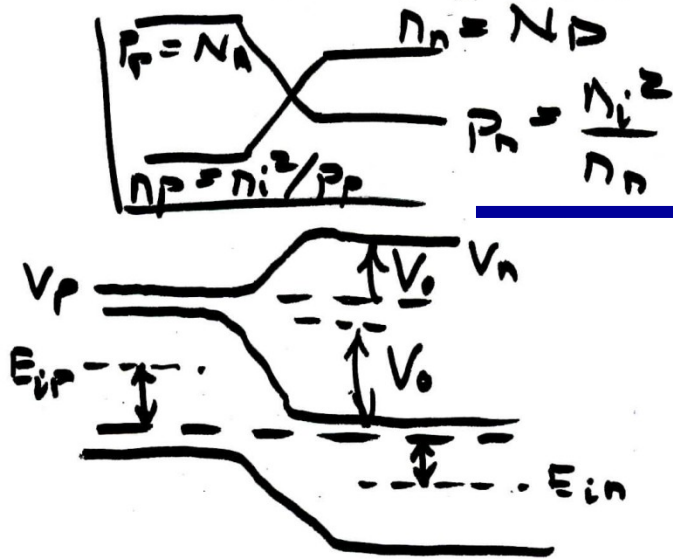


# Final Exam Review

## PN Junction

Contact Potential



$n_p p_p = n_i^2 = p_n n_n$   
in equilibrium

$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} \quad *$$

$$e^{qV_0/kT} = \frac{n_p}{p_p} = \frac{n_n}{p_n}$$

$$\text{also } p_p = n_i e^{(E_{fp} - E_F)/kT}$$

$$n_n = n_i e^{(E_F - E_{fn})/kT}$$

$$E_{fp} - E_{fn} = qV_0$$

## Space charge

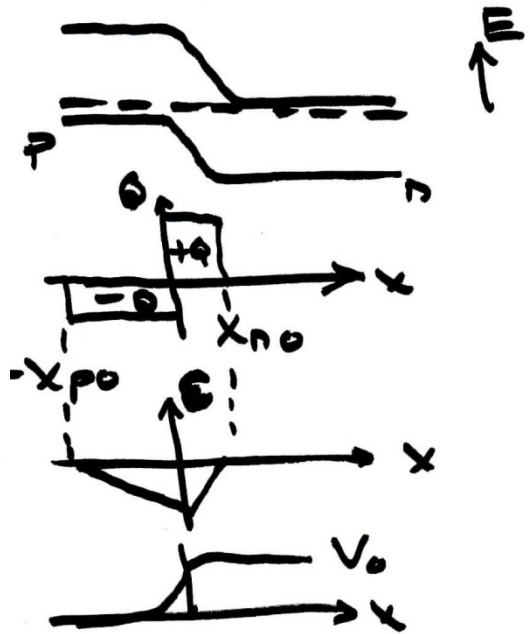
$$X_{po} N_A = X_{no} N_D$$

$$W = \left[ \frac{2\epsilon V_0}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2}$$

$$X_{no} = \frac{W N_A}{N_A + N_D}$$

$$X_{po} = \frac{W N_D}{N_A + N_D}$$

$$\epsilon_0 = \frac{q N_A X_{po}}{\epsilon}$$



## Bias



$$W = \left[ \frac{2\epsilon (V_0 - V_f)}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2}$$

$$= \left[ \frac{2\epsilon (V_0 + V_r)}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2}$$

