

Markov Regenerative Process - steady-state analysis

MVT exam

Stefano MARTINA

stefano.martina@stud.unifi.it

Tommaso PAPINI

tommaso.papini1@stud.unifi.it



Università degli Studi di Firenze

October 10, 2015



This work is licensed under a [Creative Commons Attribution-ShareAlike 4.0 International License](https://creativecommons.org/licenses/by-sa/4.0/).

Markov Regenerative Processes (MRP)

Definition

A Markov Regenerative Process (MRP) is a stochastic process that sooner or later, with probability one, will reach a **regenerative** state (will be regenerated).

Regenerative state

A state where the process loses its memory.

Markov Regenerative Processes (MRP)

Definition

A Markov Regenerative Process (MRP) is a stochastic process that sooner or later, with probability one, will reach a **regenerative** state (will be regenerated).

Regenerative state

A state where the process loses its memory.

Markov Regenerative Processes (MRP)

Definition

A Markov Regenerative Process (MRP) is a stochastic process that sooner or later, with probability one, will reach a **regenerative** state (will be regenerated).

Regenerative state

A state where the process loses its memory.



CTMC

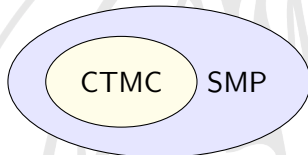
Markov Regenerative Processes (MRP)

Definition

A Markov Regenerative Process (MRP) is a stochastic process that sooner or later, with probability one, will reach a **regenerative** state (will be regenerated).

Regenerative state

A state where the process loses its memory.



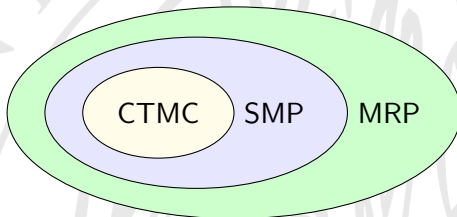
Markov Regenerative Processes (MRP)

Definition

A Markov Regenerative Process (MRP) is a stochastic process that sooner or later, with probability one, will reach a **regenerative** state (will be regenerated).

Regenerative state

A state where the process loses its memory.



The steady-state problem

Transient probabilities

The probability distribution that the process will be in a certain state, after given t time.

Steady-state

For ergodic systems, it represents the probability distribution that the process will be in a certain state, as time goes to infinity.

- ✓ ORIS current state:
 - Transient analysis for Markov Regenerative Process (MRP)
 - Steady-state analysis for everything else
- ✓ Until now! ☹
- ✓ **Warning:** we assume that the MRP is ergodic!

The steady-state problem

Transient probabilities

The probability distribution that the process will be in a certain state, after given t time.

Steady-state

For ergodic systems, it represents the probability distribution that the process will be in a certain state, as time goes to infinity.

- ✓ ORIS current state:
 - Transient analysis for Markov Regenerative Process (MRP)
 - Steady-state analysis for everything else
- ✓ Until now! ☺
- ✓ **Warning:** we assume that the MRP is ergodic!

The steady-state problem

Transient probabilities

The probability distribution that the process will be in a certain state, after given t time.

Steady-state

For ergodic systems, it represents the probability distribution that the process will be in a certain state, as time goes to infinity.

- ✓ ORIS current state:
 - Transient analysis for Markov Regenerative Process (MRP)
 - Steady-state analysis for everything else
- ✓ Until now! ☺
- ✓ **Warning:** we assume that the MRP is ergodic!

The steady-state problem

Transient probabilities

The probability distribution that the process will be in a certain state, after given t time.

Steady-state

For ergodic systems, it represents the probability distribution that the process will be in a certain state, as time goes to infinity.

- ✓ ORIS current state:
 - Transient analysis for Markov Regenerative Process (MRP)
 - Steady-state analysis for everything else
- ✓ Until now! ☺
- ✓ **Warning:** we assume that the MRP is ergodic!

The steady-state problem

Transient probabilities

The probability distribution that the process will be in a certain state, after given t time.

Steady-state

For ergodic systems, it represents the probability distribution that the process will be in a certain state, as time goes to infinity.

- ✓ ORIS current state:
 - Transient analysis for Markov Regenerative Process (MRP)
 - Steady-state analysis for everything else
- ✓ Until now! ☺
- ✓ **Warning:** we assume that the MRP is ergodic!

The steady-state problem

Transient probabilities

The probability distribution that the process will be in a certain state, after given t time.

Steady-state

For ergodic systems, it represents the probability distribution that the process will be in a certain state, as time goes to infinity.

- ✓ ORIS current state:
 - Transient analysis for Markov Regenerative Process (MRP)
 - Steady-state analysis for everything else
- ✓ Until now! ☺
- ✓ **Warning:** we assume that the MRP is ergodic!

MRP steady-state analysis - The theory

General idea:

1. Calculate the embedded DTMC steady-state on the regenerative states
2. Calculate the sojourn times in each marking, after each regenerative state
3. Combine the two above in order to calculate the MRP steady-state

Embedded DTMC
steady-state

Sojourn times

MRP steady-state

```
graph TD; A[Embedded DTMC steady-state] --> C[MRP steady-state]; B[Sojourn times] --> C;
```

The diagram illustrates the process of calculating the MRP steady-state. It features three boxes: a light blue box labeled 'Embedded DTMC steady-state', a light yellow box labeled 'Sojourn times', and a light green box labeled 'MRP steady-state'. Arrows from the first two boxes point towards the third box, indicating that the MRP steady-state is derived from these two components.

