Practice Sheet Review

2

5.) FUNCTION $P_5(n)$
```
x ← 1
i ← 1
WHILE i < n DO
    j ← 1
    WHILE j < n^3 DO
        x ← x + 1
        j ← 2j + 1
    i ← 3i
RETURN(x)
```

loop counter k	value of i
0	1
1	3
2	3^2
3	3^3
⋮	⋮
k	$i = 3^k$

$i = 3^k < n$

$k < \log_3 n$

loop ctr q	value of j
0	$1 = 1$
1	$2+1 = 3$
2	$2^2+2+1 = 7$
3	$2^3+2^2+2+1 = 15$
4	$2^4+2^3+2^2+2+1 = 31$
⋮	⋮
q	$j = 2^{q+1} - 1$

$2^{q+1} - 1 < n^2$

$2^{q+1} < n^2 + 1$

$q + 1 < \lg(n^2+1)$

$q < \lg(n^2+1) - 1$

$$r_5(n) = \sum_{k=1}^{\log_3 n} \sum_{q=1}^{\lg(n^2+1)-1} C$$

$$= C\left(\lg(n^2+1) - 1\right)\log_3 n$$

O-work

$$r_5(n) \le C \lg(n^2+1)\log_3 n \le C\lg(n^2+n^2)\log_3 n$$

$$= C\lg(2n^2)\log_3 n \ge C\lg(n \cdot n^2)\log_3 n = 3C(\lg n)(\log_3 n)$$

for $n \geq 2$ $\quad \therefore r_5(n) \in O(\log^2 n)$

Ω-work

$$r_5(n) \ge C\left(\lg(n^2) - 1\right)\log_3 n \ge C\left(\lg n^2 - \lg n\right)\log_3 n$$

for $1 \le \lg n$

$2 \le n$

$$= C\left(\lg \frac{n^2}{n}\right)\log_3 n = C(\lg n)(\log_3 n)$$

$$\therefore r_5(n) \in \Omega(\log^2 n) = \Omega\left((\log n)^2\right)$$