Relativistic Momentum

To make momentum conservation being valid in different inertial frames,

$$\vec{p} = \gamma m \vec{u} \qquad \gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

- Some physicists like to refer to the mass in the above equation as the *rest mass* m_0 and call the term $m = \gamma m_0$ the *relativistic mass*. In this manner the classical form of momentum, m, is retained. The mass is then imagined to increase at high speeds.

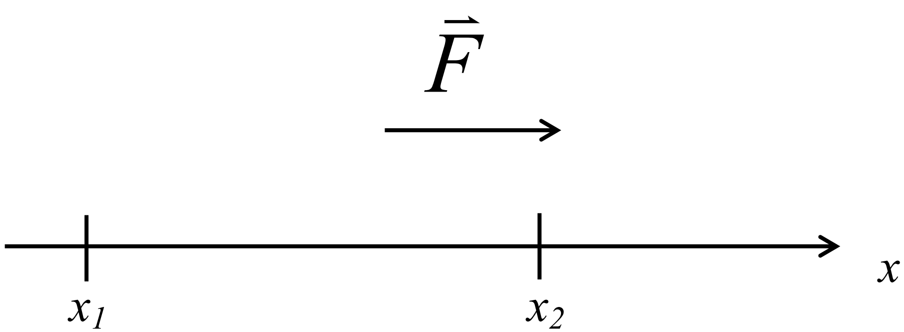
- Most physicists prefer to keep the concept of mass as an invariant, intrinsic property of an object. We adopt this latter approach and will use the term *mass* exclusively to mean *rest mass*. Although we may use the terms *mass* and *rest mass* synonymously, we will not use the term *relativistic mass*. The use of relativistic mass to often leads the student into mistakenly inserting the term into classical expressions where it does not apply.

Relativistic Energy

The work W_{12} done by a force F to move a particle from position 1 to position 2 along a path x is defined to be

$$W_{12} = \int_1^2 \vec{F} \cdot d\vec{x} = K_2 - K_1$$

where K_1 and K_2 are the kinetic energies of the particle at position 1 and 2.



Relativistic Energy

For simplicity, let the particle start from rest under the influence of the force and calculate the kinetic energy K after the work is done.

$$dx = udt$$

$$W = \int_1^2 \vec{F} \cdot d\vec{x} = \int_1^2 \frac{dp}{dt} \cdot dx = \int_1^2 \frac{d}{dt}(\gamma mu) \cdot udt = K$$

$$W = \int_1^2 \vec{F} \cdot d\vec{x} = m\int_1^2 d(\gamma u) \cdot u = m\int_0^u u \cdot (u\frac{d\gamma}{du} + \gamma) \cdot du$$

$$= m\int_0^u \frac{udu}{(1-u^2/c^2)^{3/2}} = \frac{mc^2}{\sqrt{1-u^2/c^2}} - mc^2$$

$$W = K = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$

Kinetic Energy – Classical Approximation

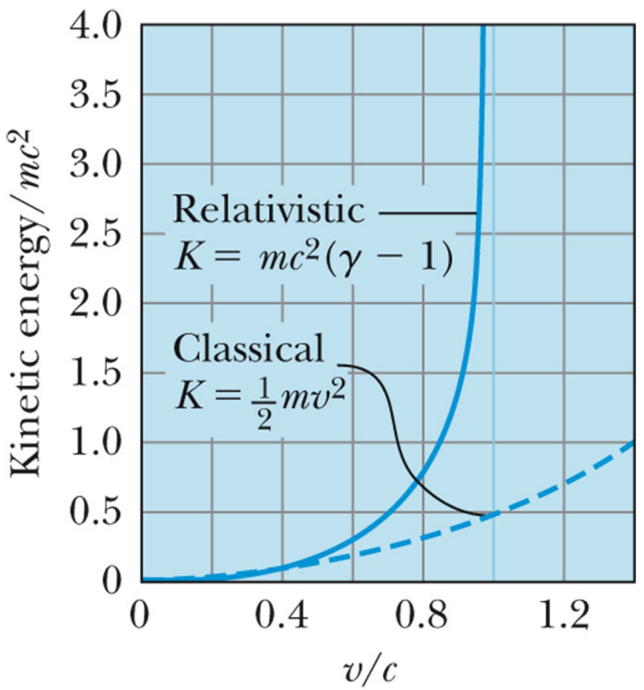
For speeds $u << c$, we expand γ in a binomial series as follows:

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2} \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$$

where we have neglected all terms of power $(u/c)^4$ and greater, because $u << c$. This gives the following equation for the relativistic kinetic energy at low speeds:

$$K = (\gamma - 1)mc^2 = \frac{1}{2}mu^2$$

which is the expected classical result.



Total Energy and Rest Energy

Consider the constant form in the energy expression:

$$K = (\gamma - 1)mc^2 = \gamma mc^2 - mc^2$$

Define Rest Energy E_R: $$E_R = mc^2$$

Total Energy = $K + E_R$ $$E = K + E_R = \gamma mc^2$$

$$p^2c^2 = \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^4 \frac{u^2}{c^2} = \gamma^2 m^2 c^4 (1 - \frac{1}{\gamma^2}) = E^2 - m^2c^4$$

$$E^2 = p^2c^2 + (mc^2)^2$$

For photons – or packets of radiation energy without rest mass: $E = pc$

Emboldened by her success in accelerating the proton, the earth observer decides to accelerate a small 100 KG micro-rocket ship to a velocity of 0.5c. To her, what is the minimum energy that she would require to do this? (Assume 100% efficiency).

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 1.155$$

(1) 16 joules

(2) 1.4×10^{18} joules

(3) 1.04×10^{19} joules

(4) 3.47×10^{10} joules

(5) None of the above