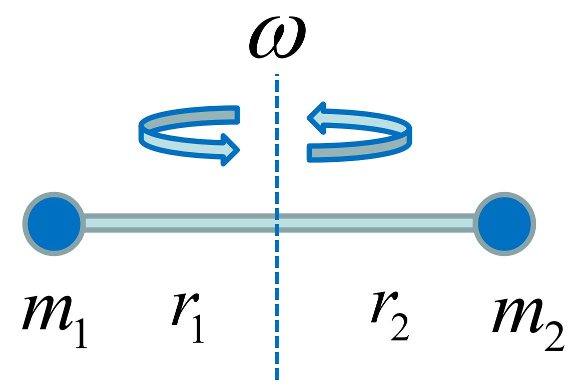
Kinetic Energy of Rotation



$$K = \sum_i \frac{1}{2} m_i V_i^2 = \sum_i \frac{1}{2} m_i (r_i \omega)^2$$

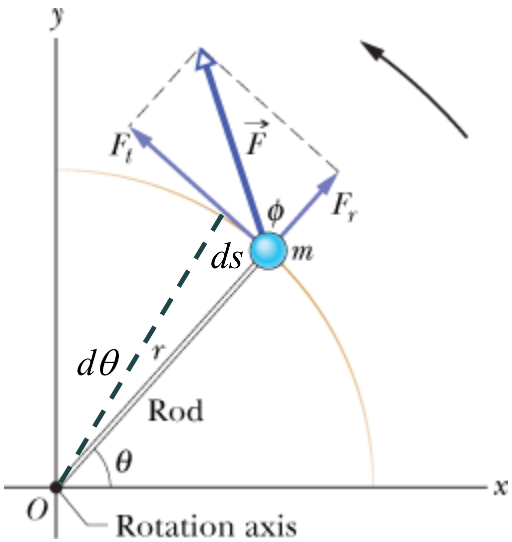
$$K = \sum_i \frac{1}{2} (m_i r_i^2) \omega^2 = \frac{1}{2} \omega^2 (\sum_i m_i r_i^2)$$

$$= \frac{1}{2} I \omega^2$$

Rotational Inertia (moment of inertia)

$$I = \sum_i m_i r_i^2 \qquad I = \int r^2 dm$$

Work and Rotational Kinetic Energy



$$dW = \vec{F}\cdot d\vec{s} = F_t ds = F_t r d\theta = \tau d\theta$$

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta = \tau(\theta_f - \theta_i)$$

$$\Delta K = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2$$

$$\omega_f^2 - \omega_i^2 = 2\alpha(\theta_f - \theta_i) = 2\frac{\tau}{I}(\theta_f - \theta_i)$$

$$\tau(\theta_f - \theta_i) = \frac{1}{2} I(\omega_f^2 - \omega_i^2) = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2$$

$$W = \Delta K$$

Rotational Kinetic Energy

$$W = \tau_{avg}(\theta_f - \theta_i)$$

$$W = \Delta K_{rot} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

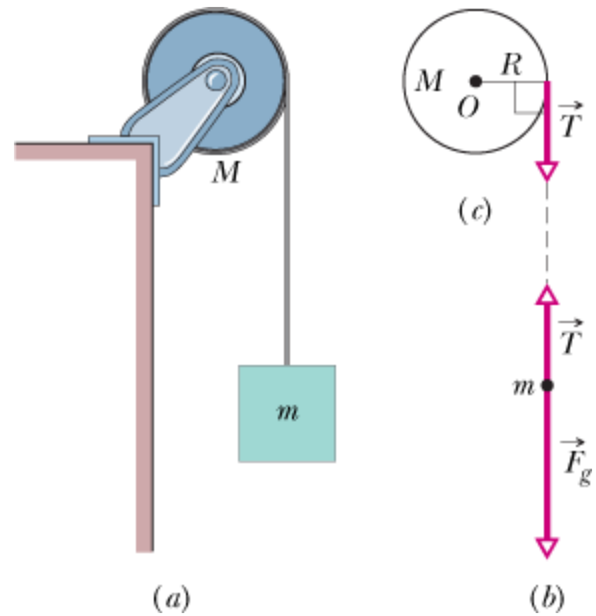
$$T = 6.0\ N \quad \tau = TR$$

$$\alpha = 24\ rad/s$$

$$R = 0.2\ m$$

$$a = \alpha R = 4.8\ m/s$$

$$\Delta\theta = \frac{s}{R} \quad s = 15\ m$$



A uniform disk, with mass M = 2.5 kg and radius R = 20 cm, mounted on a fixed horizontal axle.
A block with mass m = 1.2 kg hangs from a massless cord that is wrapped around the rim of the disk.

If block m falls 15 m from rest, what is the rotational kinetic energy of the pulley?

