CSE 2321 Foundations I Spring, 2024 Dr. Estill

Homework 7 Due: Friday, March 22

Theorem 1 (Master Theorem)**.**
Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function $f : \mathbb{N} \to \mathbb{R}^+$, and let $T(n)$ be defined on the nonnegative integers by the recurrence

$$T(n) = aT(n/b) + f(n).$$

Then $T(n)$ has the following asymptotic bounds:

1. *if $f(n) \in O(n^{\log_b a - \varepsilon})$ for some constant $\varepsilon > 0$, then $T(n) \in \Theta(n^{\log_b a})$,*
2. *if $f(n) \in \Theta(n^{\log_b a})$ then $T(n) \in \Theta(n^{\log_b a} \log n)$, and*
3. *if $f(n) \in \Omega(n^{\log_b a + \varepsilon})$ for some $\varepsilon > 0$ and if $af(n/b) \leq df(n)$ for some constant $d < 1$ and all sufficiently large n, then $T(n) \in \Theta(f(n))$.*

Use the Master Theorem above to solve the following recurrences when possible. If you need to confirm the regularity condition ($af(n/b) \leq df(n)$ for $d < 1$), work should be shown, but otherwise answers are all that is needed. (Note that not every blank needs to be filled in in every problem.):

(20 points each)

1.) $T(n) = T(n/3) + c$
$f(n) =$ ________ versus $n^{\log_b a} =$ ________
Which is growing faster: $f(n)$ or $n^{\log_b a}$? ___________
Which case of the Master Theorem does that potentially put us in? ____
If you're potentially in case one or three, is it possible to find an epsilon which makes either $f(n) \in O(n^{\log_b a - \varepsilon})$ (if you're in case one) or $f(n) \in \Omega(n^{\log_b a + \varepsilon})$ (if you're in case three) true? Choose one or show an inequality. _____
If you're potentially in case three and there is an ε, try to find a constant $d < 1$ such that $af(n/b) \leq df(n)$ for large enough n's. _______________
What can you conclude? _______________

2.) $T(n) = T(n/3) + c \log_2 n$
$f(n) =$ ________ versus $n^{\log_b a} =$ ________
Which is growing faster: $f(n)$ or $n^{\log_b a}$? ___________
Which case of the Master Theorem does that potentially put us in? ____
If you're potentially in case one or three, is it possible to find an epsilon which makes either $f(n) \in O(n^{\log_b a - \varepsilon})$ (if you're in case one) or $f(n) \in \Omega(n^{\log_b a + \varepsilon})$ (if you're in case three) true? Choose one or show an inequality. _____
If you're potentially in case three and there is an ε, try to find a constant $d < 1$ such

that $af(n/b) \leq df(n)$ for large enough n's. ________________
What can you conclude? ________________

3.) $T(n) = 4T(n/2) + cn$
$f(n) =$ _________ versus $n^{\log_b a} =$ _________
Which is growing faster: $f(n)$ or $n^{\log_b a}$? ____________
Which case of the Master Theorem does that potentially put us in? ____
If you're potentially in case one or three, is it possible to find an epsilon which makes either $f(n) \in O(n^{\log_b a - \varepsilon})$ (if you're in case one) or $f(n) \in \Omega(n^{\log_b a + \varepsilon})$ (if you're in case three) true? Choose one or show an inequality. _____
If you're potentially in case three and there is an ε, try to find a constant $d < 1$ such that $af(n/b) \leq df(n)$ for large enough n's. ________________
What can you conclude? ________________

4.) $T(n) = 4T(n/2) + cn^3$
$f(n) =$ _________ versus $n^{\log_b a} =$ _________
Which is growing faster: $f(n)$ or $n^{\log_b a}$? ____________
Which case of the Master Theorem does that potentially put us in? ____
If you're potentially in case one or three, is it possible to find an epsilon which makes either $f(n) \in O(n^{\log_b a - \varepsilon})$ (if you're in case one) or $f(n) \in \Omega(n^{\log_b a + \varepsilon})$ (if you're in case three) true? Choose one or show an inequality. _____
If you're potentially in case three and there is an ε, try to find a constant $d < 1$ such that $af(n/b) \leq df(n)$ for large enough n's. ________________
What can you conclude? ________________

5.) $T(n) = 2T(n/6) + \sqrt{n}$
$f(n) =$ _________ versus $n^{\log_b a} =$ _________
Which is growing faster: $f(n)$ or $n^{\log_b a}$? ____________
Which case of the Master Theorem does that potentially put us in? ____
If you're potentially in case one or three, is it possible to find an epsilon which makes either $f(n) \in O(n^{\log_b a - \varepsilon})$ (if you're in case one) or $f(n) \in \Omega(n^{\log_b a + \varepsilon})$ (if you're in case three) true? Choose one or show an inequality. _____
If you're potentially in case three and there is an ε, try to find a constant $d < 1$ such that $af(n/b) \leq df(n)$ for large enough n's. ________________
What can you conclude? ________________