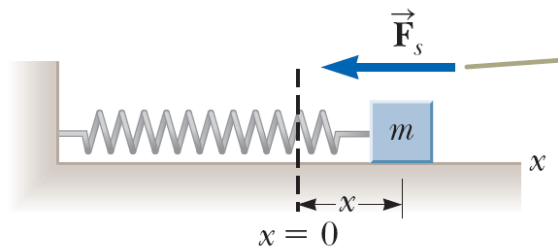


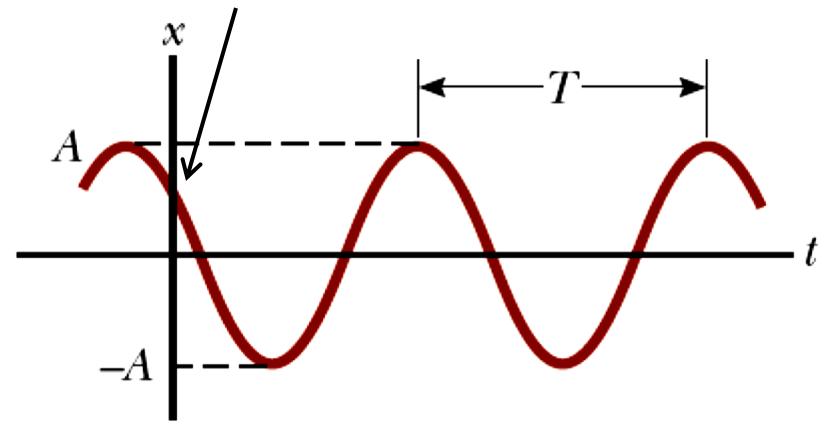
Lecture 29



$$F_{net} = -kx$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$t = 0$
 $x(0) = A \cos(0 + \phi) = A \cos(\phi)$



Important Parameters of the Motion

$$x(t) = A \cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi)$$

$t = 0$
 $x(0) = A \cos(0 + \phi) = A \cos(\phi)$
 $\phi \Rightarrow$ initial phase
 $\omega t + \phi \Rightarrow$ phase

$\omega = \sqrt{\frac{k}{m}}$
Spring

$\omega T = 2\pi \quad T = \frac{2\pi}{\omega} \quad f = \frac{1}{T} \quad A \Rightarrow$ Amplitude

At $t=0$, an oscillator is started with zero velocity at its maximum positive amplitude X_m . The equation for simple harmonic motion is:

$$x(t) = x_m \cos(\omega t + \phi)$$

What is the value of ϕ ?

(1) 0

(2) $+\pi/4$

(3) $-\pi/4$

(4) $+\pi/2$

(5) $-\pi/2$

(6) None of the above

At $t=0$, an oscillator is started with maximum negative velocity at $x(0)=0$. The equation for simple harmonic motion is:

$$x(t) = x_m \cos(\omega t + \phi) \quad V = x' = -x_m \sin(\phi)$$

What is the value of ϕ ?

