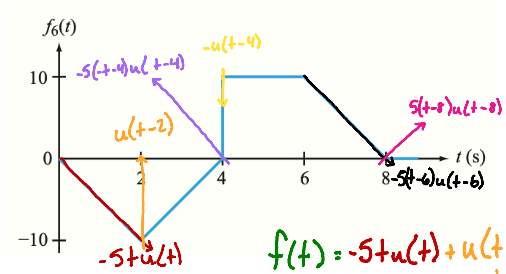
Homework 10
Due: Tuesday, April 25th, 2023 by 7PM.

Note: In order to receive full credit, you must show your work and carefully justify your answers. The correct answer without any work will receive little or no credit.

1. Use step functions to write the expression for the function below, then find the Laplace transform.



$$f(t) = -5tu(t) + u(t-2) - 5(t-4)u(t-4) - u(t-4) - 5(t-6)u(t-6) + 5(t-8)u(t-8)$$

$$\frac{-5}{s^2} \qquad \frac{e^{-2s}}{s} \qquad \frac{-5e^{4s}}{s^2} \qquad \frac{-e^{-4s}}{s} \qquad \frac{-5e^{-6s}}{s^2} \qquad \frac{5e^{-8s}}{s^2}$$

$$\boxed{f(s) = \frac{-5}{s^2} + \frac{e^{-2s}}{s} - \frac{5e^{4s}}{s^2} - \frac{e^{-4s}}{s} - \frac{5e^{-6s}}{s^2} + \frac{5e^{-8s}}{s^2}}$$

2. Obtain the inverse Laplace transform by hand.

$$\frac{100}{s^2(s+5)}$$

partial

$$-\frac{4}{s} + \frac{20}{s^2} + \frac{4}{s+5}$$

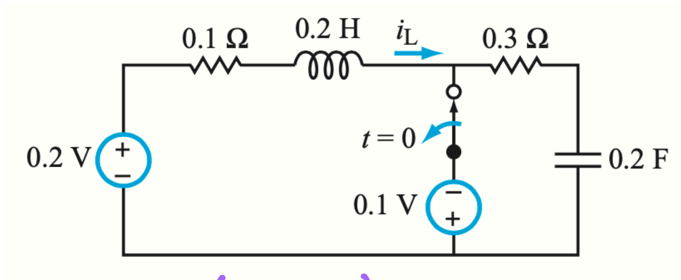
$$-L^{-1}\left(\frac{4}{s}\right) = -4u(t)$$

$$L^{-1}\left(\frac{20}{s^2}\right) = 20t$$

$$L^{-1}\left(\frac{4}{s+5}\right) = 4e^{-5t}$$

$$\boxed{L^{-1}\frac{100}{s^2(s+5)} = -4u(t) + 20t + 4e^{-5t}}$$

3. For the circuit below,



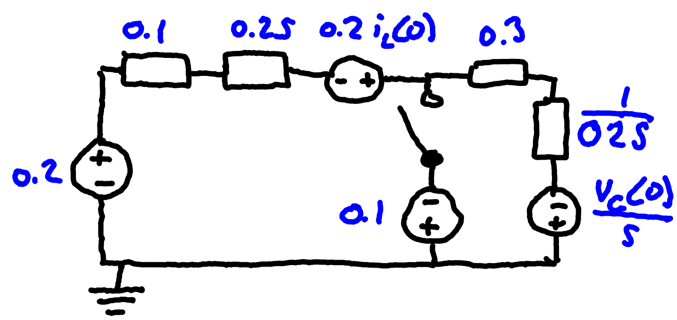
0.1 V (to −0.1V) 0 A (max 3A)

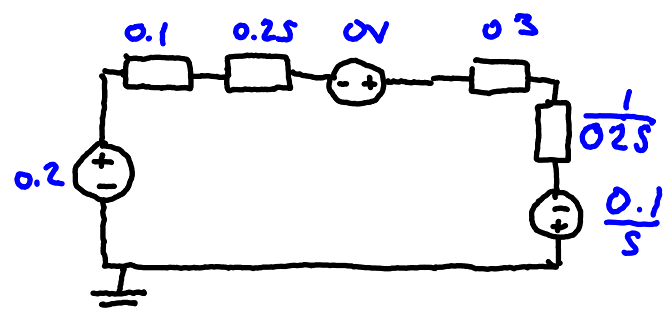
A. Find the initial voltage for the capacitor and the initial current for the inductor.

