Storyline

Chapter 24: Electric Potential

Physics for Scientists and Engineers, 10e
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BROOKS/COLE
CENGAGE Learning

Electric Potential and Potential Difference

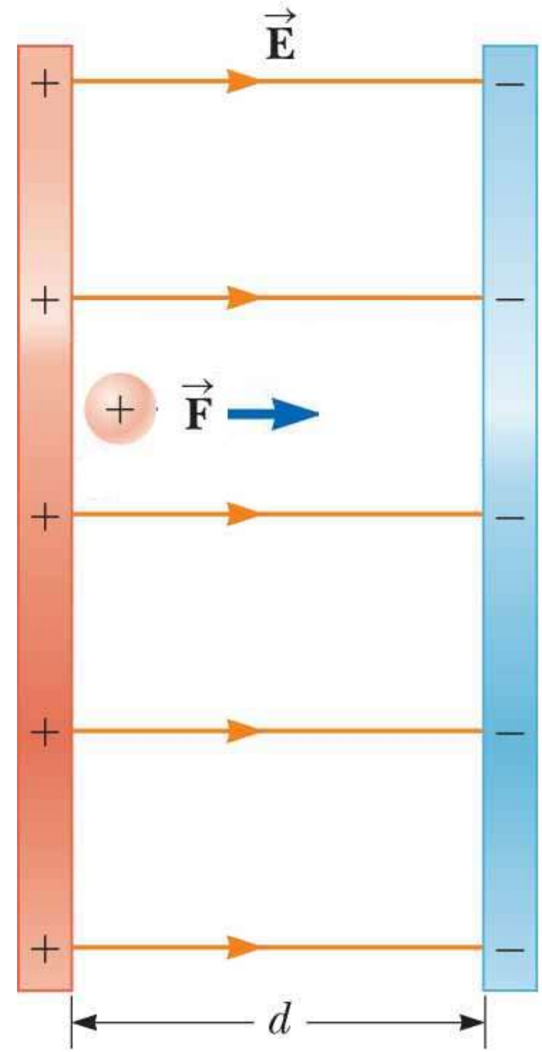
$$W_{\text{int}} = \vec{\mathbf{F}}_e \cdot d\vec{\mathbf{s}} = q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$W_{\text{int}} = -\Delta U_E$$

$$dU_E = -W_{\text{int}} = -q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$\Delta U_E = -q\int_{\text{A}}^{\text{B}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$V = \frac{U_E}{q}$$



Electric Potential and Potential Difference

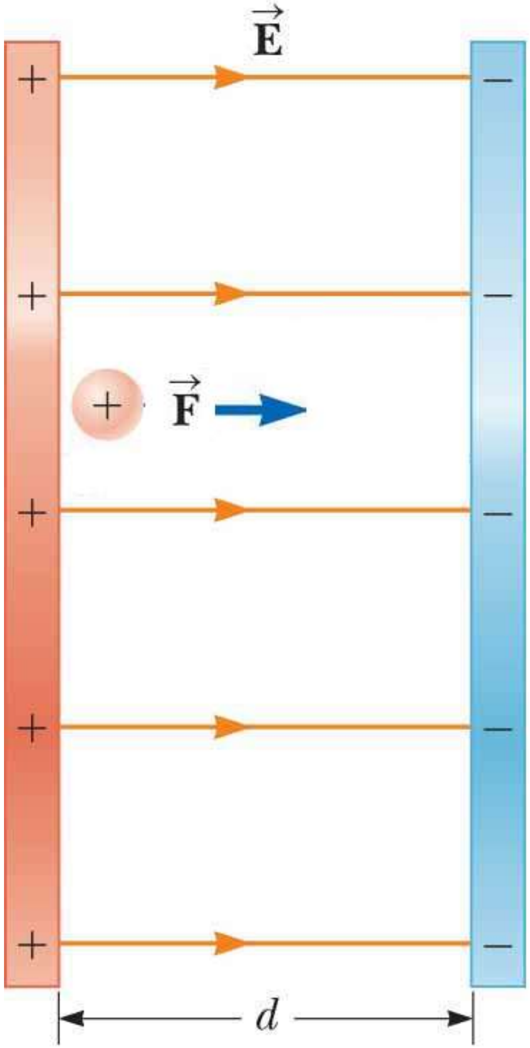
$$\Delta V \equiv \frac{\Delta U_E}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$W = q\Delta V$$

$$1\ \text{V} \equiv 1\ \text{J/C}$$

$$1\ \text{N/C} = 1\ \text{V/m}$$

The electric field is a measure of the rate of change of the electric potential with respect to position.



Electric Potential

$$1\ \mathrm{eV} = 1.60218\times10^{-19}\ \mathrm{C\cdot V} = 1.60218\times10^{-19}\ \mathrm{J}$$

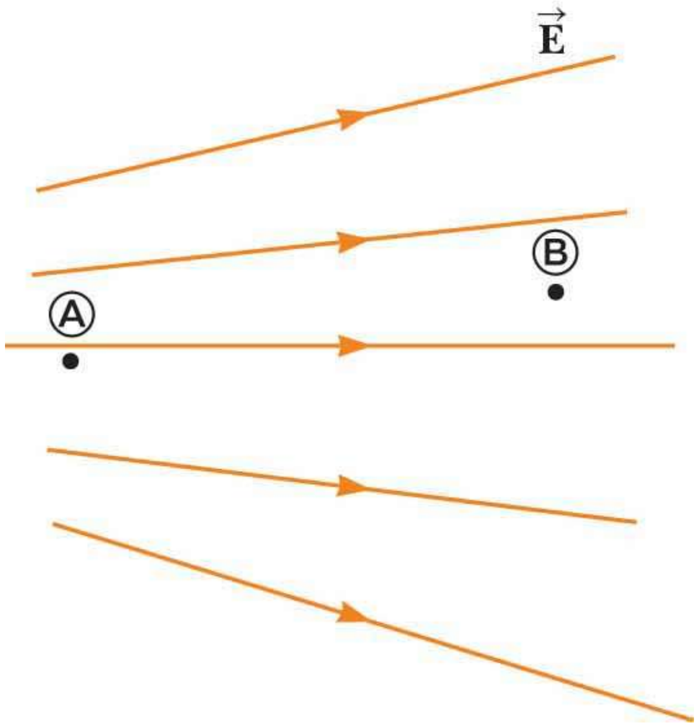
Quick Quiz 24.1 Part I

In the figure, two points A and B are located within a region in which there is an electric field. How would you describe the potential difference $\otimes V = V_B \square V_A$?

(a) It is positive.

(b) It is negative.

(c) It is zero.



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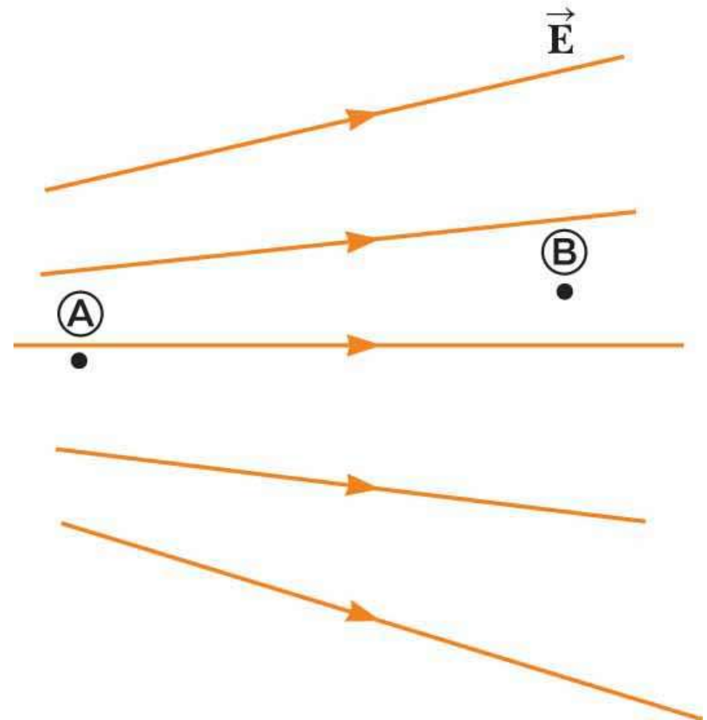
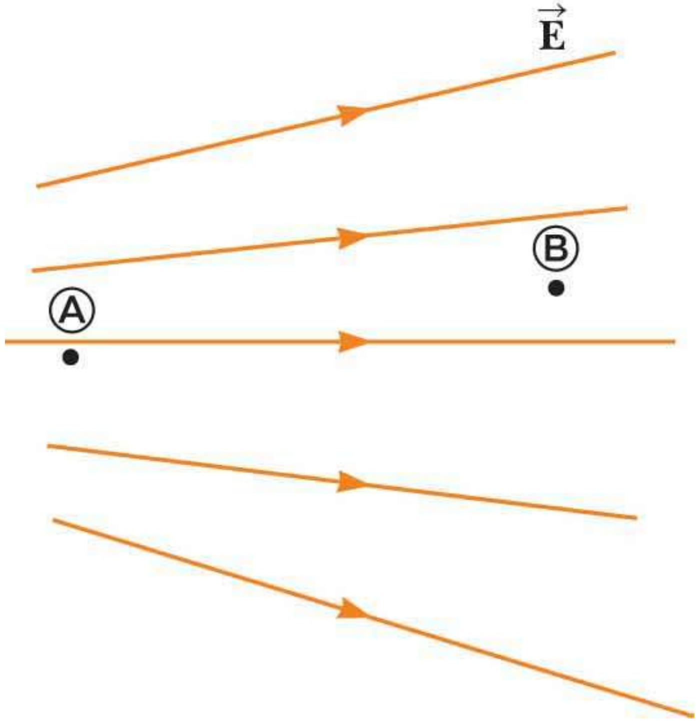


Quick Quiz 24.1 Part II

In the figure, two points A and B are located within a region in which there is an electric field. How would you describe the change in potential energy of the charge–field system for this process?

(a) It is positive.
(b) It is negative.
(c) It is zero.



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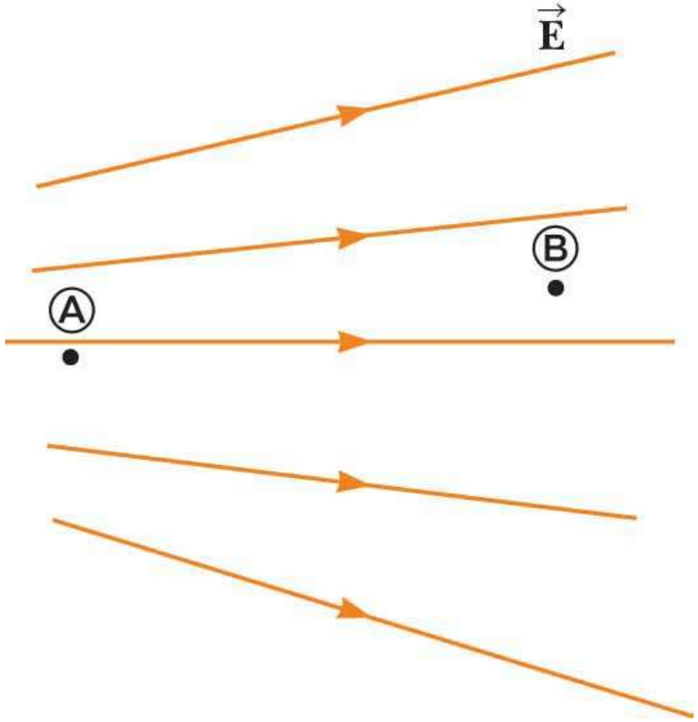


Potential Difference in a Uniform Electric Field

$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$= -\int_A^B E\,ds\left(\cos 0^\circ\right)$$