(1) 0

(2) $+\pi/4$

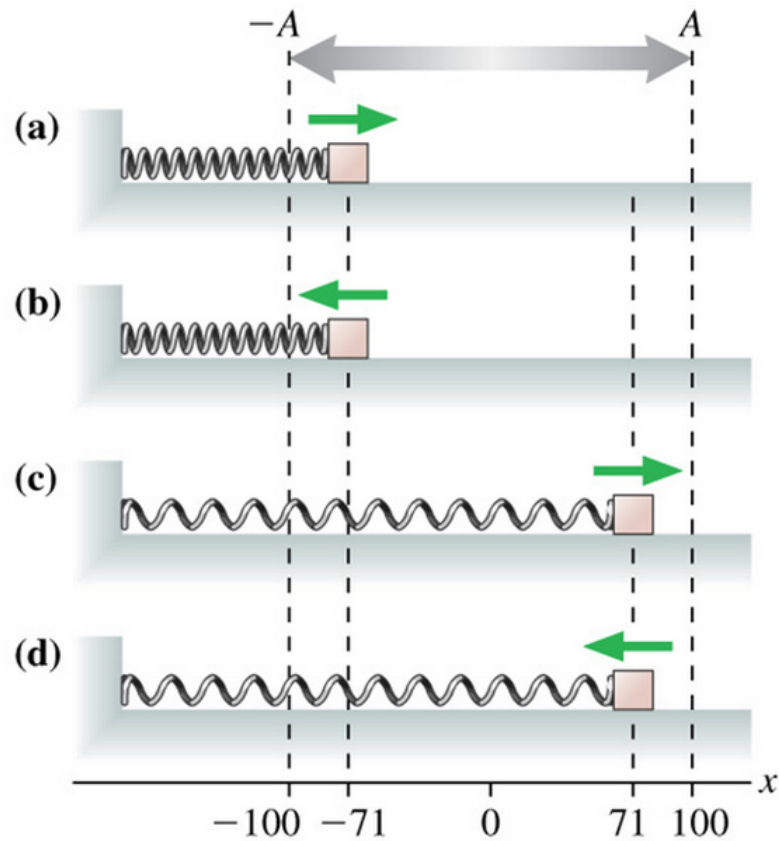
(3) $-\pi/4$

(4) $+\pi/2$

(5) $-\pi/2$

(5) None of the above

The figure shows four oscillators at $t = 0$. Which one has the phase constant $\phi_0 = +\left(\frac{\pi}{4}\right)$?



(1) (a)

(2) (b)

(3) (c)

(4) (d)

Energy of the Simple Harmonic Oscillator

$$x(t) = A \cos(\omega t + \phi)$$

$$v(t) = -\omega A \sin(\omega t + \phi)$$

Kinetic Energy $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$

Potential Energy $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$

Total Mechanics Energy $E = \frac{1}{2}kA^2$

$$E = K + U = \frac{1}{2}kA^2[\underbrace{\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)}_{=1}]$$

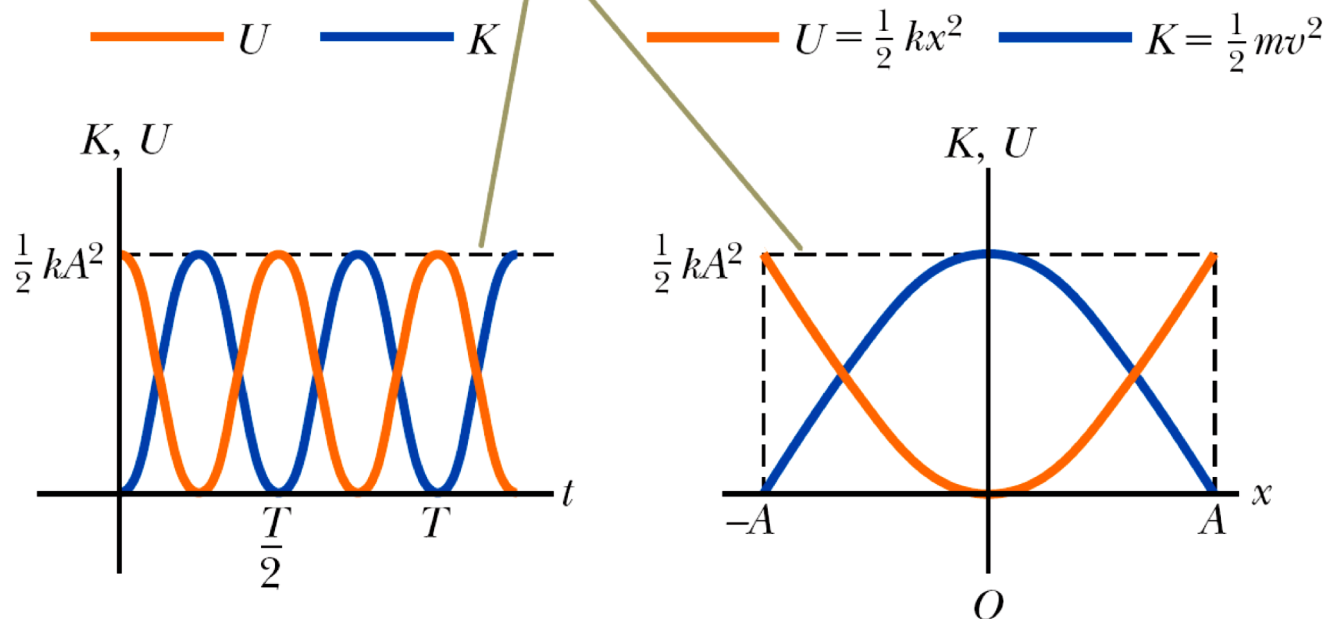
Total Mechanics Energy $E = \frac{1}{2} kA^2$

