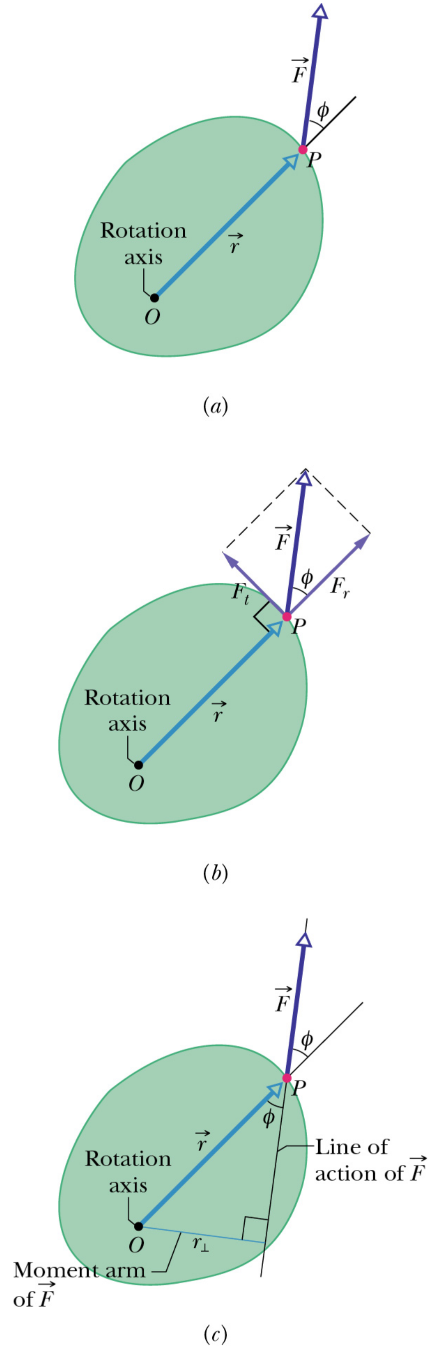
Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF\sin\phi$$

$$= (r)(F\sin\phi) = rF_{\perp}$$

$$= (F)(r\sin\phi) = r_{\perp}F$$



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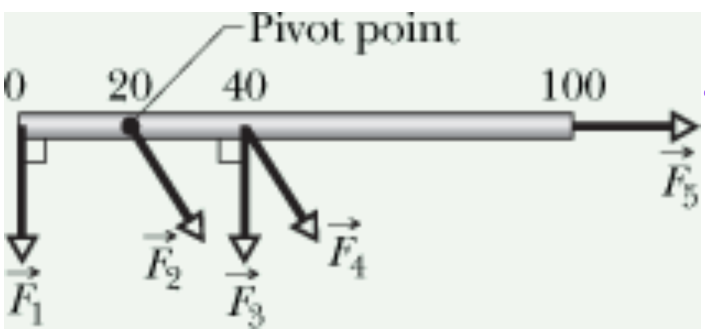
A meter stick pivot about the dot at the position marked 20 (for 20 cm).
All five forces on the stick are horizontal and have the same magnitude. Rank the forces according to the magnitude of the torque they produce, greatest first.



