

Homework 8 - Math 2568 (Autumn 2022)

Prof. Cueto

Due date: Friday November 4, 2022 (in class).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§5.3	1, 3, 4, 5, 9, 11, 13, 19, 23, 25, 27, 28, 29, 31	5, 13, 19, 23, 27
§5.4	1, 4, 5, 9, 10, 13, 15, 19, 23, 24, 25, 26, 27, 28	4, 9, 23, 25, 27
§5.7	1, 2, 3, 5, 7, 9, 10, 11	1, 2, 3, 5, 7, 9, 11

Section 5.3

$$5) \quad W = \{ p(x) \text{ in } \mathcal{P}_2 : p(0) + p(2) = 0 \}$$

$$\theta(x) = 0x^2 + 0x + 0 \rightarrow 0(0^2) + 0(0) + 0 = 0$$

S1 ✓

$$(p+q)(0) + (p+q)(2) = 0$$

S2 ✓

$$p(0) + p(2) = (a_2(0)^2 + a_1(0) + a_0) + (a_2(2)^2 + a_1(2) + a_0) = 0$$

S3 ✓

All checks passed

$$13) F = \{f(x) \text{ in } C^2[-1,1] : f''(0) = 0\}$$

$$(f+g)''(0) = f''(0) + g''(0) = 0$$

$$(cf)''(0) = cf''(0) = 0 \quad (f+g)'(0) = f'(0) + g'(0) = 0$$

F is a subspace of $C^2[-1,1]$, satisfying 53

$$19) A = \begin{bmatrix} -2 & -4 \\ 1 & 0 \end{bmatrix} = a_1 \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + a_3 \begin{bmatrix} -1 & -3 \\ 4 & -4 \end{bmatrix} + a_4 \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -3 & 2 \\ 2 & 0 & 4 & -1 \\ 1 & 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -1 & 1 & -2 \\ 0 & 1 & -3 & 2 & -4 \\ 0 & -2 & 6 & -3 & 5 \\ 0 & 1 & -3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -1 & 1 & -2 \\ 0 & 1 & -3 & 2 & -4 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & -2 & 6 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & -3 & 0 & -4 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \quad \begin{array}{l} a_1 + 2a_3 = -1 \quad a_1 = -1 - 2x \\ a_2 - 3a_3 = -4 \quad a_2 = -2 + 3x \\ a_4 = -3 \quad a_3 = x \\ \quad \quad \quad a_4 = -3 \end{array}$$

$$A = (-1-2x)B_1 + (-2+3x)B_2 + (x)B_3 + (-3)B_4, x \in \mathbb{R}$$

$$23) p(1) = p(-1) \quad p(2) = p(-2) \quad q(1) = q(-1) \quad q(2) = q(-2)$$

$$(p+q)(1) = (p+q)(-2)$$

W is subspace of P

$$(ap)(1) = (ap)(-2)$$

Spanning set of W is $\{1, x^2\}$

$$2a_1 + 8a_3 = -2a_1 - 8a_3$$

$$27) \theta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{tr}(\theta) = 0+0+0 = 0 \quad \text{S1 } \checkmark$$

$$\text{tr}(A+B) = (a_{11}+b_{11}) + (a_{22}+b_{22}) + \dots = 0+0 = 0 \quad \text{S2 } \checkmark$$

$$B_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad E_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad E_{23} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad E_{32} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$W = \text{Sp}\{B_1, B_2, E_{12}, E_{13}, E_{21}, E_{23}, E_{31}, E_{32}\}$$

