Gage Farmer

Homework 11 - Math 2568 (Autumn 2022)

Prof. Cueto

Due date: Wednesday December 7, 2022 (in class).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§4.5	1, 2, 4, 5, 6, 8, 9, 12, 16, 18, 19, 21, 22	1, 5, 6, 16, 18, 21
§4.6	1, 3, 5, 7, 9, 13, 19, 21, 23, 25, 27, 29	1, 3, 7, 19, 23, 25, 27
§4.7	1, 3, 7, 11, 13, 17, 19, 26, 27	1, 3, 11, 13, 17, 26, 27

Section 4.5

1) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ $\lambda = 3$

Alg mult = 1 Geom Mult = 1

$p(t) = (t-3)(t-1)$

$(A - \lambda I)x = \theta$ $x\left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right) = x\left(\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-x_1 - x_2 = 0$ $-x_1 = x_2$ $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

5) $C = \begin{bmatrix} -6 & -1 & 2 \\ 3 & 2 & 0 \\ -14 & -2 & 5 \end{bmatrix}$ $\lambda = -1$ Alg mult = 1

$p(t) = -(t-1)^2(t+1)$

$$(C+I)x=0 \qquad \left(\begin{bmatrix}-6 & -1 & 2\\ 3 & 2 & 0\\ -14 & -2 & 5\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)x = \begin{bmatrix}-5 & -1 & 2\\ 3 & 3 & 0\\ -14 & -2 & 6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$-5x_1 - x_2 + 2x_3 = 0 \qquad 4x_1 + 2x_3 = 0 \qquad x_3 = 2x_1$$

$$3x_1 + 3x_2 = 0 \qquad x_2 = -x_1 \qquad x = \begin{bmatrix}x_1\\ -x_1\\ 2x_1\end{bmatrix} = x_1\begin{bmatrix}1\\ -1\\ 2\end{bmatrix} \qquad \begin{bmatrix}1\\ -1\\ 2\end{bmatrix}$$

$$-14x_1 - 2x_2 + 6x_3 = 0$$

Geom Mult = 1

6) $D = \begin{bmatrix}-7 & 4 & -3\\ 8 & -3 & 3\\ 32 & -16 & 13\end{bmatrix}$ $\lambda = 1$ $p(t) = -(t-1)^3$ Alg Mult = 3

$$(D-I)x = \theta \qquad \left(\begin{bmatrix}-7 & 4 & -3\\ 8 & -3 & 3\\ 32 & -16 & 13\end{bmatrix}-\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)x = \begin{bmatrix}-8 & 4 & -3\\ 8 & -4 & 3\\ 32 & -16 & 12\end{bmatrix}x = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$-8x_1 + 4x_2 - 3x_3 = 0 \to x_1 - \tfrac{1}{2}x_2 + \tfrac{3}{8}x_3 = 0 \qquad x_1 = \tfrac{1}{2}x_2 - \tfrac{3}{8}x_3$$

$$x = x_2\begin{bmatrix}\frac{1}{2}\\ 1\\ 0\end{bmatrix} - x_3\begin{bmatrix}\frac{3}{8}\\ 0\\ 1\end{bmatrix} \qquad \left\{\begin{bmatrix}1/2\\ 1\\ 0\end{bmatrix}, \begin{bmatrix}3/8\\ 0\\ 1\end{bmatrix}\right\}$$

Geom Mult = 2

16) $\begin{bmatrix}-1 & 6 & 2\\ 0 & 5 & -6\\ 1 & 0 & -2\end{bmatrix}$ $(A-tI) = \begin{bmatrix}-1-t & 6 & 2\\ 0 & 5-t & -6\\ 1 & 0 & -2-t\end{bmatrix}$

$$\det(A-tI) = -1-t\begin{vmatrix}5-t & -6\\ 0 & -2-t\end{vmatrix} - 6\begin{vmatrix}0 & -6\\ 1 & -2-t\end{vmatrix} + 2\begin{vmatrix}0 & 5-t\\ 1 & 0\end{vmatrix}$$

$$= (-1-t)\left((5-t)(-2-t)\right) - 6(6) + 2(5-t) = (-1-t)(-10-5t+2t+t^2) - 36 + 10 - 2t$$

$= 10 + 5t - 2t - t^2 + 10t + 5t^2 - 2t^2 - t^3 - 36 + 10 - 2t$

$= -t^3 + 2t^2 + 15t - 36 \quad = -(t-3)^2(t+4) \quad \lambda = 3, -4$

$AM = 2, 1$

$(A - 3I)x = \theta$

$$\begin{bmatrix} -4 & 6 & 2 \\ 0 & 2 & -6 \\ 1 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-4x_1 + 6x_2 + 2x_3 = 0$

