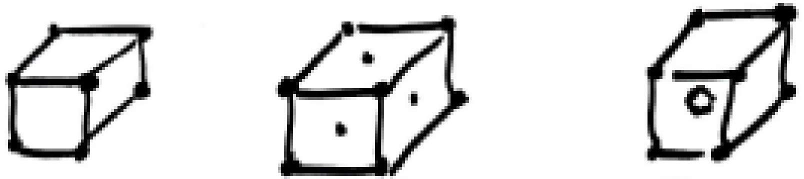
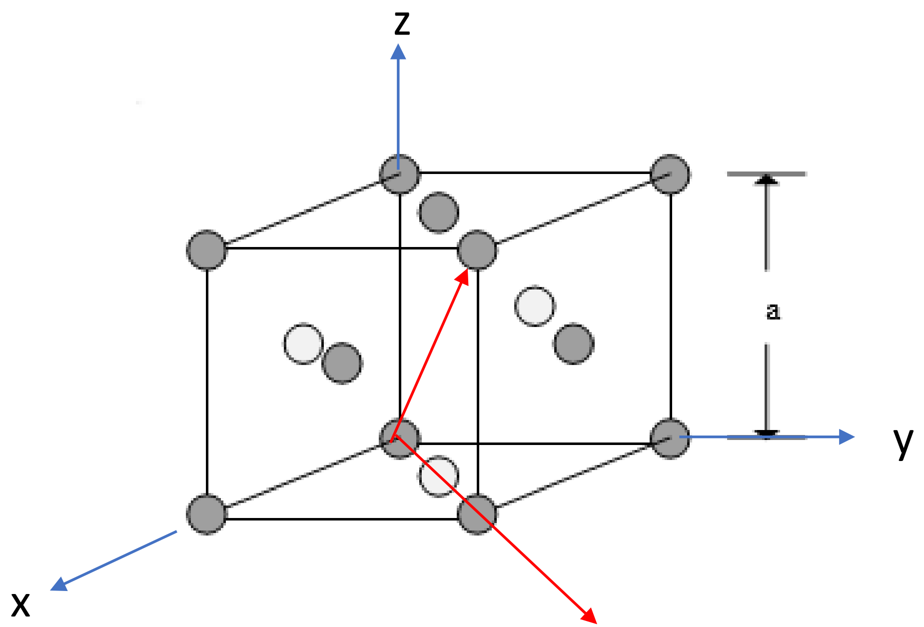
ECE 3030 Midterm 1 Review

Cubic Crystals

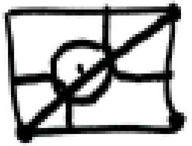


SC FCC BCC

8x1/8 = 1 8x1/8 + 6x1/2=4 8x1/8 + 1= 2

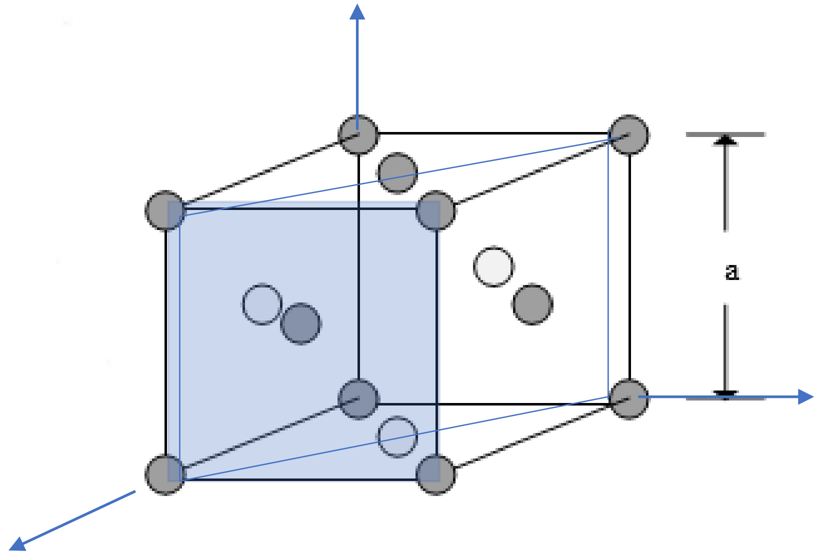


Use hard spheres for R vs. a



Calculate: Line Density
Area Density
Volume Density

along crystal directions
+ within crystal planes



- Use Miller indices to get planes from intercepts and vice-versa.
- Use R vs. a to get line, area, and volume overlaps.

