Lecture 22

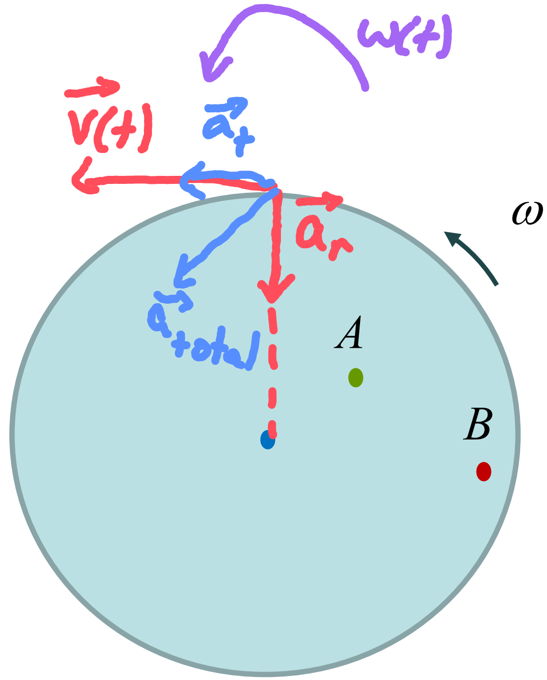
$$a_t = \alpha r \qquad v(t) = \underbrace{\omega(t)}_{\downarrow\,\alpha} r$$

$$a_{r(t)} = \frac{v_{(t)}^2}{r} = \omega_{(t)}^2 r$$

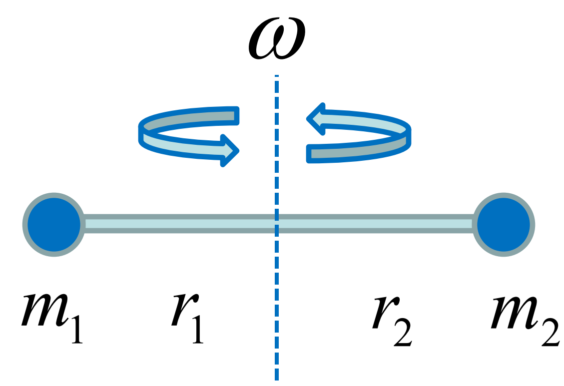
A disk is rotating CCW with an angular speed ω.

Two stickers A and B are fixed to different locations on the disk as shown.

Compare the angular speeds of the two stickers A and B; the linear speeds of two. $\rightarrow$ B > A



Kinetic Energy of Rotation



$$K = \sum_i \frac{1}{2} m_i V_i^2 = \sum_i \frac{1}{2} m_i (r_i \omega)^2$$

$$K = \sum_i \frac{1}{2} (m_i r_i^2) \omega^2 = \frac{1}{2} \omega^2 (\sum_i m_i r_i^2)$$

$$= \frac{1}{2} I \omega^2$$

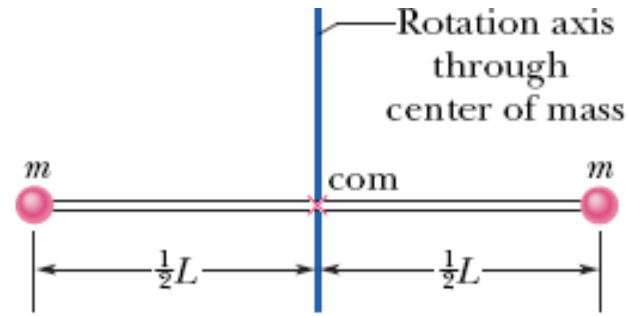
linear	rotation
x	θ
v	ω
$\frac{1}{2} m v^2$	$\frac{1}{2} I \omega^2$
m	I

Rotational Inertia (moment of inertia)

$$I = \sum_i m_i r_i^2 \qquad I = \int r^2 dm$$

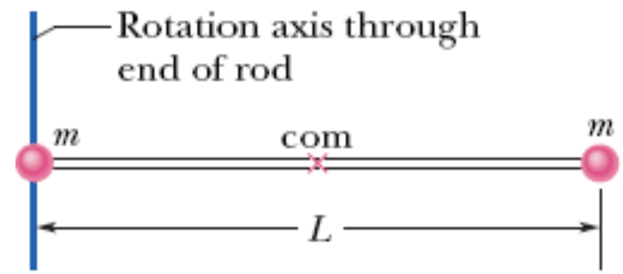
Moment of Inertia

(a)