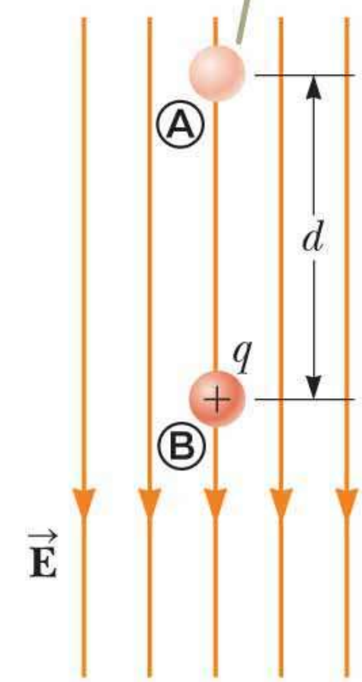
$$= -\int_A^B E\,ds$$

$$\Delta V = -E\int_A^B ds$$

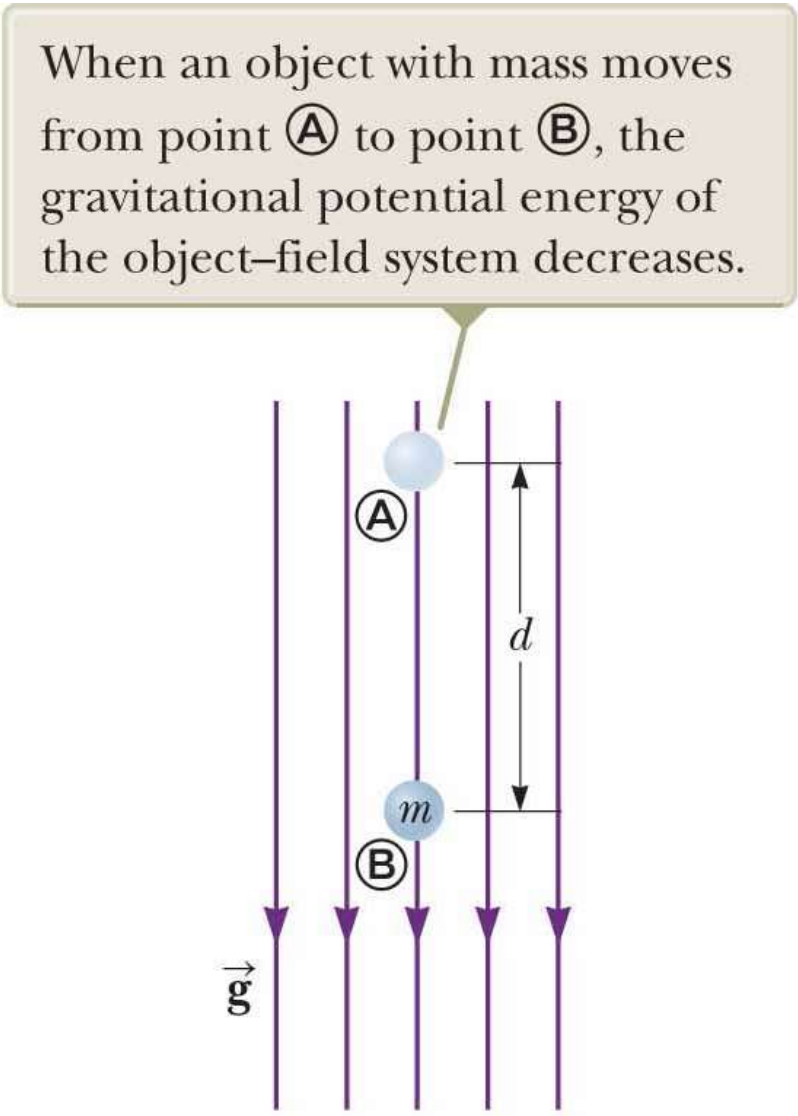
$$\boxed{\Delta V = -Ed}$$

$$\Delta U_E = q\Delta V = -qEd$$

When a positive charge moves from point Ⓐ to point Ⓑ, the electric potential energy of the charge–field system decreases.



Potential Difference in a Uniform Gravitational Field

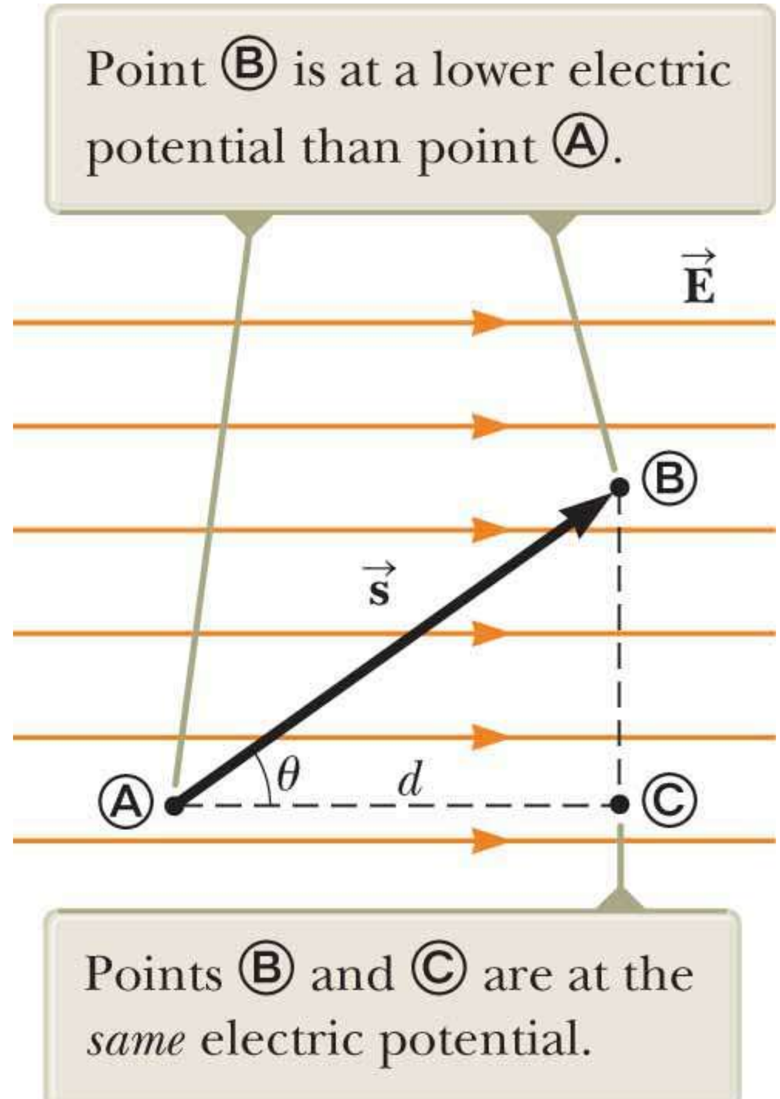


Equipotential Surfaces

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

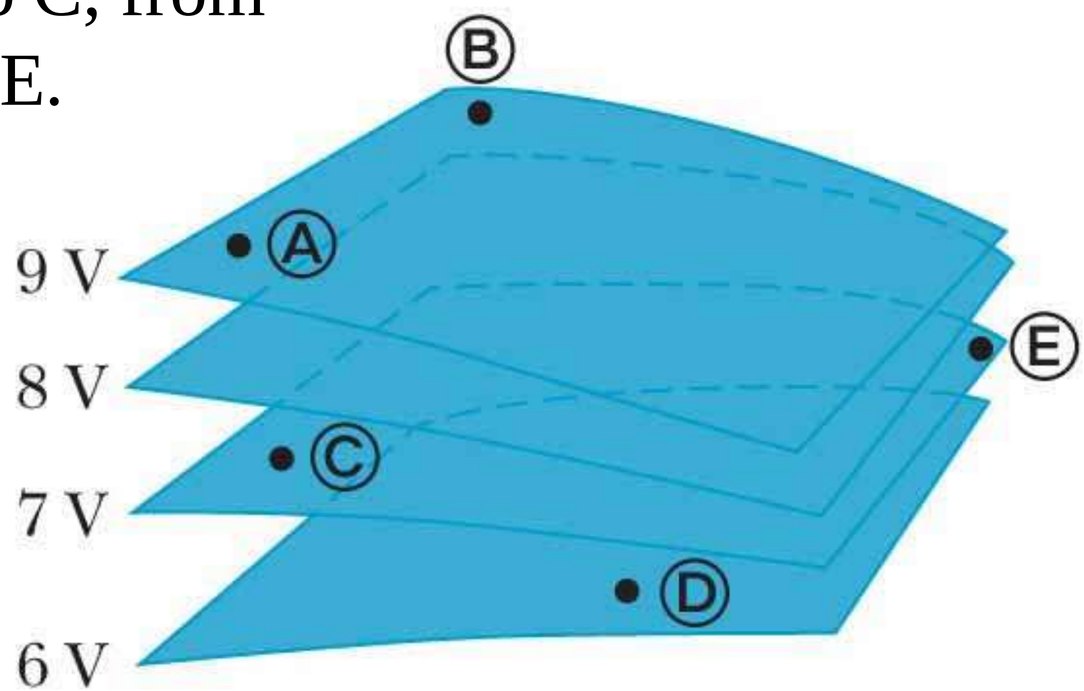
$$= -\vec{E} \cdot \int_A^B d\vec{s} = -\vec{E} \cdot \vec{s}$$

$$\Delta U_E = q\Delta V = -q\vec{E} \cdot \vec{s}$$



Quick Quiz 24.2

The labeled points in the figure are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from A to B, from B to C, from C to D, and from D to E.



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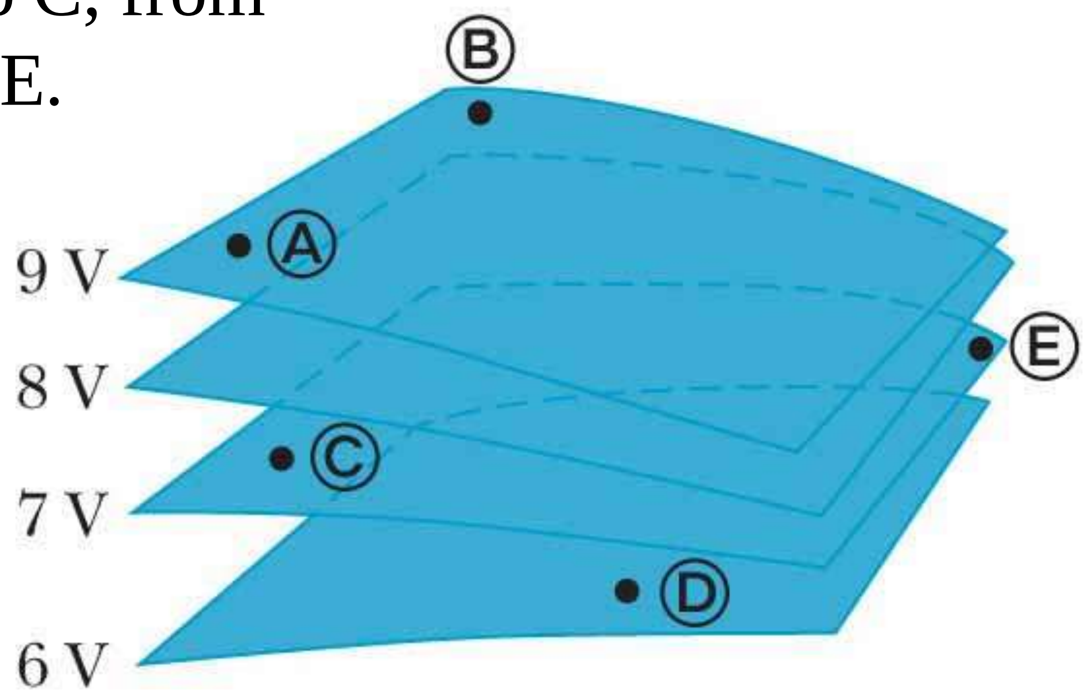
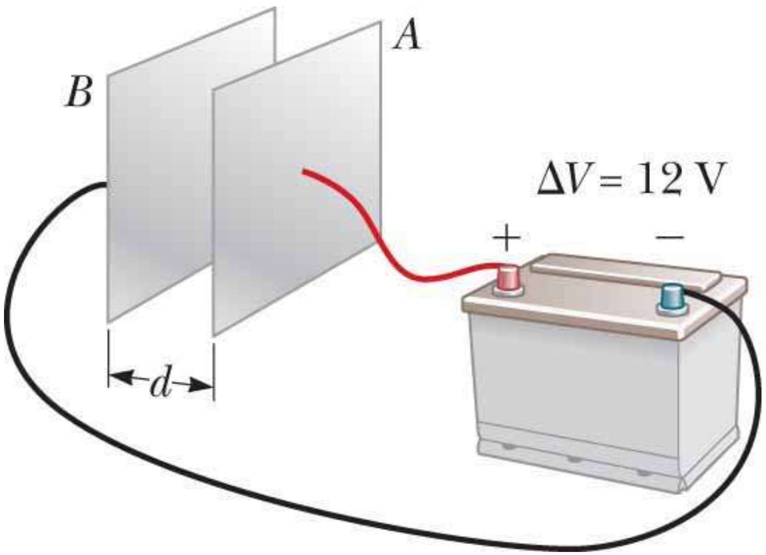
B to C,
C to D,
A to B,
D to E



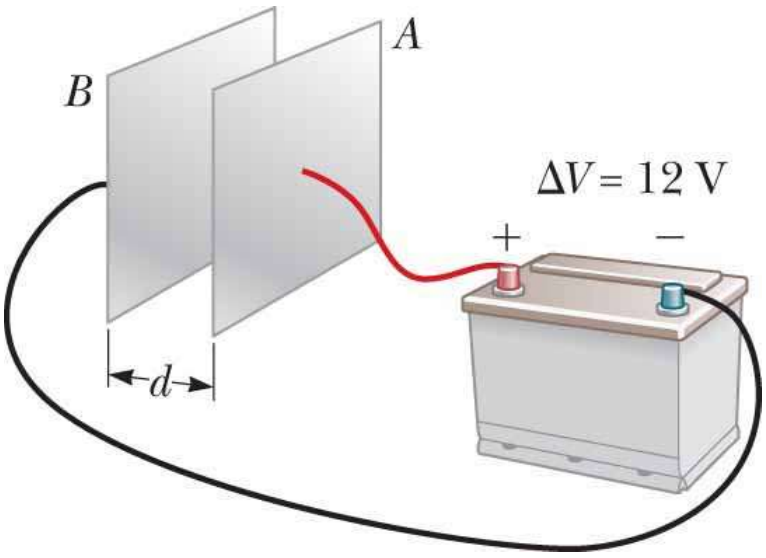
Example 24.1: The Electric Field Between Two Parallel Plates of Opposite Charge

A battery has a specified potential difference ⊗V between its terminals and establishes that potential difference between conductors attached to the terminals. A 12-V battery is connected between two parallel plates as shown in the figure. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.



