Storyline

Chapter 26: Current and Resistance

Physics for Scientists and Engineers, 10e
Raymond A. Serway
John W. Jewett, Jr.

BROOKS/COLE
CENGAGE Learning

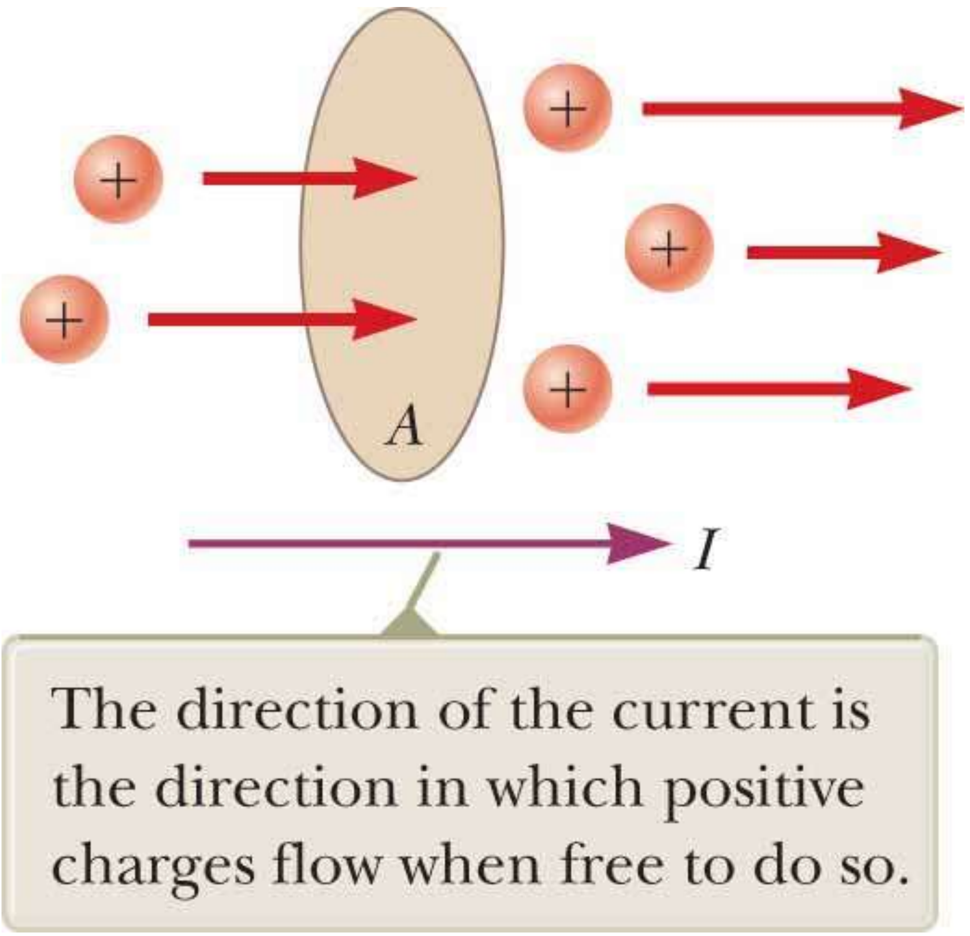
Electric Current

Electric Current

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t}$$

$$I \equiv \frac{dQ}{dt}$$

$$1\ \text{A} = 1\ \text{C/s}$$

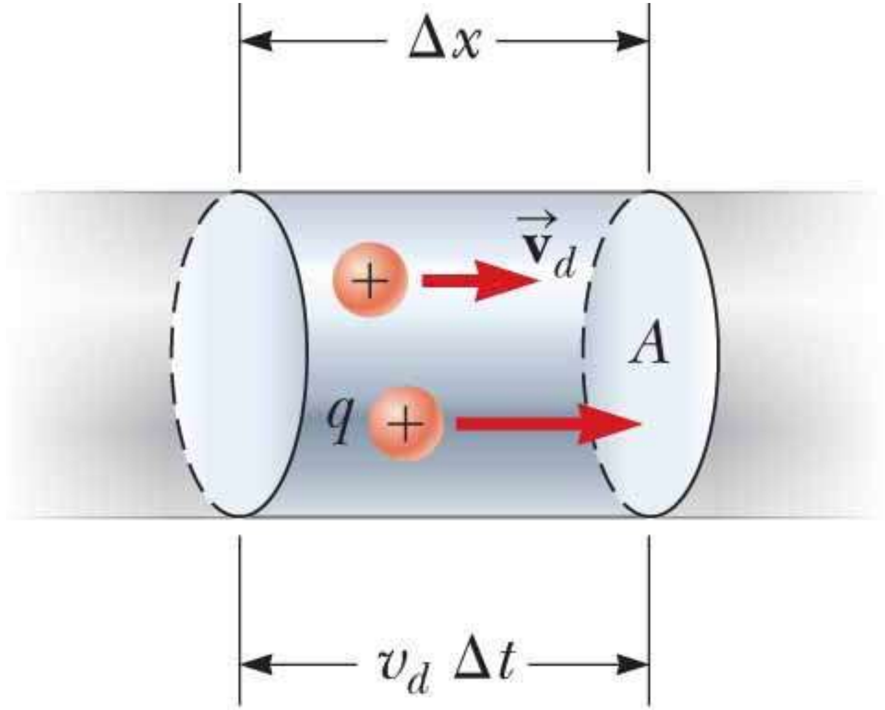


Microscopic Model of Current

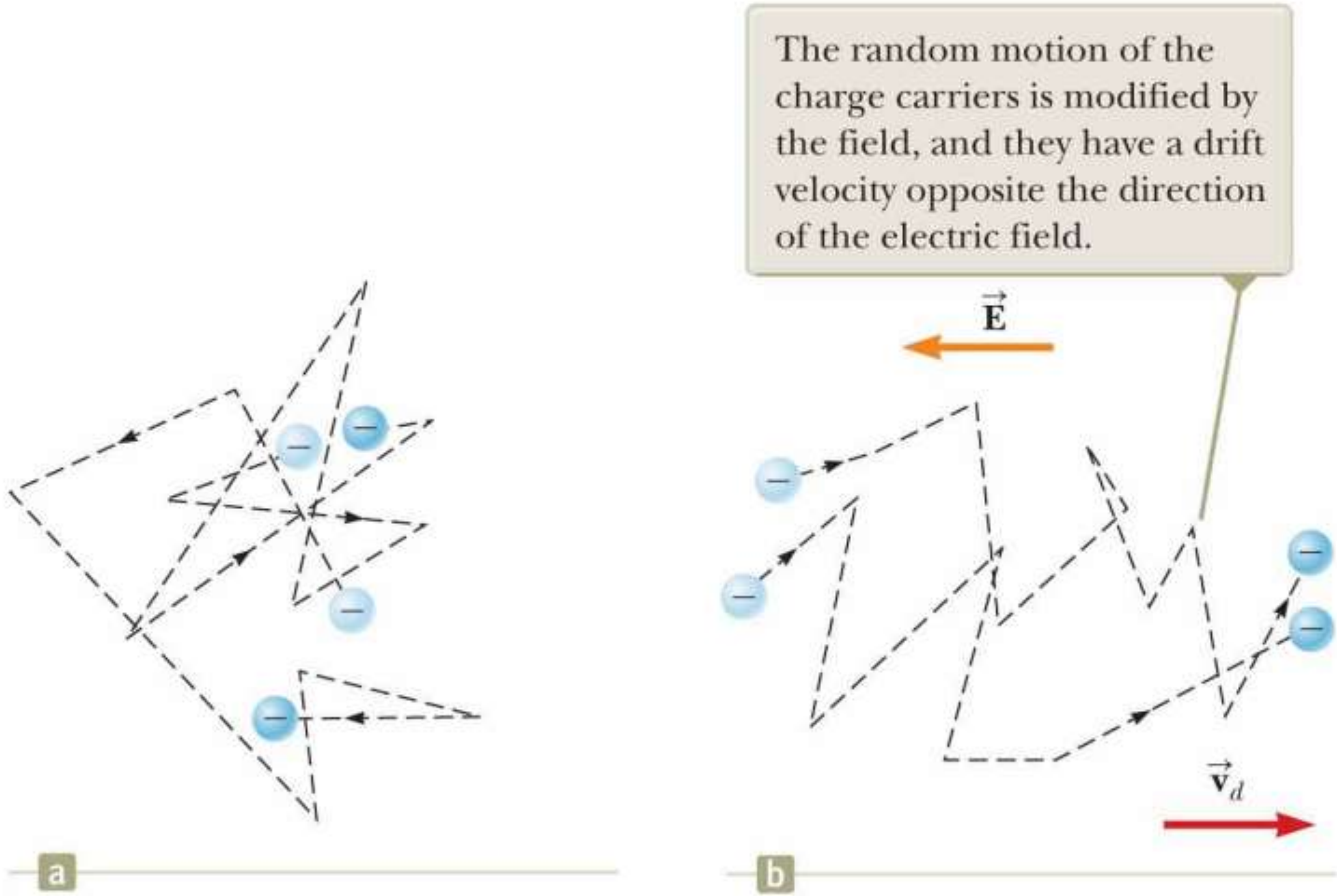
$$\Delta Q = (nA\,\Delta x)q$$

$$= (nAv_d\,\Delta t)q$$

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t} = nqv_d A$$

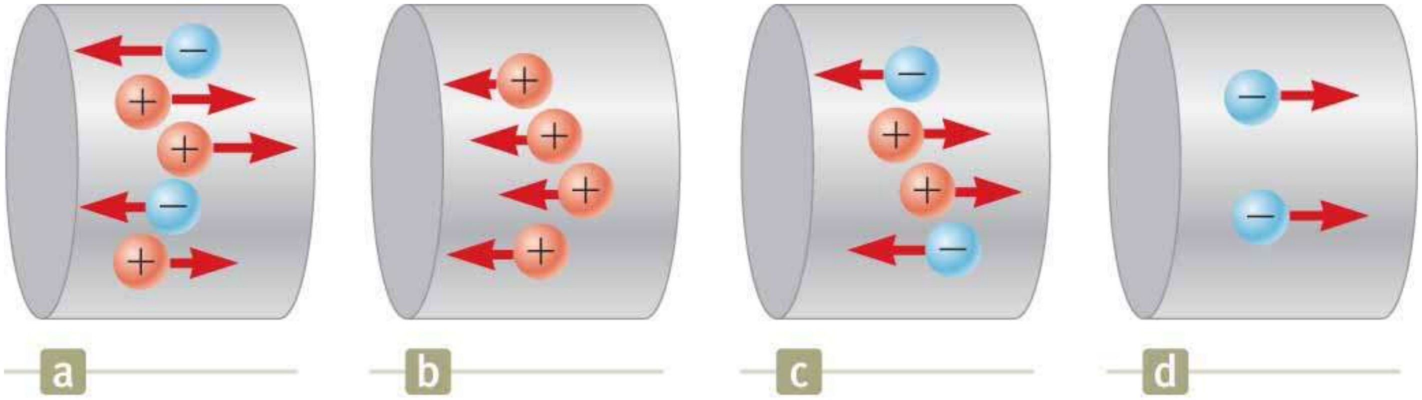


Microscopic Model of Current



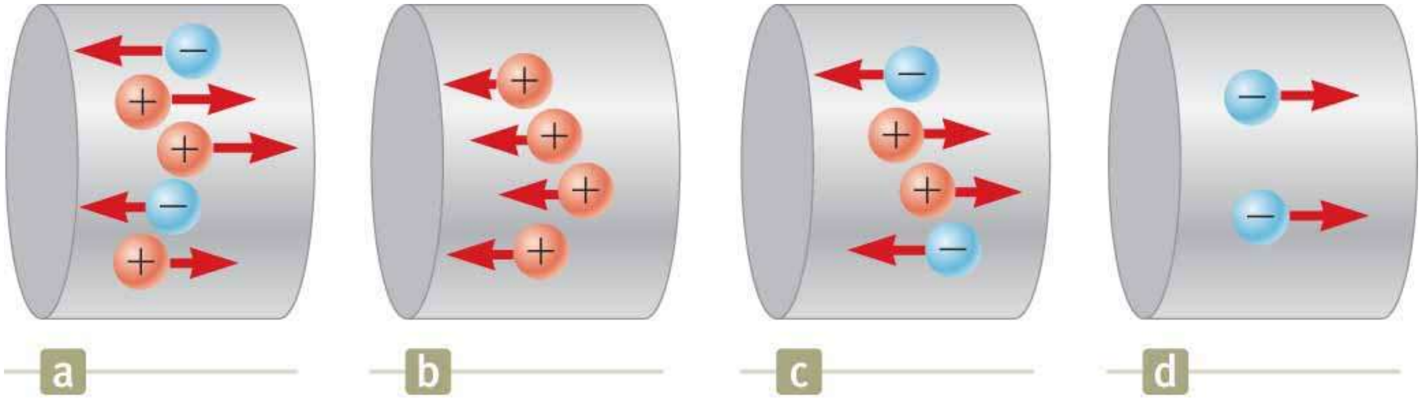
Quick Quiz 26.1

