B-Spline methods for the design of smooth spatial paths with obstacle avoidance

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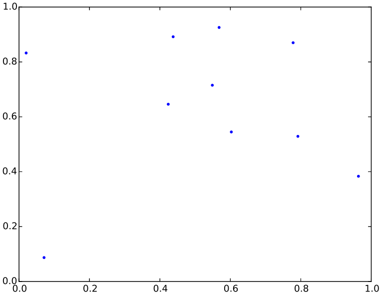
Voronoi diagrams

Input: Set of points in plane (or space) called sites

Output: partition of the plane (or space) such that each point of a region is closer to a certain site respect to others



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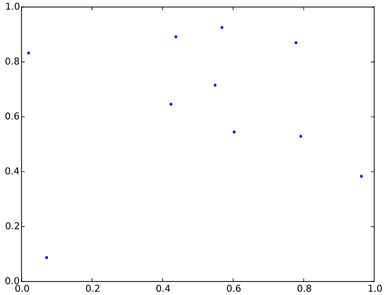
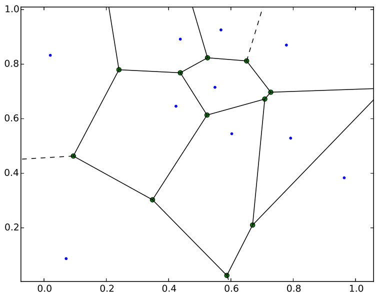
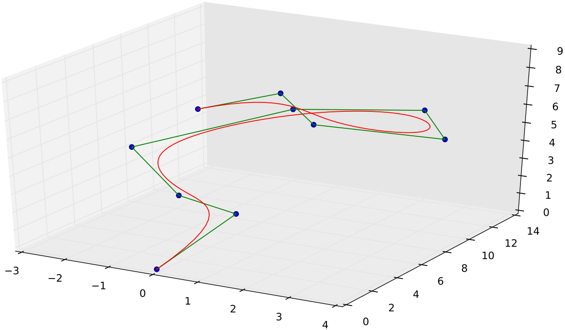




B-spline curves



- ✔ Piecewise polynomial parametric curves $\boldsymbol{S} : [a, b] \to \mathbb{E}^3$
  $\mathbf{S}(u) = \sum_{i=0}^{n} \mathbf{v_i} \cdot N_{i,m+1}(u)$
- ✔ Prescribed regularity
- ✔ Follow the shape of a control poligon
- ✔ Can interpolate the extremes of control polygon

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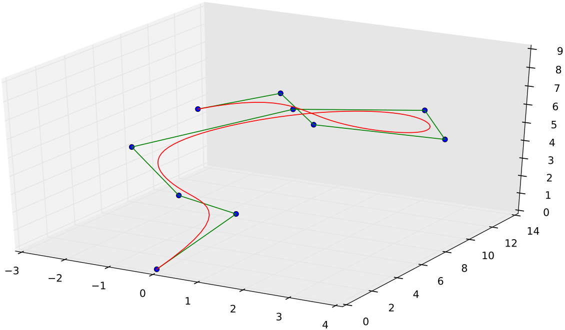

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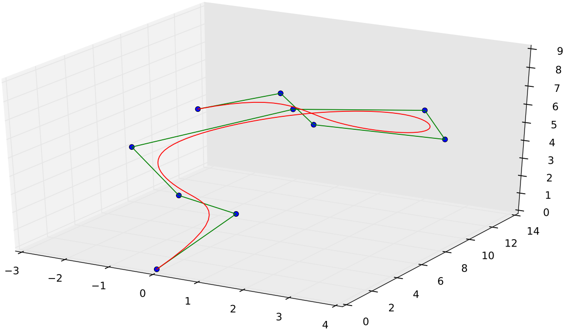

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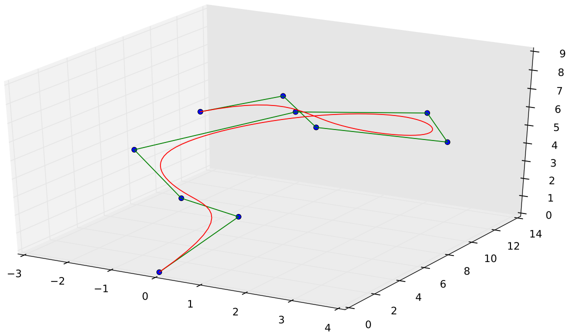

