Storyline

Chapter 25: Capacitance and Dielectrics

Physics for Scientists and Engineers, 10e
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BROOKS/COLE
CENGAGE Learning

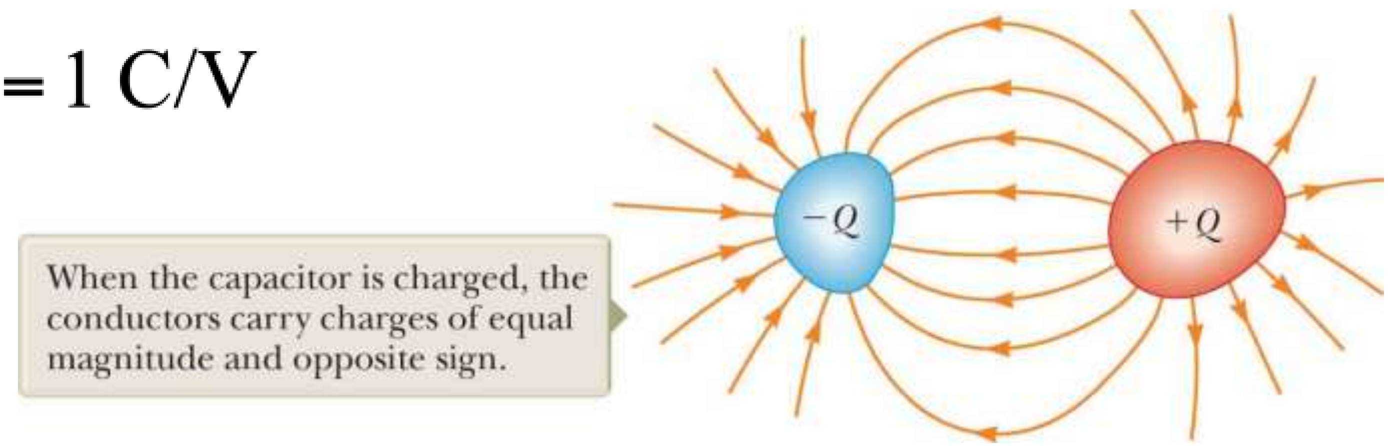
Definition of Capacitance

$$Q = C\Delta V$$

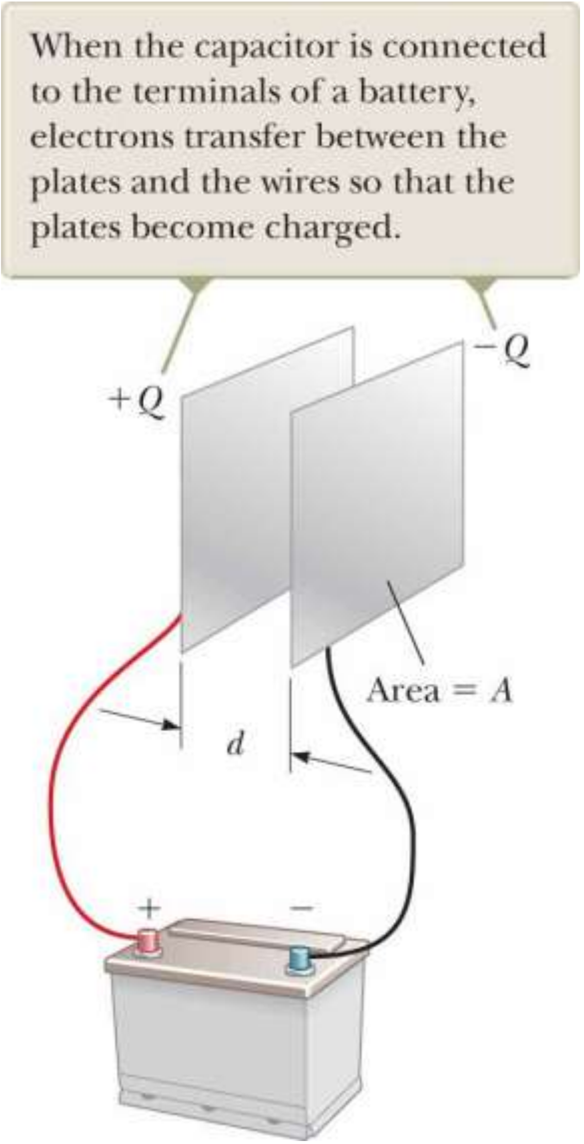
The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V}$$

$$1\ \mathrm{F} = 1\ \mathrm{C/V}$$



Definition of Capacitance



Quick Quiz 25.1

A capacitor stores charge Q at a potential difference $\otimes V$. What happens if the voltage applied to the capacitor by a battery is doubled to 2 $\otimes V$?

(a) The capacitance falls to half its initial value, and the charge remains the same.

(b) The capacitance and the charge both fall to half their initial values.

(c) The capacitance and the charge both double.

(d) The capacitance remains the same, and the charge doubles.

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Calculating Capacitance

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q/a} = \frac{a}{k_e} = 4\pi\varepsilon_0 a$$

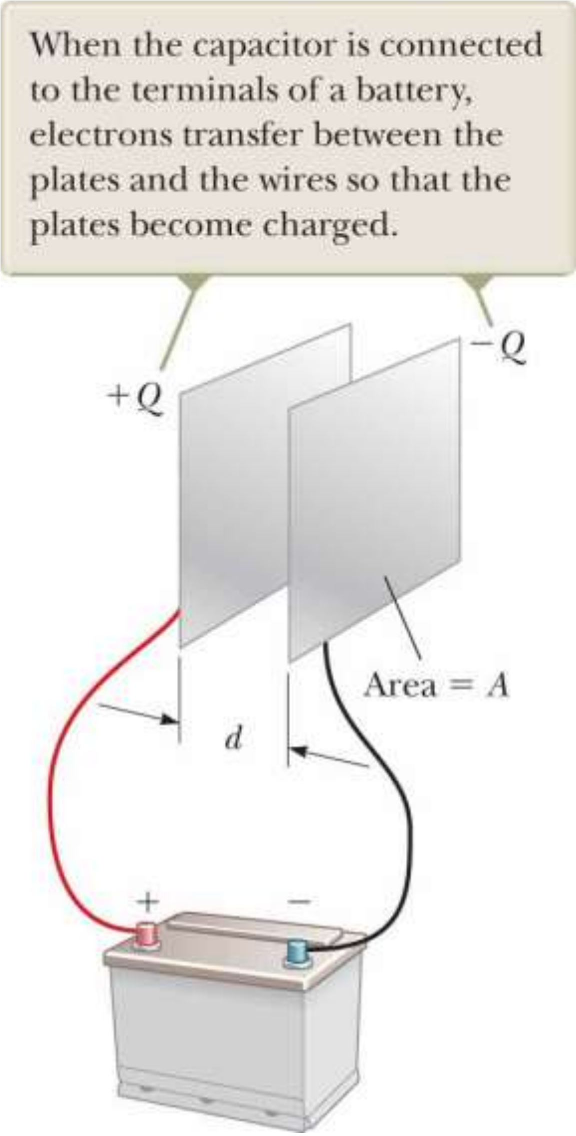
Calculating Capacitance

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\varepsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\varepsilon_0 A}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$



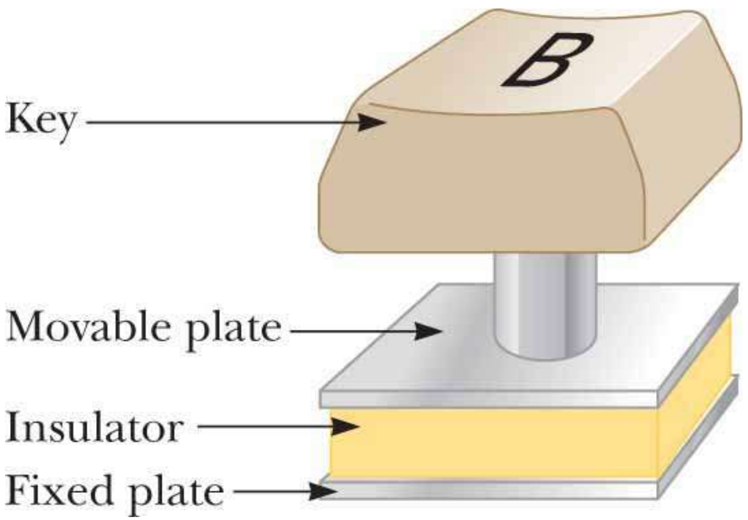
Quick Quiz 25.2

Many computer keyboard buttons are constructed of capacitors as shown in the figure. When a key is pushed down, the soft insulator between the movable plate and the fixed plate is compressed. When the key is pressed, what happens to the capacitance?

(a) It increases.

(b) It decreases.

(c) It changes in a way you cannot determine because the electric circuit connected to the keyboard button may cause a change in ⊗V.



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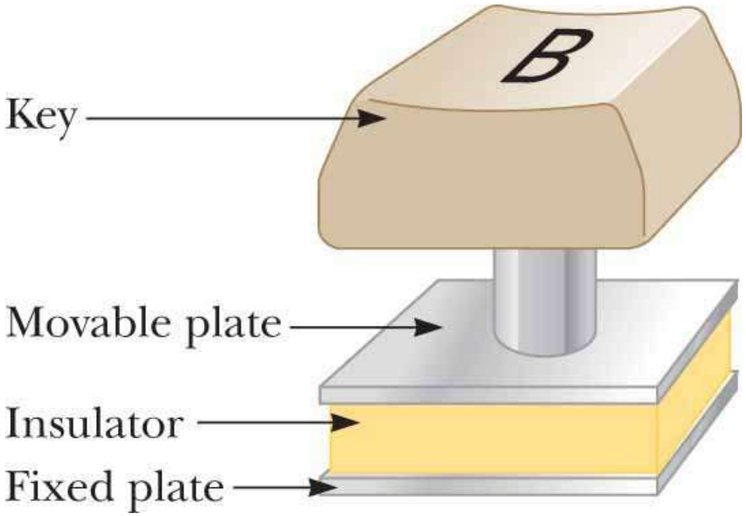
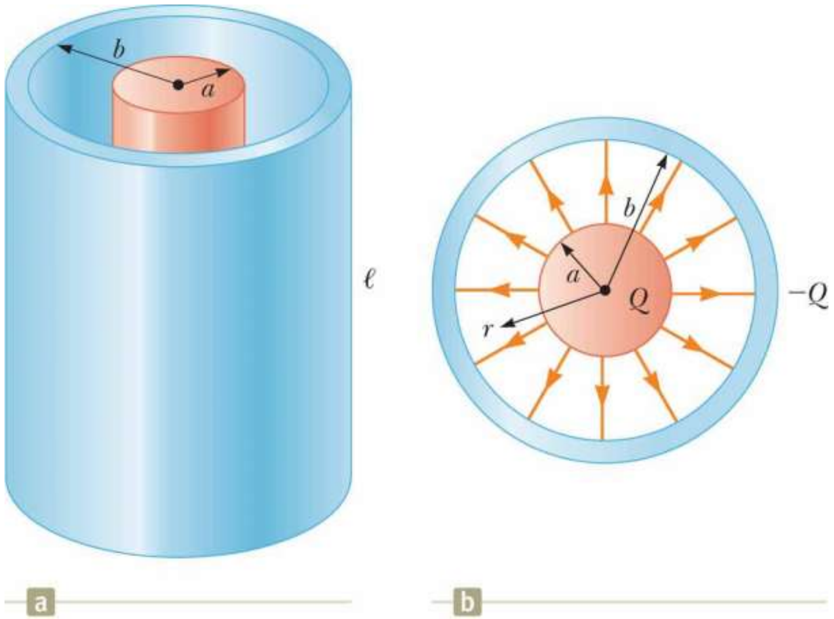


Example 25.1:
The Cylindrical Capacitor

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness and radius $b > a$. Find the capacitance of this cylindrical capacitor if its length is ℓ.

