Energy of the Simple Harmonic Oscillator

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = -\omega A\sin(\omega t + \phi)$$

Kinetic Energy $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$

Potential Energy $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$

Total Mechanics Energy $E = \frac{1}{2}kA^2$

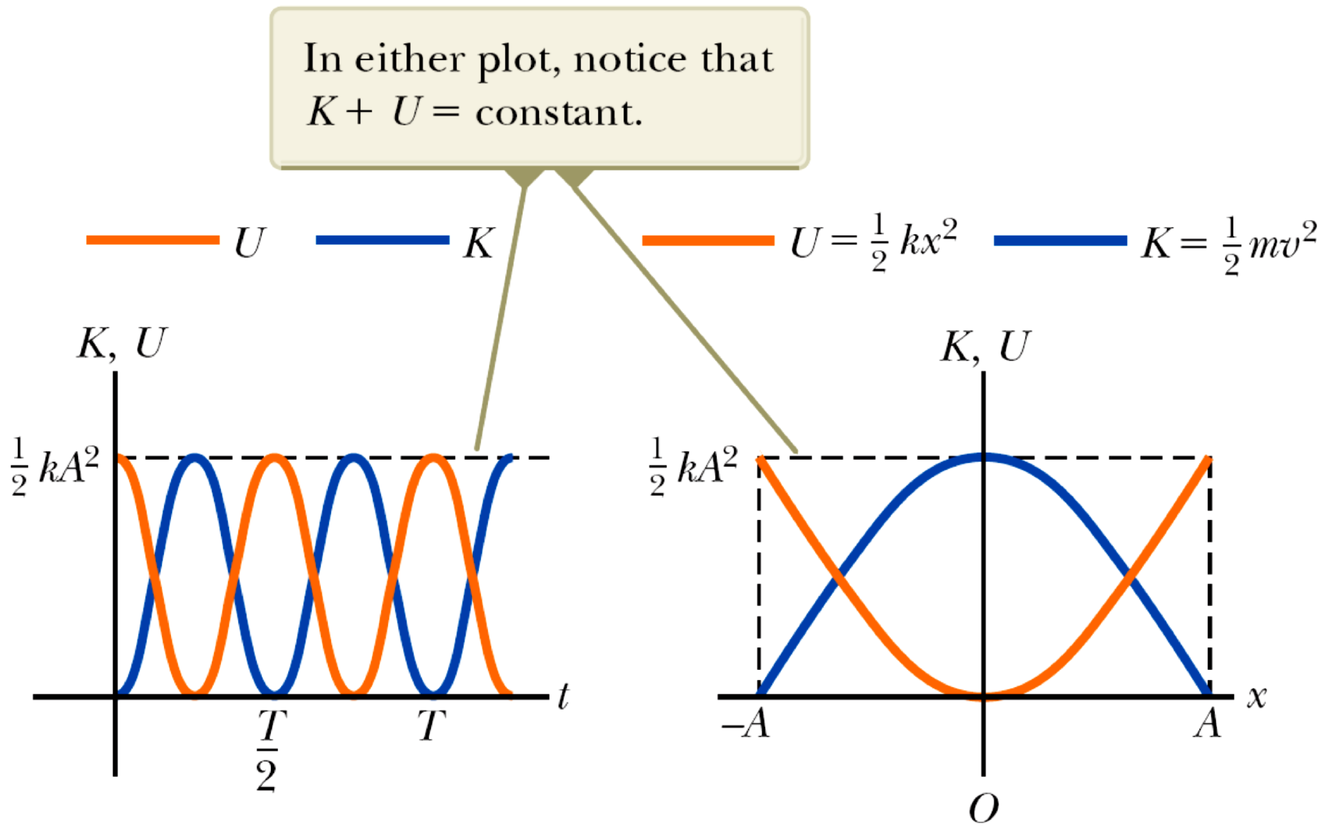
$$E = K + U = \frac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

