Lecture 22

$$a_t = \alpha r \qquad v(t) = \omega(t) r$$
$$\underset{\alpha}{\downarrow}$$

$$a_{r_{(t)}} = \frac{v_{(t)}^2}{r} = \omega_{(t)}^2 r$$
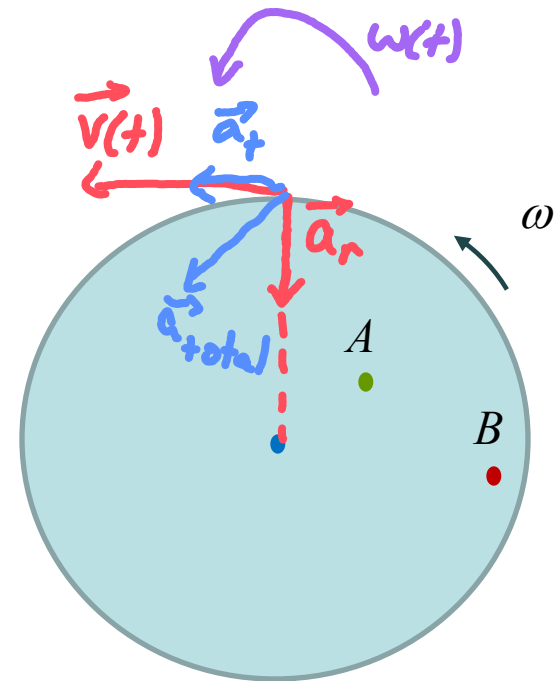
A disk is rotating CCW with an
angular speed ω.

Two stickers A and B are fixed to
different locations on the disk as
shown.

Compare the angular speeds of the
two stickers A and B; the linear
speeds of two.

$\longrightarrow$ B > A

$\omega(t)$

$\vec{v}(t)$ $\vec{a}_t$

$\vec{a}_{total}$ $\vec{a}_r$

ω

A

B

Kinetic Energy of Rotation

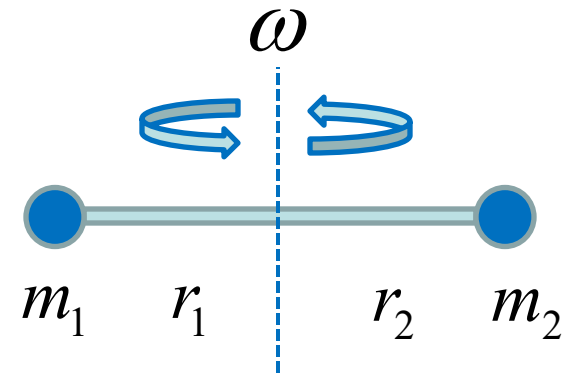$$K = \sum_i \frac{1}{2} m_i V_i^2 = \sum_i \frac{1}{2} m_i (r_i \omega)^2$$

$$K = \sum_i \frac{1}{2} (m_i r_i^2) \omega^2 = \frac{1}{2} \omega^2 \left(\sum_i m_i r_i^2 \right)$$

$$= \frac{1}{2} I \omega^2$$

Rotational Inertia (moment of inertia)

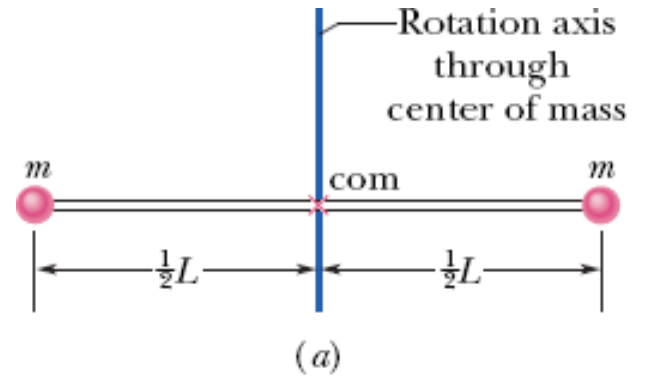$$I = \sum_i m_i r_i^2 \qquad I = \int r^2 dm$$