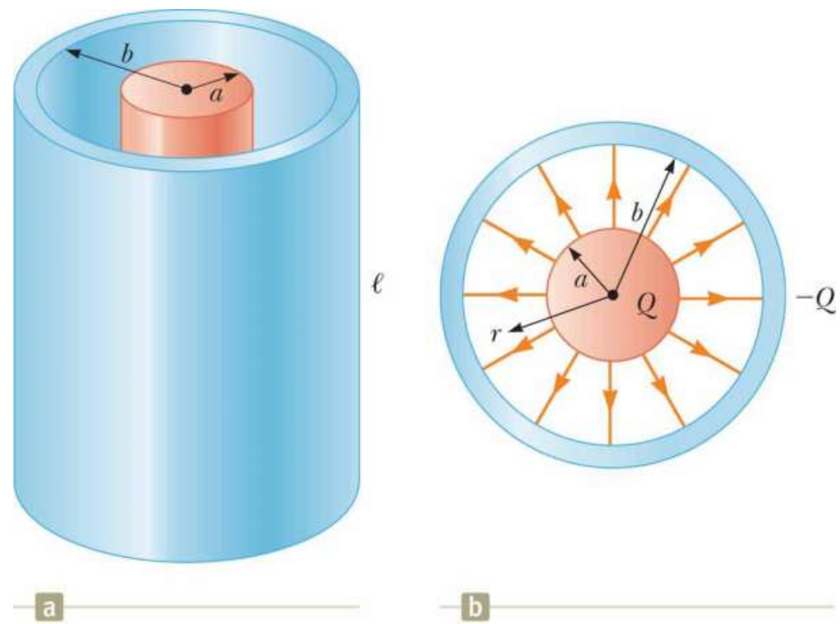


Example 25.1: The Cylindrical Capacitor

$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

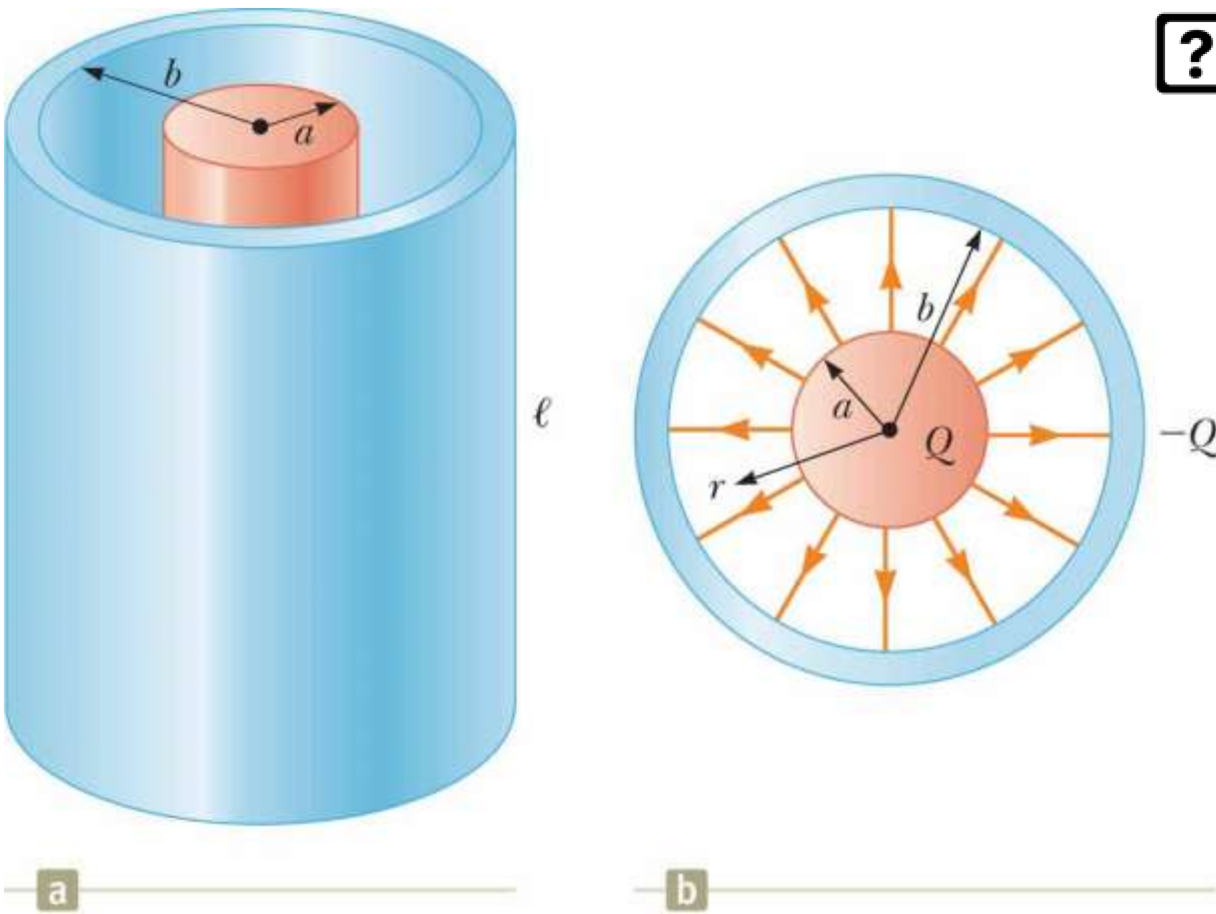
$$V_b - V_a = -\int_a^b E_r\,dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\left(2k_e Q/\boxed{?}\right)\ln\left(b/a\right)} = \boxed{\frac{\boxed{?}}{2k_e \ln\left(b/a\right)}}$$



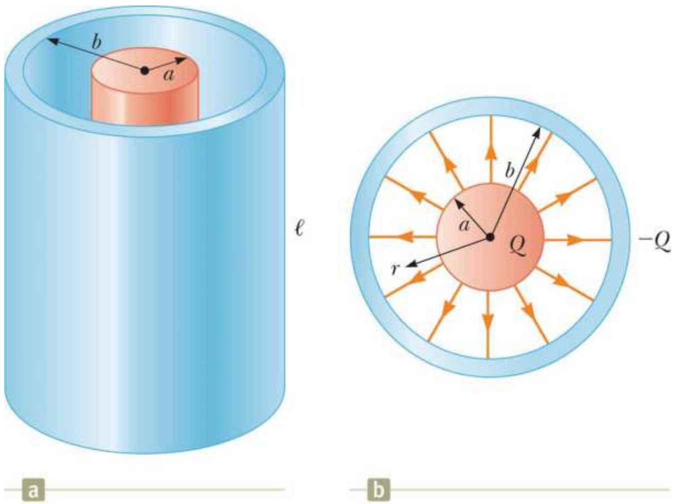
Example 25.1: The Cylindrical Capacitor

$$\frac{C}{\boxed{?}} = \frac{1}{2k_e \ln(b/a)}$$



Example 25.1:
The Cylindrical Capacitor

Suppose $b = 2.00a$ for the cylindrical capacitor. You would like to increase the capacitance, and you can do so by choosing to increase either ℓ by 10% or a by 10%. Which choice is more effective at increasing the capacitance?

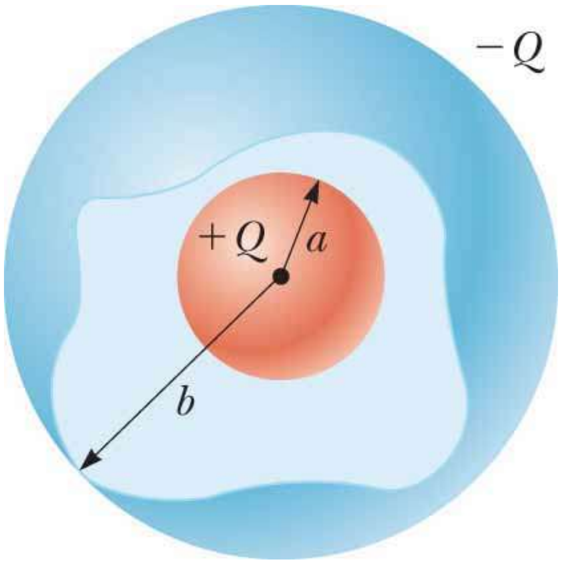


$$\frac{C'}{C} = \frac{\boxed{?}2k_e \ln(b/a)}{\boxed{?}2k_e \ln(b/a')} = \frac{\ln(b/a)}{\ln(b/a')}$$

$$\frac{C'}{C} = \frac{\ln(2.00a/a)}{\ln(2.00a/1.10a)} = \frac{\ln 2}{\ln 1.82} = 1.16$$

Example 25.2: The Spherical Capacitor

A spherical capacitor consists of a spherical conducting shell of radius *b* concentric with a smaller conducting sphere of radius *a*. Find the capacitance of this device.



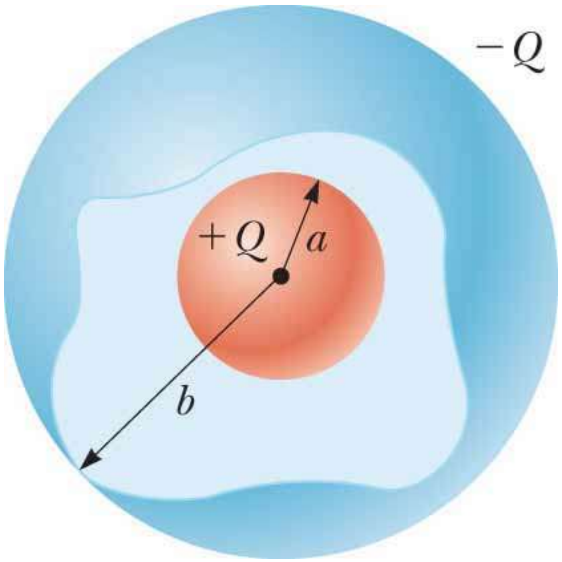
Example 25.2:
The Spherical Capacitor

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$$

$$V_b - V_a = -\int_a^b E_r\,dr = -k_e Q \int_a^b \frac{dr}{r^2} = k_e Q \left[\frac{1}{r}\right]_a^b$$

$$V_b - V_a = k_e Q \left(\frac{1}{b} - \frac{1}{a}\right) = k_e Q \frac{a-b}{ab}$$

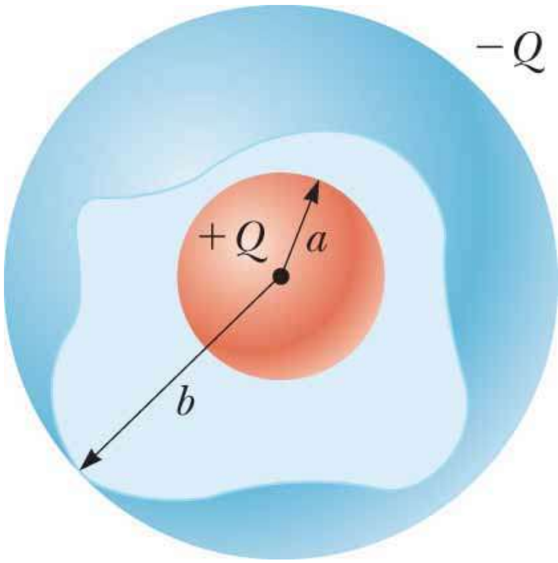
$$C = \frac{Q}{\Delta V} = \frac{Q}{|V_b - V_a|} = \boxed{\frac{ab}{k_e(b-a)}}$$



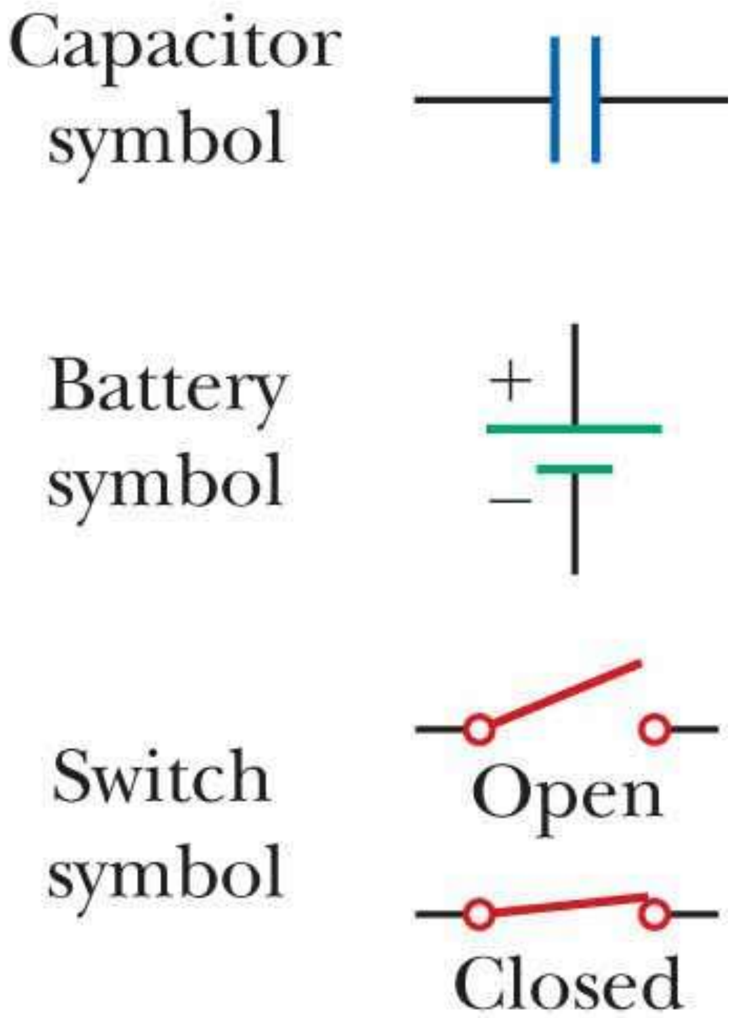
Example 25.2:
The Spherical Capacitor

If the radius *b* of the outer sphere approaches infinity, what does the capacitance become?

