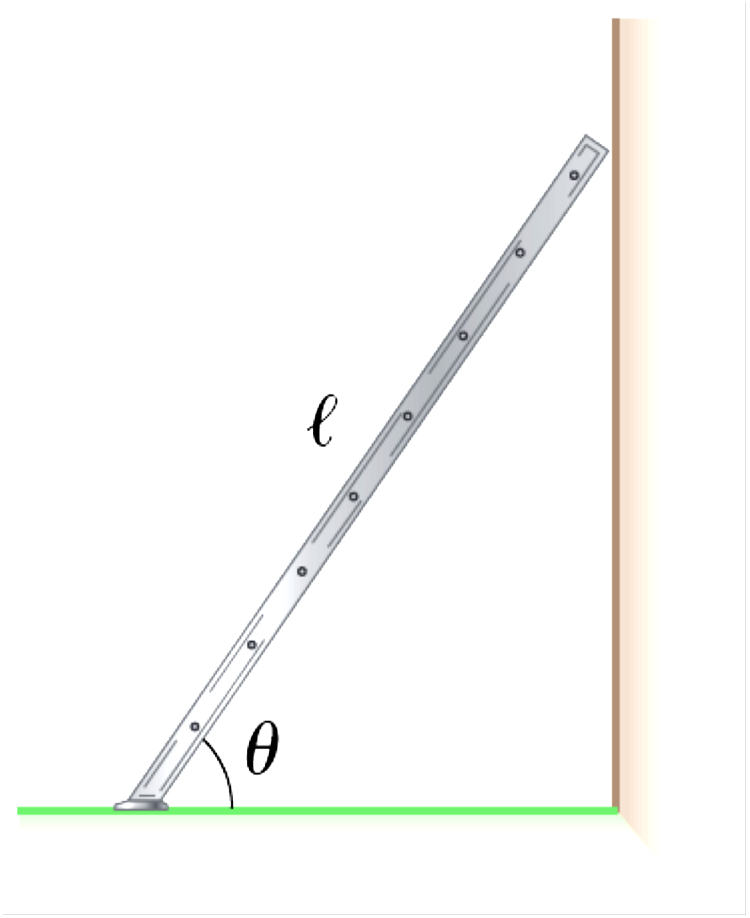
Static Equilibrium (Chapter 12)

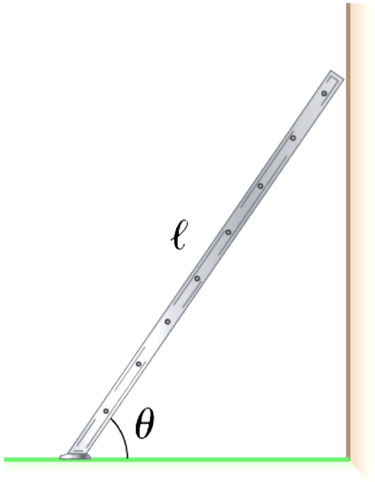
$$\sum F = 0 \quad \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$$

$$\sum \tau = 0$$

Draw a Free Body Diagram

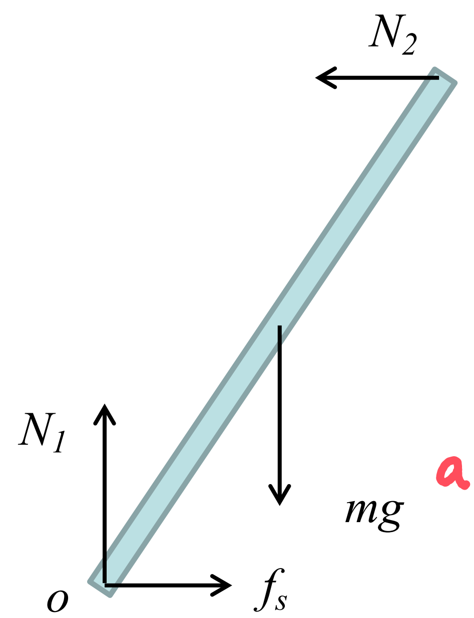


A uniform ladder of length L, rests against a smooth, vertical wall The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ_{min} at which the ladder does not slip.



$$\sum F = 0$$

$$\sum \tau = 0$$



always on c.o.m.

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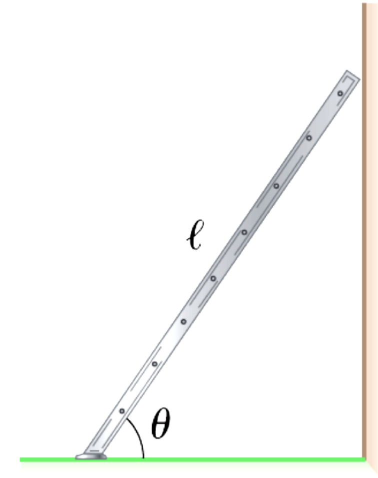


