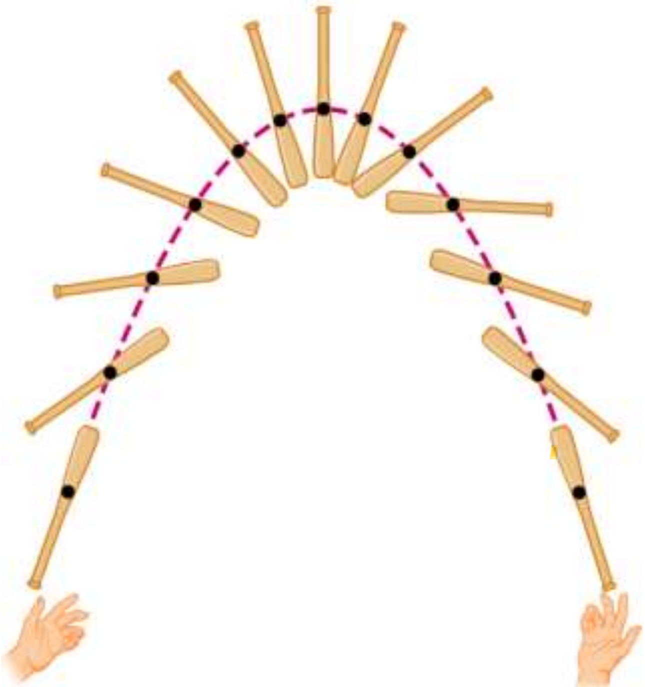
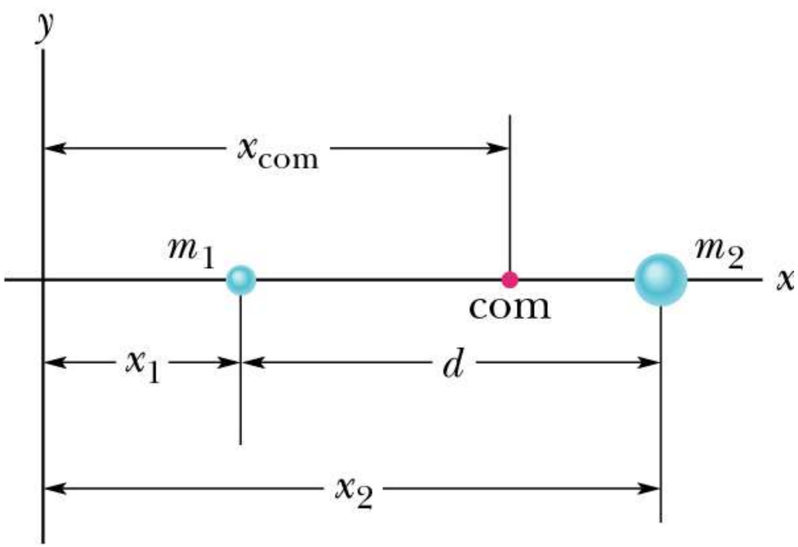
Center of Mass
(COM)



$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$If\ \ m_1 = m_2,\ x_{com} = \frac{x_1 + x_2}{2} = \bar{x}$$

$$If\ \ m_1 = 0,\ x_{com} = x_2$$



$$x_{com} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{1}{M_{total}} \sum_i m_i x_i$$

$$y_{com} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{1}{M_{total}} \sum_i m_i y_i,$$

