

CTL Model Checking

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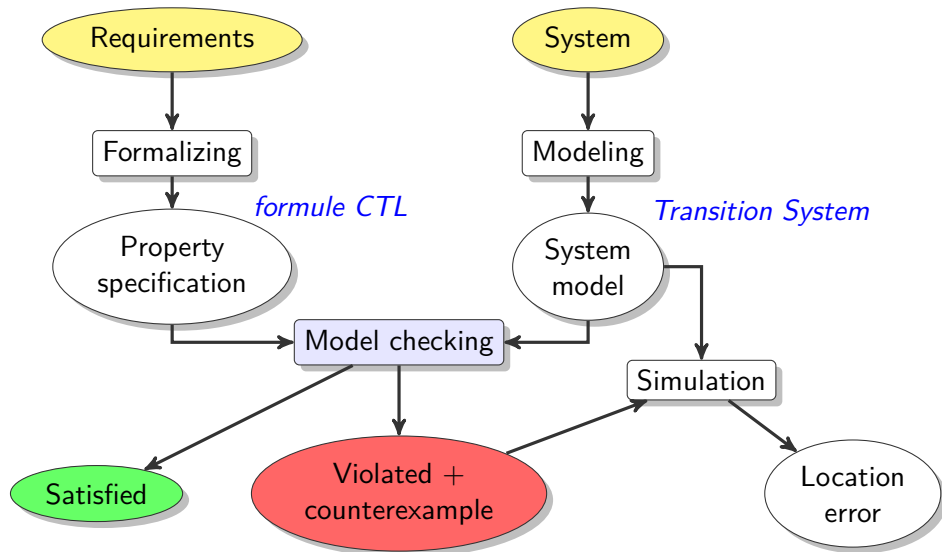
UNIVERSITÀ
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22 febbraio 2016



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Model checking



Computation Tree Logic (CTL)

Grammatica

$$\Phi ::= \text{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid \exists \varphi \mid \forall \varphi$$

$$\varphi ::= \bigcirc \Phi \mid \Phi_1 \mathbf{U} \Phi_2$$

- ✓ Φ sono “state formula”
- ✓ φ sono “path formula”

Operatori derivati

$$\exists \Diamond \Phi = \exists (\text{true} \mathbf{U} \Phi)$$

$$\forall \Diamond \Phi = \forall (\text{true} \mathbf{U} \Phi)$$

$$\exists \Box \Phi = \neg \forall \Diamond \neg \Phi$$

$$\forall \Box \Phi = \neg \exists \Diamond \neg \Phi$$

CTL Existential Normal Form (ENF)

Grammatica

$$\Phi ::= \text{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid \exists \bigcirc \Phi \mid \exists (\Phi_1 \mathbf{U} \Phi_2) \mid \exists \square \Phi$$

Equivalenze

$$\forall \bigcirc \Phi \equiv \neg \exists \bigcirc \neg \Phi$$

$$\forall \Diamond \Phi \equiv \neg \exists \square \neg \Phi$$

$$\forall \square \Phi \equiv \neg \exists \Diamond \neg \Phi = \neg \exists (\text{true} \mathbf{U} \neg \Phi)$$

✓ utile anche la formula vista prima: $\exists \Diamond \Phi = \exists (\text{true} \mathbf{U} \Phi)$

Equivalenze non banali

\forall until

$$\forall(\Phi \mathbf{U} \Psi) \equiv \neg \exists(\neg \Psi \mathbf{U} (\neg \Phi \wedge \neg \Psi)) \wedge \neg \exists \Box \neg \Psi$$

Weak until

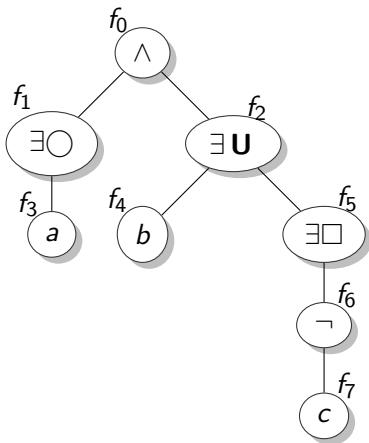
$$\begin{aligned}\exists(\Phi \mathbf{W} \Psi) &= \neg \forall((\Phi \wedge \neg \Psi) \mathbf{U} (\neg \Phi \wedge \neg \Psi)) \\ &= \exists(\Phi \mathbf{U} \Psi) \vee \exists \Box \Phi \\ &= \neg(\neg \exists(\Phi \mathbf{U} \Psi) \wedge \neg \exists \Box \Phi) \\ \forall(\Phi \mathbf{W} \Psi) &= \neg \exists((\Phi \wedge \neg \Psi) \mathbf{U} (\neg \Phi \wedge \neg \Psi))\end{aligned}$$

