Spring 2023

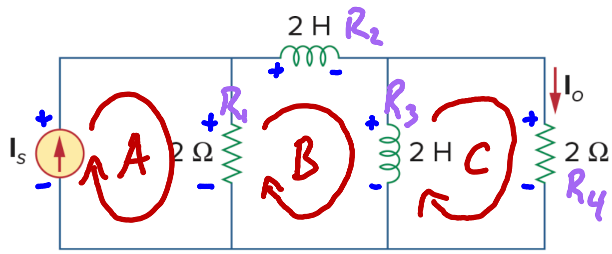
Homework 8
Due: Friday, April 7th, 2023 by 7PM.

Note: In order to receive full credit, you must show your work and carefully justify your answers. The correct answer without any work will receive little or no credit.

1. For the circuit below, find the transfer function H(jw) = Io(jw)/Is(jw).

$i_C = i_o$

$i_A = i_S$



$i_1 = i_A - i_B$

$i_2 = i_B$

$i_3 = i_B - i_C$

$i_4 = i_C$

KVL@B

$$V_1 = V_2 + V_3$$

$$2i_1 = 2j\omega i_2 + 2j\omega i_3$$

$$2i_A - 2i_B = 2j\omega i_B + 2j\omega i_B - 2j\omega i_C$$

$$2i_A - 2i_B = 4j\omega i_B - 2j\omega i_C$$

$$2i_S - 2i_B - 4j\omega i_B = -2j\omega i_o$$

$$2i_B - 4j\omega i_B = -2i_S - 2j\omega i_o$$

$$i_B - 2j\omega i_B = -i_S - j\omega i_o$$

$$(1-2j\omega)i_B = -i_S - j\omega i_o$$

$$i_B = \frac{-i_S - j\omega i_o}{1-2j\omega}$$

KVL@C

$$V_3 = V_4$$

$$2j\omega i_3 = 2i_4$$

$$2j\omega i_B - 2j\omega i_o = 2i_o$$

$$2j\omega i_B = (2+2j\omega)i_o$$

$$i_B = \frac{(2+2j\omega)i_o}{2j\omega}$$

$$\frac{(2+2j\omega)i_o}{2j\omega} = \frac{-i_S - j\omega i_o}{1-2j\omega}$$

$$\frac{(1+j\omega)i_o}{j\omega} = \frac{i_S + j\omega i_o}{2j\omega - 1}$$

$$(2j\omega-1)(1+j\omega)i_o = (i_S + j\omega i_o)j\omega$$

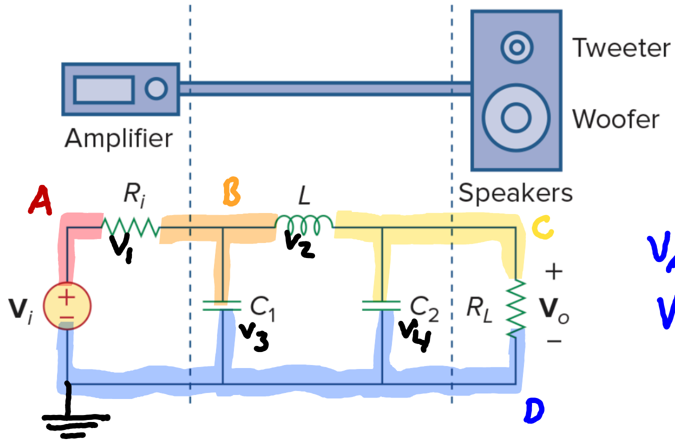
$$(2j\omega + 2j\omega^2 - j\omega - 1)i_o = j\omega i_S + j\omega^2 i_o$$

$$(2j\omega + 3j\omega^2 - j\omega - 1)i_o = j\omega i_S$$

$$\boxed{\frac{i_o}{i_S} = \frac{j\omega}{1+3j\omega+j\omega^2}}$$

2. The crossover circuit below is a low-pass filter that is connected to a woofer. Find the transfer function H(jw) = Vo(jw)/Vi(jw).

Find $\frac{V_o}{V_i}$



$V_A = V_i$

$V_D = 0$

$I_1 = I_A - I_B$

$I_2 = I_B - I_C$

$I_3 = I_B$

$I_4 = I_o = I_C$

KVL @ B

$$I_{R_1} = I_{11} + I_L$$

$$\frac{V_I - V_A}{R_i} = \frac{V_A}{\frac{1}{j\omega C_1}} + \frac{V_A}{\omega L j}$$

$$\frac{V_I}{R_i} - \frac{V_A}{R} = V_A j\omega C_1 + \frac{V_A}{j\omega L} - \frac{V_o}{j\omega L}$$

$$\frac{V_I}{R_i} + \frac{V_o}{j\omega L} = V_A \omega C_1 j + \frac{V_A}{\omega L j} + \frac{V_A}{R_i}$$

$$\frac{V_I}{R_i} + \frac{V_o}{j\omega L} = V_A\left(\omega L j + \frac{1}{\omega L j} + \frac{1}{R_i}\right)$$