$$\sum F = 0$$

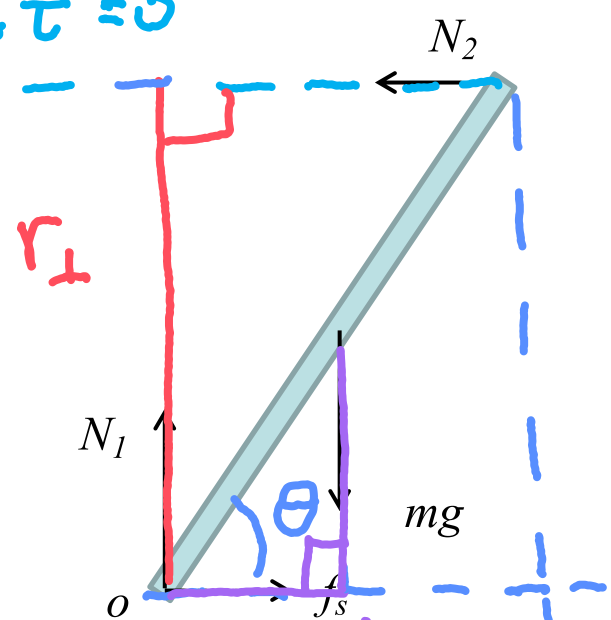
$$\sum \tau = 0$$

$$\sum F_y = 0 \quad N_1 - mg = 0 \quad N_1 = mg$$

$$\sum F_x = 0 \quad f_s - N_2 = 0 \quad N_2 = f_s$$

$$f_{s\_MAX} = \mu_s N_1 = \mu_s mg \qquad N_{2\_MAX} = \mu_s mg$$

$\Sigma \vec{\tau} = 0$

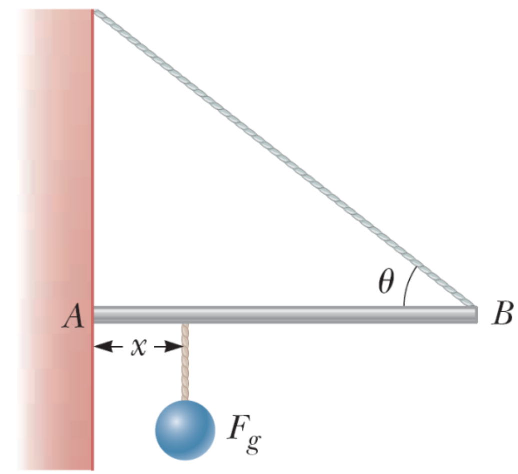


$$\sum \tau = 0 \quad mg\frac{\cancel{L}}{2}\cos\theta - N_2 \cancel{L}\sin\theta = 0$$

$$\tan\theta = \frac{mg}{2N_2} \qquad \tan\theta_{Min} = \frac{1}{2\mu_s}$$

$r_\perp = L\sin\theta$ $\qquad$ $r_\perp = \frac{L}{2}\cos\theta$

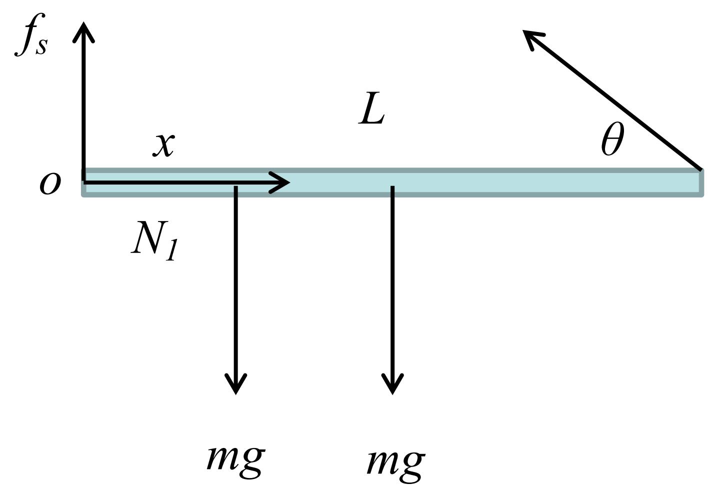
One end of a uniform 4.00-m-long rod of weight F_g is supported by a cable at an angle of $\theta = 37°$ with the rod. The other end rests against the wall, where it is held by friction as shown in the figure below. The coefficient of static friction between the wall and the rod is $\mu_s = 0.500$. Determine the minimum distance x from point A at which an additional object, also with the same weight F_g, can be hung without causing the rod to slip at point A..



$$\sum F = 0$$

$$\sum \tau = 0$$

- How many forces? Draw force diagram



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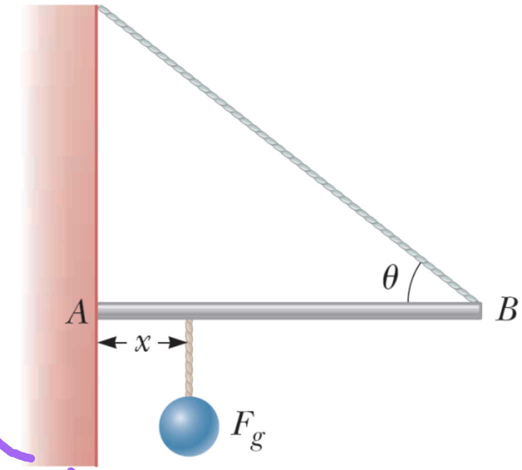


