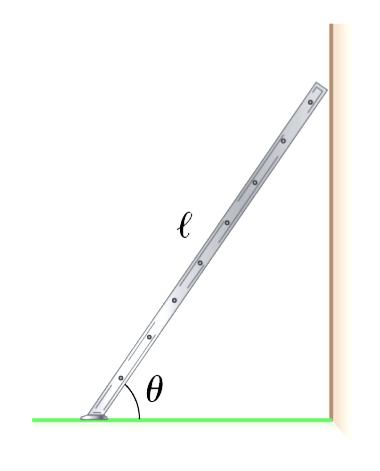
Static Equilibrium (Chapter 12)

$$\sum F = 0$$
$$\sum \tau = 0$$

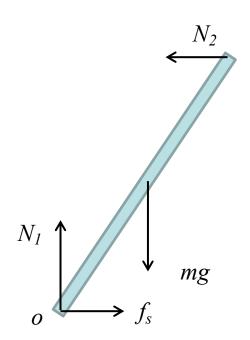
$$\sum \tau = 0$$

Draw a Free Body Diagram



A uniform ladder of length L, rests against a smooth, vertical wall The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ_{min} at which the ladder does not slip.

$$\sum F = 0$$
$$\sum \tau = 0$$





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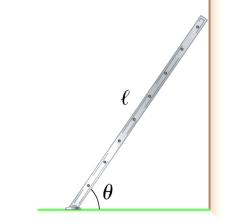
$$\sum F_{y} = 0 N_{1} - mg = 0 N_{1} = mg$$

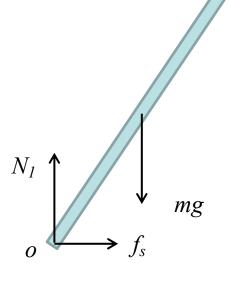
$$\sum F_{x} = 0 f_{s} - N_{2} = 0 N_{2} = f_{s}$$

$$f_{s_MAX} = \mu_{s} N_{1} = \mu_{s} mg N_{2_MAX} = \mu_{s} mg$$

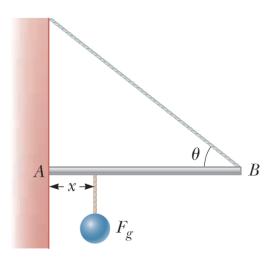
$$\sum \tau = 0 \quad mg \frac{L}{2} \cos \theta - N_2 L \sin \theta = 0$$

$$\tan \theta = \frac{mg}{2N_2} \qquad \tan \theta_{Min} = \frac{1}{2\mu_s}$$



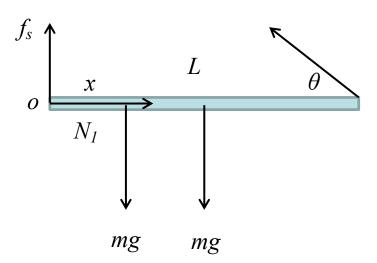


One end of a uniform 4.00-m-long rod of weight $F_{\rm g}$ is supported by a cable at an angle of $\theta=37^{\circ}$ with the rod. The other end rests against the wall, where it is held by friction as shown in the figure below. The coefficient of static friction between the wall and the rod is $\mu_{\rm s}=0.500$. Determine the minimum distance x from point A at which an additional object, also with the same weight $F_{\rm g}$, can be hung without causing the rod to slip at point A.

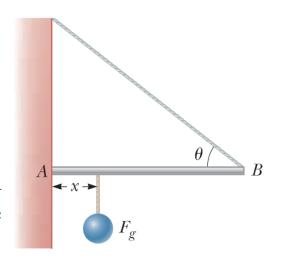


$$\sum F = 0$$

$$\sum \tau = 0$$



One end of a uniform 4.00-m-long rod of weight $F_{\rm g}$ is supported by a cable at an angle of $\theta=37^{\circ}$ with the rod. The other end rests against the wall, where it is held by friction as shown in the figure below. The coefficient of static friction between the wall and the rod is $\mu_{\rm s}=0.500$. Determine the minimum distance x from point A at which an additional object, also with the same weight $F_{\rm g}$, can be hung without causing the rod to slip at point A.



