Markov Regenerative Process - steady-state analysis

MVT exam

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Markov Regenerative Processes (MRP)

Definition

A Markov Regenerative Process (MRP) is a stochastic process that sooner or later, with probability one, will reach a regenerative state (will be regenerated).

Regenerative state

A state where the process loses its memory.

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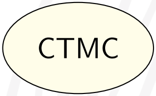

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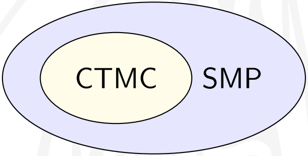

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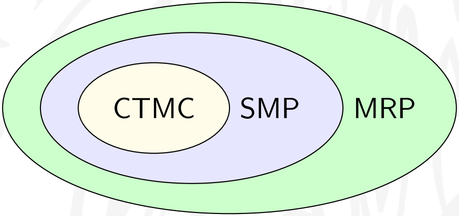

The steady-state problem

Transient probabilities

The probability distribution that the process will be in a certain state, after given t time.

Steady-state

For ergodic systems, it represents the probability distribution that the process will be in a certain state, as time goes to infinity.

- ✓ ORIS current state:
  - Transient analysis for Markov Regenerative Process (MRP)
  - Steady-state analysis for everything else
- ✓ Until now! ☺
- ✓ Warning: we assume that the MRP is ergodic!

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MRP steady-state analysis - The theory

General idea:

1. Calculate the embedded DTMC steady-state on the regenerative states
2. Calculate the sojourn times in each marking, after each regenerative state
3. Combine the two above in order to calculate the MRP steady-state

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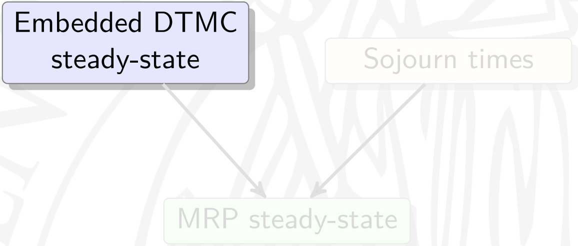

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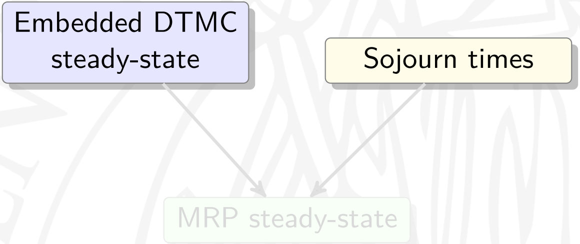

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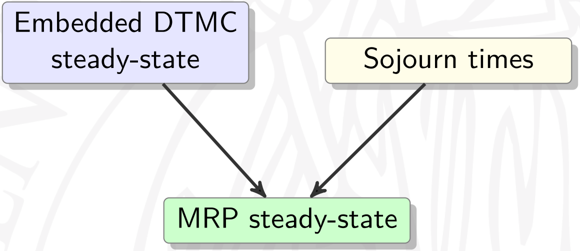

Main classes implemented

- ✓ `class EmbeddedDTMC`
  - written from scratch
  - calculate embedded DTMC steady-state
- ✓ `class RegenerativeSteadyStateAnalysis`
  - based on `class RegenerativeTransientAnalysis`
  - calculate MRP steady-state

Steady-state of the embedded DTMC on regenerative states

Steady-state in Discrete Time Markov Process (DTMC)

If the steady-state of a Discrete Time Markov Process (DTMC) exists and is unique (if is ergodic), then it's calculated by solving for v the linear system:

$$\begin{cases} v = vP \\ |v| = 1 \end{cases}$$

- ✓ We want to calculate the steady-state of the embedded DTMC of the MRP in the regenerative states
- ✓ But we don't have P! $\frown$

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Reaching probability feature

- ✓ We add a new reaching probability feature to each state: `class ReachingProbabilityFeature`
- ✓ Inside `SteadyStateInitialStateBuilder`: set it to 1
- ✓ Inside `SteadyStatePostProcessor`: multiply the parent's reaching probability by the probability to chose a certain child

If we run a transient analysis on the Petri Net (PN) we get:

- ✓ `regenerationClasses`
- ✓ `Map<DeterministicEnablingState,Map<DeterministicEnablingState,Set<State>>>`
- ✓ sum reaching probability feature of each `State` to compute elements of P

Now we can solve the linear system:

$$\begin{cases} v = vP \\ |v| = 1 \end{cases} = \begin{cases} (P' - I)v' = 0 \\ \sum_i v_i = 1 \end{cases}$$

- ✓ `RealMatrix` & `RealVector`
- ✓ QR decomposition solver
  - `DecompositionSolver solver = new QRDecomposition(coefficients).getSolver();`
  - `RealVector steadyState = solver.solve(constants);`
- ✓ Convert `steadyState` into a `Map<DeterministicEnablingState,BigDecimal>`

Sojourn time a_{ij}

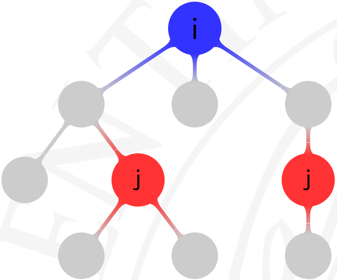
Definition

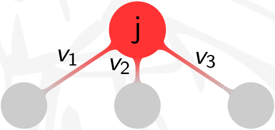
The sojourn time a_{ij} represents the average time spent in the j-th marking after the (last) i-th regeneration.

How to compute a_{ij}?

a_{ij} is:

- ✓ sum of avg time spent in marking j occurrences
  - sum of avg time before each variable fires
    - ⋆ condition each variable to be the minimum (i.e. the one that fires)
    - ⋆ compute avg time before that variable fires (thanks Marco!)





When to compute a_{ij}?

During the transient analysis!

- ✓ transient analysis generates succession trees for each regenerative state
  - regenerative state as root
  - following regenerative states as leaves
  - reachable markings as inner nodes
- ✓ during the tree generation compute and accumulate a_{ij} for each marking occurrence found

Markov Regenerative Process (MRP) steady-state

Let's combine the embedded DTMC steady-state and the sojourn times!

$$\pi_j = \frac{\sum_i v_i a_{ij}}{K}$$

- ✓ We multiply the sojourn time in the marking j after the regeneration i by the probability of reaching the i-th regeneration
- ✓ We do this for each regeneration that leads to the marking j before another regeneration
- ✓ K is a normalization factor calculated as the sum of π_j

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Test