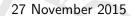
# Markov Regenerative Process - steady-state analysis

MVT exam

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Tommaso PAPINI tommaso.papini1@stud.unifi.it







#### **Definition**

A Markov Regenerative Process (MRP) is a stochastic process that sooner or later, with probability one, will reach a regenerative state (will be regenerated).

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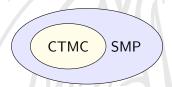
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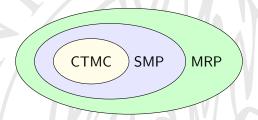
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The probability distribution that the process will be in a certain state, after given t time.

#### Steady-state

- √ ORIS current state
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  - Steady-state analysis for Continuous Time Markov Processes (CTMC
- ✓ Until how
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- 2. Calculate the expected sojourn time in each marking, after reaching a regenerative state
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MRP steady-state

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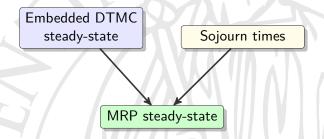
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  - written from scratch
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# Steady-state of the embedded DTMC on regenerative states

## Steady-state in Discrete Time Markov Process (DTMC)

If the steady-state of a Discrete Time Markov Process (DTMC) exists and is unique (if is ergodic), then it's calculated by solving for v the linear system:

$$\begin{cases} v = vP \\ |v| = 1 \end{cases}$$

- ✓ We want to calculate the steady-state of the embedded DTMC of the MRP in the regenerative states
- ✓ But we don't have P! ::

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- √ We add a new reaching probability feature to each state: class ReachingProbabilityFeature
- ✓ Inside SteadyStateInitialStateBuilder: set it to 1
- ✓ Inside SteadyStatePostProcessor: multiply the parent's reaching probability by the probability to chose a certain child

- √ regenerationClasses
- ✓ Map<DeterministicEnablingState,Map<DeterministicEnablingState,Set<State>>>
- √ sum reaching probability feature of each State to compute elements
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- √ RealMatrix & RealVector
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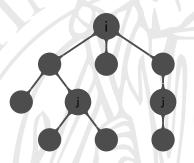
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#### **Definition**

The sojourn time aii represents the

- $\checkmark$  average time spent in the *j*-th marking
- ✓ after the (last) *i*-th regeneration.

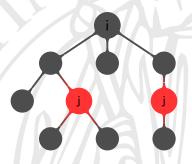


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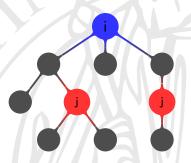


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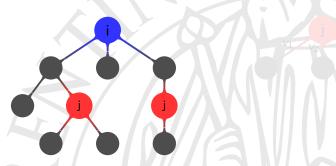
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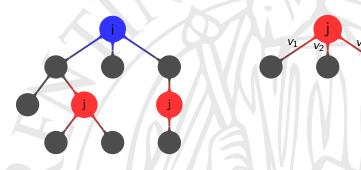
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- $\checkmark$  sum of avg time spent in marking j occurrences
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    - \* condition each variable to be the minimum (i.e. the one that fires)
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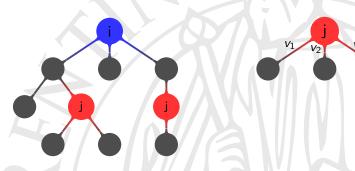
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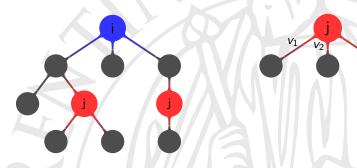
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Let's combine the embedded DTMC steady-state and the sojourn times!