Consider positive and negative charges of equal magnitude moving horizontally through the four regions shown in the figure. Rank the current in these four regions from highest to lowest.



Quick Quiz 26.1

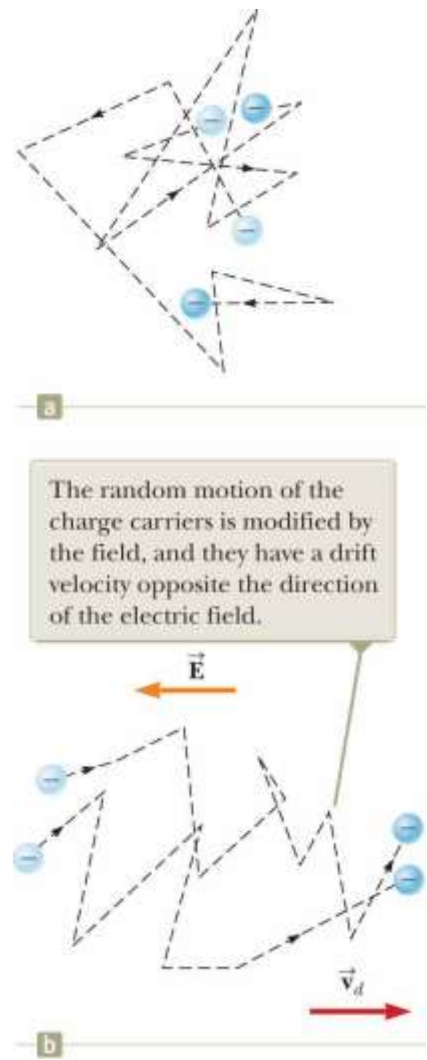
Consider positive and negative charges of equal magnitude moving horizontally through the four regions shown in the figure. Rank the current in these four regions from highest to lowest.



(a) > (b) = (c) > (d)

Example 26.1:
Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \cdot 10^{-6}\ \text{m}^2$. It carries a constant current of 10.0 A. What is the drift speed of the electrons in the wire? Assume each copper atom contributes one free electron to the current. The density of copper is $8.92\ \text{g/cm}^3$.



