Last Class…

Find f(t)

$$\boldsymbol{F}(s) = \frac{4}{(s+1)(s+2)^2}$$

$s_1 = -1$ $s_2 = -2$ $s_3 = -2$

$$F(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$A = F(s)\cdot(s+1)\Big|_{s=-1}$$

$$= \frac{4}{(s+2)^2}\Big|_{s=-1}$$

$$= \frac{4}{1} = 4$$

$A = 4$

$$C = F(s)\cdot(s+2)^2\Big|_{s=-2}$$

$$= \frac{4}{s+1}$$

$$= \frac{4}{-1} = -4$$

$C = -4$

$$F(s) = \frac{4}{s+1} + \frac{B}{s+2} - \frac{4}{(s+2)^2} = \frac{4}{(s+1)(s+2)^2}$$

$$4(s+2)^2 + B(s+1)(s+2) - 4(s+1) = 4$$

$$4(s^2+4s+4) + B(s^2+2s+2) - 4s - 4 = 4$$

$$s^2\underbrace{(4+B)}_{=0} + s\underbrace{(16+2B-4)}_{=0} + \underbrace{16+2B-4}_{=4} = 0s^2 + 0s + 4$$

$4+B=0$

$\hookrightarrow B=-4$

$$F(s) = \frac{4}{s+1} + \frac{-4}{s+2} + \frac{-4}{(s+2)^2}$$

$$F(t) = \left[4e^{-t} - 4e^{-2t} - 4te^{-2t}\right] u(t)$$

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s-Domain Circuit Elements Model

After This Lecture…

- Learning Objectives:
  - Use the Laplace transform for circuit analysis.

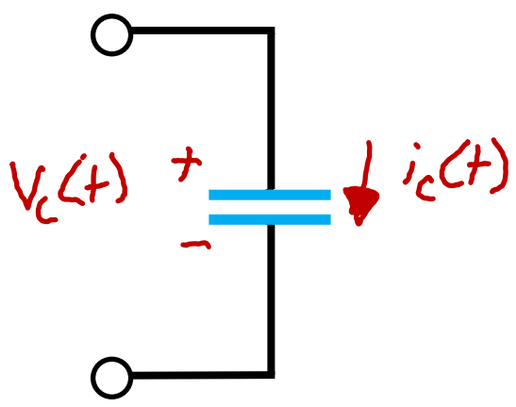
Phasor Domain

- Cosine
- move V_s / i_s -> complex number
- Find impedances
- Solve circuit
- Go back to time domain

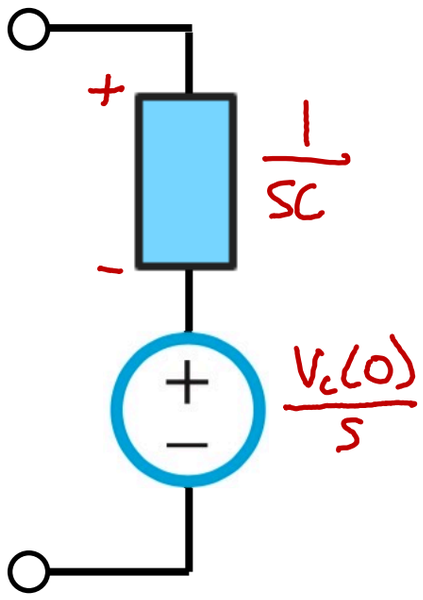
S-Domain

- Laplace transfer
  V_s / i_s -> S-Domain
- How capacitors, inductors, resistors look like in S-domain
- Solving circuit using DC analysis
- Partial fraction expansion
- Inverse laplace transfer to go back to time domain

Capacitor



Capacitor in S-Domain



Time Domain

$$i_c(t) = C \cdot \frac{dV_c(t)}{dt}$$

Frequency Domain

$$I_c(s) = C\left[sV_c(s) - v_c(0)\right]$$

Initial condition

$$I_c(s) + CV_c(0) = CsV_c(s)$$

$$V_c(s) = \underbrace{\frac{I_c(s)}{Cs}}_{\text{Voltage}} + \underbrace{\frac{CV_c(0)}{Cs}}_{\text{Voltage}}$$

