Homework 2 - Math 2568 (Autumn 2022)

Prof. Cueto

Due date: Friday September 9, 2022 (in class).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§1.3	1, 4, 6, 10, 14, 19, 21, 23, 26, 28	4, 6, 14, 23, 28
§1.5	1, 8, 14, 22, 25, 29, 31, 34, 42, 48, 54, 68	8, 14, 25, 48, 68

Extra Problem: For what values of λ does the homogeneous 2×2 linear system with coefficient matrix

$$A = \begin{pmatrix} \lambda - 4 & -1 \\ 2 & \lambda - 1 \end{pmatrix}$$

have infinitely many solutions? For those values of λ, write down the solutions to the system in vector form.

Done at the end

Section 1.3

4) $$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 1 \\ 1 & 2 & 3 & 5 & 2 \\ 2 & 4 & 6 & 1 & 1 \\ -1 & -2 & -3 & 7 & 2 \end{array}\right] \xrightarrow[\substack{R_3 - 2R_1 \\ R_4 + R_1}]{R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 1 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & 9 & 3 \end{array}\right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 1 \\ 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & 9 & 3 \end{array}\right]$$

$$\xrightarrow{R_1 + 2R_2 \quad R_3 + 3R_2 \quad R_4 - 9R_2} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\begin{array}{l} n = 4 \quad r = 2 \\ n - r = 2 \to \text{Indep. } x_2, x_3 \end{array}}$$

6) $\left[\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & b_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & b_3 \end{array}\right]$ $r = 0, 1, 2, \text{ or } 3$

3×5

Can have unique solution if the system can be put into $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & c_1 \\ 0 & 1 & 0 & 0 & c_2 \\ 0 & 0 & 1 & 0 & c_3 \end{array}\right]$

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14)

$r = 3$

$n = 4$

$4 - 3 = 1$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$

Infinite Solutions

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23) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ -1 & -1 & 1 & 0 \\ 3 & 4 & a & 0 \end{array}\right] \xrightarrow[R_3 - 3R_1]{R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -2 & a-3 & 0 \end{array}\right] \xrightarrow[R_3 + 2R_2]{} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & a+1 & 0 \end{array}\right]$

$\boxed{a = -1}$

$a + 1 = 0 \rightarrow a = -1$

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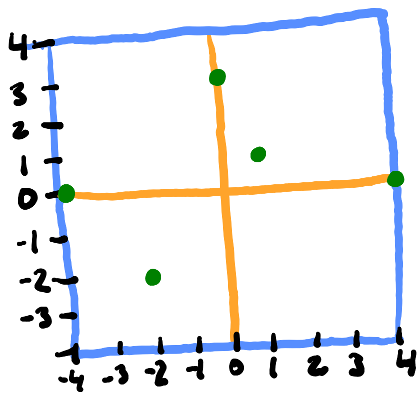
28) $-\frac{1}{16}x^2 - \frac{71}{144}xy - \frac{1}{15}y + \frac{1}{18}y^2 + \frac{1}{2} = 0$

*This was a lot of work, so I didn't write it all down here. I have it on paper elsewhere if you need to see it.



Section 1.5

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8) $r = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $s = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ $t = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ $u = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$

a) $t + s = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

b) $r + 3u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -12 \\ 18 \end{bmatrix} = \begin{bmatrix} -11 \\ 18 \end{bmatrix}$

c) $2u + 3t = \begin{bmatrix} -8 \\ 12 \end{bmatrix} + \begin{bmatrix} 3 \\ 12 \end{bmatrix} = \begin{bmatrix} -5 \\ 24 \end{bmatrix}$

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14) $a_1 r + a_2 s = u$

$a_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$

$\boxed{\begin{aligned} a_1 &= 0 \\ a_2 &= -2 \end{aligned}}$

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25) $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}$ $Z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$(A+B)C = \left(\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}\right) \times \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 2 & 6 \end{bmatrix} \times \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2(-2)+0(1) & 2(3)+0(1) \\ 2(-2)+6(1) & 2(3)+6(1) \end{bmatrix}$

$\boxed{= \begin{bmatrix} -4 & 6 \\ 2 & 12 \end{bmatrix}}$

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48) $x_1 - x_3 - x_5 - 2x_6 = 0$

$x_2 + 2x_3 + x_5 + 2x_6 = 0$

$x_4 + x_5 + x_6 = 0$

$\downarrow$

$x_1 = x_3 + x_5 + 2x_6 \qquad x_2 = -2x_3 - x_5 - 2x_6$

$x_3 = x_3 \qquad x_4 = -x_5 - x_6 \qquad x_5 = x_5 \qquad x_6 = x_6$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 2 \\ -2 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}}$$

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68)

$$\left[\begin{array}{ccccc|c} 1 & 3 & -3 & 2 & -3 & -4 \\ 3 & 9 & -10 & 10 & -14 & 2 \\ 2 & 6 & -10 & 21 & -25 & 53 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - 3R_1} \left[\begin{array}{ccccc|c} 1 & 3 & -3 & 2 & -3 & -4 \\ 0 & 0 & -1 & 4 & -5 & 14 \\ 0 & 0 & -4 & 17 & -19 & 61 \end{array}\right]$$

$$\xrightarrow{R_3 - 4R_2} \left[\begin{array}{ccccc|c} 1 & 3 & -3 & 2 & -3 & -4 \\ 0 & 0 & -1 & 4 & -5 & 14 \\ 0 & 0 & 0 & 1 & 1 & 5 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccccc|c} 1 & 3 & -3 & 2 & -3 & -4 \\ 0 & 0 & 1 & -4 & 5 & -14 \\ 0 & 0 & 0 & 1 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{R_1 + 3R_2} \left[\begin{array}{ccccc|c} 1 & 3 & 0 & -10 & 12 & 11 \\ 0 & 0 & 1 & -4 & 5 & -14 \\ 0 & 0 & 0 & 1 & 1 & 5 \end{array}\right] \xrightarrow[R_1 - 10R_3]{R_2 + 4R_3} \left[\begin{array}{ccccc|c} 1 & 3 & 0 & 0 & 22 & 4 \\ 0 & 0 & 1 & 0 & 9 & 6 \\ 0 & 0 & 0 & 1 & 1 & 5 \end{array}\right]$$

$x_1 + 3x_2 + 22x_5 = 4$

$x_3 + 9x_5 = 6$

$x_4 + x_5 = 5$

x_2 and x_5 independant

$x_1 = 4 - 3x_2 - 22x_5$

$x_3 = 6 - 9x_5$

$x_4 = 5 - x_5$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 6 \\ 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -22 \\ 0 \\ -9 \\ -1 \\ 1 \end{bmatrix}}$$

Bonus:

$$\begin{bmatrix} \lambda-4 & -1 \\ 2 & \lambda-1 \end{bmatrix}$$

$(\lambda-4)x_1 - x_2$

$2x_1 + (\lambda-1)x_2$

$$\boxed{\lambda = 2, 3}$$

$\lambda=2$ $-2x_1 - x_2$

$2x_1 + x_2$

$\lambda=3$ $-x_1 - x_2$

$2x_1 + 2x_2$