Gage Farmer.308

Homework 5 - Math 2568 (Autumn 2022)
Prof. Cueto

Due date: Monday October 3, 2022 (in class).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§2.4	1, 3, 5, 7, 9, 11, 17, 21, 23, 25	3, 5, 9, 17, 25
§3.1	1, 7, 9, 12, 13, 17, 19, 21	7, 12, 17, 19, 21
§3.2	1, 6, 7, 8, 9, 17, 18, 19, 29, 30, 31	6, 9, 18, 30, 31
§3.3	1, 11, 15, 17, 21, 23, 27, 35, 37, 45	11, 21, 27, 35, 45

Section 2.4

3) $x = t \quad y = 4-2t \quad z = 1+3t$

5) $[2 \ -1 \ 3] \quad [-4 \ 2 \ -6]$

Parallel, opposite directions

9) $x = 1+3t \quad y = 2+4t \quad z = 1-t$

17) $\vec{PQ} = [1,1,2] \quad \vec{QR} = [-1,3,-1]$

$\vec{PQ} \times \vec{QR} = [-7,-1,4]$

$-7(x-1) - 1(y-0) + 4(z-3) = 0 \rightarrow -7x + 7 - y + 4z - 12$

$\boxed{-7x - y + 4z = 5}$

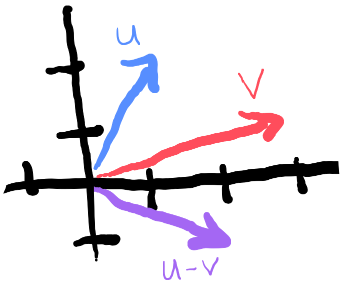
25) $[1,0,1]$ $[2,-1,3]$

$z=0$
$x=4$
$2x-y=3$
$8-y=3 \rightarrow y=5$

$x=4+t$ $y=5-t$ $z=-t$

Section 3.1

7)



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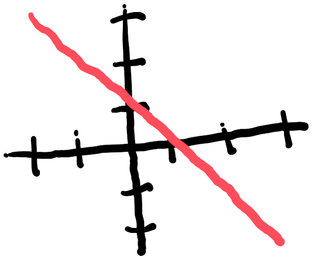
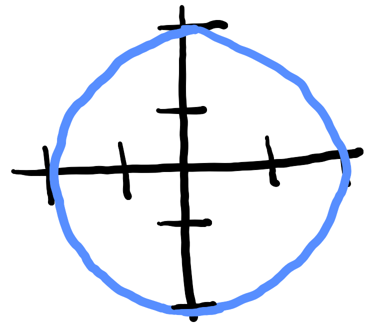
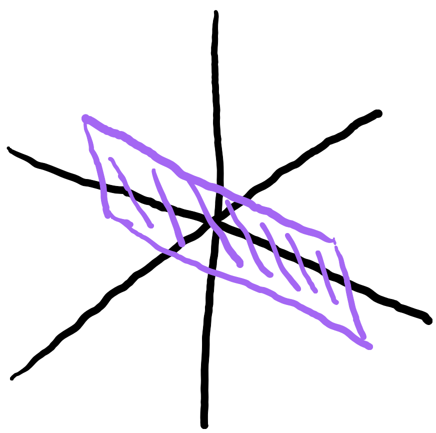


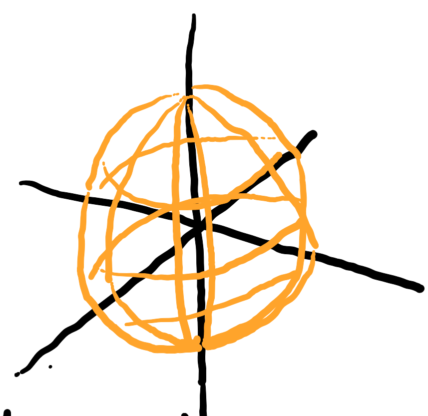
17)



19)



