



# Lecture Outline

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## Reminders to self:

- ☐ Turn on lecture recording to Cloud
- ☐ Turn on Zoom microphone

## • Last Lecture

- Finished Analysis of a Mealy machine by Transition Tables & State Graphs
- Timing charts from State Tables & Graphs
- General models for clocked sequential circuits
- Started design of clocked sequential circuits

## • Today's Lecture

- Continue analysis & design of clocked sequential circuits
  - Finish Mealy 101 sequence detector design started last lecture
  - Analysis example (of Mealy sequence detector just designed)
  - State Graph design guidelines
  - Start a larger (8 state) design example (Mustang turn signals)



# Handouts and Announcements

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- Announcements
  - Homework Problems: HW 13-2
    - Posted on Carmen yesterday morning
    - Due: 11:59pm Thursday 3/30
  - Homework Reminder
    - HW 13-1 Due: 11:25am Wednesday 3/29
  - Read for Monday: no new reading assignment  
previous assignment pages 463-472, 149-151
  - Participation Quiz 11 available 11:15am today
    - Due 11:15am tomorrow
    - Available additional 24hr with late penalty



# Handouts and Announcements

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- Announcements

- Mini-Exam 5 Reminder

- Available 5pm Monday 3/27 through 5:00pm Tuesday 3/28
- Due in Carmen PROMPTLY at 5:00pm on 3/28
- Designed to be completed in ~36 min, but you may use more
- When planning your schedule:
  - I recommend building in 10-15 min extra
  - To allow for downloading exam, signing and dating honor pledge, saving solution as pdf, and uploading to Carmen
- I also recommend not procrastinating

- Exam review topics available on Carmen

- Sample Mini-Exams 6 and 7 from Au20 also available



## Mealy Design Example: Sequence Detector

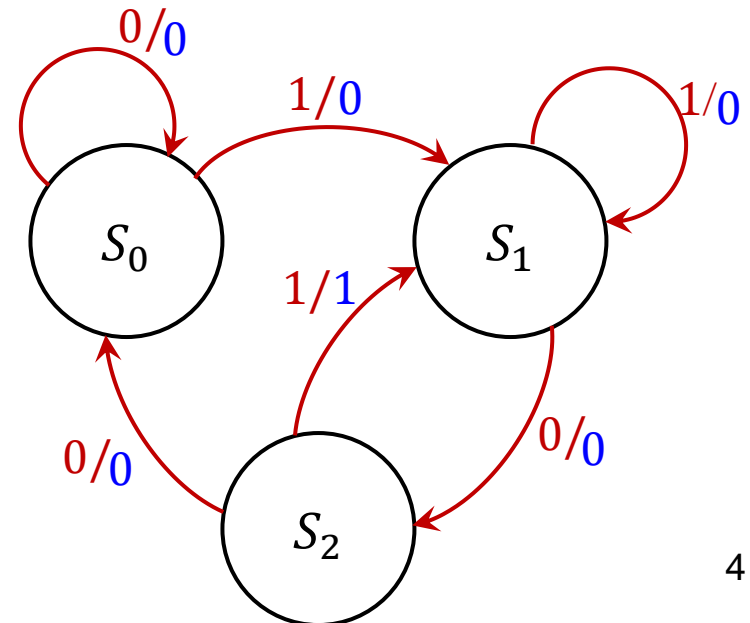
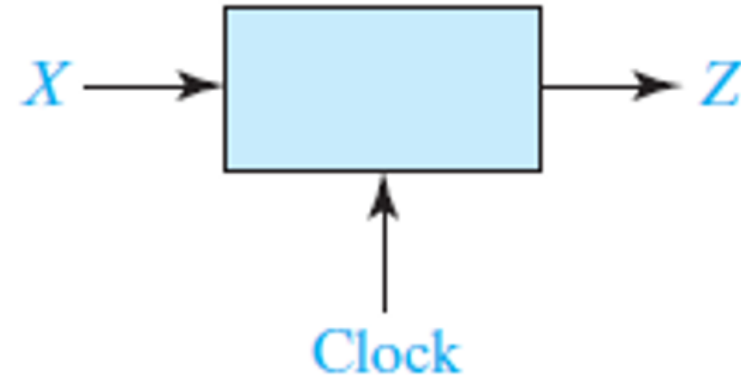
- Sequence Detector Description:

- Circuit that

- Examines a serial string of 0's and 1's applied to the  $X$  input
    - Generates an output  $Z = 1$  only when a prescribed input sequence occurs

- For this example, the prescribed input sequence is 101

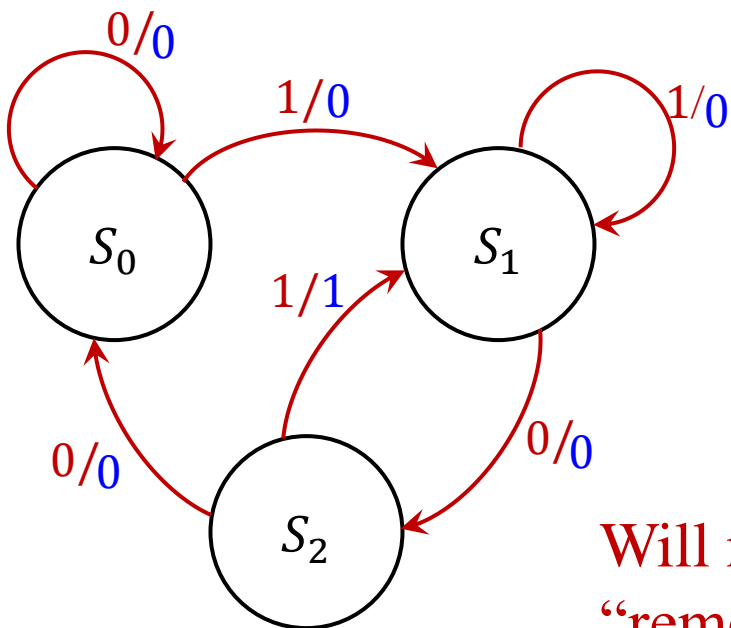
## Block Diagram



Ran out of time last lecture just after converting word description into state graph →



## Mealy Design Example: Sequence Detector



The State Table can be created from the State Graph

Present State	Next State		Present Output	
	X = 0	X = 1	X = 0	X = 1
S <sub>0</sub>	S <sub>0</sub>	S <sub>1</sub>	0	0
S <sub>1</sub>	S <sub>2</sub>	S <sub>1</sub>	0	0
S <sub>2</sub>	S <sub>0</sub>	S <sub>1</sub>	0	1

Will need two flip-flops to “remember” three states

The Transition Table can be created from the State Table

AB	A+B+		Z	
	X = 0	X = 1	X = 0	X = 1
00	00	01	0	0
01	10	01	0	0
10	00	01	0	1

Remember, output is present as soon as X changes. Present before state change at active clock edge.



