Electrons obey Fermi-Dirac Statistics: (Fact without proof)

1) Pauli Exclusion Principle holds

2) All e^- particles are indistinguishable

3) Wave Nature

Distribution of electrons over a range of allowed energy levels at thermal equilibrium is:

$f(E) =$

Probability function between 0 and 1

Absolute temperature T

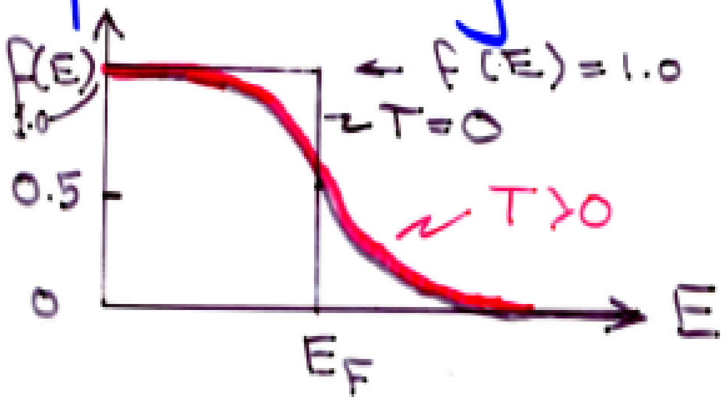
E_F = Fermi level

k_B = Boltzmann's constant
$= 1.38 \times 10^{-23}$ J/K atom
$= 8.62 \times 10^{-5}$ eV/atom K

VERY IMPORTANT QUANTITY

E_F defined by $f(E=E_F) = \frac{1}{1+e^{(E_F-E_F)/kT}} = \frac{1}{2}$

This means a state whose E has a 50% probability of containing an electron.



E_F stays constant with different T since thermal e^- = thermal h^+.

At T=0, all energy states $E<E_F$ filled; all states $E>E_F$ empty

At T>0, some probability that states $E<E_F$ are empty

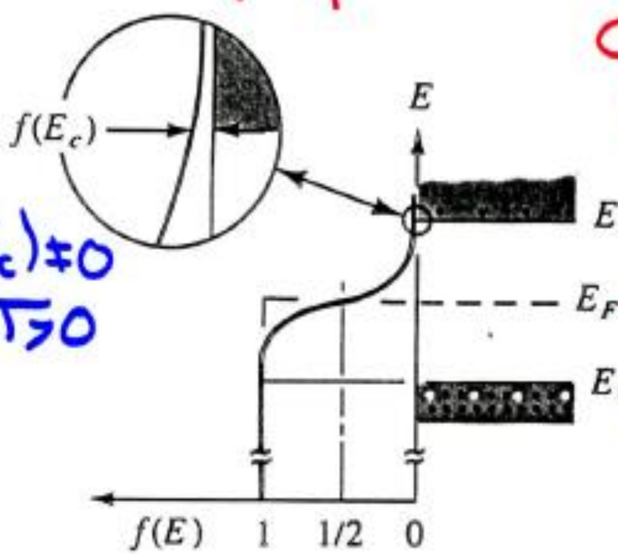
" " " " $E>E_F$ are filled.

Apply $f(E)$ to all possible states.
How many of these states have electrons or holes?

Turn $f(E)$ graph on its side to visualize overlap of $f(E)$ with bands and allowed states.

Can only have occupancy if there is a state at E to occupy.

$f(E_c) \neq 0$ at $T>0$



(a) Intrinsic

E_c ← some electrons in conduction band at $T>0$.

