Set Inference

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April 30, 2020

Part I
Concepts

In this part, we define terms and suggest notation for use in set inference.

1 Object

O is a set of knowledge objects. Each one is something we could know something about.

$$O = \{o_1, o_2, \cdots, o_n\} \tag{1}$$

These aren't necessarily objects in the usual English sense of the word. They can be immaterial, they can be impressions, they can be events. Each one is some information that is available to us.

1.1 Object Examples

These examples will be used throughout this paper to illustrate set inference.

1.1.1 Great Leader

$$O_{GreatLeader} = \{water, brie, kitten, desert, leslie, lincoln, paper\} \tag{2}$$

These objects have varying types. They're all material, but it doesn't matter. We could have done this instead:

$$O_{GreatLeaderX} = \{water, brie, physics, lastThursday, smile, skyBlue\} \tag{3}$$

1.1.2 Coin Toss

$$O_{CoinToss} = \{toss_1, toss_2, toss_3, toss_4, toss_5, toss_6\} \tag{4}$$

These objects are events. Each one is information about a coin toss.

1.1.3 Team

$$O_{Team} = \{Alice, Bob, Carol, Dave, Ed, Frank\} \tag{5}$$

Each of these objects is information about a person.

2 Object Description

A is a set of object descriptions; it is a set of sets of objects. Membership in one or more of these sets describes knowledge about an object. These sets can be empty. They can overlap.

$$A = \{A_1, A_2, \cdots, A_n\} \tag{6}$$

For each A_k there is one Boolean function a_k that determines whether an object o_i is a member of A_k.

$$a = \{a_1, a_2, \cdots, a_n\} \tag{7}$$

$$a_k\left(o_i\right) \rightarrow \{0, 1\} \tag{8}$$

Descriptive functions may impose restrictions on the objects they accept. There may be restrictions imposed by variable type in software implementations or restrictions imposed by knowledge outside the set inference context that make it impossible or senseless for a particular descriptive function a_k to operate on a particular object o_i. If for whatever reason a descriptive function can't operate on an object, it simply doesn't.

In a set of objects like the team example, there is both a *kitten* object and a *paper* object. There might be restrictions on observation such that it is difficult to determine whether one was flamable, but easy to determine the same thing about the other. It might not make sense to ask whether *brie* is a $GreatLeader$, but it might make sense to ask whether *Lincoln* was one. What is needed in set inference are some positive classifications of some objects into some descriptive sets. This style of observation lets us cleanly handle heterogenously-typed objects, objects with missing information, and otherwise sparsely- or irregularly-described objects.

Observation consists of some objects in O being tested by some functions in a to determine the sets in A to which each tested object belongs. It is not necessary to test every object with every descriptive function. It is not necessarily the case that every known object has been tested with every descriptive function. Absense from a descriptive set does not imply that the associated descriptive function was applied and returned 0.

Regardless of how sparsely or how thoroughly the objects are described,

$$o_i \in A_k \Leftrightarrow a_k\left(o_i\right) = 1 \tag{9}$$

Because the descriptive functions may have been applied sparsely, the reverse is not necessarily true. For an element o_i not present in a particular A_k, $a_k\left(o_i\right) = 0$ is not implied.

$$
\begin{aligned}
a_k\left(o_i\right) = 0 \quad &\Rightarrow \quad o_i \notin A_k && (10)\\
o_i \notin A_k \quad &\nRightarrow \quad a_k\left(o_i\right) = 0 && (11)
\end{aligned}
$$

2.1 Inference Domain

An inference domain I_k is a particular set of known objects, descriptive functions, and descriptive sets.

$$I_k = \{O_i, A_m, a_p\} \tag{12}$$

It can be thought of as the context of an inference discussion. It is specific to a particular inference problem. The choice of descriptive functions with respect to a particular set of objects is the basis for the operation of set inference. Disparate sets of descriptions used with the same set of objects will lay the groundwork for disparate inference discussions. The choice of descriptive sets in a particular inference domain suggests what, about knowledge objects, we both a) know how to figure out, and b) want to pay attention to.

2.2 The Dark Matter Set

There is another descriptive set, called Z. This is the dark matter set. Z contains all the objects in O that aren't in any of the descriptive sets in A:

$$Z \quad = \quad O - A \tag{13}$$

All objects with no membership in any descriptive set are members of the dark matter set:

$$o_i \notin A \quad \Leftrightarrow \quad o_i \in Z \tag{14}$$

The dark matter set contains objects that we know exist but are unable to describe. In a particular inference domain, based on the specific known objects, the specific descriptive functions and sets, and the thoroughness with which the descriptive functions have been applied to objects, there may or may not be objects in the dark matter set.

There will be little mention of the dark matter set even though it is subtly present throughout set inference. Its presence is implicit in the use of the cardinality of the set of all known objects, $|O|$. $|O|$ contains all the objects we know exist–both those we can describe and those we cannot–hence the use of $|O|$ incorporates the cardinality of the dark matter set.

2.3 Object Description Examples

2.3.1 Great Leader

$$\begin{aligned} A_{GreatLeader} &= \{Edible, Organism, Favorite, GreatLeader, &(15)\\ &\quad Poison, FavoriteEdibleOrganism\} &(16)\\ a_{GreatLeader} &= \{isEdible, isOrganism, isFavorite, isGreatLeader, &(17)\\ &\quad isPoison, isFavoriteEdibleOrganism\} &(18) \end{aligned}$$

Using the descriptive function $isEdible$ on the object $water$, we observe that $water$ is $Edible$:

$$isEdible\,(water) = 1 \Leftrightarrow water \in Edible \quad (19)$$

Similarly, we observe that $lincoln$ is a $Favorite\ Organism$:

$$\begin{aligned} isFavorite\,(lincoln) = 1 &\quad\Leftrightarrow\quad lincoln \in Favorite &(20)\\ isOrganism\,(lincoln) = 1 &\quad\Leftrightarrow\quad lincoln \in Organism &(21) \end{aligned}$$

As we're able, we apply descriptive functions to objects. After some additional observation, the descriptive functions might have determined these set memberships:

$$\begin{aligned} Edible &= \big\{water, brie, kitten, leslie, lincoln, &(22)\\ &\quad paper\big\} &(23)\\ Organism &= \{kitten, leslie, lincoln\} &(24)\\ Favorite &= \{brie, lincoln\} &(25)\\ GreatLeader &= \{lincoln\} &(26)\\ Poison &= \{\} &(27)\\ FavoriteEdibleOrganism &= \{lincoln\} &(28) \end{aligned}$$

We don't know that $lincoln \notin Poison$ or $brie \notin GreatLeader$. We have set memberships that positively tell us some things we know based on our descriptive functions:

$$\begin{aligned} lincoln \notin Poison &\quad\not\Rightarrow\quad isPoison\,(lincoln) = 0 &(29)\\ brie \notin GreatLeader &\quad\not\Rightarrow\quad isGreatLeader\,(brie) = 0 &(30) \end{aligned}$$

In this example, the descriptive sets overlap conceptually (they can have non-empty intersection) and they have conceptual gaps. A single object can be both $Edible$ and a $Favorite$. There's a way to measure whether an object is a $GreatLeader$ but no way to measure whether an object is a $PoorLeader$ or a $MediocreLeader$. Because these descriptive functions were chosen in a way that allows conceptual gaps, even after we describe, as fully as possible, all known objects, there may be some objects in the dark matter set.

