HW8. solution

1.

(a).

(i) $T(s) = -\frac{Z_2}{Z_1} = -\frac{R_2 // \frac{1}{sC_2}}{R_1}$

$= \frac{-R_2}{R_1} \frac{1}{1+sR_2C_2}$

$T(j\omega) = -\frac{1}{1+j\omega R_2C_2} \cdot \frac{R_2}{R_1}$

$|T(j\omega)| = \frac{R_2}{R_1} \frac{1}{\sqrt{1+\omega^2 R_2^2 C_2^2}}$

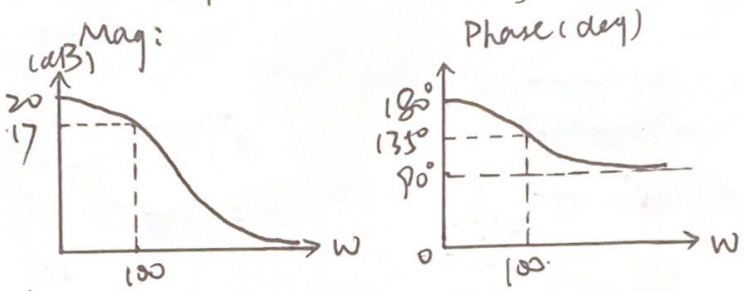
$\angle T(j\omega) = 180° - \tan^{-1}(\omega R_2 C_2)$.

Low pass Filter

(ii) $T(j\omega) = \frac{-10}{1+j\omega 10^{-2}}$. $R_2C_2 = 10^{-2}$

$|T(j\omega)| = \frac{10}{\sqrt{1+10^{-4}\omega^2}}$

$\angle T(j\omega) = 180° - \tan^{-1}(10^{-2}\omega)$



(b).

(i) $T(s) = -\frac{Z_2}{Z_1} = \frac{-R_2}{sC_1 // sL_1} = -R_2(sC_1 + \frac{1}{sL_1})$

$T(j\omega) = -R_2(j\omega C_1 + \frac{1}{j\omega L_1})$

$= R_2 j(\frac{1}{\omega L_1} - \omega C_1)$.

$|T(j\omega)| = R_2 \left|\frac{1}{\omega L_1} - \omega C_1\right|$.

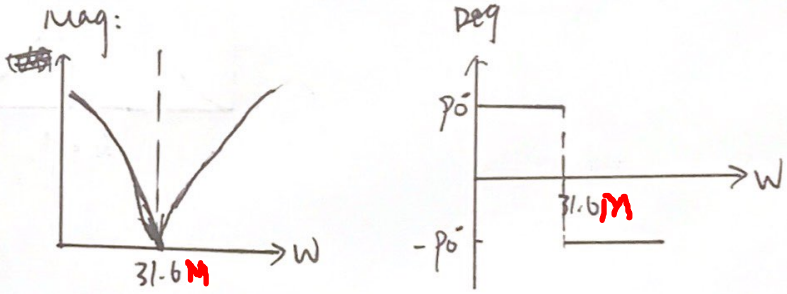
$\angle T(j\omega) = \begin{cases} 90° & \omega < \frac{1}{\sqrt{L_1C_1}} \\ -90° & \omega > \frac{1}{\sqrt{L_1C_1}} \end{cases}$

Bandstop Filter $\omega_0 = \frac{1}{\sqrt{L_1C_1}}$.

(ii). $T(j\omega) = j(\frac{10^4}{\omega} - 0.1\omega)$. $\omega_0 = \frac{1}{\sqrt{L_1C_1}} = 10^{1.5} \approx 31.6$

$|T(j\omega)| = \left|\frac{10^4}{\omega} - 0.1\omega\right|$.

$\angle T(j\omega) = \begin{cases} 90° & \omega < 31.6 \\ -90° & \omega > 31.6 \end{cases}$



(c).

(i) $T(s) = -\frac{R_2}{\frac{1}{sC_1} // R_1} = \frac{-R_2}{R_1}(1+sR_1C_1)$.

$T(j\omega) = -\frac{R_2}{R_1}(1+j\omega R_1C_1)$

$|T(j\omega)| = \frac{R_2}{R_1}\sqrt{1+(\omega R_1C_1)^2}$

$\angle T(j\omega) = 180° + \tan(\omega R_1C_1)$

$R_1C_1 = 10^{-1}$

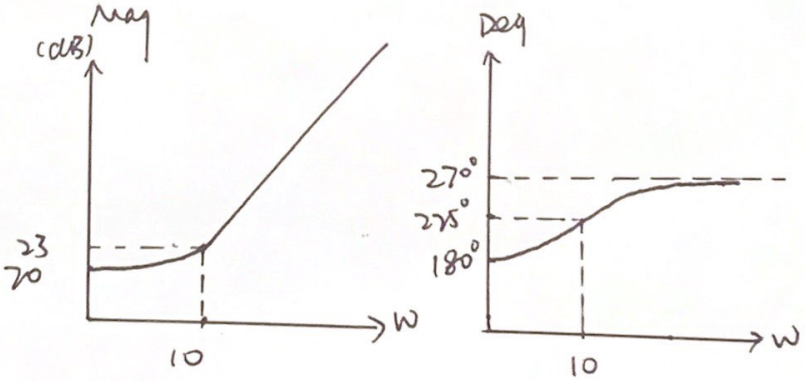
Highpass filter

(ii). $T(s) = -10(1+0.1s)$

$T(j\omega) = -10(1+0.1j\omega)$

$|T(j\omega)| = 10\sqrt{1+(0.1\omega)^2}$

$\angle T(j\omega) = 180° + \tan(0.1\omega)$



(d).

(i) $T(s) = \dfrac{-\frac{1}{sC_2} \,//\, sL_2}{R_1}$

$= \dfrac{-sL_2}{(1+s^2L_2C_2)R_1}$

Bandpass filter

$T(j\omega) = \dfrac{j\omega L_2}{(\omega^2 L_2C_2 - 1)R_1}$

$|T(j\omega)| = \dfrac{\omega L_2}{|\omega^2 L_2C_2 - 1|R_1}$

$\angle T(j\omega) = \begin{cases} -90^\circ & \omega < \frac{1}{\sqrt{L_2C_2}} \\ 90^\circ & \omega > \frac{1}{\sqrt{L_2C_2}} \end{cases}$

$\omega_0 = \dfrac{1}{\sqrt{L_2C_2}} = 10^{\frac{13}{2}} \approx 3.16e7 = 31.6M$ 3.16M

(ii).

$T(j\omega) = \dfrac{j\omega 10^{-6}}{(\omega^2 10^{-6}\cdot 10^{-7} - 1)10^4}$

$= \dfrac{j\omega}{\omega^2 10^{-3} - 10^{10}}$

$|T(j\omega)| = \dfrac{\omega}{\omega^2 10^{-3} - 10^{10}}$

$\angle T(j\omega) = \begin{cases} -90^\circ & \omega < 31.6M \\ 90^\circ & \omega > 31.6M \end{cases}$ 3.16M 3.16M