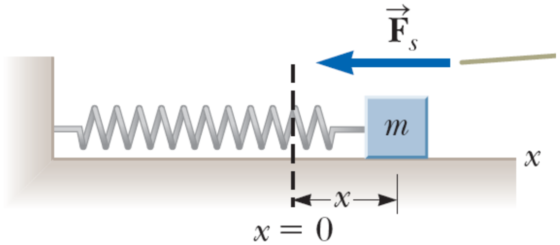
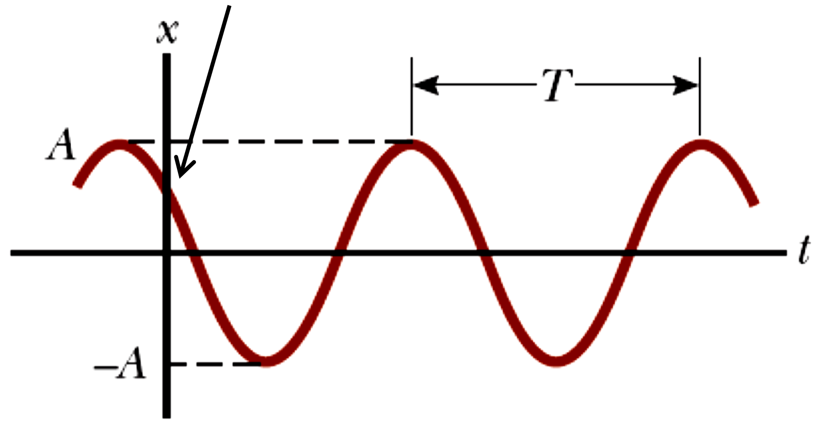
Lecture 29



$$F_{net} = -kx$$

$t = 0$

$x(0) = A\cos(0+\phi) = A\cos(\phi)$



$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Important Parameters of the Motion

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi)$$

$t = 0$

$x(0) = A\cos(0+\phi) = A\cos(\phi)$

$\phi \Rightarrow initial\ \ phase$

$\omega t + \phi \Rightarrow phase$

$$\omega = \sqrt{\frac{k}{m}} \qquad \omega T = 2\pi \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} \qquad A \Rightarrow Amplitude$$

Spring

At t=0, an oscillator is started with ***zero velocity*** at its ***maximum positive amplitude*** X_m. The equation for simple harmonic motion is:

$$x(t) = x_m \cos(\omega t + \phi)$$

What is the value of ϕ?

(1) 0

(2) $+\pi/4$

(3) $-\pi/4$

(4) $+\pi/2$

(5) $-\pi/2$

(6) None of the above

At t=0, an oscillator is started with ***maximum negative*** velocity at x(0)=0. The equation for simple harmonic motion is:

$$x(t) = x_m \cos(\omega t + \phi)$$

$V = x' = -x_m \sin(\phi)$

What is the value of ϕ?

