**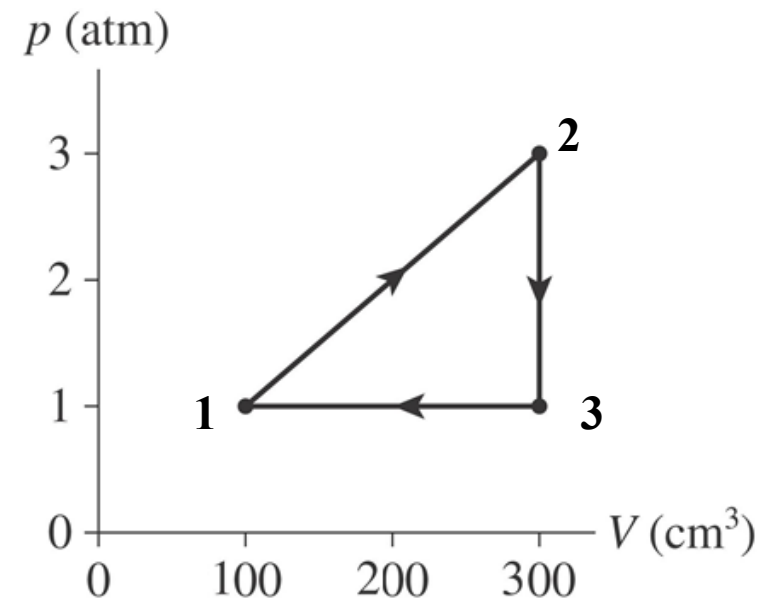Quantitative example**. 0.005 mol of a diatomic ideal gas is brought from state (1) $V_1$= 100 cm³ and $P_1$=1.0 atm to state (2) $V_2$=300 cm³ and $P_2$=3.0 atm.

a)  Draw a PV diagram for this process (assume it follows a straight line). Find Q for this process 1-2.

*Use 1 atm = 1×10⁵ Pa to simplify calculation.*

| States | P | V | T |
|--------|-----|----------|-------|
| 1 | 1 atm | 100 cm³ | $T_1$ |
| 2 | 3 atm | 300 cm³ | $T_2$ |

| Processes | Q | W | ΔU |
|-----------|---|---|-----|
| 1→2 | | | |
| | $\Delta U = Q + W$ | $W = -\int_1^2 P dV$ | $\Delta U = n C_V \Delta T$ |



*Work = −area under the curve*

$$R = 8.31 \, \text{J/mol} \cdot \text{K} \qquad R = 0.082 \, \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}$$
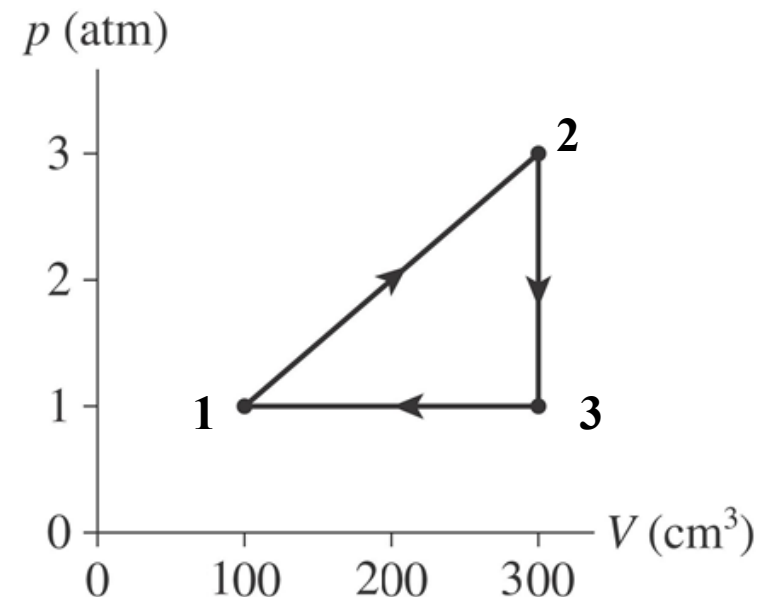
**Quantitative example**. 0.005 mol of a diatomic ideal gas is brought from state (1) $V_1$= 100 cm³ and $P_1$=1.0 atm to state (2) $V_2$=300 cm³ and $P_2$=3.0 atm.

a) Draw a PV diagram for this process (assume it follows a straight line). Find Q for this process 1-2.

