



# Lecture Outline

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## Reminders to self:

- ☐ Turn on lecture recording to Cloud
- ☐ Turn on Zoom microphone

- Last Lecture

- Started Analysis of clocked sequential circuits
  - Parity checker design example
  - Analysis by signal tracing and timing charts
  - Definitions of Moore and Mealy machines
  - Moore & Mealy machine analysis examples

- Today's Lecture

- Continue Analysis of clocked sequential circuits
  - Analysis by Transition Tables & State Graphs



# Handouts and Announcements

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- Announcements
  - Homework Problem HW 13-1
    - Posted on Carmen this morning
    - Due 11:25am Wednesday 3/29
  - Homework Reminder
    - HW 12-4 Due: 11:59pm Tuesday 3/21
    - HW 12-5 Due: 11:25am Wednesday 3/22
  - Read for Wednesday: pages 453, 457-463
  - Mini-Exam 3 regrade finished Wednesday of last week



## Basic Procedure to Find Output Sequence By Transition Tables and Graphs:

1. Determine equations for inputs to flip-flops and outputs from circuit
2. Derive next-state equations for each flip-flop from its input equations (using flip-flop next-state relations *or truth tables* )
  - D:  $Q^+ = D$
  - D-CE:  $Q^+ = D \cdot CE + Q \cdot CE'$
  - T:  $Q^+ = TQ' + T'Q = T \oplus Q$
  - S-R:  $Q^+ = S + R'Q$  (*SR=0*)
  - J-K:  $Q^+ = JQ' + K'Q$
3. Plot a next-state map for each flip-flop (*looks like k-map*)
4. Combine these maps to form the transition table that gives the next state of the flip-flop as a function of current state and circuit inputs
5. Use the transition table to form the state table  
(*names instead of next-states*)
6. Use the state table to draw the state graph

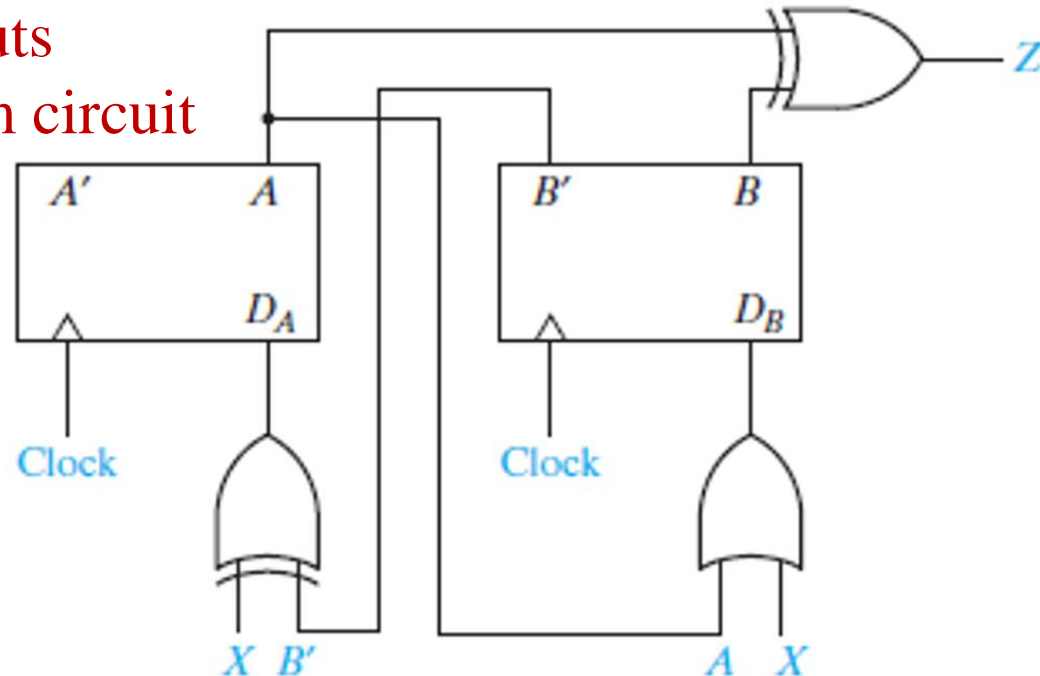


## Analysis by Transition Tables &amp; Graphs

Repeat Moore example using these techniques:

1. Determine equations for inputs to flip-flops and outputs from circuit

- $D_A = X \oplus B'$
- $D_B = X + A$
- $Z = A \oplus B$
- These are same as when worked before
- Except before we plugged in values of all variables for the initial state right away
- And then repeated as we stepped through each state





