Homework 7 - Math 2568 (Autumn 2022)

Prof. Cueto

Due date: Friday October 28, 2022 (in class).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§3.7	1, 2, 3, 4, 5, 7, 8, 10, 11, 15, 18, 19, 20, 21, 23, 25, 29, 33, 35, 36, 37, 41	2, 4, 7, 10, 11, 18, 19, 21, 23, 33, 35, 36, 37, 41
§5.2	1, 2, 3, 6, 9, 10, 19, 25, 29, 31, 33, 36	1, 2, 10, 19, 29, 33

Section 3.7

2) $A = \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix}$

a) $T\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2-2 \\ -6+6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$T\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) $T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3-1 \\ -9+3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

$T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

c) $T\left(\begin{bmatrix}2\\0\end{bmatrix}\right) = \begin{bmatrix}1 & -1\\-3 & 3\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}2+0\\-6+0\end{bmatrix} = \begin{bmatrix}2\\-6\end{bmatrix}$

$T\left(\begin{bmatrix}2\\0\end{bmatrix}\right) = \begin{bmatrix}2\\-6\end{bmatrix}$

d) $T\left(\begin{bmatrix}0\\0\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix}$

4) $\begin{bmatrix}2x_1 - 3x_2\\-x_1 + x_2\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$

$2x_1 - 3x_2 = 2 \rightarrow x_1 = \frac{3}{2}x_2 + 1$

$-x_1 + x_2 = -2 \rightarrow x_2 = x_1 - 2$

$x_1 = \frac{3}{2}(x_1 - 2) + 1 = \frac{3}{2}x_1 - 2$

$-\frac{1}{2}x_1 = -2 \rightarrow \boxed{x_1 = 4}$

$x_2 = x_1 - 2 = 4 - 2 = 2$

$X = \begin{bmatrix}4\\2\end{bmatrix}$

7) $\begin{bmatrix}1 & -1\\-3 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$ $\quad \begin{bmatrix}x_1 - x_2\\-3x_1 + 3x_2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$

$x_1 - x_2 = 1 \rightarrow x_1 = x_2$

$-3x_1 + 3x_2 = 1 \rightarrow \boxed{-3x_1 + 3x_1 = 0 \neq 1}$

10) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ 1 \end{bmatrix}$ $\quad u+v = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix}$

$$F(u+v) = \begin{bmatrix} (u_1+v_1)+(u_2+v_2) \\ 1 \end{bmatrix} = \begin{bmatrix} u_1+v_1 \\ 1 \end{bmatrix} + \begin{bmatrix} u_2+v_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (u_1+v_1)+(u_2+v_2) \\ 2 \end{bmatrix}$$

Not a linear transformation

11) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1^2 \\ x_1 x_2 \end{bmatrix}$ $\quad F(u+v) = \begin{bmatrix} (u_1+v_1)^2 \\ (u_1+v_1)(u_2+v_2) \end{bmatrix}$

$$= \begin{bmatrix} u_1^2 + v_1^2 + 2u_1v_1 \\ u_1u_2 + u_1v_2 + v_1u_2 + v_1v_2 \end{bmatrix} = \begin{bmatrix} u_1^2 \\ u_1u_2 \end{bmatrix} + \begin{bmatrix} v_1^2 \\ v_1v_2 \end{bmatrix}$$

Not a linear transformation

18) $W = \left(x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, x_2 = x_3 = 0 \right)$ $\quad W = \begin{bmatrix} x_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$T(v) = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = a$$

W is a subspace of points on the x plane

V is a point (a,b,c) in $\mathbb{R}^3$

19) $u_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $u_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

a) $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = x_1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + x_2\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ x_2 \\ -x_1 \end{bmatrix}$ $x_1 = x_2 = 1$

$= \begin{bmatrix} 1+2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$

b) $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ x_2 \\ -x_1 \end{bmatrix} = \begin{bmatrix} 2 + (-2) \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}$ $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}$

c) $T\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ x_2 \\ -x_1 \end{bmatrix} = \begin{bmatrix} 3 + 4 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ -3 \end{bmatrix}$ $T\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 7 \\ 2 \\ 3 \end{bmatrix}$

21) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = a\begin{bmatrix} 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $x_1 = a + b$, $x_2 = a - b$ $\Rightarrow$ $a = \frac{x_1 + x_2}{2}$ $b = \frac{x_1 - x_2}{2}$

$x = \frac{x_1 + x_2}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{x_1 - x_2}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{x_1 + x_2}{2}\begin{bmatrix} 2 \\ -1 \end{bmatrix} + \frac{x_1 - x_2}{2}\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$\begin{bmatrix} x_1 + x_2 \\ -\frac{1}{2}x_1 - \frac{1}{2}x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{3}{2}x_1 + \frac{3}{2}x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 - 2x_2 \end{bmatrix}$ $T(x) = \begin{bmatrix} x_1 + x_2 \\ x_1 - 2x_2 \end{bmatrix}$