MRP steady-state analysis - The theory

General idea:

1. Calculate the embedded DTMC steady-state on the regenerative states
2. Calculate the sojourn times in each marking, after each regenerative state
3. Combine the two above in order to calculate the MRP steady-state

Embedded DTMC
steady-state

Sojourn times

MRP steady-state

MRP steady-state analysis - The theory

General idea:

1. Calculate the embedded DTMC steady-state on the regenerative states
2. Calculate the sojourn times in each marking, after each regenerative state
3. Combine the two above in order to calculate the MRP steady-state

Embedded DTMC
steady-state

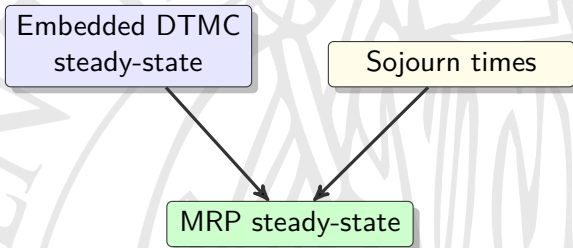
Sojourn times

MRP steady-state

MRP steady-state analysis - The theory

General idea:

1. Calculate the embedded DTMC steady-state on the regenerative states
2. Calculate the sojourn times in each marking, after each regenerative state
3. Combine the two above in order to calculate the MRP steady-state



Main classes implemented

- ✓ `class` EmbeddedDTMC
 - written from scratch
 - calculate embedded DTMC steady-state
- ✓ `class` RegenerativeSteadyStateAnalysis
 - based on `class` RegenerativeTransientAnalysis
 - calculate MRP steady-state

Steady-state of the embedded DTMC on regenerative states

Steady-state in Discrete Time Markov Process (DTMC)

If the steady-state of a Discrete Time Markov Process (DTMC) exists and is unique (if is ergodic), then it's calculated by solving for v the linear system:

$$\begin{cases} v = vP \\ |v| = 1 \end{cases}$$

- ✓ We want to calculate the steady-state of the embedded DTMC of the MRP in the regenerative states
- ✓ But we don't have P ! ☹

Steady-state of the embedded DTMC on regenerative states

Steady-state in Discrete Time Markov Process (DTMC)

If the steady-state of a Discrete Time Markov Process (DTMC) exists and is unique (if is ergodic), then it's calculated by solving for v the linear system:

$$\begin{cases} v = vP \\ |v| = 1 \end{cases}$$

- ✓ We want to calculate the steady-state of the embedded DTMC of the MRP in the regenerative states
- ✓ But we don't have P ! ☹

Steady-state of the embedded DTMC on regenerative states

Steady-state in Discrete Time Markov Process (DTMC)

If the steady-state of a Discrete Time Markov Process (DTMC) exists and is unique (if is ergodic), then it's calculated by solving for v the linear system:

$$\begin{cases} v = vP \\ |v| = 1 \end{cases}$$

- ✓ We want to calculate the steady-state of the embedded DTMC of the MRP in the regenerative states
- ✓ But we don't have P ! ☹

Reaching probability feature

- ✓ We add a new **reaching probability feature** to each state:
`class ReachingProbabilityFeature`
- ✓ Inside `SteadyStateInitialStateBuilder`: set it to 1
- ✓ Inside `SteadyStatePostProcessor`: multiply the parent's reaching probability by the probability to chose a certain child

If we run a transient analysis on the Petri Net (PN) we get:

- ✓ `regenerationClasses`
- ✓ `Map<DeterministicEnablingState, Map<DeterministicEnablingState, Set<State>>>`
- ✓ sum reaching probability feature of each State to compute elements of **P**

Now we can solve the linear system:

$$\begin{cases} v = vP \\ |v| = 1 \end{cases} = \begin{cases} (P' - I)v' = 0 \\ \sum_i v_i = 1 \end{cases}$$

✓ RealMatrix & RealVector

✓ QR decomposition solver

- DecompositionSolver solver = `new` QRDecomposition(coefficients).getSolver();
- RealVector steadyState = solver.solve(constants);

✓ Convert steadyState into a

Map<DeterministicEnablingState, BigDecimal>

Sojourn time a_{ij}

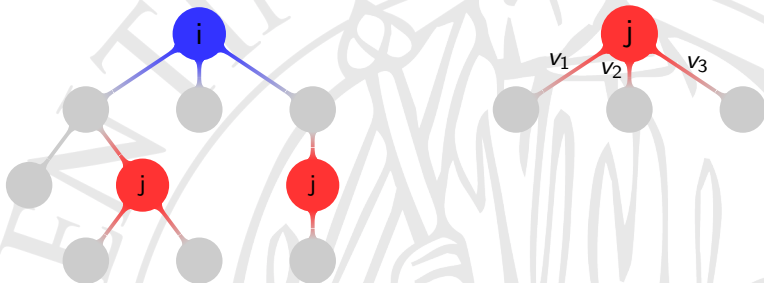
Definition

The sojourn time a_{ij} represents the average time spent in the j -th marking after the (last) i -th regeneration.

How to compute a_{ij} ?

a_{ij} is:

- ✓ sum of avg time spent in marking j occurrences
 - sum of avg time before each variable fires
 - ★ condition each variable to be the minimum (i.e. the one that fires)
 - ★ compute avg time before that variable fires (thanks Marco!)



When to compute a_{ij} ?

During the transient analysis!

