Last Class…

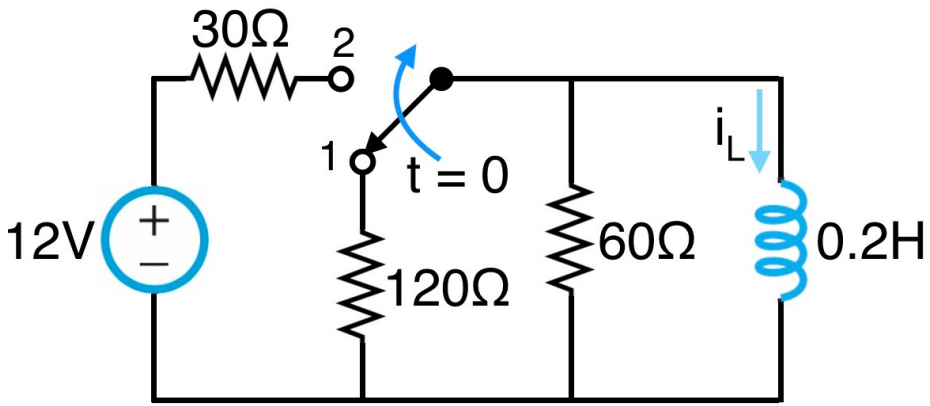
After having been in position 1 for a long time, the switch in the circuit was moved to position 2 at t = 0. Determine:

A. $i_L(0)$

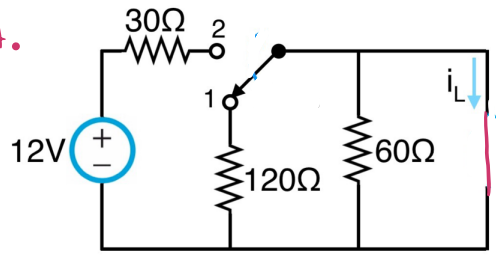
B. $i_L(\infty)$

C. $i_L(t)$ for $t \geq 0$

D. $v_L(t)$ for $t \geq 0$

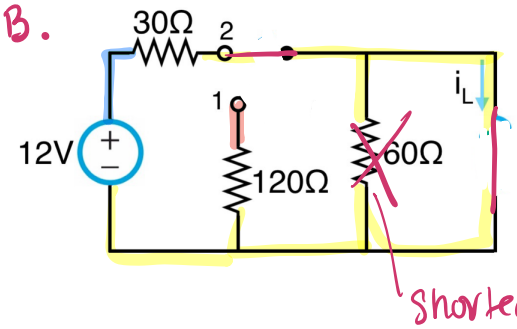


A.



$i_L(0) = 0A$

B.



