



# Lecture Outline

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## Reminders to self:

- ☐ Turn on lecture recording to Cloud
- ☐ Turn on Zoom microphone

## • Last Lecture

- Continued analysis & design of clocked sequential circuits
  - Finished Mealy 101 sequence detector design started last lecture
  - Analysis example (of Mealy sequence detector just designed)
  - State Graph design guidelines
  - Started a larger (8 state) design example (Mustang turn signals)

## • Today's Lecture

- Continue analysis & design of clocked sequential circuits
  - Finish the larger (8 state) design example (Mustang turn signals)
  - Five-variable K-Maps
  - Start larger Mealy design example



# Handouts and Announcements

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- Announcements
  - Homework Problems: No new assignment
  - Homework Reminder
    - HW 13-1 Due: 11:25am Wednesday 3/29
    - HW 13-2 Due: 11:59pm Thursday 3/30
  - Read for Wednesday: no new reading assignment  
previous assignment pages 463-472, 149-151



# Handouts and Announcements

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- Announcements

- Mini-Exam 5 Reminder

- Available 5pm Monday 3/27 through 5:00pm Tuesday 3/28
- Due in Carmen PROMPTLY at 5:00pm on 3/28
- Designed to be completed in ~36 min, but you may use more
- When planning your schedule:
  - I recommend building in 10-15 min extra
  - To allow for downloading exam, signing and dating honor pledge, saving solution as pdf, and uploading to Carmen
- I also recommend not procrastinating

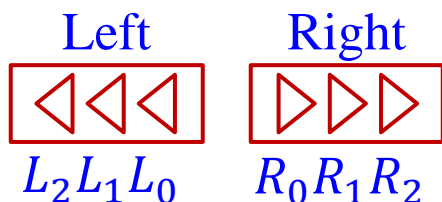
- Exam review topics available on Carmen

- Sample Mini-Exams 6 and 7 from Au20 also available



# Design Example: Mustang Sequential Turn Signals

Where we left off  
last lecture



← The six outputs

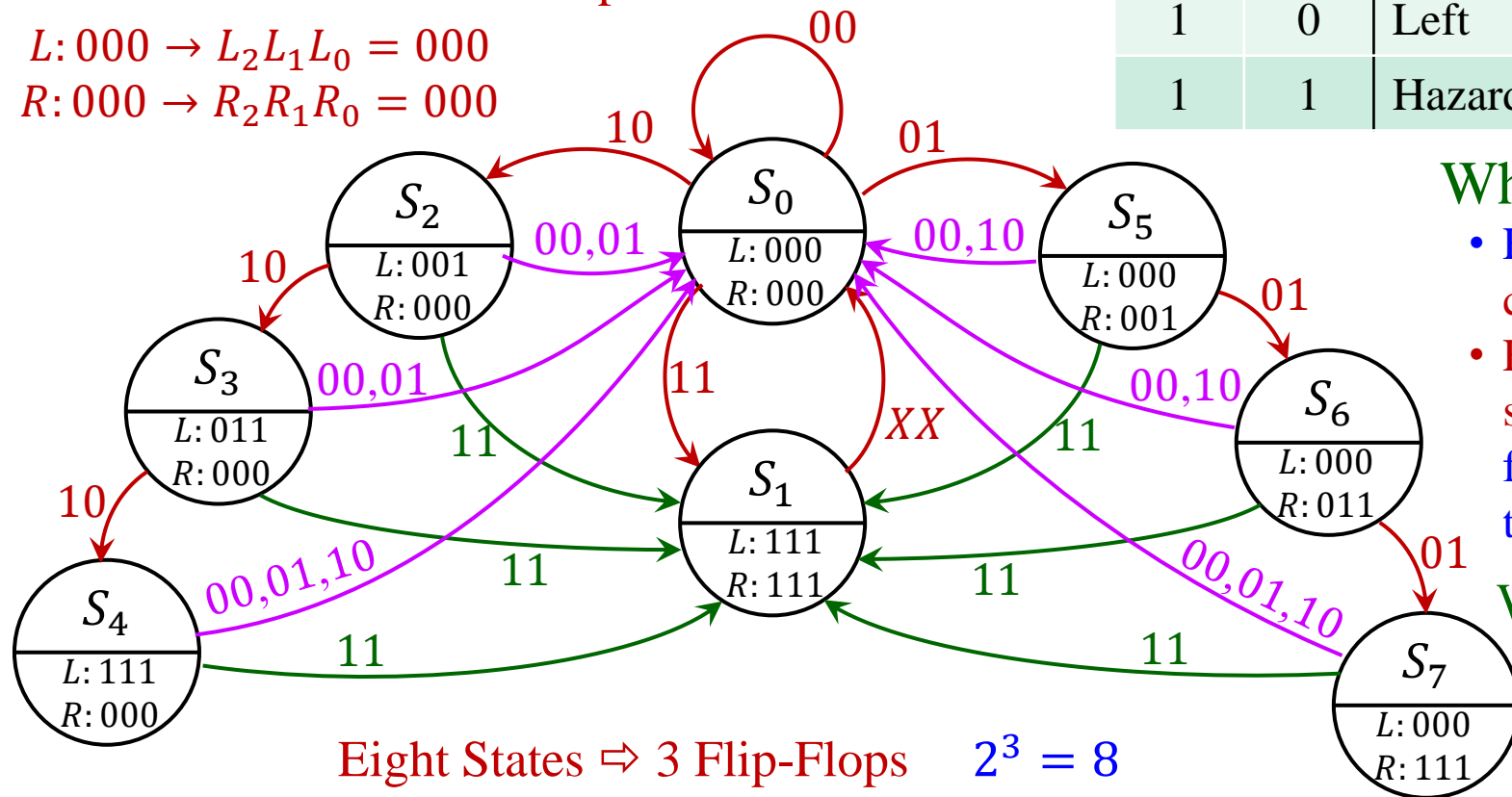
Initial State: All Off

$L: 000 \rightarrow L_2L_1L_0 = 000$

$R: 000 \rightarrow R_2R_1R_0 = 000$

Inputs labeled in LR order

Inputs: 2		
$L$	$R$	
0	0	No lights
0	1	Right
1	0	Left
1	1	Hazard



What Next?

- Four input combinations
- Each state should have four output transitions

What Next?