Section 5.4

$$4) A = \begin{bmatrix} a & a-c \\ c & 2a+c \end{bmatrix} \rightarrow B_1 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$d_1 \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + d_2 \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} d_1 & d_1-d_2 \\ d_2 & 2d_1+d_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Basis for W is

$$\{B_1, B_2\} = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} \right\}$$

$$9) p(0) = 0 \quad p'(1) = 0 \quad p''(-1) = 0$$

$$2a_2 + 6a_3(-1) + 12a_4(-1)^2 = 2a_2 - 6a_3 + 12a_4 = 0$$

$$p_1(x) = (x^3 + 3x^2 - 9x) \quad p_2(x) = (x^4 - 6x^2 + 8x)$$

$$\text{Basis of } V \quad (x^3 + 3x^2 - 9x), (x^4 - 6x^2 + 8x)$$

$$23) \begin{aligned} a+2b+3c+d &= 0 \\ 2a+5b+7c+d &= 0 \\ a+c+3d &= 0 \end{aligned} \quad \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 2 & 5 & 7 & 1 & 0 \\ 1 & 0 & 1 & 3 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} a+c+3d &= 0 \rightarrow c = \frac{-a-3d}{4} & d &= \frac{b-a}{4} & a+2b &= 0 \\ b+c-d &= 0 & & & 2a+5b &= 0 \\ & & & & a &= 0 \end{aligned}$$

Subset of $S = \{p_1(x), p_2(x)\}$ is basis for $S_p(S)$

$$25) [A_1]_B = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix} \quad [A_2]_B = \begin{bmatrix} -2 \\ 1 \\ 2 \\ -1 \end{bmatrix} \quad [A_3]_B = \begin{bmatrix} -1 \\ -1 \\ 1 \\ -3 \end{bmatrix} \quad [A_4]_B = \begin{bmatrix} -2 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

$$[V|\theta] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{2}{5} & 0 \\ 0 & 1 & 0 & \frac{6}{5} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad x_1 = -\frac{2}{5}x_4 \quad x_2 = -\frac{6}{5}x_4 \quad x_3 = 0 \quad x_4 = 1$$

$\{A_1, A_2, A_3\}$ is basis for $S_p(S)$

$$27) \begin{aligned} -a_1 + a_3 &= 1 & a_1 + a_2 + 2a_3 &= 1 & 2a_1 + 3a_2 + 8a_3 &= 1 \\ a_1 &= -4 & a_2 &= 11 & a_3 &= -3 \end{aligned} \quad [p_x]_Q = \begin{bmatrix} -4 \\ 11 \\ -3 \end{bmatrix}$$

Section 5.7

$$1) T(A+B) = \det(A) + \det(B) + (pd + as - qc - br)$$

T is not a linear transformation as $T(A+B) \neq \det(A) + \det(B)$

$$2) \quad T(A) = a + 2b - c + d \quad T(B) = p + 2q - r + s$$

$$T(A+B) = T(A) + T(B)$$

$$T(kA) = kT(A)$$

T is linear transformation

$$3) \quad T(A) = a + d \quad T(B) = p + s$$

$$T(A+B) = T(A) + T(B) \quad T(kA) = kT(A)$$

T is linear transformation

$$5) \quad T(f) = f'(0) \quad T(g) = g'(0)$$

$$T(f+g) = T(f) + T(g) \quad T(kf) = kT(f)$$

T is linear transformation

$$7) \quad T[p(x)] = (a_0+1) + (a_1+1)x + (a_2+1)x^2$$

$$T[q(x)] = (b_0+1) + (b_1+1)x + (b_2+1)x^2$$

T is not linear transformation

$$T[p(x)+q(x)] \neq T[p(x)] + T[q(x)]$$

$$9) \quad a) \quad p(x) = 3 - 2(x) + 4(x^2) \quad T[p(x)] = 11 + x^2 + 6x^3$$

$$b) \quad T(a_0 + a_1x + a_2x^2) = T[a_0(1) + a_1(x) + a_2(x^2)]$$

$$T(a_0 + a_1x + a_2x^2) = (a_0 + 2a_2) + (a_0 + a_1)x^2 + (a_2 - a_1)x^3$$

$$11) \quad A = -2 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 3 \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 4 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = -2E_{11} + 2E_{12} + 3E_{21} + 4E_{22}$$

$$T(A) = 8 + 14x - 9x^2$$