## Analysis by Transition Tables &amp; Graphs

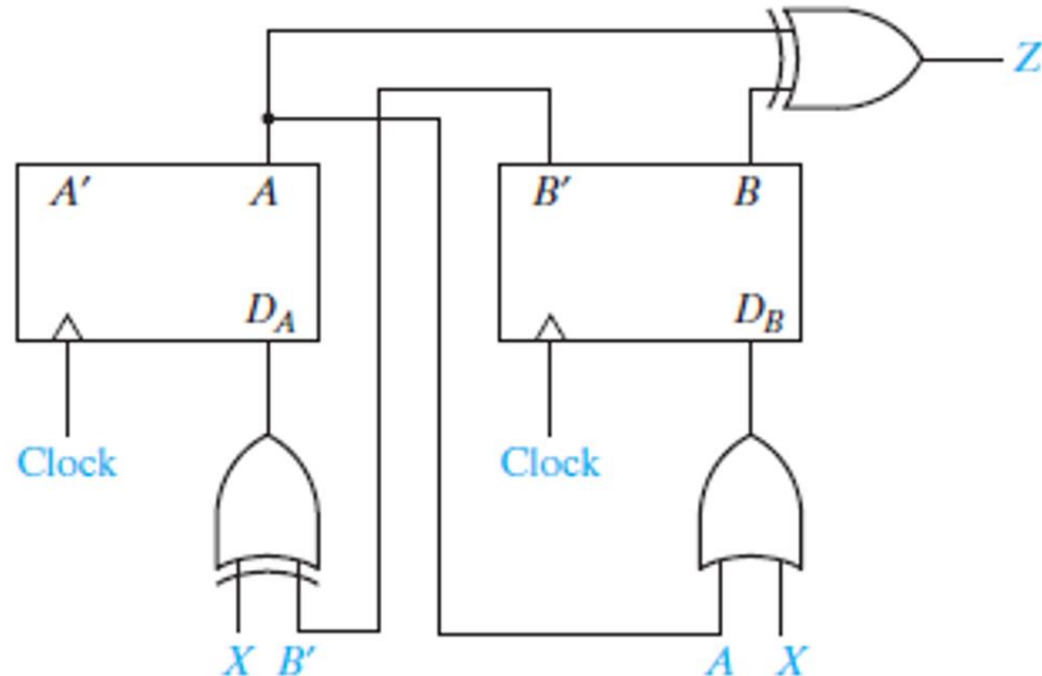
Repeat Moore example using these techniques:

- $D_A = X \oplus B'$
- $D_B = X + A$
- $Z = A \oplus B$

2. Derive next-state equations for each flip-flop from its input equations (using flip-flop next-state relations)

- D:  $Q^+ = D$

- $A^+ = X \oplus B'$
- $B^+ = X + A$





## Analysis by Transition Tables &amp; Graphs

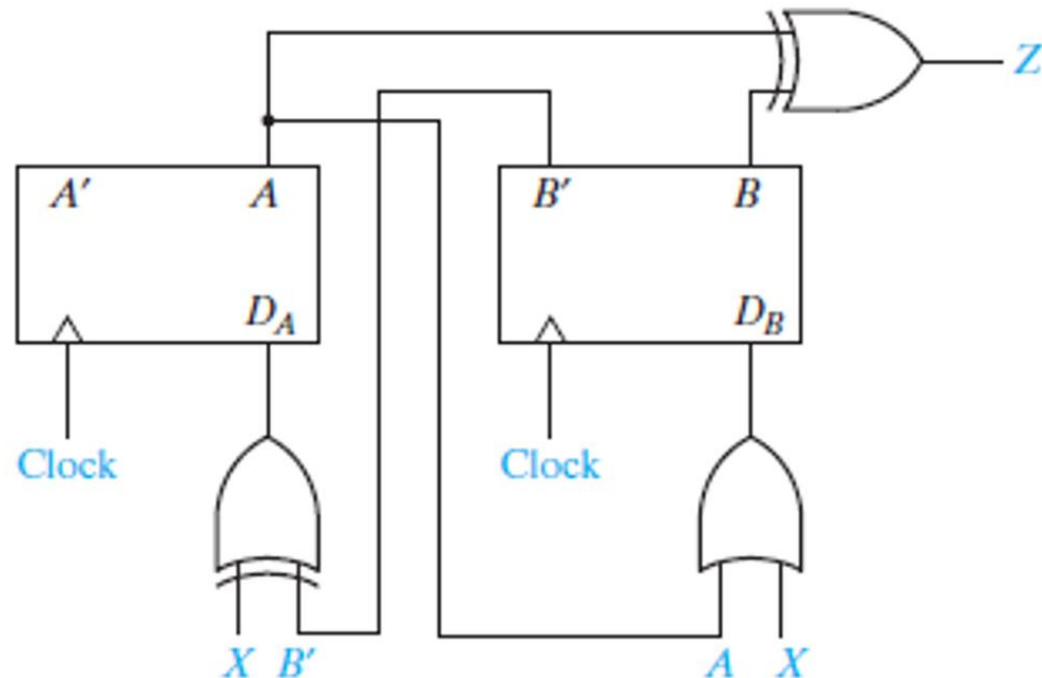
Repeat Moore example using these techniques:

- $A^+ = X \oplus B'$
- $B^+ = X + A$
- $Z = A \oplus B$

3. Plot a next-state map for each flip-flop

$A^+$ $AB \backslash X$		0	1
00			0
01		0	
11		0	
10			0

$B^+$ $AB \backslash X$		0	1
00		0	
01		0	
11			
10			





## Analysis by Transition Tables &amp; Graphs

Repeat Moore example using these techniques:

- $Z = A \oplus B$

4. Combine these maps to form the transition table that gives the next state of the flip-flop as a function of current state and circuit inputs

