BSHS

IEEJA

Number Theory (Quantitative Aptitude) (ANSWERS)

SET A

Questions: **19**
Duration: **120min**

Question Text Array Format

This section demonstrates basic questions on Number Theory using the question_text array for line breaks.

1. *What is a natural number greater than 1 that has no positive divisors other than 1 and itself?*

A. Integer
B. Perfect number
C. Composite number
D. **Prime number ***

Explanation:

A prime number is a natural number greater than 1 that is not a product of two smaller natural numbers. Its only positive divisors are 1 and itself.

2. *The Fundamental Theorem of Arithmetic states that every integer greater than 1 can be uniquely represented as a product of prime numbers.*

Which of the following is the correct prime factorization of 84?

A. **$2^2 \times 3 \times 7$ ***
B. $2 \times 6 \times 7$
C. $4 \times 3 \times 7$
D. $2 \times 3 \times 14$

Explanation:

84 can be factored as 2×42, then $2 \times 2 \times 21$, and finally $2 \times 2 \times 3 \times 7$, which is written as $2^2 \times 3 \times 7$. The other options include non-prime factors (14, 4, 6).

3. *A perfect number is a positive integer that is equal to the sum of its proper positive divisors.*

For example, the proper divisors of 6 are 1, 2, and 3, and their sum is 1 + 2 + 3 = 6.

Which of the following numbers is the next perfect number after 6?

A. **28 ***
B. 30
C. 12
D. 24

Explanation:

The proper divisors of 28 are 1, 2, 4, 7, and 14. Their sum is 1 + 2 + 4 + 7 + 14 = 28, making it the next perfect number.

LIST Placeholder Format

This section demonstrates questions on Number Theory using the LIST placeholder with a list_items array.

4. *Consider the properties of the number 36:*

i. It is a perfect square.
ii. It is a prime number.
iii. It is an abundant number.
iv. Its prime factorization is $2^2 \times 3^2$.

Which of the properties listed above are true for the number 36?

A. i and iv only
B. ii and iii only
C. All of the above
D. **i, iii, and iv only ***

Explanation:

36 is 6^2, so it's a perfect square (i). Its prime factorization is $2^2 \times 3^2$ (iv). Its proper divisors are 1, 2, 3, 4, 6, 9, 12, 18, and their sum is 55, which is greater than 36, making it an abundant number (iii). It is not prime because it has many divisors (ii is false).

5. *The Euclidean algorithm is a method for finding the greatest common divisor (GCD) of two integers.*

The steps for finding the GCD of 48 and 18 are shown below:

Step 1: $48 = 2 \times 18 + 12$
Step 2: $18 = 1 \times 12 + 6$
Step 3: $12 = 2 \times 6 + 0$

What is the GCD of 48 and 18?

A. 12
B. 48
C. 18
D. **6 ***

Explanation:

In the Euclidean algorithm, the last non-zero remainder is the greatest common divisor. In this case, the last non-zero remainder is 6.

6. *Consider the set of integers below:*

2, 9, 15, 23, 51

Which of the numbers in the list are prime?

A. 9 and 15
B. **2 and 23 ***
C. 2, 23, and 51
D. All are prime

Explanation:

A prime number has only two divisors: 1 and itself. 9 is divisible by 3, 15 is divisible by 3 and 5, and 51 is divisible by 3 and 17. Only 2 and 23 are prime.

STATEMENT/STATEMENTS Placeholder Format

This section demonstrates questions on Number Theory using STATEMENT and STATEMENTS placeholders.

7. *Consider the following statement from number theory:*

Statement:

The Twin Prime Conjecture states that there are infinitely many pairs of prime numbers that differ by 2.

Is this statement a proven theorem or an unproven conjecture?

A. It is not a valid mathematical statement.
B. It has been disproven.
C. **It is an unproven conjecture. ***
D. It is a proven theorem.

Explanation:

The Twin Prime Conjecture is one of the most famous unsolved problems in number theory. While widely believed to be true, it has not yet been proven.

8. *Review the following statements about divisibility rules:*

Statement I:

A number is divisible by 3 if the sum of its digits is divisible by 3.

