ECE 2050 Autumn 2023 Homework 11
Due 5:00 pm, Friday Dec 1, 2023
Upload to Carmen as a single PDF document

BC:11.1 A periodic sampled time function has the formula

$$f[n] = \sum_{q=-\infty}^{\infty} \left(\frac{n-7-64\cdot q}{4}\right)^2 \operatorname{rect}\left(\frac{n-7-64\cdot q}{36}\right)$$

a.) What is the period, N, for this signal?

b.)Use Matlab to display one complete cycle of f[n] in the range of

$$0 \leq n \leq (N-1)$$

c.)Use Matlab to calculate the discrete Fourier transform, $F[k]$, in the range of

$$0 \leq k \leq (N-1)$$

Plot the magnitude and phase (in degrees) of $F[k]$ for this range of k.

d.) Use the datacursor tool the evaluate the magnitude and phase of $F[0]$, $F[17]$, $F[32]$, and $F[47]$

Attach your Matlab source code for credit.

BC:11.2 A sampled time signal has a formula

$$g[n] = 0.75\cos\left(\frac{2\pi n}{10}\right) + 0.25\cos\left(\frac{3\pi n}{8}\right)$$

a.) Use Matlab to find the associated spectrum, $G_a[k]$, using a discrete Fourier transform assuming $N = 16$. Plot the time function, $g_a[n]$ used as input vs n and plot the magnitude and phase (in degrees) of the spectrum, $G_a[k]$, vs k.

b.) Use Matlab to find the associated spectrum, $G_b[k]$, using a discrete Fourier transform assuming $N = 64$. Plot the time function, $g_b[n]$ used as input vs n and plot the magnitude and phase (in degrees) of the spectrum, $G_b[k]$, vs k.

c.) Use Matlab to find the associated spectrum, $G_c[k]$, using a discrete Fourier transform assuming $N = 256$. Plot the time function, $g_c[n]$ used as input vs n and plot the magnitude and phase (in degrees) of the spectrum, $G_c[k]$, vs k.

d.) Compare the spectra from **parts a–c**: what are the k values for the corresponding spectral peaks in each of the three spectra. Is there a relationship between these values and value of N used to calculate each spectrum? What is the relationship between the magnitudes of the corresponding peaks? Is there a relationship between these values and value of N used to calculate each spectrum?

Attach your Matlab source code for credit.

BC:11.3 The data for this problem can be found on the Carmen site under

`Homework` → `Homework 11 Data`

and should be saved as a file to your own directory where you use Matlab.

Once in your directory, the data can be loaded into Matlab using the commands:

```
load('prob11_3.mat')
```

This will generate a signal vector, `xs`, that has $N = 256$. This represents the sampled data of a periodic function with sampling period $\Delta t = 3.125$ ms.

a.) Use Matlab to (i) plot the time function x_s versus time in seconds, (ii) plot the *real part* of the spectrum X_s vs frequency in Hz, and (iii) the *imaginary part* of the spectrum vs frequency in Hz. From the spectral samples, determine if there is a dc component, and identify the frequency in Hz of any cosines or sines in the time signal given.

b.) Define a Matlab representation of filter function that has sampled spectral properties:

$$H_1[k] = u[k+120] - u[k-121] = \text{rect}\left(\frac{k}{240}\right)$$

This is an ideal low pass filter. Determine the range of frequencies that are nonzero for this filter. [*Note you need to use Modulo 256 counting to represent the negative k values of this filter. Also note that Matlab does not allow an index for a vector to be zero or a negative integer.*]

Define a product $Y_1[k] = X_s[k] \cdot H_1[k]$ and show plots of (i) your H_1 vs frequency in Hz, (ii) the *real part* of Y_1 in Hz, (iii) the *imaginary part* of Y_1 in Hz, and (iv) the filtered time signal, y_1 vs time in seconds. Compare the spectra of Y_1 and the original X_s and comment on any differences or similarities. Compare the time signals y_1 with the original x_s and comment on any differences or similarities.

c.) Define a Matlab representation of filter function that has sampled spectral properties:

$$H_2[k] = u[k+83] - u[k-84] = \text{rect}\left(\frac{k}{166}\right)$$

This is another ideal low pass filter. Determine the range of frequencies that are nonzero for this filter.

Define a product $Y_2[k] = X_s[k] \cdot H_2[k]$ and show plots of (i) your H_2 vs frequency in Hz, (ii) the *real part* of Y_2 in Hz, (iii) the *imaginary part* of Y_2 in Hz, and (iv) the filtered time signal, y_2 vs time in seconds. Compare the spectra of Y_2 and the original X_s and comment on any differences or similarities. Compare the time signals y_2 with the original x_s and comment on any differences or similarities.

d.) Define a Matlab representation of filter function that has sampled spectral properties:

$$H_3[k] = u[k+14] - u[k-15] = \text{rect}\left(\frac{k}{28}\right)$$

This is another ideal low pass filter. Determine the range of frequencies that are nonzero for this filter.

Define a product $Y_3[k] = X_s[k] \cdot H_3[k]$ and show plots of (i) your H_3 vs frequency in Hz, (ii) the *real part* of Y_3 in Hz, the (iii) *imaginary part* of Y_3 in Hz, and (iv) the filtered time signal, y_3 vs time in seconds. Compare the spectra of Y_3 and the original X_s and comment on any differences or similarities. Compare the time signals y_3 with the original x_s and comment on any differences or similarities.

Attach your Matlab source code for credit.