EM field:

Ref: arxiv. 1602.02223 (Hui Li et. al.) & arxiv. 1401.3805 (Gursoy et. al.)

$$\hat{\phi} = -\sin\phi \, \hat{i} + \cos\phi \, \hat{j} \qquad \text{—— (1)}$$

$$\vec{B} = \cancel{B_r}^{0} \hat{r} + B_\phi \hat{\phi} + \cancel{B_z}^{0} \hat{z}$$

$$= B_\phi \hat{\phi}$$

$$= B_\phi \left(-\sin\phi \, \hat{i} + \cos\phi \, \hat{j}\right) \qquad \text{—— (2)}$$

$$= B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\therefore \; B_x = -B_\phi \sin\phi \, \hat{i} \qquad \text{—— (3)}$$

$$B_y = + B_\phi \cos\phi \, \hat{j} \qquad \text{—— (4)}$$

$$B_z = 0$$

$$B_y = B_\phi \cos\phi$$

$$= \frac{Q}{4\pi} \frac{v\gamma x_\perp}{\Delta^{3/2}} \left(1 + \frac{\sigma v \gamma}{2} \sqrt{\Delta}\right) e^{A} \cos\phi \qquad \text{—— (5)}$$

$$\Delta \equiv \gamma^2 (vt - z)^2 + x_T^2 \qquad \text{—— (6)}$$

$$A \equiv \frac{\sigma v \gamma}{2} \left[\gamma (vt - z) - \sqrt{\Delta}\right] \qquad \text{—— (7)}$$

$$\left.\begin{aligned} \gamma v &= \sinh y_b \\ v &= \tanh y_b \\ \gamma &= \cosh y_b \end{aligned}\right\} \; y_b \text{ is beam rapidity} \qquad \text{—— (8)}$$

Let the transverse position of charge is $\vec{x}_T'$.

The $\vec{x}_T$ will be replaced by $\vec{x}_T - \vec{x}_T'$.

$$\vec{x}_T = x_T \cos\phi \,\hat{i} + x_T \sin\phi \,\hat{j}$$
$$\vec{x}_T' = x_T' \cos\phi' \,\hat{i} + x_T' \sin\phi' \,\hat{j}$$

So; $$\vec{x}_T - \vec{x}_T' = (x_T\cos\phi - x_T'\cos\phi')\hat{i} + (x_T\sin\phi - x_T'\sin\phi')\hat{j}$$

$$\Rightarrow (\vec{x}_T - \vec{x}_T')^2 = (x_T\cos\phi - x_T'\cos\phi')^2 + (x_T\sin\phi - x_T'\sin\phi')^2$$

$$= x_T^2\cos^2\phi + x_T'^2\cos^2\phi' - 2x_T x_T'\cos\phi\cos\phi' + x_T^2\sin^2\phi + x_T'^2\sin^2\phi' - 2x_T x_T'\sin\phi\sin\phi'$$

$$= x_T^2(\cos^2\phi + \sin^2\phi) + x_T'^2(\cos^2\phi' + \sin^2\phi') - 2x_T x_T'(\cos\phi\cos\phi' + \sin\phi\sin\phi')$$

$$= x_T^2 + x_T'^2 - 2x_T x_T'\cos(\phi - \phi') \quad \text{—— (9)}$$

Again

$$\left.\begin{aligned} t &= \tau\cosh\eta_s \\ z &= \tau\sinh\eta_s \end{aligned}\right\} \quad \text{—— (10)}$$

η_s = space time rapidity

τ = proper time.

