The Satisfiability Problem

(This slide through to "NP-complete" is optional and not covered in lecture or tested.)

While propositional logic is pretty straightforward, that doesn't mean that there's nothing but triviality within. For example, consider the satisfiability problem. Given a proposition in conjunctive normal form, determine whether or not there is a choice of truth values for the atomics that makes the proposition true (i.e. satisfied).

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For example consider the proposition

$$(P \vee Q \vee R) \wedge (\neg P \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge (\neg R \vee \neg Q).$$

Is it satisfiable?

P vs. NP

The satisfiability problem is probably the biggest open problem in computer science and is part of the whole "P versus NP" question. A problem that an algorithm can give an answer to in polynomial time is considered to be in "class P". A problem whose answer can be **verified** in polynomial time is in "class NP". So, for example, a Sudoku puzzle is in class NP.

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The big question is: Are the two classes the same? Is there a polynomial solution **finding** algorithm for every problem which has a polynomial time **verifying** algorithm?

NP-complete

One idea that has been used to attack the P vs. NP problem is the idea of NP-completeness. An NP problem is NP-complete if any other NP-class problem can be modeled using the first type of problem (said model made in polynomial time). The satisfiability problem is NP-complete.

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Interestingly, so is minesweeper. (See 'Minesweeper is NP-Complete.pdf' in the 'Extra Resources / Further Reading' module on Carmen.)

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- {*The Big Lebowski*, *Six-String Samurai*, *12 Monkeys*, *Tangled*}
- $\{\{3, 10\}, \{2, 3, 17\}, \{\frac{5k-1}{2} \mid k \in \mathbb{N}\}\}$

Defining Sets With a Rule

That last set in the last set, $\{\frac{5k-1}{2} \mid k \in \mathbb{N}\}$ or $\{\frac{5k-1}{2} \in \mathbb{R} \mid k \in \mathbb{N}\}$, was a bit different.

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Set Notation

- The elements of a set are required to be distinct, but the order of the elements is not important. For example $\{x, y\} = \{y, x\}$, and if $x = y$ then it is also the same as $\{x\}$.

A set with just one element is called a "singleton set".

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- Two sets are equal if they contain exactly the same elements.
- The set containing no elements is called the empty set and is written $\emptyset$ or sometimes just $\{\,\}$.
- The cardinality of a set is the number of elements it contains and is written $|S|$.

Comparing Sets

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- As you might expect $S = T$ means that $S \subseteq T$ and $T \subseteq S$.

Combining Sets

- The union of two sets A and B (denoted $A \cup B$) is the set that contains any element that is in either set. E.g., if $A = \{1, 2, 4\}$ and $B = \{3, 4\}$ then $A \cup B = \{1, 2, 3, 4\}$.

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- The difference of A and B (denoted $A - B$ or $A \setminus B$) consists of all elements that are in A, but not in B. For our sets, $A - B = \{1, 2\}$. $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
- The Cartesian product of A and B (denoted $A \times B$) is the set of ordered pairs $\{(a, b) | (a \in A) \wedge (b \in B)\}$. For our example, $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (4, 3), (4, 4)\}$. $|C \times D| = |C| \cdot |D|$

The Power Set

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Consider $A = \{1, 2, 4\}$ from before. What is $|\mathscr{P}(A)|$?

$$\mathscr{P}(A) = \{\emptyset, \{1\}, \{1,2\}, \{2,4\}, A, \{1,4\}, \{2\}, \{4\}\}$$

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Notice that A has three elements and $\mathscr{P}(A)$ has eight. If $|B| = 10$, what do you think the cardinality (size) of $\mathscr{P}(B)$ is?

2^{10}

The size of the power set

$\mathscr{P}(S) = 2^S$

Lemma

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If $|S| = n$ that's n questions each with one of two answers, so 2^n ways of answering all the questions, or 2^n subsets.

2^S is the set of functions from S to $2 = \{0,1\}$

Venn Diagrams

A Venn diagram is a pictorial representation of **all** possible intersections between a finite number of different sets. They represents the sets as regions inside closed curves (for example, the interior of a circle) and elements as points in a plane. For example, if I wanted to confirm (*not prove*) that

$$(A \cup B) - C = (A - C) \cup (B - C)$$

then I could draw the picture for both sides and confirm that they are the same.

