ECE 3030 Spring 2025 Homework 2 Solutions

1. $E = h\upsilon = hc/\lambda$

(a) For the 5.65 eV work function of platinum, $\lambda = hc/E$ and $h = 6.625\text{x}10^{-34}$ J-s.

$E = (6.625\text{x}10^{-34}\text{ J-s})\ (3\text{x}10^{10}\text{ cm/s})/\ [(5.65\text{ eV})\ (1.6\text{x}10^{-19}\text{ J/eV})] = 2.19\text{ x}10^{-5}$ m

= 0.219 μm = 219 nm [Note conversion factor from J to eV].

(b) For the 2.90 eV work function of lithium,

$E = (6.625\text{x}10^{-34}\text{ J-s})\ (3\text{x}10^{10}\text{ cm/s})/\ [(2.90\text{ eV})\ (1.6\text{x}10^{-19}\text{ J/eV})] =0.428\text{ x}10^{-5}$ m

= 0.428 μm = 428 nm

(c) If the light wavelength λ decreases for an energy above threshold, the new energy $E' = hc/\lambda'$ increases so that more electrons are excited and ejected from the material.

(d) If the light intensity increases for a photon energy above threshold, the number of electrons increases since there are more photons with enough energy for each to excite one electron. The energy threshold does not change since the work function depends only on the material, not the number of photons.

2. For GaAs, lattice spacing a_0 = 5.65 Å (Streetman & Banarjee, Appendix III) and De Broglie and $\lambda = h/p$

$p = h/\lambda = (6.625\text{x}10^{-27}\text{ erg-s})/5.65\text{x}10^{-8}\text{ cm} = 1.17\text{x}10^{-19}$ gm-cm/s

Kinetic energy $E = ½\ mv^2 = p^2/2m = (1.17\text{x}10^{-19}\text{ gm-cm/s})^2/2\text{ x }(9.11\text{x}10^{-28}\text{ gm})$

$= 0.075\text{x}10^{-10}\text{ gm-cm}^2/\text{s}^2 = (7.5\text{x}10^{-12})\text{ erg} / (1.6\text{x}10^{-12}\text{ erg/eV}) = 4.69$ eV

3. (a) Uncertainty in position Δx = 1.7 nm. Since $\Delta x \cdot \Delta p \geq \hbar/2$,

$\Delta p \geq \hbar / 2\Delta x = 1.054 \text{ x } 10^{-34}\text{ J}/ 2\ (1.7 \text{ x}10^{-9}\text{ m}) = 3.1 \text{ x } 10^{-26}$ kg m/s.

(b) Uncertainty in energy $\Delta E = 2\text{ eV} = 3.2 \text{ x } 10^{-19}$ J.

Since $\Delta E\cdot\Delta t \geq \hbar/2$, $\Delta t \geq \hbar / 2\Delta E = 1.054 \text{ x } 10^{-34}\text{ J-s} / 2(3.2 \text{ x } 10^{-19}\text{ J}) = 1.645 \text{ x } 10^{-16}$ sec

= 0.1645 femtoseconds.

4. (a) $E_n = \hbar^2 n^2 \pi^2/2mL^2 = (1.054\ 10^{-34})^2\ n^2\ \pi^2/2\ (9.11 \text{ x}10^{-31})\ (1.7 \text{ x}10^{-9}\text{ m})^2 = n^2\ (2.082 \text{ x } 10^{-20})$ J

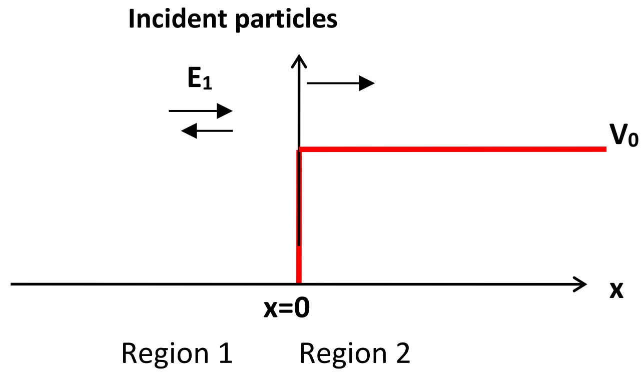
Or $E_n = n^2\ (2.082 \text{ x } 10^{-20})\text{ J} /(1.6 \text{ x } 10^{-19}\text{ J/eV}) = 0.130\text{ eV x } n^2$

Then $E_1 = 0.130$ eV
$E_2 = 0.520$ eV
$E_3 = 1.170$ eV

(b) $\lambda = hc/\Delta E$ since $E = h\upsilon$ and $\lambda\upsilon = c$

$\Delta E = E_3 – E_1 = (1.170\text{ eV} – 0.130\text{ eV})\ (1.6 \text{ x } 10^{-19}\text{ J/eV}) = 1.040 \text{ x } (1.6 \text{ x } 10^{-19})$ J
$=1.664 \text{ x } 10^{-19}$ J

$\lambda = (6.625 \text{ x } 10^{-34})\ (3\text{x}10^{8}) / 1.664 \text{ x } 10^{-19}$
$= 11.94 \text{ x } 10^{-7}$ m
= 1,1944 nm or 11,944 Å



5. Region 1: V< 0
$d^2\psi/dx^2 + (2m/\hbar^2)(E - V_0)\psi(x) =$
$d^2\psi/dx^2 + (2m/\hbar^2)E\,\psi(x) = 0$
General solution: $\psi_1(x)= A_1 e^{jk_1x} + B_1e^{-jk_1x}$
where $k_1 = [2mE/\hbar^2]^{1/2}$
Since the particle is moving in the +x direction, the first term is the forward wave and the second term is the reflected wave.