Knowing x and E to
find v

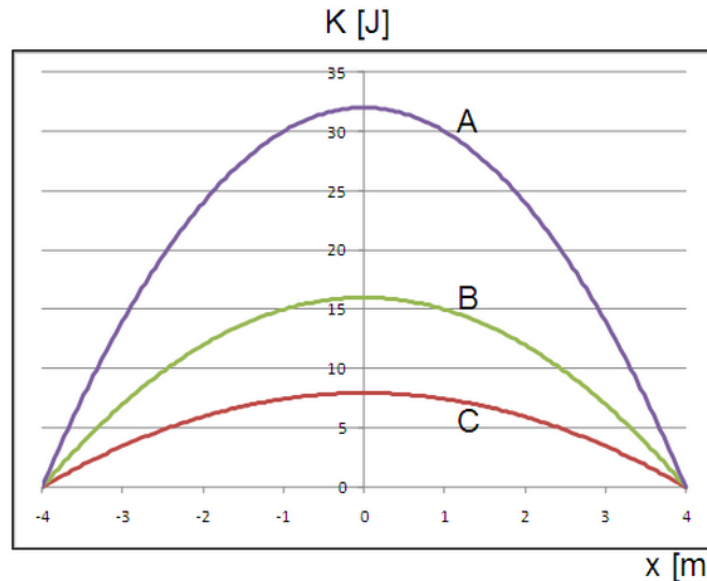
$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \pm \sqrt{\frac{k}{m}(A^2 - x^2)} = \pm \omega \sqrt{A^2 - x^2}$$

In either plot, notice that
 $K + U = \text{constant}$.



The graph below shows plots of the kinetic energy K versus position x for three harmonic oscillators with the same mass. Rank the plots according to the corresponding frequency of the oscillator.



$$K_{\max} = \frac{1}{2} m \omega^2 A^2$$

(1) $A > B > C$

(2) $A < B < C$

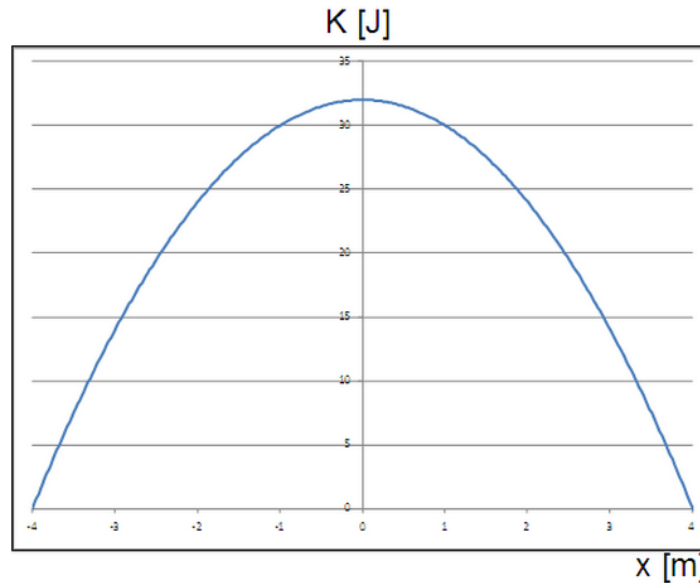
(3) $A = B = C$

(4) $B > A = C$

(5) $B < A = C$

(6) Can't tell from the info. given

The graph below shows a plot of the kinetic energy K versus position x for a spring harmonic oscillator with a 1kg mass. The maximum kinetic energy is 32 J. What is the angular frequency of this harmonic oscillator?