forward  
reverse

$$\Delta P_n = P(x_{n0}) - P_n = P_n (e^{qV/kT} - 1)$$

$$\Delta n_p = n_p(x_{p0}) - n_p = n_p (e^{qV/kT} - 1)$$



$P_n < n_p$  if  $N_D > N_A$

$$S_n(x_p) = \Delta n e^{-x_p/L_n} = n_p (e^{qV/kT} - 1) e^{-x_p/L_n}$$

$$S_p(x_n) = \Delta p e^{-x_n/L_p} = P_n (e^{qV/kT} - 1) e^{-x_n/L_p}$$

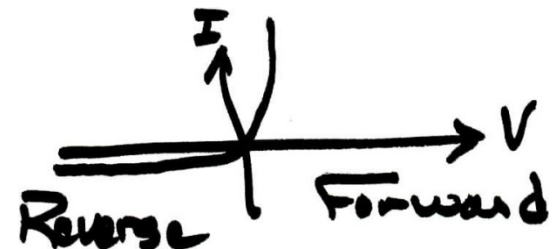
**$D/\mu = kT/q$  Einstein Relation**

**$L = [D\tau]^{1/2}$  Diffusion Length**

$$I_{total} = I_p(x_n=0) + I_n(x_p=0)$$

$$= q A D_p \frac{\Delta P_n}{L_p} + q A D_n \frac{\Delta n_p}{L_n}$$

$$= I_0 (e^{qV/kT} - 1) \quad \text{Diode Equation}^*$$



## Applications

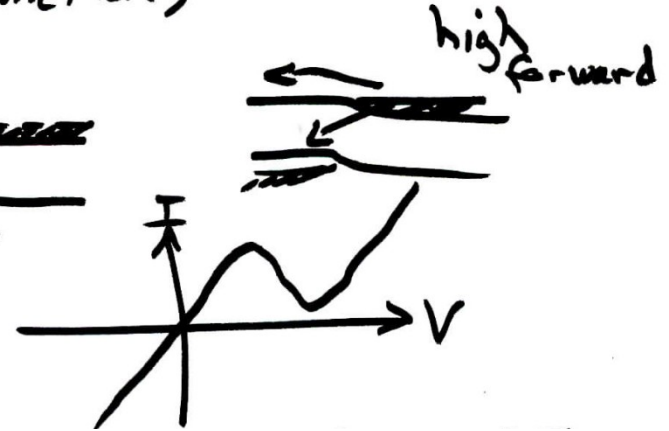
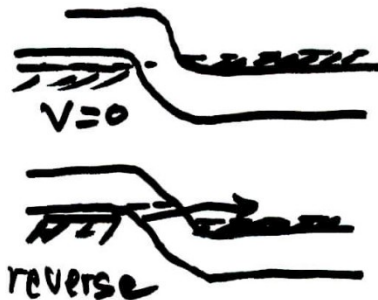
Rectifiers - Large gap, low doping,  $A$  large ( $R_s$  small),  $L$  small

Switching diodes - Very short  $\tau_n, \tau_p$  (but lower gain)

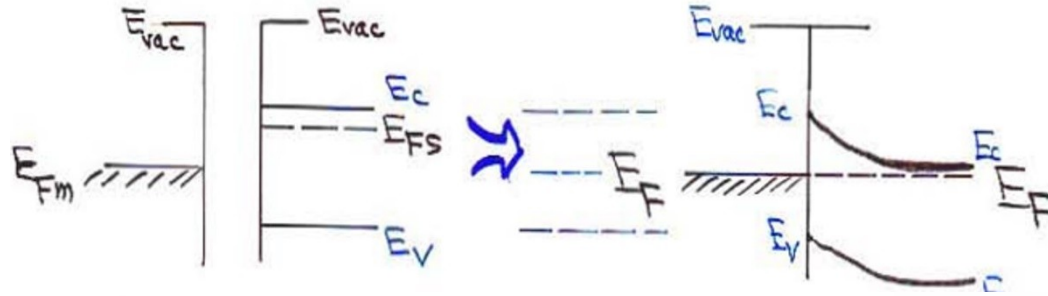
Break down Diode  $V_{br}$

Varactor  $V^{-1/2}$  for  $-V_r \gg V_0$  (abrupt junction)

Tunnel Diodes  
(Degenerate)

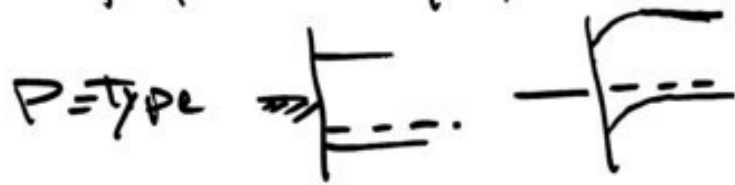


# Schottky Barriers



$$W = \sqrt{\frac{2\epsilon(V_0 - V)}{qN_D}} \quad \epsilon = \epsilon_s \epsilon_0$$

(get from P.N. eqn. for  $W$ )



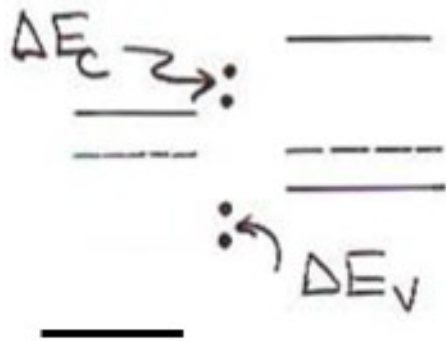
h barrier



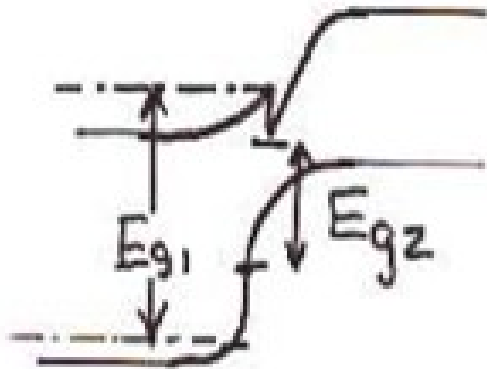
1. Join M and SC band structures by lining up vacuum levels  $E_{vac}^{metal}$  and  $E_{vac}^{SC}$ .
2.  $\Phi_M = E_{vac} - E_F^{metal}$  stays the same after contact.
3.  $\chi = E_{vac} - E_C$  stays the same at interface.
4.  $E_C - E_F$  stays the same in the SC bulk
5. Line up  $E_F^{SC}$  with  $E_F^{metal}$ .
6. Curve  $E_C - E_F$  to connect  $E_C^{surface}$  and  $E_C^{bulk}$
7. Curve valence band same way.
8. Schottky barrier  $\Phi_{SB} = E_C^{surface} - E_F^{metal}$ .
9. n-type band bending  $qV_B = \Phi_{SB} - (E_C - E_F)$
10.  $(E_C - E_F) = (E_C - E_i) - (E_F - E_i)$
11.  $n = n_i e^{(E_F - E_i)}$

$$I = I_0 (e^{qV - V_0 / kT} - 1)$$

# Heterojunctions

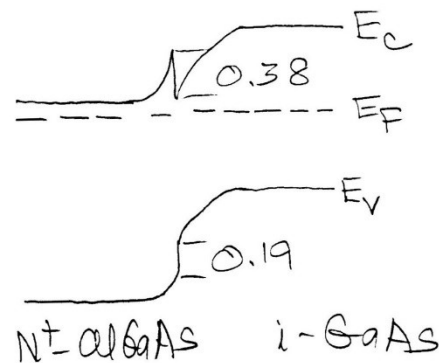


1. Align SC Fermi levels with bands flat.
2. Find  $x=0$  based on  $N_1$  vs  $N_2$  doping.
3. N-type bends up. P-type bends down.
4. Insert  $\Delta E_C$  and  $\Delta E_V$ .
5. Use 2/3 – 1/3 Rule.
6. Connect  $E_C$ 's . Connect  $E_V$ 's.
7. Keep each band gap constant.
8. No notch for longer connection.