AB	$A^+B^+$		$Z = A \oplus B$
	$X = 0$	$X = 1$	
00	10	01	0
01	00	11	1
11	01	11	0
10	11	01	1

$A^+$ $AB \backslash X$	0	1
00	1	0
01	0	1
11	0	1
10	1	0

$B^+$ $AB \backslash X$	0	1
00	0	1
01	0	1
11	1	1
10	1	1



## Analysis by Transition Tables &amp; Graphs

Repeat Moore example using these techniques:

- $Z = A \oplus B$

State Name definitions for this example

$$S_0 (AB=00) \quad S_1 (AB=01)$$

$$S_2 (AB=11) \quad S_3 (AB=10)$$

4. Combine these maps to form the transition table that gives the next state of the flip-flop as a function of current state and circuit inputs

Symbols assigned to each state

AB	$A^+B^+$		$Z = A \oplus B$
	$X = 0$	$X = 1$	
00	10	01	0
01	00	11	1
11	01	11	0
10	11	01	1

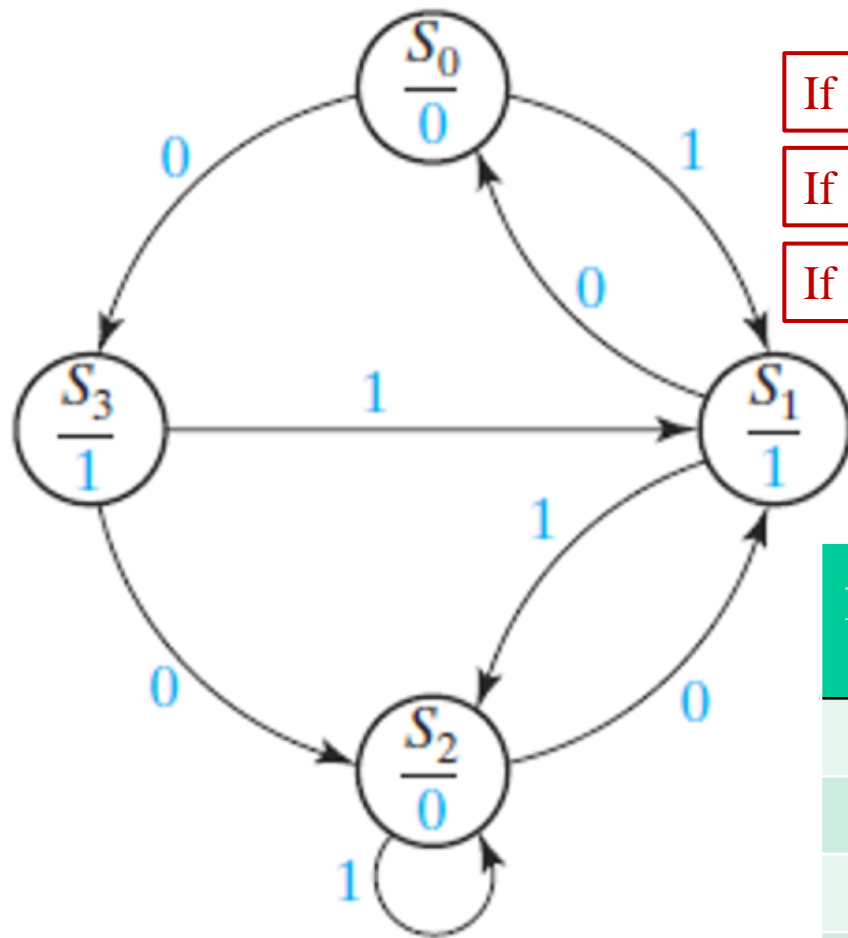
Present State	Next State		Present Output $Z = A \oplus B$
	$X = 0$	$X = 1$	
$S_0$	$S_3$	$S_1$	0
$S_1$	$S_0$	$S_2$	1
$S_2$	$S_1$	$S_2$	0
$S_3$	$S_2$	$S_1$	1





## Analysis by Transition Tables &amp; Graphs

Repeat Moore example using these techniques:



If in  $S_0$  and  $X = 0$ , go to  $S_3$  at next active clock edge

If in  $S_0$  and  $X = 1$ , go to  $S_1$  at next active clock edge

If in  $S_1$  and  $X = 1$ , go to  $S_2$  at next active clock edge

If in  $S_2$  and  $X = 0$ , go to  $S_1$  at next active clock edge ... etc.