State Table

Eight States  $\Rightarrow$  3 Flip-Flops  $2^3 = 8$

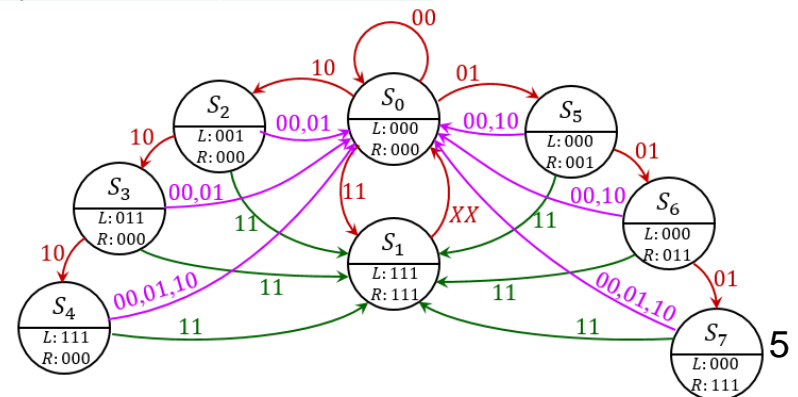


# Design Example: Mustang Sequential Turn Signals

## State Table

## What Next?

Present State	Next State				Present Outputs	
	$LR = 00$	$LR = 01$	$LR = 11$	$LR = 10$	$L_2L_1L_0$	$R_2R_1R_0$
$S_0$	$S_0$	$S_5$	$S_1$	$S_2$	000	000
$S_1$	$S_0$	$S_0$	$S_0$	$S_0$	111	111
$S_2$	$S_0$	$S_0$	$S_1$	$S_3$	001	000
$S_3$	$S_0$	$S_0$	$S_1$	$S_4$	011	000
$S_4$	$S_0$	$S_0$	$S_1$	$S_0$	111	000
$S_5$	$S_0$	$S_6$	$S_1$	$S_0$	000	001
$S_6$	$S_0$	$S_7$	$S_1$	$S_0$	000	011
$S_7$	$S_0$	$S_0$	$S_1$	$S_0$	000	111





# Design Example: Mustang Sequential Turn Signals

**Transition  
Table**  
What Next?

$ABC$	$LR = 00$	$LR = 01$	$LR = 11$	$LR = 10$	Present Outputs	
	$A^+B^+C^+$	$A^+B^+C^+$	$A^+B^+C^+$	$A^+B^+C^+$	$L_2L_1L_0$	$R_2R_1R_0$
$S_0$ 000	000	101	001	010	000	000
$S_1$ 001	000	000	000	000	111	111
$S_2$ 010	000	000	001	011	001	000
$S_3$ 011	000	000	001	100	011	000
$S_4$ 100	000	000	001	000	111	000
$S_5$ 101	000	110	001	000	000	001
$S_6$ 110	000	111	001	000	000	011
$S_7$ 111	000	000	001	000	000	111

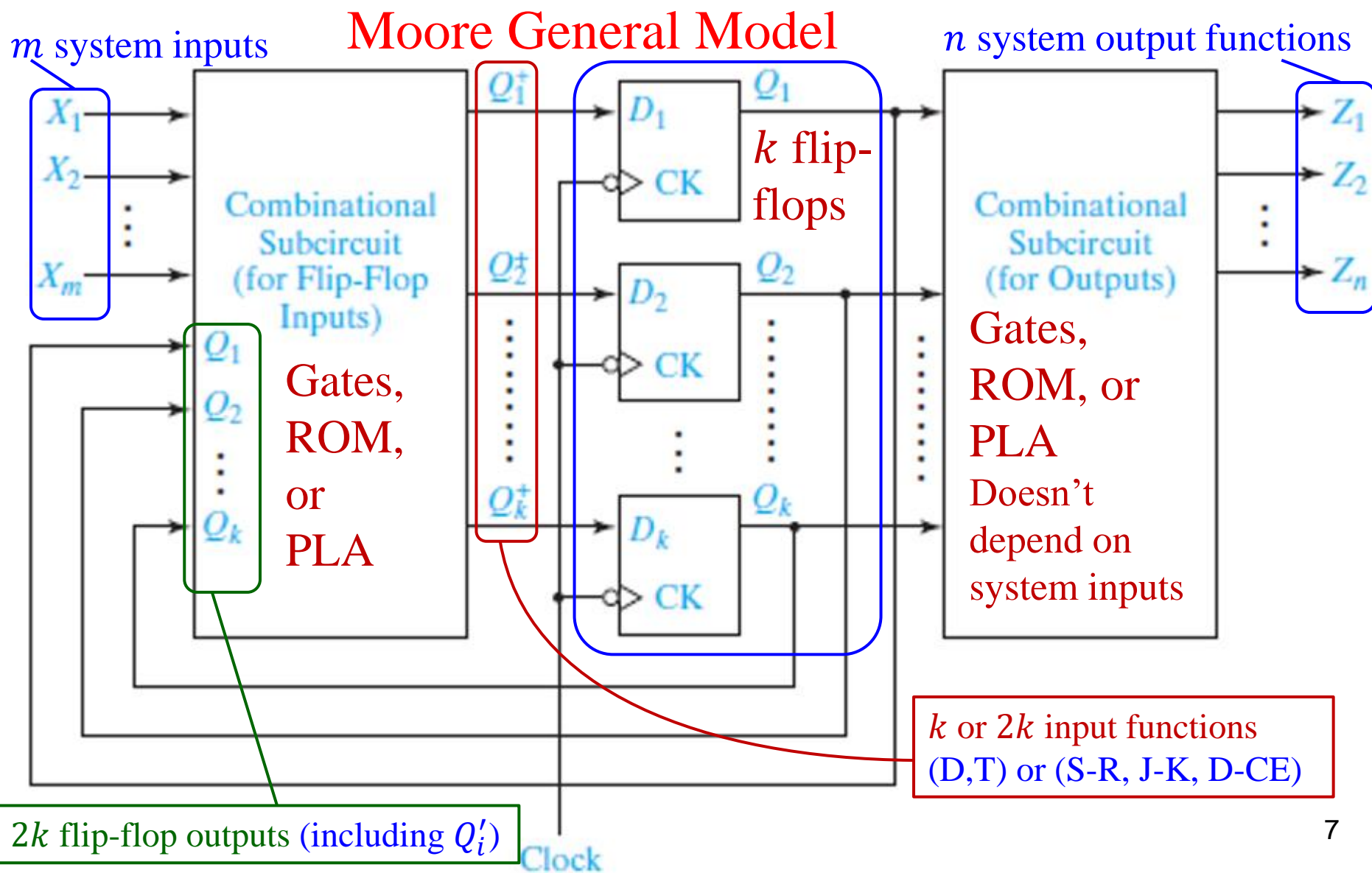
Do you see a  
new challenge?

