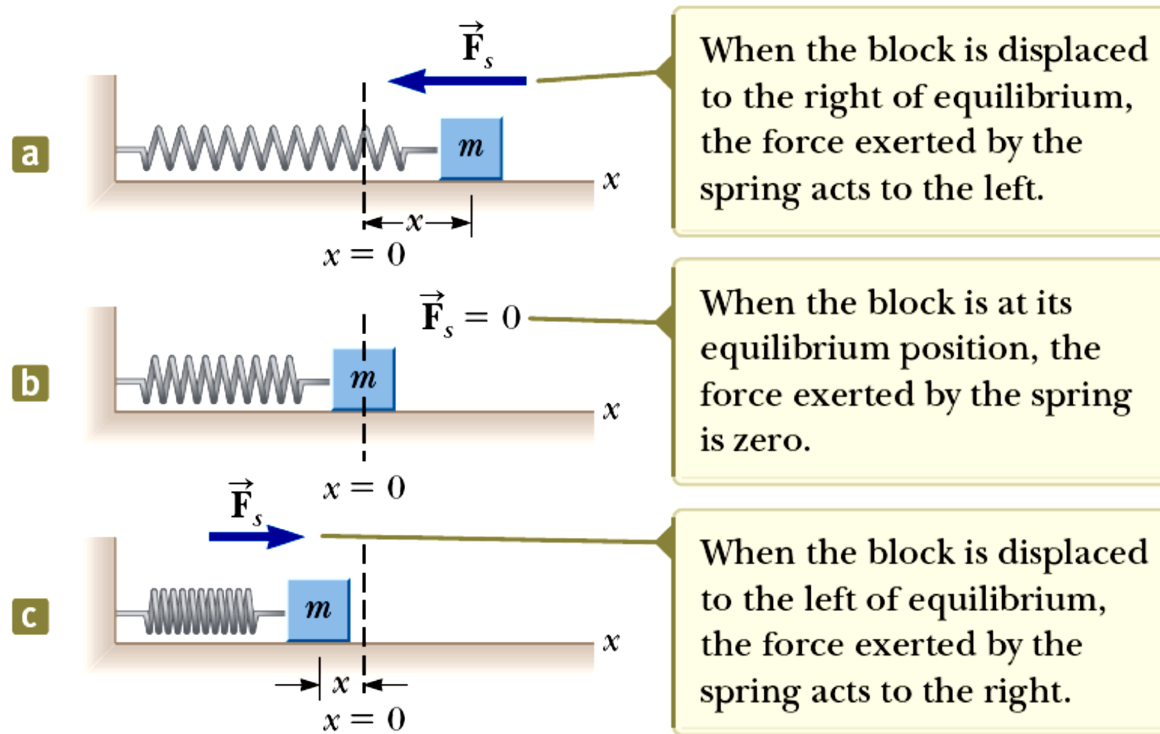
# Simple Harmonic Motion

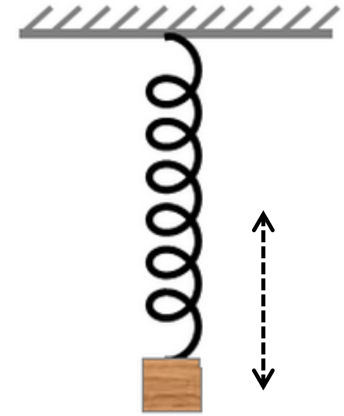
$$F_s = -kx$$

$$-kx = ma_x$$

$$a_x = -\frac{k}{m}x$$



A block is oscillating up and down. When the block reaches its lowest point, what's the velocity and net force on the block?



(1) Upward net force and velocity

(2) Upward net force and zero velocity

(3) Downward net force and velocity

(4) Downward net force and zero velocity

(5) Zero net force and zero velocity

(6) None of the above

# Model the Motion

$$F_s = -kx$$

$$-kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

Differential Equations

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Define $\omega^2 = \frac{k}{m}$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$ Differential Equations

What kind of function when taking double derivatives will return to its original form with a minus sign and some constant?

