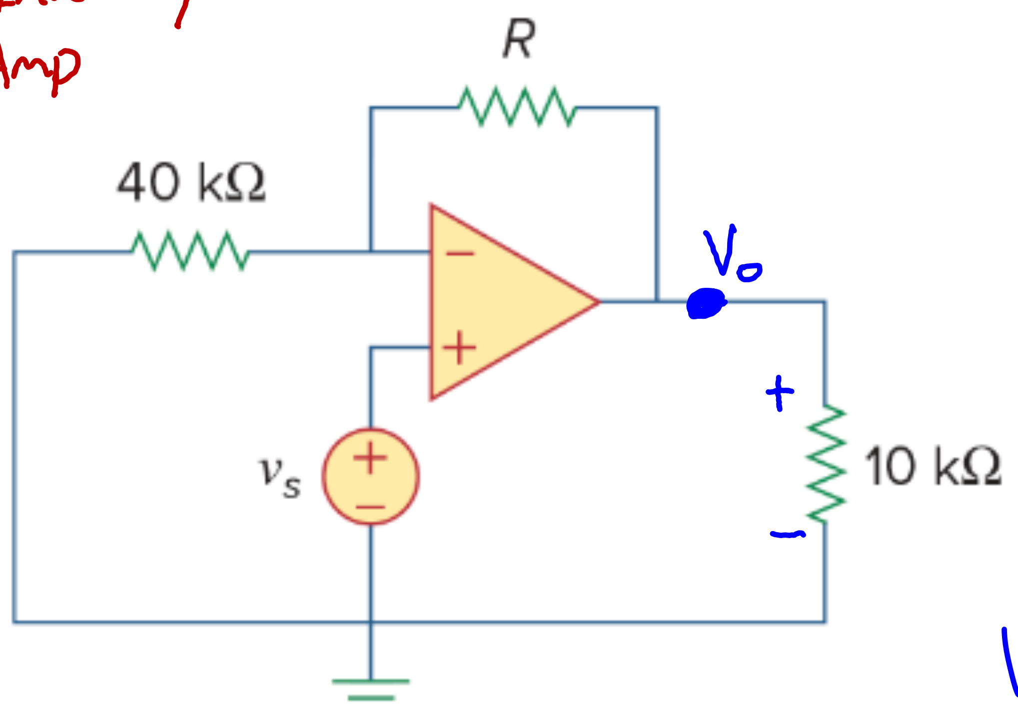
Last Class…

Find the value of R so that the power absorbed by the 10kΩ resistor is 10 mW. Assume vs = 2V.

↳ Op-amp output

Non-Inverting Op-Amp



$P = V_1 = \frac{V^2}{R}$

$10m = \frac{V_{10}^2}{10k}$

$100 = V_{10}^2$

$V_{10} = 10V = V_o$

$\frac{V_o}{V_s} = \frac{R_1 + R_2}{R_2}$

$\frac{10}{2} = \frac{R + 40k}{40k}$

$200k = R + 40k \rightarrow \boxed{R = 160k\Omega}$

THE OHIO STATE UNIVERSITY

COLLEGE OF ENGINEERING

Laplace Transform

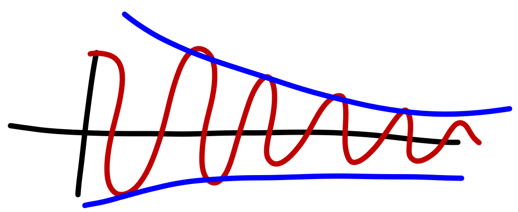
S-Domain

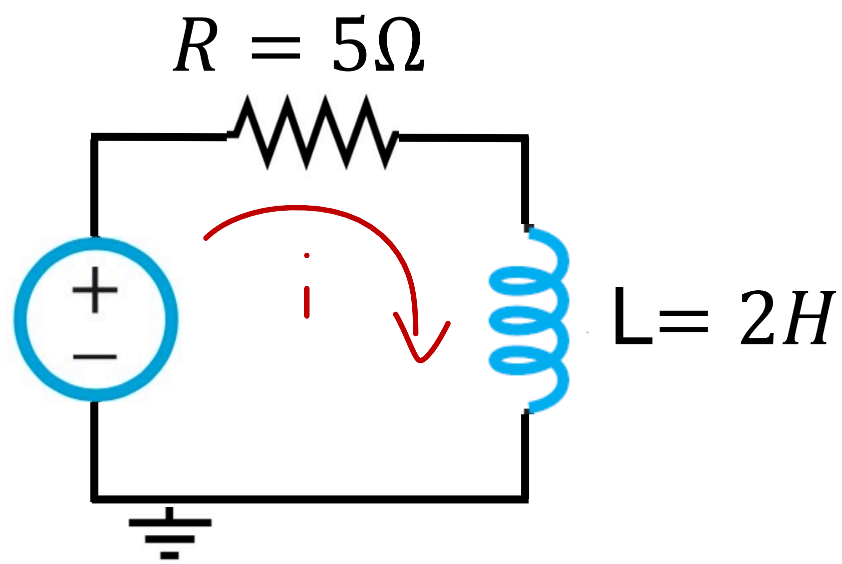
This Lecture…

- Learning Objectives:
    - Review impulse and pulse signal.
    - Compute the Laplace transform of a time-dependent function.