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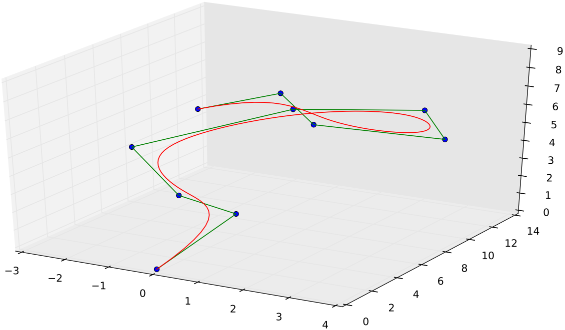

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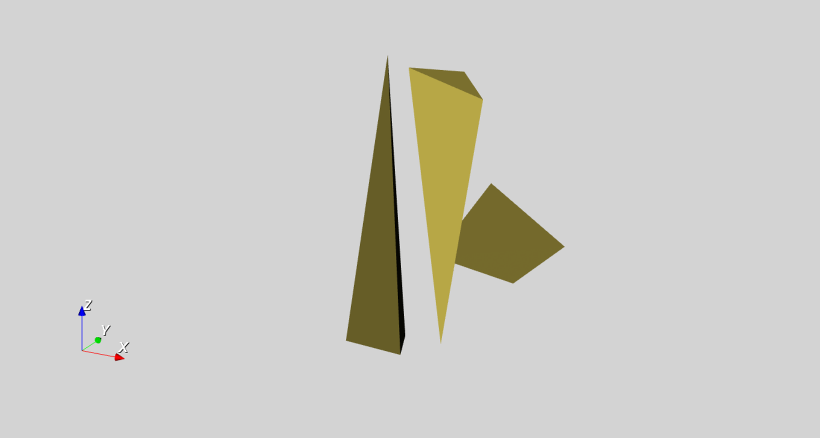

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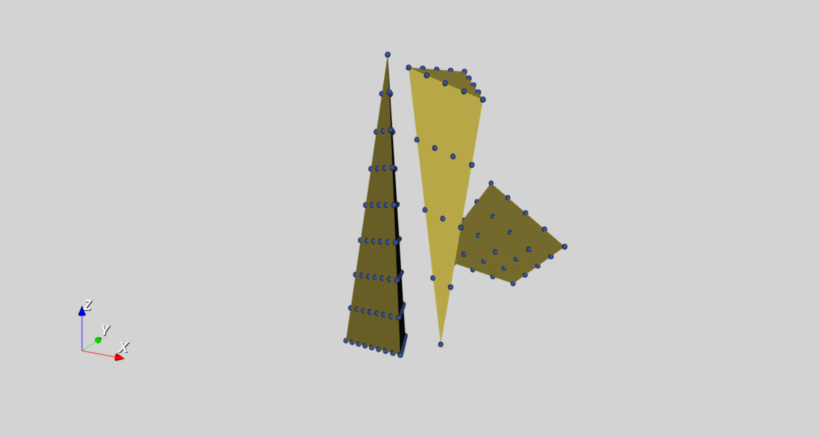

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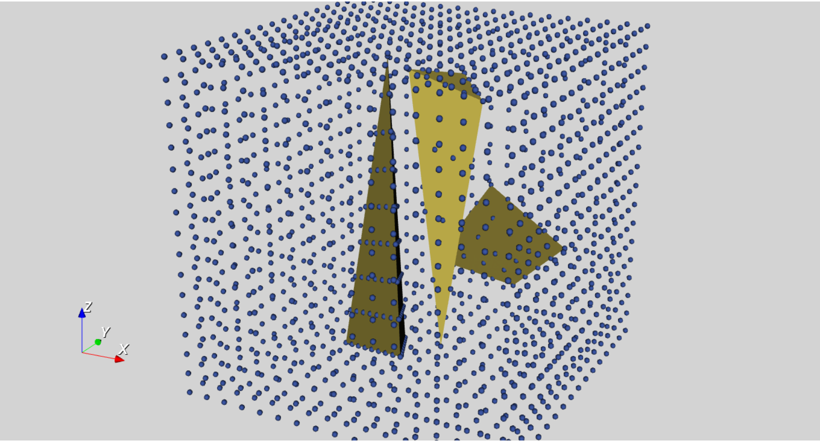