Example 26.1:
Drift Speed in a Copper Wire

29 $^2S_{1/2}$
Cu
Copper
63.546
$[Ar]3d^{10}4s$
7.7264

$$V = \frac{M}{\rho} \qquad n = \frac{N_A}{V} = \frac{N_A \rho}{M}$$

$$v_d = \frac{I_{\text{avg}}}{nqA} = \frac{I}{nqA} = \frac{IM}{qAN_A\rho}$$

$$v_d = \frac{(10.0\ \text{A})(0.0635\ \text{kg/mol})}{(1.60\times10^{-19}\ \text{C})(3.31\times10^{-6}\ \text{m}^2)(6.02\times10^{23}\text{mol}^{-1})(8920\ \text{kg/m}^3)}$$

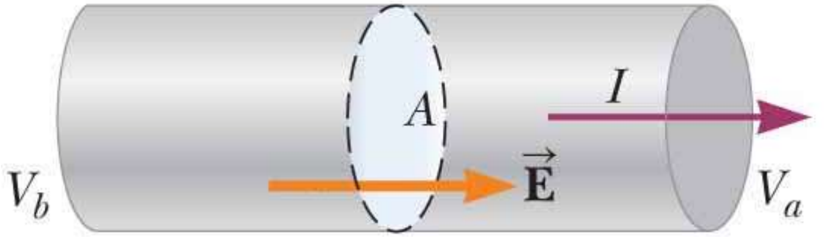
$$= \boxed{2.23\times10^{-4}\ \text{m/s}}$$

Resistance

$$I = nqv_d A$$

$$J \equiv \frac{I}{A} = nqv_d$$

$$J = \sigma E$$



For many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

Resistance

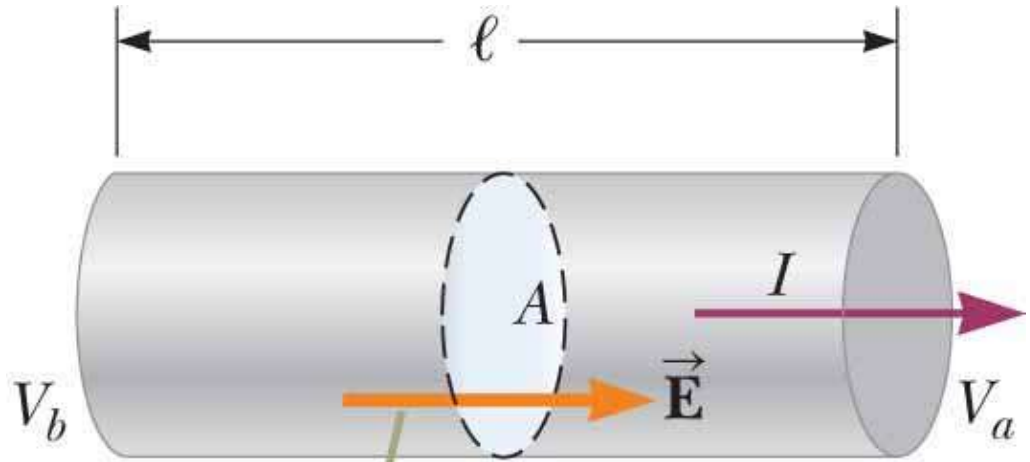
$$\Delta V = E\,\boxed{?}$$

$$\Delta V = \frac{\boxed{?}}{\sigma}$$

$$\Delta V = \left(\frac{\boxed{?}}{\sigma A}\right) I = RI$$

$$\boxed{R \equiv \frac{\Delta V}{I}}$$

$$1\ \Omega \equiv 1\ \text{V/A}$$



A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field $\vec{\mathbf{E}}$, and this field produces a current I that is proportional to the potential difference.

Resistors

The colored bands on this resistor are yellow, violet, black, and gold.

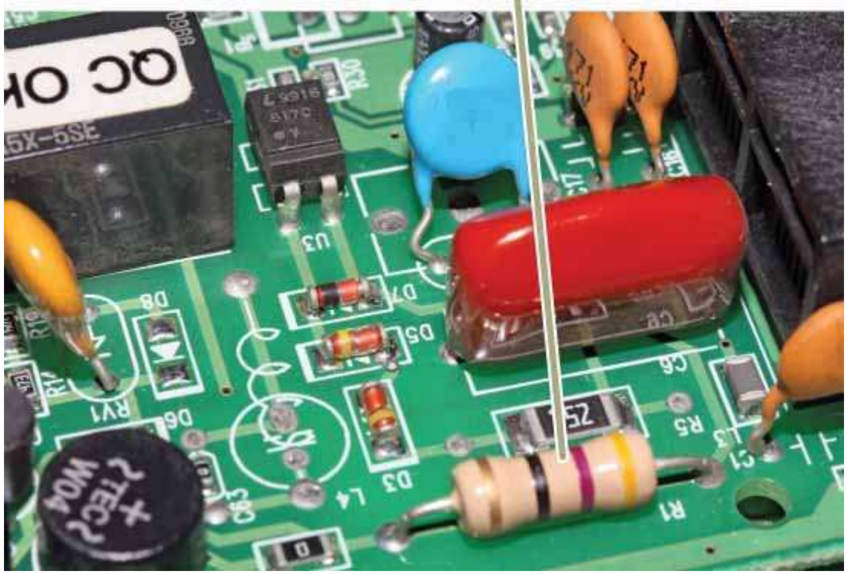


TABLE 26.1 Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

$$R = \left(10C_2 + C_1\right) \times 10^{C_3}\ \Omega$$

Resistivity

$$\rho = \frac{1}{\sigma}$$

$$R = \rho \frac{?}{A}$$

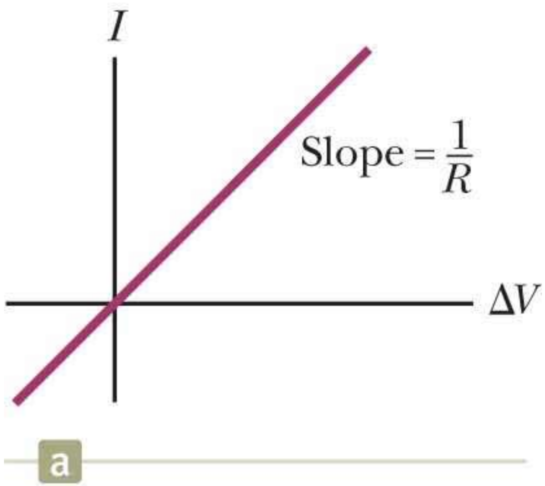
TABLE 26.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

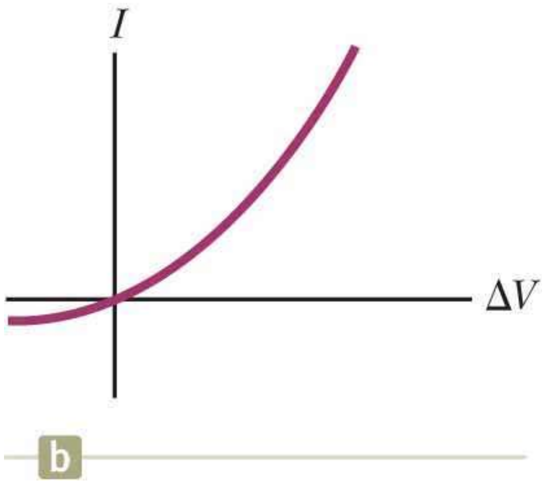
Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] α $[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 26.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Current Analogous to Water Flowing in a Pipe

Ohmic and Nonohmic Materials





Quick Quiz 26.2

A cylindrical wire has a radius r and length ℓ. If both r and ℓ are doubled, does the resistance of the wire

(a) increase,

(b) decrease, or

(c) remain the same?

Quick Quiz 26.2

A cylindrical wire has a radius *r* and length ℓ. If both *r* and ℓ are doubled, does the resistance of the wire

(a) increase,

(b) decrease, or

(c) remain the same?

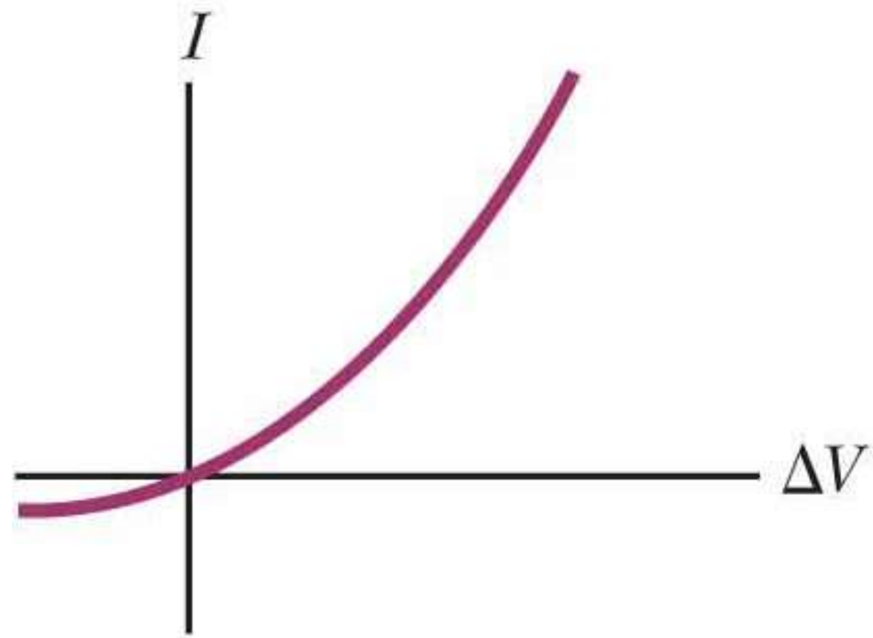
Quick Quiz 26.3

In the figure, as the applied voltage increases, does the resistance of the diode

(a) increase,

(b) decrease, or

(c) remain the same?



