ECE 2050
Prof. Clymer
Final Exam
May 2, 2023
1 hour 45 minutes

Printed Name (include .##): ____________________

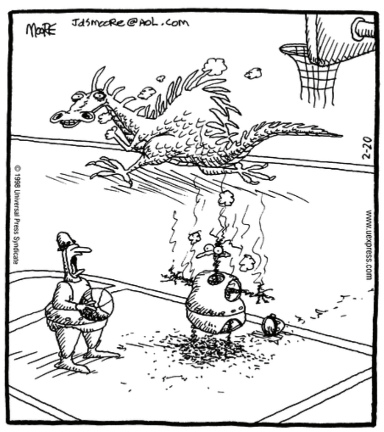
Write the Honor Code Pledge here:

Signed: ____________________

Please WRITE OUT the Honor Code Pledge:
"No aid given, received or observed," and
sign your name on the exam before your turn it in.
DO NOT OPEN THE EXAM BOOKLET UNTIL 4:00!

During the exam, no devices are permitted that facilitate communication with another party (e.g. cell phones, and other wireless electronic communication devices including laptop computers). Students may not leave the examination room without permission of the instructor (who will be sitting in the room) and while out of the examination room, no student may discuss any part of the examination with another party (except the instructor) either in person or through an electronic communications means. This is a closed book exam, with one sheet of notes allowed. A calculator may be used for the exam. You can write on the back of the pages.


"So, as an innocent prank, you jumped out of the stands and ran onto the field right in the middle of a rugby match. Then what happened?"


"You can't let a dragon go baseline, Floyd. You let a dragon go baseline and he's gonna burn you every single time."

1.) (20 pts): A continuous time function, $x(t)$, is sampled at a sampling period of $\Delta t = 25$ microseconds to form a discrete time signal, $x[n]$, which is stored in a buffer that holds 256 samples.

a.) If a discrete Fourier transform (DFT) is taken of the samples in the buffer to estimate the spectrum with the vector, $X[k]$, what is the frequency in Hz represented by the spacing between consecutive frequency samples?

b.) If the $X[k] = 320/j$ for $k = 17$ and $X[k] = -320/j$ for $k = 239$. For all other values of k in the range $0 \le k \le N-1$, $X[k] = 0$. What is $x[n]$?

c) Assuming the original time signal, $x(t)$, was oversampled to get $x[n]$, what was $x(t)$ before it was sampled?

2.) (30 pts): Let $x[n]$ be number of apples delivered to a grocery store *before the store opens* on day n and let $q[n]$ be the number of apples in the store *just after the store opens* on day n. Every day *before the store opens* 10% of the apples left over from the previous day are thrown away because of the expiration date on the apples. During day n, 75% of the apples in the store when it opens are sold to customers. Let $w[n]$ be the number of apples still in the store on day n when the store closes. The system output, $y[n]$, is the number of apples sold on day n. *Note: you do not have to complete the following parts in the order they are presented.*

a.) Write a difference equation in standard form, where y[n] is the number of apples sold on day n and x[n] is the number of apples delivered to the store on day n.

b.) Sketch a flow diagram that represents the *zero-state* system in the z-domain.

c.) Find the transfer function, $\hat{H}(z)$, for the system. Find the values of the poles and zeros for the system.

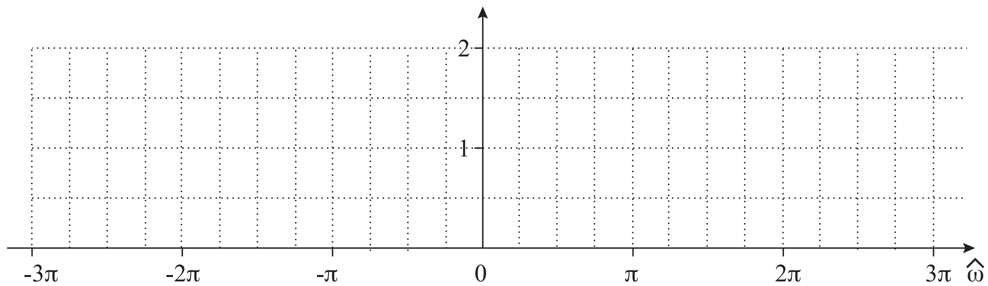
d.) If 125 apples are delivered every day beginning with day $n = 0$, specify a closed form formula for $x[n]$ and $\hat{X}(z)$.

e.) If the system can reach steady-state, specify the formula for the *steady-state* number of apples sold per day in both the z-domain, $\hat{Y}_{ss}(z)$, and time domain, $y_{ss}[n]$, including values for any constants in the formula. If the system cannot reach steady-state, state the reason it cannot. Note that you do not need to solve for the *transient* solution for the apples sold on day n.

