Since $\alpha \lesssim 1$, β can be large!

so $i_C \gg i_B \longrightarrow$ amplification

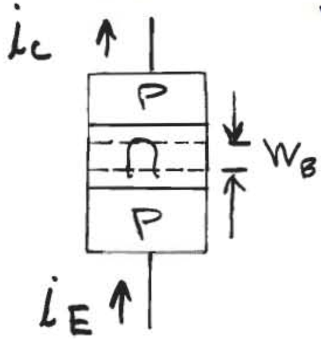
Why large gain? $B \sim 1$

$\gamma \sim 1$

whereas i_E and i_C controlled by

$$qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)$$

i_B determines i_C from lifetimes τ and charge neutrality.



Transit time of hole from emitter to collector $= \tau_t$

Hole lifetime in base $= \tau_p$

average excess hole spends τ_t in base

$$i_C = \frac{Q_p}{\tau_t}$$

average excess electron spends τ_p in base

$$i_B = \frac{Q_n}{\tau_p}$$

(assuming negligible e^- injection into emitter, $\gamma = 1$)

But so base stays neutral

so $\dfrac{i_C}{i_B} =$

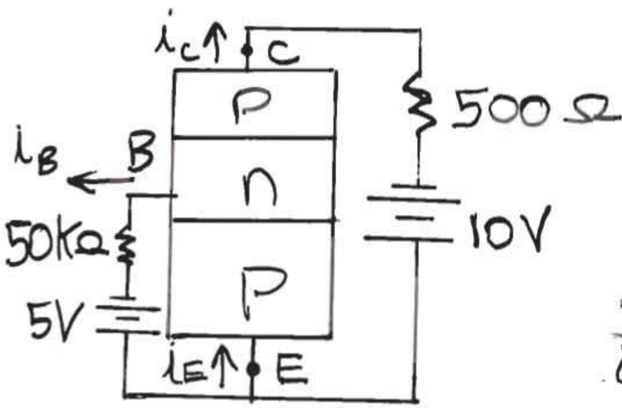
now put real numbers for τ_p and τ_t into BJT circuit:

$\tau_t = 0.1\,\mu s$ $\qquad$ $\tau_p = 10\,\mu s$

Can get $\tau_t \ll \tau_p$ by design:

Make $\qquad\qquad$ (Recall $L_p = \sqrt{D_p \tau_p}$

so recombination low in base.



$$i_B = \frac{5V}{50K\Omega} = 0.1\ mA$$

(neglecting forward biased v_{BE})

$$\frac{i_c}{i_B} = \frac{\tau_p}{\tau_t} = \frac{10\,\mu s}{0.1\,\mu s} = 100$$

note EB forward-biased
CB reverse-biased

$\hookrightarrow i_c = 10\ mA$

Common-emitter circuit: E common to both base and collector circuits

i_c determined by β and i_B,
<u>not</u> by collector circuit R and V

- If pnp were a short, $i_c = \dfrac{10V}{500\Omega} = 20mA$;

but 5V across reverse-biased BC junction
so $10-5 = \dfrac{5V}{500\Omega} = 10mA$

If i_B had an a-c component, i_c would too, <u>but</u> a<u>mplified</u> <u>by</u> <u>100</u>.

Gain comes from small base current forcing EB junction to inject large hole current (Since $\tau_p \gg \tau_t$)

Calculation of minority carrier distributions and terminal currents

Step 1) Find excess hole distribution in base

step 2) Find I_{Ep} and I_c from gradient of hole distribution

step 3) Find I_B from kirchoff's Law

Simplifying assumptions:

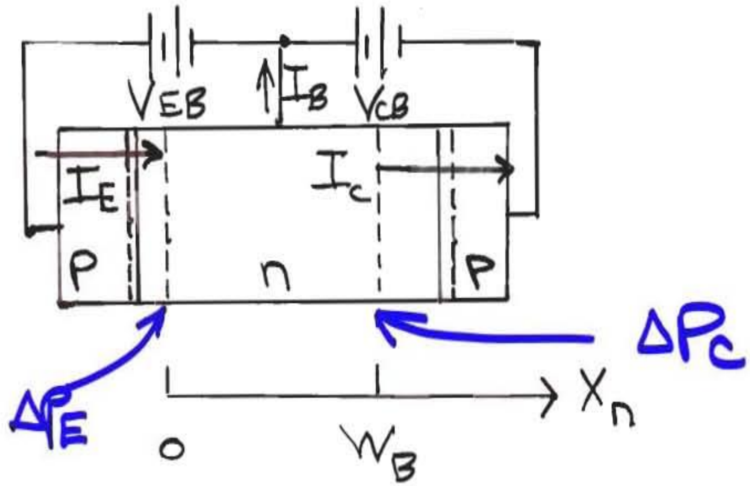
a) Neglect drift — only diffusion in base