$$= \frac{1}{sC} \cdot I_c(s)$$

$\frac{1}{sC} \rightarrow Z_c(s)$

Inductor

Time Domain

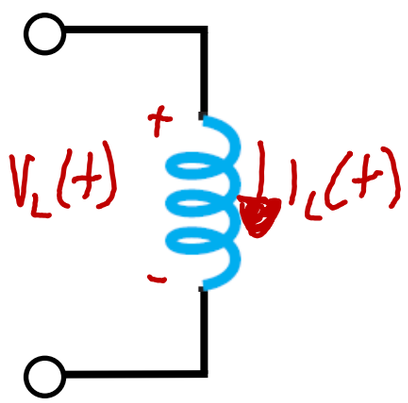
$$V_L(t) = L \cdot \frac{dI_L(t)}{dt}$$

Frequency Domain

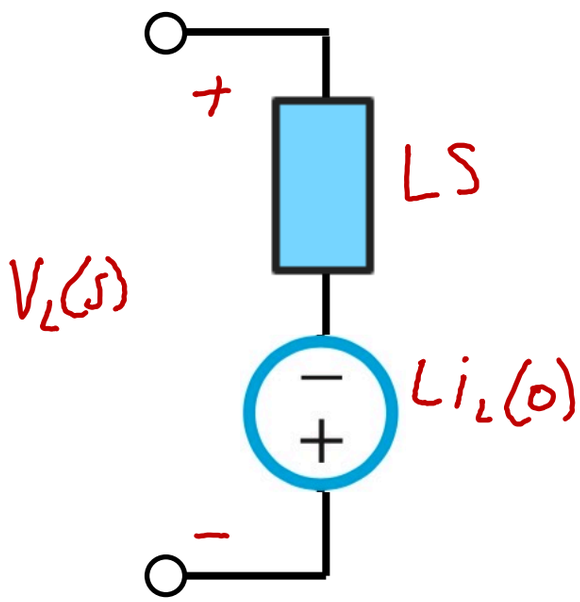
$$V_L(s) = L \cdot [sI_L(s) - i_L(0)]$$

$$V_L(s) = \underbrace{LsI_L(s)}_{\text{Voltage}} - \underbrace{L i_L(0)}_{\text{Voltage}}$$