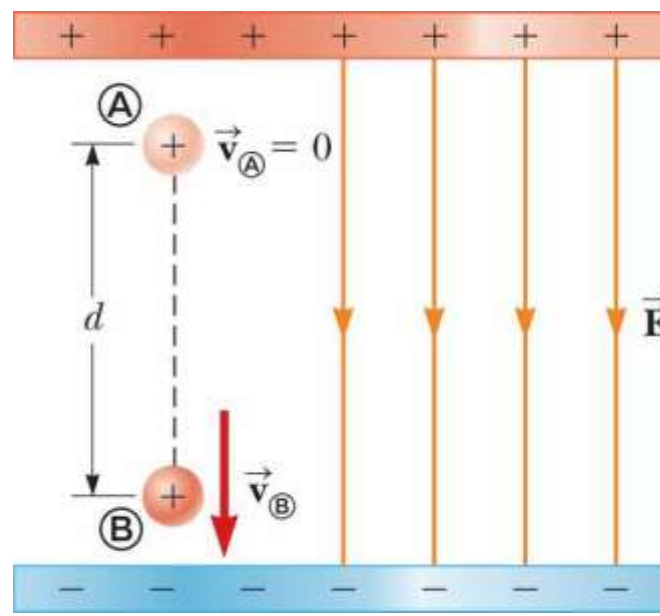
Example 24.1: The Electric Field Between Two Parallel Plates of Opposite Charge

$$E = \frac{|V_B - V_A|}{d} = \frac{12\ \text{V}}{0.30\times10^{-2}\ \text{m}} = \boxed{4.0\times10^{3}\ \text{V/m}}$$



Example 24.2:
Motion of a Proton in a Uniform Electric Field

A proton is released from rest at point A in a uniform electric field that has a magnitude of 8.0×10^{4} V/m, as shown in the figure. The proton undergoes a displacement of magnitude $d = 0.50$ m to point B in the direction of $\vec{\mathbf{E}}$. Find the speed of the proton after completing the displacement.



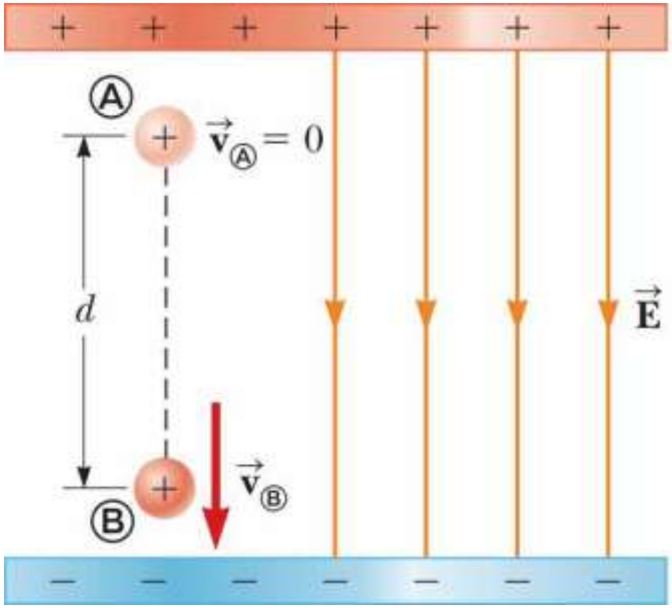
Example 24.2:
Motion of a Proton in a Uniform Electric Field

$$\Delta K + \Delta U_E = 0 \qquad \Rightarrow \left(\frac{1}{2}mv^2 - 0\right) + e\Delta V = 0$$

$$v = \sqrt{\frac{-2e\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

$$v = \sqrt{\frac{2\left(1.6\times10^{-19}\text{ C}\right)\left(8.0\times10^{4}\text{ V}\right)\left(0.50\text{ m}\right)}{1.67\times10^{-27}\text{ kg}}}$$

$$= \boxed{2.8\times10^{6}\text{ m/s}}$$



Electric Potential and Potential Energy Due to Point Charges

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

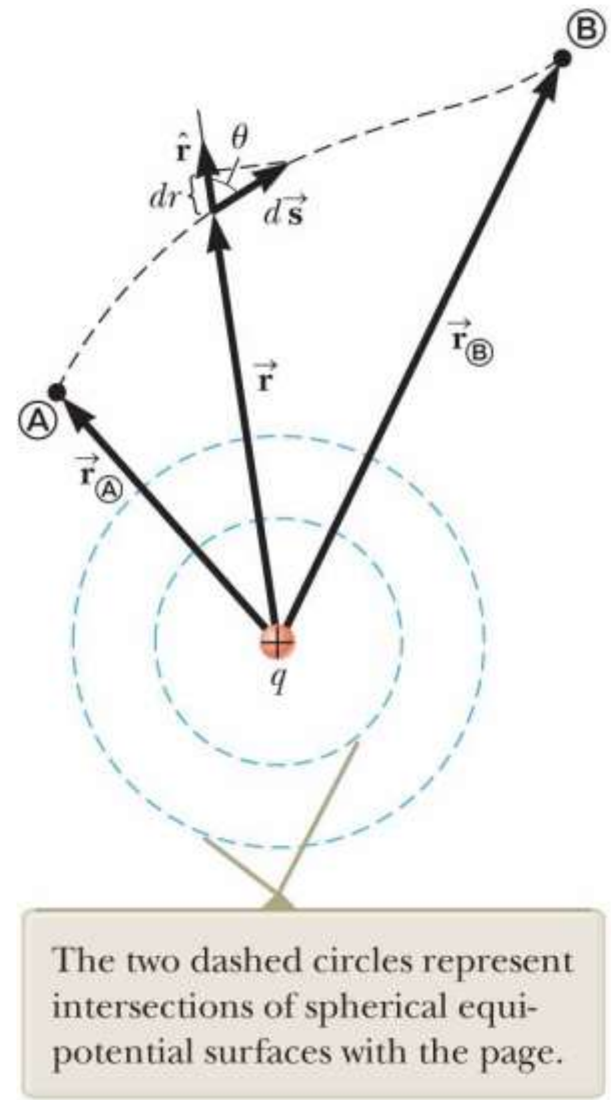
$$\vec{\mathbf{E}} = \frac{k_e q}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2}\hat{\mathbf{r}} \cdot d\vec{\mathbf{s}}$$

$$\hat{\mathbf{r}} \cdot d\vec{\mathbf{s}} = ds\cos\theta$$

$$ds\cos\theta = dr$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2} dr$$



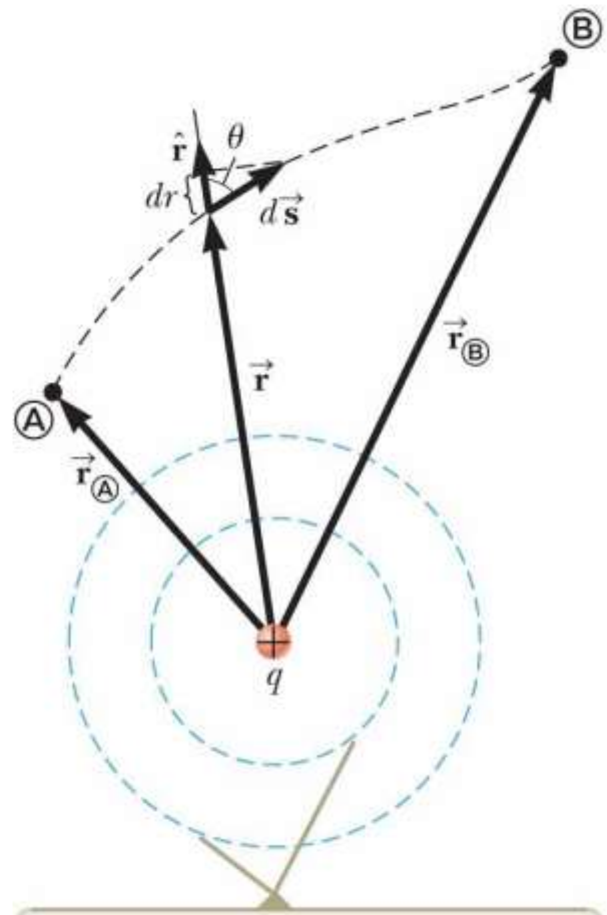
Electric Potential and Potential Energy Due to Point Charges

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = k_e \frac{q}{r}\Big|_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_A = 0 \text{ at } r_A = \infty$$

$$\boxed{V = k_e \frac{q}{r}}$$



The two dashed circles represent intersections of spherical equipotential surfaces with the page.

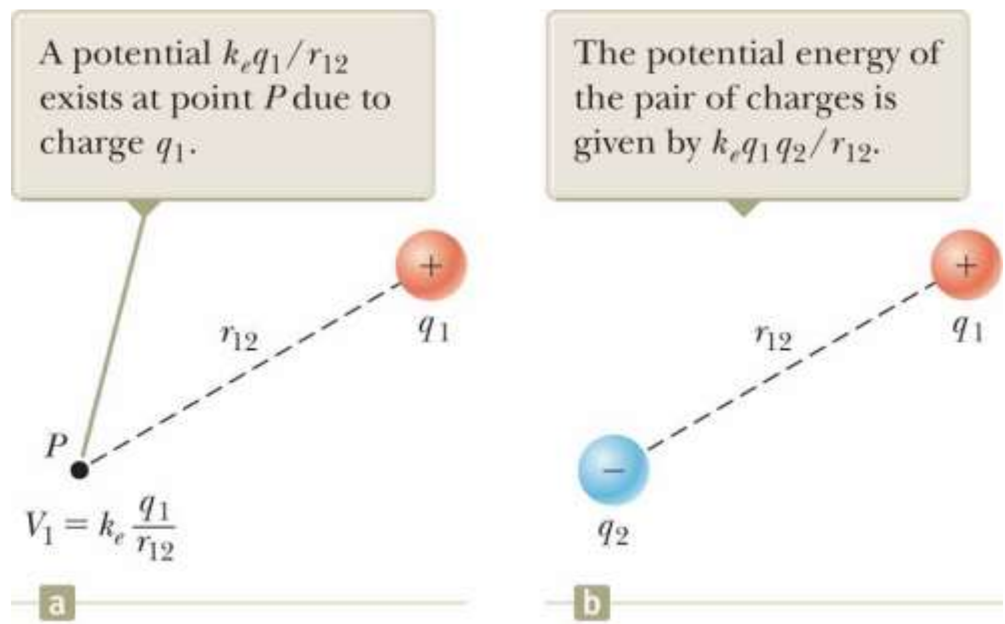
Electric Potential and Potential Energy Due to Point Charges

$$\boxed{V = k_e \sum_i \frac{q_i}{r_i}}$$

$$W_{\text{ext}} = q_2 \Delta V$$

$$\Delta U_E = W_{\text{ext}} = q_2 \Delta V \Rightarrow U_E - 0 = q_2\left(k_e \frac{q_1}{r_{12}} - 0\right)$$

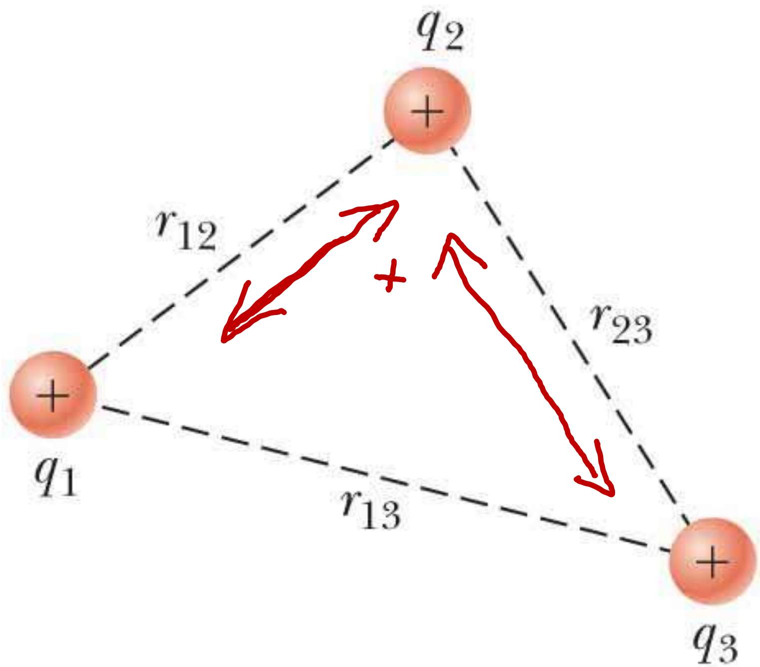
$$U_E = k_e \frac{q_1 q_2}{r_{12}}$$



Electric Potential and Potential Energy Due to Point Charges

$$U_E = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

The potential energy of this system of charges is given by Equation 24.14.