2.3.2 Coin Toss

$$
\begin{aligned}
A_{CoinToss} &= \{Heads, Tails\} && (31)\\
a_{CoinToss} &= \{isHeads, isTails\} && (32)
\end{aligned}
$$

$toss_1$ is $Heads$. $toss_2$ is $Tails$:

$$
\begin{aligned}
isHeads\,(toss_1) = 1 \quad &\Leftrightarrow \quad toss_1 \in Heads && (33)\\
isTails\,(toss_2) = 1 \quad &\Leftrightarrow \quad toss_2 \in Tails && (34)
\end{aligned}
$$

After additional observation, the descriptive functions might have determined these set memberships:

$$
\begin{aligned}
Heads &= \{toss_1, toss_3, toss_5\} && (35)\\
Tails &= \{toss_2, toss_4, toss_6\} && (36)
\end{aligned}
$$

In this example, the descriptive sets are defined such that there are counterpart sets–if it's not $Heads$ it's $Tails$ and vice versa. In this usual way of thinking about a coin toss, there can be no objects in the dark matter set. There are other descriptive functions germane to a coin toss, however. We could have used $isAQuarter$, $isNorthAmericanCurrency$, $faceValueLessThanOneDollar$, $mintedInDenver$, $isDirty$, $isNotReallyReallyDirty$, $landedOnItsEdge$, $isOneOfTheFirstThousandTossesIDidToday$, $leslieLaughedDuringTheToss$, or others.

2.3.3 Team

The descriptions and descriptive functions are:

$$
\begin{aligned}
A_{Team} &= \{Seal, NotSeal, Ranger, NotSpecial, Certified0922, && (37)\\
&= Certified9287, CertifiedAll\} && (38)\\
a_{Team} &= \{isSeal, isNotSeal, isRanger, isNotSpecial, && (39)\\
&\quad\; isCertified0922, isCertified9287, isCertifiedAll\} && (40)
\end{aligned}
$$

$alice$ is a $Seal$, she is $Certified0922$. In this case the object was tested by each descriptive function:

$$
\begin{aligned}
isSeal\,(alice) = 1 \quad &\Leftrightarrow \quad alice \in Seal && (41)\\
isNotSeal\,(alice) = 0 \quad &\Leftrightarrow \quad alice \notin NotSeal && (42)\\
isRanger\,(alice) = 0 \quad &\Leftrightarrow \quad alice \notin Ranger && (43)\\
isNotSpecial\,(alice) = 0 \quad &\Leftrightarrow \quad alice \notin NotSpecial && (44)\\
isCertified0922\,(alice) = 1 \quad &\Leftrightarrow \quad alice \in Certified0922 && (45)\\
isCertified9287\,(alice) = 0 \quad &\Leftrightarrow \quad alice \notin Certified9287 && (46)\\
isCertifiedAll\,(alice) = 0 \quad &\Leftrightarrow \quad alice \notin CertifiedAll && (47)
\end{aligned}
$$

$frank$ is a $Ranger$, he is $Certified0922$ and $Certified9287$. In this case the object was tested only by some of the descriptive functions:

$$\begin{aligned} isRanger\,(frank) = 1 \quad &\Leftrightarrow \quad frank \in Ranger && (48)\\ isNotSpecial\,(frank) = 0 \quad &\Leftrightarrow \quad frank \notin NotSpecial && (49)\\ isCertified0922\,(frank) = 1 \quad &\Leftrightarrow \quad frank \in Certified0922 && (50)\\ isCertified9287\,(frank) = 1 \quad &\Leftrightarrow \quad frank \in Certified9287 && (51)\\ isCertifiedAll\,(frank) = 1 \quad &\Leftrightarrow \quad frank \in CertifiedAll && (52) \end{aligned}$$

$carol$ is $NotSpecial$. That's all we know about her. There is no descriptive set called $Special$ that is a counterpart to $NotSpecial$, so we don't know anything about whether she's $Special$ or not. We know she's $NotSpecial$:

$$isNotSpecial\,(carol) = 1 \quad \Leftrightarrow \quad carol \in NotSpecial \tag{53}$$

ed is not $NotSpecial$. That doesn't make him a member of any of the given descriptive sets. It makes him not a member of the descriptive set $NotSpecial$:

$$isNotSpecial\,(ed) = 0 \quad \Leftrightarrow \quad ed \notin NotSpecial \tag{54}$$

If these were the only observations we made, the set memberships would be:

$$\begin{aligned} Seal &= \{alice\} && (55)\\ NotSeal &= \{\} && (56)\\ Ranger &= \{frank\} && (57)\\ NotSpecial &= \{carol\} && (58)\\ Certified0922 &= \{alice, frank\} && (59)\\ Certified9287 &= \{frank\} && (60)\\ CertifiedAll &= \{frank\} && (61) \end{aligned}$$

3 Object Type

The objects in O can be thought of as having types. In set inference, a type is a set of descriptive sets in A. The following notation indicates a type, where each A_{k_m} is a descriptive set in A:

$$[A_{k_1}, A_{k_2}, \cdots, A_{k_n}] \tag{62}$$

The following notation indicates the type of an object o_i:

$$o_i\,[A_{k_1}, A_{k_2}, \cdots, A_{k_n}] \tag{63}$$

The following means that object o_2 has a type corresponding to descriptive sets A_1 and A_3:

$$o_2\,[A_1, A_3] \tag{64}$$

$[A_1, A_3]$ is the type of o_2. That o_2 has type $[A_1, A_3]$ means $o_2 \in A_1, o_2 \in A_3$. Type, and membership in descriptive sets, are interchangeable.

$$\begin{aligned} o_i\,[A_{k_1}, A_{k_2}, \cdots, A_{k_n}] \quad &\Leftrightarrow \quad o_i \in A_{k_1}, o_i \in A_{k_2}, \cdots, o_i \in A_{k_n} && (65) \\ &\Leftrightarrow \quad o_i \in A_{k_1} \cap A_{k_2} \cap \cdots \cap A_{k_n} && (66) \end{aligned}$$

3.1 Abbreviation

When it's convenient, object type can be abbreviated like this, where each k is the subscript of a descriptive set in A:

$$[k_1, k_2, \cdots, k_n] \tag{67}$$

For example, when A_1 and A_3 are the descriptive sets, object type can be abbreviated:

$$[1, 3] \tag{68}$$

3.1.1 Great Leader

If $Edible$ and $Favorite$ are the descriptive sets on which the type is based, type can be written as:

$$[Edible, Favorite] \tag{69}$$

or:

$$[1, 3] \tag{70}$$

This is beacuse $Edible$ corresponds to A_1 and $Favorite$ to A_3 in A.