*Use 1 atm = $1\times10^5$ Pa to simplify calculation.*

| States | P | V | T |
|--------|------|---------|----------|
| 1 | 1 atm | 100 cm³ | 243.90K |
| 2 | 3 atm | 300 cm³ | 2195.12K |

| Processes | Q | W | ΔU |
|-----------|--------|----------------------------|-------------------------|
| 1→2 | 242.7 J | -40 J | 202.7 J |
| | $\Delta U = Q + W$ | $W = -\int_1^2 P dV$ | $\Delta U = nC_V \Delta T$ |



$p$ (atm)

$V$ (cm³)

$R = 8.31\,\text{J/mol}\cdot\text{K}$ 　　　 $R = 0.082\,\dfrac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}$

**Quantitative example**. 0.005 mol of a diatomic ideal gas is brought from state (1) $V_1=$ 100 cm³ and $P_1=1.0$ atm to state (2) $V_2=300$ cm³ and $P_2=3.0$ atm.
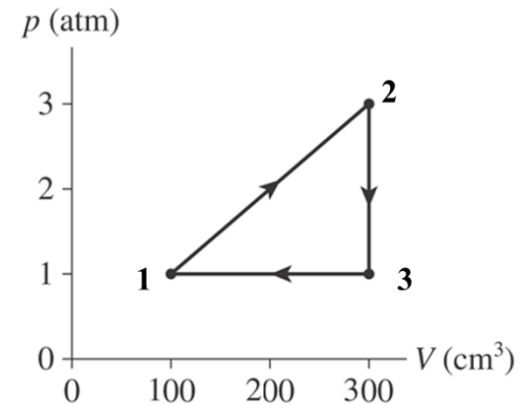
The gas is then brought to state (3) $V_3=300$ cm³ and $P_3=1.0$ atm and then back to state (1).

b) Complete the PV diagram for this whole cycle.

c) Find W, Q and ΔU for processes 2-3 and 3-1, and for the whole cycle.

| States | P | V | T |
|--------|-------|---------|----------|
| 1 | 1 atm | 100 cm³ | 243.90K |
| 2 | 3 atm | 300 cm³ | 2195.12K |
| 3 | 1 atm | 300 cm³ | |



p (atm)

V (cm³)

| Processes | Q | W | ΔU |
|-----------|--------|-------|---------|
| 1→2 | 242.7 J | -40 J | 202.7 J |
| 2→3 | | 0 | |
| 3→1 | | | |
| Cycle | | | |

$$R = 8.31 \text{ J/mol} \cdot \text{K}$$

$$R = 0.082 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}$$

$$\Delta U = Q + W \qquad W = -\int_1^2 P dV \qquad \Delta U = nC_V \Delta T \qquad\qquad \Delta U = nC_V \Delta T \qquad \Delta U = Q + W$$

**Quantitative example**. 0.005 mol of a diatomic ideal gas is brought from state (1) $V_1= 100$ cm$^3$ and $P_1=1.0$ atm to state (2) $V_2=300$ cm$^3$ and $P_2=3.0$ atm.
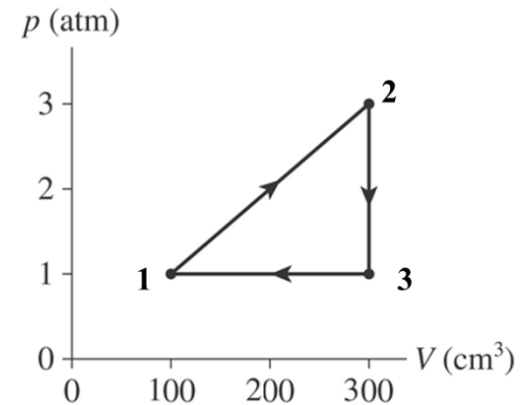
The gas is then brought to state (3) $V_3= 300$ cm$^3$ and $P_3=1.0$ atm and then back to state (1).

b) Complete the PV diagram for this whole cycle.

c) Find W, Q and $\Delta U$ for processes 2-3 and 3-1, and for the whole cycle.

| States | P | V | T |
|--------|------|---------|---------|
| 1 | 1 atm | 100 cm$^3$ | 243.90K |
| 2 | 3 atm | 300 cm$^3$ | 2195.12K |
| 3 | 1 atm | 300 cm$^3$ | 731.70 K |

| Processes | Q | W | $\Delta U$ |
|-----------|----------|---------|-----------|
| 1→2 | 242.7 J | -40 J | 202.7 J |
| 2→3 | -152.1 J | 0 | -152.1 J |
| 3→1 | -70.6 J | 20 J | -50.6 J |
| Cycle | 20 J | -20 J | 0 |



p (atm)

$R = 8.31$ J/mol·K

$R = 0.082 \dfrac{L \cdot atm}{mol \cdot K}$

$$\Delta U = Q+W \qquad W = -\int_1^2 PdV \qquad \Delta U = nC_V \Delta T \qquad\qquad \Delta U = nC_V\Delta T \qquad \Delta U = Q+W$$

Consider this complete cycle (1 → 2 → 3 → 1), the work done on the gas from the exerted force is negative. The heat transfer to the gas is positive.