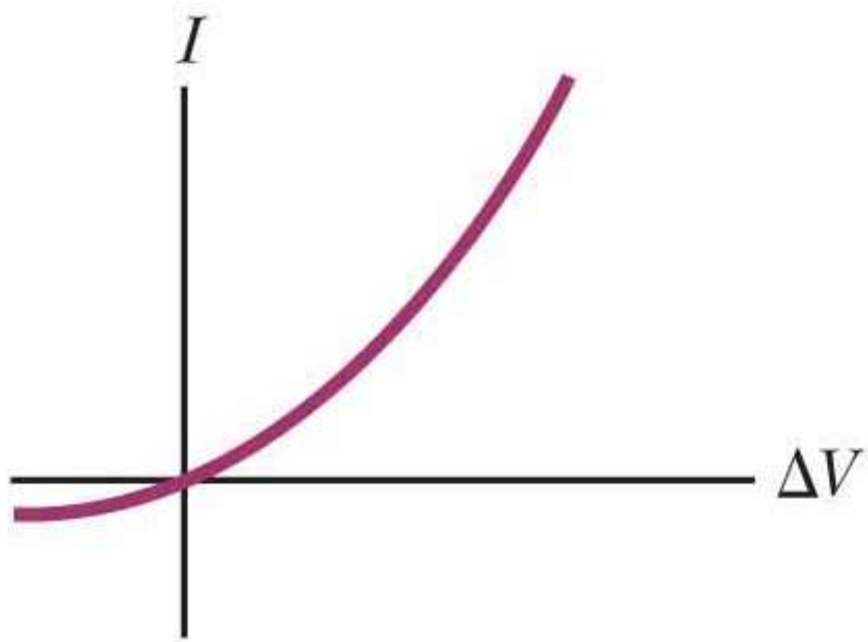
Quick Quiz 26.3

In the figure, as the applied voltage increases, does the resistance of the diode

(a) increase,

(b) decrease, or

(c) remain the same?



Example 26.2: The Resistance of Nichrome Wire

The radius of 22-gauge Nichrome wire is 0.32 mm. (A) Calculate the resistance per unit length of this wire.

TABLE 26.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot \text{m}$)	Temperature Coefficient[b] α $[(^\circ\text{C})^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 26.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and $1.50 \times 10^{-6}\ \Omega \cdot \text{m}$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

$$\frac{R}{\boxed{?}} = \frac{\rho}{A} = \frac{\rho}{\pi r^2} = \frac{1.0 \times 10^{-6}\ \Omega \cdot \text{m}}{\pi \left(0.32 \times 10^{-3}\ \text{m}\right)^2} = 3.1\ \Omega/\text{m}$$

Example 26.2:
The Resistance of Nichrome Wire

(B) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

$$I = \frac{\Delta V}{(R/\boxed{?})\boxed{?}} = \frac{10\text{ V}}{(3.1\ \Omega/\text{m})(1.0\text{ m})} = 3.2\text{ A}$$

Example 26.2: The Resistance of Nichrome Wire

What if the wire were composed of copper instead of Nichrome? How would the values of the resistance per unit length and the current change?

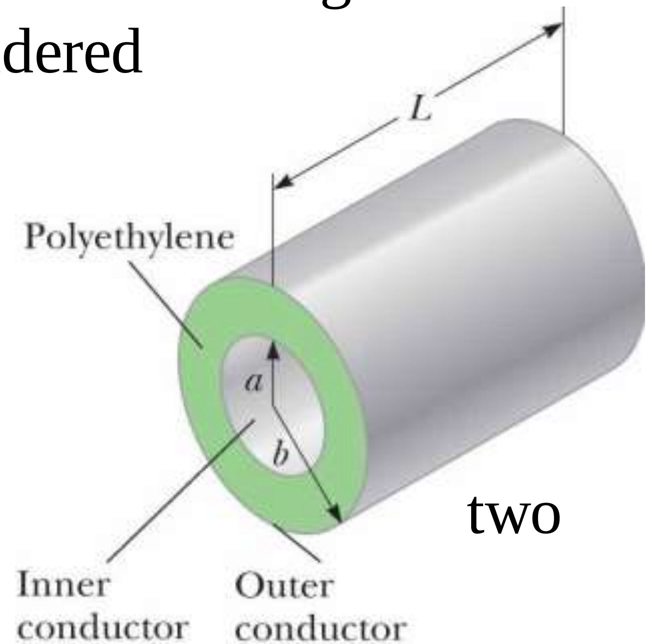
TABLE 26.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] α [$(°C)^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 26.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Example 26.3:
The Radial Resistance of a Coaxial Cable

Coaxial cables are used extensively for cable television and other electronic applications. A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with polyethylene plastic as shown in the figure. Current leakage through the plastic, in the radial direction, is unwanted. (The cable is designed to conduct current along its length, but that is not the current being considered here.) The radius of the inner conductor is a = 0.500 cm, the radius of the outer conductor is b = 1.75 cm, and the length is L = 15.0 cm. The resistivity of the plastic is 1.0 · 10^{13} ∧ ⊕m. Calculate the radial resistance of the plastic between the two conductors.



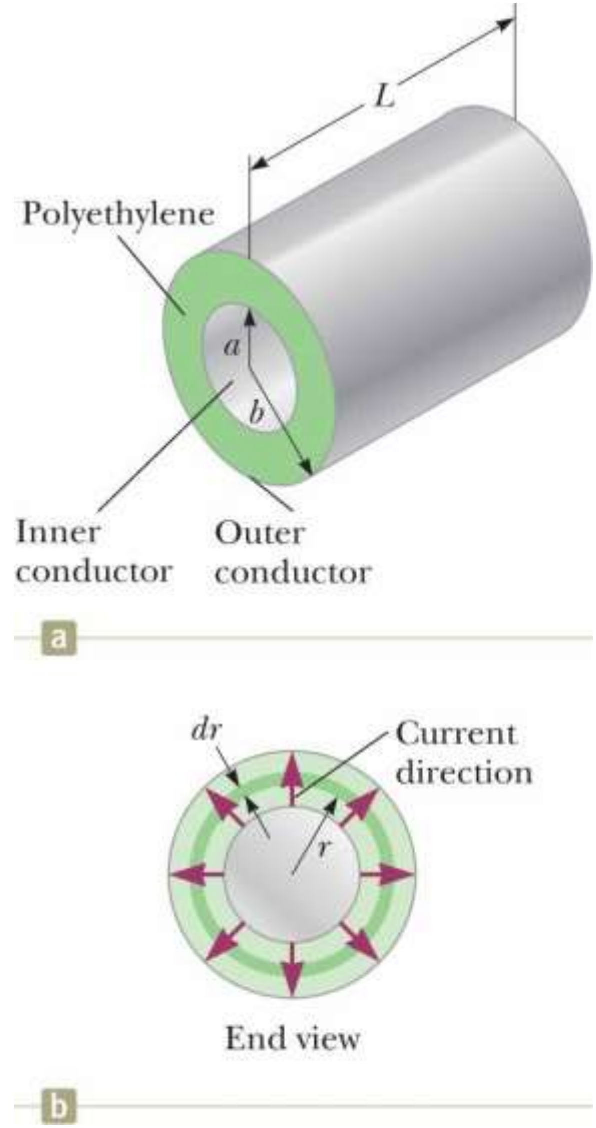
Example 26.3: The Radial Resistance of a Coaxial Cable

$$dR = \frac{\rho\, dr}{A} = \frac{\rho}{2\pi rL}dr$$

$$R = \int dR = \frac{\rho}{2\pi rL}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

$$R = \frac{1.0\times10^{13}\ \Omega\cdot\text{m}}{2\pi\,(0.150\ \text{m})}\ln\left(\frac{1.75\ \text{cm}}{0.500\ \text{cm}}\right)$$

