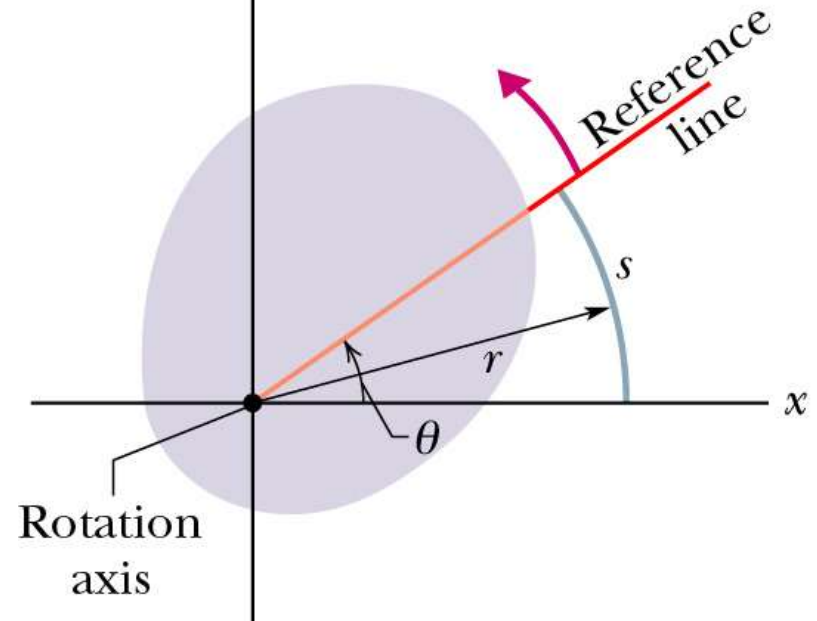
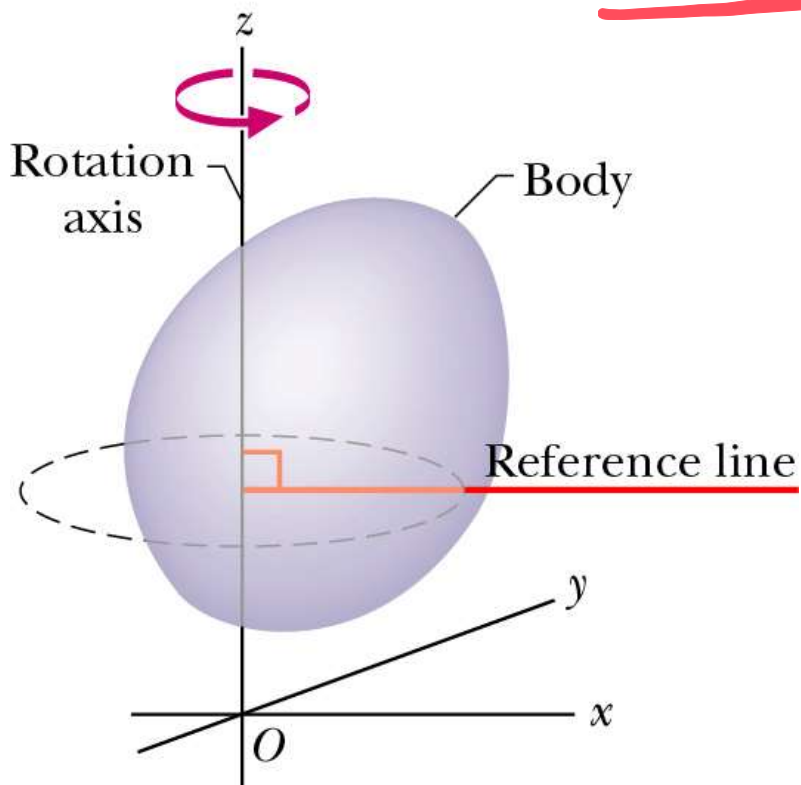


EGG !!!!!!!