$$\text{Density } \rho = \frac{n\,A}{V_c\,N_A}$$

A — atomic weight

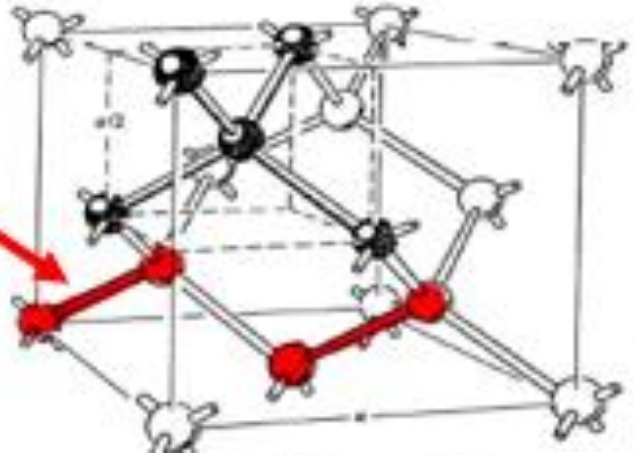
N_A — Avogadro's # $= 6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}$

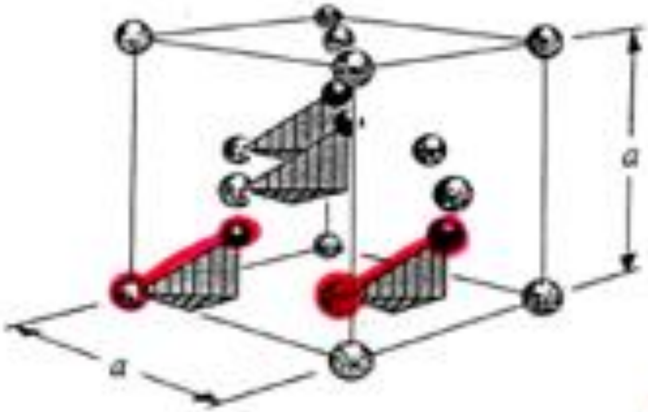
n = atoms / unit cell

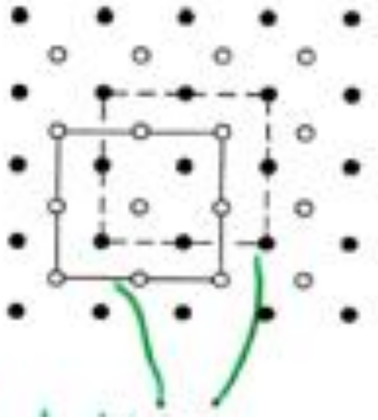
V_c = unit cell volume

- Diamond Lattice: 2 interpenetrating FCC Lattices

¼ x body diagonal $= \sqrt{3}\, a/4$







Each FCC Sublattice has:

8 corner atoms $\times \frac{1}{8}$ (shared 8 ways) $= 1$

6 face atoms $\times \frac{1}{2}$ (shared 2 ways) $= 3$

Total $\quad 4$

Two sublattices per unit cell, so $2 \times 4 = \dfrac{8 \text{ atoms}}{\text{unit cell}}$

$$\frac{8}{a^3} = \frac{8}{(5.43 \times 10^{-8} \text{cm})^3} = 5 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}$$

$$\text{Density } \rho = \frac{n}{V_c} \frac{A}{N_A} = \frac{5 \times 10^{22} \text{atoms/cm}^3 \cdot 28.1 \frac{\text{gm}}{\text{mole}}}{6.02 \times 10^{23} \text{atoms/mole}} = 2.33 \frac{\text{g}}{\text{cm}^3}$$