(a)

(b)



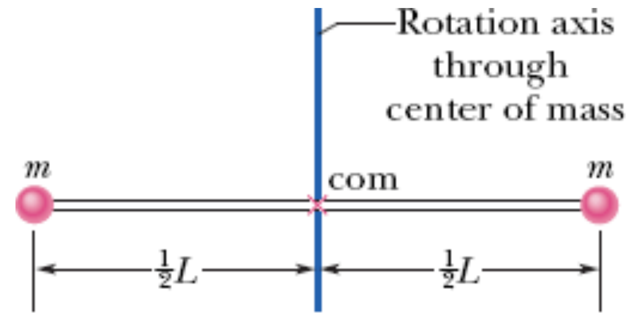
(b)

Moment of Inertia

(a)

$$I_{com} = \sum_i m_i r_i^2 = m(\frac{1}{2}L)^2 + m(\frac{1}{2}L)^2 = \frac{1}{2}mL^2$$

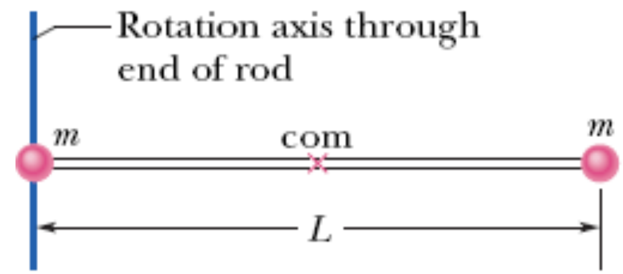
Center of mass



(a)

(b)

$$I = \sum_i m_i r_i^2 = m(0)^2 + m(L)^2 = mL^2$$



(b)

Parallel-Axis Theorem

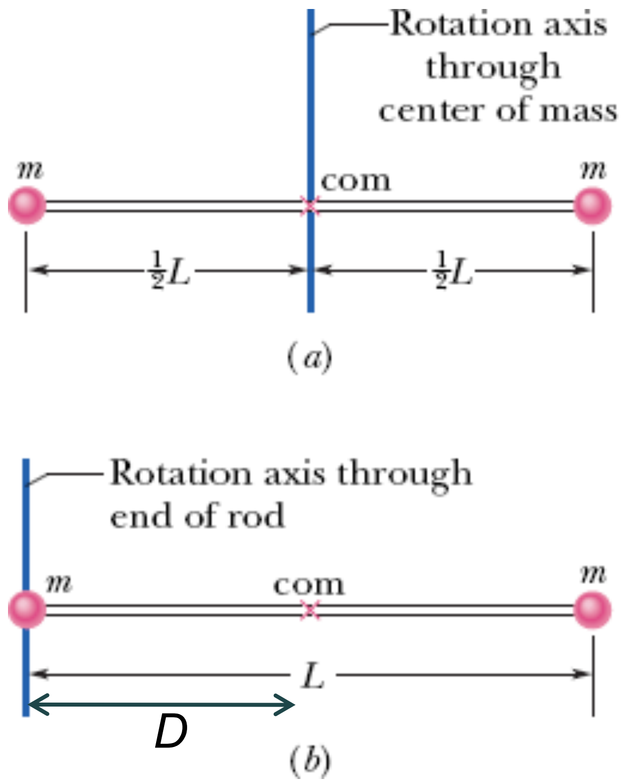
$$I = I_{com} + MD^2$$

$$I_{com} = \sum_i m_i r_i^2 = m(\frac{1}{2}L)^2 + m(\frac{1}{2}L)^2 = \frac{1}{2}mL^2$$