✓ Comportano esplosione **esponenziale** della formula

Formula example

Formula

$$\Phi = \exists \bigcirc a \wedge \exists (b \mathbf{U} \exists \square \neg c) \equiv \text{EX}(a) \ \& \ (b \ \text{EU} \ (\text{EG} \ !c))$$



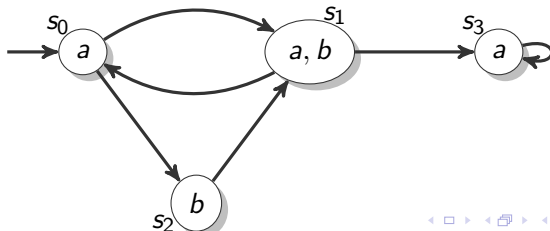
```
1  f0 &
2  f1 EX
3  f2 EU
4  f3 ap a
5  f4 ap b
6  f5 EG
7  f6 !
8  f7 ap c
9
10 f0 f1
11 f0 f2
12 f1 f3
13 f2 f4 <
14 f2 f5 >
15 f5 f6
16 f6 f7
```

Transition System (TS)

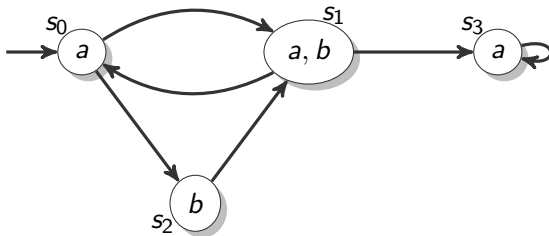
Definizione

$TS = (S, Act, \longrightarrow, I, AP, L)$

- ✓ S insieme di **stati**
- ✓ Act insieme di **azioni** (singola azione per CTL)
- ✓ $\longrightarrow \subseteq S \times Act \times S$
- ✓ $I \subseteq S$ insieme di stati **iniziali**
- ✓ AP insieme di **proposizioni atomiche**
- ✓ $L : S \rightarrow 2^{AP}$



Implementazione TS



```
1  s0 true a
2  s1 false a,b
3  s2 false b
4  s3 false a
5
6  s0 s1
7  s0 s2
8  s1 s0
9  s1 s3
10 s2 s1
11 s3 s3
```


CTL Model Checking

✓ Verificare se $TS \models \Phi$

► calcolare ricorsivamente $Sat(\Phi)$

$$Sat(true) = S$$

$$Sat(a) = \{s \in S \mid a \in L(s)\} \quad , \forall a \in AP$$

$$Sat(\Phi \wedge \Psi) = Sat(\Phi) \cap Sat(\Psi)$$

$$Sat(\neg \Phi) = S \setminus Sat(\Phi)$$

$$Sat(\exists \bigcirc \Phi) = \{s \in S \mid Post(s) \cap Sat(\Phi) \neq \emptyset\}$$

$$Sat(\exists(\Phi \mathbf{U} \Psi)) = \text{il pi\`u piccolo } T \subseteq S \text{ t.c.}$$

$$Sat(\Psi) \cup \{s \in Sat(\Phi) \mid Post(s) \cap T \neq \emptyset\} \subseteq T$$

$$Sat(\exists \Box \Phi) = \text{il pi\`u grande } T \subseteq S \text{ t.c.}$$

$$T \subseteq \{s \in Sat(\Phi) \mid Post(s) \cap T \neq \emptyset\}$$

★ formule non ENF vengono **convertite**

► $TS \models \Phi \Leftrightarrow I \subseteq Sat(\Phi)$

Complessità CTL Model Checking

Algoritmi di base $Sat(\cdot)$, formula ENF

$$\mathcal{O}((N + K) \cdot |\Phi|)$$

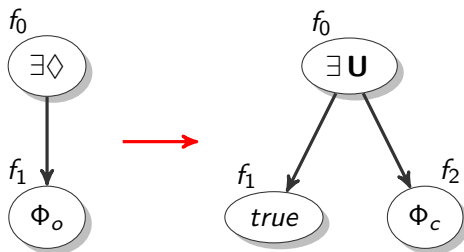
- ✓ N numero di **stati** del TS
- ✓ K numero di **transizioni** del TS
- ✓ $|\Phi|$ **lunghezza** della formula Φ

