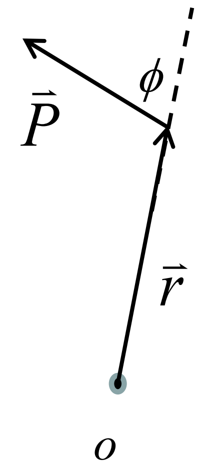
Angular Momentum

Angular Momentum

Linear Momentum

$\vec{L} = \vec{r} \times \vec{P} = \vec{r} \times m\vec{V}$ for a pointlike particle



$L = rmV \sin\phi$ $P = m\vec{V}$

$= V_t$

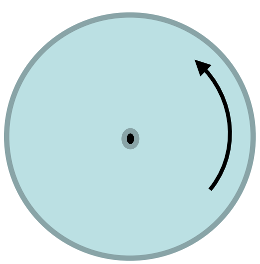
$= \omega r$

$L = mr^2\omega$

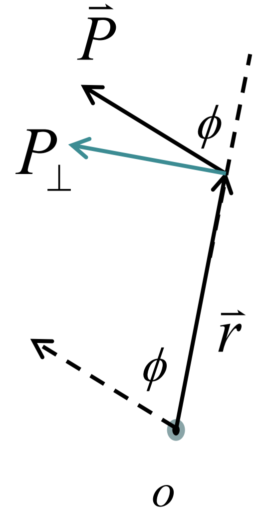
$\vec{L} = I\vec{\omega}$ for a rotating object

$L = I\omega$

$\vec{L} = I\vec{\omega}$



Angular Momentum



$$\vec{L} = \vec{r} \times \vec{P} = \vec{r} \times m\vec{V} \quad \text{for a pointlike particle}$$

$$L = rmV \sin\phi$$

$$\vec{L} = I\vec{\omega} \quad \text{for a rotating object}$$

$$L = I\omega$$

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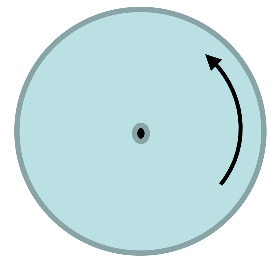
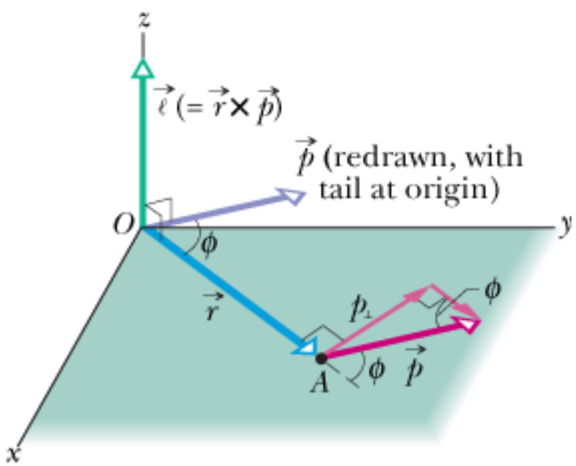
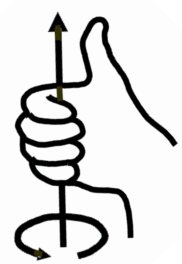




(*a*)



