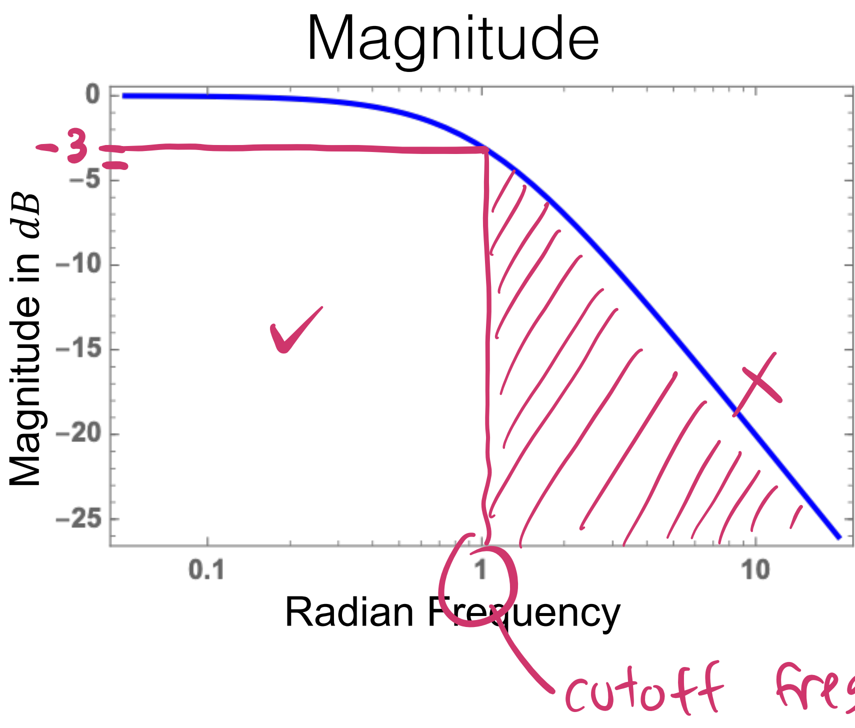


When is it filtering enough??

$$P_{out} < \frac{1}{2} P_{in}$$

Corner frequency or half-power frequency is the point where more signal is being filtered out than let through.

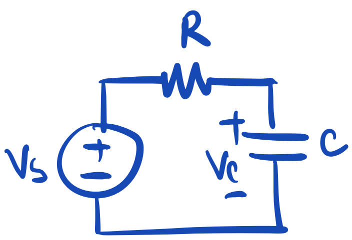
$$V_{out}^2 < \frac{1}{2} V_{in}^2$$

$$\frac{V_{out}^2}{V_{in}^2} < \frac{1}{2}$$

$$\frac{V_{out}}{V_{in}} < \frac{1}{\sqrt{2}}$$

$$\left.\frac{V_{out}}{V_{in}}\right|_{dB} < 20\log_{10}\left(\frac{1}{\sqrt{2}}\right) < -3dB$$

LPF



$$\frac{V_c}{V_s} = \frac{1}{1 + j\omega CR}$$

$$\left|\frac{V_c}{V_s}\right| = \frac{1}{\sqrt{1 + (\omega CR)^2}} < \frac{1}{\sqrt{2}}$$

$$\sqrt{2} < \sqrt{1 + (\omega CR)^2}$$

$$2 < 1 + (\omega CR)^2$$

$$1 < (\omega CR)^2$$

$$1 < \omega CR$$

all 1st order RC circuits ← $\boxed{\frac{1}{CR} < \omega}$

$$\omega_c = \frac{1}{CR}$$

Poles and Zeros

Poles: are values that make your denominator $=0$

$$\frac{1}{1+x} \rightarrow x+1=0 \Rightarrow x=-1 \leftarrow \text{pole}$$

Zeros. are values that make your numerator $=0$

$$\frac{x}{1+x} \rightarrow x=0 \leftarrow \text{zero}$$

$$\frac{1}{x^2+2x+4}$$

zeros = none

$x^2+2x+4=0$

quadratic equation

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

HPF $\frac{JWCR}{1+JWCR}$ → zeros $\cancel{JW}CR = 0$ (x)

$x = 0$ ← 1 zero

$JW = x$

poles $1+\cancel{JW}CR = 0$ (x)

$xCR = -1$

$x = -\frac{1}{CR}$

LPF $\frac{1}{1+JWCR}$

zeros = none

poles ⇒ $1+\cancel{jw}CR = 0$ (x)

$xCR = -1$

$x = -\frac{1}{CR}$ ← cut off frequency.