Present State	Next State			
	$LR = 00$	$LR = 01$	$LR = 11$	$LR = 10$
$S_0$	$S_0$	$S_5$	$S_1$	$S_2$
$S_1$	$S_0$	$S_0$	$S_0$	$S_0$
$S_2$	$S_0$	$S_0$	$S_1$	$S_3$
$S_3$	$S_0$	$S_0$	$S_1$	$S_4$
$S_4$	$S_0$	$S_0$	$S_1$	$S_0$
$S_5$	$S_0$	$S_6$	$S_1$	$S_0$
$S_6$	$S_0$	$S_7$	$S_1$	$S_0$
$S_7$	$S_0$	$S_0$	$S_1$	$S_0$

We will come back to that later. Do outputs first.



## General Models for Sequential Circuits

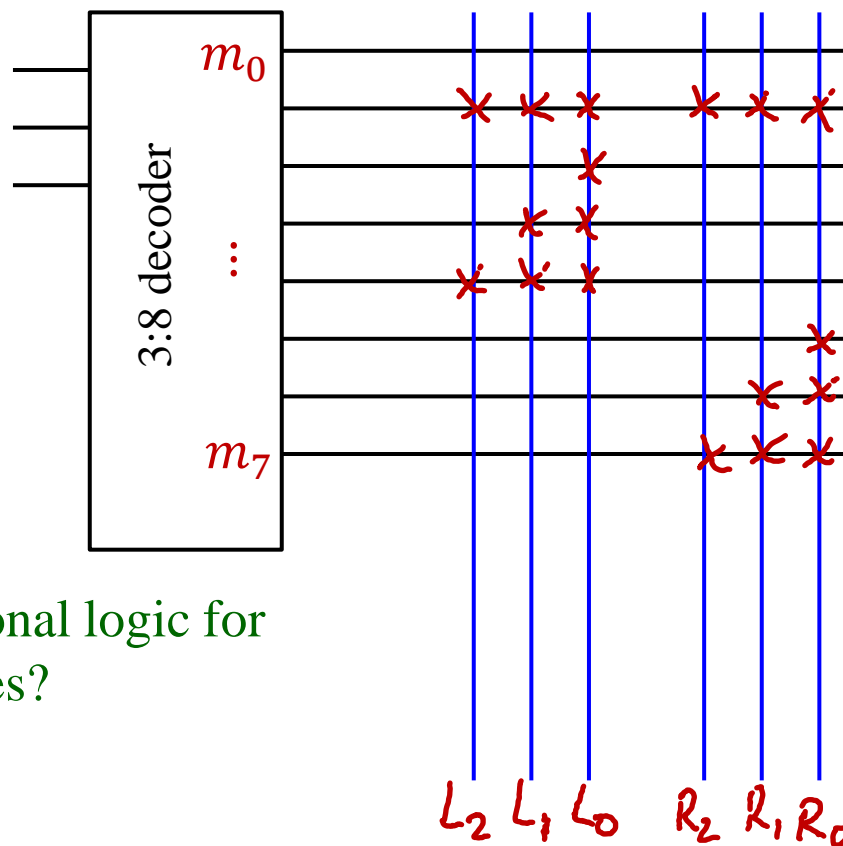




# Design Example: Mustang Sequential Turn Signals

## Outputs: Decoder and ROM

$ABC$	Present Outputs	
	$L_2L_1L_0$	$R_2R_1R_0$
$S_0$ 000	000	000
$S_1$ 001	111	111
$S_2$ 010	001	000
$S_3$ 011	011	000
$S_4$ 100	111	000
$S_5$ 101	000	001
$S_6$ 110	000	011
$S_7$ 111	000	111



What about design of combinational logic for flip-flop inputs with five variables?

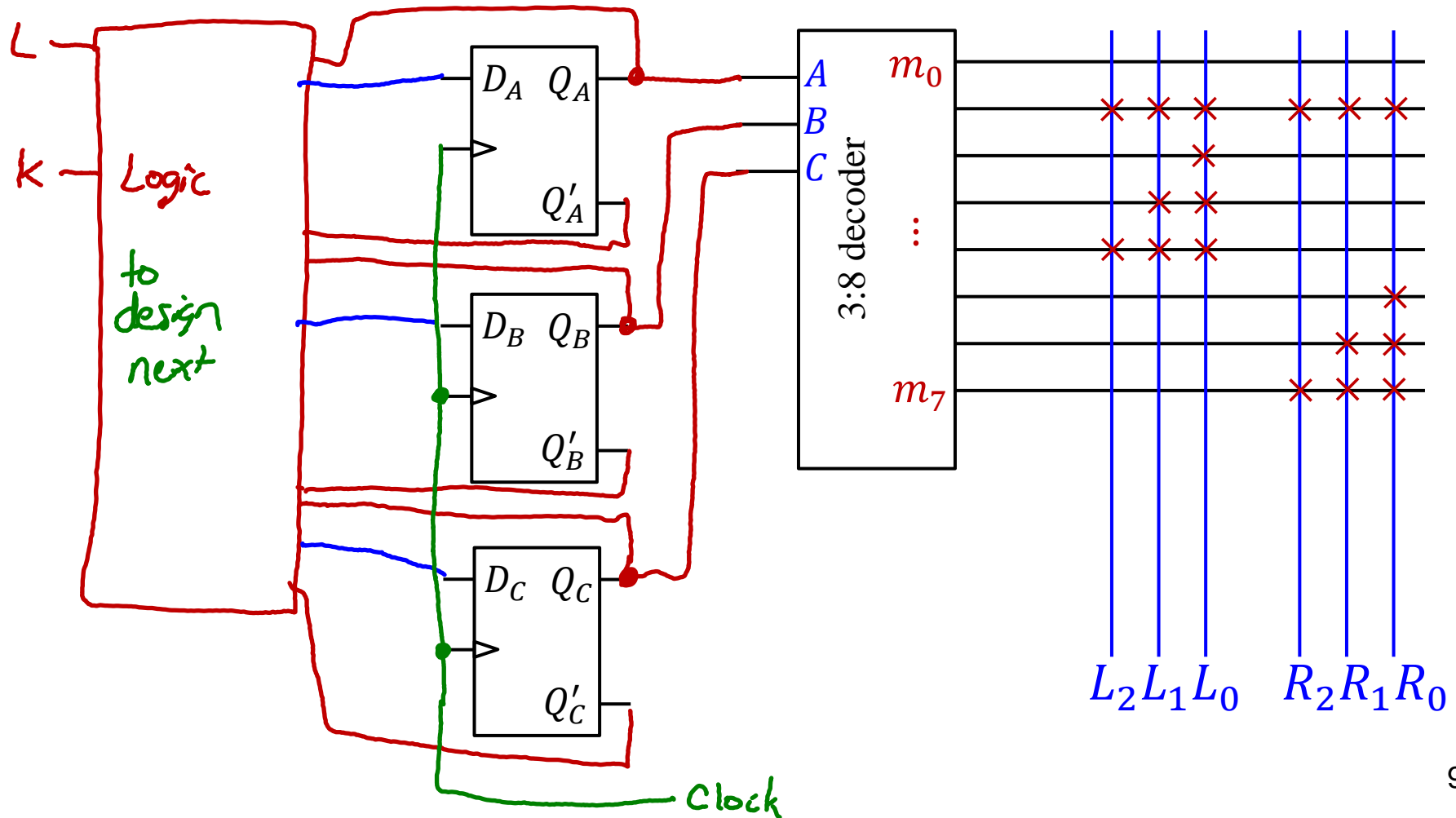
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# Design Example: Mustang Sequential Turn Signals