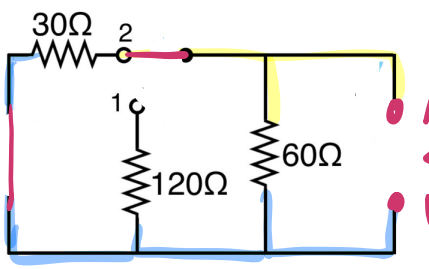
$i_L(\infty) = i_{30}$

$= \frac{V_{30}}{30} = \frac{12}{30} = 0.4A.$

C. $i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\, e^{-R/L\, t}$

$= 0.4 + [0 - 0.4]\, e^{-100t}$

$= 0.4 - 0.4\, e^{-100t}$



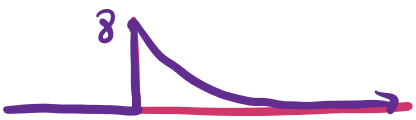
$R_N = 30 \| 60 = 20\Omega$

$\frac{R_N}{L} = \frac{20}{0.2} = 100$

D. $V_L(t) = L \cdot \frac{d\, i_L(t)}{dt}$

$= (0.2)(-0.4)\, e^{-100t}(-100)$

$= 8\, e^{-100t}$



THE OHIO STATE UNIVERSITY

COLLEGE OF ENGINEERING

Sinusoids and Complex Numbers Review

In-class participation

1. A sinusoidal waveform is characterized by three parameters. What are they, and what does each one of them specify?

2. Express the voltage $v(t) = 150\sin(300t + 60°)$ in cosine form.

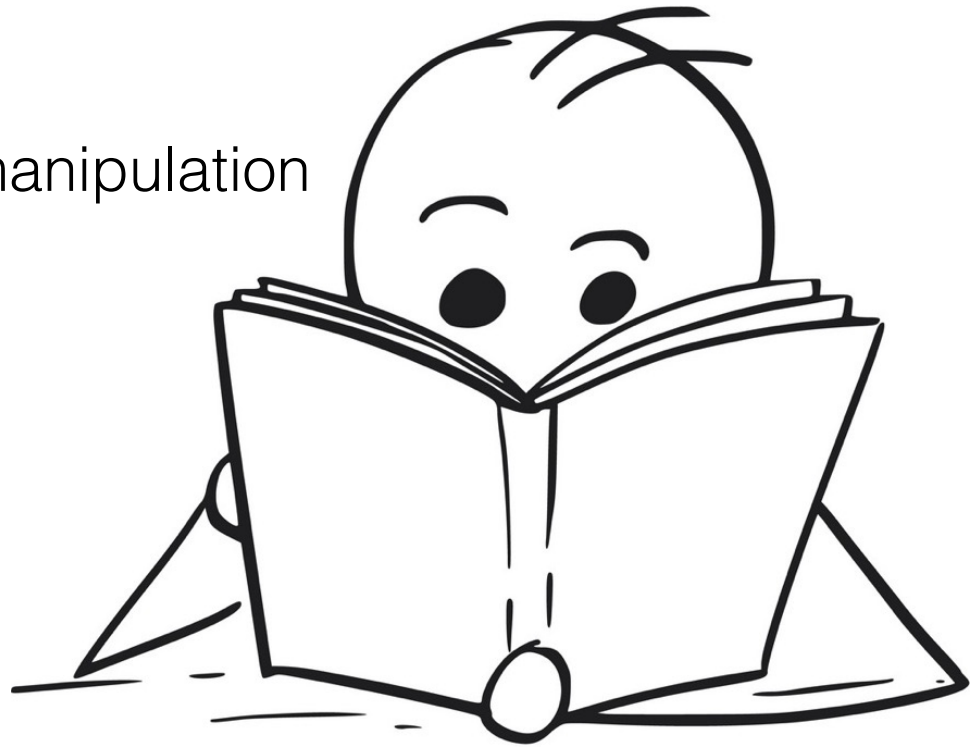
3. Find the value of ω if the frequency is 5Hz?

4. Express the following complex function in polar form: $z_1 = (4 - 3j)^2$

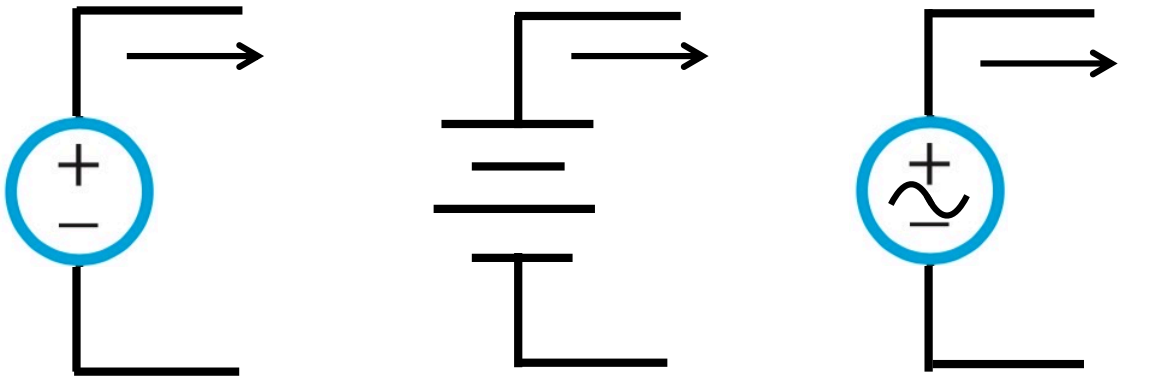
5. If two complex numbers have the same magnitude, are they necessarily equal to each other?