Statement II:

A number is divisible by 9 if the sum of its digits is divisible by 9.

Which of these statements are correct?

A. Neither statement is correct
B. **Both Statement I and Statement II ***
C. Statement I only **D.** Statement II only

Explanation:

Both statements describe standard divisibility rules in number theory. For example, for 297, the sum of digits is 18, which is divisible by both 3 and 9, so 297 is divisible by both.

9. *Consider the following assertion and reasoning:*

Assertion (A):

The numbers 15 and 28 are coprime.

Reasoning (R):

Two integers are coprime (or relatively prime) if their greatest common divisor (GCD) is 1.

Evaluate the correctness of the assertion and reasoning.

A. A is correct, but R is incorrect.
B. R is correct, but A is incorrect.
C. Both A and R are incorrect.
D. **Both A and R are correct, and R is the correct explanation for A. ***

Explanation:

The factors of 15 are 1, 3, 5, 15. The factors of 28 are 1, 2, 4, 7, 14, 28. Their only common factor is 1, so their GCD is 1, making them coprime. The reasoning correctly defines the term 'coprime'.

PARAGRAPH Placeholder Format

This section demonstrates questions on Number Theory using the PARAGRAPH placeholder.

10. *Read the following description of a famous problem in number theory:*

Goldbach's Conjecture is one of the oldest and best-known unsolved problems in number theory. It states that every even integer greater than 2 is the sum of two prime numbers.

Which of the following expresses the number 20 as a sum of two prime numbers, consistent with the conjecture?

A. **7 + 13 *** **B.** 1 + 19
C. 10 + 10 **D.** 5 + 15

Explanation:

The conjecture requires the two numbers to be prime. In the options, 1 is not prime, 15 is not prime, and 10 is not prime. Both 7 and 13 are prime numbers, and their sum is 20.

11. *Read the passage about the Fibonacci sequence:*

The Fibonacci sequence is a series of numbers where each number is the sum of the two preceding ones, usually starting with 0 and 1. The sequence begins: 0, 1, 1, 2, 3, 5, 8, 13, ...

What is the next number in this sequence?

A. 18 **B.** **21 ***
C. 20 **D.** 23

Explanation:

To find the next number in the Fibonacci sequence, you add the two previous numbers. In this case, 8 + 13 = 21.

12. *The following paragraph describes modular arithmetic:*

In modular arithmetic, we are concerned with the remainder when an integer is divided by another integer, called the modulus. The expression 'a ≡ b (mod n)' means that a and b have the same remainder when divided by n. The value of 'a mod n' is the remainder of the division a ÷ n.

Based on this definition, what is the value of 17 mod 5?

A. 3 **B.** 1
C. 4 **D.** **2 ***

Explanation:

When 17 is divided by 5, it goes in 3 times (3 × 5 = 15) with a remainder of 2 (17 - 15 = 2). Therefore, 17 mod 5 is 2.

MTF (Match The Following) Placeholder Format

This section demonstrates match-the-following questions on Number Theory using the MTF_DATA placeholder.

13. *Match the number theory terms with their correct definitions:*

Term	-	Definition
i. Prime	-	a. Two integers with a GCD of 1.
ii. Coprime	-	b. An integer equal to the sum of its proper divisors.
iii. Perfect	-	c. A natural number > 1 with only two divisors: 1 and itself.

Which option shows the correct matching?

A. i-b, ii-c, iii-a
B. i-a, ii-b, iii-c
C. **i-c, ii-a, iii-b ***
D. i-c, ii-b, iii-a

Explanation:

i. A Prime number's definition matches 'c'. ii. Coprime numbers have a GCD of 1, matching 'a'. iii. A Perfect number is the sum of its proper divisors, matching 'b'.

14. *Match the mathematical notation with its meaning in number theory:*

Notation	-	Meaning
1. a | b	-	a. The greatest common divisor of a and b.
2. gcd(a, b)	-	b. a and b have the same remainder when divided by n.
3. a ≡ b (mod n)	-	c. a divides b (b is a multiple of a).