E_F ← E_F in middle of gap

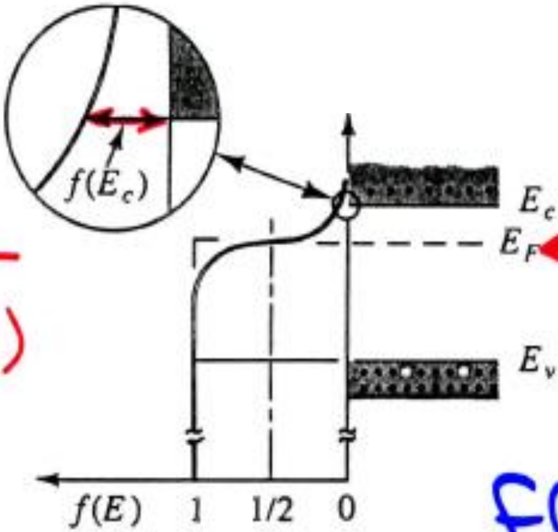
E_v ← some holes in valence band at $T>0$

$1-f(E) \neq 0$. (probability of holes)

$= 1$

Since $n_0 = p_0$, must have equal probabilities of finding electrons and holes.

Larger $f(E_c)$



(b) n-type

E_F now higher

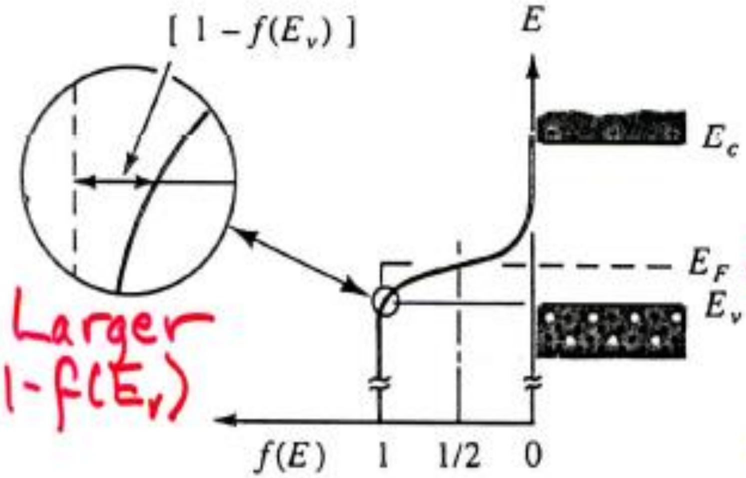
$n_o >> p_o$ more electrons in CB than holes in VB

$n_o >> n_i$

$f(E)$ retains its shape at a given temperature; just shifts <u>up</u>.

$1-f(E)$ below E_v now larger than $f(E)$ above E_c.

Larger $1-f(E_v)$



(c) p-type

E_F now lower

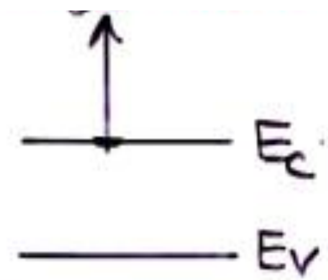
$p_o >> n_o$ more holes in VB than electrons in CB

$p_o >> n_i$

$f(E)$ retains its shape at a given Temperature; just shifts <u>down</u>.