What kind of function when taking double derivatives will return to its original form with a minus sign and some constant?

$$\frac{d^2x}{dt^2} = -\omega^2 x$$ Differential Equations

Try sin() or cos()

$$x(t) = A\cos(\omega t + \phi) \longrightarrow \frac{d^2x}{dt^2} = -\omega^2 x$$

$$\frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \phi) = -\omega A \sin(\omega t + \phi)$$

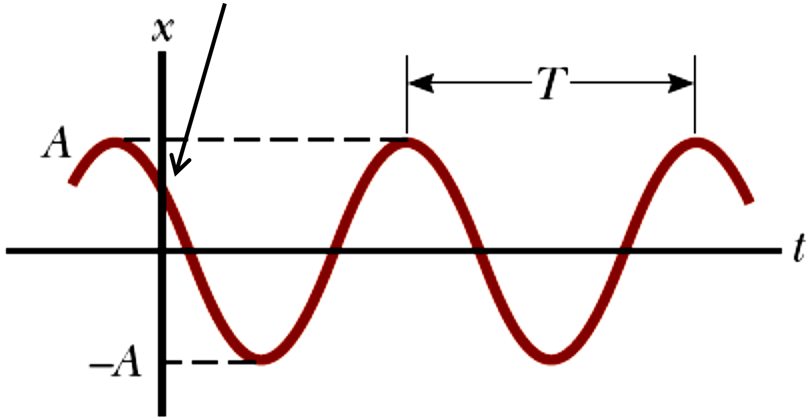
$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \phi) = -\omega^2 A \cos(\omega t + \phi)$$

What does this x-t diagram look like?

$$t = 0$$

$$x(0) = A\cos(0+\phi) = A\cos(\phi)$$

$$x(t) = A\cos(\omega t + \phi)$$




Important Parameters of the Motion

$$\omega = \sqrt{\frac{k}{m}} \qquad \omega T = 2\pi \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} \qquad A \Rightarrow Amplitude$$

$$t = 0$$

$$x(0) = A\cos(0+\phi) = A\cos(\phi)$$

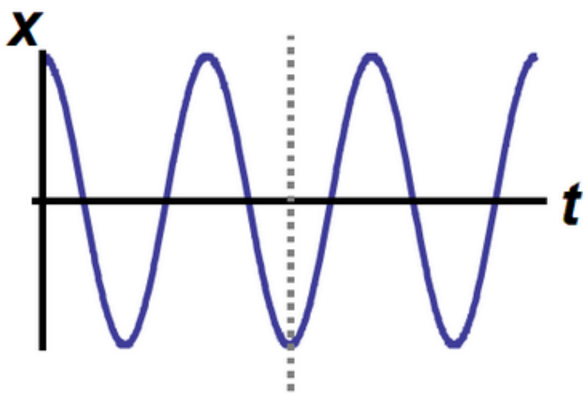
$$\phi \Rightarrow initial\ \ phase$$

$$\omega t + \phi \Rightarrow phase$$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

This is the position vs. time graph of a mass on a spring. What can you say about the velocity and the net force at the instant indicated by the dotted line?




(1) Velocity positive; force toward +x

(2) Velocity negative; force toward –x

(3) Velocity negative; force toward +x

(4) Velocity zero; force toward +x

(5) Velocity zero; force toward –x

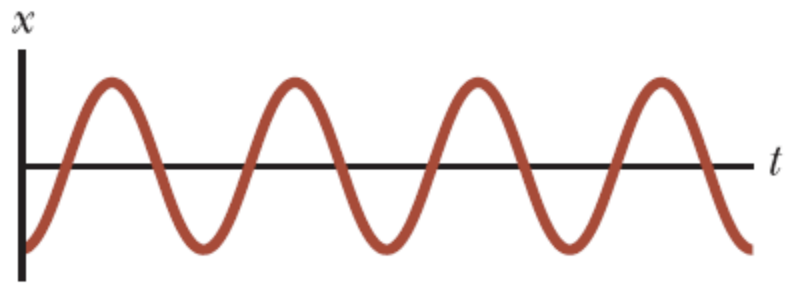
(6) None of the above

What is the initial phase of this oscillation?

