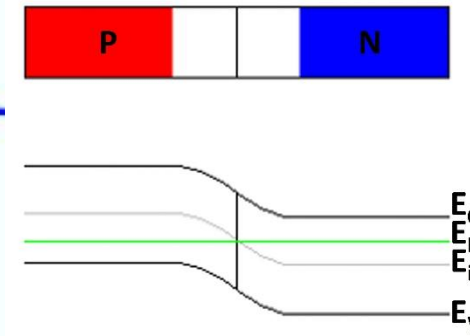
5.2.2 Equilibrium Fermi Levels



Use above expression to show that bands shift by V_0 across junction

$$\frac{p_p}{p_n} = e^{qV_0/kT} = \frac{N_v e^{-(E_{Fp}-E_{vp})/kT}}{N_v e^{-(E_{Fn}-E_{vn})/kT}}$$

But

$p = N_v e^{-(E_F - E_v)/kT}$ eq. 3-19

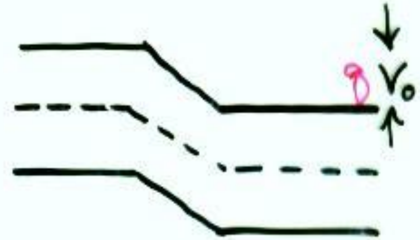
$$e^{qV_0/kT} = e^{(E_{vp} - E_{vn})/kT}$$

$$qV_0 = E_{vp} - E_{vn}$$

Likewise, $qV_0 = E_{ip} - E_{in}$

$qV_0 = E_{cp} - E_{cn}$

all 3 shift together:

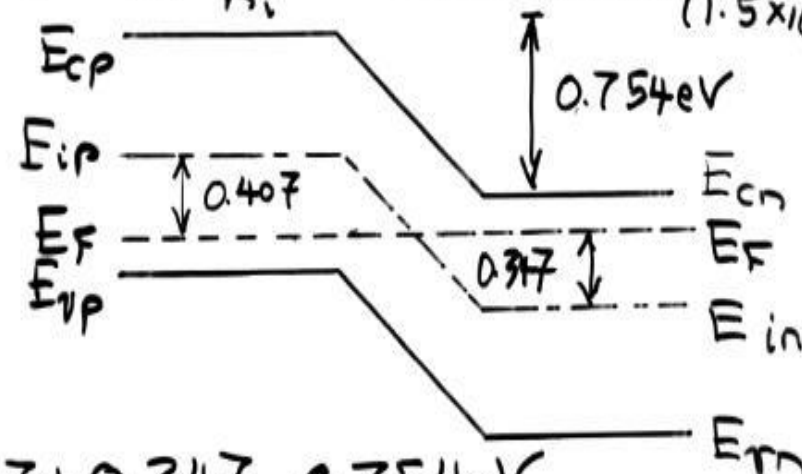


Example: Si junction has $N_a = 10^{17}\,cm^{-3}$ and $N_d = 10^{16}\,cm^{-3}$.

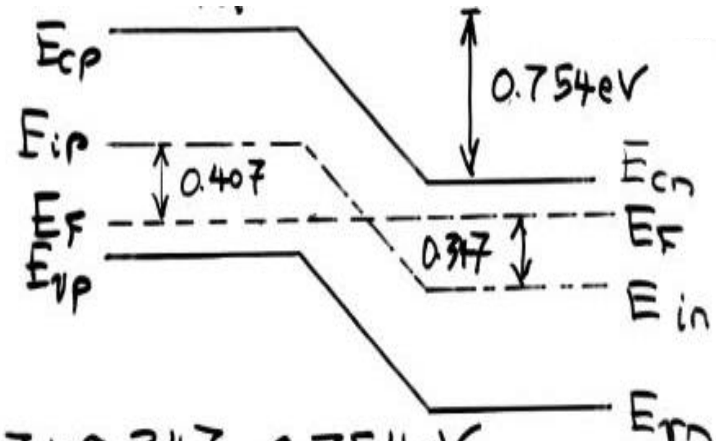
(a) Find E_F on each side. Draw the band diagram. Find V_0

$$E_{ip} - E_F = kT \ln \frac{p_p}{n_i} = 0.0259\,eV \ln \frac{10^{17}}{(1.5\times10^{10})} = 0.\underline{407}\,eV$$