$F_1 = 20F$

$F_2 = 0$

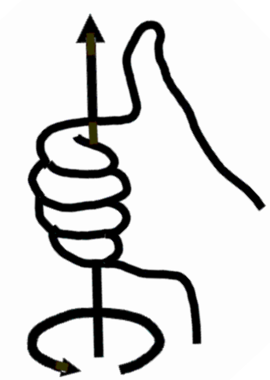
$F_3 = 20F$

$F_4 = 20F\cos\theta$

$F_5 = 0$

$F_1 = F_3 > F_4 > F_2 = F_5$

Directions of the torque:
CCW +
CW -



Newton's Second Law

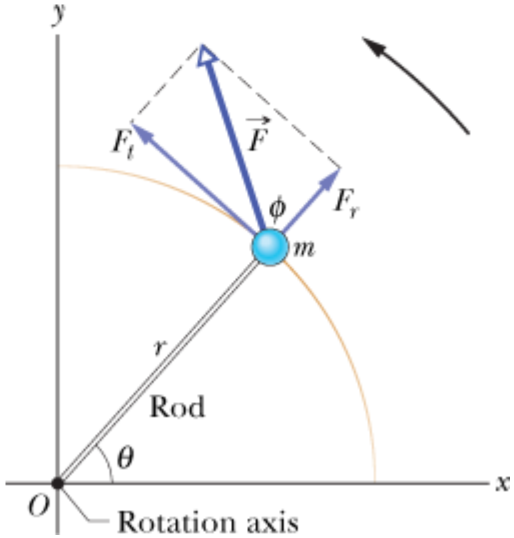
$$F_t = ma_t$$

$$F_t r = ma_t r$$

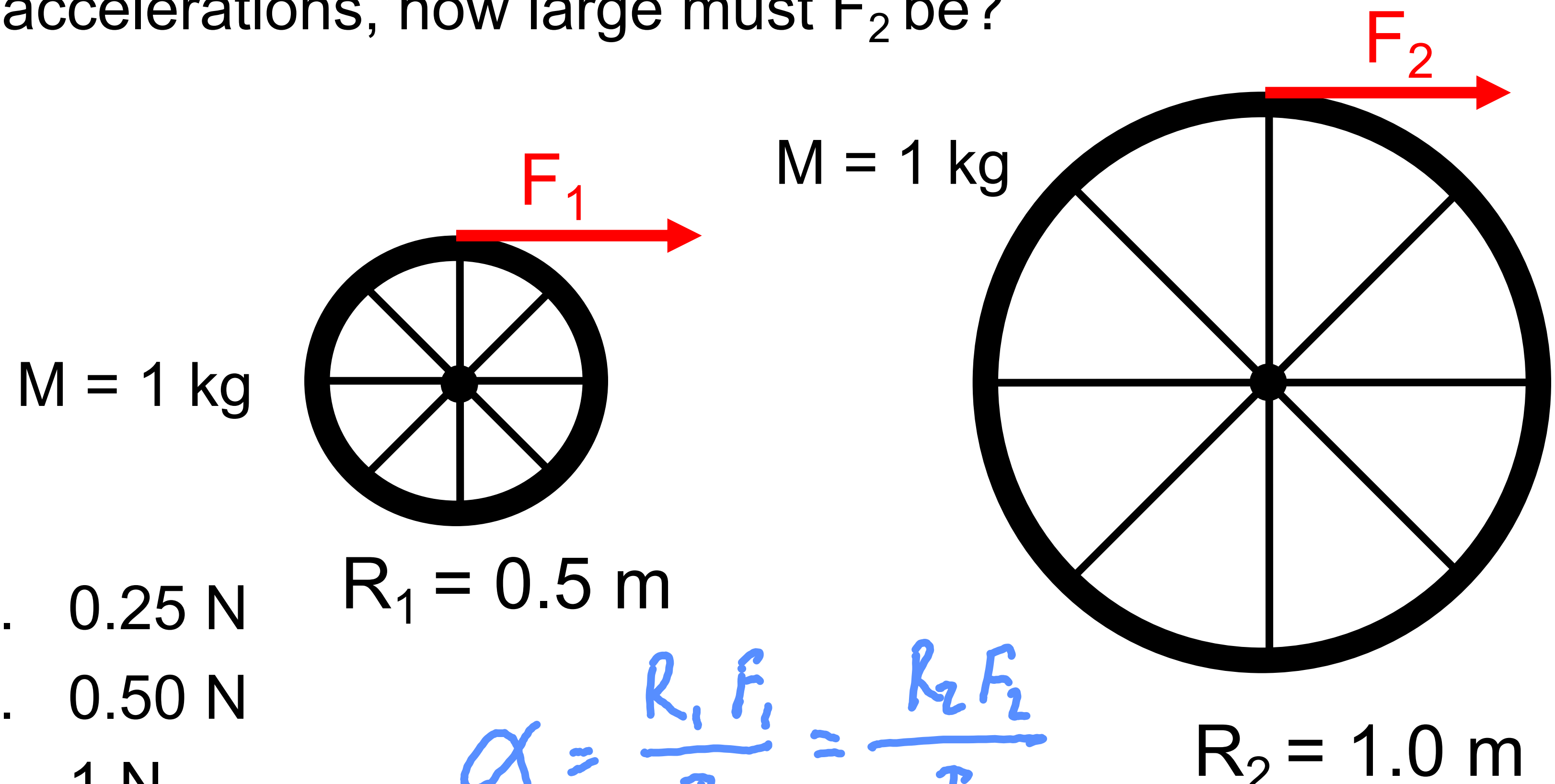
$$a_t = \alpha r$$

$$\tau = m(\alpha r)r = mr^2\alpha = I\alpha$$

$$\tau_{net} = I\alpha$$



Two 1 kg wheels with fixed hubs start from rest, and forces are applied as shown. Assume that the hub and spokes are massless, and that F_1 = 1 N. In order to impart identical angular accelerations, how large must F_2 be?