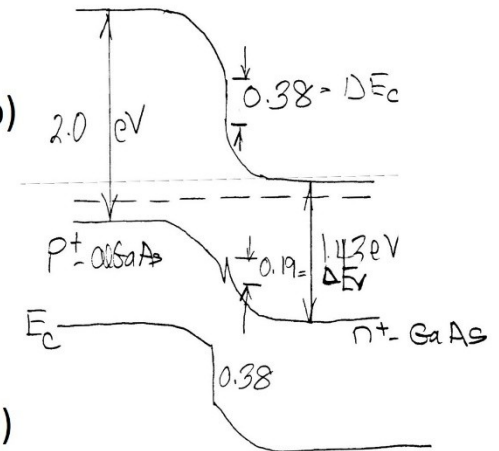


## Examples

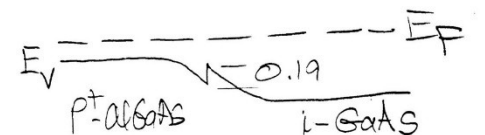
(a)



(b)

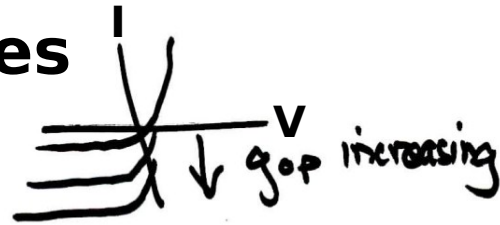
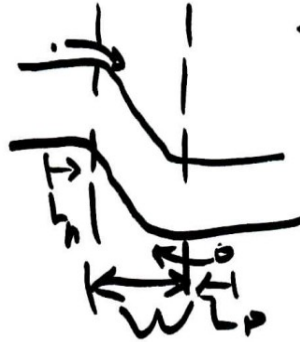


(c)





# Photodiodes



$$V_{oc} = \frac{kT}{q} \ln \left[ \frac{I_{op}}{I_{th}} + 1 \right]$$

for open circuit

$$I_{op} = q A q_{op} (L_p + L_n + W)$$

$$I_{op} = q A \left( \frac{L_p}{\tau_p} P_n + \frac{L_n}{\tau_n} n_p \left( e^{qV/kT} - 1 \right) \right)$$

$-I_{op}$

\*  $\frac{1}{\tau} = \frac{D}{L^2} \rightarrow L^2 = D\tau$ ; also  $\frac{D}{\tau} = \frac{kT}{q}$  Einstein

Reverse bias  
Photodetector

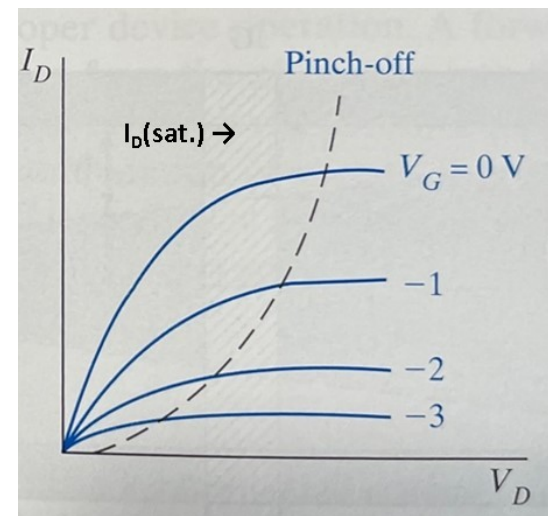
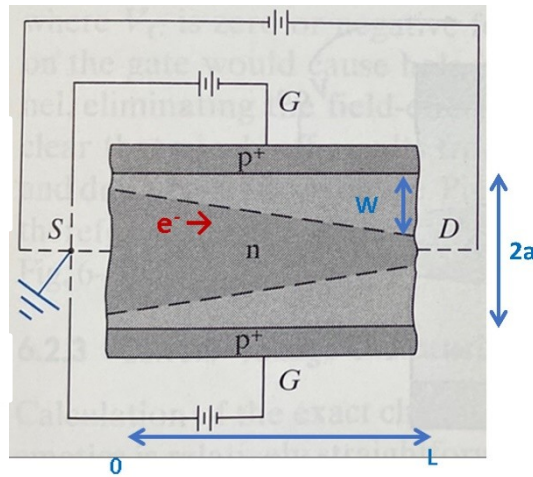


no bias  
Solar cell



Like a battery

# JFET



$$W(x=L) = \left[ \frac{2\epsilon (-V_{GD})}{q N_d} \right]^{1/2}$$

$V_{GD}$  is (-)  
negative bias to G

$$V_p = -V_{GD}(\text{pinch-off}) = -V_G + V_D$$

$$V_p = \frac{q a^2 N_d}{2\epsilon}$$

with negative  $V_G$ ,  
need less  $V_D$  to  
reach pinch-off

$$I_D = G_0 V_p \left[ \frac{V_0}{V_p} + \frac{2}{3} \left( -\frac{V_G}{V_p} \right)^{3/2} - \frac{2}{3} \left( \frac{V_D - V_G}{V_p} \right)^{3/2} \right]$$