(this page is blank for calculation)

3.) (25 pts): Design a causal FIR *high pass* filter with an impulse response that has a duration of 11 samples based on an ideal filter that passes *principal zone normalized radial frequencies* in the range of $|\hat{\omega}| > 3\pi/8$:

a.) Sketch the transfer function, $\hat{H}_{IHP}(e^{j\hat{\omega}})$, of the *ideal* filter in the range $-3\pi \leq \hat{\omega} \leq 3\pi$ in the graph below:



b.) Specify a formula for the impulse response of the *ideal* filter, $h_{IHP}[n]$. Briefly justify your answer and give values for $n = -5, n = 0$, and $n = 5$.

c.) Specify a formula for the causal FIR filter, $h_{HP11}[n]$. Give values for $n = -5, n = 0$, and $n = 5$.

4.) (25 pts): A causal discrete time filter has a z-domain description of

$$\hat{H}(z) = \frac{(z^2 + 0.8z + 0.64)(z - 1.5)}{z^3}$$

a.) Is this filter FIR or IIR? Briefly justify your answer.

b.) Is is filter stable or not? Briefly justify your answer.

c.) Find the impulse response, $h[n]$, for this filter.

d.) Find the transfer function of the filter as function of normalized radial frequency, $\hat{\omega}$.

e.) For your answer in **part d.)**, find the value of the transfer function at $\hat{\omega} = -2\pi/3$, $\hat{\omega} = 0$ and $\hat{\omega} = +2\pi/3$. If the answer is complex, specify the magnitude and angle (in degrees).

Continuous time Fourier series properties table

$g(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{j(2\pi/T_o)kt}$	$f(t) = \sum_{k=-\infty}^{\infty} \beta_k e^{j(2\pi/T_o)kt}$	$y(t) = \sum_{k=-\infty}^{\infty} \gamma_k e^{j(2\pi/T_o)kt}$	T_o same for all periodic signals

Property		Property	
Time Signal	Coefficients	Time Signal	Coefficients
$y(t) = Ag(t) + Bf(t)$	$\gamma_k = A\alpha_k + B\beta_k$	$f(t) = g(t - t_o)$	$\beta_k = \alpha_k e^{-j(2\pi/T_o)kt_o}$
$f(t) = \frac{d}{dt}g(t)$	$\beta_k = jk\frac{2\pi}{T_o}\alpha_k$	$f(t) = \int_{-\infty}^{t} g(\tau)d\tau$ where $\alpha_o = 0$	$\beta_k = \frac{\alpha_k}{jk}\left(\frac{T_o}{2\pi}\right)$, $\beta_o = \frac{1}{T_o}\int_{T_o} f(t)dt$
$g(t)$ is real	$\alpha_k = \alpha_{-k}^*$	$g(t) = A$	$\alpha_0 = A$, $(\alpha_k = 0, k \neq 0)$
$g(t) = g(-t)$	$\alpha_k = \alpha_{-k}$	$g(t) = -g(-t)$	$\alpha_k = -\alpha_{-k}$

