



Lecture Outline

Reminders to self:

- ☐ Turn on lecture recording to Cloud
- ☐ Turn on Zoom microphone

- Last Lecture

- Continued Counters

- Finished 3-bit Gray-Code counter from last Friday's lecture
 - Up and Down counter
 - Counter design with other types of flip-flops (SR)

- Today's Lecture

- Continue Counters

- Finish counter design with other types of flip-flops (SR and JK)
 - Analysis of Counter Circuits – verifying design does the expected



Handouts and Announcements

- Announcements

- Homework Problem 12-5

- Posted on Carmen yesterday
- Due: 11:25am Wednesday 3/22

- Homework Reminder

- HWs 12-2 & 12-3 Due: 11:59pm Thursday 3/9
- HW 12-4 Due: 11:59pm Tuesday 3/21

- Read for Friday: pages 425-432

- Mini-Exam 3 regrade continuing

- A-some C regraded
- Some P-Z regraded



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Counters

4-bit Excess-3 counter using SR flip-flops

Q	Q ⁺	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

1 gate
3 inputs

DC BA	00	01	11	10
00	x			x
01	x		x	x
11		1	x	x
10	x		x	x

$$S_D = ABC$$

DC BA	00	01	11	10
00	x	x	1	
01	x	x	x	
11	x		x	
10	x	x	x	

1 gate
2 inputs

$$R_D = B'c$$

or $R_D = CD$
or $R_D = A'c$



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Counters

4-bit Excess-3 counter using SR flip-flops

D	C	B	A	D ⁺	C ⁺	B ⁺	A ⁺	S _D	R _D	S _C	R _C	S _B	R _B	S _A	R _A
0	0	0	0	-	-	-	-	X	X	X	X	X	X	X	X
0	0	0	1	-	-	-	-	X	X	X	X	X	X	X	X
0	0	1	0	-	-	-	-	X	X	X	X	X	X	X	X
0	0	1	1	0	1	0	0	0	X	1	0				
0	1	0	0	0	1	0	1	0	X	X	0				
0	1	0	1	0	1	1	0	0	X	X	0				
0	1	1	0	0	1	1	1	0	X	X	0				
0	1	1	1	1	0	0	0	1	0	0	1				
1	0	0	0	1	0	0	1	X	0	0	X				
1	0	0	1	1	0	1	0	X	0	0	X				
1	0	1	0	1	0	1	1	X	0	0	X				
1	0	1	1	1	1	0	0	X	0	1	0				
1	1	0	0	0	0	1	1	0	1	0	1				
1	1	0	1	-	-	-	-	X	X	X	X	X	X	X	X
1	1	1	0	-	-	-	-	X	X	X	X	X	X	X	X
1	1	1	1	-	-	-	-	X	X	X	X	X	X	X	X