This Lecture…

- Learning Objectives:
  - Identify the general form or a sinusoidal signal.
  - Understand the geometric interpretation of complex numbers and the relationship between the polar and rectangular form.
  - Perform basic algebraic manipulation with complex numbers.



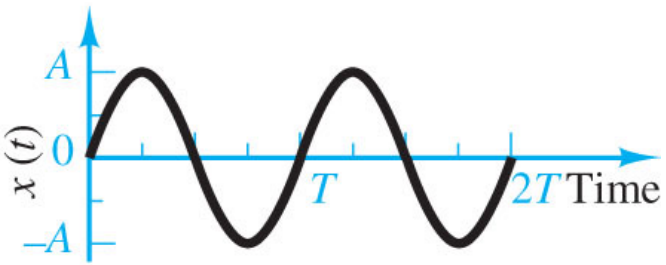
AC Circuits



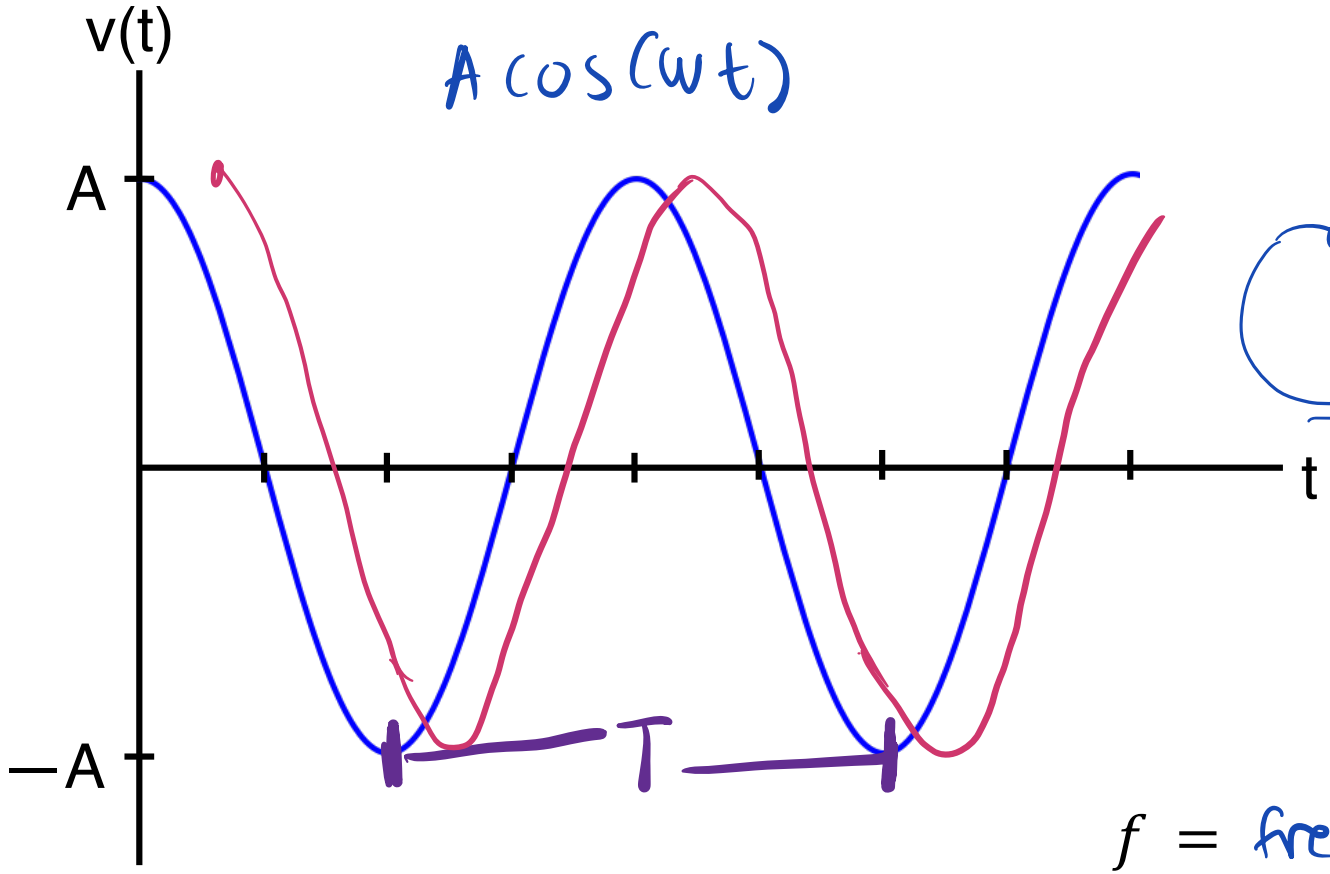
Generic DC voltage AC voltage

Alternative Current