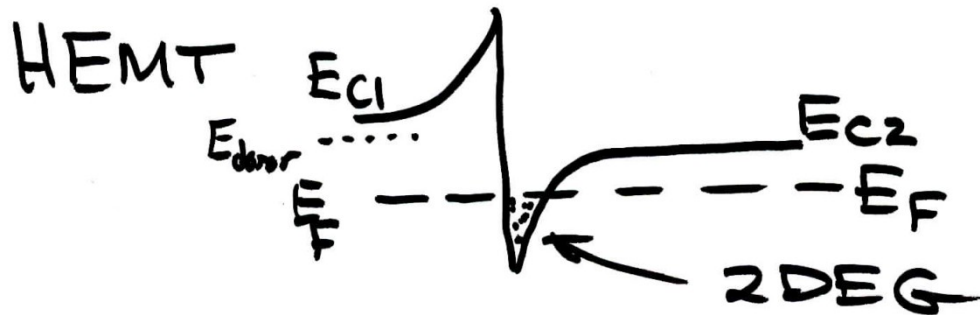
$$G_0 = \text{conductance} = \frac{2aZ}{\rho L}$$

$$I_D(\text{sat.}) = G_0 V_p \left[ \frac{V_0}{V_p} + \frac{2}{3} \left( -\frac{V_G}{V_p} \right)^{3/2} + \frac{1}{3} \right]$$

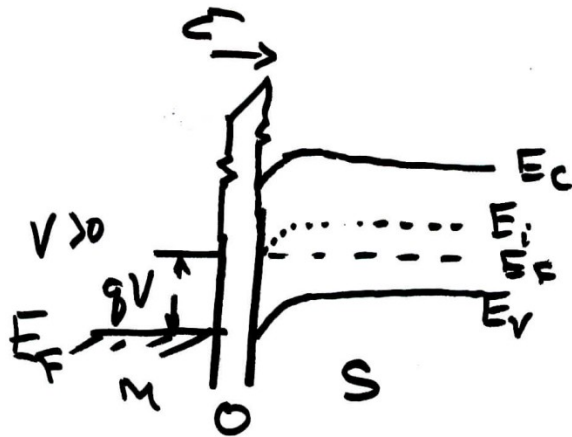
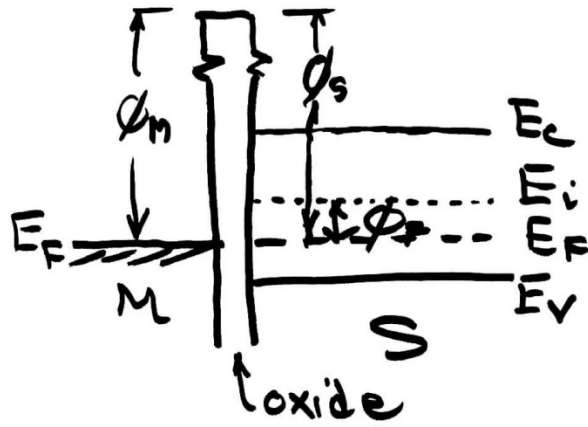


$g_m = \text{mutual transconductance} \equiv \frac{\partial I_D}{\partial V_G}$

$$g_m(\text{sat.}) = \frac{\partial I_D(\text{sat.})}{\partial V_G} = G_0 \left[ 1 - \left( -\frac{V_a}{V_P} \right)^{1/2} \right]$$

