Definition of $\mathcal{E}$-field

$$\mathcal{E}(x) = -\frac{d}{dx}\mathcal{V}(x)$$

$\mathcal{V}(x)$ = electron potential

$=$

E = energy

Band diagrams indicate electron energies

Electrons drift

Holes drift

$\mathcal{E}$-field produces band tilt or "bending"

so $\mathcal{E}$-field exerts a force and sets up gradient

and

carrier gradient causes diffusion (also exerts a force)

Now ready for Einstein Relation ! * #

Very simple but very fundamental:

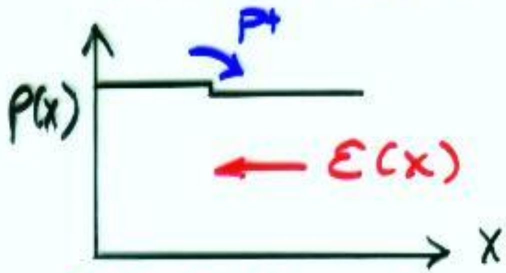
At equilibrium (i.e., no external force such as $\vec{\mathcal{E}}$

$\longrightarrow$ No Current

So Drift and Diffusion currents

must

Imagine a fluctuation in potential that causes a carrier gradient:



carriers diffuse toward lower concentration

But this sets up $\mathcal{E}$-field to oppose more diffusion.

Result: $J_{Total} = 0$ at equilibrium
(not steady-state)

For holes,

$$J_p = 0 = q\mu_p p(x)\mathcal{E}(x) - qD_p \frac{dp(x)}{dx}$$

so $$q\mu_p p(x)\mathcal{E}(x) = qD_p \frac{dp(x)}{dx}$$

$$\mathcal{E}(x) = \frac{D_p}{\mu_p}\,\frac{1}{p(x)}\,\frac{dp(x)}{dx}$$

Recall eq. 3-25b: $p(x) = n_i\, e^{(E_i - E_F)/kT}$

at equilibrium

$$\mathcal{E}(x) = \frac{D_p}{\mu_p}\cdot\frac{1}{kT}\left(\underbrace{\frac{dE_i}{dx}}_{=q\mathcal{E}(x)} - \underbrace{\frac{dE_F}{dx}}_{=0 \text{ by definition}}\right)$$

$$\cancel{\mathcal{E}(x)} = \frac{D_p}{\mu_p} \frac{1}{kT} \cancel{\mathcal{E}(x)}\, q$$

Einstein Relation

Relates D and μ

analogous expression for electrons: $\dfrac{D_n}{\mu_n} = \dfrac{kT}{q}$

We'll use this a lot.

Can verify experimentally – Table 4-1 data

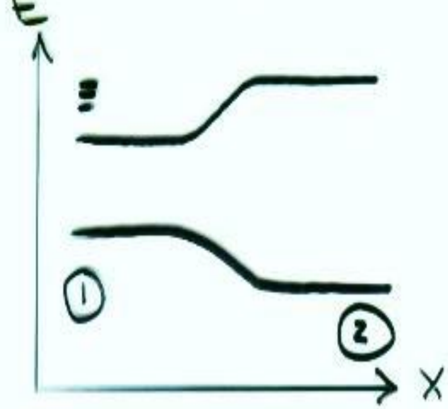
$D/\mu \simeq 0.026\,V$ for both n and p, for different semiconductors

Again, diffusion of carriers sets up new field which counterbalances diffusion current with drift current

This is 2nd kind of "balance" we've talked about so far... first kind was

Gradients in Energy have built-in fields

(preview of what's to come)



Get $\frac{dE}{dx} \neq 0$ by:

- Changes in alloy composition
- Changes in doping

(very important)

Example 4-5: Change in doping

$$N_d = N_0 e^{-ax}$$

$$N_d = N_0 e^{-ax}$$

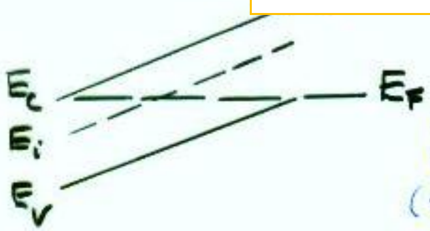
What is $\mathcal{E}(x)$ for $N_d \gg n_i$?

$n(x)$ is not $= n_i e^{(E_i - E_F)/kT}$ any more

$$J = 0 = q\mu_n n(x)\mathcal{E}(x) + qD_n \frac{dn(x)}{dx}$$

$$\mathcal{E}(x) = -\frac{D_n}{\mu_n} \frac{dn(x)}{dx} \Big/ n(x)$$

$$=$$



$\mathcal{E}(x)$ = constant!
Doesn't depend on N_0 or x.
(we'll see this again later).