From the external view point, the work done to the external world is Positive!

That is, the external world can give heat to the gas and make it run in cycles. The gas can then do work to the external world – so this is a "heat engine".

| Processes | Q | W | ΔU |
|---|---|---|---|
| 1→2 | 242.7 J | -40 J | 202.7 J |
| 2→3 | -152.1 J | 0 | -152.1 J |
| 3→1 | -70.6 J | 20 J | -50.6 J |
| Cycle | 20 J | -20 J | 0 |

Work done by the gas to the external world is 20 J.
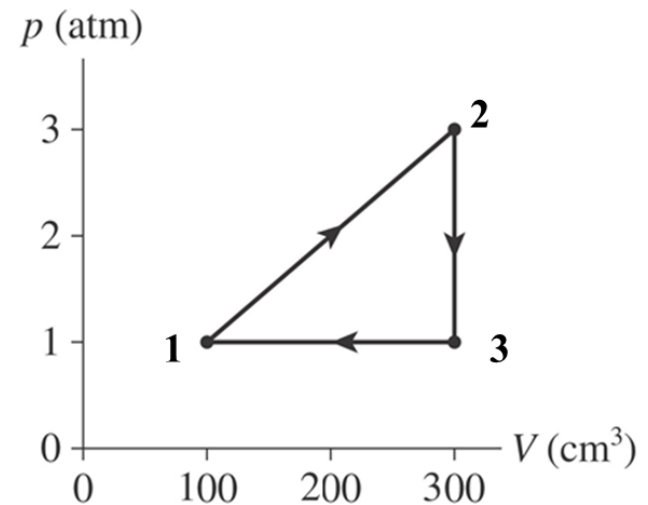
$$efficiency = \frac{|W|}{Heat}$$

Heat engine: A cycle of thermal processes that produces work in exchange for heat.

# Another way to write 1st Law of Thermodynamics
## Heat Engine

$$\Delta U = W_{\text{onGas}} + Q$$

$$Q = W_{\text{by gas}} + \Delta U$$

Any energy transferred into a system as **heat** is either used to **do work** or **stored within the system as an increased internal energy**.
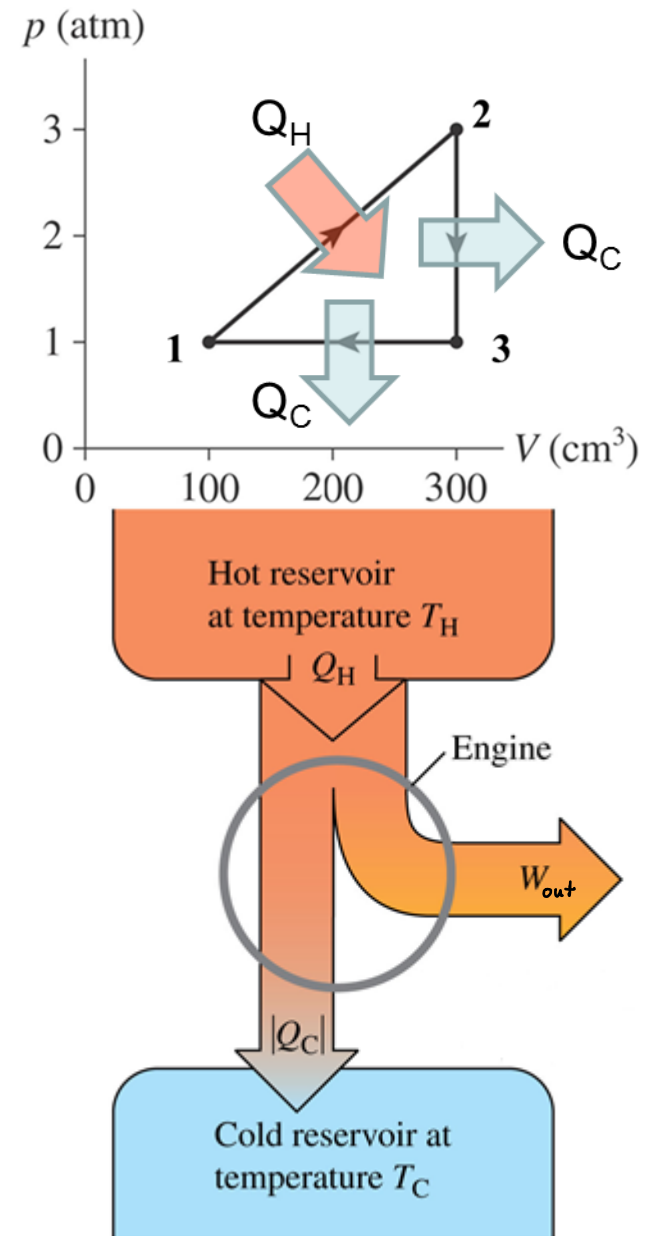
# Heat Engine