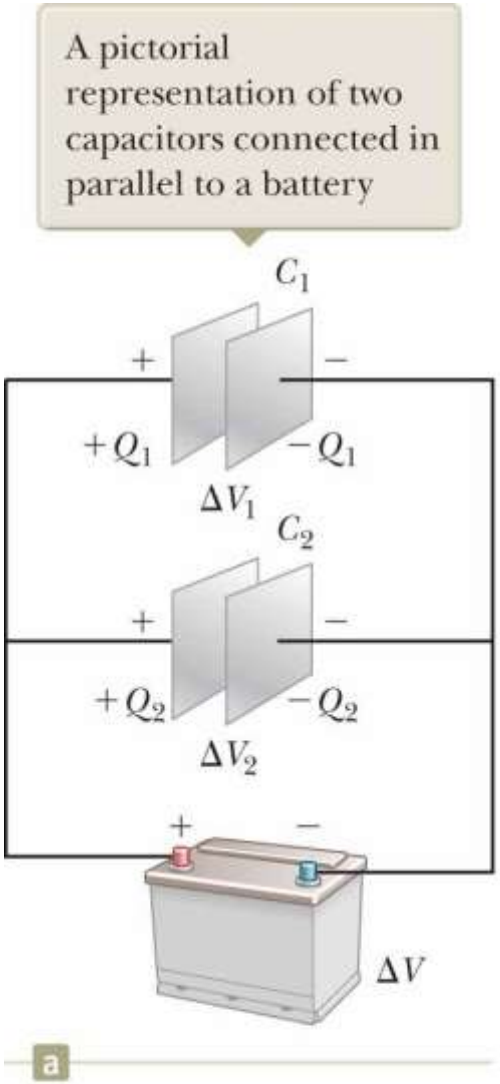
$$C = \lim_{b \to \infty} \frac{ab}{k_e(b-a)} = \frac{ab}{k_e(b)} = \frac{a}{k_e} = 4\pi\varepsilon_0 a$$



Combinations of Capacitors

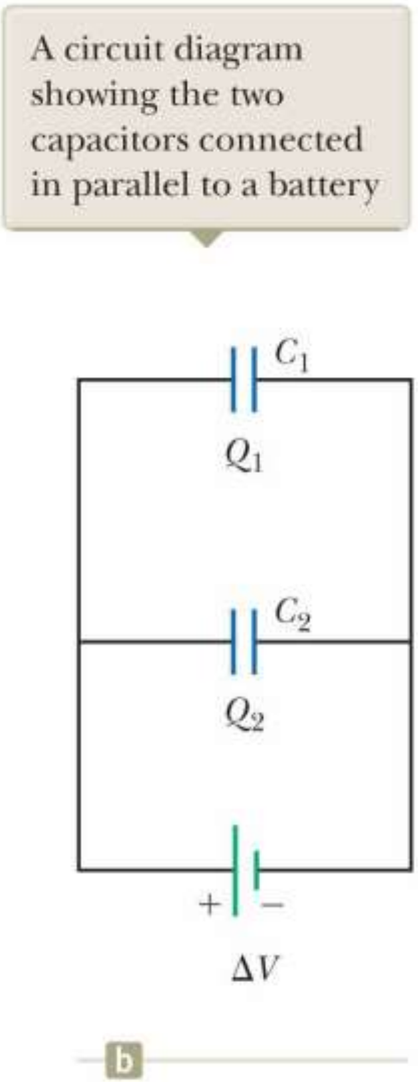


Parallel Combination



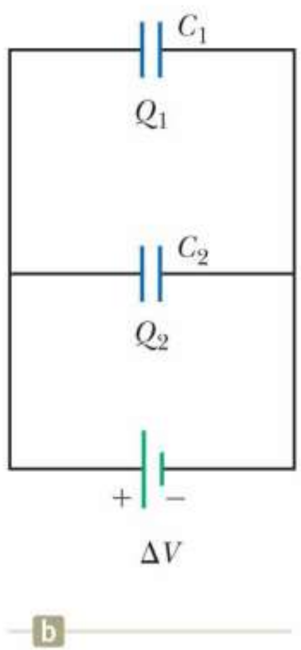
$$\Delta V_1 = \Delta V_2 = \Delta V$$

$$\begin{aligned} Q_{\text{tot}} &= Q_1 + Q_2 \\ &= C_1 \Delta V_1 + C_2 \Delta V_2 \end{aligned}$$



Parallel Combination

A circuit diagram showing the two capacitors connected in parallel to a battery

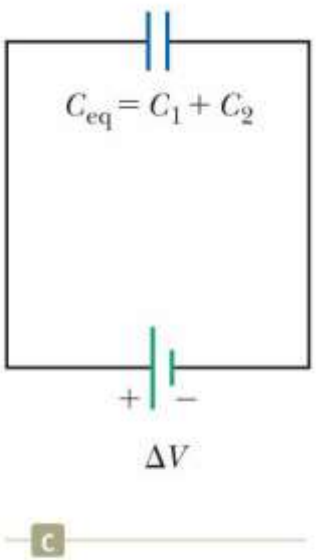


$$Q_{\text{tot}} = C_{\text{eq}} \Delta V$$

$$\begin{aligned} C_{\text{eq}} \Delta V &= C_1 \Delta V_1 + C_2 \Delta V_2 \\ &= (C_1 + C_2) \Delta V \\ &\equiv C_{\text{eq}} \Delta V \\ C_{\text{eq}} &= C_1 + C_2 \end{aligned}$$

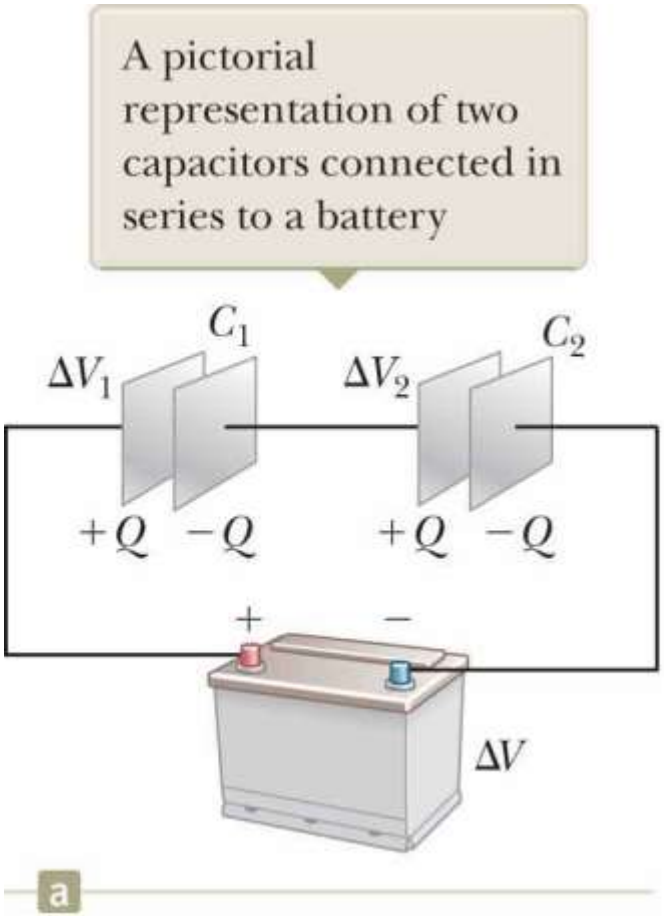
$$(\text{parallel combination})$$

A circuit diagram showing the equivalent capacitance of the capacitors in parallel



$$C_{\text{eq}} = C_1 + C_2 + C_3 + \boxed{?} \quad (\text{parallel combination})$$

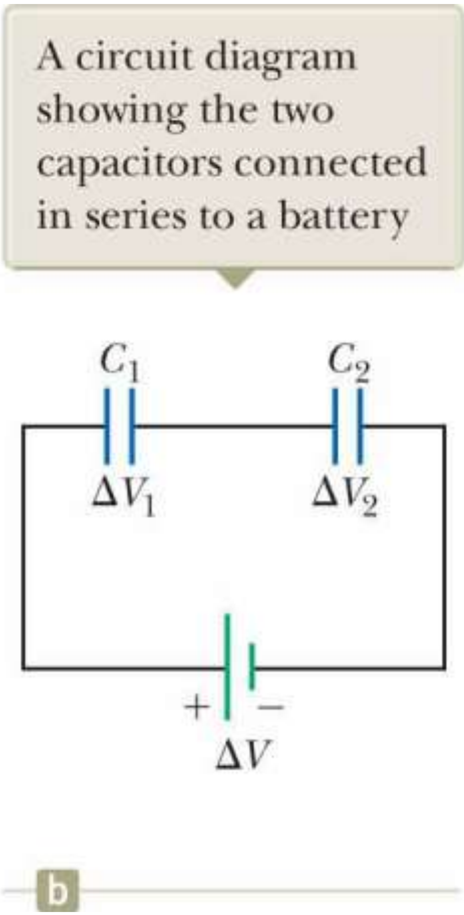
Series Combination



$$Q_1 = Q_2 = Q$$

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2$$

$$= \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$



Series Combination

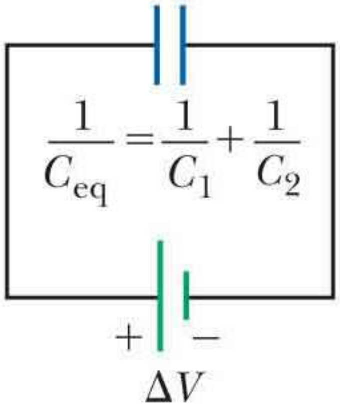
$$\Delta V_{tot} = \frac{Q}{C_{eq}}$$

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(series combination)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \boxed{?} \quad \text{(series combination)}$$

A circuit diagram showing the equivalent capacitance of the capacitors in series



Quick Quiz 25.3

Two capacitors are identical. They can be connected in series or in parallel. If you want the smallest equivalent capacitance for the combination,
how should you connect them?