Total Mechanics Energy $E = \frac{1}{2}kA^2$

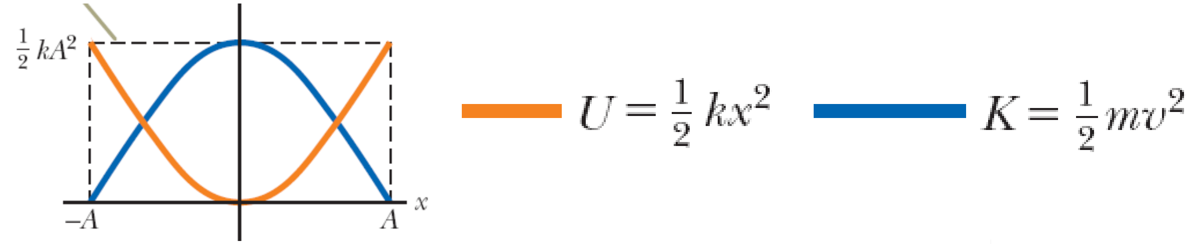
Knowing x and E to find v

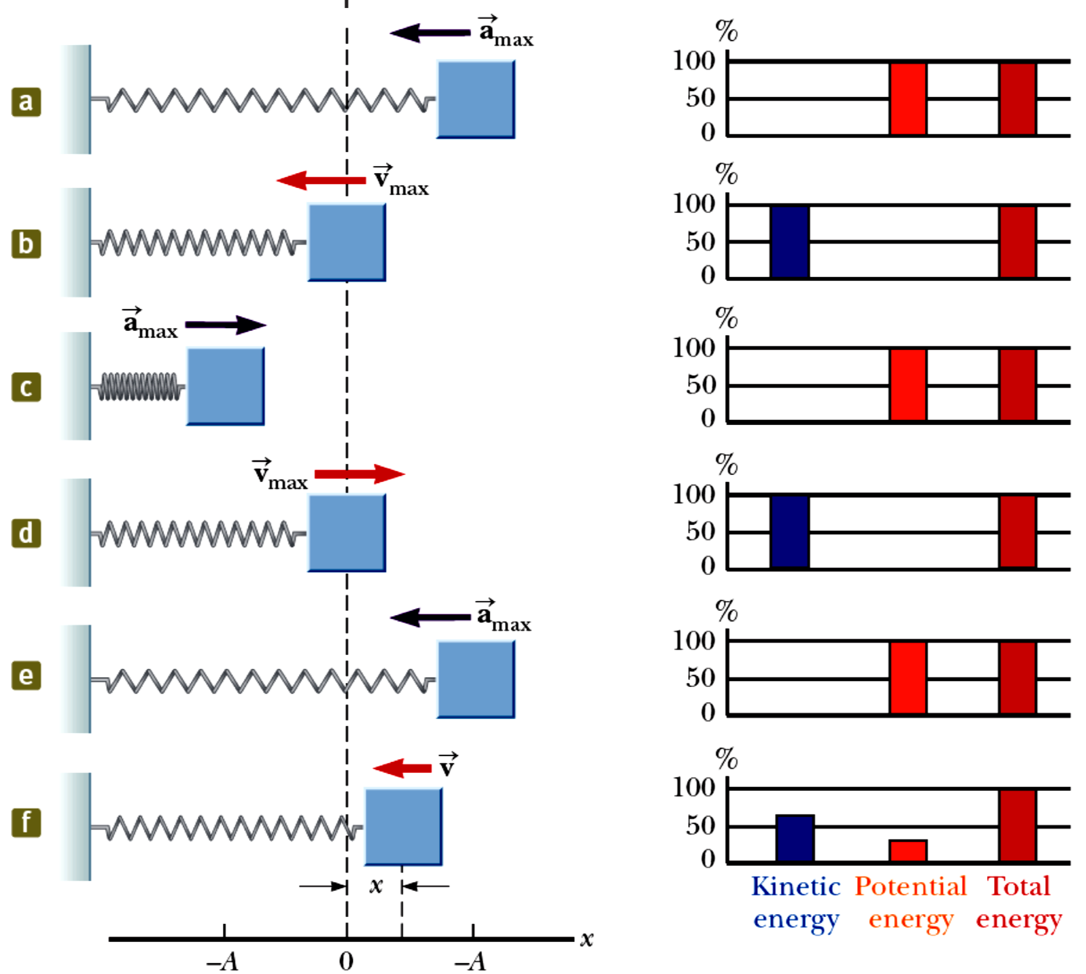
$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{A^2 - x^2}$$



