

ECE 3030 Homework 2

100 - 22 + 10

wavelength of light?

(88)

1) a) photon = 5.65 eV $\rightarrow 5.65 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV} = 9.04 \times 10^{-19} \text{ J}$ X -3

b) lithium = 2.90 eV $\rightarrow 2.90 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV} = 4.64 \times 10^{-19} \text{ J}$ X -3

c) As light wavelength λ increases, more electrons are excited and ejected. ✓

d) The number of electrons increases, and the energy threshold does not change. ✓

2) GaAs $\lambda_0 = 5.65 \text{ \AA}$ $\rightarrow \lambda = \frac{h}{p} = 5.65 \text{ \AA}$ $h = 6.63 \times 10^{-34} \text{ Jsec}$

$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.65 \text{ \AA}} = 1.17 \times 10^{-24} \text{ kg m/s} \rightarrow 1.17 \times 10^{-29} \text{ g cm/s}$ -1

$E = \frac{p^2}{2m} = \frac{1.17 \times 10^{-29} \text{ g cm/s}}{2 \times 9.11 \times 10^{-28} \text{ g}} = 6.42 \times 10^{-3} \text{ eV}$ X -2

3) a) $\Delta x \Delta p \geq \frac{\hbar}{2} \rightarrow \Delta p \geq \frac{\hbar}{2\Delta x} \rightarrow \frac{1.054 \times 10^{-34}}{2 \times 1.7 \times 10^{-7}} = \Delta p = 3.1 \times 10^{-21} \text{ g cm/s}$ ✓

b) $\Delta E \Delta t \geq \frac{\hbar}{2} \rightarrow \Delta t \geq \frac{\hbar}{2\Delta E} \rightarrow \frac{1.054 \times 10^{-34}}{2 \times 3.204 \times 10^{-19}} = \Delta t = 1.64 \times 10^{-16} \text{ s}$ ✓
 $2 \text{ eV} \rightarrow 3.204 \times 10^{-19} \text{ J}$

4) a) $E_1 = \frac{\pi^2 \hbar^2}{2mL^2} = 2.08 \times 10^{-20} \text{ eV}$ X -1 $\hbar = 1.054 \times 10^{-34}$
 $E_2 = \frac{2^2 \pi^2 \hbar^2}{2mL^2} = 8.32 \times 10^{-20} \text{ eV}$ X -1 $m = 9.109 \times 10^{-31}$
 $E_3 = \frac{3^2 \pi^2 \hbar^2}{2mL^2} = 18.72 \times 10^{-20} \text{ eV} \rightarrow 1.87 \times 10^{-19} \text{ eV}$ X -1 $L = 17 \text{ \AA} = 17 \times 10^{-10} \text{ m}$

b) $\Delta E = E_3 - E_1 = 18.72 - 2.08 = 16.64 \times 10^{-20} \text{ eV}$

$\lambda = \frac{hc}{\Delta E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{16.64 \times 10^{-20}} = 1.19 \times 10^{-6} \text{ m}$ ✓

5) $\psi(x) = A e^{ik_1 x} + B_1 e^{-ik_1 x}$ (for + and - x directions)

$$k_1 = [2m(E - V)/\hbar^2]^{1/2}$$

$$E_1 > V = 0$$



a) $k_1 = \sqrt{\frac{2mE_1}{\hbar^2}}$ Since $V=0$

b) $\psi(x < 0) = A_1 e^{ik_1 x} + B_1 e^{-ik_1 x}$

c) $\psi(x > 0) = A_2 e^{ik_2 x} + B_2 e^{-ik_2 x}$ where $k_2 = \sqrt{\frac{2m(E - V)}{\hbar^2}}$

d) $\psi(x=0) = A_1 e^{ik_1 x} + B_1 e^{-ik_1 x} = A_2 e^{ik_2 x} + B_2 e^{-ik_2 x}$

$$ik_1 A_1 - ik_1 B_1 = ik_2 A_2$$

Since $\left. \frac{\partial \psi_1}{\partial x} \right|_{x=0} = \left. \frac{\partial \psi_2}{\partial x} \right|_{x=0}$

$A_2 = ?$
 $B_1 = ?$
 $B_2 = ?$

1st derivative is continuous at $x=0$

e) The wave will travel faster with $x > 0$.

There are reflected waves for both $x > 0$ and $x \leq 0$.

6) $\rho \propto e^{-2kx}$ $E_k = 3.3 \text{ eV} = 5.2866 \times 10^{-19} \text{ J}$ $V_0 = 4 \text{ eV} = 6.408 \times 10^{-19} \text{ J}$

a) $k = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \sqrt{\frac{2m(1.12 \times 10^{-19})}{\hbar^2}} = 4.28 \times 10^9$

$$5 \text{ \AA} = 5 \cdot 10^{-10} \text{ m}$$

$\rho \propto e^{-2(4.28 \times 10^9)(5 \times 10^{-10})} = e^{-4.28}$

b) $\rho \propto e^{-8.56 \times 10^9 (10 \times 10^{-10})} = e^{-8.56}$

c) $\rho \propto e^{-8.56 \times 10^9 (40 \times 10^{-10})} = e^{-34.24}$

- 1 simplify to numeric values

7) a) $m = 0.067 m_0$ $V_0 = 0.8 \text{ eV} = 1.2816 \times 10^{-19} \text{ J}$ $W = 15 \times 10^{-10} \text{ m}$

$E_k = 0.2 \text{ eV} = 3.204 \times 10^{-20} \text{ J}$

$T \approx \ln\left(\frac{E}{V_0}\right) \left(1 - \frac{E}{V_0}\right) e^{-2k_2 W}$

$k_2 = \sqrt{\frac{2(0.067)m(9.612 \times 10^{-20})}{\hbar^2}} = 1.027 \times 10^9$

$T \approx \ln\left(\frac{3.204 \times 10^{-20}}{1.2816 \times 10^{-19}}\right) \left(1 - \frac{3.204 \times 10^{-20}}{1.2816 \times 10^{-19}}\right) e^{-2(1.027 \times 10^9)(15 \times 10^{-10})} = 0.13784$

b) $k_2 = \sqrt{\frac{2(1.08)m(9.612 \times 10^{-20})}{\hbar^2}} = 4.1223 \times 10^9$

$T \approx \ln(\dots) \left(1 - \dots\right) e^{-2(4.1223 \times 10^9)(15 \times 10^{-10})} = 1.277 \times 10^{-5}$

The particle with a greater effective mass ($1.08 m_0$) tunnels much less than the particle with a lower mass.

8) $T_M = 5 \times 10^{-6}$ $E = 0.08 \text{ eV} = 1.2816 \times 10^{-20} \text{ J}$ $V_0 = 0.8 \text{ eV} = 1.2816 \times 10^{-19} \text{ J}$

$5 \times 10^{-6} \leq \ln\left(\frac{1.2816 \times 10^{-20}}{1.2816 \times 10^{-19}}\right) \left(1 - \frac{1.2816 \times 10^{-20}}{1.2816 \times 10^{-19}}\right) e^{-2(4.345 \times 10^9)W}$

$5 \times 10^{-6} \leq 1.44 e^{-8.2446 \times 10^9 \cdot W}$

$k_2 = \sqrt{\frac{2m(1.15 \times 10^{-19})}{\hbar^2}} = 4.345 \times 10^9$

$W \geq -1.8267 \times 10^{-9} \text{ m}$