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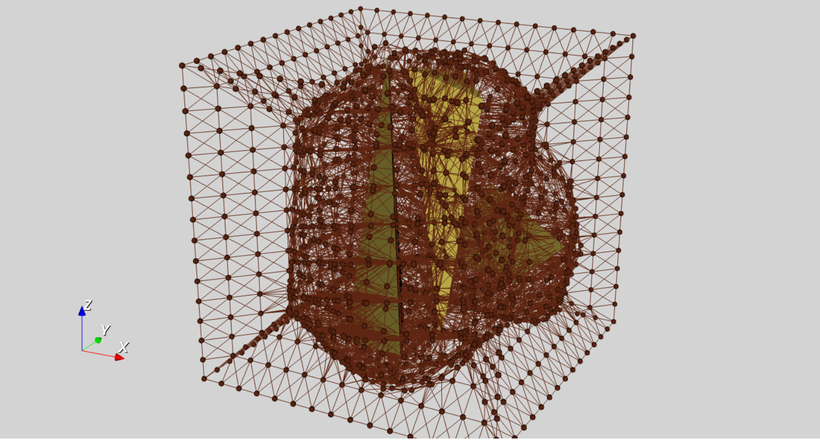

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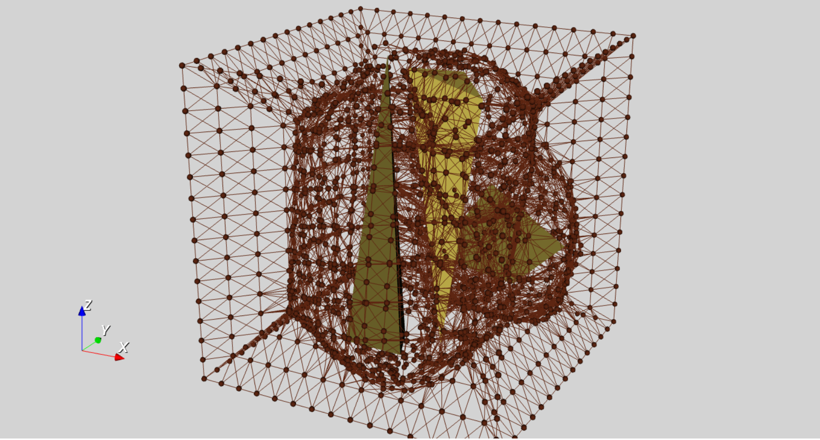

Improvement

Idea

Smoother curve instead of polygonal chain

- ✔ Use a B-Spline that
  - interpolate the start and end
  - Shortest path as control polygon

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✔ Control polygon obstacle-free by construction

- (because is pruned of arcs that cross obstacles)

✔ Curve may intersect an obstacle

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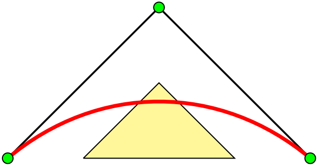

Solution

- A B-Spline of degree m is contained inside the union of convex hulls of $m + 1$ consecutive vertices

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- Use a quadratic B-Spline to smooth the path
- Achieve convex hulls free from intersection with obstacles

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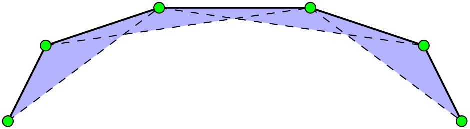

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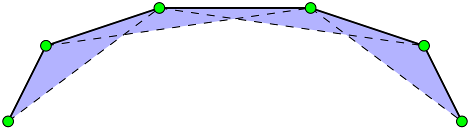

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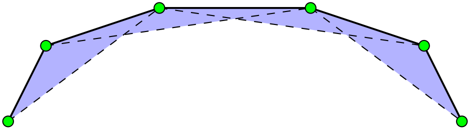

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First implementation

Graph transformation ($G \to G_t$)

- ✔ Triples (a, b, c) of neighboring nodes in G become nodes in G_t
- ✔ Arcs in G_t between triples in the form $(a, b, c) \to (b, c, d)$
  - weighted with the distance of the edge $a \leftrightarrow b$ in G