3.2 Object Type Examples

3.2.1 Great Leader

The objects are abbreviated:

$$brie \quad = \quad o_2 \tag{71}$$

The descriptions are abbreviated:

$$
\begin{aligned}
Edible &= A_1 && (72)\\
Favorite &= A_3 && (73)
\end{aligned}
$$

$brie$ has the type $[Edible, Favorite]$:

$$
\begin{aligned}
brie\,[Edible, Favorite] &\Leftrightarrow brie \in Edible, brie \in Favorite && (74)\\
&\Leftrightarrow brie \in Edible \cap Favorite && (75)
\end{aligned}
$$

Abbreviated,

$$
\begin{aligned}
o_2\,[1,3] &\Leftrightarrow o_2 \in A_1, o_2 \in A_3 && (76)\\
&\Leftrightarrow o_2 \in A_1 \cap A_3 && (77)
\end{aligned}
$$

3.2.2 Coin Toss

The objects are abbreviated:

$$
\begin{aligned}
toss_1 &= o_1 && (78)\\
toss_2 &= o_2 && (79)
\end{aligned}
$$

The descriptions are abbreviated:

$$
\begin{aligned}
Heads &= A_1 && (80)\\
Tails &= A_2 && (81)
\end{aligned}
$$

$coin_1$ has the type $[Heads]$. $coin_2$ has the type $[Tails]$:

$$
\begin{aligned}
toss_1\,[Heads] &\Leftrightarrow toss_1 \in Heads && (82)\\
toss_2\,[Tails] &\Leftrightarrow toss_2 \in Tails && (83)
\end{aligned}
$$

Abbreviated,

$$
\begin{aligned}
o_1\,[1] &\Leftrightarrow o_1 \in A_1 && (84)\\
o_2\,[3] &\Leftrightarrow o_2 \in A_2 && (85)
\end{aligned}
$$

3.2.3 Team

The objects are abbreviated:

$$alice = o_1 \tag{86}$$

The descriptions are abbreviated:

$$Seal = A_1 \tag{87}$$
$$Certified0922 = A_3 \tag{88}$$

$alice$ has the type $[Seal, Certified0922]$:

$$alice\,[Seal, Certified0922] \Leftrightarrow alice \in Seal, alice \in Certified0922 \tag{89}$$
$$\Leftrightarrow alice \in Seal \cap Certified0922 \tag{90}$$

Abbreviated,

$$o_1\,[1,3] \Leftrightarrow o_1 \in A_1, o_1 \in A_3 \tag{91}$$
$$\Leftrightarrow o_1 \in A_1 \cap A_3 \tag{92}$$

4 Subtype and Supertype

A subtype is based on any subset of the descriptive sets in a type. $[1,2]$, $[2,3]$, $[1]$, and $[1,2,3]$ are subtypes of $[1,2,3]$. A supertype is based on any superset of the descriptive sets in a type. $[1,2,3,7]$ and $[1,2,3,7,12]$ are supertypes of $[1,2,3]$.

4.1 Subtype Examples

4.1.1 Great Leader

$[Edible]$ is a subtype of $[Edible, Favorite]$. $[Edible, Favorite]$ and $\left[Edible, Favorite, Poison\right]$ are supertypes of $[Edible]$, $[Favorite]$ and $[Edible, Favorite]$.

5 Set Type

All the objects in a typed set share a common set of descriptions. Each object may also have other descriptions not shared among all objects in the typed set. To indicate the type of a set, use:

$$A_m\,[A_{k_1}, A_{k_2}, \cdots, A_{k_n}] \tag{93}$$

$$A_m\,[A_{k_1}, A_{k_2}, \cdots, A_{k_n}] = \{o_i : o_i \in A_{k_1}, o_i \in A_{k_2}, \cdots, o_i \in A_{k_n}\} \quad (94)$$

$$= A_{k_1} \cap A_{k_2} \cap \cdots \cap A_{k_n} \quad (95)$$

5.1 Abbreviation

Set type can be abbreviated in the same way as object type. $A_m\,[k_1, k_2, \cdots, k_n]$ means the same thing as $A_m\,[A_{k_1}, A_{k_2}, \cdots, A_{k_n}]$. For example, when A_1 and A_3 are the descriptive sets on which the type of set A_m is based, that can be indicated simply by:

$$A_m\,[1, 3] \quad (96)$$

5.2 Set Type Examples

5.2.1 Great Leader

$$A_p\,[Edible, Favorite] = \{o_i : o_i \in Edible, o_i \in Favorite\} \quad (97)$$

$$= Edible \cap Favorite \quad (98)$$

$$= A_p\,[1, 3] \quad (99)$$

Part II
An Example of Inference: Inferred Membership

In this part, we look at a specific method for inferring the membership of an object in a target set.

6 Target Set

A target set is a subset A_k of A. When inferring membership of an object, we infer its membership in a target set.

6.1 Target Set Examples

6.1.1 Great Leader

$GreatLeader$ might be a target set. We might calculate the inferred membership of the $kitten$ object in the $GreatLeader$ target set.

6.1.2 Coin Toss

We might calculate the inferred membership of $toss_5$ in the target set $Tails$.

6.1.3 Team

We might calculate the inferred membership of $alice$ in the $Certified9287$ target set.

7 Indicator Set

An indicator set is a subset A_m of A. When calculating indication, we calculate the indication of a target set by an indicator set.

7.1 Indicator Set Examples

7.1.1 Great Leader

We might calculate the set indication of the target set $GreatLeader$ by the indicator set $Edible$. We might calculate the set indication of the target set $Edible$ by the indicator set $Poison$.

8 Indication

The indication of a target set A_k by an indicator set A_m is written $A_k \cap \cdot A_m$. It is calculated like this:

$$A_k \cap \cdot A_m = \frac{|A_k \cap A_m|}{|A_m|} \tag{100}$$

Indication is the frequency with which objects in the indicator set (or objects described by the indicator set) have also been members of (or have been described by) the target set.

Indication is conditional probability based on the known objects. Indication, conditional probability, and Bayes' theorem, though they start with different ingredients, produce the same measurement:

$$
\begin{aligned}
P\left(A|B\right) &= \frac{P\left(A \cap B\right)}{P\left(B\right)} && (101)\\
&\rightsquigarrow \frac{\frac{|A \cap B|}{|O|}}{\frac{|B|}{|O|}} && (102)\\
&= \frac{|A \cap B|}{|B|} && (103)\\
P\left(A|B\right) &= \frac{P\left(B|A\right)P\left(A\right)}{P\left(B\right)} && (104)\\
&= \frac{\frac{P(A \cap B)}{P(A)}P\left(A\right)}{P\left(B\right)} && (105)\\
&\rightsquigarrow \frac{\frac{\left|\frac{A \cap B}{O}\right|}{\left|\frac{A}{O}\right|} \cdot \left|\frac{A}{O}\right|}{\frac{|B|}{|O|}} && (106)\\
&= \frac{|A \cap B|}{|O|} \cdot \frac{|O|}{|A|} \cdot \frac{|A|}{|O|} \cdot \frac{|O|}{|B|} && (107)\\
&= \frac{|A \cap B|}{|B|} && (108)
\end{aligned}
$$

8.1 Indication Examples

8.1.1 Great Leader

The object descriptions are the same as initially stated. For easy reference,

$$Edible = \{water, brie, kitten, leslie, lincoln, \tag{109}$$

$$
\begin{aligned}
& paper\} && (110)\\
Organism &= \{kitten, leslie, lincoln\} && (111)\\
Favorite &= \{brie, lincoln\} && (112)\\
GreatLeader &= \{lincoln\} && (113)\\
Poison &= \{\} && (114)\\
FavoriteEdibleOrganism &= \{lincoln\} && (115)
\end{aligned}
$$

Indication is calculated:

$$
\begin{aligned}
GreatLeader \cap \cdot Edible &= \frac{|GreatLeader \cap Edible|}{|Edible|} = \frac{1}{6} \approx 0.166 && (116)\\
Edible \cap \cdot Poison &= \frac{|Edible \cap Poison|}{|Poison|} = \frac{0}{0}\,(undefined) && (117)\\
Organism \cap \cdot Edible &= \frac{|Organism \cap Edible|}{|Edible|} = \frac{3}{6} = \frac{1}{2} = 0.500 && (118)\\
Edible \cap \cdot Organism &= \frac{|Organism \cap Edible|}{|Organism|} = \frac{3}{3} = 1.000 && (119)
\end{aligned}
$$

9 Rarity

The rarity of a set A_m is written $\odot A_m$. It is calculated like this:

$$\odot A_m = \frac{|O|}{|A_m|} \tag{120}$$

Rarity is the inverse frequency, oddity, unusualness, or distinction of the occurrence of a particular description. A rarity of n means there would have to be n descriptions of this rarity if they were to completely, evenly, and without overlapping, describe every known object.