- PV diagram
  - Clockwise cycle: heat engine
  - Counterclockwise cycle: refrigerator
- Schematic representation
  Energy-flow diagram

$$W_{out} = Q_H - Q_C$$

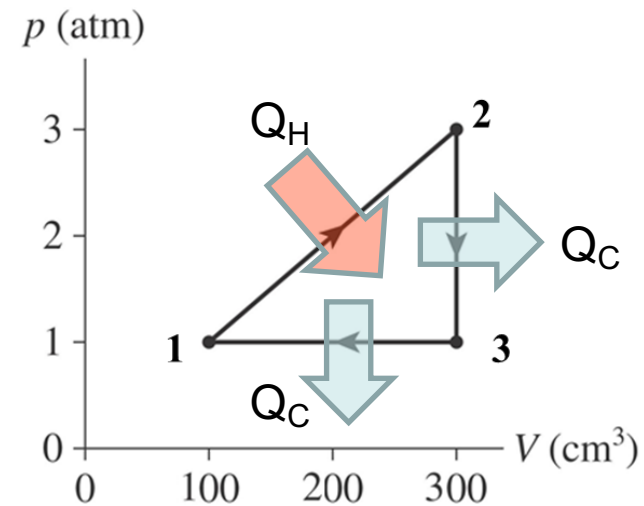- Efficiency: Fraction of heat you put in that's converted to work

$$e = \frac{W_{out}}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

# Thermal Efficiency

$$e = \frac{W_{out}}{Q_H} = \frac{\text{Work output}}{\text{Heat supplied}} = \frac{\text{What you get}}{\text{What you had to pay}}$$

| Processes | Q | W | $\Delta U$ |
|-----------|---|---|-----|
| 1→2 | 242.7 J | -40 J | 202.7 J |
| 2→3 | -152.1 J | 0 | -152.1 J |
| 3→1 | -70.6 J | 20 J | -50.6 J |
| Cycle | 20 J | -20 J | 0 |



Efficiency?     e = 20/242.7=8.2 %

$$e = \frac{W_{out}}{Q_H} = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \frac{|Q_C|}{|Q_H|}$$

What is the thermal efficiency of this heat engine?
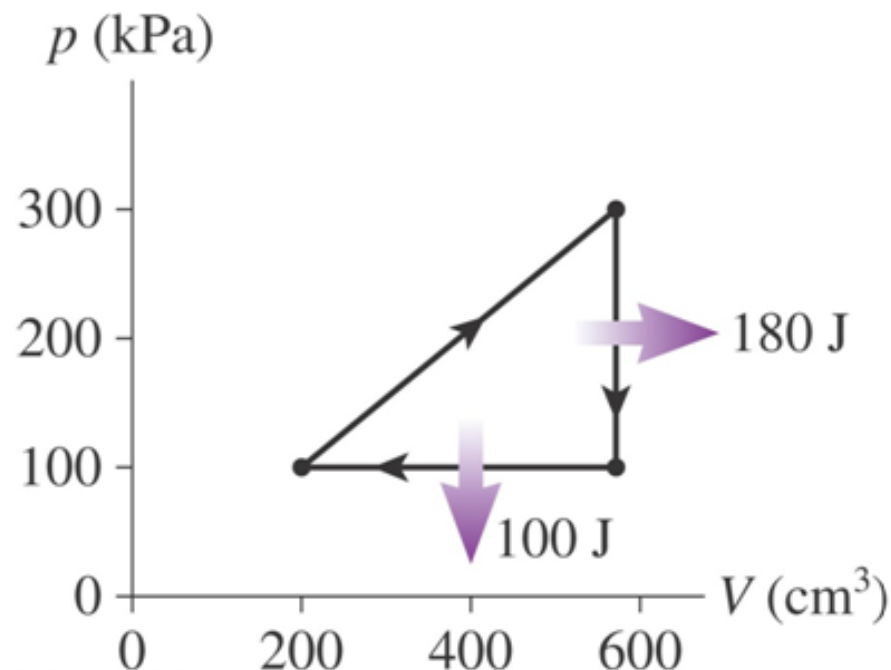
A. 10.0%
B. 12.5%
C. 14.5%
D. 40.0%
E. 55.5%
F. None of the above.



$$e = \frac{W_{out}}{Q_H} = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \frac{|Q_C|}{|Q_H|}$$

What is the thermal efficiency of this heat engine?