- ✔ Prune all the triples that intersect an obstacle
- ✔ Shortest path in the remaining triples

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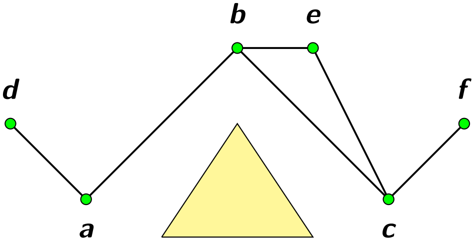

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Creation of G	$\mathcal{O}(\|O\| \log \|O\|)$
Pruning of G	$\mathcal{O}(k\|O\|^2)$
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Routing in G_t	$\mathcal{O}(k^3\|O\| + k^2\|O\| \log(k^2\|O\|))$
Total	$\mathcal{O}(k^2\|O\|^2 + k^3\|O\|)$
Total (k constant)	$\mathcal{O}(\|O\|^2)$

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- ✔ Scene construction
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Second implementation

Reason

- ✔ First implementation interesting
- ✘ but rejects many paths

- ✔ Shortest path on G
- ✔ Add aligned control vertices when an obstacle intersects a triple

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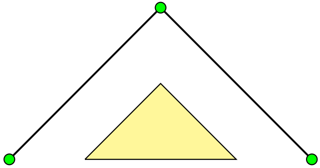

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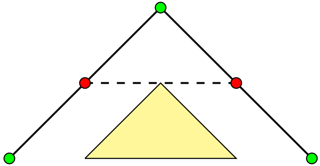

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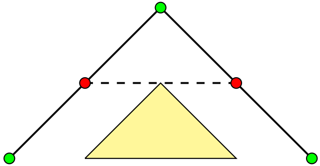

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✘ If we increase the B-Spline degree → convex hull not planar anymore

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- ✘ If we increase the B-Spline degree $\rightarrow$ convex hull not planar anymore
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Solution

- ✔ Add aligned vertices in control polygon
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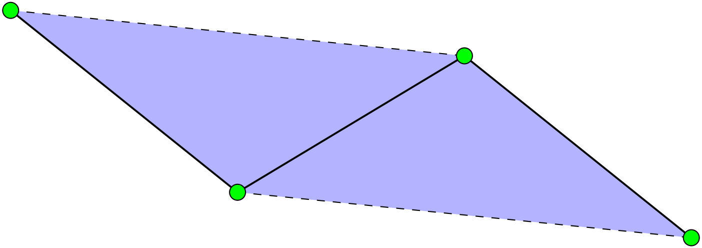

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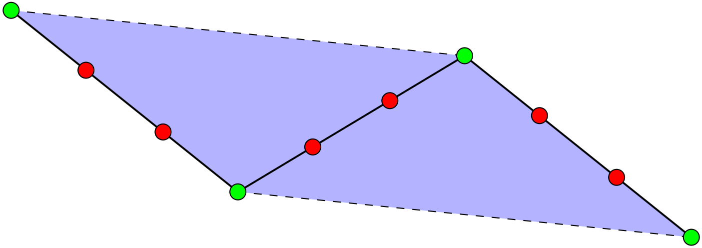

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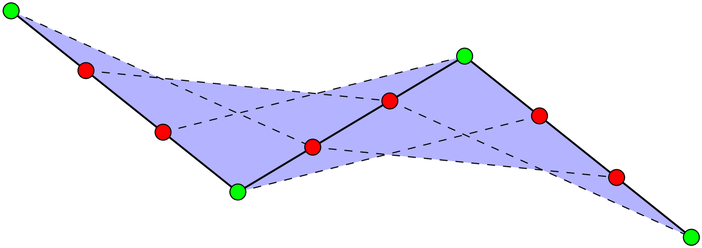

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Purpose

- ✔ Simplify the control polygon
- ✔ Remove useless turns

- ✔ For each triple $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ of consecutive points in path
- ✔ If no obstacles intersect the triangle $\rightarrow$ the triple is simplified to a single edge $(\boldsymbol{a}, \boldsymbol{c})$

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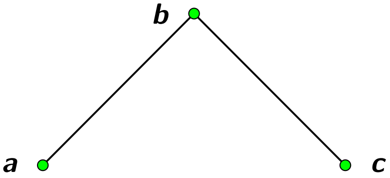