ω

$m_1 \qquad r_1 \qquad\qquad r_2 \qquad m_2$

linear
x
v
$\frac{1}{2} m v^2$
m

rotation
θ
ω
$\frac{1}{2} I \omega^2$
I

$\int dm \, r^2$
↑
inertia

Moment of Inertia

(a)

(b)

Rotation axis through center of mass

m

com

m

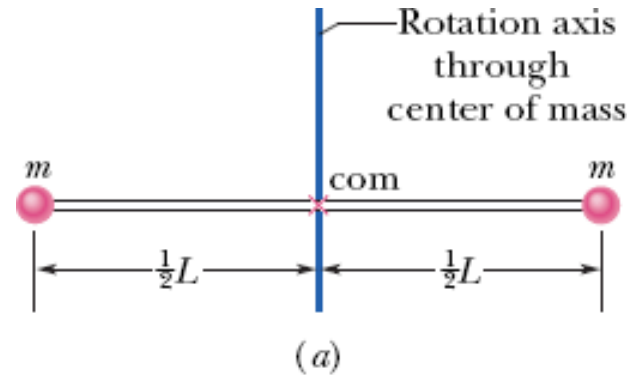$\frac{1}{2}L$ $\frac{1}{2}L$

(a)

Rotation axis through end of rod

m

com

m

L

(b)
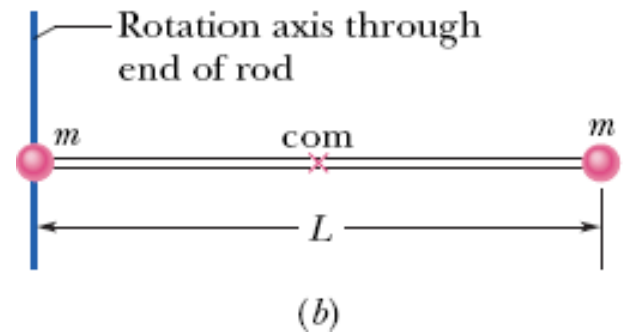
Moment of Inertia

(a)

$$I_{com} = \sum_i m_i r_i^2 = m(\frac{1}{2}L)^2 + m(\frac{1}{2}L)^2 = \frac{1}{2}mL^2$$

Center of mass

Rotation axis through center of mass

m com m

$\frac{1}{2}L$ $\frac{1}{2}L$

(a)

(b)

$$I = \sum_i m_i r_i^2 = m(0)^2 + m(L)^2 = mL^2$$

Rotation axis through end of rod

m com m

L

(b)
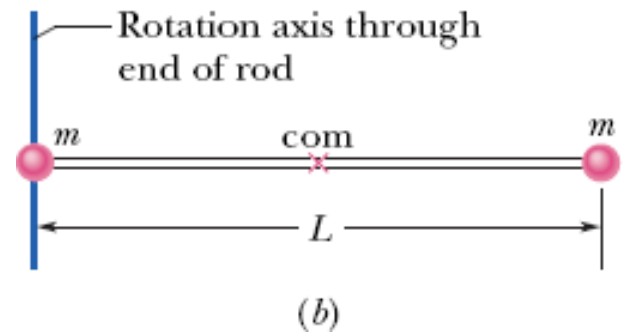
Parallel-Axis Theorem

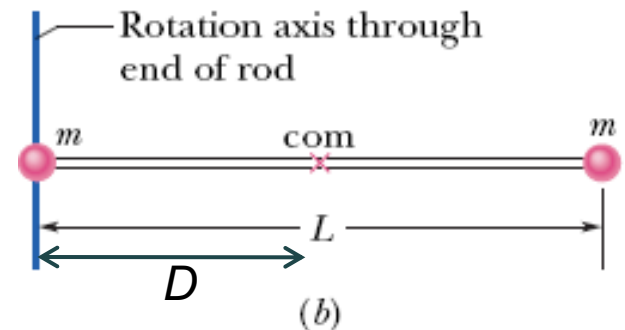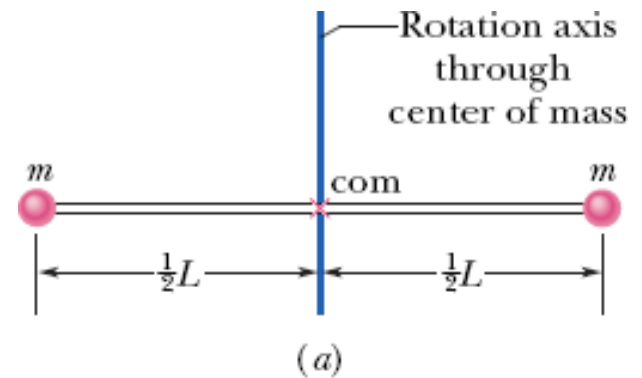$$I = I_{com} + MD^2$$

