Practice Sheet Review

1.) FUNCTION $P_1(n)$

```
IF n < 1 THEN RETURN(1)
    x ← 1
    t ← 0
    WHILE x ≤ n DO
        t ← t + x
        x ← x + 2
    RETURN(P_1(n − 2) + 1)
```

- Base Case? $n=0$
- # recursive calls? 1
- input size? $n-2$
- non-recursive?

loop ctr k	value of x
0	1
1	$1+2$
2	$1+2\cdot 2$
3	$1+2\cdot 3$
$\vdots$	$\vdots$
k	$1+2k=x$

$$x = 1+2k \le n$$

$$k \le \frac{n-1}{2}$$

$$\sum_{k=1}^{\frac{n-1}{2}} c = \frac{c}{2}(n-1) \in \Theta(n)$$

$$T_1(n) = T_1(n-2) + cn$$

$$T_1(\text{input}) = c\cdot\text{input} + T_1(\text{input}-2)$$

$$n - 2M = 0$$

$$2M = n$$

$$M = \frac{n}{2}$$

$$T_1(n) = cn + T_1(n-2)$$

$$= cn + c(n-2) + T_1(n-2\cdot 2)$$

$$= cn + c(n-2) + c(n-2\cdot 2) + T_1(n-2\cdot 3)$$

$$= \cdots \approx c\sum_{k=0}^{n/2}(n-2k) + T_1(0)$$

$$c\sum_{k=0}^{n/2}(n-2k) \le c\sum_{k=0}^{n/2} n \approx cn\frac{n}{2} \in \Theta(n^2)$$

O-work

Assume $T_1(k) \le ak^2$ for $n_0 \le k < n$ $(a>0)$

$T_1(n) = cn + T_1(n-2) \le cn + a(n-2)^2$ WANT $\le an^2$

$= cn + an^2 - 4an + 4a$

$= an^2 + (c-4a)n + 4a$

If $c-4a \le 0$ then $(c-4a)n+4a$ will be ≤ 0 for large enough n's (i.e. $a \geq \frac{c}{4}$)

try $n=2$

$2c - 8a + 4a \le 0$

$c \le 2a$

$a \geq \frac{c}{2}$

if $a \geq \frac{c}{2}$ $(c-4a)n + 4a \le 0$ for $n \geq 2$

$\cancel{T_1(0) \le a0^2}$

$T_1(1) \le a \cdot 1^2$

$T_1(2) \le a \cdot 2^2$

$a \geq \max\left\{\frac{c}{2}, T_1(1), \frac{T_1(2)}{4}\right\}$

Ω-work

Assume $T_1(k) \geq bk^2$ for $1 \le k < n$ (with $b>0$)

$T_1(n) = cn + T_1(n-2) \geq cn + b(n-2)^2$

$$= bn^2 + (c-4b)n + 4b \geqslant bn^2 + (c-4b)n$$

$$\geqslant bn^2 \quad \text{if } c-4b \geqslant 0, \text{ i.e. } b \leq \frac{c}{4}$$

$T_1(1) \leq b1^2$, $T_1(2) \leq b2^2$

$$0 < b \leq \min\left\{\frac{c}{4}, T_1(1), \frac{T_1(2)}{4}\right\}$$