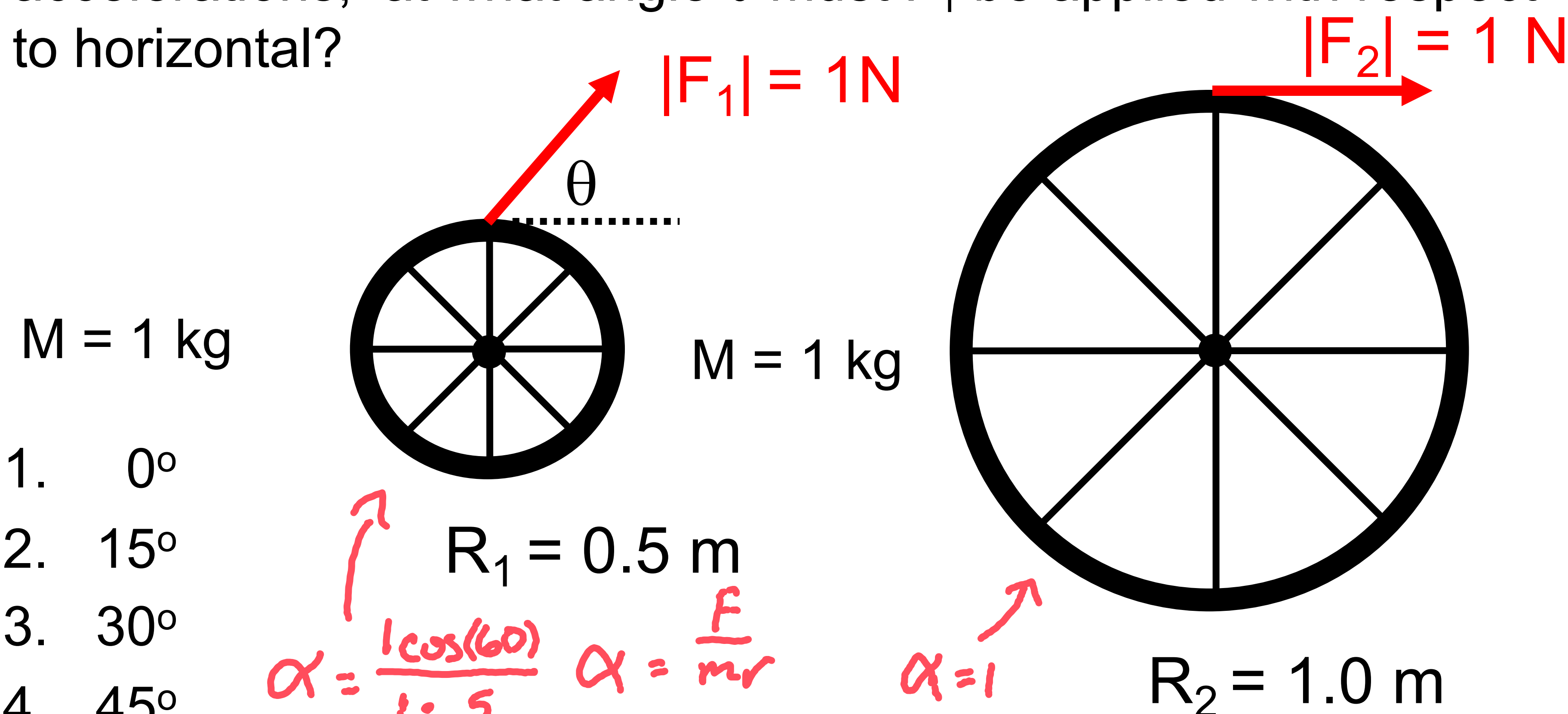
1. 0.25 N
2. 0.50 N
3. 1 N
4. 2 N
5. 4 N

$$\alpha = \frac{R_1 F_1}{I_1} = \frac{R_2 F_2}{I_2}$$

$$I = MR^2 \qquad \alpha = \frac{F}{MR}$$

$$F_2 = F_1\left(\frac{R_1}{R_2}\right) = 2\,N$$

Two 1 kg wheels with fixed hubs start from rest and equal-magnitude 1 N forces are applied to each. The hub and spokes are virtually massless. In order to impart identical angular accelerations, at what angle θ must F_1 be applied with respect to horizontal?