Which option correctly matches the notations to their meanings?

A. 1-b, 2-c, 3-a
B. **1-c, 2-a, 3-b ***
C. 1-a, 2-b, 3-c
D. 1-c, 2-b, 3-a

Explanation:

1. 'a | b' means 'a divides b'. 2. 'gcd(a, b)' stands for the greatest common divisor. 3. 'a ≡ b (mod n)' is the notation for congruence, meaning they have the same remainder when divided by n.

15. *Match each number with its unique prime factorization:*

Number	-	Prime Factorization
A. 56	-	1. $3^2 \times 11$
B. 99	-	2. $2^3 \times 3 \times 5$
C. 120	-	3. $2^3 \times 7$

Which pairing is correct?

A. A-3, B-2, C-1
B. A-1, B-2, C-3
C. A-2, B-3, C-1
D. **A-3, B-1, C-2 ***

Explanation:

A. $56 = 8 \times 7 = 2^3 \times 7$. B. $99 = 9 \times 11 = 3^2 \times 11$. C. $120 = 12 \times 10 = (2^2 \times 3) \times (2 \times 5) = 2^3 \times 3 \times 5$.

Mixed Placeholder Combinations

This section demonstrates complex questions on Number Theory by combining multiple placeholders.

16. *Read the definition of amicable numbers:*

Amicable numbers are two different positive integers such that the sum of the proper divisors of each is equal to the other number. A proper divisor of a number is a positive divisor other than the number itself.

Consider the following pairs of numbers:

i. (110, 150)
ii. (220, 284)
iii. (300, 310)

Which pair from the list is amicable?

A. **Pair ii ***
B. Pair iii
C. Pair i
D. None of the pairs are amicable

Explanation:

The proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, which sum to 284. The proper divisors of 284 are 1, 2, 4, 71, 142, which sum to 220. This fits the definition of amicable numbers.

17. *The Chinese Remainder Theorem helps solve systems of congruences.*

The Chinese Remainder Theorem can find an integer x that satisfies multiple remainder conditions simultaneously. For example, finding an integer x that leaves a remainder of 2 when divided by 3, and a remainder of 3 when divided by 5.

Consider the following system of congruences:

Condition I:

$x \equiv 2 \pmod 3$

Condition II:

$x \equiv 3 \pmod 5$

Which number from the list below is the smallest positive integer solution?

A. 11
B. **8 ***
C. 5
D. 13

Explanation:

We need a number that fits both conditions. For x=8: 8 ÷ 3 gives a remainder of 2 (Condition I is met). 8 ÷ 5 gives a remainder of 3 (Condition II is met). Therefore, 8 is a valid solution.

18. *Consider the numbers in the list below:*

12, 17, 25, 30

Review the following properties:

Statement I:

The number is a composite number.

Statement II:

The number is a deficient number (the sum of its proper divisors is less than the number).

Which number from the list satisfies both Statement I and Statement II?

A. 17 **B.** 12
C. 30 **D.** **25 ***

Explanation:

17 is prime. 12 is composite but abundant (1+2+3+4+6=16>12). 30 is composite but abundant (1+2+3+5+6+10+15=42>30). 25 is composite (5x5) and deficient (proper divisors are 1, 5; sum=6<25). Thus, 25 is the only number that satisfies both conditions.

19. *Read the definition of a Mersenne Prime:*

A Mersenne prime is a prime number that can be written in the form 2^p − 1, where p must also be a prime number. Not all numbers of this form with a prime p are themselves prime.

Now evaluate the following statements:

Assertion (A):

The number 31 is a Mersenne prime.

Reasoning (R):

31 can be expressed as 2^5 − 1, and 5 is a prime number.

Based on the definition provided, what is the most logical conclusion?

A. Both A and R are incorrect
B. A is correct, but R is incorrect
C. R is correct, but A is incorrect
D. **Both A and R are correct, and R explains A ***

Explanation:

The assertion is correct because 31 is a prime number. The reasoning is also correct because 2^5 - 1 = 32 - 1 = 31, and the exponent (5) is a prime number. This fits the definition of a Mersenne prime perfectly.

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