$$x(t) = A\cos(\omega t + \phi)$$

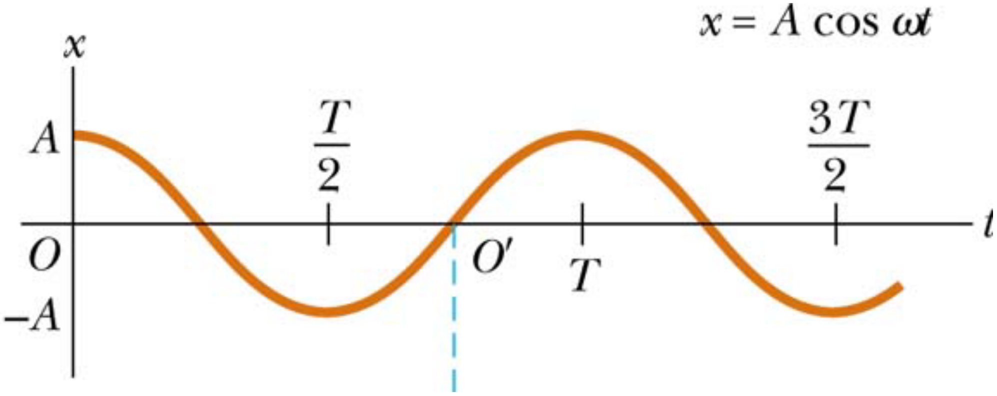
$$t = 0$$

$$x(0) = A\cos(0 + \phi) = A\cos(\phi)$$




What is the initial phase of this oscillation? $\phi = ?$

# Simple Harmonic Motion




$$x(t) = A\cos(\omega t + \phi)$$
$$v(t) = -\omega A\sin(\omega t + \phi)$$
$$a(t) = -\omega^2 A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad \omega T = 2\pi \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} \qquad A \Rightarrow Amplitude$$

$\omega$, $f$, $T$ determined by mass and spring constant

$A$, $\phi$ determined by initial conditions: $x(0)$, $v(0)$

An object undergoing simple harmonic motion follows the expression,

$$x(t) = 4 + 2\cos[\pi(t-3)]$$

Where x will be in cm if t is in seconds

The amplitude of the motion is:

a) 1 cm
b) 2 cm
c) 3 cm
d) 4 cm
e) -4 cm

**An object undergoing simple harmonic motion follows the expression,**

$$x(t) = 4 + 2\cos[\pi(t-3)]$$

**Here, x will be in cm if t is in seconds**

**The angular frequency of the motion is:**

**a) 1/3 rad/s**
**b) 1/2 rad/s**
**c) 1 rad/s**
**d) 2 rad/s**
**e) $\pi$ rad/s**

An object undergoing simple harmonic motion follows the expression,

$$x(t) = 4 + 2\cos[\pi(t-3)]$$

Here, x will be in cm if t is in seconds

The period of the motion is:

a) 1/3 s
b) 1/2 s
c) 1 s
d) 2 s
e) $2/\pi$ s

An object undergoing simple harmonic motion follows the expression,

$$x(t) = 4 + 2\cos[\pi(t-3)]$$

Here, x will be in cm if t is in seconds

The frequency of the motion is:

a) 1/3 Hz
b) 1/2 Hz
c) 1 Hz
d) 2 Hz
e) $\pi$ Hz

$$x(t) = 4 + 2\cos[\pi(t - 3)]$$

Here, x will be in cm if t is in seconds

The object will pass through the equilibrium position at the times, t = ______ seconds

a) …, -2, -1, 0, 1, 2 …
b) …, -1.5, -0.5, 0.5, 1.5, 2.5, …
c) …, -1.5, -1, -0.5, 0, 0.5, 1.0, 1.5, …
d) …, -4, -2, 0, 2, 4, …
e) …, -2.5, -0.5, 1.5, 3.5,