$$I_{com} = \sum_i m_i r_i^2 = m(\frac{1}{2}L)^2 + m(\frac{1}{2}L)^2 = \frac{1}{2}mL^2$$
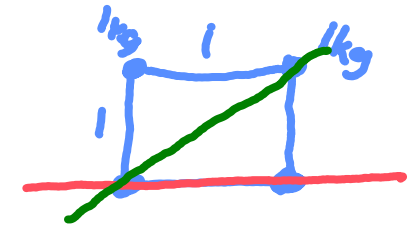
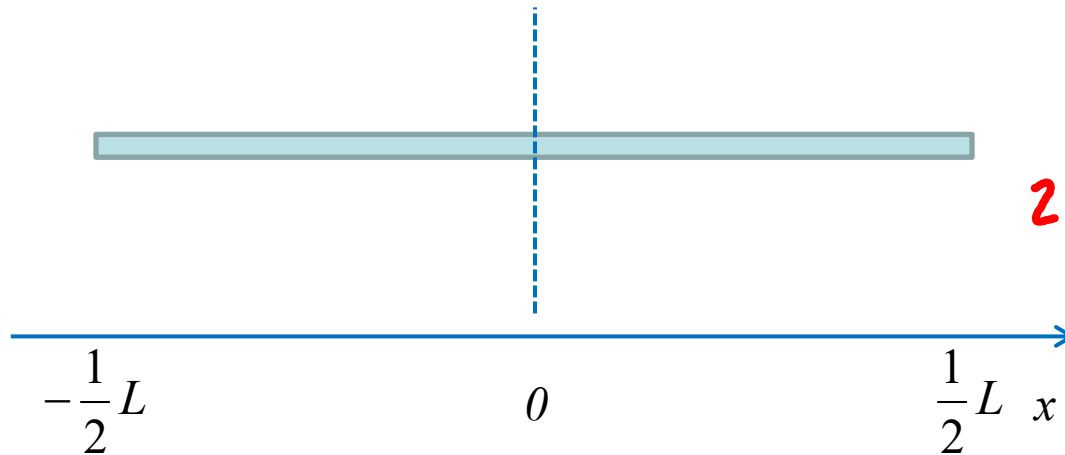$$I = I_{com} + MD^2 = \frac{1}{2}mL^2 + (2m)(\frac{1}{2}L)^2$$

$$= \frac{1}{2}mL^2 + \frac{1}{2}mL^2 = mL^2$$

$$I = \sum_i m_i r_i^2 = m(0)^2 + m(L)^2 = mL^2$$

Rotation axis through center of mass

m com m

$\frac{1}{2}L$ $\frac{1}{2}L$

(a)

Rotation axis through end of rod

m com m

L

D

(b)

Calculating Moment of Inertia

$$2(1^2) + 2(0^2) = 2$$

$$1\left(\frac{\sqrt{2}}{2}^2\right) + 1\left(\frac{\sqrt{2}}{2}^2\right) = 1$$

$$-\frac{1}{2}L \qquad 0 \qquad \frac{1}{2}L \quad x$$

A mass M is uniformly distributed over the length L of a thin rod. Find its moment of inertia around an axis perpendicular to its length and going through its center of mass.

