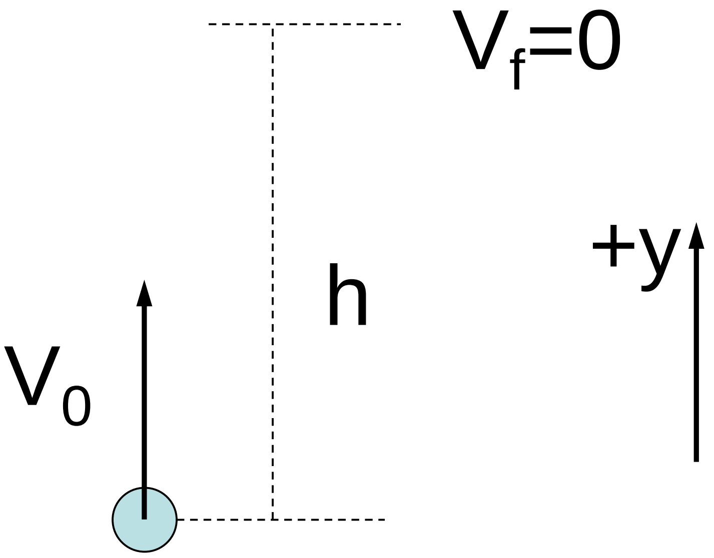
Potential Energy $U=mgy$



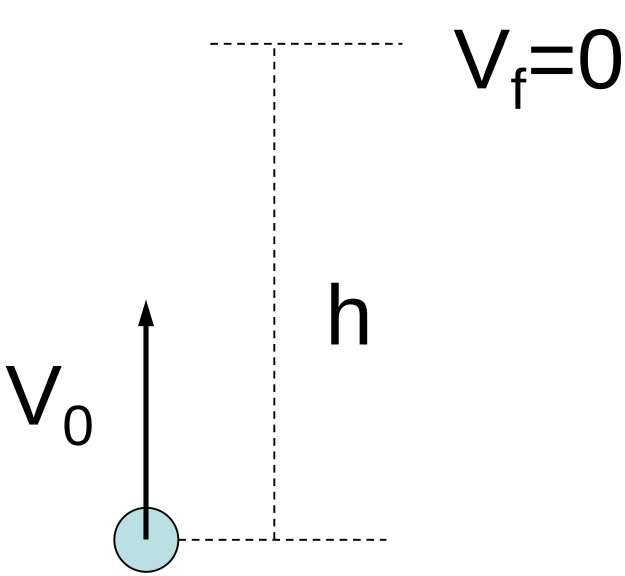
$F=-mg$

$W=-mgh=\Delta K$

Initial velocity V_0
What is h ?

$mgh=mgy_2-mgy_1=U_2-U_1$
$=\Delta U$

$W=-\Delta U=\Delta K$



F=-mg

W=-mgh=ΔK

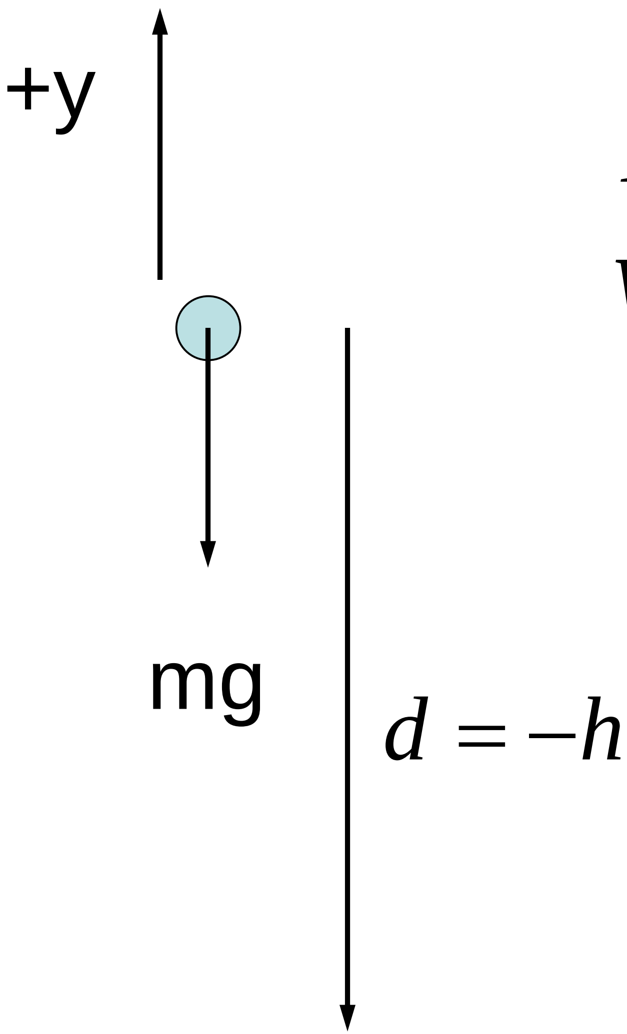
$$\Delta K = \frac{1}{2}mV_f^2 - \frac{1}{2}mV_0^2 = -\frac{1}{2}mV_0^2$$