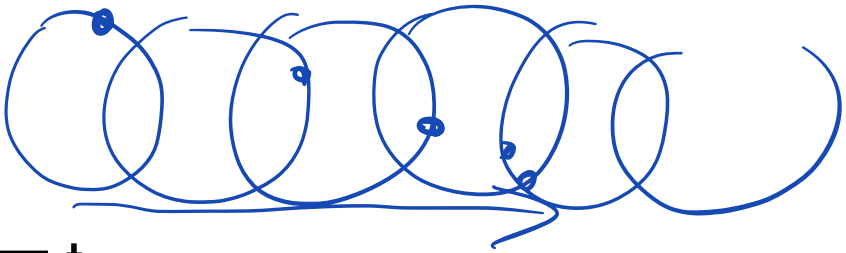
- Time-Dependent Sources.
- Electric power delivered in the form of periodic voltages and currents.
- Alternative Current



Sinusoidal Signals



$$v_{AC} = A\cos(\omega t + \varphi)$$



A = amplitude [positive]

v_{pp} = voltage peak to peak $2A$

T = period [seconds]

f = frequency $= \frac{1}{T}$ [Hz]

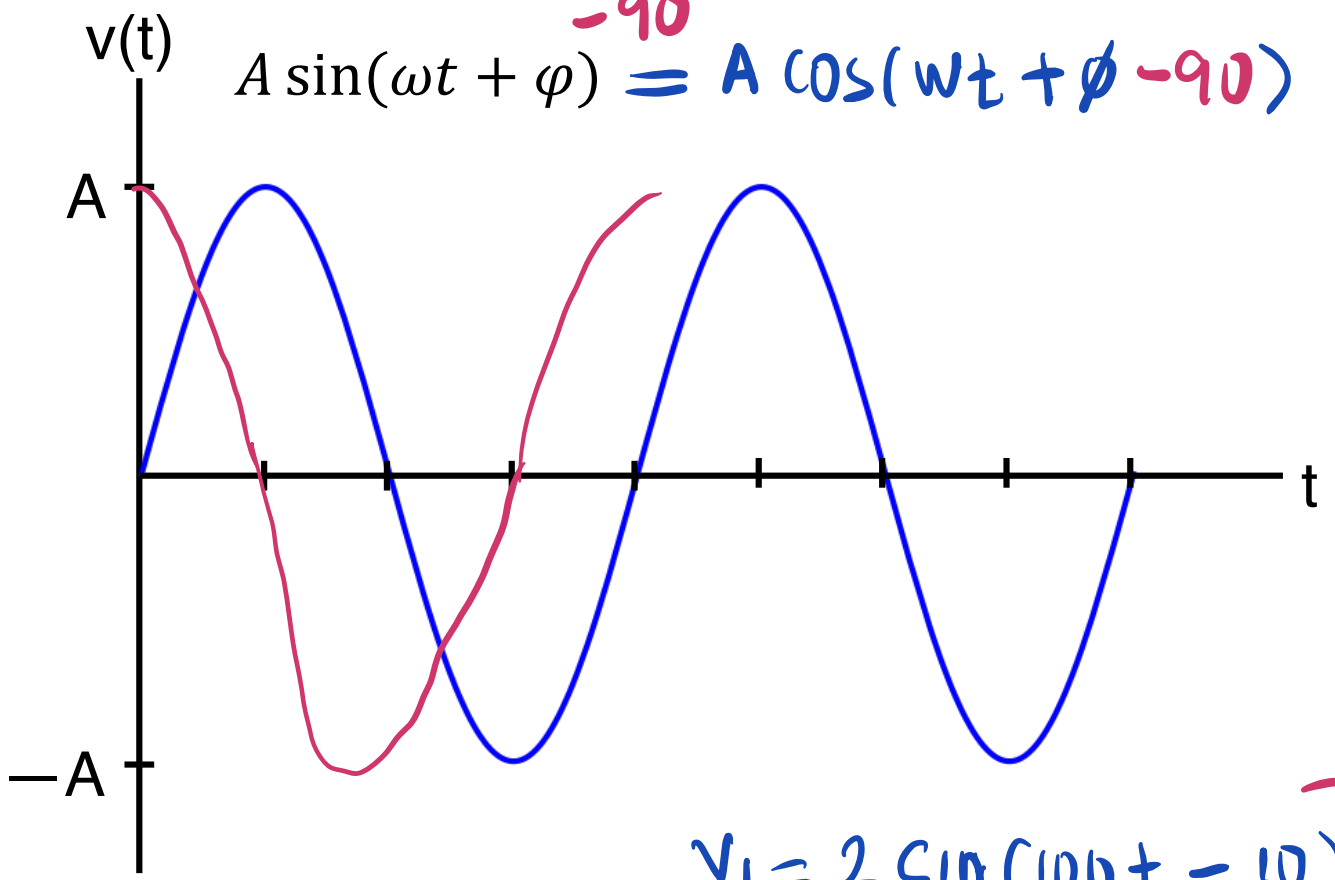
ω = angular frequency [rad/sec]

