ECE 3030 **Spring 2025**

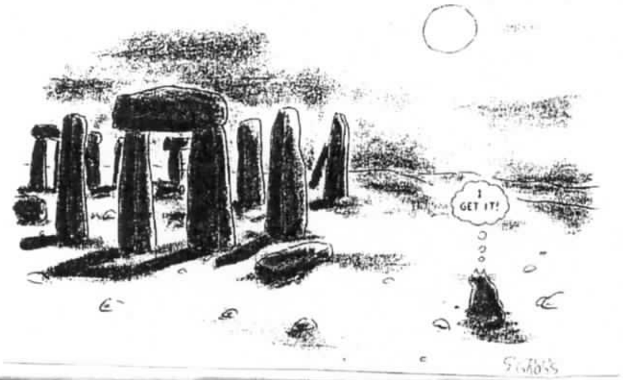
MIDTERM EXAMINATION 1

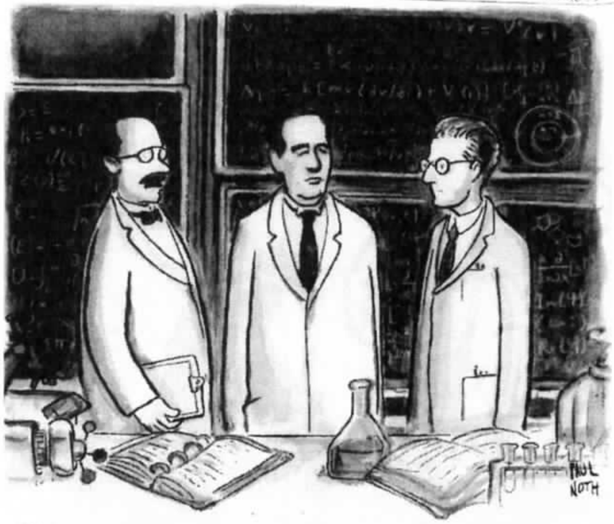
- Closed Book and Notes
- You are allowed to use two double-sided 8.5"x11" sheets of notes.
- A sheet of information (more than you need) is attached at the back of this exam.
- Make Sure You Have All of the Pages. Budget your time wisely.
- Show All Work for Full Credit

ECE Honor Code Pledge: *"No aid given, received or observed."*

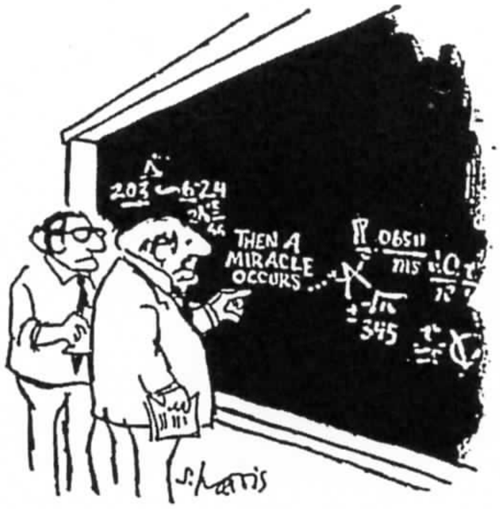
Signature: ______________________________

Print Name Here: Solutions



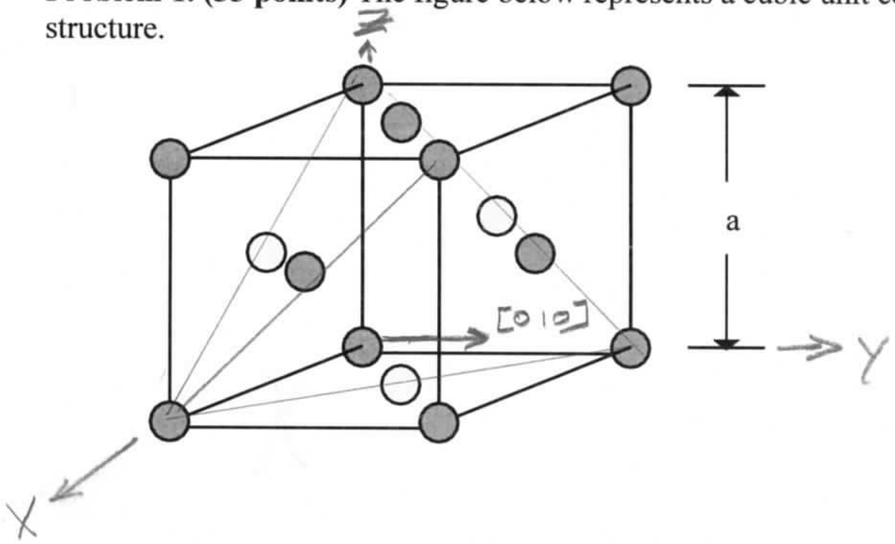


"We've agreed to count it as both a wave and a particle for tax purposes."



