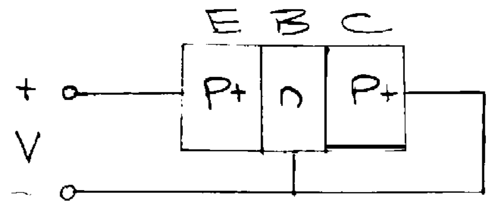
- Now we have I_E, I_C, and I_B in terms of materials parameters: W_b, ΔP_E and ΔP_C
  ↳ $D_p(\mu_p)$ and $L_p(\tau_p)$

- ΔP_E, ΔP_C relate to V_{BE}, V_{CB}
  so can get performance (α, β, γ) from I's under different biasing.

- I_E, I_C, I_B <u>equations are general</u>
  - <u>all</u> biasing conditions *

Example



(hole current since pnp)

$$V_{BC} = 0$$

$$\Delta P_C = 0$$

$$I_E = q\frac{AD_p}{L_p}\,\Delta P_E \,\operatorname{ctnh}\left(\frac{W_b}{L_p}\right)$$

$$I_C = q\frac{AD_p}{L_p}\,\Delta P_E \,\operatorname{csch}\left(\frac{W_b}{L_p}\right)$$

$$I_B = q\frac{AD_p}{L_p}\,\Delta P_E \,\tanh\left(\frac{W_b}{2L_p}\right)$$

Get ΔP_E from V

Or If V_{BE} shorted, then $\Delta P_E = 0$
etc.

For a *pnp* device with $I_{Ep} = 1$ mA, $I_{En} = 0.01$ mA, $I_{Cp} = 0.98$ mA, and $I_{Cn} = 0.1$ μA, calculate

(a) Base transport factor B

(b) Emitter injection efficiency γ

(c) α_{dc} and β_{dc}; the value of I_B

(e) If $I_{Cp} = 0.99$ mA, calculate β_{dc} and I_B.

(f) If $I_{Cp} = 0.99$ mA and $I_{En} = 0.005$ mA, calculate β_{dc} and I_B.

(g) How will β_{dc} change if I_{En} is increased?

Example

$I_{Ep} = 1$ mA , $I_{En} = 0.01$ mA .

$I_{cp} = 0.98$ mA . $I_{cn} = 0.1$ μA

(a) $B = I_{cp} / I_{Ep} = \boxed{}$

(b) $\gamma = \dfrac{I_{Ep}}{I_{En} + I_{Ep}} = \boxed{}$

(c)

$$\alpha_{dc} = \boxed{\quad} = \boxed{\quad}$$

$$\beta_{dc} = \boxed{\quad} = \boxed{\quad}$$

$$I_B = I_E - I_c = (\boxed{\quad}) - (\boxed{\quad})$$

$$= \boxed{\quad}\ \underline{\underline{\mu A}}$$

(e) $I_{cp} = 0.99\ mA$

$$\beta = I_{cp}/I_{Ep} = 0.99\ , \quad \alpha_{dc} = 0.9802$$

$$\beta_{dc} = \boxed{\quad}$$

$$I_B = \boxed{\quad}$$

(f) $I_{cp} = 0.99\, mA\ , \quad I_{En} = 0.005\ mA$

$$\beta_{dc} = \boxed{\quad}\ , \quad I_B = \boxed{\quad}$$

(g) $I_{En} \uparrow \rightarrow P \downarrow \quad \therefore\ \alpha_{dc} \downarrow \rightarrow \beta_{dc} \downarrow \Rightarrow I_B \uparrow$

Approximations of the Terminal Currents

Can simplify general equations for I_E, I_C, and I_B for normal transistor biasing

Thus, for collector reverse-biased, $\Delta p_C = -p_n$ and for small p_n, can neglect Δp_C.

For $\gamma = 1$ (very efficient emitter injection),

$$I_E = I_{Ep} \cong qA \frac{D_p}{L_p} \Delta p_E \operatorname{ctnh} \frac{W_b}{L_p}$$

$$I_C \simeq qA \frac{D_p}{L_p} \Delta p_E \operatorname{csch} \frac{W_b}{L_p}$$

$$I_B \simeq qA \frac{D_p}{L_p} \Delta p_E \tanh \frac{W_b}{2L_p}$$

For small values of W_b/L_p, can expand Hyperbolic trig. functions and keep only lowest order:

$$\text{sech}\,(y) = 1 - y^2/2 + 5y^4/24 - \ldots$$

$$\text{ctnh}\,(y) = 1/y + y/3 - y^3/45 + \ldots$$

$$\text{csch}\,(y) = 1/y - y/6 + 7y^3/360 - \ldots$$

$$\tanh\,(y) = y - y^3/3 + \ldots$$

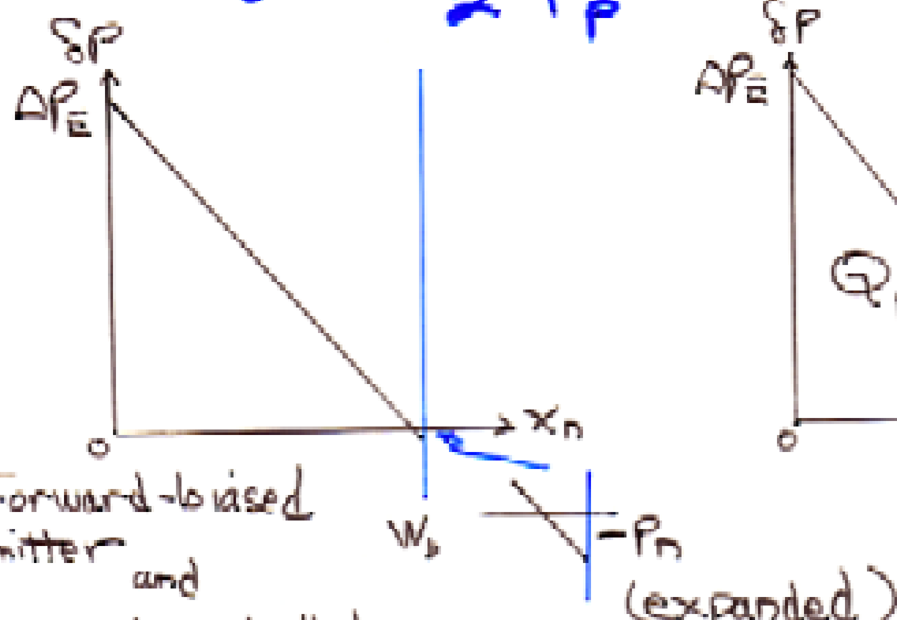
I_E only slightly larger than I_C.

$$\frac{qAD_p\Delta p_E}{L_p}\left[\left(\frac{1}{y}+\frac{y}{3}\right)-\left(\frac{1}{y}-\frac{y}{6}\right)\right]\simeq I_E -$$ [redacted]$_c$