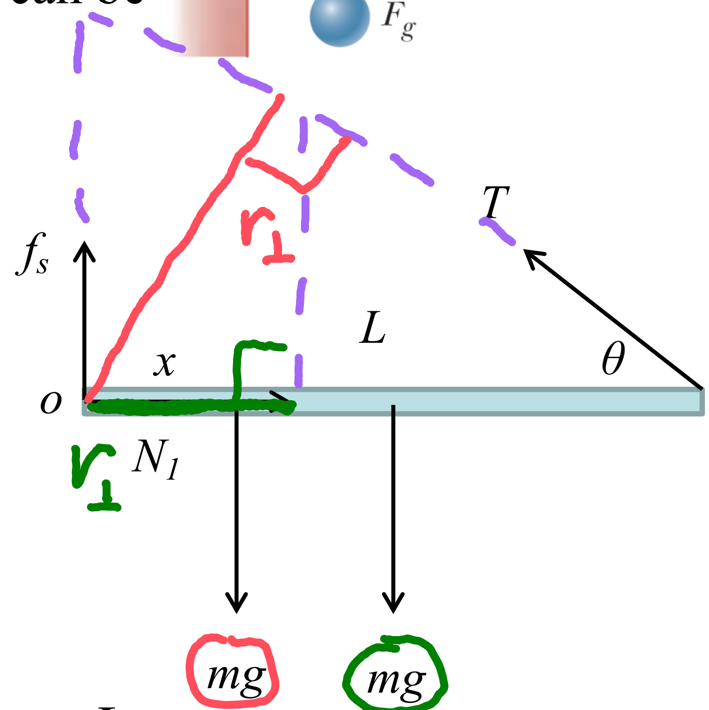
Split forces into comps.

$$\sum F_x = 0 \qquad N_1 - T\cos\theta = 0$$

$$\sum F_y = 0 \qquad f_s + T\sin\theta - 2mg = 0$$

$$f_{s\_MAX} = \mu_s N_1 = \mu_s T\cos\theta$$

$\tau = r_\perp F$



solve for T and x

$$\sum \tau = 0 \qquad mgx + mg\frac{L}{2} - TL\sin\theta = 0$$

$$\mu_s T\cos\theta + T\sin\theta = 2mg$$

$$T = \frac{2mg}{\mu_s\cos\theta + \sin\theta} \qquad x = \frac{2L\sin\theta}{\mu_s\cos\theta + \sin\theta} - \frac{L}{2}$$

$V = \omega R$ $\quad a = \alpha R$

Rotational Kinetic Energy around P

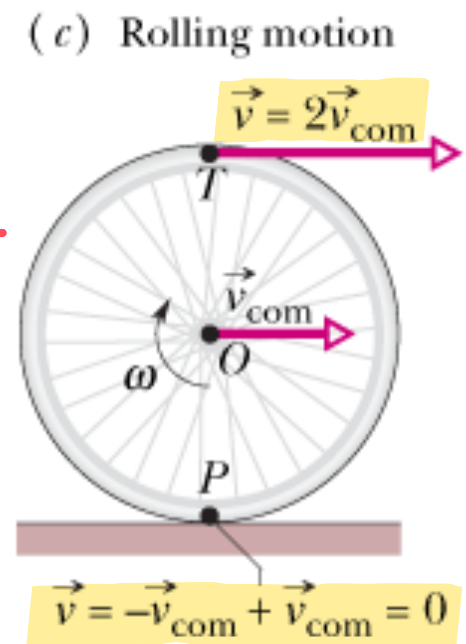
$$K = \frac{1}{2} I_P \omega^2$$

$$I_P = I_{com} + MR^2$$

$$K = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} MR^2 \omega^2$$

$$K = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} MV_{com}^2$$

Velocity different at every point



Travelling 1 rotation → linear distance $= 2\pi R = V_{com} \cdot T$

$$S = 2\pi R = \omega R T$$

Kinetic energy of rolling:

$$K_{TOTAL} = K_{ROTATIONAL} + K_{TRANSLATIONAL}$$

$$K_{TOTAL} = \frac{1}{2} I_{COM} \omega^2 + \frac{1}{2} M v_{COM}^2$$

Conservation of energy including rotational energy:

$$U_{gf} + K_{Rf} + K_{Tf} = U_{gi} + K_{Ri} + K_{Ti}$$

Moment of inertia:

$$I = \sum m_i r_i^2 \quad (\text{point particles})$$

$$I_{COM} = MR^2 \quad (ring)$$

$$I_{COM} = \frac{1}{2}MR^2 \quad (disk)$$

$$I_{COM} = \frac{2}{5}MR^2 \quad (sphere)$$

$$I_{COM} = \frac{1}{12}ML^2 \quad (rod)$$

$$I = I_{COM} + Mh^2 \quad (\text{axis displaced from COM by } h)$$

$$K_R = \frac{1}{2}I\omega^2 \quad (\text{rotational kinetic energy})$$

$$W_R = \int \tau d\theta \quad (\text{rotational work})$$

$$\tau = rF\sin(\theta) \quad (torque)$$

$$K_{rot} = \frac{1}{2}I\omega^2$$

$$E_{mech} = K_{rot\_com} + K_{trans} + U$$

$$= \frac{1}{2}I_{com}\omega^2 + \frac{1}{2}mV_{com}{}^2 + U$$

Find the V_{com} at the bottom of the ramp.