φ = phase shift [degrees]

[-180, 180]

+ −

Sinusoidal Signals



$A\sin(\omega t + \varphi) = A\cos(\omega t + \varphi - 90)$

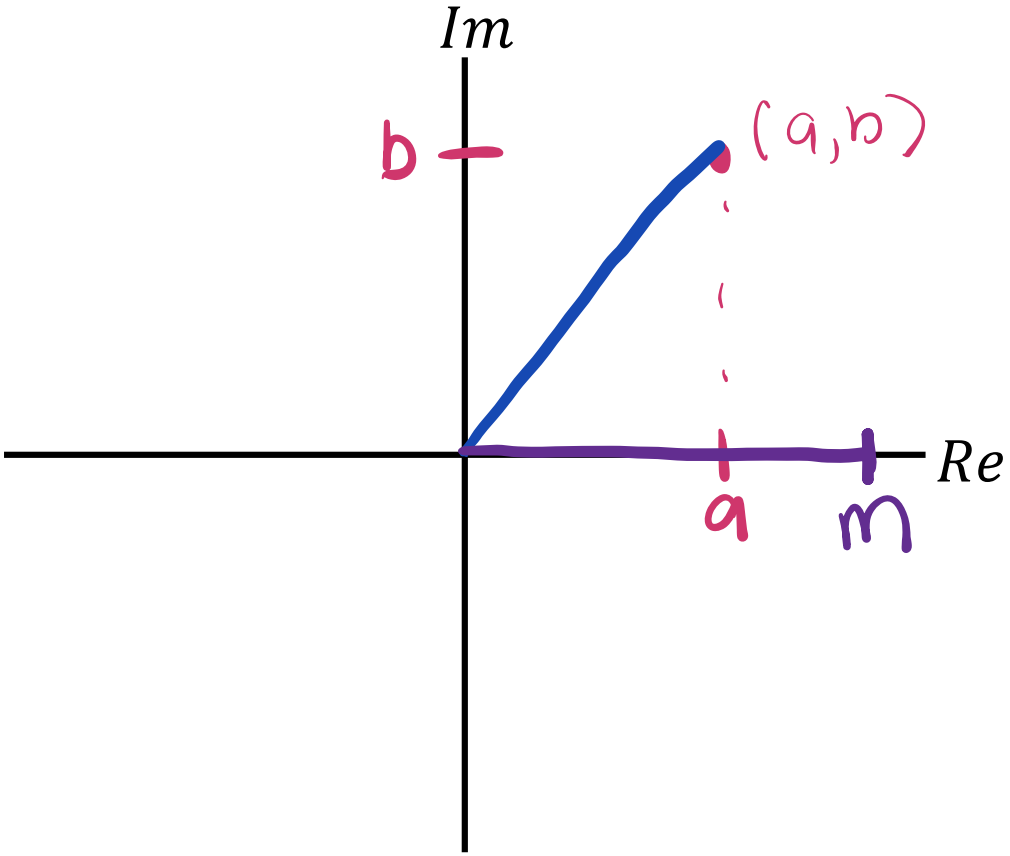
$$v_{AC} = A\cos(\omega t + \varphi)$$

$v_1 = 2\sin(100t - 10)$

$= 2\cos(100t - 100)$

Phase shift (φ) must be defined between $180°$ and $-180°$ degress.

Complex Numbers Review



