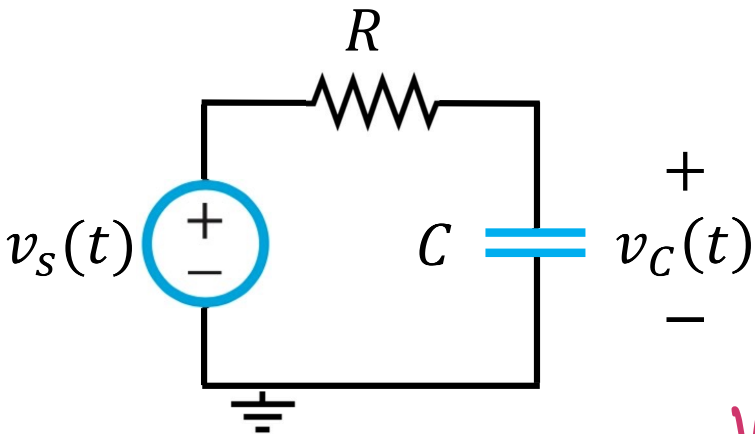
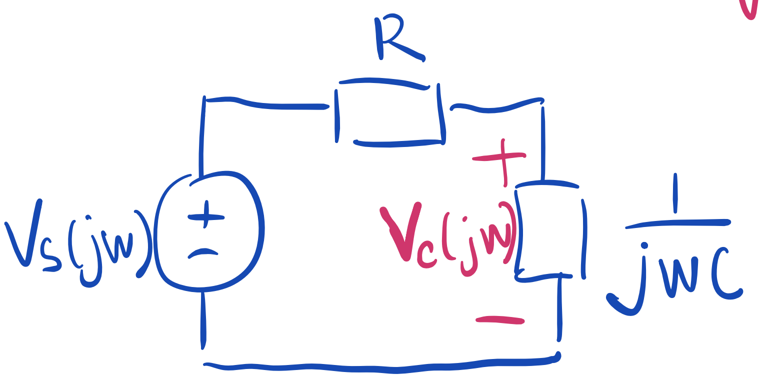
Last Class…

Find $V_c(j\omega)$





Voltage Division

$$V_c(jw) = \frac{\frac{1}{jwC}}{R + \frac{1}{jwC}} \cdot V_s(jw)$$

$$= \frac{1}{jwCR + 1} V_s(jw)$$

THE OHIO STATE UNIVERSITY

COLLEGE OF ENGINEERING

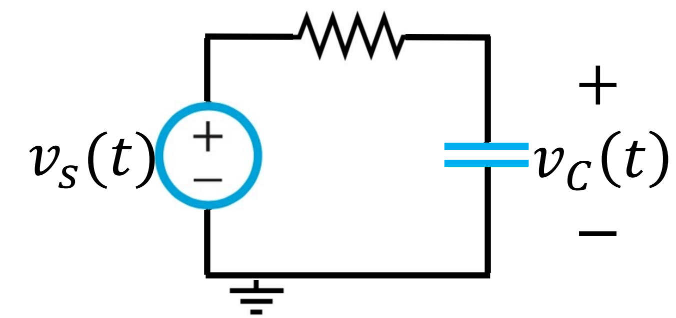
Transfer Functions

NO CLASS ON MONDAY,

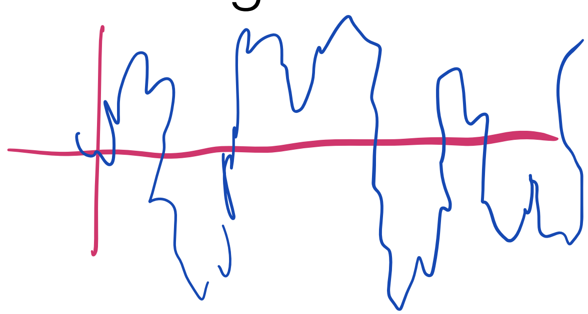
MARCH 20TH

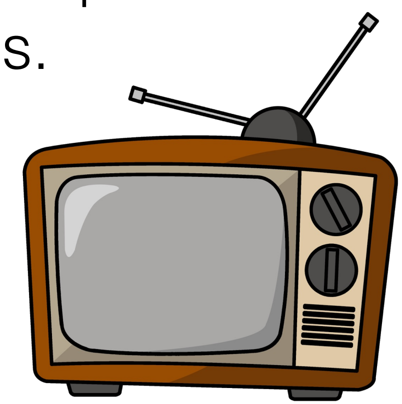
This Lecture…

- Learning Objectives:
  - Derive the transfer function of an AC Circuit.