$$-mgh = \Delta K = \frac{1}{2}mV_f^2 - \frac{1}{2}mV_0^2 = -\frac{1}{2}mV_0^2$$

$$V_0^2 = 2gh \quad (h = \frac{V_0^2}{2g})$$

-ΔU=ΔK

Ex: Drop a mass (gravity does work)

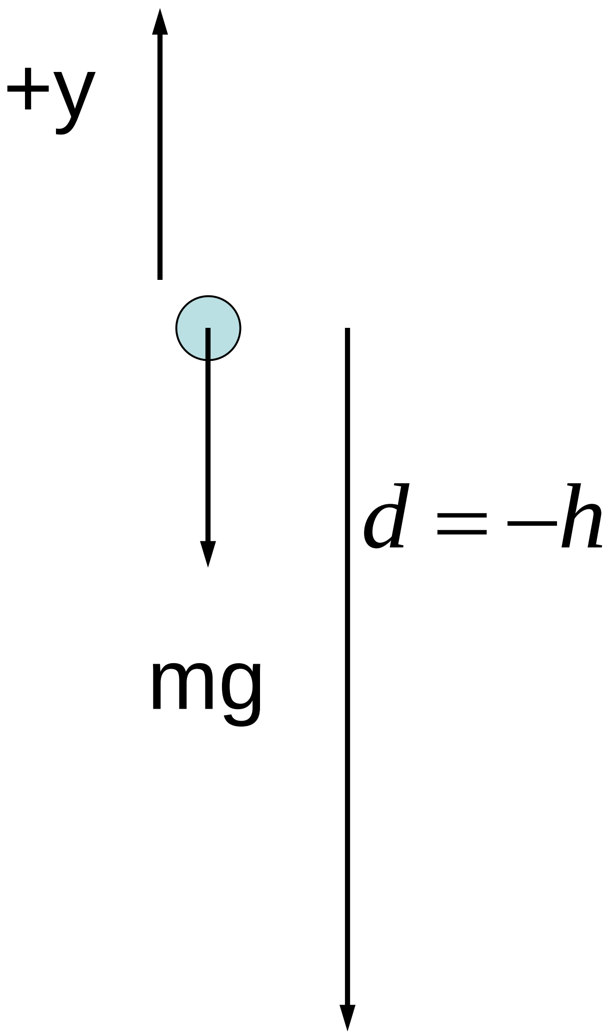


$$F = -mg\,,\, d = -h$$

$$W = (-mg)(-h) = mgh = \Delta K$$

$$v_0^2 = 2gh$$

Ex: Drop a mass (gravity does work)