$$\sum F_{x} = 0 \quad N_{1} - T \cos \theta = 0$$

$$\sum F_{y} = 0 \quad f_{s} + T \sin \theta - 2mg = 0$$

$$\int_{s_{-MAX}} F_{s} = \mu_{s} N_{1} = \mu_{s} T \cos \theta$$

$$\sum \tau = 0 \quad mgx + mg \frac{L}{2} - TL \sin \theta = 0$$

$$\mu_{s} T \cos \theta + T \sin \theta = 2mg$$

$$T = \frac{2mg}{\mu_{s} \cos \theta + \sin \theta} \qquad x = \frac{2L \sin \theta}{\mu_{s} \cos \theta + \sin \theta} - \frac{L}{2}$$

$$mg \quad mg$$

Rotational Kinetic Energy around P

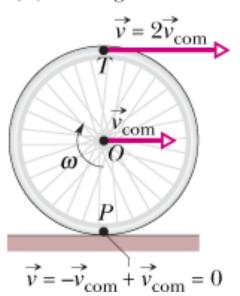
$$K = \frac{1}{2}I_P\omega^2$$

$$I_P = I_{com} + MR^2$$

$$K = \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}MR^2\omega^2$$

$$K = \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}MV_{com}^2$$

(c) Rolling motion



Kinetic energy of rolling:

$$K_{TOTAL} = K_{ROTATIONAL} + K_{TRANSLATIONAL}$$
$$K_{TOTAL} = \frac{1}{2} I_{COM} \omega^2 + \frac{1}{2} M v_{COM}^2$$

Conservation of energy including rotational energy:

$$U_{gf} + K_{Rf} + K_{Tf} = U_{gi} + K_{Ri} + K_{Ti}$$

Moment of inertia:

$$I = \sum m_i r_i^2$$
 (point particles)

$$I_{COM} = MR^2$$
 (ring)

$$I_{COM} = \frac{1}{2}MR^2 \quad (disk)$$

$$I_{COM} = \frac{2}{5}MR^2$$
 (sphere)

$$I_{COM} = \frac{1}{12} ML^2 \quad (rod)$$

$$I = I_{COM} + Mh^2$$
 (axis displaced from COM by h)

$$K_R = \frac{1}{2}I\omega^2$$
 (rotational kinetic energy)

$$W_R = \int \tau d\theta \quad (rotational \ work)$$

$$\tau = rF\sin(\theta)$$
 (torque)

$$K_{rot} = \frac{1}{2}I\omega^2$$

$$E_{mech} = K_{rot_com} + K_{trans} + U$$

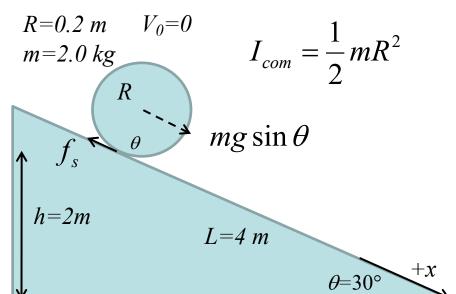
$$1 \quad 1 \quad 2 \quad 1 \quad 3$$

$$= \frac{1}{2}I_{com}\omega^{2} + \frac{1}{2}mV_{com}^{2} + U$$

Find the V_{com} at the bottom of the ramp.

$$\boldsymbol{U}_f + \boldsymbol{K}_{rot_f} + \boldsymbol{K}_{com_f} = \boldsymbol{U}_i + \boldsymbol{K}_{rot_i} + \boldsymbol{K}_{com_i}$$

$$K = \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}MV_{com}^2$$



$$\alpha R = a_{com}$$

$$\omega R = V_{com}$$

Find the V_{com} at the bottom of the ramp.

$$U_f + K_{rot_f} + K_{com_f} = U_i + K_{rot_i} + K_{com_i}$$

$$K = \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}mV_{com}^2$$

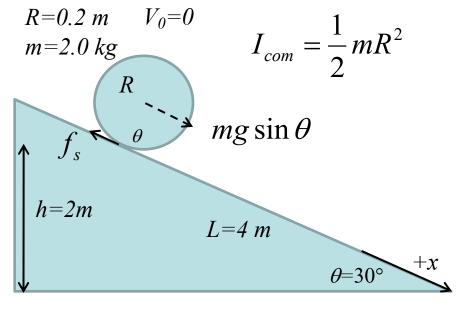
$$mgh = \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}mV_{com}^2$$

$$= \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}mV_{com}^2 = \frac{1}{2}I_{com}\frac{V_{com}^2}{R^2} + \frac{1}{2}mV_{com}^2$$

$$=\frac{1}{2}\left(\frac{I_{com}}{R^2}+m\right)V_{com}^2$$

$$2gh = \left(\frac{I_{com}}{mR^2} + 1\right)V_{com}^2$$

$$V_{com}^2 = \frac{2gh}{\left(\frac{I_{com}}{mR^2} + 1\right)}$$



$$\alpha R = a_{com}$$

$$\omega R = V_{com}$$

Motion under External Force and Torque

$$F_{net} = ma_{com}$$

$$\tau_{net} = Fr = I_{com}\alpha$$

$$I_{com} = \frac{1}{2} mR^2$$

$$F$$

Static Equilibrium (Chapter 12)

