Problem 1

In physics, an equation is like a sentence describing how some physical quantities are related to other physical quantities. Units are labels that tell you what the quantities are and aid you in understanding what the equation is describing. Like sentences, there are rules for equations and units, three of which are below.

Rule 1 - the unit balance rule: Units must be the same on both sides of the equals sign (because no amount of kilograms is equal to 1 second!).

$$4\,\mathrm{kg} = 4\,\mathrm{s} \quad \textbf{Incorrect}$$

Rule 2 - the addition and subtraction rule: When two or more physical quantities are added together or subtracted from each other, they must have the same units (if you're trying to add something with units of meters to something with units of force, there is a problem somewhere).

$$2\,\mathrm{kg} + 3\,\mathrm{m} = \ ? \quad \textbf{Incorrect}$$

Rule 3 - the multiplication and division rule: When you multiply or divide one quantity by another, the units behave the way symbolic variables do in algebra. This means that if you multiply, say, something in kilograms by something in meters, you get kg·m, or if you multiply something in seconds by something else in seconds you get s^2. Here are some division examples.

$$\mathrm{m} \cdot \frac{1}{\mathrm{s}} = \frac{\mathrm{m}}{\mathrm{s}}$$

$$\frac{\mathrm{m}}{\mathrm{s}^2} \times \mathrm{s} = \frac{\mathrm{m}}{\mathrm{s}}$$

$$\frac{4\,\mathrm{kg}}{2\,\mathrm{kg}} = 2 \ (\textit{no units})$$

Rule 4 - dimensionless arguments for certain functions: The argument of an exponential function (for example: 2^x, e^x, or 10^x) must be dimensionless. The argument of a logarithmic function (for example: $\log_2 x$, $\ln x$, or $\log_{10} x$) must be dimensionless. The argument of a trigonometric function (for example: $\sin x$, $\cos x$, or $\tan x$) must be dimensionless. Concerning the trig functions, recall that angles measured in radians are dimensionless since a radian is just a ratio between arc length and circle radius, so the dimension of radians is: $^{\text{Length}}/_{\text{Length}}$ = dimensionless. And since you can switch from radians to degrees with a conversion factor, degrees are also dimensionless. Here are some more examples.

$$\log_{10}\left(\frac{500\,\text{meters}}{5\,\text{meters}}\right) = \log_{10} 100 = 2 \ (\textit{no units})$$

$$\tan\left(\frac{\pi}{4}\ \text{radians}\right) = \tan\left(\frac{\pi}{4}\ \frac{\text{meters}}{\text{meters}}\right) = 1 \ (\textit{no units})$$

$$\cos\left(15\,\mathrm{m/s}\right) = \ ? \quad \textbf{Incorrect}$$

Each of the examples below breaks at least one of the rules above. Explain what is wrong with each example.

(A) $4\,\mathrm{m} \cdot 3\,\mathrm{m} = 12\,\mathrm{m}$

Rule 3: m*m should equal m²

(B) $12\,\mathrm{m/s} + 3\,\mathrm{m/s^2} = 15\,\mathrm{m/s}$

Rule 2: Cannot add/subtract unlike units

(C) $(5\,\mathrm{kg})\sin\left(\frac{\pi}{2}\ \mathrm{m/s}\right) = 5\,\mathrm{kg}$

Rule 4: sin functions are dimensionless (cannot have a unit)

(D) $4\,\mathrm{m/s} + (6\,\mathrm{m/s^2})(2\,\mathrm{s}) = 16\,\mathrm{kg}$

Rule 1: m/s + m/s = m/s, not kg

(E) $7\,\mathrm{s} + \frac{6\,\mathrm{m}}{3\,\mathrm{m/s}} = (5\,\mathrm{m/s})(2\,\mathrm{s}) - (1\,\mathrm{m/s^2})(1\,\mathrm{s^2})$

Rule 1: Balance rule, s ≠ m

(F) $-1\frac{\mathrm{kg\,m}}{\mathrm{s^2}} + 7\frac{\mathrm{kg\,m}}{\mathrm{s^2}} - (4\,\mathrm{kg}\ \times 2\,\mathrm{m/s}) = -2\frac{\mathrm{kg\,m}}{\mathrm{s^2}}$

Rule 2: Cannot subtract unlike units

(G) $(5\,\mathrm{m}) \times \ln(1\,\mathrm{m/s^2}) = 0\,\mathrm{m}$

Rule 4: Natural Log is dimensionless (can't have a unit)

(H) $\frac{8\,\mathrm{kg\,s}}{4\,\mathrm{kg\,m/s}} = 2\,\mathrm{m/s}$

Rule 1: s²/m ≠ m/s

Problem 2

Alice jumps down the rabbit hole to follow the white rabbit into Wonderland and falls for 10 seconds before reaching the bottom. As she is falling she wonders whether the rabbit hole is very deep, or if she is just falling very slowly (which would mean that acceleration due to gravity is less than $9.8\,\mathrm{m/s^2}$ in Wonderland).

(a) Suppose that she falls from rest and accelerates at the usual rate due to gravity on Earth ($9.8\,\mathrm{m/s^2}$). How deep is the rabbit hole?

$$x = v_0 t + \frac{1}{2} a t^2$$
$$= \frac{1}{2}(9.8)(10^2)$$
$$\boxed{= 490\,m}$$

(b) Suppose that she falls from rest and with constant acceleration, but that the rabbit hole is only 3.0 m deep. What was Alice's acceleration as she fell?

$$x = v_0 t + \frac{1}{2} a t^2$$
$$\frac{1}{2} a t^2 = x - v_0 t$$
$$a = \frac{2(x - v_0 t)}{t^2}$$
$$= \frac{2(3-0)}{10^2}$$
$$= \frac{6}{100} \boxed{= 0.06\,m/s^2}$$

(c) Consider the same scenario as in part (b). What is Alice's speed when she reaches the bottom?

$$v = v_0 + at$$
$$= 0 + (.06)(10)$$
$$\boxed{= 0.6\,m/s}$$