Homework 9 - Math 2568 (Autumn 2022)

Prof. Cueto

Due date: Wednesday November 16, 2022 (in class).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§5.7	12, 13, 14, 15, 19, 25, 26, 27	12, 13, 14, 15, 27
§5.9	1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 14, 17, 19, 23	1, 2, 8, 13, 14, 17
§6.2	1, 7, 11, 13, 17, 19, 21, 29	1, 7, 11, 17
§6.3	1, 5, 7, 9, 13, 17, 19, 20, 21, 22, 23, 24	7, 9, 13, 17, 24

Section 5.7

12) $V = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+2d \\ b-c \end{bmatrix}$

a) $\underline{T\left(x\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)} = \begin{bmatrix} xa + 2xd \\ xb - xc \end{bmatrix} = \begin{bmatrix} x(a+2d) \\ x(b-c) \end{bmatrix} = \underline{xT\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)}$

b) $R^2 = \begin{bmatrix} a+2d \\ b-c \end{bmatrix}$ $\quad T(V) = R^2$ therefore $T(\theta_V) = \theta_{R^2}$

$N(T) = \left\{ V \text{ in } R^2 : V = \begin{bmatrix} -2d & c \\ c & d \end{bmatrix} \right\}$

c) $\begin{bmatrix} a+2d \\ b-c \end{bmatrix} \to \left\{ \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$

d) $\text{rank}(T) + \text{nullity}(T) = \dim(V) = 4$

$\text{rank}(T) = 2 \quad \text{nullity}(T) = 2$

e) $R(T)$ has the same dimensions and is a subspace of R^2

f) $A = \begin{bmatrix} x & y \\ 0 & 0 \end{bmatrix} \quad T(A) = \begin{bmatrix} x \\ y \end{bmatrix}$

$T(A) = v$ and $v \in R(T)$ so $R(T) = R^2$

13) $T(p) = p''(x) \quad T: P_4 \to P_2$

a) $T(1) = \frac{d^2}{dx^2}(1) = 0 \to T(x) = \frac{d^2}{dx^2}(x) = 0$

$\to T(x^2) = \frac{d^2}{dx^2}(x^2) = 2 \to T(x^3) = \frac{d^2}{dx^2}(x^3) = 6x$

$\to T(x^4) = \frac{d^2}{dx^2}(x^4) = 12x^2$

$A = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 12 \end{bmatrix} \to$ Basis of $R(T) = \{2, 6x, 12x^2\}$

b) $\dim(P_4) = \text{rank}(T) + \text{nullity}(T) \to 5 = 3 + \text{null}(T)$

$\text{nullity}(T) = 2 \neq 0 \to$ not one to one

c) $\int p(x)\,dx = \int (a_0 + a_1 x + a_2 x^2)\,dx = a_0 x + a_1\left(\frac{x^2}{2}\right) + a_2\left(\frac{x^3}{3}\right) + C = r(x)$

$T(r(x)) = p(x)$ so $R(T) = P_2$

14) $T: P_4 \to P_3$ $\quad C = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$

$$C = \begin{bmatrix} 1 & -1 & 2 & -1 & 1 \\ -1 & 3 & -2 & 3 & -1 \\ 2 & -3 & 5 & -1 & 1 \\ 3 & -1 & 7 & 2 & 2 \end{bmatrix} \quad C^T = \begin{bmatrix} 1 & -1 & 2 & 3 \\ -1 & 3 & -3 & -1 \\ 2 & -2 & 5 & 7 \\ -1 & 3 & -1 & 2 \\ 1 & -1 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & -.5 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & -1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \quad D^T = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$p(x) = 1 - x$

$q(x) = x$

$r(x) = x^2$ $\quad s(x) = x^3$

Basis $R(T) = \{p(x), q(x), r(x), s(x)\}$

nullity$(T) = 1$ not one of one

15) $N(T) = \{ a_0 + a_1x + a_2x^2 \in P_2 \mid a_0 + 2a_1 + 4a_2 = 0 \}$

$R(T) = \mathbb{R}^1$

27) $V = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad T: V \to V$ by $T(A) = A^T$

a)