Damped Sinusoid

$v_s(t) = 25\, e^{-t} cos(2t)$





KVL.

$V_s(t) = V_R + V_L$

$V_s(t) = i_R + L\frac{di}{dt}$

diff eqns

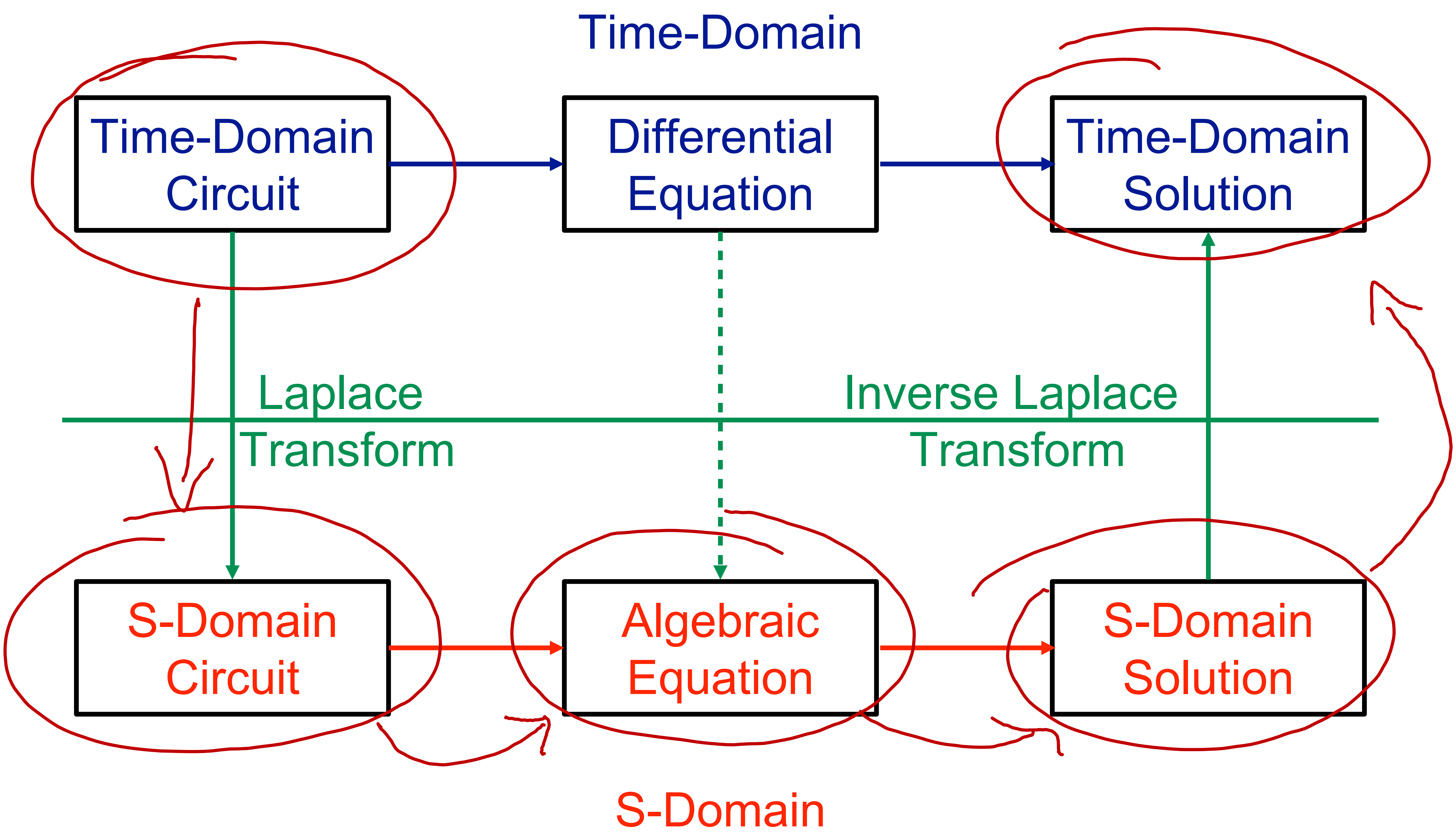
On time-domain:

$V_s(t) = Ae^{\sigma t}\cos(\omega t + \phi)$

↳ $\sigma = 0 \Rightarrow V_s(t) = A\cos(\omega t + \phi) \leftarrow AC$

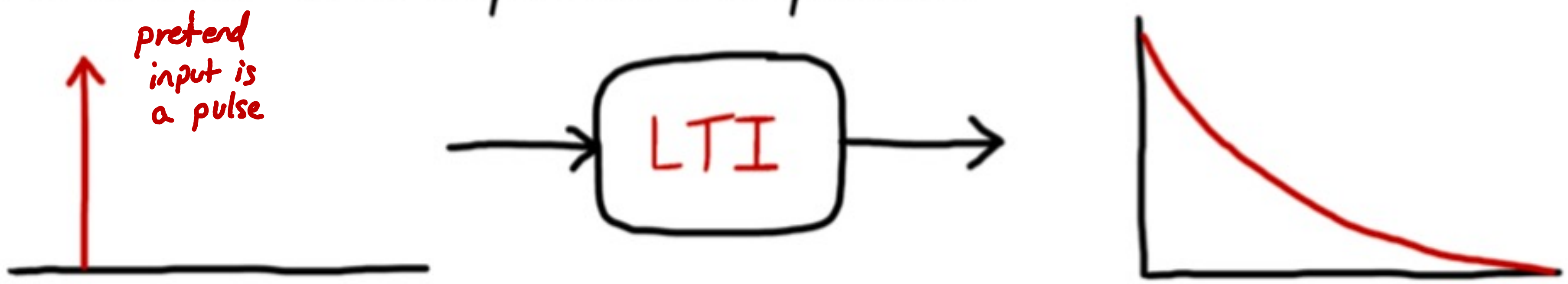
↳ $\omega = 0 \Rightarrow V_s(t) = Ae^{\sigma t}\cos(\phi)$

↳ $\sigma = 0,\ \omega = 0 \Rightarrow V_s(t) = A\cos(\phi) \leftarrow DC$

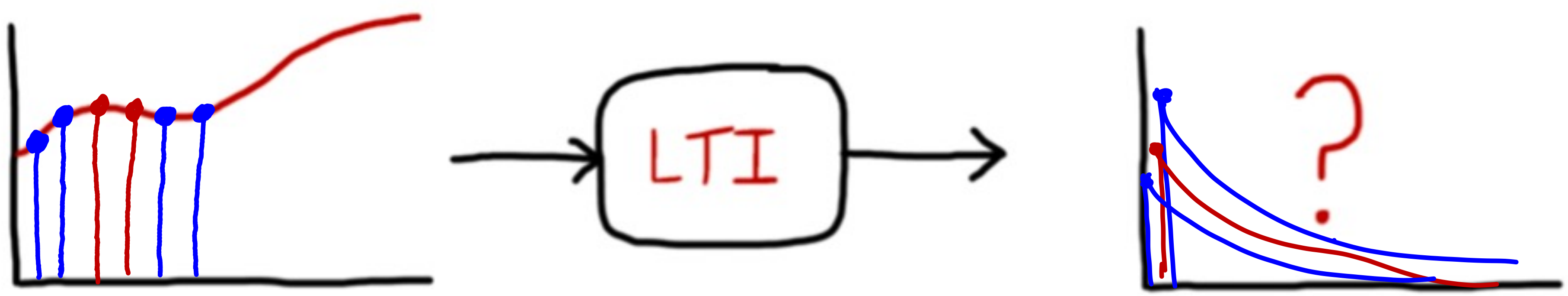

Time-Domain
Time-Domain Circuit
Differential Equation
Time-Domain Solution
Laplace Transform
Inverse Laplace Transform
S-Domain Circuit
Algebraic Equation
S-Domain Solution
S-Domain