$2x_2 - 6x_3 = 0 \quad x_2 = 3x_3$

$x_1 - 5x_3 = 0 \quad x_1 = 5x_3$

$\begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \quad GM = 1$

For $\lambda = 3$, $AM = 2$ $GM = 1$, so it is defective

18) $A = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix} \quad x = \begin{bmatrix} 0 \\ 9 \end{bmatrix}$

$\det(A - tI) = \begin{vmatrix} 4-t & -2 \\ 5 & -3-t \end{vmatrix} = (4-t)(-3-t) + 10 = -12 + 3t - 4t + t^2 + 10 = t^2 - t - 2$

$= (t-2)(t+1) \quad \lambda = 2, -1$

$(A - 2I)x = \theta \quad \begin{bmatrix} 2 & -2 \\ 5 & -5 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad 2x_1 - 2x_2 = 0 \quad x_1 = x_2 \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$(A + I) = \theta \quad \begin{bmatrix} 5 & -2 \\ 5 & -2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad 5x_1 - 2x_2 = 0 \quad 5x_1 = 2x_2 \quad \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$x_2 = \frac{2}{5} x_1$

$$\begin{bmatrix} 0 \\ 9 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} a + 2b \\ a + 5b \end{bmatrix} \quad \begin{matrix} a = -6 \\ b = 3 \end{matrix}$$

$$A^{10} x = (-6)(2)^{10} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (3)(-1)^{10} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -6144 \\ -6144 \end{bmatrix} + \begin{bmatrix} 6 \\ 15 \end{bmatrix} = \begin{bmatrix} -6138 \\ -6129 \end{bmatrix}$$

21) $P^2 = P \rightarrow P^{-1} \times P^2 = P^{-1} \times P \rightarrow P^{-1} \times (P \times P) = P^{-1} \times P$

$(P^{-1} \times P) \times P = P^{-1} \times P \quad P^{-1} \times P = I$ so $I \times P = I \rightarrow P = I$

Section 4.6

1) $u = 3 - 2i$ $\qquad \bar{u} = 3 + 2i$

3) $v = 4 + i$ $\qquad \bar{v} = 4 - i$

$u + \bar{v} = 7 - 3i$

7) $v\bar{v} = 16 - i$

19) $\begin{bmatrix} 6 & 8 \\ -1 & 2 \end{bmatrix}$ $\det(A - tI) = \begin{bmatrix} 6-t & 8 \\ -1 & 2-t \end{bmatrix} = (6-t)(2-t) + 8 = t^2 - 8t + 20 = 0$

$-2t \quad t^2$

$12 \quad -6t$

$\dfrac{8 \pm \sqrt{8^2 - 4(1)(20)}}{2}$ $\qquad \lambda = 4+2i,\ 4-2i$

$(A - (4+2i)I)x = \theta$ $\quad \begin{bmatrix} 2-2i & 8 \\ -1 & -2-2i \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2+2i \\ 0 & 0 \end{bmatrix}$ $\quad x_1 + (2+2i)x_2 = 0$, $\quad x_1 = (-2-2i)x_2$ $\quad \begin{bmatrix} -2-2i \\ 1 \end{bmatrix}$

$(A - (4-2i)I)x = \theta$ $\quad \begin{bmatrix} 2+2i & 8 \\ -1 & -2+2i \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2-2i \\ 0 & 0 \end{bmatrix}$ $\quad x_1 + (2-2i)x_2 = 0$, $\quad x_1 = (2-2i)x_2$ $\quad \begin{bmatrix} 2-2i \\ 1 \end{bmatrix}$

23) $\begin{bmatrix} 1 & -4 & -1 \\ 3 & 2 & 3 \\ 1 & 1 & 3 \end{bmatrix}$ $\det(A - tI) = \begin{bmatrix} 1-t & -4 & -1 \\ 3 & 2-t & 3 \\ 1 & 1 & 3-t \end{bmatrix}$

$= (1-t)\begin{vmatrix} 2-t & 3 \\ 1 & 3-t \end{vmatrix} + 4\begin{vmatrix} 3 & 3 \\ 1 & 3-t \end{vmatrix} - \begin{vmatrix} 3 & 2-t \\ 1 & 1 \end{vmatrix} = (1-t)\big((2-t)(3-t) - 3\big) + 4(9 - 3t - 3) - (3 - 2 - t)$

$= (1-t)(3 - 5t + t^2) + 23 - 13t = 3 - 5t + t^2 - 3t + 5t^2 - t^3 + 23 - 13t$

$= -t^3 + 6t^2 - 21t + 26 = 0$ $\qquad -(t-2)(t^2 - 4t + 13) = 0$ $\qquad t - 2 = 0$, $\quad t^2 - 4t + 13 = 0$