$1 + jw/w_c$

← cutt off frequency.

$\cancel{1} + JWCR = \cancel{1} + JW/w_c$

$\cancel{JW}CR = \cancel{JW}/w_c$

$CR = \frac{1}{w_c} \Rightarrow w_c = \frac{1}{CR}$

THE OHIO STATE UNIVERSITY

COLLEGE OF ENGINEERING

Bode Plot

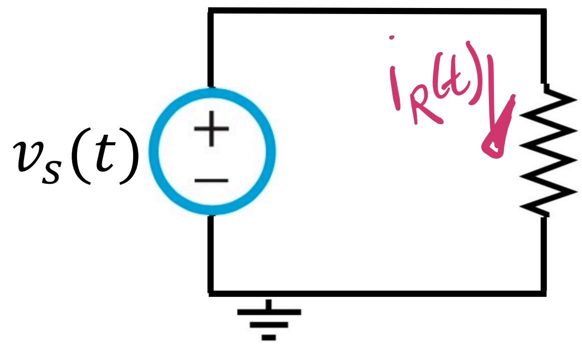
This Lecture…

- Learning Objectives:
    - Generate magnitude frequency plots for high and low pass-filters.

Sketch a Bode Plot

Simples Transfer Function (constant):

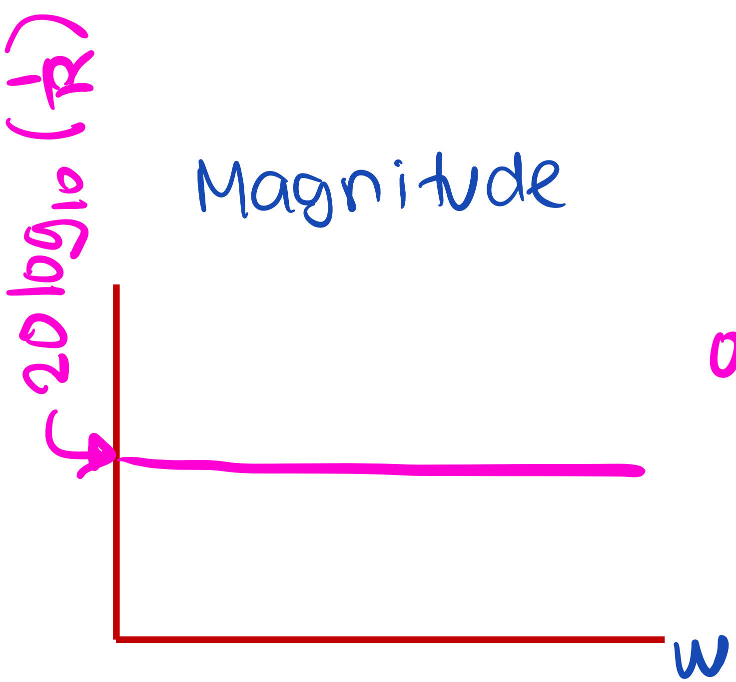
- $H(s) = K = \frac{1}{R} \angle 0^\circ$



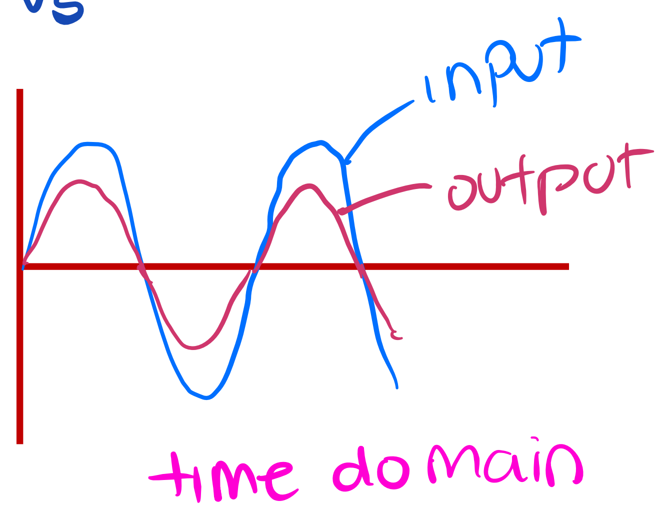
$$I_R = \frac{V_S}{R}$$

$$\frac{I_R}{V_S} = \frac{1}{R}$$

Magnitude

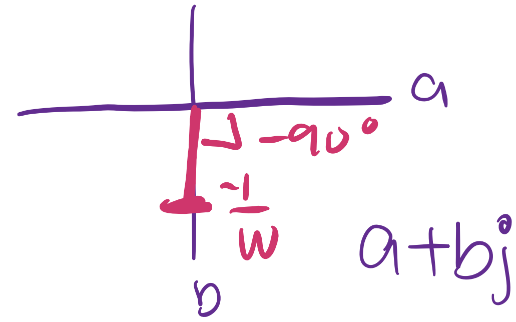


Phase.