Present State	Next State		Present Output $Z = A \oplus B$
	$X = 0$	$X = 1$	
$S_0$	$S_3$	$S_1$	0
$S_1$	$S_0$	$S_2$	1
$S_2$	$S_1$	$S_2$	0
$S_3$	$S_2$	$S_1$	1



## Moore Machine Design Example

## Traffic Signal – Left Turn Arrow (Moore Machine)

## • Two outputs:

- Green Arrow
- Yellow Arrow
- No Arrow

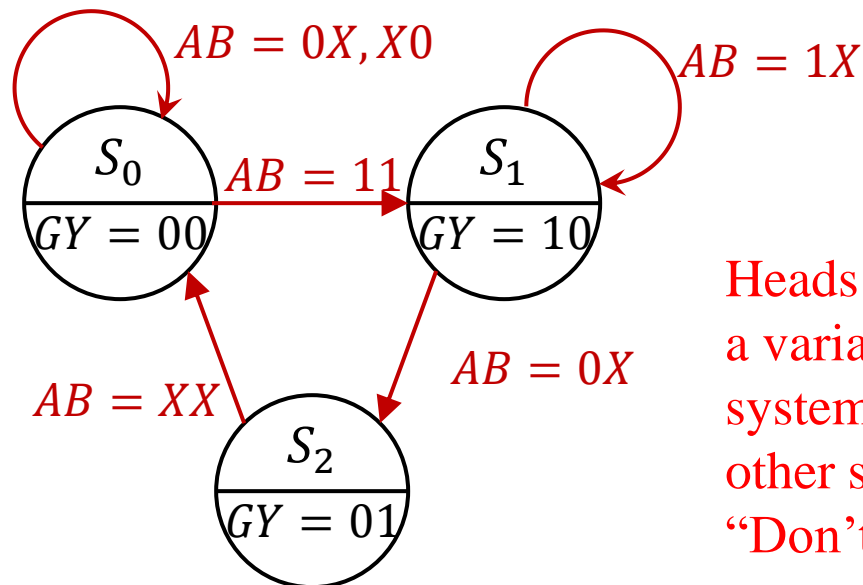
2 Flip Flops: Will use D

## • Two control inputs:

- $A$  = Time for the arrow to be on
- $B$  = There is a car waiting to make a left turn

Heads Up: I labeled all outputs  $GY = xy$ . If only labeled with values, provide a key

Heads Up: I labeled all transitions  $AB = xy$ . If only labeled with values, provide a key



Heads Up: If  $X$  is a variable in a system, use some other symbol for “Don’t Care”!



## Moore Machine Design Example

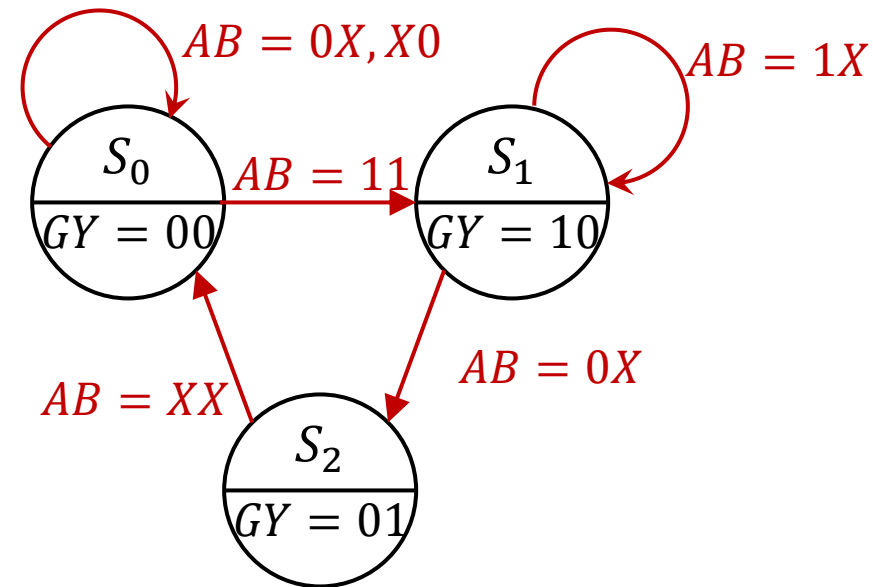
## Traffic Signal – Left Turn Arrow (Moore Machine)

$A$  = time for turn arrow to be on

$B$  = car is waiting to make turn

**Note:**

- For complex systems one might need state graph  $\rightarrow$  state table  $\rightarrow$  transition table
- This problem is simple enough to skip directly to transition table



