ECE 3020
Introduction to Electronics

Section 4: Filters

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Dept. of Electrical & Computer Engineering

The Ohio State University

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  - Prof. Nima Ghalichechian
  - Prof. Asimina Kiourti
  - Prof. Ayman Fayed
  - Prof. George Valco

Topics Covered in this Course

- Section 1: Basic Concepts
- Section 2: Operational Amplifiers (Op-Amps)
- Section 3: Introduction to Feedback
- **Section 4: Filters**
- Section 5: Diodes and Applications
- Section 6: Field Effect Transistors (FETs) and Applications
- Section 7: Bipolar Junction Transistors (BJTs) and Applications
- Section 8: Digital and Mixed-Signal Circuits
- Section 9: Circuit Simulation Software

Reading Assignment

- Text → pp 35-41, 1290-1300, 1307-1322

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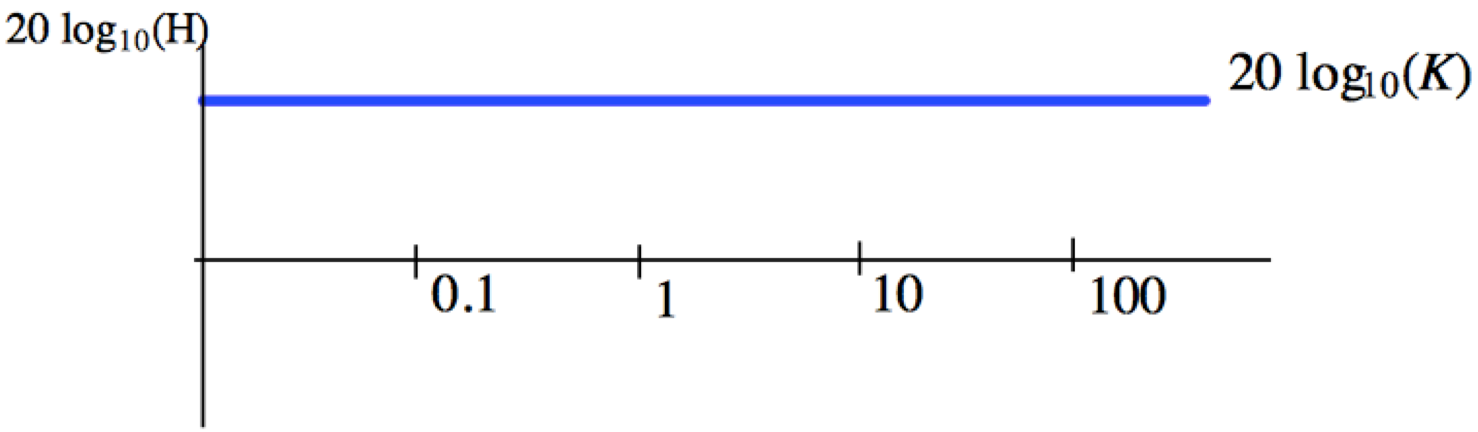
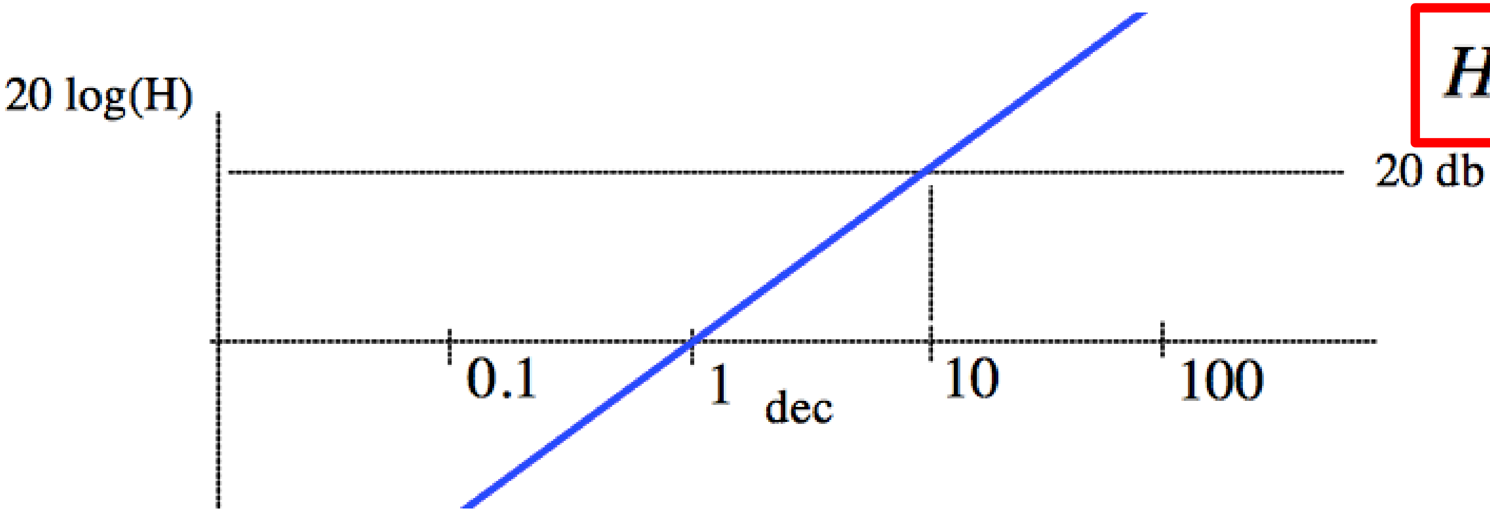
A SIMPLE METHOD TO DRAW THE TRANSFER FUNCTION