Hence;

$$\Delta = \gamma^2 (vt-z)^2 + \vec{x}_T^{\,2}$$

$$= \gamma^2 (vt-z)^2 + (\vec{x}_T - \vec{x}_T')^2$$

$\left\{ \vec{x}_T \text{ is replaced by } \vec{x}_T - \vec{x}_T' \right.$

$$= \cosh^2 y_b \left(\tanh y_b \,\tau \cosh \eta_s - \tau \sinh \eta_s\right)^2 + (\vec{x}_T - \vec{x}_T')^2$$

$$= \left(\cosh y_b \tanh y_b \,\tau \cosh \eta_s - \tau \cosh y_b \sinh \eta_s\right)^2 + (\vec{x}_T - \vec{x}_T')^2$$

$$= \left(\tau \sinh y_b \cosh \eta_s - \tau \cosh y_b \sinh \eta_s\right)^2 + (\vec{x}_T^{\,2} - \vec{x}_T')^2$$

$$= \tau^2 \sinh^2 (y_b - \eta_s) + x_T^2 + x_T'^2 - 2 x_T x_T' \cos(\phi - \phi') \qquad (11)$$

and

$$A = \frac{\sigma v \gamma}{2} \left[\gamma (vt - z) - \sqrt{\Delta}\right]$$

$$= \frac{\sigma}{2} \sinh y_b \left[\cosh y_b \left(\tau \tanh y_b \cosh \eta_s - \tau \sinh \eta_s\right) - \sqrt{\Delta}\right]$$

$$= \frac{\sigma}{2} \sinh y_b \left[\tau \sinh y_b \cosh \eta_s - \tau \cosh y_b \sinh \eta_s - \sqrt{\Delta} \right]$$

$$= \frac{\sigma}{2} \sinh y_b \left[\tau \sinh (y_b - \eta_s) - \sqrt{\Delta} \right]$$

$$= \frac{\sigma}{2} \left[\tau \sinh y_b \sinh (y_b - \eta_s) - \sinh y_b \sqrt{\Delta} \right] \quad (12)$$

Then;

$$B_y = \frac{Q}{4\pi} \frac{v \gamma x_T \cos\phi}{\Delta^{3/2}} \left(1 + \frac{\sigma v \gamma}{2} \sqrt{\Delta}\right) e^{A}$$

Let the charge is at $\vec{x}' = (x_T', \phi')$

$$= \frac{Q}{4\pi} \frac{\gamma v \left(x_T \cos\phi - x_T' \cos\phi'\right)}{\Delta^{3/2}} \left(1 + \frac{\sigma v \gamma}{2} \sqrt{\Delta}\right) e^{A}$$

$$= \frac{Q}{4\pi} \frac{\sinh y_b \left(x_T \cos\phi - x_T' \cos\phi'\right)}{\Delta^{3/2}} \left(1 + \frac{\sigma \sinh y_b}{2} \sqrt{\Delta}\right) e^{A}$$

let the charge is c times the ~~electron~~ Proton charge.

$$Q = ce$$

$$B_y = \frac{ce}{4\pi} \sinh y_b \left(x_T \cos\phi - x_T' \cos\phi'\right) \frac{\left(1 + \frac{\sigma \sinh y_b}{2} \sqrt{\Delta}\right) e^{A}}{\Delta^{3/2}}$$

$$eB_y = c\alpha \sinh y_b \left(x_T \cos\phi - x_T' \cos\phi'\right) \frac{\left(1 + \frac{\sigma \sinh y_b}{2} \sqrt{\Delta}\right) e^{A}}{\Delta^{3/2}}$$

where; $\alpha = e^2/4\pi$

Again;

$$B_x = -B_\phi \sin\phi$$

$$= -\frac{Q}{4\pi}\frac{\gamma v\, x_T \sin\phi}{\Delta^{3/2}}\left(1+\frac{\sigma v \gamma}{2}\sqrt{\Delta}\right) e^{A}$$

$$= \frac{Q}{4\pi}\frac{\gamma v\left(x_T' \sin\phi' - x_T \sin\phi\right)}{\Delta^{3/2}}\left(1+\frac{\sigma v \gamma}{2}\sqrt{\Delta}\right) e^{A}$$

$$\therefore eB_x = c\,\alpha \sinh y_b\left(x_T' \sin\phi' - x_T \sin\phi\right)\frac{\left(1+\frac{\sigma(\sinh y_b)}{2}\sqrt{\Delta}\right)}{\Delta^{3/2}} e^{A}$$

