Welcome!

- CSE 2321: Foundations I: Discrete Structures
- Dr. Charles Estill (He/Him)

Class Conventions

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- In the pseudocode, indentation will be used to indicate nested code. For example:

  FUNCTION $Total(n)$

      $x \leftarrow 0$

      FOR $i \leftarrow 1$ TO n DO

          $x \leftarrow x + i$

      RETURN(x)

Sigma Summation, $\sum$

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means to add up all the terms of the form $f(k)$ (or f_k) where k starts at an integer a, goes up by one each time, and finishes at an (other) integer b. Or, more explicitly,

$$f(a) + f(a+1) + f(a+2) + \cdots + f(b-2) + f(b-1) + f(b).$$

$$\sum_{k=a}^{a} f(k) = f(a) \quad \text{and} \quad \sum_{k=3}^{1} 2 = 0$$

Propositions

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- $8 + 3 = 10$
- Any field of characteristic zero contains a copy of the rational numbers.
- The earth is 90% cheese.

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- The whole number x is a perfect square.

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  (There is no reasonable way to interpret this except as poetry.)
- The whole number x is a perfect square.

  (We'll return to this, but the main issue is that the truth of the statement depends on what x is.)

Logical Connectives

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The first of these methods are the logical connectives. For example, the English word “and” is somewhat equivalent to the logical $\wedge$, where $P \wedge Q$ is true when both P and Q are true.

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dictionary definition

other (unstated) definition

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In addition, in natural language there are multiple words sharing a denotation but with different connotations. E.g., in translating from English to math, the word "BUT" would usually translate to $\wedge$, but it frequently suggests surprise on the part of the speaker: "$\underbrace{\text{Joe Montana threw for five touchdowns}}_{M}$ but $\underbrace{\text{the 49ers lost}}_{L}$".

$M \wedge L$

Loss of Meaning

Minor trigger warning: I use being nauseous and its effects in an example here

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Similarly in other natural languages, there will be ambiguity in how one is to interpret the equivalent translations.

Truth Tables

In order to avoid this ambiguity we introduce the main tool of propositional logic: truth tables.

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P	Q	$P \vee Q$
0	0	0
0	1	1
1	0	1
1	1	1

Note that this is what is sometimes called the inclusive-or.

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P	Q	R	f(P,Q,R)
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

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If there are n inputs, how many rows (not including the header) should there be in the truth table?

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If there are n inputs, how many rows (not including the header) should there be in the truth table? 2^n, because there's two possibilities for each of the n variables and a choice for one doesn't affect the choice for the others.

The Commonly Used Connectives

So far we've talked about $\wedge$ and $\vee$:

P	Q	$P \wedge Q$
0	0	0
0	1	0
1	0	0
1	1	1

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0	0	0
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$\sim P$

P	$\neg P$
0	1
1	0

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P	$\neg P$
0	1
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The other common logical connectives are $\neg$ (called "not"), the conditional $\Rightarrow$ ("P implies Q"), and the biconditional $\Leftrightarrow$, which have the following truth tables:

$P \Rightarrow Q$
and
$P \Leftarrow Q$ $(Q \Rightarrow P)$

P	$\neg P$
0	1
1	0

P	Q	$P \Rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

P	Q	$P \Leftrightarrow Q$
0	0	1
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The Conditional

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With some exceptions the English/math translations tend to be as follows:

if P then Q	$P \Rightarrow Q$
P if Q	$P \Leftarrow Q$
P only if Q	$P \Rightarrow Q$ or $Q \Rightarrow P$
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But not always. The conditional is especially troublesome to translate.

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P: "no shoes"

Q: "no service"

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As an example consider the common sign in restaurants "No shoes, no shirt, no service." interpreted as "If you're not wearing shoes and a shirt then we will not serve you." (Or, simpler: "No shoes implies no service".)

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"If you were served, you were wearing shoes"

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"no service implies no shoes"

Why is $0 \Rightarrow P$ Always True?

The Agreement (not an actual agreement)

Suppose I tell you "If you study for ten hours a day, you will pass the class." We can translate this sentence into logic as follows:

$P =$ "You study 10 hours a day."	$Q =$ "You'll pass the class"	$P \Rightarrow Q$
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If we think of the original sentence as an agreement between us, we see that the agreement isn't violated by the first two rows.

In addition, consider the following truth table:

P	Q	$P \Rightarrow Q$	$\neg Q$	$\neg P$	$(\neg Q) \Rightarrow (\neg P)$
0	0	1	1	1	1
0	1	1	0	1	1
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Finally, consider that if you knew that $A \Rightarrow B$ and $B \Rightarrow C$, what else might you consider to be true? That $A \Rightarrow C$ is one pretty reasonable thing. Or, in other words, you'd want $((A \Rightarrow B) \wedge (B \Rightarrow C)) \Rightarrow (A \Rightarrow C)$ to be always true. It turns out that the truth table for $\Rightarrow$ is the only choice that makes that work.

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3. (or similarly constructed strings with any connectives that we may define in the future), or
4. of the form $(thing1)$ where $thing1$ is a WFS.

An Example

Consider the example

$$((Q \Rightarrow P) \wedge \big(((\neg P) \vee Q) \wedge (R \Rightarrow R)\big)) \Leftrightarrow P.$$

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			A	B	C	D	
P	Q	R	$Q \Rightarrow P$	$((\neg P) \vee Q)$	$R \Rightarrow R$	$A \wedge (B \wedge C)$	$D \Leftrightarrow P$
0	0	0	1	1	1	1	0
0	0	1	1	1	1	1	0
0	1	0	0	1	1	0	1
0	1	1	0	1	1	0	1
1	0	0	1	0	1	0	0
1	0	1	1	0	1	0	0
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1