Rotating Flux (Complex Vector)

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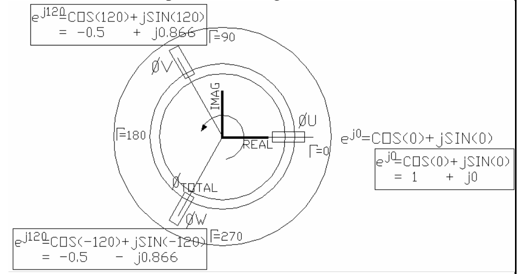

a.) The Stator Windings are oriented on the Real-Imaginary (Complex) Plane

$[e^{j(\theta)}] = \cos(\theta) + j\sin(\theta)$

Winding-A [U] at $[e^{j(0)}] = \cos(0) + j\sin(0) = (1) + j(0) = 1\measuredangle(0)$

Winding-B [V] at $[e^{j(120)}] = \cos(120) + j\sin(120) = (-0.5) + j(0.866) = 1\measuredangle(120)$

Winding-C [W] at $[e^{j(-120)}] = \cos(-120) + j\sin(-120) = (-0.5) + j(-0.866) = 1\measuredangle(-120)$

b.) Phase Currents

> The 3-Phase Currents are given by
> $I_U = I_0 \cos(\boldsymbol{\omega t})$
> $I_V = I_0 \cos(\boldsymbol{\omega t} - 120)$
> $I_W = I_0 \cos(\boldsymbol{\omega t} + 240)$

c.) The Complex Vector [Current Magnitude and Coil Direction in Complex Plane] for each Phase Current in its Coil

a.) $(\bar{I}_A) = \{(I_A)\}\,[e^{j(0)}] = \{(I_0)\cos(\boldsymbol{\omega t})\}\,[1 + j0] = (I_0)\,\{\cos(\boldsymbol{\omega t}) + j(0)\}$

b.) $(\bar{I}_B) = \{(I_B)\}\,[e^{j(120)}] = \{(I_0)\cos(\boldsymbol{\omega t} - 120)\}\,[-0.5 + j0.866]$

$\cos(\boldsymbol{\omega t} - 120) = \cos(\omega t)\cos(-120) - \sin(\omega t)\sin(-120)$

$= \{(-0.5)\cos(\omega t) - (-0.866)\sin(\omega t)\}$

$= \{(-0.5)\cos(\omega t) + (0.866)\sin(\omega t)\}$

$\{(I_B)\}\,[e^{j(120)}] = (I_0)\,\{(-0.5)\cos(\omega t) + (0.866)\sin(\omega t)\}\,[-0.5 + j0.866]$

$= (I_0)\,\{\,[(-0.5)\cos(\omega t) * (-0.5)] + [(-0.5)\cos(\omega t) * (j0.866)] + [(0.866)\sin(\omega t) * (-0.5)] + [(0.866)\sin(\omega t) * (j0.866)]\,\}$

$= (I_0)\,\{\,[(0.25)\cos(\omega t)] + [(-j0.433)\cos(\omega t)] + [(-0.433)\sin(\omega t))] + [(j0.75)\sin(\omega t)]\,\}$

$(\bar{I}_B) = (I_0)\,\{\,[(0.25)\cos(\omega t) - (0.433)\sin(\omega t)] + j[(-0.433)\cos(\omega t) + (0.75)\sin(\omega t)]\,\}$

c.) $(\bar{I}_C) = \{(I_C)\}\,[e^{j(-120)}] = \{(I_0)\cos(\boldsymbol{\omega t} + 120)\}\,[-0.5 - j0.866]$

$\cos(\boldsymbol{\omega t} + 120) = \cos(\omega t)\cos(120) - \sin(\omega t)\sin(120)$

$= \{(-0.5)\cos(\omega t) - (0.866)\sin(\omega t)\}$

$= \{(-0.5)\cos(\omega t) - (0.866)\sin(\omega t)\}$

$\{(I_C)\}\,[e^{j(-120)}] = (I_0)\,\{(-0.5)\cos(\omega t) - (0.866)\sin(\omega t)\}\,[-0.5 - j0.866]$

$= (I_0)\,\{\,[(-0.5)\cos(\omega t) * (-0.5)] + [(-0.5)\cos(\omega t) * (-j0.866)] + [(-0.866)\sin(\omega t) * (-0.5)] + [(-0.866)\sin(\omega t) * (-j0.866)]\,\}$

$= (I_0)\,\{\,[(0.25)\cos(\omega t)] + [(j0.433)\cos(\omega t)] + [(0.433)\sin(\omega t))] + [(j0.75)\sin(\omega t)]\,\}$

$(\bar{I}_C) = (I_0)\,\{\,[(0.25)\cos(\omega t) + (0.433)\sin(\omega t)] + j[(0.433)\cos(\omega t) + (0.75)\sin(\omega t)]\,\}$

d.) The Total Current Vector $(\bar{I}_{ABC}) = (\bar{I}_A) + (\bar{I}_B) + (\bar{I}_C)$

$(\bar{I}_{ABC}) = (I_0)\,\{\cos(\boldsymbol{\omega t}) + j(0)\}$

$+ (I_0)\,\{\,[(0.25)\cos(\omega t) - (0.433)\sin(\omega t)] + j[(-0.433)\cos(\omega t) + (0.75)\sin(\omega t)]\,\}$

$+ (I_0)\,\{\,[(0.25)\cos(\omega t) + (0.433)\sin(\omega t)] + j[(0.433)\cos(\omega t) + (0.75)\sin(\omega t)]\,\}$

$(\bar{I}_{ABC}) = (I_0)\,\{\,[\cos(\boldsymbol{\omega t}) + 0.25\cos(\omega t) + 0.25\cos(\omega t)] + j\,(I_0)\,[(0.75)\sin(\omega t) + (0.75)\sin(\omega t)]\,\}$

$(\bar{I}_{ABC}) = (I_0)\,\{\,(1.5)\,[\cos(\boldsymbol{\omega t})] + j\,(I_0)\,[(1.5)\sin(\omega t)]\,\}$

$(\bar{I}_{ABC}) = [(1.5)(I_0)]\,\{\,[\cos(\boldsymbol{\omega t})] + j\,[\sin(\omega t)]\,\}$

$\{e^{j(\omega t)}\} = \{\,\cos(\omega t) + j\sin(\omega t)\,\}$

$(\bar{I}_{ABC}) = [(1.5)(I_0)]\,\{\,e^{j(\omega t)}\,\} = [(1.5)(I_0)]\,\measuredangle(\omega t)$

The Magnitude of the Total Current-Complex Vector is $(1.5)(I_0)$
The Orientation of the Total Current-Complex Vector rotates at the Rotational Velocity (ω) equal to the Electrical System Frequency $[(\omega) = 2\pi f]$

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>
> 1. Complex vector $\bar{I}_{abc}$ has a constant magnitude $|\bar{I}_{abc}| = 1.5\,I_m$
> 2. Complex vector $\bar{I}_{abc}$ rotates in space at the same frequency as the three-phase variables, ω_e
>
> $i_a = I_m\cos(\omega_e t)$
> $i_b = I_m\cos\left(\omega_e t - \frac{2\pi}{3}\right)$
> $i_c = I_m\cos\left(\omega_e t + \frac{2\pi}{3}\right)$

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