9.1 Rarity Examples

9.1.1 Great Leader

The object descriptions are the same as above. Rarity is calculated:

$$
\begin{aligned}
\odot Edible &= \frac{7}{6} \approx 1.166 && (121)\\
\odot Organism &= \frac{7}{3} \approx 2.333 && (122)\\
\odot GreatLeader &= \frac{7}{1} = 7.000 && (123)\\
\odot Poison &= \frac{7}{0}\,(undefined) && (124)
\end{aligned}
$$

Edible is a common description of objects in $O_{GreatLeader}$; its rarity is low relative to the rarity of *GreatLeader*. *GreatLeader* has not often described objects in $O_{GreatLeader}$; its rarity is high.

9.1.2 Coin Toss

The object descriptions are the same as initially stated. For easy reference,

$$Heads = \{toss_1, toss_3, toss_5\} \tag{125}$$
$$Tails = \{toss_2, toss_4, toss_6\} \tag{126}$$

Rarity is calculated:

$$\odot Heads = \frac{6}{3} = \frac{2}{1} = 2.000 \tag{127}$$
$$\odot Tails = \frac{6}{3} = \frac{2}{1} = 2.000 \tag{128}$$

A rarity of 2 means there would have to be 2 descriptions of this rarity if they were to completely, evenly, and without overlapping, describe every known object. In the usual way of thinking about a coin toss, this is the case: there are two descriptions, each of rarity 2, that completely, evenly, and without overlapping, describe every known object.

10 Inferred Membership

The inferred membership of an object o_i in set A_k is written $A_k \between o_i$. It is calculated like this:

$$\begin{aligned} A_k \between o_i &= \sum_{m:o_i \in A_m} (A_k \cap \cdot A_m)(\odot A_m) && (129) \\ &= \sum_{m:o_i \in A_m} \left(\frac{|A_k \cap A_m|}{|A_m|}\right)\left(\frac{|O|}{|A_m|}\right) && (130) \\ &= \sum_{m:o_i \in A_m} \frac{|O| \cdot |A_k \cap A_m|}{|A_m|^2} && (131) \end{aligned}$$

?:

$$\begin{aligned} A_k \between o_i &= \sum_{m:o_i \in A_m} (A_k \cap \cdot A_m)(\odot A_m) && (132) \\ &= \sum_{m:o_i \in A_m} \left(\frac{|A_k \cap A_m|}{|A_m|}\right)\left(\frac{|O|}{|A_m|}\right) && (133) \\ ? &= \frac{\sum_{m:o_i \in A_m}^{n} \frac{|O| \cdot |A_k \cap A_m|}{|A_m|^2}}{n|O|} && (134) \end{aligned}$$

You can calculate inferred membership of any object, no matter how partially or fully described. (Of course, description helps.)

Typically, inferred membership of an object in a set will be calculated with respect to a set in which the object has not been observed, but this doesn't have to be the case.

10.1 Inferred Membership Examples

10.1.1 Great Leader

The object descriptions are the same as initially stated. For easy reference,

$$\begin{aligned}
Edible &= \big\{water, brie, kitten, leslie, lincoln, && (135)\\
&\quad paper\big\} && (136)\\
Organism &= \{kitten, leslie, lincoln\} && (137)\\
Favorite &= \{brie, lincoln\} && (138)\\
GreatLeader &= \{lincoln\} && (139)\\
Poison &= \{\} && (140)\\
FavoriteEdibleOrganism &= \{lincoln\} && (141)
\end{aligned}$$

Inferred membership is calculated as follows. We sum a series of products of indication of the target set by the indicator set and rarity of the indicator set, for each indicator set containing the object in question:

$$\begin{aligned}
GreatLeader \between water &= (GreatLeader \cap \cdot Edible)\,(\odot Edible) && (142)\\
&= \frac{7\cdot 1}{6^2} = \frac{7}{36} && (143)\\
&\approx 0.194 && (144)
\end{aligned}$$

$$\begin{aligned}
GreatLeader \between kitten &= (GreatLeader \cap \cdot Edible)\,(\odot Edible) && (145)\\
&\quad + (GreatLeader \cap \cdot Organism) && (146)\\
&\quad (\odot Organism) && (147)\\
&= \frac{7\cdot 1}{6^2} + \frac{7\cdot 1}{3^2} = \frac{7}{36} + \frac{7}{9} = \frac{7}{36} + \frac{28}{36} = \frac{35}{36} && (148)\\
&\approx 0.972 && (149)
\end{aligned}$$

$$\begin{aligned}
GreatLeader \between leslie &= (GreatLeader \cap \cdot Edible)\,(\odot Edible) && (150)\\
&\quad + (GreatLeader \cap \cdot Organism) && (151)\\
&\quad (\odot Organism) && (152)\\
&= GreatLeader \between kitten && (153)
\end{aligned}$$

$$
\begin{aligned}
GreatLeader \between brie &= (GreatLeader \cap \cdot Edible)\,(\odot Edible) && (154)\\
&\quad + (GreatLeader \cap \cdot Favorite) && (155)\\
&\quad (\odot Favorite) && (156)\\
&= \frac{7\cdot 1}{6^2} + \frac{7\cdot 1}{2^2} = \frac{7}{36} + \frac{7}{4} = \frac{7}{36} + \frac{63}{36} = \frac{70}{36} && (157)\\
&\approx 1.944 && (158)
\end{aligned}
$$

$$
\begin{aligned}
GreatLeader \between lincoln &= (GreatLeader \cap \cdot Edible)\,(\odot Edible) && (159)\\
&\quad + (GreatLeader \cap \cdot Organism) && (160)\\
&\quad (\odot Organism) && (161)\\
&\quad + (GreatLeader \cap \cdot Favorite) && (162)\\
&\quad (\odot Favorite) && (163)\\
&\quad + (GreatLeader \cap \cdot GreatLeader) && (164)\\
&\quad (\odot GreatLeader) && (165)\\
&= \frac{7\cdot 1}{6^2} + \frac{7\cdot 1}{3^2} + \frac{7\cdot 1}{2^2} + \frac{7\cdot 1}{1^2} && (166)\\
&= \frac{7}{36} + \frac{28}{36} + \frac{63}{36} + \frac{252}{36} = \frac{350}{36} && (167)\\
&\approx 9.722 && (168)
\end{aligned}
$$

11 Meaning of Inferred Membership

The meaning of $A_k \between o_i$ has everything to do with how the descriptive functions a are defined. In general, though, the quantity $A_k \between o_i$ is a certainty about the idea that $o_i \in A_k$:

$$
A_k \between o_i \begin{cases} = 0 & \textit{completely uncertain} \\ (0,1) & \textit{somewhat uncertain, somewhat certain} \\ = 1 & \textit{certain} \\ > 1 & \textit{multiply certain} \end{cases} \tag{169}
$$

12 Ratio of Inferred Memberships

Comparing the inequality of two inferred memberships is meaningful. So is comparing the the ratio of two inferred memberships. That one inferred membership is larger than another is meaningful, but differently so is the idea that one is, for example, twice the quantity of the other.

12.1 Great Leader

In the great leader example, we're 5 times more certain that $lincoln$ is a $GreatLeader$ than we are certain of the same with $brie$ and we're equally certain that $kitten$ and $leslie$ are $GreatLeader$s.