Rectangular form

$$z = a + bj$$

$$\text{Real}\{z\} = a$$

$$\text{Imaginary}\{z\} = b.$$

Polar form

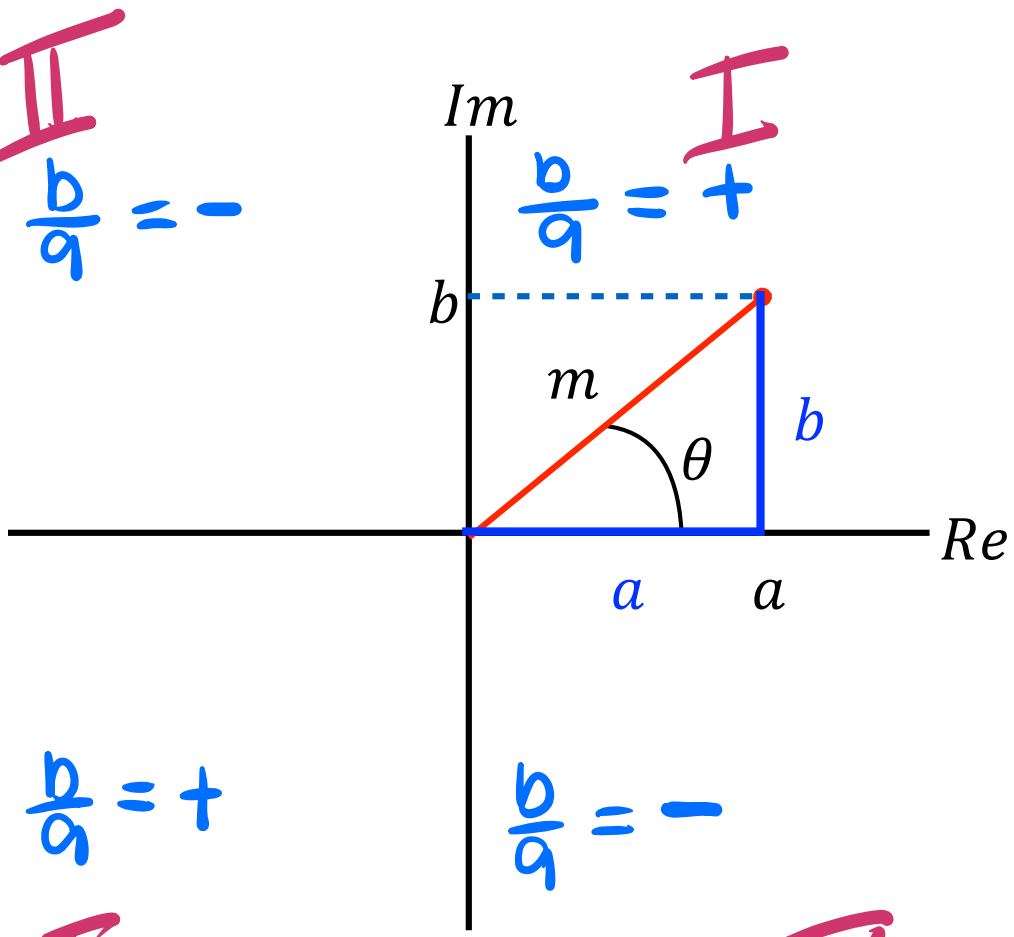
$$z = m \angle \theta$$

$$= m e^{\theta j}$$

$$\text{magnitude}\{z\} = m$$

$$\text{phase}\{z\} = \theta$$

Rectangular to Polar Form



Rectangular form

$$z = a + jb$$

magnitude:

$$m = \sqrt{a^2 + b^2}$$

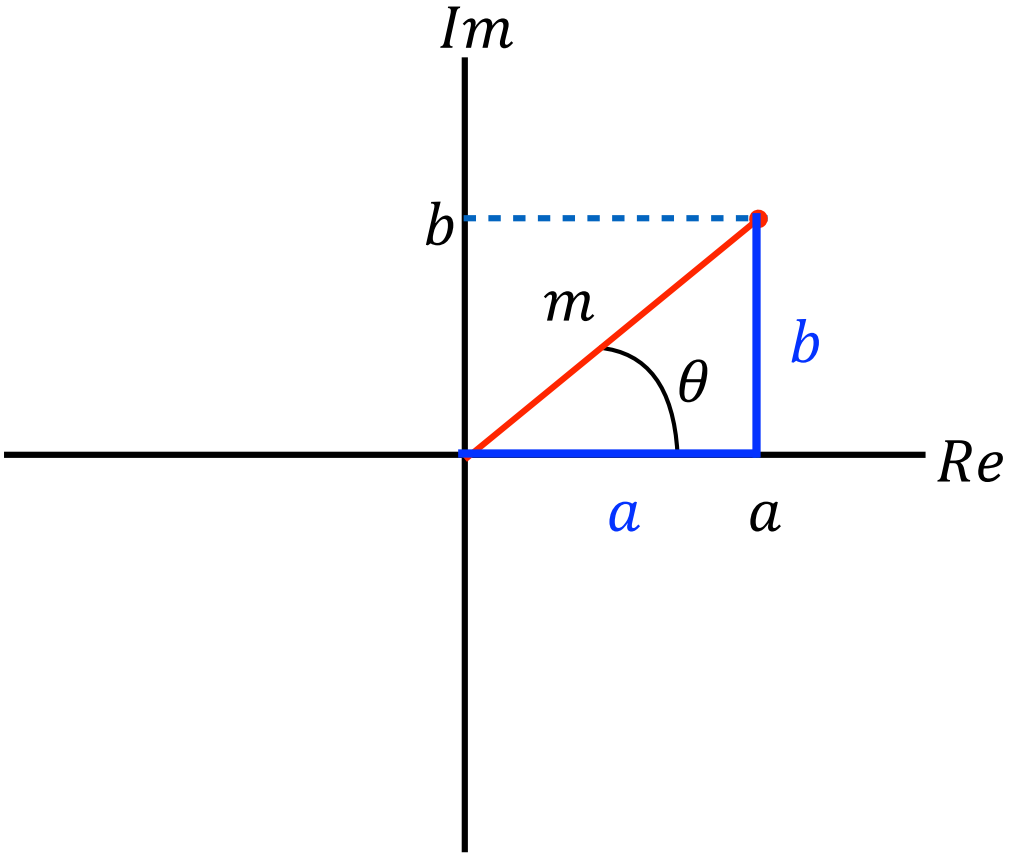
phase:

$$\tan(\theta) = \frac{b}{a}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) \pm 180$$

(+ → II, − → III)