$$\vec{r}_{com} = \frac{1}{M_{total}} \sum_i m_i \vec{r}_i$$

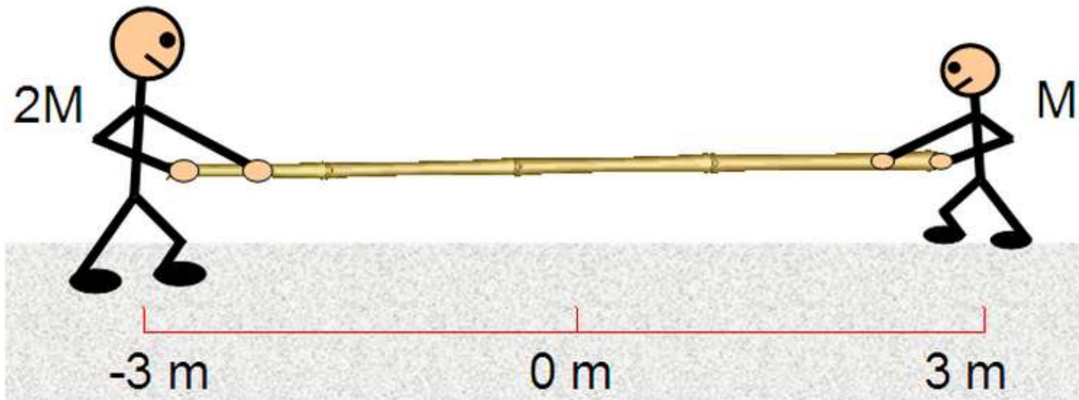
$$M_{total} = \sum_i m_i$$

$$If\ \ m_1 = 1.0kg\ \ \ m_2 = 2.0kg$$

$$If\ \ x_1 = 2\ (m),\ x_2 = 6\ (m)$$

$$x_{com} = ?$$

A large skinny guy with mass 2M and a smaller guy with mass M are holding onto a ***massless*** pole while standing on ***frictionless*** ice, as shown below. If the big guy pulls himself toward the little guy, where would they meet?



(1) -3 m

(2) -2 m

(3) -1 m

(4) 0 m

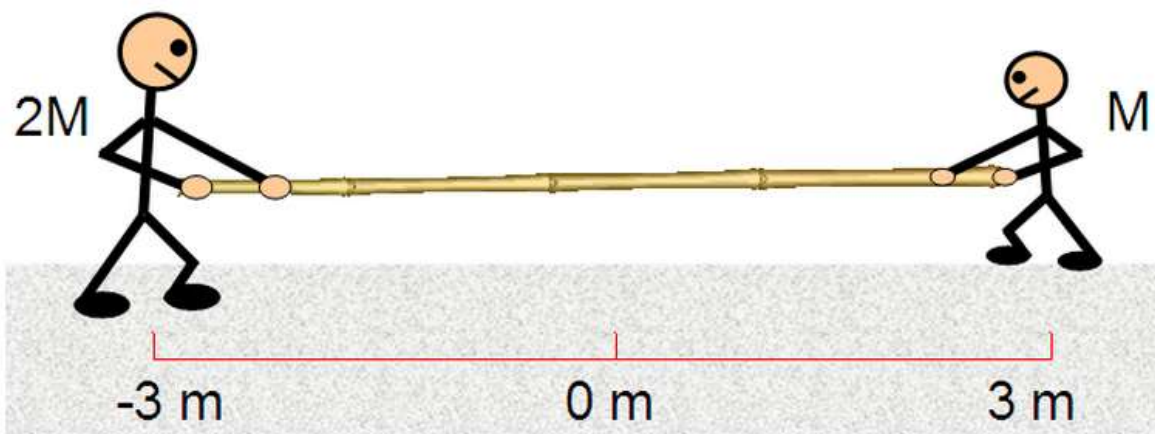
(5) 1 m

(6) 2 m

(7) 3 m

(8) None of the above

A large skinny guy with mass 2M and a smaller guy with mass M are holding onto a ***massless*** <u>pole while standing on</u> ***frictionless*** <u>ice, as shown below. If the little guy pulls himself toward the big guy, where would they meet?</u>