Z transform table

Transform Pairs		Transform Properties	
Time Domain	z Domain	Time Domain	z Domain
$A\delta[n]$	A	$\alpha f[n] + \beta g[n]$	$\alpha\hat{F}(z) + \beta\hat{G}(z)$
$A\delta[n-q]$ $(q \geq 0$ & integer)	Az^{-q}	$f[n-q]$ $(q \geq 1$ & integer)	$z^{-q}\hat{F}(z) + \sum_{p=1}^{q} f[-p]z^{(p-q)}$
$u[n]$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$	$f[n+q]$ $(q \geq 1$ & integer)	$z^{+q}\hat{F}(z) - \sum_{p=0}^{q-1} f[p]z^{q-p}$
$u[n] - u[n-L]$ $(L > 0$ & integer)	$\frac{z^L - 1}{z^{L-1}(z-1)}$	$nf[n]$	$-z\frac{d}{dz}\hat{F}(z)$
$a^n u[n]$ $(\lvert a\rvert \leq 1)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$	$a^n f[n]$	$\hat{F}\left(\frac{z}{a}\right)$
$nu[n]$	$\frac{z^{-1}}{(1-z^{-1})^2} = \frac{z}{(z-1)^2}$	$(f[n] \cdot u[n]) * (g[n] \cdot u[n])$	$\hat{F}(z)\hat{G}(z)$
$\cos(\Omega_o n)u[n]$	$\frac{0.5z}{z-e^{-j\Omega_o}} + \frac{0.5z}{z-e^{+j\Omega_o}} = \frac{z^2 - z\cos(\Omega_o)}{z^2 - 2z\cos(\Omega_o) + 1}$	$\cos(\Omega_o n)f[n]$	$\frac{1}{2}\left[\hat{F}(ze^{j\Omega_o}) + \hat{F}(ze^{-j\Omega_o})\right]$
$\sin(\Omega_o n)u[n]$	$\frac{0.5jz}{z-e^{-j\Omega_o}} - \frac{0.5jz}{z-e^{+j\Omega_o}} = \frac{z\sin(\Omega_o)}{z^2 - 2z\cos(\Omega_o) + 1}$	$\sin(\Omega_o n)f[n]$	$\frac{1}{2j}\left[-\hat{F}(ze^{j\Omega_o}) + \hat{F}(ze^{-j\Omega_o})\right]$
$a^n\cos(\Omega_o n)u[n]$ $(\lvert a\rvert \leq 1)$	$\frac{0.5z}{z-ae^{-j\Omega_o}} + \frac{0.5z}{z-ae^{+j\Omega_o}} = \frac{z^2 - za\cos(\Omega_o)}{z^2 - 2za\cos(\Omega_o) + a^2}$	$f^*[n]$	$\hat{F}^*(z)$
$a^n\sin(\Omega_o n)u[n]$ $(\lvert a\rvert \leq 1)$	$\frac{0.5jz}{z-ae^{-j\Omega_o}} - \frac{0.5jz}{z-ae^{+j\Omega_o}} = \frac{az\sin(\Omega_o)}{z^2 - 2za\cos(\Omega_o) + a^2}$	$f[n] - f[n-1]$	$(1-z^{-1})\hat{F}(z) - f[-1]$
$e^{j\Omega_o n}u[n]$	$\frac{1}{1-e^{j\Omega_o}z^{-1}} = \frac{z}{z-e^{j\Omega_o}}$	$e^{j\Omega_o n}f[n]$	$\hat{F}(ze^{-j\Omega_o})$

