

## STAT 3470 Midterm 1 Review Questions by Chapter

This review consists of questions over the material from Chapters 1-5, so it covers the same content that is on the first midterm. This doesn't mean that all test questions will be exactly the same as a question listed here. This is not necessarily a comprehensive review of all concepts. Nor is it a practice test. There are considerably more questions on this review than will be on the actual test. This should be used as a study tool along with examples from the lecture notes, reading the book and doing extra practice problems there, and reviewing WebAssign problems.

The solutions that are posted that go along with this practice set are meant to help you understand the amount of work that needs to be shown (which is one reason they are handwritten and not typed). To be safe, show at least as much work on your midterm problems as are shown on the solutions for these problems.

### Chapter 2

- Each of a sample of four home mortgages is classified as fixed rate (F) or variable rate (V).
  - What are the 16 outcomes in  $S$ ?
  - Which outcomes are in the event that exactly three of the selected mortgages are fixed rate?
  - Which outcomes are in the event that all four mortgages are of the same type?
  - Which outcomes are in the event that at most one of the four is a variable-rate mortgage?
  - What is the union of the events in parts (c) and (d), and what is the intersection of these two events? Draw Venn diagrams to represent these sets.
  - What are the union and intersection of the two events in parts (b) and (c)? Draw Venn diagrams to represent these sets.

- A certain factory operates three different shifts. Over the last year, 200 accidents have occurred at the factory. Some of these can be attributed at least in part to unsafe working conditions, whereas the others are unrelated to working conditions. The accompanying

		Unsafe Conditions	Unrelated to Conditions
Shift	Day	10%	35%
	Swing	8%	20%
	Night	5%	22%

table gives the percentage of accidents falling in each type of accident– shift category.

- What is the probability that the selected accident was attributed to unsafe conditions?
  - What is the probability that the selected accident did not occur on the day shift?
  - What is the probability that the selected accident was due to unsafe conditions and occurred on the night shift?
  - What is the probability that the selected accident was on the swing shift or unrelated to conditions?
  - Suppose the randomly selected accident occurred on the night shift. What is the probability it was due to unsafe conditions?
  - What is the probability that an accident unrelated to conditions occurred at a time other than the night shift?
  - Are shift and cause independent?
- At a certain gas station, 40% of the customers use regular gas, 35% use plus gas, and 25% use premium gas. Of those using regular gas, only 30% fill their tanks, compared to 60% of those using plus gas and 50% of those using premium gas.

- a. Draw a tree diagram representing this scenario.
  - b. What is the probability that a randomly selected customer fills their tank with plus gas?
  - c. What is the probability that a randomly selected customer doesn't fill their tank?
  - d. Suppose the person in front of you filled her tank. What is the probability she used premium gas?
4. a. Suppose the probability of a certain baseball player getting a hit in an at-bat is 0.27. Assume his results are independent from at-bat to at-bat, and that he gets 5 at-bats in a game. What is the probability he gets a hit in at least one of his at-bats?
- b. Now suppose his results are not independent across at-bats. Rather, each time he gets a hit he gains more confidence and his probability increases by 0.10. Each time he fails to get a hit he loses confidence and his probability decreases by 0.05. If his probability of a hit in his first at-bat is 0.27, what is the probability he gets a hit in only the third of his five at-bats.

### Chapter 3

5. Let  $X$  be a random variable such that  $E(X)=10$  and  $\text{Var}(X)=25$ .
- a. What is  $\sigma_X$ ?
  - b. What is  $E(3X+5)$ ?
  - c. What is  $\text{Var}(2X-7)$ ?
  - d. What is  $\sigma_{4X}$ ?

6. A mail order computer business has six telephone lines. Let  $X$  denote the number of lines in use at a specified time. Suppose the pmf of  $X$  is as given in the accompanying table.

$x$	0	1	2	3	4	5	6
$p(x)$	.10	.15	.20	.25	.20	.06	.04

- a. What is the probability that at most 3 lines are in use?
  - b. What is the probability that between 1 and 4 lines, inclusive, are in use?
  - c. Calculate the cdf of  $X$ .
  - d. What are the mean, variance, and standard deviation of the number of lines in use?
  - e. Someone at the company has developed the following statistic to measure line usage: each line being used, up to three lines, counts for one point. After that, each additional line being used is worth one more point than the line before it. What is the expected value of this statistic?
  - f. Someone else at the company has developed a different statistic to measure line usage: the number of lines in use is squared, and that number is added to the number of lines in use to give the total number of points. What is the expected value of this statistic?
  - g. Why might the company want to use one of the statistics in either parts (e) or (f) rather than just reporting the number of lines in use?
7. Suppose that, Daryl can make 40% of shots behind the 3-point line on a basketball court and that each shot is independent of the next.
- a. What is the probability that in a game where he shoots 15 shots, he makes exactly 4 of them? At least 2 of them? Less than 7 of them?
  - b. How many shots would you expect him to make in a season in which he shoots 300 shots? What is the standard deviation of the number of shots he makes in such a season?
  - c. What is the expected value and variance of the number of shots it takes him to hit his first shot?