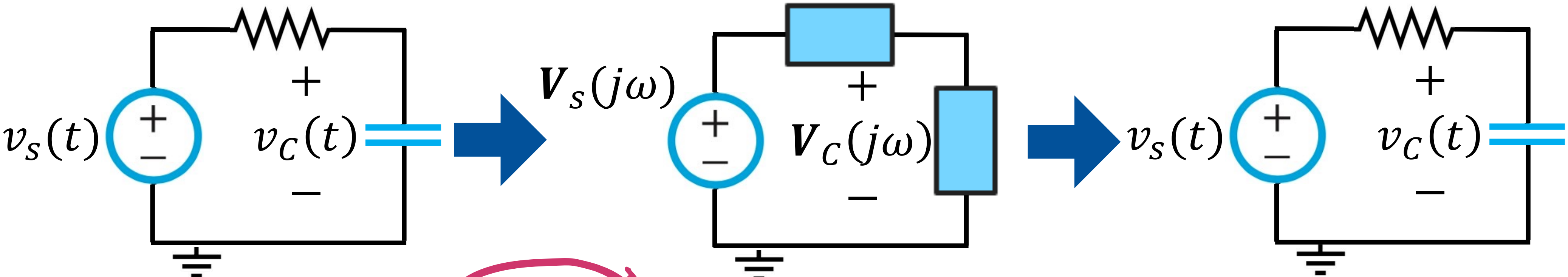


- Often, the input signal is a superposition of many sinusoidal signals at different frequencies.





- How to design RLC circuits that can filter in (pass through) the range of frequencies of interest and filter out (reject) the range of frequencies of signals that are either problematic or not of interest.



$V_s(t) = A\cos(\omega t + \theta)$

INPUT

$V_s(j\omega) = A \angle \theta$

$V_c(j\omega) = B \angle \alpha$

OUTPUT

$V_c(t) = B\cos(\omega t + \alpha)$

$B = \beta A$

- At a given angular frequency ω:
  - Load voltage is a sinusoid with the same frequency as the source voltage.

- $\boldsymbol{V}_C(j\omega)$ is a phase-shifted and amplitude-scaled version of $\boldsymbol{V}(j\omega)$.

Transfer Function

- Frequency Response: Measures how circuit responds to sinusoidal inputs of arbitrary frequency.

$$\text{input} \longrightarrow \boxed{\text{system}} \longrightarrow \text{output}$$

$$V_s(j\omega) \qquad\qquad TF \qquad\qquad V_c(j\omega)$$

- Transfer function: describes the output response to an input excitation as a function of the angular frequency ω.

$$TF = \frac{\text{output}}{\text{input}} = \frac{V_c(j\omega)}{V_s(j\omega)}$$

$$\text{output} = TF \cdot \text{input}$$

For example:

$$TF = 2w \angle 45^\circ$$

$$\text{input} = 10\cos(100t - 15^\circ)$$

$$\Longrightarrow$$

$$\text{output} = (200\angle 45^\circ)\cdot(10\angle -15^\circ)$$

$$= 2000\angle 30^\circ$$

$$= 2000\cos(100t + 30^\circ)$$

Transfer Function

Voltage Gain:

$$H_V = \frac{V_{out}}{V_{in}}$$

Transfer Impedance:

$$H_Z = \frac{V_{out}}{I_{in}}$$

Current Gain:

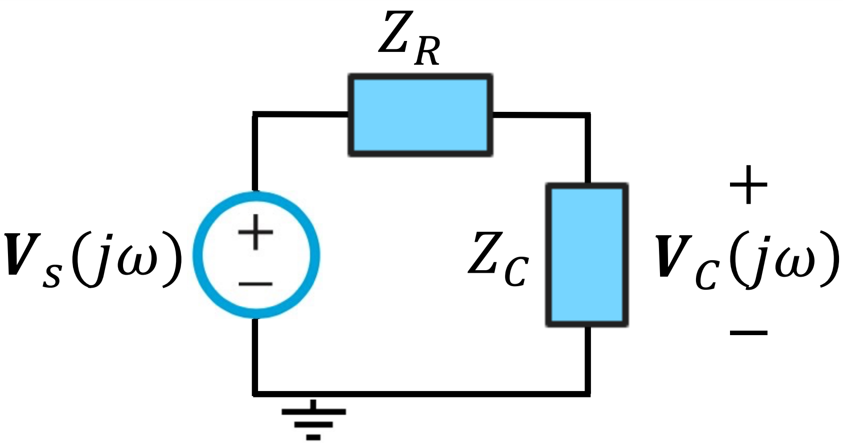
$$H_I = \frac{I_{out}}{I_{in}}$$

Transfer Admittance:

$$H_Y = \frac{I_{out}}{V_{in}}$$

- V_{in} and I_{in} are often chosen to be independent voltage and current sources.
- Outputs V_{out} and I_{out} are freely chosen and represent the load in a circuit.

Example 1



$= \dfrac{1}{jwCR+1} V_s(jw)$

Same as first slide.

