ECE 3030 Homework 2

1) a) platinum = 5.65 eV $\rightarrow$ $5.65\,\text{eV} \times 1.6\times10^{-19}\,\text{J/eV} = \boxed{9.04\times10^{-19}\,\text{J}}$

b) lithium = 2.90 eV $\rightarrow$ $2.90\,\text{eV} \cdot 1.6\times10^{-19}\,\text{J/eV} = \boxed{4.64\times10^{-19}\,\text{J}}$

c) As light wavelength λ increases, more electrons are excited and ejected.

d) The number of electrons increases, and the energy threshold does not change.

2) GaAs $a_0 = 5.65\,\text{Å}$ $\lambda = \frac{h}{p} = 5.65\,\text{Å}$ $h = 6.63\times10^{-34}\,\text{J sec}$

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{5.65\,\text{Å}} = 1.17\times10^{-34}\,\text{kg m/s} \rightarrow \boxed{1.17\times10^{-29}\,\text{g cm/s}}$$

$$E = \frac{p^2}{2m} = \frac{1.17\times10^{-29}\,\text{g cm/s}}{2\cdot 9.11\times10^{-28}\,\text{g}} = \boxed{6.42\times10^{-3}\,\text{eV}}$$

3) a) $\Delta x \Delta p \geq \frac{\hbar}{2} \rightarrow \Delta p \geq \frac{\hbar}{2\Delta x} \rightarrow \frac{1.054\times10^{-34}}{2\times1.7\times10^{-7}}$ $\boxed{\Delta p = 3.1\times10^{-21}\,\text{g cm/s}}$

b) $\Delta E \Delta t \geq \frac{\hbar}{2} \rightarrow \Delta t \geq \frac{\hbar}{2\Delta E} \rightarrow \frac{1.054\times10^{-34}}{2\times3.204\times10^{-19}}$ $\boxed{\Delta t = 1.64\times10^{-16}\,\text{s}}$

$2\,\text{eV} \rightarrow 3.204\times10^{-19}\,\text{J}$

4) a) $E_1 = \frac{\pi^2\hbar^2}{2mL^2} = \boxed{2.08\times10^{-20}\,\text{eV}}$

$\hbar = 1.054\times10^{-34}$

$m = 9.109\times10^{-31}$

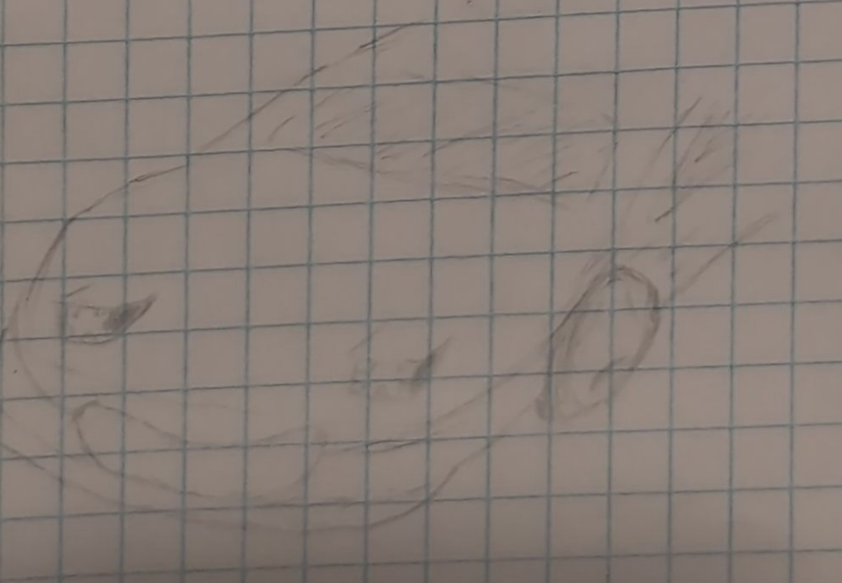
$L = 17\,\text{Å} = 17\times10^{-10}\,\text{m}$

$E_2 = \frac{2^2\pi^2\hbar^2}{2mL^2} = \boxed{8.32\times10^{-20}\,\text{eV}}$

$E_3 = \frac{3^2\pi^2\hbar^2}{2mL^2} = 18.72\times10^{-20}\,\text{eV} \rightarrow \boxed{1.87\times10^{-19}\,\text{eV}}$

