ECE 3030 HW10 Solutions

1. *A Si p^+-n-p BJT has $N_d = 10^{16} cm^{-3}$ in the base and $N_a = 10^{15} cm^{-3}$ in the collector. Find χ_{no} at the collector junction for V_{CB} = -2 and -10 V. Is the Early effect significant if W_b = 1 µm.*

For the collector junction

$V_0 = 0.0259 \ln [10^{31}/(2.25 \times 10^{20})] = 0.635V$

The width of the depletion region on the n-side is

$\chi_{no} = [(2\varepsilon (Vo + V_r)/q)(N_a/N_d(N_a + N_d))]^{1/2}$

$= [(2 \times 11.8 \times 8.85 \times 10^{-14} (0.635 + V_r)/1.6 \times 10^{-19}) (10^{15}/10^{16}(1.1 \times 10^{16}))]^{1/2}$

For V_r = - 2V, this is 0.18 μm

V_r = - 10V, this is 0.35 μm

The intrusion of the depletion region into the base is a significant fraction of W_b. Therefore, we expect a large Early effect.

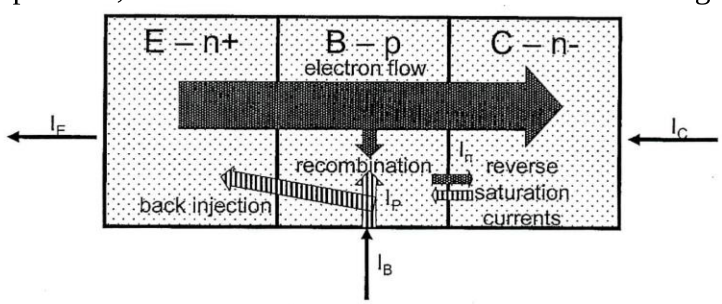
2. From S&B Eq. 7-14 and Fig. 7-7(a), plot δp across the base of a p-n-p BJT with W_b/L_p = 1 and 0.1.

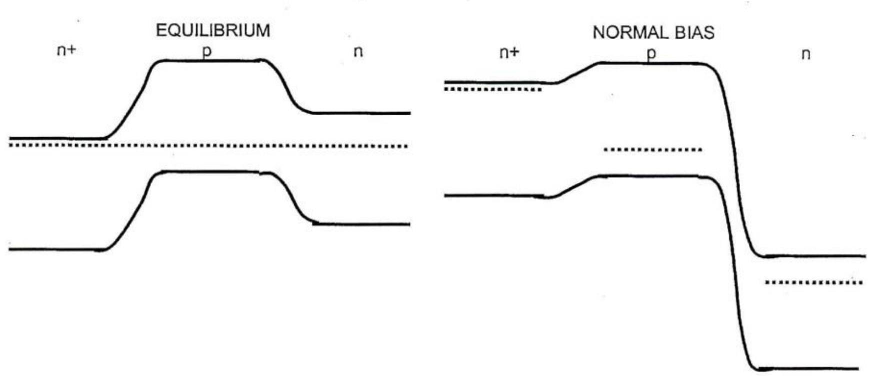
$\Delta p/\Delta p_E = M_1 e^{-xn/Lp} - M_2 e^{xn/Lp}$

For W_b/L_p = 1 : M_1 = 1.157 , M_2 = 0.157

W_b/L_p = 0.1 : M_1 = 5.52 , M_2 = 4.52

3. For a n+-p-n BJT, show the current contributions and band diagram.





4. *A Si* p^+*-n-p BJT with area* 2 x 10^{-4} cm^{-2} *and base width of one micron has an emitter with* $N_a = 10^{18}$. *In the base* $N_d = 10^{16}$, *and* $\tau_p = 1\ \mu s$.