Effect of Constant Terms

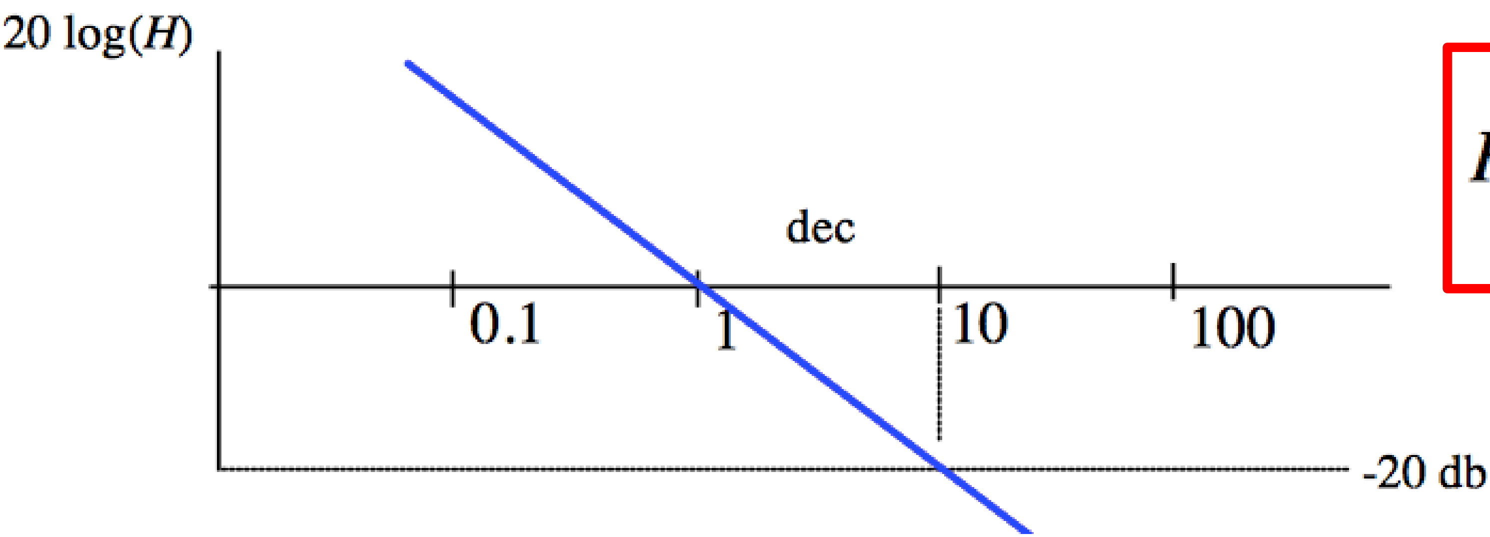


$$\boxed{H = \mathrm{K}}$$

Effect of Poles and Zeros at 0

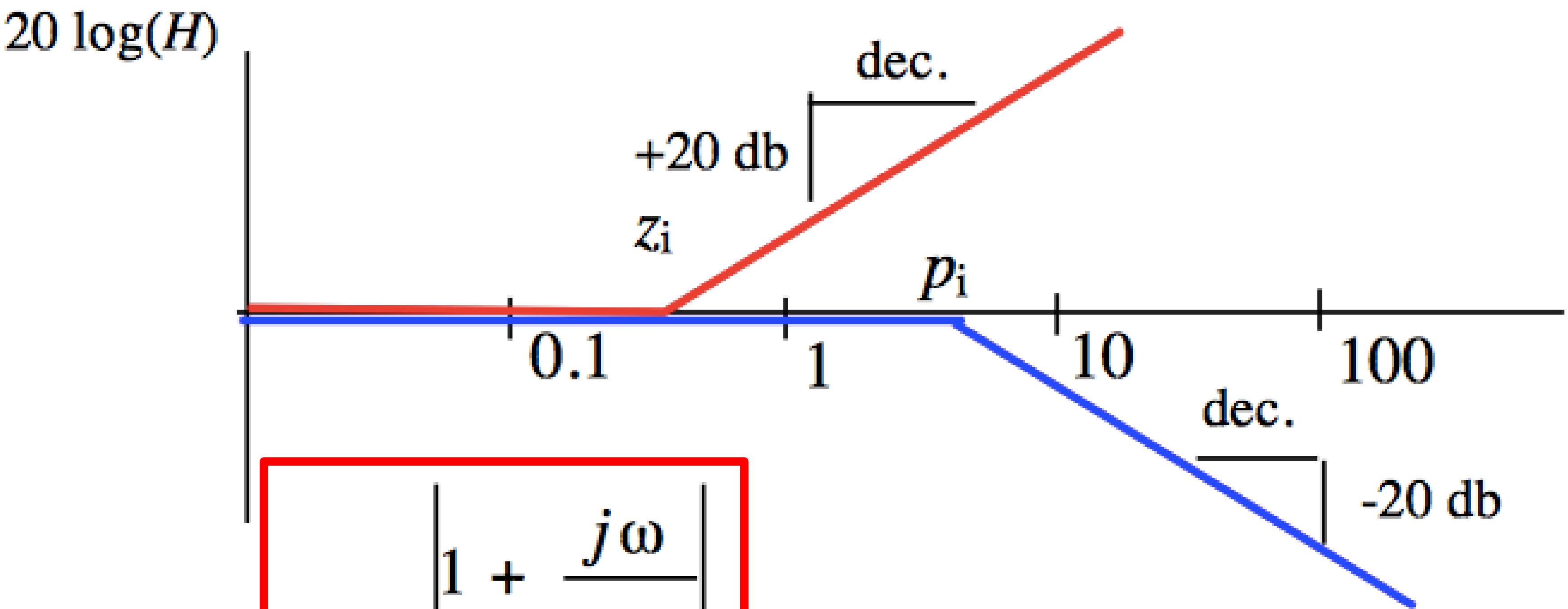


$$H = j\omega$$



$$H = \frac{1}{j\omega}$$

Effect of Poles and Zeros



$$H = \frac{\left|1 + \dfrac{j\omega}{z_i}\right|}{\left|1 + \dfrac{j\omega}{p_i}\right|}$$

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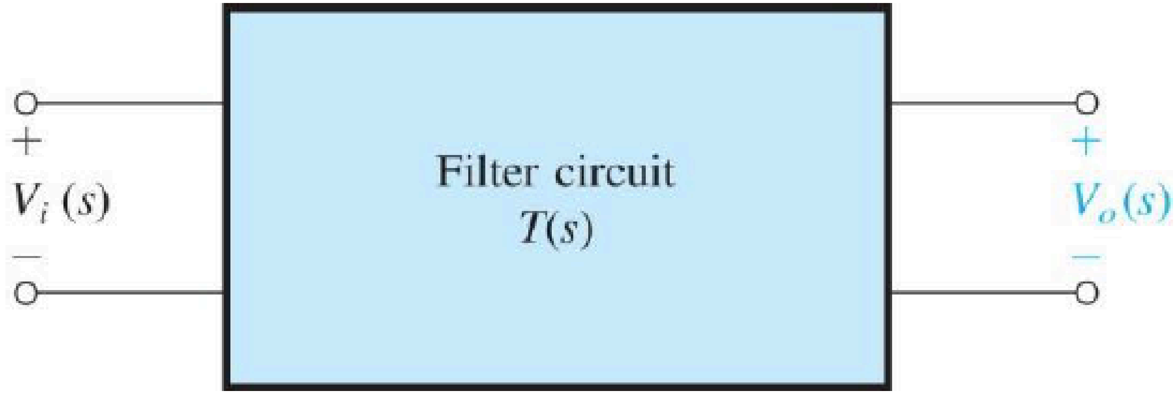
FILTER BASICS

Analog Filters: Definitions and Types

Linear circuits represented by general two-port network

$$T(s) \equiv \frac{V_o(s)}{V_i(s)}$$



For physical frequencies $s = j\omega$ filter transmission

$$T(j\omega) = |T(j\omega)|e^{j\phi(\omega)}$$

Alternate expressions for magnitude of transmission

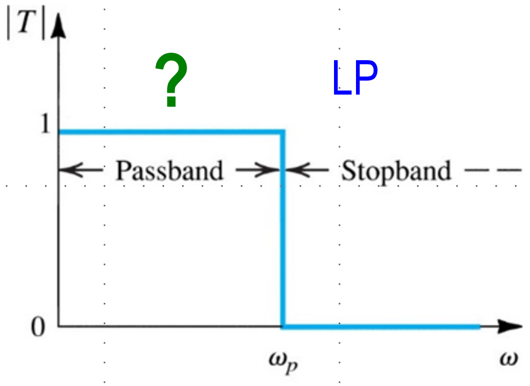
- Gain function $G(\omega) \equiv 20\log|T(j\omega)|$ in dB
- Attenuation function $A(\omega) \equiv -20\log|T(j\omega)|$ in dB

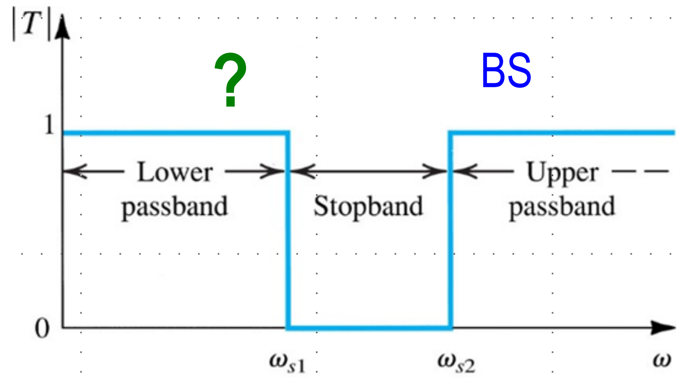
Analog Filters: Definitions and Types

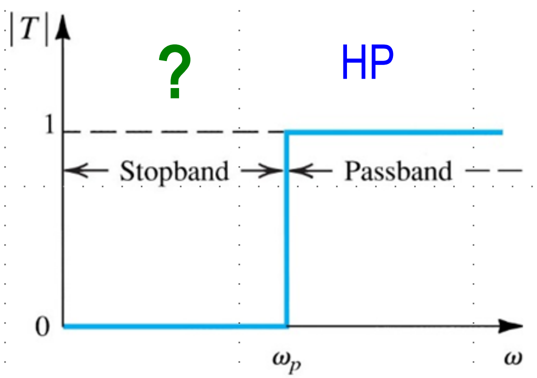
Ideal filters
- Unity transmission magnitude in passband
- Zero transmission in stopband
- Vertical (brick-wall) transitions between

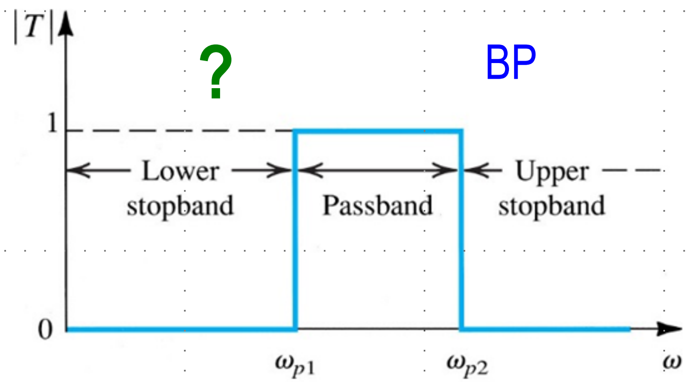
Assign one of the following to each of these filter types

a) Low-pass (LP)
b) High-pass (HP)
c) Bandpass (BP)
d) Bandstop (BS)
   a.k.a. band-reject