$R=0.2\ m$ $\quad V_0=0$

$m=2.0\ kg$

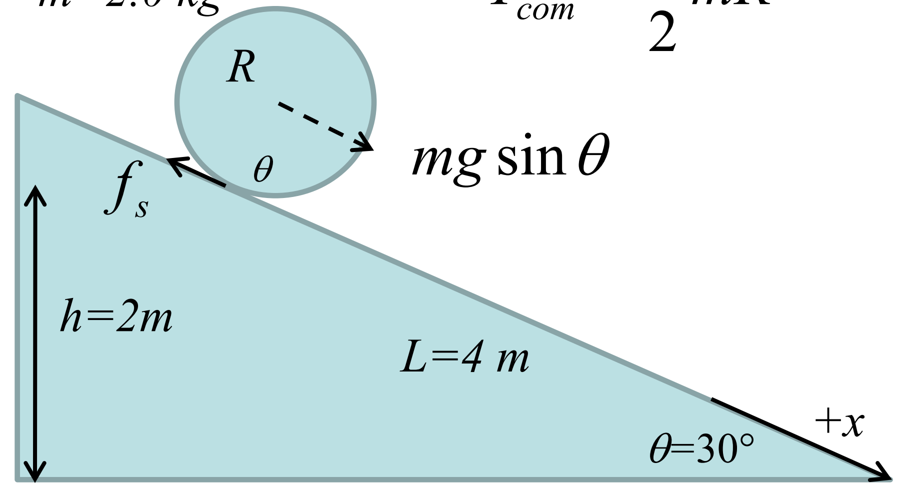
$$I_{com} = \frac{1}{2} mR^2$$

$$U_f + K_{rot\_f} + K_{com\_f} = U_i + K_{rot\_i} + K_{com\_i}$$

$$K = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} MV_{com}^2$$

use energy conservation

rot. energy + tang. energy



$$\alpha R = a_{com}$$

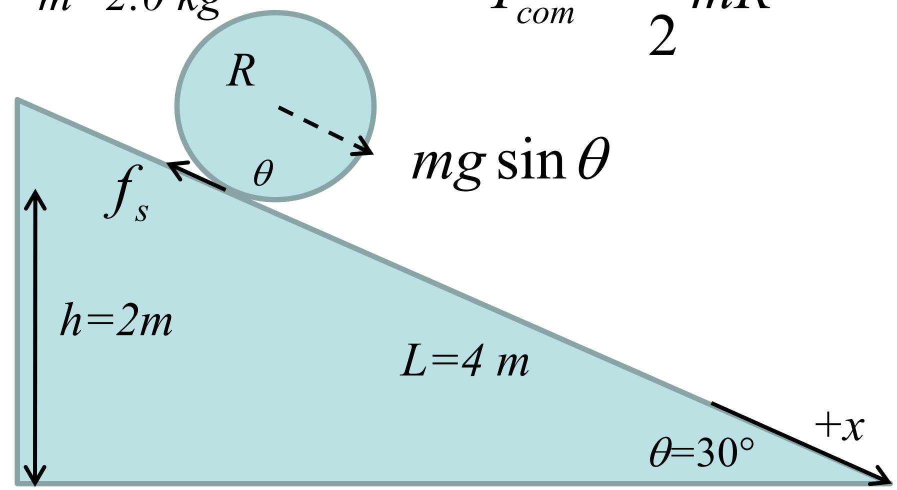
$$\omega R = V_{com}$$

Find the V_{com} at the bottom of the ramp.

$R=0.2\ m$ $\quad V_0=0$

$m=2.0\ kg$

$$I_{com}=\frac{1}{2}mR^2$$



$$U_f+K_{rot\_f}+K_{com\_f}=U_i+K_{rot\_i}+K_{com\_i}$$

$$K=\frac{1}{2}I_{com}\omega^2+\frac{1}{2}mV_{com}^2$$

$$mgh=\frac{1}{2}I_{com}\omega^2+\frac{1}{2}mV_{com}^2$$

$$=\frac{1}{2}I_{com}\omega^2+\frac{1}{2}mV_{com}^2=\frac{1}{2}I_{com}\frac{V_{com}^2}{R^2}+\frac{1}{2}mV_{com}^2$$

$$=\frac{1}{2}\left(\frac{I_{com}}{R^2}+m\right)V_{com}^2$$

$$\alpha R=a_{com}$$

$$\omega R=V_{com}$$

$$2gh=\left(\frac{I_{com}}{mR^2}+1\right)V_{com}^2$$

$$V_{com}^2=\frac{2gh}{\left(\frac{I_{com}}{mR^2}+1\right)}$$

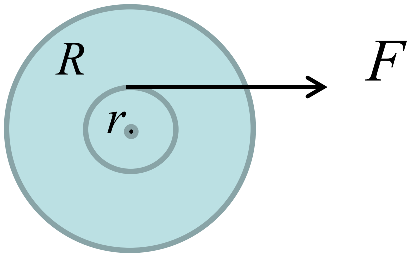
$\tau = FR = I\alpha$

Motion under External Force and Torque

$$I_{com} = \frac{1}{2} mR^2$$

$$F_{net} = ma_{com}$$

$$\tau_{net} = Fr = I_{com}\alpha$$



Static Equilibrium (Chapter 12)

