A Proof of Theorem 2

Proof. For any edge (v_i, v_j), it consumes the privacy budget $\alpha\epsilon_1$ when $l_{i,j} = 1$ or $\epsilon_2 - (1-\alpha)\epsilon_1$ when $l_{i,j} = 2$ in the process of the Randomized Response. And in the second round, each edge uniformly consumes the privacy budget $\beta(1-\alpha)\epsilon_1$ to compute the user's noisy degree and consumes the remaining privacy budget $(1-\beta)(1-\alpha)\epsilon_1$ to publish the triangle count. Following the sequential combination shown in Theorem 1, we complete the proof.

B Proof of Theorem 3

Proof. The proof is presented here in two cases, first for the user v_i whose edges' privacy levels are not all 2:

$$\begin{aligned}
\mathbb{E}[\hat{w}_i] &= \mathbb{E}[w_i + Lap(\frac{K_i/(2p_1-1)}{(1-\beta)(1-\alpha)\epsilon_1})] \\
&= \mathbb{E}[w_i^{(1)} + w_i^{(2)}] \\
&= \mathbb{E}[\frac{t_i^{(1)} - (1-p_1)s_i^{(1)}}{2p_1-1} + \frac{t_i^{(2)} - (1-p_2)s_i^{(2)}}{2p_2-1}] \\
&= \frac{1}{2p_1-1}\mathbb{E}[t_i^{(1)} - (1-p_1)s_i^{(1)}] + \frac{1}{2p_2-1}\mathbb{E}[t_i^{(2)} - (1-p_2)s_i^{(2)}].
\end{aligned} \tag{8}$$

Let $s_i^{*(g)} (g \in \{1,2\})$ be the count of the 2-star counts of user v_i excluding the triangle part, i.e., $s_i^{*(g)} = |\{(v_i, v_j, v_k) : i < j < k, a_{i,j} = a_{i,k} = 1, (v_j, v_k) \notin E\}|$. For simplicity, it is assumed that the user does not perform an edge clipping operation. T_i denotes the real triangle counts in the local graph for user v_i. $t_i^{(g)}$ is the noisy triangle counts, i.e., $t_i = |\{(v_i, v_j, v_k) : i < j < k, a_{i,j} = a_{i,k} = 1, (v_j, v_k) \in E'\}|$. $s_i^{(g)}$ is the true 2-star counts for user v_i, then we have $s_i^{(g)} = s_i^{*(g)} + T_i^{(g)}$. By the properties of RR:

$$\mathbb{E}[t_i^{(g)}] = T_i^{(g)} p + s_i^{*(g)}(1-p), \tag{9}$$

so,

$$\begin{aligned}
\mathbb{E}[\hat{w}_i] &= \frac{1}{2p_1-1}\mathbb{E}[t_i^{(1)} - (1-p_1)s_i^{(1)}] + \frac{1}{2p_2-1}\mathbb{E}[t_i^{(2)} - (1-p_2)s_i^{(2)}] \\
&= \frac{1}{2p_1-1}\mathbb{E}[T_i^{(1)}p_1 + s_i^{*(1)}(1-p_1) - (1-p_1)s^{(1)}] \\
&+ \frac{1}{2p_2-1}\mathbb{E}[T_i^{(2)}p_2 + s_i^{*(2)}(1-p_2) - (1-p_2)s^{(2)}] \\
&= \frac{(2p_1-1)T_i^{(1)}}{2p_1-1} + \frac{(2p_2-1)T_i^{(2)}}{2p_2-1} \\
&= T_i^{(1)} + T_i^{(2)} \\
&= T_i.
\end{aligned} \tag{10}$$

For the user v_i whose edges all have privacy level 2:

$$
\begin{aligned}
\mathbb{E}[\hat{w}_i] &= \mathbb{E}[w_i + Lap(\frac{K_i/(2p_2-1)}{(1-\beta)(1-\alpha)\epsilon_1})] \\
&= \mathbb{E}[w_i^{(2)}] \\
&= \frac{1}{2p_2-1}\mathbb{E}[t_i^{(2)} - (1-p_2)s_i^{(2)}] \\
&= \frac{1}{2p_2-1}\mathbb{E}[T_i^{(2)}p_2 + s_i^{*(2)}(1-p_2) - (1-p_2)s^{(2)}] \\
&= \frac{(2p_2-1)T_i^{(2)}}{2p_2-1} \\
&= T_i^{(2)} \\
&= T_i.
\end{aligned}
\tag{11}
$$

C Proof of Theorem 4

For a user v_i whose associated edges all have a privacy level of 2:

Proof.

$$
\begin{aligned}
Var[\hat{w}_i] &= Var[w_i^{(2)} + Lap(\frac{K_i/(2p_2-1)}{(1-\beta)(1-\alpha)\epsilon_1})] \\
&= Var[\frac{t_i^{(2)} - (1-p_2)s_i^{(2)}}{2p_2-1} + Lap(\frac{K_i/(2p_2-1)}{(1-\beta)(1-\alpha)\epsilon_1})] \\
&= \frac{1}{(2p_2-1)^2}Var[t_i^{(2)}] + 2(\frac{K_i/(2p_2-1)}{(1-\beta)(1-\alpha)\epsilon_1})^2 \\
&= \frac{p_2q_2(T_i^{(2)} + s_i^{*(2)})}{(2p_2-1)^2} + 2(\frac{K_i/(2p_2-1)}{(1-\beta)(1-\alpha)\epsilon_1})^2 \\
&= \frac{p_2q_2s_i^{(2)}}{(2p_2-1)^2} + 2(\frac{K_i/(2p_2-1)}{(1-\beta)(1-\alpha)\epsilon_1})^2 \\
&\leq \frac{p_2q_2}{(2p_2-1)^2}\frac{K_i(K_i-1)}{2} + 2(\frac{K_i/(2p_2-1)}{(1-\beta)(1-\alpha)\epsilon_1})^2.
\end{aligned}
\tag{12}
$$

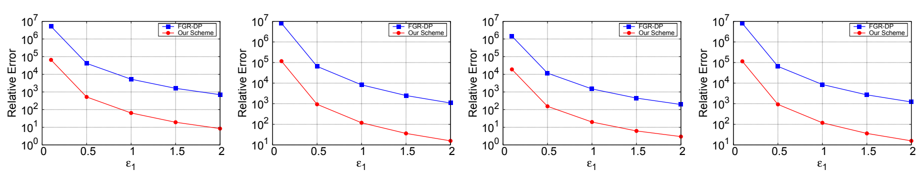
For a user v_i whose associated edges' privacy levels are not all 2:

Proof.

$$
\begin{aligned}
Var[\hat{w}_i] &= Var[w_i^{(1)} + w_i^{(2)} + Lap(\frac{K_i/(2p_1-1)}{(1-\beta)(1-\alpha)\epsilon_1})] \\
&= Var[\frac{t_i^{(1)} - (1-p_1)s_i^{(1)}}{2p_1-1} + \frac{t_i^{(2)} - (1-p_2)s_i^{(2)}}{2p_2-1} + Lap(\frac{K_i/(2p_1-1)}{(1-\beta)(1-\alpha)\epsilon_1})] \\
&= \frac{1}{(2p_1-1)^2} Var[t_i^{(1)}] + \frac{1}{(2p_2-1)^2} Var[t_i^{(2)}] + 2(\frac{K_i/(2p_1-1)}{(1-\beta)(1-\alpha)\epsilon_1})^2 \\
&= \frac{p_1 q_1 (T_i^{(1)} + s_i^{*(1)})}{(2p_1-1)^2} + \frac{p_2 q_2 (T_i^{(2)} + s_i^{*(2)})}{(2p_2-1)^2} + 2(\frac{K_i/(2p_1-1)}{(1-\beta)(1-\alpha)\epsilon_1})^2 \\
&= \frac{p_1 q_1 s_i^{(1)}}{(2p_1-1)^2} + \frac{p_2 q_2 s_i^{(2)}}{(2p_2-1)^2} + 2(\frac{K_i/(2p_1-1)}{(1-\beta)(1-\alpha)\epsilon_1})^2 \\
&\le \frac{p_1 q_1 (s_i^{(1)} + s_i^{(2)})}{(2p_1-1)^2} + 2(\frac{K_i/(2p_1-1)}{(1-\beta)(1-\alpha)\epsilon_1})^2 \\
&\le \frac{p_1 q_1}{(2p_1-1)^2} \frac{K_i(K_i-1)}{2} + 2(\frac{K_i/(2p_1-1)}{(1-\beta)(1-\alpha)\epsilon_1})^2.
\end{aligned}
\tag{13}
$$

D Evaluations on the Entire Algorithm

We also report the relative errors of our algorithm and FGR-DP in Fig. 8. It is observed that our algorithm outperforms the competitor by nearly 2 orders of magnitude on the relative error metric.



(a) Orkut (b) LJDB (c) DBLP (d) Youtube

Fig. 8. Evaluating the performance of the entire algorithm with relative error by varying ϵ_1.