(1) 0

(2) $+\pi/4$

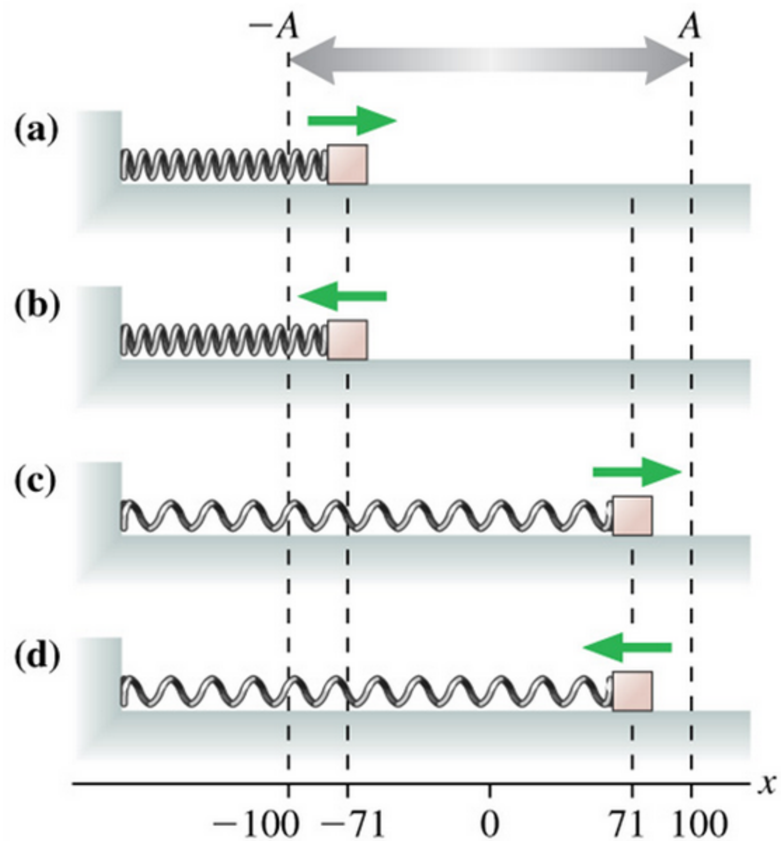
(3) $-\pi/4$

(4) $+\pi/2$

(5) $-\pi/2$

(5) None of the above

The figure shows four oscillators at t = 0. Which one has the phase constant $\phi_0 = +\left(\pi/4\right)$?



(1) (a)

(2) (b)

(3) (c)

(4) (d)

Energy of the Simple Harmonic Oscillator

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = -\omega A\sin(\omega t + \phi)$$

Kinetic Energy $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi)$

Potential Energy $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$

Total Mechanics Energy $E = \frac{1}{2}kA^2$

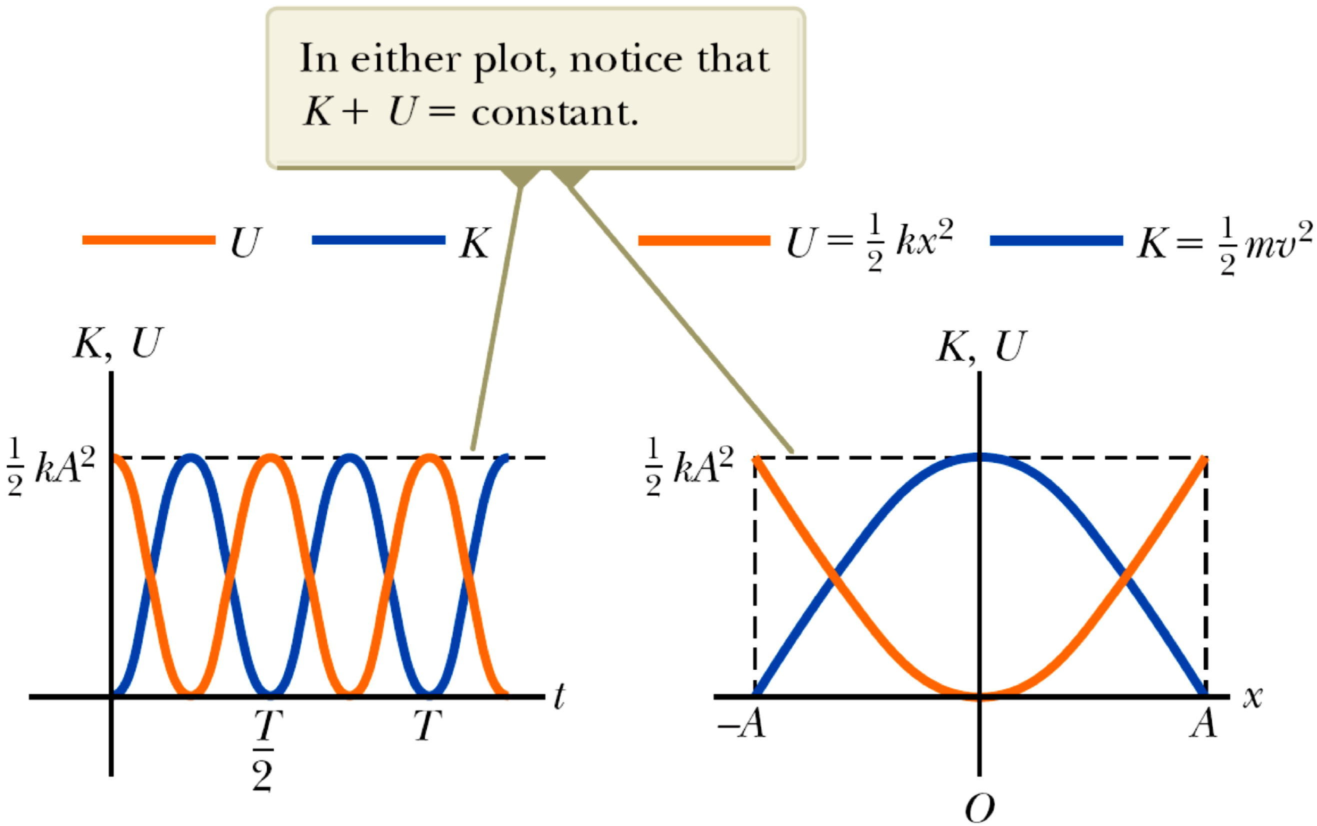
$$E = K + U = \frac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

