Gage Farmer

Homework 8 - Math 2568 (Autumn 2022)

Prof. Cueto

Due date: Friday November 4, 2022 (in class).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§5.3	1, 3, 4, 5, 9, 11, 13, 19, 23, 25, 27, 28, 29, 31	5, 13, 19, 23, 27
§5.4	1, 4, 5, 9, 10, 13, 15, 19, 23, 24, 25, 26, 27, 28	4, 9, 23, 25, 27
§5.7	1, 2, 3, 5, 7, 9, 10, 11	1, 2, 3, 5, 7, 9, 11

Section 5.3

5) $W = \{ p(x) \text{ in } P_2 : p(0) + p(2) = 0 \}$

$\theta(x) = 0x^2 + 0x + 0 \rightarrow 0(0^2) + 0(0) + 0 = 0$

S1 ✓

$(p+q)(0) + (p+q)(2) = 0$

S2 ✓

$p(0) + p(2) = (a_2(0)^2 + a_1(0) + a_0) + (a_2(2)^2 + a_1(2) + a_0) = 0$

S3 ✓ All checks passed

13) $F = \{f(x) \text{ in } C^2[-1,1] : f''(0) = 0\}$

$$(f+g)''(0) = f''(0) + g''(0) = 0$$

$$(cf)''(0) = cf''(0) = 0 \qquad (f+g)^{*}(0) = f^{*}(0) + g^{*}(0) = 0$$

F is a subspace of $C^2[-1,1]$, satisfying S3

19) $A = \begin{bmatrix} -2 & -4 \\ 1 & 0 \end{bmatrix} = a_1 \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + a_3 \begin{bmatrix} -1 & -3 \\ 4 & -4 \end{bmatrix} + a_4 \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -3 & 2 \\ 2 & 0 & 4 & -1 \\ 1 & 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ 1 \\ 0 \end{bmatrix} \longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & 1 & -3 & 2 & -4 \\ 0 & -2 & 6 & -3 & 5 \\ 0 & 1 & -3 & 0 & 2 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & 1 & -3 & 2 & -4 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & -2 & 6 \end{array}\right]$$

$$\to \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -3 & 0 & -4 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 6 \end{array}\right] \qquad \begin{aligned} a_1 + 2a_3 &= -1 \\ a_2 - 3a_3 &= -4 \\ a_4 &= -3 \end{aligned} \qquad \begin{aligned} a_1 &= -1 - 2x \\ a_2 &= 2 + 3x \\ a_3 &= x \\ a_4 &= -3 \end{aligned}$$

$A = (1-2x)B_1 + (2+3x)B_2 + (x)B_3 + (-3)B_4,\ x \in \mathbb{R}$

23) $p(1) = p(-1) \quad p(2) = p(-2) \quad q(1) = q(-1) \quad q(2) = q(-2)$

$(p+q)(1) = (p+q)(-2)$

$(ap)(1) = (ap)(-2)$

$2a_1 + 8a_3 = -2a_1 - 8a_3$

W is subspace of P

Spanning Set of W is $\{1, x^2\}$

27) $\theta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $tr(\theta) = 0+0+0 = 0$ S1 ✓

$tr(A+B) = (a_{11}+b_{11}) + (a_{22}+b_{22}) + \dots = 0+0=0$ S2 ✓

$B_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $B_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $E_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$E_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $E_{23} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $E_{31} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $E_{32} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$W = Sp\{B_1, B_2, E_{12}, E_{13}, E_{21}, E_{23}, E_{31}, E_{32}\}$

Section 5.4

4) $A = \begin{bmatrix} a & a-c \\ c & 2a+c \end{bmatrix} \longrightarrow B_1 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ $B_2 = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

$d_1 \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + d_2 \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} d_1 & d_1 - d_2 \\ d_2 & 2d_1 + d_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Basis for W is
$\{B_1, B_2\} = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} \right\}$

9) $p(0) = 0$ $p'(1) = 0$ $p''(-1) = 0$

$2a_2 + 6a_3(-1) + 12a_4(-1)^2 = 2a_2 - 6a_3 + 12a_4 = 0$

$p_1(x) = (x^3 + 3x^2 - 9x)$ $p_2(x) = (x^4 - 6x^2 + 8x)$

Basis of V $(x^3 + 3x^2 - 9x), (x^4 - 6x^2 + 8x)$

23) $a+2b+3c+d=0$
$2a+5b+7c+d=0$
$a+c+3d=0$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 2 & 5 & 7 & 1 & 0 \\ 1 & 0 & 1 & 3 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$a+c+3d=0$
$b+c-d=0$ $\rightarrow$ $c=\dfrac{-a-3b}{4}$ $d=\dfrac{b-a}{4}$

$a+2b=0$
$2a+5b=0$
$a=0$

Subset of $S=\{p_1(x), p_2(x)\}$ is basis for $Sp(S)$

25) $[A_1]_B=\begin{bmatrix}1\\2\\-1\\3\end{bmatrix}$ $[A_2]_B=\begin{bmatrix}-2\\1\\2\\-1\end{bmatrix}$ $[A_3]_B=\begin{bmatrix}-1\\-1\\1\\-3\end{bmatrix}$ $[A_4]_B=\begin{bmatrix}-2\\2\\2\\0\end{bmatrix}$

$$[V|\Theta]=\begin{bmatrix}1 & 0 & 0 & \frac{2}{5} & 0\\ 0 & 1 & 0 & \frac{6}{5} & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$x_1=-\frac{2}{5}x_4$ $x_2=-\frac{6}{5}x_4$ $x_3=0$ $x_4=1$

$\{A_1, A_2, A_3\}$ is basis for $S_p(S)$

27) $-a_1+a_3=1$ $a_1+a_2+2a_3=1$ $2a_1+3a_2+8a_3=1$

$a_1=-4$ $a_2=11$ $a_3=-3$ $[p_x]_Q=\begin{bmatrix}-4\\11\\-3\end{bmatrix}$

Section 5.7

1) $T(A+B)=\det(A)+\det(B)+(pd+as-qc-br)$

T is not a linear transformation as $T(A+B)\neq \det(A)+\det(B)$

2) $T(A) = a + 2b - c + d$ $\quad T(B) = p + 2q - r + s$

$T(A+B) = T(A) + T(B)$

$T(kA) = kT(A)$ $\quad$ T is linear transformation

3) $T(A) = a + d$ $\quad T(B) = p + s$

$T(A+B) = T(A) + T(B)$ $\quad T(kA) = kT(A)$

T is linear transformation

5) $T(f) = f'(0)$ $\quad T(g) = g'(0)$

$T(f+g) = T(f) + T(g)$ $\quad T(kf) = kT(f)$

T is linear transformation

7) $T[p(x)] = (a_0+1) + (a_1+1)x + (a_2+1)x^2$

$T[q(x)] = (b_0+1) + (b_1+1)x + (b_2+1)x^2$ $\quad$ T is not linear transformation

$T[p(x) + q(x)] \neq T[p(x)] + T[q(x)]$

9) a) $p(x) = 3 - 2(x) + 4(x^2)$ $\quad T[p(x)] = 11 + x^2 + 6x^3$

b) $T(a_0 + a_1x + a_2x^2) = T[a_0(1) + a_1(x) + a_2(x^2)]$

$T(a_0 + a_1x + a_2x^2) = (a_0 + 2a_2) + (a_0 + a_1)x^2 + (a_2 - a_1)x^3$

11) $A = -2\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} + 2\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} + 3\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix} + 4\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$

$A = -2E_{11} + 2E_{12} + 3E_{21} + 4E_{22}$

$T(A) = 8 + 14x - 9x^2$