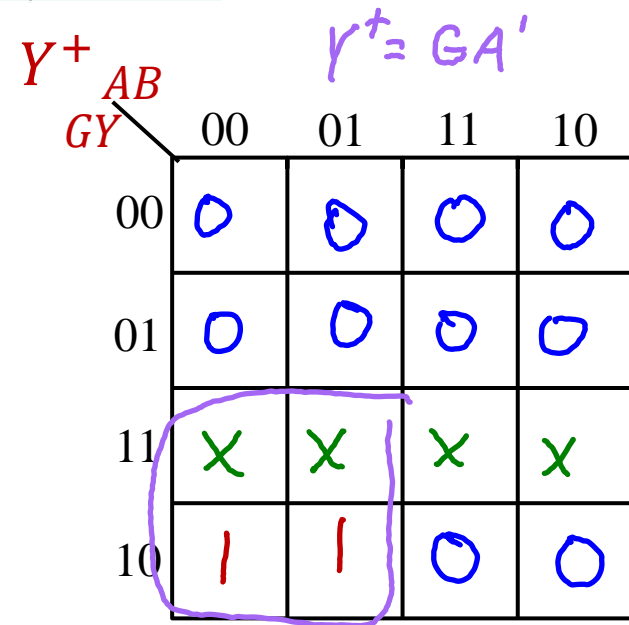
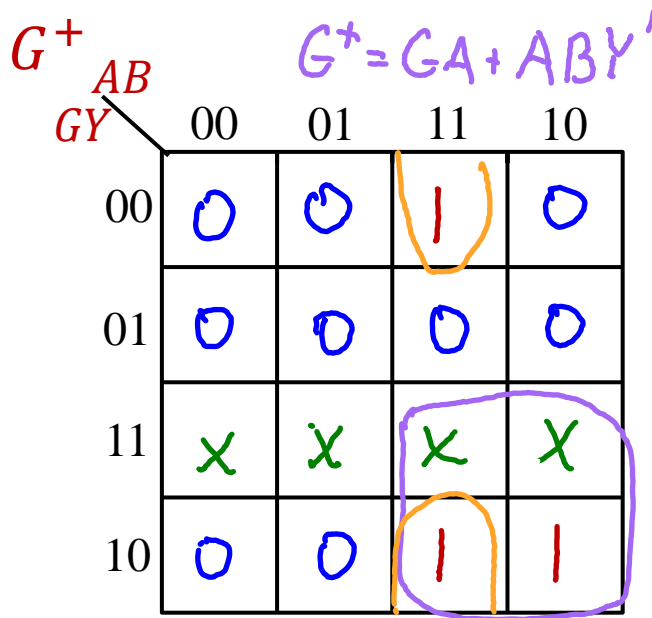
Present	Next $G^+Y^+$				Output
$GY$	$AB = 00$	$AB = 01$	$AB = 11$	$AB = 10$	$GY$
00	00	00	10	00	00
01	00	00	00	00	01
11	XX	XX	XX	XX	11
10	01	01	10	10	10



## Moore Machine Design Example

## Traffic Signal – Left Turn Arrow (Moore Machine)

Present	Next $G^+Y^+$				Output
$GY$	$AB = 00$	$AB = 01$	$AB = 11$	$AB = 10$	$GY$
00	00	00	10	00	00
01	00	00	00	00	01
11	XX	XX	XX	XX	11
10	01	01	10	10	10

