Last Class…

A current source in a linear circuit has i = 15 cos(25πt +25)

A. What is the amplitude of the current? $A = 15$

B. What is the angular frequency? $\omega = 25\pi$

C. Find the frequency of the current. $f = \frac{\omega}{2\pi} = 12.5\ Hz$

D. What is the phase? $\phi = 25^{\circ}$

$\omega = 2\pi f$

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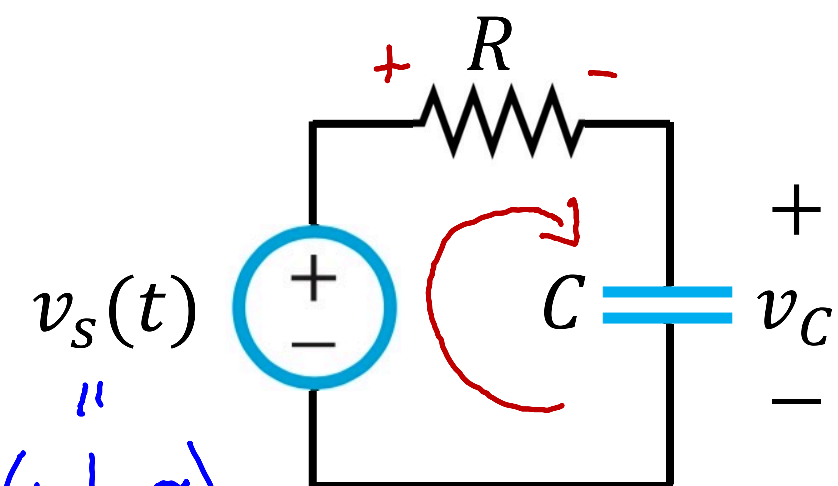
Phasor Domain

This Lecture…

- Learning Objectives:
    - Transform time-varying sinusoidal functions to the phasor domain and vice versa.

On time domain:

- AC circuit with capacitors or inductors challenging
  - i-v relationships are time dependent



$$= A\cos(\omega t + \phi)$$

series, same current

$$i_c(t) = C \cdot \frac{dV_c(t)}{dt} = i_R$$

KVL

$$A\cos(\omega t + \phi) = V_R + V_c(t)$$

$$A\cos(\omega t + \phi) = R \cdot i_R + V_c(t)$$

$$A\cos(\omega t + \phi) = RC\frac{dV_c(t)}{dt} + V_c(t)$$

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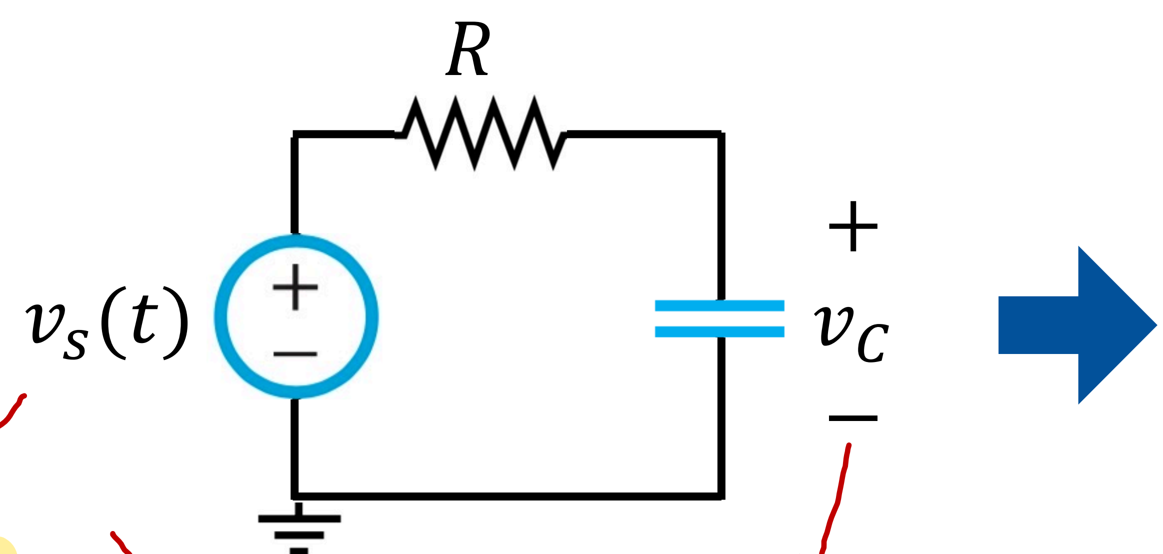
AC Circuits

