Homework 1 - Math 2568 (Autumn 2022)

Prof. Cueto

Due date: Friday September 2, 2022 (in class).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§1.1	1, 5, 6, 7, 11, 15, 16, 24, 31, 32	11, 15, 16, 24, 32
§1.2	1, 8, 12, 13, 22, 31, 36, 41, 47, 50, 52	13, 31, 36, 41, 50

Extra Problem: Write down the following information from four different people in your class (not including yourself):

1. Name Will
2. Favorite movie, book or song. Shawshank Redemption
3. Fun fact about them (hobbies, took a selfie with someone famous, travelled to an exotic location, visited other states, etc.) Been to Africa and Europe

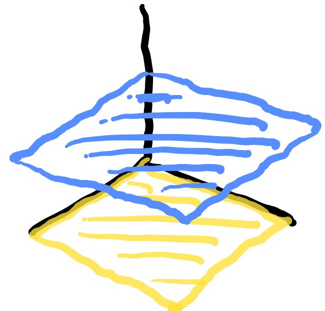
Section 1.1

11.) $2x + y = 5$
$x - y = 1$

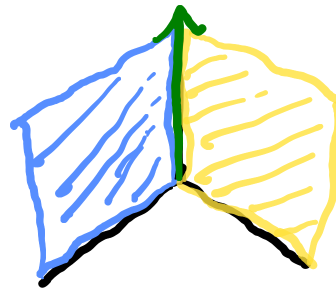
$\boxed{\begin{array}{l} x = 2 \\ y = 1 \end{array}}$ Unique solution

15.)

a) No Solution



b) Infinite Solutions



A (2x3) system of linear equations could have a unique solution, and it would consist of two planes only meeting at a single point, such as along an edge or at a corner.

16.)

$$2x_1 + x_2 + x_3 = 3$$
$$-2x_1 + x_2 - x_3 = 1$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ -2 & 1 & -1 & 1 \end{array}\right]$$

$R_1 \to R_1 + R_2$
$R_2 \to R_2 + R_1$

$$\left[\begin{array}{ccc|c} 0 & 2 & 0 & 4 \\ 0 & 2 & 0 & 4 \end{array}\right]$$

$\boxed{x_3 = t}$

$2x_2 = 4$ $\boxed{x_2 = 2}$

$2x_1 + 2 + t = 3$ $2x_1 = 1 - t$ $\boxed{x_1 = \frac{1-t}{2}}$

Their intersection is a line

24.)

$$x_1 - x_2 = -1$$
$$x_1 + x_2 = 3$$

$$\boxed{A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \qquad B = \left[\begin{array}{cc|c} 1 & -1 & -1 \\ 1 & 1 & 3 \end{array}\right]}$$

32.) $x_2 + x_3 = 4$
$x_1 - x_2 + 2x_3 = 1$
$2x_1 + x_2 - x_3 = 6$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 4 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & -1 & 6 \end{array}\right]$$

Swap $R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & 4 \\ 2 & 1 & -1 & 6 \end{array}\right] \xrightarrow[R_3-2R_1]{\text{Combine}} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 3 & -5 & 4 \end{array}\right]$$

$$\boxed{\begin{array}{r} x_1 - x_2 + 2x_3 = 1 \\ x_2 + x_3 = 4 \\ 3x_2 - 5x_3 = 4 \end{array}}$$

Section 1.2

13.) $$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 \end{array}\right] \xrightarrow[R_1-2R_2]{} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -2 \\ 0 & 1 & 3 & 1 \end{array}\right]$$

$x_1 - 5x_3 = -2$
$x_2 + 3x_3 = 1$

$$\boxed{\begin{array}{l} x_3 \text{ is arbitrary} \\ x_2 = 1 - 3x_3 \\ x_1 = -2 + 5x_3 \end{array}}$$

31.) $x_1 + x_3 + x_4 - 2x_5 = 1$
$2x_1 + x_2 + 3x_3 - x_4 + x_5 = 0$
$3x_1 - x_2 + 4x_3 + x_4 + x_5 = 1$

$$\rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 1 & -2 & 1 \\ 2 & 1 & 3 & -1 & 1 & 0 \\ 3 & -1 & 4 & 1 & 1 & 1 \end{array}\right]$$

R_2-2R_1 / R_3-3R_1

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 1 & -2 & 1 \\ 0 & 1 & 1 & -3 & 5 & -2 \\ 0 & -1 & 1 & -2 & 7 & -2 \end{array}\right] \xrightarrow[R_3+R_2]{} \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 1 & -2 & 1 \\ 0 & 1 & 1 & -3 & 5 & -2 \\ 0 & 0 & 2 & -5 & 12 & -4 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 1 & -2 & 1 \\ 0 & 1 & 1 & -3 & 5 & -2 \\ 0 & 0 & 1 & -\frac{5}{2} & 6 & -2 \end{array}\right] \xrightarrow[R_1-R_3,\ R_2-R_3]{} \left[\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & \frac{7}{2} & -8 & 3 \\ 0 & \boxed{1} & 0 & -\frac{1}{2} & -1 & 0 \\ 0 & 0 & \boxed{1} & -\frac{5}{2} & 6 & -2 \end{array}\right]$$

$$x_1 + \frac{7}{2}x_4 - 8x_5 = 3$$
$$x_2 - \frac{1}{2}x_4 - x_5 = 0$$
$$x_3 - \frac{5}{2}x_4 + 6x_5 = -2$$

$$\boxed{\begin{array}{ll} x_1 = 4 & x_4 = 2 \\ x_2 = 2 & x_5 = 1 \\ x_3 = -3 & \end{array}}$$