$$\pi_j = \frac{\sum_i v_i a_{ij}}{7}$$

- We multiply the sojourn time in the marking patter the regeneration by the probability of reaching the 1-th regeneration
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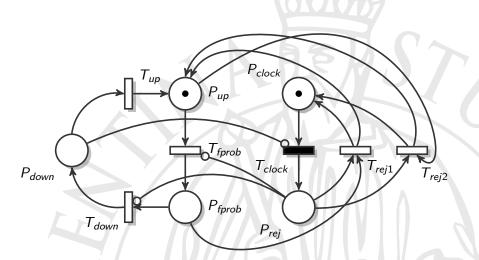
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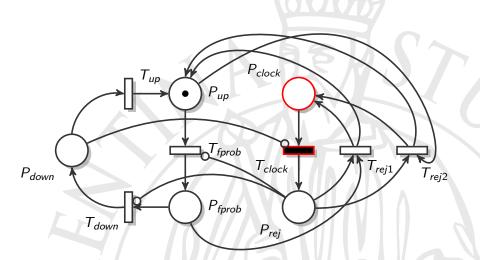
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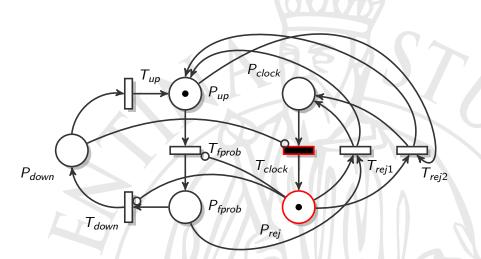
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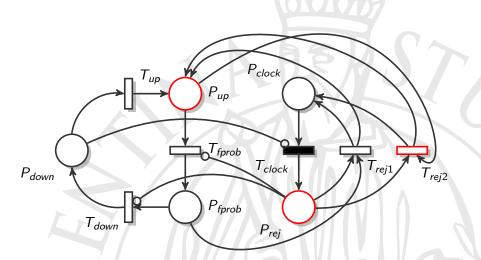
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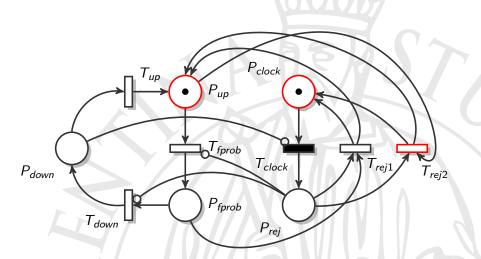
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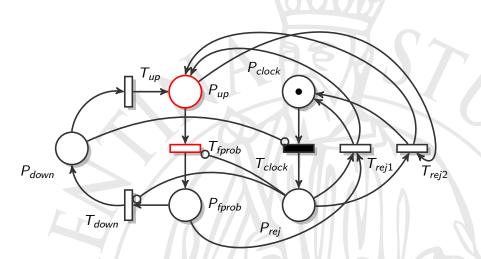