- AC circuit: Circuit with a sinusoidal source.
- AC circuit with capacitors or inductors is described by a differential equation.
  - May be challenging to solve because i-v relationships are time dependent.



$A\cos(\omega t + \phi)$

$B\cos(\omega t + \gamma)$

Phasor domain

- Sinusoidal signals can be represented as complex numbers.
- Differential equations get converted into linear equations with no sinusoidal functions.

Phasor Domain

- The phasor-analysis technique transforms equations from the time domain to the phasor domain.

Time domain:

time is lowercase

$$v(t) = A\cos(\omega t + \varphi)$$

$$= \text{Real}\left\{Ae^{(\omega t + \phi)j}\right\}$$

$$= \text{Real}\left\{\underbrace{Ae^{\phi j}}_{\text{Real}} + e^{\omega t j}\right\}$$

same for everything

same ω

Phasor domain:

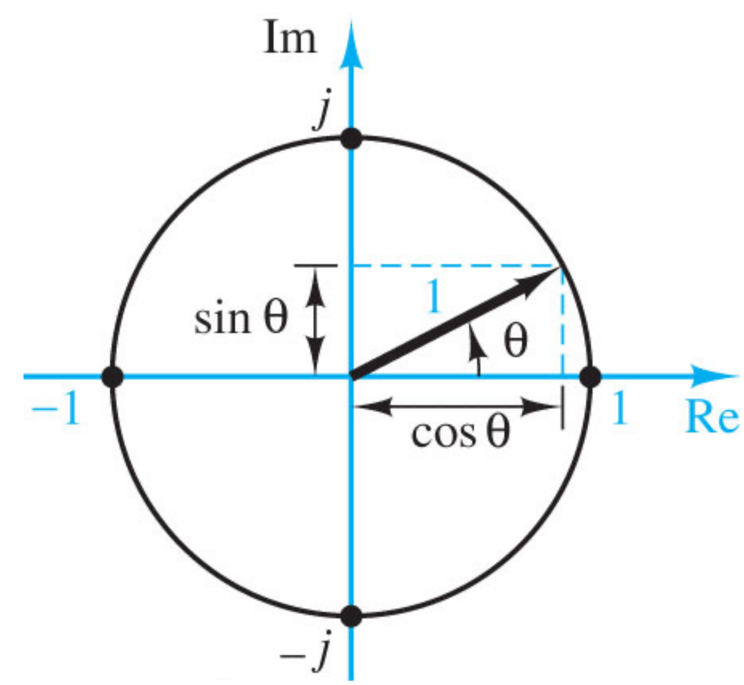
phasor is bold

$$\mathbf{V}(j\omega) = Ae^{\theta j}$$

Frequency is common to all voltages and currents, so it is avoid in the phasor form.

Euler's identity: $z = me^{\theta j}$

$$z = \underbrace{m\cos(\theta)}_{\text{real}} + \underbrace{m\sin(\theta)j}_{\text{imaginary}}$$



Phasor Domain

Time domain:

$v_1(t) = A\cos(\omega t + \varphi) = 10\cos(5t+45)$

$v_2(t) = A\cos(\omega t) = \cos(100t)$

$v_3(t) = A\cos(\omega t - 90°) = 5\cos(t-90)$

$v_4(t) = A\sin(\omega t) = 20\sin(2t)$

$\hookrightarrow 20\cos(2t-90)$

$$\frac{dv(t)}{dt}$$

$$\int v(t)\,dt$$

$V_c(t) = 10\cos(10t+15) \leftarrow V_5(10j) = 10e^{15j}$

Phasor domain:

$V_1(5j) = 10e^{45j}$

$V_2(100j) = 1\cancel{e^{0j}} = 1$

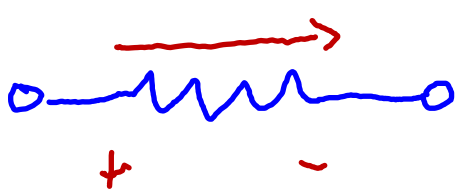
$V_3(1j) = 5e^{-90j} = -5j$

$V_4(2j) = 20e^{-90j} = -20j$

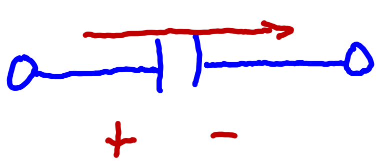
Impedance

- *i-v* relationships of resistors, inductors, and capacitors can be expressed in phasor notation.
- Phasors and impedance simplify AC circuit analysis.
  - Allow use of same solution methods as DC circuits.

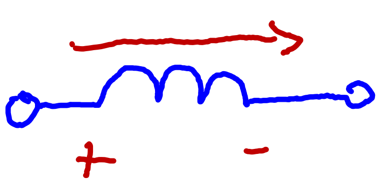
time domain



$V_R = R \cdot i_R$

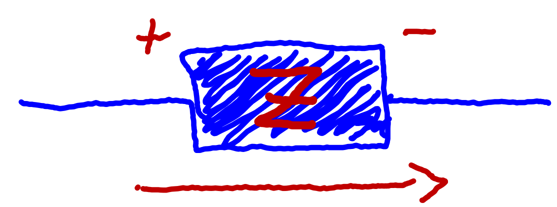


$i_C(t) = C \cdot \frac{dV_C(t)}{dt}$



$V_L(t) = L \cdot \frac{di_L(t)}{dt}$

phasor domain



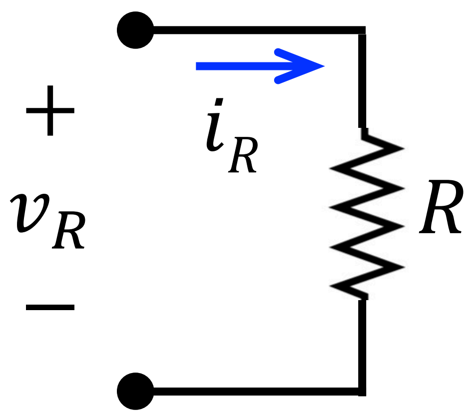
$\mathbf{V} = Z\mathbf{I}$

$Z = \frac{\mathbf{V}}{\mathbf{I}}$

Impedance

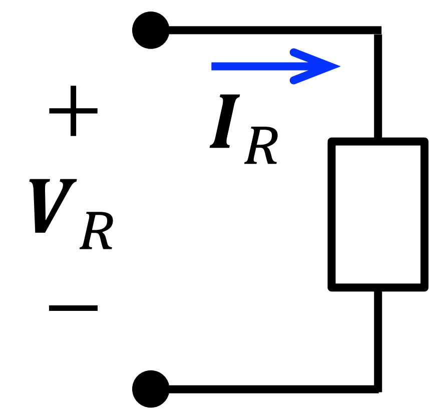
i-v relationships of resistors, inductors, and capacitors can be expressed in phasor notation.

Time domain:



Impedance: Ratio of phasor voltage to phasor current.

Phasor domain:



$$i_R = I_m \cos(\omega t + \theta_I)$$

$$V_R = R \cdot i_R$$
$$= R I_m \cos(\omega t + \theta_I)$$

$$I_R(j\omega) = I_m e^{\theta_I j}$$

$$V_R(j\omega) = R I_m e^{\theta_I j}$$
$$= R I_R(j\omega)$$

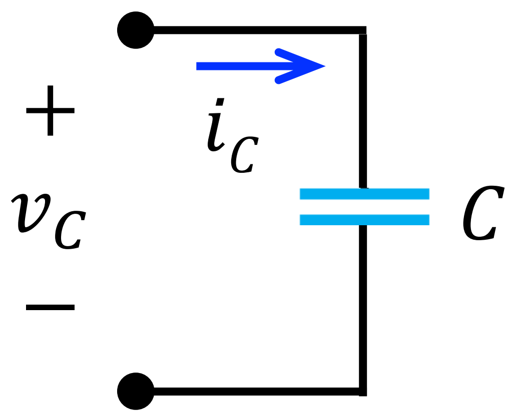
$$\frac{V_R(j\omega)}{I_R(j\omega)} = R = Z_R$$