(1) -3 m

(2) -2 m

(3) -1 m

(4) 0 m

(5) 1 m

(6) 2 m

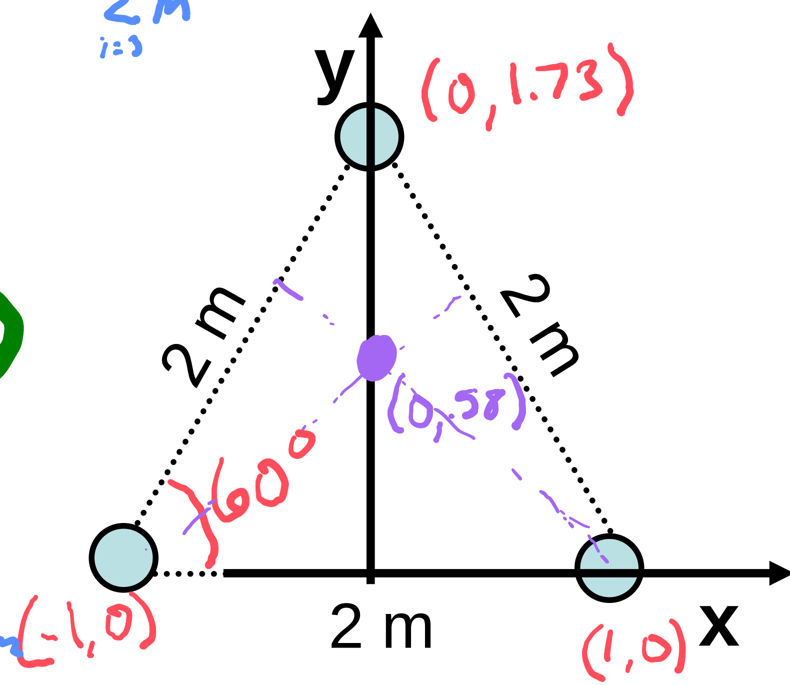
(7) 3 m

(8) None of the above

Three tiny equal-mass magnets are placed on a horizontal frictionless surface at the corners of an equilateral triangle (all sides 2 m and all angles 60^0). When the magnets are released, they attract and quickly slide to a single point. ***What are the x and y coordinates of that point?*** (Before release, the y-axis passed through the top ball and the x-axis passed through the bottom balls.)

$$X_{cen} = \frac{\sum x_i m}{\sum_{i=3} m} = \frac{-1m + 1m + 0m}{3m} = 0$$

	x	y
1.	0 m	0 m
2.	0 m	1 m
3.	0 m	.58 m
4.	0 m	.87 m
5.	.67 m	.67 m
6.	0 m	.67 m



$2\sin 60 = \sqrt{3}$

$$y_{cen} = \frac{\sum y_i m}{\sum_{i=3} m} = \frac{0m + 0m + 1.73m}{3m} = \boxed{0.58}$$

$$x_{com} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{1}{M_{total}} \sum_i m_i x_i$$

$$y_{com} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{1}{M_{total}} \sum_i m_i y_i ,$$