(a) *Find* I_E, *and* I_C *with* $V_{EB} = 0.6\ V$ *and large reverse bias on the collector junction.*

In the base:

$\mu_p = 400\ cm^2/V\ s$

$p_n = n_i^2/n_n = (1.5 \times 10^{10})^2/10^{16} = 2.25 \times 10^4$

$D_p = 400(0.0259) = 10.36$ so that $L_p = (10.36 \times 10^{-6})^{1/2} = 3.2 \times 10^{-3}$

$W_b/L_p = 10^{-4}/(3.2 \times 10^{-3}) = 3.11 \times 10^{-2}$

We will assume $\gamma \sim 1$
From S&B Eq.7-8 and 7-9, $\Delta p_E = p_n e^{qV_{EB}/kT}$, $\Delta p_C \sim 0$

$\Delta p_E = 2.25 \times 10^4 \times e^{(0.6/0.0259)} = 2.59 \times 10^{14}$

From slide 16, Lecture 36 or S&B Eq. 7—18(a), 18(b), and 19,

$I_E = qA(D_p/L_p)\Delta p_E\, ctnh(W_b/L_p)$

$= (1.6 \times 10^{-19})(2 \times 10^{-4})(10.36/3.2 \times 10^{-3})(2.59 \times 10^{14})\ \mathrm{ctnh} 3.11 \times 10^{-2}$

$= 0.86\mathrm{mA}$

$I_C = I_E \mathrm{sech}(W_b/L_p) = 0.86 \times 0.9995 = 0.86\mathrm{mA}$

(b) *Find* I_B *by two methods*.

We can't easily subtract $I_E - I_C$ directly, but we can **use the expression for $I_E - I_C$ on slide 18, Lecture 36:**

$I_B = qA(D_p/L_p)\Delta p_E \tanh(W_b/2L_p) = 2.68 \times 10^{-5} \tanh(1.56 \times 10^{-2})$

$= 0.417\mu\mathrm{A}$

Comparing this with the Charge Control Approximation, **slide 10, Lecture 36:**

$I_B = Q_b/\tau_p = qAW_b\Delta p_E/2\,\tau_p = 0.414\ \mu\mathrm{A}$

5. For the given BJT, calculate β in terms of B and γ and using the charge control model.

In emitter, $L_n^E = \sqrt{\mu_n \cdot \frac{kT}{q} \cdot \tau_n} = \sqrt{150 \frac{cm^2}{V \cdot s} \cdot 0.0259V \cdot 10^{-10}s} = 1.97 \cdot 10^{-5}cm = 0.197\mu m$

In base,

$$L_p^B = \sqrt{D_p \tau_p} = \sqrt{\mu_p \cdot \frac{kT}{q} \cdot \tau_p} = \sqrt{400 \frac{cm^2}{V \cdot s} \cdot 0.0259V \cdot 25 \cdot 10^{-10}s} = 1.61 \cdot 10^{-4}cm = 1.61\mu m$$

$$\gamma = \left[1 + \frac{\mu_n^E \cdot N_D^B \cdot W_b}{\mu_p^B \cdot N_A^E \cdot L_n^E}\right]^{-1} = \left[1 + \frac{150 \frac{cm^2}{V \cdot s} \cdot 10^{16} \frac{1}{cm^3} \cdot 0.2\mu m}{400 \frac{cm^2}{V \cdot s} \cdot 5 \cdot 10^{18} \frac{1}{cm^3} \cdot 0.197\mu m}\right]^{-1} = 0.9992$$

$$B = 1 - \frac{W_b^2}{2 \cdot L_p^2} = 1 - \frac{(0.2\mu m)^2}{2 \cdot (1.61\mu m)^2} = .9961$$

$$\beta = \frac{B \cdot \gamma}{1 - B \cdot \gamma} = 213$$

Charge control approach

$$\tau_t = \frac{W_b^2}{2 \cdot D_p} = 0.514 \cdot 10^{-11}s$$

$$\beta = \frac{\tau_p}{\tau_t} = 486$$

These differ because the charge control approach assumes γ = 1.