(a) in series
(b) in parallel
(c) either way because both combinations have the same capacitance

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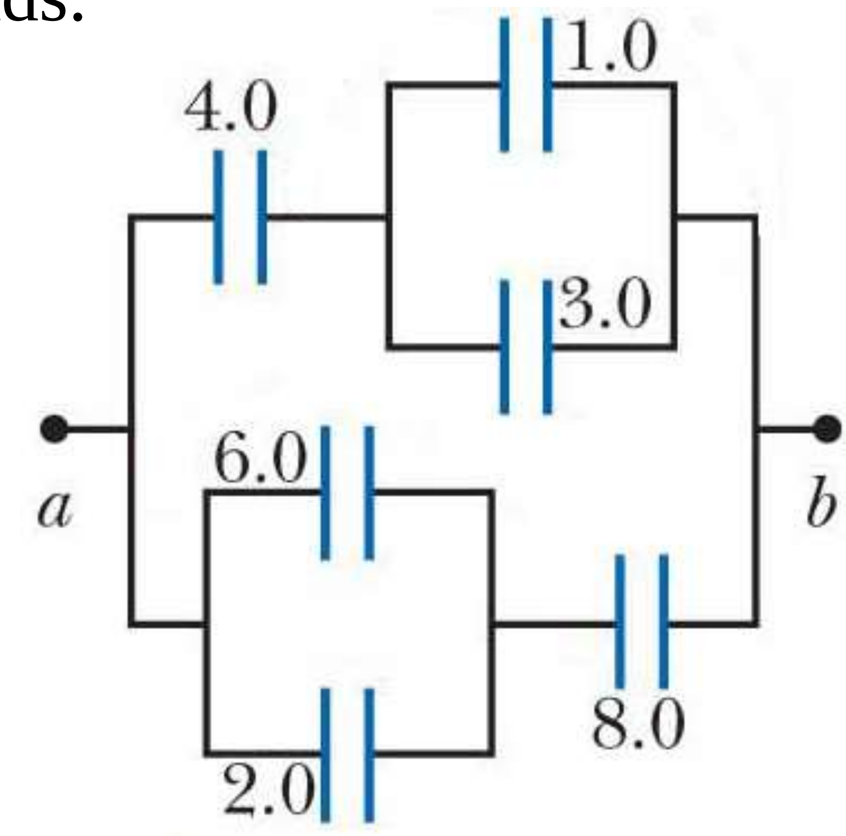
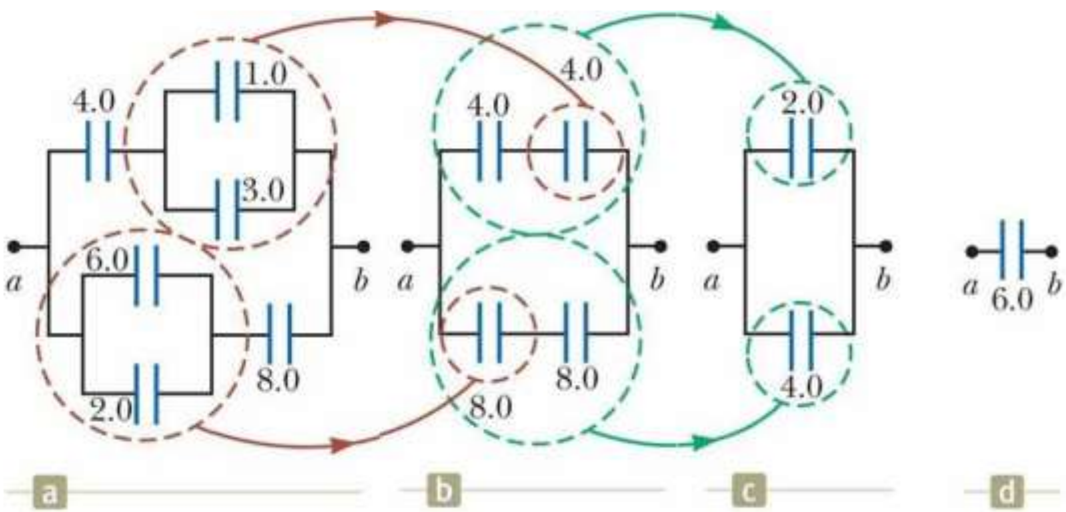
Example 25.3:
Equivalent Capacitance

Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in the figure. All capacitances are in microfarads.



Example 25.3: Equivalent Capacitance



$$C_{eq} = C_1 + C_2 = 4.0\ \mu F$$

$$C_{eq} = C_1 + C_2 = 8.0\ \mu F$$

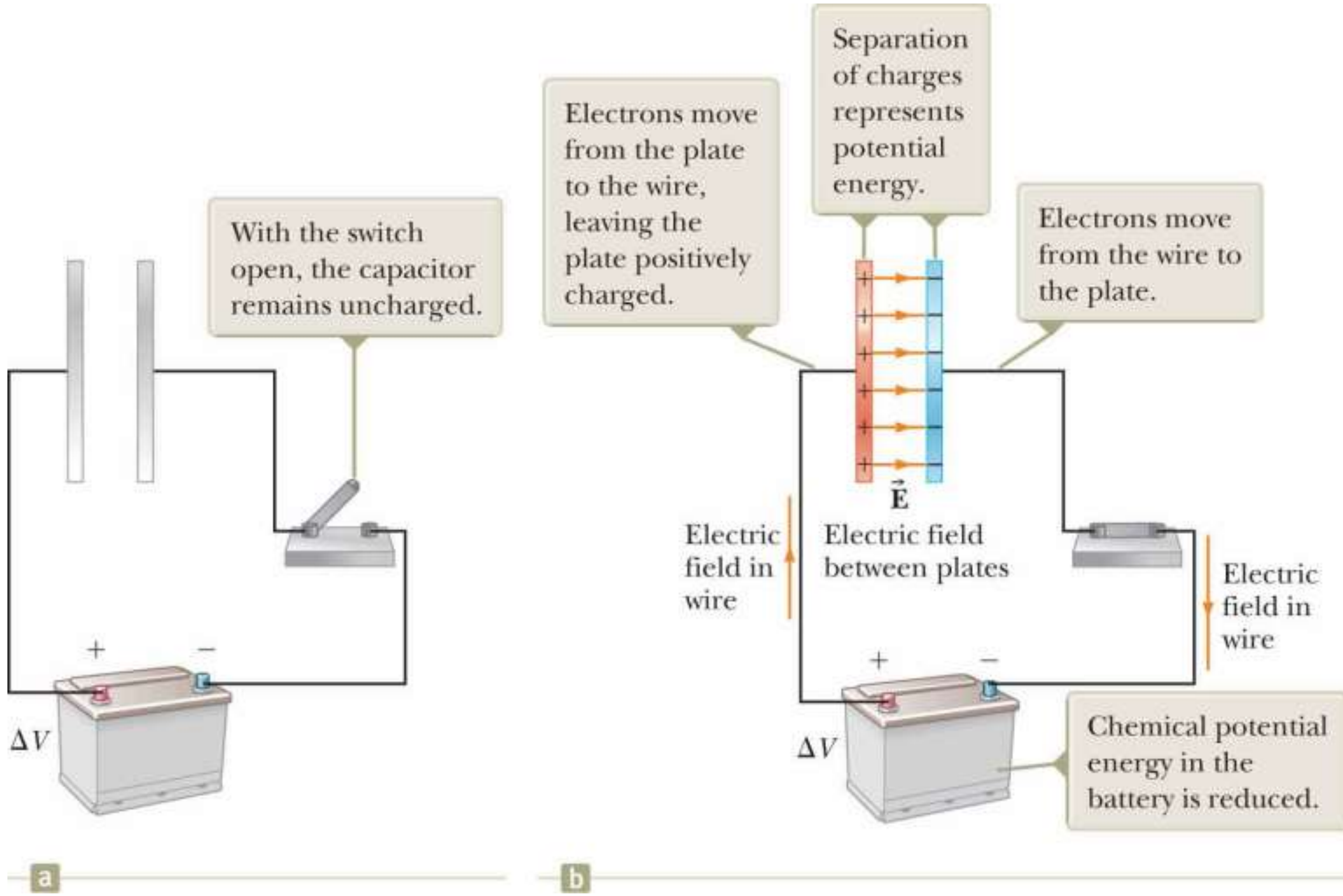
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F} \rightarrow C_{eq} = 2.0\ \mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8.0\ \mu F} + \frac{1}{8.0\ \mu F} = \frac{1}{4.0\ \mu F} \rightarrow C_{eq} = 4.0\ \mu F$$

$$C_{eq} = C_1 + C_2 = 6.0\ \mu F$$

Energy Stored in a Charged Capacitor

Energy Stored in a Charged Capacitor



Energy Stored in a Charged Capacitor

$$dW = \Delta V\, dq = \frac{q}{C} dq$$

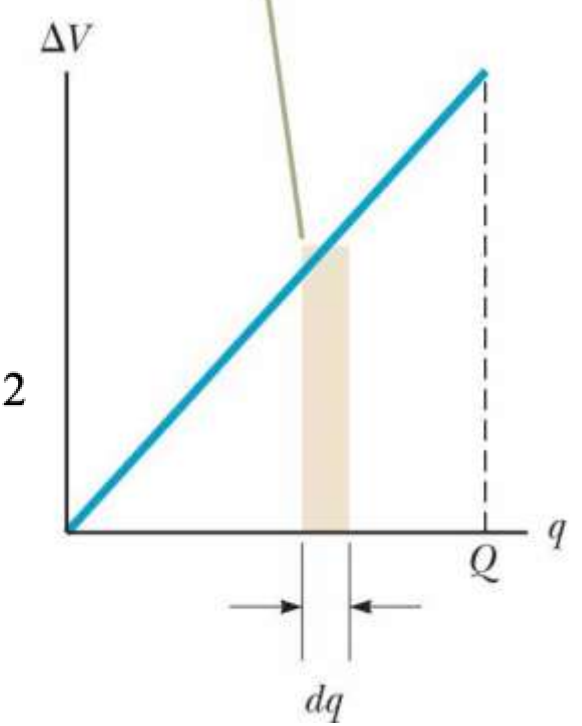
$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

$$\boxed{U_E = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2}$$

$$U_E = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}(\varepsilon_0 A d)E^2$$

$$\boxed{u_E = \frac{1}{2}\varepsilon_0 E^2}$$

The work required to move charge *dq* through the potential difference ΔV across the capacitor plates is given approximately by the area of the shaded rectangle.



Quick Quiz 25.4

You have three capacitors and a battery. In which of the following combinations of the three capacitors is the maximum possible energy stored when the combination is attached to the battery?

(a) series
(b) parallel
(c) no difference because both combinations store the same amount of energy

Quick Quiz 25.4

You have three capacitors and a battery. In which of the following combinations of the three capacitors is the maximum possible energy stored when the combination is attached to the battery?

(a) series

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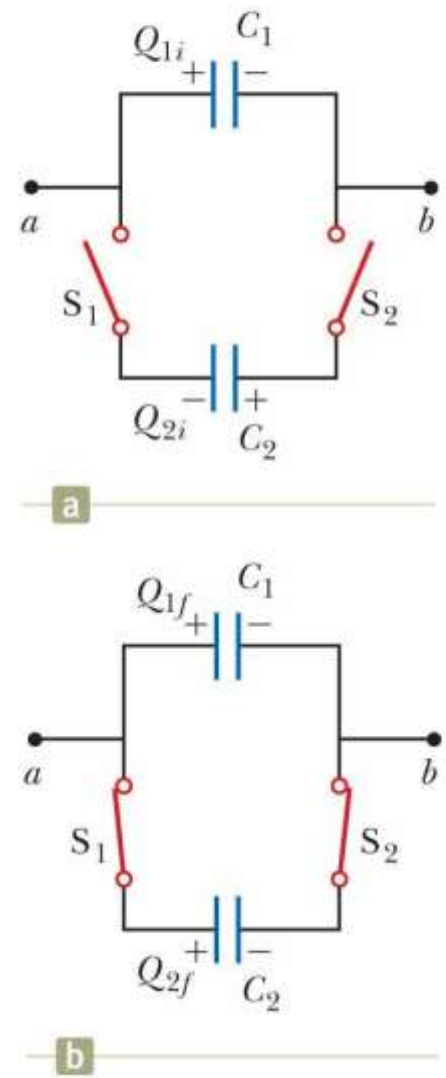
Example 25.4:
Rewiring Two Charged Capacitors

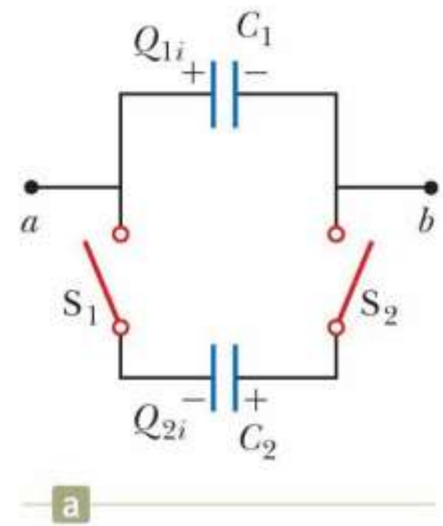
Two capacitors C_1 and C_2 (where $C_1 > C_2$) are charged to the same initial potential difference $\otimes V_i$. The charged capacitors are removed from the battery, and their plates are connected with opposite polarity as in the top figure. The switches S_1 and S_2 are then closed as in the bottom figure.