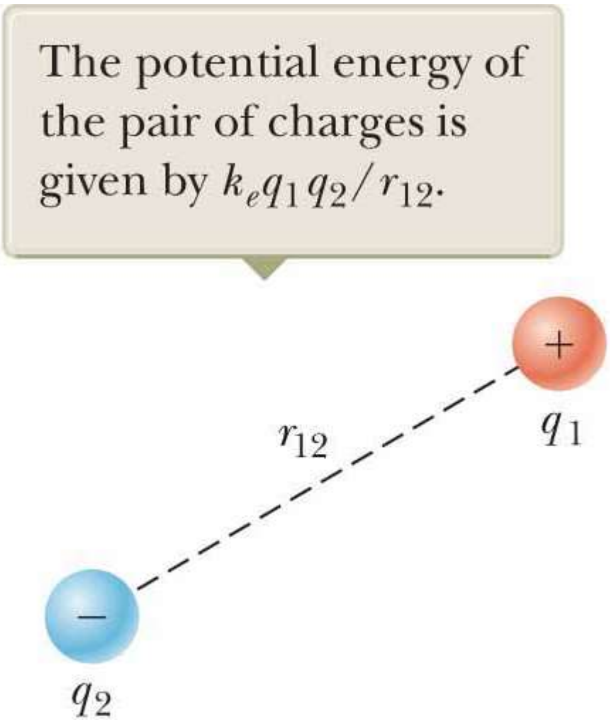
Quick Quiz 24.3 Part I

In the figure, take q_2 to be a negative source charge and q_1 to be a second charge whose sign can be changed. If q_1 is initially positive and is changed to a charge of the same magnitude but negative, what happens to the potential at the position of q_1 due q_2?

(a) It increases.

(b) It decreases.

(c) It remains the same.



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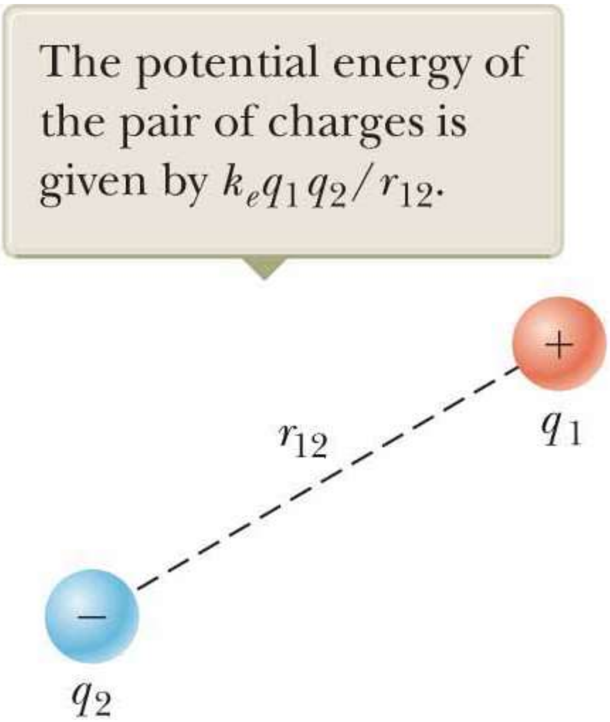


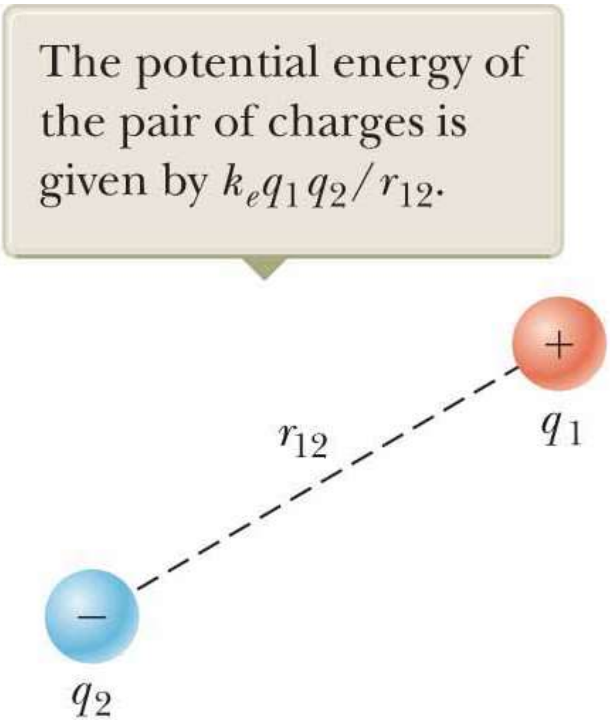
Quick Quiz 24.3 Part II

In the figure, take q_2 to be a negative source charge and q_1 to be a second charge whose sign can be changed. When q_1 is changed from positive to negative, what happens to the potential energy of the two-charge system?

(a) It increases.

(b) It decreases.

(c) It remains the same.



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(a) It increases.
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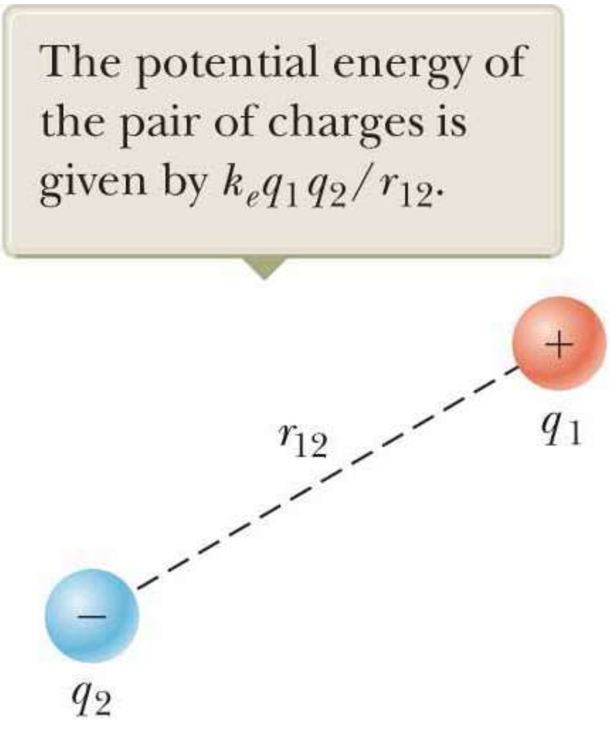


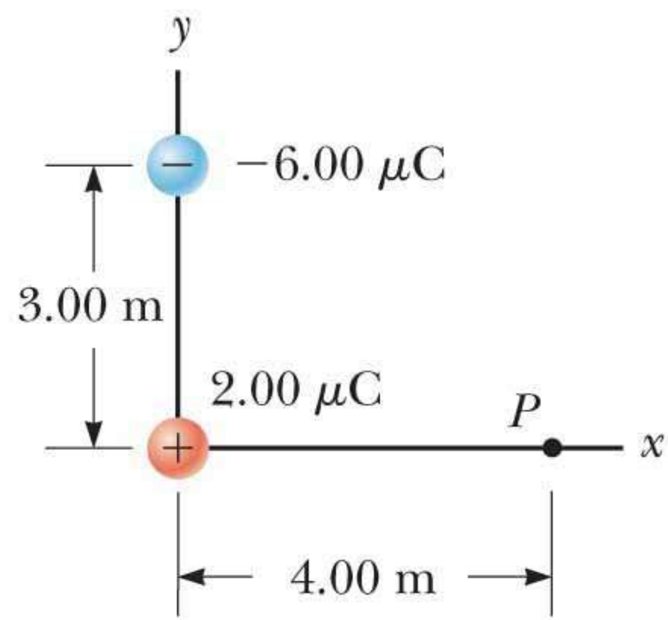
Example 24.3: The Electric Potential Due to Two Point Charges

As shown in the figure, a charge q_1 = 2.00 ⌠C is located at the origin and a charge q_2 = □6.00 ⌠C is located at (0, 3.00) m.

(A) Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = \left(8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)$$
$$\times\left(\frac{2.00\times10^{6}\ \mathrm{C}}{4.00\ \mathrm{m}} + \frac{-6.00\times10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$
$$= \boxed{-6.29\times10^{3}\ \mathrm{V}}$$

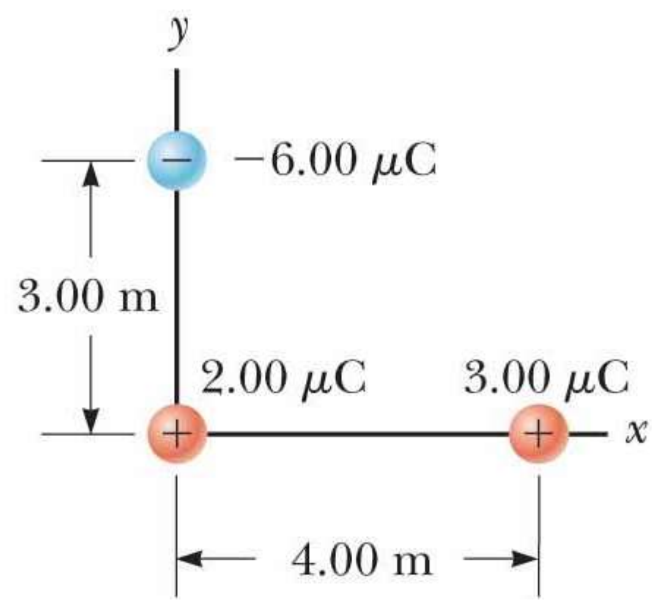


Example 24.3: The Electric Potential Due to Two Point Charges

(B) Find the change in potential energy of the system of two charges plus a third charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point *P*.

$$U_f = q_3 V_P$$

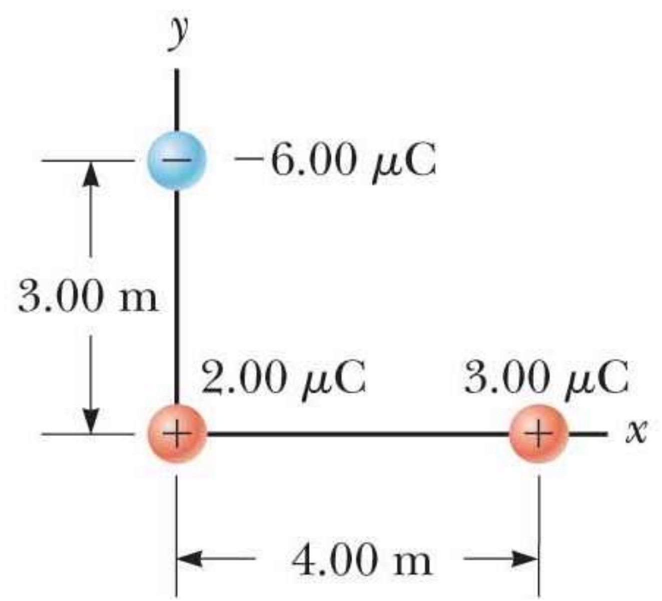
$$\begin{aligned}\Delta U_E &= U_f - U_i \\ &= q_3 V_P - 0 \\ &= \left(3.00\times10^{-6}\ \text{C}\right)\left(-6.29\times10^{3}\ \text{V}\right) \\ &= \boxed{-1.89\times10^{-2}\,\text{J}}\end{aligned}$$



Example 24.3: The Electric Potential Due to Two Point Charges

You are working through this example with a classmate and she says, "Wait a minute! In part (B), we ignored the potential energy associated with the pair of charges q_1 and q_2!" How would you respond?

$$U_E = k_e\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$



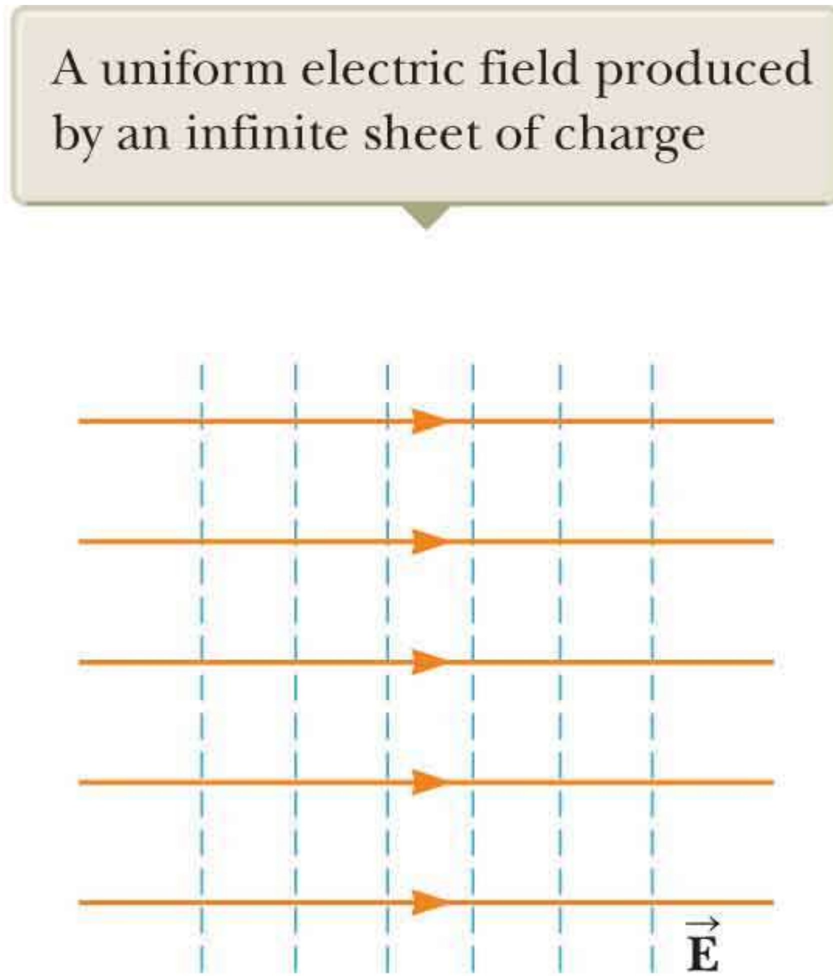
Obtaining the Value of the Electric Field from the Electric Potential