	x	y
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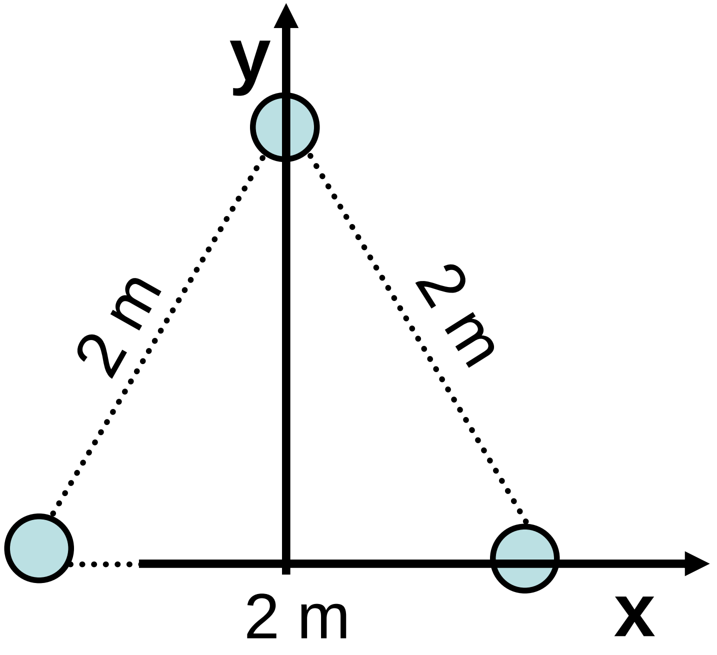
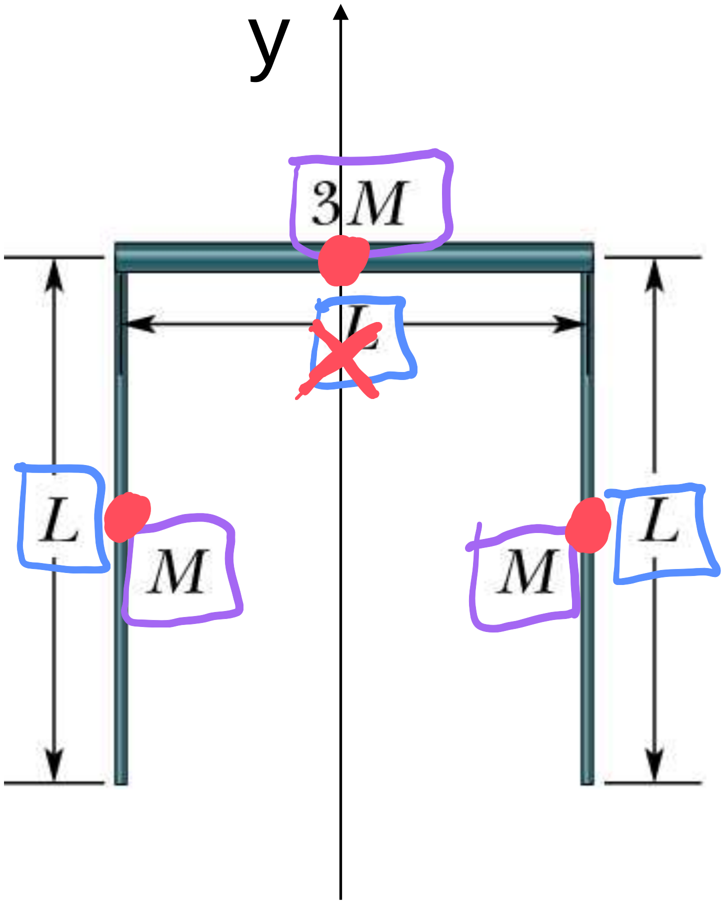




Symmetry: $X_{com}=L/2$

$$y_{com} = \left(M\frac{L}{2} + M\frac{L}{2} + 3ML\right)/(M+M+3M)$$

$$= \frac{4ML}{5M} = \boxed{0.8L}$$

Newton's 2nd Law for a system of particles

$$\vec{F}_{net} = M_{total}\vec{a}_{com}$$

$$\vec{r}_{com} = \frac{1}{M}\sum_i m_i \vec{r}_i$$

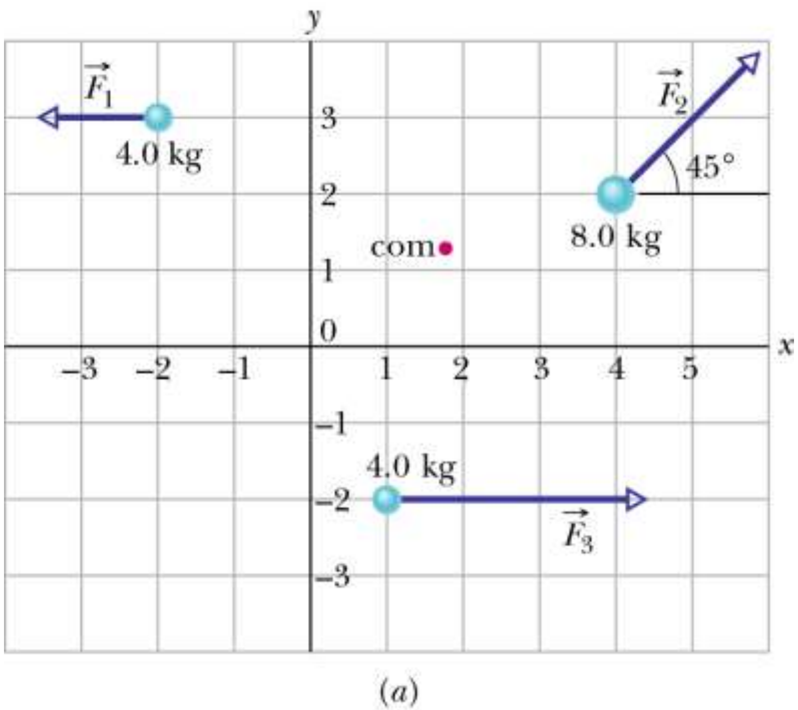
$$M\vec{V}_{com} = \sum_i m_i \vec{V}_i$$