(A) Find the final potential difference $\otimes V_f$ between *a* and *b* after the switches are closed.

Example 25.4: Rewiring Two Charged Capacitors

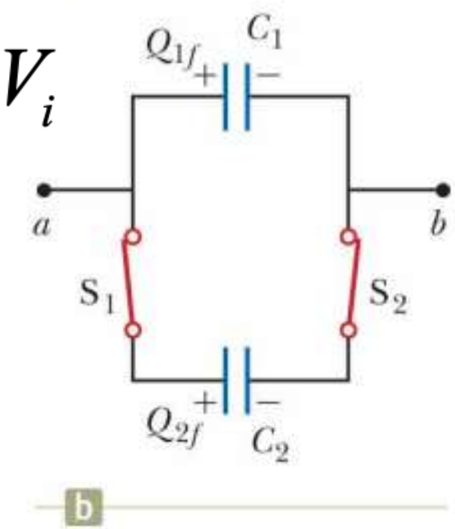
$$Q_i = Q_{1i} + Q_{2i} = C_1 \Delta V_i - C_2 \Delta V_i$$
$$= \left(C_1 - C_2\right) \Delta V_i$$



$$Q_f = Q_{1f} + Q_{2f} = C_1 \Delta V_f + C_2 \Delta V_f$$
$$= \left(C_1 + C_2\right) \Delta V_f$$

$$Q_f = Q_i \rightarrow \left(C_1 + C_2\right) \Delta V_f = \left(C_1 - C_2\right) \Delta V_i$$

$$\Delta V_f = \boxed{\left(\frac{C_1 - C_2}{C_1 + C_2}\right) \Delta V_i}$$

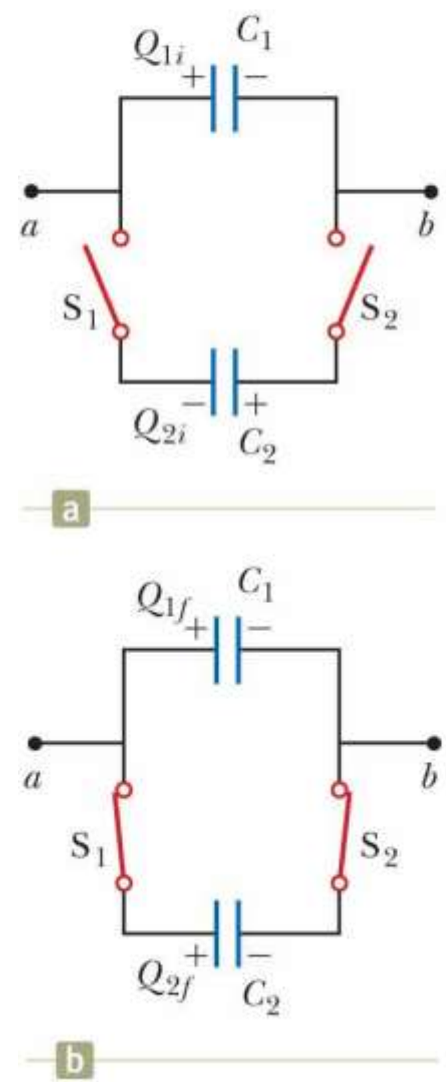


Example 25.4: Rewiring Two Charged Capacitors

(B) Find the total energy stored in the capacitors before and after the switches are closed and determine the ratio of the final energy to the initial energy.

$$U_i = \frac{1}{2}C_1(\Delta V_i)^2 + \frac{1}{2}C_2(\Delta V_i)^2$$

$$= \boxed{\frac{1}{2}(C_1 + C_2)(\Delta V_i)^2}$$



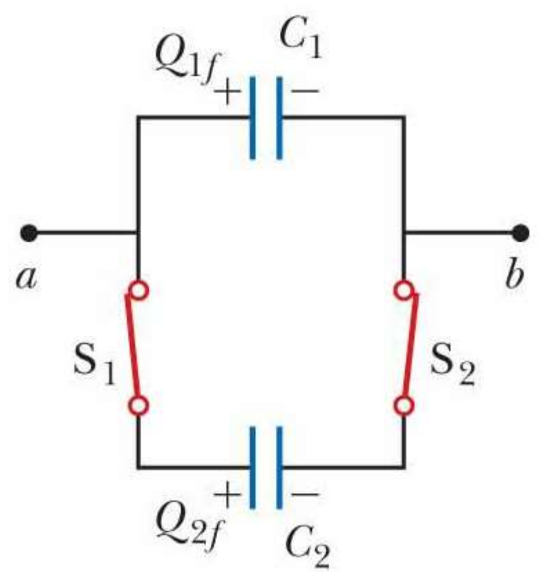
Example 25.4:
Rewiring Two Charged Capacitors

$$U_f = \frac{1}{2}C_1\left(\Delta V_f\right)^2 + \frac{1}{2}C_2\left(\Delta V_f\right)^2 = \frac{1}{2}\left(C_1 + C_2\right)\left(\Delta V_f\right)^2$$

$$U_f = \frac{1}{2}\left(C_1 + C_2\right)\left[\left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i\right]^2 = \boxed{\frac{1}{2}\frac{\left(C_1 - C_2\right)^2\left(\Delta V_i\right)^2}{C_1 + C_2}}$$

$$\frac{U_f}{U_i} = \frac{\frac{1}{2}\left(C_1 - C_2\right)^2\left(\Delta V_i\right)^2 / \left(C_1 + C_2\right)}{\frac{1}{2}\left(C_1 + C_2\right)\left(\Delta V_i\right)^2}$$

$$= \boxed{\left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2}$$



Example 25.4: Rewiring Two Charged Capacitors

What if the two capacitors have the same capacitance? What would you expect to happen when the switches are closed?

$$Q_i = (C_1 - C_2)\Delta V_i \rightarrow Q_i = 0$$

$$\Delta V_f = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i \rightarrow \Delta V_f = 0$$

$$U_f = \frac{1}{2}\frac{(C_1 - C_2)^2(\Delta V_i)^2}{C_1 + C_2} \rightarrow U_f = 0$$

Portable Defibrillator

Earth's Atmosphere as a Capacitor

Capacitors with Dielectrics

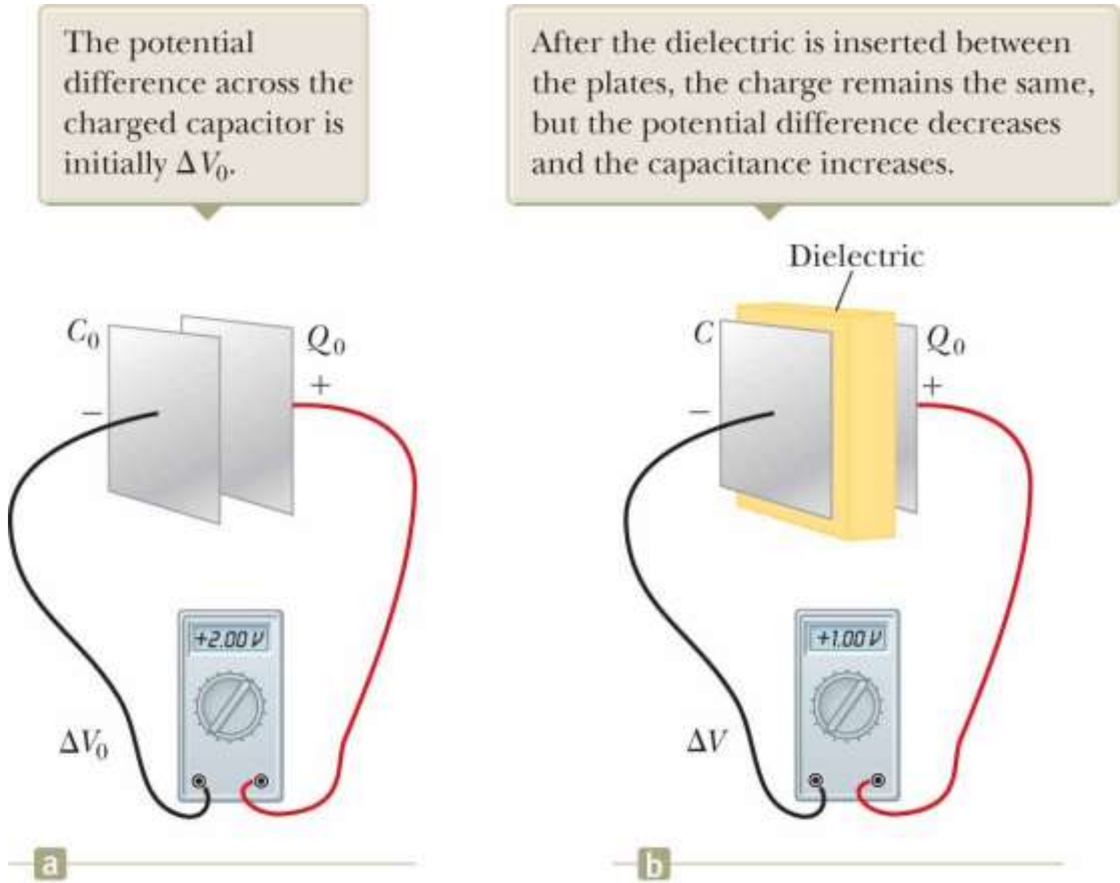
$$\Delta V = \frac{\Delta V_0}{\kappa}$$

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa}$$

$$= \kappa \frac{Q_0}{\Delta V_0}$$

$$\boxed{C = \kappa C_0}$$

$$C = \kappa \frac{\varepsilon_0 A}{d}$$



Capacitors with Dielectrics

$$C = \kappa \frac{\varepsilon_0 A}{d}$$

TABLE 25.1 Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polyethylene	2.30	18
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—

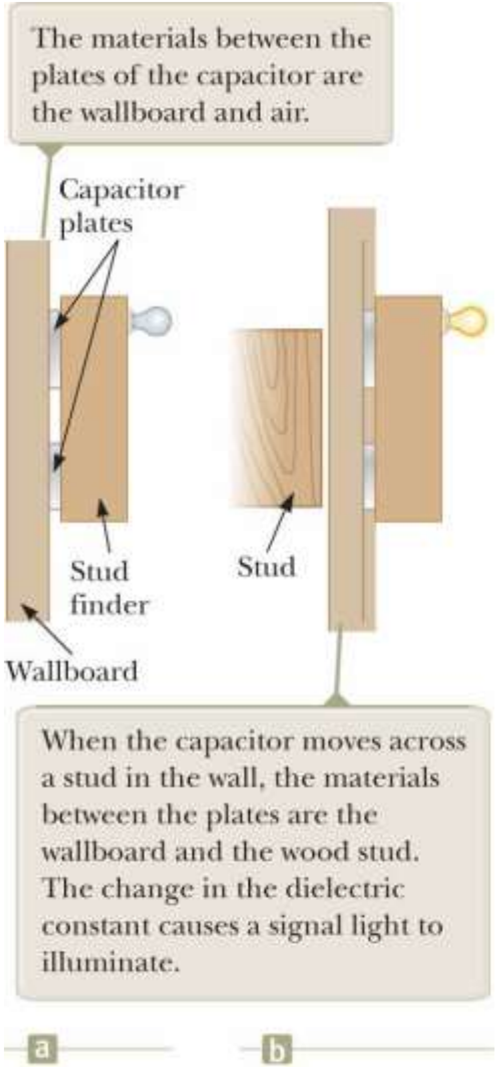
[a]The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

Quick Quiz 25.5

If you have ever tried to hang a picture or a mirror, you know it can be difficult to locate a wooden stud in which to anchor your nail or screw. A carpenter's stud finder is a capacitor with its plates arranged side by side instead of facing each other as shown in the figure. When the device is moved over a stud, does the capacitance

(a) increase or

(b) decrease?