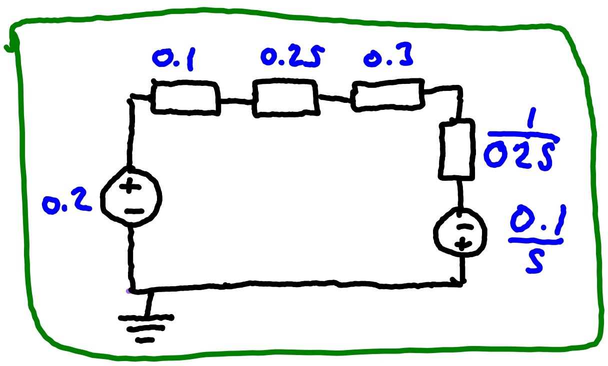
B. Find the s-domain circuit equivalent.

C. Find IL(s).

D. Find iL(t).







$$0.1 + 0.2s + 0.3 + \frac{5}{s}$$

$$0.4 + 0.2s + \frac{5}{s}$$

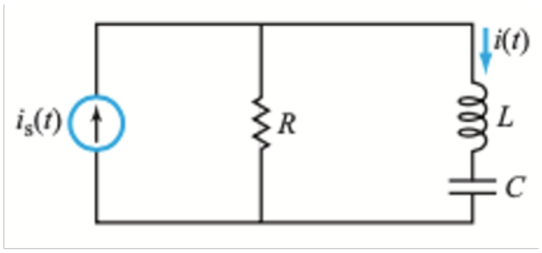
$$\boxed{I_L(s) = \frac{0.2}{0.4 + 0.2s + \frac{5}{s}}}$$

$$\frac{0.2}{\left((0.2s^2 + 0.8s + 5s)/s\right)} \rightarrow \frac{0.2s}{0.2s^2 + 5.8s} \rightarrow \frac{0.0345}{s + 2.708} + \frac{0.1655}{0.2s + 0.85}$$

$$0.0345e^{-2.708t} \qquad 0.8275e^{-4.25t}$$

$$\boxed{I_L(t) = 0.0345e^{-2.708t} + 0.8275e^{-4.25t}}$$

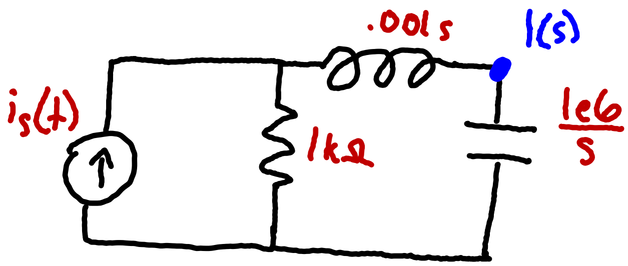
4. On the circuit below, assume all initial conditions are 0 and that R = 1kΩ, L = 1mH, and C = 1uF.
   a) Find the transfer function I(s)/Is(s).
   b) Assume s = jw. What type of filter is it?
   c) Sketch by hand the magnitude bode plot of the system. Clearly explain your reasoning.



$L = SL = 001s$

$C = \frac{1}{SC} = \frac{1}{.000001S} = \frac{1e6}{S}$

$R =$



KCL@ i(s)

$$\frac{i_s(s) - i(s)}{.001s - 1000} = \frac{I(s)}{\frac{1e6}{s}}$$

$$\frac{i_s(s)}{.001s-1000} = i(s)\left(\frac{s}{1e6} + \frac{1}{.001s-1000}\right)$$

$$\frac{i_s(s)}{\cancel{.001s-1000}} = i(s)\left(\frac{.001s^2-1000s+1e6}{1e6\,\cancel{(.001s-1000)}}\right)$$

$$i_s(s) = i(s)\left(\frac{.001s^2-1000s+1e6}{1e6}\right)$$

$$\frac{i_s(s)}{i(s)} = \left(\frac{.001s^2-1000s+1e6}{1e6}\right) \rightarrow \frac{i(s)}{i_s(s)} = \frac{1e6 \times 1000}{001s^2-1000s+1e6 \times 1000}$$

$$\boxed{\frac{i(s)}{i_s(s)} = \frac{1e9s}{s^2-1e6s+1e9}}$$

Band Pass Filter

Zeros: 0

Poles: ~~998999~~ 1000000, 1001

(via quadratic formula)