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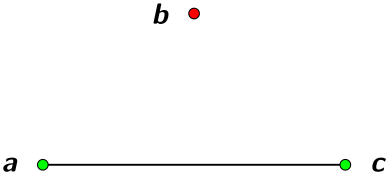

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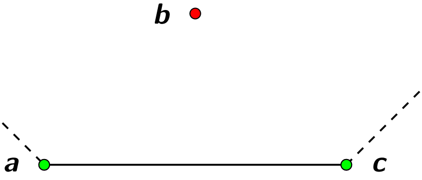

Optimization method

Problem

$$\underset{P}{\text{minimize}} \quad \alpha \cdot \max_u[\kappa_{\boldsymbol{S}}(u)] + \beta \cdot \max_u[\tau_{\boldsymbol{S}}(u)] + \gamma \cdot len(\boldsymbol{S})$$

$$\text{subject to} \quad \left| \boldsymbol{S}(u) \cap \bigcup_{i \in I} obstacle_i \right| = 0$$

- ✔ Relax constraint: $L(P, \lambda) = gain(P) + \lambda \cdot constraint(P)$
- ✔ Saddle point $L(P^*, \lambda) \leq L(P^*, \lambda^*) \leq L(P, \lambda^*)$
- ✔ Simulated annealing finds saddle point that minimizes *gain*

Drawbacks

- ✘ slower respect to the other methods
- ✘ *gain* and *constraint* are calculated in a discrete way

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python™ 3

SciPy

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python™ 3

SciPy

NetworkX

Technologies

python 3

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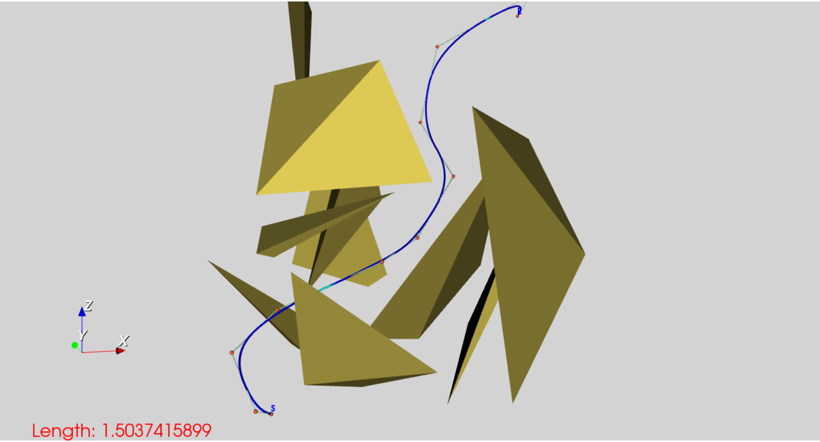
Visualization Toolkit

Example



✔ Method 1, no post processing

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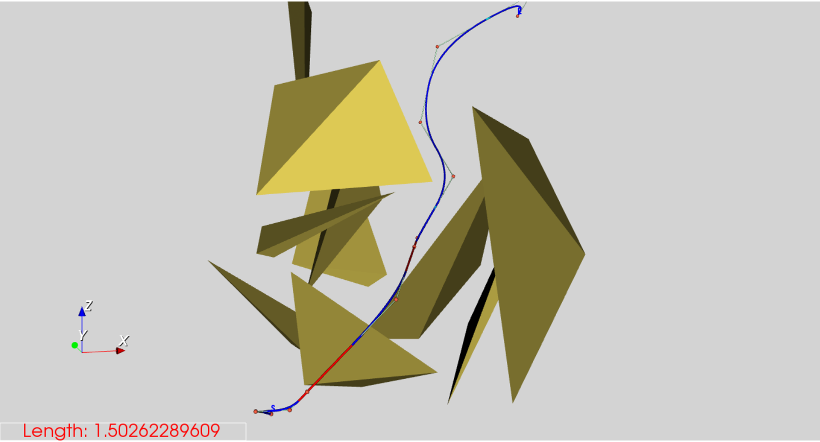