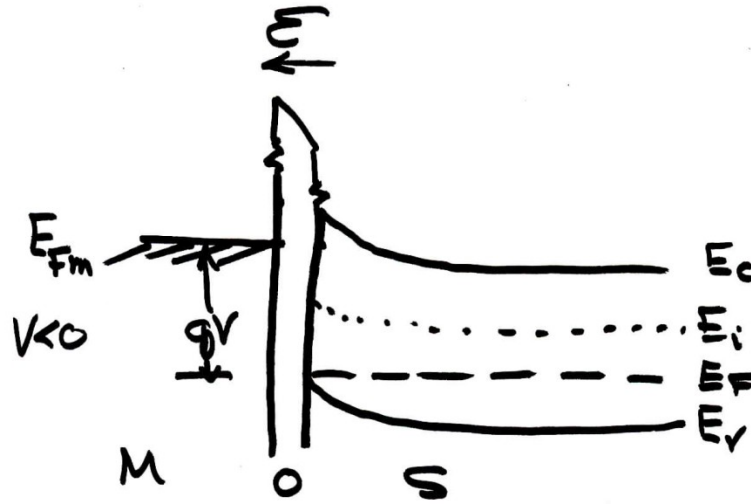


# MOSFET



Depletion

Positive voltage to metal



Accumulation

$$E(x) = \frac{1}{q} \frac{dE_i}{dx}$$

$$P = n_i e^{(E_i - E_F)/kT}$$

$$\phi_s (\text{Inverted}) = 2\phi_F = \frac{2kT}{q} \ln \frac{N_A}{n_i}$$

$$\rho = q(N_d^+ - N_a^- + P - n)$$

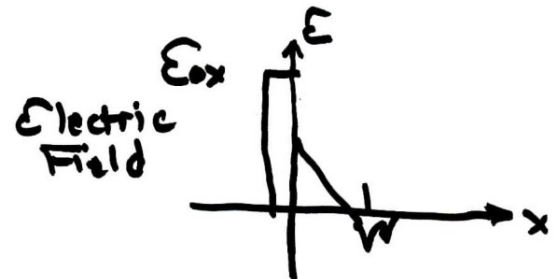
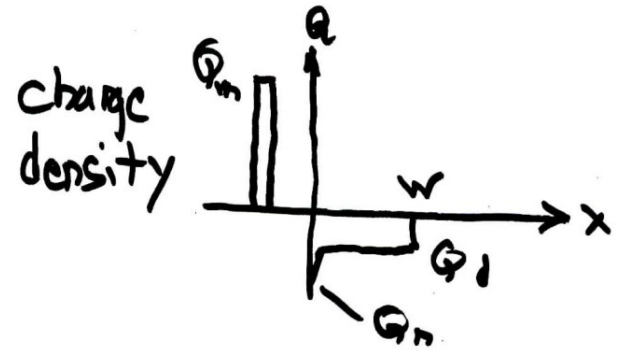
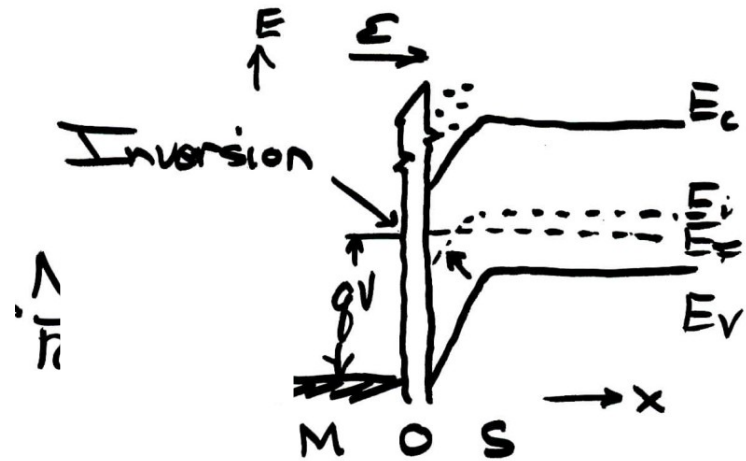
$$Q_m = -Q_s = \underbrace{q N_a W}_{Q_d} - Q_n$$

$$V = V_i + \phi_s$$

$$V_i = -\frac{Q_{sd}}{\epsilon_i} = -\frac{Q_s}{C_i}$$

$$W = \left[ \frac{2\epsilon_s \phi_s}{q N_a} \right]^{1/2} \quad \text{like pt-n junction}$$

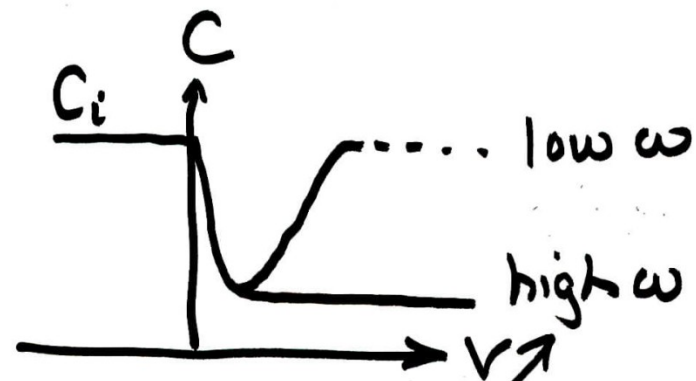
$$W_m = \left[ \frac{2^2 \epsilon_s kT \ln(N_a/n_i)}{q^2 N_a} \right]^{1/2}$$



$$W_m = \left[ \frac{2^2 \epsilon_s kT \ln(N_a/n_i)}{q^2 N_a} \right]^{1/2}$$

$$V_T = -\frac{Q_d}{C_i} + 2\phi_F$$

$$C_i = \frac{\epsilon_i}{d} \quad C_d = \frac{\epsilon_s}{W}$$



$$C = \frac{C_i C_d}{C_i + C_d}$$

# Real MOS Transistors

$$\Phi_{ms} = \Phi_M - \Phi_S \neq 0 \quad V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i}$$