A mass M is uniformly distributed over the length L of a thin rod. The mass inside a short element dx is given by:
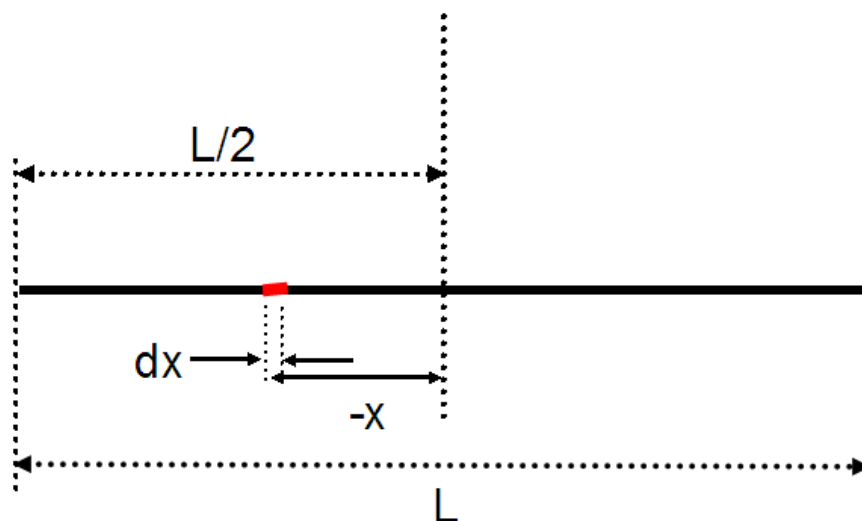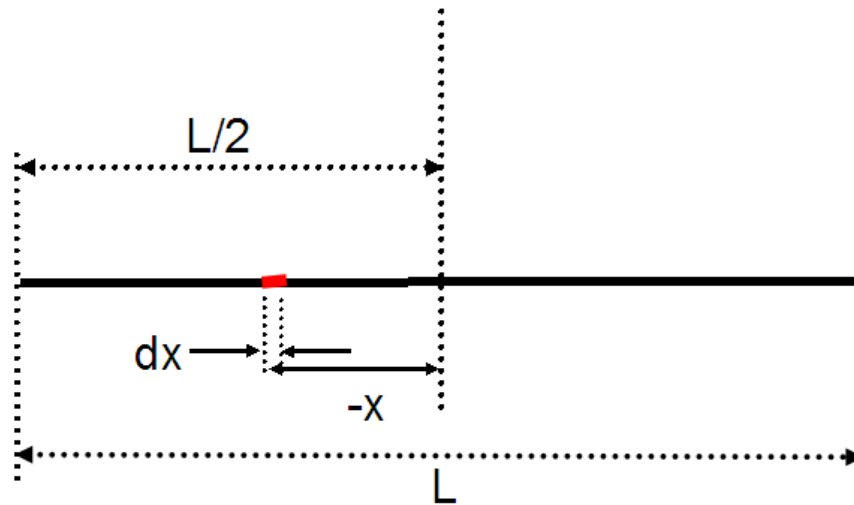

(1)

$$\frac{M}{L}$$

(2)

$$\left(\frac{M}{L}\right)dx$$

(3)

$$\frac{L}{M}$$

(4) Mdx

(5) None of the above

A mass M is uniformly distributed over the length L of a thin rod. The contribution to the moment of inertia by a short element dr is given by:
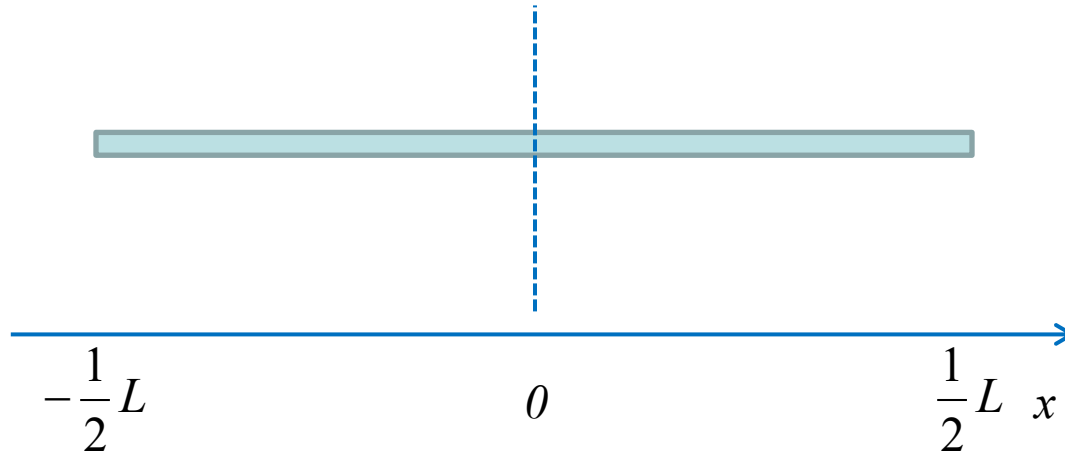