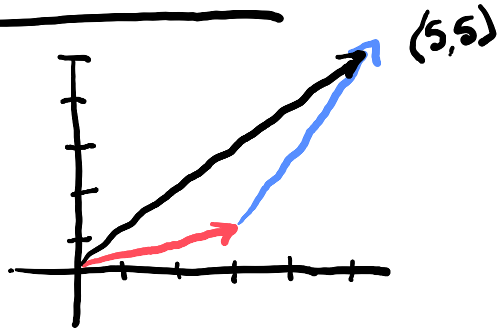
21)



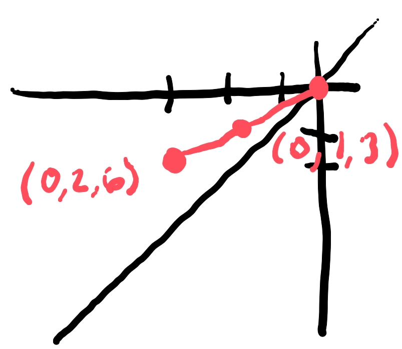
★ I am bad at drawing ★

Section 3.2

6)



9)



18) (S3) is in subspace of $\mathbb{R}^3$

30) $ku = \begin{bmatrix} ku_1 \\ ku_2 \\ ku_3 \end{bmatrix}$

$k(u_1 + v_1) = k(at + bt)$
$= 2kat = 2ka(t)$

same with $2kb(t)$ & $2kc(t)$ so ku is in W

W is a subspace of $\mathbb{R}^3$

31) (S1) U & V are both subspaces, so 0 vector is in both

(S2) x and y are vectors in U and V, and their sum is in U and V in $U \cap V$

so $x + y \in V \cap V$

(S3) scalar multiplication stays consistent between U and V, and $U \cap V$

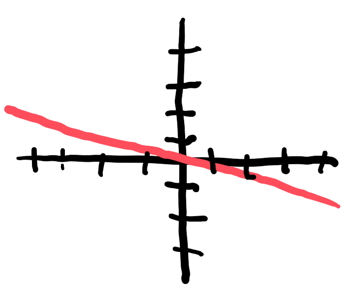
$U \cap V$ is subspace in $\mathbb{R}^3$

Section 3.3

11) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = y_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + y_2 \begin{bmatrix} -2 \\ 2 \end{bmatrix} + y_3 \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_1 = y_1 - 2y_2$
$x_2 = -y_1 + 2y_2$

$$\begin{bmatrix} 1 & -2 & 0 & x_1 \\ -1 & 2 & 0 & x_2 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & -2 & 0 & x_1 \\ 0 & 0 & 0 & x_1 + x_2 \end{bmatrix}$$



21)

a) $u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $u \neq y_1 v + y_2 x$ not in sp(S)

b) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = y_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + y_2 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $y_2 = 1$ Is in sp(S)

$y_1 + y_2 = 1$ $2y_1 + y_2 = 1$ $y_1 = 0$

c) $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = y_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + y_2 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $y_1 + y_2 = 1$

$2y_1 + y_2 = 2$ Is in sp(S)

$y_2 = 0$

$y_1 = 0$

$u = v$

d) $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = y_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + y_2 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $y_1 + y_2 = 2$ Not in sp(S)

$2y_1 + y_2 = 3$ doesn't satisfy

$y_2 = -1$ $2 \times 3 - 1 = 6 - 1 = 5$

$y_1 = 3$

e) $\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = y_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + y_2 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $y_1 + y_2 = -1$ $y_2 = -4$

$2y_1 + y_2 = 2$ $y_1 = 3$

Is in sp(S)

f) $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = y_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + y_2 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $y_1 + y_2 = 1$

$2y_1 + y_2 = 1$ Not in sp(S)

$y_2 = -3$

27) $c\begin{bmatrix}1\\-2\end{bmatrix}$ c is any real num

35) $A = \begin{bmatrix}1 & 2 & 3\\1 & 3 & 1\\2 & 2 & 10\end{bmatrix} \rightarrow N(A) = \left\{\begin{bmatrix}-7\\2\\1\end{bmatrix}\right\}$

$\hookrightarrow R(A) = \left\{\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}2\\3\\2\end{bmatrix}\right\}$

45) when $Sp(S) = R(A)$

$$A = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$