b) Neglect electron emission current: $\gamma = 1$
(all holes)

c) Neglect collector saturation current

d) Uniform cross section A: 1-D flow

e) All currents and voltages steady state



Find $\delta p(x_n)$

Use $\Delta P_E =$

and $\Delta P_C =$

If emitter strongly forward-biased,
$V_{EB} \gg kT/q$

If collector strongly reverse-biased,
$V_{CB} \ll 0$

Then $\Delta p_E \simeq$

$\Delta p_C \simeq$

(equilibrium carrier concentration depleted)

Solve diffusion equation

S&B Eqs. 4-34

$$\frac{d^2 \delta p(x_n)}{dx^2} = \frac{\delta p(x_n)}{L_p^2}$$

Again, general solution is:

$$\delta p(x_n) = C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}$$

$L_p =$

Now boundary conditions can't eliminate a constant since $\delta p(x_n)$ doesn't go to zero for large x_n.

In fact, W_b short versus L_p

→ most carriers reach collector

→ x_n/L_p is small.

Solve for C_1 and C_2.

$$C_1 + C_2 = \Delta P_E$$

$$C_1 e^{W_b/L_p} + C_2 e^{-W_b/L_p} = \Delta P_c$$

(see next page for details)

$$C_1 = \frac{\Delta P_c - \Delta P_E e^{-W_b/L_p}}{\left(e^{W_b/L_p} - e^{-W_b/L_p}\right)}$$

$$C_2 = \frac{\Delta P_E e^{W_b/L_p} - \Delta P_c}{\left(e^{W_b/L_p} - e^{-W_b/L_p}\right)}$$

$$C_1 + C_2 = \Delta P_E$$

$$C_1 e^{W_b/L_p} + C_2 e^{-W_b/L_p} = \Delta P_c$$

$$C_1 e^{W_b/L_p} + (\Delta P_E - C_1) e^{-W_b/L_p} = \Delta P_c$$

$$C_1 (e^{W_b/L_p} - e^{-W_b/L_p}) = \Delta P_c - \Delta P_E e^{-W_b/L_p}$$

$$C_1 = \frac{\Delta P_c - \Delta P_E e^{-W_b/L_p}}{(e^{W_b/L_p} - e^{-W_b/L_p})}$$

$$C_2 = \Delta P_E - C_1$$

$$= \frac{\Delta P_E (e^{W_b/L_p} - e^{-W_b/L_p}) - \Delta P_c + \Delta P_E e^{-W_b/L_p}}{(e^{W_b/L_p} - e^{-W_b/L_p})}$$

$$= \frac{\Delta P_E e^{W_b/L_p} - \Delta P_c}{(e^{W_b/L_p} - e^{-W_b/L_p})}$$

$$= \frac{\Delta P_E e^{W_b/L_p} - \Delta P_c}{\left(e^{W_b/L_p} - e^{-W_b/L_p}\right)}$$

For strong reverse bias, $\Delta P_c \simeq -P_n$

and assume $\Delta P_c \simeq 0$ compared to ΔP_E

Then $C_1 = -\dfrac{\Delta P_E e^{-W_b/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}}$ and $C_2 = \dfrac{\Delta P_E e^{W_b/L_p}}{\left(e^{W_b/L_p} - e^{-W_b/L_p}\right)}$

so $\delta p(x_n) = \Delta P_E \dfrac{\left(e^{(W_b - x_n)/L_p} - e^{-(W_b - x_n)/L_p}\right)}{e^{W_b/L_p} - e^{-W_b/L_p}}$

(for $\Delta P_c \simeq 0$)

Put C_1 and C_2 into

$$\delta p(x_n) = C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}$$

One simplified (but very useful) solution:

strongly reversed biased collector ( )