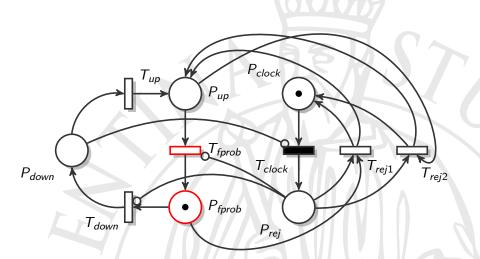


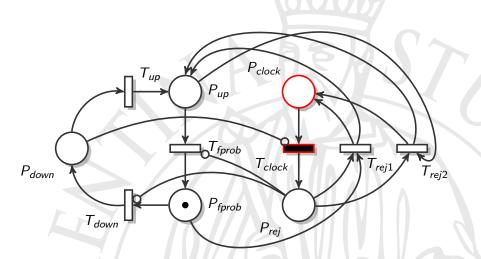


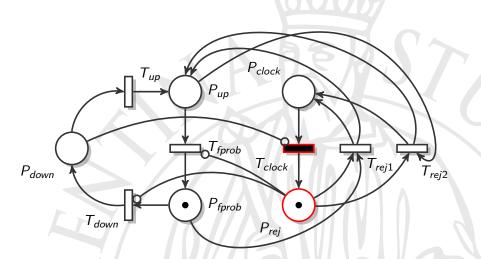


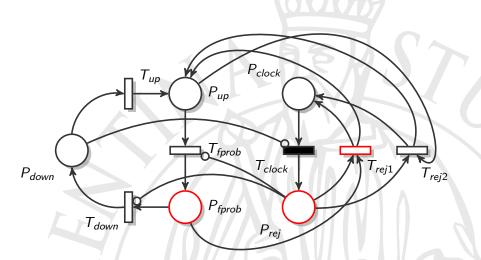


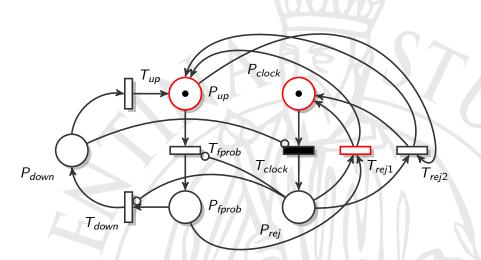


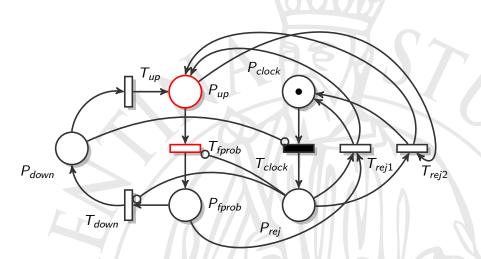


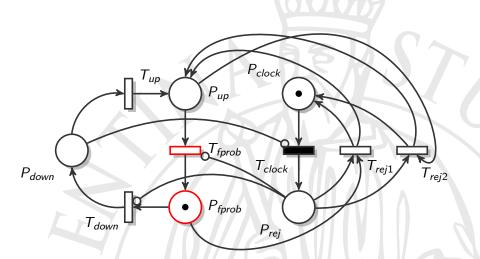


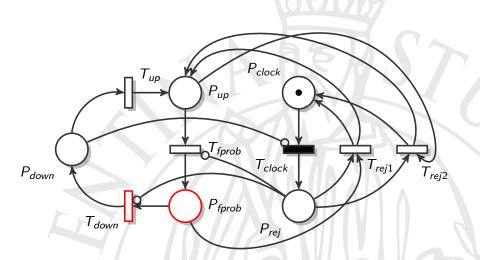


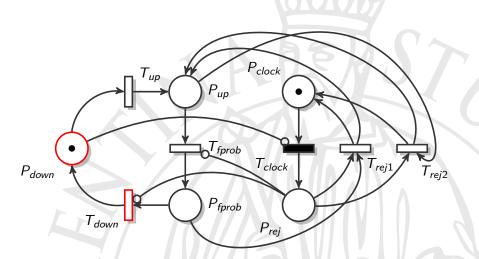


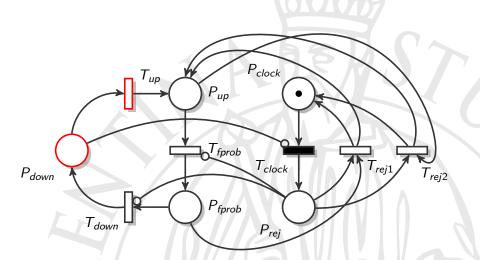


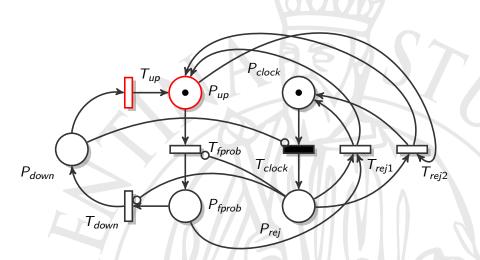




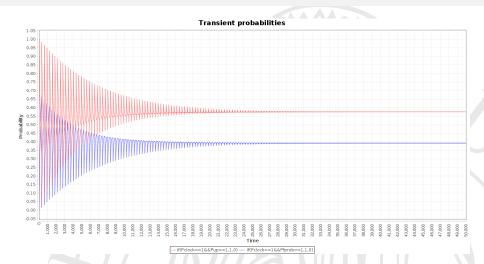






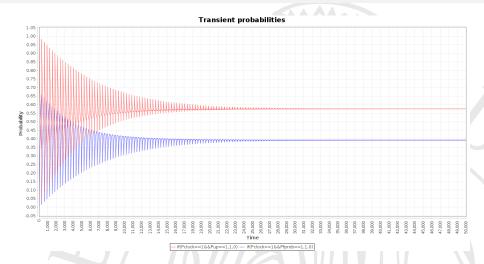


## Transient analysis



Steady-state estimation using transient analysis  $Prob(P_{lock} = 1 \mid P_{lock} = 1) \approx 0.58$ 

### Transient analysis



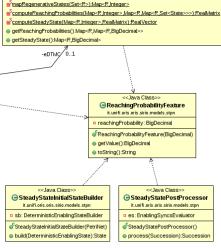
- ✓ Steady-state estimation using transient analysis:
  - Prob $(P_{clock} = 1 \land P_{up} = 1) \approx 0.58$
  - $Prob(P_{clock} = 1 \land P_{fprob} = 1) \approx 0.40$

### Steady state analysis

## Steady state analysis



#### << Java Class>> RegenerativeSteadyStateAnalysis<R> it .unifi.oris .oris .sirio .models .stpn reachableMarkings: Set<Marking> alwaysRegenerativeMarkings: Set<Marking> neverRegenerativeMarkings: Set<Marking> regenerativeAndNotRegenerativeMarkings: Set<Marking> EmbeddedDTMC() sojournMap: Map<R,Map<Marking,BigDecimal>> steadyState: Map<Marking.BigDecimal> p initialRegeneration; R