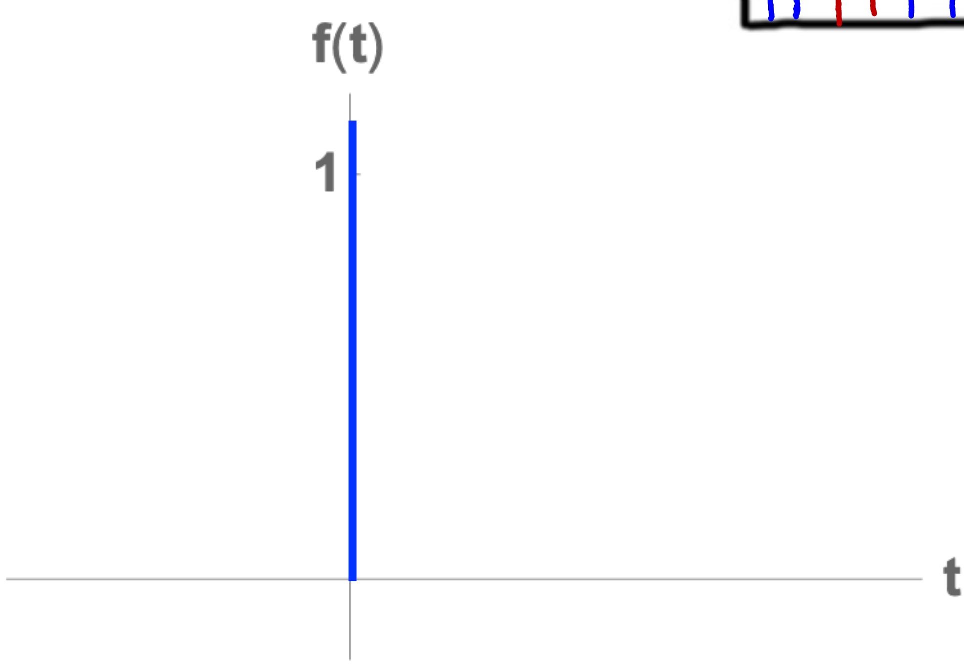
Delta Function

if we know the impulse response



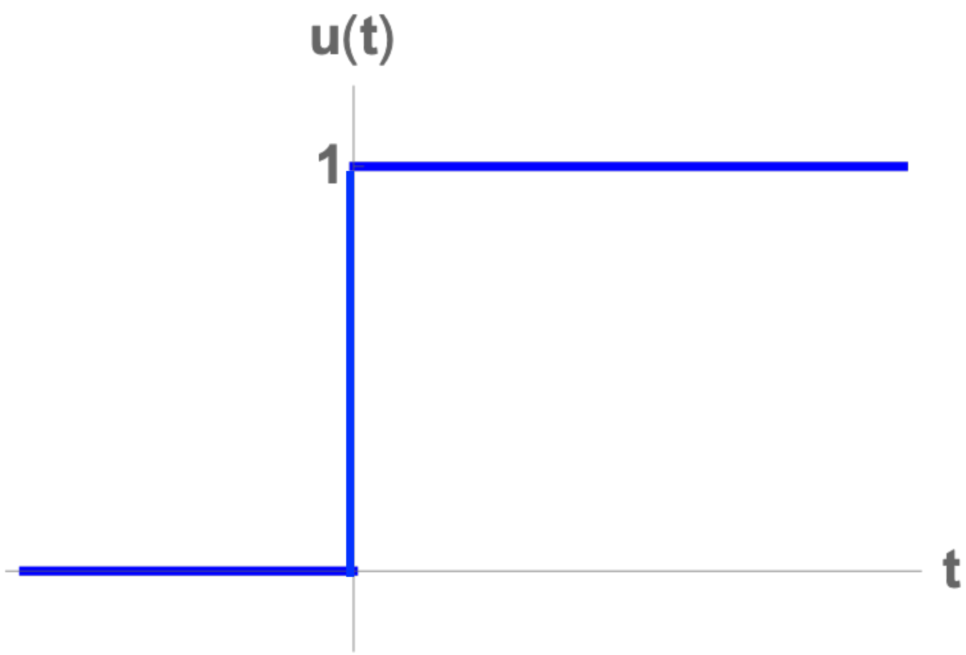
what can we say about an arbitrary input?



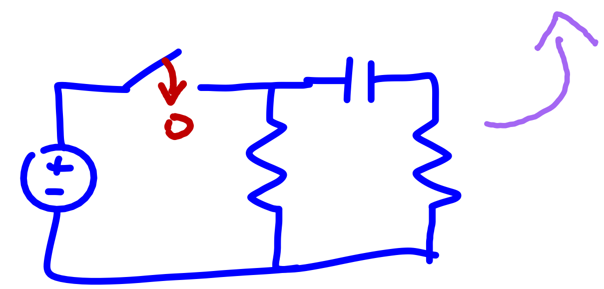


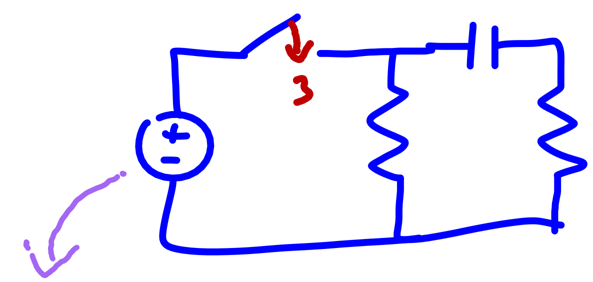
$$d(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

