, B-1, C-2
C. A-2, B-3, C-1
D. A-3, B-2, C-1

15. Match the number theory terms with their correct definitions:

Term	-	Definition
i. Prime	-	a. Two integers with a GCD of 1.
ii. Coprime	-	b. An integer equal to the sum of its proper divisors.
iii. Perfect	-	c. A natural number > 1 with only two divisors: 1 and itself.

Which option shows the correct matching?

- A. i-c, ii-a, iii-b
B. i-a, ii-b, iii-c
C. i-b, ii-c, iii-a
D. i-c, ii-b, iii-a

Mixed Placeholder Combinations

This section demonstrates complex questions on Number Theory by combining multiple placeholders.

16. The Chinese Remainder Theorem helps solve systems of congruences.

The Chinese Remainder Theorem can find an integer x that satisfies multiple remainder conditions simultaneously. For example, finding an integer x that leaves a remainder of 2 when divided by 3, and a remainder of 3 when divided by 5.

Consider the following system of congruences:

Condition I:
 $x \equiv 2 \pmod{3}$

Condition II:
 $x \equiv 3 \pmod{5}$

Which number from the list below is the smallest positive integer solution?

- A. 5
B. 11
C. 8
D. 13

17. Read the definition of a Mersenne Prime:

A Mersenne prime is a prime number that can be written in the form $2^p - 1$, where p must also be a prime number. Not all numbers of this form with a prime p are themselves prime.

Now evaluate the following statements:

Assertion (A):
The number 31 is a Mersenne prime.

Reasoning (R):
31 can be expressed as $2^5 - 1$, and 5 is a prime number.

Based on the definition provided, what is the most logical conclusion?

- A. Both A and R are incorrect
B. A is correct, but R is incorrect
C. Both A and R are correct, and R explains A
D. R is correct, but A is incorrect

18. Read the definition of amicable numbers:

Amicable numbers are two different positive integers such that the sum of the proper divisors of each is equal to the other number. A proper divisor of a number is a positive divisor other than the number itself.

Consider the following pairs of numbers:

- i. (110, 150)
ii. (220, 284)
iii. (300, 310)

Which pair from the list is amicable?

- A. None of the pairs are amicable
B. Pair ii
C. Pair i
D. Pair iii

19. Consider the numbers in the list below:

12, 17, 25, 30

Review the following properties:

Statement I:
The number is a composite number.

Statement II:
The number is a deficient number (the sum of its proper divisors is less than the number).

Which number from the list satisfies both Statement I and Statement II?

- A. 12
B. 17
C. 30
D. 25

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