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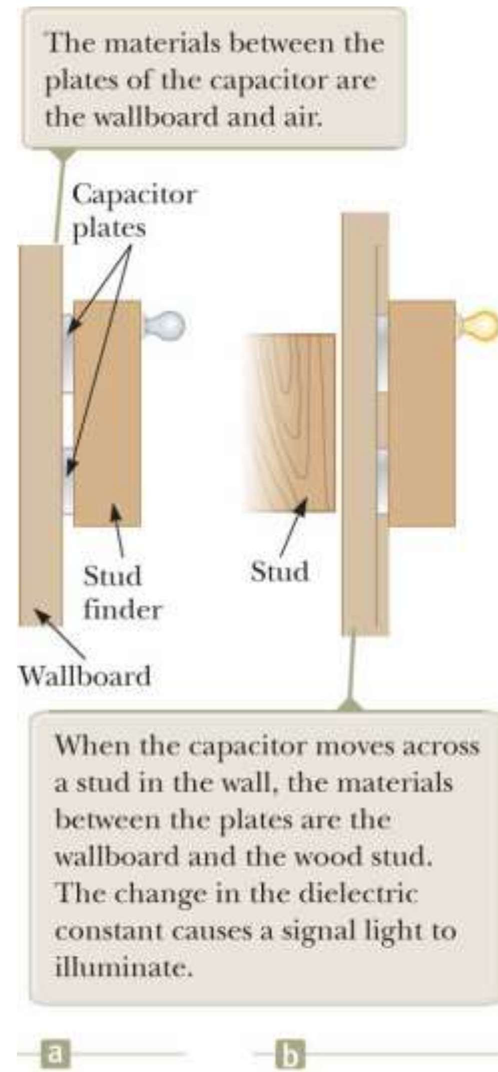


Example 25.5: Energy Stored Before and After

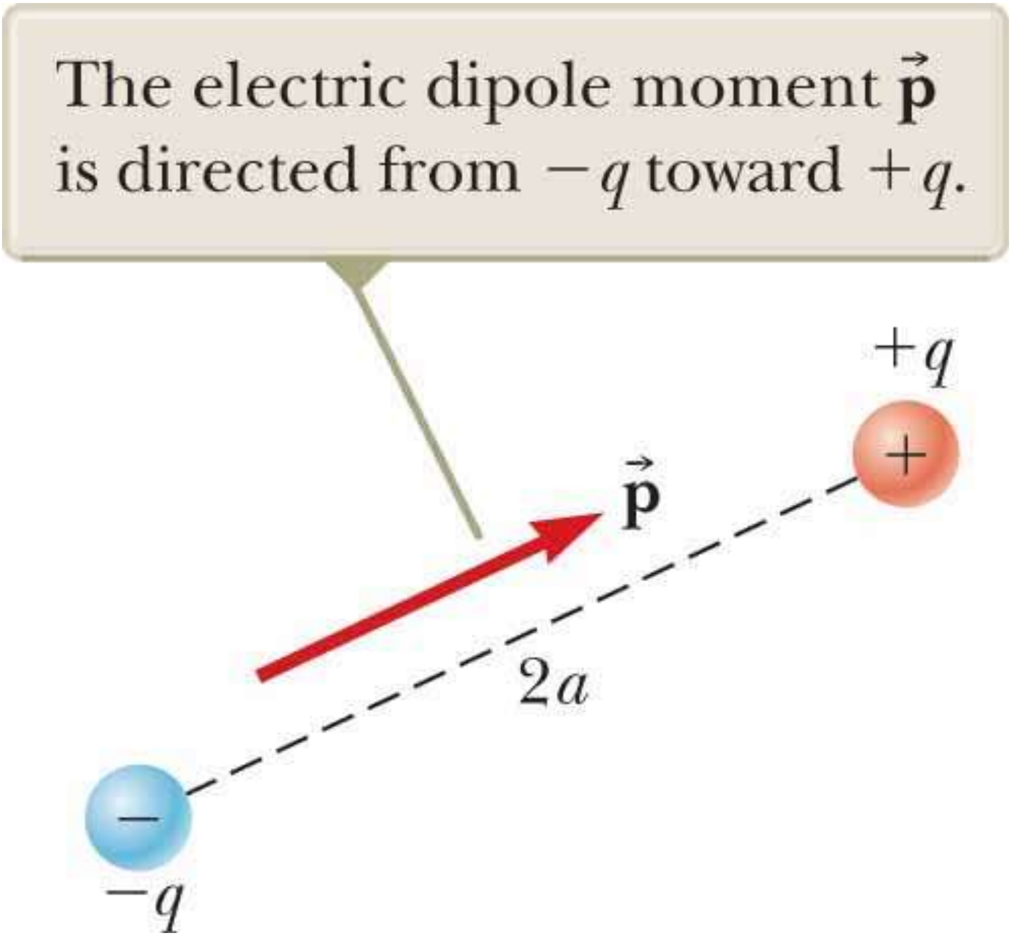
A parallel-plate capacitor is charged with a battery to a charge Q_0. The battery is then removed, and a slab of material that has a dielectric constant | is inserted between the plates. Identify the system as the capacitor and the dielectric. Find the energy stored in the system before and after the dielectric is inserted.

$$U_0 = \frac{Q_0^{\,2}}{2C_0} \qquad\qquad U_E = \frac{Q_0^{\,2}}{2C}$$

$$U_E = \frac{Q_0^{\,2}}{2\kappa C_0} = \frac{U_0}{\kappa}$$

Electric Dipole in an Electric Field

$$p \equiv 2aq$$



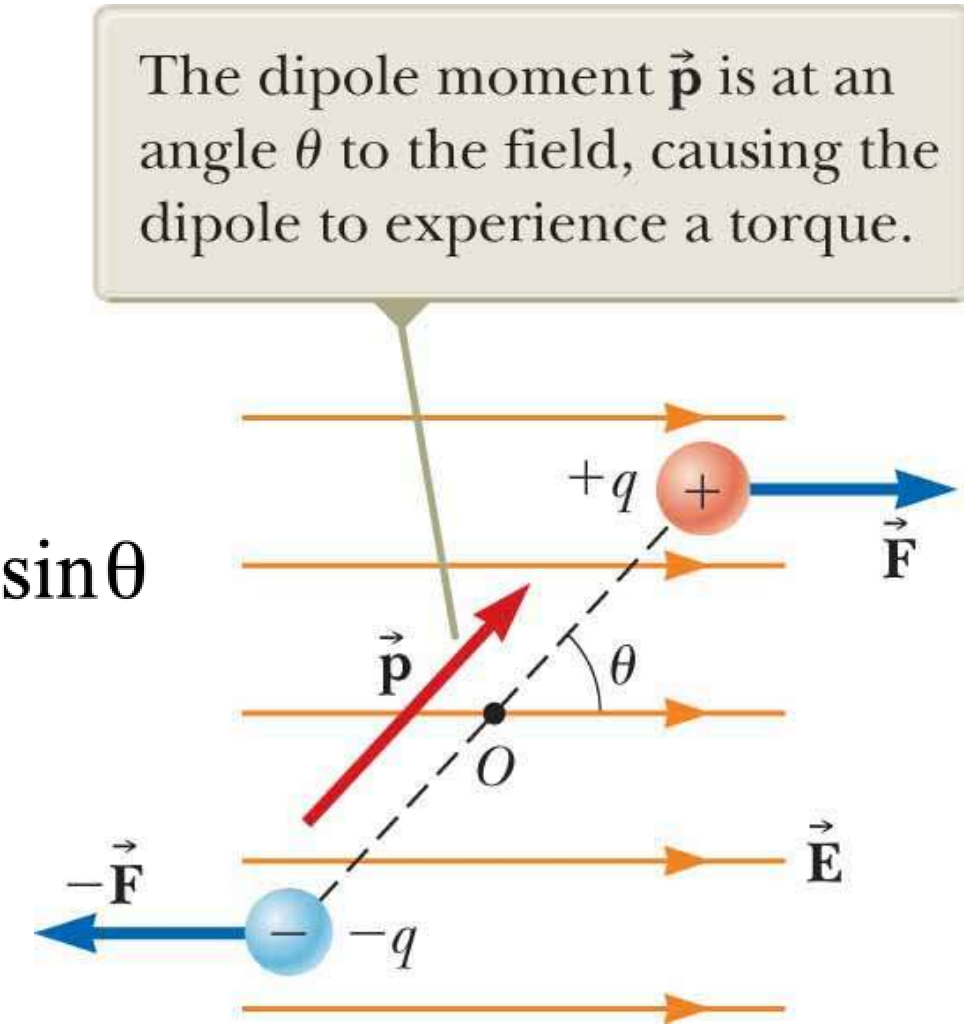
Electric Dipole in an Electric Field

$$Fa\sin\theta$$

$$\tau = 2Fa\sin\theta$$

$$\tau = 2aqE\sin\theta = pE\sin\theta$$

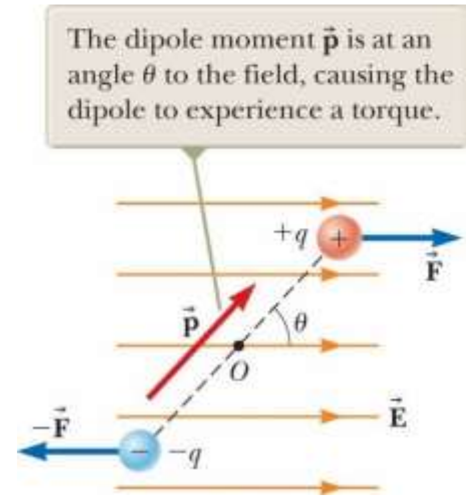
$$\vec{\tau} = \vec{p} \times \vec{E}$$



Energy of Dipole

$$dW = \tau\, d\theta$$

$$\tau = pE\sin\theta$$



$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau\, d\theta = \int_{\theta_i}^{\theta_f} pE\sin\theta\, d\theta = pE\int_{\theta_i}^{\theta_f} \sin\theta\, d\theta$$

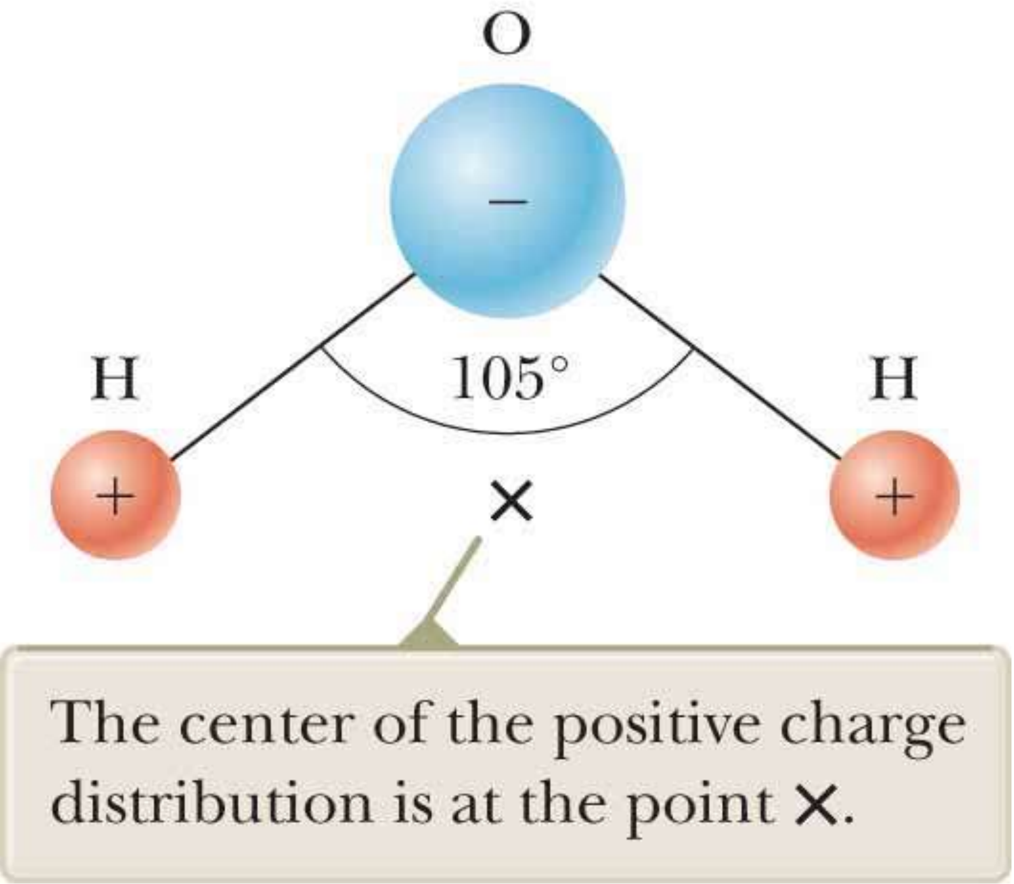
$$= pE\left[-\cos\theta\right]_{\theta_i}^{\theta_f} = pE\left(\cos\theta_i - \cos\theta_f\right)$$