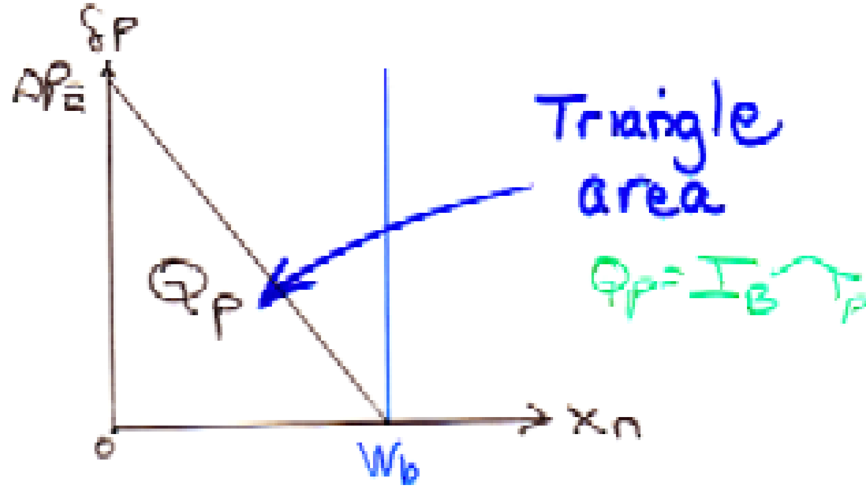
To lowest order, $\tanh y \simeq y$, so

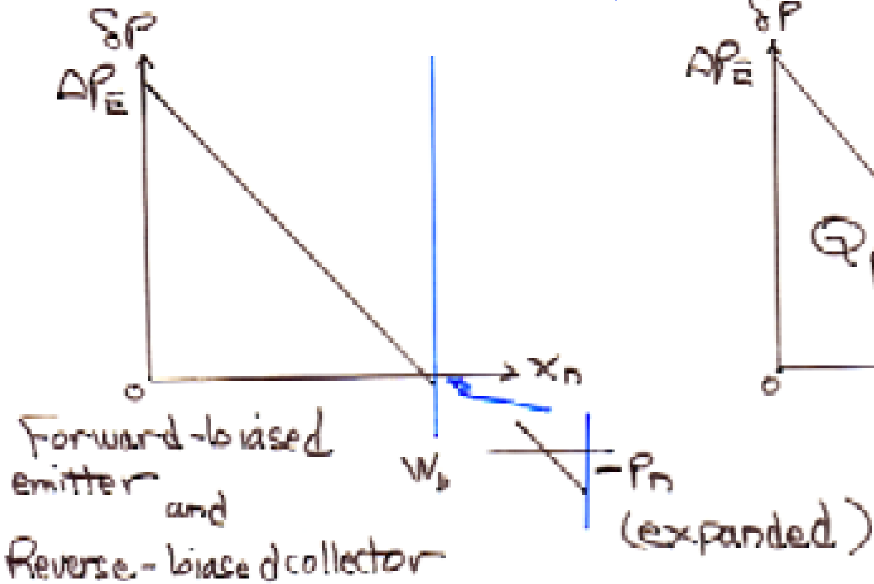
$$I_B \simeq qA\frac{D_p}{L_p}\Delta p_E\frac{W_b}{2L_p}$$

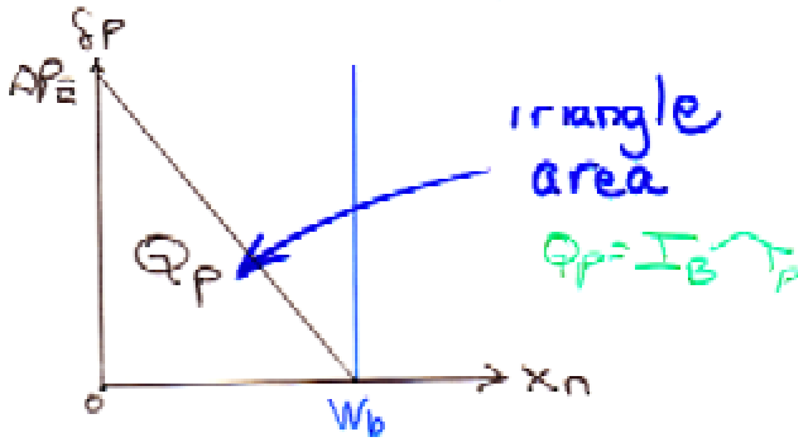
$$= qA\frac{\Delta p_E W_b}{2\tau_p}$$ since $L_p^2 = D_p\tau_p$



Forward-biased emitter and Reverse-biased collector







Triangle area = charge in base = $\Delta p_E \dfrac{W_b}{2}$
(assuming nearly straight line)

$$Q_p \equiv \Delta p_E \frac{W_b}{2} \cdot q A$$

Current = charge replaced per second

$$= q A \Delta p_E \frac{W_b}{2 \tau_p}$$

Charge Control Approximation

Comments

- Just an approximation : slope must differ between E and C. otherwise, $I_B = 0$

But for normal biasing, see basic relationships:

- $\frac{I_C}{I_B}$ = gain increases as I_B decreases.

$$\frac{I_C}{I_B} \sim \frac{\operatorname{csch} \frac{W_b}{L_p}}{\tanh \frac{W_b}{2L_p}} \approx \frac{\frac{1}{y}}{y/2} = \frac{1}{y^2/2} = \frac{2L_p^2}{W_b^2}$$

as W_b gets smaller ("narrow base"), gain increases.

$$\frac{I_C}{I_B} \sim \frac{\operatorname{csch}\frac{W_b}{L_p}}{\tanh\frac{W_b}{2L_p}} \approx \frac{\frac{1}{y}}{y/2} = \frac{1}{y^2/2} = \frac{2L_p^2}{W_b^2}$$

as W_b gets smaller ("narrow base"), gain increases.