- ✓ transient analysis generates succession trees for each regenerative state
 - regenerative state as root
 - following regenerative states as leaves
 - reachable markings as inner nodes
- ✓ during the tree generation compute and accumulate a_{ij} for each marking occurrence found

Markov Regenerative Process (MRP) steady-state

Let's combine the embedded DTMC steady-state and the sojourn times!

$$\pi_j = \frac{\sum_i v_i a_{ij}}{K}$$

- ✓ We multiply the sojourn time in the marking j after the regeneration i by the probability of reaching the i -th regeneration
- ✓ We do this for each regeneration that leads to the marking j before another regeneration
- ✓ K is a normalization factor calculated as the sum of π_j

Markov Regenerative Process (MRP) steady-state

Let's combine the embedded DTMC steady-state and the sojourn times!

$$\pi_j = \frac{\sum_i v_i a_{ij}}{K}$$

- ✓ We multiply the sojourn time in the marking j after the regeneration i by the probability of reaching the i -th regeneration
- ✓ We do this for each regeneration that leads to the marking j before another regeneration
- ✓ K is a normalization factor calculated as the sum of π_j

Markov Regenerative Process (MRP) steady-state

Let's combine the embedded DTMC steady-state and the sojourn times!

$$\pi_j = \frac{\sum_i v_i a_{ij}}{K}$$

- ✓ We multiply the sojourn time in the marking j after the regeneration i by the probability of reaching the i -th regeneration
- ✓ We do this for each regeneration that leads to the marking j before another regeneration
- ✓ K is a normalization factor calculated as the sum of π_j

Markov Regenerative Process (MRP) steady-state

Let's combine the embedded DTMC steady-state and the sojourn times!

$$\pi_j = \frac{\sum_i v_i a_{ij}}{K}$$

- ✓ We multiply the sojourn time in the marking j after the regeneration i by the probability of reaching the i -th regeneration
- ✓ We do this for each regeneration that leads to the marking j before another regeneration
- ✓ K is a normalization factor calculated as the sum of π_j

Markov Regenerative Process (MRP) steady-state

Let's combine the embedded DTMC steady-state and the sojourn times!

$$\pi_j = \frac{\sum_i v_i a_{ij}}{K}$$

- ✓ We multiply the sojourn time in the marking j after the regeneration i by the probability of reaching the i -th regeneration
- ✓ We do this for each regeneration that leads to the marking j before another regeneration
- ✓ K is a normalization factor calculated as the sum of π_j

Test

Unit test

- ✓ Class `SteadyStateTest` with **JUnit** tests
- ✓ Three different models:
 - `TestCaseSMP`
 - `TestCase2ParallelTasks`
 - `TestCaseRejuvenation`
- ✓ For each test:
 1. launch MRP steady state **analysis**
 2. check if the result is comparable to the **expected** value (with a tolerance)

Test

Unit test

- ✓ Class `SteadyStateTest` with **JUnit** tests
- ✓ Three different models:
 - `TestCaseSMP`
 - `TestCase2ParallelTasks`
 - `TestCaseRejuvenation`
- ✓ For each test:
 1. launch MRP steady state **analysis**
 2. check if the result is comparable to the **expected** value (with a tolerance)

Test

Unit test

- ✓ Class `SteadyStateTest` with **JUnit** tests
- ✓ Three different models:
 - `TestCaseSMP`
 - `TestCase2ParallelTasks`
 - `TestCaseRejuvenation`
- ✓ For each test:
 1. launch MRP steady state **analysis**
 2. check if the result is comparable to the **expected** value (with a tolerance)

Test

Unit test

- ✓ Class `SteadyStateTest` with **JUnit** tests
- ✓ Three different models:
 - `TestCaseSMP`
 - `TestCase2ParallelTasks`
 - `TestCaseRejuvenation`
- ✓ For each test:
 1. launch MRP steady state **analysis**
 2. check if the result is comparable to the **expected** value (with a tolerance)

Test

Unit test

- ✓ Class SteadyStateTest with JUnit tests
- ✓ Three different models:
 - TestCaseSMP
 - TestCase2ParallelTasks
 - TestCaseRejuvenation
- ✓ For each test:
 1. launch MRP steady state analysis
 2. check if the result is comparable to the expected value (with a tolerance)

Test

Unit test

- ✓ Class `SteadyStateTest` with **JUnit** tests
- ✓ Three different models:
 - `TestCaseSMP`
 - `TestCase2ParallelTasks`
 - `TestCaseRejuvenation`
- ✓ For each test:
 1. launch MRP steady state **analysis**
 2. check if the result is comparable to the **expected** value (with a tolerance)

Test

Unit test

- ✓ Class SteadyStateTest with JUnit tests
- ✓ Three different models:
 - TestCaseSMP
 - TestCase2ParallelTasks
 - TestCaseRejuvenation
- ✓ For each test:
 1. launch MRP steady state analysis
 2. check if the result is comparable to the expected value (with a tolerance)

Test

Unit test

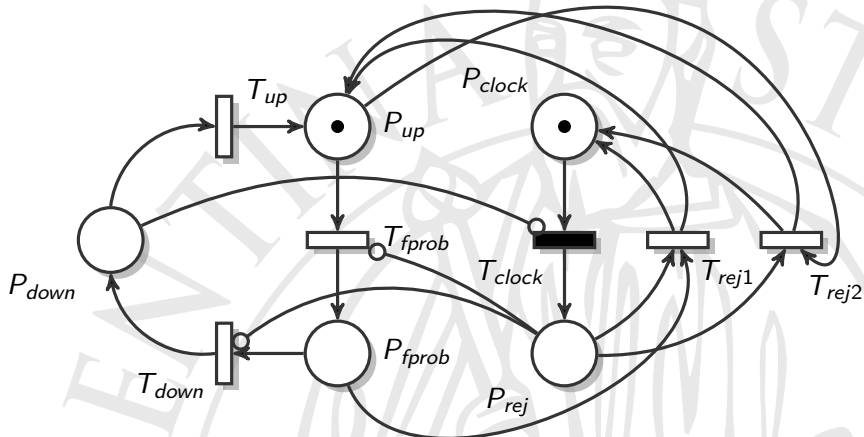
- ✓ Class SteadyStateTest with JUnit tests
- ✓ Three different models:
 - TestCaseSMP
 - TestCase2ParallelTasks
 - TestCaseRejuvenation
- ✓ For each test:
 1. launch MRP steady state analysis
 2. check if the result is comparable to the expected value (with a tolerance)