A. 10.0%

B. 12.5%

C. 14.5%

D. 40.0%

E. 55.5%

F. None of the above.

$p$ (kPa)

300

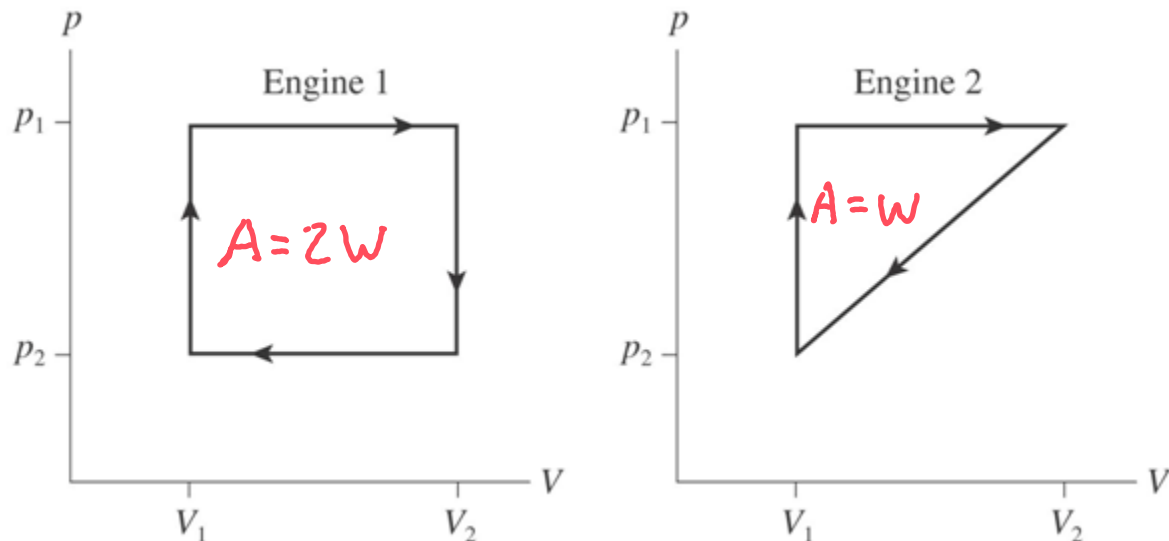200 — 180 J

100

100 J

0

0 200 400 600

$V$ (cm³)
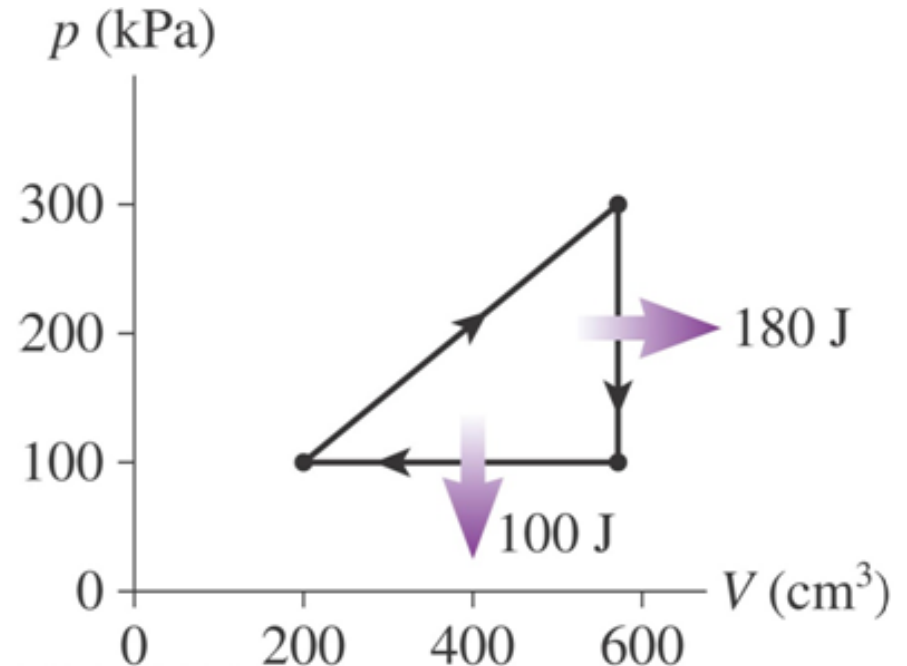
$W = area\ of\ triangle$

$= \dfrac{1}{2} \times 400 \times 10^{-6} \times 200 \times 10^{3} = 40J$

$Q_H - Q_C = W \implies Q_H = 40 + 180 + 100 = 320J$

$e = \dfrac{W}{Q_H} = \dfrac{1}{8}$

$e = \dfrac{W_{out}}{Q_H} = \dfrac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \dfrac{|Q_C|}{|Q_H|}$

Which of the heat engine has the larger thermal efficiency?



