CSE 2321 Foundations I Spring 2024 Dr. Estill

Homework 3 Due: Friday, February 16

1.) (40 points) Let's set $P(x)$ to be the predicate "x is a dragon",
$Q(x)$ to be the predicate "x breathes fire",
and $x = y$ to be the predicate "x and y are the exact same object".

Rewrite the following English statements in symbolic language using the predicates P, Q, and R, universal and existential quantifiers and logical connectives.

(a) There are no dragons.
(b) Not everything is a dragon.
(c) There is at least one dragon.
(d) There are at least two dragons.
(e) There is at most one dragon.
(f) There is exactly one dragon.
(g) Dragons breathe fire.
(h) Only dragons breathe fire.

2.) Using only symbols $=$, $\neq$, $\leq$, $+$, $\times$, logical connectives, the numbers 0, 1 and 2, variables, brackets (or braces or parentheses or ...) and quantifiers, write the following predicates (a.k.a. Boolean functions of the variables the sentence is talking about). The simpler your answer, the better. The first one, part (a), is solved as an example. These are all regarding the natural numbers, $\mathbb{N} = \{0, 1, 2, 3, \dots\}$.

(a) (0 point example)"x is a sum of two squares of natural numbers."
SOLUTION: $(\exists y \in \mathbb{N})(\exists z \in \mathbb{N})[x = y \times y + z \times z]$
Notice, as this is a predicate about x, x remains unquantified.
(b) (10 points)"x is a common multiple of y and z."
(c) (10 points extra credit)"x is the least common multiple of y and z."
(d) (5 or 10 points[1]) "x is an odd prime number."

3.) (40 points) Prove, using induction, that

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

(Remember that $\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2$.)

[1]You can get a maximum of five points if you use a set of primes, P. If you instead correctly use a definition of primes in your expression you can get ten points.