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

Problem 1. (35 points) The figure below represents a cubic unit cell with FCC lattice structure.



A. (5 Points) Draw and label the standard coordinate system axes (x, y, z) on the figure. Draw the vector corresponding to the [010] direction.

B. 10 Points) Calculate the number of atoms/unit length along this direction in cm^{-1}, given that a= 5.43 x 10^{-8} cm. What is the line density along this direction?

1 atom / a = 1/5.43 Å = 0.184 × 10^{+8} atoms/cm
= 1.84 × 10^7 atoms/cm

Solve for R vs a:

$4R = \sqrt{2}a$, $R = \frac{a}{2\sqrt{2}}$

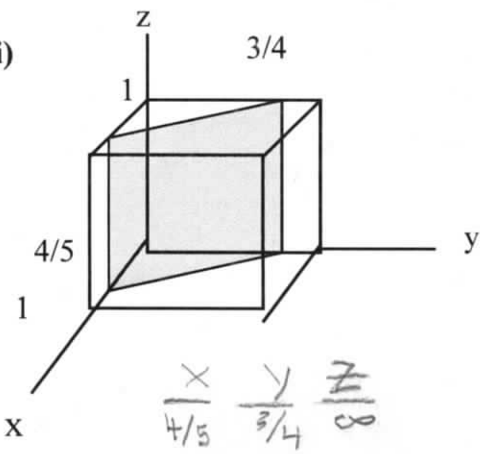
2R along [010]

Line Density = $\frac{2R}{a} = \frac{a/\sqrt{2}}{a} = \frac{1}{\sqrt{2}} = 0.707$

C. (5 Points) Sketch the (111) plane on the unit cell figure above.

C. (8 Points) Determine the Miller indices for the planes in the cubic unit cells below:

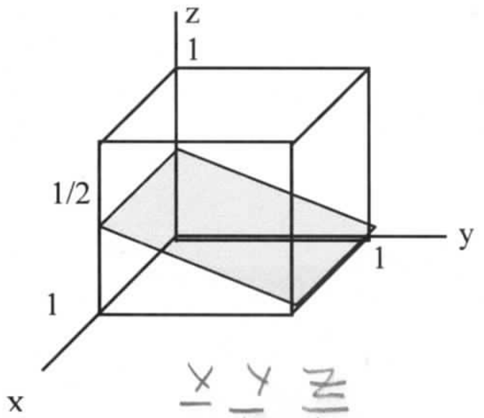
(i)



x	y	z
4/5	3/4	∞
$\frac{5}{4}$	$\frac{4}{3}$	0

(15, 16 0)

(ii)



x	y	z
∞	1	½
0	1	2

(012)

D. (10 Points) A common metal is known to have a cubic unit cell with an edge length a = 4.95 x 10^{-8} cm. If this metal has a density of 11.35 g/cm^3 and an atomic weight of 207.2 g/mol, calculate how many atoms there are in its unit cell.

$$\rho = \frac{n}{V_c}\frac{A}{N_A} \rightarrow \frac{n}{V_c} = \frac{N_A \rho}{A} = \frac{(6.02\times10^{23}\ \text{atoms/mol})(11.35\ \text{g/cm}^3)}{207.2\ \text{g/mol}}$$

$$\frac{n}{V_c} = 0.329 \times 10^{23}\ \text{atoms/cm}^3$$

$$V_c = a^3 = (4.95\times10^{-8}\ \text{cm})^3$$

$$n = (0.329\times10^{23}\ \text{atoms/cm}^3) \times (4.95\times10^{-8})^3$$

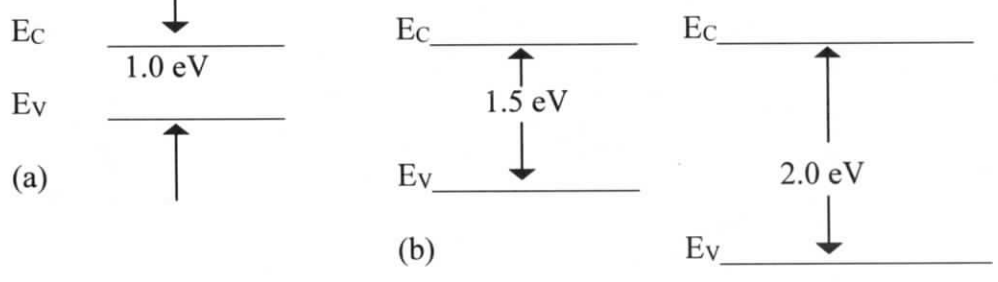
$$= \underline{4\ \text{atoms}}$$

E. (2 Points) Based on this calculation, which crystal structure does this metal have - SC, BCC, or FCC?

FCC $\left(6\times\frac{1}{2} + 8\times\frac{1}{8} = 4\right)$

Problem 2. (30 points)