$$F_{net} = ma_{com} = 0$$

$$\tau_{net} = Fr = I_{com}\alpha = 0$$

Motion under External Force and Torque

$$I_{com} = \frac{1}{2} mR^2$$

$$F_{net} = ma_{com}$$

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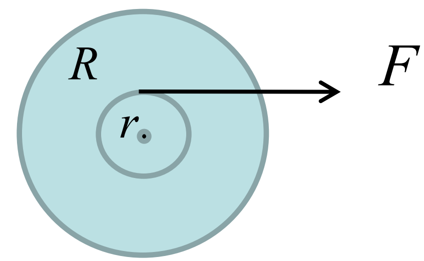


What is the V_{com} after Δt ?

$$V_{com} = \overset{0}{V_0} + a_{com}\Delta t = a_{com}\Delta t = \frac{F}{m}\Delta t$$

$$F\Delta t = mV_{com}$$

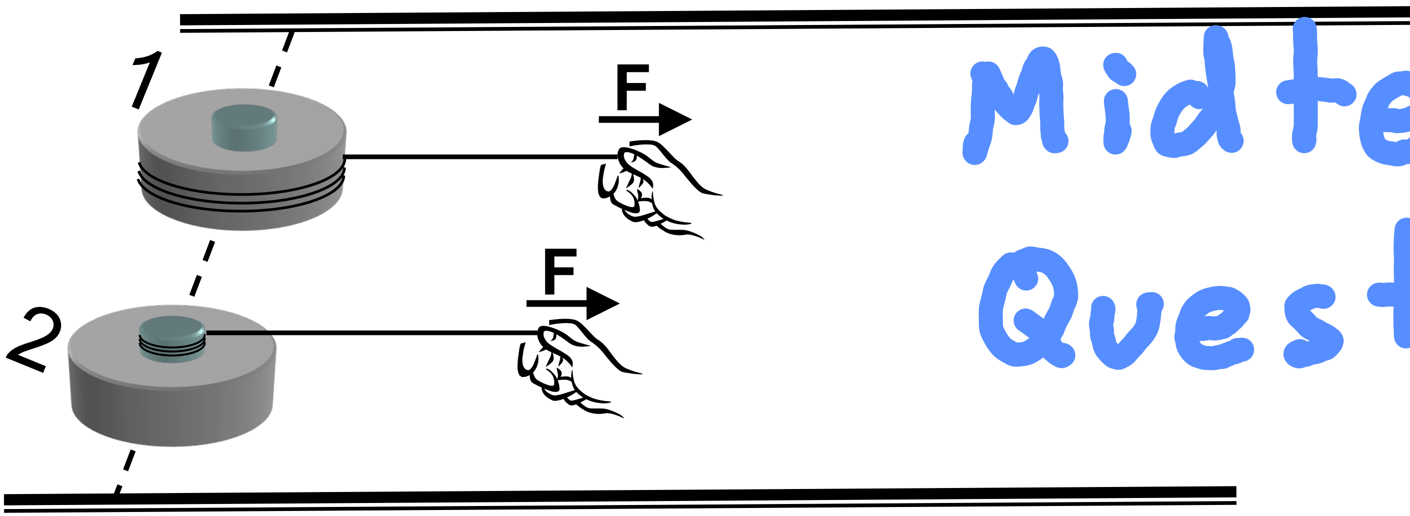
What is the ω after Δt ?

$$\omega = \overset{0}{\omega_0} + \alpha\Delta t = \alpha\Delta t = \frac{Fr}{I_{com}}\Delta t$$

$$\tau\Delta t = I\omega$$

$$\omega = \frac{\tau\Delta t}{I_{com}} = \frac{Fr\Delta t}{I_{com}}$$

Strings are wound around two ***identical*** pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force ***F***. Both pucks start to move on a frictionless surface. 5 seconds later, which puck has greater center-of-mass speed?