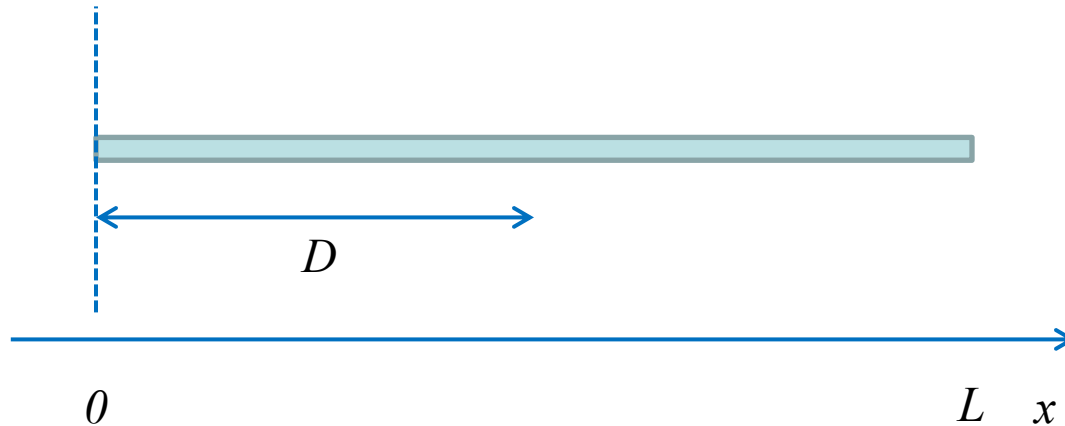(1)
xMdx

(2)
$x^2(M/L)dx$

(3)
$x^2(M/L)$

(4)
x^2M

(5)
x^2Mdx

A mass M is uniformly distributed over the length L of a thin rod. Find its moment of inertia around an axis perpendicular to its length and going through its center of mass.


$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \, dm = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \left(\frac{M}{L}\right) dx = \left(\frac{M}{L}\right) \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \, dx$$

$$= \left(\frac{M}{L}\right) \frac{1}{3} x^3 \Big|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{M}{3L} \left(\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right) = \frac{M}{3L} \cdot \frac{L^3}{4} = \frac{ML^2}{12}$$

A mass M is uniformly distributed over the length L of a thin rod. Find its moment of inertia around an axis perpendicular to its length and going through one end.

$$I = I_{CM} + MD^2 = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{1}{3}ML^2$$

A mass M is uniformly distributed over the circumference of a thin ring with radius r. The moment of inertia for this ring when rotating about its center is:

r
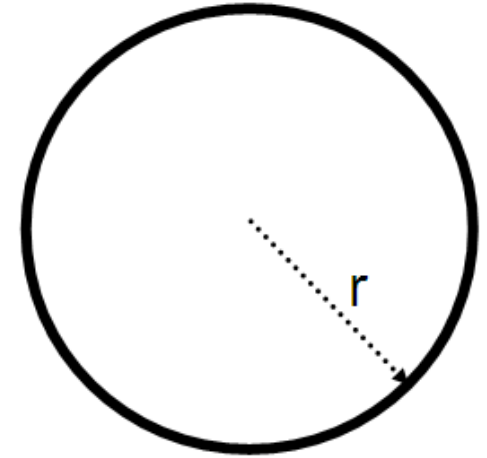
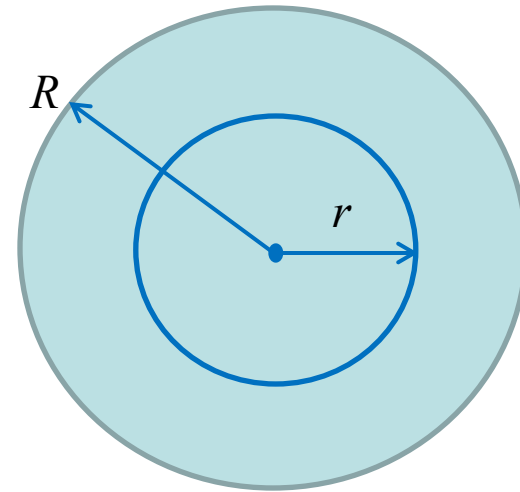(1) Cannot be determined without integrating.