## Mealy Design Example: Sequence Detector

D Flip-Flop Next-State Maps and the Output Map can be created from the Transition Table

Transition Table

AB	$A^+B^+$		Z	
	X = 0	X = 1	X = 0	X = 1
00	00	01	0	0
01	10	01	0	0
10	00	01	0	1

AB \ X	0	1
AB		
00	0	0
01	1	0
11	X	X
10	0	0

$$A^+ = X'B$$

AB \ X	0	1
AB		
00	0	1
01	0	1
11	X	X
10	0	1

$$B^+ = X$$

AB \ X	0	1
AB		
00	0	0
01	0	0
11	X	X
10	0	1

$$Z = XA$$

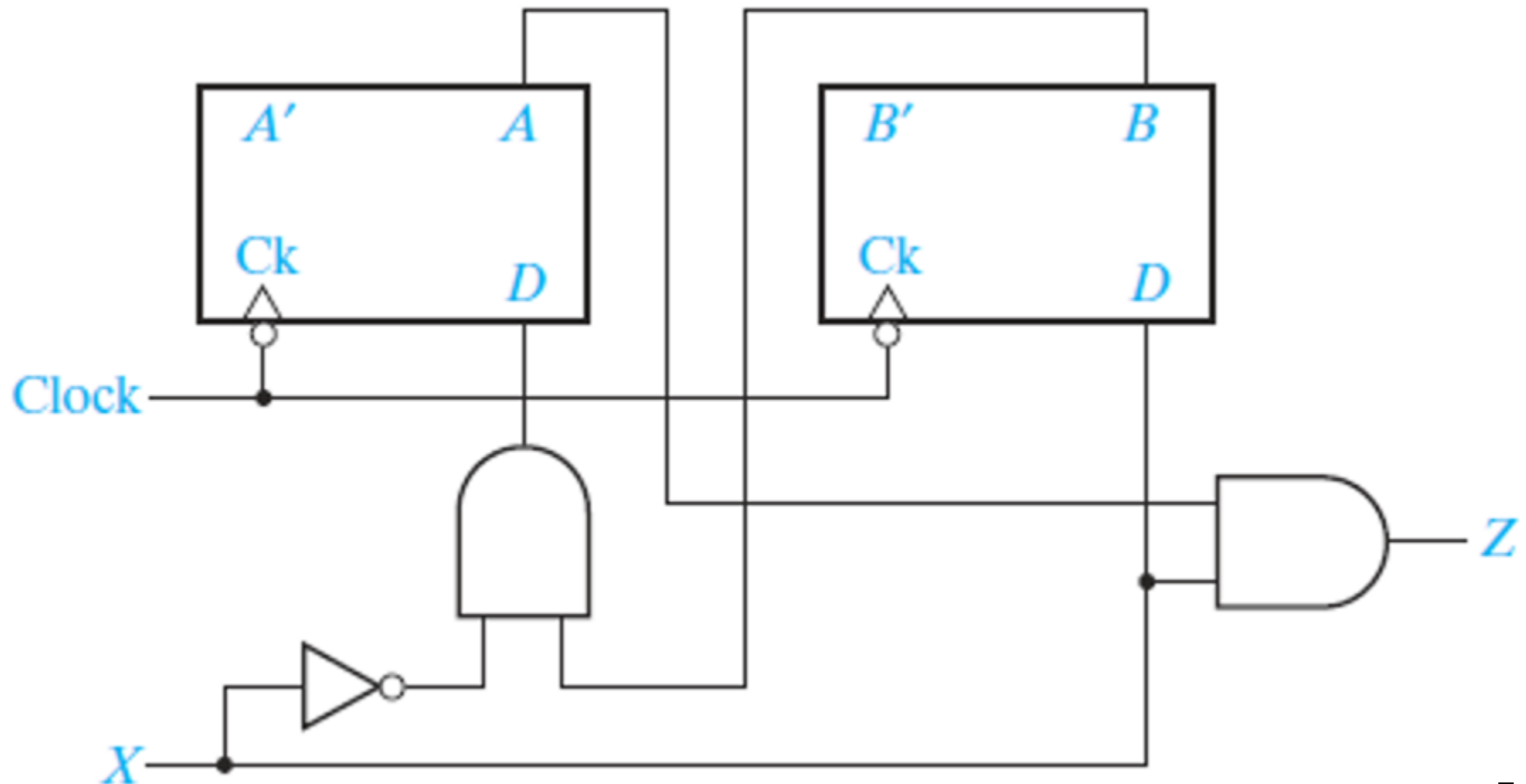


## Mealy Design Example: Sequence Detector

$$A^+ = X'B$$

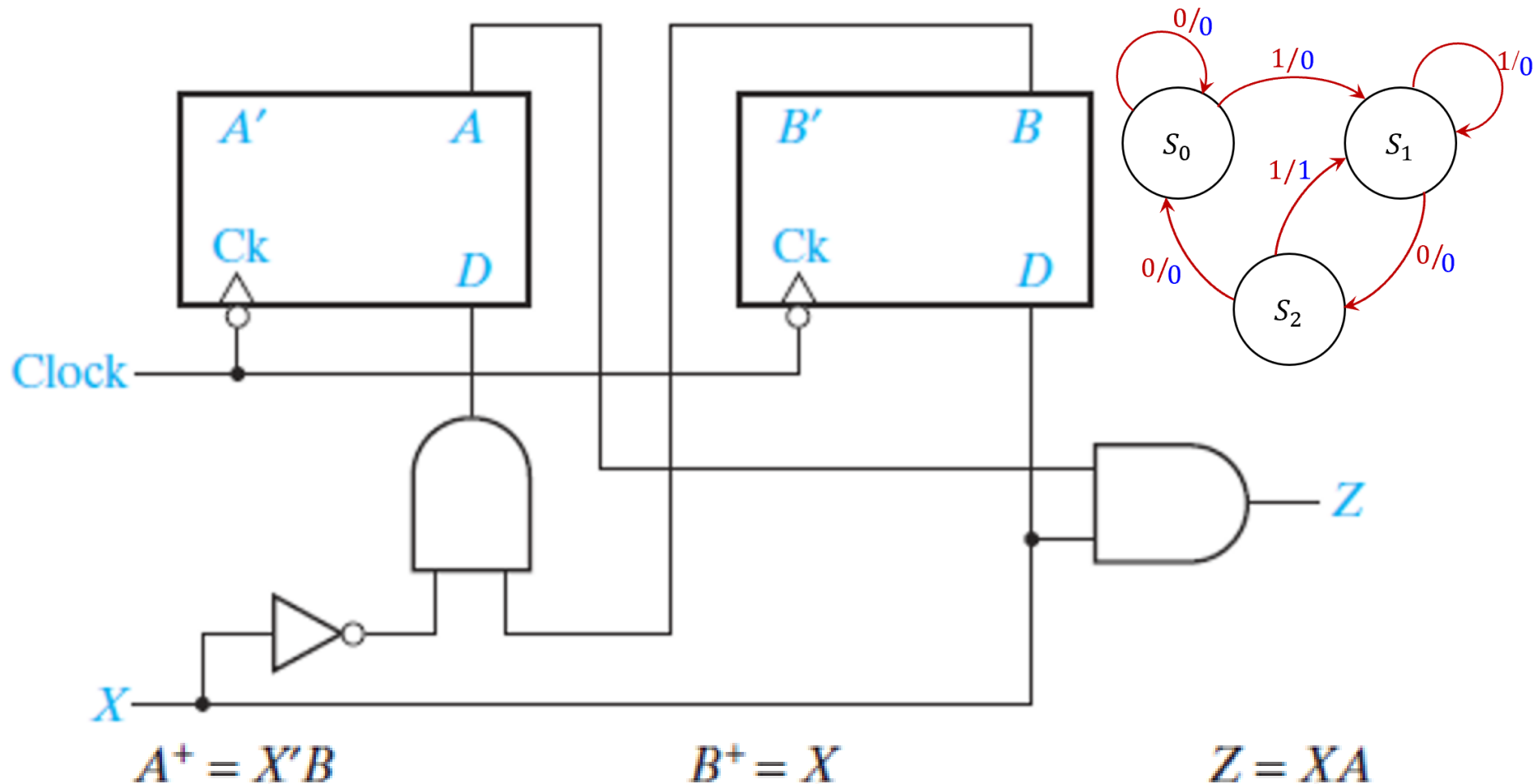
$$B^+ = X$$

$$Z = XA$$





## Mealy Design Example: Sequence Detector



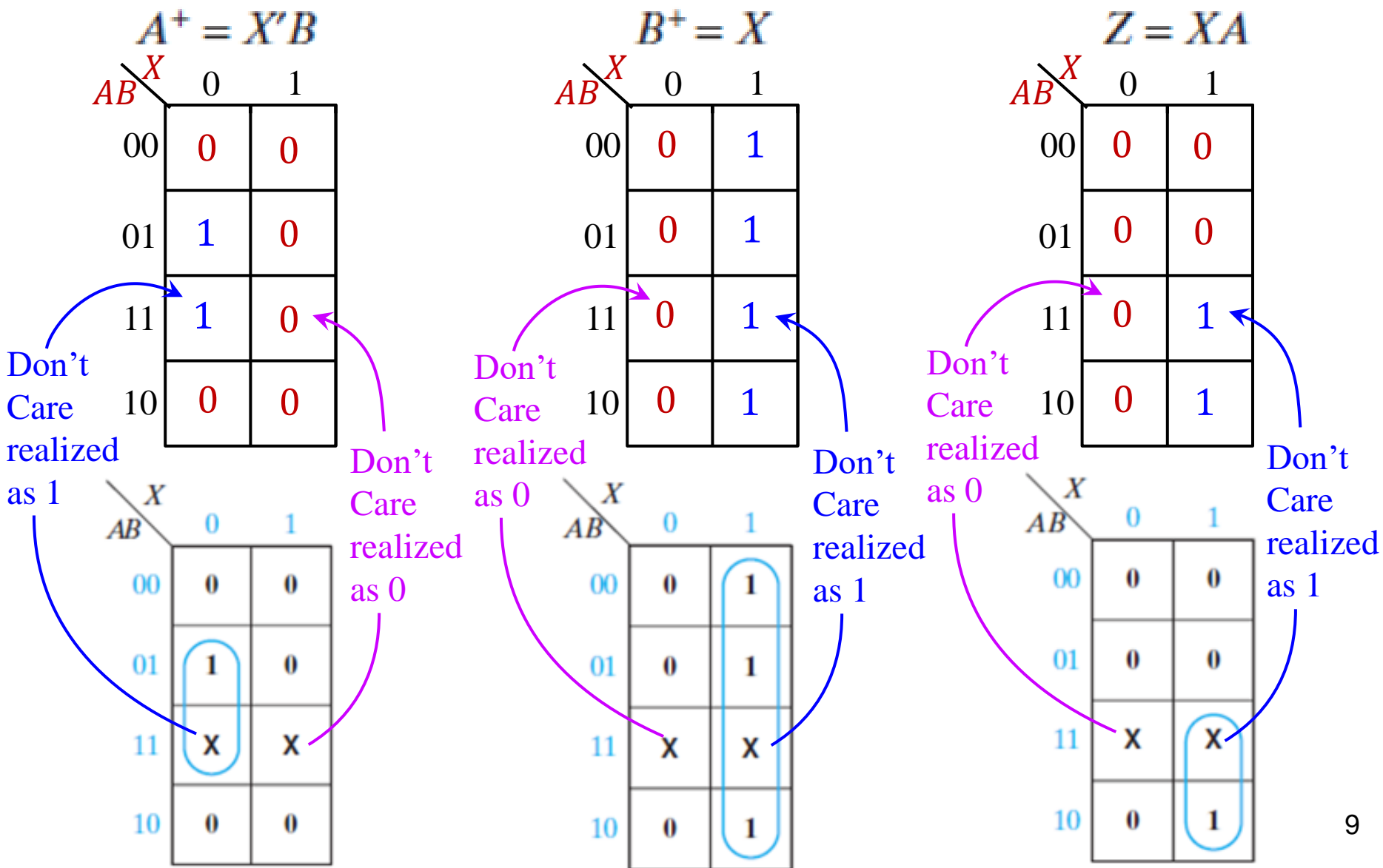
Sequence detector for “101” designed as a 3-state machine last lecture

- But with two flip-flops circuit realizes 4 states
- Let's analyze and compare to design goals at each step





## Mealy Analysis Example: Sequence Detector



Mealy **Analysis** Example: Sequence Detector
$$A^+ = X'B$$

$\begin{matrix} X \\ AB \end{matrix}$	0	1
00	0	0
01	1	0
11	1	0
10	0	0

$$B^+ = X$$

$\begin{matrix} X \\ AB \end{matrix}$	0	1
00	0	1
01	0	1
11	0	1
10	0	1

$$Z = XA$$

$\begin{matrix} X \\ AB \end{matrix}$	0	1
00	0	0
01	0	0
11	0	1
10	0	1

AB	$A^+B^+$		Present Z	
	X = 0	X = 1	X = 0	X = 1
00	00	01	0	0
01	10	01	0	0
11	10	01	0	1
10	00	01	0	1