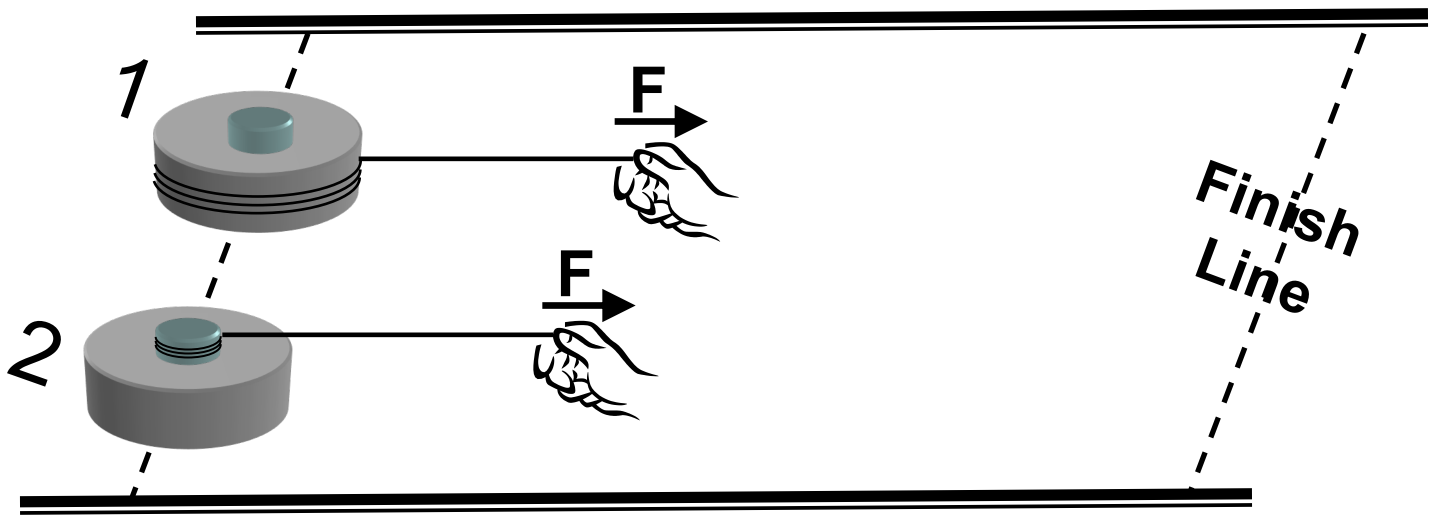


Midterm Question

1. Puck 1
2. Puck 2
3. Both have the same C.O.M speed
4. Not enough info. to determine

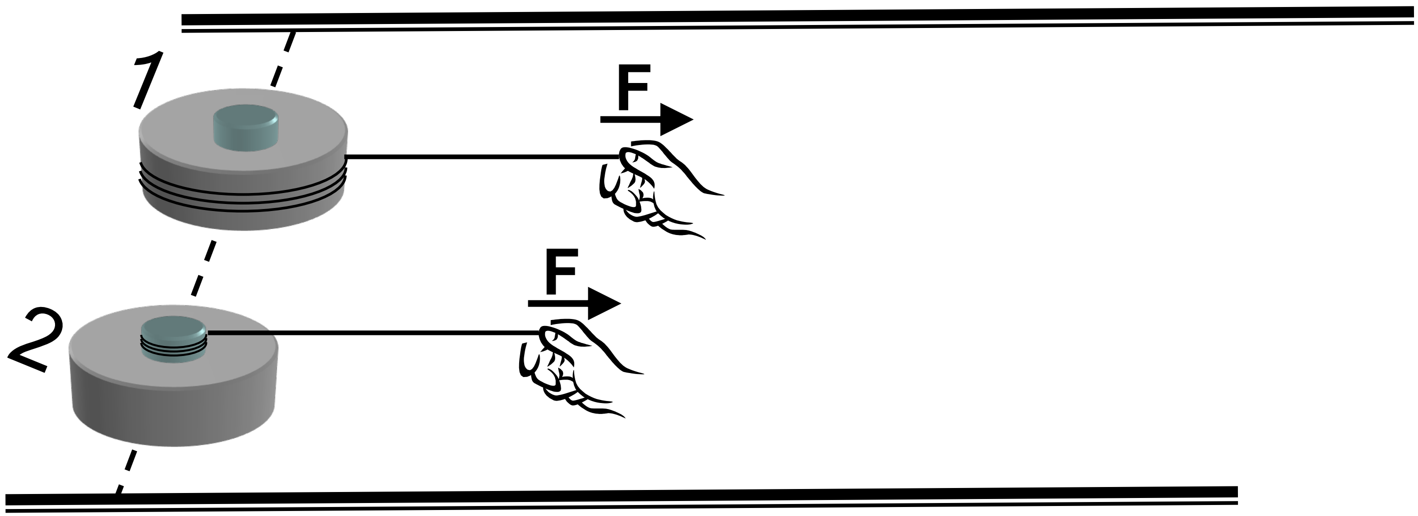
$v_{com} = \frac{F}{m}\Delta t$

Strings are wound around two ***identical*** pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force ***F***. Both pucks start to move on a frictionless surface. Which puck arrives at the finish line first?