EE3030 Review

Wave nature of particles $\lambda = \frac{h}{p}$

Heisenberg Uncertainty Principle $\Delta x \Delta p \geq \hbar$

$E = h\nu = \frac{hc}{\lambda}$

Schroedinger's Equation: $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = -\frac{\hbar}{j}\frac{\partial \psi}{\partial t}$

and $\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}\left[E - V(x)\right]\psi(x) = 0$ (Time-independent)

Free electron wave solution

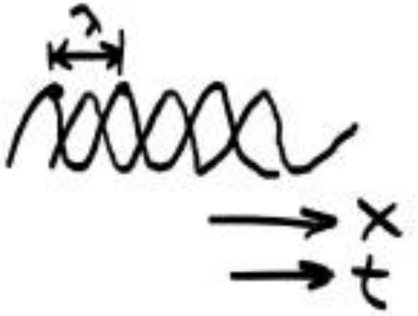
$k = \frac{2\pi}{\lambda}$ **k =[2m(E-V)/ħ²]$^{1/2}$** $\psi(x) \propto e^{-jkx}$ or e^{jkx}

$e^{j(kx-\omega t)}$ $\omega = \frac{E}{\hbar}$

Free electron wave solution

$E = \hbar^2 k^2/2m$, $k^2 = 2mE/\hbar^2$, $k = [2mE/\hbar^2]^{1/2}$ $\psi(x) \propto e^{jkx}$ or e^{jkx}

$e^{j(kx-\omega t)}$ $\omega = \frac{E}{\hbar}$



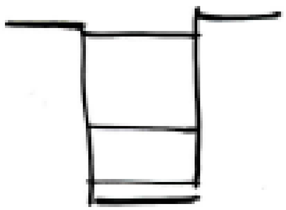
Phase velocity $v_p = \frac{x}{t} = \frac{E}{\hbar k}$

group velocity $\frac{dx}{dt} = \frac{1}{\hbar}\frac{dE}{dk}$

$p = \hbar k$ momentum

Standing wave solution (quantum well)

$\sin kx$, $\cos kx$ $\frac{e^{jkx} - e^{-jkx}}{2j}$, $\frac{e^{jkx} + e^{-jkx}}{2}$



$k = \frac{n\pi}{L}$ from boundary conditions

$\rightarrow E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$

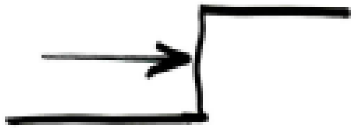
$\frac{\pi^2 \hbar^2}{2m_0} = 38\,eV\text{-}\mathring{A}^2$

$E - V = \hbar^2 k^2/2m$

$k^2 = 2m(E-V)/\hbar^2$

$k = [2m(E-V)/\hbar^2]^{1/2}$

Step Potential Wave solution



Region 1   2

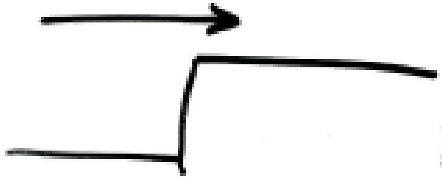
$E < V$

$j\sqrt{\frac{2m(E-V)}{\hbar^2}}$ negative under $\sqrt{\ }$

$\rightarrow e^{-kx}$

so traveling wave, then decaying wave

$e^{jk_1 x}$ $e^{-k_2 x}$



$E > V$

so Both traveling waves but different k_1, k_2

$k = [2m(E-V)/\hbar^2]^{1/2}$

$k = [(-1)2m(V-E)/\hbar^2]^{1/2}$

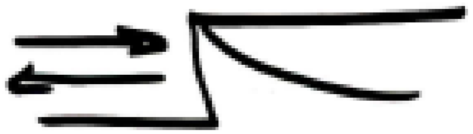
$k = j[(V-E)/\hbar^2]^{1/2}$

so Both traveling waves

but different k_1, k_2

$$j\sqrt{\frac{2m(E-V)}{\hbar^2}} \rightarrow e^{jkx} \qquad j^2=-1$$

Reflected waves only with interfaces



or

$$R = \frac{v_r}{v_i}\frac{|B_1|^2}{|A_1|^2} \qquad T = \frac{v_t}{v_i}\frac{|A_2|^2}{|A_1|^2}$$

