Last Class…

A current source in a linear circuit has i = 15 cos(25πt +25)

A. What is the amplitude of the current? $A = 15$

B. What is the angular frequency? $\omega = 25\pi$

C. Find the frequency of the current. $f = \frac{\omega}{2\pi} = 12.5\ Hz$

D. What is the phase? $\phi = 25^\circ$

$\omega = 2\pi f$

THE OHIO STATE UNIVERSITY

COLLEGE OF ENGINEERING

Phasor Domain

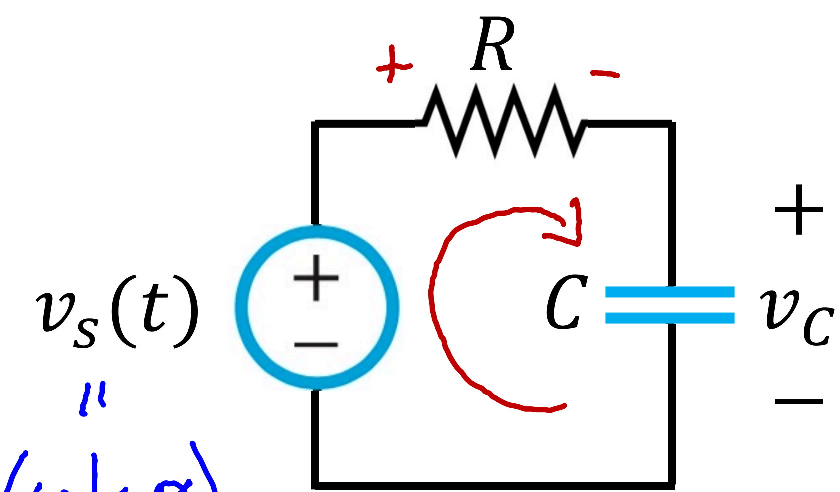
This Lecture…

- Learning Objectives:
  - Transform time-varying sinusoidal functions to the phasor domain and vice versa.

AC Circuits

On time domain:

- AC circuit with capacitors or inductors challenging
  - i-v relationships are time dependent



$$= A\cos(\omega t + \phi)$$

series, same current

$$i_C(t) = C \cdot \frac{dV_c(t)}{dt} = i_R$$

KVL

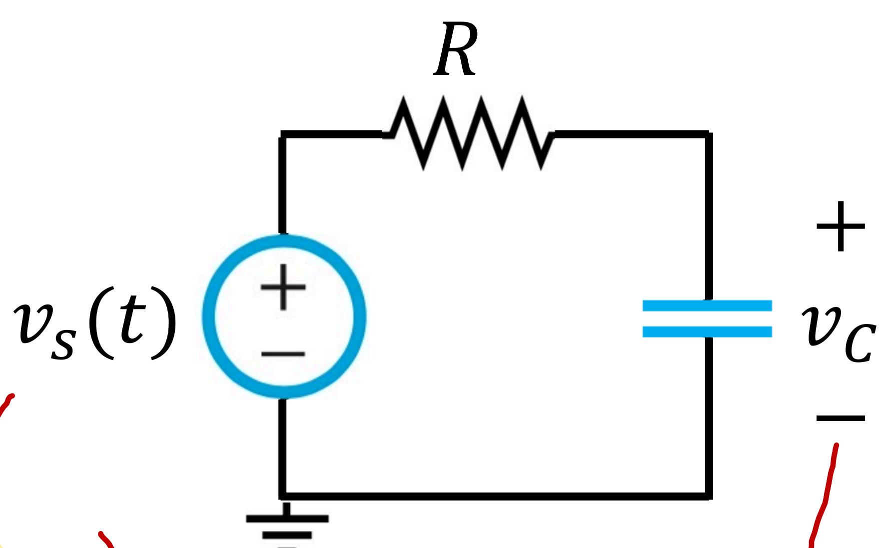
$$A\cos(\omega t + \phi) = V_R + V_c(t)$$

$$A\cos(\omega t + \phi) = R \cdot i_R + V_c(t)$$

$$A\cos(\omega t + \phi) = RC\,\frac{dV_c(t)}{dt} + V_c(t)$$

AC Circuits

- AC circuit: Circuit with a sinusoidal source.
- AC circuit with capacitors or inductors is described by a differential equation.
  - May be challenging to solve because i-v relationships are time dependent.