$$\Delta V = -\int_{A}^{B} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_x dx$$

$$E_x = -\frac{dV}{dx}$$



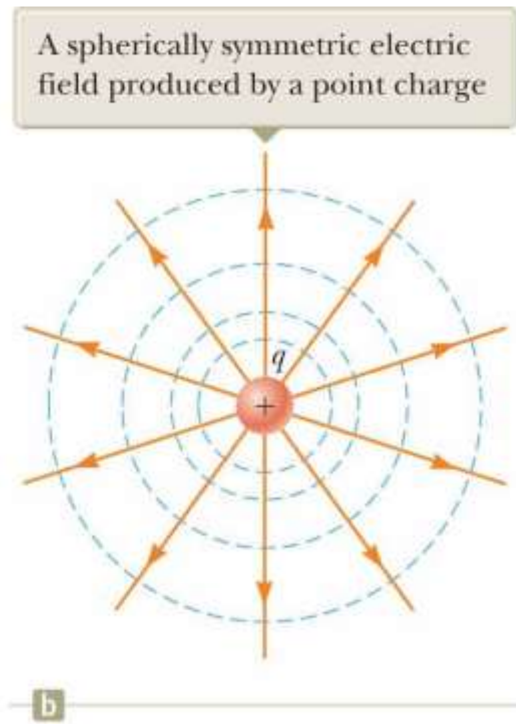
Obtaining the Value of the Electric Field from the Electric Potential

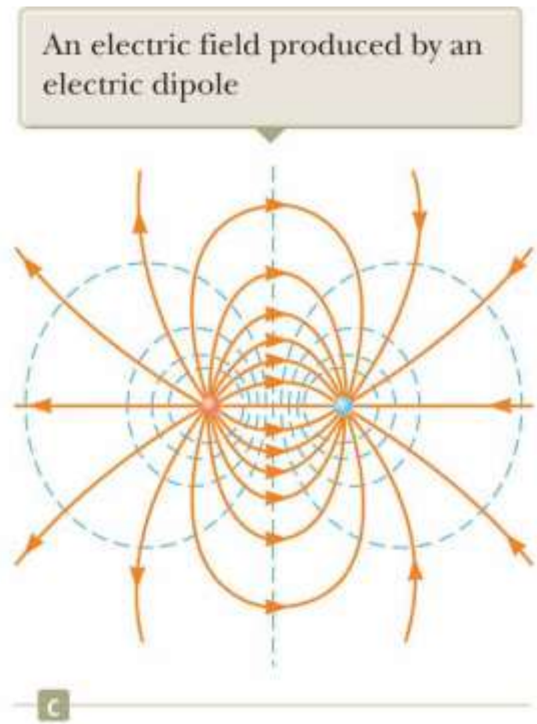
$$\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = E_r\,dr \qquad dV = -E_r\,dr \qquad E_r = -\frac{dV}{dr}$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$





Quick Quiz 24.4 Part I

In a certain region of space, the electric potential is zero everywhere along the x axis. From this information, you can conclude that the x component of the electric field in this region is

(a) zero,

(b) in the positive x direction, or

(c) in the negative x direction.

Quick Quiz 24.4 Part I

In a certain region of space, the electric potential is zero everywhere along the *x* axis. From this information, you can conclude that the *x* component of the electric field in this region is

(a) zero,

(b) in the positive *x* direction, or

(c) in the negative *x* direction.

Quick Quiz 24.4 Part II

Suppose the electric potential is +2 V everywhere along the x axis. From this information, you can conclude that the x component of the electric field in this region is

(a) zero,

(b) in the positive x direction, or

(c) in the negative x direction.

Quick Quiz 24.4 Part II

Suppose the electric potential is +2 V everywhere along the *x* axis. From this information, you can conclude that the *x* component of the electric field in this region is

(a) zero,

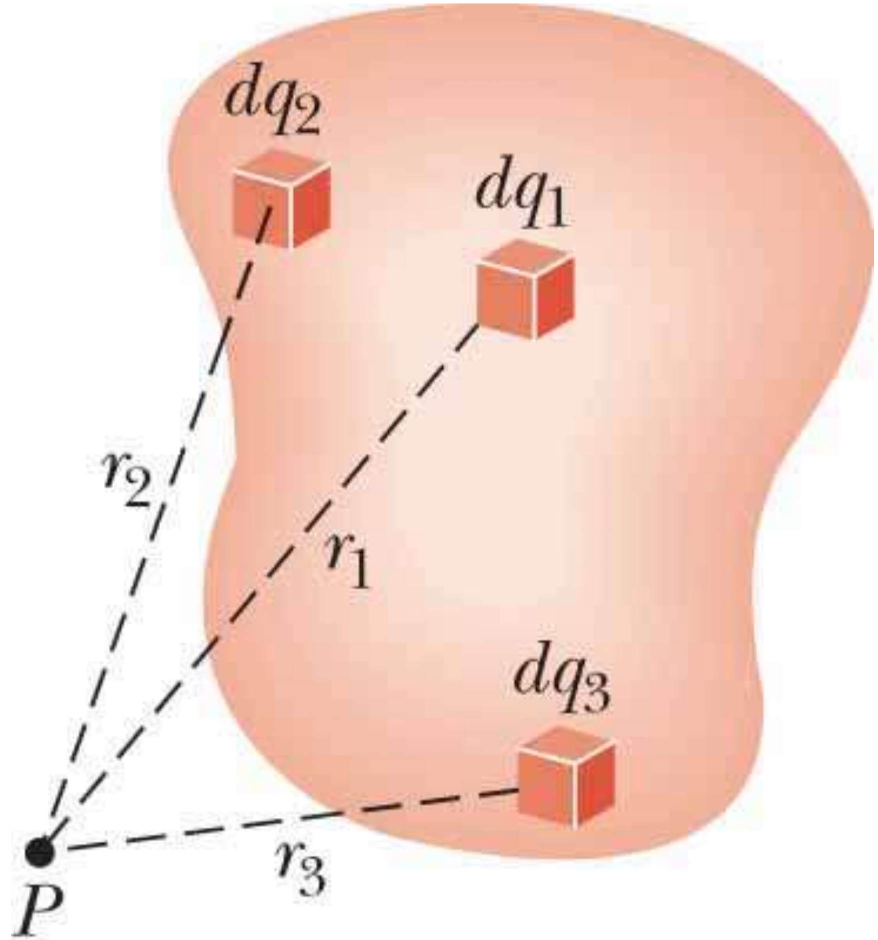
(b) in the positive *x* direction, or

(c) in the negative *x* direction.

Electric Potential Due to Continuous Charge Distributions

$$dV = k_e \frac{dq}{r}$$

$$\boxed{V = k_e \int \frac{dq}{r}}$$



Problem-Solving Strategy: Calculating Electric Potential

1. Conceptualize.
2. Categorize
3. Analyze
4. Finalize

Example 24.4: The Electric Potential Due to a Dipole

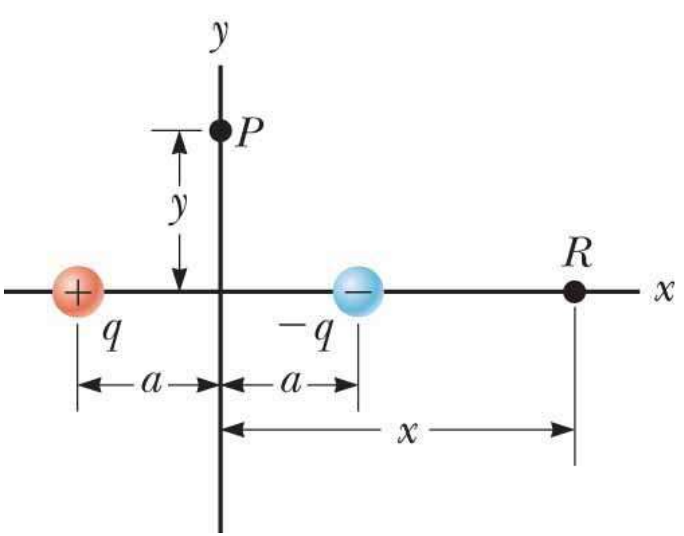
An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance 2*a* as shown in the figure. The dipole is along the *x* axis and is centered at the origin.

(A) Calculate the electric potential at point *P* on the *y* axis.

$$V_P = k_e \sum_i \frac{q_i}{r_i}$$

$$= k_e \left(\frac{q}{\sqrt{a^2 + y^2}} + \frac{-q}{\sqrt{a^2 + y^2}} \right)$$

$$= \boxed{0}$$



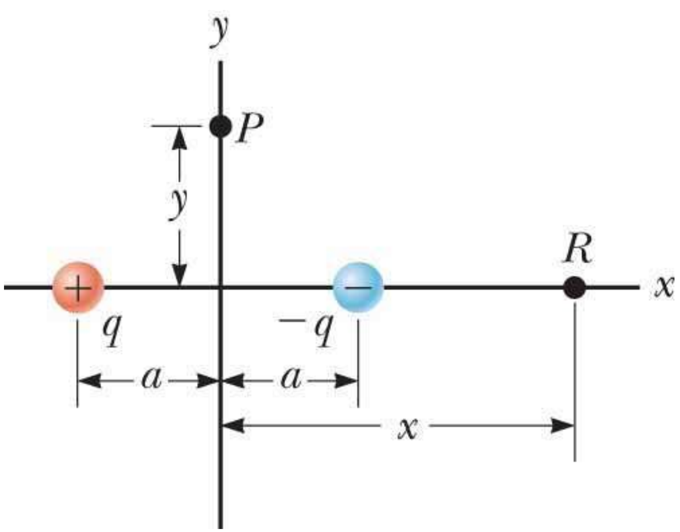
Example 24.4: The Electric Potential Due to a Dipole

(B) Calculate the electric potential at point *R* on the positive *x* axis.

$$V_R = k_e \sum_i \frac{q_i}{r_i}$$

$$= k_e \left(\frac{-q}{x-a} + \frac{q}{x+a} \right)$$

$$= \boxed{-\frac{2k_e qa}{x^2 - a^2}}$$



Example 24.4: The Electric Potential Due to a Dipole

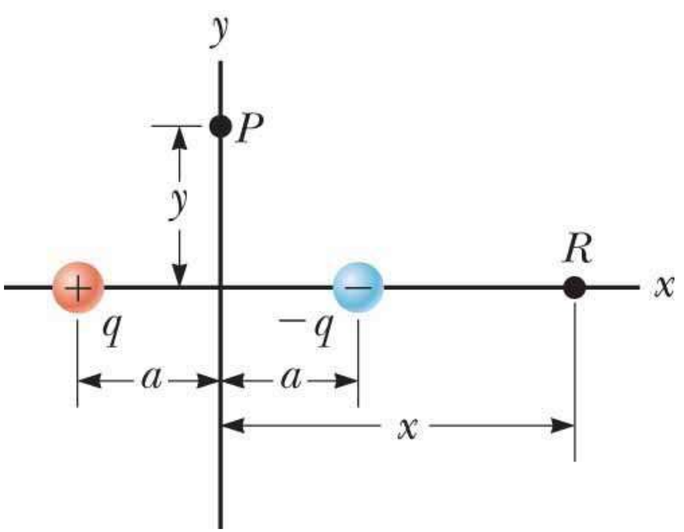
(C) Calculate V and E_x at a point on the x axis far from the dipole.

$$V_R = \lim_{x>>a}\left(-\frac{2k_e qa}{x^2 - a^2}\right) \approx \boxed{-\frac{2k_e qa}{x^2} \quad (x >> a)}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qa}{x^2}\right)$$

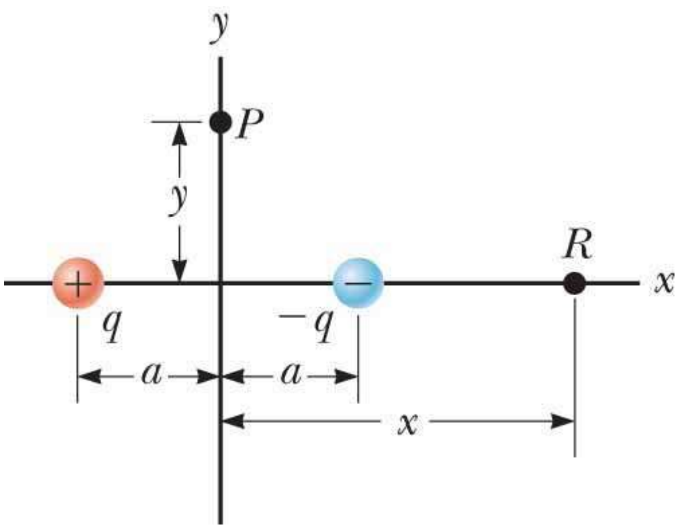
$$= 2k_e qa \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= \boxed{-\frac{4k_e qa}{x^3} \quad (x >> a)}$$



Example 24.4: The Electric Potential Due to a Dipole

Suppose you want to find the electric field at a point *P* on the y axis. In part (A), the electric potential was found to be zero for all values of. Is the electric field zero at all points on the *y* axis?