truncationPolicy: EnumerationPolicy absorbingCondition: MarkingCondition absorbingMarkings: Set<Marking> localClasses: Map<R\_Map<Marking.Set<State>>> regenerationClasses: Map<R, Map<R, Set<State>>> regenerations: Set<R> aeteDTMC():EmbeddedDTMC<R> ⊚ getSoiournMap():Map<R.Map<Marking.BigDecimal>> getSteadyState():Map<Marking.BigDecimal> getReachableMarkings():Set<Marking> getAlwaysRegenerativeMarkings();Set<Marking> getNeverRegenerativeMarkings():Set<Marking> getRegenerativeAndNotRegenerativeMarkings():Set<Marking> getInitialRegeneration() getRegenerations():Set<R> getPetriNet():PetriNet getTruncationPolicy():EnumerationPolicy



<<.lava Class>>

● EmbeddedDTMC <R > it unifi oris oris sirio models ston

reachingProbabilities: Map<R\_Map<R\_BigDecimal>>

Scompute(Set<R>.Map<R.Map<R.Set<State>>>):EmbeddedDTMC<R>

steadyState: Map<R.BigDecimal>

getAbsorbingCondition():MarkingCondition

getLocalClasses():Map<R.Map<Marking.Set<State>>>

getRegenerationClasses():Map<R,Map<R,Set<State>>>

ScanAnalyze(PetriNet.ValidationMessageCollector):boolean

CalculateSteadvState(RegenerativeSteadvStateAnalvsis<R>):Map<Marking.BigDecimal> Scompute(PetriNet,R,StateBuilder<R>,SuccessionProcessor,EnumerationPolicy,MarkingCo

getAbsorbingMarkings():Set<Marking>

FregenerativeSteadyStateAnalysis()

petriNet: PetriNet



Questions? Thank you.



Questions?



Questions? Thank you!