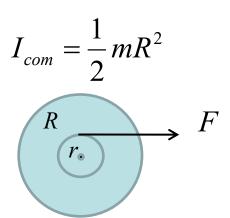
$$F_{net} = ma_{com} = 0$$

$$\tau_{net} = Fr = I_{com}\alpha = 0$$

Motion under External Force and Torque

$$F_{net} = ma_{com}$$

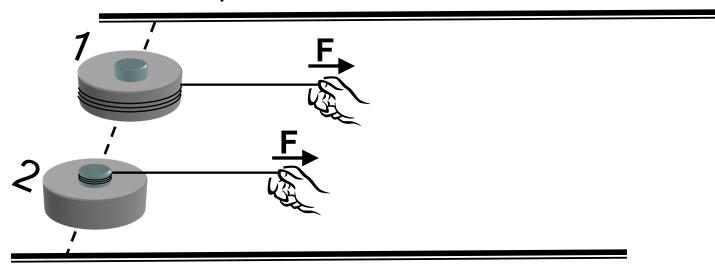
$$\tau_{net} = Fr = I_{com}\alpha$$



What is the V_{com} after Δt ?

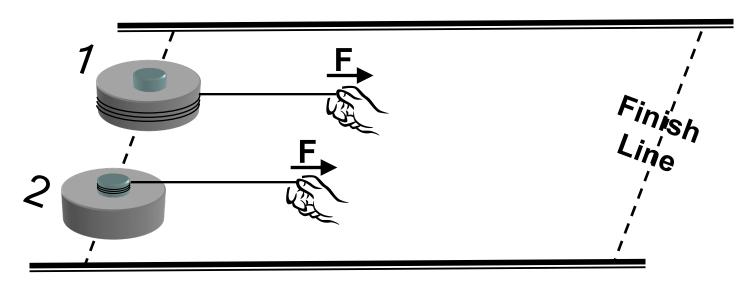
What is the ω after Δt ?

Strings are wound around two <u>identical</u> pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force **F**. Both pucks start to move on a frictionless surface. 5 seconds later, which puck has greater center-of-mass speed?



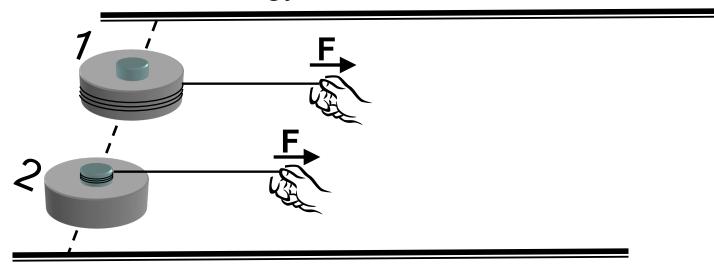
- 1. Puck 1
- 2. Puck 2
- 3. Both have the same C.O.M speed
- 4. Not enough info. to determine

Strings are wound around two <u>identical</u> pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force **F**. Both pucks start to move on a frictionless surface. Which puck arrives at the finish line first?



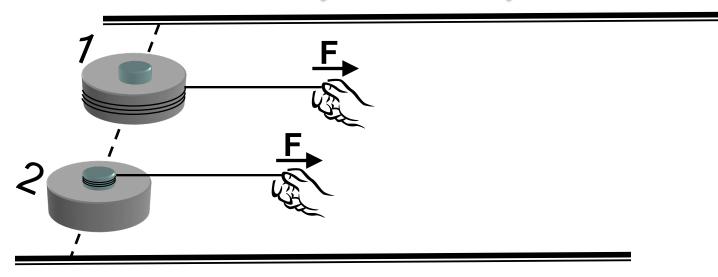
- Puck 1
- 2. Puck 2
- 3. Both arrive at the same time
- 4. Not enough info. to determine

Strings are wound around two <u>identical</u> pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force **F**. Both pucks start to move on a frictionless surface. 5 seconds later, which puck has greater rotational kinetic energy?