Test

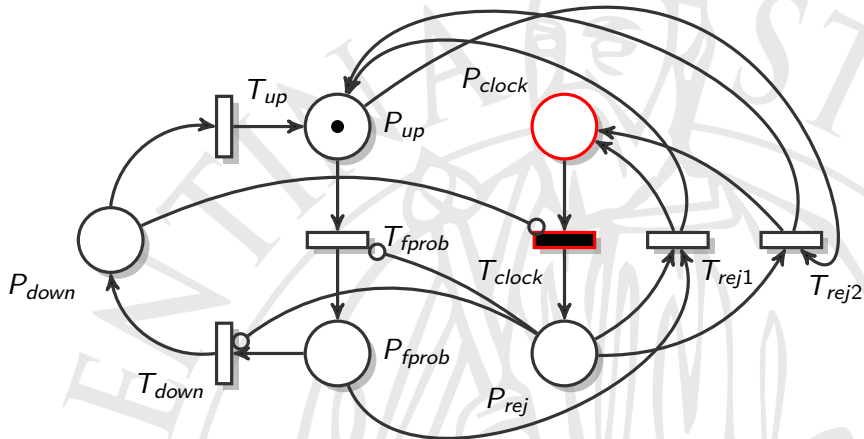
Unit test

- ✓ Class SteadyStateTest with JUnit tests
- ✓ Three different models:
 - TestCaseSMP
 - TestCase2ParallelTasks
 - TestCaseRejuvenation
- ✓ For each test:
 1. launch MRP steady state analysis
 2. check if the result is comparable to the expected value (with a tolerance)