Sketch overall system





# Design Example: Mustang Sequential Turn Signals

Picking  $A$  as the outlier; Demonstrating for  $A^+$ , D Flip-flops

$A = 0$

$LR \backslash BC$	00	01	11	10
00		1		
01				
11				1
10				

$A = 1$

$LR \backslash BC$	00	01	11	10
00				
01		1		
11				
10		1		

$$A^+ = B'C'L'RA' + BCLR'A' + B'CL'RA + BC'L'RA$$

**K-maps: Try to group neighboring 1s**

4-variable: Each cell can have 4 neighbors

3-variable: Each cell can have 3 neighbors

5-variable: Each cell can have 5 neighbors – Layered vertically

This case: none of the 1s overlap – no further reduction

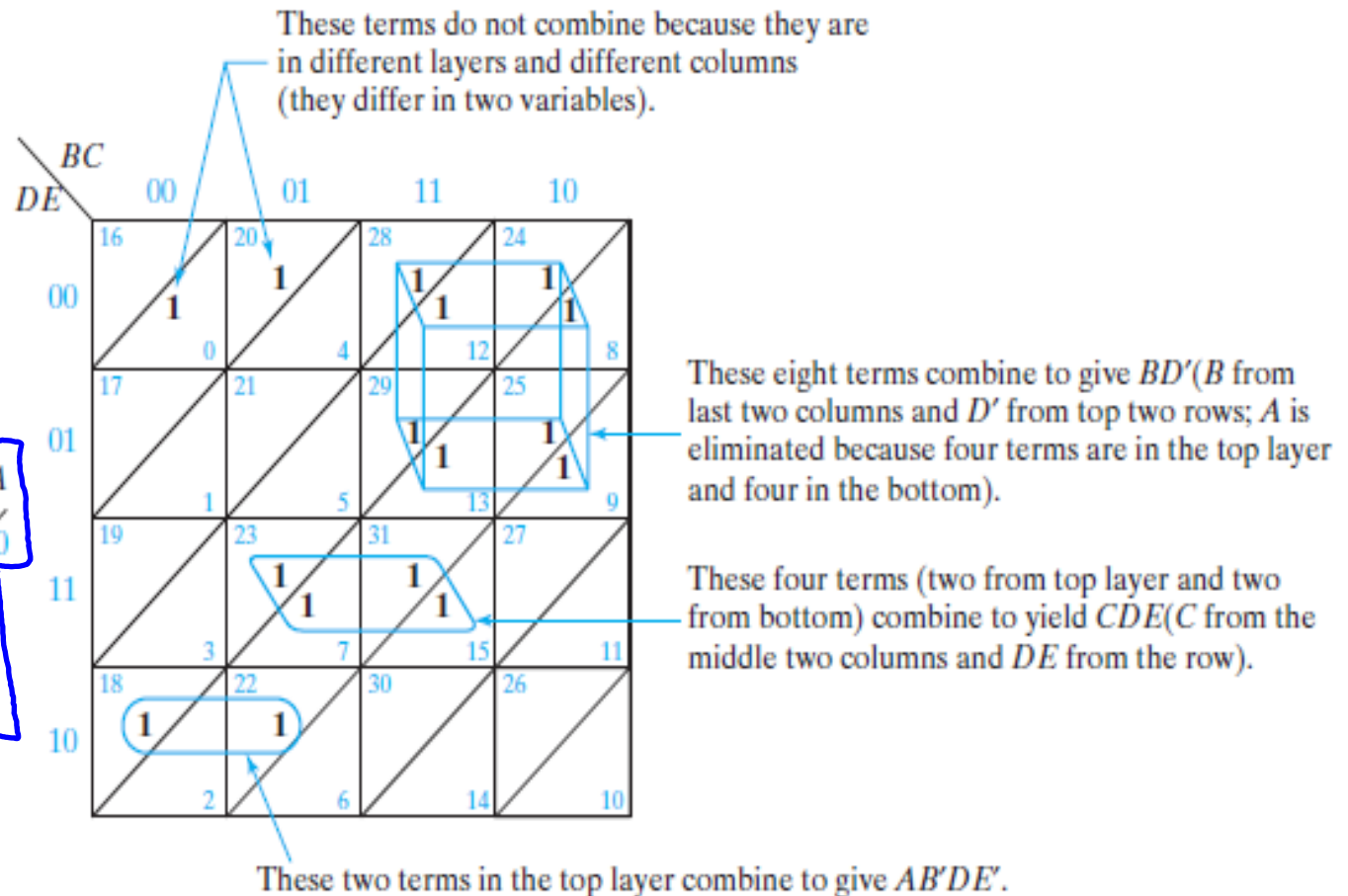
## Design Example: Mustang Sequential Turn Signals

FIGURE 5-21

## A Five-Variable Karnaugh Map

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- Add diagonals across each cell for top and bottom layers
- Top layer:
  - $A = 1$
  - $m_{16} - m_{31}$
- Bottom layer:
  - $A = 0$
  - $m_0 - m_{15}$





# Design Example: Mustang Sequential Turn Signals

## Transition Table

What Next?

