****Note: this group work focuses on topics that students in the past have found difficult. These problems do not necessarily reflect the conceptual content of the final exam.****

Problem 1

A block with mass $m_1 = 3.5\,\text{kg}$ sits at rest on a horizontal, frictionless surface. It is connected to the wall by means of a spring. After being pulled horizontally $15\,\text{cm}$ away from its equilibrium position and released, it oscillates back and forth with angular frequency $\omega = 22\,\text{rad/s}$.

(a) What is the period T of the oscillation? What is the frequency f?

$$T = \frac{2\pi}{\omega} = \frac{\pi}{11} = 0.286\text{ s}$$

$$f = \frac{1}{T} = \frac{1}{.286} = 3.5\text{ Hz}$$

(b) What is the spring constant k_s?

$$\omega = \sqrt{\frac{k}{m}} \qquad k = \omega^2 m = 22^2 \cdot 3.5 = 1694$$

(c) What is the maximum speed of the block?

$$v = \omega x = 22 \cdot 15 = 33\text{ m/s}$$

(d) Just as block m_1 is passing through its equilibrium position, another block with mass $m_2 = 1.7\,\text{kg}$ is placed just barely above block m_1 and released from rest. The two blocks stick together. What is the maximum speed of oscillation now?

$$m_1 v_1 = m_2 v_2 \qquad v_2 = \frac{m_1 v_1}{m_2} = \frac{3.5 \cdot 3.3}{5.2} = 2.22\text{ m/s}$$

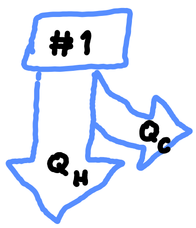
(e) What is the maximum distance by which the spring is compressed now?

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \qquad mv^2 = kx^2 \qquad x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{5.2 \cdot 2.22^2}{1694}} = 0.123\text{ m}$$

Problem 2

Suppose you have two heat engines that are in contact with the same hot/cold reservoirs. Engine #1 absorbs $Q_H = 250\,\text{J}$ from the hot reservoir and gives up $Q_C = 150\,\text{J}$ to the cold reservoir each cycle. Engine #2 absorbs the same amount of heat each cycle as engine #1 does, but it only produces half as much waste heat.

(a) Sketch a diagram of this configuration, and calculate the efficiency of each engine. (Recall that efficiency is the ratio between the work performed and the energy input: $e = {}^{W}/_{Q}$.)



(b) Suppose we thought of these two engines operating in parallel with each other (side-by-side, as described above) as two components of a larger, combined engine. Sketch a diagram of this configuration, and calculate the overall efficiency of the combined engine.

(c) Suppose that engine #2's input side is disconnected from the hot reservoir, and then attached to the output side of engine #1. In other words, the engines now operate in series, so that the heat output of engine #1 becomes the heat input of engine #2. Assume that the individual efficiencies of each engine remain the same as in part (a), and that engine #1 still produces 150 J of waste heat each cycle. Sketch a diagram of this configuration, and calculate the overall efficiency of the combined engine.

Problem 3

Freddie Mercury is on a rocket ship on his way to Mars. He's traveling at a speed of $v = 0.9c$ (relative to Earth) when he sings a few songs for the crew onboard. He also uses a radio transmitter to broadcast the performance back to his fans on Earth. According to the ship's crew, his performance lasts for 15 min.

(a) Who measures the proper time for this performance? Explain your reasoning without making any calculations.

Freddie does because it is time relative to him

(b) According to his fans back on Earth, how long did the performance last?

$$t = \frac{t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{900}{\sqrt{1 - .9^2}} = 2065.15 \text{ sec} \rightarrow 34.42 \text{ min}$$

(c) Who measures the proper length for the amount of distance covered by Freddie's ship during the performance? Explain your reasoning without making any calculations.

(d) According to the fans back on Earth, how much distance did the ship cover during the performance? State your answer in the units of light-minutes (i.e., the distance traveled by a ray of light during one minute).

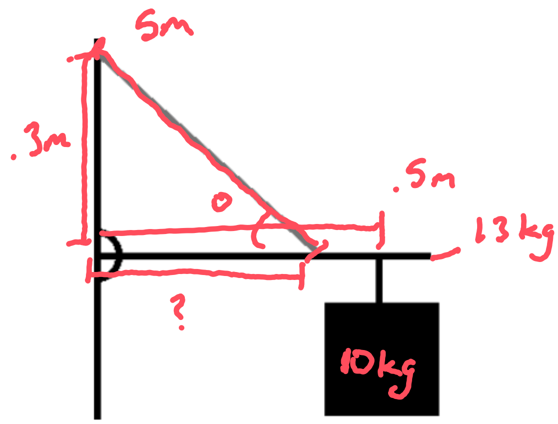
$$x = vt = .9 \cdot 34.42 = 30.978 \text{ light-mins}$$

(e) According to Freddie, how much distance did the ship cover during the performance? State your answer in the units of light-minutes. (Hint: you can calculate this using either the proper time or the proper length.)

$$x = vt = .9 \cdot 15 = 13.5 \text{ light-mins}$$

Problem 4

A bakery owner is hanging a sign outside their shop. The sign has a mass of $m = 10.0\,\text{kg}$ and is placed $0.10\,\text{m}$ from the end of an $L = 0.60\,\text{m}$ long metal rod. The other end of the rod is attached to the building, and the rod has a mass of $M = 13.0\,\text{kg}$. A cable with length $l = 0.50\,\text{m}$ and negligible mass is used to support the rod. The cable attaches to the wall $0.3\,\text{m}$ above the point where the rod attaches to the wall.



(a) Given the geometry of this problem, where is the cable attached to the rod?

$.5^2 = x^2 + .3^2$

$x^2 = .5^2 - .3^2$

$x = \sqrt{.5^2 - .3^2} = 0.4\,m$ from the wall

(b) What angle does the cable make with respect to the horizontal rod?

$\cos\theta = \frac{.4}{.5}$ $\theta = \cos^{-1}\left(\frac{.4}{.5}\right) = 36.87°$

(c) What is the tension in the cable?

$F_T = F_m + F_r = m_m g + m_r g = (10 \cdot 9.8) + (13 \cdot 9.8) = 225.4 \cos(36.87)$

$= 180.32\,N$