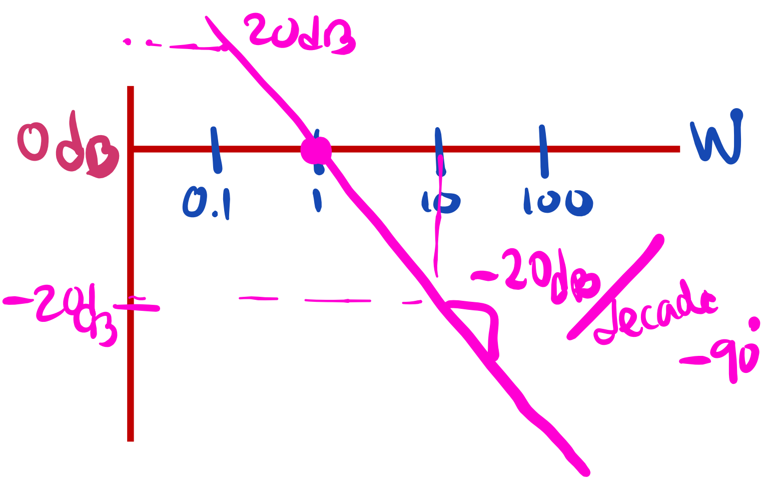
Sketch a Bode Plot

Pole @ the origin:

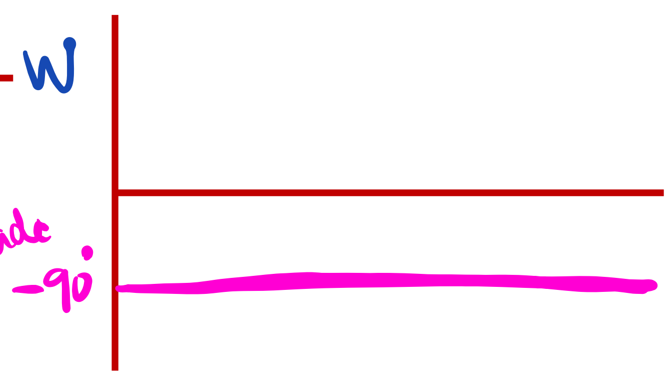


- $H(s) = \frac{1}{j\omega} = \frac{-j}{w} = \frac{1}{w} \angle -90^\circ$

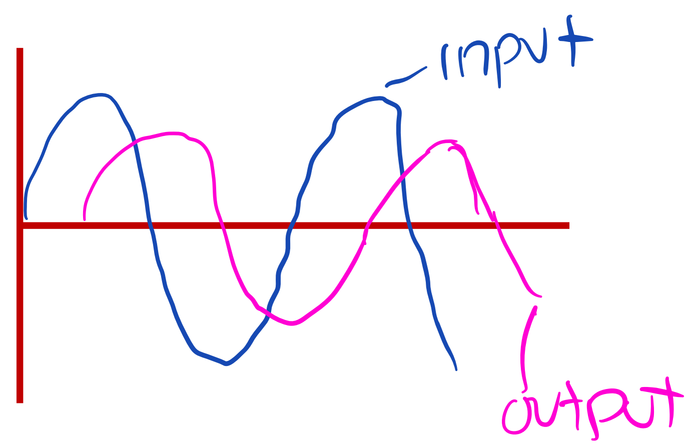
$$20\log_{10}\left(\frac{1}{w}\right) \begin{cases} w = 0.1 \rightarrow 20\log_{10}(10) = 20\text{dB} \\ w = 1 \rightarrow 20\log_{10}(1) = 0\text{dB} \\ w = 10 \rightarrow 20\log_{10}(0.1) = -20\text{dB} \end{cases}$$