8. The number of typographical errors on a page of a textbook follows a Poisson process, with an average of .05 errors per page.
  - a. What is the probability that a 300 page book contains at most 18 errors?
  - b. What is the probability that a 150 page book contains no errors?
  - c. What are the mean and standard deviation of the number of errors in a 400 page book?
  - d. Now suppose that there are 10 textbooks that all have 100 pages, in which the number of typographical errors are independent of each other. What is the probability that fewer than 9 of the books have at least two errors? (Hint: Define a random variable  $X$  as the number of errors in each textbook, and a random variable  $Y$  as the number of books that have at least two errors).

#### Chapter 4

9. Let  $X$  be a continuous random variable with pdf:  $f(x) = kx^3$  for  $2 < X < 4$  and 0 otherwise.
  - a. Find the value of  $k$  that makes this a legitimate pdf.
  - b. Find the cdf of  $X$ .
  - c. Find  $P(2.5 < X < 3.5)$
  - d. Find the 20<sup>th</sup> percentile of  $X$ .
  - e. Find the median of  $X$ .
  - f. Find the mean of  $X$
  - g. Find the standard deviation of  $X$ .
10. Suppose the scores on a certain quiz are uniformly distributed from 10 to 20.
  - a. Write down the pdf and cdf of the scores.
  - b. Calculate the probability that a randomly chosen quiz score is between 12 and 14.
  - c. What are the mean and variance of the scores?
  - d. Suppose that the scores are curved in such a way that the original score is increased by a factor of 20% and then an additional 5 points are added to it. What will the mean and variance of the curved scores be?
11. Suppose the scores on a certain test are normally distributed with mean 80 and variance 36.
  - a. What is the probability a randomly chosen test score is between 72 and 81?
  - b. What is the probability a randomly chosen test score is above 83?
  - c. What is the probability a randomly chosen test score is below 85?
  - d. If you want to be in the top 10% of scores, what is the minimum score you need?
  - e. What is the interquartile range of test scores? [Hint: find the test scores at the 25<sup>th</sup> percentile and the 75<sup>th</sup> percentile and subtract]
  - f. Suppose on the following test, the mean stays the same but the standard deviation changes. If you scored an 85 and are at the 70<sup>th</sup> percentile, what is the new standard deviation?
12. Suppose the stress range ( $X$ ) in certain bridge connections can be modeled as an exponential distribution with mean 6 MPa.
  - a. Write the pdf and cdf of  $X$ .
  - b. Find the variance of  $X$ .
  - c. Find the median of  $X$ .
  - d. Find  $P(X > 10)$
  - e. Find  $P(5 < X < 10)$ .

- f. What is the probability that the stress range is more than two standard deviations away from the mean?
  - g. Now suppose you have 10 such connections that are independent of one another. What is the probability that no more than 3 of them have a stress range more than 2 standard deviations away from the mean?
13. Suppose  $X \sim \text{Gamma}(2, 4)$ .  
Find the mean and standard deviation of  $X$ .

## Chapter 5

14. A local supermarket has two checkout lines. Let  $X$  be the number of customers in the first checkout line and  $Y$  be the number of customers in the second checkout line. The probability distribution of  $X$  and  $Y$  is given.

p(x,y)	y			
		0	1	2
x	0	.04	.10	.02
	1	.16	.10	.20
	2	.06	.22	.10

- a. What is the probability that there both lines have at least one person in them?
  - b. What is  $P(X > 1 \text{ and } Y = 2)$ ?
  - c. What is  $P(X = 2 \text{ or } Y = 0)$ ?
  - d. What is the marginal distribution of the number of customers in the first line?
  - e. Are  $X$  and  $Y$  independent?
  - f. What is the expected value of the total number of customers in both lines?
  - g. What is  $E(|X - 2Y|)$ ?
  - h. What is  $E(XY)$ ?
  - i. What is  $\text{Cov}(X, Y)$ ?
  - j. What is the correlation between  $X$  and  $Y$ ?
15. Suppose  $X$  and  $Y$  are jointly dist. with pdf  $f(x, y) = \begin{cases} k(2x^2y + xy^2) & \text{if } 0 \leq x \leq 3, 1 \leq y \leq 2 \\ \text{otherwise} \end{cases}$
- a. Find the value of  $k$ .
  - b. Calculate the marginal distributions of  $X$  and  $Y$ .
  - c. Are  $X$  and  $Y$  independent?
  - d. Calculate  $P(X < 2, Y < 1.5)$
  - e. Calculate  $E(X^2 + 2Y)$