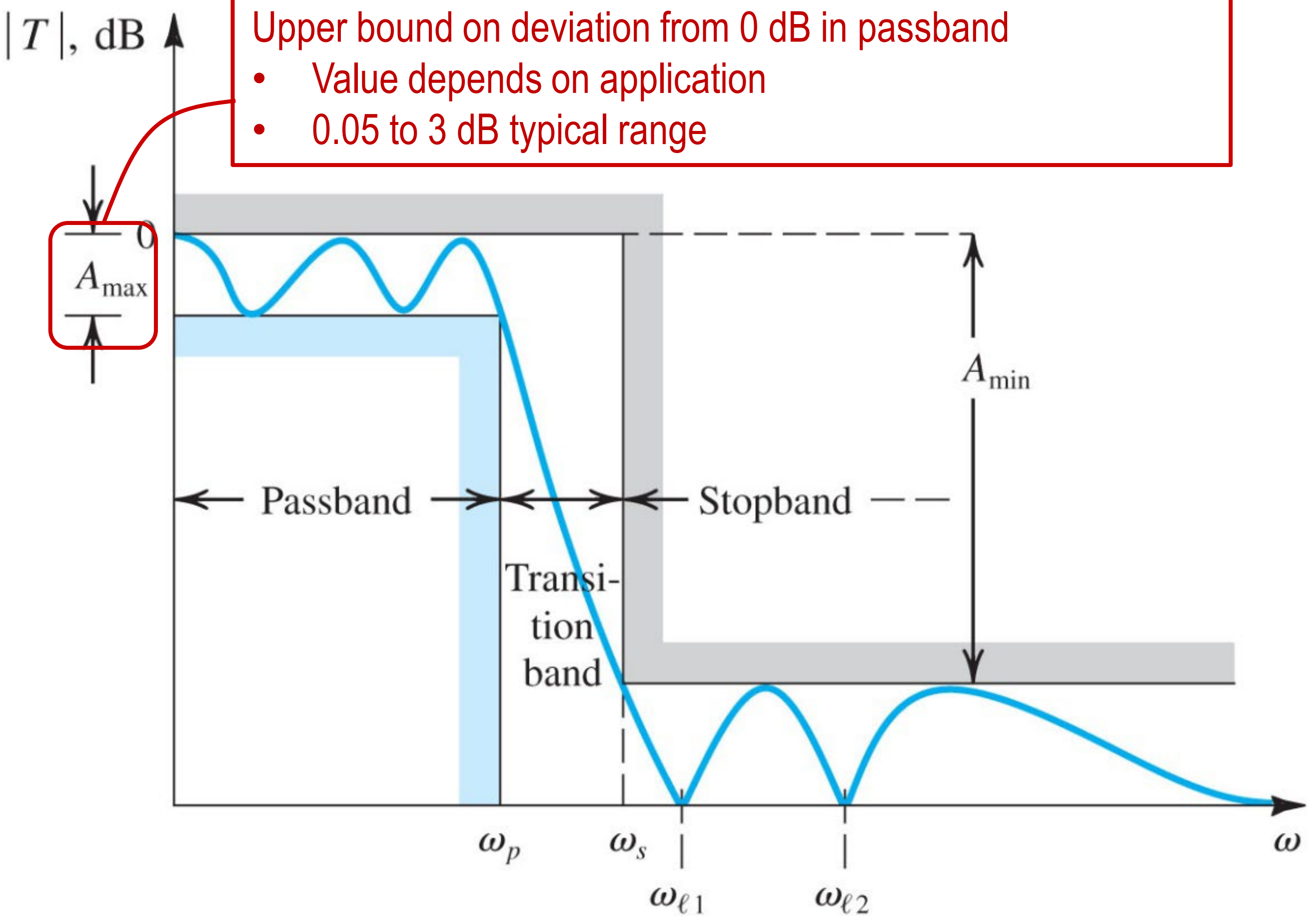


Filter Specifications – LPF Example

- Real physical circuits cannot attain ideal characteristics
- Specifications for a filter design need to be realistic
- Use **low-pass** and bandpass as examples

- Example shows 0 dB as the reference
- If filter has gain, reference might be different dB value

Upper bound on deviation from 0 dB in passband
- Value depends on application
- 0.05 to 3 dB typical range



Filter Specifications – LPF Example

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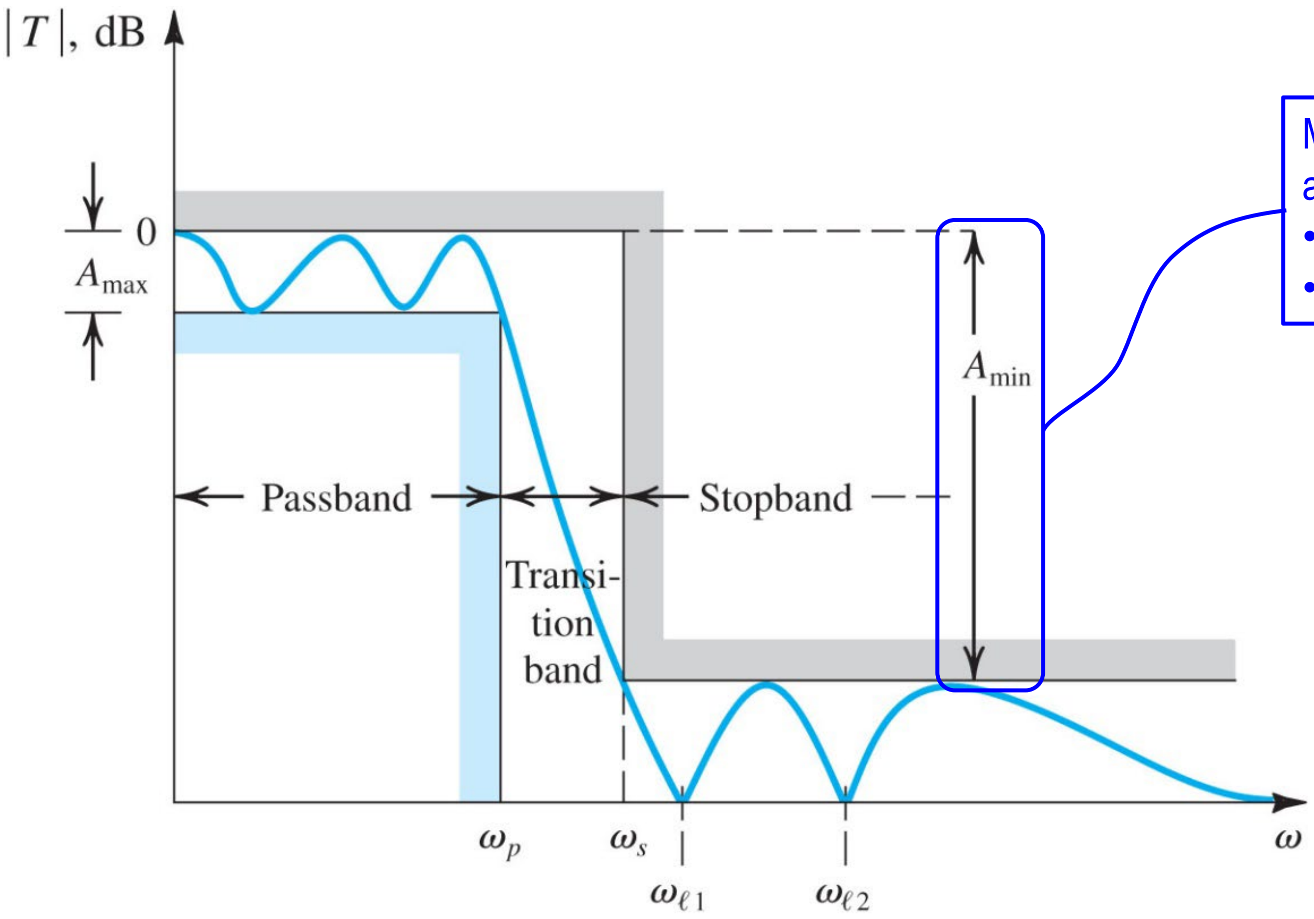

Minimum specification for attenuation in stopband

- Value depends on application
- 20 to 100dB typical range

Filter Specifications – LPF Example

- Real physical circuits cannot attain ideal characteristics
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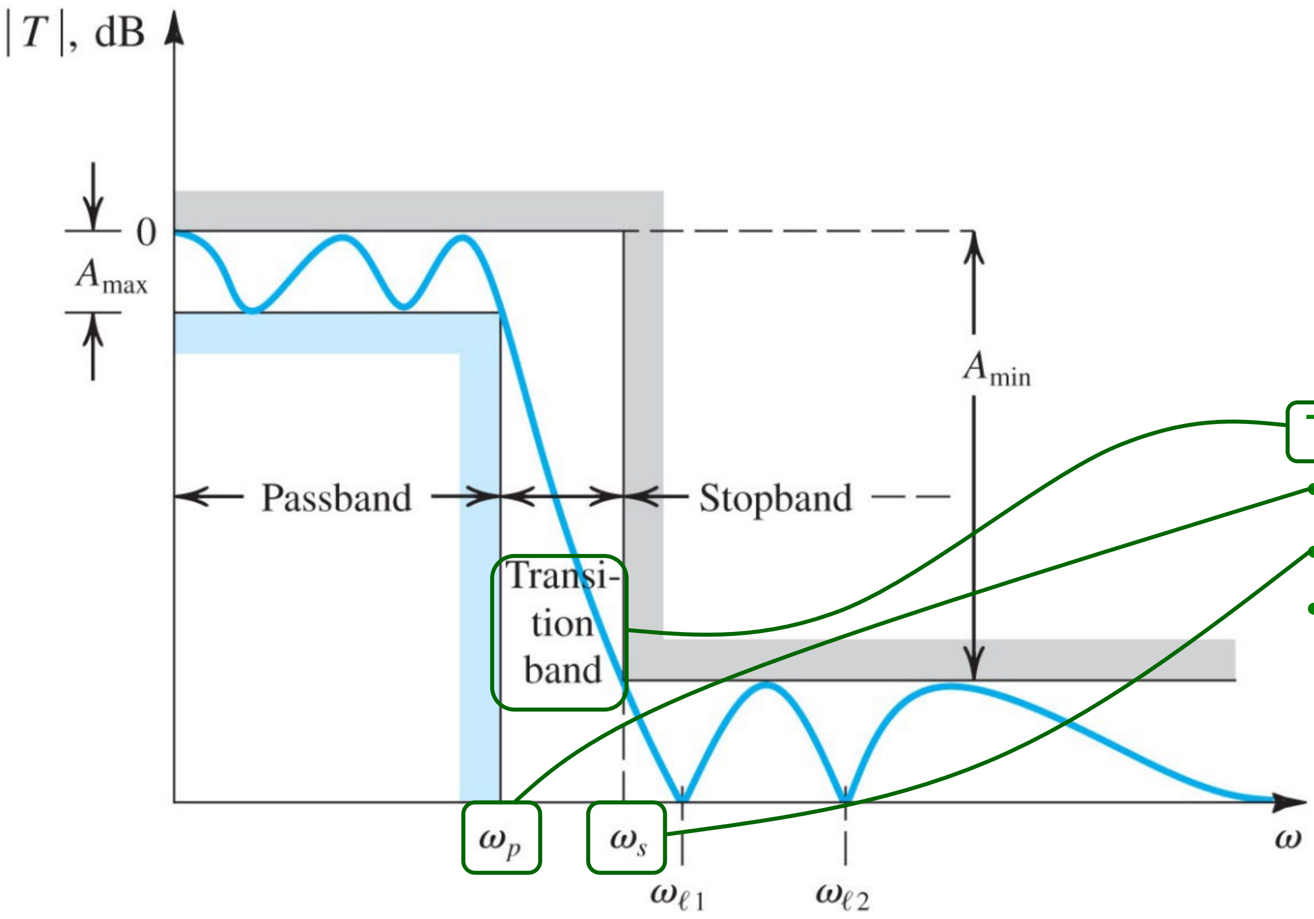
- Use **low-pass** and bandpass as examples