Next State

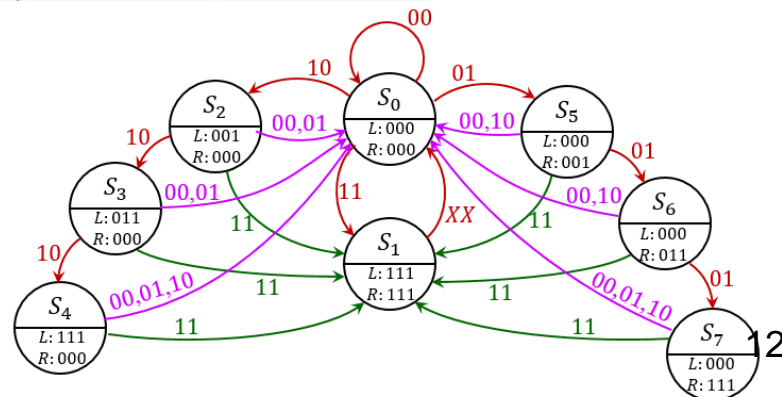
Maps for each  
flip-flop

$ABC$	$LR = 00$	$LR = 01$	$LR = 11$	$LR = 10$	Present Outputs	
	$A^+B^+C^+$	$A^+B^+C^+$	$A^+B^+C^+$	$A^+B^+C^+$	$L_2L_1L_0$	$R_2R_1R_0$
$S_0$ 000	000	101	001	010	000	000
$S_1$ 001	000	000	000	000	111	111
$S_2$ 010	000	000	001	011	001	000
$S_3$ 011	000	000	001	100	011	000
$S_4$ 100	000	000	001	000	111	000
$S_5$ 101	000	110	001	000	000	001
$S_6$ 110	000	111	001	000	000	011
$S_7$ 111	000	000	001	000	000	111

$A^+$ : Already done

$B^+$ : Four 1s

$C^+$ : Ten 1s





# Design Example: Mustang Sequential Turn Signals

$B^+$ , D Flip Flops

$LR$ $BC$					
		00	01	11	10
$A^{1/0}$	00				1
	01		1		
	11				
	10		1		1

$LR$ $BC$					
		00	01	11	10
$A^{1/0}$	00		1	1	
	01			1	
	11			1	1
	10		1	1	1

$$B^+ = B'CL'RA + BC'L'RA + C'LR'A'$$

$$C^+ = B'C'RA' + BC'LA' + BC'RA + LRA + BLR$$



# Design Example: Mustang Sequential Turn Signals

$$A^+ = B'C'L'RA' + BCLR'A' + B'CL'RA + BC'L'RA$$

$$B^+ = B'CL'RA + BC'L'RA + C'LR'A'$$

$$C^+ = B'C'RA' + BC'LA' + BC'RA + LRA + BLR$$

A	B	C	L	R	A <sup>+</sup>	B <sup>+</sup>	C <sup>+</sup>
0	0	0	0	1	1	0	0
0	1	1	1	0	1	0	0
1	0	1	0	1	1	1	0
1	1	0	0	1	1	1	0
0	-	0	1	0	0	1	0
0	0	0	-	1	0	0	1
0	1	0	1	-	0	0	1
1	1	0	-	1	0	0	1
1	-	-	1	1	0	0	1
-	1	-	1	1	0	0	1

What is this  
table for?