✔ Method 2, no post processing

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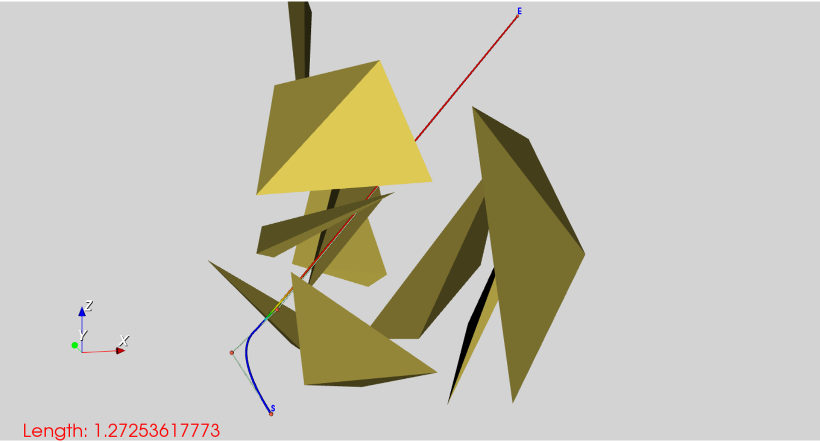

✔ Method 1, with post processing

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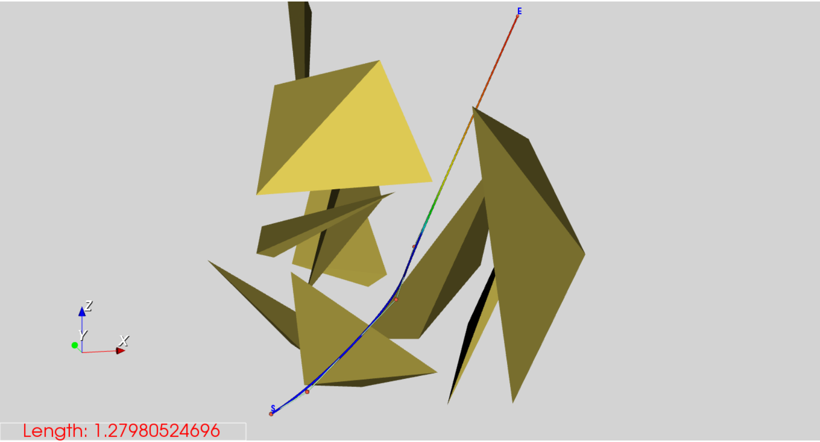

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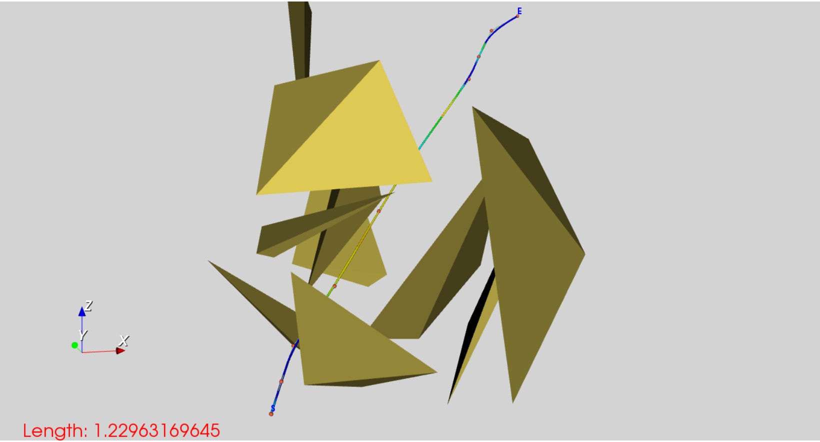

✔ Method 3

Future improvements

- ✔ Change underlying structure
  - different attach point
  - visibility graph
  - rapidly exploring random tree (RRT)
  - other ...
- ✔ Improve degree increase
  - without aligned vertices
  - like second solution but with quadruple/quintuples of vertices
- ✔ Improve post processing
  - make a symmetric algorithm
- ✔ Another optimization process
  - output of other solutions as initial state
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Questions? Thank you!