- Example shows 0 dB as the reference
- If filter has gain, reference might be different dB value

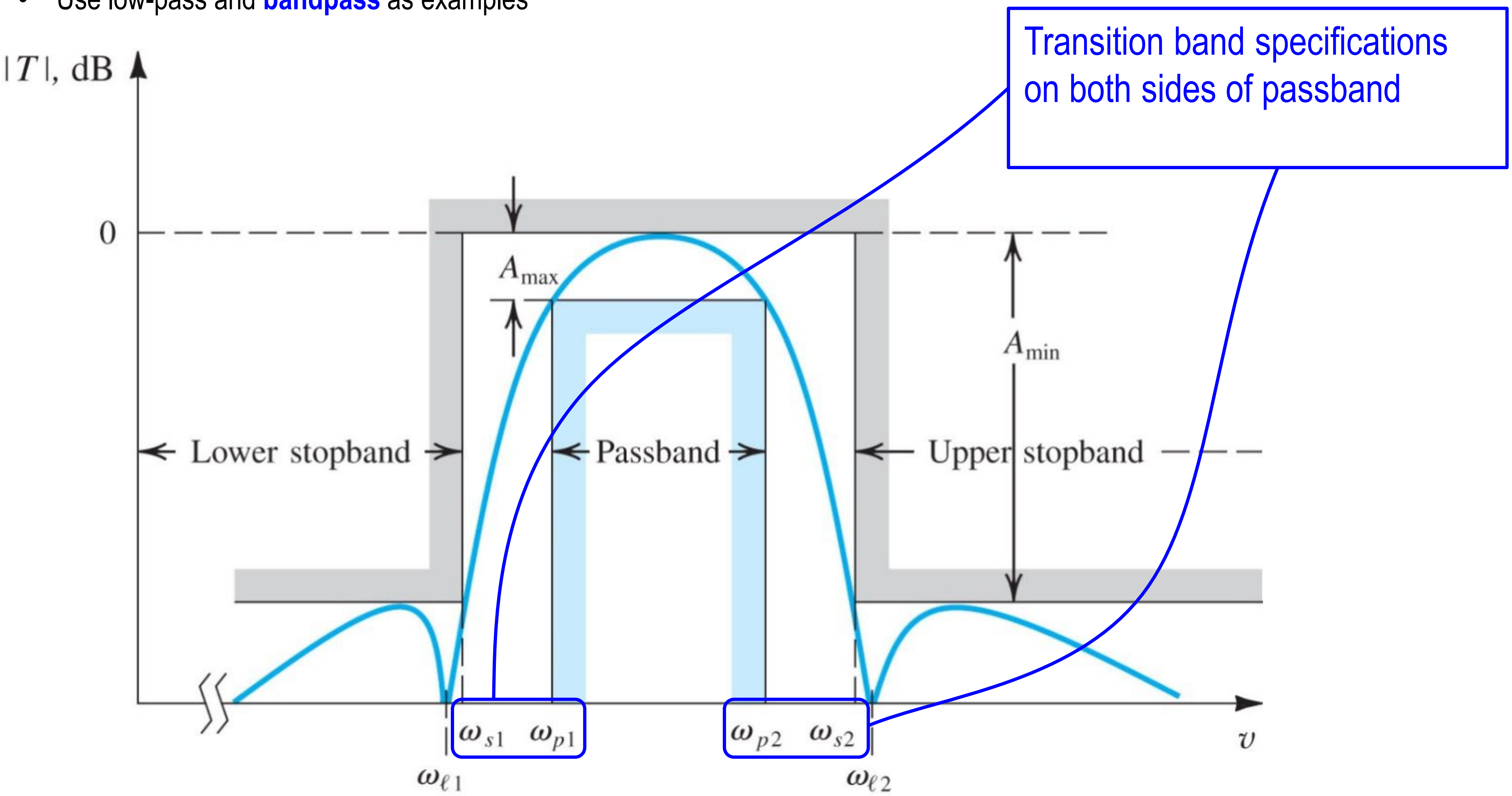


Transition band extends from

- Passband edge (ω_p) to
- Stopband edge (ω_s)
- Selectivity factor ω_s/ω_p measures sharpness of transition

Filter Specifications – BPF Example

- Real physical circuits cannot attain ideal characteristics
- Specifications for a filter design need to be realistic
- Use low-pass and **bandpass** as examples



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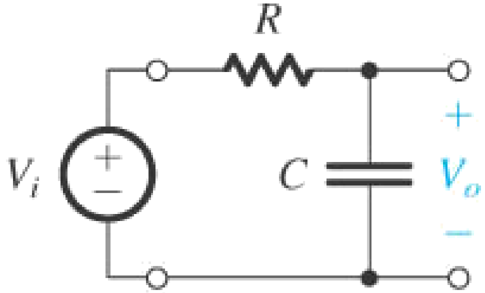
FREQUENCY RESPONSE: FIRST-ORDER

Single Time Constant Network – LOW PASS

$$\tau = RC = \frac{1}{\omega_0}$$



Low-pass network

$$T(s) = \frac{K}{1 + \frac{s}{\omega_0}}$$

Magnitude and Phase Response of Low-pass STC Network

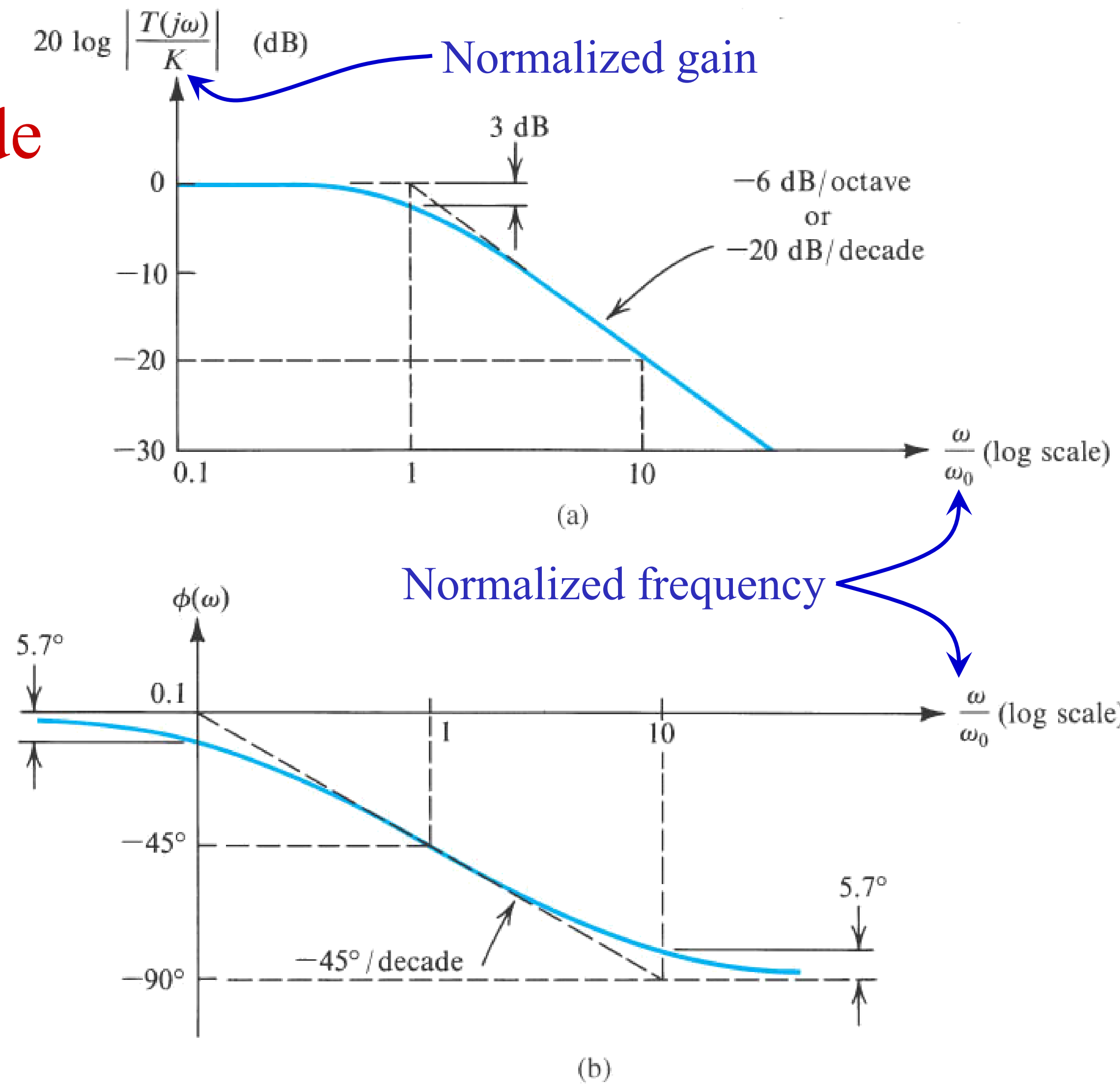
Magnitude

$$|T(j\omega)|^2 = \frac{|K|^2}{1+\left(\frac{\omega}{\omega_0}\right)^2}$$

Sinusoidal steady-state response; s replaced by $j\omega$

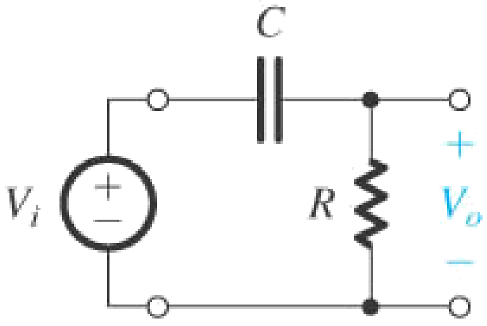
Phase

$$-\tan^{-1}\left[\frac{\omega}{\omega_0}\right]$$



Single Time Constant Network – HIGH PASS

$$\tau = RC = \frac{1}{\omega_0}$$



(b)