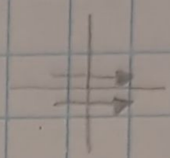
b) $\Delta E = E_3 - E_1 = 18.72 - 2.08 = 16.64\times10^{-20}\,\text{eV}$

$$\lambda = \frac{hc}{\Delta E} = \frac{6.626\times10^{-34}\times3\times10^{8}}{16.64\times10^{-20}} = \boxed{1.19\times10^{-6}\,\text{m}}$$



5) $\psi(x) = A e^{jk x} + B_1 e^{-jk_1 x}$ (for + and − x directions)

$k_1 = [2m(E-V)/\hbar^2]^{1/2}$ $\quad E_1 > V = 0$



a) $\boxed{k_1 = \sqrt{\frac{2mE_1}{\hbar^2}}}$ since $V=0$

b) $\boxed{\psi(x<0) = A_1 e^{jk_1 x} + B_1 e^{jk_1 x}}$

c) $\boxed{\psi(x>0) = A_2 e^{-k_2 x} + B_2 e^{k_2 x}}$ where $\boxed{k_2 = \sqrt{\frac{2m(-E)}{\hbar^2}}}$

d) $\psi(x=0) = A_1 e^{jk_1 x} + B_1 e^{-jk_1 x} = A_2 e^{-k_2 x} + B_2 e^{k_2 x}$

$\boxed{jk_1 A_1 - jk_1 B_1 = -k_2 A_2}$ since $\left.\frac{\partial \psi_1}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_2}{\partial x}\right|_{x=0}$

1st derivative is continuous at $x=0$

e) The wave will travel faster with $x>0$.

There are reflected waves for both $x>0$ and $x \leq 0$.

6) $P \propto e^{-2kx}$ $\quad E_k = 3.3\,eV = 5.2866 \times 10^{-19}\,J \quad V_0 = 4\,eV = 6.408 \times 10^{-19}\,J$

a) $k = \sqrt{\frac{2m(V_0-E)}{\hbar^2}} = \sqrt{\frac{2m(1.12\times10^{-19})}{\hbar^2}} = 4.28 \times 10^{9}$

$5\,Å = 5 \cdot 10^{-10}\,m$

$P \propto e^{-2(4.28\times10^9)(5\times10^{-10})} = \boxed{e^{-4.28}}$

b) $P \propto e^{-8.56\times10^9(10\times10^{-10})} = \boxed{e^{-8.56}}$

c) $P \propto e^{8.56\times10^9(40\times10^{-10})} = \boxed{e^{-34.24}}$

7) a) $m = 0.067 m_o$ $V_o = 0.8\,eV = 1.2816\times10^{-19}\,J$ $w = 15\times10^{-10}\,m$

$E_k = 0.2\,eV = 3.204\times10^{-20}\,J$

$T \cong 16\left(\frac{E}{V_o}\right)\left(1-\frac{E}{V_o}\right)e^{-2k_2 w}$ $k_2 = \sqrt{\frac{2(0.067)m(9.612\times10^{-20})}{\hbar^2}} = 1.027\times10^{9}$

$T \cong 16\left(\frac{3.204\times10^{-20}}{1.2816\times10^{-19}}\right)\left(1-\frac{3.204\times10^{-20}}{1.2816\times10^{-19}}\right)e^{-2(1.027\times10^{9})(15\times10^{-10})} = \boxed{0.13784}$

b) $k_2 = \sqrt{\frac{2(1.08)m(9.612\times10^{-20})}{\hbar^2}} = 4.1223\times10^{9}$

$T \cong 16(\sim)(1-\sim)e^{-2(4.1223\times10^{9})(15\times10^{-10})} = \boxed{1.277\times10^{-5}}$

The particle with a greater effective mass ($1.08\,m_o$) tunnels much less than the particle with a lower mass.

8) $T_M = 5\times10^{6}$ $E = 0.08\,eV = 1.2816\times10^{-20}\,J$ $V_o = 0.8\,eV = 1.2816\times10^{-19}\,J$

$5\times10^{6} \le 16\left(\frac{1.2816\times10^{-20}}{1.2816\times10^{-19}}\right)\left(1-\frac{1.2816\times10^{-20}}{1.2816\times10^{-19}}\right)e^{-2(4.345\times10^{9})w}$

$5\times10^{6} \le 1.44\,e^{-8.2446\times10^{9}\cdot w}$

$k_2 = \sqrt{\frac{2m(1.15\times10^{-19})}{\hbar^2}} = 4.345\times10^{9}$

$\boxed{w \ge -1.8267\times10^{-9}\,m}$