AB	$A^+B^+$		Z	
	X = 0	X = 1	X = 0	X = 1
00	00	01	0	0
01	10	01	0	0
10	00	01	0	1
11	xx	xx	x	x

Mealy **Analysis** Example: Sequence Detector

AB	$A^+B^+$		Present Z	
	X = 0	X = 1	X = 0	X = 1
00	00	01	0	0
01	10	01	0	0
11	10	01	0	1
10	00	01	0	1

AB	$A^+B^+$		Present Z	
	X = 0	X = 1	X = 0	X = 1
$S_0$	$S_0$	$S_1$	0	0
$S_1$	$S_2$	$S_1$	0	0
$S_3$	$S_2$	$S_1$	0	1
$S_2$	$S_0$	$S_1$	0	1

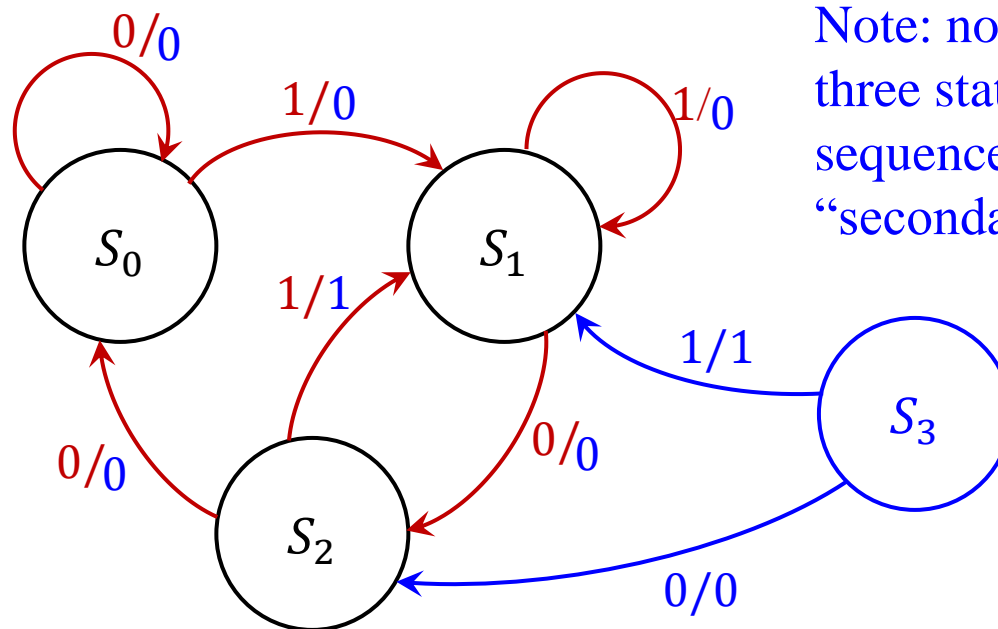
Present State	Next State		Present Output	
	X = 0	X = 1	X = 0	X = 1
$S_0$	$S_0$	$S_1$	0	0
$S_1$	$S_2$	$S_1$	0	0
$S_2$	$S_0$	$S_1$	0	1



# Mealy Analysis Example: Sequence Detector

AB	$A^+B^+$		Present Z	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
$S_0$	$S_0$	$S_1$	0	0
$S_1$	$S_2$	$S_1$	0	0
$S_3$	$S_2$	$S_1$	0	1
$S_2$	$S_0$	$S_1$	0	1

Present State	Next State		Present Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
$S_0$	$S_0$	$S_1$	0	0
$S_1$	$S_2$	$S_1$	0	0
$S_2$	$S_0$	$S_1$	0	1



Note: no transitions from three states needed for sequence detector into “secondary” state  $S_3$



## State Graph Guidelines

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Note that these are guidelines (helpful steps, not “The Law”)

1. Start by identifying sample input and output sequences. Doing this also helps you understand the problem statement.
2. Determine an initial state and any condition that causes a reset to that state (if there are any)
3. If the output is mostly zero, identify the few states that cause non-zero output and start with those (partial state graph)
4. Another way to start is to determine sequences or groups of sequences that must be remembered by the circuit, and set up states for them
5. Can transition arrows go to existing states? Add a new state only when you really have to.
6. Once graph is complete, make sure each input combination leaves each state only once
7. Test graph using input-output combinations found in step (1)



## State Graph Guidelines

## 101sequence detector

Note that these are guidelines (helpful steps, not “The Law”)

1. Start by identifying sample input and output sequences. Doing this also helps you understand the problem statement.

7 8 9 10

Desired sequence, tested both alone and in succession

Desired pattern after 00

Desired pattern after 11

$X =$  0 0 1 1 0 1 1 0 0 1 0 1 0 1 0 0  
 $Z =$  0 0 0 0 0 1 0 0 0 0 0 1 0 1 0 0  
(time: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15)

Repeated 1s tested after repeated 0s  
and after desired pattern

Repeated 0s tested initially, after repeated 1s, and after desired pattern



## 101sequence detector

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Note that these are guidelines (helpful steps, not “The Law”)

1. Start by identifying sample input and output sequences. Doing this also helps you understand the problem statement.
2. Determine an initial state and any condition that causes a reset to that state (if there are any)

From the problem statement “Circuit will not reset when a 1 output occurs”

3. If the output is mostly zero, identify the few states that cause non-zero output and start with those (partial state graph)
4. Another way to start is to determine sequences or groups of sequences that must be remembered by the circuit, and set up states for them

The sequence detector outputs 1 in only a single situation, but...