Unit Step Function



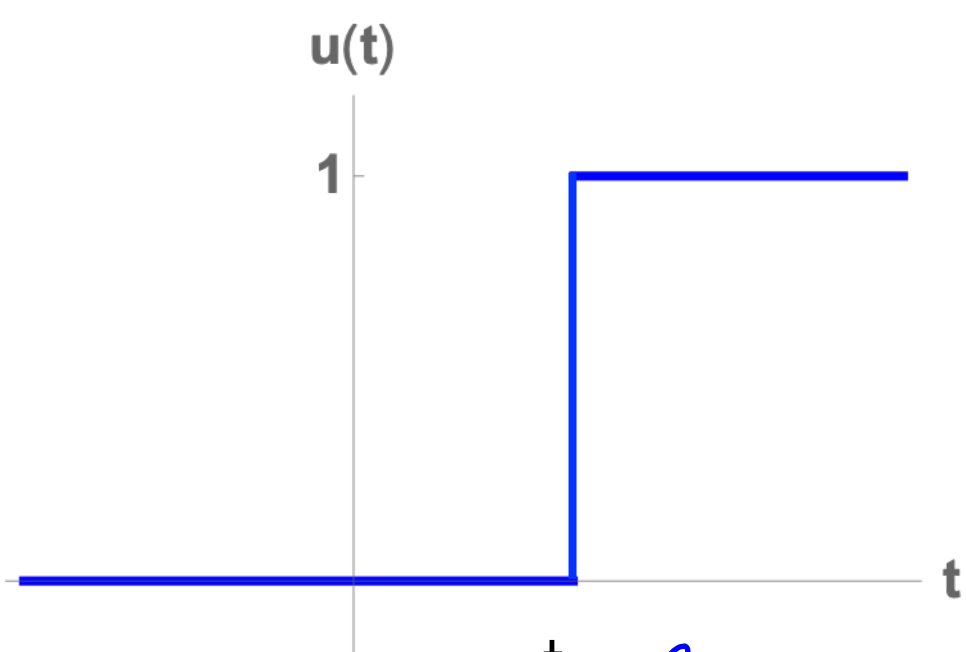
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$





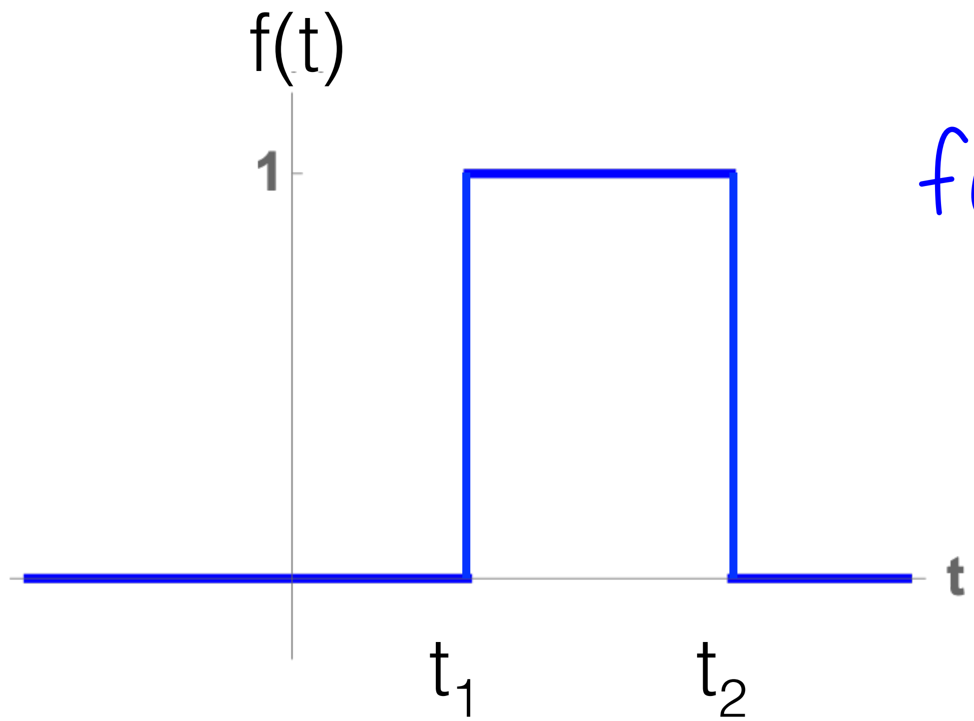
Use to model switches in circuits.