Engine 1

$A = 2W$

Engine 2

$A = W$

$e = \dfrac{W}{Q_H}$

A. Engine 1
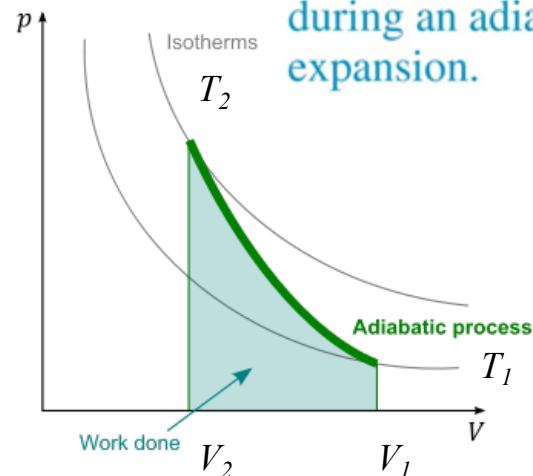B. Engine 2
C. The same
D. Can't tell
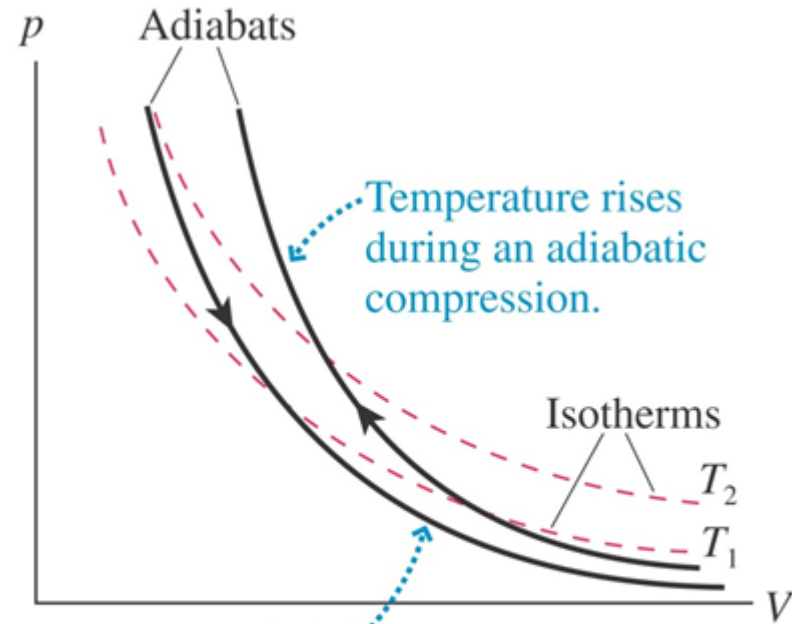
# Adiabatic Process

$$\Delta U = W_{onGas} + Q = W$$

$$W = nC_V \Delta T$$

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

$$P_f V_f^{\gamma} = P_i V_i^{\gamma}$$



$p$

Adiabats

Temperature rises during an adiabatic compression.

Isotherms

$T_2$

$T_1$

$V$

Temperature falls during an adiabatic expansion.



$p$

Isotherms

$T_2$

Adiabatic process

$T_1$

$V$

Work done    $V_2$    $V_1$

In the "compression stroke" of an internal combustion engine, air-fuel mixture at 20 °C and 1.0 atm is compressed until its volume is 1/10 of its original volume. The compression is so rapid that it can be assumed to be adiabatic. Find the final pressure and temperature.