13 Metaphor of Inferred Membership as Inferring Related Types

Inferred membership can be seen as inferring properties of an object where only a subtype of the object is known. It can be seen as inferring the supertype of an object of a certain type.

14 Inferred Membership as Ensemble Classification

To the degree to which the descriptions in A can be seen as classifications, inferred membership can be seen as an ensemble classification.

Part III

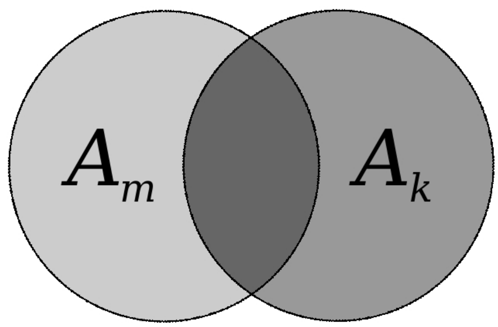
General Inference

In this part, we look more generally at the inference concepts implicit in the example of inferred membership.

15 Simple Ratios of Cardinalities Involving Two Sets

Here we look at some simple ratios of cardinalities involving two sets. Since types and sets are interchangeable, these are also ratios between the cardinalities of types.

Consider two sets A_m and A_k:

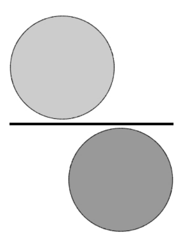


When it is appropriate to think of these as an indicator set and a target set, A_m is the indicator set and A_k is the target set.

15.1 $A_k :: A_m$ Indicator Frequency

The frequency of an indicator set A_m with respect to a target set A_k is written $A_k :: A_m$ It is calculated like this:

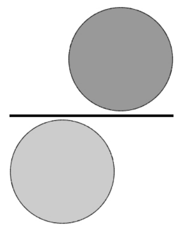
$$A_k :: A_m = \frac{|A_m|}{|A_k|} \tag{170}$$



15.2 $A_k \cdot \cdot A_m$ Target Frequency

The frequency of a target set A_k with respect to an indicator set A_m is written $A_k \cdot \cdot A_m$ It is calculated like this:

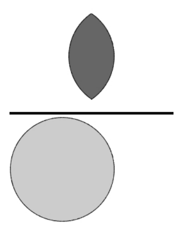
$$A_k \cdot \cdot A_m = \frac{|A_k|}{|A_m|} \tag{171}$$



15.3 $A_k \cap \cdot A_m$ Indicator Overlap (Indication)

The indicator overlap, or indication, given a target set A_k and an indicator set A_m is written $A_k \cap \cdot A_m$. It is calculated like this:

$$A_k \cap \cdot A_m = \frac{|A_k \cap A_m|}{|A_m|} \tag{172}$$

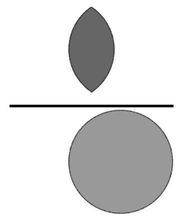


This can be thought of as a degree to which the indicator set is indicating the target set.

15.4 $A_k \cap : A_m$ Target Overlap (Targeting)

The target overlap, or targeting, given a target set A_k and an indicator set A_m is written $A_k \cap : A_m$. It is calculated like this:

$$A_k \cap : A_m = \frac{|A_k \cap A_m|}{|A_k|} \tag{173}$$

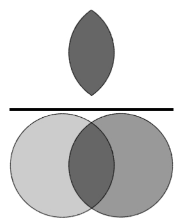


This can be thought of as a degree to which the target set is being indicated by, or targeted by, the indicator set.

15.5 $A_k \cap\cup A_m$ Overlap

The overlap of sets A_k and A_m is written $A_k \cap\cup A_m$. It is calculated like this:

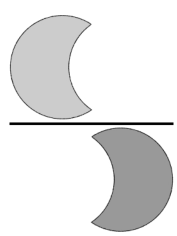
$$A_k \cap\cup A_m = \frac{|A_k \cap A_m|}{|A_k \cup A_m|} \tag{174}$$



15.6 $A_k = -A_m$ Indicator Mismatch

The indicator mismatch of a target set A_k and an indicator set A_m is written $A_k = -A_m$. It is calculated like this:

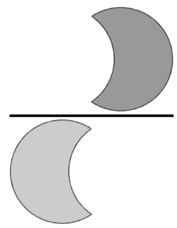
$$A_k = -A_m = \frac{|A_m - A_k|}{|A_k - A_m|} \tag{175}$$



15.7 $A_k- = A_m$ Target Mismatch

The target mismatch of a target set A_k and an indicator set A_m is written $A_k- = A_m$. It is calculated like this:

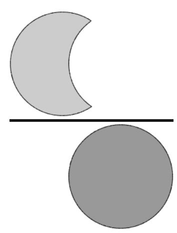
$$A_k- = A_m = \frac{|A_k - A_m|}{|A_m - A_k|} \tag{176}$$



15.8 $A_k =: A_m$ Indicator Impertinence

Indicator impertinence, given an indicator set A_m and a target set A_k is written $A_k =: A_m$. It is calculated like this:

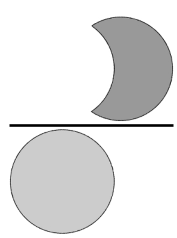
$$A_k =: A_m = \frac{|A_m - A_k|}{|A_k|} \tag{177}$$



15.9 $A_k - \cdot A_m$ Target Impertinence

Target impertinence, given an indicator set A_m and a target set A_k is written $A_k - \cdot A_m$. It is calculated like this:

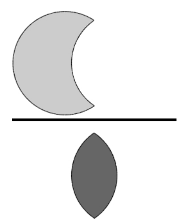
$$A_k - \cdot A_m = \frac{|A_k - A_m|}{|A_m|} \tag{178}$$



15.10 $A_k = \cap A_m$ Indicator Opacity

Indicator opacity, given an indicator set A_m and a target set A_k is written $A_k = \cap A_m$. It is calculated like this:

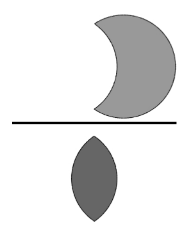
$$A_k = \cap A_m = \frac{|A_m - A_k|}{|A_k \cap A_m|} \tag{179}$$



15.11 $A_k - \cap A_m$ Target Opacity

Target opacity, given an indicator set A_m and a target set A_k is written $A_k - \cap A_m$. It is calculated like this:

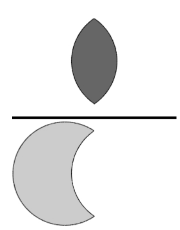
$$A_k - \cap A_m = \frac{|A_k - A_m|}{|A_k \cap A_m|} \tag{180}$$



15.12 $A_k \cap = A_m$ Indicator Transparency

The indicator transparency of a target set A_k and an indicator set A_m is written $A_k \cap = A_m$. It is calculated like this:

$$A_k \cap = A_m = \frac{|A_k \cap A_m|}{|A_m - A_k|} \tag{181}$$

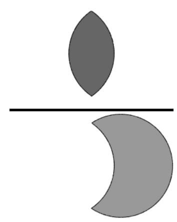


This can be thought of as an extent to which, or a thoroughness with which, the target set invades the indicator set. It can be thought of as a degree to which the target set disects the indicator set.