Region 2: $V > V_0$
$d^2\psi/dx^2 + (2m/\hbar^2)(E - V_0)\psi(x) = 0$
General solution is $\psi_2(x)= A_2 e^{jk_2x} + B_2e^{-jk_2x}$
Where $k_2 = [2m(E-V_0)/\hbar^2]^{1/2}$

Since the transmitted particle/wave is moving in the +x direction, the first term is the forward wave and the second term corresponds to a reflected wave. But no reflection is possible for the wave moving to the right after it passes x = 0. Therefore, there is no wave moving to the left inside Region II. Therefore, $B_2 = 0$ and $\psi_{II}(x)= A_2 e^{jk_2x}$

Continuity of ψ across the interface at x = 0 requires: $A_1\cdot(1) + B_1\cdot(1) = A_2\cdot(1)$ since $e^0 = 1$.
Continuity of $d\psi/dx$ across the interface at x = 0 requires: $A_1jk_1\cdot(1) - B_1jk_1\cdot(1) = A_2jk_2\cdot(1)$ so that $k_1\cdot(A_1 - B_1) = k_2\cdot A_2$ and finally
$B_1 = A_1\cdot(k_1 - k_2)/(k_1+k_2)$ and $A_2 = A_1(2k_1/(k_1+k_2))$

Note that $p = mv = \hbar k$ so that $v = \hbar k/m$. Since $k_1 > k_2$, then the waves in Region I propagate faster than in Region II.

6. Here, $E-V_0$ is negative inside square root, which introduces an extra j, which changes $\exp(jk_2x)$ to $\exp(-k_2x)$ and $V_0 - E$ inside square root is > 0.

$$\psi_2(x) = A_2\exp(-k_2x)$$
$$P = \frac{|\psi(x)|^2}{A_2A_2^*} = \exp(-2k_2x)$$
$$\text{where}\quad k_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$$
$$= \frac{\sqrt{2(9.11\times10^{-31})(4.0-3.3)(1.6\times10^{-19})}}{1.054\times10^{-34}}$$
$$k_2 = 4.286\times10^{9}\ \mathrm{m}^{-1}$$

(a) For $x = 5\,\mathring{A} = 5\times10^{-10}$ m
$$P = \exp(-2k_2x)$$
$$= \exp[-2(4.2859\times10^{9})(5\times10^{-10})]$$
$$= 0.0138$$

(b) For $x = 10\,\mathring{A} = 10\times10^{-10}$ m,
$$P = \exp[-2(4.2859\times10^{9})(10\times10^{-10})]$$
$$= 1.89\times10^{-4}$$

(c) For $x = 40\,\overset{o}{A} = 40 \times 10^{-10}$ m,

$$P = \exp\left[-2\left(4.2859 \times 10^{9}\right)\left(40 \times 10^{-10}\right)\right]$$
$$= 1.29 \times 10^{-15}$$

7. $T \cong 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp\left(-2k_2 a\right)$

(a) For $m = (0.067)m_o$

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(0.067)\left(9.11 \times 10^{-31}\right)(0.8 - 0.2)\left(1.6 \times 10^{-19}\right)}{\left(1.054 \times 10^{-34}\right)^2}\right\}^{1/2}$$

or

$$k_2 = 1.027 \times 10^{9} \text{ m}^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1 - \frac{0.2}{0.8}\right) \times \exp\left[-2\left(1.027 \times 10^{9}\right)\left(15 \times 10^{-10}\right)\right]$$

or

$$T = 0.138$$

(b) For $m = (1.08)m_o$, $k_2 =$

$$\left\{\frac{2(1.08)\left(9.11 \times 10^{-31}\right)(0.8 - 0.2)\left(1.6 \times 10^{-19}\right)}{\left(1.054 \times 10^{-34}\right)^2}\right\}^{1/2}$$

or

$$k_2 = 4.124 \times 10^{9} \text{ m}^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1 - \frac{0.2}{0.8}\right) \times \exp\left[-2\left(4.124 \times 10^{9}\right)\left(15 \times 10^{-10}\right)\right]$$

or

$T = 1.27 \times 10^{-5}$ Much lower tunneling probability than in GaAs.

8. Bonus (15 pts)

$T \cong 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right) exp(-2k_2 W)$ and $k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$ with V_0 = 0.8 eV and E = 0.08 eV

k_2 = [2(9.11x10^{-31})(0.8 – 0.08) (1.6 x 10^{-19})]^{½} /(1.054 x 10^{-34}) = 4.517 x10^{9}/1.054 = 4.346 x 10^{9} m^{-1}

5 x 10^{-6} = 16 (0.1)(0.90) exp[-2k$_2$W) = 1.44 exp[-2(4.346x 10^{9})W] ^{½}
ln(5x10^{-6} / 1.44) = -2(4.346 x 10^{9})W $\rightarrow$
W = 12.57/-2(4.285 x 10^{9}) = 1.446 x 10^{-9} m minimum