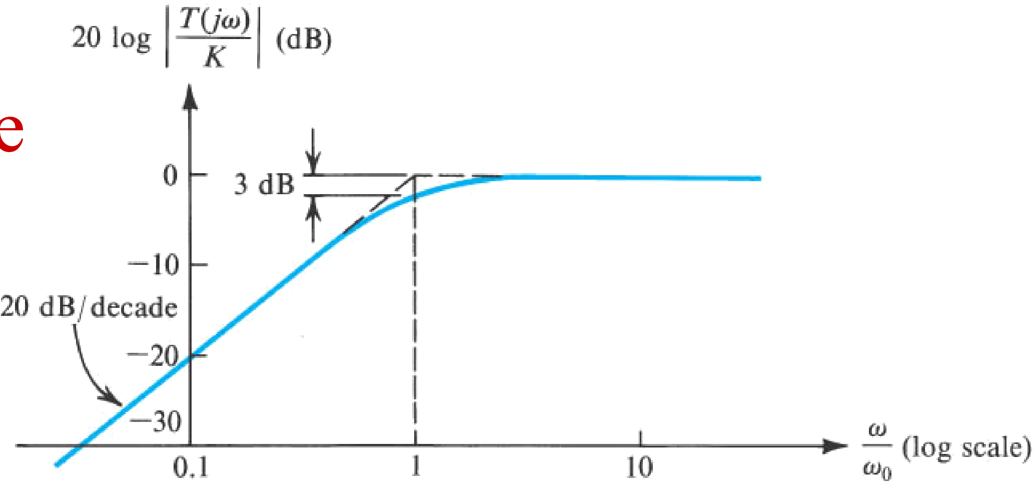
High-pass network

$$T(s) = \frac{Ks}{s + \omega_0}$$

Magnitude and Phase Response of High-pass STC Network

Magnitude

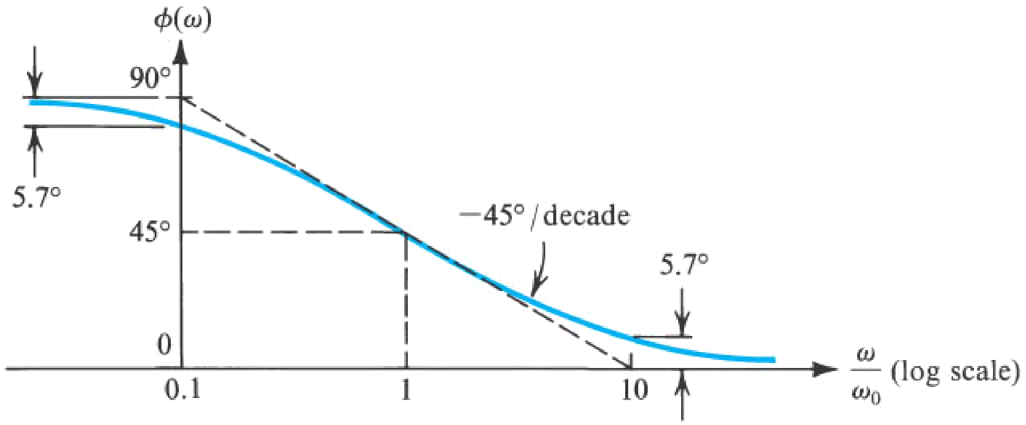
$$|T(j\omega)|^2 = \frac{|K|^2}{1+\left(\frac{\omega_0}{\omega}\right)^2}$$



(a)

Phase

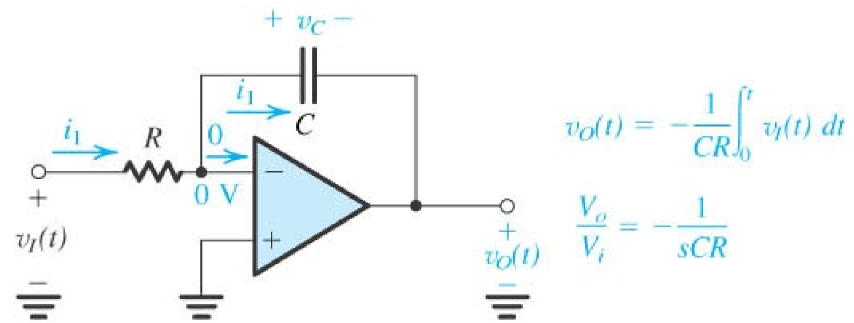
$$\tan^{-1}\left[\frac{\omega_0}{\omega}\right]$$



(b)

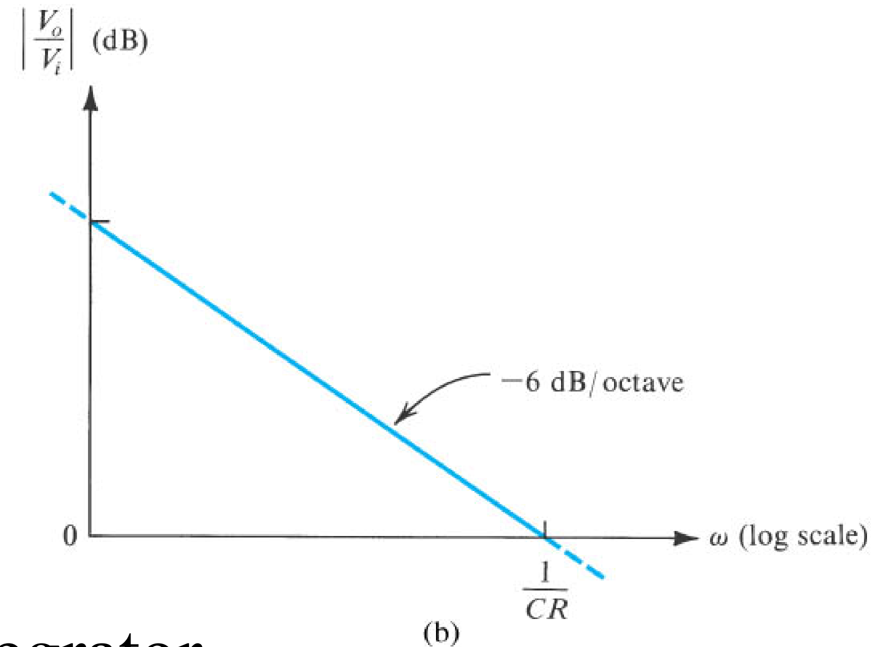
The Miller or Inverting Integrator

Using $Z_1 = R$ and $Z_2 = \dfrac{1}{Cs}$



we get

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{1}{RCs}$$



Frequency response of the integrator.

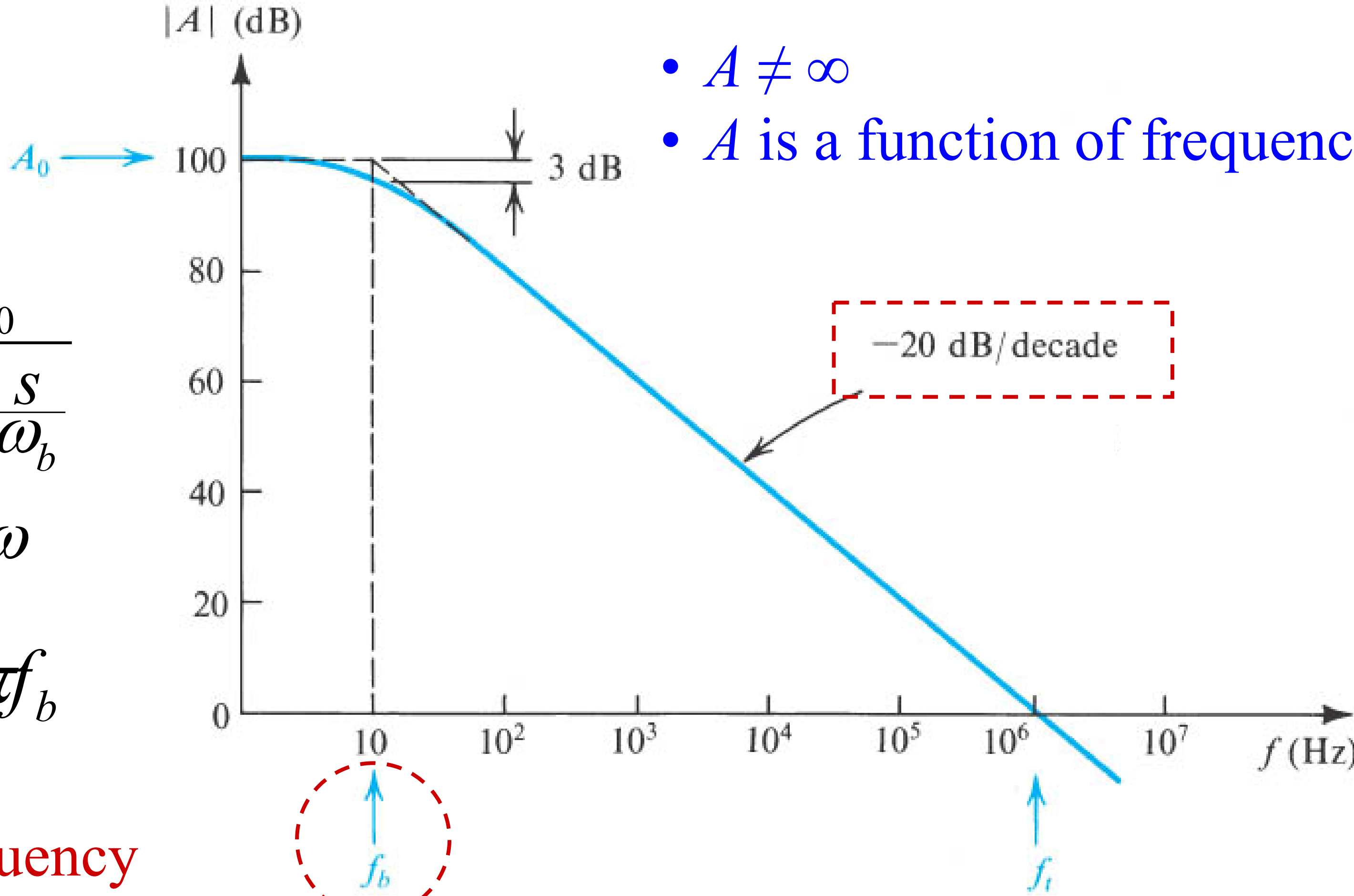
Effect of Finite Open-Loop Gain & Bandwidth on Op-Amps