$$E_F - E_{in} = kT \ln \frac{n_n}{n_i} = 0.0259\,eV \ln \frac{10^{16}}{(1.5\times10^{10})} = 0.\underline{347}\,eV$$



$$qV_0 = 0.407 + 0.347 = 0.754\,eV$$



$$qV_0 = 0.407 + 0.347 = 0.754\,eV$$

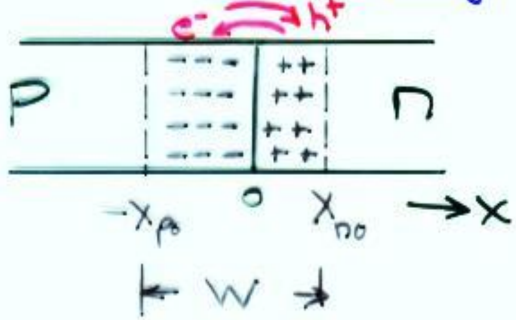
(b) Find V_0 from

$$qV_0 = kT \ln \frac{N_a N_d}{n_i^2} = 0.0259\,eV \ln \frac{10^{33}}{2.25\times10^{20}}$$

$$= \underline{0.754\,eV}$$

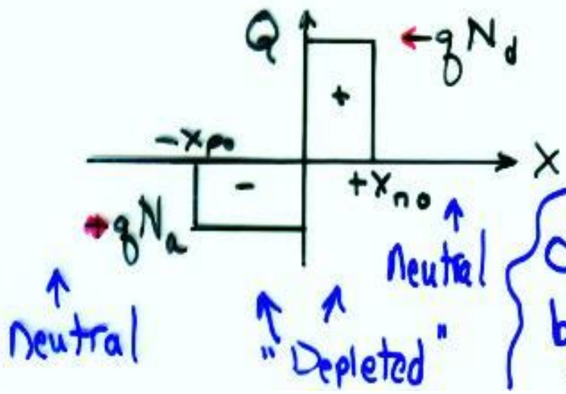
Same result.

5.2.3 Space Charge at a Junction



e^-s diffuse from n-side, leaving N_d^+

h^+s diffuse from p-side, leaving N_a^-



"Depletion Approximation": Charge swept out of transition region by $\mathcal{E}$ field.

Net Charge $= 0$

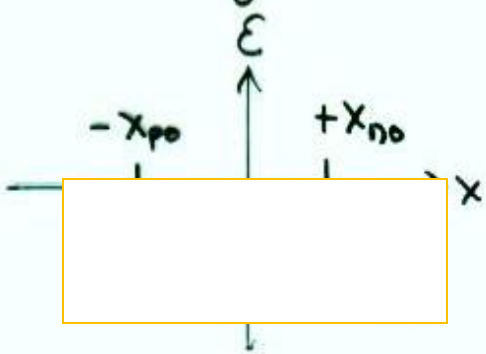
$Q_+ =$ Charge density $\times$ volume

$= qN_d A x_{no}$

Flux lines begin and end on charges in transition region

$Q_- = qN_a A x_{po}$

Therefore, $|Q_+| = |Q_-|$



$$qN_d A x_{no} = qN_a A x_{po}$$

$\mathcal{E}$ is negative since $\oplus$ to $\ominus$ charge in $-x$ direction.

$\mathcal{E} = 0$ outside W.

$\mathcal{E}$ is maximum at interface (most flux lines).

Get $\mathcal{E}$ from Poisson's equation

$\nabla^2 \phi = \frac{Q}{\epsilon}$ or

$$\frac{d\mathcal{E}(x)}{dx} = \frac{q}{\epsilon}\left(\underset{0}{p} - \underset{0}{n} + N_d^+ - N_a^-\right)$$