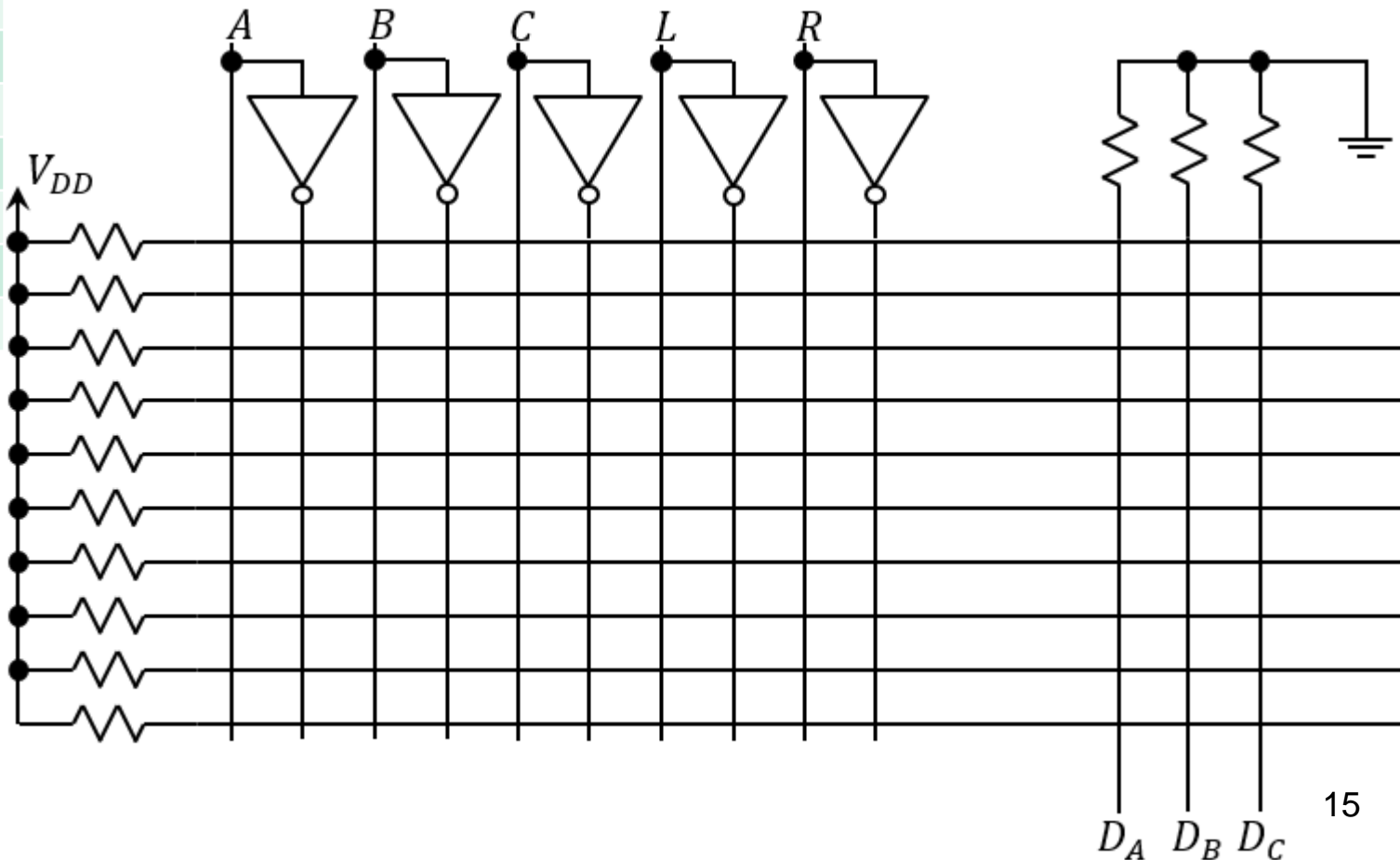


# Design Example: Mustang Sequential Turn Signals

Present state outputs  
from flip-flops – to  
PLA inputs

Inputs to  
the system

A	B	C	L	R	A <sup>+</sup>	B <sup>+</sup>	C <sup>+</sup>
0	0	0	0	1	1	0	0
0	1	1	1	0	1	0	0
1	0	1	0	1	1	1	0
1	1	0	0	1	1	1	0
0	-	0	1	0	0	1	0
0	0	0	-	1	0	0	1
0	1	0	1	-	0	0	1
1	1	0	-	1	0	0	1
1	-	-	1	1	0	0	1
-	1	-	1	1	0	0	1





# Mealy Design Example: Detector of 110 or 1010

## • Sequence Detector Description:

- Circuit that

- Examines a serial string of 0's and 1's applied to the  $X$  input
- Generates an output  $Z = 1$  when either of two prescribed input sequences occur

- Input  $X$  synchronized with clock pulses

- For this example, the prescribed input sequences are 110 or 1010

- Circuit will not automatically reset when a 1 output occurs

- From our guidelines

1. Start by identifying sample input and output sequences. Doing this also helps you understand the problem statement.

$X$ :      001101000111011010100

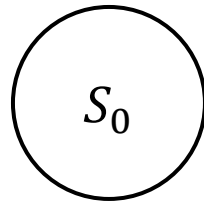
$Z$ :      000010100000100101010





# Mealy Design Example: Detector of 110 or 1010

- Determine an initial state and any condition that causes a reset to that state (if there are any)

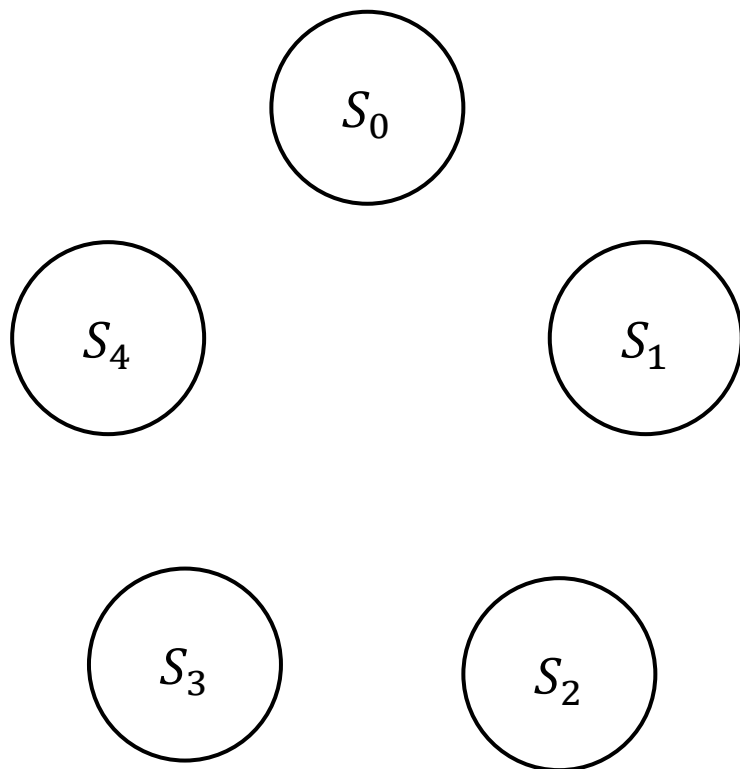


- If the output is mostly zero, identify the few states that cause non-zero output and start with those (partial state graph)
- Another way to start is to determine sequences or groups of sequences that must be remembered by the circuit, and set up states for them