$$\phi_F = \frac{kT}{q} \ln \frac{N_A}{n_i}$$

$$W_m = \sqrt{\frac{2}{q} \left[ \frac{E_s \phi_F}{N_A} \right]^{1/2}}$$

$$C_i = \frac{C_i}{d}$$

$$Q_D = -q N_A W_m$$

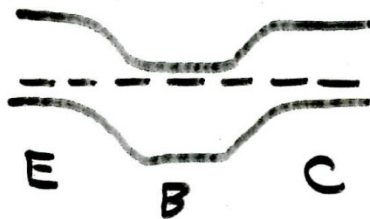
$$C_d = \frac{E_s}{W_m}$$

$$C_{min} = \frac{C_i C_d}{C_i + C_d}$$

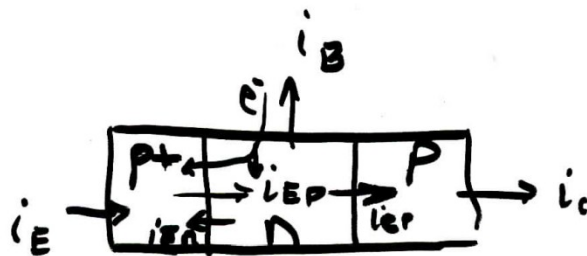
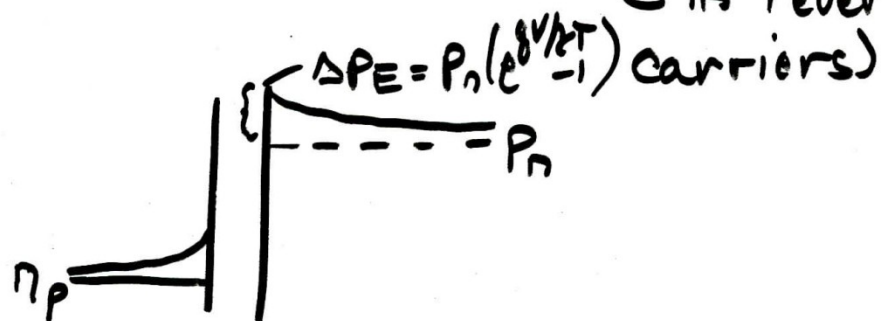
$$V_T = \Phi_{ms} - \frac{Q_i}{C_i} - \frac{Q_D}{C_i} + 2\phi_F$$



# Bipolar Junction Transistor



"on" : E in forward bias  
(lower the barrier),  
C in reverse bias (sweep out all



$$i_E = i_{EP} + i_{EN}$$

$$\frac{i_{EP}}{i_{EP} + i_{EN}} = \gamma$$

$$i_{EN} + i_{EP}$$

Emitter  
Injection Efficiency

$$i_C = \beta i_{EP}$$

Base Transport  
Factor

$\beta \approx \alpha$  Current Transfer Ratio

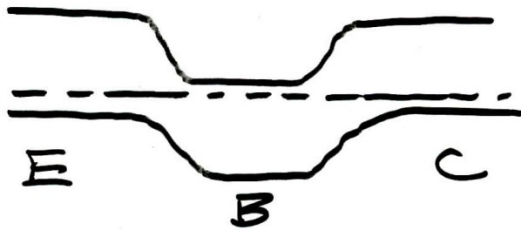
$$\beta = \frac{\alpha}{1 - \alpha}$$

Base-to-collector Current Amplification factor (gain)

Hope  $\alpha \sim 1$

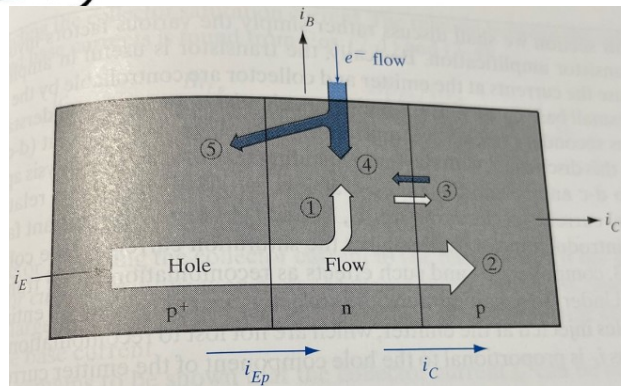
Know how to calculate  $\Delta P_E$  and  $\Delta P_C$ . You need it for  $I_E$ ,  $I_C$ , and  $I_B$ .