Find $\dfrac{V_c(jw)}{V_s(jw)}$ ← output

$$\frac{V_c(jw)}{V_s(jw)} = \frac{1}{jwCR+1}$$

How do I find the output?

$V_c(jw) = ?$

Denominator:

$1 + jw\,\square$

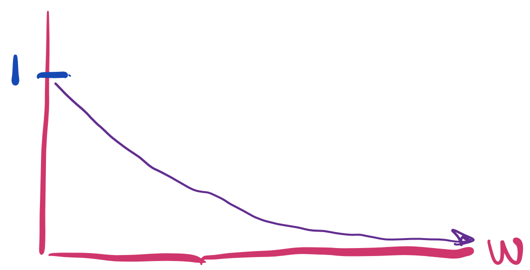
$1 + jw\,\square + (jw)^2\,\square$

$$\frac{V_c(j\omega)}{V_s(j\omega)} = \frac{1}{j\omega CR + 1} = M(\omega) \angle \theta(\omega)$$

$$= \frac{1 \angle 0}{\sqrt{1+(\omega CR)^2} \angle \tan^{-1}(\omega CR)}$$

$$= \frac{1}{\sqrt{1+(\omega CR)^2}} \angle -\tan^{-1}(\omega CR)$$

Magnitude



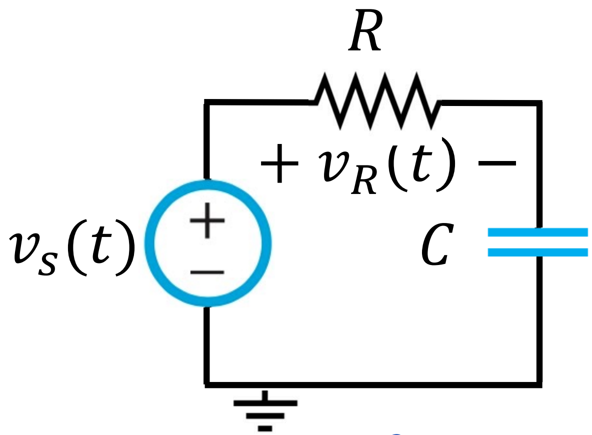
$$\lim_{\omega \to 0} M(\omega) = 1$$

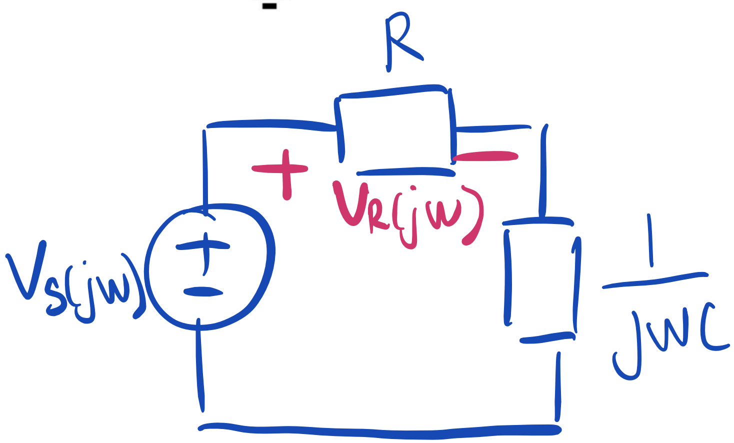
$$\lim_{\omega \to \infty} M(\omega) = 0$$

output = TF • input

LOW PASS FILTER

Example 2





Find $\dfrac{V_R(j\omega)}{V_s(j\omega)}$ ← output.

Voltage Division:

$$V_R(j\omega) = \frac{R}{R + \frac{1}{j\omega C}} \cdot V_s(j\omega)$$

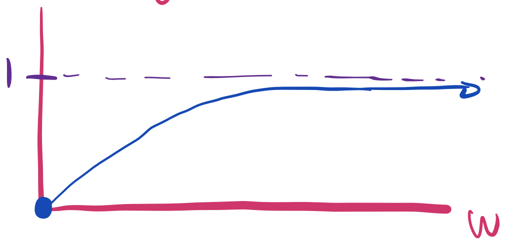
$$\frac{V_R(j\omega)}{V_s(j\omega)} = \frac{j\omega CR}{j\omega CR + 1}$$

$$\frac{V_R(jw)}{V_S(jw)} = \frac{jwCR}{jwCR+1}$$

$$= \frac{wCR \angle 90^\circ}{\sqrt{1+(wCR)^2} \angle \tan^{-1}(wCR)}$$

$$= \frac{wCR}{\sqrt{1+(wCR)^2}} \angle 90 - \tan^{-1}(wCR)$$

Magnitude



$$\lim_{w \to 0} M(w) = 0$$

$$\lim_{w \to \infty} M(w) = 1$$

HIGH PASS FILTER