$\dfrac{4 \pm \sqrt{16 - 52}}{2}$ $= 2 + 3i$, $\ = 2 - 3i$ $\qquad \lambda = 2,\ 2+3i,\ 2-3i$

$$(A-2I)x=\theta \quad \begin{bmatrix}-1&-4&-1\\3&0&3\\1&1&1\end{bmatrix} \to \begin{bmatrix}1&4&1\\0&-12&0\\0&-3&0\end{bmatrix} \to \begin{bmatrix}1&0&1\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$x_1+x_3=0 \quad x_1=-x_3$$
$$x_2=0$$

$$\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

$$(A-(2+3i)I)x=\theta \quad \begin{bmatrix}-1-3i&-4&-1\\3&-3i&3\\1&1&1-3i\end{bmatrix} \to \begin{bmatrix}-1-3i&-4&-1\\0&-\frac{6}{5}+\frac{3}{5}i&\frac{27}{10}+\frac{9}{10}i\\0&\frac{3}{5}+\frac{6}{5}i&\frac{9}{10}-\frac{27}{10}i\end{bmatrix} \to \begin{bmatrix}-1-3i&-4&-1\\0&\frac{3}{5}+\frac{6}{5}i&\frac{9}{10}-\frac{27}{10}i\\0&0&0\end{bmatrix}$$

$$\to \begin{bmatrix}1&0&\frac{5}{2}-\frac{3}{2}i\\0&1&-\frac{3}{2}-\frac{3}{2}i\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$x_1+\left(\frac{5}{2}-\frac{3}{2}i\right)x_3=0 \quad x_1=\left(-\frac{5}{2}+\frac{3}{2}i\right)x_3$$
$$x_2-\left(\frac{3}{2}+\frac{3}{2}i\right)x_3=0 \quad x_2=\left(\frac{3}{2}+\frac{3}{2}i\right)x_3$$

$$\begin{bmatrix}-\frac{5}{2}+\frac{3}{2}i\\\frac{3}{2}+\frac{3}{2}i\\1\end{bmatrix}$$

$$(A-(2-3i)I)x=\theta \quad \begin{bmatrix}-1+3i&-4&-1\\3&3i&3\\1&1&1+3i\end{bmatrix} \to \begin{bmatrix}-1-3i&-4&-1\\0&-\frac{6}{5}+\frac{3}{5}i&\frac{27}{10}+\frac{9}{10}i\\0&\frac{3}{5}+\frac{6}{5}i&\frac{9}{10}-\frac{27}{10}i\end{bmatrix} \to \begin{bmatrix}-1-3i&-4&-1\\0&\frac{3}{5}+\frac{6}{5}i&\frac{9}{10}-\frac{27}{10}i\\0&0&0\end{bmatrix}$$

$$\to \begin{bmatrix}1&0&\frac{5}{2}+\frac{3}{2}i\\0&1&-\frac{3}{2}+\frac{3}{2}i\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$x_1+\left(\frac{5}{2}+\frac{3}{2}i\right)x_3=0 \quad x_1=\left(-\frac{5}{2}-\frac{3}{2}i\right)x_3$$
$$x_2-\left(\frac{3}{2}-\frac{3}{2}i\right)x_3=0 \quad x_2=\left(\frac{3}{2}-\frac{3}{2}i\right)x_3$$

$$\begin{bmatrix}-\frac{5}{2}-\frac{3}{2}i\\\frac{3}{2}-\frac{3}{2}i\\1\end{bmatrix}$$

25) $(1+i)x+iy=5+4i \qquad (1-i)x-4y=-11+5i$

$$(1+i)x=5+4i-iy \quad x=\frac{5+4i-iy}{1+i} \quad x=\frac{(5+4i-iy)(1-i)}{(1+i)(1-i)}=\frac{5+4i-iy-5i+4i^2+i^2y}{1-i^2}$$

$$=\frac{5+4i-iy-5i-4-y}{2}=\frac{9-i-i(y-1)}{2}=\frac{-y+9}{2}+\frac{-y-1}{2}$$

$$(1-i)\left(\frac{-y+9}{2}+\frac{-y-1}{2}\right)-4y=-11+5i=(-y+4-5i)-4y=-y+4-5i-4y=-5y-5i+4$$

$$-5y-5i+4=-11+5i \quad -5y-5i=-15+5i \quad -5y=-15+10i \quad y=-3+2i$$

$$x=\frac{9-(3-2i)}{2}+\frac{-1-(3-2i)}{2}=3-i-1-2i=2-i$$

$$x=2-i$$
$$y=3-2i$$

27) $x = \begin{bmatrix} 1+i \\ 2 \end{bmatrix}$ $\|x\| = x^T x = \begin{bmatrix} 1-i & 2 \end{bmatrix} \begin{bmatrix} 1+i \\ 2 \end{bmatrix}$

$= (1-i)(1+i) + (2 \cdot 2) = 1 - i^2 + 4 = 1+1+4 = 6$

Section 4.7