Also must remember sequence of serial inputs received in 101 pattern

We used approach from statement 4 last lecture



# State Graph Guidelines

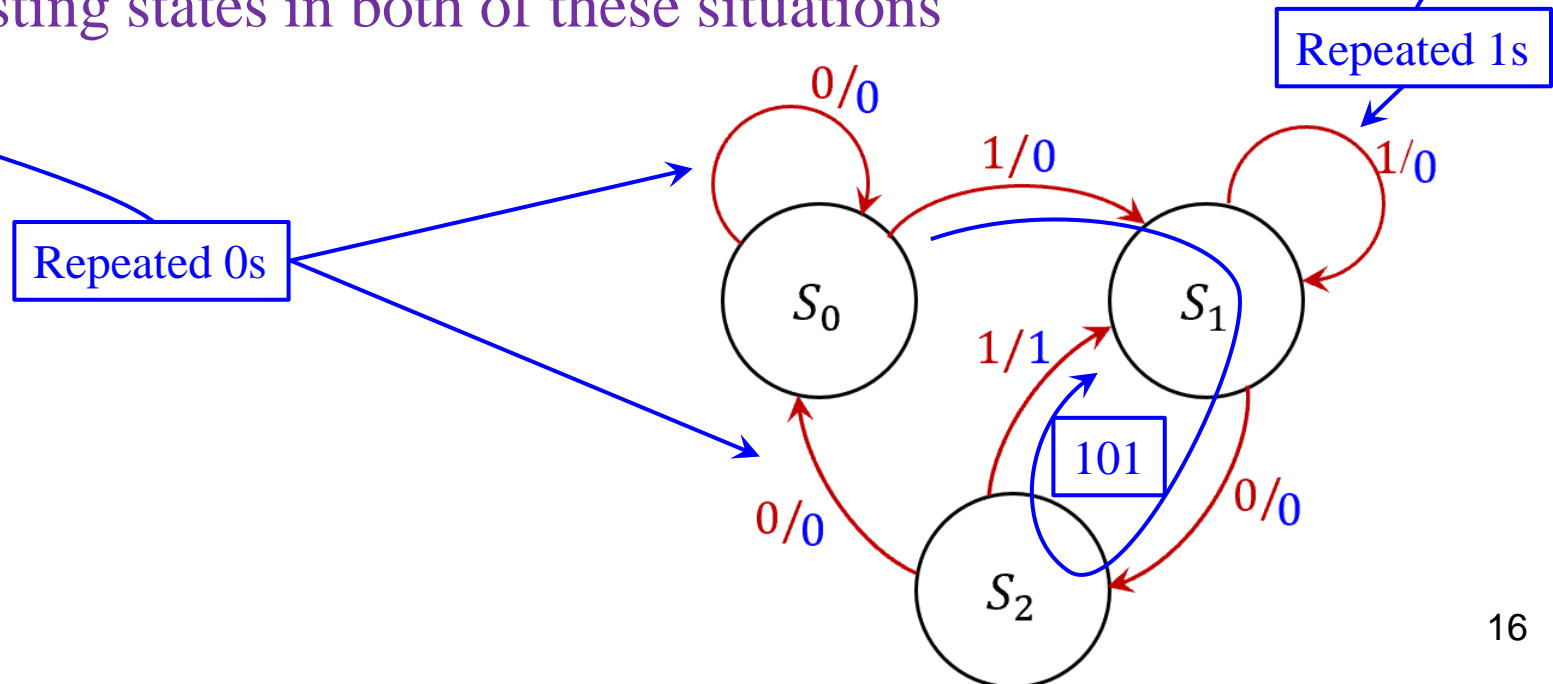
## 101 sequence detector

Note that these are guidelines (helpful steps, not “The Law”)

5. Can transition arrows go to existing states? Add a new state only when you really have to.

- Partway through development of state graph

- If in  $S_1$  and 1 received, still have only first “1” bit in “101” pattern. Stay in  $S_1$
- If in  $S_2$  and 0 received, “00” isn’t desired pattern, nor first “1”. Return to  $S_0$
- Used existing states in both of these situations





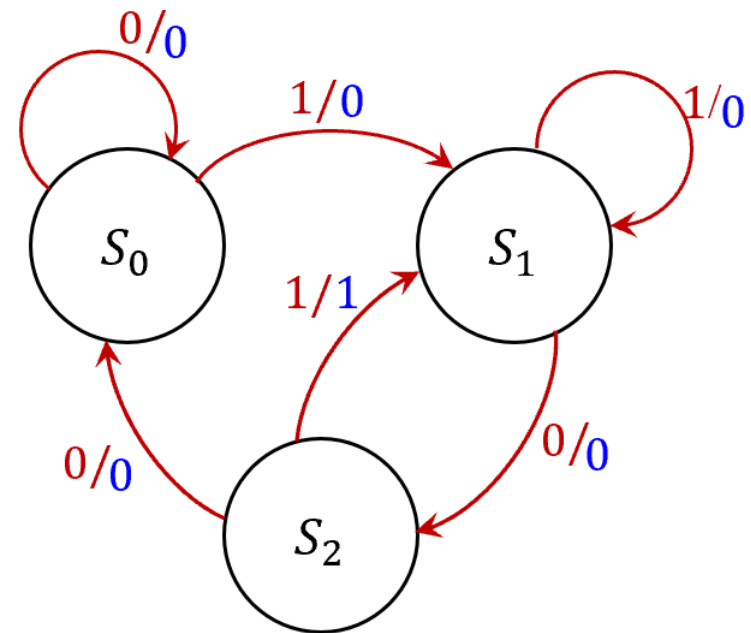


# State Graph Guidelines

## 101 sequence detector

Note that these are guidelines (helpful steps, not “The Law”)

6. Once graph is complete, make sure each input combination leaves each state only once.
7. Test graph using input-output combinations found in step (1)

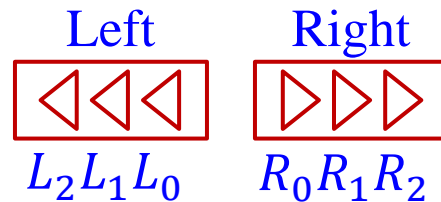


$X =$	0	0	1	1	0	1	1	0	0	1	0	1	0	1	0	0
$Z =$	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0
(time:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15)



# Design Example: Mustang Sequential Turn Signals

- Left and right sequential signals
- Hazard lights



When turning left:  $L_0 \rightarrow L_0 L_1 \rightarrow L_0 L_1 L_2 \rightarrow \text{all off}$

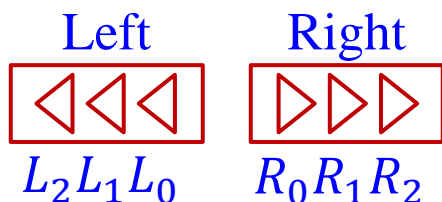
When turning right:  $R_0 \rightarrow R_1 R_0 \rightarrow R_2 R_1 R_0 \rightarrow \text{all off}$