$$v = \frac{\hbar k}{m}$$

Probability $P \propto \psi^*(x)\,\psi(x)$

For $E < V$, $\psi \propto e^{-kx} \rightarrow P \propto e^{-2kx}$

Relative $P = \dfrac{P(x)}{P(0)} = e^{-2kx}$

Tunnel Barrier Wave Solution

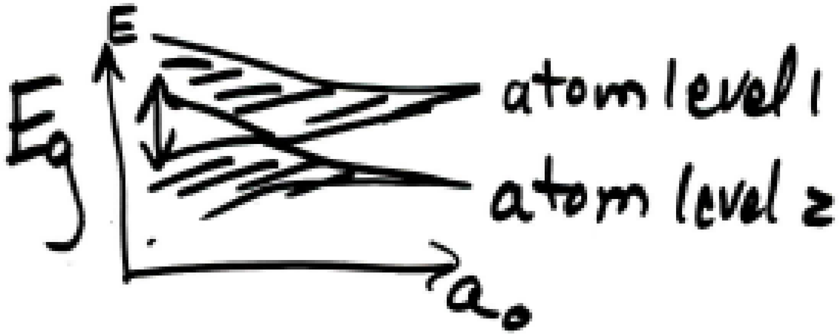
$$T \cong 16\left(\frac{E}{V_0}\right)\left(1-\frac{E}{V_0}\right)e^{-2k_2 W}$$

Tunneling at very short distances

Wave overlap

Wave overlap

— → = splitting



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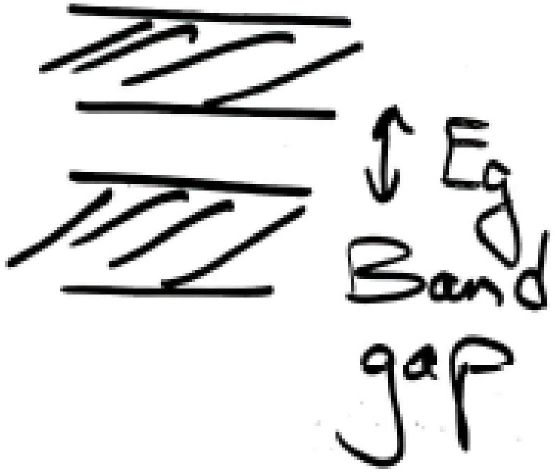
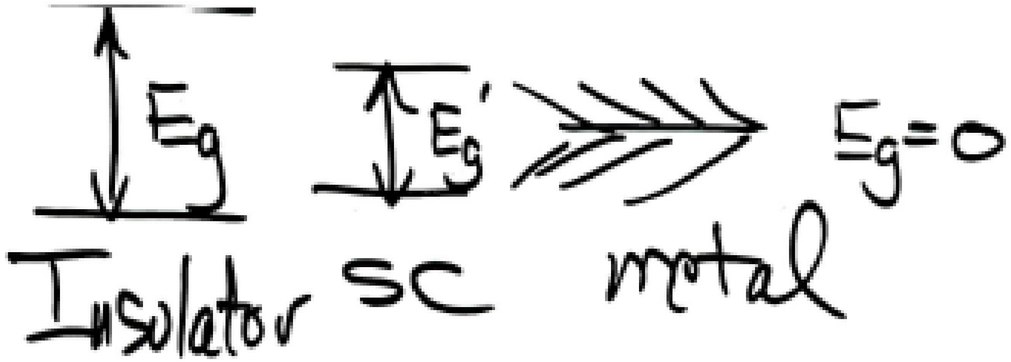
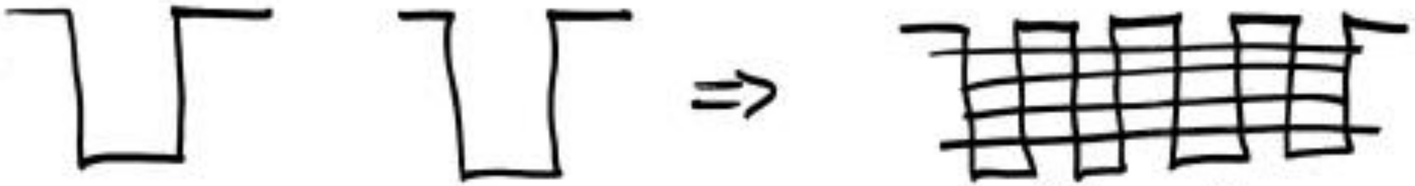
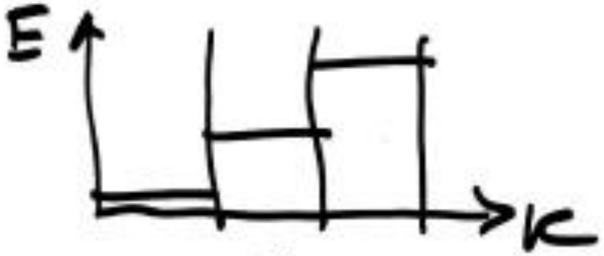