Impedance

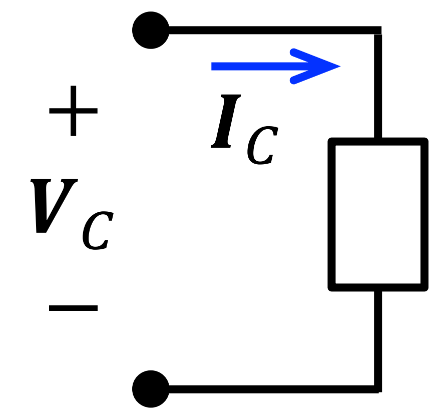
Impedance: Ratio of phasor voltage to phasor current.

$$\boxed{\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}}}$$

Time domain:



Phasor domain:



$$v_c(t) = V_m \cos(\omega t + \theta_v)$$

$$i_c(t) = C \cdot \frac{dv_c(t)}{dt}$$

$$= -CV_m \sin(\omega t + \theta_v) \cdot \omega$$

$$= -\omega C V_m \cos(\omega t + \underbrace{\theta_v - 90}_{\text{phase}})$$

$$V_c(j\omega) = V_m e^{\theta_v j}$$

$$I_c(j\omega) = -\omega C V_m e^{(\theta_v - 90)j}$$

$$= -\omega C V_m e^{\theta_v j} e^{-90j}$$

$$= j\omega C V_m e^{\theta_v j}$$

same

$$= j\omega C \boldsymbol{V}_c(j\omega)$$

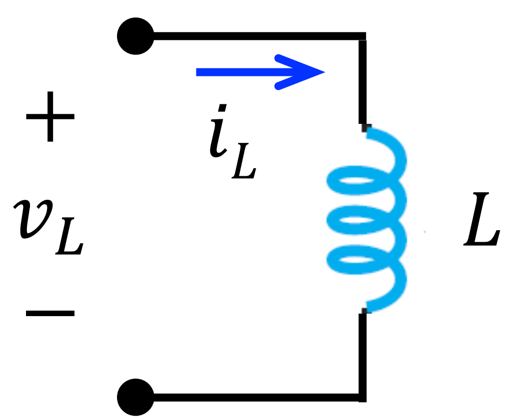
$$\frac{\boldsymbol{V}_c(j\omega)}{\boldsymbol{I}_c(j\omega)} = \frac{1}{j\omega C} = Z_c$$

Impedance

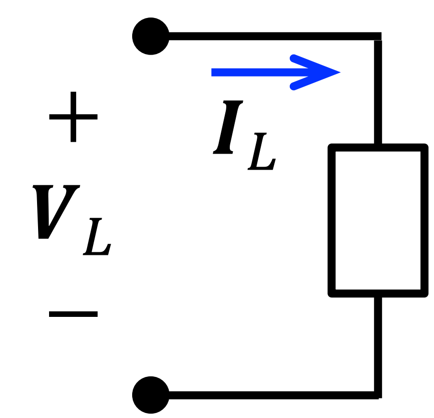
Impedance: Ratio of phasor voltage to phasor current.

$$\boxed{\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}}}$$

Time domain:



Phasor domain:



$i_L(t) = I_m \cos(\omega t + \theta_I) \longrightarrow I_L(j\omega) = I_m e^{\theta_I j}$

$v_L(t) = L \cdot \frac{di_L(t)}{dt} \longrightarrow V_L(j\omega) = L \cdot j\omega \cdot I_L(j\omega)$

Impedance

Rectangular form   Polar form

$$\boxed{V = I\,Z}$$

Resistor: $Z_R = R = R + 0j$

Capacitor: $Z_C = \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C}$

Inductor: $Z_L = j\omega L = 0 + j\omega L$