A rigid body rotates along a fixed axis

$$\theta = \frac{s}{r} \quad (\text{radian measure})$$

$s = \text{arc of the circle}$

* circumference : $s = 2\pi r$

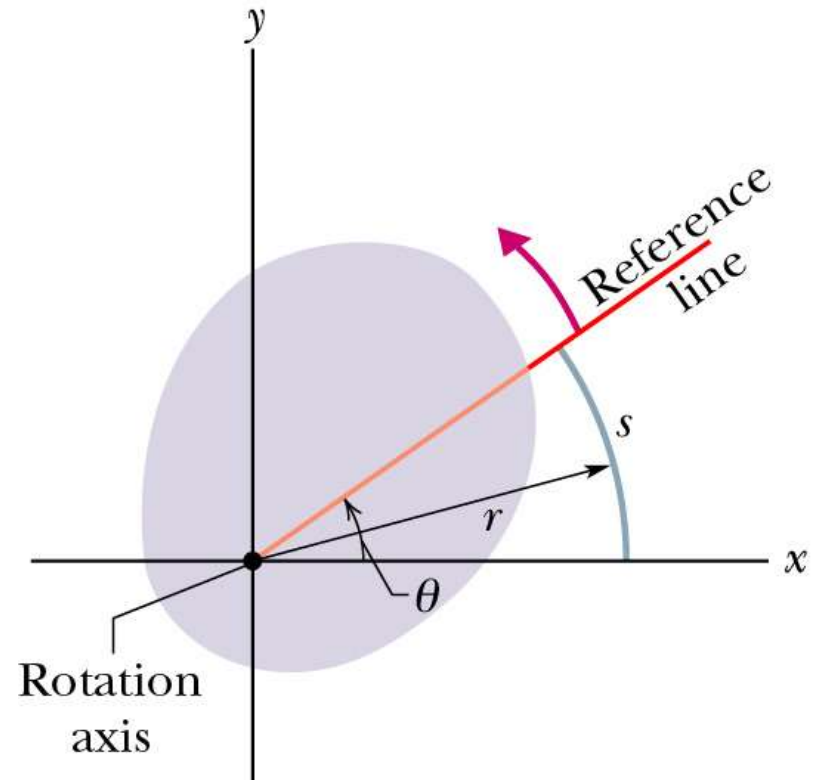
* 1 rev : $\Delta\theta = 2\pi = 360^\circ$

* 1 rad = $\frac{360^\circ}{2\pi} = 57.3^\circ$

$$s = r\theta$$

for $\theta = 1 \text{ rad}$

$$s = r$$



θ angular position

angular displacement :

$$\Delta\theta = \theta_2 - \theta_1$$

θ angular position

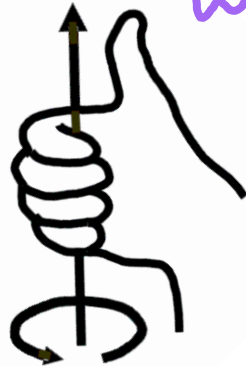
$\Delta\theta = \theta_2 - \theta_1$ angular displacement

* angular velocity:

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} \quad \omega = \frac{d\theta}{dt}$$