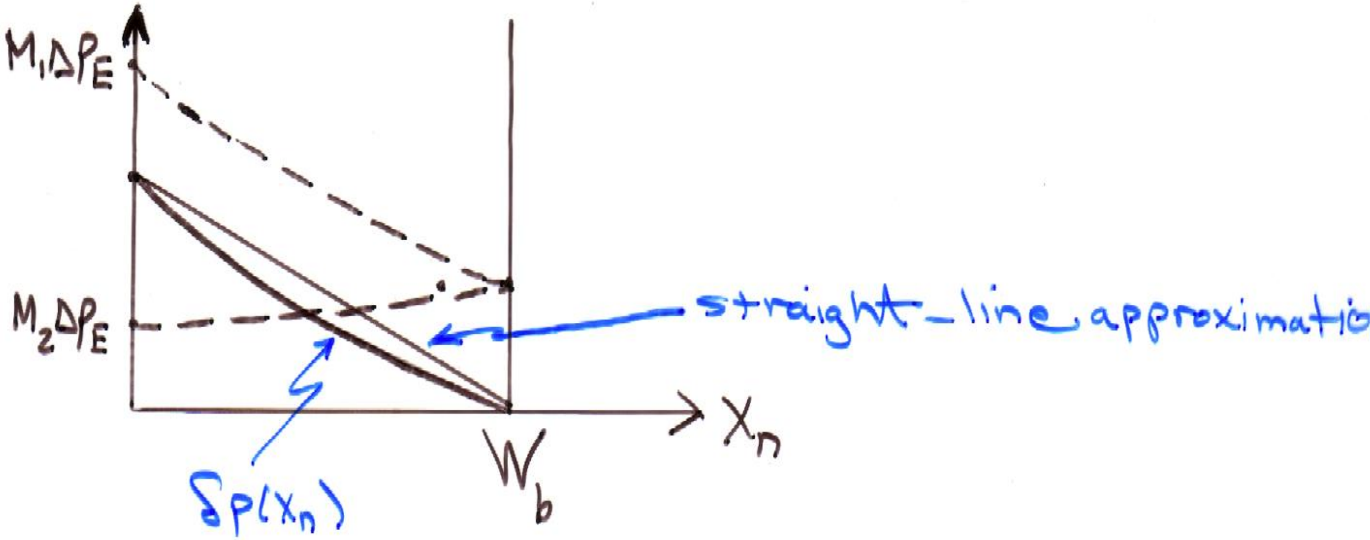
and ΔP_E ΔP_c (ΔP_c)

Then C_1 and C_2 simplify and

$$\delta p(x_n) = M_1 \Delta P_E e^{-x_n/L_p} + M_2 \Delta P_E e^{x_n/L_p}$$

where $M_1 = e^{W_b/L_p}$, and $M_2 = e^{-W_b/L_p}$

Graphically,



will see deviation from linearity due to base recombination.

We've got $\delta p(x_n)$ now.

Step 2. Find emitter and collector currents

$$I_p(x_n) = \int J_p \, dA =$$

Eq. 4-22 b for holes

Use C_1 and C_2 to make calculation easier. Substitute later.

$$\delta p(x_n) = C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}$$

$$\frac{d\,\delta p(x_n)}{dx_n} = \frac{C_1}{L_p} e^{x_n/L_p} - \frac{C_2}{L_p} e^{-x_n/L_p}$$

$$I_p(x_n=0) = qA\frac{D_p}{L_p}(C_2 - C_1) \quad \text{at } E$$

$$I_p(x_n=W_b) = qA\frac{D_p}{L_p}\left(C_2 e^{-W_b/L_p} - C_1 e^{W_b/L_p}\right) \quad \text{at } C$$

Now substitute C_1 and C_2 back in.

