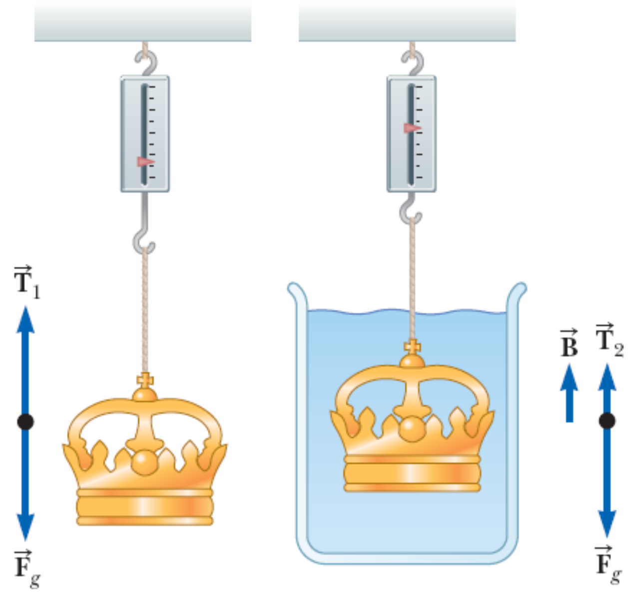
Application of Archimedes's Principle

Determine if the crown is made of pure gold by finding the density of it.



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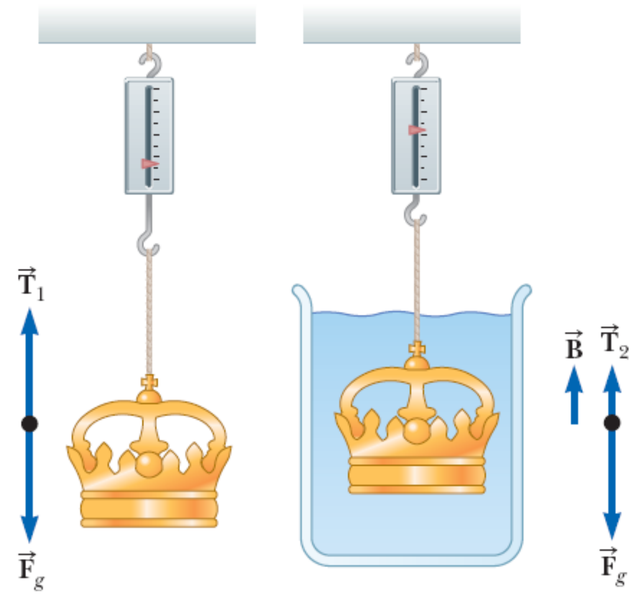


$$\rho_{crown} = \frac{m_c}{V_c} = \frac{F_g / g}{V_c}$$

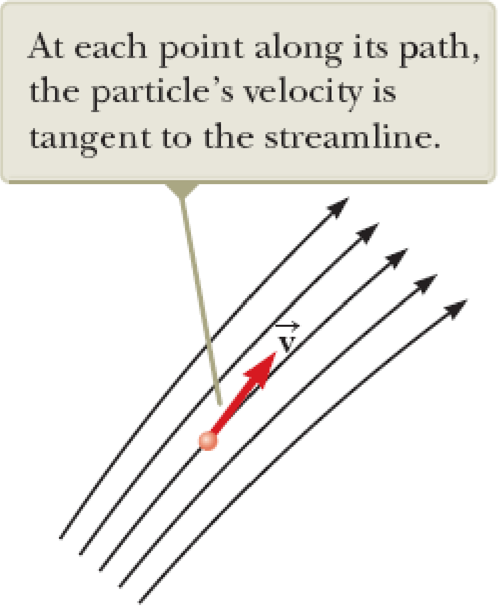
$$F_b = \rho_{water} g V_{disp} \qquad V_{disp} = V_c = \frac{F_b}{\rho_{water} g}$$