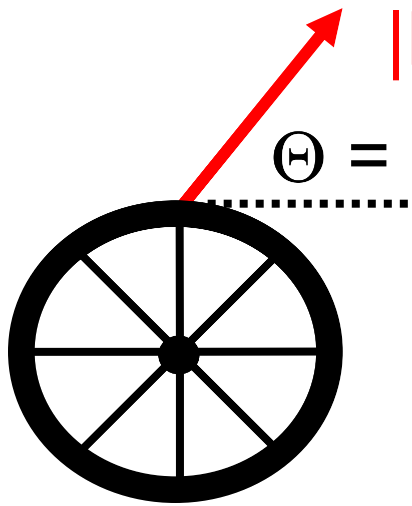


1. 0^o
2. 15^o
3. 30^o
4. 45^o
5. 60^o
6. 90^o

$\alpha = \frac{1\cos(60)}{1 \cdot .5}$ $\alpha = \frac{F}{mr}$ $\alpha = 1$

$\alpha = \frac{.5}{.5} = 1$

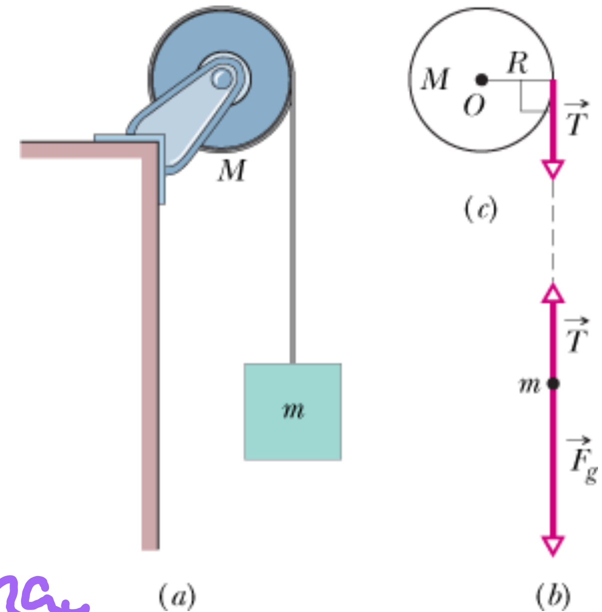
You are given the same small wheel and force as in the previous problem. As before, the spokes and hub are virtually massless. What is the magnitude of the wheel's angular acceleration?



$$\alpha = \frac{F}{MR} = \frac{1\cos(60)}{1\cdot .5} = \frac{.5}{.5} = 1 \text{ R/sec}^2$$

1. ¼ rad/sec²
2. ½ rad/sec²
3. 1 rad/sec²
4. 2 rad/sec²
5. 4 rad/sec²

A uniform disk, with mass M = 2.5 kg and radius R = 20 cm, mounted on a fixed horizontal axle.
A block with mass m = 1.2 kg hangs from a massless cord that is wrapped around the rim of the disk.
Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord.
(The cord does not slip, and there is no friction at the axle.)



$$\sum \tau = I\alpha$$

$$RT = I\alpha$$

$$T = \frac{I}{R}\alpha = \frac{I}{R^2}a$$

$$T = \frac{1}{2}Ma$$

$$\sum F_y = ma_y$$

$$mg - T = ma$$

$$mg - \frac{M}{2}a = ma$$

$$a = \frac{mg}{\left(m + \frac{M}{2}\right)}$$

A uniform disk, with mass *M* = 2.5 kg and radius *R* = 20 cm, mounted on a fixed horizontal axle.
A block with mass *m* = 1.2 kg hangs from a massless cord that is wrapped around the rim of the disk.
Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord.
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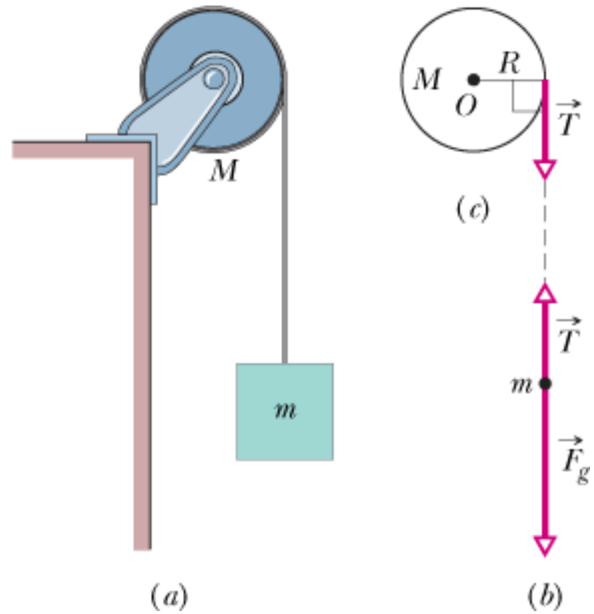