Complessità $TS \models \Phi$

- ✓ La trasformazione da formula CTL generica a CTL ENF sarebbe **esponenziale**, però:
 - ▶ Esistono algoritmi **lineari** analoghi a quelli per formule ENF
 - ▶ Implementata conversione/calcolo **lineare** di $Sat(\cdot)$ per formule generiche
- ✓ La complessità totale rimane **invariata**

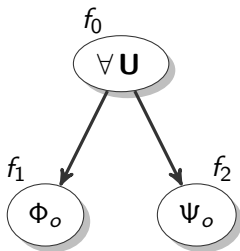
Formule non ENF

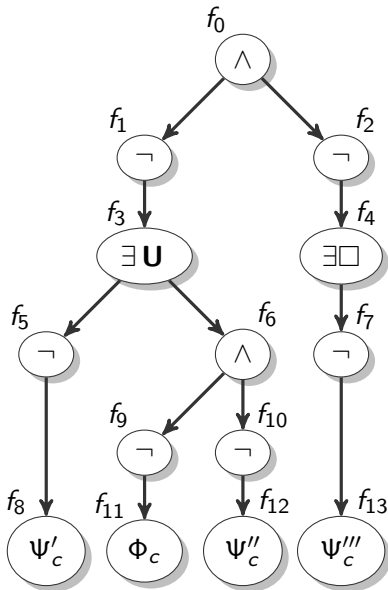
- ✓ Alberi di **conversione** generici
 - Foglie speciali per agganciarci alla formula originale



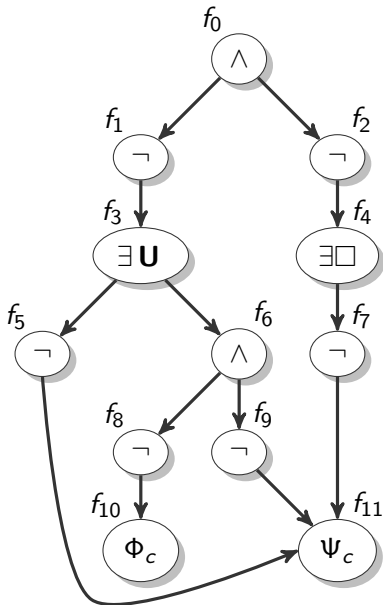
1. **Calcolo** di $Sat(\Phi_o)$
2. **Salva** risultato in nodo speciale Φ_c
 - Evita esplosione della formula $\forall(\Phi \mathbf{U} \Psi)$
3. **Calcolo** di $Sat(\exists \mathbf{U})$ su albero di conversione

- ✓ Formula da convertire:





✓ $Sat(\Psi)$ da calcolare **tre** volte



✓ $Sat(\Psi)$ calcolata **una** volta

The End.



Questions? Thank you!

Implementazione costruttore

```
10 def __init__(self, transitionSystem):
11     """
12     A ctlChecker is linked to a certain transition system. When
13     you build a CtlChecker you need to pass a TransitionSystem to
14     it.
15     """
16     self._syntax = syntax.Syntax()
17     self._conv = conversions.Conversions()
18     self._ts = transitionSystem
19
20     self._callDic = {
21         self._syntax.true          : self._satTrue,
22         self._syntax.ap            : self._satAp,
23         self._syntax.land          : self._satAnd,
24         self._syntax.lor           : self._satConversionTwoSons,
25         self._syntax.lnot          : self._satNot,
26         self._syntax.implies       : self._satConversionTwoSonsOrdered,
27         self._syntax.equals        : self._satConversionTwoSons,
28         self._syntax.exNext        : self._satExNext,
29         self._syntax.exUntil       : self._satExUntil,
30         self._syntax.exAlways      : self._satExAlways,
31         self._syntax.exEventually : self._satConversionOneSon,
32         self._syntax.faNext        : self._satConversionOneSon,
33         self._syntax.faUntil       : self._satConversionTwoSonsOrdered,
34         self._syntax.faAlways      : self._satConversionOneSon,
35         self._syntax.faEventually : self._satConversionOneSon,
36         self._syntax.exWeakUntil   : self._satConversionTwoSonsOrdered,
37         self._syntax.faWeakUntil   : self._satConversionTwoSonsOrdered,
38         self._syntax.phiNode       : self._satPhi,
39     }
```


Implementazione *Sat*(.)

```
200 def sat(self, ctlFormule):
201     """
202     The function that compute the satisfaction set of a formula
203     and is callable by outside the class. Basically it calls _sat
204     initializing the current node with the root of the formula.
205     """
206     return self._sat(ctlFormule.graph.copy(), [s for s,a in
        ↪     ctlFormule.graph.nodes(data=True) if a['root'] ==True][0])
```

```
191 def _sat(self, tree, currNode):
192     """
193     The generic function for the calculus of the satisfaction set
194     of a formula. It uses a dictionary for calling the right
195     function for the type of formula.
196     """
197     if (tree.node[currNode]['form'] in self._callDic.keys()) :
198         return self._callDic[tree.node[currNode]['form']](tree, currNode)
```