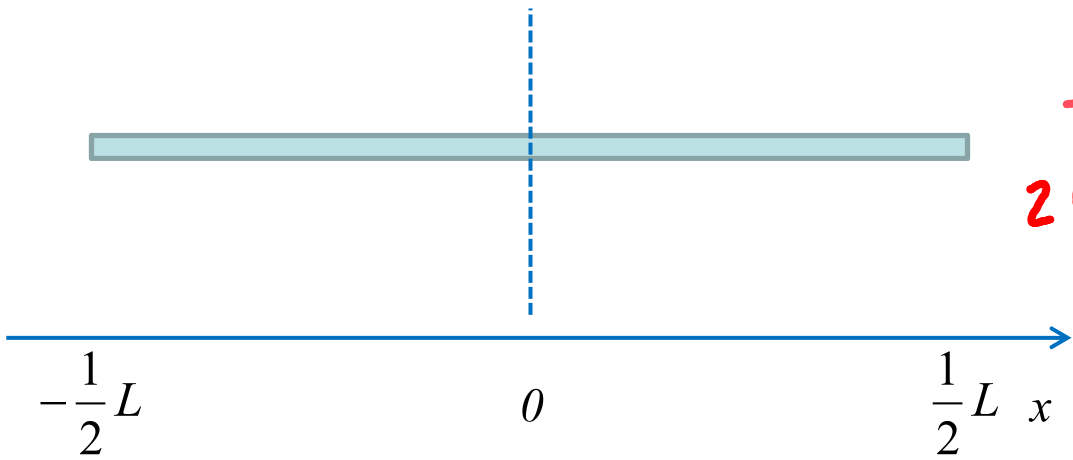
$$I = I_{com} + MD^2 = \frac{1}{2}mL^2 + (2m)(\frac{1}{2}L)^2$$

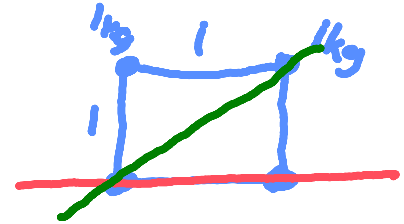
$$= \frac{1}{2}mL^2 + \frac{1}{2}mL^2 = mL^2$$

$$I = \sum_i m_i r_i^2 = m(0)^2 + m(L)^2 = mL^2$$



Calculating Moment of Inertia



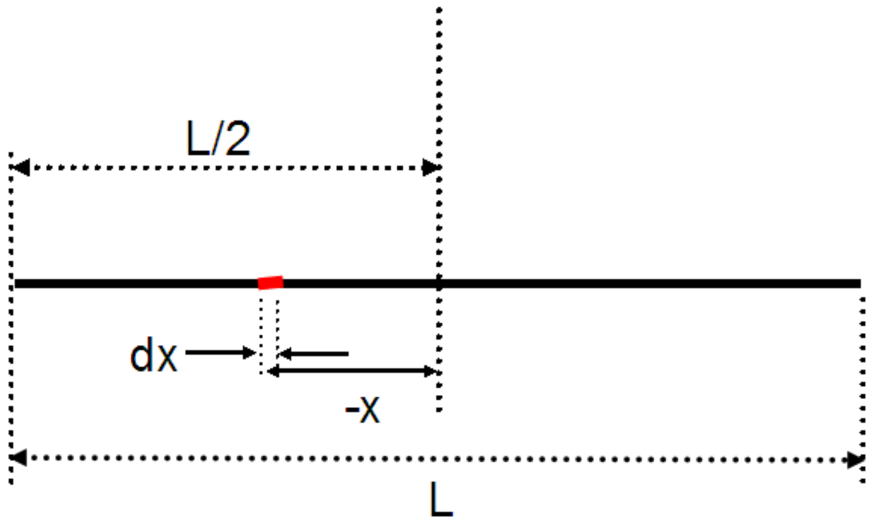


$2(1^2) + 2(0^2) = 2$

$1\left(\frac{\sqrt{2}}{2}^2\right) + 1\left(\frac{\sqrt{2}}{2}^2\right) = 1$

A mass M is uniformly distributed over the length L of a thin rod. Find its moment of inertia around an axis perpendicular to its length and going through its center of mass.