1. Puck 1
2. Puck 2
3. Both arrive at the same time
4. Not enough info. to determine

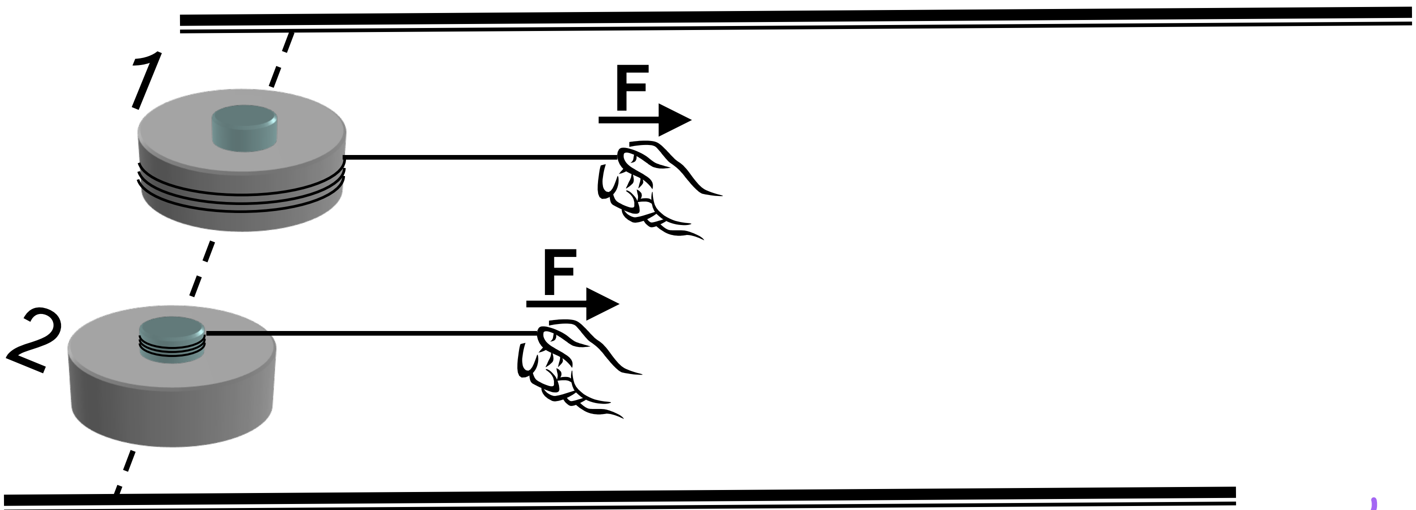
Strings are wound around two ***identical*** pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force ***F***. Both pucks start to move on a frictionless surface. 5 seconds later, which puck has greater rotational kinetic energy?



1. Puck 1
2. Puck 2
3. Both have the same kinetic energy
4. Not enough info. to determine

$\tau = rF\sin\theta$

Strings are wound around two ***identical*** pucks: one is around its outer rim; the other is around its axle. You pull both pucks from rest by using the same force ***F***. Both pucks start to move on a frictionless surface. 5 seconds later, which puck has greater total kinetic energy? (*Do you know why?*) yes



1. Puck 1
2. Puck 2
3. Both have the same kinetic energy
4. Not enough info. to determine

$$K = \underbrace{\tfrac{1}{2} I_{com} \omega^2}_{1>2} + \underbrace{\tfrac{1}{2} M v^2}_{\text{same}}$$

$W = \Delta K$

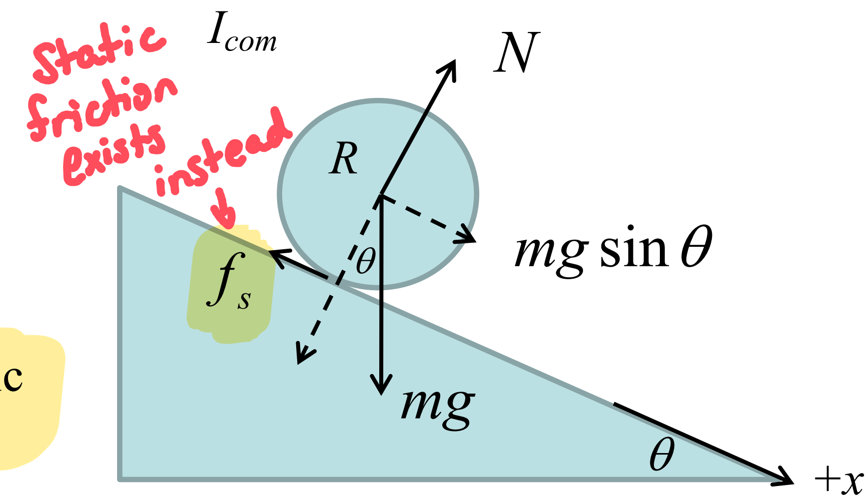
Rolling Object

Forces and torques

What if there is no friction?

No slipping so there is a static frictional force.

Find a_{com}, the acceleration of the center of the mass.

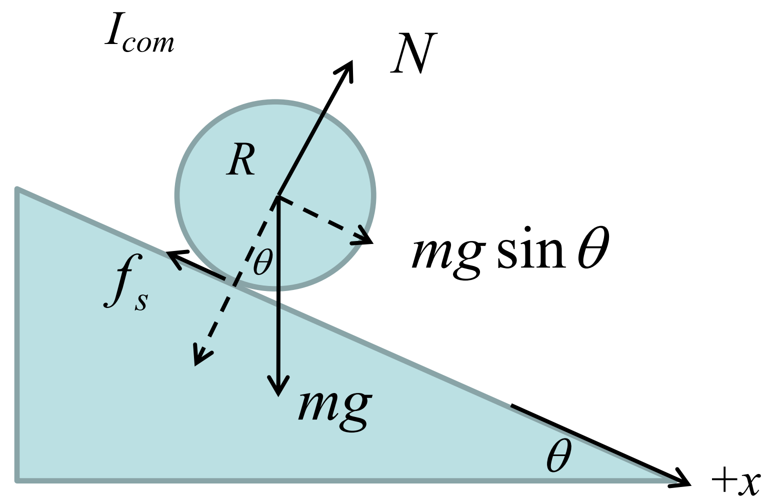


$$\alpha R = a_{com}$$

$$\omega R = V_{com}$$

$a = g\sin\theta$

Find a_{com}, the acceleration of the center of the mass.