(see next page for details)

$$I_p(x_n=0) = I_{Ep} = q\frac{AD_p}{L_p}\left(\Delta p_E \operatorname{ctnh}\frac{W_b}{L_p} - \Delta p_c \operatorname{csch}\frac{W_b}{L_p}\right)$$

$$I_p(x_n=W_b) = I_c = q\frac{AD_p}{L_p}\left(\Delta p_E \operatorname{csch}\frac{W_b}{L_p} - \Delta p_c \operatorname{ctnh}\frac{W_b}{L_p}\right)$$

Now get I_B from I_{Ep} and I_c. = Step 3

$$I_p(X_n=0) = I_{Ep} = \frac{qAD_p}{L_p}(C_2 - C_1)$$

$$= \frac{qAD_p}{L_p}\left(\frac{\Delta P_E e^{W_b/L_p} - \Delta P_c - \Delta P_c + \Delta P_E e^{-W_b/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}}\right)$$

$$= \frac{qAD_p}{L_p}\left(\frac{\Delta P_E (e^{W_b/L_p} + e^{-W_b/L_p}) - 2\Delta P_c}{e^{W_b/L_p} - e^{-W_b/L_p}}\right)$$

Easier to write as hyperbolic functions

$$\sinh(W_b/L_p) = \frac{e^{W_b/L_p} - e^{-W_b/L_p}}{2}$$

$$\cosh(W_b/L_p) = \frac{e^{W_b/L_p} + e^{-W_b/L_p}}{2}$$

$$\tanh(W_b/L_p) = \frac{\sinh(\ )}{\cosh(\ )}$$

$$\mathrm{csch}(\ ) = \frac{1}{\sinh(\ )}$$

$$\mathrm{sech}(\ ) = \frac{1}{\cosh(\ )}$$

$$\mathrm{ctnh}(\ ) = \frac{1}{\tanh(\ )}$$

$$\underline{\underline{I_{Ep} = \frac{qAD_p}{L_p}\left(\Delta P_E \,\mathrm{ctnh}\left(\frac{W_b}{L_p}\right) - \Delta P_c \,\mathrm{csch}\left(\frac{W_b}{L_p}\right)\right)}}$$

$$I_p(x_n = W_b) = I_{Cp} = q\frac{AD_p}{L_p}\left(C_2 e^{-W_b/L_p} - C_1 e^{+W_b/L_p}\right)$$

$$= q\frac{AD_p}{L_p}\left(\frac{\Delta P_E e^{W_b/L_p} \cdot e^{-W_b/L_p} - \Delta P_c e^{W_b/L_p} - \Delta P_c e^{W_b/L_p} + \Delta P_E e^{-W_b/L_p} \cdot e^{\frac{W_b}{L_p}}}{e^{W_b/L_p} - e^{-W_b/L_p}}\right)$$

$$= q\frac{AD_p}{L_p}\left(\frac{2\Delta P_E - \Delta P_c\left(e^{-W_b/L_p} + e^{W_b/L_p}\right)}{e^{W_b/L_p} - e^{-W_b/L_p}}\right)$$

$$= q\frac{AD_p}{L_p}\left(\Delta P_E \operatorname{csch}\left(\frac{W_b}{L_p}\right) - \Delta P_c \operatorname{ctnh}\left(\frac{W_b}{L_p}\right)\right)$$

Now get I_B from I_{Ep} and I_{Cp} : <u>Step 3</u>

$$I_B = I_E - I_C$$

$$I_B = q\frac{AD_p}{L_p}\left[(\Delta P_E + \Delta P_c)\tanh\frac{W_b}{2L_p})\right]$$

see next page for details

$$I_B = I_E - I_C$$

$$= qA\frac{D_p}{L_p}\left[(\Delta P_E + \Delta P_C)\left(\operatorname{ctnh}\left(\frac{W_b}{L_p}\right) - \operatorname{csch}\left(\frac{W_b}{L_p}\right)\right)\right]$$

$$I_B = qA\frac{D_p}{L_p}\left[(\Delta P_E + \Delta P_C)\left(\frac{e^{\frac{W_b}{2L_p}} - e^{-\frac{W_b}{2L_p}}}{e^{W_b/2L_p} + e^{-W_b/2L_p}}\right)\right]$$

$$\operatorname{ctnh}(x) = \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}} \; ; \quad \operatorname{csch}(x) = \frac{2}{e^{x}-e^{-x}}$$

$$\operatorname{ctnh}(x) - \operatorname{csch}(x) = \frac{e^{x}+e^{-x}-2}{e^{x}-e^{-x}} = \frac{\left(e^{\frac{x}{2}}-e^{-\frac{x}{2}}\right)^2}{\left(e^{x/2}-e^{-x/2}\right)\left(e^{x/2}+e^{-x/2}\right)}$$