DC BA	00	01	11	10
00	x	X		
01	x	X	x	
11	1		x	1
10	x	X	x	

$$S_C = ABC'$$

1 gate
3 inputs

DC BA	00	01	11	10
00	x		1	X
01	x		x	X
11		1	x	
10	x		x	X

$$R_C = CD + ABC$$

reuse R_D reuse S_D

$$\text{or } R_C = B'D + ABC$$

$$\text{or } R_C = A'D + ABC$$

1 new gate
2 new inputs



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Counters

4-bit Excess-3 counter using SR flip-flops

Q	Q ⁺	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

DC BA	00	01	11	10
00	x			
01	x	1	x	1
11			x	
10	x	x	x	x

$$S_B = C'D + AB'$$

reuse R_D

2 new gates
4 new inputs

DC BA	00	01	11	10
00	x	x		x
01	x		x	
11	1	1	x	1
10	x		x	

$$R_B = AB$$

1 new gate
2 new inputs

D	C	B	A	D ⁺	C ⁺	B ⁺	A ⁺	S _D	R _D	S _C	R _C	S _B	R _B	S _A	R _A
0	0	0	0	-	-	-	-	X	X	X	X	X	X	X	X
0	0	0	1	-	-	-	-	X	X	X	X	X	X	X	X
0	0	1	0	-	-	-	-	X	X	X	X	X	X	X	X
0	0	1	1	0	1	0	0	0	X	1	0	0	1		
0	1	0	0	0	1	0	1	0	X	X	0	0	X		
0	1	0	1	0	1	1	0	0	X	X	0	1	0		
0	1	1	0	0	1	1	1	0	X	X	0	X	0		
0	1	1	1	1	0	0	0	1	0	0	1	0	1		
1	0	0	0	1	0	0	1	X	0	0	X	0	X		
1	0	0	1	1	0	1	0	X	0	0	X	1	0		
1	0	1	0	1	0	1	1	X	0	0	X	X	0		
1	0	1	1	1	1	0	0	X	0	1	0	0	1		
1	1	0	0	0	0	1	1	0	1	0	1	1	0		
1	1	0	1	-	-	-	-	X	X	X	X	X	X	X	X
1	1	1	0	-	-	-	-	X	X	X	X	X	X	X	X
1	1	1	1	-	-	-	-	X	X	X	X	X	X	X	X



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Counters

4-bit Excess-3 counter using SR flip-flops

Q	Q ⁺	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

DC BA	00	01	11	10
00	x	1	1	1
01	x		x	
11			x	
10	x	1	x	1

$$S_A = A'$$

DC BA	00	01	11	10
00	x			
01	x	1	x	1
11	1	1	x	1
10	x		x	

$$R_A = A$$

Total
7 gates
16 inputs



Counters

Counter design using J-K flip-flops

- Very similar to S-R
- Except now $J = K = 1$ is allowed, for the Toggle operation

Circuit Analysis TT

J	K	Q	Q^+
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Q	Q^+	J	K
0	0	{ 0	0
		{ 0	1
0	1	{ 1	0
		{ 1	1
1	0	{ 0	1
		{ 1	1
1	1	{ 0	0
		{ 1	0

Circuit Design TT

Q	Q^+	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Toggle for $J = K = 1$
allows change of these
from 0 to Don't Care



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Counters

4-bit Excess-3 counter using JK flip-flops

Q	Q ⁺	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

1 gate
3 inputs

DC BA	00	01	11	10
00	x		x	x
01	x		x	x
11		1	x	x
10	x		x	x

$$J_0 = ABC$$

DC BA	00	01	11	10
00	x	x	1	
01	x	x	x	
11	x	x	x	
10	x	x	x	

$$K_0 = C$$

D	C	B	A	D ⁺	C ⁺	B ⁺	A ⁺	J _D	K _D	J _C	K _C	J _B	K _B	J _A	K _A
0	0	0	0	-	-	-	-	X	X	X	X	X	X	X	X
0	0	0	1	-	-	-	-	X	X	X	X	X	X	X	X
0	0	1	0	-	-	-	-	X	X	X	X	X	X	X	X
0	0	1	1	0	1	0	0	0	X						
0	1	0	0	0	1	0	1	0	X						
0	1	0	1	0	1	1	0	0	X						
0	1	1	0	0	1	1	1	0	X						
0	1	1	1	1	0	0	0	1	X						
1	0	0	0	1	0	0	1	X	0						
1	0	0	1	1	0	1	0	X	0						
1	0	1	0	1	0	1	1	X	0						
1	0	1	1	1	1	0	0	X	0						
1	1	0	0	0	0	1	1	X	1						
1	1	0	1	-	-	-	-	X	X	X	X	X	X	X	X
1	1	1	0	-	-	-	-	X	X	X	X	X	X	X	X
1	1	1	1	-	-	-	-	X	X	X	X	X	X	X	X



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Counters

4-bit Excess-3 counter using JK flip-flops

Q	Q ⁺	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

1 gate
2 inputs

DC BA	00	01	11	10
00	x	x	X	
01	x	x	x	
11	1	x	x	1
10	x	x	x	

$$J_C = AB$$

DC BA	00	01	11	10
00	x		1	X
01	x		x	x
11	X	1	x	x
10	x		x	X

$$K_C = D + AB$$

reuse J_C

1 new gate
2 new inputs

D	C	B	A	D ⁺	C ⁺	B ⁺	A ⁺	J _D	K _D	J _C	K _C	J _B	K _B	J _A	K _A
0	0	0	0	-	-	-	-	X	X	X	X	X	X	X	X
0	0	0	1	-	-	-	-	X	X	X	X	X	X	X	X
0	0	1	0	-	-	-	-	X	X	X	X	X	X	X	X
0	0	1	1	0	1	0	0	0	X	1	X				
0	1	0	0	0	1	0	1	0	X	X	0				
0	1	0	1	0	1	1	0	0	X	X	0				
0	1	1	0	0	1	1	1	0	X	X	0				
0	1	1	1	1	0	0	0	1	X	X	1				
1	0	0	0	1	0	0	1	X	0	0	X				
1	0	0	1	1	0	1	0	X	0	0	X				
1	0	1	0	1	0	1	1	X	0	1	X				
1	0	1	1	1	1	0	0	X	0	1	X				
1	1	0	0	0	0	1	1	X	1	X	1				
1	1	0	1	-	-	-	-	X	X	X	X	X	X	X	X
1	1	1	0	-	-	-	-	X	X	X	X	X	X	X	X
1	1	1	1	-	-	-	-	X	X	X	X	X	X	X	X