15.13 $A_k \cap -A_m$ Target Transparency

The target transparency of a target set A_k and an indicator set A_m is written $A_k \cap -A_m$. It is calculated like this:

$$A_k \cap -A_m = \frac{|A_k \cap A_m|}{|A_k - A_m|} \tag{182}$$



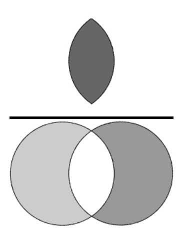
This can be thought of as an extent to which, or a thoroughness with which, the indicator set invades the target set. It can be thought of as a degree to which the indicator set disects the target set.

15.14 $A_k \cap \pitchfork A_m$ Transparency

The transparency of two sets, A_m and A_k is written $A_k \cap \pitchfork A_m$. It is calculated like this:

$$\begin{aligned} A_k \pitchfork A_m &= \{A_k - A_m\} \cup \{A_m - A_k\} && (183) \\ &= \{A_k \cup A_m\} - \{A_k \cap A_m\} && (184) \end{aligned}$$

$$A_k \cap \pitchfork A_m = \frac{|A_k \cap A_m|}{|A_k \pitchfork A_m|} \tag{185}$$



15.15 Notation

For a simple ratio of cardinalities involving two sets, write $A_k\, X\, Y\, A_m$, where X and Y are two symbols that indicate the operation and operands in each part of the ratio.

The symbols are each one of $\{\cdot, :, -, =, \cap, \cup, \pitchfork\}$.

The first symbol, X, refers to the operation occuring in the ratio's numerator. The second symbol, Y, refers to the operation occuring in the ratio's demoninator.

By default, A_k is the first operand in each operation and A_m is the second. Variations on certain symbols reverse the default ordering.

By default, when there is a single term in the numerator it is A_k. By default, when there is a single term in the denominator it is A_m. Variations on certain operators make it so that when there is a single operator in the numerator, the alternate, A_m, is used. The same operator variations make it so that when there is a single operator in the denominator, the alternate, A_k, is used.

$\cdot$ means use the default operand by itself. This is either A_k or A_m depending on whether the . symbol is, respectively, in the first or second position (the X or the Y position).

: means use the alternate operand by itself. This is either A_m or A_k depending on whether the . symbol is, respectively, in the first or second position (the X or the Y position). ($\cdot$ and : are counterparts.)

$-$ means subtract A_m from A_k.

$=$ means subtract A_k from A_m. ($-$ and $=$ are counterparts.)

$\cap$ means use the intersection of A_k and A_m.

$\cup$ means use the union of A_k and A_m.

$\pitchfork$ means use the union of A_k and A_m, minus the intersection of A_k and A_m.

So,

$$A_k\, X\, Y\, A_m \quad : \quad \frac{X}{Y} \tag{186}$$

$$A_k \cdot \cdot A_m \quad : \quad \frac{A_k}{A_m} \tag{187}$$

$$A_k : \cdot A \quad : \quad \frac{A_m}{A_m} \tag{188}$$

$$A_k :: A_m \quad : \quad \frac{A_m}{A_k} \tag{189}$$

$$A_k - \cdot A_m \quad : \quad \frac{A_k - A_m}{A_m} \tag{190}$$

$$A_k = - A_m \quad : \quad \frac{A_m - A_k}{A_k - A_m} \tag{191}$$

$$A_k \cap - A_m \quad : \quad \frac{A_k \cap A_m}{A_k - A_m} \tag{192}$$

Reversing the order of X and Y means swapping the numerator and denominator of the ratio:

$$A_k\,X\,Y\,A_m = \frac{1}{A_k\,Y\,X\,A_m} \tag{193}$$

15.16 A Systematic Look

To reduce clutter, cardinality (||) symbols are omitted.

$$\begin{array}{cc|ccccccc}
 & & & & & Y & & & \\
 & & \cdot & : & - & = & \cap & \cup & \pitchfork \\
 & \cdot & \frac{A_k}{A_m} & \frac{A_k}{A_k} & \frac{A_k}{A_k-A_m} & \frac{A_k}{A_m-A_k} & \frac{A_k}{A_k\cap A_m} & \frac{A_k}{A_k\cup A_m} & \frac{A_k}{A_k\pitchfork A_m} \\
 & : & \frac{A_m}{A_m} & \frac{A_m}{A_k} & \frac{A_m}{A_k-A_m} & \frac{A_m}{A_m-A_k} & \frac{A_m}{A_k\cap A_m} & \frac{A_m}{A_k\cup A_m} & \frac{A_m}{A_k\pitchfork A_m} \\
 & - & \frac{A_k-A_m}{A_m} & \frac{A_k-A_m}{A_k} & \frac{A_k-A_m}{A_k-A_m} & \frac{A_k-A_m}{A_m-A_k} & \frac{A_k-A_m}{A_k\cap A_m} & \frac{A_k-A_m}{A_k\cup A_m} & \frac{A_k-A_m}{A_k\pitchfork A_m} \\
X & = & \frac{A_m-A_k}{A_m} & \frac{A_m-A_k}{A_k} & \frac{A_m-A_k}{A_k-A_m} & \frac{A_m-A_k}{A_m-A_k} & \frac{A_m-A_k}{A_k\cap A_m} & \frac{A_m-A_k}{A_k\cup A_m} & \frac{A_m-A_k}{A_k\pitchfork A_m} \\
 & \cap & \frac{A_k\cap A_m}{A_m} & \frac{A_k\cap A_m}{A_k} & \frac{A_k\cap A_m}{A_k-A_m} & \frac{A_k\cap A_m}{A_m-A_k} & \frac{A_k\cap A_m}{A_k\cap A_m} & \frac{A_k\cap A_m}{A_k\cup A_m} & \frac{A_k\cap A_m}{A_k\pitchfork A_m} \\
 & \cup & \frac{A_k\cup A_m}{A_m} & \frac{A_k\cup A_m}{A_k} & \frac{A_k\cup A_m}{A_k-A_m} & \frac{A_k\cup A_m}{A_m-A_k} & \frac{A_k\cup A_m}{A_k\cap A_m} & \frac{A_k\cup A_m}{A_k\cup A_m} & \frac{A_k\cup A_m}{A_k\pitchfork A_m} \\
 & \pitchfork & \frac{A_k\pitchfork A_m}{A_m} & \frac{A_k\pitchfork A_m}{A_k} & \frac{A_k\pitchfork A_m}{A_k-A_m} & \frac{A_k\pitchfork A_m}{A_m-A_k} & \frac{A_k\pitchfork A_m}{A_k\cap A_m} & \frac{A_k\pitchfork A_m}{A_k\cup A_m} & \frac{A_k\pitchfork A_m}{A_k\pitchfork A_m}
\end{array} \tag{194}$$

TODO: make this table in pitcures as well

15.17 (?) Identities

TODO?: lay out the equivalent of trig identities...stuff like abc = 1/xyz

16 Some Other Ratios Involving Cardinalities

Of course there are other ratios than the ones cataloged above. These examples are meant to suggest the possibilities. In each case, $\otimes$ means SOME UNNAMED OPERATOR.