* ω has a "direction"—right hand rule
counterclockwise – positive

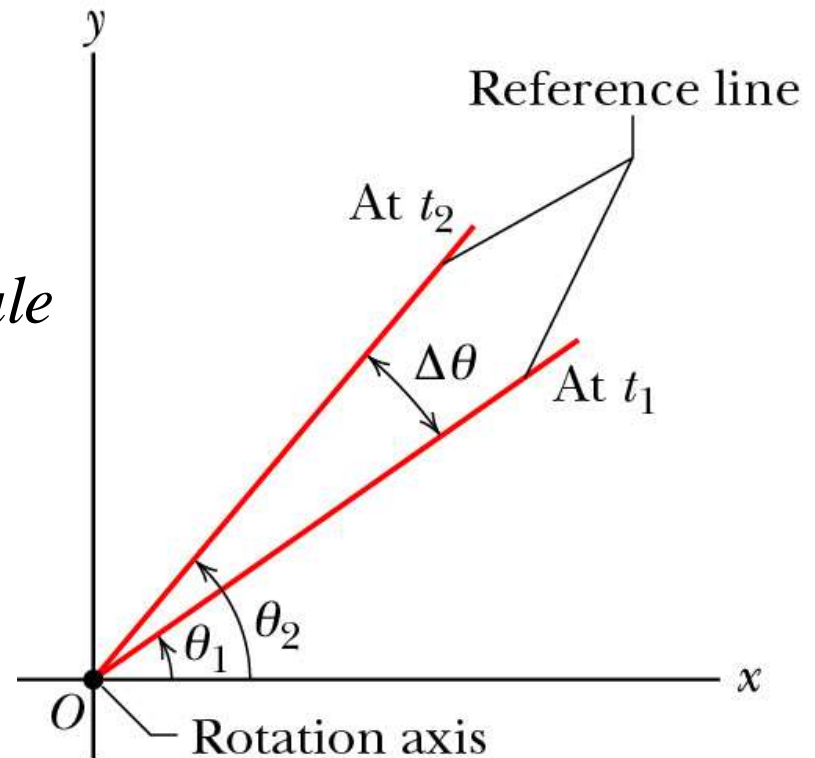
clockwise – negative



$$\omega(t) = \frac{d\theta(t)}{dt}$$

$$v(t) = \frac{dx(t)}{dt}$$

$$a = \frac{dv(t)}{dt}$$



θ angular position

$\Delta\theta = \theta_2 - \theta_1$ angular displacement

* angular velocity:

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega(t)}{dt}$$

* angular acceleration:

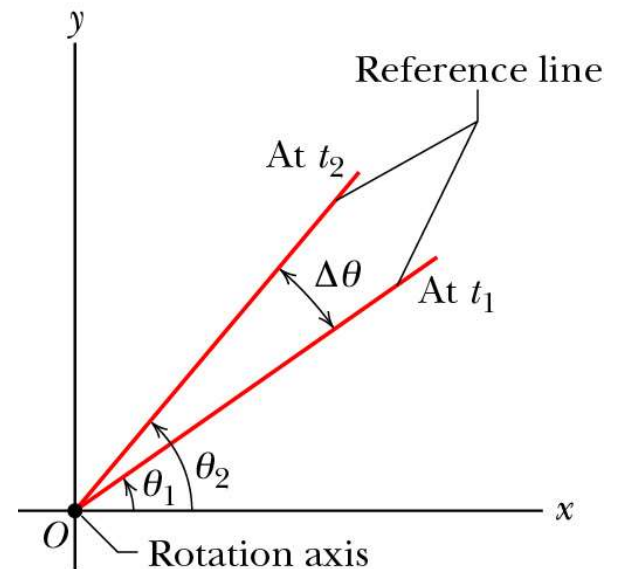
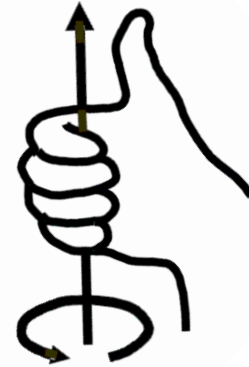
$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$$

$$\alpha = \frac{d\omega}{dt}$$

α is also a vector (right-hand rule)

* for counterclockwise-rotation in $x-y$ plane

$$\vec{\omega} = \omega \vec{k}, \quad \vec{\alpha} = \alpha \vec{k}$$



*for constant angular acceleration :

