

MCQ Examination

Roll no.: _____

- Fill OMR sheet with blue/black pen.
- Fill circles completely.
- No stray marks.
- Enter Name, Class, Section.

Questions: **9**
Duration: **120min**

This section covers fundamental and advanced topics in differential and integral calculus.

- $$\int_0^{\pi/2} \sin^2(x) \, dx$$

A. $\pi/4$ B. 0
C. $\pi/2$ D. 1

- If $F'(x) = f(x)$ for all x in $[a, b]$, then
 $\int_a^b f(x) \, dx = F(b) - F(a)$.

If f is continuous on $[a, b]$, then the function $G(x) = \int_a^x f(t) \, dt$ is differentiable on (a, b) and $G'(x) = f(x)$.

A. Statement I only
B. Statement II only
c. Both Statement I and Statement II
D. Neither statement is correct

- Let $f(x, y)$ be a differentiable function defined as $f(x, y) = e^{xy} \cdot \cos(x^2 + y^2)$.

A. e B. 1
C. -1 D. 0

Linear Algebra: Matrices and Vector Spaces

This section tests knowledge of matrix properties, linear transformations, and vector spaces.

4. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation defined by the rule $T(x, y, z) = (x + 2y - z, 3x + y + 4z)$. Based on the transformation T , evaluate the following statements:

Statement I:
The kernel (null space) of T has a dimension greater than zero.

Statement II:
The transformation T is surjective (onto).

Which assessment of the statements is correct?

- A. Both statements are false
 - B. Statement I is true, Statement II is false
 - C. Both statements are true
 - D. Statement I is false, Statement II is true
5. For any two $n \times n$ matrices A and B , which of the following properties is NOT always true?
- i. $\det(A) = \det(A)$
 - ii. $\det(AB) = \det(A)\det(B)$
 - iii. $\det(A + B) = \det(A) + \det(B)$
 - iv. If A is invertible, $\det(A^{-1}) = 1/\det(A)$
- A. Property ii
 - B. Property iv
 - C. Property i
 - D. Property iii

6. Match each type of matrix in Column A with its defining property in Column B.

Column A: Matrix Type	-	Column B: Property
1. Orthogonal	-	a. $A = A$
2. Symmetric	-	b. $A^2 = A$
3. Skew-Symmetric	-	c. $A = -A$
4. Idempotent	-	d. $A^{-1} = A$

Which option represents the correct matching?

- A. 1-a, 2-d, 3-b, 4-c
- B. 1-b, 2-a, 3-d, 4-c
- C. 1-d, 2-a, 3-c, 4-b
- D. 1-d, 2-c, 3-a, 4-b

Complex Analysis and Series

Questions on complex numbers, Euler's formula, and the convergence of infinite series.

7. Consider the infinite series $S = \sum_{n=1}^{\infty} ((-1)^n * n) / (n^2 + 1)$. Determine the nature of the convergence of the series S .
- A. The series does not converge or diverge
 - B. Divergent
 - C. Absolutely convergent
 - D. Conditionally convergent

8. For a complex number $z = a + ib$, where $i^2 = -1$, consider the following statements:

Assertion (A):
The modulus $|z|$ is given by the formula $\sqrt{a^2 + b^2}$.

Reason (R):
 $|z|$ geometrically represents the distance of the point (a, b) from the origin in the Argand (complex) plane.

Which of the following is true regarding statements (A) and (R)?

- A. Both (A) and (R) are true, and (R) is the correct explanation of (A).
 - B. (A) is true but (R) is false.
 - C. (A) is false but (R) is true.
 - D. Both (A) and (R) are true, but (R) is not the correct explanation of (A).
9. Euler's formula states that $e^{i\theta} = \cos(\theta) + i \sin(\theta)$, linking complex exponentials with trigonometric functions. Using this formula, which step in the following simplification of $e^{i\pi} + 1$ contains an error?
- Step 1: Substitute $\theta = \pi$ into the formula, yielding $e^{i\pi} = \cos(\pi) + i \sin(\pi)$.
- Step 2: Evaluate the trigonometric functions: $\cos(\pi) = -1$ and $\sin(\pi) = 0$.
- Step 3: Conclude that $e^{i\pi} = -1 + i(0) = -1$.
- Step 4: Perform the final addition: $e^{i\pi} + 1 = -1 + 1 = 0$.
- A. Step 2
 - B. Step 3
 - C. Step 4
 - D. Step 1

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