$$Q = 0$$

| States | P | V | T |
|--------|-------|-----------|----------|
| $i$ | 1 atm | $V_0$ | 293.15K |
| $f$ | ? | $V_0/10$ | ? |
|  |  |  |  |

$$P_f V_f^{\gamma} = P_i V_i^{\gamma}$$

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$
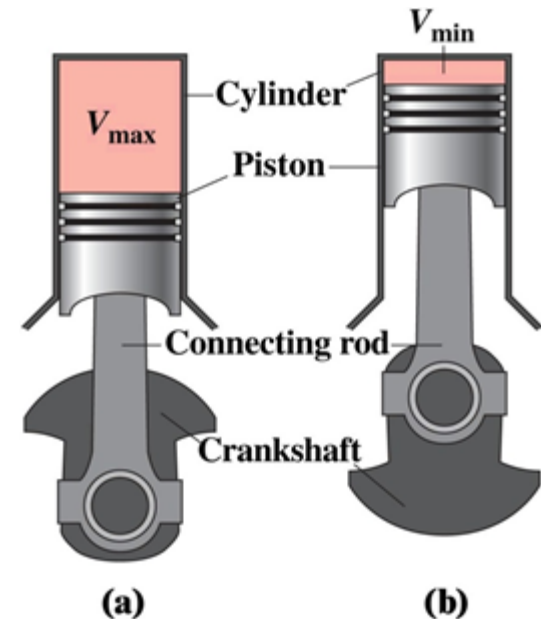


$V_{min}$

Cylinder

$V_{max}$

Piston

Connecting rod

Crankshaft

(a)                    (b)

In the "compression stroke" of an internal combustion engine, air-fuel mixture at 20 °C and 1.0 atm is compressed until its volume is 1/10 of its original volume. The compression is so rapid that it can be assumed to be adiabatic. Find the final pressure and temperature.

Treat the gas mixture as approximately diatomic gasses:

$$C_V = \tfrac{5}{2}R \quad C_P = \tfrac{7}{2}R$$

$$\gamma = \tfrac{7}{5} = 1.4$$
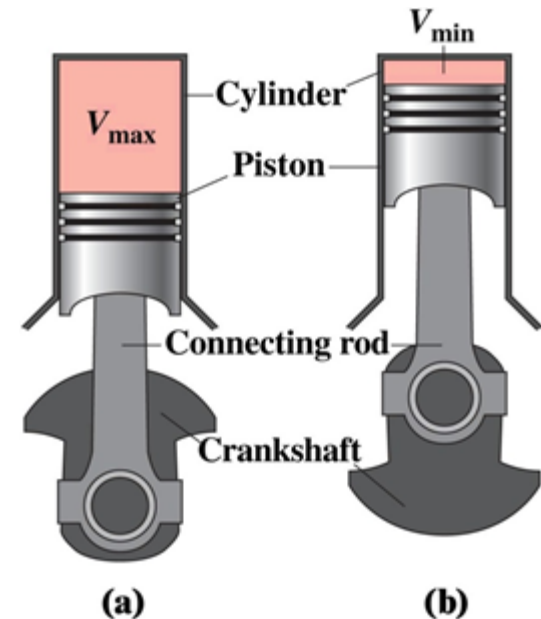
$$P_f V_f^{\gamma} = P_i V_i^{\gamma}$$

$$\frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^{\gamma} = 10^{\gamma} = 10^{1.4} = 25.12$$
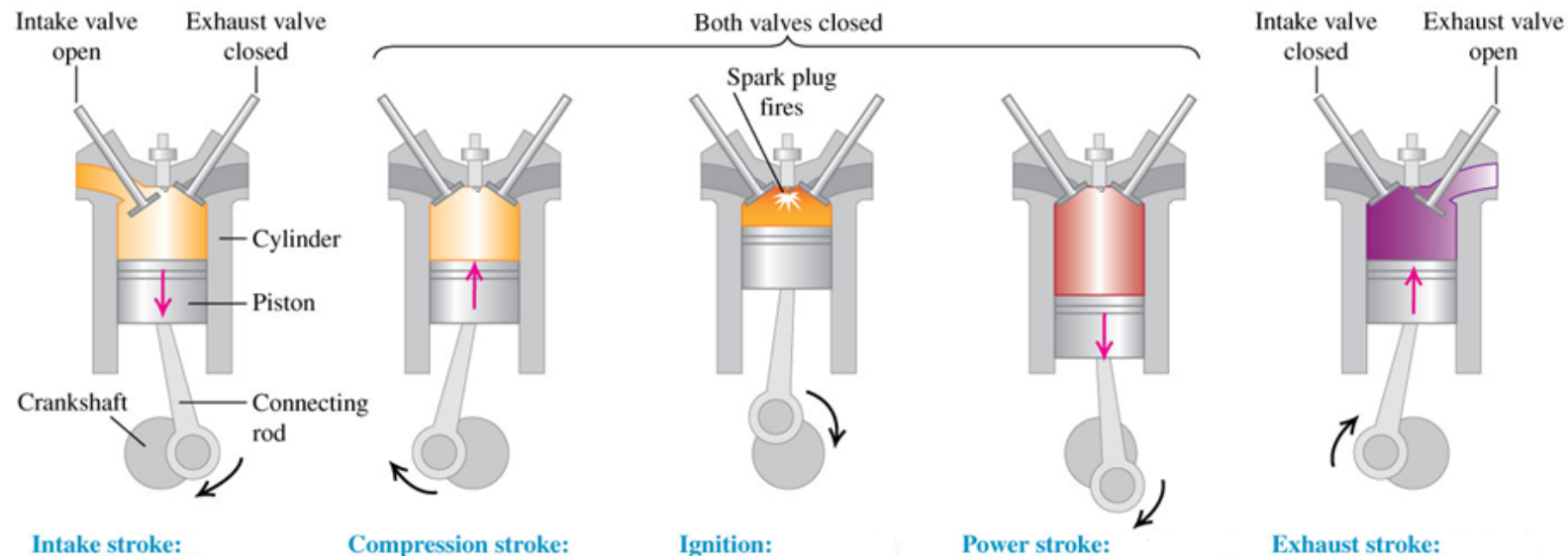
$$P_f = 25.12 \; atm$$