More Examples

$$N_d(x) = 10^{16} - 10^{19} \cdot x \quad (cm^{-3}) \quad \text{between } 0 \le x \le 1\,\mu m$$

$$\frac{dN_d(x)}{dx} = -10^{19} \quad (cm^{-4})$$

$\mathcal{E}(x) =$

using Einstein Relation

$=$ at $x=0$

$=$

$\sim\!\!\!\#$

$$\frac{D_p}{\mu_p} = \frac{kT}{q}$$

Determine D_p if $\mu_p = 1000\ cm^2/V\text{-}sec$ at $T = 300°K$

$$D_p = 0.0259 \frac{eV}{q} \left(1000 \frac{cm^2}{V\text{-}sec}\right)$$

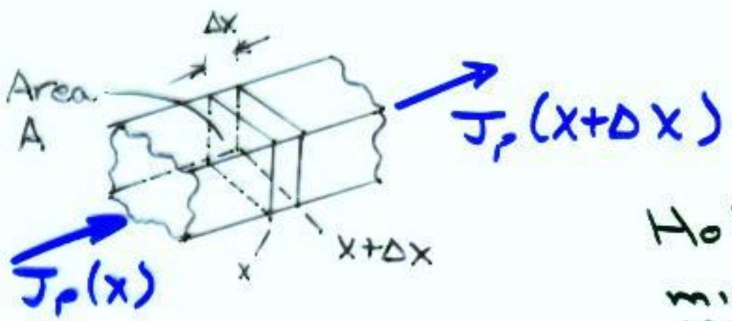
$$= 25.9 \frac{cm^2}{sec}$$

\#

Easy energy / temperature conversion: $kT = 0.0259\,eV$ at $300°K$

Continuity Equation

Add in recombination, relate to D.



Holes in = Holes out minus Recombination in differential volume

$$\left.\frac{\partial p}{\partial t}\right|_{x \to x+\Delta x} = \frac{1}{q}\,\frac{J_p(x) - J_p(x+\Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$

hole buildup — net holes in volume $A\Delta x$/sec — Recombination Rate

Use such an equation for laser, EL diode

$$\left.\frac{\partial p}{\partial t}\right|_{x \to x+\Delta x} = \frac{1}{q}\,\frac{J_p(x) - J_p(x+\Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$

hole buildup — net holes in volume $A\Delta x$/sec — Recombination Rate

Use such an equation for laser, EL diode

As $\Delta x \to 0$, change to derivatives

$$\boxed{\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}}$$

Continuity Equation for Holes

$$\boxed{\frac{\partial \delta n}{\partial t} = +\frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}}$$

continuity Equation for electrons

(opposite sign of q)

$$J_{Total} = J(\text{drift}) + J(\text{diffusion})$$

For negligible drift, $J = J(\text{diffusion})$

$$= q D_n \frac{dn(x)}{dx} = q D_n \frac{\partial \delta n}{\partial x}$$

inside a differential volume

Then

Diffusion Equation for Electrons

and

Diffusion Equation for Holes

Use to solve transient problems of diffusion with recombination

Example:

Steady-state case: excess carrier distribution maintained as a constant

$$\rightarrow \quad \frac{\partial \delta p}{\partial t} = 0 \qquad \frac{\partial \delta n}{\partial t} = 0$$

Diffusion equations now:

$$\frac{d^2\delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} \equiv$$

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} \equiv$$

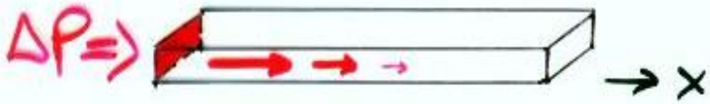
Steady State

where $L_n =$

= "diffusion length"

Big Result! D and τ now related to a length!

Illustrate significance with example:



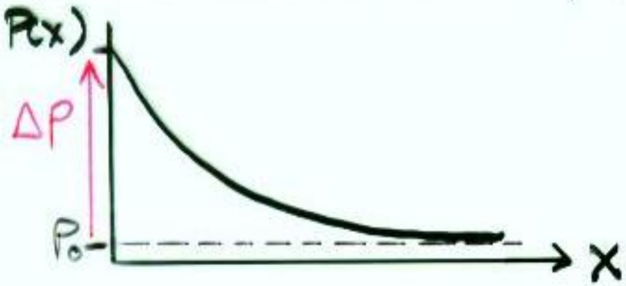
Excess holes Δp injected somehow at $x=0$

$\delta p = \Delta p$ at $x=0$

Concentration δp at $x=0$ maintained constant

Carriers diffuse into bar.

Recombination takes place.



What happens to $p(x)$?