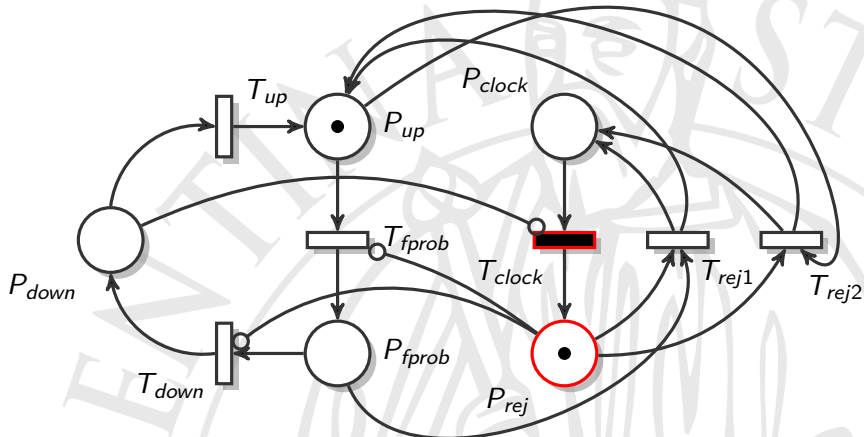
Rejuvenation



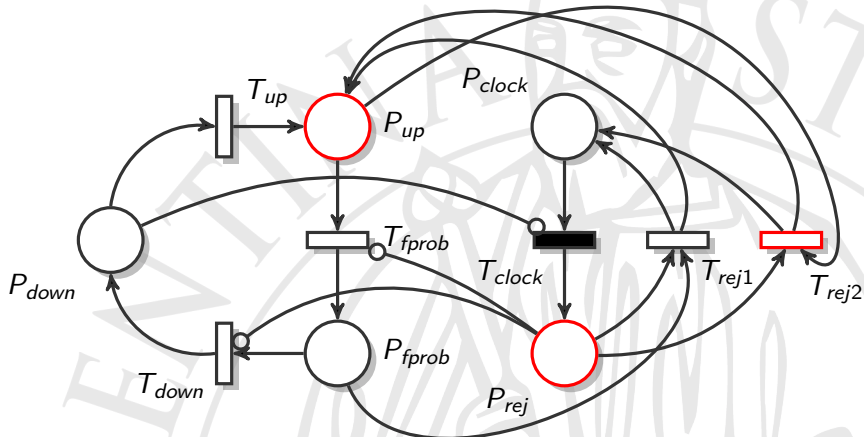
Rejuvenation



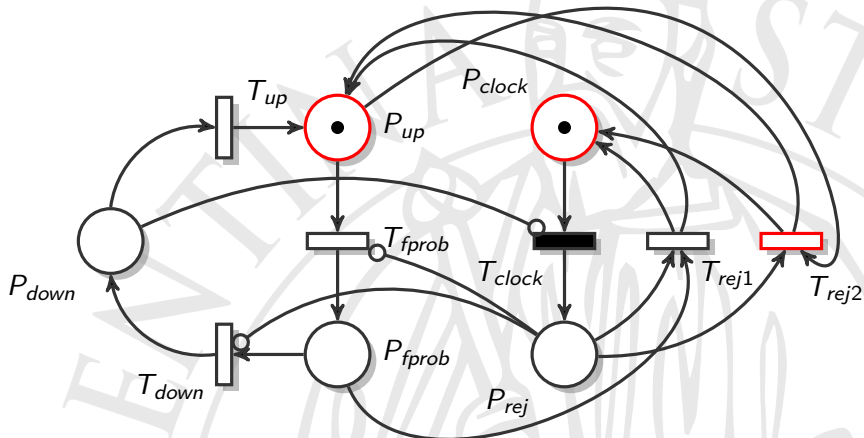
Rejuvenation



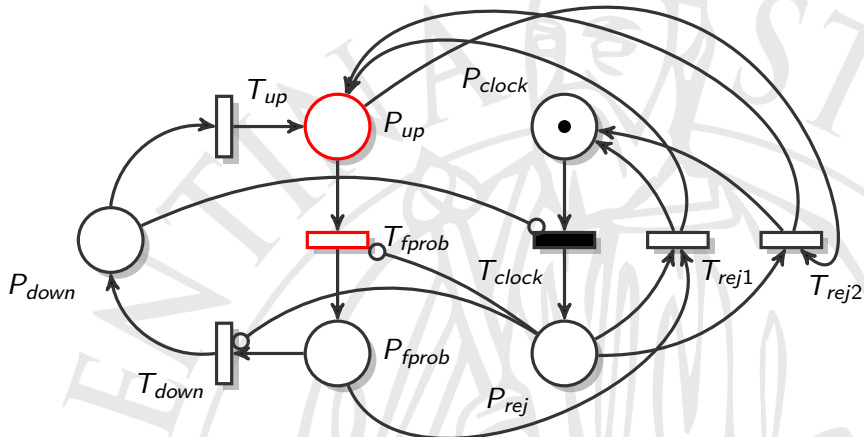
Rejuvenation



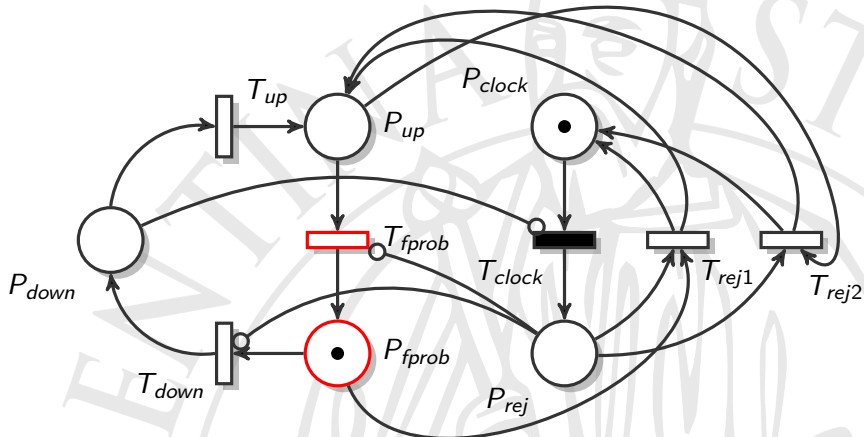
Rejuvenation



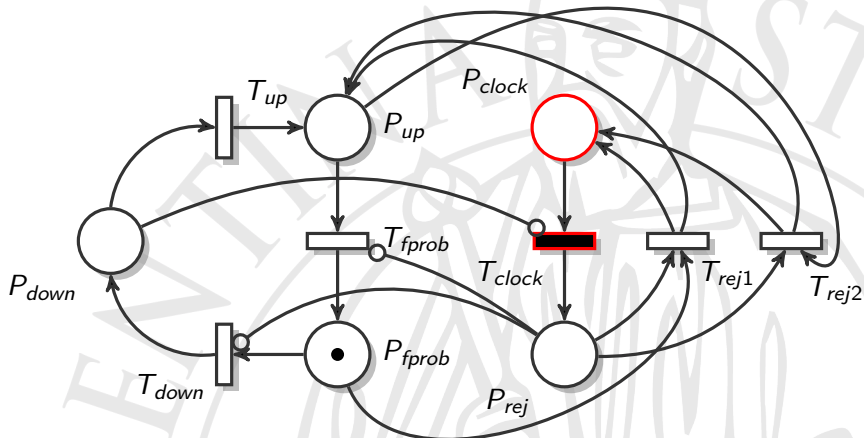
Rejuvenation



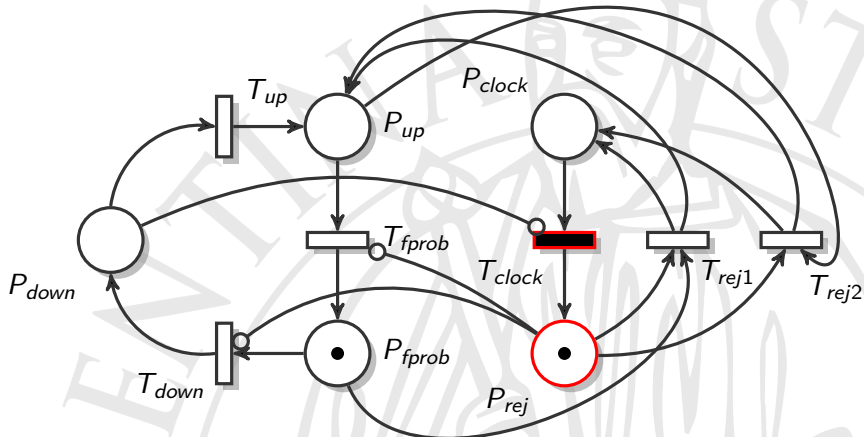