Total Mechanics Energy $E = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

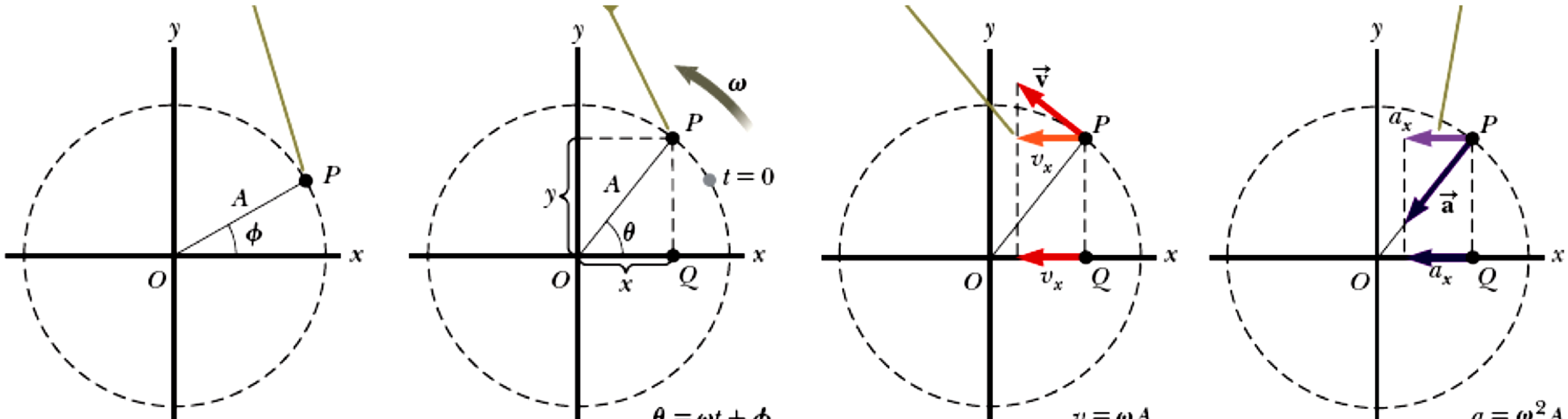




Simple Harmonic Motion and Circular Motion

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = -\omega A\sin(\omega t + \phi)$$