$$\frac{V_I}{R_i} + \frac{V_o}{j\omega L} = \frac{V_o\left(j\omega C + \frac{1}{R_2} + \frac{1}{\omega L j}\right)\left(\omega C_1 j + \frac{1}{\omega L j} + \frac{1}{R_i}\right)}{j\omega L} - \frac{V_o}{j\omega L}$$

$$\frac{V_I}{R_i} + \frac{V_o}{j\omega L} = V_o\left(\frac{\left(j\omega C + \frac{1}{R_2} + \frac{1}{\omega L j}\right)\left(\omega C_1 j + \frac{1}{\omega L j} + \frac{1}{R_i}\right)}{j\omega L} - \frac{1}{j\omega L}\right)$$

$$\frac{V_I}{\frac{\left(j\omega C_2 + \frac{1}{R_L} + \frac{1}{\omega L j}\right)\left(j\omega C_1 + \frac{1}{\omega L j} + \frac{1}{R_i}\right) - \frac{1}{j\omega L}}{\omega L j}} = V_o R_i$$

KVL @ C

$$I_L = I_{C2} + I_{RC}$$

$$\frac{V_A - V_o}{\omega L j} = \frac{V_o}{\frac{1}{j\omega C_2}} + \frac{V_o}{R_L}$$

$$\frac{V_A}{j\omega L} - \frac{V_o}{j\omega L} = V_o j\omega C_2 + \frac{V_o}{R_L}$$

$$\frac{V_A}{j\omega L} = V_o j\omega C_2 + \frac{V_o}{R_L}$$

$$\frac{V_A}{j\omega L} = V_o\left(j\omega C_2 + \frac{1}{R_2} + \frac{1}{j\omega L}\right)$$

$$V_A = \frac{V_o\left(j\omega C_2 + \frac{1}{R_L} + \frac{1}{j\omega L}\right)}{j\omega L}$$

$$\frac{V_o(j\omega)}{V_I(j\omega)} = R_i \frac{1}{\left(\frac{\left(\omega C_2 + \frac{1}{R_L} + \frac{1}{j\omega L}\right)\left(j\omega C_1 + \frac{1}{j\omega L} + \frac{1}{R_i}\right)}{j\omega L} - \frac{1}{j\omega L}\right)}$$

3. Sketch by hand the bode plot of the transfer function below. Clearly explain your reasoning.

$$\mathbf{H}(\omega) = \frac{0.2\,(10 + j\omega)}{j\omega\,(2 + j\omega)}$$

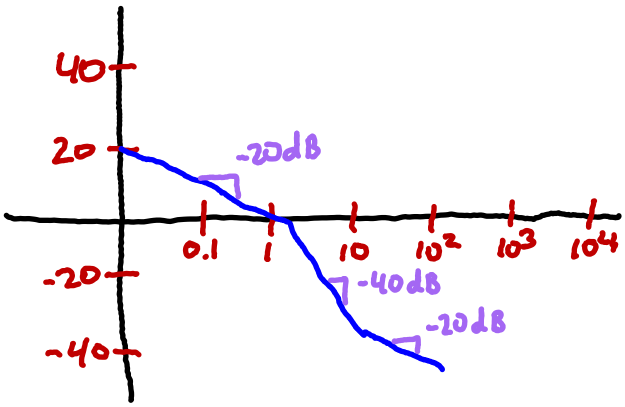
Poles

$j\omega = -2, 0$

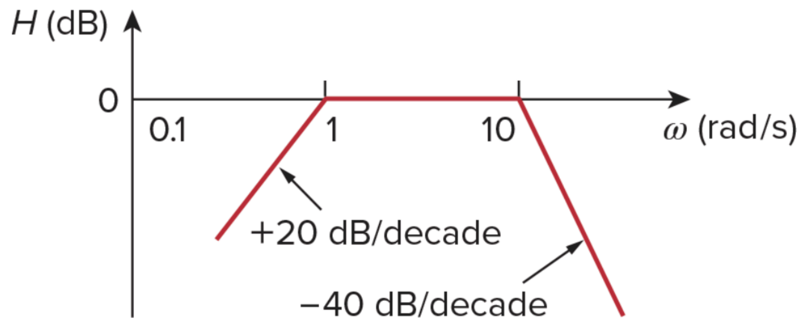
Zeros

$j\omega = -10$

Magnitude



4. For the bode plot below:
   A. What is the transfer function that represents this plot?
   B. Design low and high pass RC filters connected in series for the transfer function found in part A. Clearly specify the circuit drawing and pick realistic values for the resistors and capacitors.



A. $k = 20\log(1) = 0$

$$j\omega \cdot \frac{1}{1+j\omega} \cdot \left(\frac{1}{1+\frac{j\omega}{10}}\right)^2 = \frac{j\omega RC}{(1+j\omega RC)} \cdot \left(\frac{1}{1+j\omega RC}\right)^2$$

$C = 10^{-6} \Rightarrow RC = 0.1$

$R = 1m\Omega$

$$= \frac{j\omega(10^{-6})(1m\Omega)}{1+j\omega(10^{-6})(1m\Omega)} \cdot \left(\frac{1}{1+j\omega 0.1}\right)^2$$

B

HPF → LPF → LPF