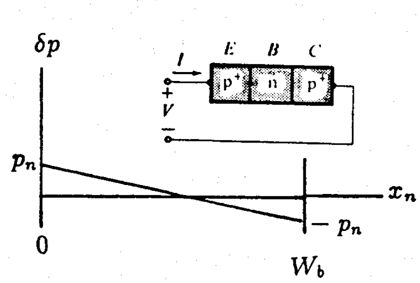
6. For the BJT in Problem 5, calculate the charge stored in the base when V_{CB} = 0 V and V_{EB} = 0.7 V. Find f_T if the base transit time is the dominant delay component.

$$Q_b \approx \frac{1}{2} \cdot q \cdot A \cdot W_b \cdot p_n \cdot e^{\frac{qV_{EB}}{kT}} = \frac{1}{2} \cdot 1.6 \cdot 10^{-19}C \cdot 10^{-4}cm^2 \cdot 0.2 \cdot 10^{-4}cm \cdot \frac{\left(1.5 \cdot 10^{10} \frac{1}{cm^3}\right)^2}{10^{16} \frac{1}{cm^3}} \cdot e^{\frac{0.7}{.026}} = 1.968 \cdot 10^{-12}C$$

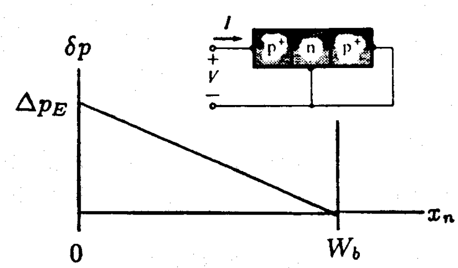
$$\tau_t = \frac{W_b^2}{2 \cdot D_p} = \frac{(0.2 \cdot 10^{-4}cm)^2}{2 \cdot 10.36 \frac{cm^2}{s}} = 1.93 \cdot 10^{-11}s$$

$$f_T = \frac{1}{2\pi \cdot \tau_t} = 8.24 \cdot 10^9 Hz$$

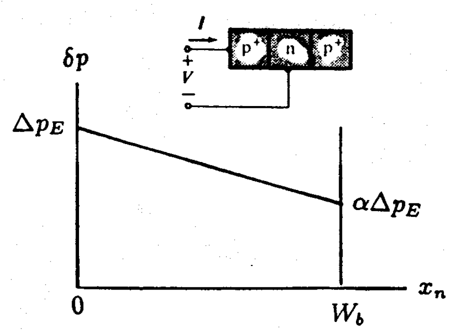
7. *For the diode connections shown, sketch δp in the base. Which is the best diode?*



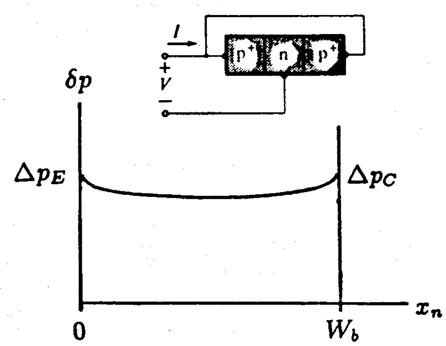
(a) $I_E = I_C$, $I_B = 0$. Since V is large, the collector is strongly reverse biased, $\Delta p_C = -p_n$. Since $I_E = I_C$, $\Delta p_E = -\Delta p_C$ $= p_n$ The area under $\delta p(x_n)$ is zero.



(b) $V_{CB} = 0$, thus $\Delta p_C = 0$. Notice that this is the narrow-base diode distribution.



(c) *Since* $I_C = 0$, $\Delta p_C = \alpha \Delta p_E$ *from Eq.(7-34b).*



(d) $V_{EB} = V_{CB} = V$. *Thus* $\Delta p_C = \Delta p_E$

Connection (b) gives the best diode since the stored charge is least; (a) is not a good diode since the current is small and symmetrical about $V = 0$.