$$u(t-3) = \begin{cases} 1 & t > 3 \\ 0 & t < 3 \end{cases}$$

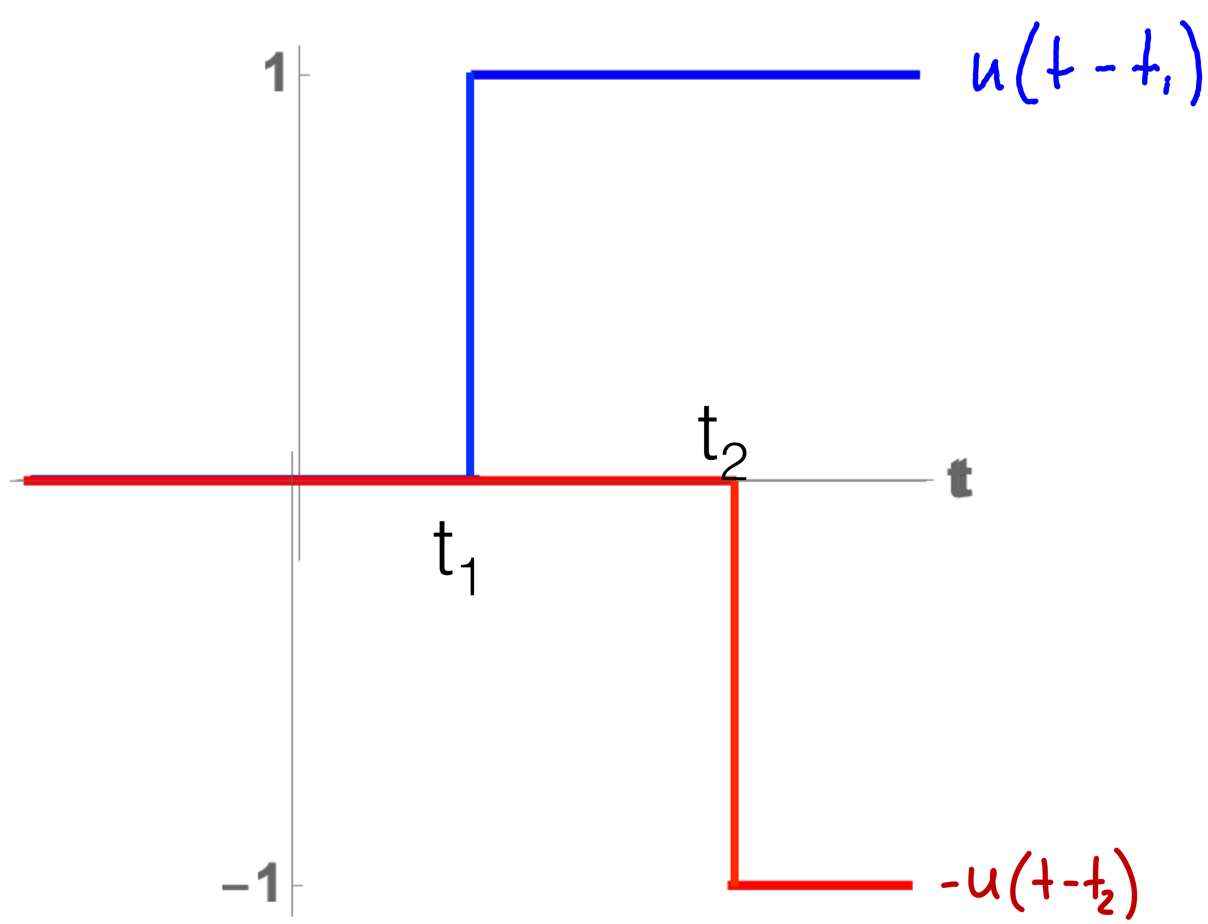


$t_0 = 3$

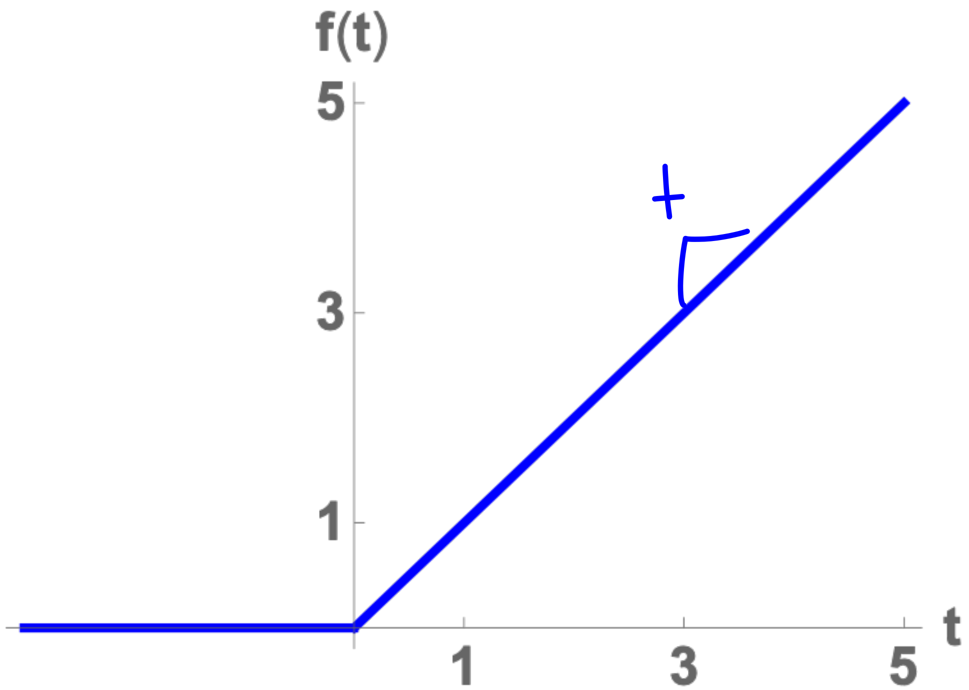
Rectangular Pulse



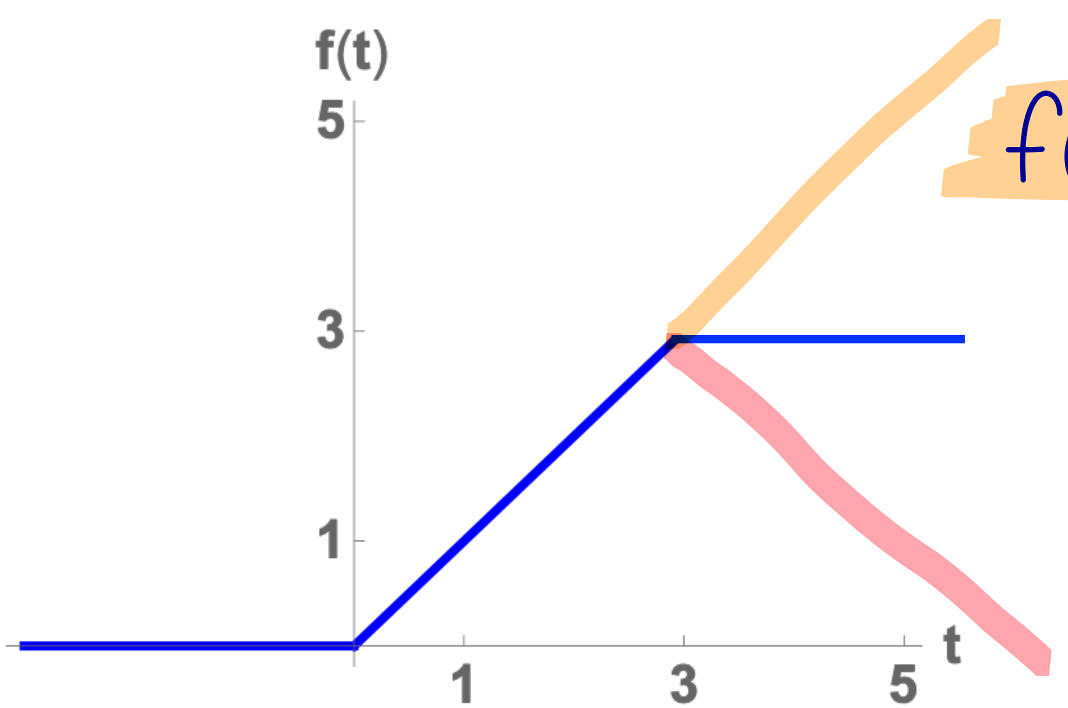
$f(t) = u(t-t_1) - u(t-t_2)$



Ramp Function



$r(t) = t\,u(t)$



$f(t) = t\,u(t)$

$f(t) = t\,u(t) - (t-3)\,u(t-3)$

Laplace transform Definition

$$\mathcal{L}\{f(t)\} = \boldsymbol{F}(s) \qquad\qquad f(t) \leftrightarrow \boldsymbol{F}(s)$$

$\underline{s = \sigma} + j\omega$ represents the Laplace variable.

Complex Frequency

$$\boldsymbol{F}(s) = \int_0^\infty f(t)e^{-st}\,dt$$

Time domain
↳ s Domain

- Rule for convergence: magnitude of $\boldsymbol{F}(s)$ must be finite.

Unit Step Function: $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

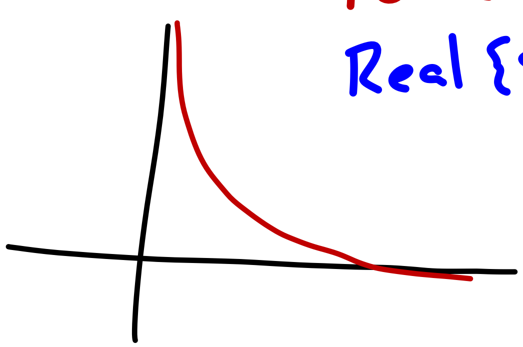
Laplace Transform: $\boldsymbol{U}(s) = \int_0^{\infty} u(t) e^{-st}\, dt = \int_0^{\infty} 1 e^{-st}\, dt$