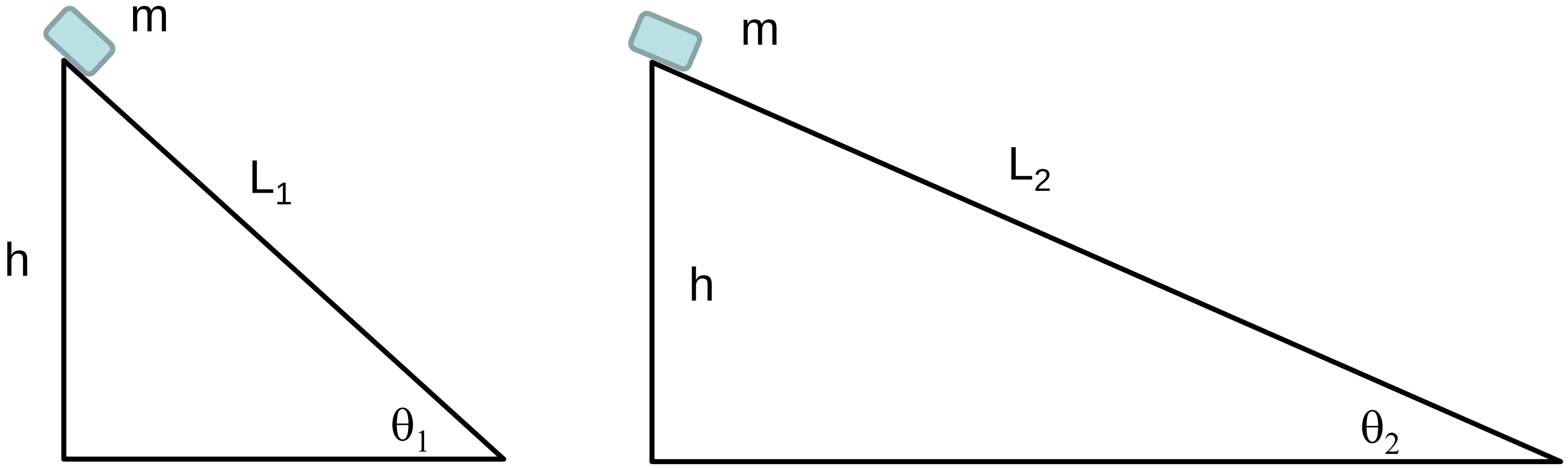
$$F = -mg\,,\; d = -h$$

$$W = \vec{F} \cdot \vec{d} = +mgh = K_f - K_i$$

$$= \frac{1}{2} m V_f^2 - \frac{1}{2} m V_0^2$$

W=-ΔU=ΔK

No Friction. Initially at rest.



V_{1f}=?

V_{2f}=?

$$U_1 + K_1 = U_2 + K_2$$

$$U = mgy$$

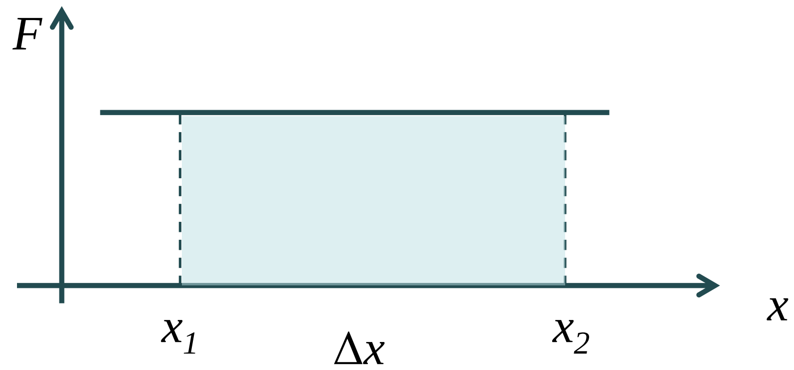
$V_{1f} = V_{2f}$

since 'h' is the same

Work by Changing forces:

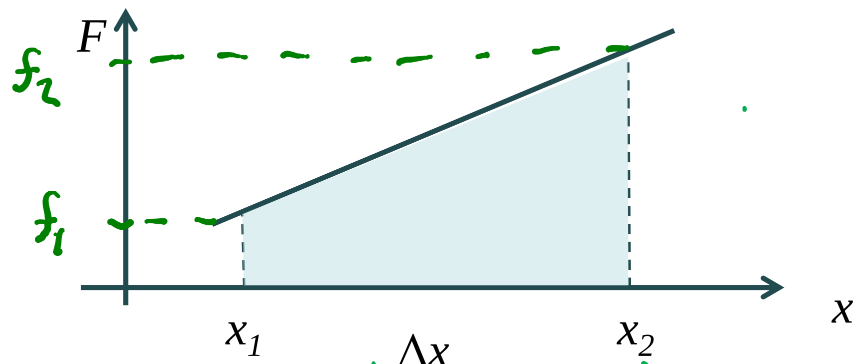
Work done by a constant force

$$W = \vec{F} \cdot \Delta\vec{x}$$



Work done by a changing force

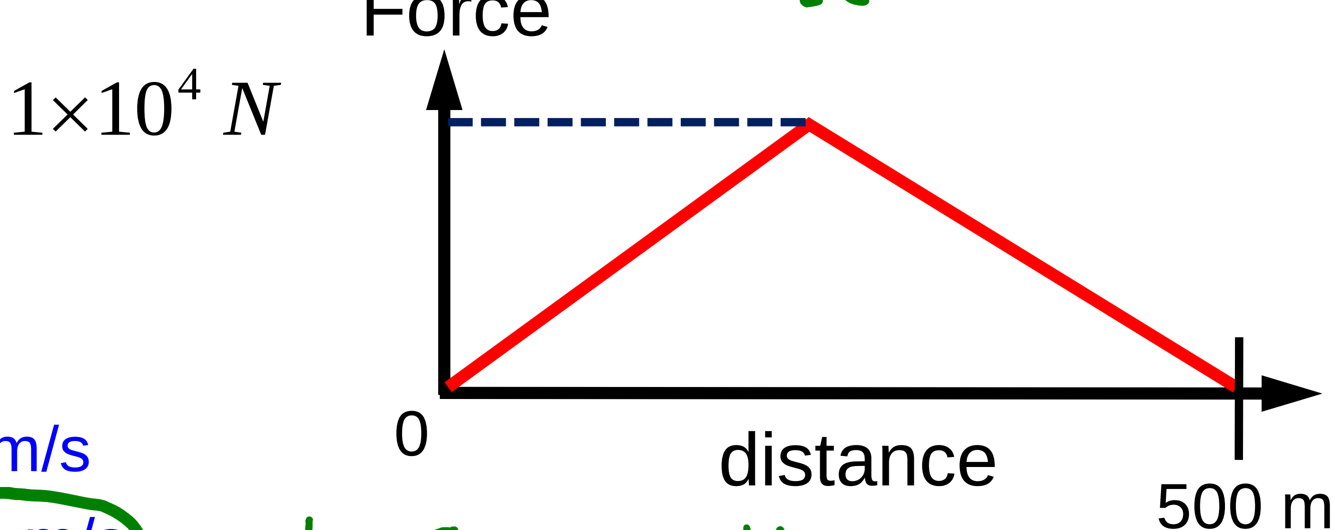
$$W = \int_1^2 F(x)\,dx$$



$W = f_1(x_2 - x_1) + \frac{1}{2}(x_2 - x_1)(f_2 - f_1)$

The engine of a 1000 kg sports car rotates the tires, creating a net forward pushing force F on the tires of the car that varies as a function of distance. The force is shown below. If the car starts at rest, what is the speed of the car after traveling 500 meters?

