

A PROOF OF THEOREM 4.1

PROOF. For any edge (v_i, v_j) , it consumes the privacy budget $\alpha\epsilon_1$ when $l_{i,j} = 1$ or $\epsilon_2 - (1 - \alpha)\epsilon_1$ when $l_{i,j} = 2$ in the process of the Randomized Response. And in the second round, each edge uniformly consumes the privacy budget $\beta(1 - \alpha)\epsilon_1$ to compute the user's noisy degree and consumes the remaining privacy budget $(1 - \beta)(1 - \alpha)\epsilon_1$ to publish the triangle count. Following the sequential combination shown in Theorem 3.4, we complete the proof. \square

B PROOF OF THEOREM 4.2

PROOF. The proof is presented here in two cases, first for the user v_i whose edges' privacy levels are not all 2:

$$\begin{aligned}\mathbb{E}[\hat{w}_i] &= \mathbb{E}[w_i + \text{Lap}(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\ &= \mathbb{E}[w_i^{(1)} + w_i^{(2)}] \\ &= \mathbb{E}[\frac{t_i^{(1)} - (1 - p_1)s_i^{(1)}}{2p_1 - 1} + \frac{t_i^{(2)} - (1 - p_2)s_i^{(2)}}{2p_2 - 1}] \\ &= \frac{1}{2p_1 - 1}\mathbb{E}[t_i^{(1)} - (1 - p_1)s_i^{(1)}] + \frac{1}{2p_2 - 1}\mathbb{E}[t_i^{(2)} - (1 - p_2)s_i^{(2)}].\end{aligned}\tag{8}$$

Let $s_i^{*(g)}$ ($g \in \{1, 2\}$) be the count of the 2-star counts of user v_i excluding the triangle part, i.e., $s_i^{*(g)} = |\{(v_i, v_j, v_k) : i < j < k, a_{i,j} = a_{i,k} = 1, (v_j, v_k) \notin E\}|$. For simplicity, it is assumed that the user does not perform an edge clipping operation. T_i denotes the real triangle counts in the local graph for user v_i . $t_i^{(g)}$ is the noisy triangle counts, i.e., $t_i = |\{(v_i, v_j, v_k) : i < j < k, a_{i,j} = a_{i,k} = 1, (v_j, v_k) \in E'\}|$. $s_i^{(g)}$ is the true 2-star counts for user v_i , then we have $s_i^{(g)} = s_i^{*(g)} + T_i^{(g)}$. By the properties of RR:

$$\mathbb{E}[t_i^{(g)}] = T_i^{(g)}p + s_i^{*(g)}(1 - p),\tag{9}$$

so,

$$\begin{aligned}\mathbb{E}[\hat{w}_i] &= \frac{1}{2p_1 - 1}\mathbb{E}[t_i^{(1)} - (1 - p_1)s_i^{(1)}] + \frac{1}{2p_2 - 1}\mathbb{E}[t_i^{(2)} - (1 - p_2)s_i^{(2)}] \\ &= \frac{1}{2p_1 - 1}\mathbb{E}[T_i^{(1)}p_1 + s_i^{*(1)}(1 - p_1) - (1 - p_1)s_i^{(1)}] \\ &\quad + \frac{1}{2p_2 - 1}\mathbb{E}[T_i^{(2)}p_2 + s_i^{*(2)}(1 - p_2) - (1 - p_2)s_i^{(2)}] \\ &= \frac{(2p_1 - 1)T_i^{(1)}}{2p_1 - 1} + \frac{(2p_2 - 1)T_i^{(2)}}{2p_2 - 1} \\ &= T_i^{(1)} + T_i^{(2)} \\ &= T_i.\end{aligned}\tag{10}$$

For the user v_i whose edges all have privacy level 2:

$$\begin{aligned}\mathbb{E}[\hat{w}_i] &= \mathbb{E}[w_i + \text{Lap}(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\ &= \mathbb{E}[w_i^{(2)}] \\ &= \frac{1}{2p_2 - 1}\mathbb{E}[t_i^{(2)} - (1 - p_2)s_i^{(2)}] \\ &= \frac{1}{2p_2 - 1}\mathbb{E}[T_i^{(2)}p_2 + s_i^{*(2)}(1 - p_2) - (1 - p_2)s_i^{(2)}] \tag{11} \\ &= \frac{(2p_2 - 1)T_i^{(2)}}{2p_2 - 1} \\ &= T_i^{(2)} \\ &= T_i.\end{aligned}$$

\square

C PROOF OF THEOREM 4.3

For a user v_i whose associated edges all have a privacy level of 2:

PROOF.

$$\begin{aligned}\text{Var}[\hat{w}_i] &= \text{Var}[w_i^{(2)} + \text{Lap}(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\ &= \text{Var}[\frac{t_i^{(2)} - (1 - p_2)s_i^{(2)}}{2p_2 - 1} + \text{Lap}(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\ &= \frac{1}{(2p_2 - 1)^2}\text{Var}[t_i^{(2)}] + 2(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\ &= \frac{p_2q_2(T_i^{(2)} + s_i^{*(2)})}{(2p_2 - 1)^2} + 2(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\ &= \frac{p_2q_2s_i^{(2)}}{(2p_2 - 1)^2} + 2(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\ &\leq \frac{p_2q_2}{(2p_2 - 1)^2}\frac{K_i(K_i - 1)}{2} + 2(\frac{K_i/(2p_2 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2.\end{aligned}\tag{12}$$

\square

For a user v_i whose associated edges' privacy levels are not all 2:

PROOF. □

$$\begin{aligned}
Var[\hat{w}_i] &= Var[w_i^{(1)} + w_i^{(2)} + Lap(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\
&= Var[\frac{t_i^{(1)} - (1 - p_1)s_i^{(1)}}{2p_1 - 1} + \frac{t_i^{(2)} - (1 - p_2)s_i^{(2)}}{2p_2 - 1} \\
&\quad + Lap(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})] \\
&= \frac{1}{(2p_1 - 1)^2} Var[t_i^{(1)}] + \frac{1}{(2p_2 - 1)^2} Var[t_i^{(2)}] \\
&\quad + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&= \frac{p_1 q_1 (T_i^{(1)} + s_i^{*(1)})}{(2p_1 - 1)^2} + \frac{p_2 q_2 (T_i^{(2)} + s_i^{*(2)})}{(2p_2 - 1)^2} \quad (13) \\
&\quad + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&= \frac{p_1 q_1 s_i^{(1)}}{(2p_1 - 1)^2} + \frac{p_2 q_2 s_i^{(2)}}{(2p_2 - 1)^2} + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&\leq \frac{p_1 q_1 (s_i^{(1)} + s_i^{(2)})}{(2p_1 - 1)^2} + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2 \\
&\leq \frac{p_1 q_1}{(2p_1 - 1)^2} \frac{K_i(K_i - 1)}{2} + 2(\frac{K_i/(2p_1 - 1)}{(1 - \beta)(1 - \alpha)\epsilon_1})^2.
\end{aligned}$$

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