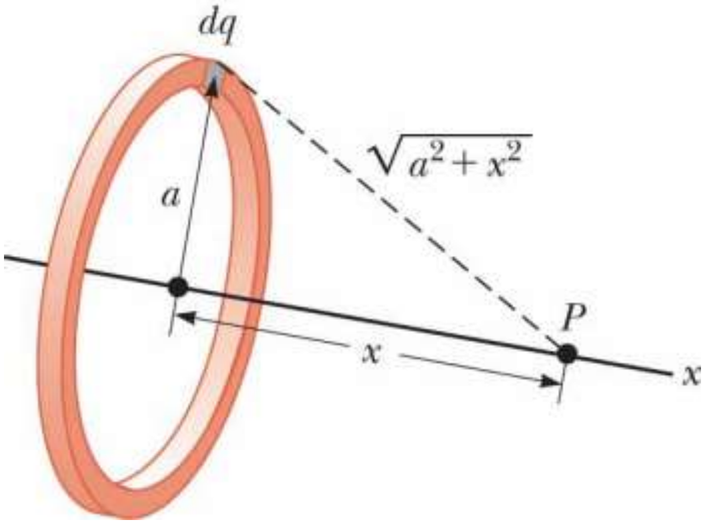


Example 24.5: Electric Potential Due to a Uniformly Charged Ring

(C) Find an expression for the electric potential at a point *P* located on the perpendicular central axis of a uniformly charged ring of radius a and total charge *Q*.

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{a^2 + x^2}}$$

$$V = \frac{k_e}{\sqrt{a^2 + x^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{a^2 + x^2}}}$$



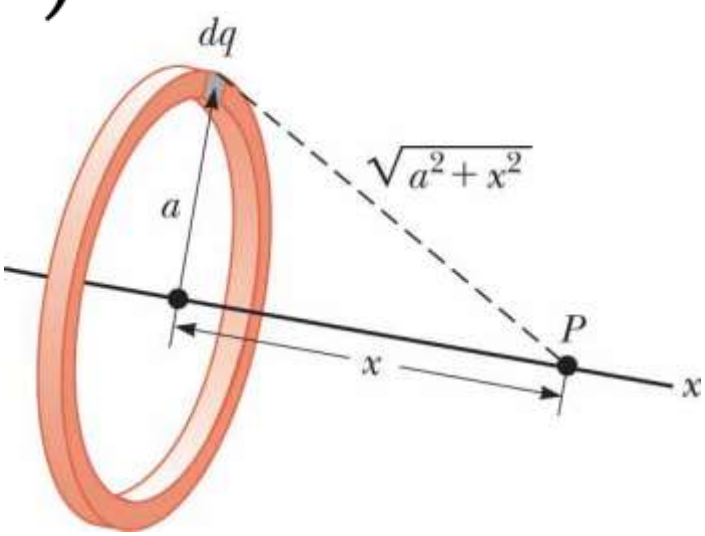
Example 24.5: Electric Potential Due to a Uniformly Charged Ring

(B) Find an expression for the magnitude of the electric field at point *P*.

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}\left(a^2 + x^2\right)^{-1/2}$$

$$= -k_e Q\left(-\frac{1}{2}\right)\left(a^2 + x^2\right)^{-3/2}(2x)$$

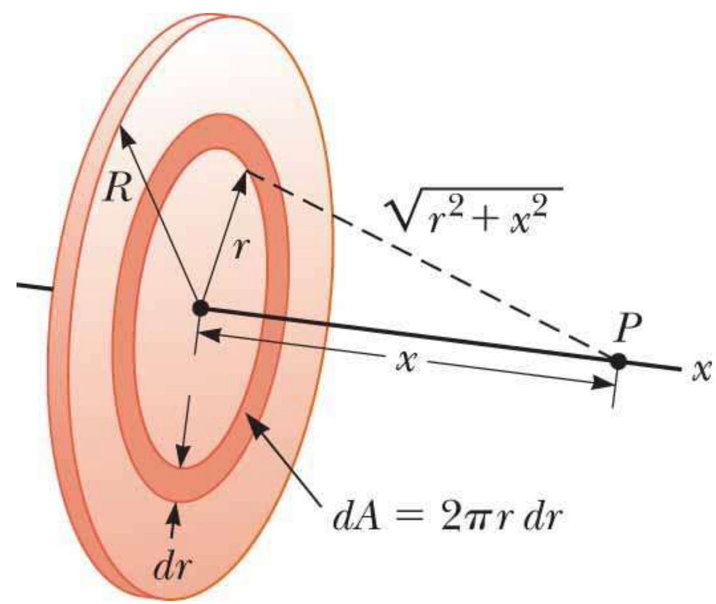
$$E_x = \boxed{\frac{k_e x}{\left(a^2 + x^2\right)^{3/2}} Q}$$



Example 24.6: Electric Dipole Due to a Uniformly Charged Disk

A uniformly charged disk has radius *R* and surface charge density ∫.

(A) Find the electric potential at a point *P* along the perpendicular central axis of the disk.



Example 24.6: Electric Dipole Due to a Uniformly Charged Disk

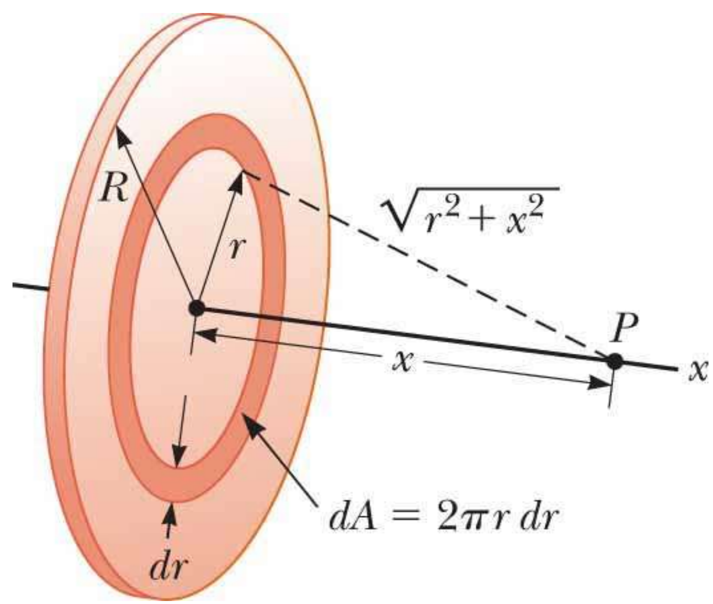
$$dq = \sigma\, dA = \sigma\left(2\pi r\, dr\right) = 2\pi\sigma r\, dr$$

$$dV = \frac{k_e dq}{\sqrt{r^2 + x^2}} = \frac{k_e 2\pi\sigma r\, dr}{\sqrt{r^2 + x^2}}$$

$$V = \pi k_e \sigma \int_0^R \frac{2r\, dr}{\sqrt{r^2 + x^2}}$$

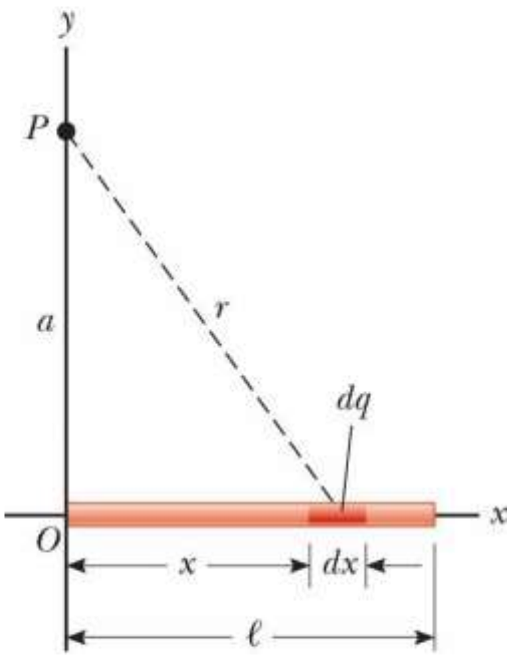
$$= \pi k_e \sigma \int_0^R \left(r^2 + x^2\right)^{-1/2} 2r\, dr$$

$$V = \boxed{2\pi k_e \sigma \left[\left(R^2 + x^2\right)^{1/2} - x\right]}$$



Example 24.7: Electric Potential Due to a Finite Line of Charge

A rod of length , located along the *x* axis has a total charge *Q* and a uniform linear charge density ⎣ . Find the electric potential at a point *P* located on the *y* axis a distance a from the origin

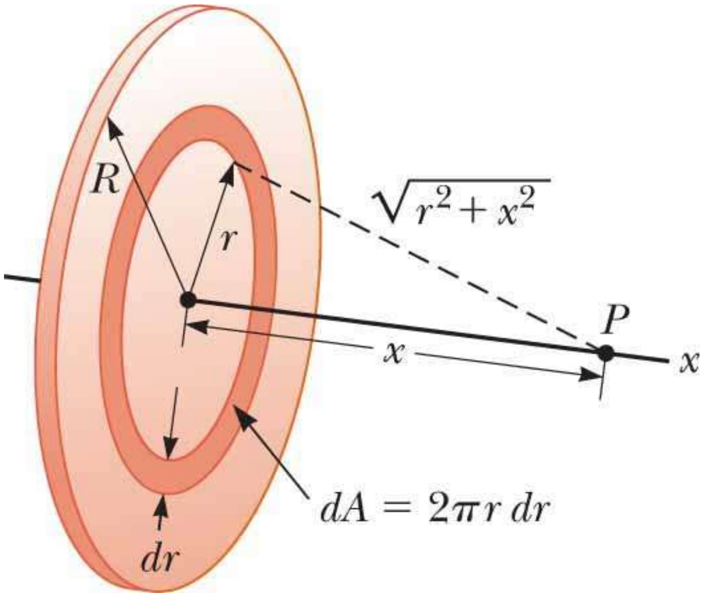


Example 24.6: Electric Dipole Due to a Uniformly Charged Disk

(B) Find the x component of the electric field at a point P along the perpendicular central axis of the disk.

$$E_x = -\frac{dV}{dx}$$

$$= \boxed{2\pi k_e \sigma \left[1 - \frac{x}{\left(R^2 + x^2\right)^{1/2}}\right]}$$



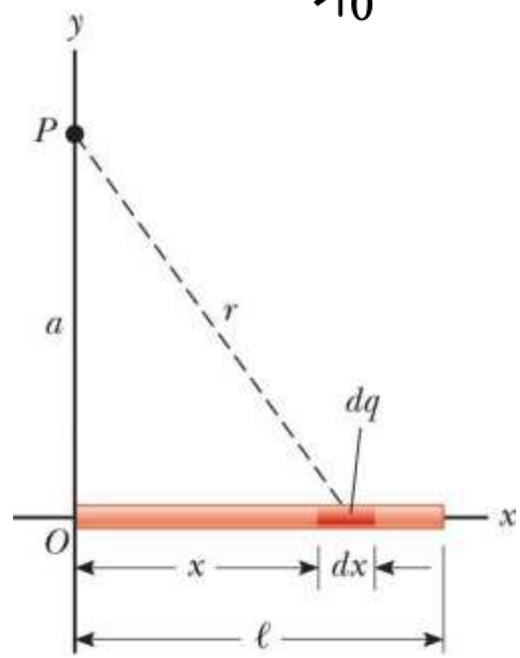
Example 24.7: Electric Potential Due to a Finite Line of Charge

$$dV = k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}} \qquad \to V = \int_0^{\boxed{?}} k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}}$$

$$V = k_e \lambda \int_0^{\boxed{?}} \frac{dx}{\sqrt{a^2 + x^2}} = k_e \frac{Q}{\boxed{?}} \ln\left(x + \sqrt{a^2 + x^2}\right)\Big|_0^{\boxed{?}}$$

$$V = k_e \frac{Q}{\boxed{?}}\left[\ln\left(\boxed{?} + \sqrt{a^2 + \ ^2}\boxed{?}\right) - \ln a\right]$$

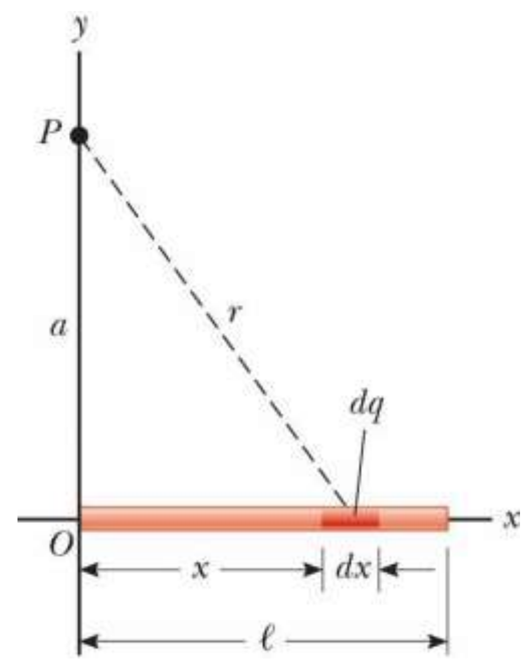
$$= \boxed{k_e \frac{Q}{\boxed{?}} \ln\left(\frac{\boxed{?} + \sqrt{a^2 + \ ^2}\boxed{?}}{a}\right)}$$