$$F_b = T_1 - T_2$$

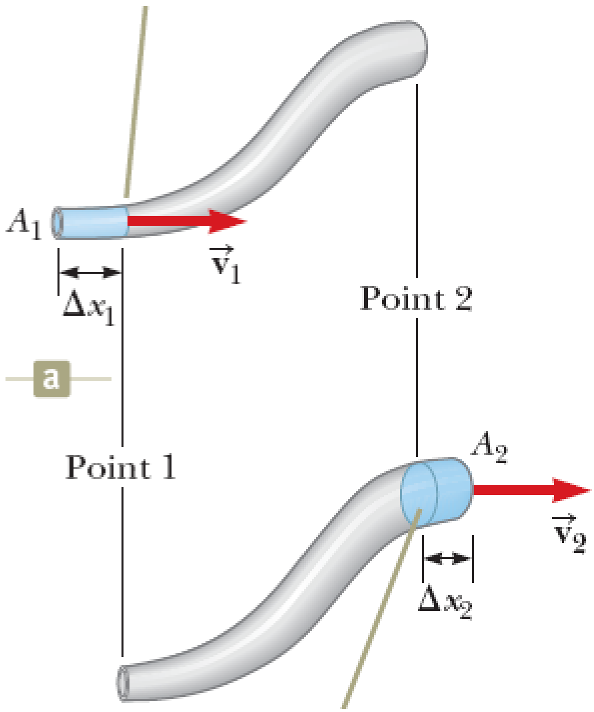
Fluid Dynamics

Ideal fluid flow

1. **nonviscous --** internal friction is neglected
2. **steady** -- all particles passing through a point have the same velocity.
3. **incompressible** -- the density is constant.
4. **irrotational** -- no angular momentum



Fluid Dynamics -- Continuity



$$m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$$

$$m_2 = \rho A_2 \Delta x_2 = \rho A_2 v_2 \Delta t$$

$$m_1 = m_2 \qquad A_1 \Delta x_1 = A_2 \Delta x_2 = V$$

$$A_1 v_1 = A_2 v_2 = \text{constant} \quad \leftarrow \textit{Continuity}$$

Watering a Garden

$$m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t = m_2 = \rho A_2 v_2 \Delta t$$

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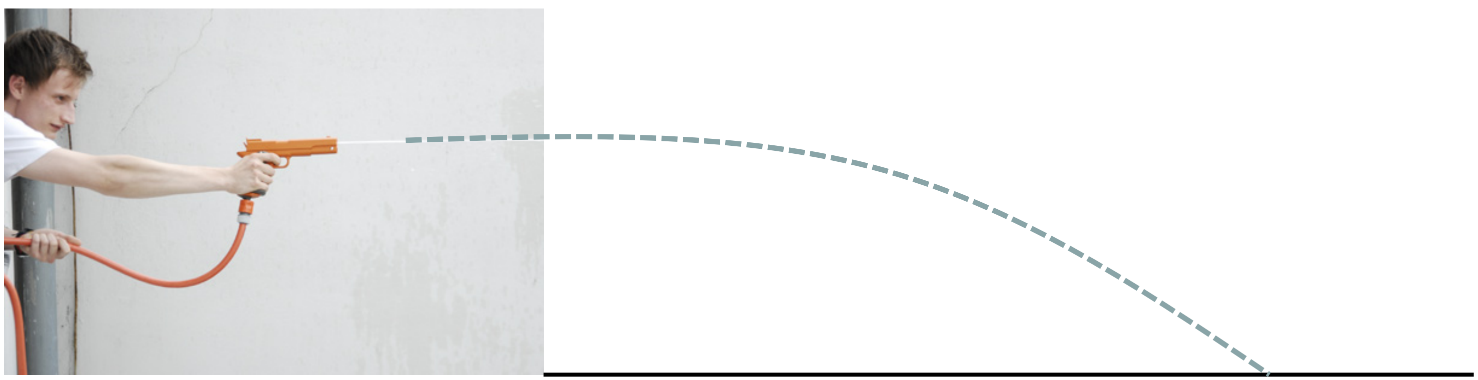
A gardener uses a water hose 2.50 cm in diameter to fill a 30.0-L bucket. The gardener notes that it takes 1.00 min to fill the bucket.
A nozzle with an opening of cross-sectional area 0.500 cm^2 is then attached to the hose. The nozzle is held so that water is projected horizontally from a point 1.00 m above the ground. Over what horizontal distance can the water be projected?