Hazards on:  $L_2 L_1 L_0 R_2 R_1 R_0 \rightarrow \text{all off}$

Highest priority: Leave any state  $\rightarrow$  Hazard State Directly



# Design Example: Mustang Sequential Turn Signals



← The six outputs

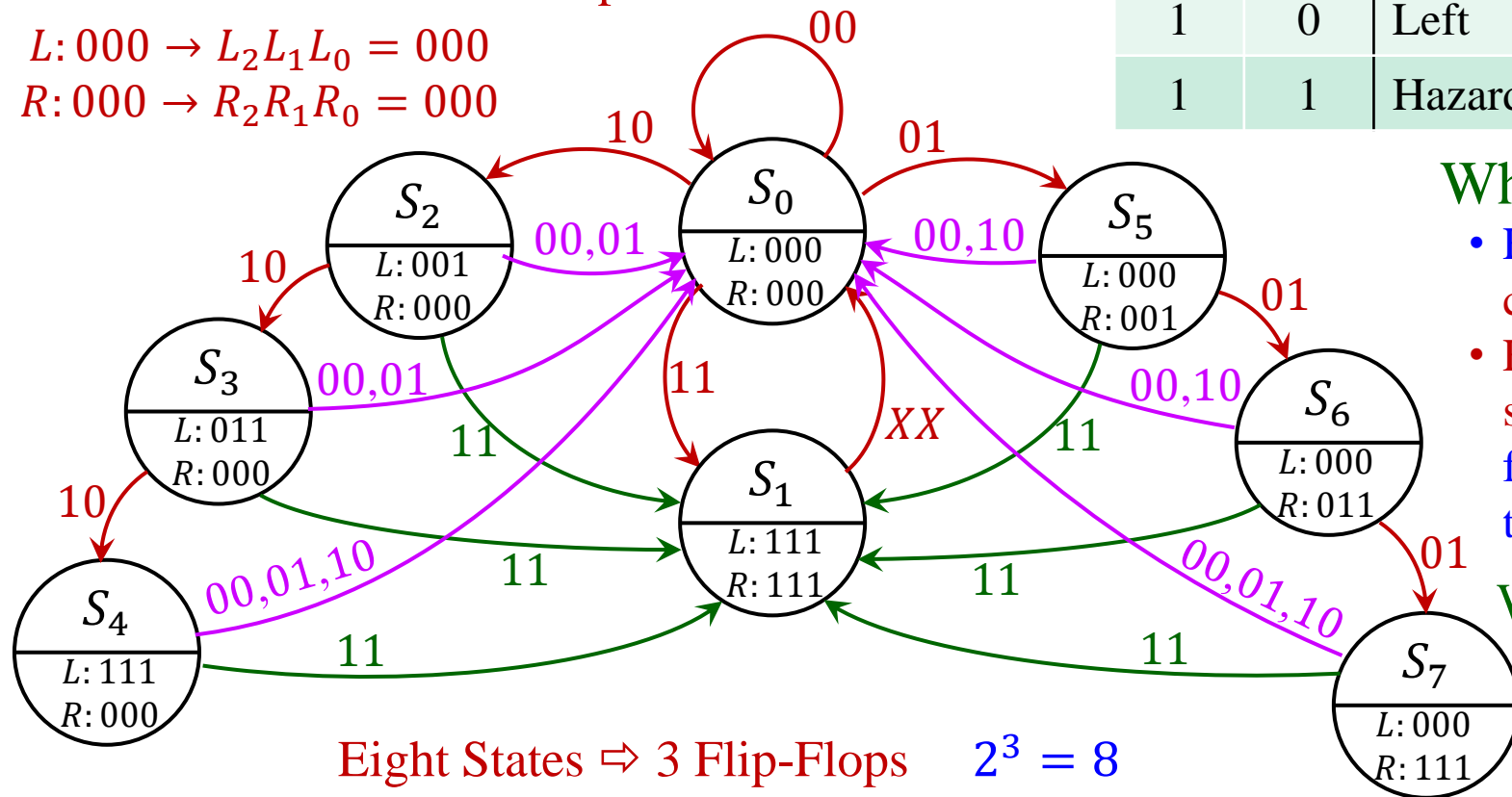
Initial State: All Off

Inputs labeled in *LR* order

$L: 000 \rightarrow L_2 L_1 L_0 = 000$

$R: 000 \rightarrow R_2 R_1 R_0 = 000$

Inputs: 2		
<i>L</i>	<i>R</i>	
0	0	No lights
0	1	Right
1	0	Left
1	1	Hazard



What Next?

- Four input combinations
- Each state should have four output transitions

What Next?

State Table



# Design Example: Mustang Sequential Turn Signals

**Present  
State****Next State****Present Outputs** $LR = 00$  $LR = 01$  $LR = 11$  $LR = 10$  $L_2L_1L_0$  $R_2R_1R_0$  $S_0$  $S_0$  $S_5$  $S_1$  $S_2$ 

000

000

 $S_1$  $S_0$  $S_0$  $S_0$  $S_0$ 

111

111

 $S_2$  $S_0$  $S_0$  $S_1$  $S_3$ 

001

000

 $S_3$  $S_0$  $S_0$  $S_1$  $S_4$ 

011

000

 $S_4$  $S_0$  $S_0$  $S_1$  $S_0$ 

111

000

 $S_5$  $S_0$  $S_6$  $S_1$  $S_0$ 

000

001

 $S_6$  $S_0$  $S_7$  $S_1$  $S_0$ 

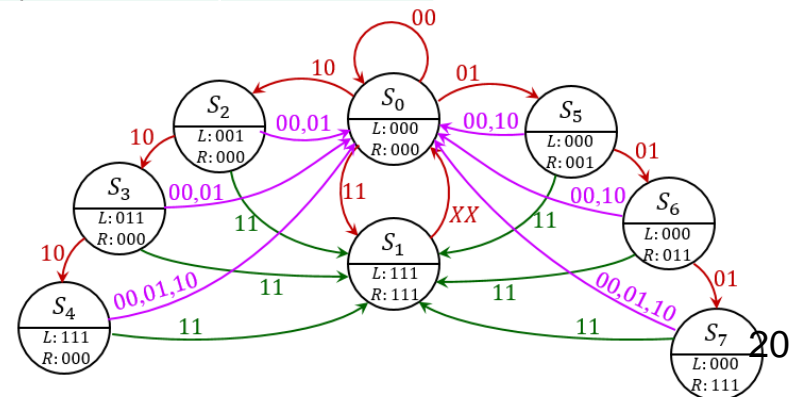
000

011

 $S_7$  $S_0$  $S_0$  $S_1$  $S_0$ 

000

111

**State Table****What Next?****Transition  
Table**



# Design Example: Mustang Sequential Turn Signals

## Transition Table

What Next?