$$K_{\max} = \frac{1}{2} m \omega^2 A^2$$

$$32 = \frac{1}{2} (1) \omega^2 (2)^2$$

$$\omega = \sqrt{\frac{32}{16}} = 2$$

(1) 32 rad/s

(2) 8 rad/s

(3) 4 rad/s

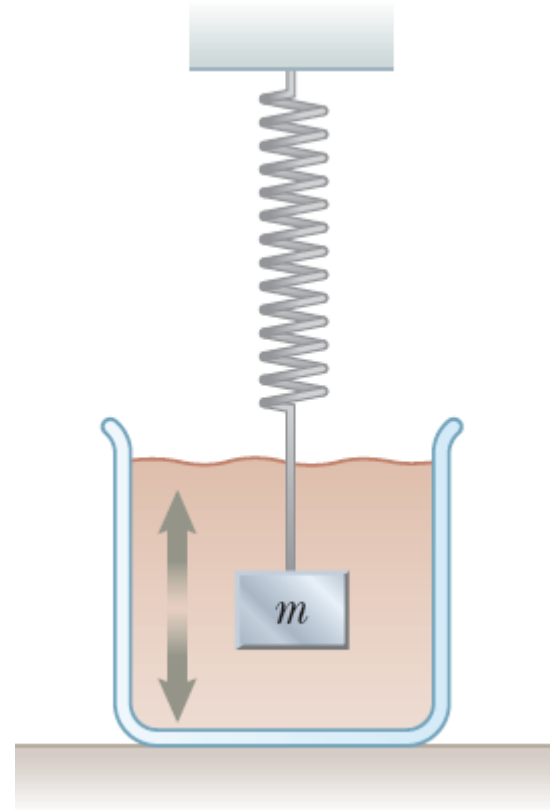
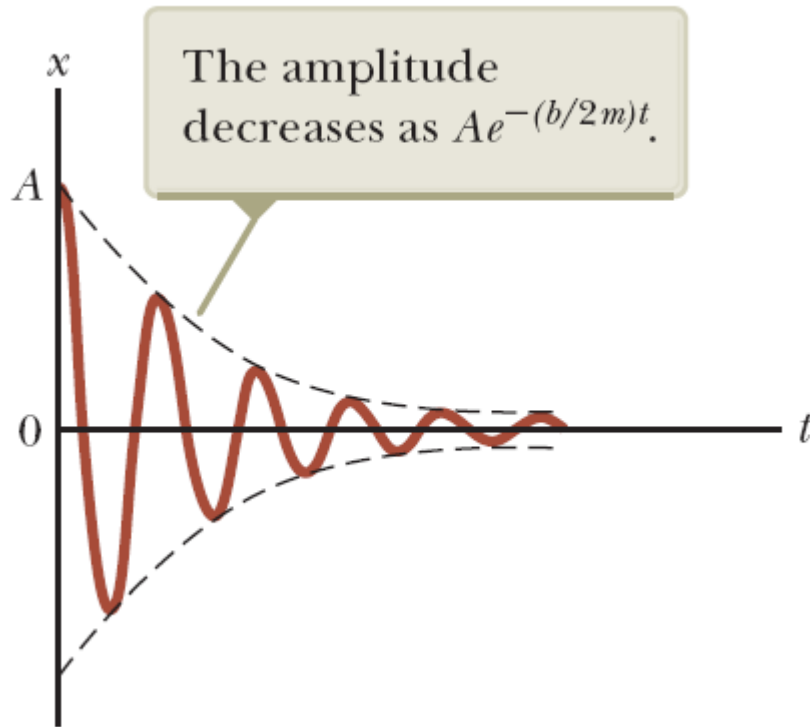
(4) 2 rad/s

