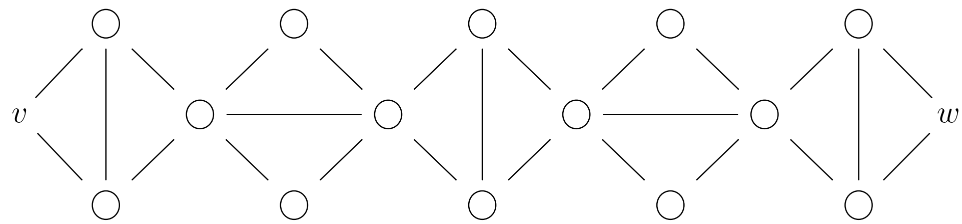
CSE 2321 - Foundations I - Spring 2024 - Dr. Estill

Homework 8 - Due: Tuesday, April 2

Definition 1. *A path in a graph is called* simple *if all of its vertices are distinct (but not all vertices in the graph have to be visited).*

1.) (20 points) In the graph depicted below, how many simple paths are there from v to w? Briefly justify your answer.



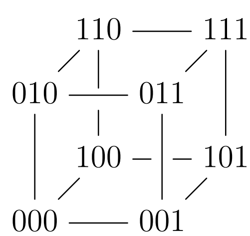
2.) (40 points)The *Hamming cube of dimension* n is defined as the undirected graph $H_n = (V_n, E_n)$ where $V_n = \{v : v \text{ is a binary string of length } n\}$ and

$$E_n = \big\{\{v, w\} \subseteq V_n : v \text{ and } w \text{ differ by one character}\big\}.$$

For example, H_2 looks like

$$\begin{tikzcd}[arrows=-] 00 \arrow[r] \arrow[d] & 01 \arrow[d] \\ 10 \arrow[r] & 11 \end{tikzcd}$$

and H_3 looks like



(a) What is $|V_n|$?

(b) What is $|E_n|$? *Justify your answer.*

(c) Does H_{17} have an Eulerian cycle? *Justify your answer.*

(d) Does H_{42} have an Eulerian cycle? *Justify your answer.*

(e) Find a Hamiltonian cycle in H_2. List the sequence of vertices.

(f) Find a Hamiltonian cycle in H_3. List the sequence of vertices.

(g) Find a Hamiltonian cycle in H_4. List the sequence of vertices.

Definition 2. *Define the* complement *of a graph,* $G = (V, E)$*, (either directed or undirected) as the graph* $\overline{G} = (V, \overline{E})$ *which has the same vertex set and where for every pair of distinct vertices,* v *and* w*,* $(v, w) \in \overline{E} \Leftrightarrow (v, w) \notin E$*. That is to say, there's an edge in* $\overline{G}$ *if and only if there isn't an edge in* G*. Or, to look at another way: take the adjacency matrix for* G *and (except on the main diagonal) change each one to a zero and each zero to a one. The resulting matrix is the adjacency matrix for* $\overline{G}$*.*

3.) (20 points) Give an example of a simple undirected graph on four vertices which is isomorphic to its complement. A picture of your graph and its complement.are enough for your answer.

4.) (20 points) For $n \geq 3$, let C_n be the undirected graph consisting of a single simple cycle of length n. I.e., $C_n = (V_n, E_n)$ where

$$V_n = \{0, 1, 2, \ldots, n-1\} \text{ and}$$
$$E_n = \{\{i, (i+1 \mod n)\} \mid 0 \leq i \leq n-1\}.$$

in simpler terms, one picture of C_n would be an n-sided regular polygon, with the edges being the sides. Find all values of n such that C_n is isomorphic to its complement. Justify your answer.