A mass M is uniformly distributed over the length L of a thin rod. The mass inside a short element dx is given by:



(1) $\frac{M}{L}$

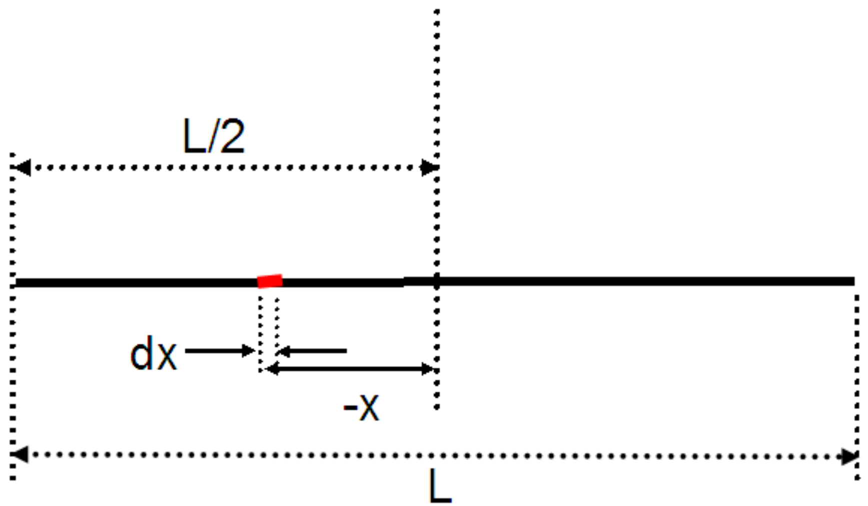
(2) $\left(\frac{M}{L}\right)dx$

(3) $\frac{L}{M}$

(4) Mdx

(5) None of the above

A mass M is uniformly distributed over the length L of a thin rod. The contribution to the moment of inertia by a short element dr is given by:



(1)
$xMdx$

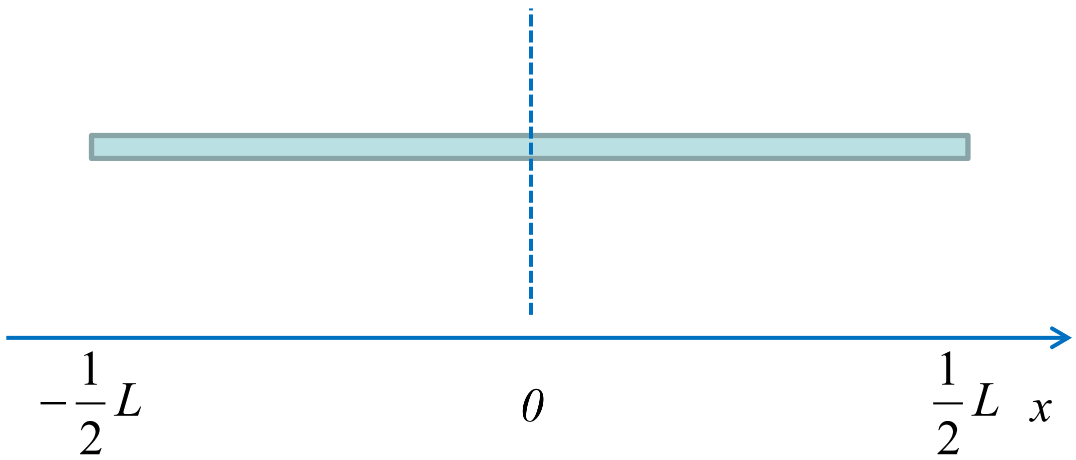
(2)
$x^2(M/L)dx$

(3)
$x^2(M/L)$

(4)
x^2M

(5)
x^2Mdx

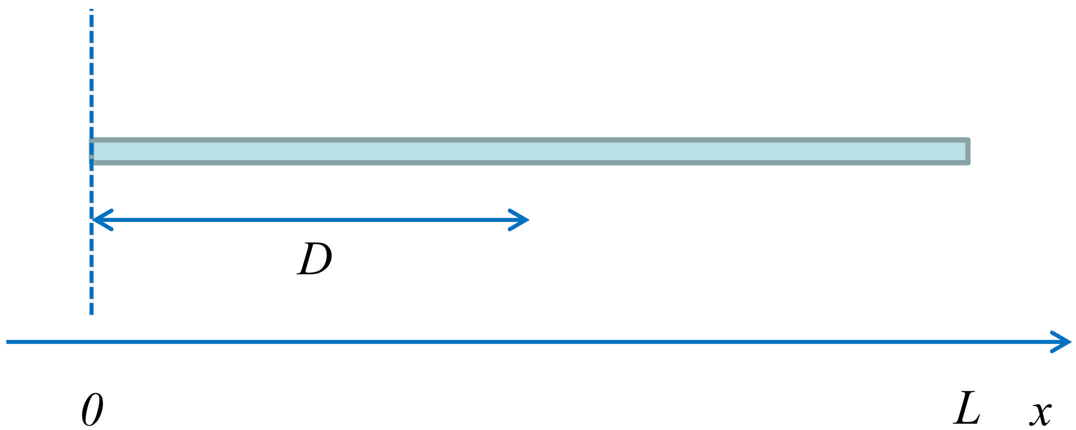
A mass M is uniformly distributed over the length L of a thin rod. Find its moment of inertia around an axis perpendicular to its length and going through its center of mass.