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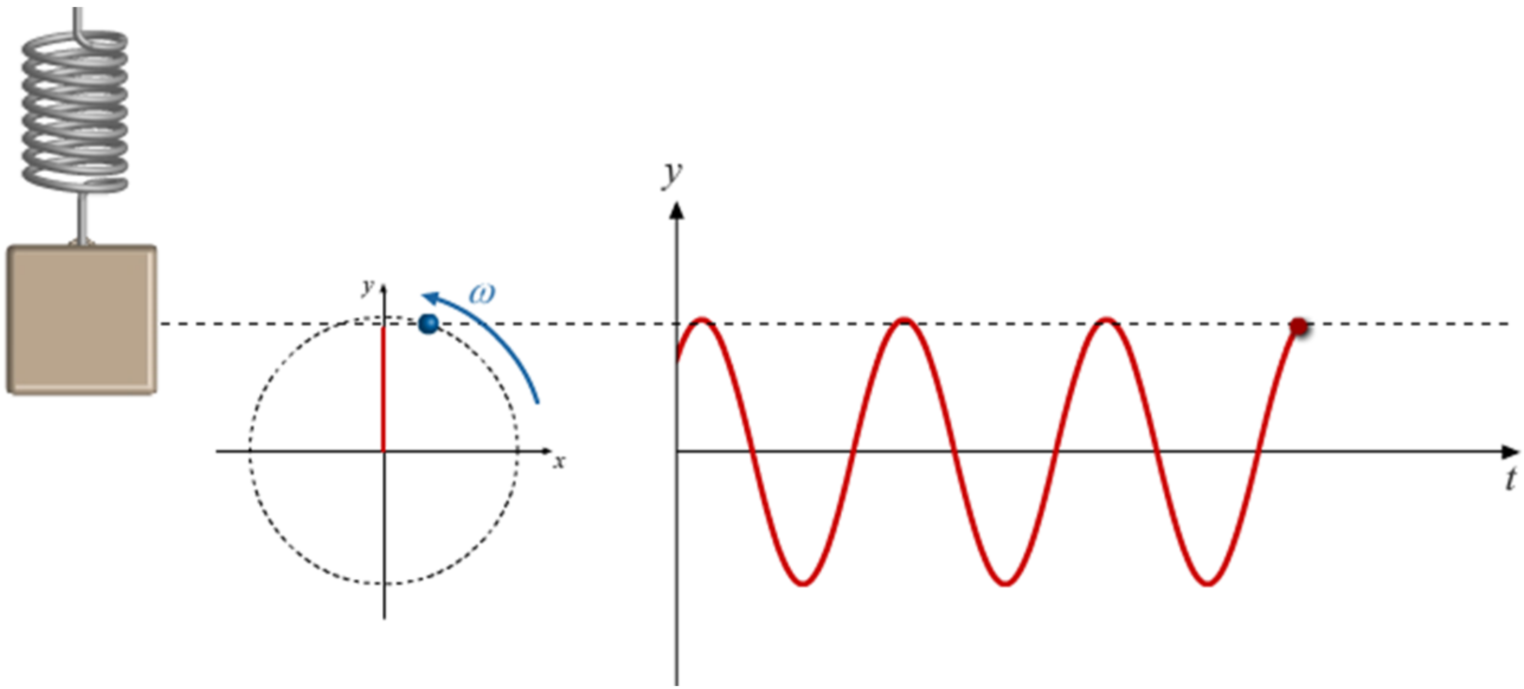
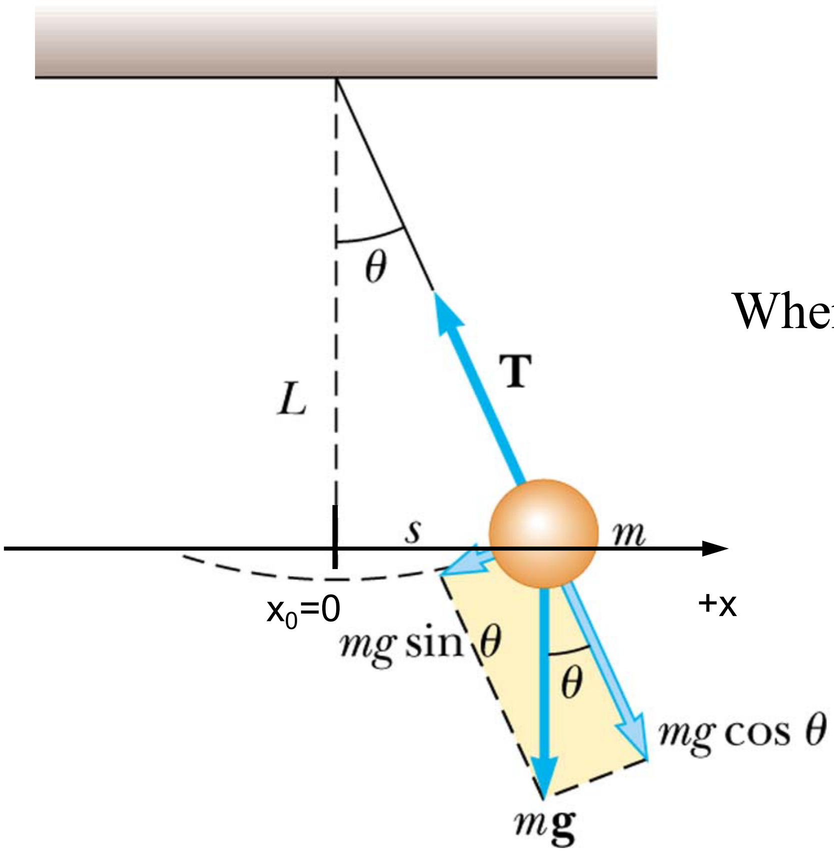