$$\vec{L} = I\vec{\omega} \quad \text{for a rotating object}$$

$$I = \frac{1}{2}MR^2 \quad \text{for a disk}$$

$$L = I\omega = \frac{1}{2}MR^2\omega$$

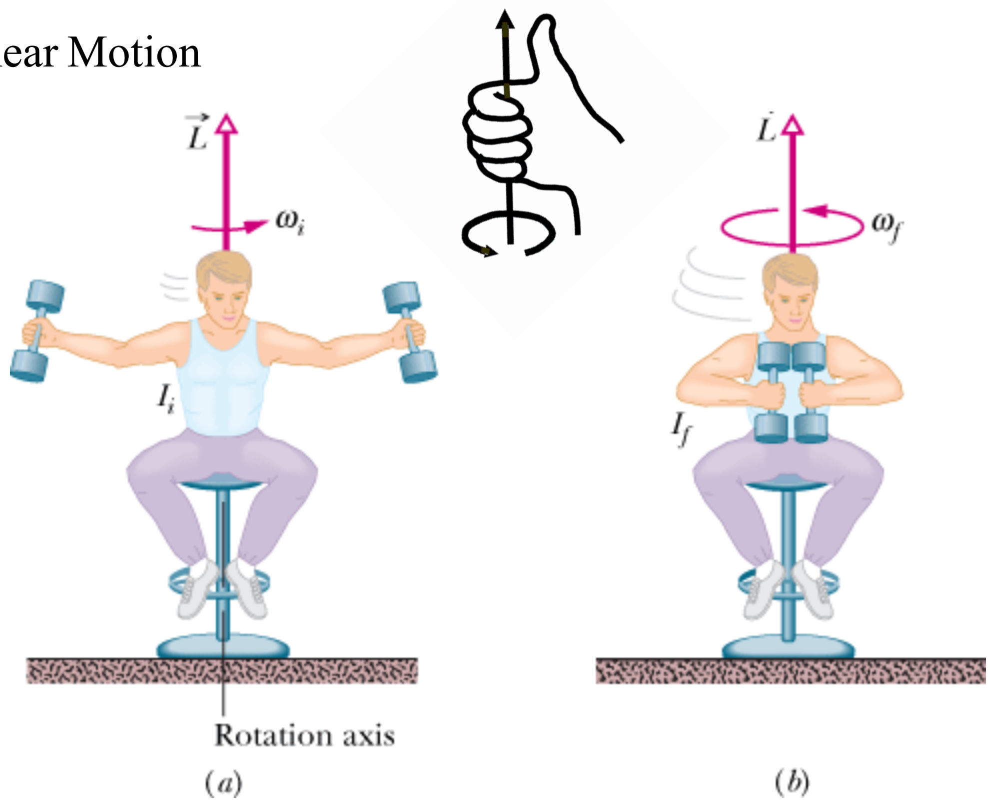
Conservation of Angular Momentum

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$ Newton's 2nd Law in Angular Form

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$ Newton's 2nd Law in Linear Motion