$$\frac{Ls \cdot I_L(s)}{\rightarrow Z_L(s)}$$



Inductor in s-Domain

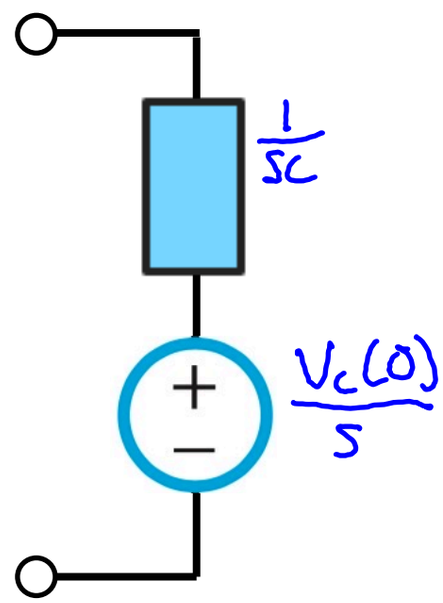


$Ls \leftarrow V_A - V_C$

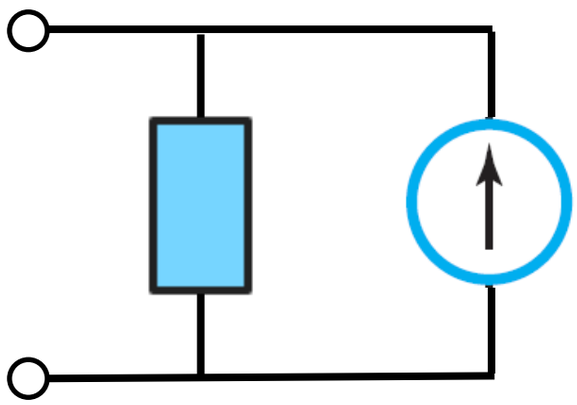
$Li_L(0) \leftarrow V_B - V_C$

$$V_A - \cancel{V_C} - (V_B - \cancel{V_C}) = V_A - V_B$$

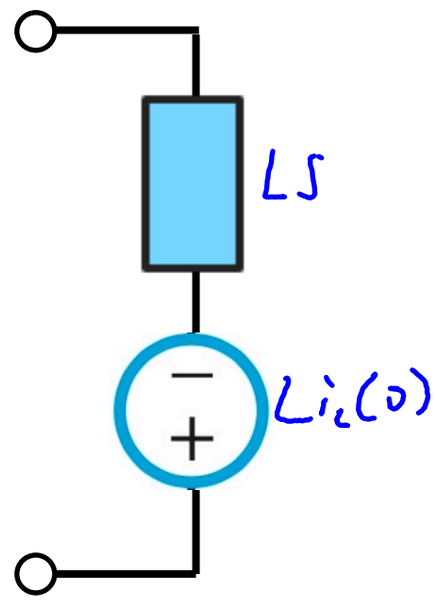
Capacitor



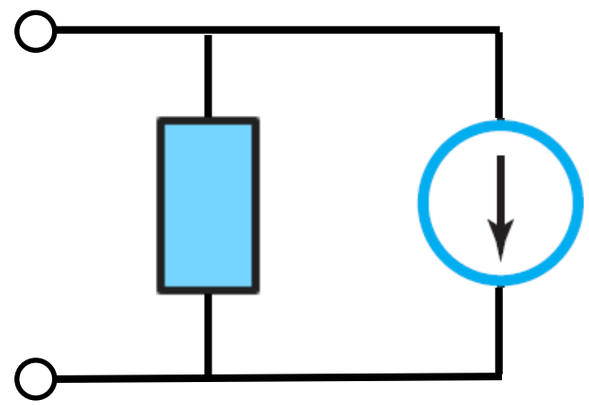
Same thing

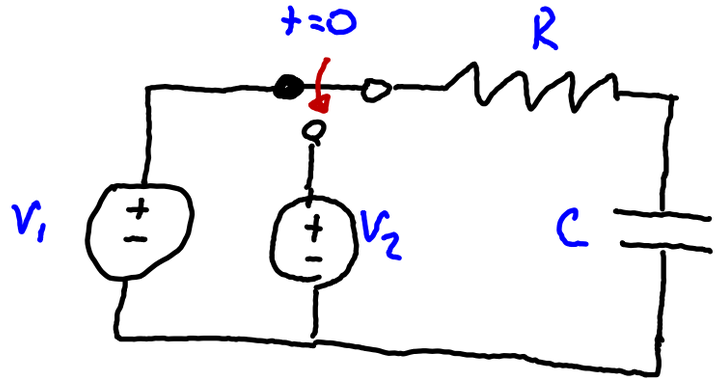


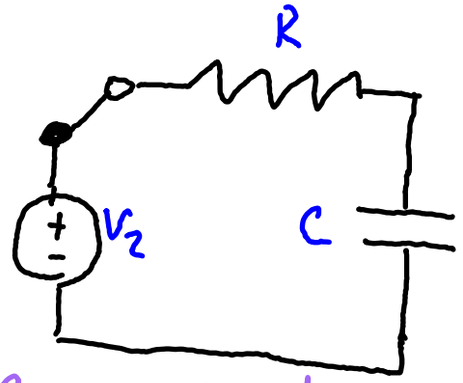
Inductor



Same thing

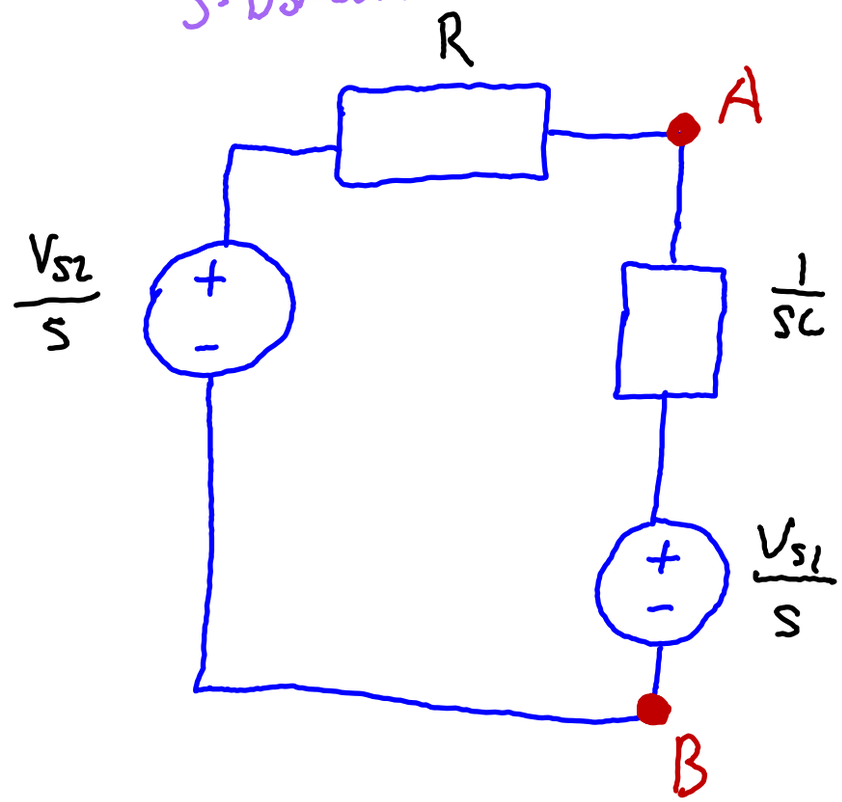






Can only do S-domain for $t>0$

S-Domain



$$V_c(0) = V_{S1}$$