MATH-2415, Ordinary and Partial Differential Equations **Name:**
Summer 2023
Problem Set 6
Due July 16, 2023 by midnight

Directions: You can either

(I) Print this sheet and show all work on the sheet itself, or

(II) Use separate paper to work out the problems. The paper must be plain white paper. No notebook paper, no lined paper. Engineering paper is acceptable.

For either selection, clearly show all work that leads to your final answer. Don't make me hunt for your steps! And DON'T turn in **any** scratched out work! Your final product must be clear and legible, or it will be returned, ungraded.

You can scan your work and save the pages as a single pdf. Or you can take pictures of your work, add the pictures to Word or Powerpoint and export the pages to a single pdf. You will submit the pdf to me via email.

1. A mass weighing 2 N stretches a spring 6 cm. If the mass is pulled down an additional 3 cm and then released, determine the position u of the mass at any time t if there is no damping. Find the frequency, period, and amplitude of the motion.

$k = \frac{W}{x} = \frac{2}{.06} = \frac{100}{3}$ N/m

$m = \frac{G}{W} = \frac{2}{10} = \frac{1}{5}$ kg

$X(0) = .03 \quad \frac{dx}{dt} = 0$

$\frac{1}{5}\frac{d^2u}{dt^2} + 5ku = 0$

$r^2 + \frac{500}{3} = 0$

$r = \pm\sqrt{\frac{500}{3}}\, i = \pm 12.91i$

$u(t) = C_1 \cos 12.91t + C_2 \sin 12.91t$

$u(0) = C_1 = 0.03 \quad C_2 = 0$

$u(t) = .03\cos 12.9t$

$A = 3cm$

$\omega = 12.91$ rad/sec

$$T = 2\frac{\pi}{\omega} = 2\frac{\pi}{12.91} = 2.05471\,\text{Hz}$$

2. A mass of 300 g stretches a spring 4 cm. If the mass is pulled down an additional 2 cm and given an upward velocity of 2 m/s, determine the position u of the mass at any time t if there is no damping. Also find the frequency, period, amplitude, and phase.

$$M\frac{d^2v}{dt^2} + kv = 0$$

$v_0 = .04m \quad m = .3\,kg$

$$\frac{d^2v}{dt^2} + \frac{k}{m}v = 0 \qquad v_0 + \Delta v = .06m \qquad \frac{dv_0}{dt} = 2m/s$$

$$kv = mg \rightarrow \frac{k}{m} = \frac{g}{v} = \frac{9.8}{4e\text{-}2} = 245 \rightarrow \sqrt{\frac{k}{m}} = 15.65$$

$$D = \pm\sqrt{\frac{k}{m}};$$

$$u(t) = C_1 \cos 15.65t + C_2 \sin 15.65t$$

$$u(0) = C_1 = 0.06 \qquad C_2 = \frac{2}{15.65} = 0.13$$

$u(t) = .06\cos 15.65t + .13\sin 15.65t$

$\omega = 15.65$

$T = 2.45\,Hz$

$A = \sqrt{.13^2 + .06^2} = 0.14$

$\text{phase} = \tan^{-1}\left(\frac{.06}{.13}\right) = 0.43\ rad$