A. (5 points) Here are 3 semiconductor band gaps. Which semiconductor has the SMALLEST value of intrinsic carrier concentration at 300 K? (Mark one.) Give a reason and an equation that explains why.



(c) ✓ Largest band gap allows smallest number of electrons excited from valence to conduction band

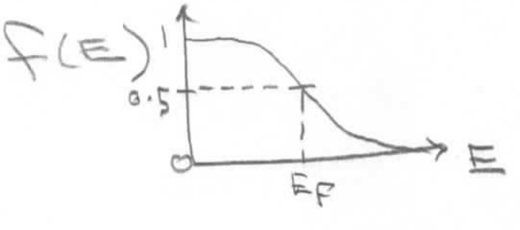
$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2k_B T}$$

B. (10 Points) Define the Fermi level E_F in words and draw a diagram with the Fermi-Dirac distribution function at temperature T > 0 K. Label the vertical and horizontal axes with values.

Fermi level equals energy at which a state has exactly 50% probability of occupancy

$$f(E = E_F) = 0.5$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$



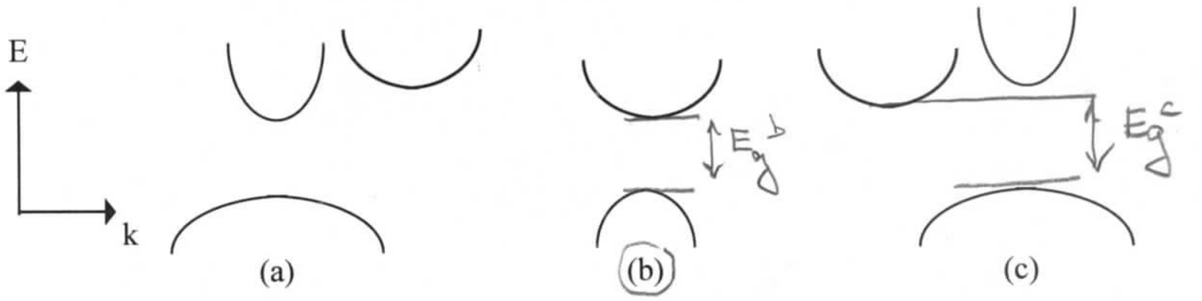
C. (5 Points) If the E_F is positioned exactly at E_C (i.e., $E_F = E_C$), calculate the probability of finding an electron in an energy state at $E_C + kT$.

$E = E_c + kT \quad E_F = E_c$

So $f(E = E_c + kT) = \dfrac{1}{1 + e^{(E_c + kT - E_c)/kT}} = \dfrac{1}{1 + e^{kT/kT}}$

$= \dfrac{1}{1+e^{1}} = \dfrac{1}{1+2.72} = 0.269$ or 26.9%

D. (3 Points) For the E versus k diagrams shown below, which case has the smallest valence band effective mass? Write the equation for effective mass and explain why.



$m^* = \dfrac{\hbar^2}{\partial^2 E / dk^2}$

Case (b) has the most valence band curvature and therefore the smallest m^*.

D. (3 Points) In the diagram above, which of the semiconductor band diagrams has the highest electron mobility, (a) or (b)? Write the equation for mobility to explain why.

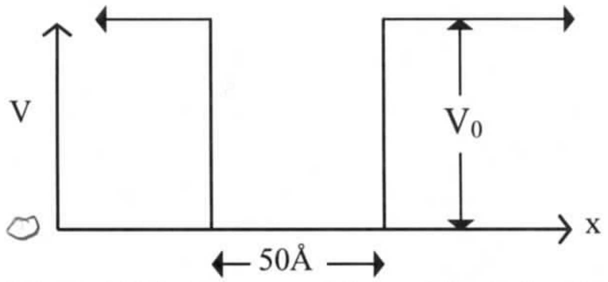
(a) has the highest electron mobility since it has a conduction band with the largest curvature of the lowest conduction band. $\mu = \dfrac{q\langle \tau \rangle}{m^*}$ Higher μ for lower m^*

E. (2 Points) Draw the minimum band gap between conduction band and valence band in (b) and in (c) above. See above.

F. (2 Points) Which band structure would not be useful for a laser? Explain why.

(c) because it has an indirect bandgap. Momentum transfer is required for electrons in the conduction band to recombine with valence band holes.