DTFT table

Transform Pairs		Transform Properties	
Time Domain	Frequency Domain	Time Domain	Frequency Domain
$A\delta[n]$	A	$Af[n]+Bg[n]$	$A\hat{F}(e^{j\hat{\omega}})+B\hat{G}(e^{j\hat{\omega}})$
$A\delta[n-n_o]$	$Ae^{-j\hat{\omega}n_o}$	$f[n-n_o]$	$e^{-j\hat{\omega}n_o}\hat{F}(e^{j\hat{\omega}})$
A	$2\pi A\sum_{k=-\infty}^{\infty}\delta(\hat{\omega}-2\pi k)$	$f[-n]$	$\hat{F}(e^{-j\hat{\omega}})$
$Ae^{j\hat{\omega}_o n}$	$2\pi A\sum_{k=-\infty}^{\infty}\delta(\hat{\omega}-\hat{\omega}_o-2\pi k)$	$e^{j\hat{\omega}_o n}f[n]$	$\hat{F}(e^{j(\hat{\omega}-\hat{\omega}_o)})$
$\cos(\hat{\omega}_o n)$	$\pi\sum_{k=-\infty}^{\infty}\{\delta(\hat{\omega}-\hat{\omega}_o-2\pi k)+\delta(\hat{\omega}+\hat{\omega}_o-2\pi k)\}$	$\cos(\hat{\omega}_o n)f[n]$	$\frac{1}{2}\left[\hat{F}(e^{j(\hat{\omega}+\hat{\omega}_o)})+\hat{F}(e^{j(\hat{\omega}-\hat{\omega}_o)})\right]$
$\cos(\hat{\omega}_o n+\theta)$	$\pi\sum_{k=-\infty}^{\infty}\left\{e^{j\theta}\delta(\hat{\omega}-\hat{\omega}_o-2\pi k)+e^{-j\theta}\delta(\hat{\omega}+\hat{\omega}_o-2\pi k)\right\}$	$\sin(\hat{\omega}_o n)f[n]$	$\frac{j}{2}\left[\hat{F}(e^{j(\hat{\omega}+\hat{\omega}_o)})-\hat{F}(e^{j(\hat{\omega}-\hat{\omega}_o)})\right]$
$\sin(\hat{\omega}_o n)$	$\frac{\pi}{j}\sum_{k=-\infty}^{\infty}\{\delta(\hat{\omega}-\hat{\omega}_o-2\pi k)-\delta(\hat{\omega}+\hat{\omega}_o-2\pi k)\}$	$\sum_{p=-\infty}^{n}f[p]$	$\frac{1}{1-e^{-j\hat{\omega}}}\hat{F}(e^{j\hat{\omega}})+\pi\hat{F}(e^{j0})\sum_{k=-\infty}^{\infty}\{\delta(\hat{\omega}-2\pi k)\}$
$a^n u[n]\quad(\lvert a\rvert<1)$	$\frac{1}{1-ae^{-j\hat{\omega}}}$	$f^*[n]$	$\hat{F}^*(e^{-j\hat{\omega}})$
$\text{rect}\left(\frac{n}{2N_m}\right)$	$\frac{\sin(\hat{\omega}(N_m+1/2))}{\sin(\hat{\omega}/2)}$	$f[n]-f[n-1]$	$(1-e^{-j\hat{\omega}})\hat{F}(e^{j\hat{\omega}})$
$\frac{\sin(\hat{\omega}_m n)}{\pi n}=\frac{\hat{\omega}_m}{\pi}\text{sinc}\left(\frac{\hat{\omega}_m n}{\pi}\right)$	$\sum_{k=-\infty}^{\infty}\text{rect}\left(\frac{\hat{\omega}-2\pi k}{2\hat{\omega}_m}\right)$	$f[n]*g[n]$	$\hat{F}(e^{j\hat{\omega}})\hat{G}(e^{j\hat{\omega}})$
$\sum_{k=\langle N\rangle}a_k e^{j2\pi kn/N}$	$2\pi\sum_{k=-\infty}^{\infty}a_k\delta\left(\hat{\omega}-\frac{2\pi k}{N}\right)$	$f[n]g[n]$	$\frac{1}{2\pi}\int_{2\pi}\hat{F}(e^{j\hat{\theta}})\hat{G}(e^{j(\hat{\omega}-\hat{\theta})})d\hat{\theta}$
		$nf[n]$	$j\frac{d\hat{F}(e^{j\hat{\omega}})}{d\hat{\omega}}$

DFT properties			
Transform Properties		Transform Properties	
Time Domain	Frequency Domain	Time Domain	Frequency Domain
$\alpha f[n]+\beta g[n]$	$\alpha F[k]+\beta G[k]$	$g[(-n)_{\text{Mod N}}]$	$G[(-k)_{\text{Mod N}}]$
$g[(n-q)_{\text{Mod N}}]$	$G[k]\ e^{-j(2\pi/N)qk}$	$g[n]\ e^{j(2\pi/N)qn}$	$G[(k-q)_{\text{Mod N}}]$
$f[n]\ \circledN\ g[n]=\sum_{\lambda=0}^{N-1}f[\lambda]g[(n-\lambda)_{\text{Mod N}}]$	$F[k]\ G[k]$	$f[n]g[n]$	$\frac{1}{N}F[k]\ \circledN\ G[k]=\frac{1}{N}\sum_{\lambda=0}^{N-1}F[\lambda]G[(k-\lambda)_{\text{Mod N}}]$