$$U_E = -pE\cos\theta$$

$$\boxed{U_E = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}}$$

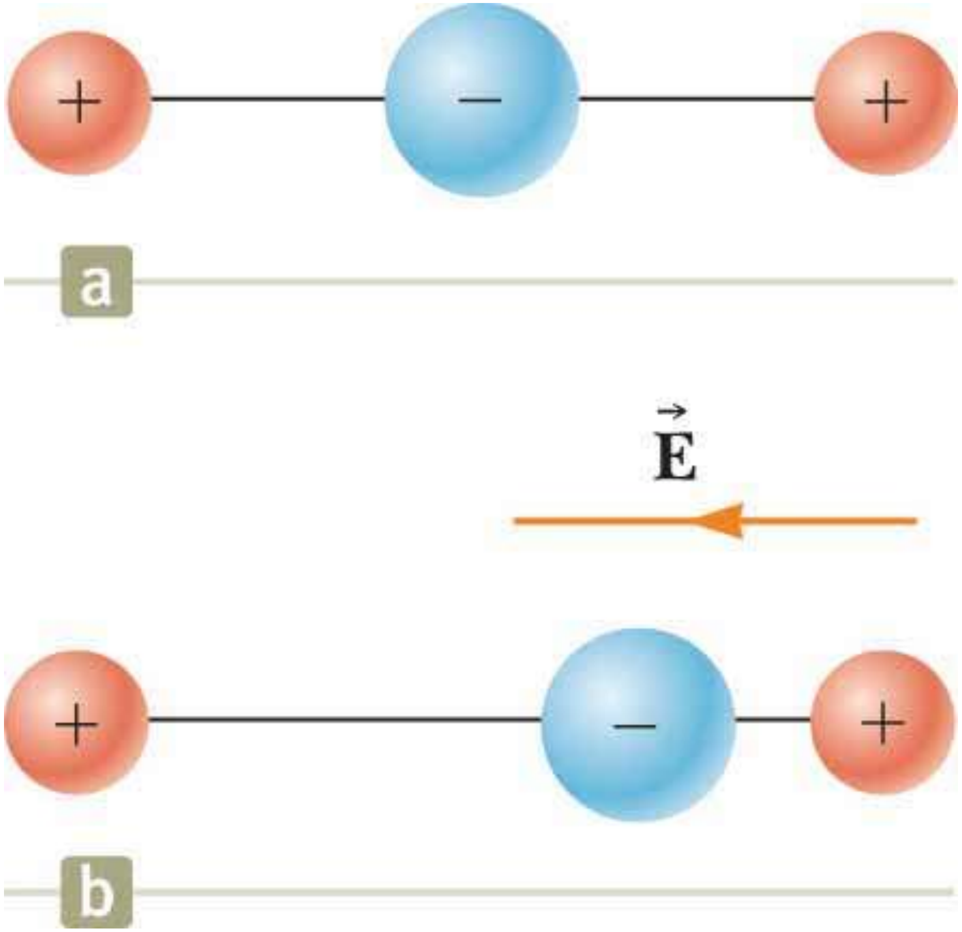
$$U_g = mgy$$

Polar Molecules



Soap and the Dipole Structure of Water

Induced Polarization



Example 25.6:
The H_2O Molecule

The water (H_2O) molecule has an electric dipole moment of $6.3 \cdot 10^{-30}$ C·m. A sample contains 10^{21} water molecules, with the dipole moments all oriented in the direction of an electric field of magnitude $2.5 \cdot 10^5$ N/C. How much work is required to rotate the dipoles from this orientation ($\theta = 0°$) to one in which all the moments are perpendicular to the field ($\theta = 90°$)?

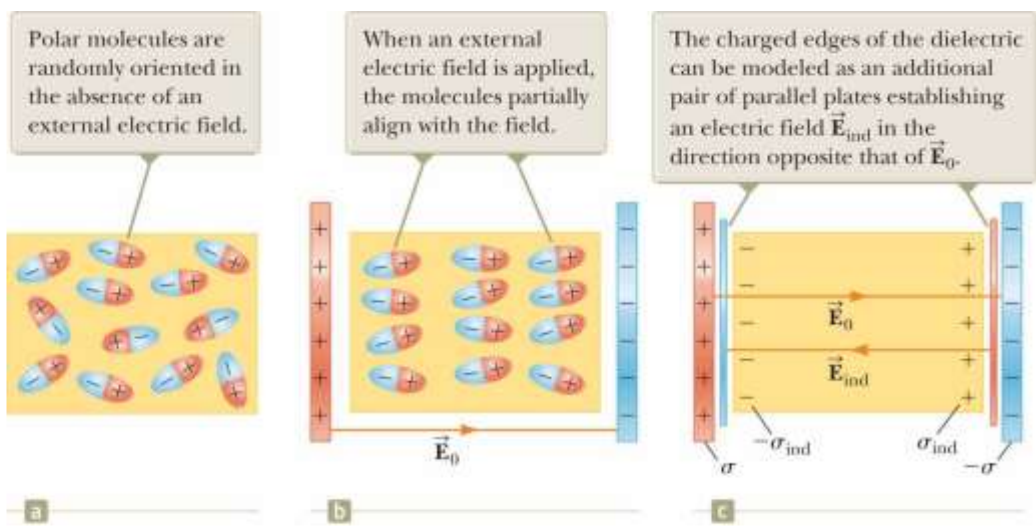
$$\Delta U_E = W$$

$$\begin{aligned} W &= U_{90°} - U_{0°} = \left(-NpE\cos 90°\right) - \left(-NpE\cos 0°\right) \\ &= NpE = \left(10^{21}\right)\left(6.3\times 10^{-30}\ \mathrm{C\cdot m}\right)\left(2.5\times 10^{5}\ \mathrm{N/C}\right) \\ &= 1.6\times 10^{-3}\ \mathrm{J} \end{aligned}$$

An Atomic Description of Dielectrics

$$\mathbf{E} = \frac{\mathbf{E}_0}{\kappa} \qquad\qquad E = E_0 - E_{\text{ind}}$$

$$\frac{\sigma}{\kappa\varepsilon_0} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_{\text{ind}}}{\varepsilon_0} \Rightarrow \sigma_{\text{ind}} = \left(\frac{\kappa - 1}{\kappa}\right)\sigma$$



Example 25.7:
Effect of a Metallic Slab

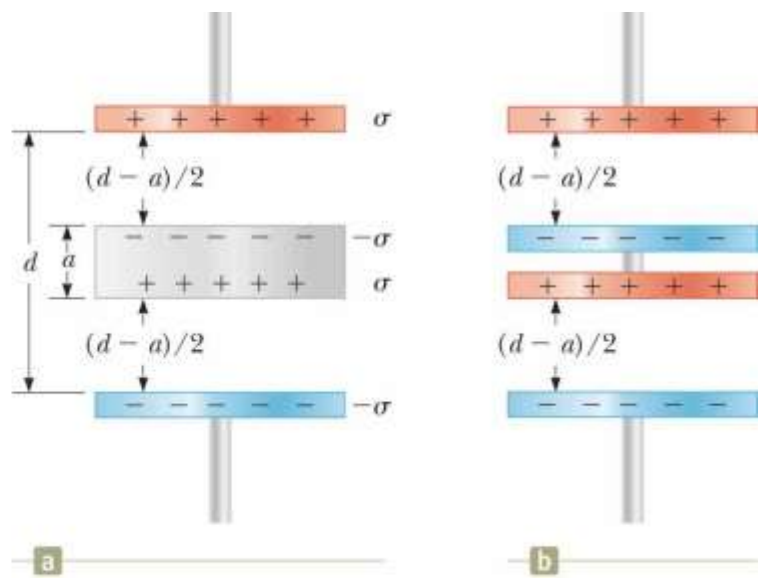
A parallel-plate capacitor has a plate separation d and plate area *A*. An uncharged metallic slab of thickness *a* is inserted midway between the plates.

(A) Find the capacitance of the device.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{1/\varepsilon_0 A}{(d-a)/2} + \frac{1/\varepsilon_0 A}{(d-a)/2}$$

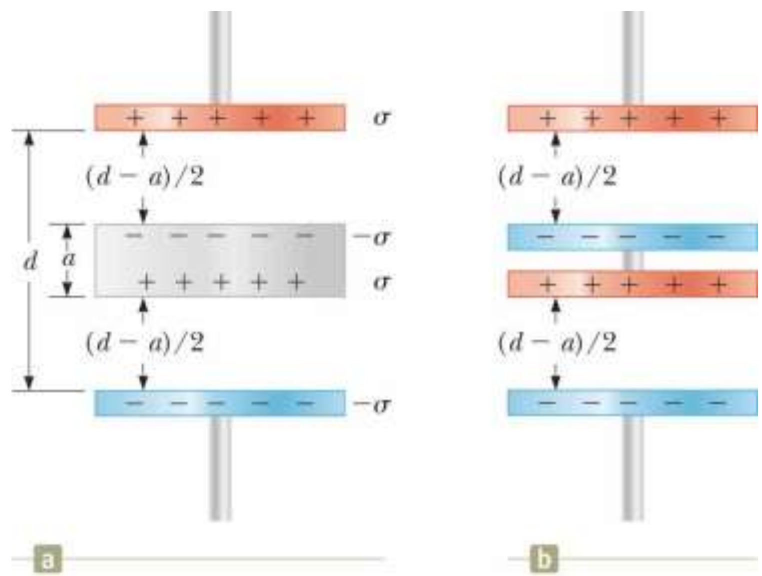
$$C = \frac{\varepsilon_0 / A}{d-a}$$



Example 25.7:
Effect of a Metallic Slab

(B) Show that the capacitance of the original capacitor is unaffected by the insertion of the metallic slab if the slab is infinitesimally thin.

$$C = \lim_{a \to \infty} \left(\frac{\varepsilon_0 / A}{d - a} \right) = \frac{\varepsilon_0 A}{d}$$



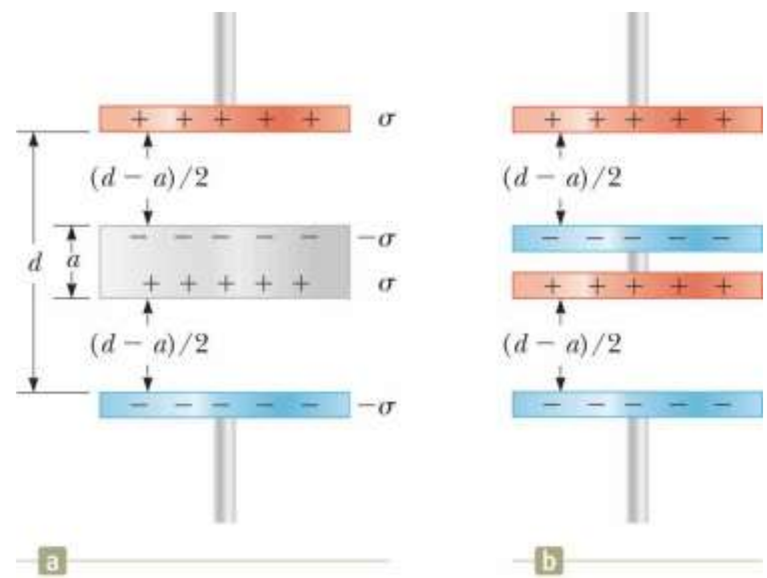
Example 25.7: Effect of a Metallic Slab

What if the metallic slab in part (A) is not midway between the plates? How would that affect the capacitance?

