CSE 2321 Homework 2 Template

Problem 1

Let $P(x)$ be the predicate "x is a dragon."
Let $Q(x)$ be the predicate "x breathes fire."
Let $R(x, y)$ be the predicate "x and y are the same object."
Let S be an arbitrary nonempty set.

a

i $\forall x \in S, \neg P(x)$

ii $\exists x \in S, \neg P(x)$

iii $\exists x \in S, P(x)$

iv $\exists x, y \in S, (P(x) \wedge P(y)) \Rightarrow \neg R(x, y)$

v $\exists x, y \in S, (P(x) \wedge P(y)) \Rightarrow R(x, y)$

vi $(\exists x \in S, P(x)) \wedge (\exists x, y \in S, P(x) \wedge P(y) \Rightarrow R(x, y))$

vii $\forall x \in S, (P(x) \Rightarrow Q(x))$

viii $\forall x \in S, (Q(x) \iff P(x))$

b

i Everything in S is a dragon.

ii There exists at least one dragon in S such that if it is a dragon, it breathes fire.

iii If it breathes fire, it is a dragon.

iv If everything in S breathes fire, everything in S must be a dragon.

v There is at least one dragon in S that breathes fire.

vi For everything that exists in S, there is another object that is the same object.

Problem 2

a

$\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, (x \leq y)$

b

$\forall x, y \in \mathbb{N}, ((x\%2 = 1) \wedge (y\%2 = 1) \Rightarrow ((x + y)\%2 = 0)$