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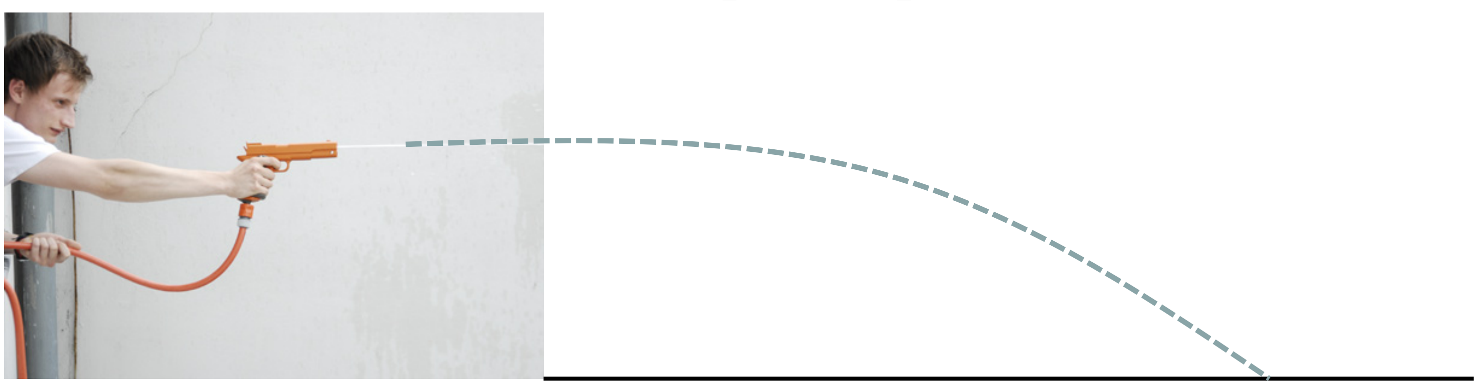
$$A_1 v_1 \Delta t = Volume$$

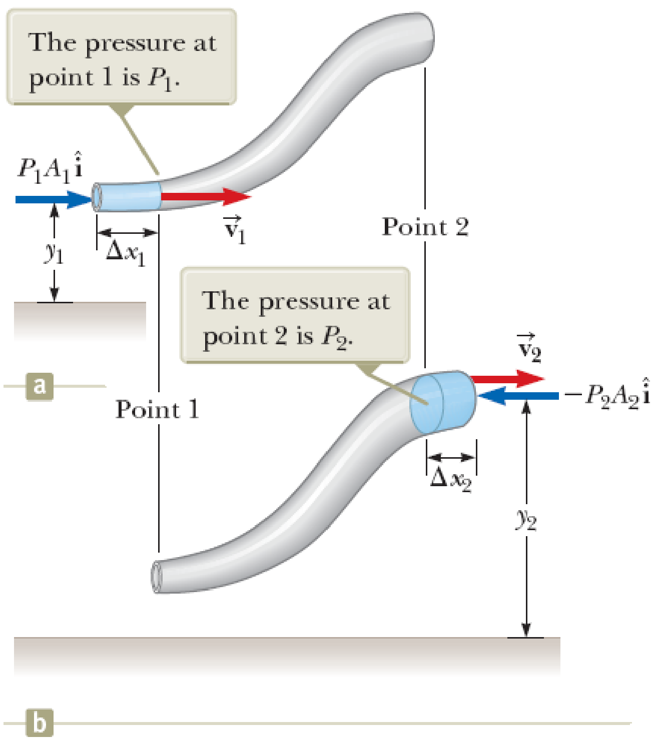
$$v_1 = \frac{Volume}{A_1 \Delta t} = \frac{0.03}{\pi(0.0125)^2 \cdot 60} = 1.02\ m/s$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi R_1^2}{\pi R_2^2} v_1 = \frac{\pi(1.25)^2}{0.500} v_1 = 9.82 v_1 = 10.01\ m/s$$



Bernoulli's Equation

The relationship between
- fluid speed (KE)
- Pressure (force, work)
- and elevation (mg, U)



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$$A_1 v_1 = A_2 v_2 = \text{constant} \quad \leftarrow \; \textit{Continuity}$$

$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 V$$

$$W_2 = -F_2 \Delta x_2 = -P_2 A_2 \Delta x_2 = -P_2 V$$

$$\Delta x = v \Delta t$$

The net work during Δt is

$$W_{net} = W_1 + W_2 = (P_1 - P_2)V$$

$$\Delta K = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2 \qquad \Delta U = mgy_2 - mgy_1$$

$$W_{net} = \Delta K + \Delta U$$

$$(P_1 - P_2)V = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2 + mgy_2 - mgy_1$$