$(A \cup B) - C = (A - C) \cup (B - C)$

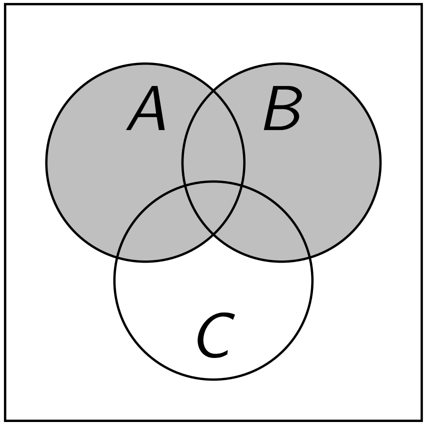


Figure: $A \cup B$

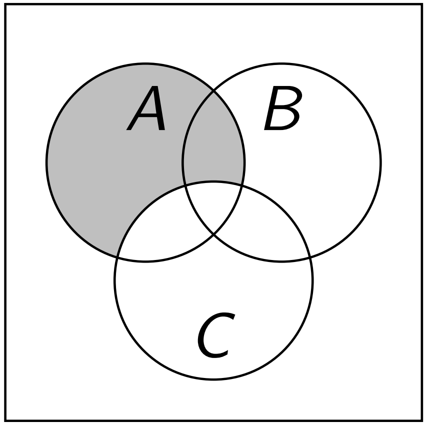


Figure: $A - C$

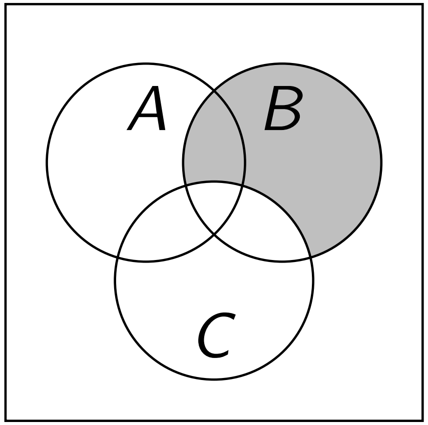


Figure: $B - C$

$(A \cup B) - C = (A - C) \cup (B - C)$

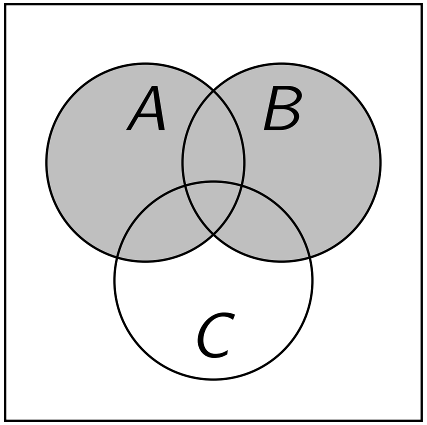


Figure: $A \cup B$

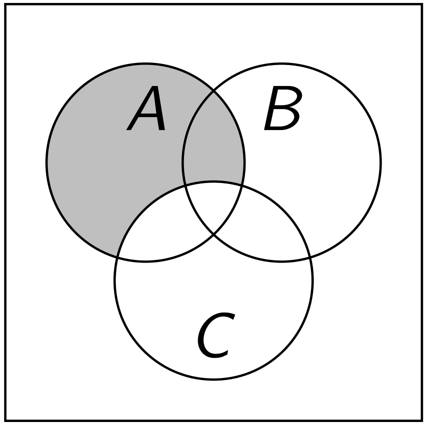


Figure: $A - C$

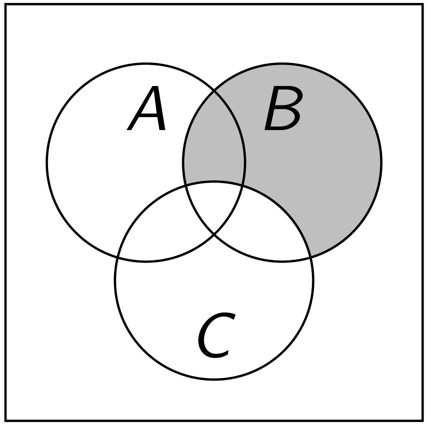


Figure: $B - C$

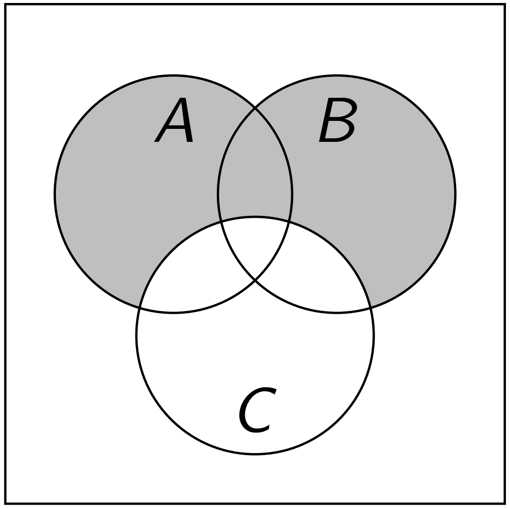


Figure: $(A \cup B) - C$

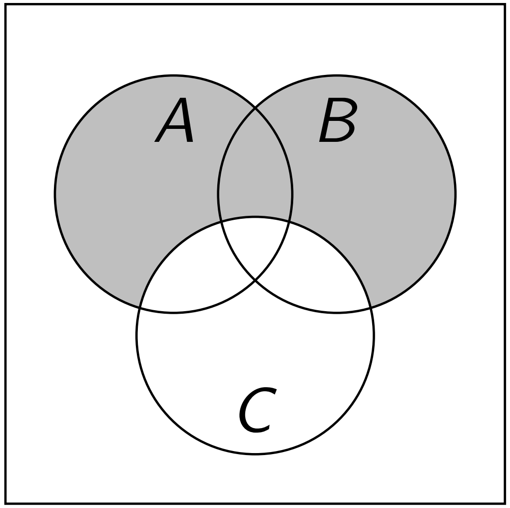
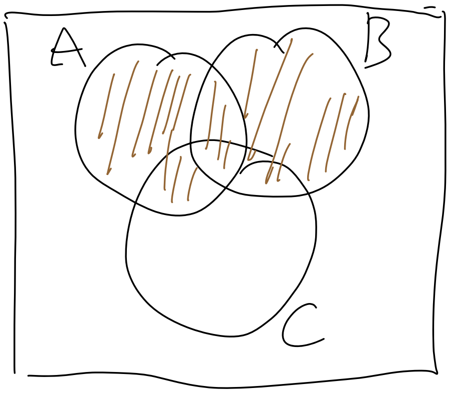
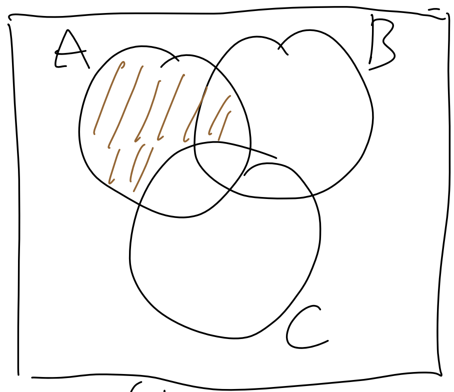


Figure: $(A - C) \cup (B - C)$

$(A \cup B) - C \stackrel{?}{=} (A - C) \cup B$



$(A \cup B)$



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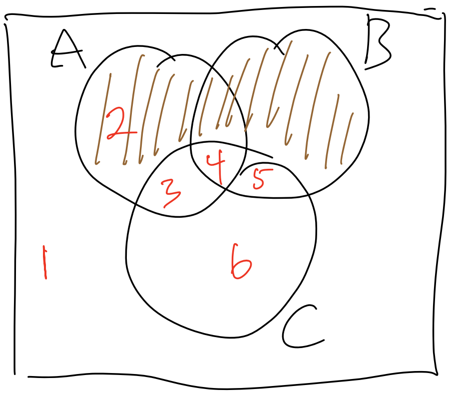
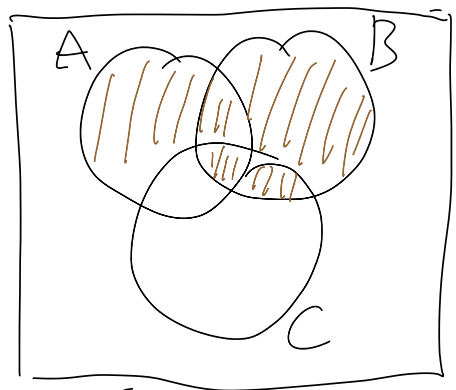


$(A \cup B) - C$



$(A - C) \cup B$

$A = \{2, 3, 4\}$, $B = \{4, 5\}$, $C = \{3, 4, 5, 6\}$

$(A \cup B) - C = \{2, 3, 4, 5\} - C = \{2\}$

$(A - C) \cup B = \{2\} \cup B = \{2, 4, 5\}$

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- $(f_n) = (1, 1, 2, 3, 5, 8, 13, \dots)$

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Like propositions, predicates are generally named with a letter, but because of the dependence on variables, a function-like notation is used. For example, naming the above predicate P tells us that $P(4)$ and $P(9)$ are true but $P(5)$ and $P(3.14159)$ are false.

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In other words, predicates are simply functions that return Boolean values.

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There are two common quantifiers: $\forall$ and $\exists$.

The Universal Quantifier $\forall$

We use the universal quantifier when we want to assert that a predicate is true for every element in a given set. So

$$(\forall x \in S)[P(x)]$$

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