Problem 3. (35 Points) You are given a potential well with a well width of 50 Angstroms (1 Å = 1 x 10^{-8} cm.), as shown.



A. (5 Points) Write the general form of the Schrodinger equation expression for the region *inside* the well. Do not solve! For $V_0 = \infty$, are the wave function solutions of this equation traveling waves or standing waves with these boundary conditions?

$$\frac{d^2\psi(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$ since $V = 0$ inside well.

Standing waves since can't travel.

B. (15 Points) If the infinite potential well solution that we derived in class holds when the well is made to have finite barriers of $V_0 = 0.15$ eV, determine the number of energy levels which are confined within the quantum well. What are the values of these energies? (Assume electron mass $m_0 = 9.1 \times 10^{-31}$ kg or $h^2/8m_0 = 38$ eV Å^2.)

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{38\ \text{eV-Å}^2 n^2}{L^2} = \frac{38\ \text{eV-Å}^2 n^2}{(50\text{Å})^2}$$

$= 0.0152$ eV for $n=1$

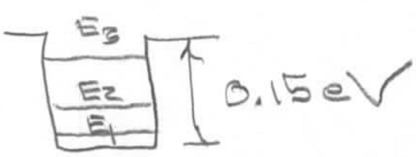
$E_1 = 0.0152$ eV for $n=1$ so $E_1 < V_0$

$E_2 = 0.061$ eV for $n=2$ $\quad E_2 < V_0$

$E_3 = 0.137$ eV for $n=3$ $\quad E_3 < V_0$

$E_4 = 0.243$ eV for $n=4$ $\quad E_4 > V_0$

so the maximum number of levels inside the well is 3.

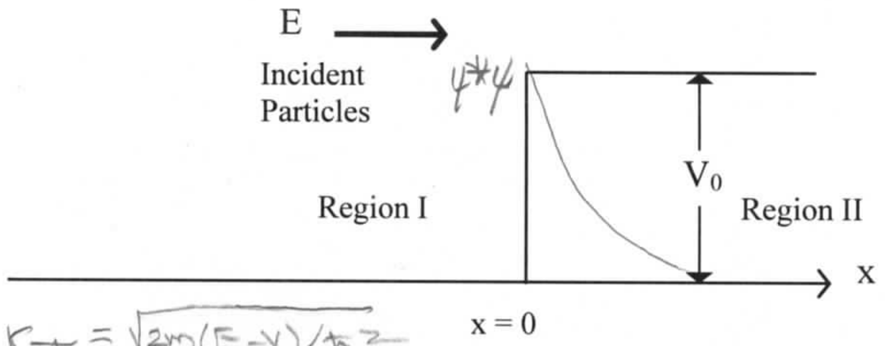


C. (5 Points) For part B, does the number of energy levels confined in the quantum well increase or decrease as the well width is increased? Why? Explain with the quantum well energy level equation.

$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$ decreases as L increases.

So more levels can fit inside the well.

D. (10 points) Now consider a step potential as shown.
- Draw the sinusoidal wavefunctions (/\/\/\/→) and (←/\/\/\/) for particles with energy E traveling in the +x direction for x <0 and for x > 0 with E > V_0. Your sinusoids should have different wavelengths and amplitudes depending on their energy and k values.
- Write expressions for their wavelengths based on their k values.
- Will there be a reflected wave in region I? Will there be a reflected wave in region II?
- Draw the relative probability $|\psi^*\psi|$ of finding the particle in region II starting from the interface if E <V_0.



$K_I = \sqrt{2mE/\hbar^2}$ $\quad K_{II} = \sqrt{2m(E-V)/\hbar^2}$

$K_I > K_{II}$

$P = \frac{h}{\lambda} \rightarrow \lambda = \frac{h}{p} = \frac{h}{\hbar K} = \frac{2\pi}{K}$

Larger K → smaller λ.

Region I: Reflected wave at X=0.

Region II: No interface at X>0 to reflect.

$P = |\psi^*\psi| = 1$ at $X=0$

$P = e^{-2KX}$ for $X>0$ $\quad$ See above.

****Extra Credit****

EC1. (5 Points) What is the relative probability of an electron with incident energy E = 3.7 eV penetrating to a position 3 x 10^{-10} meters inside the step potential (region II) of Problem 3 above for V_0 = 6.0 eV? Use the free electron mass m_0. (Hint: the probability can be taken as equal to 1.0 at x = 0.)