$\frac{1}{2}(x_2-x_1)(f_2-f_1)$



1. 0 m/s
2. 71 m/s
3. 100 m/s
4. 141 m/s
5. 200 m/s

$= \frac{1}{2} \times 500 \times (1\times10^4)$

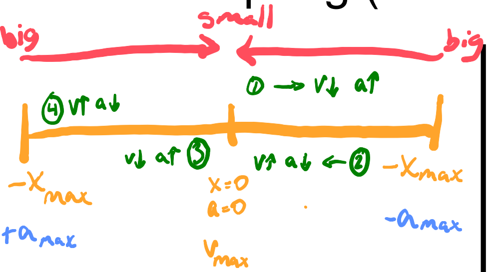
$= \frac{1}{2} \times 1000 \times v^2$

$= 71$ m/s

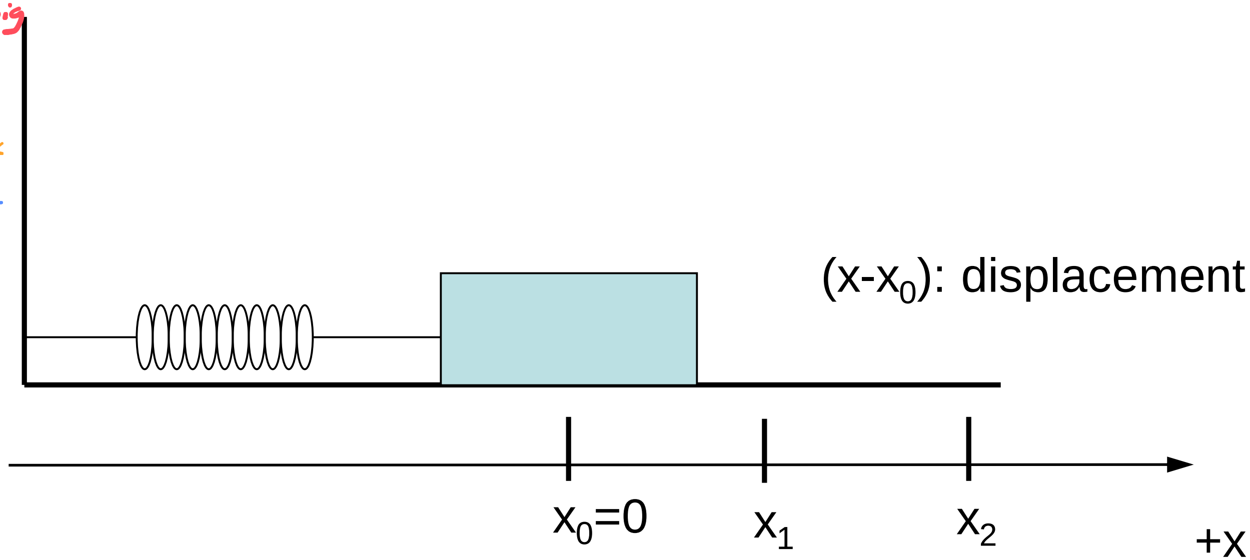
Work done by the spring force

No Friction

Spring (Hooke's law) $F=-k(x-x_0)$ *Force of spring on mass m.*



$F = -k(x_1 - x_0)$



$(x-x_0)$: displacement

x_0 is where the spring is fully relaxed or balanced, which is usually set to be 0. Ignore all frictions.

$$W_{12} = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2} -kxdx = -\frac{1}{2}kx^2 \Big|_{x1}^{x2}$$

$$= \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 \quad (Set\ x_0 = 0)$$

$\frac{1}{2}mv^2 + \frac{1}{2}kx_1^2$

$= \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$

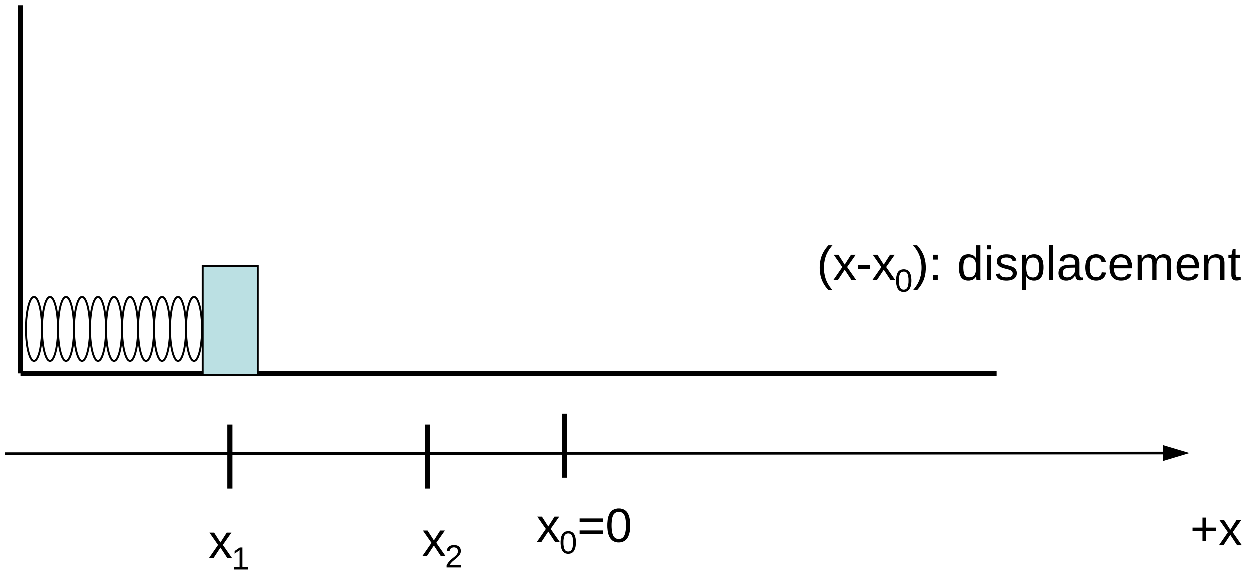
<0 <0

$W = \Delta k$

$E_p = \frac{1}{2}kx^2$

The spring is relaxed at x_0. The block is pushed against the spring to x_1 and held stationary (the spring is compressed.) Then the block is released. What is the block's velocity at x_2?