Next State

Maps for each  
flip-flop

$ABC$	$LR = 00$	$LR = 01$	$LR = 11$	$LR = 10$	Present Outputs	
	$A^+B^+C^+$	$A^+B^+C^+$	$A^+B^+C^+$	$A^+B^+C^+$	$L_2L_1L_0$	$R_2R_1R_0$
$S_0$ 000	000	101	001	010	000	000
$S_1$ 001	000	000	000	000	111	111
$S_2$ 010	000	000	001	011	001	000
$S_3$ 011	000	000	001	100	011	000
$S_4$ 100	000	000	001	000	111	000
$S_5$ 101	000	110	001	000	000	001
$S_6$ 110	000	111	001	000	000	011
$S_7$ 111	000	000	001	000	000	111

Do you see a  
new challenge?

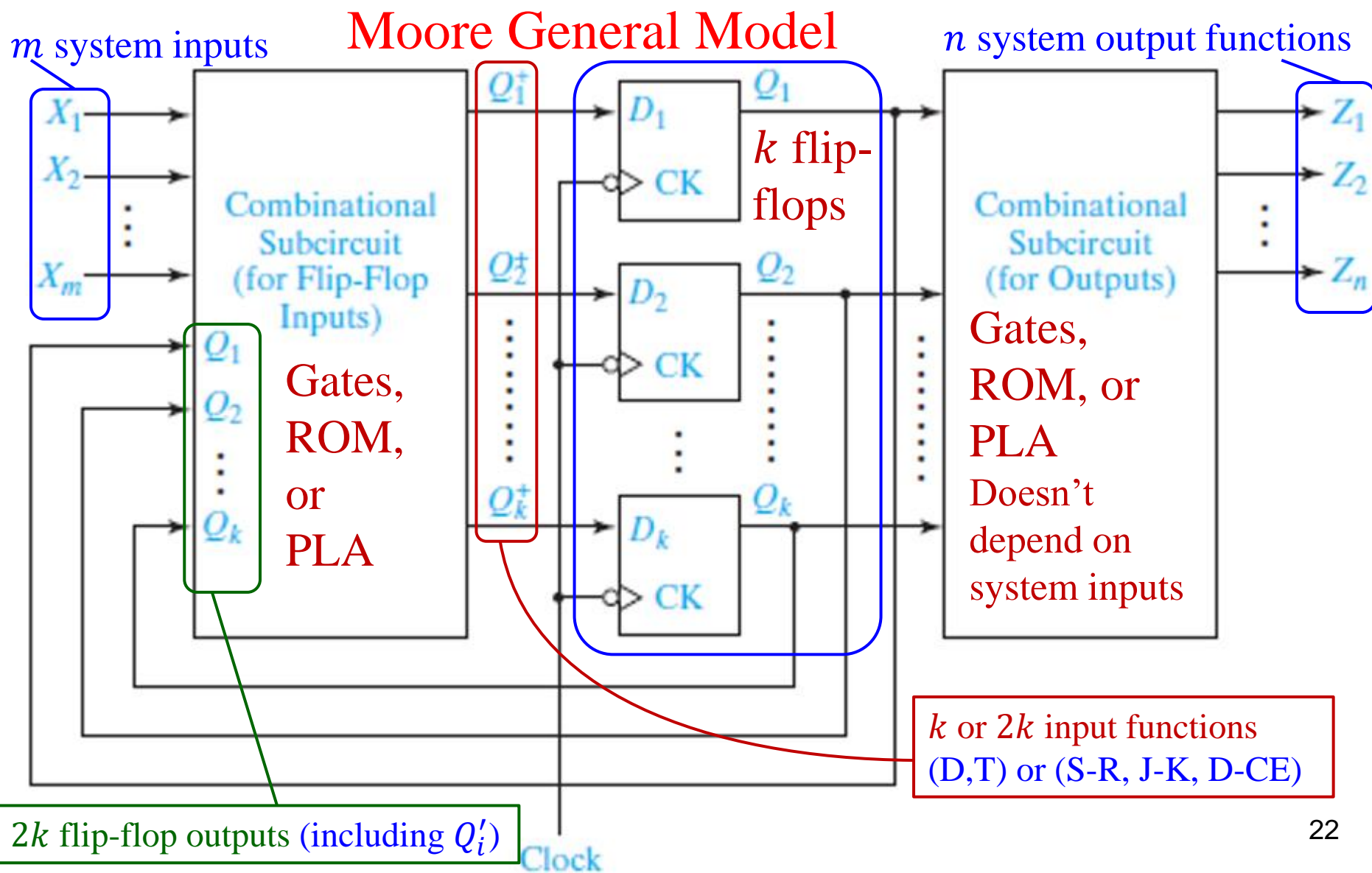
Five variables:  $A, B, C, L, R$

We will come back to that later. Do outputs first.

Present State	Next State			
	$LR = 00$	$LR = 01$	$LR = 11$	$LR = 10$
$S_0$	$S_0$	$S_5$	$S_1$	$S_2$
$S_1$	$S_0$	$S_0$	$S_0$	$S_0$
$S_2$	$S_0$	$S_0$	$S_1$	$S_3$
$S_3$	$S_0$	$S_0$	$S_1$	$S_4$
$S_4$	$S_0$	$S_0$	$S_1$	$S_0$
$S_5$	$S_0$	$S_6$	$S_1$	$S_0$
$S_6$	$S_0$	$S_7$	$S_1$	$S_0$
$S_7$	$S_0$	$S_0$	$S_1$	$S_0$