(5) 1 rad/s

(6) Can't tell from the info. given

Damped Oscillations

$$E_{\text{mech}} = K + U = \frac{1}{2}kA(t)^2 \quad \text{decreases in time}$$



Damped Oscillations

$$\vec{f} = -b\vec{v}$$

↑ or ↓

$$F_b = \rho V g$$

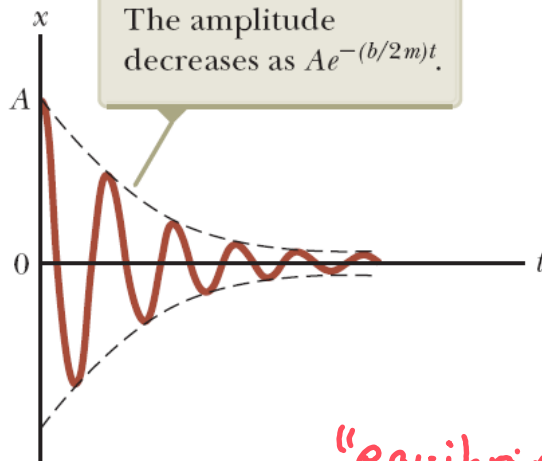
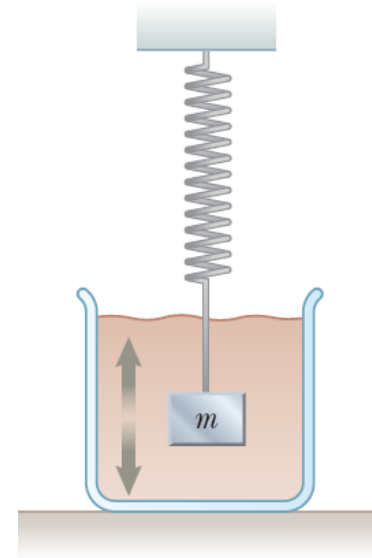
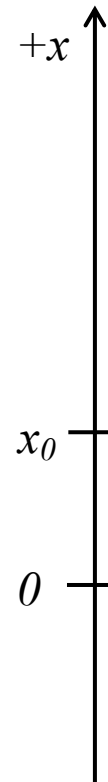


$$F_s = -k(x - x_0)$$

$$mg$$

$$F_{net} = -kx - bv = ma$$

$$m \frac{d^2 x}{dt^2} = -b \frac{dx}{dt} - kx$$



"equilibrium"

$$m \frac{d^2 x}{dt^2} = -b \frac{dx}{dt} - kx \quad \frac{d^2 x}{dt^2} = -\frac{b}{m} \frac{dx}{dt} - \frac{k}{m} x$$

$$x = Ae^{-(b/2m)t} \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

Forced Oscillations

$$E_{mech} = K + U = \frac{1}{2} k A(t)^2 \quad \text{depends on the driving action}$$

$$F_{hand}(t) = F_0 \sin \omega t$$

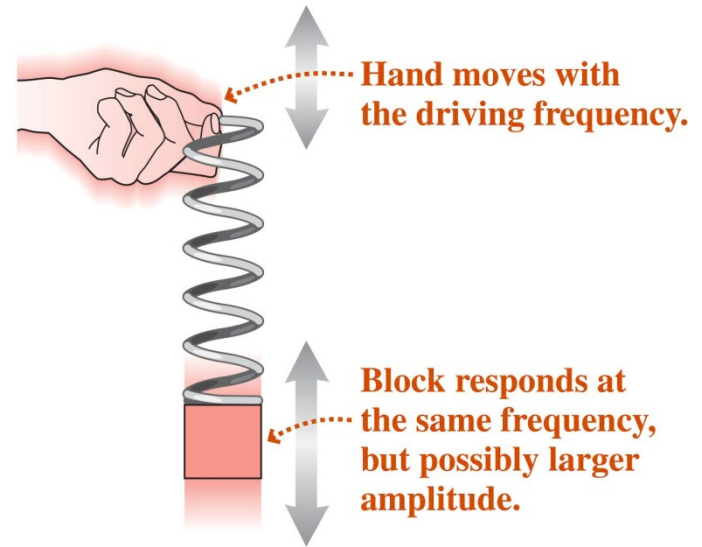
$$F_{net} = F_0 \sin \omega t - kx - bv = ma$$

