Homework 8

8.1 $f(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{j(\frac{2\pi}{T_0})kt}$

a) Fundamental Frequency $= f_0 = \frac{1}{T_0} = \frac{1}{125ns} =$ **8 MHz**

b) $f(t) = -15 + 12.5\cos(16\pi - 13°) + 12.5\cos(16\pi + 13°)$
$+ 3\cos(16\pi + 7°) + 3\cos(16\pi - 7°)$

8.2 $g(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{j(\frac{2\pi}{T_0})kt}$

$$\alpha_k = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} g(t)\, e^{-j(\frac{2\pi}{T_0})kt}\, dt$$

$$\alpha_0 = \frac{1}{100}\left[\int_0^{50} \sin(20\pi t)\, dt + \int_{50}^{100} 0\, dt\right]$$

$$\alpha_0 = \frac{1}{100}[0] = 0$$

$$\alpha_k = \frac{1}{100}\left[\int_0^{50} \sin(20\pi t)\, e^{j(\frac{2\pi}{100})kt}\, dt\right]$$

$\alpha_0 = 0$

$\alpha_1 = 0.032 \angle -1.54\times10^{-7}{}^{\circ}$

$\alpha_{-1} = 0.032 \angle 1.54\times10^{-7}{}^{\circ}$

$\alpha_2 = 5.991 \angle -147.82°$

$\alpha_{-2} = 5.991 \angle 147.82°$

HW8-2

8.3 a) $f(t) = \sum_{k=-\infty}^{\infty} (\alpha_k - 0.05)\, e^{j\left(\frac{2\pi}{T_0}\right)kt}$

$$\boxed{\alpha_k = -\beta_k = \alpha_k - 0.05}$$

b) $\frac{2\pi}{T_0} = \Delta\omega \qquad \Delta\omega \cdot k = \omega$

$$(\beta_k \cdot T_0) = \int_{T_0} f(t)\, e^{j\omega t}\, dt$$

$$\lim_{T_0 \to \infty} = \int_{-\infty}^{\infty} f(t)\, e^{j\omega t}\, dt = F(j\omega)$$

$$\boxed{\beta_k = \frac{1}{100}\left[\int_{50}^{100} -\sin(20\pi t)\, e^{-j\left(\frac{2\pi}{T_0}\right)kt}\, dt\right]}$$

$$f(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \underbrace{((\alpha_k - 0.05)\cdot T_0)}_{F(j\omega)} e^{j\Delta\omega kt} \underbrace{\frac{2\pi}{T_0}}_{\Delta\omega}$$

c).

$$\beta_0 = 0$$

$$\beta_1 = -0.018 \measuredangle -1.541 \times 10^{-7}{}^{\circ}$$
$$\beta_{-1} = -0.018 \measuredangle 1.54 \times 10^{-7}{}^{\circ}$$
$$\beta_2 = 5.941 \measuredangle -147.82^{\circ}$$
$$\beta_{-2} = 5.941 \measuredangle 147.82^{\circ}$$

d) $x(t) = g(t) + f(t) = \sum_{k=-\infty}^{\infty} \sigma_k\, e^{j\left(\frac{2\pi}{T_0}\right)kt}$

$$\sigma_k = \alpha_k + \beta_k = 2\alpha_k - 0.05$$

$$\boxed{\sigma_k = \frac{1}{50}\left[\int_0^{50} \sin(20\pi t)\, e^{-j\left(\frac{2\pi}{T_0}\right)kt}\, dt\right] - 0.05}$$

HW8-3

8.4 a) $y(t) = \sum_{k=-\infty}^{\infty} Y_k e^{j(\frac{2\pi}{T_0})kt}$

$\lim_{T_0 \to \infty} = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt \rightarrow g(t) = y(t) e^{j\omega t}$

IDk!!!

HW8-4

8.5

a) $876.75 = M \cdot f_0$ $\quad \frac{M}{P} = \frac{876.75}{1878.75} = \frac{7}{15} P$

b) $1503 = N \cdot f_0$ $\quad \frac{N}{P} = \frac{1503}{1878.75} = \frac{4}{5} = \frac{12}{15} P$

$1878.75 = P \cdot f_0$ $\quad \frac{P}{P} = \frac{1878.75}{1878.75} = 1P$

$P = 15 \rightarrow M = 7 \quad N = 12$

$f_0 = \frac{1}{T_0} = \frac{1}{0.049 - 0.024}$

$f_0 = \frac{1}{0.025} = 40$

$f_0 = 40\,Hz \quad T_0 = 25\,ms$

smallest $f_0 = 7\,Hz$

$T_0 = \frac{1}{7}\,s$

c)

$\alpha_7 = \frac{61}{2} e^{j17^\circ} \quad / \alpha_{-7} = \frac{61}{2} e^{-j17^\circ}$

$\alpha_{12} = \frac{29}{2} e^{j39^\circ} \quad / \alpha_{-12} = \frac{29}{2} e^{j39^\circ}$

$\alpha_{15} = \frac{17}{2} e^{j90^\circ} \quad / \alpha_{-15} = \frac{17}{2} e^{-j90^\circ}$

$\alpha_0 = 0$