$P = P(0)\,e^{-2k'x} \qquad k' = \sqrt{2m(E-V)/\hbar^2} = \sqrt{2m(6-3.7)/\hbar^2}$

$k' = \left[2(9.11\times10^{-31}\,kg)(2.3\,eV)(1.6\times10^{-19}\,J/eV)\right]^{1/2} / 1.06\times10^{-34}\,J/s$

$= \left[67.05\times10^{18}\right]^{1/2}/1.06 = 7.72\times10^{9}\,m^{-1}$

$P = e^{-2k'x} = e^{-2\cdot 7.72\times10^{9}\cdot 3\times10^{-10}\,m} = e^{-4.632} = \underline{9.74\times10^{-3}}$

$= \underline{0.97\,\%}$

EC2. (5 Points) For intrinsic Ge, μ_p = 1900 cm²/V-s, n_i = 2.5 x 10^{13} cm^{-3}, and its conductivity is 2.32 x 10^{-2} (Ω-cm)$^{-1}$, Calculate μ_n.

$\sigma = q(\mu_e n + \mu_p P)$

$\sigma_i = q(\mu_e + \mu_p)\,n_i$

$= 1.602\times10^{-19}\,C\,(\mu_e + 1900\,cm^2/V\text{-}s)(2.5\times10^{13}\,cm^{-3}) = 2.32\times10^{-2}(\Omega\text{-}cm)^{-1}$

$\mu_e + 1900 = \dfrac{2.32\times10^{-2}}{(1.6\times10^{-19}\,C)(2.5\times10^{13}\,cm^{-3})}\,cm^2/V\text{-}s$

$= 5.8\times10^{3}\,cm^2/V\text{-}s$

$\mu_e = 5{,}800 - 1{,}900 = \underline{3{,}900}\,cm^2/V\text{-}s$

Note $V = IR \Rightarrow V = C\,\Omega$

Some Useful Information and Equations (more than you need)

$N_A = 6.02 \times 10^{23}$ atoms/mole = Avogadro's number

$1 \text{ eV} = 1.602 \times 10^{-19}$ Joule

$h/2\pi = \hbar = 1.06 \times 10^{-34}$ J-s

$q = 1.6 \times 10^{-19}$ C

$m_0 = 9.11 \times 10^{-31}$ kg $= 9.11 \times 10^{-28}$ gm

$k_B = 1.38 \times 10^{-23}$ J/atom-K $= 8.62 \times 10^{-5}$ eV/atom-K

$k_BT == 0.0259$ eV at room temperature (RT) =300°K

k_BT at any temperature = 0.0259 eV x(T/300)

T °C = 0 in Celsius = 273 +°C in Kelvin

$\rho = nA/V_CN_A$ where n = atoms/unit cell, A = atomic weight, V_C = unit cell volume

❶ $n_0\, p_0 = n_i^2$ Law of Mass Action

❷ $N_A + n_o = N_D + p_0$ Charge Neutrality Law (assuming complete ionization, i.e., $N_D \sim N_D^+$ and $N_A \sim N_A^-$)

Intrinsic carrier concentration

$n_i(T) = (N_CN_V)^{1/2}\, e^{-Eg/2kT}$ (Assumes E_g independent of T, good to 1st approximation)

$n_0 = N_C\ e^{-(E_C - E_F)/kT} = n_i\ e^{(E_F - E_i)/kT}$

$p_0 = N_V\ e^{-(E_F - E_V)/kT} = n_i\ e^{(E_i - E_F)/kT}$

n-type: $n_0 = (N_D - N_A) + p_0 \simeq N_D$
$p_0 = n_i^2/n_0$

p-type: $p_0 = (N_A - N_D) + n_0 \simeq N_A$
$n_0 = n_i^2/p_0$

Exact solution for arbitrary N_A and N_D (e.g., compensated): Use ❶ And ❷ together.

Si data at 300 K: $N_C = 2.82 \times 10^{19}$ cm^{-3}, $N_V = 1.04 \times 10^{19}$ cm^{-3}, $n_i = 1.5 \times 10^{10}$ cm^{-3},

Si for arbitrary temperature T in degrees K:

$N_C(T) = 2.82 \times 10^{19}\, (T/300)^{3/2}$; $N_V(T) = 1.83 \times 10^{19}\, (T/300)^{3/2}$

$E_g = 1.11$ eV, and (for intrinsic Si) $\mu_n = 1350$ cm^2/V-sec, $\mu_p = 480$ cm^2/V-sec.

Ge band gap $E_g = 0.67$ eV at T = 300 K.