$$\Delta\omega = \omega - \omega_0 = \alpha t$$

$$\Delta\theta = \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$

**Rigid Body Under Constant
Angular Acceleration**

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

$$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$$

**Particle Under Constant
Acceleration**

$$v_f = v_i + at$$

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

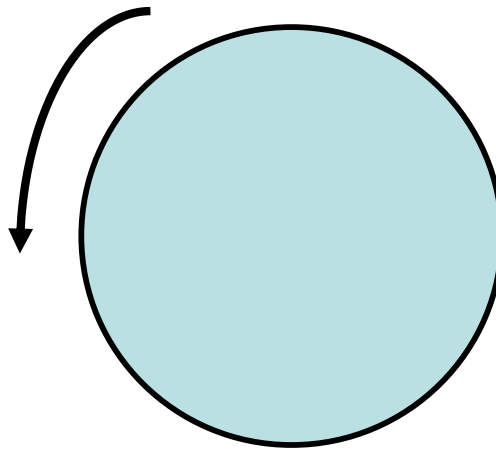
$$x_f = x_i + \frac{1}{2}(v_i + v_f)t$$

A disk that initially spins at 2 revolutions/sec is braked uniformly to a stop in $\frac{1}{2}$ second. What is its initial angular speed ω_0 ?

$$V_i = 2 \text{ r/s}$$

$$V_o = 0 \text{ r/s}$$

$$t = 0.5 \text{ s}$$



1. π / s

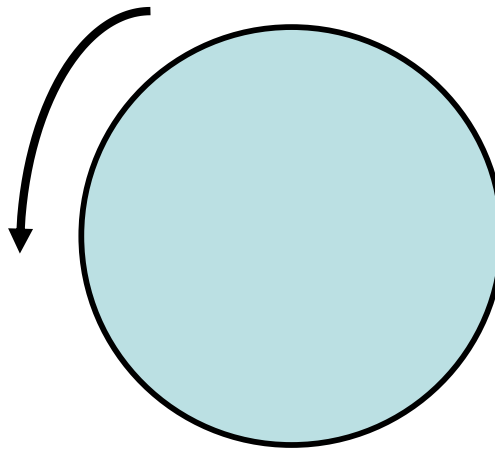
2. $2\pi / s$

3. $4\pi / s$

4. $8\pi / s$

$$\omega_o = V_i \cdot 2\pi = 2 \cdot 2\pi = 4\pi / s$$

A disk that initially spins at 2 revolutions/sec is braked uniformly to a stop in $\frac{1}{2}$ second. What is the value of angular acceleration α ? (Negative α means stopping)



$$\alpha = \frac{\Delta\omega}{\Delta t}$$
$$\frac{-4\pi}{0.5}$$
$$-8\pi/s^2$$

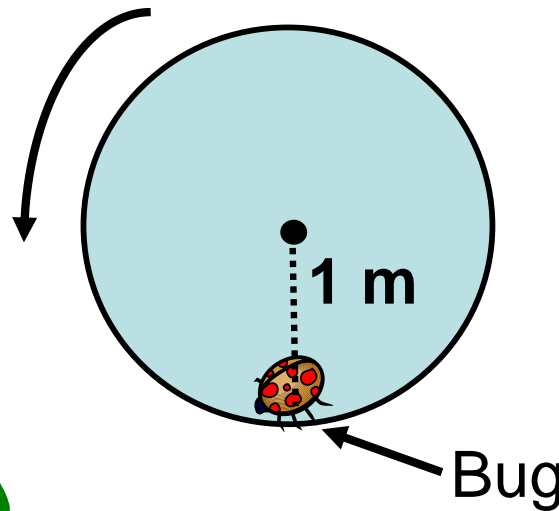
1. $-4\pi/s^2$

2. $4\pi/s^2$

3. $-8\pi/s^2$

4. $8\pi/s^2$

A disk that initially spins at 2 revolutions/sec is braked uniformly to a stop in $\frac{1}{2}$ second. A bug is stuck on the outer edge 1 meter from the center. How many revolutions does the bug go during the time it takes the disk to stop?



$$\begin{aligned}\theta &= \omega_i t + \frac{1}{2} \alpha t^2 \\ &= 4\pi(.5) + \frac{1}{2}(-8\pi)(.25) \\ &= 2\pi - \pi \\ &= \pi = 0.5 \text{ rev.}\end{aligned}$$

1. 0.25 revolution
2. 0.5 revolution
3. 1 revolution
4. 1.5 revolution
5. 2 revolution



A flywheel turns 40 revolutions as it slows from $\omega_0 = 1.5 \text{ rad/s}$ to a stop. How long does it take?