Total Mechanics Energy $E = \frac{1}{2}kA^2$

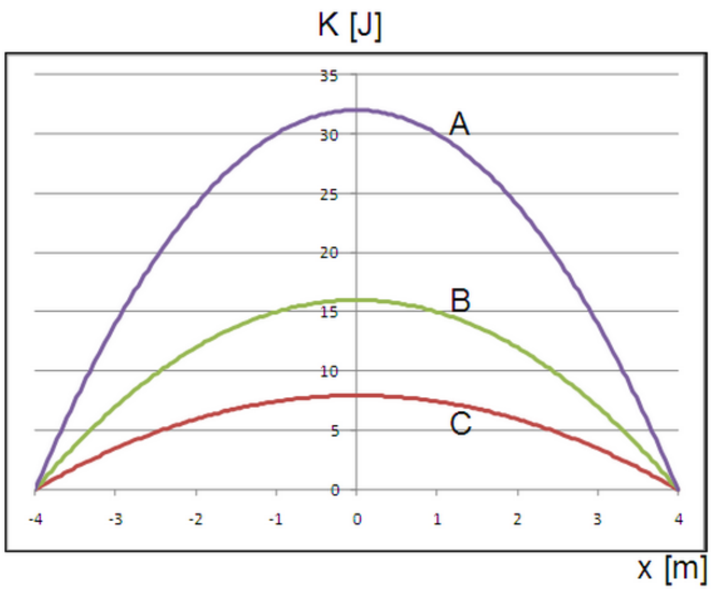
Knowing x and E to find v

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{A^2 - x^2}$$



The graph below shows plots of the kinetic energy K versus position x for three harmonic oscillators with the same mass. Rank the plots according to the corresponding frequency of the oscillator.



(1) A > B > C

(2) A < B < C

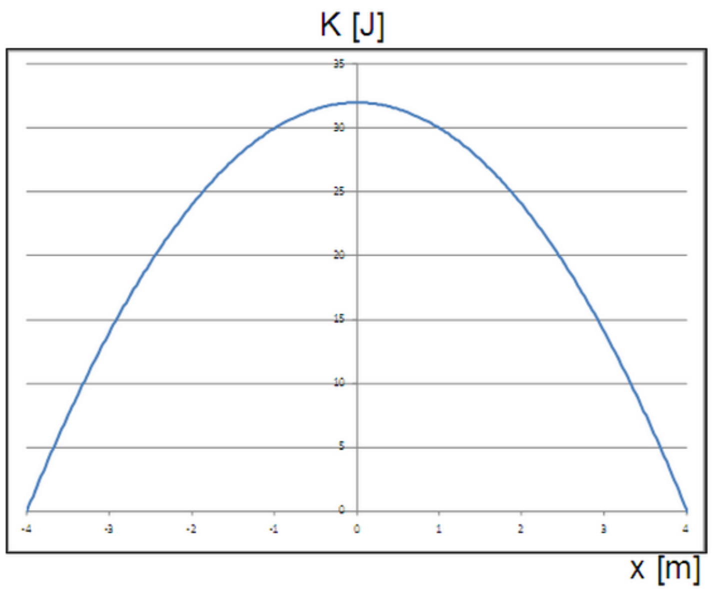
(3) A = B = C

(4) B > A = C

(5) B < A = C

(6) Can't tell from the info. given

The graph below shows a plot of the kinetic energy K versus position x for a spring harmonic oscillator with a 1kg mass. The maximum kinetic energy is 32 J. What is the angular frequency of this harmonic oscillator?