$$m \frac{d^2 x}{dt^2} = F_0 \sin \omega t - b \frac{dx}{dt} - kx$$

$$x = A \cos(\omega t + \phi)$$

$$A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}} \quad \omega_0 = \sqrt{k/m}$$

$$\omega = \omega_0 \quad \text{resonance}$$

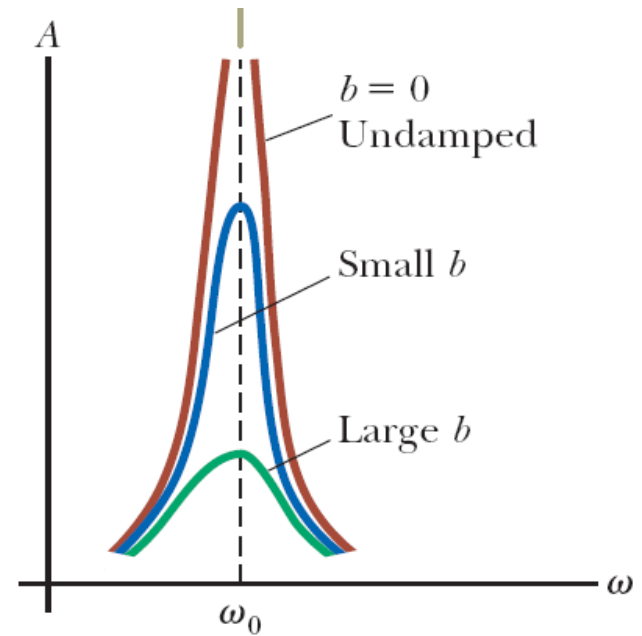


Forced Oscillations

$$x = A \cos(\omega t + \phi)$$

$$A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

$$\omega_0 = \sqrt{k / m}$$



$\omega = \omega_0$ resonance

<http://www.youtube.com/watch?v=j-zczJXSxnw>

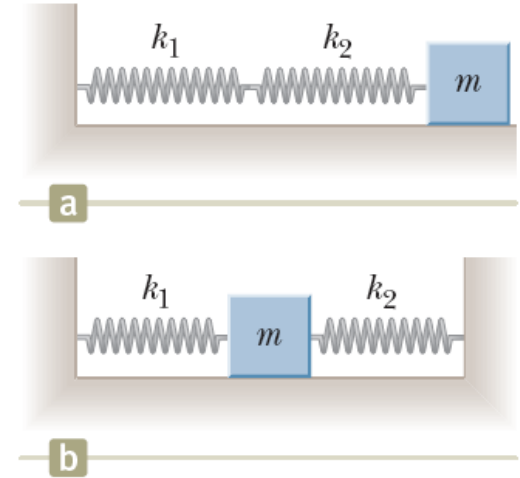


AP Images



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A block of mass m is connected to two (massless) springs of force constants k_1 and k_2 in two ways as shown in Figure. In both cases, the block moves on a frictionless table after it is displaced from equilibrium and released. Show that in the two cases the block exhibits simple harmonic motion with periods



$$(a) \quad T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}} \quad \text{and} \quad (b) \quad T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

Spring in series:

$$F_s = -k_s(\Delta x_1 + \Delta x_2) = -k_1\Delta x_1 = -k_2\Delta x_2$$

$$\frac{\Delta x_2}{\Delta x_1} = \frac{k_1}{k_2} \quad k_1 = k_s \left(1 + \frac{\Delta x_2}{\Delta x_1}\right) = k_s \left(1 + \frac{k_1}{k_2}\right)$$

$$k_s = \frac{k_1 k_2}{k_1 + k_2} \quad \omega = \sqrt{\frac{k_s}{m}} \quad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

Spring in parallel:

$$F_p = -k_p \Delta x = -k_1 \Delta x - k_2 \Delta x = -(k_1 + k_2) \Delta x$$

$$k_s = k_1 + k_2 \quad \omega = \sqrt{\frac{k_s}{m}} \quad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$$