(A flywheel stores rotational energy. Need them in places such as generators with a varying electrical load)



A flywheel turns 40 revolutions as it slows from

$\omega_0 = 1.5 \text{ rad/s}$ to a stop. How long does it take?

(A flywheel stores rotational energy. Need them in places such as generators with a varying electrical load)

First method:

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

$$\alpha = \left(\frac{\omega^2 - \omega_0^2}{2\theta} \right) = \left(\frac{0 - (1.5 \text{ rad/s})^2}{2(40 \text{ rev})(2\pi \text{ rad/rev})} \right) = -4.48 \times 10^{-3} \text{ rad/s}^2$$

$$\omega - \omega_0 = \alpha t$$

$$t = \left(\frac{\omega - \omega_0}{\alpha} \right) = \left(\frac{0 - 1.5 \text{ rad/s}}{-4.48 \times 10^{-3} \text{ rad/s}^2} \right) = 335 \text{ sec}$$

Second method:

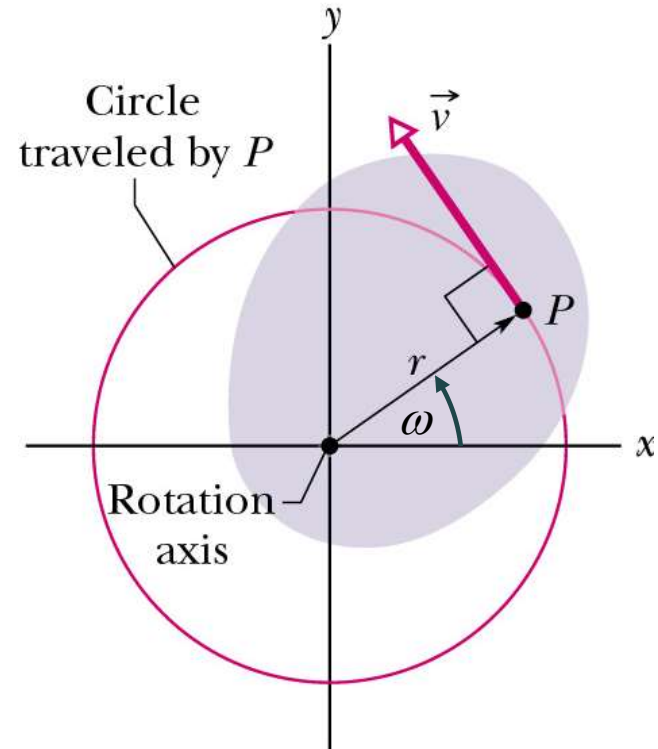
$$t = \left(\frac{\theta}{\omega_{AV}} \right) = \left(\frac{(40 \text{ rev})(2\pi \text{ rad/rev})}{0.75 \text{ rad/s}} \right) = 335 \text{ sec}$$