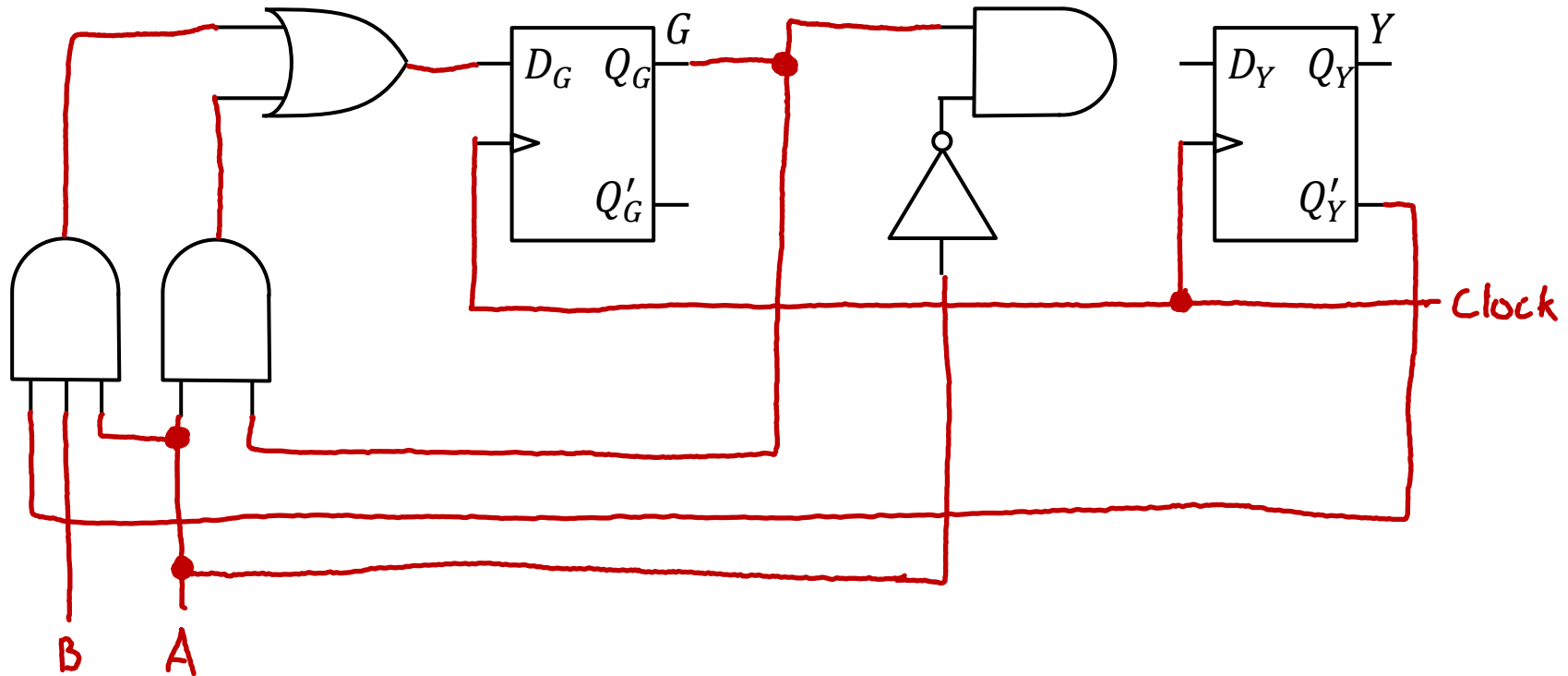


# Moore Machine Design Example

# Traffic Signal – Left Turn Arrow (Moore Machine)

$$G^+ = GA + \textcolor{violet}{ABY'}$$

$$Y^+ = GA'$$



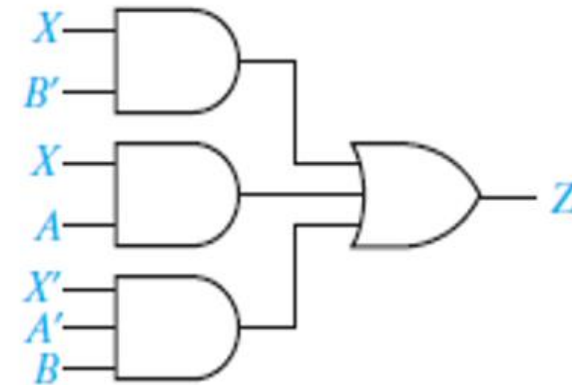
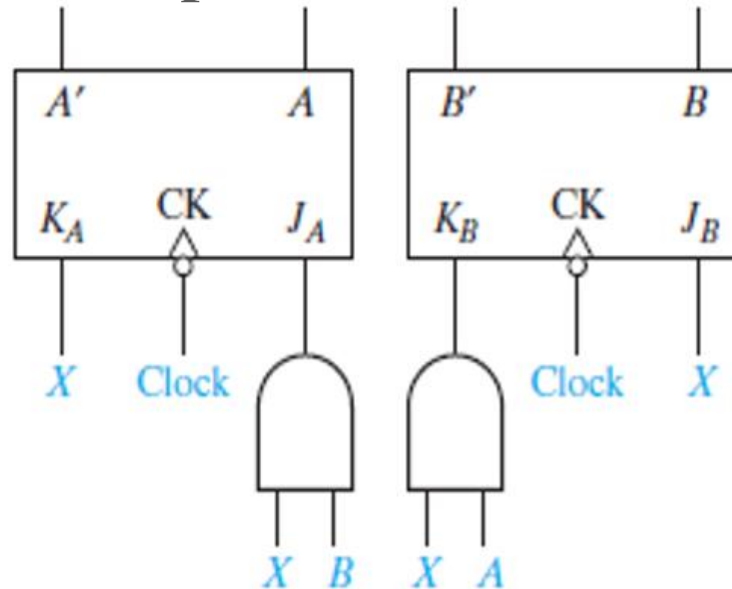


## Analysis by Transition Tables &amp; Graphs

Repeat Mealy example (last lecture) using these techniques:

1. Determine equations for inputs to flip-flops & outputs from circuit

- $J_A = XB$
- $K_A = X$
- $J_B = X$
- $K_B = XA$
- $Z = XB' + XA + X'A'B$
- These are same as when worked last lecture, except last lecture we plugged in values of all variables for the initial state right away and then repeated as we stepped through each state





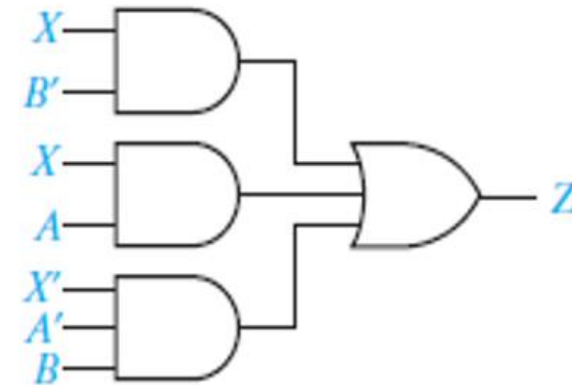
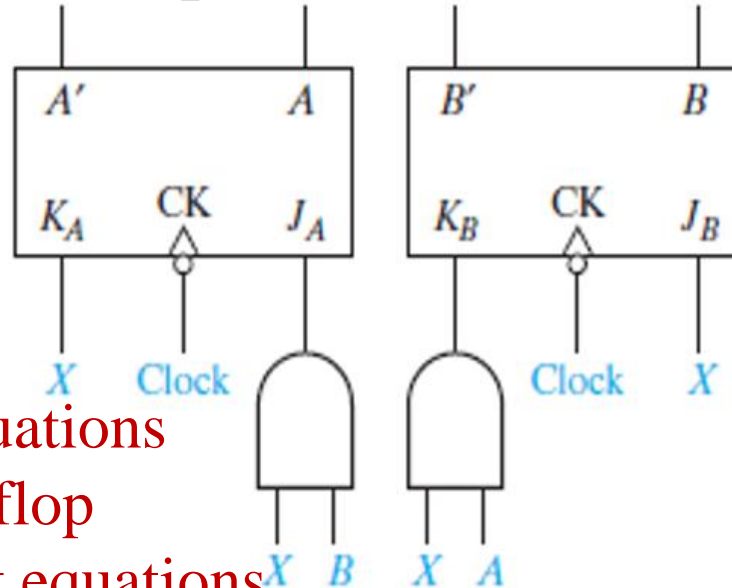
## Analysis by Transition Tables &amp; Graphs