$$mg\sin\theta - f_s = ma_{com}$$

$$\tau_{net} = f_s R = I_{com}\alpha$$

$$\alpha R = a_{com}$$

$$f_s R = I_{com}\frac{a_{com}}{R} \Rightarrow f_s = I_{com}\frac{a_{com}}{R^2}$$

$$mg\sin\theta - I_{com}\frac{a_{com}}{R^2} = ma_{com}$$

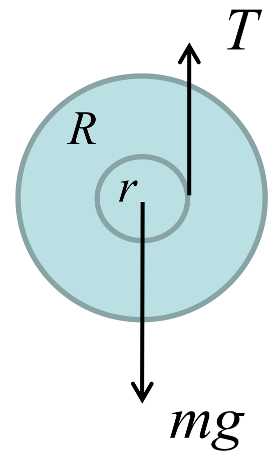
$$a_{com} = \frac{g\sin\theta}{1+\dfrac{I_{com}}{mR^2}}$$

$$\alpha R = a_{com}$$

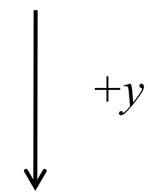
$$\omega R = V_{com}$$

Yo-Yo

Find a_{com}, the acceleration of the center of the mass.

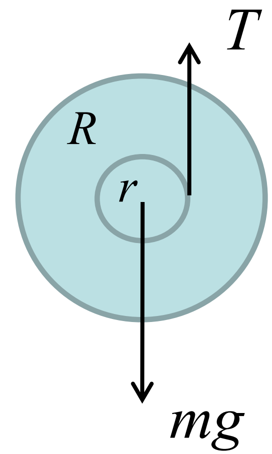


$$I_{com} = \frac{1}{2} m R^2$$



Yo-Yo

Find a_{com}, the acceleration of the center of the mass.



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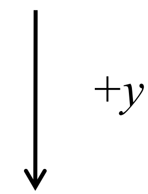


$$mg - T = ma_{com}$$

$$\tau_{net} = Tr = I_{com}\alpha$$

$$\alpha r = a_{com}$$

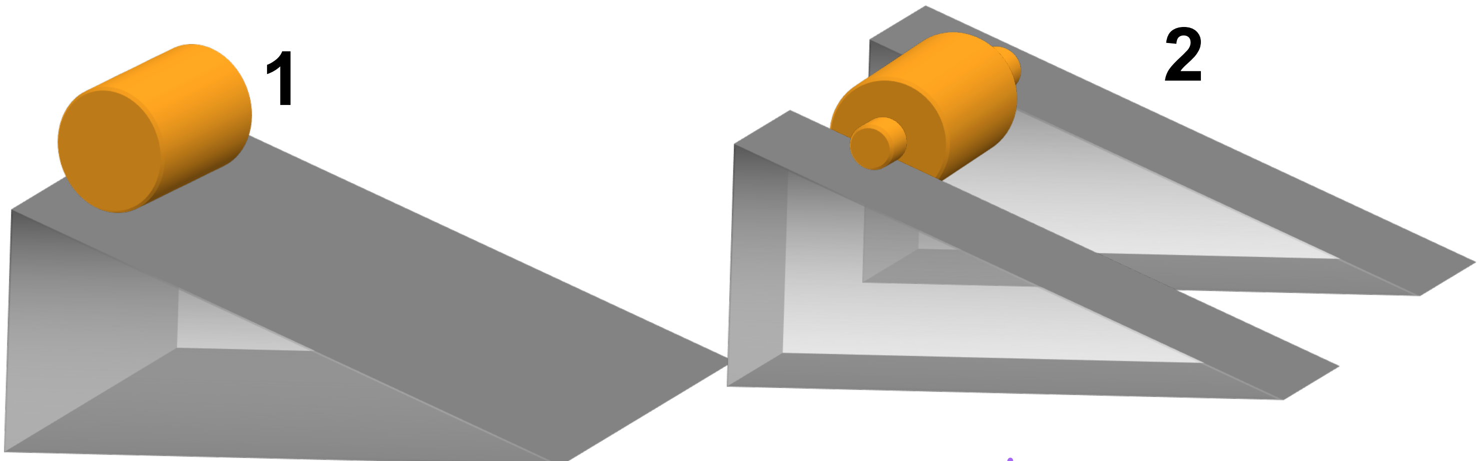
$$Tr = I_{com}\frac{a_{com}}{r} \Rightarrow T = I_{com}\frac{a_{com}}{r^2}$$

$$mg - I_{com}\frac{a_{com}}{r^2} = ma_{com}$$

$$mg = (m + \frac{I_{com}}{r^2})a_{com}$$

$$a_{com} = \frac{g}{1 + \frac{I_{com}}{mr^2}} = \frac{g}{1 + \frac{1}{2}\frac{R^2}{r^2}}$$

Two cylinders with the same radius and mass start from rest and roll down (without slipping) two hills from the same height. Cylinder 2 has a massless axle with a smaller radius. There is a groove in the hill, so that only the axle touches. Which object reaches the bottom first?



1. 1
2. 2
3. They arrive at the same time.
4. Need to know the axle's radius.

Friction applies to larger radius so more torque, and more angular acceleration