Simple Pendulum

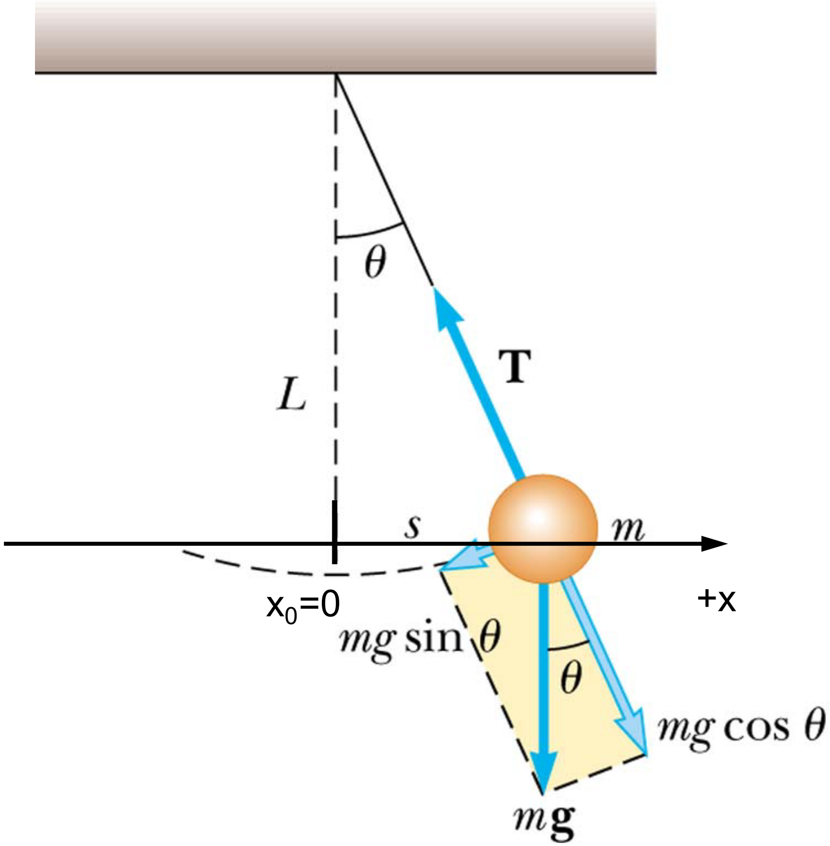
When θ is small

$$F = -mg\sin\theta$$

$$\sin\theta = \frac{x}{\sqrt{x^2+L^2}} \approx \frac{x}{L}$$

$$F \approx -\frac{mg}{L}x$$

Looks like Hooke's law (k → mg/L)



When θ is small

$$s \approx x, \quad \sin\theta \approx \theta, \quad s = L\theta,$$

$$-mg\sin\theta \approx -mg\theta = -\frac{mg}{L}s$$

$$-\frac{mg}{L}s = m\frac{d^2s}{dt^2} \quad \Leftarrow F_t = ma$$

$$-mg\theta = mL\frac{d^2\theta}{dt^2} \quad \Leftarrow \frac{s}{L} = \theta$$

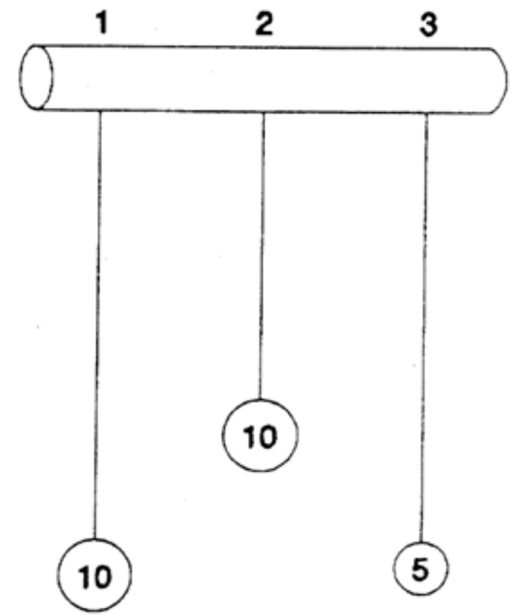
$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

$$\theta(t) = \theta_{\max}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{g}{L}}, \quad T = \frac{2\pi}{\omega}$$

Pendulum Experiment

On the right is a drawing of three strings hanging from a bar. The three strings have metal weights attached to their ends.



String 1 and String 3 are the same length. String 2 is shorter.

