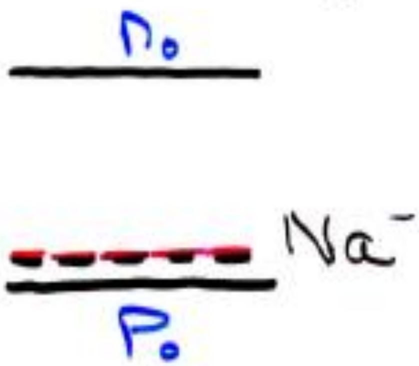
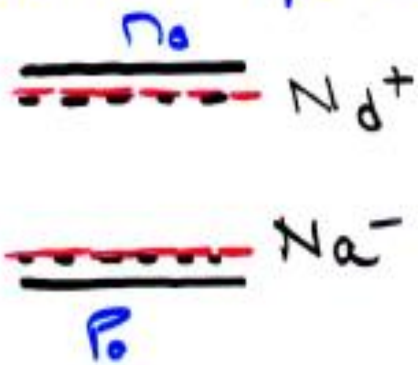
and, for p-type semiconductor,



$$\Rightarrow \quad p_o = N_a^- + n_o \simeq N_a + n_o$$

(full ionization)

What if we have **both** donors **and** acceptors?



$$p_o + N_d^+ = n_o + N_a^-$$

(full ionization)

Charge Neutrality Law

3 cases:

1) $N_a >> N_d$

$p_o = n_o + (N_a - N_d) \sim N_a$ still p-type

and $n_o = n_i^2 / N_a$

2) $N_d >> N_a$

$n_o = p_o + (N_d - N_a) \sim N_d$ still n-type

and $p_o = n_i^2 / N_d$

3) $N_d \sim N_a$

They compensate each other.

Example: If $N_a = N_d$, then $n_o = p_o$

Then perfectly compensated.

In general, $n_o + N_a = p_o + N_d$

and for $N_a \sim N_d$, substitute $n_o p_o = n_i^2$

and solve:

$N_d > N_a$: $$n_o + N_a = \frac{n_i^2}{n_o} + N_d$$

Put in terms of n_o and <u>not</u> p_o since expect n_o larger than p_o.

$$n_o^2 + n_o(N_a - N_d) - n_i^2 = 0$$

First, find n_o

Then, find $p_o = n_i^2 / n_o$

$N_a > N_d$: $\quad p_o + N_d = \dfrac{n_i^2}{p_o} + N_a$

put in terms of p_o (not n_o) since expect p_o larger

$$p_o^2 + p_o(N_d - N_a) - n_i^2 = 0$$

First, find p_o

Then find $n_o = n_i^2/p_o$

Example I: P-Type Dopant

$N_a = 10^{15}$ Al in Si at Room Temperature

(Al has 3 valence electrons versus 4 in Si, so Al is an acceptor)

After Al accepts an e^-, $N_a \rightarrow N_a^-$

$n_0 + N_a = p_0 + N_d$ Charge Neutrality Equation

$n_0 p_0 = n_i^2$ Law of Mass action ($n_i = 1.5 \times 10^{10}$)

Since doped with acceptors, then $p_0 > n_0 \rightarrow$ Solve for p_0.

$$\frac{n_i^2}{p_o} + N_a = p_o + N_d$$

$$N_d = 0$$

$$\frac{n_i^2}{p_o} + N_a = p_o$$

$$p_o^2 - N_a p_o - n_i^2 = 0$$

$$\left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{for } ax^2 + bx + c = 0 \right)$$

$$\text{so } p_o = \frac{N_a \pm \sqrt{N_a^2 + 4n_i^2}}{2}$$

Negligible since $n_i \ll N_a$

$$p_o \cong \frac{N_a \pm N_a}{2} = N_a = \boxed{\quad}$$

(ignore (-) solution)

Example II. Compensating Dopants

$N_a = 10^{15}$ Al in Si (acceptor)

$N_d = 5\times10^{14}$ As in Si (donor)

and $T = 500K$, so $n_i = 3\times10^{14}\,cm^{-3}$

(See Streetman graph)

Since slightly more acceptors than donors, expect $p_o \gtrsim n_o$

again, solve for majority carrier $\rightarrow p_o$.

$$p_o^2 + (N_d - N_a)p_o - n_i^2 = 0$$

$$p_o = \frac{(N_a - N_d) \pm \sqrt{(N_a - N_d)^2 + 4n_i^2}}{2}$$

$$p_o = \frac{5\times10^{14} \pm \sqrt{5^2\times(10^{14})^2 + (4\times9)\times(10^{14})^2}}{2}$$

$$p_o = \frac{5\times10^{14} \pm \sqrt{61\times10^{28}}}{2}$$

$$p_o = \frac{(5 \pm 7.81)}{2}\times10^{14} =$$

Note: $n_i < p_o < N_a$

$$n_o = \frac{n_i^2}{p_o} = \frac{9\times10^{28}}{6.4\times10^{14}} =$$

Carrier Motion in Electric Fields

Mobility = ease of motion in crystal of electron or hole

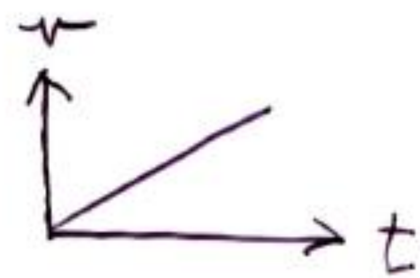
Consider conductivity of electrons in $\vec{\mathcal{E}}$ field

