Practice Sheet Review

3.) FUNCTION $P_3(A[a,\ldots,b])$

IF $b-a \le 1$ THEN RETURN(1)

$root = \lfloor \sqrt{b-a+1} \rfloor$

$total \leftarrow 0$

$k \leftarrow 0$

WHILE $k \le b-a+1$

$total \leftarrow total + P_3(A[a+k,\ldots,a+k+root]) + 1$

$k \leftarrow k + root$

FOR $i \leftarrow a+1$ TO $b-1$ DO

$A[i] \leftarrow \frac{A[i-1]+A[i+1]}{2}$

RETURN($total$)

1

- Base Case? $b-a \le 1 \Rightarrow b-a+1 = n \le 2$
- #recursive calls? $\sqrt{n}$

ℓ, loop itr	value of k
0	0
1	$\sqrt{n}$
2	$2\sqrt{n}$
$\vdots$	$\vdots$
ℓ	$k = \ell\sqrt{n}$

$\ell\sqrt{n} \le n$

$\ell \le \sqrt{n}$

$T_3(n) = cn + \sqrt{n}\,T_3(\sqrt{n})$

$T_3(\text{input}) = c(\text{input}) + \sqrt{\text{input}}\,T_3(\text{input}^{1/2})$

- recursive call size? $\sqrt{n}$
- non-recursive work?

$\sqrt{n} + \sum_{i=a+1}^{b-1} c = \sqrt{n} + (b-1-(a+1)+1)$

$= \sqrt{n} + (n-2) \in \Theta(n)$

$$T_3(n) = cn + \sqrt{n}\,T_3(\sqrt{n})$$

$$= cn + \sqrt{n}\left(c\sqrt{n} + \sqrt{\sqrt{n}}\,T_3\left(n^{1/2^2}\right)\right)$$

$n^{1/4}$

$$= cn + cn + n^{3/4}\,T_3\left(n^{\frac{1}{2^2}}\right)$$

$$= cn + cn + n^{3/4}\left(cn^{1/4} + n^{1/8}\,T_3\left(n^{1/2^3}\right)\right)$$

$$= cn + cn + cn + n^{7/8}\,T_3\left(n^{1/2^3}\right)$$

$$= \cdots \approx cn\lg(\lg n) + n^{\frac{\lg(\lg n)-1}{\lg(\lg n)}}\,T_3(2)$$

$n^{1/2^m} = 2$

$n = 2^{(2^m)}$

$2^m = \lg n$

$m = \lg(\lg n)$

Guess: $T_3(n) \in \Theta(n \log(\log n))$

O-work $\quad T_3(n) = cn + \sqrt{n}\, T_3(\sqrt{n})$

Assume $n_0 \le k < n$ $\quad T_3(k) \le a k \lg(\lg k)$ with $a > 0$

$T_3(n) = cn + \sqrt{n}\, T_3(\sqrt{n}) \le cn + \sqrt{n}\left(a\sqrt{n} \lg(\lg \sqrt{n})\right)$

$= cn + an\left(\lg\left(\frac{1}{2}\lg n\right)\right) = cn + an\left(\lg(\lg n) - \lg 2\right)$

$= an\lg(\lg n) + cn - an \le an\lg(\lg n)$

True if $c - a \le 0$

$a \ge c$

$T_3(2) \le a\, 2\lg(\lg 2) = 2a \lg 1 = 0$

Sub. base cases: 3, 4, 5, 6, 7, 8

$T_3(3) \le a \cdot 3 \lg(\lg 3) \Rightarrow a \ge \dfrac{T_3(3)}{3\lg(\lg 3)}$

$\vdots$

$T_3(8) \le \quad \dots \quad \Rightarrow a \ge \dfrac{T_3(8)}{8\lg(\lg 8)}$