$A\cos(\omega t + \emptyset)$

$B\cos(\omega t + \gamma)$

Phasor domain

- Sinusoidal signals can be represented as complex numbers.
- Differential equations get converted into linear equations with no sinusoidal functions.

Phasor Domain

- The phasor-analysis technique transforms equations from the time domain to the phasor domain.

Time domain:

time is lowercase

$$v(t) = A\cos(\omega t + \varphi)$$

$$= \text{Real}\left\{Ae^{(\omega t + \phi)j}\right\}$$

$$= \text{Real}\left\{\underbrace{Ae^{\phi j}}_{\text{Real}} + \underbrace{e^{\omega t j}}_{\text{same for everything}}\right\}$$

same ω

Phasor domain:

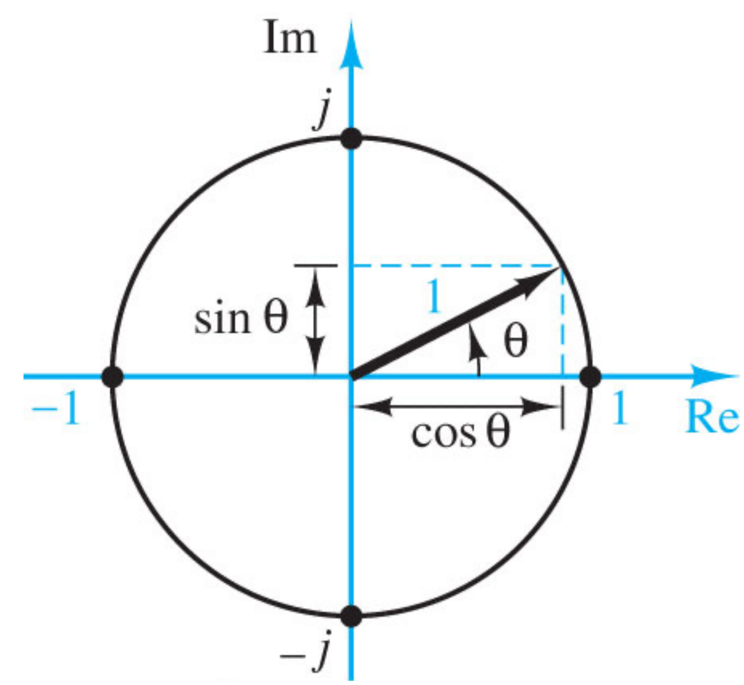
phasor is bold

$$\mathbf{V}(j\omega) = Ae^{\theta j}$$

Frequency is common to all voltages and currents, so it is avoid in the phasor form.

Euler's identity: $Z = me^{\theta j}$

$$Z = \underbrace{m\cos(\theta)}_{\text{real}} + \underbrace{m\sin(\theta)j}_{\text{imaginary}}$$



Phasor Domain

Time domain:

$v_1(t) = A\cos(\omega t + \varphi) = 10\cos(5t+45)$

$v_2(t) = A\cos(\omega t) = \cos(100t)$

$v_3(t) = A\cos(\omega t - 90°) = 5\cos(t-90)$

$v_4(t) = A\sin(\omega t) = 20\sin(2t)$

$\hookrightarrow 20\cos(2t-90)$

$$\frac{dv(t)}{dt}$$

$$\int v(t)\,dt$$

Phasor domain:

$V_1(5j) = 10e^{45j}$

$V_2(100j) = 1\cancel{e^{0j}} = 1$

$V_3(1j) = 5e^{-90j} = -5j$

$V_4(2j) = 20e^{-90j} = -20j$

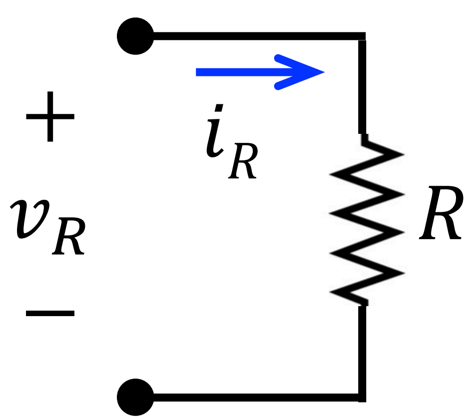
Impedance

- *i-v* relationships of resistors, inductors, and capacitors can be expressed in phasor notation.
- Phasors and impedance simplify AC circuit analysis.
  - Allow use of same solution methods as DC circuits.