- $I_B = qA\frac{\Delta p_E W_b}{2\tau_p}$

as τ_p gets larger, I_B decreases, gain

can increase τ_p by light doping in base.
(improves emitter efficiency too)

But tradeoff: longer $\tau_p \rightarrow$ lower bandwidth and frequency response

Current Transfer Ratio

▷ Emitter Injection Efficiency

can write in terms of emitter and base properties:

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}}$$

$$= \left[1 + \frac{L_p^n \, n_n \, \mu_n^p}{L_n^p \, p_p \, \mu_p^n} \tanh \frac{W_b}{L_p^n} \right]^{-1}$$

$$\simeq \left[1 + \frac{W_b \, n_n \, \mu_n^p}{L_n^p \, p_p \, \mu_p^n} \right]^{-1}$$

Homework problem:

$$\frac{I_{Ep}}{I_{Ep}+I_{En}} = \left[1 + \frac{I_{En}}{I_{Ep}} \right]^{-1}$$

Make γ large: ~ 1

For p-n-p transistor, make p_p $\boxed{}$ n_n

and L_n^p $\boxed{}$ W_B

▷ Base Transport Factor

$$B = \frac{I_C}{I_{Ep}} = \frac{\operatorname{csch} W_b/L_p}{\operatorname{ctnh} W_b/L_p} = \operatorname{sech} \frac{W_b}{L_p}$$

for $\frac{W_b}{L_p}$ small, $\operatorname{sech} \frac{W_b}{L_p} \sim 1$

Current Transfer Ratio

$\alpha = B\,\gamma$ efficiency × transfer ratio

Plug in for B and γ from above expressions

$$\alpha = B\gamma = \operatorname{sech} \frac{W_b}{L_p} \cdot \left[1 + \frac{L_p^n\, n_n\, \mu_n^p}{L_n^p\, p_p\, \mu_p^n} \tanh \frac{W_b}{L_p}\right]^{-1}$$

$$= \left[\cosh \frac{W_b}{L_p^n} + \frac{L_p^n\, n_n\, \mu_n^p}{L_n^p\, p_p\, \mu_p^n} \sinh \frac{W_b}{L_p}\right]^{-1}$$

Example

p-n-p transistor

$p_p = 10\, n_n \qquad \mu_n^p = \frac{1}{2}\mu_p^n \qquad W_b = \frac{1}{10} L_p^n$

carrier lifetimes $\tau_n = \tau_p$

Calculate α and β

$L = \sqrt{D\tau}\ ; \qquad \frac{D}{\mu} = \frac{kT}{q}\ ; \qquad L = \sqrt{\mu kT\tau/q}$

$L_p^n / L_n^p = \sqrt{\mu_p^n/\mu_n^p}$ for equal lifetimes

$$\alpha = \left[\cosh 0.1 + \sqrt{2}\,(0.1)(0.5)\sinh 0.1\right]^{-1}$$

$$= \left[1.005 + 0.0707\,(0.1)\right]^{-1} = \boxed{}$$

and $\beta = \frac{\alpha}{1-\alpha} = \boxed{}$

Heterojunction Bipolar Transistors

Useful to improve device properties

Want high emitter efficiency $\gamma \rightarrow$ high α and β

$$\gamma \simeq \left[1 + \frac{W_b}{L_p}\frac{n_n}{p_p}\frac{\mu_n^p}{\mu_p^n}\right]^{-1}$$ for p-n-p

so keep

But also want low base resistance (lower voltage drop)

→ keep

And want low emitter junction capacitance (higher ω)

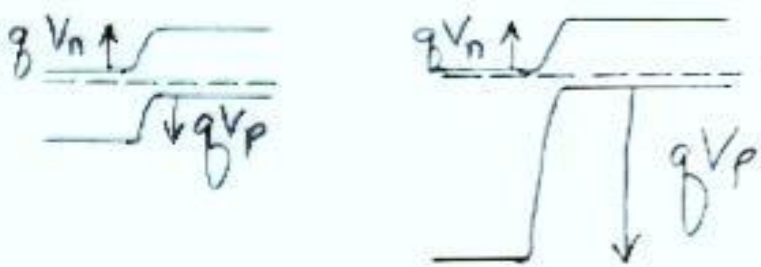
→ keep ($C \propto N^{1/2}$)

Need new mechanism to control relative injection across emitter junction

Solution :

→

Heterojunction Barrier Differences



Much bigger hole barrier so electron emitter γ is high.

Homojunction

Heterojunction

Exponential Effect. So even small ΔE_g has huge effect.

So could now choose heavily doped base & lightly doped emitter.

Example shows that:

Amplification depends on

- doping (p_p, n_n)
- basewidth (W_b)
- diffusion length (L_p^n, L_n^p)
- mobility (μ_n^p, μ_p^n)
- lifetime (τ_n, τ_p)

$\longrightarrow$ Fundamental physical parameters and design parameters