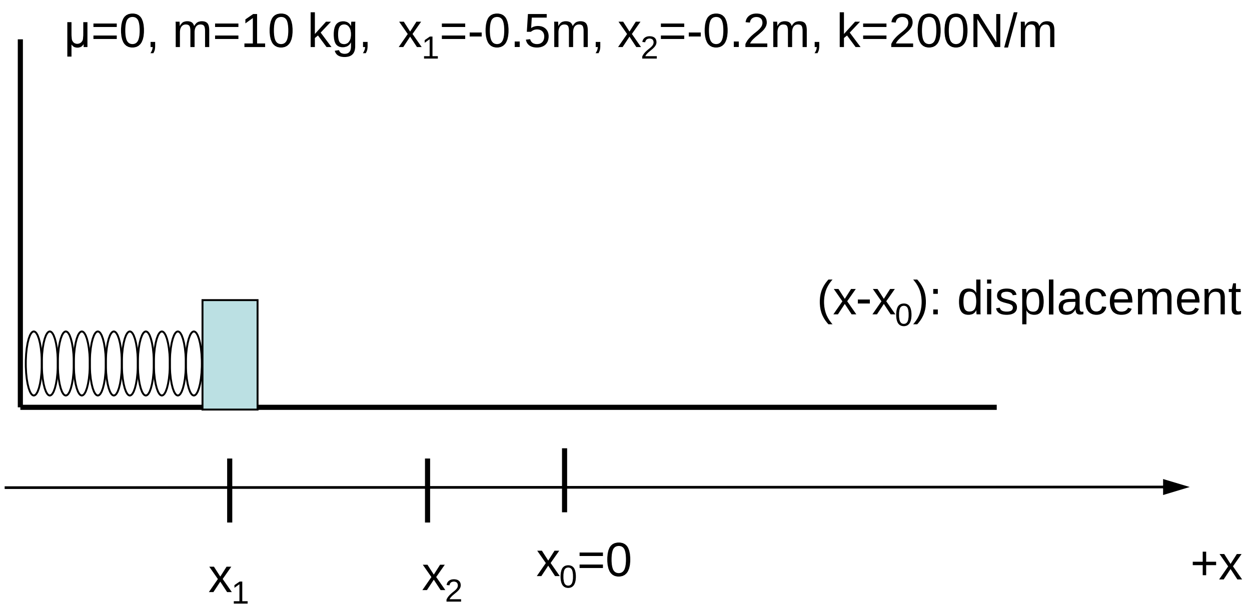
$F=-k(x-x_0)$ $\mu=0$, $m=10$ kg, $x_1=-0.5$m, $x_2=-0.2$m, $k=200$N/m



$$W_{12}=\frac{1}{2}kx_1^2-\frac{1}{2}kx_2^2=\frac{1}{2}k(x_1^2-x_2^2)=\frac{1}{2}200(0.5^2-0.2^2)=21(J)$$

$$W_{net}=\Delta K=K_2-K_1=\frac{1}{2}mV_2^2-\frac{1}{2}mV_1^2$$

The spring is relaxed at x_0. The block is pushed against the spring to x_1 and held stationary (the spring is compressed.) Then the block is released. What is the block's velocity at x_2?



$$W_{12} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = \frac{1}{2}k(x_1^2 - x_2^2) = \frac{1}{2}200(0.5^2 - 0.2^2) = 21J$$

$$W_{net} = \Delta K = K_2 - K_1 = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2 = \frac{1}{2}mV_2^2 = 21J$$

$$V_2 = \sqrt{\frac{2\times 21J}{10kg}} = 2.02\, m/s$$

Work done by the spring force