Calculation of electron and hole concentrations at equilibrium

$$n_0 = \int_{E_c}^{\infty} f(E)\, N(E)\, dE$$



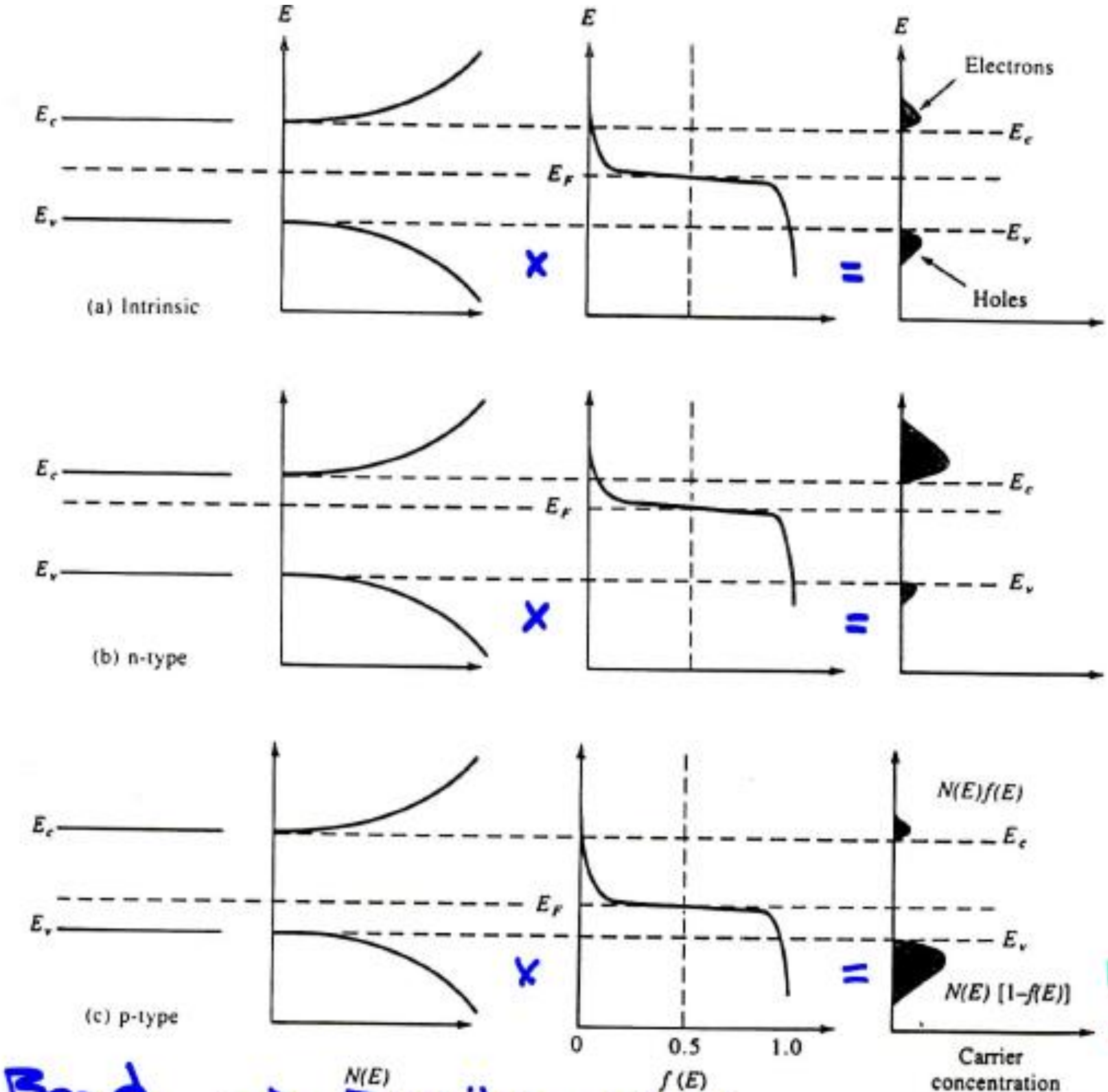
= probability of occupancy per state · Total # of states / energy-volume

integrated over all energies from E_c on up.

= Total # of filled states in conduction band

$N(E)dE$ = density of states in energy interval dE
(concentration per dE and per cm^{-3})

$$N(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2} E^{1/2}$$



$n_0 = p_0 = n_i$

$n_0 \gg p_0$

$p_0 \gg n_0$

Checkout:
http://jas2.eng.buffalo.edu/applets/index.html

Band Structure → Density of states × Fermi Distribution = carrier concentration

$n_0 = N_C f(E_c)$ "effective" density of states and integrating above for N_C

$$= \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}\int_{E_c}^{\infty} \frac{E^{1/2}\, dE}{1+e^{(E-E_F)/kT}}$$

$$f(E_c) \cong \frac{1}{1+e^{(E_c-E_F)/kT}} \approx e^{-(E_c-E_F)/kT}$$

for $E_c - E_F \gg kT$

In general, a good approximation

in general

$3kT = 3 \cdot 0.026\,eV$ at room temp.

E_c, E_F: $> 3kT$

where $N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$ = effective density of states in conduction band

Likewise, for holes, $p_o = \int_{-\infty}^{E_v} [1 - f(E)]\, N(E)\, dE$

$$p_o = N_v [1 - f(E)]$$ (probability of state being empty × # of states)

in general

where $N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$ = effective density of states in valence band

For intrinsic material, $E_F = E_i$ since expressions for n_o and p_o valid, whether or not "doped."