Implementazione *Sat*(.)

```
41 def _satTrue(self, tree, currNode):
42     """
43     Return satisfaction set for 'true', that is all the nodes of
44     the transition system.
45     """
46     return set(self._ts.graph.nodes())
```

```
48 def _satAp(self, tree, currNode):
49     """
50     Return satisfaction set for an atomic proposition, that is all
51     the nodes of the transition system that contain that atom.
52     """
53     retSet = set()
54     for stato, att in self._ts.graph.nodes(data=True):
55         if tree.node[currNode]['val'] in att['att']:
56             retSet.add(stato)
57
58     return retSet
```

```
60 def _satAnd(self, tree, currNode):
61     """
62     Return satisfaction set for 'phi and psi', that is the
63     intersection of the satisfaction sets of phi and psi.
64     """
65     sonA = tree.successors(currNode)[0]
66     sonB = tree.successors(currNode)[1]
67
68     return self._sat(tree, sonA).intersection(self._sat(tree, sonB))
```

Implementazione $Sat(\cdot)$

```
70 def _satNot(self, tree, currNode):
71     """
72     Return satisfaction set for 'not phi', that is the complement
73     of the satisfaction set of phi.
74     """
75     return set(self._ts.graph.nodes()).difference(self._sat(tree,
        ↳ tree.successors(currNode)[0]))
```

```
77 def _satExNext(self, tree, currNode):
78     """
79     Return satisfaction set for 'EX phi', that is the set of nodes that
80     have a successor that satisfy phi.
81     """
82     retSet = set()
83     satPhi = self._sat(tree, tree.successors(currNode)[0])
84
85     for stato in self._ts.graph.nodes():
86         if set(self._ts.graph.successors(stato)).intersection(satPhi): #true if
            ↳ not empty
87             retSet.add(stato)
88
89     return retSet
```

Implementazione $Sat(\cdot)$

```
91 def _satExUntil(self, tree, currNode):
92     """
93     Return satisfaction set for 'E(phi U psi)', that is the set of nodes that
94     have a track that satisfy phi ended by a state that satisfy
95     psi. This set is calculated going backward starting from the
96     states that satisfy psi, adding the states that satisfy phi
97     and have an edge to the already found states.
98     """
99     leftSon = [x for x in tree[currNode] if tree[currNode][x]['son'] ==
100                ↳ self._syntax.leftSon][0]
101     rightSon = [x for x in tree[currNode] if tree[currNode][x]['son'] ==
102                 ↳ self._syntax.rightSon][0]
103
104     S = self._sat(tree, leftSon)
105     E = self._sat(tree, rightSon)
106     T = E.copy()
107
108     while E: #while not empty
109         r = E.pop()
110         for s in self._ts.graph.predecessors(r):
111             if s in S.difference(T):
112                 E.add(s)
113                 T.add(s)
114
115     return T
```

Implementazione $Sat(\cdot)$

```
115 def _satExAlways(self, tree, currNode):
116     """
117     Return satisfaction set for 'EG phi', that is the set of nodes
118     that have a track that satisfy always phi. This set is
119     calculated using counters that start with the numbers of
120     neighbours for each state that satisfy phi, and then remove a
121     state from the set only if every neighbour don't satisfy phi.
122     """
123
124     T = self._sat(tree, tree.successors(currNode)[0])
125     E = set(self._ts.graph.nodes()).difference(T)
126     count = dict()
127     for s in T:
128         count[s] = len(self._ts.graph.successors(s))
129
130     while E:
131         r = E.pop()
132         for s in self._ts.graph.predecessors(r):
133             if s in T:
134                 count[s] = count[s]-1
135                 if count[s] == 0:
136                     T.remove(s)
137                     E.add(s)
138
139     return T
```