From forces:

$$mg - T = ma$$

From torques:

$$\tau_{net} = I\alpha \qquad a = \alpha R$$

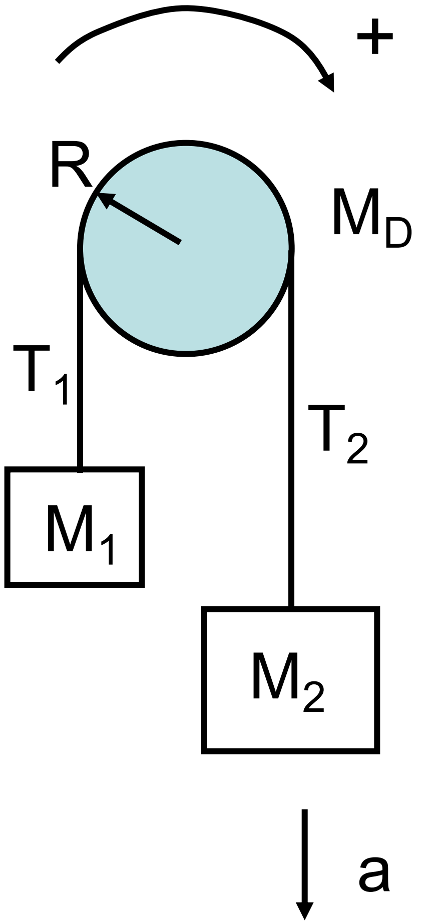
$$RT = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$$

$$T = \frac{1}{2}Ma$$

$$a = \left(\frac{m}{m + \frac{1}{2}M}\right)g$$

Atwood's machine:

$M_2 > M_1$
Find the expression for a.



Atwood's machine:

$$\text{From forces:}$$

$$T_1 - M_1 g = M_1 a$$

$$M_2 g - T_2 = M_2 a$$

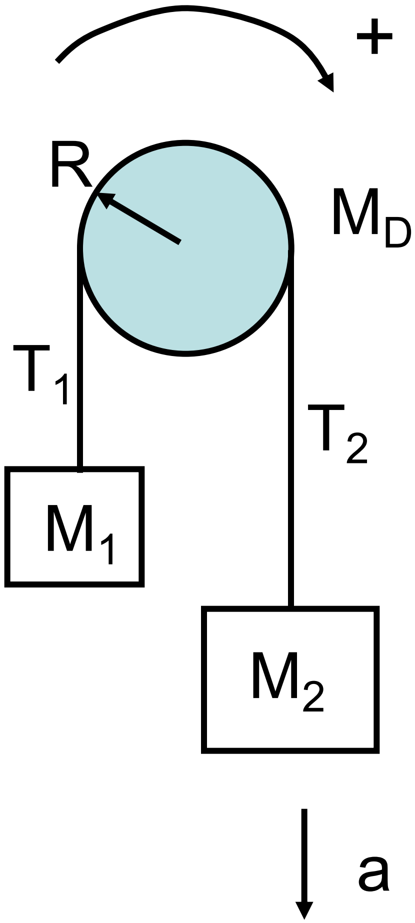
$$\text{From torques:}$$

$$\sum \tau = \tau_{net} = I\alpha$$

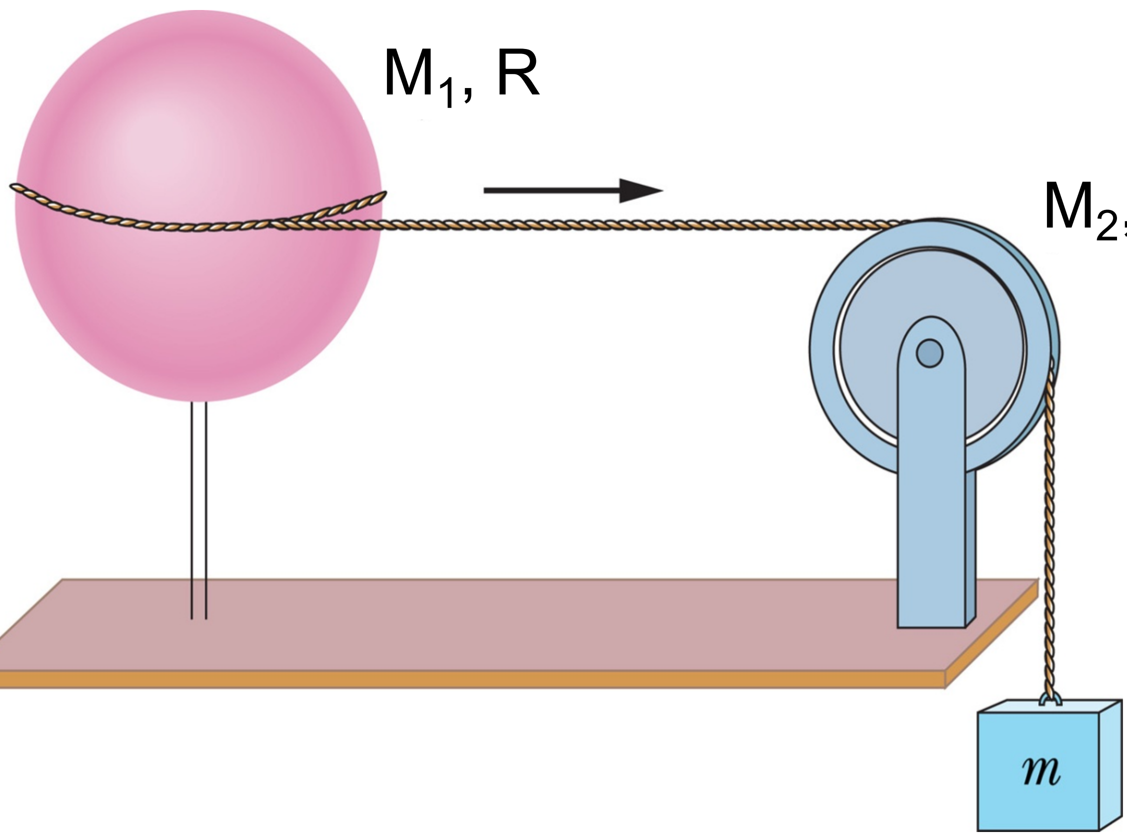
$$RT_2 - RT_1 = \left(\frac{1}{2} M_D R^2\right)\left(\frac{a}{R}\right)$$

$$T_2 - T_1 = \frac{1}{2} M_D a$$

$$a = \left(\frac{(M_2 - M_1)g}{M_1 + M_2 + \frac{1}{2} M_D}\right)$$



A uniform sphere of mass M_1 and radius R can rotate about a vertical axis on frictionless bearings. A massless cord is wound around the equator of the sphere and passes over a frictionless uniform disk pulley of mass M_2 and radius r. The cord doesn't slip on the sphere or the pulley. What is the acceleration of the hanging mass m?



$$Sphere\ (rotational):$$

$$\tau_1 = I\alpha = \left(\frac{2}{5}M_1R^2\right)\left(\frac{a}{R}\right) = RT_1$$

$$T_1 = \frac{2}{5}M_1a$$

$$Disk\ (rotational):$$

$$\tau_2 = \left(\frac{1}{2}M_2r^2\right)\left[\frac{a}{r}\right] = rT_2 - rT_1$$

$$T_2 = \left(\frac{2}{5}M_1 + \frac{1}{2}M_2\right)a$$

$$Hanging\ mass\ (translational):$$

$$mg - T_2 = ma$$

$$a = \frac{mg}{\left(\frac{2}{5}M_1 + \frac{1}{2}M_2 + m\right)}$$

$$Moment\ of\ inertia:$$

$$I = \sum m_i r_i^2 \quad (point\ particles)$$

$$I_{COM} = MR^2 \quad (ring)$$

$$I_{COM} = \frac{1}{2}MR^2 \quad (disk)$$

$$I_{COM} = \frac{2}{5}MR^2 \quad (sphere)$$

$$I_{COM} = \frac{1}{12}ML^2 \quad (rod)$$

$$I = I_{COM} + Mh^2 \quad (axis\ displaced\ from\ COM\ by\ h)$$

$$K_R = \frac{1}{2}I\omega^2 \quad (rotational\ kinetic\ energy)$$

$$W_R = \int \tau d\theta \quad (rotational\ work)$$

$$\tau = rF\sin(\theta) \quad (torque)$$