36.) $\begin{array}{l} x_1 + 2x_2 = -3 \\ ax_1 - 2x_2 = 5 \end{array} \rightarrow \left[\begin{array}{cc|c} 1 & 2 & -3 \\ a & -2 & 5 \end{array}\right] \xrightarrow[R_2 \to R_2+R_1]{} \left[\begin{array}{cc|c} 1 & 2 & -3 \\ a+1 & 0 & 2 \end{array}\right]$

$$\xrightarrow{R_2 \to \frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & 2 & -3 \\ \frac{a+1}{2} & 0 & 1 \end{array}\right] \rightarrow \left(\frac{a+1}{2}\right)x_1 + 0x_2 = 1 \rightarrow \frac{a+1}{2} = 0 \rightarrow \boxed{a = -1}$$

41.) $\begin{array}{l} 2\cos\alpha + 4\sin\beta = 3 \\ 3\cos\alpha - 5\sin\beta = -1 \end{array} \rightarrow \begin{array}{l} 2x_1 + 4x_2 = 3 \\ 3x_1 - 5x_2 = -1 \end{array} \rightarrow \left[\begin{array}{cc|c} 2 & 4 & 3 \\ 3 & -5 & -1 \end{array}\right]$

$$\xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 2 & \frac{3}{2} \\ 3 & -5 & -1 \end{array}\right] \xrightarrow[R_2-3R_1]{} \left[\begin{array}{cc|c} 1 & 2 & \frac{3}{2} \\ 0 & -11 & -\frac{11}{2} \end{array}\right] \xrightarrow{\left(-\frac{1}{11}\right)R_2} \left[\begin{array}{cc|c} 1 & 2 & \frac{3}{2} \\ 0 & 1 & \frac{1}{2} \end{array}\right]$$

$$\xrightarrow{R_1-2R_2} \left[\begin{array}{cc|c} \boxed{1} & 0 & \frac{1}{2} \\ 0 & \boxed{1} & \frac{1}{2} \end{array}\right] \rightarrow$$

$$\cos\alpha = \frac{1}{2}$$
$$\alpha = \cos^{-1}\left(\frac{1}{2}\right) \rightarrow \boxed{\alpha = \frac{\pi}{3}}$$
$$\alpha = 2\pi - \frac{\pi}{3} \rightarrow \boxed{\alpha = \frac{5\pi}{3}}$$

$\sin B = \frac{1}{2} \rightarrow B = \sin^{-1}\left(\frac{1}{2}\right) \rightarrow \boxed{B = \frac{\pi}{6}} \rightarrow B = \pi - \frac{\pi}{6} \rightarrow \boxed{B = \frac{5\pi}{6}}$

50.) $(-1,6) \rightarrow 6 = a(-1)^2 + b(-1) + c \rightarrow 6 = a - b + c$

$(1,4) \rightarrow 4 = a(1)^2 + b(1) + c \rightarrow 4 = a + b + c$

$(2,9) \rightarrow 9 = a(2)^2 + b(2) + c \rightarrow 9 = 4a + 2b + c$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 1 & 1 & 1 & 4 \\ 4 & 2 & 1 & 9 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 4 & 2 & 1 & 9 \\ 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & 6 \end{array}\right]$$

$$\xrightarrow[R_3 \rightarrow 4R_3 - R_1]{R_2 \rightarrow 4R_2 - R_1} \left[\begin{array}{ccc|c} 4 & 2 & 1 & 9 \\ 0 & 2 & 3 & 7 \\ 0 & -6 & 3 & 15 \end{array}\right] \xrightarrow[R_3 \rightarrow R_1 + 3R_2]{R_1 \rightarrow R_1 - R_2} \left[\begin{array}{ccc|c} 4 & 0 & -2 & 2 \\ 0 & 2 & 3 & 7 \\ 0 & 0 & 12 & 36 \end{array}\right]$$

$$\xrightarrow{R_3 \rightarrow \frac{1}{12}R_3} \left[\begin{array}{ccc|c} 4 & 0 & -2 & 2 \\ 0 & 2 & 3 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2 \rightarrow R_2 - 3R_3]{R_1 \rightarrow R_1 + 2R_3} \left[\begin{array}{ccc|c} 4 & 0 & 0 & 8 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2 \rightarrow \frac{1}{2}R_2]{R_1 \rightarrow \frac{1}{4}R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$1a + 0b + 0c = 2 \rightarrow a = 2$

$0a + 1b + 0c = -1 \rightarrow b = -1$

$0a + 0b + 1c = 3 \rightarrow c = 3$

$\xrightarrow{y = ax^2 + bx + c} \boxed{y = 2x^2 - x + 3}$