Example 24.7: Electric Potential Due to a Finite Line of Charge

What if you were asked to find the electric field at point *P* ? Would that be a simple calculation?

$$V = k_e \frac{Q}{\boxed{?}} \ln\left(\frac{\boxed{?} + \sqrt{a^2 + \,^2}\,\boxed{?}}{a}\right)$$

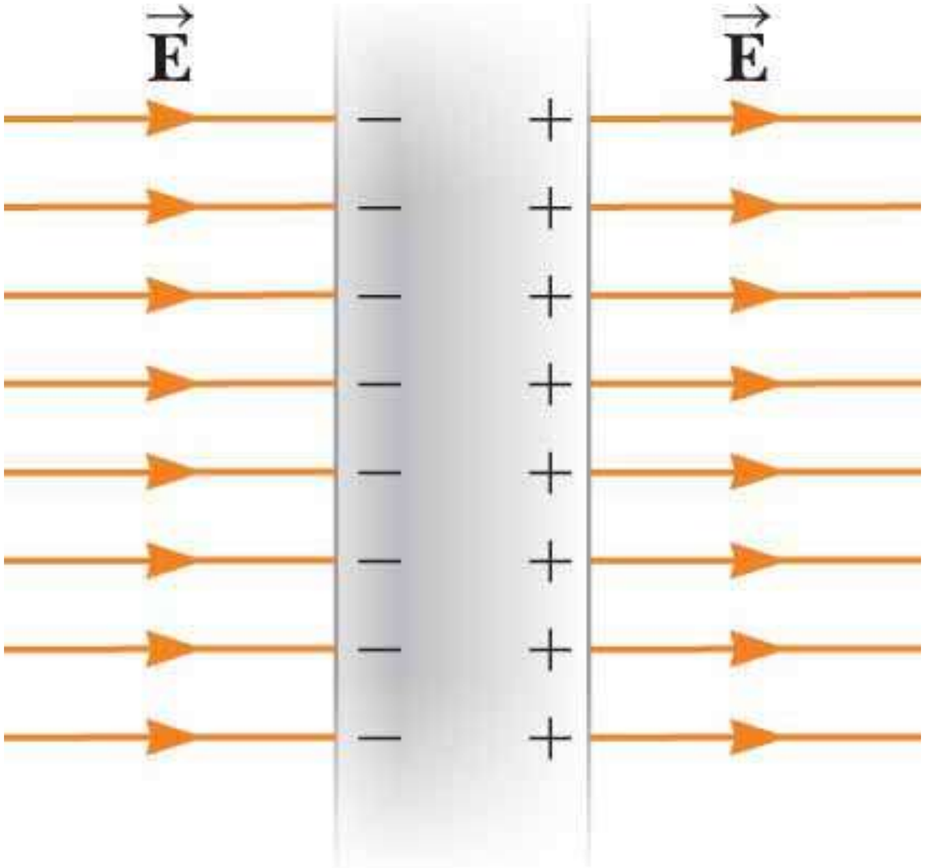


Conductors in Electrostatic Equilibrium

1. $E = 0$ inside conductor
2. Charge resides on surface of isolated conductor
3. E at point just outside conductor, perpendicular to surface, has magnitude σ/ε_0
4. Irregularly shaped conductor: σ greatest where radius of curvature smallest

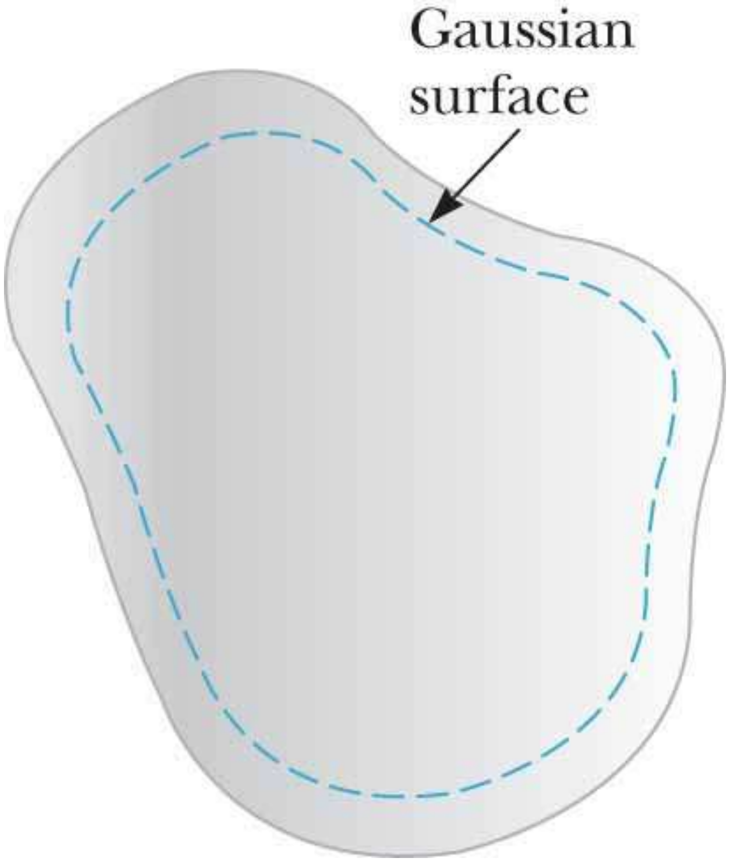
Conductors in Electrostatic Equilibrium

$E = 0$ inside conductor



Conductors in Electrostatic Equilibrium

Charge resides on surface of isolated conductor

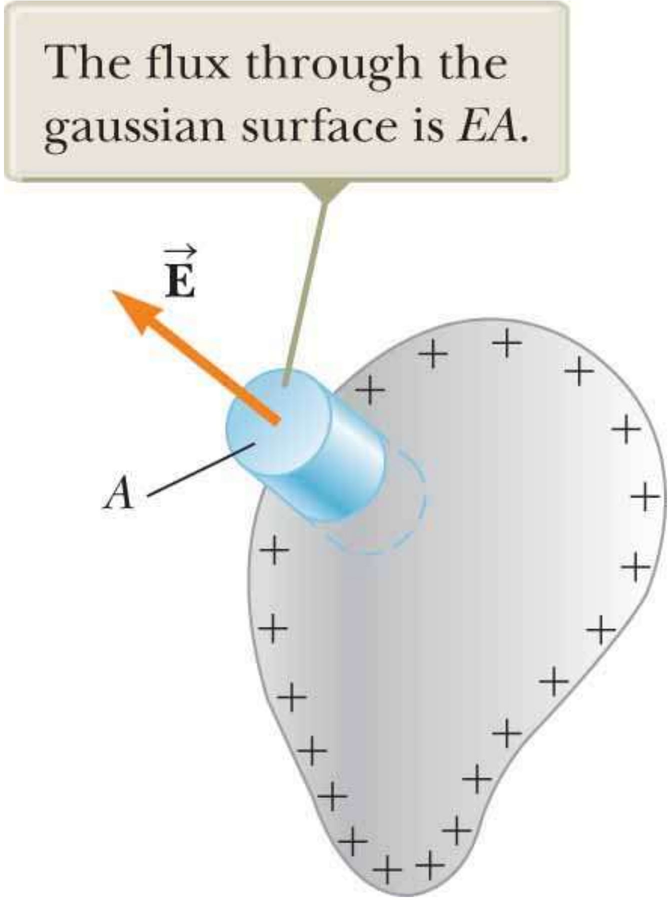


Electric Fields and Charged Conductors

E at point just outside conductor, perpendicular to surface, has magnitude $\int/\Sigma_0$

$$\Phi_E = \oint E\,dA = EA = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

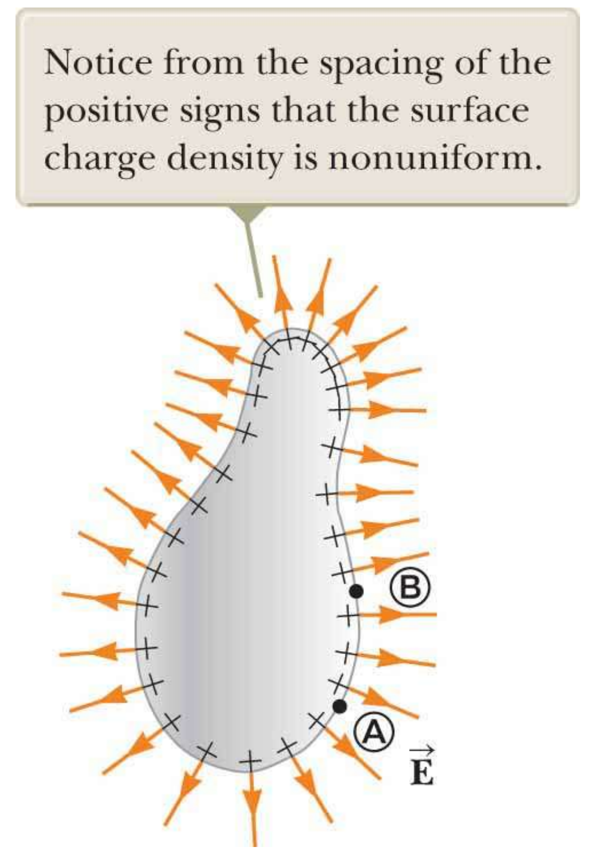


Electric Fields and Charged Conductors

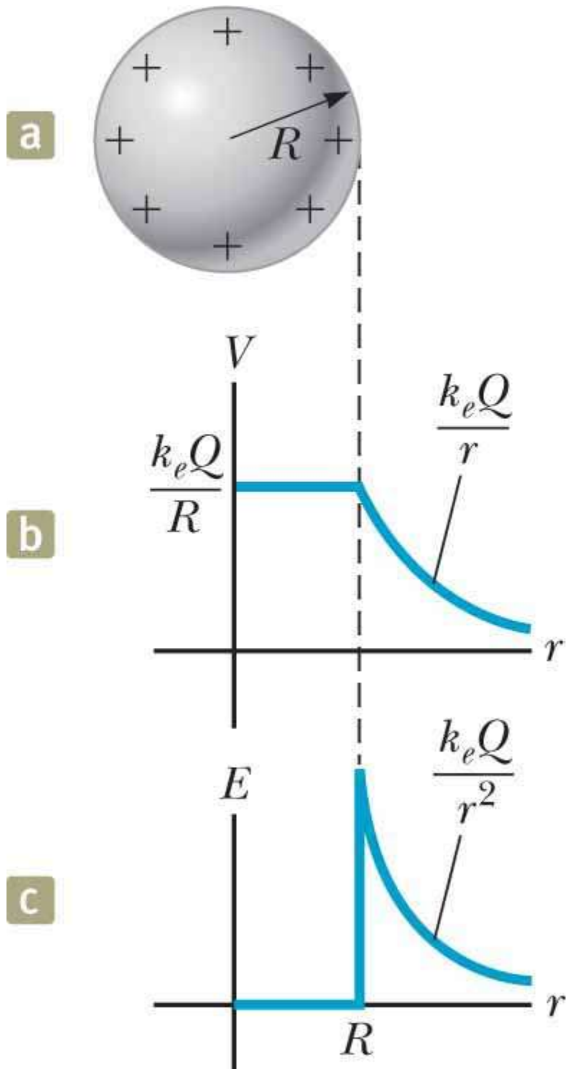
Irregularly shaped conductor: ʃ greatest where radius of curvature smallest

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

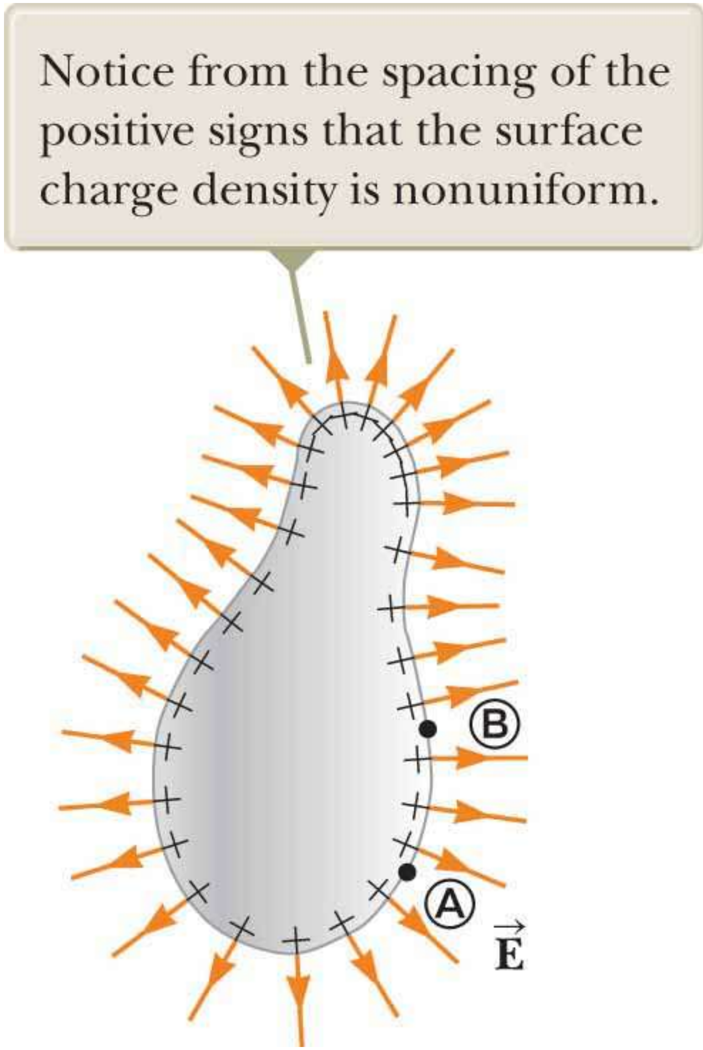
The surface of any charged conductor in electrostatic equilibrium is an equipotential surface: every point on the surface of a charged conductor in equilibrium is at the same electric potential. Furthermore, because the electric field is zero inside the conductor, the electric potential is constant everywhere inside the conductor and equal to its value at the surface.