BJT

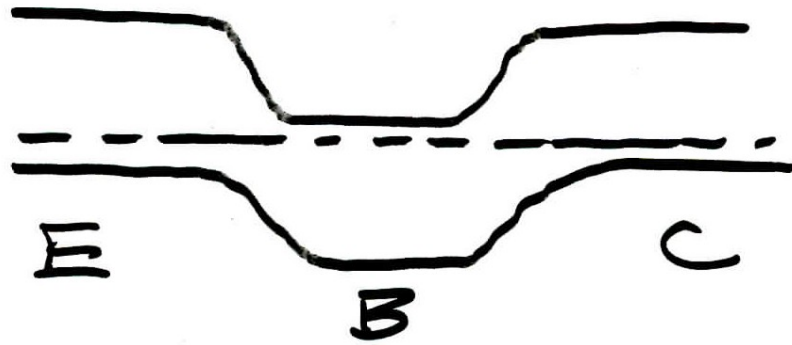


"on" — lower E-B barrier; C is reverse: Sweep out carriers

$$\Delta P_E = P_n (e^{qV_{EB}/kT} - 1)$$

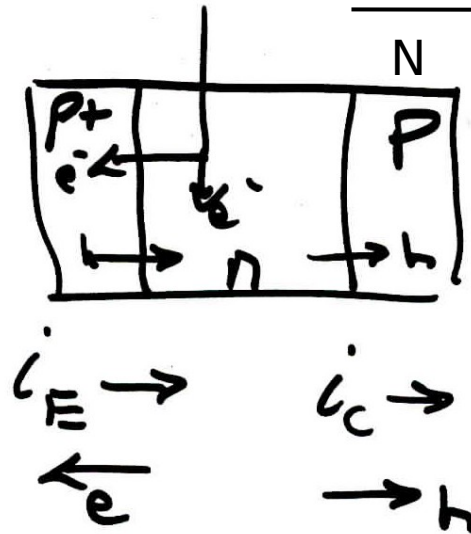
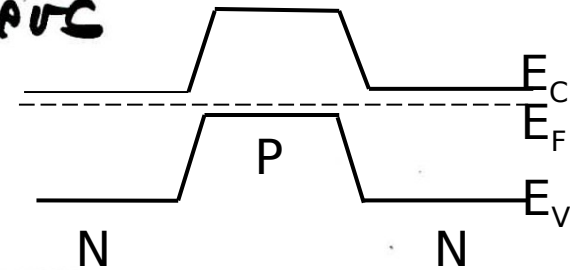
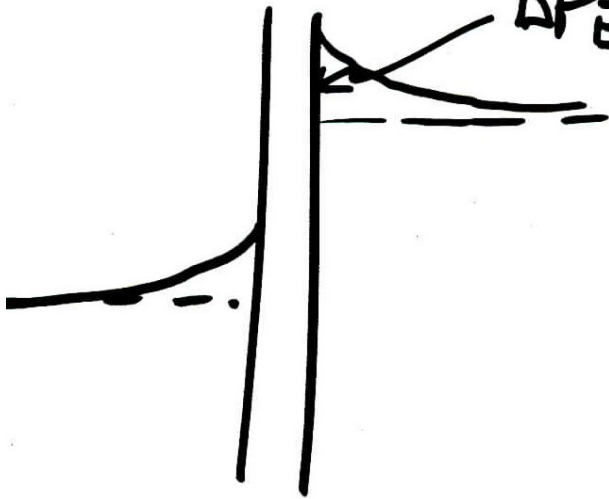


BJT



"on" — lower E-B barrier; C is reverse: Sweepout carriers

$$A_{PE} = P_n / (e^{qV/kt} - 1)$$



$$I_{EP} = g \frac{ADP}{L_p} (\Delta P_E \coth \frac{W_b}{L_p} - \Delta P_c \operatorname{csch} \frac{W_b}{L_p})$$

$$I_{CP} = g \frac{ADP}{L_p} (\Delta P_E \operatorname{csch} \frac{W_b}{L_p} - \Delta P_c \coth \frac{W_b}{L_p})$$

$$I_B = I_E - I_C = g \frac{ADP}{L_p} \left[ (\Delta P_E + \Delta P_c) \tanh \frac{W_b}{2L_p} \right]$$



$$Q_P = \frac{1}{2} g A W_b \Delta P_E \quad \text{replaced every } \tau_p$$

$$\text{so } I_B = \frac{Q_P}{\tau_p} = \frac{1}{2} g A W_b \frac{\Delta P_E}{\tau_p}$$

$$\gamma = \frac{I_{EP}}{I_{EP} + I_{EN}}$$

$$B = \frac{I_C}{I_{EP}}$$

$$\alpha = B\gamma$$

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\gamma = \left[ 1 + \frac{L_p^2}{L_n^2} \frac{n_n \mu_n^p}{p_p \mu_p^n} \tanh \frac{W_b}{L_p} \right]^{-1} \approx \left[ 1 + \frac{W_b}{L_n^2} \frac{n_n \mu_n^p}{p_p \mu_p^n} \right]^{-1}$$