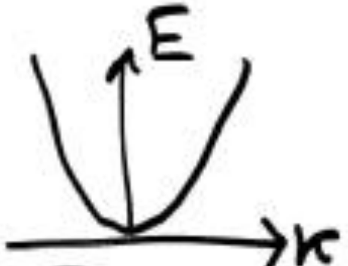




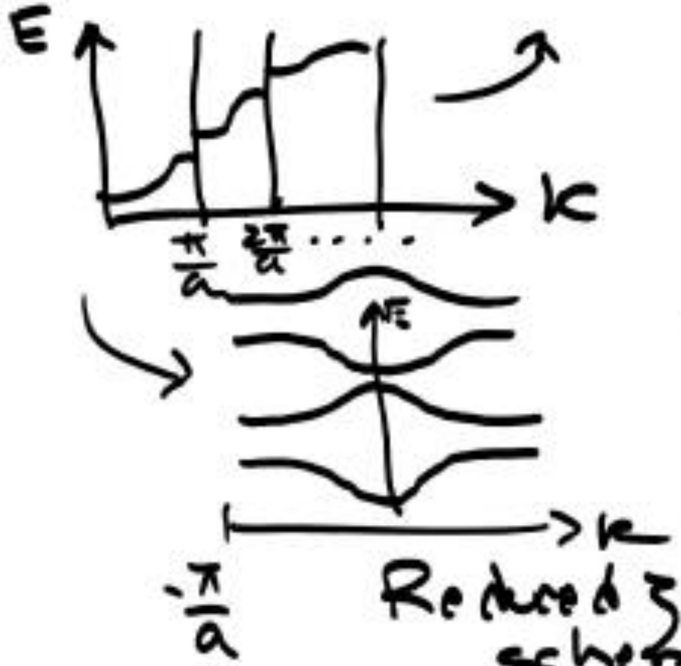
minibands



like QW.

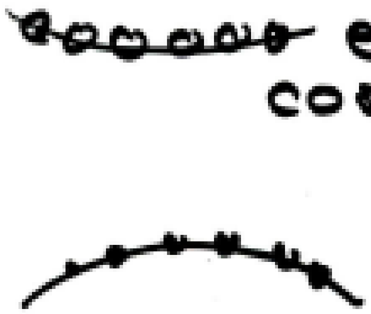


freeelectron



Reduced zone scheme

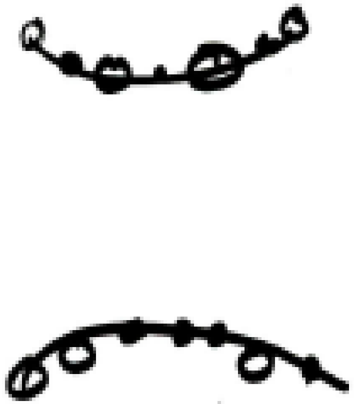
electron in periodic potential (lattice)