So $n_i = N_c e^{-(E_c - E_i)/kT} = N_v e^{-(E_i - E_v)/kT} = p_i$

$$n_i p_i = N_c N_v e^{-(E_c - E_v)/kT}$$

$$= N_c N_v e^{-E_g/kT} =$$

From general expressions for n_o and p_o,

$$n_o p_o = N_c N_v e^{-E_g/kT}$$

so

Important - We use extensively!

Law of Mass Action (in equilibrium)

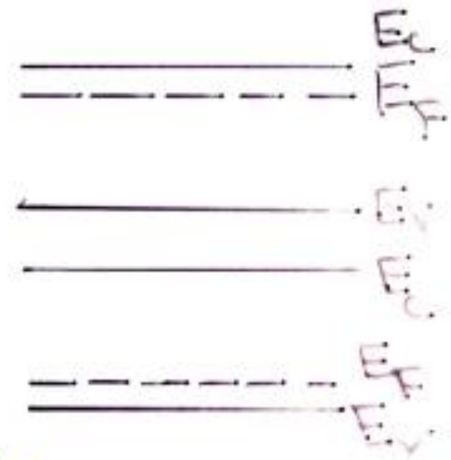
Can also write:

$n_o =$

$p_o =$

For $E_F > E_i$, $n_o \gg p_o$ → n-type

For $E_F < E_i$, $n_o \ll p_o$ → p-type



$n_i =$

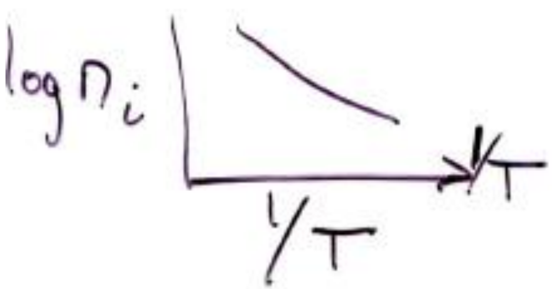
function of purely materials parameters

Example: Si — $E_g = 1.12\,eV$ at 300K

$m_n^* = 1.1\,m_o$

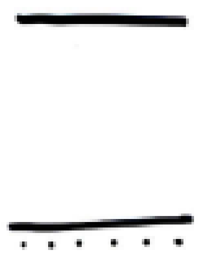
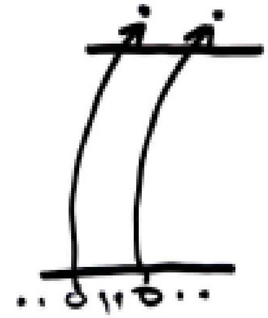
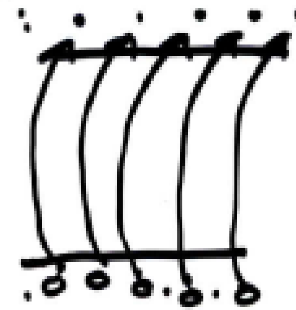
$m_p^* = 0.56\,m_o$