Electric field:

$$\vec{B} = E_r\hat{r} + \cancel{E_\phi}\hat{\phi} + E_z\hat{z}$$

$$= E_r\hat{r} + E_z\hat{z}$$

$$= E_r\left(\cos\phi\,\hat{i} + \sin\phi\,\hat{j}\right) + E_z\hat{k}$$

$$= E_r\cos\phi\,\hat{i} + E_r\sin\phi\,\hat{j} + E_z\hat{k}$$

$$= E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$E_x = E_r\cos\phi$$

$$E_y = E_r\sin\phi$$

$$E_x = E_n \cos\phi$$

$$= \frac{Q}{4\pi}\left\{ \frac{\gamma x_T}{\Delta^{3/2}}\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right) - \frac{\sigma}{v x_T}\, e^{-\sigma\left(t-\frac{z}{v}\right)}\left[1+\frac{\gamma(vt-z)}{\sqrt{\Delta}}\right]\right\} e^{A} \cos\phi$$

$$= \frac{Q}{4\pi}\left\{ \frac{\gamma x}{\Delta^{3/2}}\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right) - \frac{\sigma x}{v x_T^2}\, e^{-\sigma v(vt-z)}\left[1+\frac{\gamma(vt-z)}{\sqrt{\Delta}}\right]\right\} e^{A}$$

Let the charge is at (x', y', z').

$$= \frac{Q}{4\pi}\left\{ \frac{\gamma (x-x')}{\Delta^{3/2}}\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right) - \frac{\sigma (x-x')}{v\{(x-x')^2+(y-y')^2\}}\, e^{-\sigma v(vt-z)}\left[1+\frac{\gamma(vt-z)}{\sqrt{\Delta}}\right]\right\} e^{A}$$

$$= \cancel{\frac{Q}{4\pi}\left\{ \frac{\cosh y_b\,(x_T\cos\phi - x_T'\cos\phi')}{\Delta^{3/2}}\left(1+\frac{\sigma \sinh y_b}{2}\sqrt{\Delta}\right)\right.}$$

$$= \frac{Q}{4\pi}\left\{ \frac{\gamma}{\Delta^{3/2}}\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right) - \frac{\sigma}{v\lambda}\, e^{-\sigma v(vt-z)}\left[1+\frac{\gamma(vt-z)}{\sqrt{\Delta}}\right]\right\} e^{A}\left(x_T\cos\phi - x_T'\cos\phi'\right)$$

where;

$$\lambda = x_T^2 + x_T'^2 - 2x_T x_T' \cos(\phi-\phi')$$

$$\therefore \frac{F}{x} = \frac{Q}{4\pi}\left\{\frac{\gamma}{\Delta^{3/2}}\left(1+\frac{\sigma v \gamma}{2}\sqrt{\Delta}\right) - \frac{\sigma}{v\lambda} e^{-\frac{\sigma}{v}(vt-z)}\left[1+\frac{\gamma(vt-z)}{\sqrt{\Delta}}\right]\right\}$$

$$e^{A}\left(x_T\cos\phi - x_T'\cos\phi'\right)$$

$$= \frac{Q}{4\pi}\left\{\frac{\cosh y_b}{\Delta^{3/2}}\left(1+\frac{\sigma\sinh y_b}{2}\sqrt{\Delta}\right) - \frac{\sigma}{\tanh y_b\,\lambda} e^{-\sigma\left(\frac{1}{\tanh y_b}\right)(\tanh y_b\,\tau\cosh\eta_s - \tau\sinh\eta_s)}\right.$$

$$\left.\left[1+\frac{\cosh y_b(\tanh y_b\,\tau\cosh\eta_s - \tau\sinh\eta_s)}{\sqrt{\Delta}}\right]\right\}$$

$$e^{A}\left(x_T\cos\phi - x_T'\cos\phi'\right)$$

$$= \frac{Q}{4\pi}\left\{\frac{\cosh y_b}{\Delta^{3/2}}\left(1+\frac{\sigma\sinh y_b}{2}\sqrt{\Delta}\right) - \frac{\sigma}{\tanh y_b\,\lambda} e^{-\sigma\tau\left(\cosh\eta_s - \frac{\sinh\eta_s}{\tanh y_b}\right)}\right.$$

$$\left.\left[1+\frac{\tau\sin(y_b-\eta_s)}{\sqrt{\Delta}}\right]\right\} e^{A}\left(x_T\cos\phi - x_T'\cos\phi'\right)$$