Counters

4-bit Excess-3 counter using JK flip-flops

Q	Q ⁺	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

2 new gates
4 new inputs

DC BA	00	01	11	10
00	x		1	0
01	x	1	x	1
11	x	x	x	x
10	x	x	x	x

$$J_B = A + C_0$$

DC BA	00	01	11	10
00	x	x	x	x
01	x	x	x	x
11	1	1	x	1
10	x		x	

$$K_B = A$$



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Counters

4-bit Excess-3 counter using JK flip-flops

Q	Q ⁺	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

DC BA	00	01	11	10
00	x	1	1	1
01	x	X	x	X
11	X	X	x	X
10	x	1	x	1

$$J_A = 1$$

DC BA	00	01	11	10
00	x	X	X	X
01	x	1	x	1
11	1	1	x	1
10	x	X	x	X

$$K_A = 1$$

Total
5 gates
11 inputs



4-bit Excess-3 counter using J-K flip-flops vs T or SR

- Using T flip-flops: 6 gates, 13 inputs
- Using S-R flip-flops: 7 gates, 16 inputs
- Using J-K flip-flops: 5 gates, 11 inputs

J-K is not always minimal

Different types of flip-flops will yield minimal designs for different count sequences

Watch for things like this in Homework 12-4



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Counters

4-bit Excess-3 counter using J-K flip-flops

$$J_D = ABC$$

$$K_D = C$$

$$J_C = AB$$

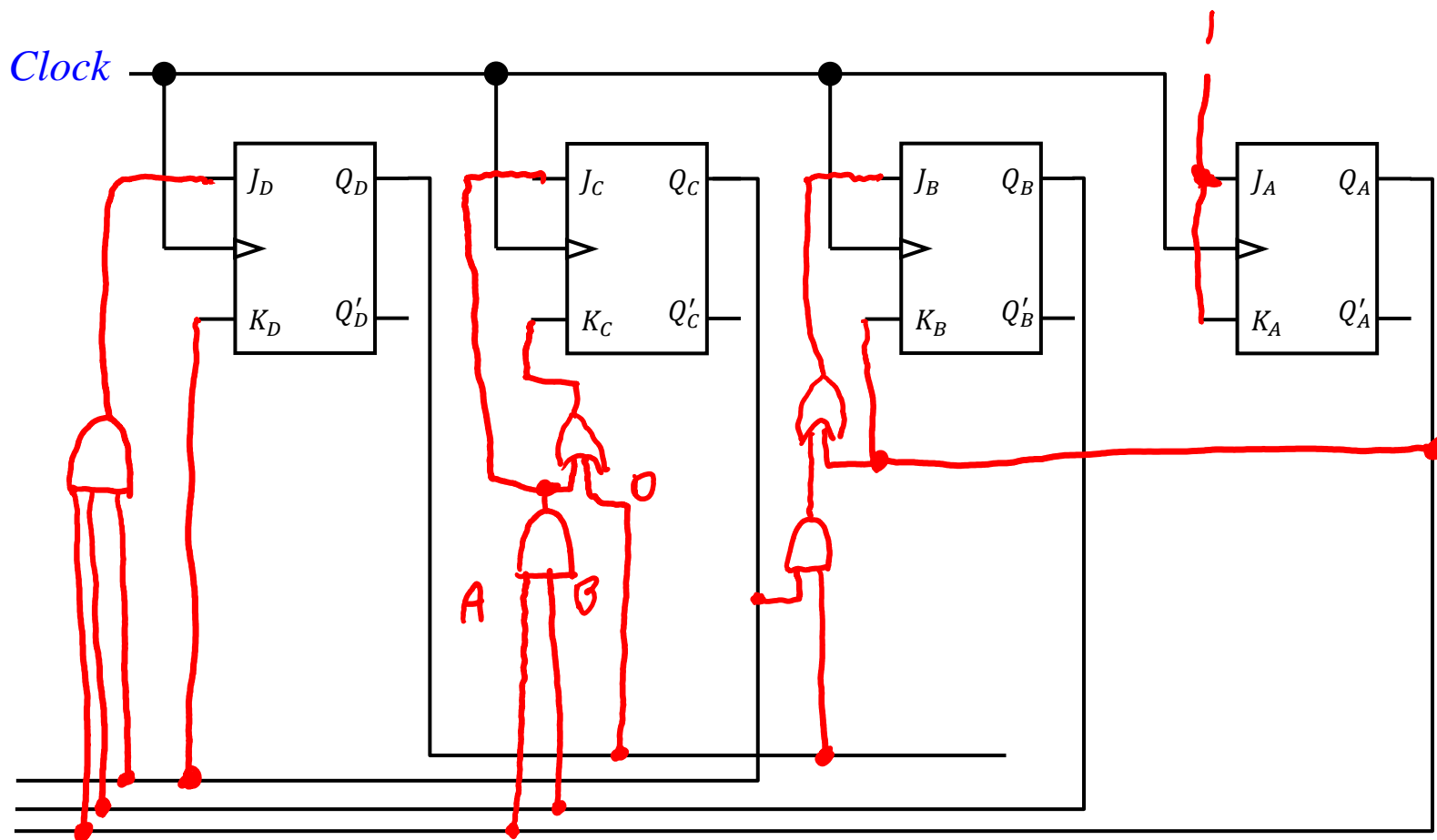
$$K_C = AB + D$$

$$J_B = A + CD$$

$$K_B = A$$

$$J_A = 1$$

$$K_A = 1$$

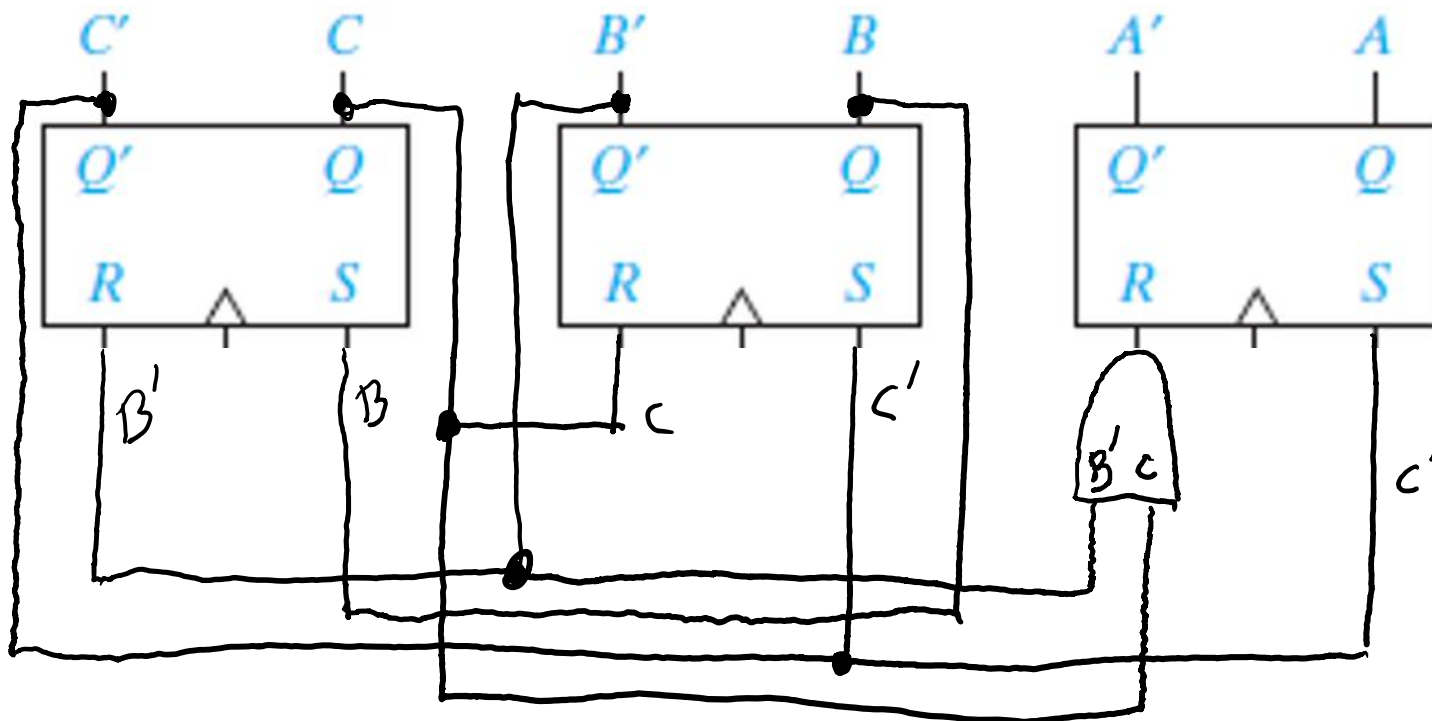
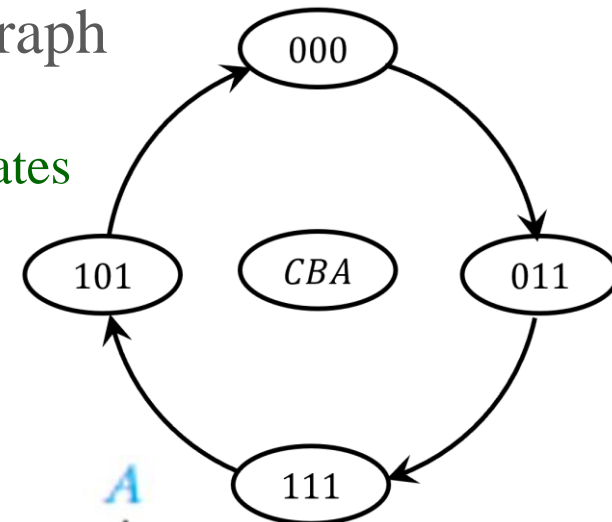