$n_i(300K) = 1.5 \times 10^{10}\,cm^{-3}$ for Si



very T-dependent

Intrinsic Semiconductor : n_i versus T

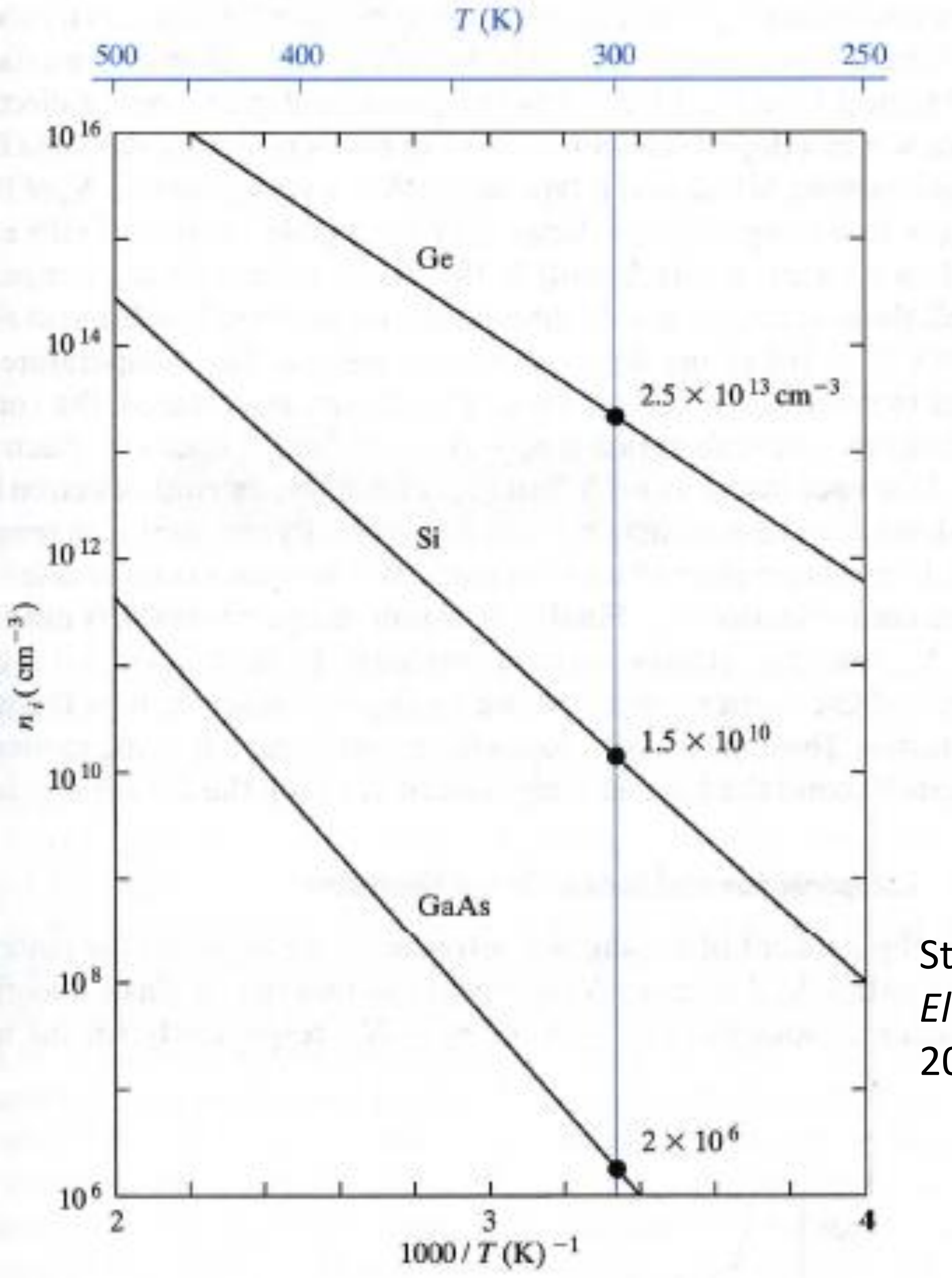
Low T:  Mid T:  High T: 

Intrinsic Semiconductor: n_i versus E_g

Larger E_g, lower n_i

Example: GaAs – $E_g = 1.42\,eV$, $n_i = 2 \times 10^6\,cm^{-3}$

Ge – E_g $0.67\,eV$, $n_i = 2.5 \times 10^{13}\,cm^{-3}$



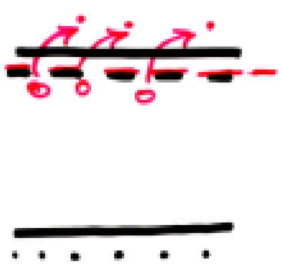
Streetman & Banarjee, *Solid State Electronic Devices*, 7th ed. (Pearson, 2015) p. 98.