When $\vec{\tau}_{net} = 0$,

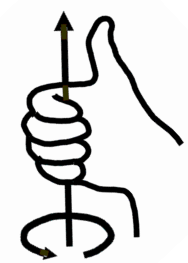
$$\vec{L}_i = \vec{L}_f$$

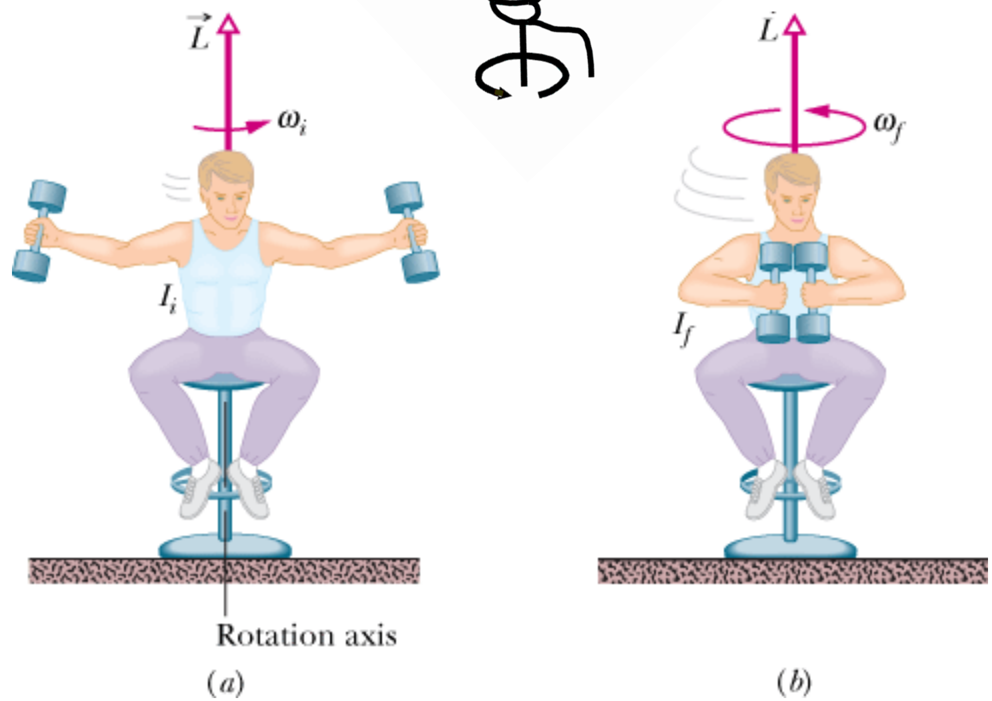


Conservation of Angular Momentum

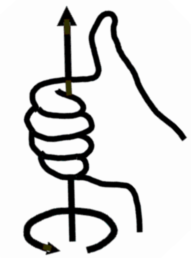
$$\vec{L}_i = \vec{L}_f$$

$$\vec{L}_i = I_i \vec{\omega}_i \qquad \vec{L}_f = I_f \vec{\omega}_f$$



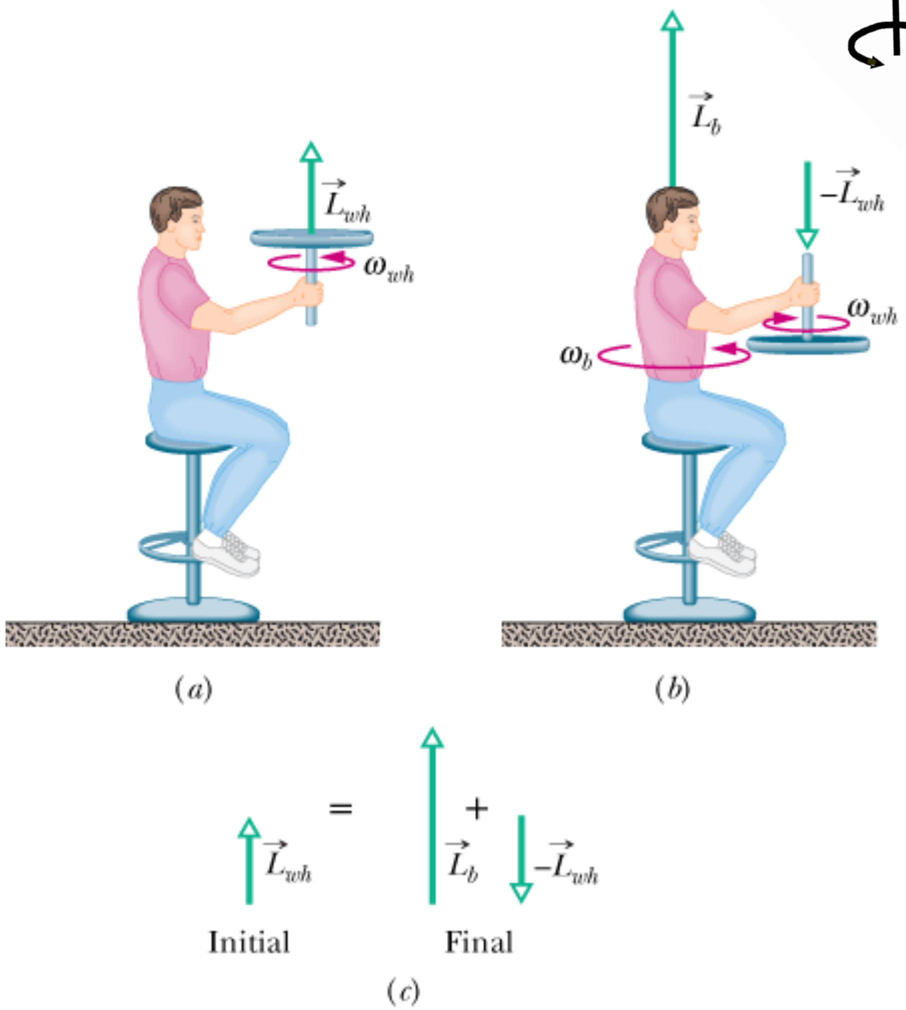


Conservation of Angular Momentum



$$\vec{L}_i = \vec{L}_f$$

$$\vec{L}_{bi} + \vec{L}_{wi} = \vec{L}_{bf} + \vec{L}_{wf}$$



Conservation of Angular Momentum

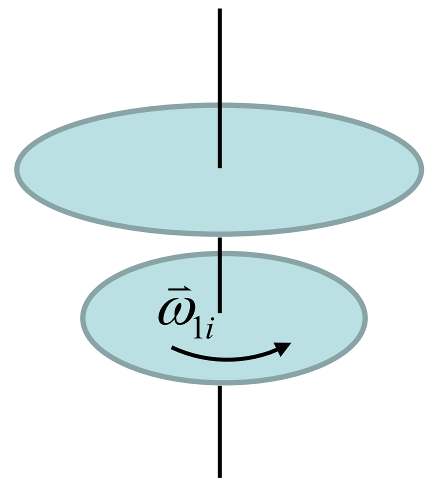
A wheel is rotating freely with an angular speed of 700 rev/min on a shaft whose rotational inertia is negligible. A second wheel, initially at rest and with twice the rotational inertia of the first, is suddenly coupled to the same shaft.

(a) What is the angular speed of the resultant combination of the shaft and two wheels?