empty conduction band

filled valence band

T = 0 K



T > 0K

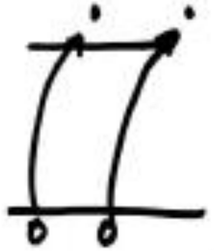
Electrons and holes

direct gap

indirect gap

Phonon required

Effective mass $m^* = \dfrac{\hbar^2}{\partial^2 E/\partial k^2}$



electrons + holes created in pairs

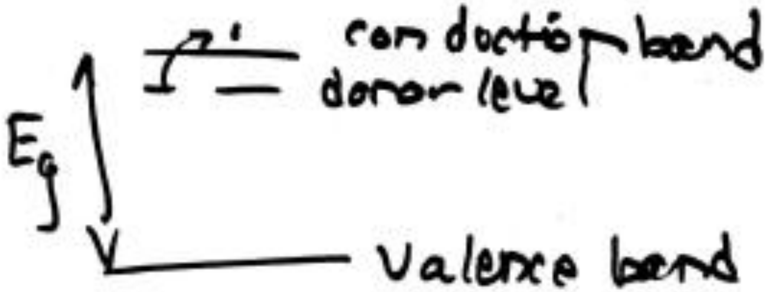
$n = p = n_i$ intrinsic carrier concentration

$n_i = (N_c N_v)^{1/2} e^{-E_g/k_BT}$

n-type dopant atoms increase n_0 (free electron concentration)

p – " " " " p_0 (free hole concentration)

Si = Si = Si
‖ ‖ ‖
Si = As = Si e^-
‖
Si



easier to get e^- into conduction band

donor, acceptor atoms

k_BT at any Temperature (in Kelvin) = 0.0259 eV x (T/300)

Fermi-Dirac

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

$$n_0 = N_C \, e^{-(E_C - E_F)/kT}$$

$$p_0 = N_V \, e^{-(E_F - E_V)/kT}$$

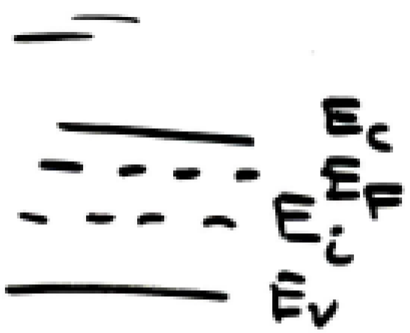
$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$N_V = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