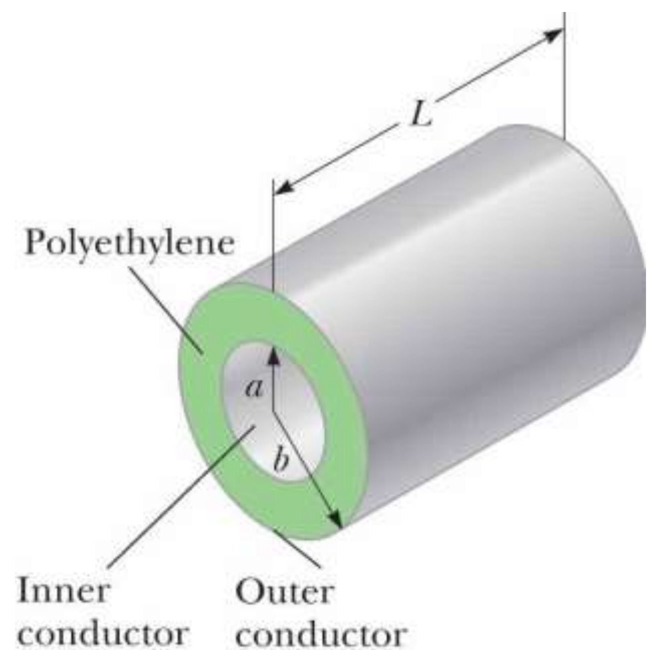
$$= \boxed{1.33\times10^{13}\ \Omega}$$



Example 26.3: The Radial Resistance of a Coaxial Cable

$$R_{\mathrm{Cu}} = \rho \frac{\boxed{?}}{A} = \left(1.7\times10^{-8}\ \Omega\cdot\mathrm{m}\right)\left[\frac{0.150\ \mathrm{m}}{\pi\left(5.00\times10^{-3}\ \mathrm{m}\right)^2}\right]$$

$$= 3.2\times10^{-5}\ \Omega$$

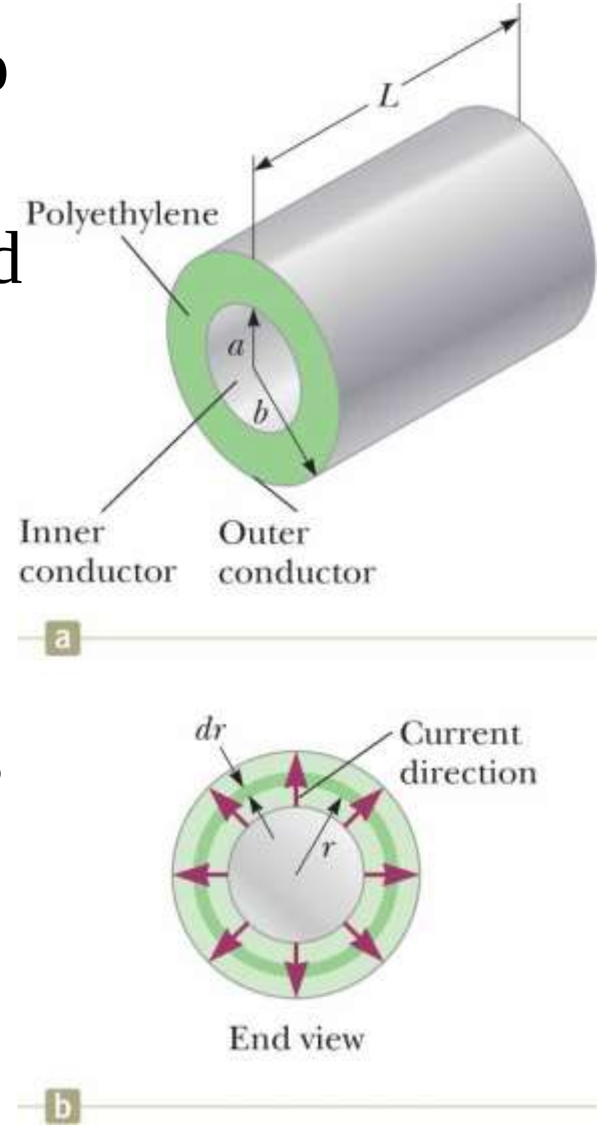


Example 26.3:
The Radial Resistance of a Coaxial Cable

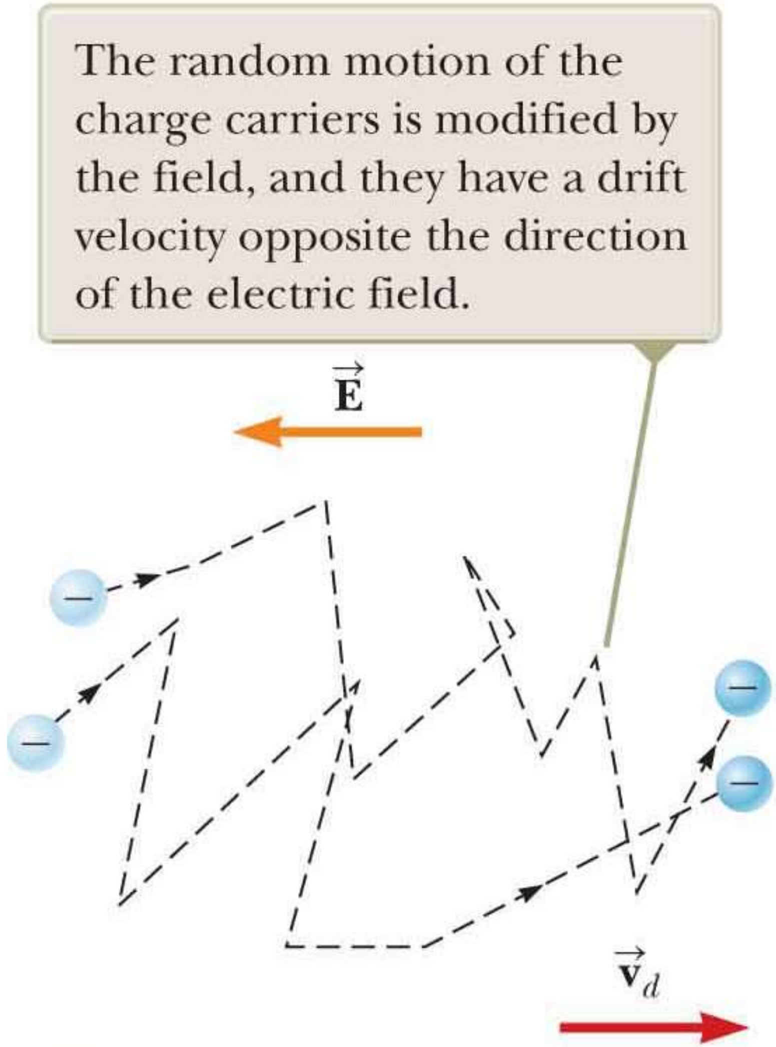
Suppose the coaxial cable is enlarged to twice the overall diameter with two possible choices: (1) the ratio b/a is held fixed, or (2) the difference *b* – *a* is held fixed.