(b) What fraction (written as a decimal) of the original rotational kinetic energy is lost?

$$\vec{L}_i = \vec{L}_f$$

$$\vec{L}_i = I_i\vec{\omega}_i \qquad \vec{L}_f = I_f\vec{\omega}_f$$



$$I_2 = 2I_1$$

$$I_1$$

$$\vec{L}_i = I_i\vec{\omega}_i = I_1\vec{\omega}_{1i} + I_2 \cdot 0$$

$$\vec{L}_f = I_f\vec{\omega}_f = (I_1 + I_2)\vec{\omega}_f$$

$K_i = \frac{1}{2} I_1 \omega_{1i}^2$

$K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$

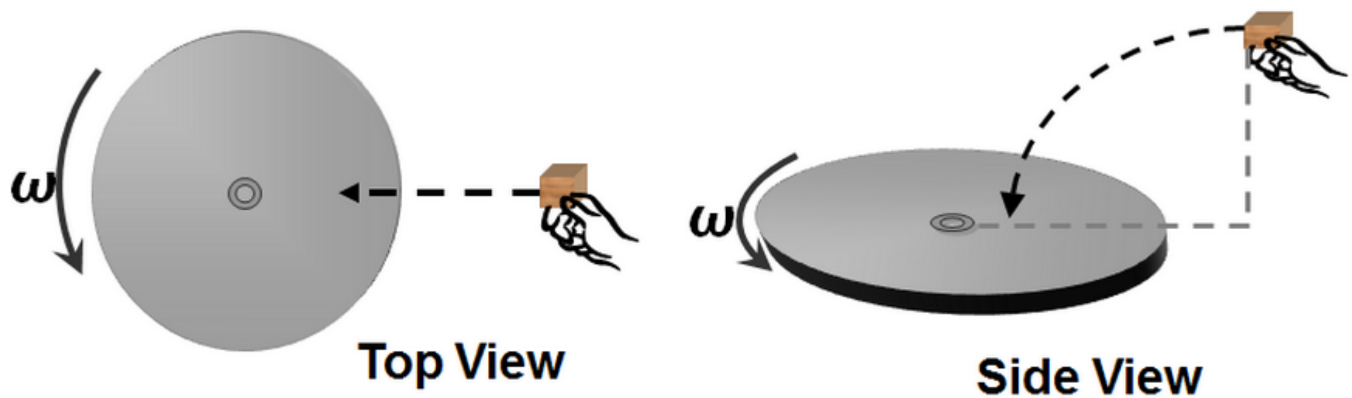
$= \frac{1}{2}(3I_1)\cdot(\frac{1}{3}\omega_{1i})^2$

$= \frac{1}{3}(\frac{1}{2} I_1 \omega_{1i}^2)$

$\frac{2}{3} K_i$ lost

$\omega_f = \frac{1}{3}\omega_i$

The magnitude of the angular momentum for a freely rotating disk around its center is L. You drop a heavy block onto the disk along the direction as depicted below, and the block then stays on the disk. Now the magnitude of the angular momentum for the disk-block system is:



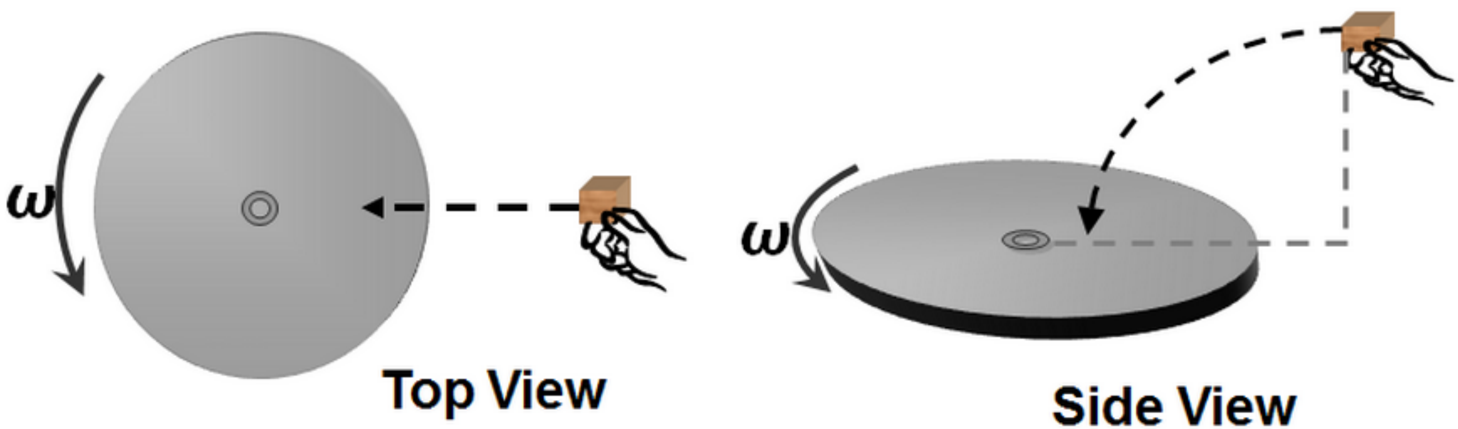
(1) > L

(2) < L

(3) = L

(4) Not enough information to determine

The angular speed of a freely rotating disk around its center is ω. You drop a heavy block onto the disk along the direction as depicted below, and the block then stays on the disk. The angular speed of the disk-block system now:



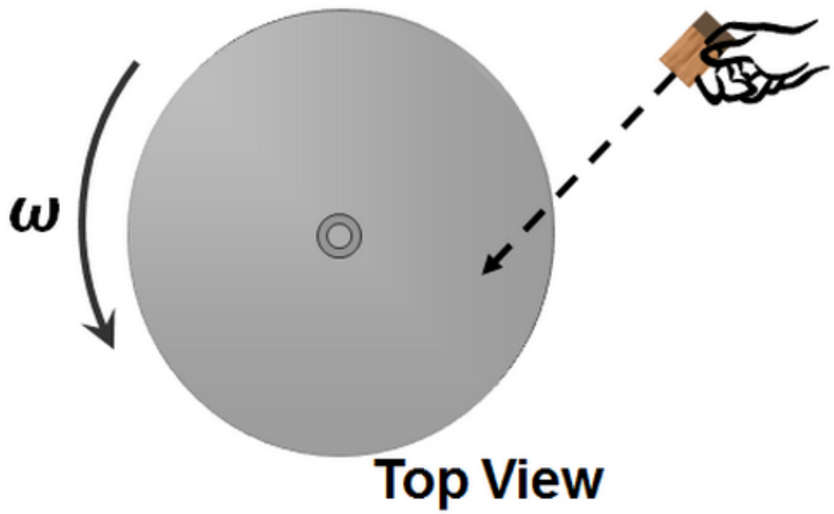
(1) decreases

(2) increases

(3) remains the same

(4) Not enough information to determine

The magnitude of the angular momentum for a freely rotating disk around its center is L. You drop a heavy block onto the disk along the direction as depicted below, and the block then stays on the disk. Now the magnitude of the angular momentum for the disk-block system is:



(1) > L

(2) < L

(3) = L

(4) Not enough information to determine

Conservation of Angular Momentum

$$\vec{L}_i = \vec{L}_f$$

$$\vec{L}_i = I_i \vec{\omega}_i \qquad \vec{L}_f = I_f \vec{\omega}_f$$

A 0.005-kg bullet traveling horizontally with speed 1000 m/s strikes an 18.0-kg door, imbedding itself 10.0 cm from the side opposite the hinges. The 1.00-m wide door is free to swing on its frictionless hinges.