(1) 32 rad/s

(2) 8 rad/s

(3) 4 rad/s

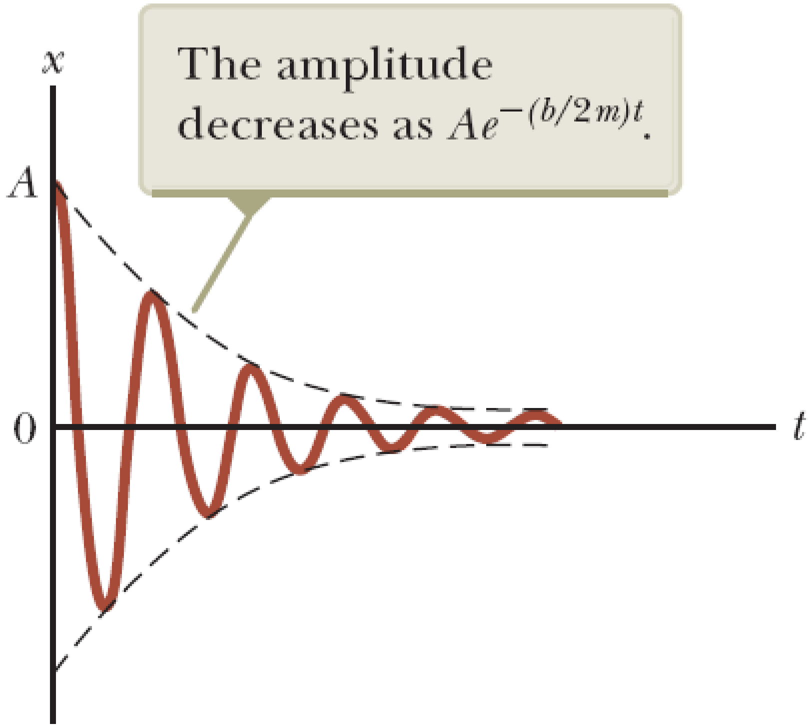
(4) 2 rad/s

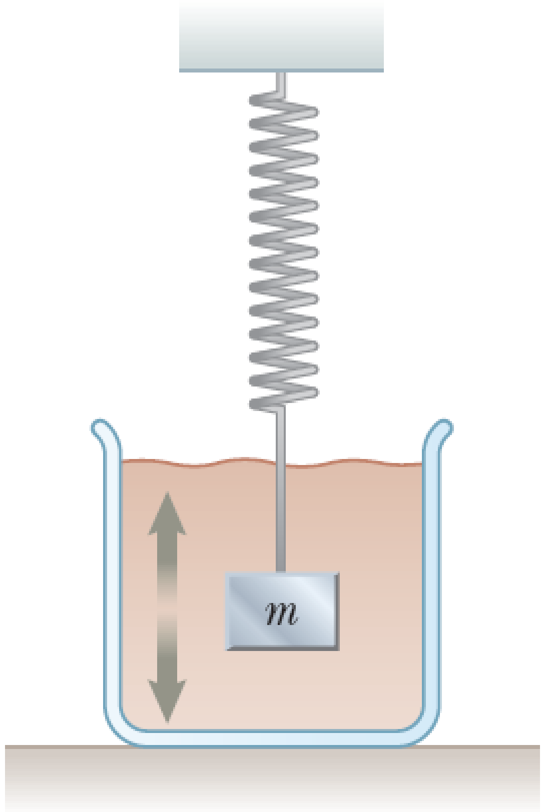
(5) 1 rad/s

(6) Can't tell from the info. given

Damped Oscillations

$$E_{mech} = K + U = \frac{1}{2} k A(t)^2 \quad \textit{decreases in time}$$



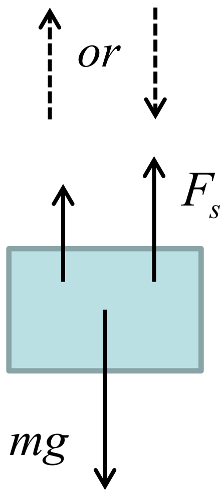


Damped Oscillations

$$\vec{f} = -b\vec{v}$$

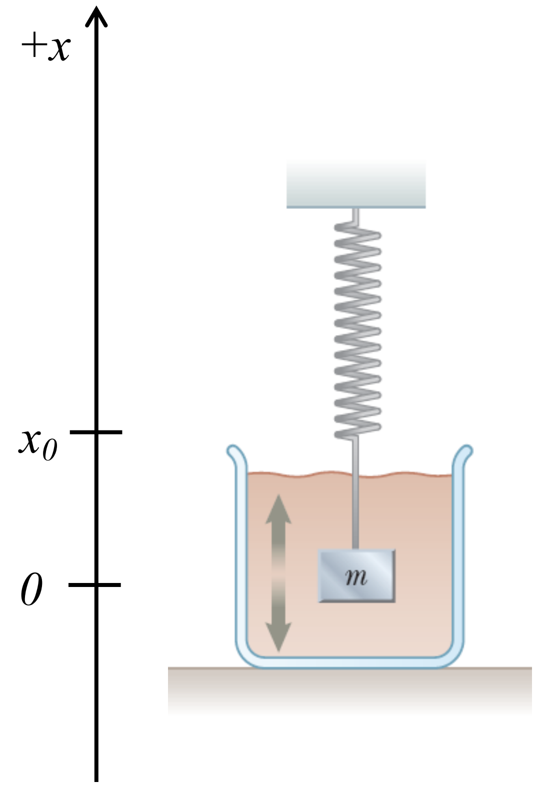
or

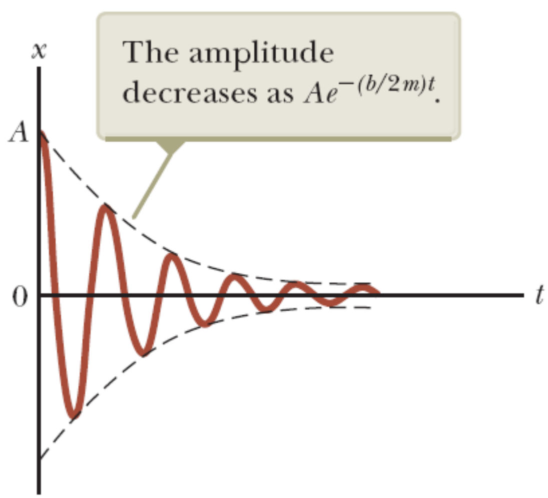