(depletion approximation)

so $$\frac{d\mathcal{E}}{dx} = \frac{q}{\epsilon} N_d^+ \qquad 0 < x < x_{no}$$

$$\frac{d\mathcal{E}}{dx} = \frac{-q}{\epsilon} N_a^- \qquad -x_{po} < x < 0$$

assume $N_d^+ = N_d$ and $N_a^- = N_a$

Get $\mathcal{E}_0$ by integrating

$$\int_{\mathcal{E}_0}^{0} d\mathcal{E} = \frac{q}{\epsilon} N_d \int_0^{x_{no}} dx \qquad 0 < x < x_{no}$$

$$-\mathcal{E}_0 = q \frac{N_d}{\epsilon} x_{no}$$

and $$\int_0^{\mathcal{E}_0} d\mathcal{E} = -\frac{q}{\epsilon} N_a \int_{-x_{po}}^{0} dx \qquad \text{similarly}$$

$$\mathcal{E}_0 = -\frac{q}{\epsilon} N_a x_{po}$$

Again, $\boxed{N_a x_{po} = N_d x_{no}}$

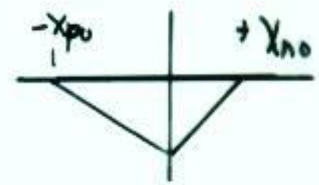
Relate $\mathcal{E}$ to V_0 now

$$\mathcal{E}(x) = -\frac{dV(x)}{dx} \quad \text{or} \quad V_0 = -\int_{-x_{po}}^{x_{no}} \mathcal{E}(x)\,dx$$

V_0 = area under triangle (!)

$$= -\frac{1}{2}\underbrace{\frac{qN_a x_{po}}{\epsilon}}_{\mathcal{E}_0} \cdot x_{po} - \frac{1}{2}\underbrace{\frac{qN_d x_{no}}{\epsilon}}_{\mathcal{E}_0} \cdot x_{no}$$

$$= -\frac{1}{2}\mathcal{E}_0 \left(x_{po} + x_{no}\right)$$



V_0 already in terms of doping densities.

Now V_0 in terms of widths and ϵ

(Good ol' Poisson!)

$$V_o = \frac{1}{2} q \frac{1}{\epsilon} N_d X_{no} \cdot W$$

$\mathcal{E}_o$ a negative value

Use $N_d X_{no} = N_a X_{po}$

and $X_{no} + X_{po} = W$

$$X_{no} + \frac{N_d}{N_a} X_{no} = W$$

$$X_{no} = W \Big/ \left(1 + \frac{N_d}{N_a}\right) = \frac{N_a W}{N_d + N_a}$$

and $X_{po} = \dfrac{W N_d}{N_d + N_a}$

(note: X_{no} and X_{po} add up to W.)

To Get: $$\underline{V_o = \frac{1}{2\epsilon} q \frac{N_d N_a W^2}{(N_d + N_a)}}$$

and finally

$W =$

Get W in terms of V_0, doping, ϵ, and q

$$W \propto V_0^{1/2}$$

$$W \propto \frac{1}{N^{1/2}}$$

Later on will see W also varies with applied Voltage.

This p-n junction is central to micro- and opto-electronics. Apply voltage: rectifier
(apply external +V to p-side, get electrons in, holes out.)
(apply −V to p-side, get no current flow.)

→ rectifier
→ lots of applications

- V-variable capacitor
- photocells
- light emitters

put 2 together → transistors + controlled switches

Example (Just to get a feeling for scale):

$N_a = 10^{18}\,\text{cm}^{-3}$, $\quad N_d = 10^{17}\,\text{cm}^{-3}$ $\quad$ for Si

$$V_0 = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2} = 0.0259\ \text{V}\ \ln\frac{10^{35}}{2.25\times10^{20}}$$

$$= 0.0259\ \text{V}(33.73) = \underline{0.87\ \text{V}}$$

$$W =$$

$$= \left[\frac{2(11.8)(8.85\times10^{-14})}{1.6\times10^{-19}}(0.87)(10^{-17}+10^{-18})\right]^{1/2}$$

$$= 11.18\times10^{-6}\ \text{cm} = 0.11\times10^{-4}\ \text{cm} = \underline{0.11\ \mu\text{m}}$$

$$x_{no} = \frac{N_a}{N_a+N_d} \cdot W =$$

$$x_{po} = \frac{N_d}{N_a+N_d} \cdot W =$$

$$\mathcal{E}_o = \quad - \frac{(1.6\times10^{-19})(10^{18})(0.01\times10^{-4}\,\text{cm})}{(11.8)(8.85\times10^{-14})}$$

$$=$$