Rejuvenation



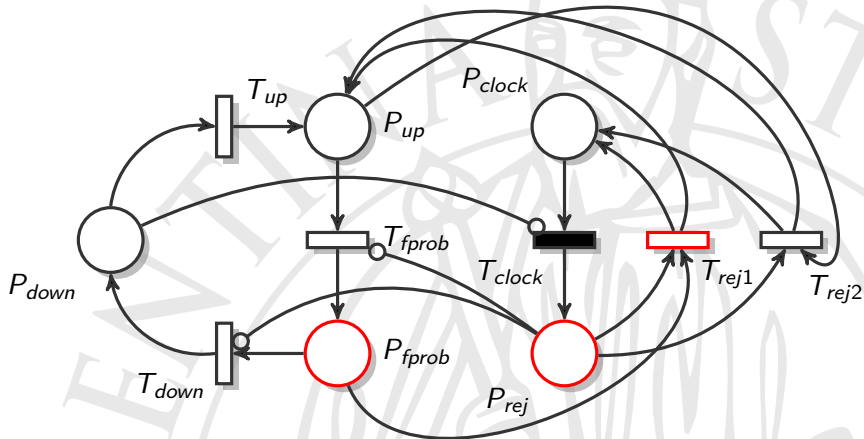
Rejuvenation



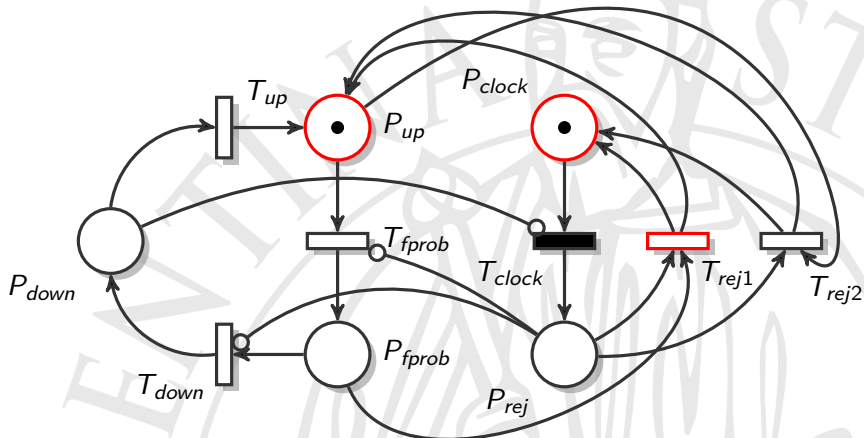
Rejuvenation



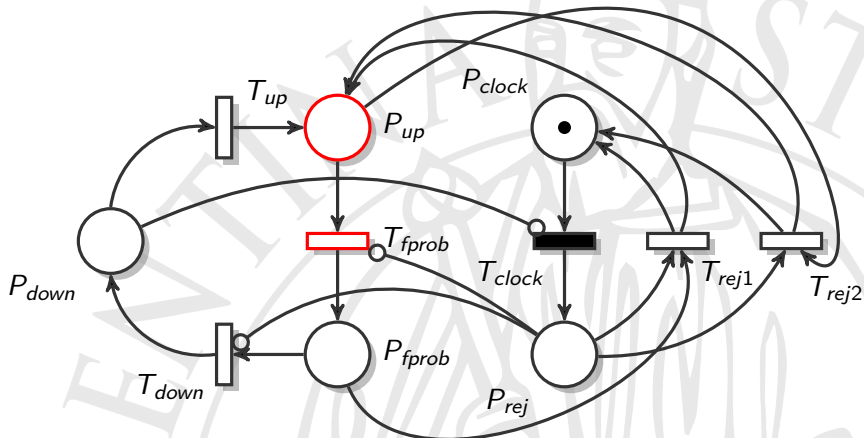
Rejuvenation



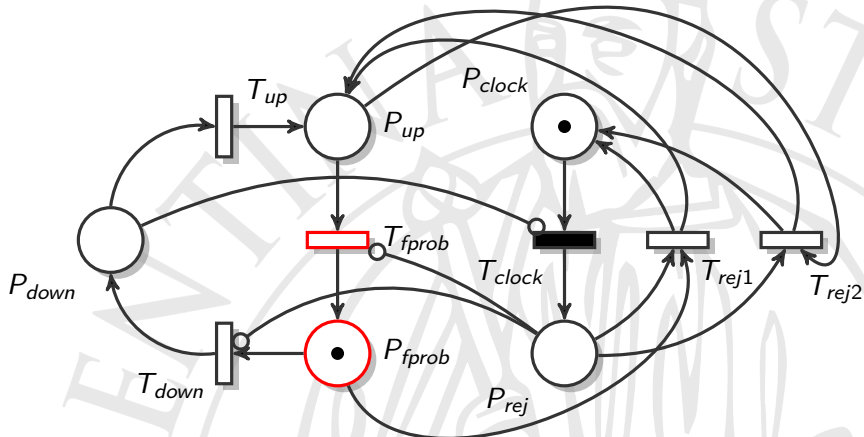
Rejuvenation



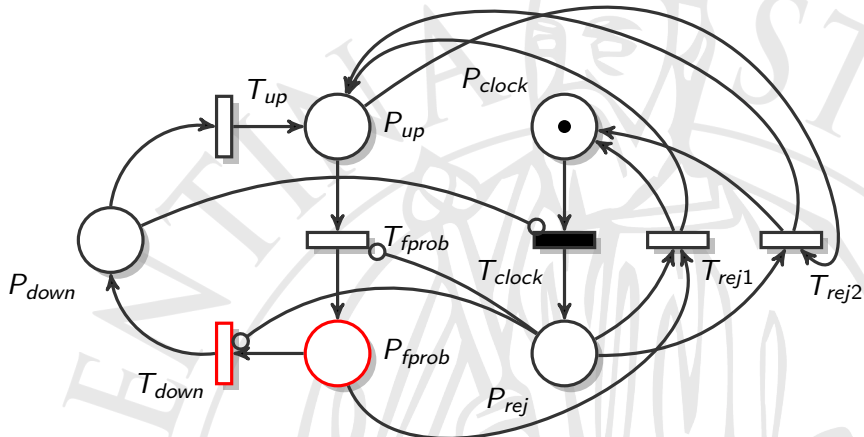
Rejuvenation



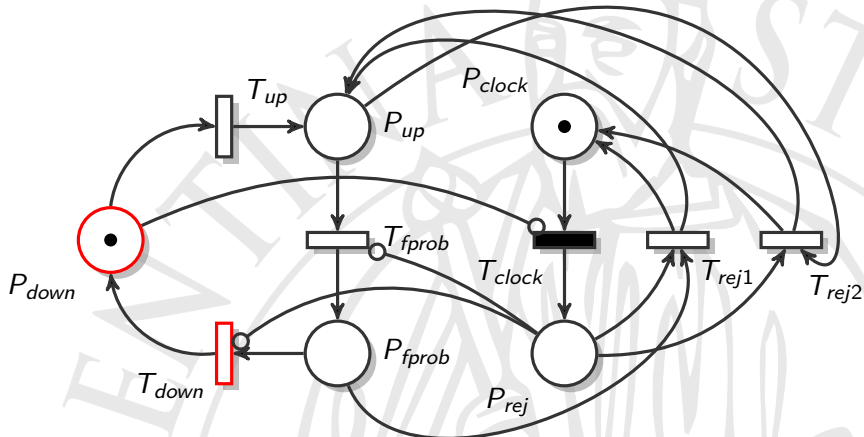
Rejuvenation



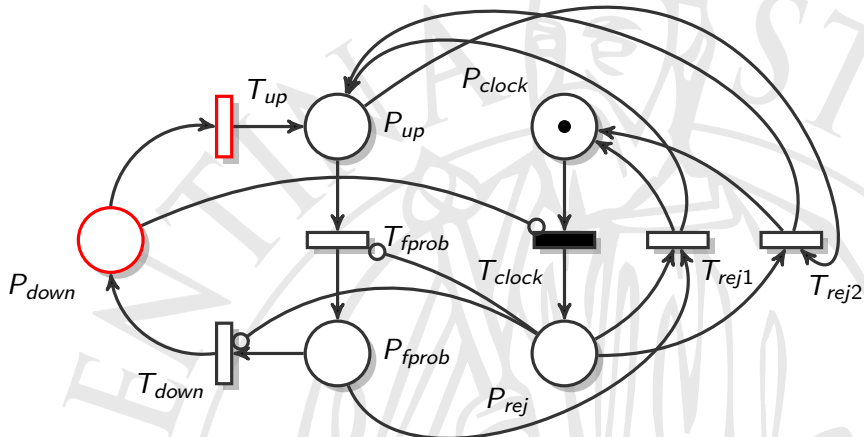