$$s = r\theta$$

$$\frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$V_t = \omega r$$

$$T = \frac{2\pi r}{V_t} = \frac{2\pi r}{\omega r} = \frac{2\pi}{\omega}$$



$$V_t = \omega r$$

velocity & omega
time dependent

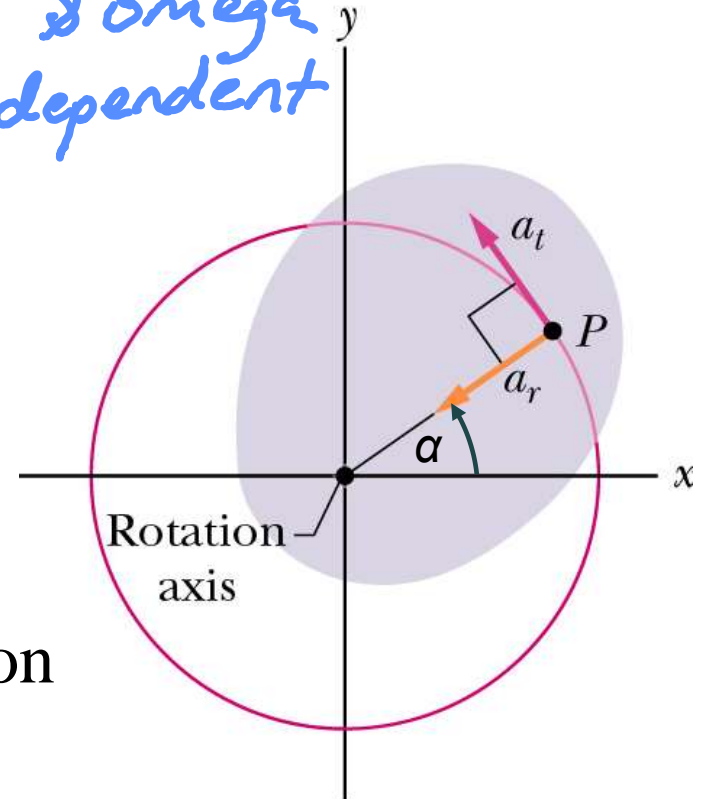
tangential acceleration

$$a_t = \frac{dV_t}{dt} = \frac{d\omega}{dt} r = \alpha r$$

$$V_t = \omega r$$

"centripetal" (radial) acceleration

$$a_r = \frac{V_t^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$$



(b)

$$s = r\theta$$

$$V_t = \omega r$$

$$a_t = \alpha r$$

$$a_r = \omega^2 r$$

A disk that initially spins at 2 revolutions/sec is braked uniformly to a stop in $\frac{1}{2}$ second. A bug is stuck on the outer edge 1 meter from the center.

What is the initial angular speed of the bug?

$$\omega = 4\pi / \text{sec}$$

What is the initial tangential speed of the bug?

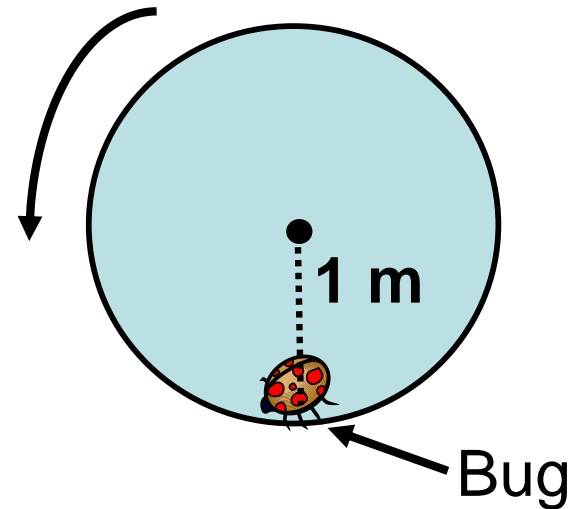
$$v = \omega r = 4\pi \cdot 1 = 12.6 \text{ m/s}$$

What is the angular acceleration of the bug?
(magnitude and direction **CW** or CCW)

$$\alpha = -8\pi / \text{sec}^2$$

What is the tangential acceleration of the bug?
(magnitude and direction **CW** or CCW)

$$a_t = \alpha r = -8\pi \cdot 1 = -25.1 \text{ m/s}^2$$



A disk that initially spins at 2 revolutions/sec is braked uniformly to a stop in $\frac{1}{2}$ second. A bug is stuck on the outer edge 1 meter from the center.

What is the initial angular speed of the bug?

$4\pi/\text{s}$

What is the initial tangential speed of the bug?

12.6 m/s

What is the angular acceleration of the bug?
(magnitude and direction CW or CCW)

CW, $8\pi/\text{s}$

What is the tangential acceleration of the bug?
(magnitude and direction CW or CCW)

CW, 25.1 m/s