Electric Potential and Electric Field of Charged Conductor



Surface Charge Density on Charged Conductior



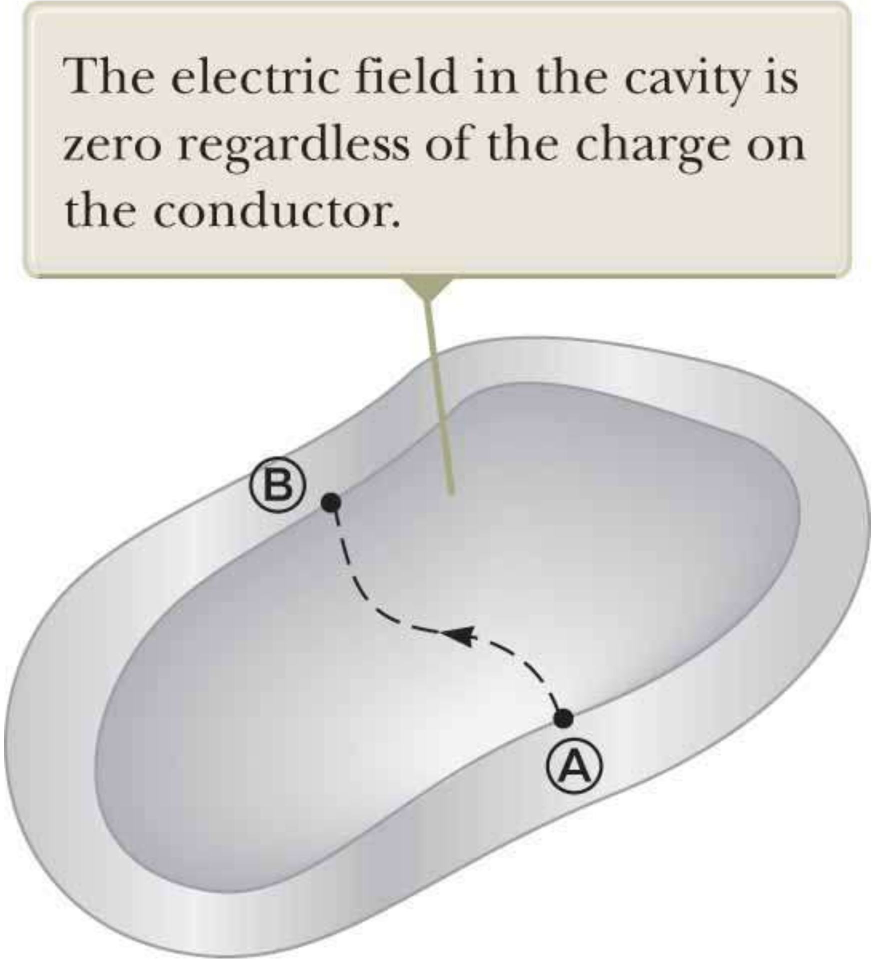
Surface Charge Density on Charged Conductior

$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$$

$$\frac{E_1}{E_2} = \frac{k_e \dfrac{q_1}{r_1^{\,2}}}{k_e \dfrac{q_2}{r_2^{\,2}}} = \frac{\dfrac{1}{r_1} V}{\dfrac{1}{r_2} V} = \frac{r_2}{r_1}$$

A Cavity Within a Conductor

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$



Faraday Cage

Quick Quiz 24.3

Your younger brother likes to rub his feet on the carpet and then touch you to give you a shock. While you are trying to escape the shock treatment, you discover a hollow metal cylinder in your basement, large enough to climb inside. In which of the following cases will you not be shocked?

(a) You climb inside the cylinder, making contact with the inner surface, and your charged brother touches the outer metal surface.

(b) Your charged brother is inside touching the inner metal surface and you are outside, touching the outer metal surface.

(c) Both of you are outside the cylinder, touching its outer metal surface but not touching each other directly.

Quick Quiz 24.3

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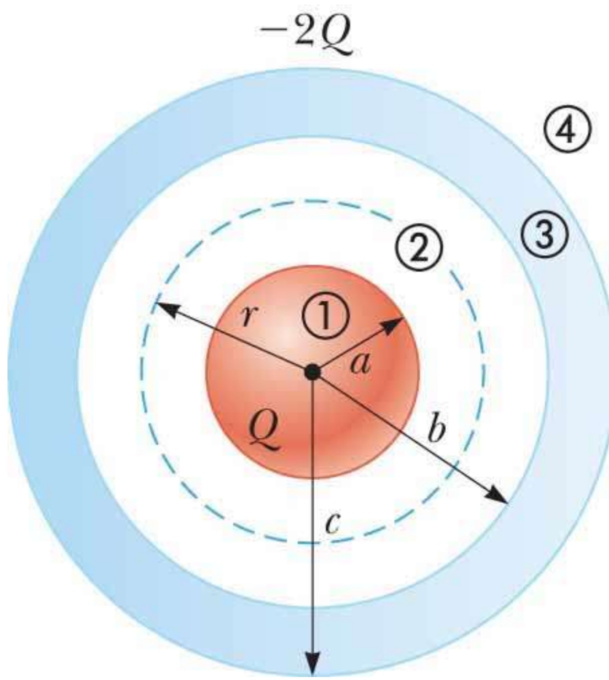
(c) Both of you are outside the cylinder, touching its outer metal surface but not touching each other directly.

Example 24.8: A Sphere Inside a Spherical Shell

A solid insulating sphere of radius a carries a net positive charge Q uniformly distributed throughout its volume. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge □2*Q*. Using Gauss’s law, find the electric field in the regions labeled 1, 2, 3, and 4. The figure and the charge distribution on the shell when the entire system is in electrostatic equilibrium.



Example 24.8: A Sphere Inside a Spherical Shell

$$E_2 = k_e \frac{Q}{r^2} \quad (\text{for } a < r < b)$$

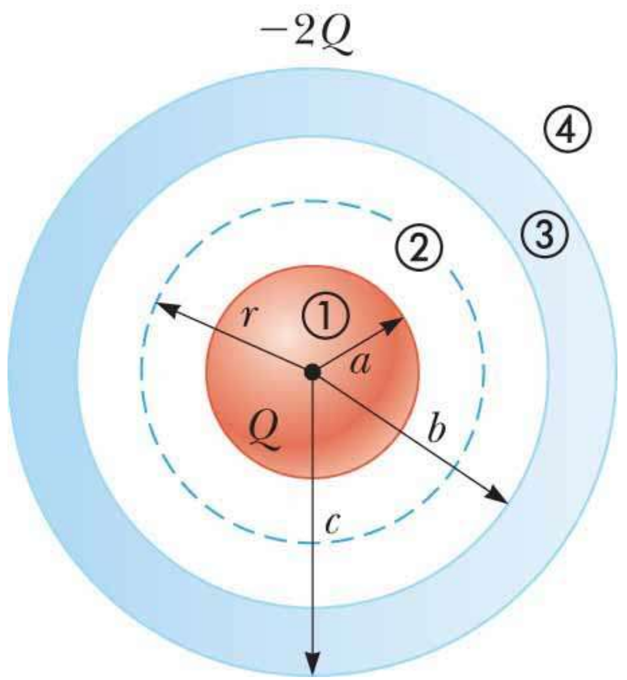
$$E_1 = k_e \frac{Q}{a^3} r \quad (\text{for } r < a)$$

$$E_4 = -k_e \frac{Q}{r^2} \quad (\text{for } r > c)$$

$$E_3 = 0 \quad (\text{for } b < r < c)$$

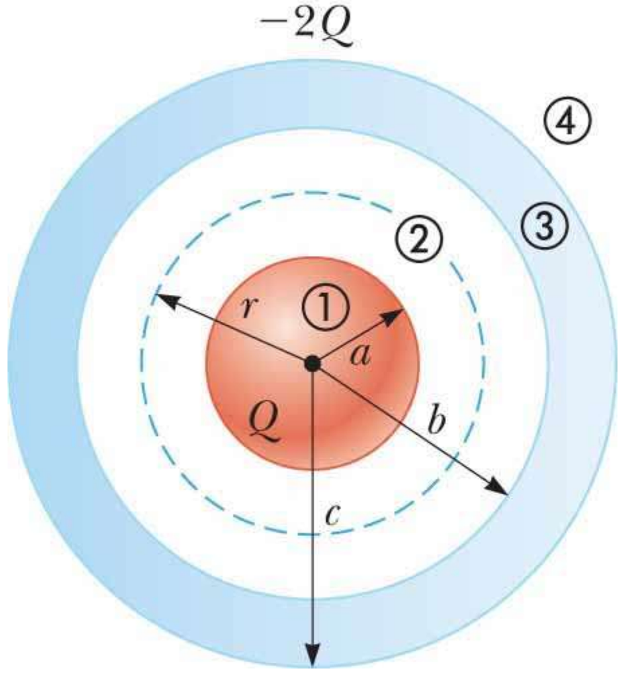
$$q_{\text{in}} = q_{\text{sphere}} + q_{\text{inner}}$$

$$q_{\text{inner}} = q_{\text{in}} - q_{\text{sphere}} = 0 - Q = -Q$$



Example 24.8: A Sphere Inside a Spherical Shell

How would the results of this problem differ if the sphere were conducting instead of insulating?



Assessing to Learn

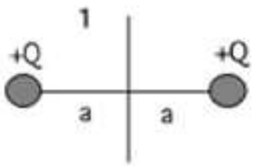
The electric potential at two points in space is V_1 = 200 volts and V_2 = 300 volts. Which of the following statements is true for moving a point charge q from point 1 to 2?

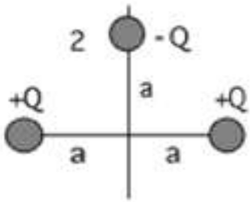
A. The work done by an external agent to move q from point 1 to 2 is positive.
B. We can't determine the work done because we don't know the direction of V at the two points.
C. The work done by the electric force exerted on q in moving it from point 1 to 2 is $W = -q(100\text{ V})$.

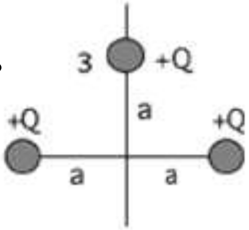
1. A 2. B 3. C 4. A and B 5. A and C
6. B and C 7. A, B, and C 8. None of the above

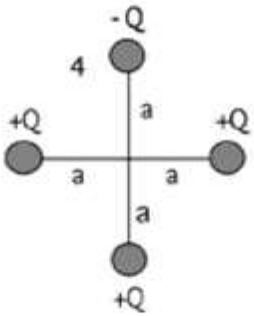
Assessing to Learn

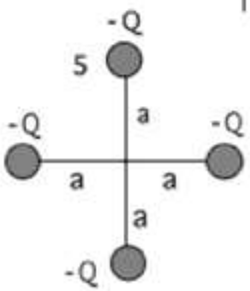
Which of the following charge distributions has the lowest potential energy?

1.


2.


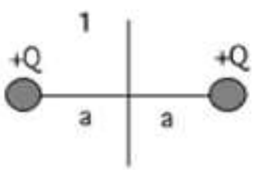
3.


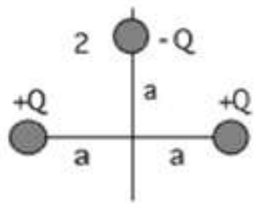
4.


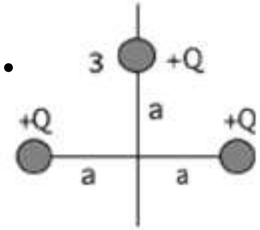
5.


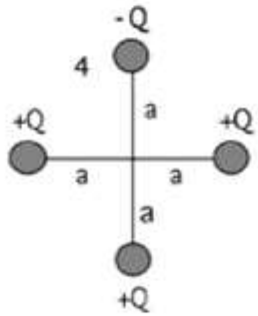
Assessing to Learn

Which of the following charge distributions has the lowest electric potential at the origin?

1. 

2. 

3. 

4. 

5.