Impedance

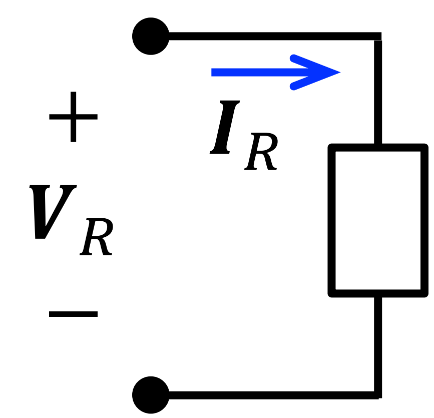
i-v relationships of resistors, inductors, and capacitors can be expressed in phasor notation.

Time domain:



Impedance: Ratio of phasor voltage to phasor current.

Phasor domain:

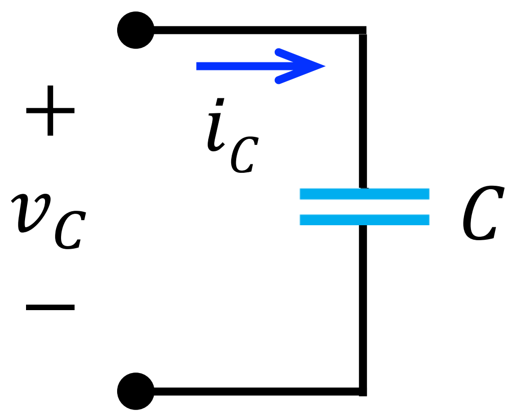


Impedance

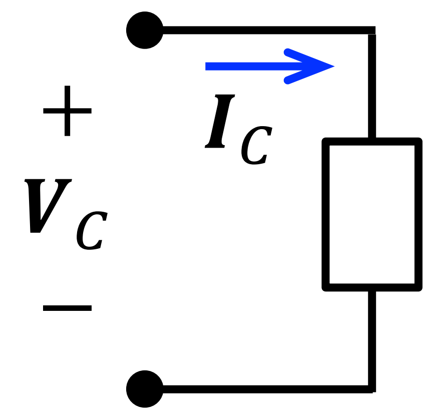
Impedance: Ratio of phasor voltage to phasor current.

$$\boxed{\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}}}$$

Time domain:



Phasor domain:



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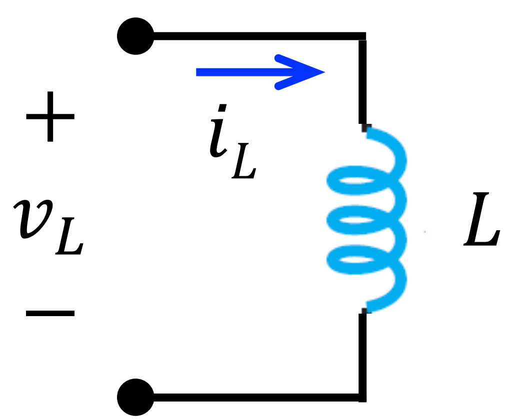
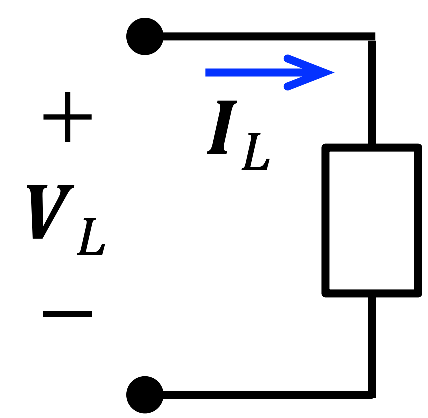


Phasor domain:



Impedance

Rectangular form   Polar form

$$\boxed{\mathbf{V} = \mathbf{I}\,Z}$$

Resistor: $Z_R = R$

Capacitor: $Z_C = \dfrac{1}{j\omega C}$

Inductor: $Z_L = j\omega L$