(a) At what angular speed does the door swing open immediately after the collision?

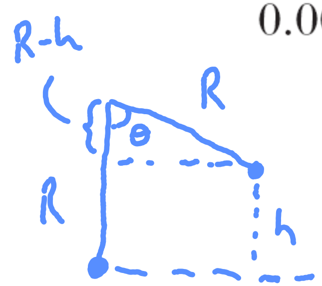
$$mV_i = (m+M)V_f$$

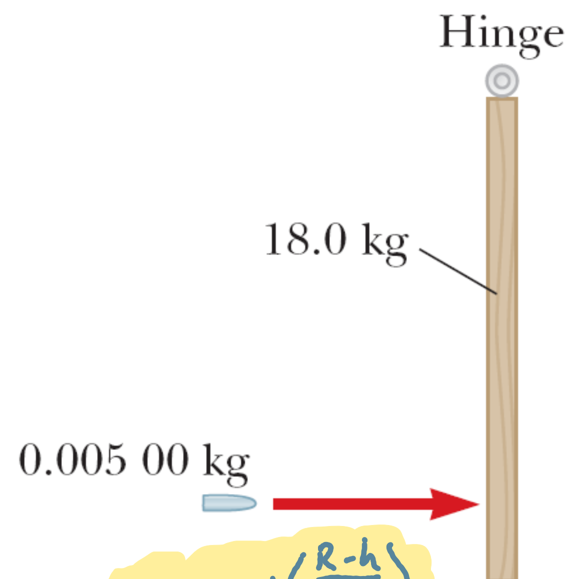
$$(m+M)gh = \frac{1}{2}(m+M)V_f^2$$

$$h = \frac{V_f^2}{2g}$$

$$\vec{L}_i = I_i \vec{\omega}_i = I_1 \vec{\omega}_{1i} + I_2 \cdot 0$$

$$\vec{L}_f = I_f \vec{\omega}_f = (I_1 + I_2)\vec{\omega}_f$$





$$\theta = \cos^{-1}\left(\frac{R-h}{R}\right)$$

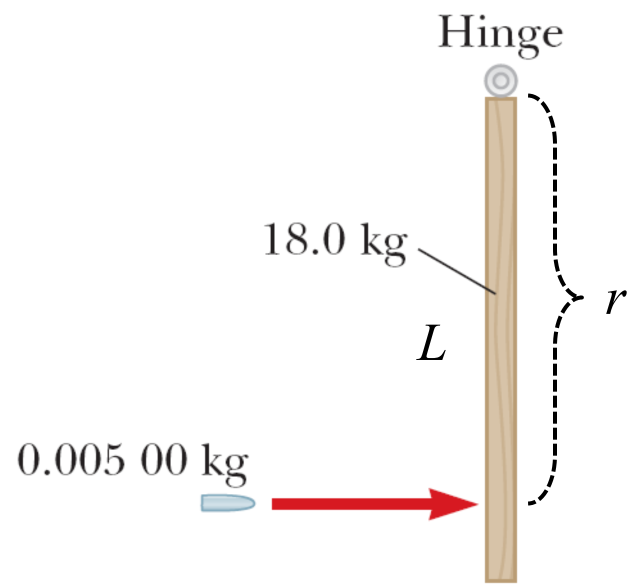
Conservation of Angular Momentum

A 0.005-kg bullet traveling horizontally with speed 1000 m/s strikes an 18.0-kg door, imbedding itself 10.0 cm from the side opposite the hinges. The 1.00-m wide door is free to swing on its frictionless hinges.

$$\vec{L}_i = \vec{L}_f$$

$$\vec{L}_i = I_i \vec{\omega}_i \qquad \vec{L}_f = I_f \vec{\omega}_f$$

(a) At what angular speed does the door swing open immediately
after the collision?



$$\vec{L}_i = \vec{r} \times \vec{p} + I_2 \cdot 0 \Rightarrow L_i = m_1 v_{1i} r$$

$$L_f = I_f \omega_f = (I_1 + I_2)\omega_f$$

$$= (m_1 r^2 + \frac{1}{3} m_2 L^2)\omega_f$$

$$(m_1 r^2 + \frac{1}{3} m_2 L^2)\omega_f = m_1 v_{1i} r$$