Rejuvenation



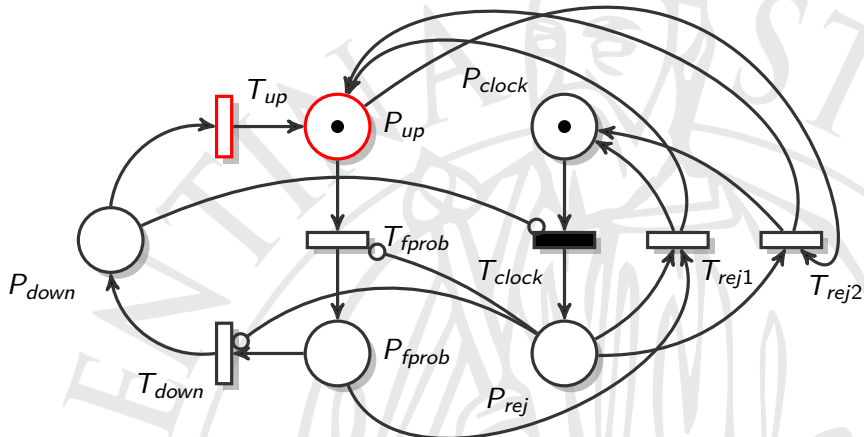
Rejuvenation



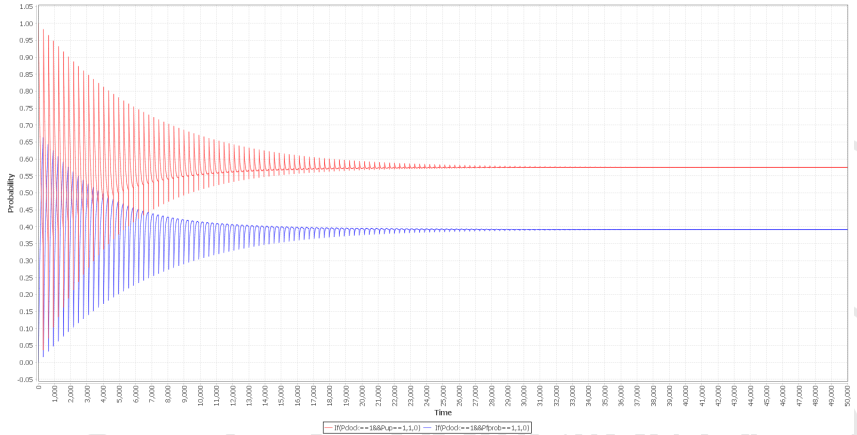
Rejuvenation



Rejuvenation

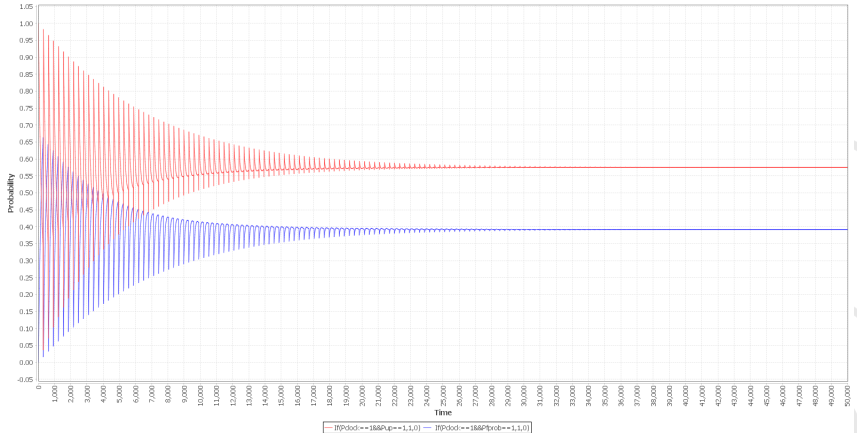


Transient probabilities



- ✓ Steady-state analysis results:
- $\text{Prob}(P_{\text{clock}} P_{\text{Pup}}) \approx 0.58$
 - $\text{Prob}(P_{\text{clock}} P_{\text{Pprob}}) \approx 0.41$