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{\varepsilon_0 A/b}+\frac{1}{\varepsilon_0 A/\left(d-b-a\right)}$$

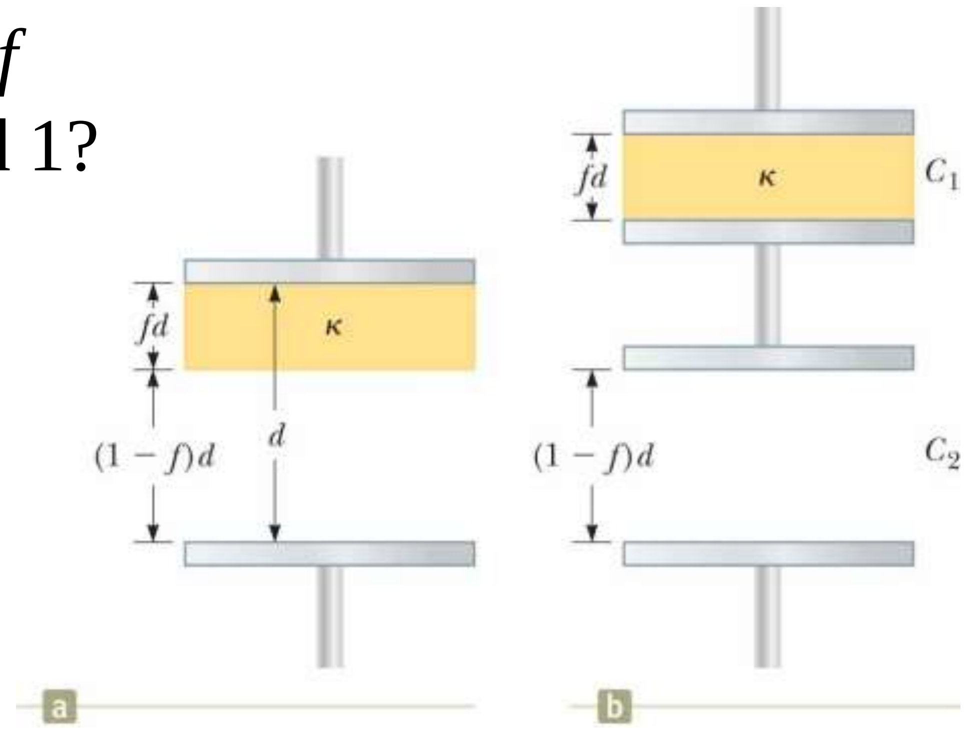
$$=\frac{b}{\varepsilon_0 A}+\frac{d-b-a}{\varepsilon_0 A}$$

$$\Rightarrow C=\frac{\varepsilon_0 A}{d-a}$$



Example 25.8:
A Partially Filled Capacitor

A parallel-plate capacitor with a plate separation *d* has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant | and thickness *fd* is inserted between the plates, where *f* is a fraction between 0 and 1?



Example 25.8: A Partially Filled Capacitor

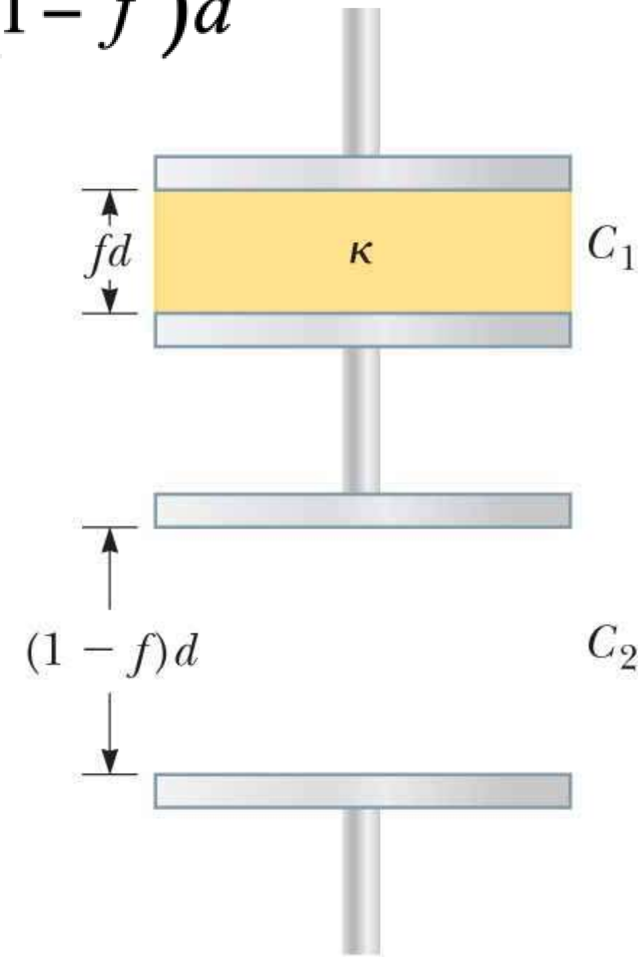
$$C_1 = \frac{\kappa\varepsilon_0 A}{fd} \quad \text{and} \quad C_2 = \frac{\varepsilon_0 A}{(1-f)d}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{fd}{\kappa\varepsilon_0 A} + \frac{(1-f)d}{\varepsilon_0 A}$$

$$\frac{1}{C} = \frac{fd}{\kappa\varepsilon_0 A} + \frac{\kappa(1-f)d}{\kappa\varepsilon_0 A}$$

$$= \frac{f + \kappa(1-f)}{\kappa}\frac{d}{\varepsilon_0 A}$$

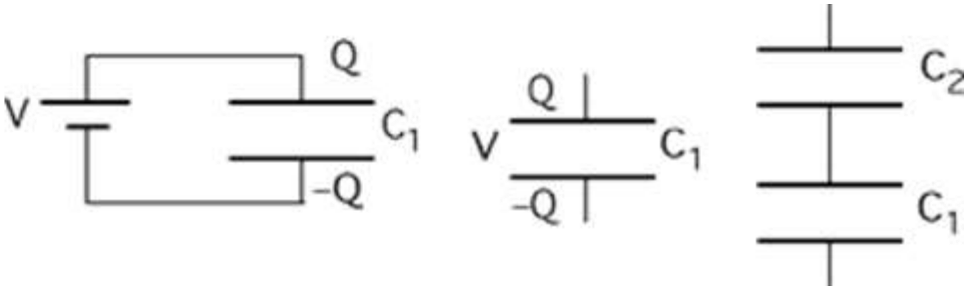
$$C = \frac{\kappa}{f + \kappa(1-f)} C_0$$



Assessing to Learn

A capacitor, C_1, is connected to a battery until charged, and then disconnected from the battery. A second capacitor, C_2, is connected in series to the first capacitor. What changes occur in capacitor C_1 after C_2 is connected as shown?

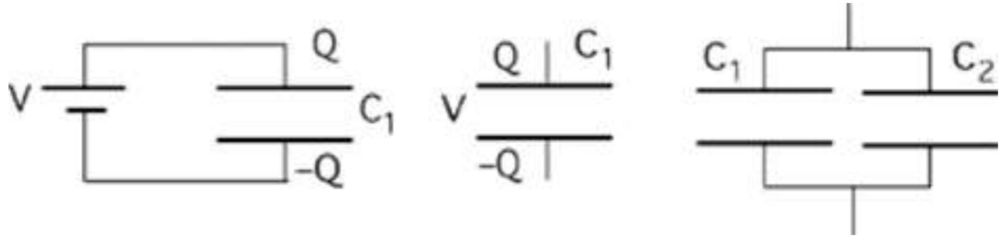
1. ΔV same, Q increases, U increases
2. ΔV same, Q decreases, U same
3. ΔV increases, Q decreases, U increases
4. ΔV decreases, Q same, U decreases
5. ΔV decreases, Q decreases, U decreases
6. None of the above
7. Cannot be determined



Assessing to Learn

A capacitor having, C_1, is connected to a battery until charged, then disconnected from the battery. A second capacitor, C_2, is connected in parallel to the first capacitor. Which statements below are true?

1. Charge on C_1 decreases.
2. Total charge on C_1 and C_2 is the same as the original Q.
3. The total energy stored in both capacitors is the same as the original U stored in C_1.
4. The potential difference (Voltage) across C_1 decreases.
5. All of the above.
6. Only 1, 2, and 3 are true.
7. Only 1, 2, and 4 are true.
8. None of the above.

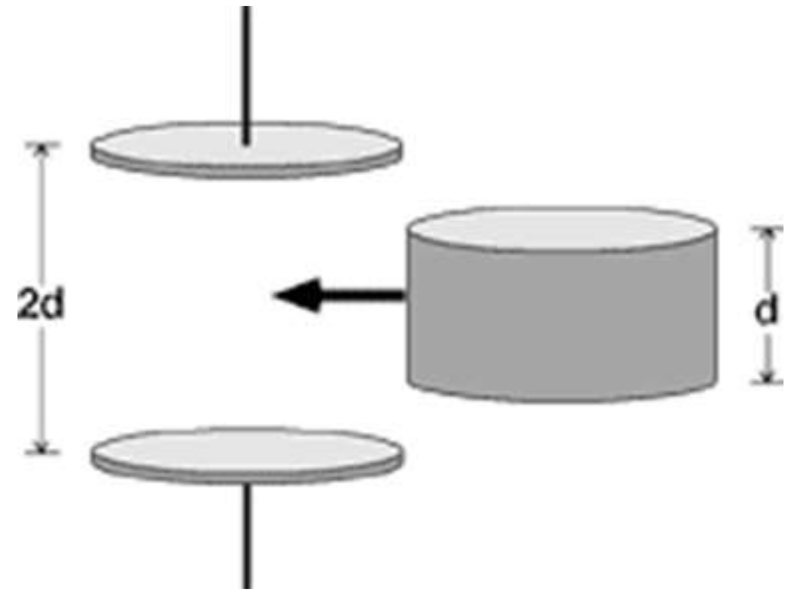


Assessing to Learn

Two parallel conducting plates form a capacitor. It is isolated and a charge Q is placed on it. A metal cylinder of length half the plate separation is then inserted between the plates.

HOW MANY of the quantities C, ΔV, Q, E, and U change?

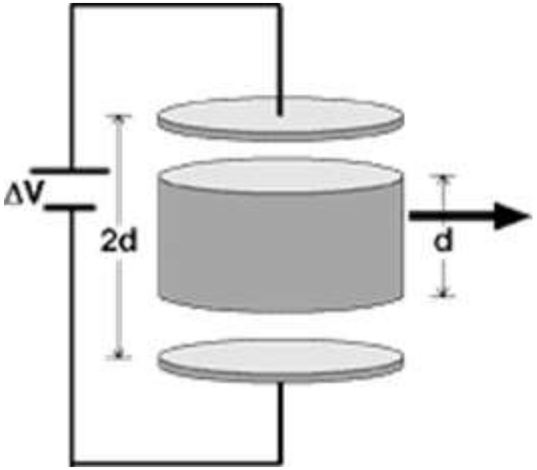
1. 0 2. 1 3. 2
4. 3 5. 4 6. 5
7. Impossible to determine



Assessing to Learn

Two parallel conducting plates form a capacitor. With a metal cylinder of length half the plate separation inserted between the plates, it is connected to a battery with potential ΔV. The cylinder is now removed. HOW MANY of the quantities C, ΔV, Q, E, and U change?

1. 0   2. 1   3. 2
4. 3   5. 4   6. 5
7. Impossible to determine



Assessing to Learn

A capacitor with capacitance C is connected to a battery until charged, then disconnected from the battery. A dielectric having constant κ is inserted in the capacitor. What changes occur in the charge, potential and stored energy of the capacitor after the dielectric is inserted?

1. ΔV stays same, Q increases, U increases
2. ΔV stays same, Q decreases, U stays same
3. ΔV increases, Q decreases, U increases
4. ΔV decreases, Q stays same, U decreases
5. None of the above
6. Cannot be determined