$$B = \operatorname{sech} \frac{W_b}{L_p}$$

**Sech  $y \sim 1$  Ctnh  $y \sim 1/y$**

**Csch  $y \sim 1/y$**

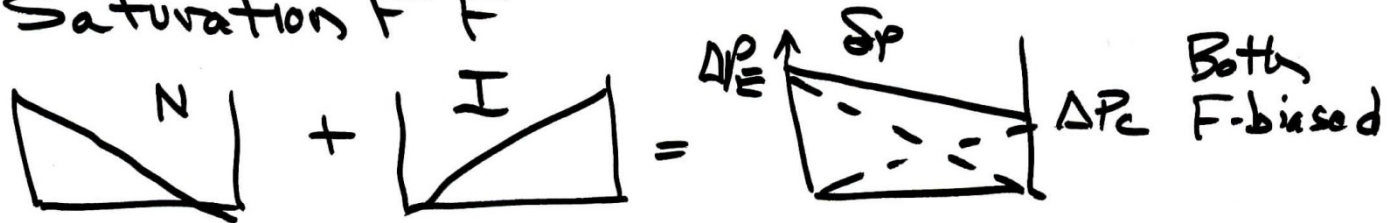
**Tanh  $y \sim$**

# P-n-p coupled diodes



4 Options:

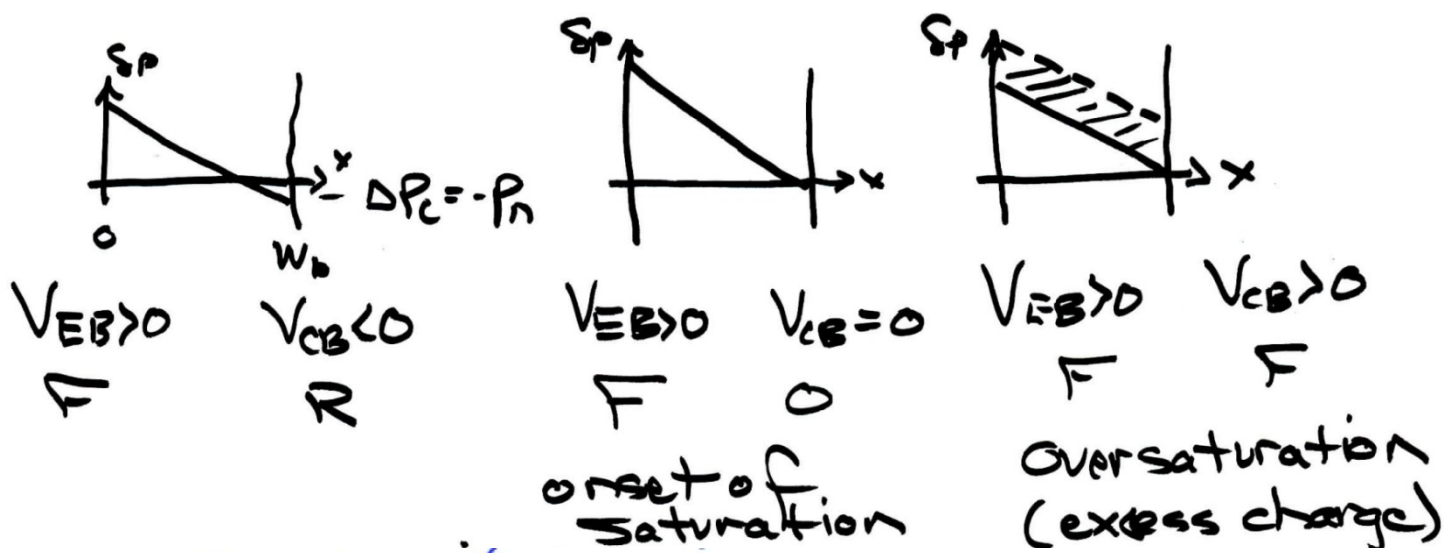
Normal	F	P
Inverted	P	P
Cutoff	P	P
Saturation	F	F



$$I_{EP} = q A D_P \left( \frac{I_V}{L_P} \right) \text{ (emitter - collector diode)}$$

$$I_{CP} = \text{ " " ( " " )}$$





$$Q_p \equiv \Delta P_E \frac{W_b}{2} \cdot q A$$

$$\begin{aligned} \text{Current} &= \text{charge replaced per second} \\ &= q A \frac{\Delta P_E W_b}{2 \tau_p} \end{aligned}$$

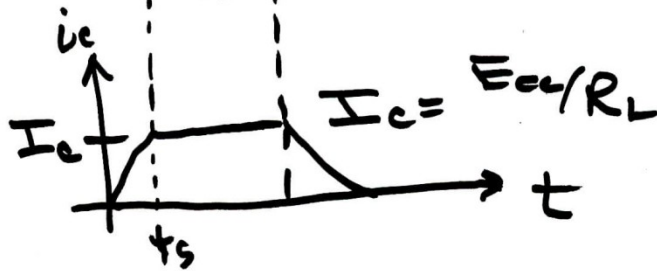
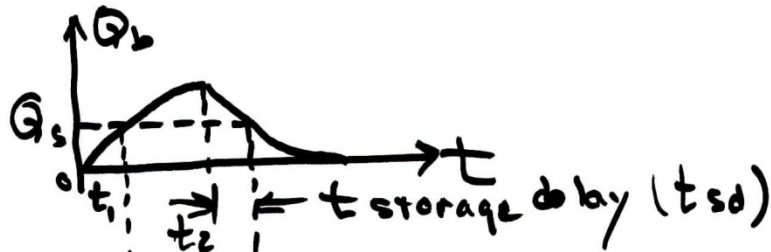
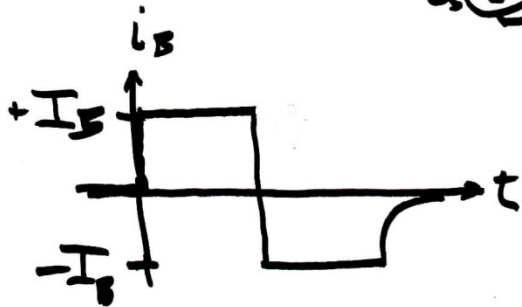
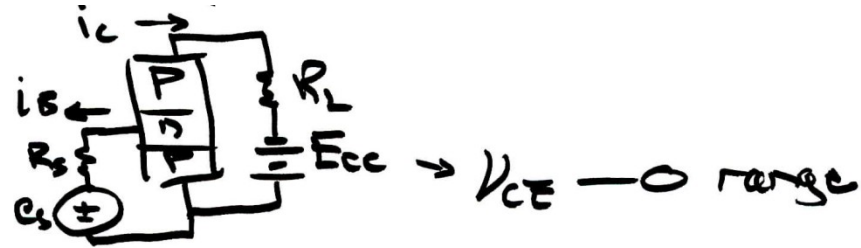
Charge Control Approximation

Transit time across the base often limits high frequency.

$$\beta \approx \frac{\cosh W_b/L_p}{\tanh W_b/2L_p} = \frac{2L_p^2}{W_b^2} = \frac{2D_p \tau_p}{W_b^2} \equiv \frac{\tau_p}{\tau_t}$$

$$\tau_t = \frac{W_b^2}{2D_p}$$

# Switching



$$I_C = \beta I_B$$

increase  $i_B$  to increase  $i_C$   
but not above saturation

$I_C$  fixed by  $V_{CC}$  and  $R_L$

(can only draw so much current)

# Early Effect (base narrowing)