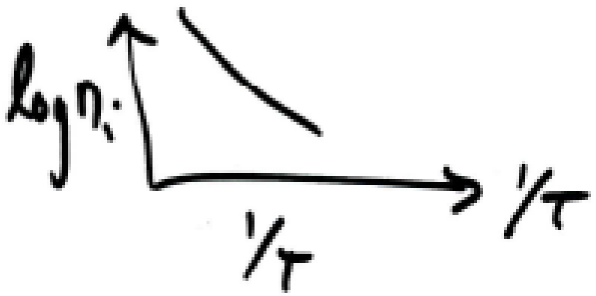
Law of mass action $n_0 p_0 = n_i^2$

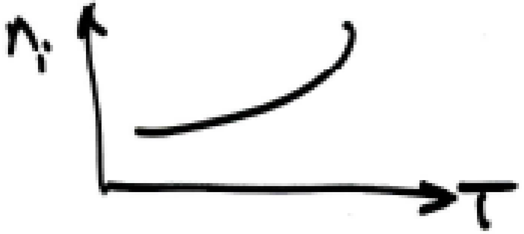
$$n_0 = n_i e^{(E_F - E_i) kT}$$

$$p_0 = n_i e^{(E_i - E_F)/kT}$$

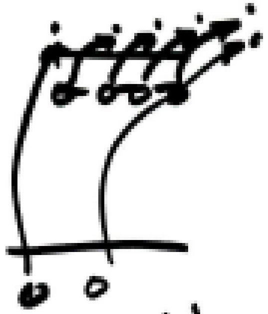


For Si at 300 k, $n_i = 1.5 \times 10^{10}\ cm^{-3}$

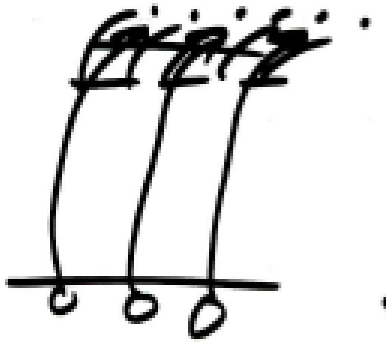




T low



T mid.
complete ionization



Hi T
Intrinsic dominates

$1/T$

or p_0 vs. $1/T$

Charge neutrality equation

$p_0 + N_d = n_0 + N_a$ for $N_d^+ = N_d$
$+ N_A^- = N_A$
complete ionization

use $n_o p_o = n_i^2$ to get minority carrier

Solve Quadratic if $N_a \sim N_d$ or $N \sim n_i$

$$\left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ for } ax^2 + bx + c = 0 \right)$$

$$v = -q \frac{t\,\mathcal{E}}{m^*}$$

$$\langle v_n \rangle = -\mu_n \mathcal{E}$$

mobility

Current density $\quad J = q(n\mu_n + p\mu_p)\mathcal{E}$

Conductivity $J = \sigma \mathcal{E}$

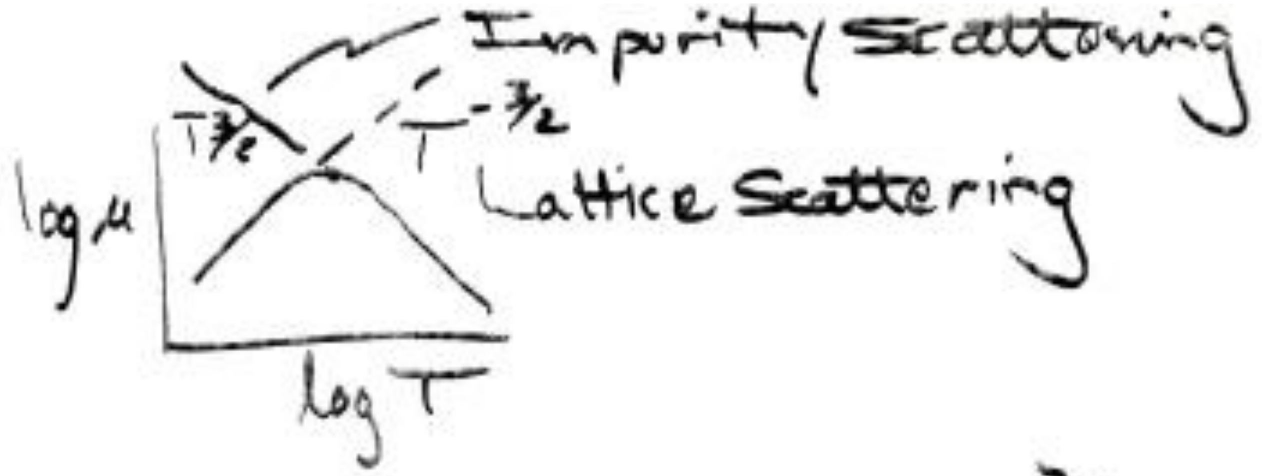
Resistance $R = \frac{\rho L}{A} = \frac{V}{I}$

Resistivity $= \rho = \frac{1}{\sigma}$

use $n_o p_o = n_i^2$ to get minority carrier

Solve Quadratic if $N_a \sim N_d$

or $N \sim n_i$



$$N_v, N_c \propto \left(\frac{T}{300}\right)^{3/2}$$

$$E/k_B T = \frac{E}{0.0259\,\text{eV}\left(\frac{T}{300}\right)}$$