For brevity, we write $A_{k_1}, A_{k_2}, \cdots, A_{k_n}$ as $A_{k_{[1..n]}}$ and $A_{k_1} \otimes A_{k_2} \otimes \cdots \otimes A_{k_n}$ as $A_{k_{\otimes[1..n]}}$.

$$\otimes A_p = \frac{|A_p|}{|O|} \tag{196}$$

$$A_k \otimes A_{m_{[1..n]}} = \frac{\left|A_k \cap A_{m_{\cap[1..n]}}\right|}{\left|A_{m_{\cap[1..n]}}\right|} \tag{197}$$

These are names for the ratios (fill in all of them? flesh out, or reduce, examples above?):

$$
\begin{array}{cc|ccccccc}
 & & & & & Y & & & \\
 & & \cdot & : & - & = & \cap & \cup & \pitchfork \\
 & \cdot & t.\,frequency & \frac{A_k}{A_k} & \frac{A_k}{A_k - A_m} & \frac{A_k}{A_m - A_k} & \frac{A_k}{A_k \cap A_m} & \frac{A_k}{A_k \cup A_m} & \frac{A_k}{A_k \pitchfork A_m} \\
 & : & \frac{A_m}{A_m} & i.\,frequency & \frac{A_m}{A_k - A_m} & \frac{A_m}{A_m - A_k} & \frac{A_m}{A_k \cap A_m} & \frac{A_m}{A_k \cup A_m} & \frac{A_m}{A_k \pitchfork A_m} \\
 & - & t.\,impertinence & \frac{A_k - A_m}{A_k} & \frac{A_k - A_m}{A_k - A_m} & t.\,mismatch & t.\,opacity & \frac{A_k - A_m}{A_k \cup A_m} & \frac{A_k - A_m}{A_k \pitchfork A_m} \\
X & = & \frac{A_m - A_k}{A_m} & i.\,impertinence & i.\,mismatch & \frac{A_m - A_k}{A_m - A_k} & i.\,opacity & \frac{A_m - A_k}{A_k \cup A_m} & \frac{A_m - A_k}{A_k \pitchfork A_m} \\
 & \cap & i.\,overlap & t.\,overlap & t.\,transparency & i.\,transparency & \frac{A_k \cap A_m}{A_k \cap A_m} & overlap & transparency \\
 & \cup & \frac{A_k \cup A_m}{A_m} & \frac{A_k \cup A_m}{A_k} & \frac{A_k \cup A_m}{A_k - A_m} & \frac{A_k \cup A_m}{A_m - A_k} & \frac{A_k \cup A_m}{A_k \cap A_m} & \frac{A_k \cup A_m}{A_k \cup A_m} & \frac{A_k \cup A_m}{A_k \pitchfork A_m} \\
 & \pitchfork & \frac{A_k \pitchfork A_m}{A_m} & \frac{A_k \pitchfork A_m}{A_k} & \frac{A_k \pitchfork A_m}{A_k - A_m} & \frac{A_k \pitchfork A_m}{A_m - A_k} & \frac{A_k \pitchfork A_m}{A_k \cap A_m} & \frac{A_k \pitchfork A_m}{A_k \cup A_m} & \frac{A_k \pitchfork A_m}{A_k \pitchfork A_m}
\end{array}
\tag{195}
$$

TODO: name all the ratios and make an almanac of them, with a name, formula, and picture for each

$$A_k \otimes A_{m_{[1..n]}} = \frac{|O| \cdot \left|A_k - A_{m_{\cup[1..n]}}\right|}{\left|A_{m_{\cap[1..n]}}\right|} \tag{198}$$

$$A_{k_{[1..n]}} \otimes A_m = \frac{\left|A_{k_{\cup[1..n]}} \cap A_m\right|}{|O| - |A_m|^3} \tag{199}$$

$$A_{k_{[1..n]}} \otimes A_{m_{[1..p]}} = \frac{\left|A_{k_{\cap[1..n]}} \pitchfork A_{m_{\cap[1..p]}}\right|}{\left|A_{m_{\cap[1..p]}}\right|} \tag{200}$$

$$A_{k_{[1..n]}} \otimes A_{m_{[1..p]}} = \frac{\left|A_{k_{\cup[1..n]}} \cap A_{m_{\cup[1..p]}}\right|}{\left|A_{m_{\pitchfork[1..p]}}\right|} \tag{201}$$

TODO: show some meaningful examples and say what they do

17 Relationships Between an Object to an Object

$$\ominus (o_i, o_k) = \cdots \tag{202}$$

TODO: show some meaningful examples and say what they do

18 Relationships Between a Set and an Object

$\ominus$ means SOME UNNAMED OPERATOR distinct from $\otimes$. $r(\cdots)$ is some function controlling the ranges over which $\otimes$ operates. $f(\cdots)$ is some function that is aggregated by $\otimes$.

$$A_x \ominus o_i = \bigotimes_{r(\cdots)} f(\cdots) \tag{203}$$

Each of the following relationships is based on one of the simple ratios of cardinalities involving two sets, above.

18.1 Indicator Frequency

$$A_m \ominus o_i = \bigotimes_{k:o_i \in A_k} (A_k :: A_m)(\odot A_k) \tag{204}$$

$$= \bigotimes_{k:o_i \in A_k} \frac{|O| \cdot |A_m|}{|A_k|^2} \tag{205}$$

18.2 Target Frequency

$$A_k \ominus o_i = \bigotimes_{m:o_i \in A_m} (A_k \cdot\cdot A_m)(\odot A_m) \tag{206}$$

$$= \bigotimes_{m:o_i \in A_m} \frac{|O| \cdot |A_k|}{|A_m|^2} \tag{207}$$

18.3 Indicator Overlap (Indication)

Functions based on indicator overlap are functions of inferred indication. These are measures of indication from many indicator sets to one target set.

$$\begin{aligned} A_k \ominus o_i &= \bigotimes_{m:o_i \in A_m} (A_k \cap \cdot A_m)\,(\odot A_m) &(208)\\ &= \bigotimes_{m:o_i \in A_m} \frac{|O| \cdot |A_k \cap A_m|}{|A_m|^2} &(209)\end{aligned}$$

Inferred membership of an object in a set, $\between$, is based on indicator overlap. It is an aggregate function of inferred indication.

18.4 Target Overlap (Targeting)

Functions based on target overlap are functions of inferred targeting. These are measures of targeting from many target sets to one indicator set.

$$\begin{aligned} A_m \ominus o_i &= \bigotimes_{k:o_i \in A_k} (A_k \cap : A_m)\,(\odot A_k) &(210)\\ &= \bigotimes_{k:o_i \in A_k} \frac{|O| \cdot |A_k \cap A_m|}{|A_k|^2} &(211)\end{aligned}$$

This can be thought of as a measure of promiscuity, or overtness, or obviousness of o_i, due to its having been described by A_m, with respect to the available target sets. What is being measured is the obviousness of the indication of the pertinent target sets in A, given that the party doing inference has described o_i as having membership in A_m.

TODO: this, with others here, will have a corollary in the set-to-set-relationship section, as they'll want to choose a subset of the pertinent target sets to calculate this with respect to (right?)