Polar to Rectangular Form



Polar form

$$z = m\angle\theta = me^{j\theta}$$

Real

$$\cos(\theta) = \frac{a}{m}$$

$$a = m\cos(\theta)$$

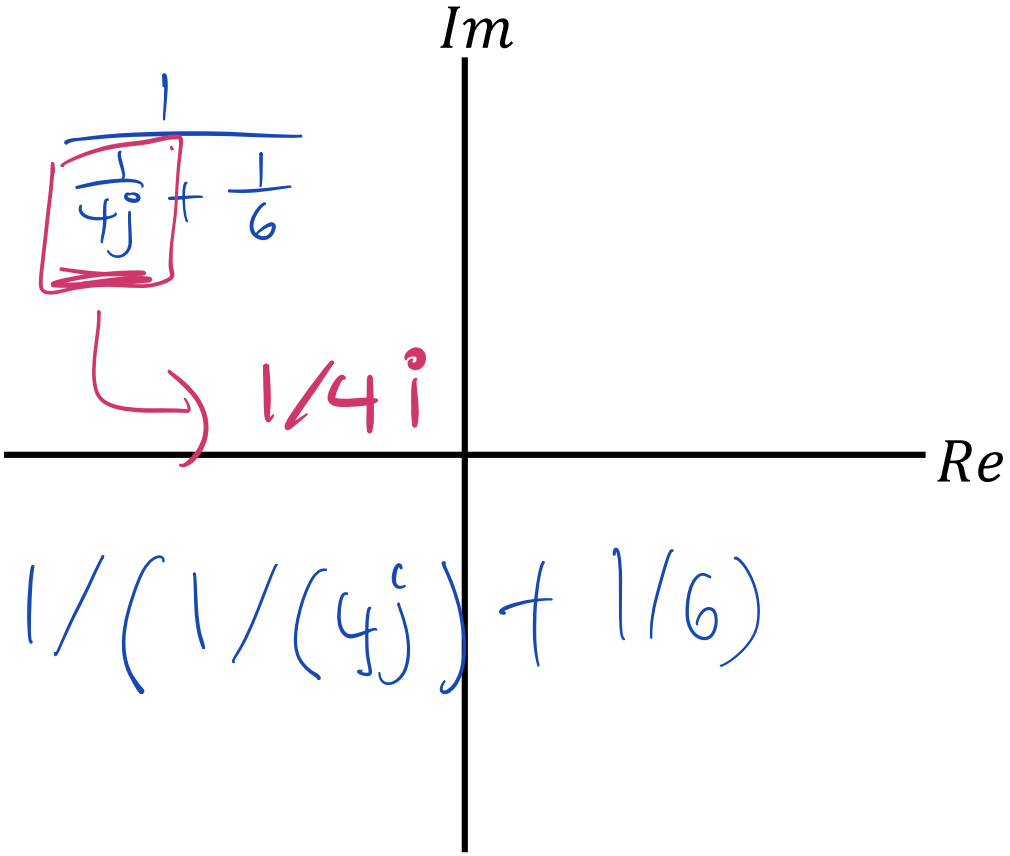
Imaginary

$$\sin(\theta) = \frac{b}{m}$$

$$b = m\sin(\theta)$$

Complex Numbers Review

$$\frac{1}{\frac{1}{4}+\frac{1}{6}}$$



$$\frac{1}{\frac{1}{4j}+\frac{1}{6}}$$

1/(1/(4j) + 1/6)

1/4 i = $\frac{1}{4}$ i

1/(4i) = $\frac{1}{4i}$

$$j = \sqrt{-1}$$

$$j^2 = -1$$

$$j^3 = j\,j^2 = -j$$

$$j^4 = j^2\,j^2 = (-1)(-1) = 1$$

$$j^{-1} = \frac{1}{j} \times \frac{j}{j} = \frac{j}{j^2} = -j$$

$$\frac{1}{100j} = \frac{1}{100}(-j) = \frac{-j}{100}$$

Complex Numbers Review

Rectangular form

$z_1 = 1 + j\,2 \qquad z_2 = 3 + j\,4$

Addition:

$z_1 + z_2 = (1+3) + (4j+2j)$

$= 4 + 6j$

$z_1 - z_2 = (1-3) + (2j-4j)$

$= -2 - 2j$

Polar form

$z_1 = 5e^{j30^\circ} \qquad z_2 = 2e^{-j15^\circ}$

Multiplication:

$z_1 z_2 = 5(2)\, e^{30j-15j}$

$= 10\, e^{15j}$

Division:

$\frac{z_1}{z_2} = \frac{5}{2}\, e^{30j-(-15j)}$

$= 2.5\, e^{45j}$