## General Models for Sequential Circuits

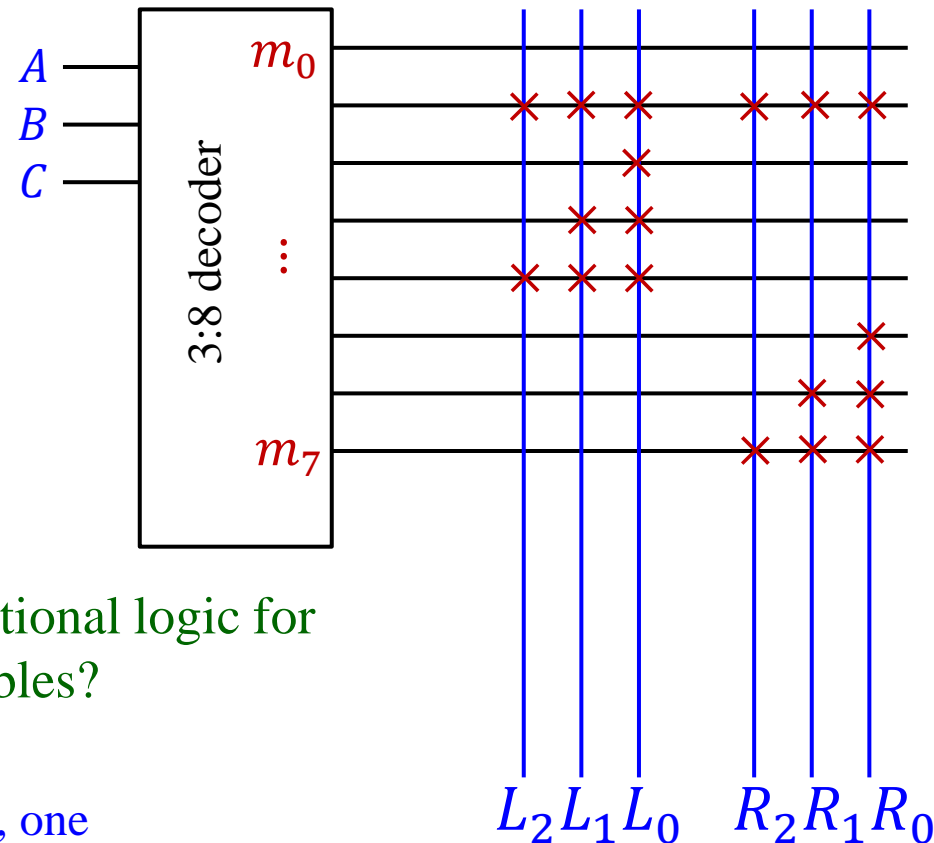




# Design Example: Mustang Sequential Turn Signals

## Outputs: Decoder and ROM

$ABC$	Present Outputs	
	$L_2L_1L_0$	$R_2R_1R_0$
$S_0$ 000	000	000
$S_1$ 001	111	111
$S_2$ 010	001	000
$S_3$ 011	011	000
$S_4$ 100	111	000
$S_5$ 101	000	001
$S_6$ 110	000	011
$S_7$ 111	000	111



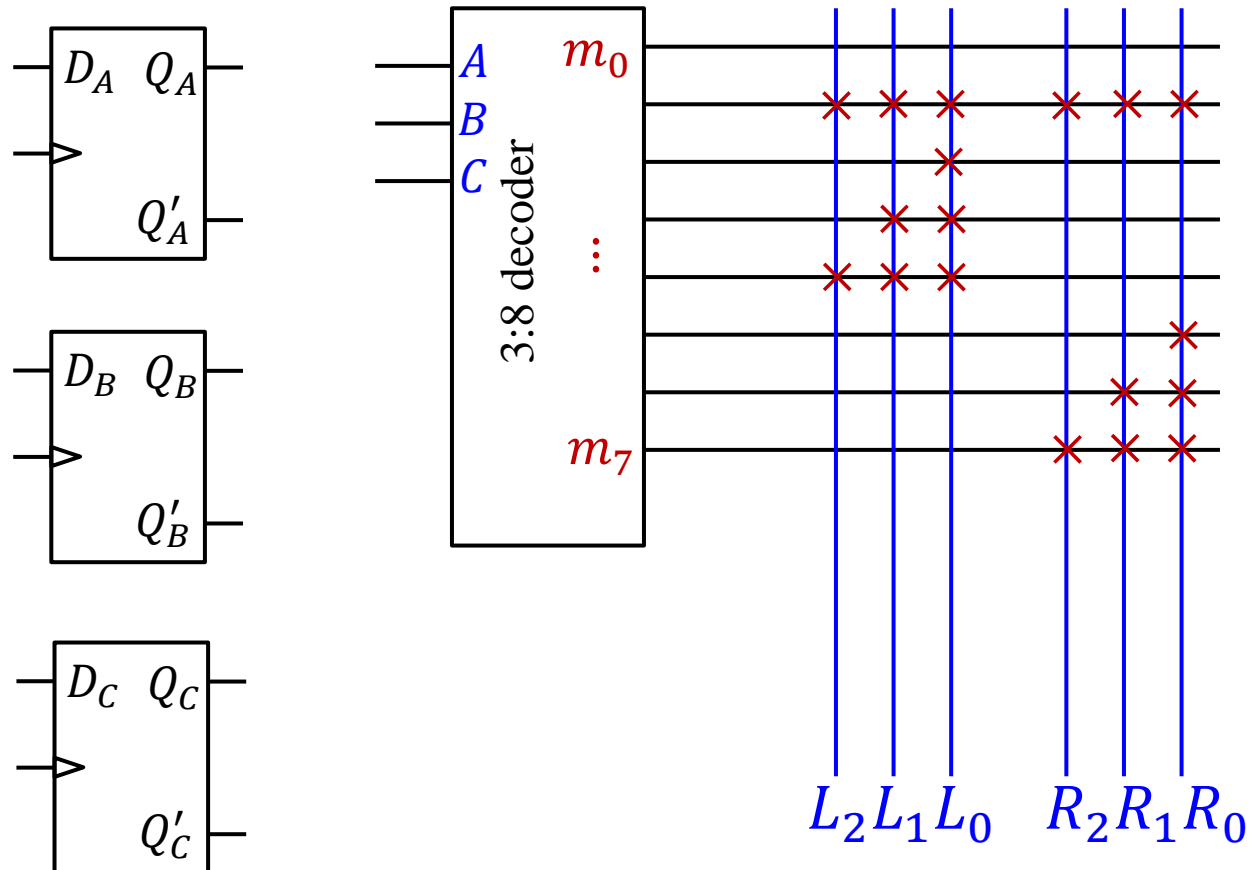
What about design of combinational logic for flip-flop inputs with five variables?

- Pick one variable as “outlier”
- Make two 4-variable K-maps, one for each of the “outlier’s” values



# Design Example: Mustang Sequential Turn Signals

Sketch overall system







# Design Example: Mustang Sequential Turn Signals

Picking  $A$  as the outlier; Demonstrating for  $A^+$ , D Flip-flops

$A = 0$

$LR \backslash BC$	00	01	11	10
00		1		
01				
11				1
10				

$A = 1$

$LR \backslash BC$	00	01	11	10
00				
01		1		
11				
10		1		

$$A^+ = B'C'L'RA' + BCLR'A' + B'CL'RA + BC'L'RA$$

**K-maps: Try to group neighboring 1s**

4-variable: Each cell can have 4 neighbors

3-variable: Each cell can have 3 neighbors

5-variable: Each cell can have 5 neighbors – think layers, top & bottom

This case: none of the 1s overlap – no further reduction



# Design Example: Mustang Sequential Turn Signals

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The design for  $B^+$  and  $C^+$ , D Flip-flops will be completed next lecture

Before wrapping up for the day:

- Remember to complete Participation Quiz 11
- Available from 11:15am today through 11:15am 3/26
- Due at 11:15am tomorrow (late penalty starts then)