$$A(s) = \frac{A_0}{1+\frac{s}{\omega_b}}$$

with $s = j\omega$

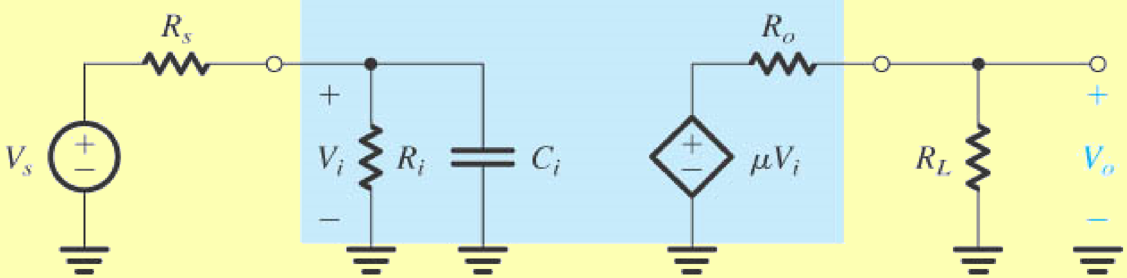
$$\omega_b = 2\pi f_b$$

break frequency



- $A \neq \infty$
- A is a function of frequency

Exercise #1



1) What is the voltage gain as a function of frequency?

2) What is the DC voltage gain?

3) What is the 3dB frequency?

4) How can I create an extra zero?

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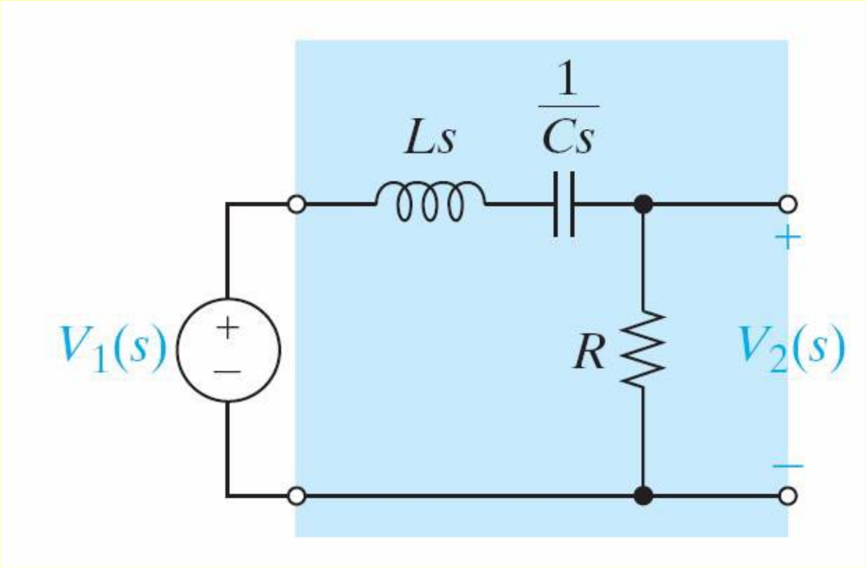
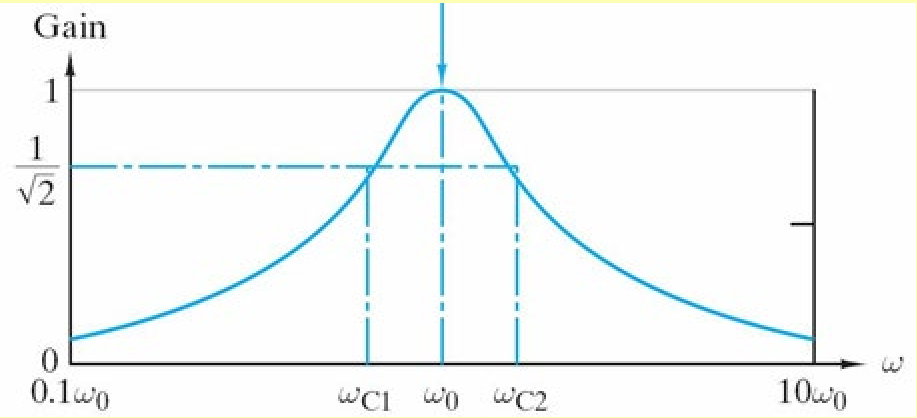
FREQUENCY RESPONSE: SECOND-ORDER

Exercise #2: Second Order Frequency Response





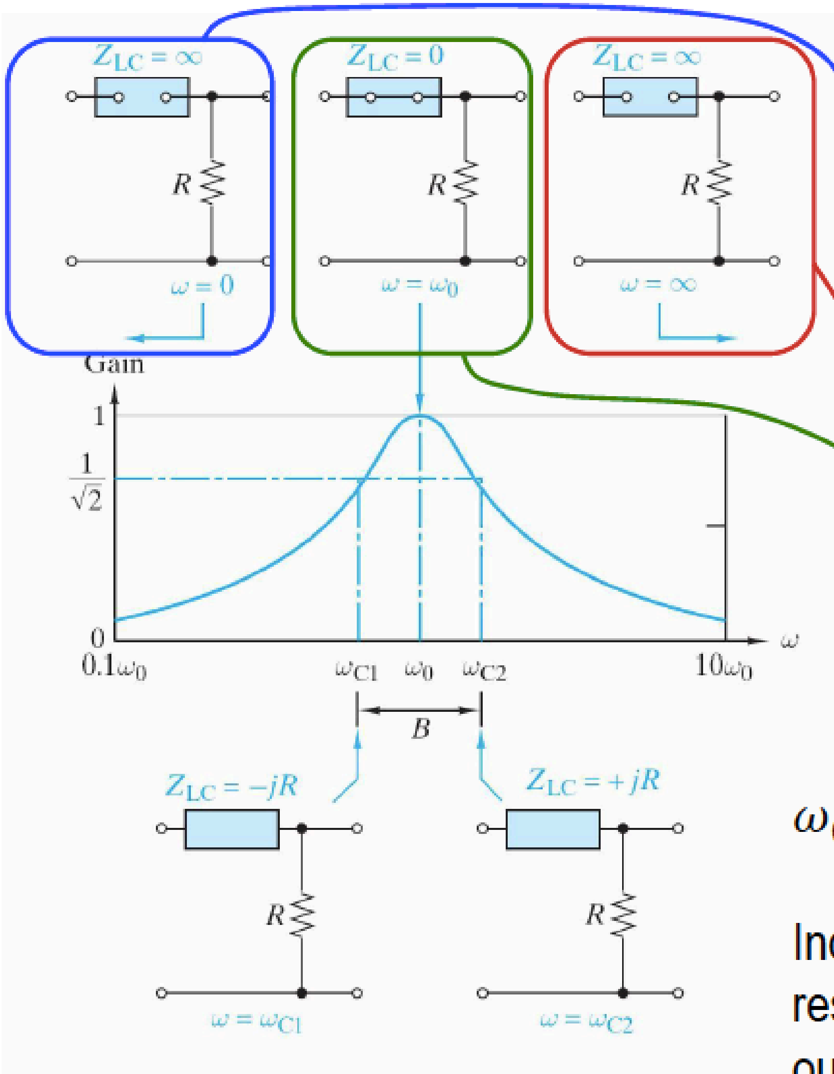
1) **What is the transfer function of this filter?**

$$T(s) = \frac{R}{R + Ls + 1/Cs} = \frac{R}{R + Z_{LC}(s)}$$

2) **Can I easily predict the type of this filter just by looking at its transfer function?**