State	Sequence Received



# Mealy Design Example: Detector of 110 or 1010



State	Sequence Received

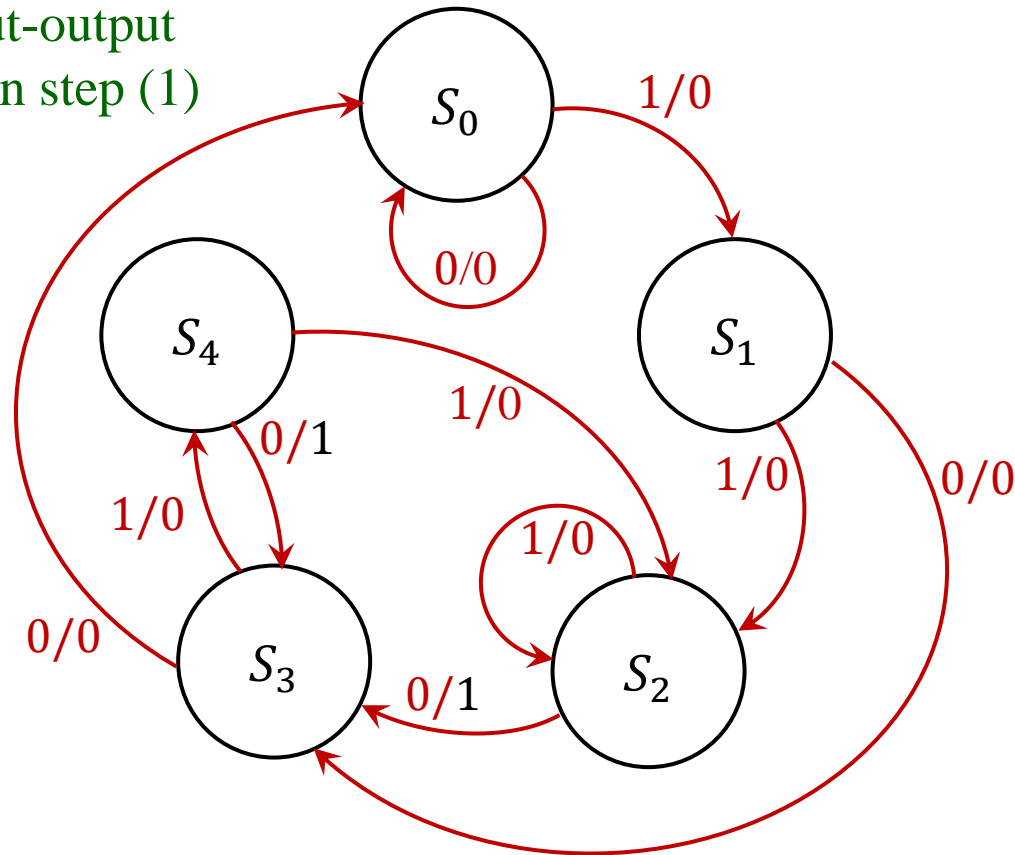
State	Sequence <b>Ending In</b>

5. Can transition arrows go to existing states?  
Add a new state only when you really have to.
6. Once graph is complete, make sure each input combination leaves each state only once.



# Mealy Design Example: Detector of 110 or 1010

## 7. Test graph using input-output combinations found in step (1)



Input X: 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 1 0 1 0 0

Output Z:

State:  $S_0$



# Review of Steps to Complete the Design

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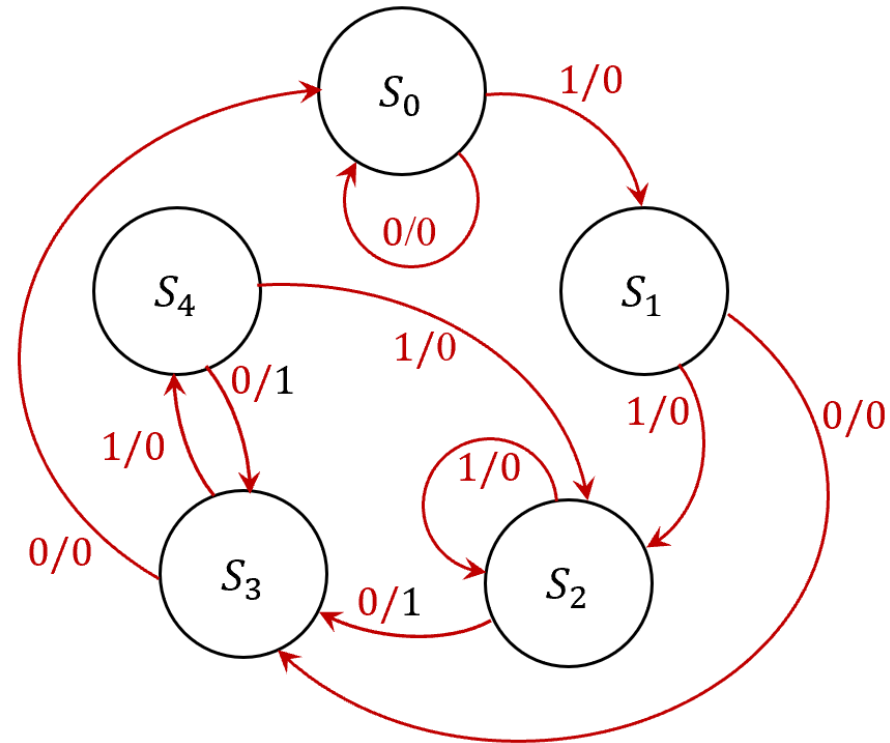
1. Develop a State Graph (Done)
2. Fill in a State Table
3. Fill in a Transition Table
4. Generate Flip-Flop Next State and Output Maps
5. Determine the SOP Expressions for the circuit output(s) and flip-flop input(s)
6. Design the logic circuits for the circuit output(s) and flip-flop input(s)



# Mealy Design Example: Detector of 110 or 1010

## 2. Fill in a State Table

Present State	Next State		Present Z	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
$S_0$				
$S_1$				
$S_2$				
$S_3$				
$S_4$				





# Mealy Design Example: Detector of 110 or 1010

## 3. Fill in a Transition Table

Why did I choose these *ABC* codes for the states, instead of the obvious binary values of decimal subscripts?

Present State	Next State		Present Z	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
$S_0$	$S_0$	$S_1$	0	0
$S_1$	$S_3$	$S_2$	0	0
$S_2$	$S_3$	$S_2$	1	0
$S_3$	$S_0$	$S_4$	0	0
$S_4$	$S_3$	$S_2$	1	0

<i>ABC</i>	$A^+B^+C^+$		Present Z	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
100			0	0
010			0	0
001			1	0
000			0	0
011			1	0



# Mealy Design Example: Detector of 110 or 1010

4. Generate Flip-Flop Next State and Output Maps

5. Determine SOP Expressions for circuit output(s) and flip-flop input(s)

Don't Cares for  $ABCX = 101$

$ABC$	$A^+B^+C^+$		Present $Z$	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
100	1 0 0	0 1 0	0	0
010	0 0 0	0 0 1	0	0
001	0 0 0	0 0 1	1	0
000	1 0 0	0 1 1	0	0
011	0 0 0	0 0 1	1	0