$$M\vec{a}_{com} = \sum_i m_i \vec{a}_i$$

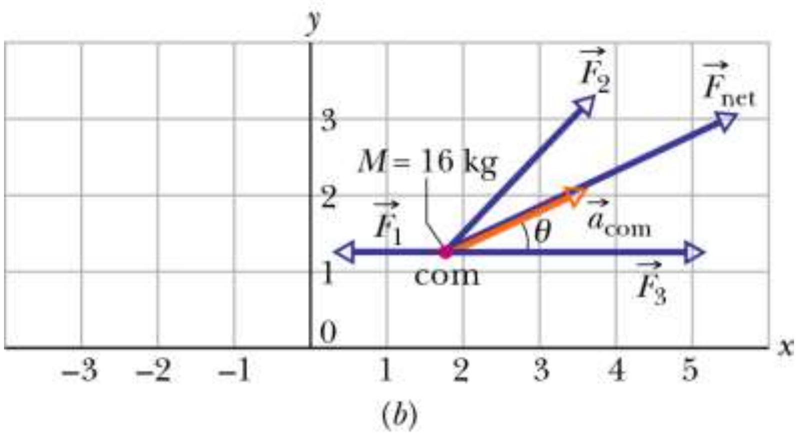
$$\vec{P}_{com} = \sum_i \vec{P}_i$$

$$\vec{F}_{net} = \sum_i \vec{F}_i$$

Ex: P.208



(a)



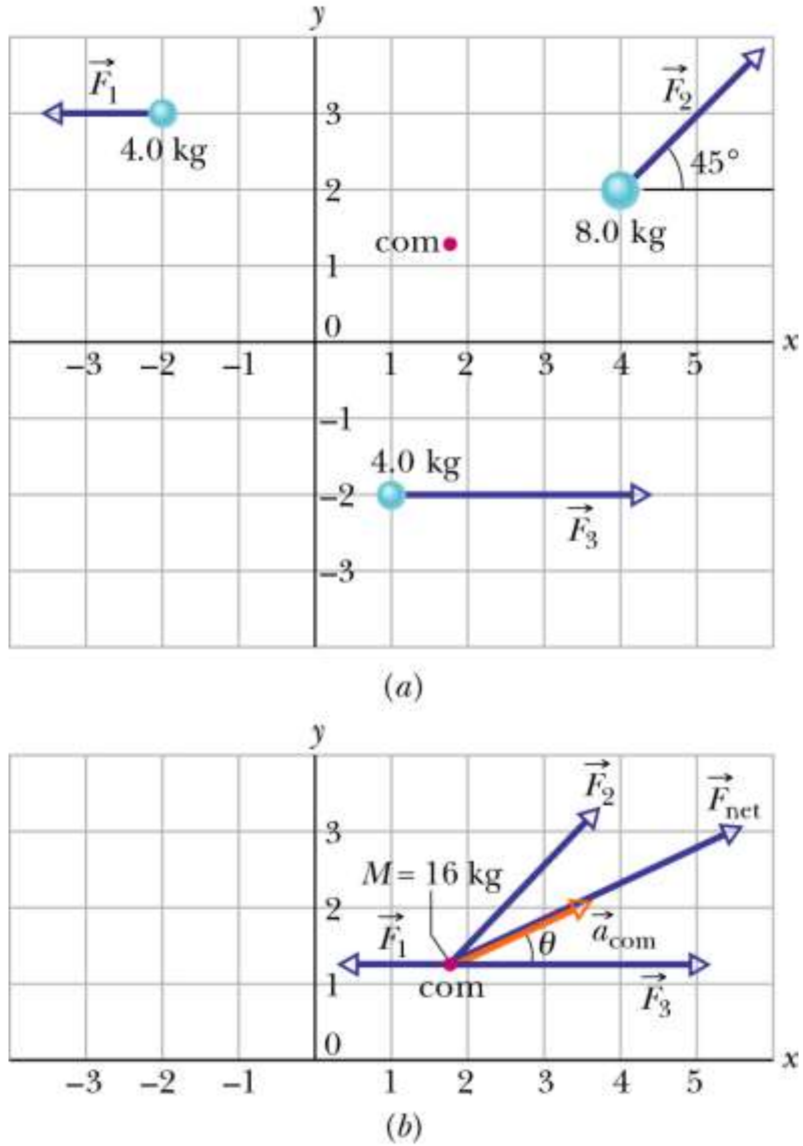
(b)

