CSE 2321 Foundations I Spring 2024 Dr. Estill

Homework 2 Due: Friday, February 2

1.) (5 pts each) For each of the following propositions (also known as"Boolean expressions"), show, by means of a truth table, whether it is a *tautology* (i.e., it's always true), a *contradiction* (it's never true), or a *contingency* (its truth changes, depending on the truth of the variables). There is no guarantee that all three types will be present.

(a) $\neg(P \wedge Q) \Leftrightarrow (\neg P \wedge \neg Q)$
(b) $\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$
(c) $((P \wedge Q) \Leftrightarrow Q) \Leftrightarrow (Q \Rightarrow P)$
(d) $((P \Rightarrow Q) \Rightarrow R) \oplus ((P \wedge \neg Q) \vee R)$
(e) $((P \vee Q) \wedge R) \Rightarrow (P \wedge Q) \vee (P \wedge R)$
(f) $((P \vee Q) \Rightarrow (P \wedge Q)) \wedge (P \oplus Q)$

Reminder: $\oplus$ is the exclusive or from the first homework.

2.) (10 pts each) Use truth tables to decide whether each of the following arguments are valid or invalid. If it is invalid, what is the truth values of the atomic propositions that show that the argument is invalid? (I.e., $P = 0, A = 1$, and so on is the row that isn't crossed out or with a true conclusion.)

(a)
$$\begin{array}{l} P \Rightarrow Q \\ Q \Rightarrow R \\ \hline \therefore P \Rightarrow R \end{array}$$

(b)
$$\begin{array}{l} (A \vee B) \Rightarrow C \\ A \vee \neg C \\ B \vee \neg C \\ A \Leftrightarrow \neg B \\ \hline \therefore C \wedge \neg C \end{array}$$

(c)
$$\begin{array}{l} (X \vee Y) \wedge (X \vee Z) \\ Y \wedge Z \\ \hline \therefore X \end{array}$$

3.) (25 points) We will define the binary Boolean function NOR, written $\downarrow$, as in the following truth table

P	Q	$P \downarrow Q$
0	0	1
0	1	0
1	0	0
1	1	0

Notice that $P \downarrow Q \equiv \neg(P \vee Q)$. Prove that {NOR} is universal.

4.) (15 points) Let $U = \{a, b, c, d\}$ and $V = \{b, d, f\}$. Remember that $\mathscr{P}(S)$ is the power set of the set S, i.e. the set of all subsets of S.

(a) What are the elements of $\mathscr{P}(U) \cap \mathscr{P}(V)$?

(b) What is the cardinality of $\mathscr{P}(U \cup V)$?

(c) What is $|\mathscr{P}(U) \cup \mathscr{P}(V)|$?