Answer – 90 (J)

Useful Equations in Rotational and Translational Motion

Rotational Motion About a Fixed Axis	Translational Motion
Angular speed $\omega = d\theta/dt$	Translational speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Translational acceleration $a = dv/dt$
Net torque $\Sigma\tau_{\text{ext}} = I\alpha$	Net force $\Sigma F = ma$
If $\alpha =$ constant $\begin{cases} \omega_f = \omega_i + \alpha t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If $a =$ constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$	Work $W = \int_{x_i}^{x_f} F_x \, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $P = \tau\omega$	Power $P = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

Total Mechanical Energy

$$K_{rot} = \frac{1}{2} I\omega^2$$

$$E_{mech} = K_{rot} + K_{trans} + U$$

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} mV^2 + U$$

$$E_1 = E_2 \quad Conservation\ of\ E_{mech}$$

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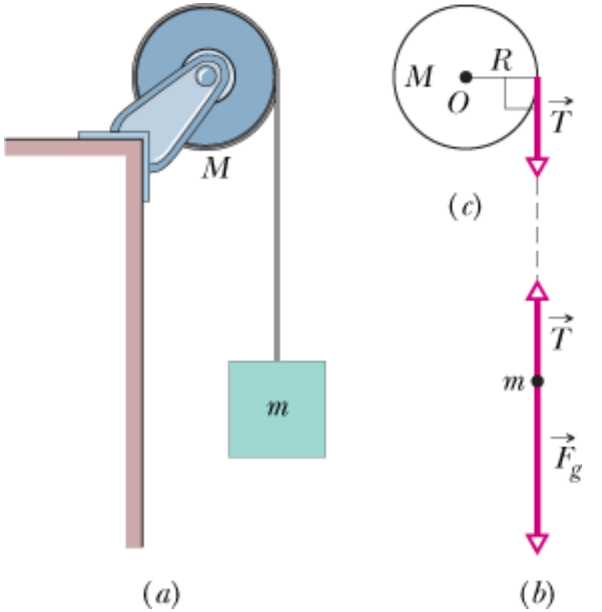
If block m falls 15 m from rest, what is the linear velocity?



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$$\frac{1}{2} I\omega^2 + \frac{1}{2} mV^2 = mgh$$

$$V = \omega r$$

$$\frac{1}{2} \frac{Mr^2}{2} \frac{V^2}{r^2} + \frac{1}{2} mV^2 = mgh$$

$$V = \left(\frac{mgh}{\frac{1}{4} M + \frac{1}{2} m}\right)^{\frac{1}{2}}$$

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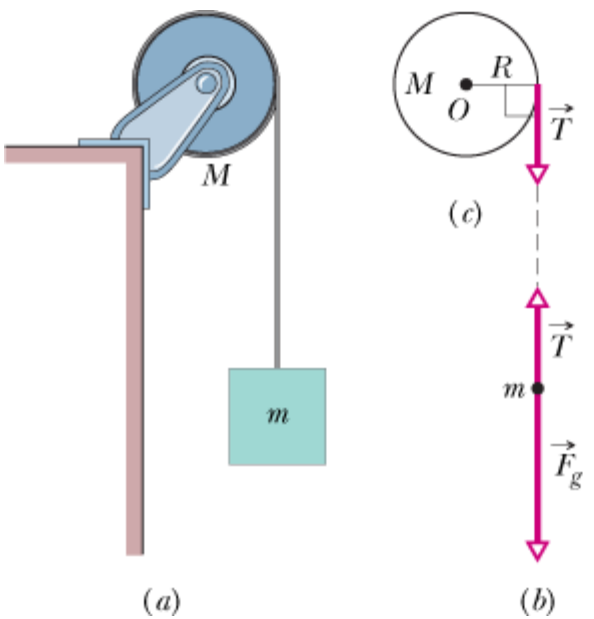

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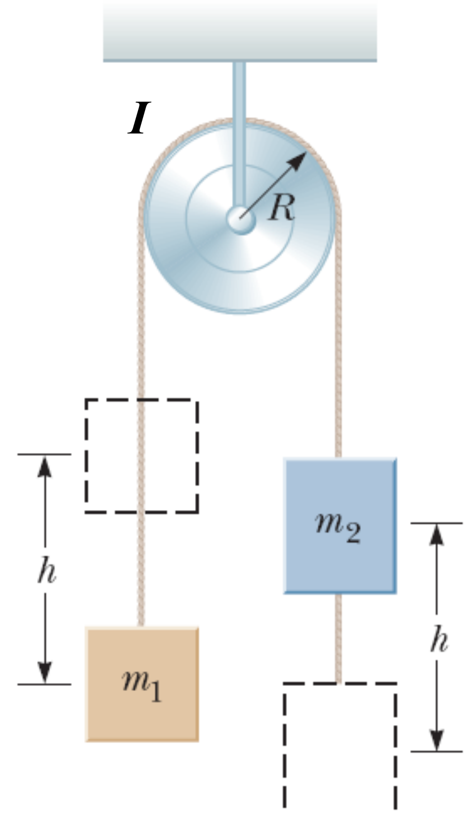
Initially the system is stationary.
What is the linear velocity?



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$$E_1 = E_2 \quad Conservation\ of\ E_{mech}$$

$$K_f + U_f = K_i + U_i$$

$$\left(\tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 + \tfrac{1}{2}I\omega_f^2\right) + (m_1 gh - m_2 gh) = 0 + 0$$

$$\tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 + \tfrac{1}{2}I\frac{v_f^2}{R^2} = m_2 gh - m_1 gh$$

$$\tfrac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v_f^2 = m_2 gh - m_1 gh$$

$$(1) \quad v_f = \left[\frac{2(m_2 - m_1)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}$$

$$\omega_f = \frac{v_f}{R} = \frac{1}{R}\left[\frac{2(m_2 - m_1)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}$$