- 1. Puck 1
- 2. Puck 2
- 3. Both have the same kinetic energy
- 4. Not enough info. to determine

Strings are wound around two <u>identical</u> pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force **F**. Both pucks start to move on a frictionless surface. 5 seconds later, which puck has greater total kinetic energy? (*Do you know why?*)



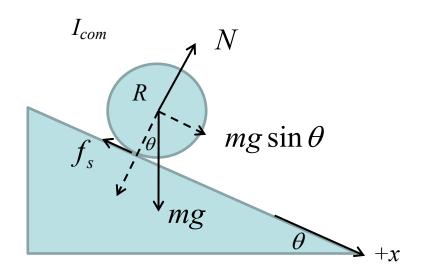
- 1. Puck 1
- Puck 2
- 3. Both have the same kinetic energy
- 4. Not enough info. to determine

Rolling Object

Forces and torques

What if there is no friction?

No slipping so there is a static frictional force.



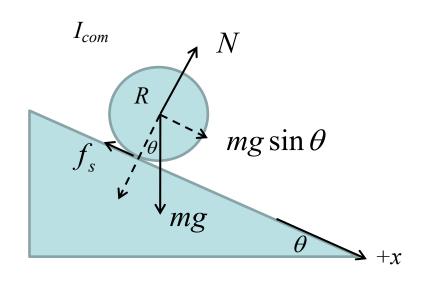
$$\alpha R = a_{com}$$
$$\omega R = V_{com}$$

$$mg \sin \theta - f_s = ma_{com}$$

$$\tau_{net} = f_s R = I_{com} \alpha$$

$$\alpha R = a_{com}$$

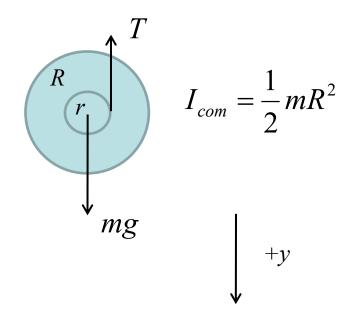
$$\begin{split} f_s R &= I_{com} \, \frac{a_{com}}{R} \Rightarrow f_s = I_{com} \, \frac{a_{com}}{R^2} \\ mg \sin \theta - I_{com} \, \frac{a_{com}}{R^2} &= ma_{com} \\ a_{com} &= \frac{g \sin \theta}{1 + \frac{I_{com}}{mR^2}} \end{split}$$



$$\alpha R = a_{com}$$

$$\omega R = V_{com}$$

Yo-Yo



Yo-Yo

$$mg - T = ma_{com}$$

$$\tau_{net} = Tr = I_{com}\alpha$$

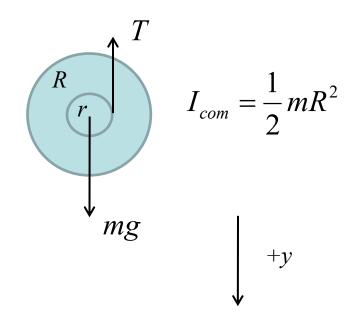
$$\alpha r = a_{com}$$

$$Tr = I_{com} \frac{a_{com}}{r} \Rightarrow T = I_{com} \frac{a_{com}}{r^2}$$

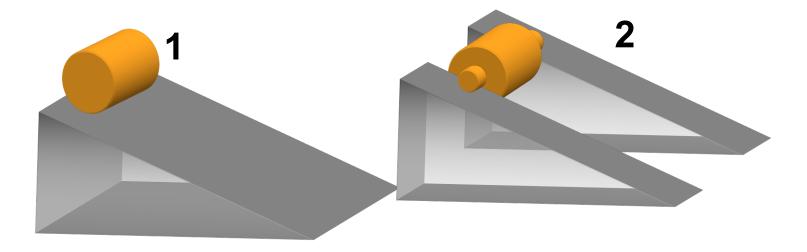
$$mg - I_{com} \frac{a_{com}}{r^2} = ma_{com}$$

$$mg = (m + \frac{I_{com}}{r^2})a_{com}$$

$$a_{com} = \frac{g}{1 + \frac{I_{com}}{mr^2}} = \frac{g}{1 + \frac{1}{2}\frac{R^2}{r^2}}$$



Two cylinders with the same radius and mass start from rest and roll down (without slipping) two hills from the same height. Cylinder 2 has a massless axle with a smaller radius. There is a groove in the hill, so that only the axle touches. Which object reaches the bottom first?



- **1**. 1
- 2. 2
- 3. They arrive at the same time.
- Need to know the axle's radius.