CSE 2321 Homework 6

Problem 1

$$\begin{aligned} T(n) &= \sum_{k=0}^{n-1} ar^k && \text{Geometric series is really useful!} \\ &= a\left(\frac{1-r^n}{1-r}\right) && \text{If } r \neq 1 \end{aligned}$$

1A

$$\begin{aligned} T(n) &= \sum_{a=1}^{n^2}(\sum_{b=1}^{n^2}(\sum_{c=1}^{b} 1)) \\ &= \sum_{a=1}^{n^2}(\sum_{b=1}^{n^2} b) \\ &= \sum_{a=1}^{n^2}(\frac{n^2(n^2+1)}{2}) \\ &= \sum_{a=1}^{n^2}(\frac{n^2(n^2+1)}{2}) \\ &= \sum_{a=1}^{n^2} *n^4 \\ &= n^2 * n^4 = n^6 \\ T(n) &= \Theta(n^6) \end{aligned}$$

1B

$$
\begin{aligned}
T(n) &= \sum_{a=1}^{n^2} (\sum_{b=1}^{n^3} 1) \\
&= \sum_{a=1}^{n^2} *n^3 \\
&= n^2 * n^3 = n^5 \\
T(n) &= \Theta(n^5)
\end{aligned}
$$

1C

$$
\begin{aligned}
T(n) &= \sum_{a=1}^{n^2} (\sum_{b=1}^{n/5} (\sum_{c=1}^{5b} 1)) \\
&= \sum_{a=1}^{n^2} (\sum_{b=1}^{n/5} 5b) \\
&= \sum_{a=1}^{n^2} n \\
&= \frac{n^2(n^2+1)}{2} = Cn^4 + ... \\
T(n) &= \Theta(n^4)
\end{aligned}
$$

1D

$$
\begin{aligned}
T(n) &= \sum_{a=1}^{n^2} (\sum_{b=1}^{3log_2(a)-1} 1) \\
&= \sum_{a=1}^{n^2} 3log_2(a) - 1 \\
&= ((3log_2(n) - 1)(3log_2(n) - 1 + 1))/(2) \\
T(n) &= \Theta(6log(n))
\end{aligned}
$$

1E

$$
\begin{aligned}
T(n) &= \sum_{a=1}^{log_3(5n^2)} \left(\sum_{b=1}^{log_5(10/3)n^3} 1 \right) \\
&= \sum_{a=1}^{log_3(5n^2)} log_5(10/3)n^3 \\
&= log_3(5n^2) * log_5((10/3)n^3) \\
T(n) &= \Theta(log(n))
\end{aligned}
$$

1F

$$
\begin{aligned}
T(n) &= \sum_{a=1}^{n}(\sum_{b=a}^{n^2}(\sum_{c=1}^{n^3} 1)) \\
&= \sum_{a=1}^{n}(\sum_{b=a}^{n^2} *n^3) \\
&= \sum_{a=1}^{n} n^2 * n^3 \\
&= \sum_{a=1}^{n} n^5 \\
&= n * n^5 = n^6 \\
T(n) &= \Theta(n^6)
\end{aligned}
$$

1G

$$
\begin{aligned}
T(n) &= \sum_{a=1}^{n/2} (\sum_{b=a}^{a^2} 1) \\
&= \sum_{a=1}^{n/2} a^2 \\
&= \frac{(n/2)(n/2+1)(2(n/2)+1)}{6} \\
&= \frac{Cn^3 + ...}{6} \\
&= n^3 \\
T(n) &= \Theta(n^3)
\end{aligned}
$$

1H

$$
\begin{aligned}
T(n) &= \sum_{a=1}^{n} 1 \\
&= n \\
T(n) &= \Theta(n)
\end{aligned}
$$

Problem 2

2A

$$
\begin{aligned}
T(n) &= T(n/2) + 5 \\
T(1) &= 1 \\
&= T'(n/4) + n/2 + 5 + 5 \\
&= T'(n/8) + n/4 + n/2 + 5 + 5 + 5 \\
T'(n) &= T'(n/(2^k + 1)) + 5k \\
k &= \log_2(n) - 1 \\
T'(n) &= T'(n/2^{log_2(n)}) + 5(log_2(n) - 1) \\
T'(n) &= n + 5(log_2(n)) - 5 \\
T'(n) &= n + (log_2(n^5)) \\
T'(n) &= \Theta(n)
\end{aligned}
$$

2B

$$
\begin{aligned}
T(n) &= T(n/2) + n \\
T(1) &= 1 \\
&= T'(n/4) + n/2 + n \\
&= T'(n/8) + n/4 + n/2 + n \\
T'(n) &= T'(n/(2^k + 1)) + n \\
k &= \log_2(n) - 1 \\
T'(n) &= T'(n/2^{log_2(n)}) + n \\
&= n + n = 2n \\
T(n) &= \Theta(n)
\end{aligned}
$$