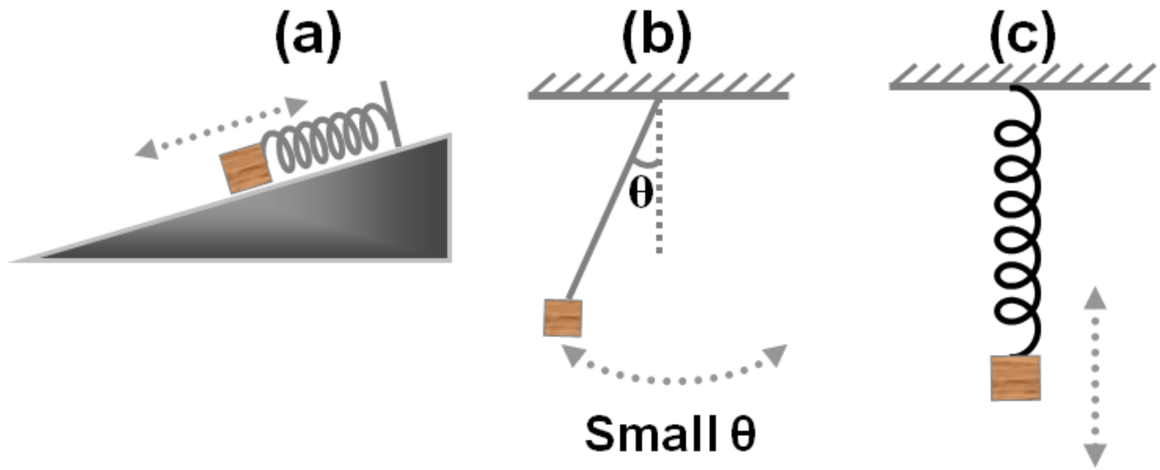
A 10-unit weight is attached to the end of String 1.
A 10-unit weight is also attached to the end of String 2.
A 5-unit weight is attached to the end of String 3.

The strings (and attached weights) can be swung back and forth and the time it takes to make a swing can be timed.

Suppose you want to find out whether the length of the string has an effect on the time it takes to swing back and forth.
Which strings would you use to find out?

a. only one string
b. all three strings
c. strings 2 and 3
d. strings 1 and 3
e. strings 1 and 2

The following diagrams show an object in different motions. Which of the following demonstrate(s) simple harmonic motion? (Neglect frictional effects.)



(1) (a)

(2) (b)

(3) (c)

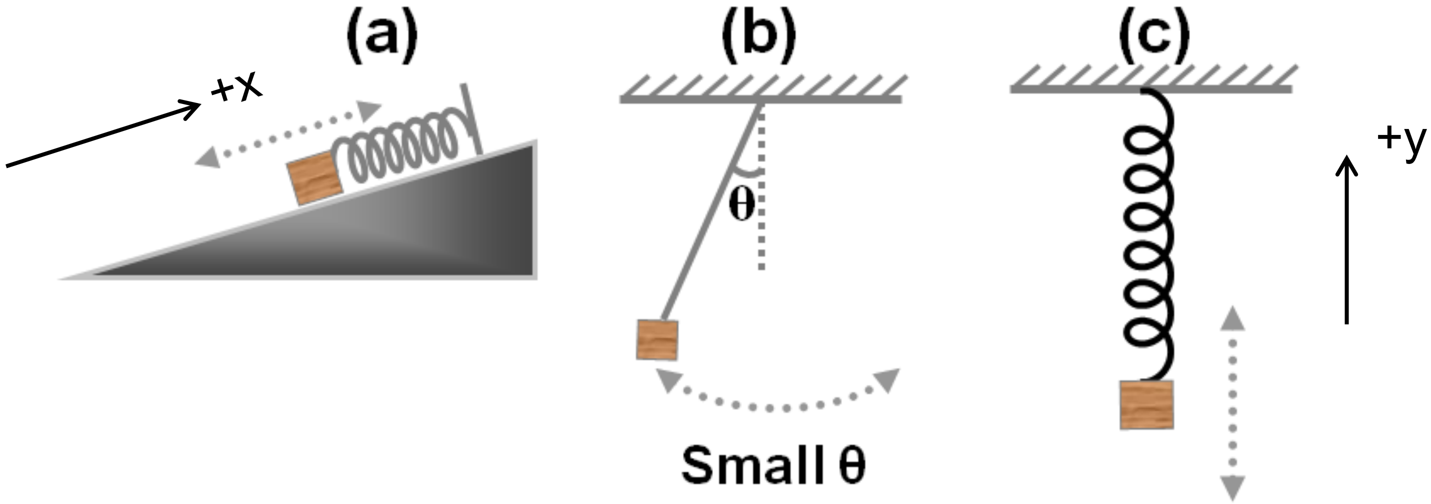
(4) (a) & (b)

(5) (a) & (c)

(6) All of the above

The following diagrams show an object in different motions. Which of the following demonstrate(s) simple harmonic motion? (Neglect frictional effects.)

$F_{net} = -kx$ $\quad$ $F_{net} \approx -mg\theta$ $\quad$ $F_{net} = -ky$



(1) (a) $\qquad$ (2) (b)

(3) (c) $\qquad$ (4) (a) & (b)

(5) (a) & (c) $\qquad$ (6) All of the above