Three particles are initially at rest. Each experiences an external force as shown.

What is the acceleration of the COM?

Lots of trig

Ex: P.208



$$\vec{F}_{net} = M\vec{a}_{com}$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\vec{a}_{com} = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3)/M$$

$$a_{com,x} = \frac{F_{1x} + F_{2x} + F_{3x}}{M}$$

$$= \frac{-6.0 + 12\cos 45^0 + 14}{16} = 1.03\,m/s^2$$

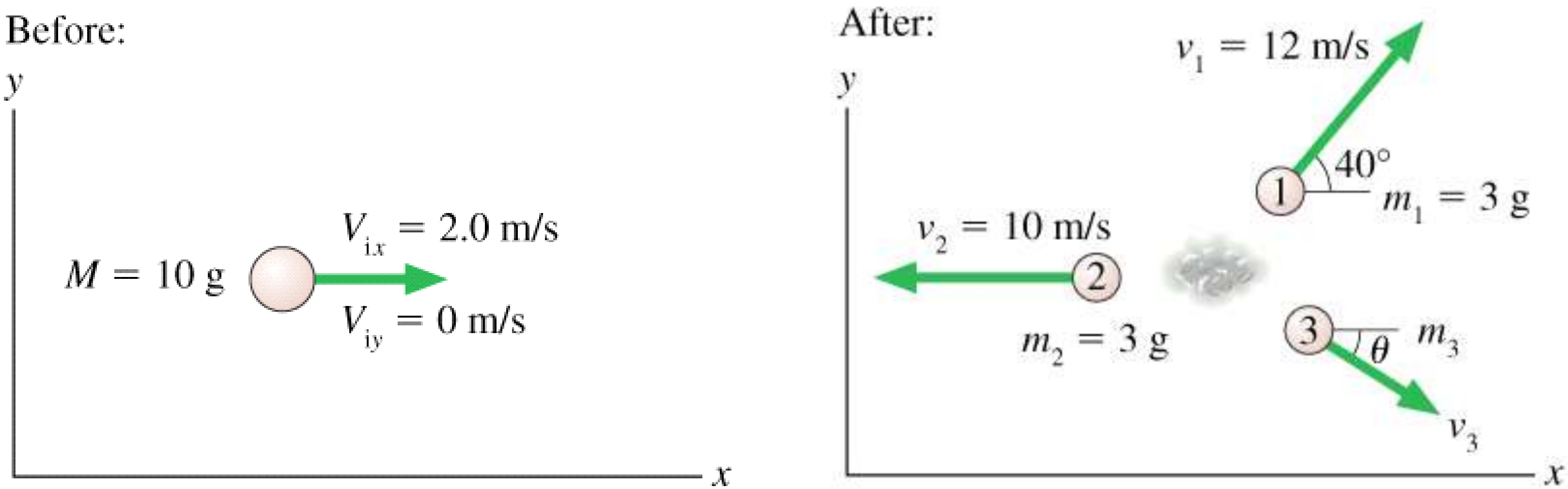
$$a_{com,y} = \frac{F_{1y} + F_{2y} + F_{3y}}{M}$$