$$= \left. \frac{e^{-st}}{-s} \right|_0^{\infty}$$

$$= \frac{e^{-s\infty}}{-s} - \frac{e^{0}}{-s}$$

For convergence

Real{s}>0



$\boxed{\text{See the tables}}$

Property	$f(t)$		$\mathbf{F}(s) = \mathcal{L}[f(t)]$
1. Multiplication by constant	$K\ f(t)$	$\longleftrightarrow$	$K\ \mathbf{F}(s)$
2. Linearity	$K_1\ f_1(t) + K_2\ f_2(t)$	$\longleftrightarrow$	$K_1\ \mathbf{F}_1(s) + K_2\ \mathbf{F}_2(s)$
3. Time scaling	$f(at), \quad a > 0$	$\longleftrightarrow$	$\frac{1}{a}\mathbf{F}\left(\frac{s}{a}\right)$
4. Time shift	$f(t-T)\ u(t-T)$	$\longleftrightarrow$	$e^{-Ts}\ \mathbf{F}(s)$
5. Frequency shift	$e^{-at}\ f(t)$	$\longleftrightarrow$	$\mathbf{F}(s+a)$
6. Time 1st derivative	$f' = \frac{df}{dt}$	$\longleftrightarrow$	$s\ \mathbf{F}(s) - f(0^-)$
7. Time 2nd derivative	$f'' = \frac{d^2 f}{dt^2}$	$\longleftrightarrow$	$s^2\mathbf{F}(s) - s f(0^-) - f'(0^-)$
8. Time integral	$\int_0^t f(t)\ dt$	$\longleftrightarrow$	$\frac{1}{s}\ \mathbf{F}(s)$
9. Frequency derivative	$t\ f(t)$	$\longleftrightarrow$	$-\frac{d}{ds}\mathbf{F}(s) = -\mathbf{F}'(s)$
10. Frequency integral	$\frac{f(t)}{t}$	$\longleftrightarrow$	$\int_s^\infty \mathbf{F}(s)\ ds$
11. Initial value	$f(0^+)$	$=$	$\lim_{s\to\infty} s\ \mathbf{F}(s)$
12. Final value	$f(\infty)$	$=$	$\lim_{s\to 0} s\ \mathbf{F}(s)$
13. Convolution	$f_1(t) * f_2(t)$	$\longleftrightarrow$	$\mathbf{F}_1(s)\ \mathbf{F}_2(s)$

$5u(t)$

$$5\,u(t) \downarrow 5\frac{1}{s} = \frac{5}{s}$$

Laplace Transform Pairs

	$f(t)$		$\mathbf{F}(\mathbf{s}) = \mathcal{L}[f(t)]$
1	$\delta(t)$	$\leftrightarrow$	1
1a	$\delta(t-T)$	$\leftrightarrow$	$e^{-T\mathbf{s}}$
2	1 or $u(t)$	$\leftrightarrow$	$\frac{1}{\mathbf{s}}$
2a	$u(t-T)$	$\leftrightarrow$	$\frac{e^{-T\mathbf{s}}}{\mathbf{s}}$
3	$e^{-at}\,u(t)$	$\leftrightarrow$	$\frac{1}{\mathbf{s}+a}$
3a	$e^{-a(t-T)}\,u(t-T)$	$\leftrightarrow$	$\frac{e^{-T\mathbf{s}}}{\mathbf{s}+a}$
4	$t\,u(t)$	$\leftrightarrow$	$\frac{1}{\mathbf{s}^2}$
4a	$(t-T)\,u(t-T)$	$\leftrightarrow$	$\frac{e^{-T\mathbf{s}}}{\mathbf{s}^2}$
5	$t^2\,u(t)$	$\leftrightarrow$	$\frac{2}{\mathbf{s}^3}$
6	$te^{-at}\,u(t)$	$\leftrightarrow$	$\frac{1}{(\mathbf{s}+a)^2}$
7	$t^2e^{-at}\,u(t)$	$\leftrightarrow$	$\frac{2}{(\mathbf{s}+a)^3}$
8	$t^{n-1}e^{-at}\,u(t)$	$\leftrightarrow$	$\frac{(n-1)!}{(\mathbf{s}+a)^n}$
9	$\sin\omega t\,u(t)$	$\leftrightarrow$	$\frac{\omega}{\mathbf{s}^2+\omega^2}$
10	$\sin(\omega t+\theta)\,u(t)$	$\leftrightarrow$	$\frac{\mathbf{s}\sin\theta+\omega\cos\theta}{\mathbf{s}^2+\omega^2}$
11	$\cos\omega t\,u(t)$	$\leftrightarrow$	$\frac{\mathbf{s}}{\mathbf{s}^2+\omega^2}$
12	$\cos(\omega t+\theta)\,u(t)$	$\leftrightarrow$	$\frac{\mathbf{s}\cos\theta-\omega\sin\theta}{\mathbf{s}^2+\omega^2}$
13	$e^{-at}\sin\omega t\,u(t)$	$\leftrightarrow$	$\frac{\omega}{(\mathbf{s}+a)^2+\omega^2}$
14	$e^{-at}\cos\omega t\,u(t)$	$\leftrightarrow$	$\frac{\mathbf{s}+a}{(\mathbf{s}+a)^2+\omega^2}$
15	$2e^{-at}\cos(bt-\theta)\,u(t)$	$\leftrightarrow$	$\frac{e^{j\theta}}{\mathbf{s}+a+jb}+\frac{e^{-j\theta}}{\mathbf{s}+a-jb}$
16	$\frac{2t^{n-1}}{(n-1)!}e^{-at}\cos(bt-\theta)\,u(t)$	$\leftrightarrow$	$\frac{e^{j\theta}}{(\mathbf{s}+a+jb)^n}+\frac{e^{-j\theta}}{(\mathbf{s}+a-jb)^n}$