Implementazione $Sat(\cdot)$ formule non ENF

```
141 def _satConversionOneSon(self, tree, currNode):
142     """
143     Return satisfaction set for every conversion tree that has only
144     one son phi. That is calculated saving the satisfaction set of
145     phi in a special node of the conversion tree before calling
146     the calculus.
147     """
148     form = tree.node[currNode]['form']
149     son = tree.successors(currNode)[0]
150
151     self._conv.trees[form].node[self._conv.phis[form]]['sat'] = self._sat(tree,
152     ↪ son)
153     return self._sat(self._conv.trees[form], self._conv.roots[form])
```

```
154 def _satConversionTwoSons(self, tree, currNode):
155     """
156     Return satisfaction set for every conversion tree that has two
157     sons phi and psi. That is calculated saving the satisfaction
158     sets of phi and psi in special nodes of the conversion tree
159     before calling the calculus.
160     """
161     form = tree.node[currNode]['form']
162     sonA = tree.successors(currNode)[0]
163     sonB = tree.successors(currNode)[1]
164
165     self._conv.trees[form].node[self._conv.phis[form]]['sat'] = self._sat(tree,
166     ↪ sonA)
167     self._conv.trees[form].node[self._conv.psis[form]]['sat'] = self._sat(tree,
168     ↪ sonB)
169     return self._sat(self._conv.trees[form], self._conv.roots[form])
```

Implementazione $Sat(\cdot)$ formule non ENF

```
169     def _satConversionTwoSonsOrdered(self, tree, currNode):
170         """
171         Return satisfaction set for every conversion tree that has two
172         sons phi and psi with a proper order. That is calculated
173         saving the satisfaction sets of phi and psi in special nodes of
174         the conversion tree before calling the calculus.
175         """
176         form = tree.node[currNode]['form']
177         leftSon = [x for x in tree[currNode] if tree[currNode][x]['son'] ==
178                    ↪ self._syntax.leftSon][0]
179         rightSon = [x for x in tree[currNode] if tree[currNode][x]['son'] ==
180                     ↪ self._syntax.rightSon][0]
181
182         self._conv.trees[form].node[self._conv.phis[form]]['sat'] = self._sat(tree,
183                    ↪ leftSon)
184         self._conv.trees[form].node[self._conv.psis[form]]['sat'] = self._sat(tree,
185                    ↪ rightSon)
186         return self._sat(self._conv.trees[form], self._conv.roots[form])
```

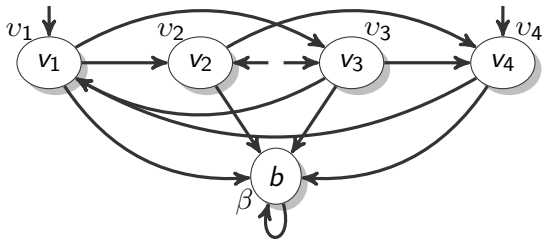
Implementazione $TS \models \Phi$

```
208     def check(self, ctlFormule):
209         """
210         The function that do the model checking. It check if all the
211         initial states of the transition system are contained inside
212         the calculated satisfaction set.
213         """
214         sats = self.sat(ctlFormule)
215         initials = set([s for s,a in self._ts.graph.nodes(data=True) if
216             ↪ a['initial'] ==True])
217         return initials.issubset(sats)
```


Complessità rispetto a Linear Temporal Logic (LTL)

1. Problema Hamiltoniano (HP) è **NP-Completo**
 - ▶ Trovare un percorso in un certo grafo G che passa esattamente una volta per ogni nodo
- 2a. è possibile descrivere TS_G e una formula **LTL** Φ_{LTL} di lunghezza **polinomiale** nel numero di stati del grafo del problema
 - ▶ tali che $TS_G \not\models \Phi_{LTL} \Leftrightarrow G$ contiene un percorso hamiltoniano
- 2b. il model checking di Linear Temporal Logic (LTL) ha complessità **esponenziale** in $|\Phi|$
- 3a. è possibile descrivere un TS_G e una formula **CTL** Φ_{CTL} di lunghezza **esponenziale** nel numero di stati del grafo
 - ▶ tali che $TS_G \not\models \neg\Phi_{CTL} \Leftrightarrow G$ contiene un percorso hamiltoniano
- 3b. il model checking di CTL ha complessità **lineare** in $|\Phi|$
4. se $P \neq NP$ **non** esiste una formula Φ_{CTL} equivalente a Φ_{LTL} e **non** esponenzialmente più lunga.

Implementazione TS_G , Φ_{LTL} , Φ_{CTL}



$$\Phi_{LTL} = \neg \bigwedge_{v \in V} (\Diamond v \wedge \Box (v \rightarrow \bigcirc \Box \neg v))$$

$$\Phi_{CTL} = \bigvee_{(i_1, \dots, i_n)} \Psi(v_{i_1}, \dots, v_{i_n}) \in \mathcal{O}(n!)$$

dove n è il numero di stati, e

$$\Psi(v_i) = v_i; \quad \Psi(v_{i_1}, v_{i_2}, \dots, v_{i_n}) = v_{i_1} \wedge \exists \bigcirc \Psi(v_{i_2}, \dots, v_{i_n}).$$