Now consider Extrinsic Semiconductor

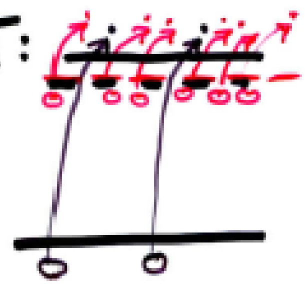
Donors = $N_d \longrightarrow N_d^{+} + e^{-}$

Acceptors = $N_a \longrightarrow N_a^{-} + h^{+}$

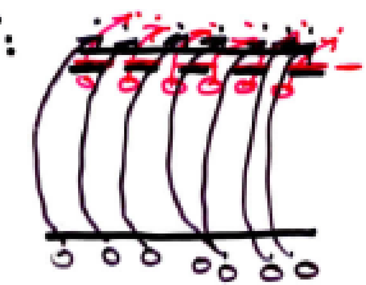
Low T:



Mid T:



High T:



Partial ionization of donors

$n_o = N_d^{+} + n_i$

$n_i \sim N_d^{+}$

Complete ionization of donors

$N_d \gg n_i$

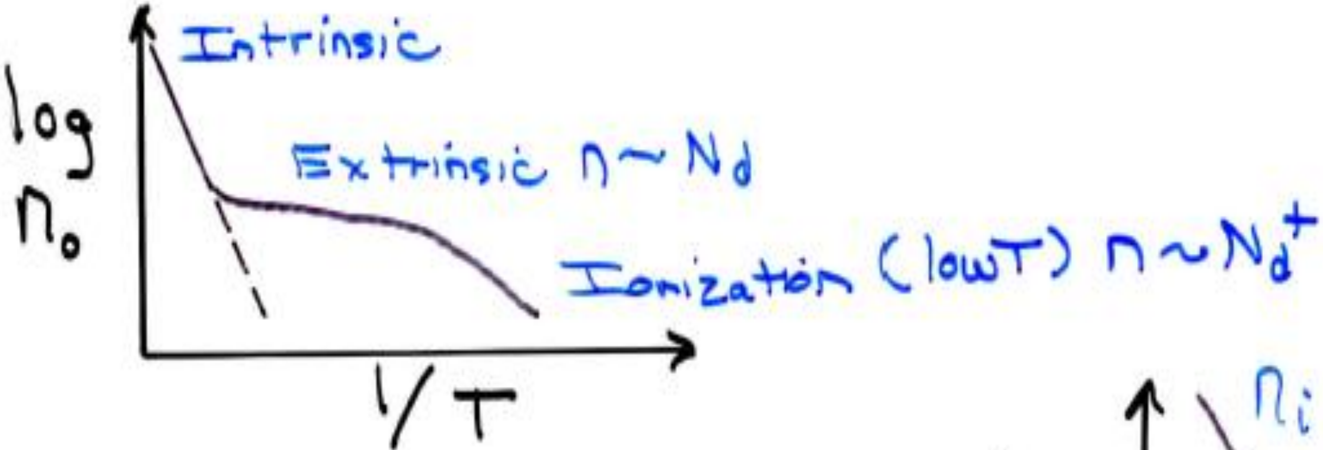
Extrinsic dominates

$n_o = N_d + n_i(T)$

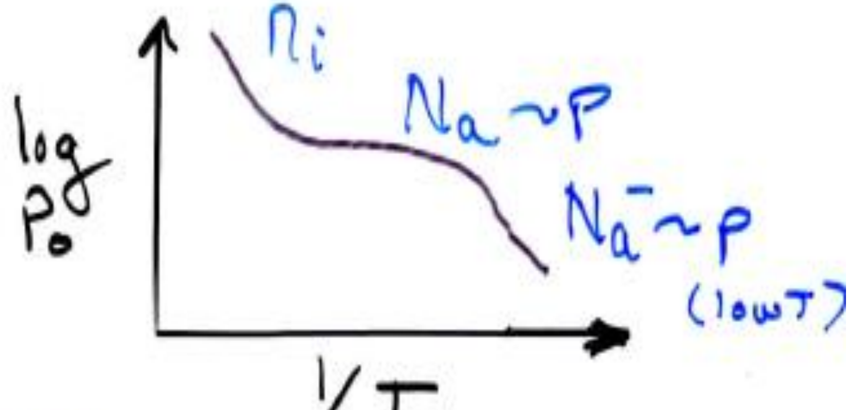
$n_i \gg N_d$

Intrinsic dominates

$n_o = N_d + n_i(T)$



Also:



