

ECE 2050 Autumn 2023 Homework 2
Due 5:00 pm, Friday Sept 8, 2023
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BC:2.1 For each of the continuous time sinusoids below, sampled at spacing $T_s = \Delta t = 2.5 \mu s$, find the values for A , $\hat{\omega}_o$ (in radians) and θ_o (in degrees) for the sampled version of the signal that has the form:

$$f[n] = A \cos(\hat{\omega}_o n + \theta_o)$$

where $\hat{\omega}_o$ represents the (positive) normalized radial frequency (in radians) of the *principal zone (principal alias)* representation of the sampled sinusoid.

- a.) $f(t) = 13 \cos(1300000\pi t - 30^\circ)$
- b.) $f(t) = 0.5 \cos(960000\pi t - 15^\circ)$
- c.) $f(t) = 200 \cos(720000\pi t + 45^\circ)$
- d.) $f(t) = 125 \cos(540000\pi t - 25^\circ)$
- e.) $f(t) = 0.001 \cos(120000\pi t - 37^\circ)$

BC:2.2 Given the continuous time signal,

$$f(t) = 140 \cos(200\pi t - 25^\circ)$$

for each of the sampling periods in parts (a–e) listed below, sketch the two sided spectrum vs the normalized radial frequency, $\hat{\omega}$, including all aliases between -4π and 4π . Include the complex coefficients for each spectral line as magnitude and phase angle (in degrees). For each of parts (a–e) below, identify the normalized radial frequency, $\hat{\omega}_o$ (in radians) and corresponding angle, θ_o (in degrees), for the *principal zone (principal alias)* description of the sampled cosine that has the form $f[n] = A \cos(\hat{\omega}_o n + \theta_o)$. For each case, identify whether the sampled signal is oversampled or undersampled and justify your choice.

- a.) $T_s = \Delta t = 18 \text{ ms}$
- b.) $T_s = \Delta t = 13.5 \text{ ms}$
- c.) $T_s = \Delta t = 7.5 \text{ ms}$
- d.) $T_s = \Delta t = 3.75 \text{ ms}$
- e.) $T_s = \Delta t = 1.5 \text{ ms}$

BC:2.3 Express the following sampled signals using a sum of weighted *impulses (delta functions)*. If the weights are complex valued (not purely real), then express the weights in polar form with the angle in degrees. (You can use the “summation” symbol: Σ and specify limits of the summation.)

a.)

$$f_a[n] = \begin{cases} \sin(0.55\pi n + 17^\circ) & \text{for } |n| \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

b.)

$$f_b[n] = \begin{cases} \sin(0.55\pi n + 17^\circ) & \text{for } -3 \leq n \leq 8 \\ 0 & \text{otherwise} \end{cases}$$

c.)

$$f_c[n] = \begin{cases} (-0.25j/(0.5 - 0.5j))^{n-1} & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

d.)

$$f_d[n] = \begin{cases} \left(\frac{2}{1+j\sqrt{3}}\right)^{n-1} & \text{for } n \geq -5 \\ 0 & \text{otherwise} \end{cases}$$

e.)

$$f_e[n] = \begin{cases} \left(0.5 + \frac{\sqrt{3}j}{2}\right)^{1-n} & \text{for } n \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

BC:2.4 Use weighted *unit step functions* of the form $u[n]$ and/or $u[n - n_o]$ to express the following sampled signals. If the weights are complex valued (not purely real), then express the weights in polar form with the angle in degrees.

a.)

$$f_a[n] = \begin{cases} \sin(0.55\pi n + 17^\circ) & \text{for } |n| \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

b.)

$$f_b[n] = \begin{cases} \sin(0.55\pi n + 17^\circ) & \text{for } -3 \leq n \leq 8 \\ 0 & \text{otherwise} \end{cases}$$

c.)

$$f_c[n] = \begin{cases} \left(-0.25j/(0.5 - 0.5j)\right)^{n-1} & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

d.)

$$f_d[n] = \begin{cases} \left(\frac{2}{1+j\sqrt{3}}\right)^{n-1} & \text{for } n \geq -5 \\ 0 & \text{otherwise} \end{cases}$$

e.)

$$f_e[n] = \begin{cases} \left(0.5 + \frac{\sqrt{3}j}{2}\right)^{1-n} & \text{for } n \geq 3 \\ 0 & \text{otherwise} \end{cases}$$