$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 dm = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \left(\frac{M}{L}\right) dx = \left(\frac{M}{L}\right) \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 dx$$

$$= \left(\frac{M}{L}\right) \frac{1}{3} x^3 \Big|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{M}{3L}\left(\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right) = \frac{M}{3L} \cdot \frac{L^3}{4} = \frac{ML^2}{12}$$

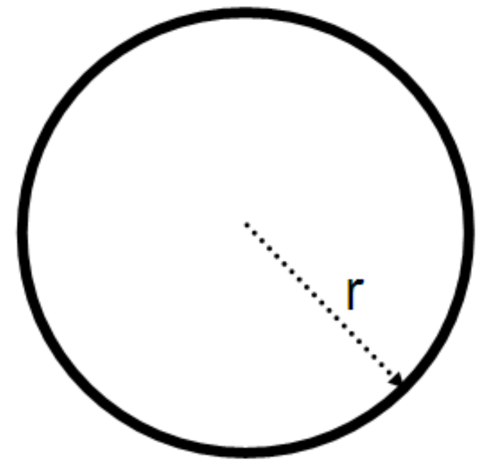
A mass M is uniformly distributed over the length L of a thin rod. Find its moment of inertia around an axis perpendicular to its length and going through one end.



$$I = I_{CM} + MD^2 = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{1}{3}ML^2$$

A mass M is uniformly distributed over the circumference of a thin ring with radius r. The moment of inertia for this ring when rotating about its center is:



(1) Cannot be determined without integrating.

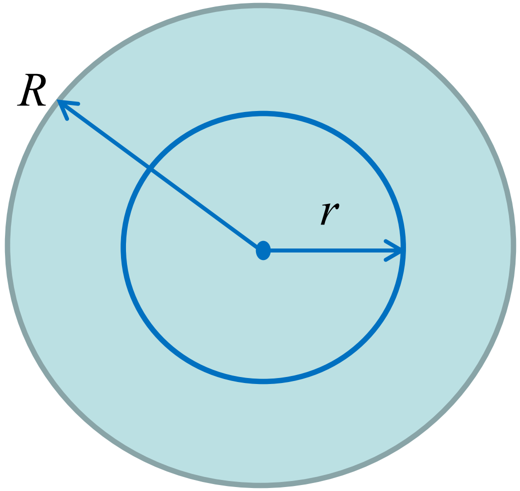
(2) Mr

(3) ½ Mr

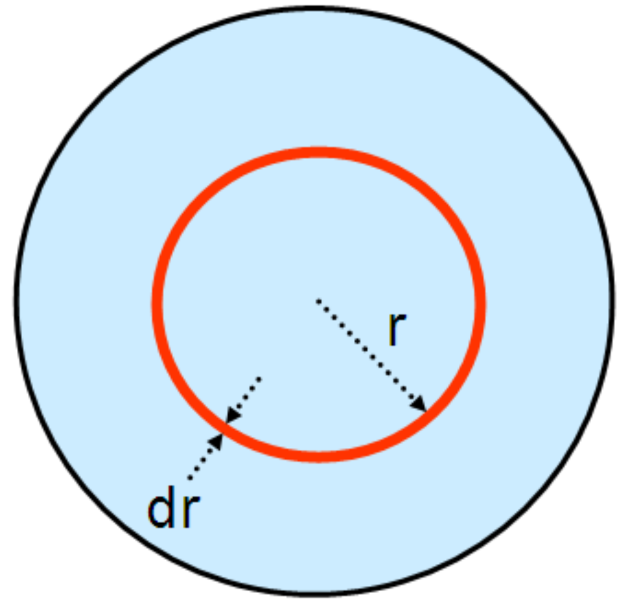
(4) Mr^2

(5) ½ Mr^2

A mass M is uniformly distributed over a disk. Find its moment of inertia around an axis perpendicular to the disk and going through its center of mass.