$$= \frac{0 + 12\sin 45^0 + 0}{16} = 0.53\,m/s^2$$

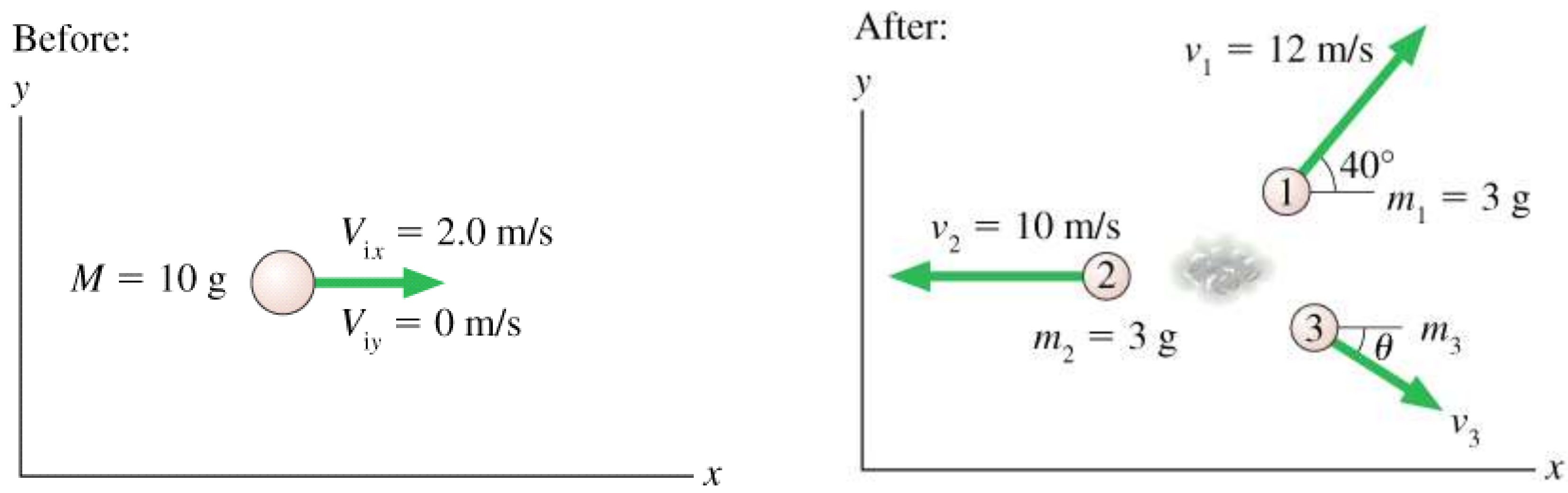
$$a_{com} = \sqrt{(a_{com,x})^2 + (a_{com,y})^2} = 1.16\,m/s^2$$

$$\theta = \tan^{-1}\frac{a_{com,y}}{a_{com,x}} = 27^0$$

An object was initially traveling toward +x direction with V=2.0 m/s. It explodes into three pieces (see diagram). Find V_3 and θ.



Find: v_3 and θ



Find: v_3 and θ

$$P_i = P_f \quad P_{ix} = P_{fx} \quad P_{iy} = P_{fy}$$

$$MV_{ix} = m_1 V_{1x} + m_2 V_{2x} + m_3 V_{3x}$$

$$MV_{iy} = m_1 V_{1y} + m_2 V_{2y} + m_3 V_{3y}$$

$\theta = 45.9^{\circ}$ $V_3 = 8.05$ m/s