(2) Mr

(3) ½ Mr
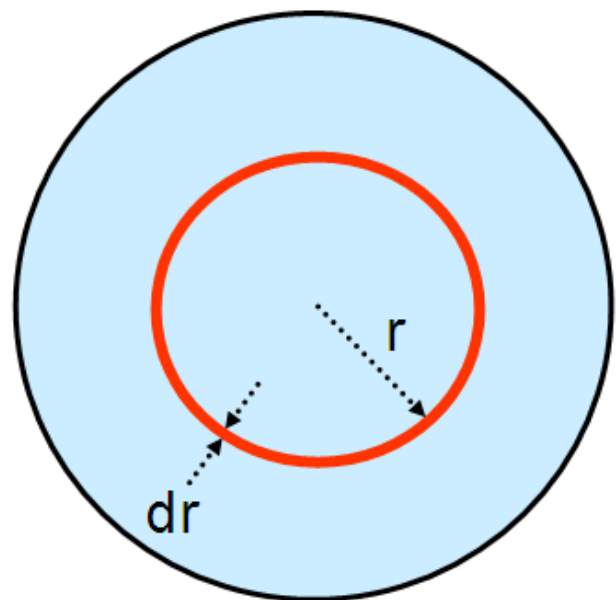
(4) Mr^2

(5) ½ Mr^2

A mass M is uniformly distributed over a disk. Find its moment of inertia around an axis perpendicular to the disk and going through its center of mass.

A mass M is uniformly distributed over a disk of radius R and area πR^2. The area of a thin ring inside the disk with radius r and thickness dr is:
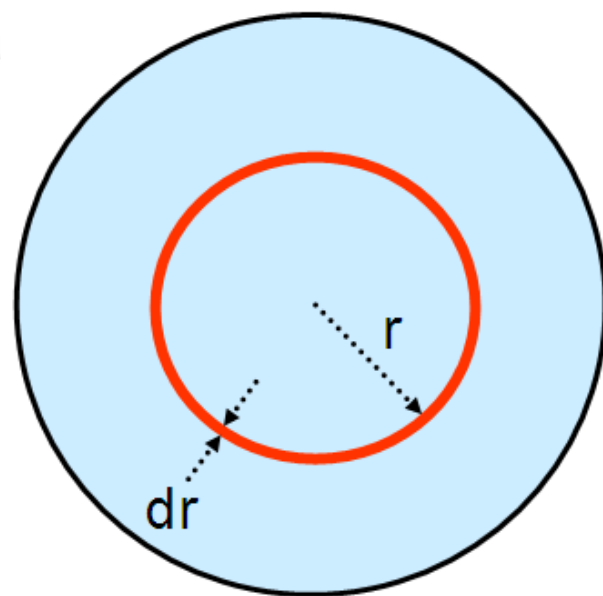

(1)

$$2\pi r dr$$

(2)

$$r dr$$

(3)

$$\pi r^2$$

(4)

$$\pi r^2 dr$$

(5)

$$r^2$$

A mass M is uniformly distributed over a disk of radius R. The mass contained in a thin ring with radius r and thickness dr inside the disk is given by:
(Remember to use a ratio of the ring area to the total area of the disk.)

(1)

$$\left(\frac{M}{\pi R^2}\right) r^2 dr$$

(2)

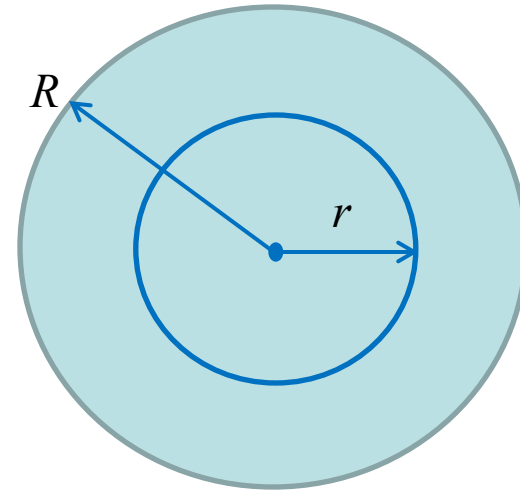$$\left(\frac{M}{R}\right) r$$

(3)

$$\left(\frac{MR^2}{rdr}\right)$$

(4)

$$\left(\frac{2M}{R^2}\right) rdr$$

(5) None of the above

A mass M is uniformly distributed over a disk. Find its moment of inertia around an axis perpendicular to the disk and going through its center of mass.