$$F_e = m^* \frac{dv}{dt} = -q\mathcal{E}$$ force on electron

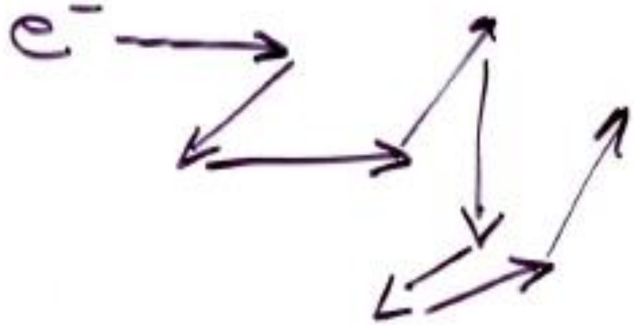
$$\int_0^v dv = -\int_0^t \frac{q}{m^*} \mathcal{E}\, dt$$

$$v =$$

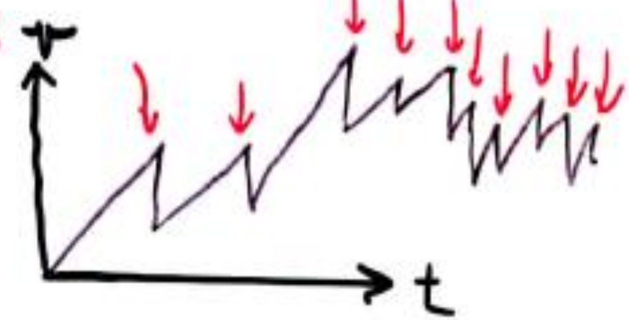
So for constant $\mathcal{E}$, expect:



But can't accelerate electron forever:



scattering:



so constant net velocity
= net drift in presence of $\mathcal{E}$

Average velocity $\langle v_n \rangle =$

where $\langle t \rangle$ = average time between scattering events

$\langle v_n \rangle = -\mu_n \mathcal{E}$ leads to a "mobility" $\left(\frac{cm^2}{V\text{-}sec} \right)$

μ_n for electrons

Drift velocity for electrons

$$v_{dn} = -\mu_n \mathcal{E} = -\frac{q\langle t\rangle}{m^*}\mathcal{E}$$

so $\mu_n = \frac{q\langle t\rangle}{m_n^*}$ for electrons (electron mobility)

and $\mu_p = \frac{q\langle t\rangle}{m_p^*}$ for holes by analogy (hole mobility)

Current Density

$$J_n = -q n v_{dn} = \boxed{\phantom{q n \mu_n \mathcal{E}}} =$$ electron drift current density

$$q \times \frac{e^-}{cm^3} \times \frac{cm}{sec} = \frac{q}{cm^2\text{-}sec} = \frac{Amp}{cm^2}$$

$J_p = q\,p\,v_{dp} = +q\,\mu_p\,p\,\mathcal{E}$ = hole drift current density

$J_{Total} = q\,n\,\mu_n\,\mathcal{E} + q\,p\,\mu_p\,\mathcal{E} =$

$\leftarrow e^-$

$h^+ \longrightarrow$

$\xrightarrow[\mathcal{E}]{} +x$

J_n(drift) $\longrightarrow$ since $\mu_n = -\dfrac{\langle v_x\rangle}{\mathcal{E}_x}$

J_p(drift) $\longrightarrow$ since $\mu_p = +\dfrac{\langle v_x\rangle}{\mathcal{E}_x}$

Mobility very important for device

$\mu_n = q\dfrac{\langle t\rangle}{m^*}$ and since $m^* = \dfrac{\hbar^2}{d^2E/dk^2}$, $\leftarrow$ band curvature

then mobility depends on band structure.

Sharp band curvature:  means small m^*, high μ.
broad " "  means large m^*, small μ.

Conductivity

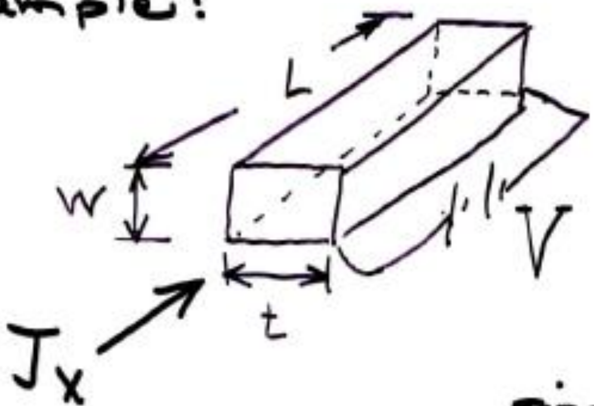
$J =$

$\sigma =$ conductivity $\left(\frac{1}{\Omega\text{-cm}}\right)$

$\sigma =$ (from expression for J_{Total})

Now consider resistance:

example:



$$J_x = \frac{I}{A} = \frac{I}{Wt} = \sigma \frac{V}{L}$$

since $\frac{V}{L} = \mathcal{E}$

$$\frac{I}{Wt} = \boxed{}$$

so $\frac{V}{I} = \frac{L}{Wt} \cdot \frac{1}{\sigma}$

$$R = \frac{V}{I} = \frac{L}{Wt} \cdot \frac{1}{\sigma} = \frac{L}{A} \cdot \frac{1}{\sigma} = \rho \frac{L}{A}$$ since $\rho = \frac{1}{\sigma}$

ρ = Resistivity (Ω-cm)

$$R = \rho \frac{L}{A}$$

$$\rho = \frac{1}{\sigma} = \boxed{}$$