$AB$ $CX$	00	01	11	10
00				
01				
11				
10				

$A^+ =$

$AB$ $CX$	00	01	11	10
00				
01				
11				
10				

$B^+ =$

$AB$ $CX$	00	01	11	10
00				
01				
11				
10				

$C^+ =$

$AB$ $CX$	00	01	11	10
00				
01				
11				
10				

$Z =$

Note: All don't cares realized as 0 in next-state maps  $\rightarrow$  extra states  $S_5, S_6$ , and  $S_7$  all transition to  $S_3$  ( $ABC = 000$ ) for  $X = 0$  or  $X = 1$  in full 8-state graph



# Mealy Design Example: Detector of 110 or 1010

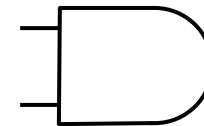
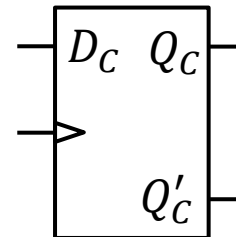
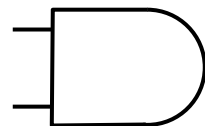
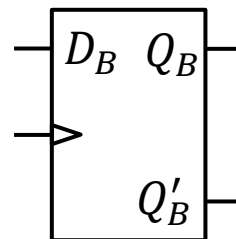
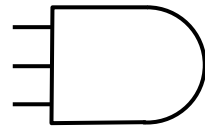
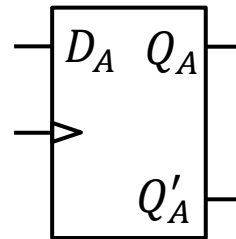
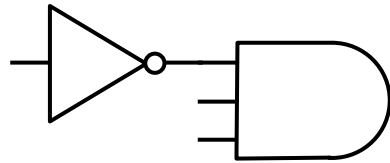
6. Design the logic circuits for circuit output(s) and flip-flop input(s)

$$A^+ = B'C'X'$$

$$B^+ = B'C'X$$

$$C^+ = A'X$$

$$Z = CX'$$







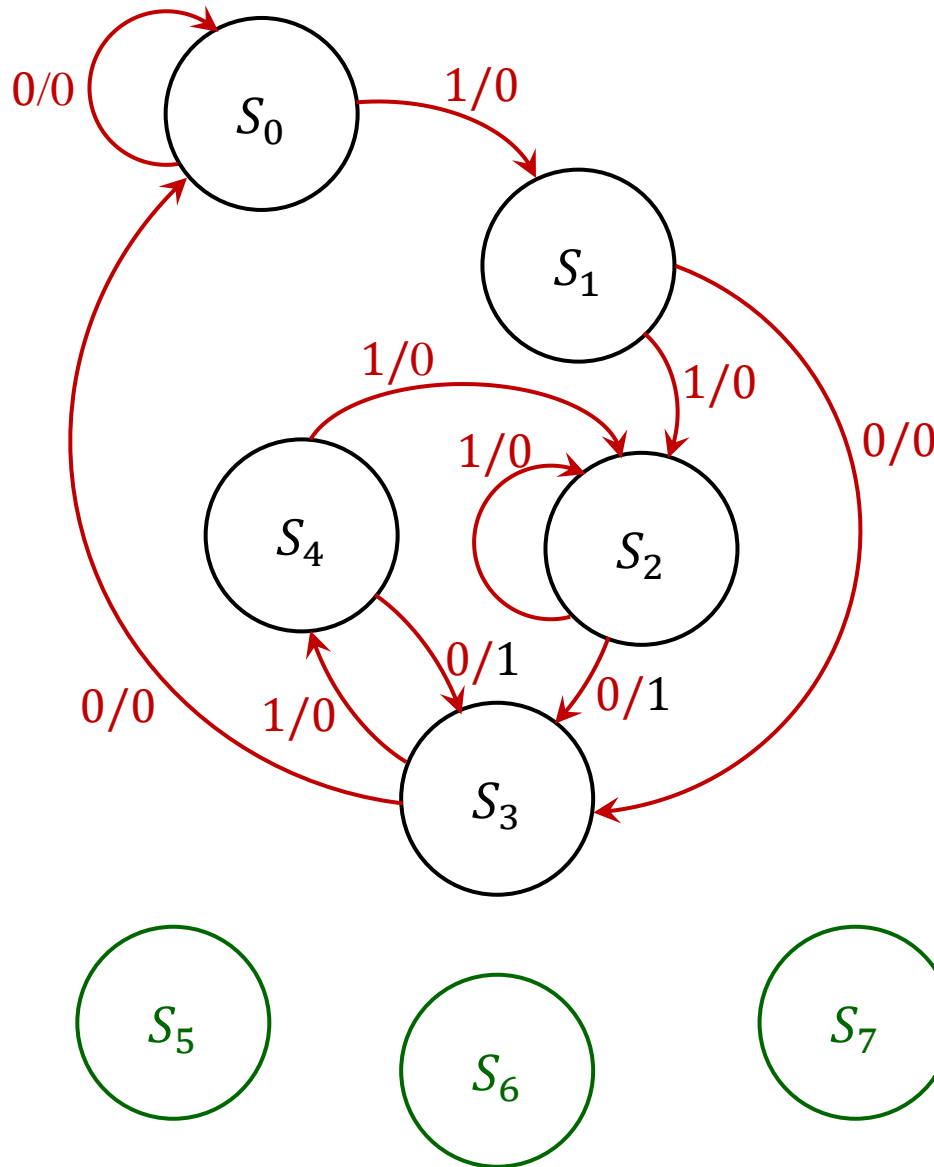
## Steps to Analyze a State Machine Circuit

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1. Study circuits used to generate the Flip-Flop input(s) and circuit output(s). Write out their Boolean expressions (e.g. in SOP form).
2. Use the expressions to fill in Flip-Flop Next State and Output Maps
3. Use them to fill in a Transition Table
4. Fill in a State Table based on the Transition Table
5. Draw the State Graph based on the State Table



# Mealy Design Example: Detector of 110 or 1010



$S_0$

$S_1$

$S_2$

$S_3$

$S_4$

$S_5$

$S_6$

$S_7$

$ABC$	$A^+B^+C^+$		Present $Z$	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
100	1 0 0	0 1 0	0	0
010	0 0 0	0 0 1	0	0
001	0 0 0	0 0 1	1	0
000	1 0 0	0 1 1	0	0
011	0 0 0	0 0 1	1	0
101	0 0 0	0 0 0	1	0
110	0 0 0	0 0 0	0	0
111	0 0 0	0 0 0	1	0

$$Z = CX'$$