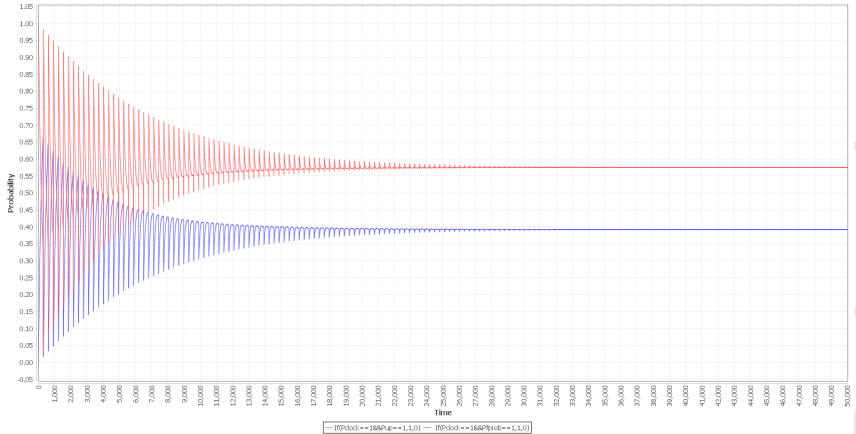
Transient probabilities



✓ Steady-state analysis results:

- Prob(Pclock Pup) \approx 0.58
- Prob(Pclock Pfprob) \approx 0.41

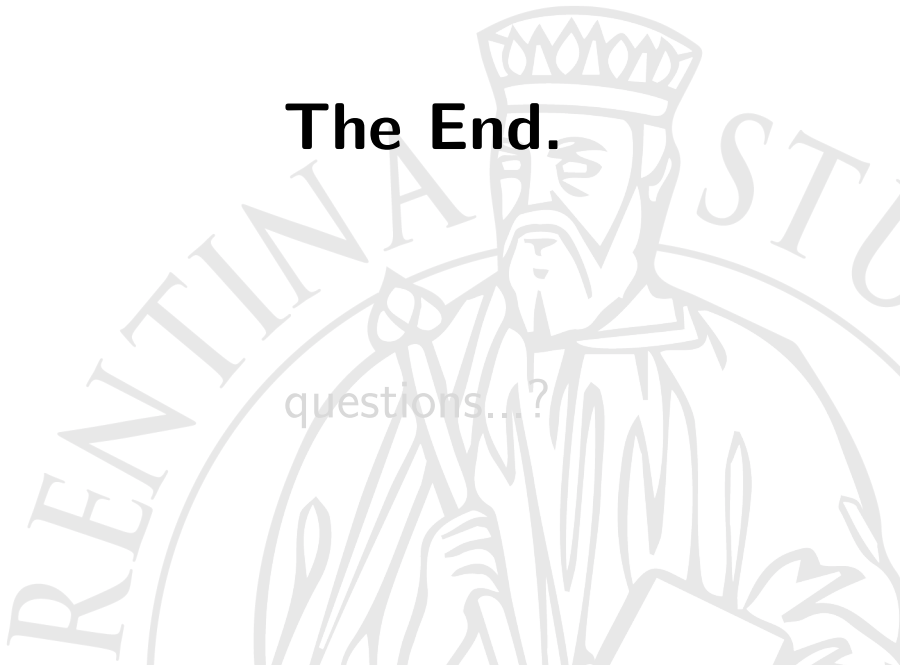
Transient probabilities



- ✓ Steady-state analysis results:
- Prob(**P**clock **P**up) ≈ 0.58
 - Prob(**P**clock **P**fprob) ≈ 0.41

The End.

questions...?



The End.

questions...?

