Gage Farmer

Homework 11 - Math 2568 (Autumn 2022)
Prof. Cueto

Due date: Wednesday December 7, 2022 (in class).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§4.5	1, 2, 4, 5, 6, 8, 9, 12, 16, 18, 19, 21, 22	1, 5, 6, 16, 18, 21
§4.6	1, 3, 5, 7, 9, 13, 19, 21, 23, 25, 27, 29	1, 3, 7, 19, 23, 25, 27
§4.7	1, 3, 7, 11, 13, 17, 19, 26, 27	1, 3, 11, 13, 17, 26, 27

Section 4.5

1) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ $\lambda = 3$

Alg mult = 1 Geom Mult = 1

$p(t) = (t-3)(t-1)$

$(A - \lambda I)x = \theta$ $x\left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right) = x\left(\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-x_1 - x_2 = 0$ $-x_1 = x_2$ $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

5) $C = \begin{bmatrix} -6 & -1 & 2 \\ 3 & 2 & 0 \\ -14 & -2 & 5 \end{bmatrix}$ $\lambda = -1$ Alg mult = 1

$p(t) = -(t-1)^2(t+1)$

$$(C+I)x=0 \quad \left(\begin{bmatrix}-6&-1&2\\3&2&0\\-14&-2&5\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right)x=\begin{bmatrix}-5&-1&2\\3&3&0\\-14&-2&6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$-5x_1-x_2+2x_3=0 \qquad 4x_1+2x_3=0 \qquad x_3=2x_1$$

$$3x_1+3x_2=0 \qquad x_2=-x_1 \qquad x=\begin{bmatrix}x_1\\-x_1\\2x_1\end{bmatrix}=x_1\begin{bmatrix}1\\-1\\2\end{bmatrix} \qquad \begin{bmatrix}1\\-1\\2\end{bmatrix}$$

$$-14x_1-2x_2+6x_3=0$$

Geom Mult = 1

6) $D=\begin{bmatrix}-7&4&-3\\8&-3&3\\32&-16&13\end{bmatrix}$ $\lambda=1$ $p(t)=-(t-1)^3$ Alg Mult = 3

$$(D-I)x=0 \quad \left(\begin{bmatrix}-7&4&-3\\8&-3&3\\32&-16&13\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\right)x=\begin{bmatrix}-8&4&-3\\8&-4&3\\32&-16&12\end{bmatrix}x=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$-8x_1+4x_2-3x_3=0 \to x_1-\tfrac{1}{2}x_2+\tfrac{3}{8}x_3=0 \qquad x_1=\tfrac{1}{2}x_2-\tfrac{3}{8}x_3$$

$$x=x_2\begin{bmatrix}\frac{1}{2}\\1\\0\end{bmatrix}-x_3\begin{bmatrix}\frac{3}{8}\\0\\1\end{bmatrix} \qquad \left\{\begin{bmatrix}1/2\\1\\0\end{bmatrix},\begin{bmatrix}3/8\\0\\1\end{bmatrix}\right\}$$

Geom Mult = 2

16) $\begin{bmatrix}-1&6&2\\0&5&-6\\1&0&-2\end{bmatrix}$ $(A-tI)=\begin{bmatrix}-1-t&6&2\\0&5-t&-6\\1&0&-2-t\end{bmatrix}$

$$\det(A-tI)=-1-t\begin{vmatrix}5-t&-6\\0&-2-t\end{vmatrix}-6\begin{vmatrix}0&-6\\1&-2-t\end{vmatrix}+2\begin{vmatrix}0&5-t\\1&0\end{vmatrix}$$

$$=(-1-t)((5-t)(-2-t))-6(6)+2(5-t)=(-1-t)(-10-5t+2t+t^2)-36+10-2t$$

$= 10 + 5t - 2t - t^2 + 10t + 5t^2 - 2t^2 - t^3 - 36 + 10 - 2t$

$= -t^3 + 2t^2 + 15t - 36 \quad = -(t-3)^2(t+4) \quad \lambda = 3, -4$

$AM = 2, 1$

$(A - 3I)x = \theta$

$$\begin{bmatrix} -4 & 6 & 2 \\ 0 & 2 & -6 \\ 1 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-4x_1 + 6x_2 + 2x_3 = 0$

$2x_2 - 6x_3 = 0 \quad x_2 = 3x_3$

$x_1 - 5x_3 = 0 \quad x_1 = 5x_3$

$\begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$ $GM = 1$

For $\lambda = 3$, $AM = 2$ $GM = 1$, so it is defective

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