Magnitude



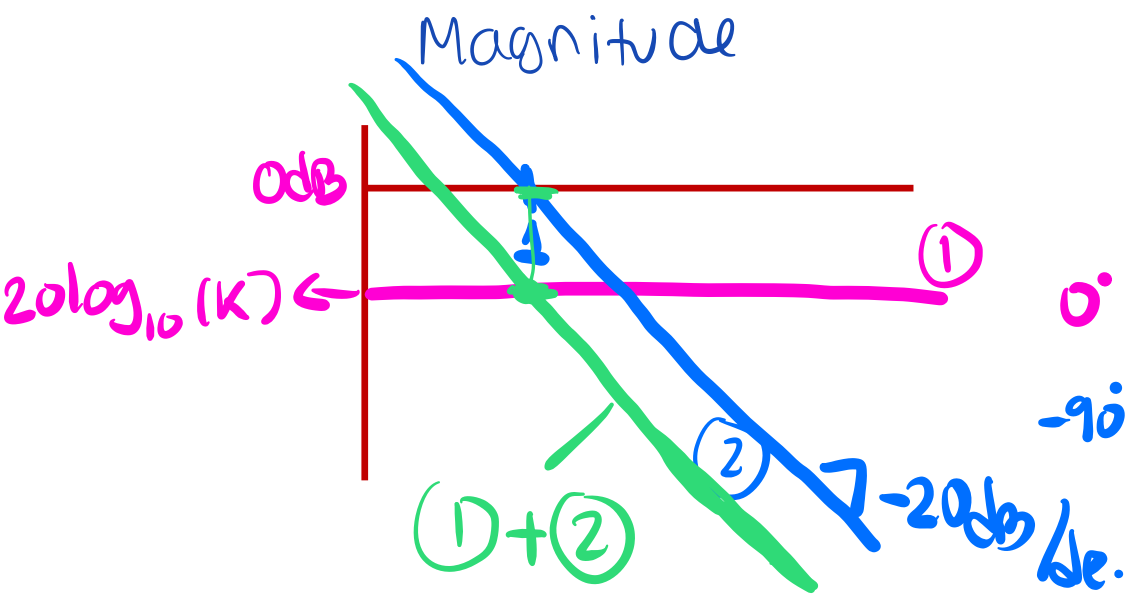
Phase.

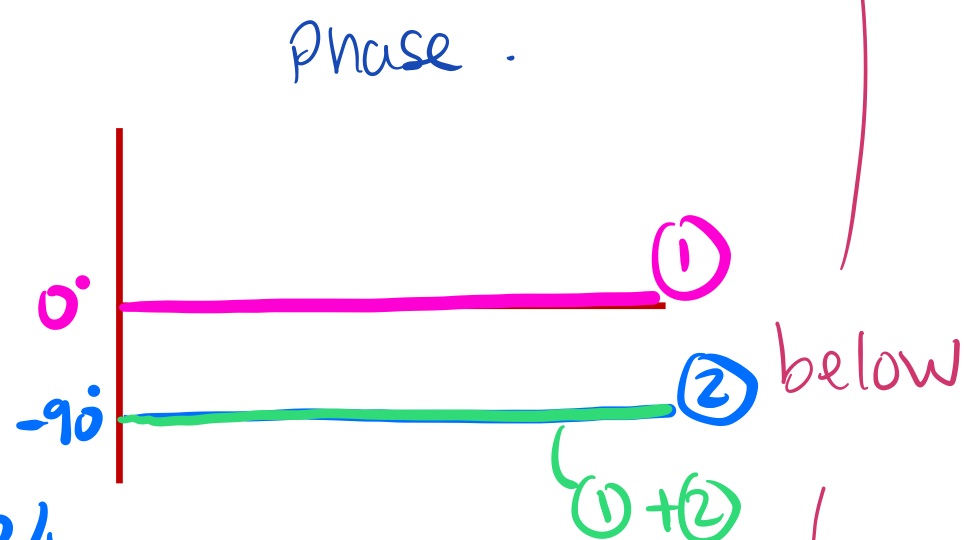


Sketch a Bode Plot

Pole @ the origin:

$$H(s) = \frac{K}{j\omega} = \underbrace{K}_{①} \cdot \underbrace{\frac{1}{jw}}_{②} \qquad G(s) = \frac{K}{(j\omega)^2} = K \cdot \frac{1}{jw} \cdot \frac{1}{jw}$$





$$z_1 = A \angle \alpha \qquad z_2 = B \angle \beta$$

$$P(s) = z_1 z_2 = \underbrace{AB}_{\text{magnitude}} \angle \underbrace{\alpha + \beta}_{\text{phase}}$$

$$20 \log_{10}(A \cdot B) = 20\log_{10}(A) + 20\log_{10}(B)$$