$$F_b = \rho V g \qquad F_s = -k(x - x_0)$$



$$F_{net} = -kx - bv = ma$$

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx$$





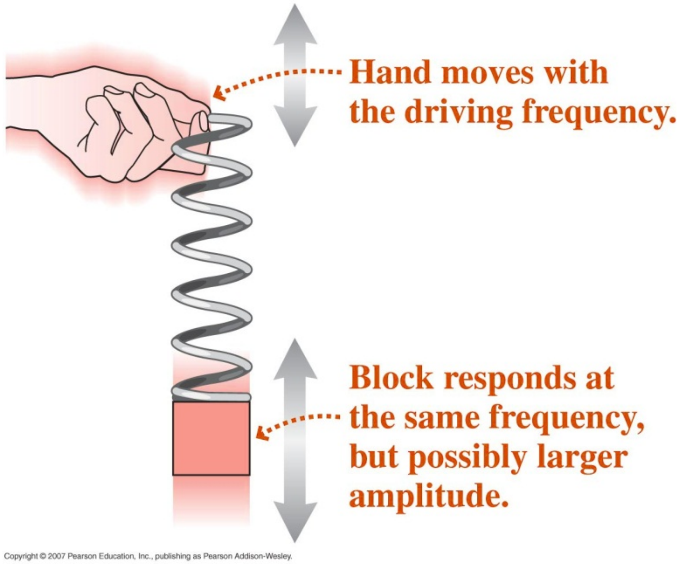
$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx \qquad \frac{d^2x}{dt^2} = -\frac{b}{m}\frac{dx}{dt} - \frac{k}{m}x$$

$$x = Ae^{-(b/2m)t}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

Forced Oscillations

$$E_{mech} = K + U = \frac{1}{2}kA(t)^2 \quad \textit{depends on the driving action}$$