For which choice does the leakage current between the inner and outer conductors increase when the voltage is applied between them?

$$R = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$



Drude Model for Electrical Conduction



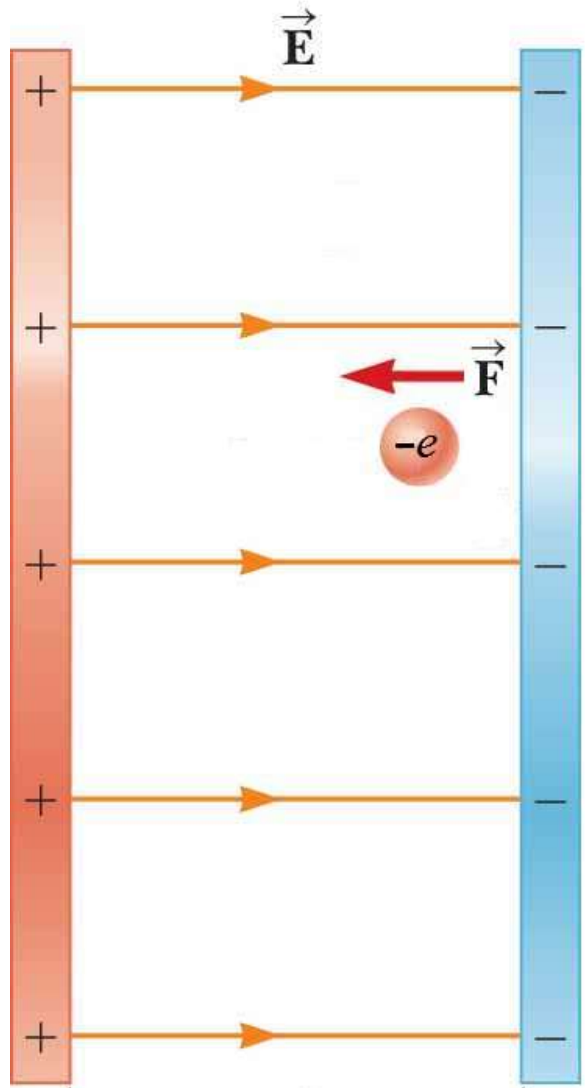
Drude Model for Electrical Conduction

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{q\vec{\mathbf{E}}}{m_e}$$

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \vec{\mathbf{v}}_i + \frac{q\vec{\mathbf{E}}}{m_e}t$$

$$\vec{\mathbf{v}}_{f,\text{avg}} = \vec{\mathbf{v}}_d = \frac{q\vec{\mathbf{E}}}{m_e}\tau$$

$$I_{\text{avg}} = nq\left(\frac{qE}{m_e}\tau\right)A = \frac{nq^2\tau A}{m_e}E$$



Drude Model for Electrical Conduction

$$J = \frac{nq^2\tau}{m_e} E \qquad\qquad J = \sigma E$$

$$\sigma = \frac{nq^2\tau}{m_e}$$

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau}$$

$$\tau = \frac{?_{\text{avg}}}{v_{\text{avg}}}$$

Resistance and Temperature

$$\rho = \rho_0 \left[1 + \alpha \left(T - T_0\right)\right]$$

$$\alpha = \frac{\Delta\rho / \rho_0}{\Delta T}$$

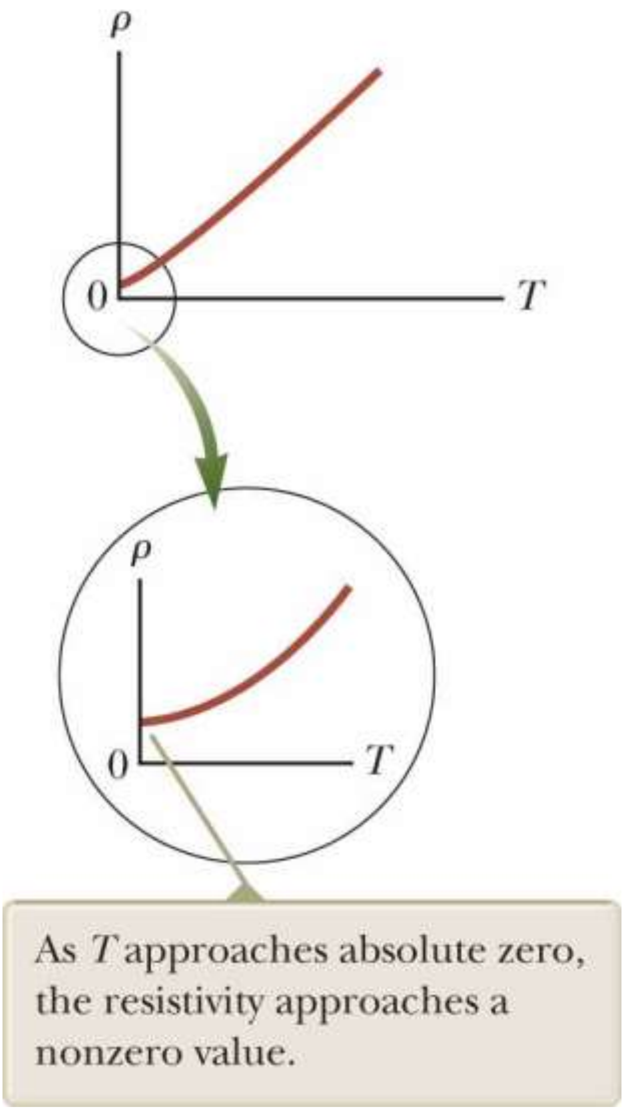
$$R = R_0 \left[1 + \alpha \left(T - T_0\right)\right]$$

TABLE 26.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] α [$(°C)^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 26.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Resistance and Temperature



Quick Quiz 26.4

When does an incandescent lightbulb carry more current,

(a) immediately after it is turned on and the glow of the metal filament is increasing or

(b) after it has been on for a few milliseconds and the glow is steady?

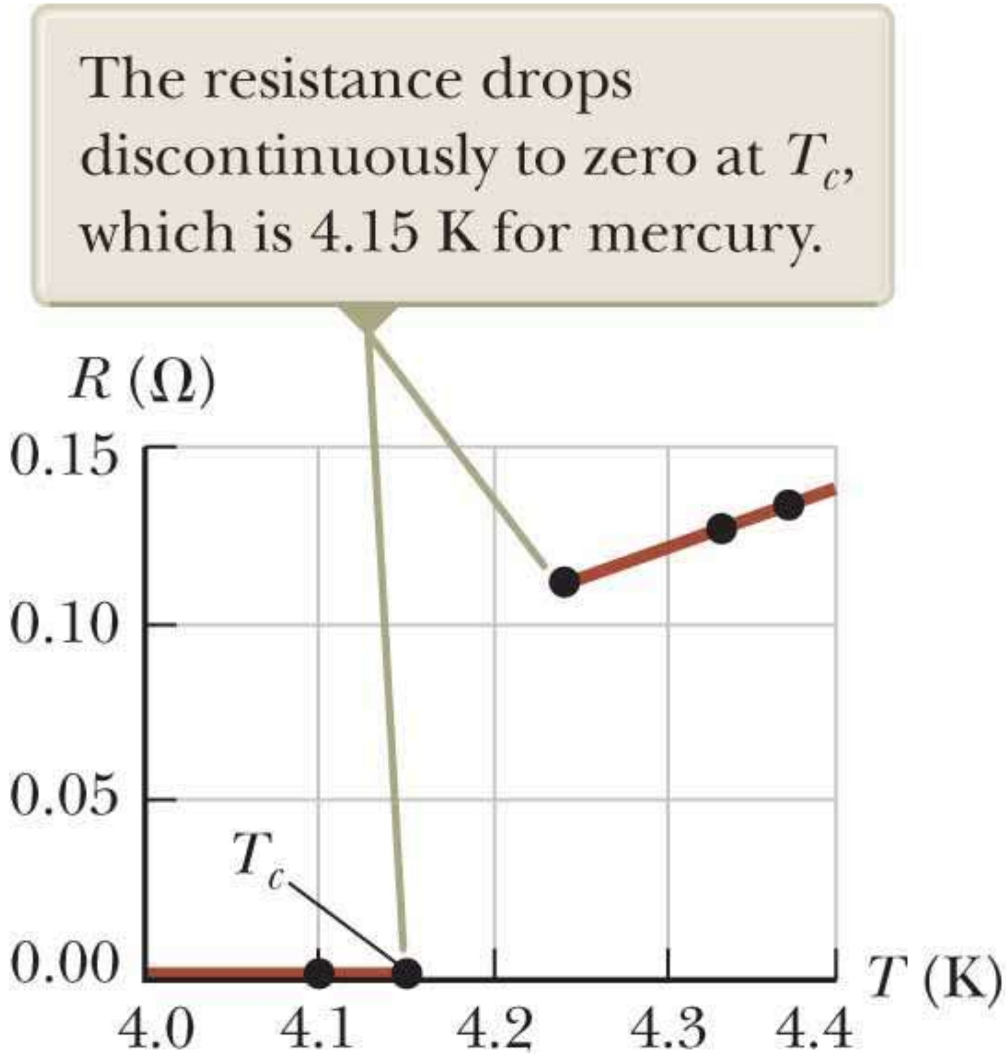
Quick Quiz 26.4

When does an incandescent lightbulb carry more current,

(a) immediately after it is turned on and the glow of the metal filament is increasing or

(b) after it has been on for a few milliseconds and the glow is steady?