3) **What is the filter's resonant frequency, ω_0?**

A) Calculation of: Resonant Frequency



$$Z_{LC}(j\omega) = j(\omega L - 1/\omega C)$$

At DC, capacitor open, $T(0) = 0$

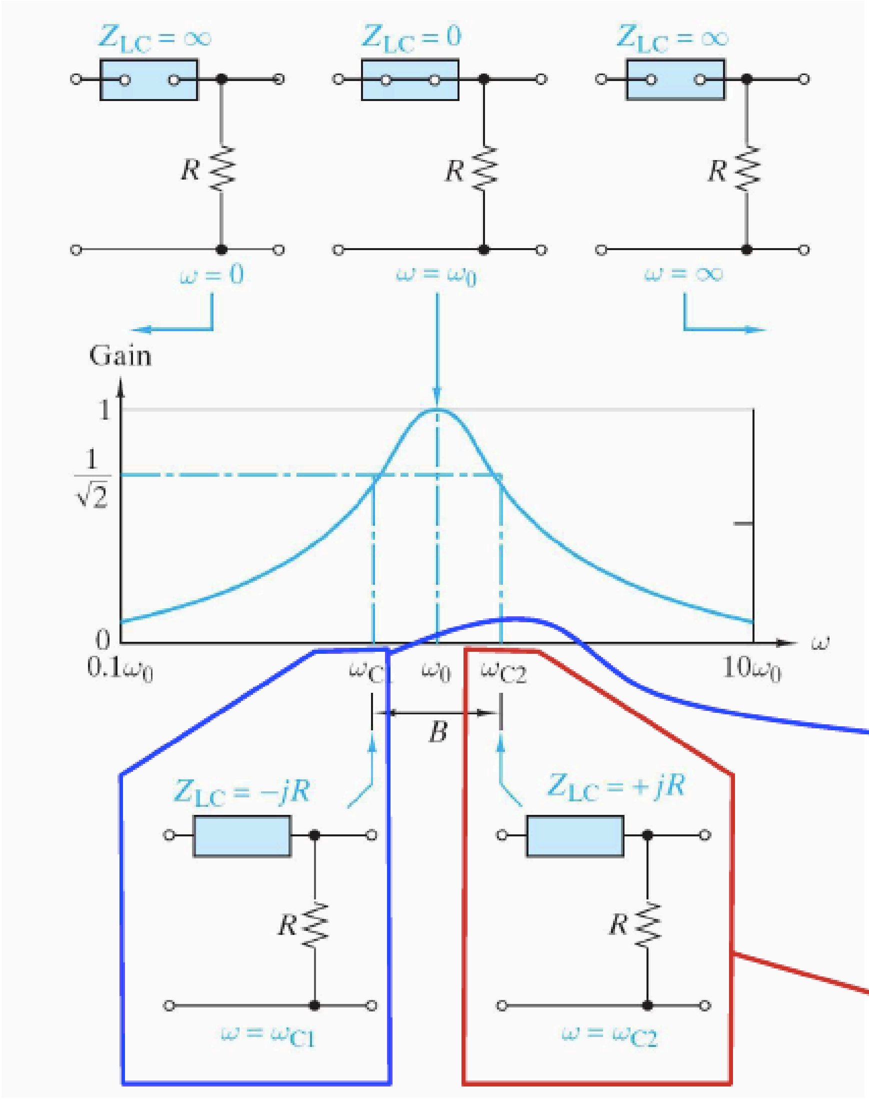
At high frequency, inductor open, $T(\infty) = 0$

At $\omega = \omega_0 = 1/\sqrt{LC}$,
$Z_{LC} = j\left(\sqrt{L/C} - \sqrt{L/C}\right) = 0, T(j\omega_0) = 1$

$\omega_0 \equiv$ center frequency or resonant frequency

Inductor and capacitor interact to produce a resonant short circuit at ω_0 connecting input to output

B) Calculation of: Cut-off Frequencies



$$T(j\omega) = \frac{R}{R + Z_{LC}(j\omega)}$$

The 3dB or cutoff frequencies occur when

$Z_{LC} = \pm jR$

$$|T(j\omega)| = \left|\frac{R}{R \pm jR}\right| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} T_{max}$$

$$Z_{LC}(j\omega_C) = j(\omega_C L - 1/\omega_C C) = \pm jR$$

algebra $\quad LC\omega_C^2 \pm RC\omega_C - 1 = 0$

The physically valid roots are

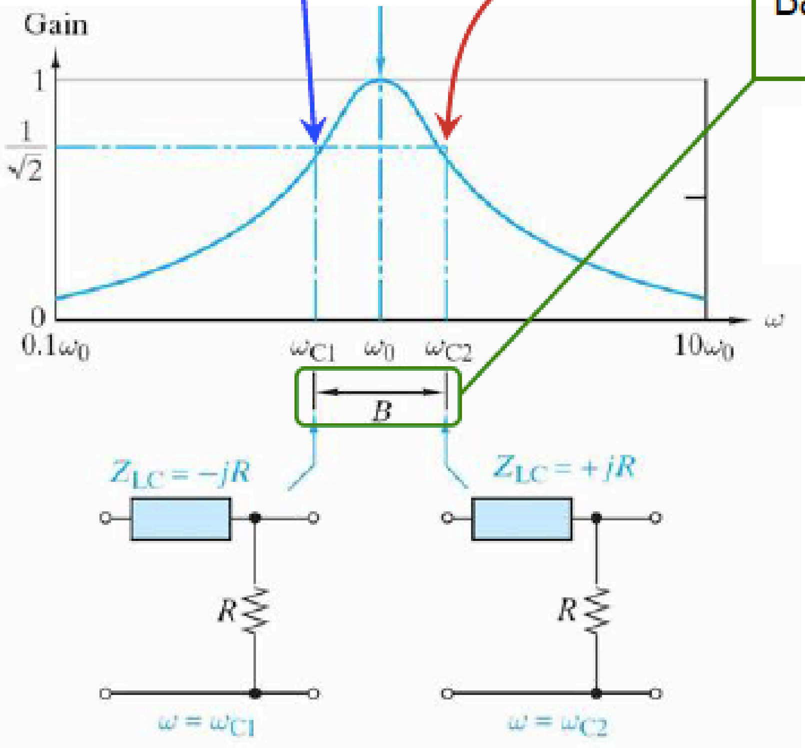
$$\omega_{C1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{C2} = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{C1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{C2} = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

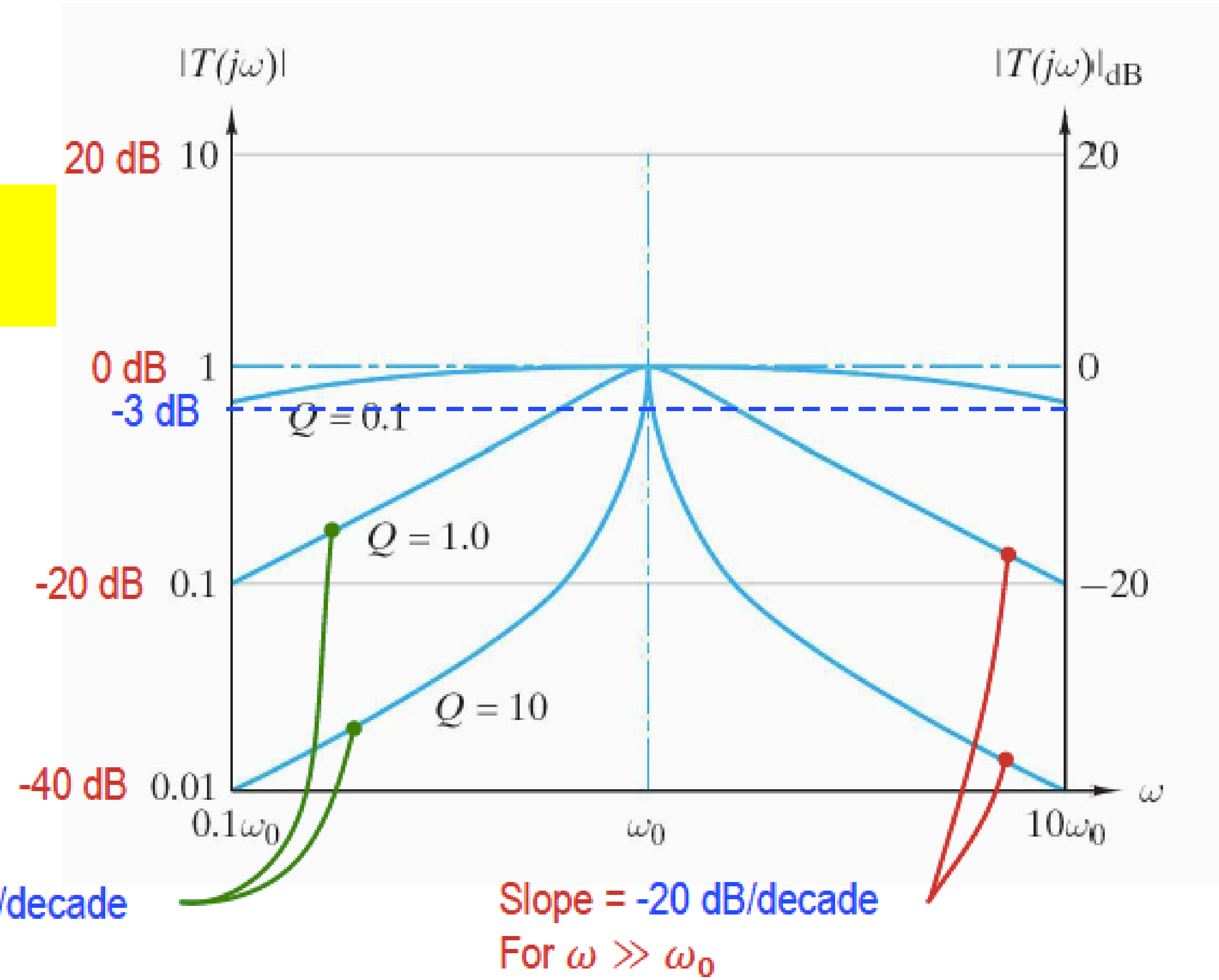
Bandwidth $\equiv B = \omega_{C2} - \omega_{C1} = \frac{R}{L}$



D) Calculation of: Quality Factor

$$Q = \frac{\omega_0}{B} = \frac{\sqrt{L/C}}{R}$$

High Q yields narrow band, or tuned, filters.