18.5 Overlap

$$\begin{aligned} A_k \ominus o_i &= \bigotimes_{m:o_i \in A_m} (A_k \cap \cup A_m)\,(\odot A_m) &(212)\\ &= \bigotimes_{m:o_i \in A_m} \frac{|O| \cdot |A_k \cap A_m|}{|A_m| \cdot |A_k \cup A_m|} &(213)\end{aligned}$$

$$
\begin{aligned}
A_k \ominus o_i &= \prod_{m:o_i \in A_m} A_k \cap \cup A_m && (214)\\
&= \prod_{m:o_i \in A_m} \frac{|A_k \cap A_m|}{|A_k \cup A_m|} && (215)
\end{aligned}
$$

18.6 Indicator Mismatch

$$
\begin{aligned}
A_m \ominus o_i &= \bigotimes_{k:o_i \in A_k} (A_k = -A_m)\,(\odot A_k) && (216)\\
&= \bigotimes_{k:o_i \in A_k} \frac{|O| \cdot |A_m - A_k|}{|A_k| \cdot |A_k - A_m|} && (217)
\end{aligned}
$$

18.7 Target Mismatch

$$
\begin{aligned}
A_k \ominus o_i &= \bigotimes_{m:o_i \in A_m} (A_k - = A_m)\,(\odot A_m) && (218)\\
&= \bigotimes_{m:o_i \in A_m} \frac{|O| \cdot |A_k - A_m|}{|A_m| \cdot |A_m - A_k|} && (219)
\end{aligned}
$$

18.8 Indicator Impertinence

$$
\begin{aligned}
A_m \ominus o_i &= \bigotimes_{k:o_i \in A_k} (A_k =: A_m)\,(\odot A_k) && (220)\\
&= \bigotimes_{k:o_i \in A_k} \frac{|O| \cdot |A_m - A_k|}{|A_k|^2} && (221)
\end{aligned}
$$

18.9 Target Impertinence

$$
\begin{aligned}
A_k \ominus o_i &= \bigotimes_{m:o_i \in A_m} (A_k - \cdot A_m)\,(\odot A_m) && (222)\\
&= \bigotimes_{m:o_i \in A_m} \frac{|O| \cdot |A_k - A_m|}{|A_m|^2} && (223)
\end{aligned}
$$

18.10 Indicator Opacity

$$
\begin{aligned}
A_k \ominus o_i &= \bigotimes_{m:o_i \in A_m} (A_k = \cap A_m)\,(\odot A_m) && (224)\\
&= \bigotimes_{m:o_i \in A_m} \frac{|O| \cdot |A_m - A_k|}{|A_m| \cdot |A_k \cap A_m|} && (225)
\end{aligned}
$$

18.11 Target Opacity

$$A_m \ominus o_i = \bigotimes_{k:o_i \in A_k} (A_k - \cap A_m)\,(\odot A_k) \tag{226}$$

$$= \bigotimes_{k:o_i \in A_k} \frac{|O| \cdot |A_k - A_m|}{|A_k| \cdot |A_k \cap A_m|} \tag{227}$$

18.12 Indicator Transparency

$$A_k \ominus o_i = \bigotimes_{m:o_i \in A_m} (A_k \cap = A_m)\,(\odot A_m) \tag{228}$$

$$= \bigotimes_{m:o_i \in A_m} A_k \cap = A_m \frac{|O| \cdot |A_k \cap A_m|}{|A_m| \cdot |A_m - A_k|} \tag{229}$$

18.13 Target Transparency

$$A_m \ominus o_i = \bigotimes_{k:o_i \in A_k} (A_k \cap -A_m)\,(\odot A_k) \tag{230}$$

$$= \bigotimes_{k:o_i \in A_k} \frac{|O| \cdot |A_k \cap A_m|}{|A_k| \cdot |A_k - A_m|} \tag{231}$$

18.14 Transparency

$$A_k \ominus o_i = \bigotimes_{m:o_i \in A_m} (A_k \cap \pitchfork A_m)\,(\odot A_m) \tag{232}$$

$$= \bigotimes_{m:o_i \in A_m} \frac{|O| \cdot |A_k \cap A_m|}{|A_m| \cdot |A_k \pitchfork A_m|} \tag{233}$$

$$A_k \ominus o_i = \prod_{m:o_i \in A_m} A_k \cap \pitchfork A_m \tag{234}$$

$$= \prod_{m:o_i \in A_m} \frac{|A_k \cap A_m|}{|A_k \pitchfork A_m|} \tag{235}$$

19 Relationships Between Two Sets

Relationships between a target set A_k and a source set A_p quantify a relationship between the two sets.

Sometimes their form is to aggregate object-to-set relationships between objects in the source set and the target set:

$$A_k \ominus A_p = \bigotimes_{r(\cdots)} f(\cdots) \tag{236}$$

TODO?: look at other forms they could take

19.1 Aggregate Functions of Inferred Membership

There are uses for functions that aggregate inferred membership in a set A_k of all the objects in a set A_p. Where inferred membership calculates a measure of inference from an object to a target set, an aggregate function of $\between$ quantifies an inference-based relationship from a source set to a target set.

19.1.1 Sum

$$sumIM\left(A_k, A_p\right) = \sum_{i:o_i \in A_p} A_k \between o_i \tag{237}$$

sumIM might be appropriate in measuring something like a team rank in a multiplayer game that isn't critically team-oriented–that is, a game that is a games of sides composed of multiple players but in which teamwork isn't critical for team success.

19.1.2 Product

$$productIM\left(A_k, A_p\right) = \prod_{i:o_i \in A_p} A_k \between o_i \tag{238}$$

productIM might be appropriate in measuring team viability in situations where A_x (as either an indicator set or a target set) is a description of something like trust, such that the lack of A_x in an individual object o_i has a devastating effect on the description of A_x for a set of objects.

19.1.3 Average

$$averageIM\left(A_k, A_p\right) = \frac{1}{|A_p|} \sum_{i:o_i \in A_p} A_k \between o_i \tag{239}$$

averageIM can be thought of as providing a measure of inferred set inference from one set to another, or from one type to another.

19.2 Ratio of Aggregate Functions of Inferred Membership

In the same way that not only the inequality but the ratio of inferred memberships is meaningful, with the *productIM* and the *averageIM* aggregate functions, the ratio of two aggregate values is meaningful. This might be true for other aggregate functions of inferred membership not mentioned here. It would certainly be true of other types of products and averages.

19.3 Inferred Membership of a Set in a Set

The inferred membership in a set A_k of a set A_p is expressed $A_k \left(\between\right) A_p$ and is the same as *productIM*:

$$A_k \left(\between\right) A_p = \prod_{i:o_i \in A_p} A_k \between o_i \tag{240}$$

20 Complex Mixed Relationships

Complex mixed relationships are relationships from a set of objects and/or sets to a set of objects and/or sets.

$$\ominus \left(o_{i_1}, o_{i_2}, \cdots, o_{i_n}; A_{p_1}, A_{p_2}, \cdots, A_{p_q}; o_r, A_t, \cdots; \cdots\right) = \bigotimes_{r(\cdots)} f\left(\cdots\right) \tag{241}$$

TODO: show some meaningful examples and say what they do

TODO: talk about the interplay between multiple inference domains (as is the case, essentially, with complex mixed relationships

21 Cardinality of Multisets

TODO: say what it means to use multisets and show examples of places to use them

22 (?) Nitpicky Inferred Membership

TODO?: write out my exclusionary subset method for calculating inferred membership (if we do this, it might be better placed after the inferred membership definition)

Part IV
Mechanics

In this part, we discuss mechanical details that might be useful when doing set inference.

23 Scaling Functions

TODO: define some scaling functions to normalize the 0, (0,1), 1, >1 range

24 Finding Descriptive Functions

The descriptive functions in a don't have to be designed. They can be bred genetically, or searched for using other methods, producing a set of native/cryptic descriptive functions that serve as an ensemble of classifiers.

25 Lookup Table of Precomputed Average Inferred Memberships

An estimate of $A_k \between o_i$ can be found quickly using a lookup table of average inferred memberships. Construct a two-dimensional table $averageIMTable$ containing $averageIM\left(A_k, A_p\right)$ for each $\{A_k, A_p\}$ pair, based on O at the time of the lookup table construction. Then, when a new object o_i is added to O, $A_k \between o_i$ can be estimated by:

$$A_k \between o_i \approx \sum_{p: o_i \in A_p} averageIMTable\left\{A_k, A_p\right\} \tag{242}$$

TODO: generalize this to talk about lookup tables for aggregate functions