Superconductors



Superconductors

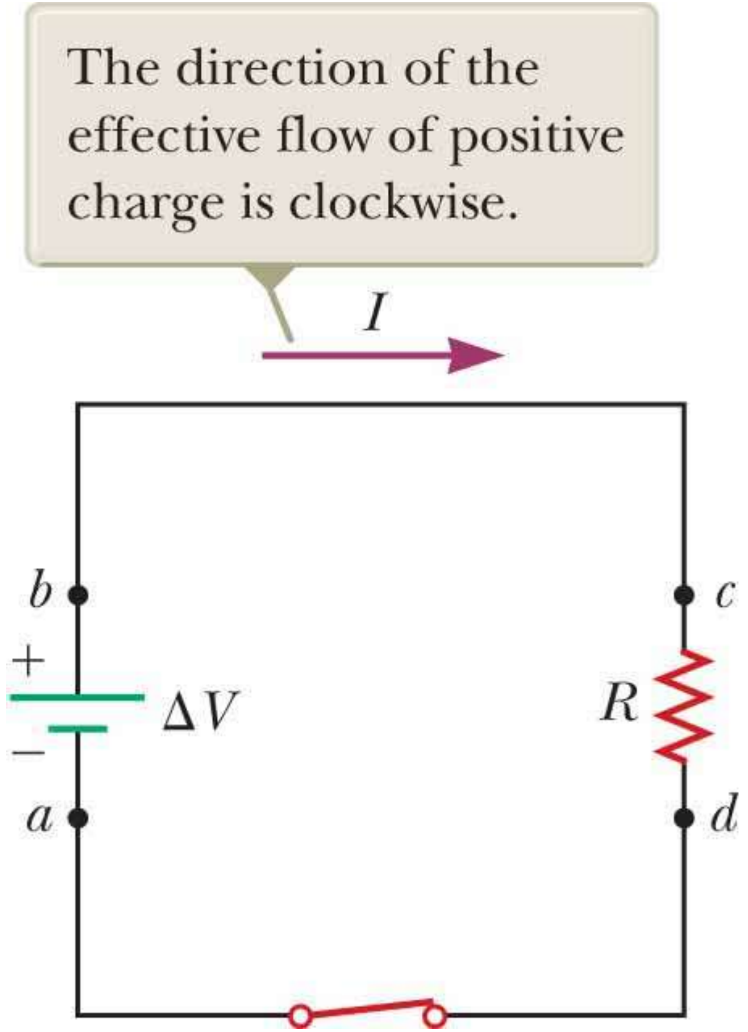
TABLE 26.3 Critical Temperatures for Various Superconductors

Material	T_c (K)
$HgBa_2Ca_2Cu_3O_8$	134
Tl—Ba—Ca—Cu—O	125
Bi—Sr—Ca—Cu—O	105
$YBa_2Cu_3O_7$	92
Nb_3Ge	23.2
Nb_3Sn	18.05
Nb	9.46
Pb	7.18
Hg	4.15
Sn	3.72
Al	1.19
Zn	0.88

