1.

    a)   Not (P and Q) cross-implies (Not P and Not Q)

        Contingency

    b)   Not (P or Q) cross-implies (Not P and Not Q)

        Tautology

    c)   ((P and Q) cross-implies Q) cross-implies (Q implies P)

                                    00 or 10 or 11

        Tautology

    d)   P implies Q implies R – OR – P and Not Q or R

                                    111 or 100 or 001

        Contingency

    e)   (P or Q) and R - implies – (P and Q) or (P or R)

        Tautology

    f)   (P or Q – implies – P and Q) and (P xor Q)

        Contradiction

2.

    a)

P	Q	R	P => Q	Q => R	P => R
1	1	1	1	1	1
1	1	0	1	0	0
1	0	1	0	1	1
1	0	0	0	1	0
0	1	1	1	1	1
0	1	0	1	0	1
0	0	1	1	1	1
0	0	0	1	1	1

    Argument is invalid because conclusion can be true when only one premise is true

b)

A	B	C	(A v B) => C	A v ¬C	B v ¬C	A <=> ¬B	C ∧ ¬C
1	1	1	1	1	1	0	0
1	1	0	0	1	1	0	0
1	0	1	1	1	1	0	0
1	0	0	0	1	0	0	0
0	1	1	1	0	1	0	0
0	1	0	1	1	1	1	0
0	0	1	1	0	1	1	0
0	0	0	1	1	1	1	0

Argument is invalid because conclusion is always false, even when all premises are true.

c)

X	Y	Z	(X v Y) ∧ (X v Z)	Y ∧ Z	X
1	1	1	1	1	1
1	1	0	1	0	1
1	0	1	1	0	1
1	0	0	1	0	1
0	1	1	1	1	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

Argument is valid because whenever all premises are true, conclusion is true.

3.

NOR is universal because it can be used to demonstrate each of the other logical operations

NOT written with NOR

NOT P = P NOR P

$\neg P = P \downarrow P$

P	¬P	P ↓ P
1	0	0
0	1	1

AND written with NOR

$$P \text{ AND } Q = NOR(NOR(P,P), NOR(Q,Q))$$

P	Q	P ↓ P	Q ↓ Q	(P ↓ P) ↓ (Q ↓ Q)	P ∧ Q
0	0	1	1	0	0
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	1	1

OR written with NOR

$$P \text{ OR } Q = NOR(NOR(P,Q), NOR(P,Q))$$

P	Q	P ↓ Q	(P ↓ Q) ↓ (P ↓ Q)	P ∨ Q
0	0	1	0	0
0	1	0	1	1
1	0	0	1	1
1	1	0	1	1

4.

a)

{}, {b}, {d}, {b, d}

b)

2^5 = 32

c)

P(U) ∪ P(V)={{},{a},{b},{c},{d},{f},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c}, {a,b,d},{a,c,d},{b,c,d},{b,d,f},{a,b,c,d},{a,b,d,f},{a,c,d,f},{b,c,d,f},{a,b,c,d,f}}

|P(U) ∪ P(V)| = 22