We will assume complete ionization unless otherwise stated.

i.e., $N_d = N_d^+$ and $N_a = N_a^-$

n-type Semiconductor :

$n_o \gg n_i$, $n_o \gg p_o$

$n_o = N_d$

are the majority carriers

$p_o = n_i^2/n_o = n_i^2/N_d$

are the minority carriers

p-type semiconductor :

$p_o \gg n_i$, $p_o \gg n_o$

$p_o = N_a$

are majority carriers

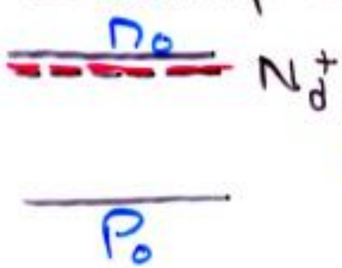
$n_o = n_i^2/p_o = n_i^2/N_a$

are minority carriers

In extrinsic region

In general, must have charge neutrality for equilibrium : + charges = − charges at any point

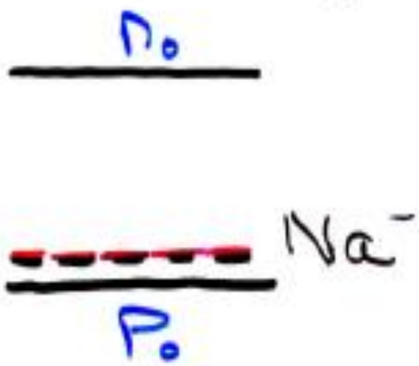
Example: n-type semiconductor



$$\Rightarrow n_o = p_o + N_d^+ \simeq p_o + N_d$$

(full ionization)

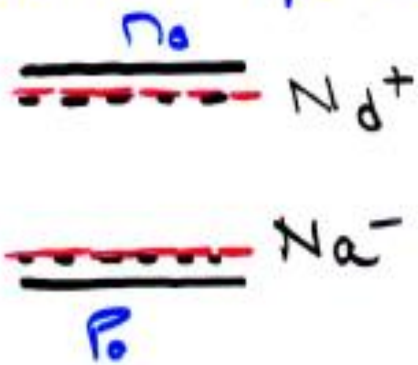
and, for p-type semiconductor,



$$\Rightarrow \quad p_0 = N_a^- + n_0 \simeq N_a + n_0$$

(full ionization)

What if we have <u>both</u> donors <u>and</u> acceptors?



$$p_0 + N_d^+ = n_0 + N_a^-$$

(full ionization)

Charge Neutrality Law

3 cases:

1) $N_a >> N_d$

$p_o = n_o + (N_a - N_d) \sim N_a$ still p-type

and $n_o = n_i^2 / N_a$

2) $N_d >> N_a$

$n_o = p_o + (N_d - N_a) \sim N_d$ still n-type

and $p_o = n_i^2 / N_d$

3) $N_d \sim N_a$

They compensate each other.

Example: If $N_a = N_d$, then $n_o = p_o$

Then perfectly compensated.

In general, $n_o + N_a = p_o + N_d$

and for $N_a \sim N_d$, substitute $n_o p_o = n_i^2$

and solve:

$N_d > N_a$: $$n_o + N_a = \frac{n_i^2}{n_o} + N_d$$

Put in terms of n_o and <u>not</u> p_o since expect n_o larger than p_o.

$$n_o^2 + n_o(N_a - N_d) - n_i^2 = 0$$

First, find n_o

Then, find $p_o = n_i^2/n_o$

$N_a > N_d$: $\quad p_o + N_d = \frac{n_i^2}{p_o} + N_a$

put in terms of p_o (<u>not</u> n_o) since expect p_o larger

$$p_o^2 + p_o(N_d - N_a) - n_i^2 = 0$$

First, find p_o

Then find $n_o = n_i^2/p_o$