Repeat **Mealy** example (last lecture) using these techniques:

- $J_A = XB$
- $K_A = X$
- $J_B = X$
- $K_B = XA$

2. Derive

next-state equations  
for each flip-flop  
from its input equations  
(using flip-flop next-state relations)



$$Z = XB' + XA + X'A'B$$

$$\text{J-K: } Q^+ = JQ' + K'Q$$

- $A^+ = XBA' + X'A$
- $B^+ = XB' + (XA)'B = XB' + X'B + A'B$



## Analysis by Transition Tables &amp; Graphs

Repeat Mealy example (last lecture) using these techniques:

3. Plot a next-state map  
for each flip-flop

$$Z = XB' + XA + X'A'B$$

- $A^+ = XBA' + X'A$

- $B^+ = XB' + (XA)'B = XB' + X'B + A'B$

$X \backslash AB$		$X$	
		0	1
00	0	0	
01	0	1	
11	1	0	
10	1	0	

 $A^+$ 

$X \backslash AB$		$X$	
		0	1
00	0	1	
01	1	1	
11	1	0	
10	0	1	

 $B^+$ 

$X \backslash AB$		$X$	
		0	1
00	0	1	
01	1	0	
11	0	1	
10	0	1	

 $Z$





## Analysis by Transition Tables &amp; Graphs

Repeat Mealy example (last lecture) using these techniques:

4. Combine these maps to form the transition table that gives the next state of the flip-flop as a function of current state and circuit inputs

		X	
		0	1
AB	00	0	0
	01	0	1
	11	1	0
	10	1	0

$A^+$

		X	
		0	1
AB	00	0	1
	01	1	1
	11	1	0
	10	0	1

$B^+$

		X	
		0	1
AB	00	0	1
	01	1	0
	11	0	1
	10	0	1

$Z$

AB	$A^+B^+$		$Z$	
	X = 0	1	X = 0	1
00	00	01	0	1
01	01	11	1	0
11	11	00	0	1
10	10	01	0	1



Present State	Next State		Present Output	
	X = 0	1	X = 0	1
$S_0$	$S_0$	$S_1$	0	1
$S_1$	$S_1$	$S_2$	1	0
$S_2$	$S_2$	$S_0$	0	1
$S_3$	$S_3$	$S_1$	0	1

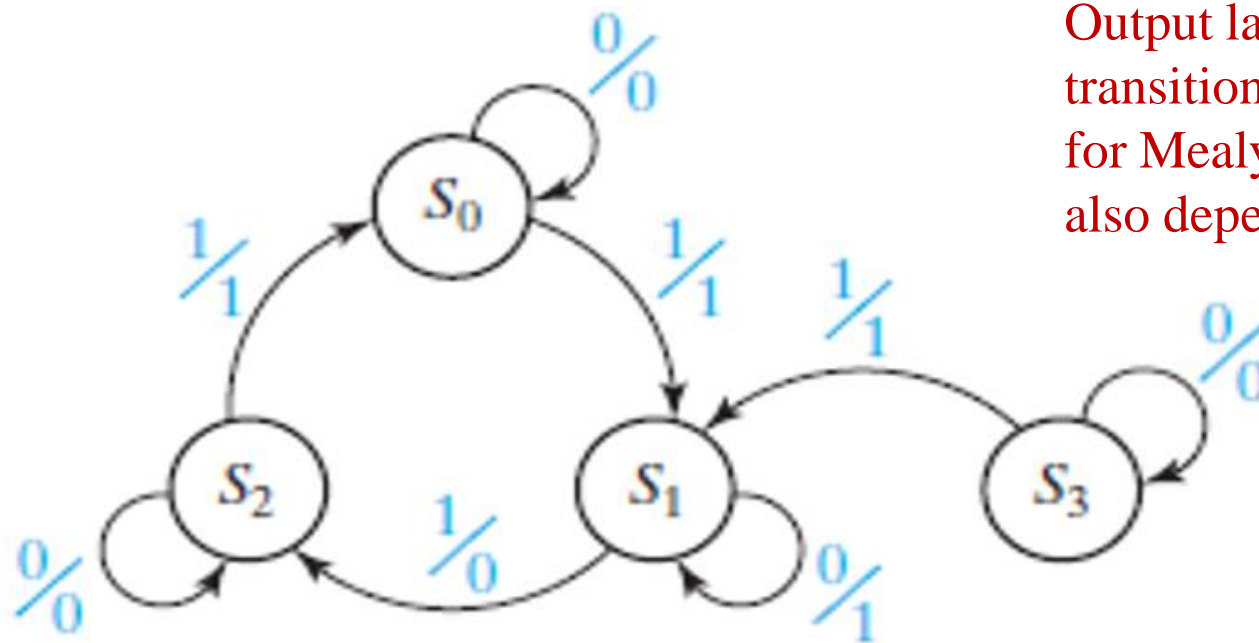


## Analysis by Transition Tables &amp; Graphs

Repeat Mealy example (last lecture) using these techniques:

AB	$A^+B^+$		Z	
	X = 0	1	X = 0	1
00	00	01	0	1
01	01	11	1	0
11	11	00	0	1
10	10	01	0	1

Present State	Next State		Present Output	
	X = 0	1	X = 0	1
$S_0$	$S_0$	$S_1$	0	1
$S_1$	$S_1$	$S_2$	1	0
$S_2$	$S_2$	$S_0$	0	1
$S_3$	$S_3$	$S_1$	0	1



Output labeled on transition arrows since for Mealy machine output also depends on input

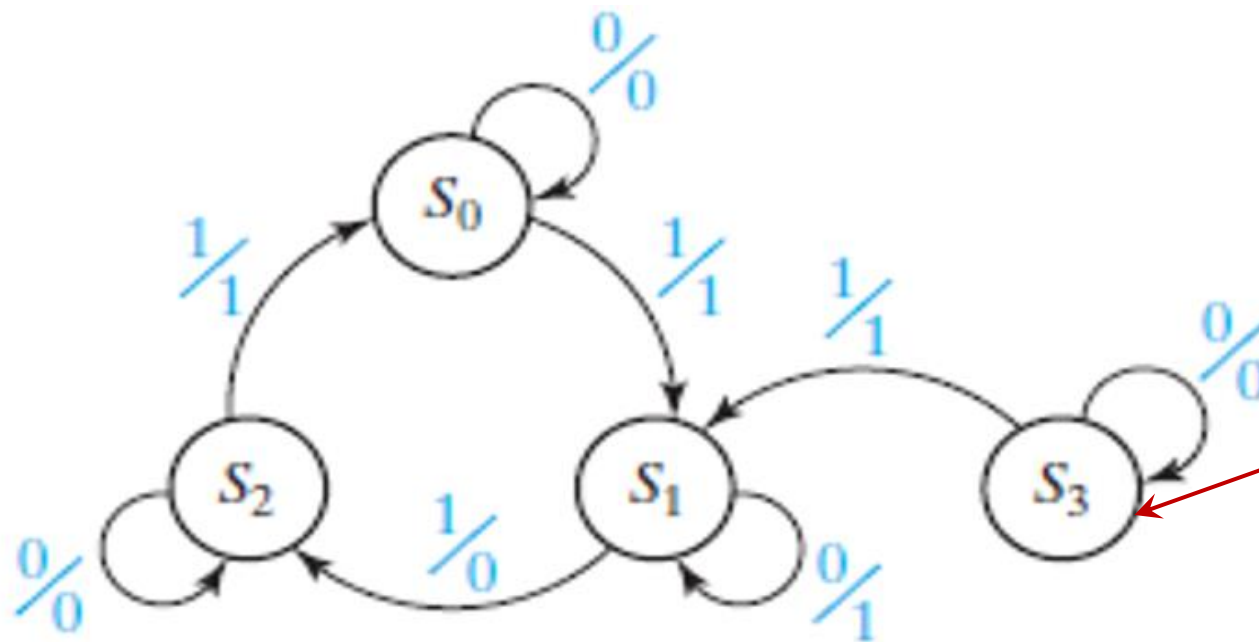


## Analysis by Transition Tables &amp; Graphs

Repeat Mealy example (last lecture) using these techniques:

AB	$A^+B^+$		Z	
	X = 0	1	X = 0	1
00	00	01	0	1
01	01	11	1	0
11	11	00	0	1
10	10	01	0	1

Present State	Next State		Present Output	
	X = 0	1	X = 0	1
$S_0$	$S_0$	$S_1$	0	1
$S_1$	$S_1$	$S_2$	1	0
$S_2$	$S_2$	$S_0$	0	1
$S_3$	$S_3$	$S_1$	0	1

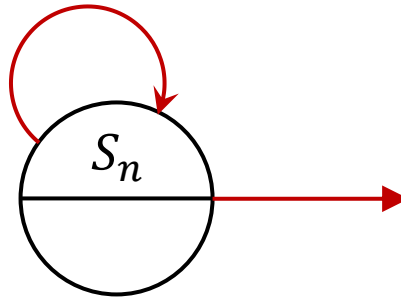


But look at  $S_3$  out here without any transitions pointing into it

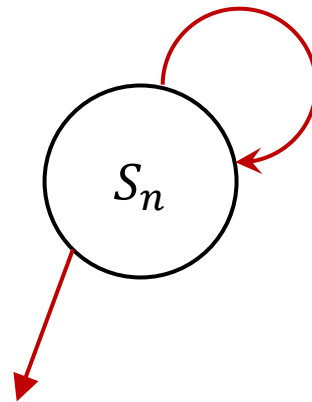


## Comparison of Moore vs Mealy State Graphs

Moore:



Mealy:



- This labeling assumes that output is only on active clock edge
- Recall that doing so eliminates errors due to
- Thus no provision for extra input changes that produce false outputs included on state graphs