Let $Q = ce$

$$\Rightarrow e\frac{F}{x} = c\alpha\left\{\frac{\cosh y_b}{\Delta^{3/2}}\left(1+\frac{\sigma\sinh y_b}{2}\sqrt{\Delta}\right) - \frac{\sigma}{\tanh y_b\,\lambda} e^{-\sigma\tau\left(\cosh\eta_s - \frac{\sinh\eta_s}{\tanh y_b}\right)}\right.$$

$$\left.\left[1+\frac{\tau\sinh(y_b-\eta_s)}{\sqrt{\Delta}}\right]\right\} e^{A}\left(x_T\cos\phi - x_T'\cos\phi'\right)$$

Similarly;

$$eE_y = c\alpha \left\{ \frac{\cosh y_b}{\Delta^{3/2}} \left(1 + \frac{\sigma \sinh y_b}{2} \sqrt{\Delta}\right) - \frac{\sigma}{\lambda \tanh y_b} e^{-\tau\sigma\left(\cosh\eta_s - \frac{\sinh\eta_s}{\tanh y_b}\right)} \left[1 + \frac{\tau \sinh(y_b - \eta_s)}{\sqrt{\Delta}}\right] \right\} e^{A} \left(x_\perp \sin\phi - x_\perp' \sin\phi'\right)$$

Then;

$$E_z = \frac{Q}{4\pi} \left\{ -e^{A} \frac{1}{\Delta^{3/2}} \left[\gamma (vt - z) + A\sqrt{\Delta} + \frac{\sigma\gamma}{v} \Delta\right] + \frac{\sigma^2}{v^2} e^{-\sigma\left(t - \frac{z}{v}\right)} \Gamma(0, -A) \right\}$$

$$= \frac{Q}{4\pi} \left\{ -e^{A} \Delta^{-3/2} \left[\cosh y_b \left(\tau \tanh y_b \cosh\eta_s - \tau \sinh\eta_s\right) + A\sqrt{\Delta} + \frac{\sigma \cosh y_b}{\tanh y_b} \Delta\right] + \frac{\sigma^2}{\tanh^2 y_b} e^{-\frac{\sigma}{\tanh y_b}\left(\tau \tanh y_b \cosh\eta_s - \tau \sinh\eta_s\right)} \Gamma(0, -A) \right\}.$$

$$= \frac{Q}{4\pi} \left\{ -e^{A} \Delta^{-3/2} \left[\tau \sinh(y_b - \eta_s) + A\sqrt{\Delta} + \frac{\sigma \cosh^2 y_b}{\sinh y_b} \Delta\right] + \frac{\sigma^2}{\tanh^2 y_b} e^{\frac{-\sigma\tau}{\tanh y_b}\left(\tanh y_b \cosh\eta_s - \sinh\eta_s\right)} \Gamma(0, -A) \right\}$$

Let; $Q = ce$

$$\therefore \quad e E_z = c\alpha \left\{ -e^{A} \Delta^{-3/2} \left[\tau \sinh(y_b - \eta_s) + A\sqrt{\Delta} + \frac{\sigma \cosh^2 y_b}{\sinh y_b} \Delta \right] \right.$$

$$\left. + \frac{\sigma^2}{\tanh^2 y_b} e^{\frac{-\sigma\tau}{\tanh y_b}(\tanh y_b \cosh\eta_s - \sinh\eta_s)} \Gamma(0, -A) \right\}$$

$\Gamma(0, -A)$ is the incomplete gamma function defined as;

$$\Gamma(a, z) = \int_z^\infty dt\, t^{a-1} \exp(-t).$$