$$I = \int_0^R r^2 dm = \int_0^R r^2 (\frac{M}{\pi R^2}) 2\pi r dr = \int_0^R \frac{2M}{R^2} r^3 dr$$

$$= \frac{2M}{4R^2} r^4 \Big|_0^R = \frac{M}{2R^2}(R^4 - 0) = \frac{1}{2}MR^2$$
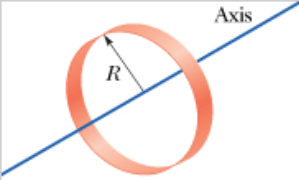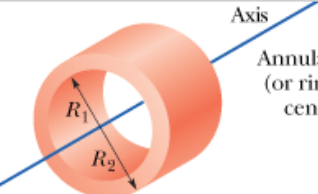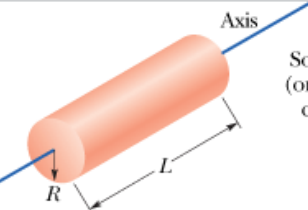
Moment of inertia :

$$I = \sum m_i r_i^2$$

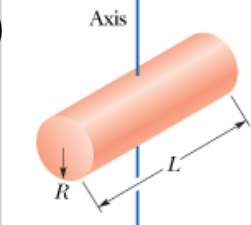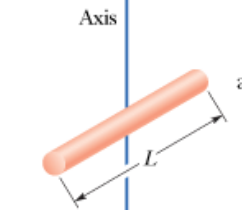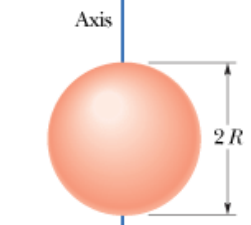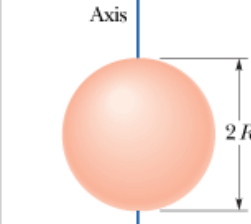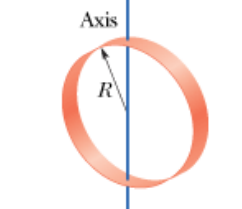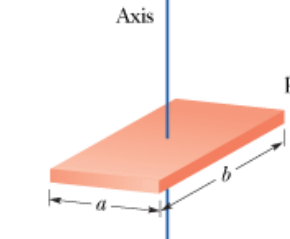$$I_{COM} = MR^2 \quad (ring)$$

$$I_{COM} = \frac{1}{2}MR^2 \quad (disk)$$

$$I_{COM} = \frac{2}{5}MR^2 \quad (sphere)$$

$$I_{COM} = \frac{1}{12}ML^2 \quad (rod)$$

$$I = I_{COM} + Mh^2$$

Axis

Hoop about
central axis

R

$I = MR^2$ (a)

Axis

Annular cylinder
(or ring) about
central axis

R_1

R_2

$I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)

Axis

Solid cylinder
(or disk) about
central axis

L

R

$I = \frac{1}{2}MR^2$ (c)

Axis

Solid cylinder
(or disk) about
central diameter

L

R

$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (d)

Axis

Thin rod about
axis through center
perpendicular to
length

L

$I = \frac{1}{12}ML^2$ (e)

Axis

Solid sphere
about any
diameter

$2R$

$I = \frac{2}{5}MR^2$ (f)

Axis

Thin
spherical shell
about any
diameter

$2R$

$I = \frac{2}{3}MR^2$ (g)

Axis

Hoop about any
diameter

R

$I = \frac{1}{2}MR^2$ (h)

Axis

Slab about
perpendicular
axis through
center

a

b

$I = \frac{1}{12}M(a^2 + b^2)$ (i)