

1)

a)

$$\Sigma(i = 4 \rightarrow n^2) O(i \log i)$$

$$\Theta(n^2 \log n^2) = \Theta(n^2 \log n)$$

b)

$$\Sigma(i = 4 \rightarrow \lfloor n^{3/20} \rfloor) O(i^2)$$

$$\Theta(\lfloor n^{3/20} \rfloor^3) = \Theta(n^{9/20})$$

c)

$$\Sigma(i = \lfloor n/5 \rfloor \rightarrow n) O(n - i)$$

$$\Theta((n - \lfloor n/5 \rfloor + 1)^2) = \Theta(n^2)$$

d)

$$\Sigma(i = n \rightarrow 2n^2) O(1)$$

$$\Theta(n^2)$$

e)

$$\text{Outer loop} - \Theta(n^3)$$

$$\text{Middle loop} - \Theta(n) \rightarrow \Theta(n^7)$$

$$\text{Inner Loop} - \Theta(n^4)$$

$$\text{Total} - \Theta(n^3) * \Theta(n^7) * \Theta(n^4) = \Theta(n^{14})$$

2)

a)

$$f_a(n) = \text{sum of } G_p = \frac{3}{4} + \frac{3}{4}^2 + \dots + \frac{3}{4}^n$$

$$= \left(\frac{3}{4} * (\frac{3}{4} n^2 - 1) \right) / (\frac{3}{4} - 1)$$

$$f_a(n) = O\left(\left(\frac{3}{4}\right) n^2\right)$$

b)

$$\log_4(17) = O(1), 1 \ll \log_4(n)$$

$$f_b(n) = O(\log(n))$$

c)

$$\begin{aligned}\text{sum of } A_p &= 2 * n/2 * (2 * 1 + (2 * 1 + (2n + 1 - 1) * 4) - 1) \\ &= n * (2 + 8n) - 1\end{aligned}$$

$$\text{hence } n \ll n^2$$

$$f_c(n) = \mathbf{O(n^2)}$$

d)

$$6^{13} = O(1)$$

$$f_d(n) = \mathbf{O(1)}$$

e)

$$n^{0.6} \ll n^{0.7}$$

$$f_e(n) = \mathbf{O(n^{0.7})}$$

f)

$$\sqrt{n} \ll \sqrt{n^2} \ll \sqrt{n^3}$$

$$f_f(n) = \mathbf{O(\sqrt{n^3})}$$

g)

$$\log_3(n) \ll n \log_3(n)$$

$$f_g(n) = \mathbf{O(n \log_3(n))}$$