A mass M is uniformly distributed over a disk of radius R and area πR^2. The area of a thin ring inside the disk with radius r and thickness dr is:



(1) $2\pi r dr$

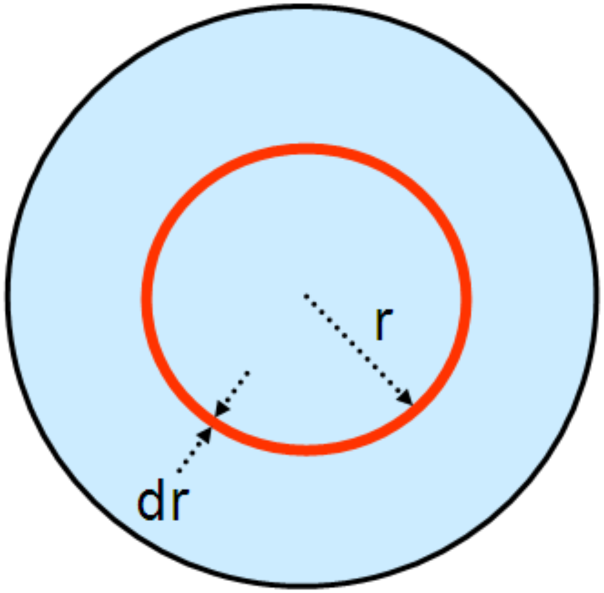
(2) $r dr$

(3) πr^2

(4) $\pi r^2 dr$

(5) r^2

A mass M is uniformly distributed over a disk of radius R. The mass contained in a thin ring with radius r and thickness dr inside the disk is given by:
(Remember to use a ratio of the ring area to the total area of the disk.)



(1)
$\left(\frac{M}{\pi R^2}\right) r^2 dr$

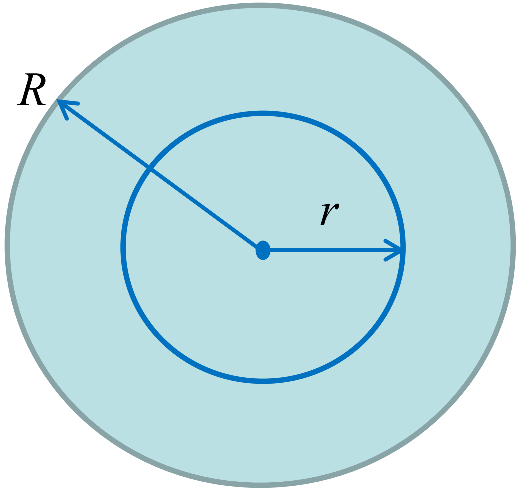
(2)
$\left(\frac{M}{R}\right) r$

(3)
$\left(\frac{MR^2}{rdr}\right)$

(4)
$\left(\frac{2M}{R^2}\right) rdr$

(5) None of the above

A mass M is uniformly distributed over a disk. Find its moment of inertia around an axis perpendicular to the disk and going through its center of mass.



$$I = \int_0^R r^2 dm = \int_0^R r^2 \left(\frac{M}{\pi R^2}\right) 2\pi r dr = \int_0^R \frac{2M}{R^2} r^3 dr$$

$$= \frac{2M}{4R^2} r^4 \Big|_0^R = \frac{M}{2R^2}(R^4 - 0) = \frac{1}{2} M R^2$$

Moment of inertia:

$$I = \sum m_i r_i^2$$

$$I_{COM} = MR^2 \quad (ring)$$

$$I_{COM} = \frac{1}{2} MR^2 \quad (disk)$$

$$I_{COM} = \frac{2}{5} MR^2 \quad (sphere)$$

$$I_{COM} = \frac{1}{12} ML^2 \quad (rod)$$

$$I = I_{COM} + Mh^2$$