Counters

A 3-bit counter was designed for this transition graph

1. Verify the design counts the correct sequence
2. And what happens if the counter starts in one of the states not in this sequence? (Secondary transition graph?)





Counters

C	B	A	S_C	R_C	S_B	R_B	S_A	R_A	C^+	B^+	A^+
0	0	0	0	1	1	0	1	0	0	1	1
0	0	1	0	1	1	0	1	0	0	1	1
0	1	0	1	0	1	0	1	0	1	1	1
0	1	1	1	0	1	0	1	0	1	1	1
1	0	0	0	1	0	1	0	1	0	0	0
1	0	1	0	1	0	1	0	1	0	0	0
1	1	0	1	0	0	1	0	0	1	0	0
1	1	1	1	0	0	1	0	0	1	0	1

3-bit counter: verification of count

sequence

$$S_C(C, B, A) = B$$

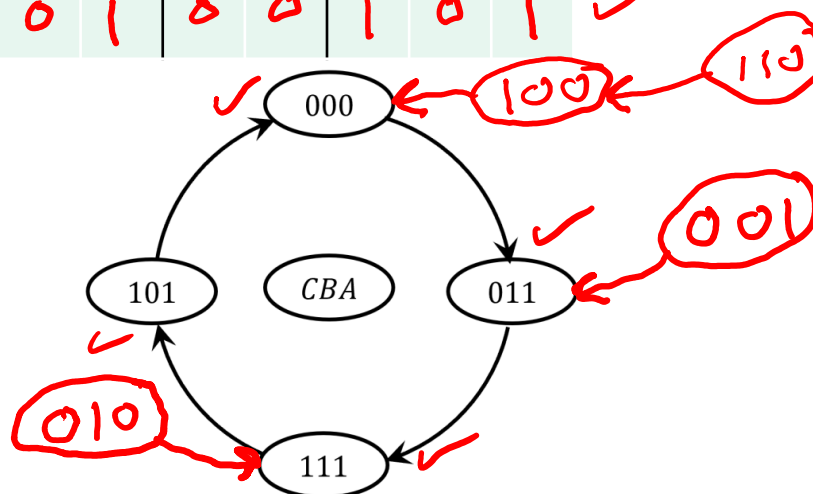
$$R_C(C, B, A) = B'$$

$$S_B(C, B, A) = C'$$

$$S_A(C, B, A) = C'$$

$$R_B(C, B, A) = C$$

$$R_A(C, B, A) = B'C$$





Counters

- Is possible that Don't Cares during design result in $R = S = 1$ for one or more bits of states not in count sequence
- Shows up in analysis of all states
- Could leave such a state by more than one path
- Not predictable which path will be followed
- But for correctly designed counter either path eventually leads to main count loop