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

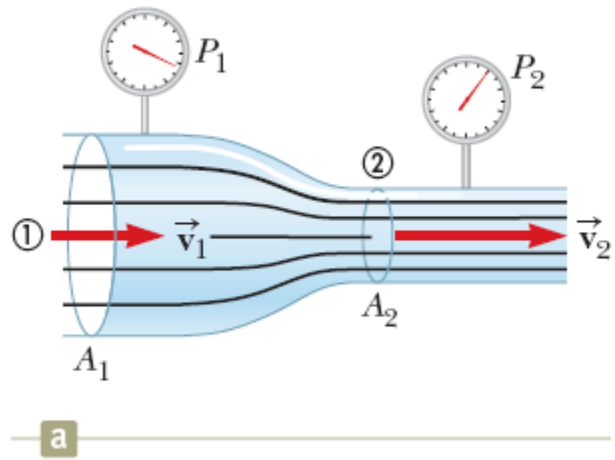
Bernoulli's Equation Applications

Venturi Tube

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

$$v_1 = \frac{A_2 v_2}{A_1}$$



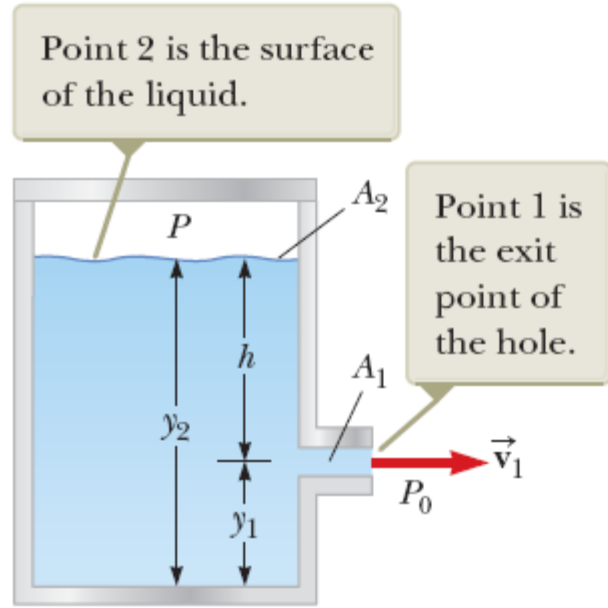
$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$P_1 + \tfrac{1}{2}\rho\left(\frac{A_2}{A_1}v_2\right)^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2)$$

$$v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

Bernoulli's Equation Applications



$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

$$P_0 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

$$\tfrac{1}{2}\rho v_1^2 = P - P_0 + \rho g y_2 - \rho g y_1$$

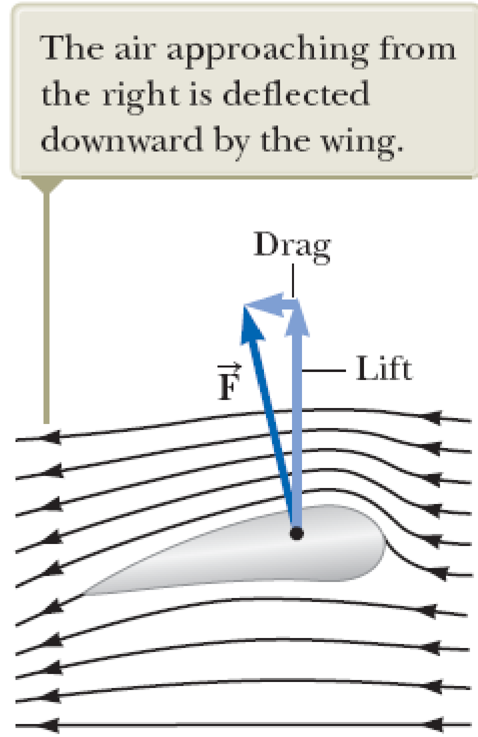
$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

Applications of Fluid Dynamics

How does the wing of an airplane work?

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

$v \uparrow \qquad P \downarrow \qquad F \downarrow$



Same for the spray gun