Second Order Frequency Response Review

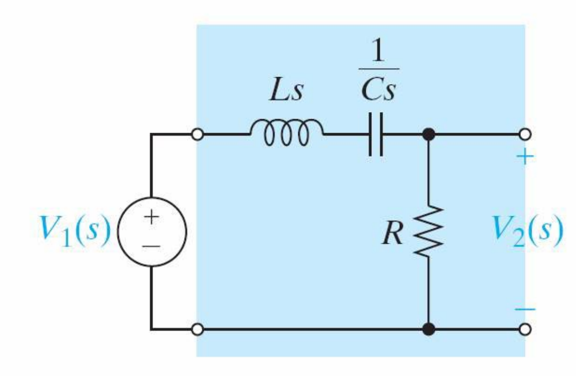
$$T(s) = \frac{R}{R + Ls + 1/Cs} = \frac{s\,\boxed{R/L}}{s^2 + s\,R/L + 1/LC}$$

$$T(s) = \frac{sB}{s^2 + s\,\boxed{\left(\frac{1}{\sqrt{LC}}\right)}R\sqrt{LC}/L + \boxed{1/LC}}$$

$$T(s) = \frac{sB}{s^2 + s\omega_0\boxed{\left(\frac{R}{\sqrt{L/C}}\right)} + \omega_0^2}$$

$$T(s) = \frac{sB}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

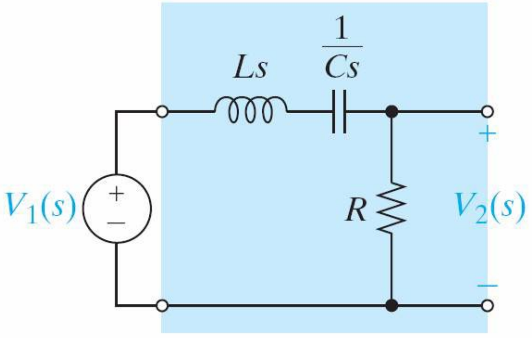
Exercise #3: Second Order Frequency Response



1) **What is the transfer function if the output is taken across the capacitor?**

$$T(s) = \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Exercise #4: Second Order Frequency Response



1) **What is the transfer function if the output is taken across the inductor?**

$$T(s) = \frac{s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

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FILTER POLES AND ZEROS

Analog Filters: Transfer Function

In general transfer functions of analog filters can be written as ratio of polynomials

$$T(s) = \frac{a_M s^M + a_{M-1} S^{M-1} + \cdots + a_0}{s^N + b_{N-1} S^{N-1} + \cdots + b_0}$$

- Degree of denominator: filter order
- For a stable filter $M \leq N$
- The coefficients of the polynomials are real numbers

The polynomials can be factored

$$T(s) = \frac{a_M (s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)}$$

- Numerator roots, z_1, z_2, z_M, are transfer function zeros, a.k.a transmission zeros
- Denominator roots, p_1, p_2, p_N, are transfer function poles, a.k.a natural modes
- Poles and zeros can be real numbers or complex numbers
- Any complex poles or zeros always occur in conjugate pairs

Exercise #5

A linear system is described by the transfer functions below

$H_1(s) = 1 / (2s+100)$.   H1(s) = (2s+1) / (s2+5s+6).

Find the system poles and zeros and plot the Bode plot

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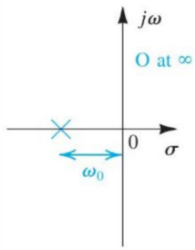
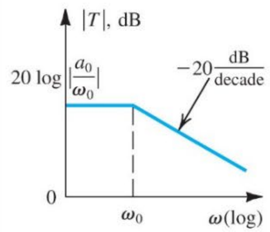
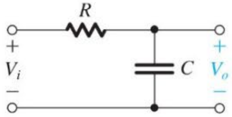
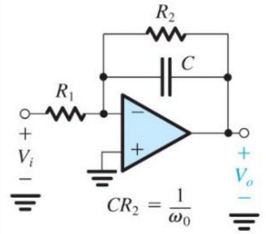
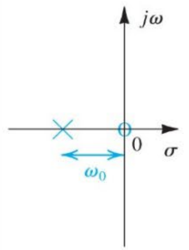
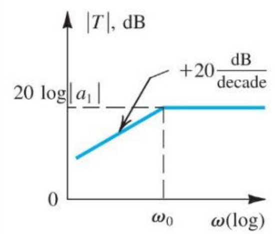
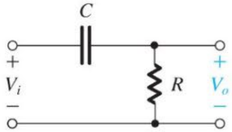
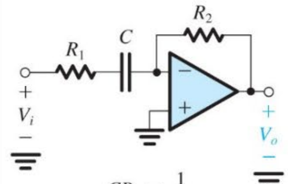
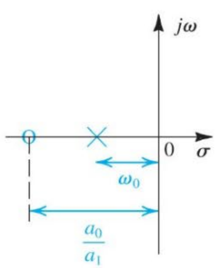
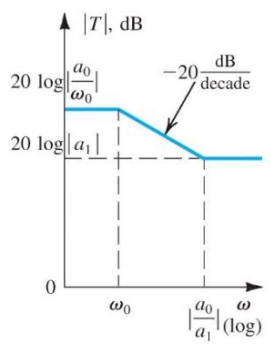
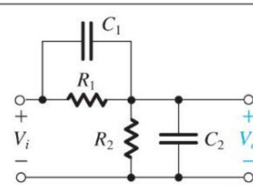
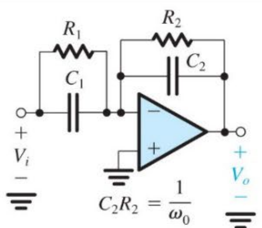
REVIEW WITHIN THE FRAMEWORK OF ACTIVE FILTER REALIZATIONS

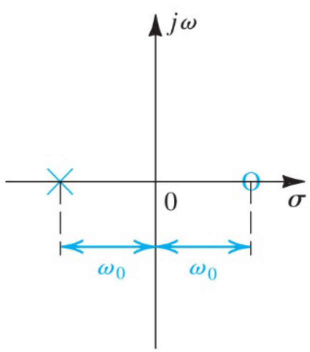
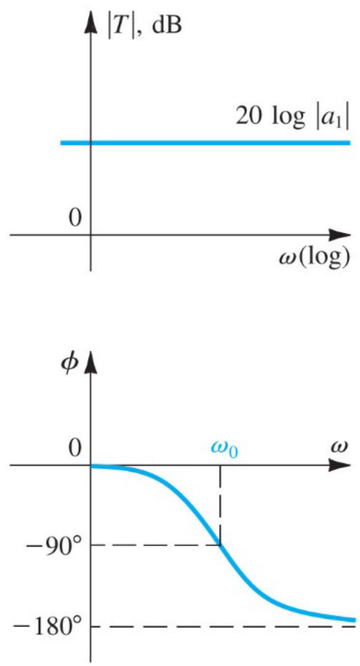
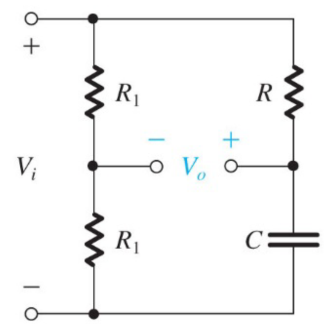
First Order Filters (Summary)

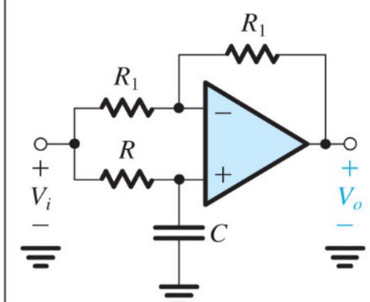
Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\lvert T \rvert$	Passive Realization	Op Amp–RC Realization
(a) Low pass (LP) $T(s) = \dfrac{a_0}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ DC gain $= 1$	 $CR_2 = \dfrac{1}{\omega_0}$ DC gain $= -\dfrac{R_2}{R_1}$
(b) High pass (HP) $T(s) = \dfrac{a_1 s}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ High-frequency gain $= 1$	 $CR_1 = \dfrac{1}{\omega_0}$ High-frequency gain $= -\dfrac{R_2}{R_1}$
(c) General $T(s) = \dfrac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 // R_2) = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_1}{a_0}$ DC gain $= \dfrac{R_2}{R_1 + R_2}$ HF gain $= \dfrac{C_1}{C_1 + C_2}$	 $C_2 R_2 = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_1}{a_0}$ DC gain $= -\dfrac{R_2}{R_1}$ HF gain $= -\dfrac{C_1}{C_2}$

First order All-Pass Filter

<table>
<tr><th>T(s)</th><th>Singularities</th><th>|T| and φ</th><th>Passive Realization</th><th>Op Amp–RC Realization</th></tr>
<tr>
<td>All pass (AP)

$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$

$a_1 > 0$</td>
<td>
</td>
<td>
</td>
<td>

$CR = 1/\omega_0$
Flat gain $(a_1) = 0.5$</td>
<td>

$CR = 1/\omega_0$
Flat gain $(a_1) = 1$

$\left|\frac{V_o}{V_i}\right| = 1$
$\phi(\omega) = -2 \tan^{-1}(\omega CR)$</td>
</tr>
</table>

Exercise #6

a) Derive an expression of the transfer function $T(s)=Vo/V_{IN}$ in terms of R1, R2, and C.

b) Determine the order of T(s) and the number of poles and zeros.

c) Write an expression for the location of each pole and zero in T(s).

d) Write an expression for the magnitude of T(s) at DC and at infinite frequency in dB-scale.

e) Sketch the magnitude of T(s) (in dB-scale) versus ω. Indicate on the sketch the expressions for the DC gain, gain at infinity, and the location of any poles and zeros.