$$F_{hand}(t) = F_0 \sin \omega t$$

$$F_{net} = F_0 \sin \omega t - kx - bv = ma$$

$$m\frac{d^2x}{dt^2} = F_0 \sin \omega t - b\frac{dx}{dt} - kx$$

$$x = A\cos(\omega t + \phi)$$

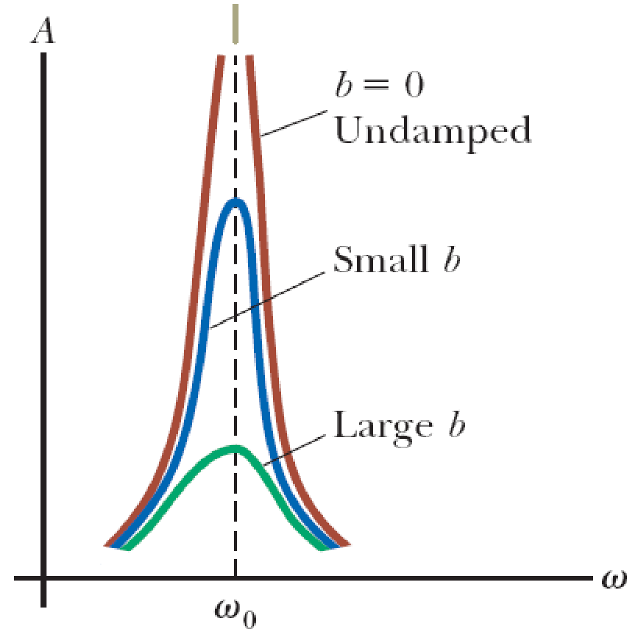
$$A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}} \qquad \omega_0 = \sqrt{k/m}$$

$$\omega = \omega_0 \quad \textit{resonance}$$

Forced Oscillations

$$x = A\cos(\omega t + \phi)$$

$$A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}} \qquad \omega_0 = \sqrt{k/m}$$

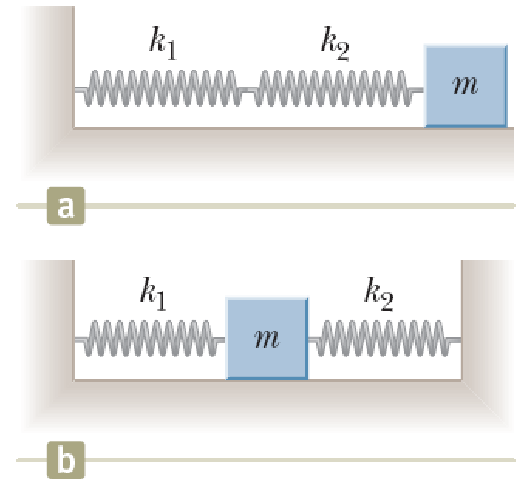


$\omega = \omega_0$ *resonance*

http://www.youtube.com/watch?v=j-zczJXSxnw

AP Images

A block of mass m is connected to two (massless) springs of force constants k_1 and k_2 in two ways as shown in Figure. In both cases, the block moves on a frictionless table after it is displaced from equilibrium and released. Show that in the two cases the block exhibits simple harmonic motion with periods



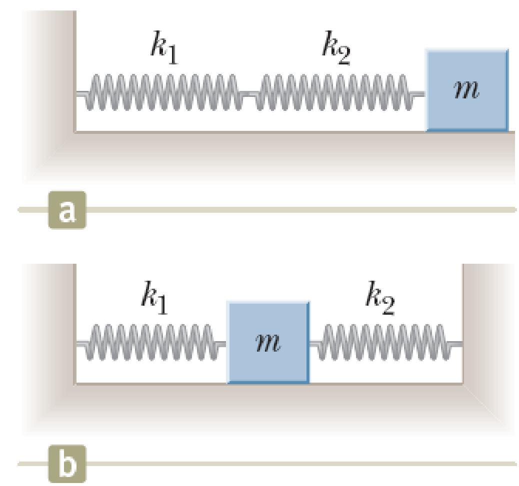
(a) $T = 2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1 k_2}}$ and (b) $T = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$

Spring in series:

$$F_s = -k_s(\Delta x_1 + \Delta x_2) = -k_1 \Delta x_1 = -k_2 \Delta x_2$$

$$\frac{\Delta x_2}{\Delta x_1} = \frac{k_1}{k_2} \qquad k_1 = k_s(1 + \frac{\Delta x_2}{\Delta x_1}) = k_s(1 + \frac{k_1}{k_2})$$

$$k_s = \frac{k_1 k_2}{k_1 + k_2} \qquad \omega = \sqrt{\frac{k_s}{m}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$



Spring in parallel:

$$F_p = -k_p \Delta x = -k_1 \Delta x - k_2 \Delta x = -(k_1 + k_2)\Delta x$$

$$k_s = k_1 + k_2 \qquad \omega = \sqrt{\frac{k_s}{m}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$