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

$$\frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} = 10^{\gamma-1} = 10^{0.4} = 2.51$$

$$T_f = T_i \times 2.51 = 293.15 \times 2.51 = 736 \; K°$$



$V_{max}$   $V_{min}$   Cylinder   Piston   Connecting rod   Crankshaft

(a)    (b)

# Practical Heat Engine: Otto Cycle (ICE)



Intake valve open
Exhaust valve closed

Both valves closed

Spark plug fires

Intake valve closed
Exhaust valve open

Cylinder
Piston
Crankshaft
Connecting rod

Intake stroke:
Compression stroke:
Ignition:
Power stroke:
Exhaust stroke:

Process
ab: adiabatic compression
bc: isochoric heating
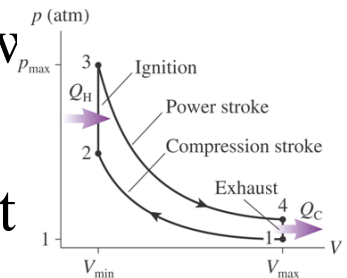cd: adiabatic expansion
da: isochoric cooling
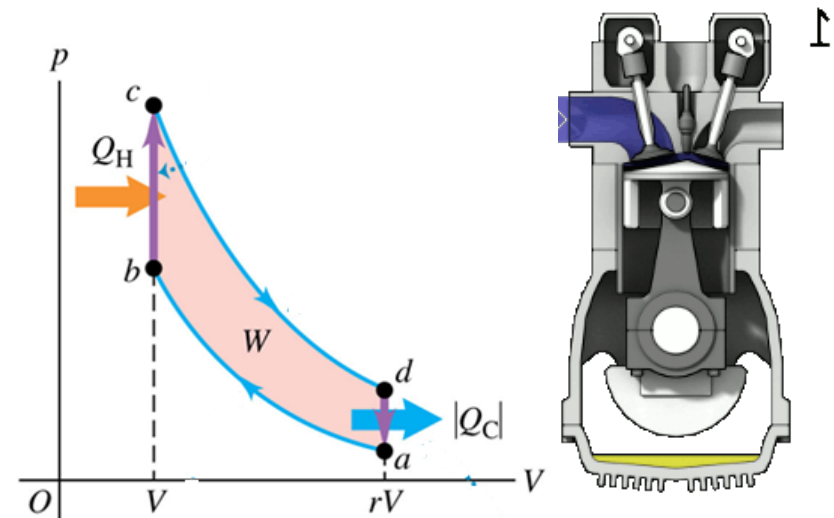
# Gasoline Engine—Otto Cycle

A fuel-air mixture is sprayed into the cylinder at point 1, where the piston is at its farthest distance from the spark plug. This mixture is compressed as the piston moves tov the spark plug during the adiabatic *compression stroke.*

The spark plug fires at point 2, releasing heat energy that been stored in the gasoline. The fuel burns so quickly that the piston doesn't have time to move, so the heating is an isochoric process.

The hot, high-pressure gas then pushes the piston outward during the *power stroke.*

Finally, an exhaust value opens to allow the gas temperature and pressure to drop back to their initial values before starting the cycle over again.

# A Heat Engine Example



Isobaric heating and expansion

Heat in

Gas

*M*

*M*

Heat in

Pin

*M*

$\vec{F}_{\text{ext}}$

Heat out

Isothermal compression

Heat out

Constant-volume cooling

*p*

Isotherm

$Q_{\text{H}}$

The mass is lifted in an isobaric expansion.

Mass removed

1

2

$Q_{\text{C}}$

The gas undergoes constant-volume cooling.

$Q_{\text{C}}$

3

*V*

An external force compresses the gas back to its initial conditions.