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B-spline curves details

- ✔ Degree m
- ✔ Extended partition (of parametric space $[a, b]$)

$$T = \{t_0, \ldots, t_{m-1}, t_m, \ldots, t_{n+1}, t_{n+2}, \ldots, t_{n+m+1}\}$$

$$t_0 \leq \cdots \leq t_{m-1} \leq t_m (\equiv a) < \cdots < t_{n+1} (\equiv b) \leq t_{n+2} \leq \cdots \leq t_{n+m+1}$$

- ✔ $n + 1$ basis (of $S_{m,\tau} = P_{m,\tau} \cap C^{m-1}$)

$$N_{i,1}(u) = \begin{cases} 1, & \text{if} \quad t_i \leq t < t_{i+1} \\ 0, & \text{otherwise} \end{cases} \qquad i = 0, \ldots, n+m$$

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- ✔ Interpolates extremes if $t_0 = \cdots = t_m$ and $t_{n+1} = \cdots = t_{n+m+1}$
- ✔ Continuity C^{m-1} between polynomials
- ✔ Contained in convex hulls of $m+1$ consecutive vertices

- ✔ Touches segment between m aligned vertices
- ✔ Lays in segment between $m+1$ aligned vertices

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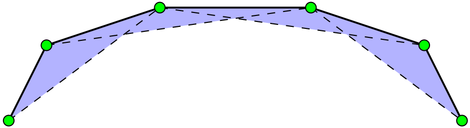

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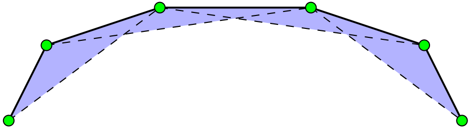
- ✔ Continuity C^{m-1} between polynomials
- ✔ Contained in convex hulls of $m+1$ consecutive vertices



- ✔ Touches segment between m aligned vertices
- ✔ Lays in segment between $m+1$ aligned vertices

$$\kappa(u) = \frac{\left\| \dot{\mathbf{S}}(u) \wedge \ddot{\mathbf{S}}(u) \right\|_2}{\left\| \dot{\mathbf{S}}(u) \right\|_2^3}$$

$$\tau(u) = \frac{\det\left[\dot{\mathbf{S}}(u), \ddot{\mathbf{S}}(u), \dddot{\mathbf{S}}(u) \right]}{\left\| \dot{\mathbf{S}}(u) \wedge \ddot{\mathbf{S}}(u) \right\|_2} = \frac{\left(\dot{\mathbf{S}}(u) \wedge \ddot{\mathbf{S}}(u) \right) \cdot \dddot{\mathbf{S}}(u)}{\left\| \dot{\mathbf{S}}(u) \wedge \ddot{\mathbf{S}}(u) \right\|_2}$$

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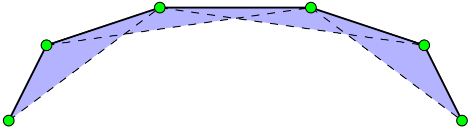

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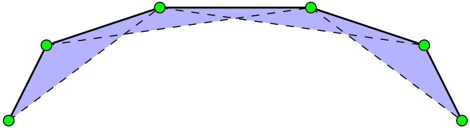

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Main problem

Path planning from a start point to an end point in 3D space with obstacles using Voronoi diagrams.

1. Distribute points in obstacles surfaces
   - and bounding box
2. Voronoi diagram using those points
3. Transform Voronoi diagram in graph
   - cells vertices $\rightarrow$ nodes
   - cells edges $\rightarrow$ arcs (infinite edges ignored)
4. Prune arcs that cross obstacles
5. Attach start and end
6. Shortest path from start to end
   - Dijkstra's algorithm

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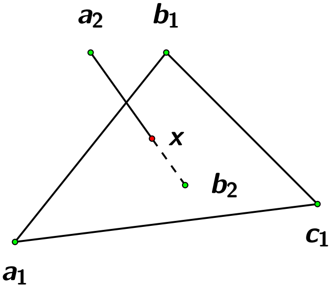
Intersection segment-triangle



$$\begin{cases} \alpha \boldsymbol{a_2} + \beta \boldsymbol{b_2} = \gamma \boldsymbol{a_1} + \delta \boldsymbol{b_1} + \zeta \boldsymbol{c_1} \\ \alpha + \beta = 1 \\ \gamma + \delta + \zeta = 1 \end{cases}$$

$$\begin{cases} \alpha \geq 0 \\ \beta \geq 0 \\ \gamma \geq 0 \\ \delta \geq 0 \\ \zeta \geq 0. \end{cases}$$