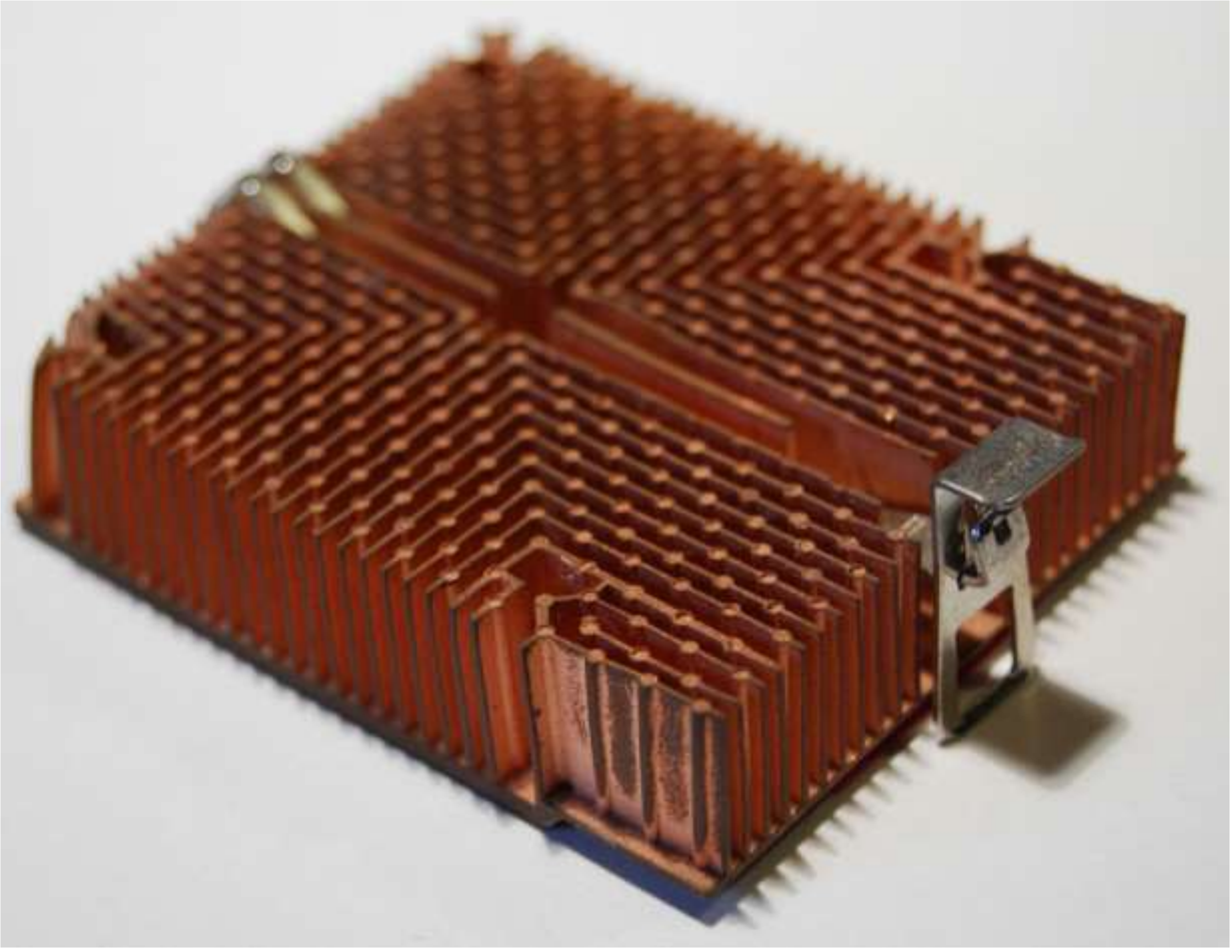
Electrical Power

$$\Delta E_{\text{int}} = Q + T_{\text{ET}} + T_{\text{ER}}$$

$$\Delta U_{\text{c}} + \Delta E_{\text{int}} = Q + T_{\text{ER}}$$



Heat Sink

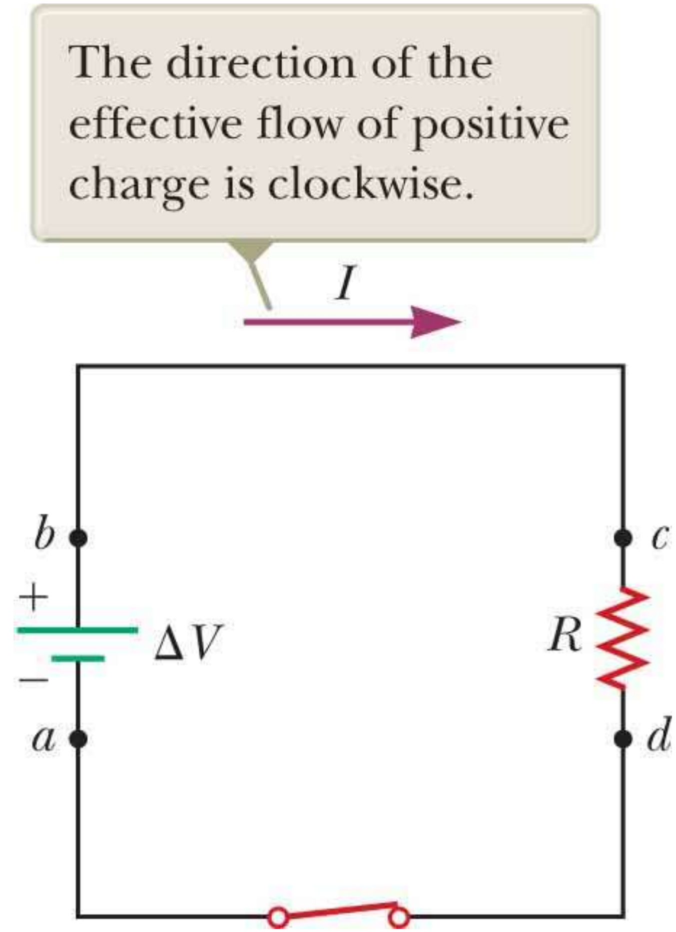


Electrical Power

$$\frac{dU_E}{dt} = \frac{d}{dt}(Q\Delta V) = \frac{dQ}{dt}\Delta V = I\Delta V$$

$$\boxed{P = I\Delta V}$$

$$\boxed{P = I^2 R = \frac{(\Delta V)^2}{R}}$$



Electrical Power

Example 26.4:
Power in an Electric Heater

An electric heater is constructed by applying a potential difference of 120 V across a Nichrome wire that has a total resistance of 8.00 ∧. Find the current carried by the wire and the power rating of the heater.

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega}$$

$$P = I^2 R = (15.0\ \text{A})^2 (8.00\ \Omega) = 1.80 \times 10^3\ \text{W}$$

Example 26.4:
Power in an Electric Heater

What if the heater were accidentally connected to a 240-V supply? (That is difficult to do because the shape and orientation of the metal contacts in 240-V plugs are different from those in 120-V plugs.) How would that affect the current carried by the heater and the power rating of the heater, assuming the resistance remains constant?

$$R = \frac{\Delta V}{I} \qquad\qquad P = I^2 R$$

Example 26.5:
Linking Electricity and Thermodynamics

An immersion heater must increase the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V.

(A) What is the required resistance of the heater?

$$P = \frac{(\Delta V)^2}{R} = \frac{Q}{\Delta t}$$

$$\frac{(\Delta V)^2}{R} = \frac{mc\Delta T}{\Delta t} \rightarrow R = \frac{(\Delta V)^2 \Delta t}{mc\Delta T}$$

$$R = \frac{(110\ \mathrm{V})^2 (600\ \mathrm{s})}{(1.50\ \mathrm{kg})(4186\ \mathrm{J/kg \cdot {}^\circ C})(50.0^\circ\mathrm{C} - 10.0^\circ\mathrm{C})} = \boxed{28.9\ \Omega}$$

Example 26.5: Linking Electricity and Thermodynamics

(B) Estimate the cost of heating the water.

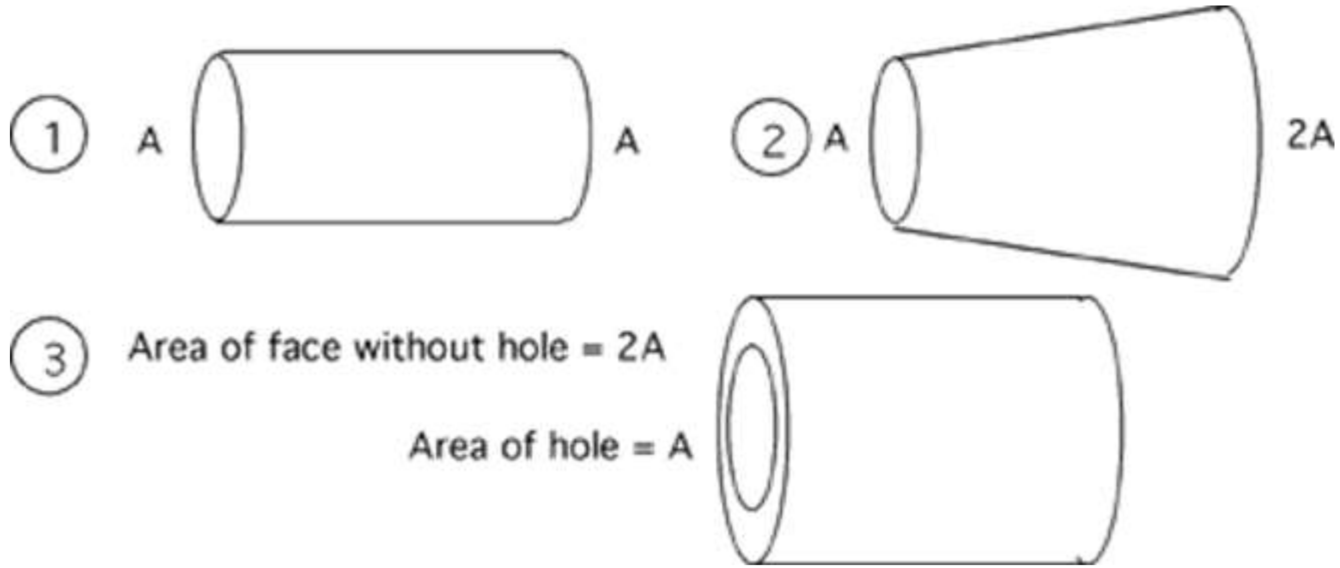
$$T_{\text{ET}} = P\Delta t = \frac{(\Delta V)^2}{R}\Delta t = \frac{(110\ \text{V})^2}{28.9\ \Omega}(10.0\ \text{min})\left(\frac{1\ \text{h}}{60.0\ \text{min}}\right)$$

$$= 69.8\ \text{Wh} = 0.0698\ \text{kWh}$$

$$\text{Cost} = (0.0698\ \text{kWh})(\$0.11/\text{kWh}) = \$0.008 = \boxed{0.8\ \text{cents}}$$

Assessing to Learn

Which object below has the lowest resistance? All three have length L and are made out of the same material.

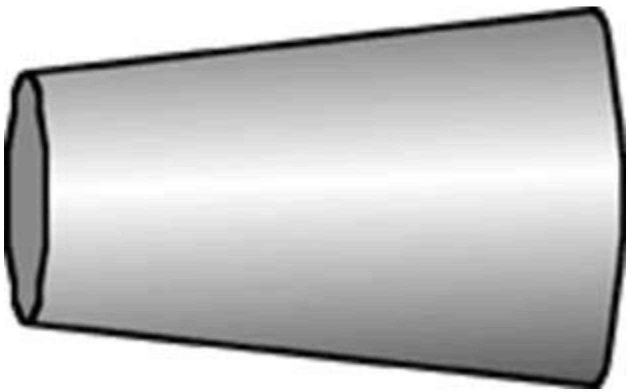


1. #1
2. #2
3. #3
4. Both #1 and #3 have the same, and theirs is less than the resistance of #2.

Assessing to Learn

An ohmic conductor is carrying a current. The cross-sectional area of the wire changes from one end of the wire to the other. Which of the following quantities vary along the wire?

A. The resistivity
B. The current
C. The current density
D. The electric field



1. A only 2. B only 3. C only 4. D only
5. A and B only 6. C and D only
7. A, B, C, and D 8. None of the above