

Homework 4 - Math 2568 (Autumn 2022)

Prof. Cueto

Due date: Friday September 23, 2022 (on Carmen).

The sections and problem numbers refer to the course's textbook (L.W. Johnson, R.D. Riess, J.T. Arnold: *Introduction to Linear Algebra*, 5th edition, Pearson.)

Section	Assigned Problems	Problems to be turned in
§2.1	1, 5, 6, 10, 15, 19, 26, 28, 30, 35	5, 6, 19, 26, 35
§2.2	1, 5, 8, 12, 22, 25, 28, 30, 33	5, 12, 22, 28, 33
§2.3	1, 4, 8, 13, 16, 19, 23, 32, 34, 42, 48	4, 8, 16, 23, 48

Section 2.1

5) $A = (-3, 5)$ $B = (2, 2)$
 $C = (3, 4)$ $D = (-2, 7)$

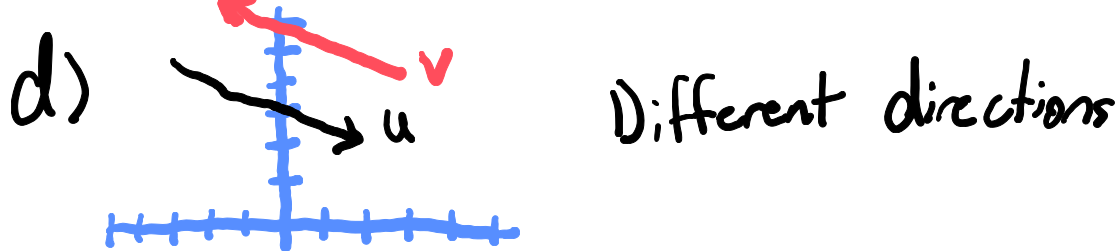
a) $u = \overrightarrow{AB}$ $x \rightarrow 2 - (-3) = 5$ $y \rightarrow 2 - 5 = -3$ $5 + (-3) = 2$

$v = \overrightarrow{CD}$ $x \rightarrow -2 - 3 = -5$ $y \rightarrow 7 - 4 = 3$ $(-5) + 3 = 2$
 $2 = 2 \checkmark$

b) $u = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$
 $v = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

c) $b_1 - a_1 = 2 - (-3) = 5$ $b_2 - a_2 = 2 - 5 = -3$
 $d_1 - c_1 = -2 - 3 = -5$ $d_2 - c_2 = 7 - 4 = 3$

$5 \neq -5$
 $-3 \neq 3$



6) $A = (-1, -3)$ $B = (3, 2)$ $C = (0, 2)$

$AB = (3 - (-1), 2 - (-3)) = (4, 5)$

$D = C + AB = (4, 5) + (0, 2) = (4, 7)$

$D = (4, 7)$

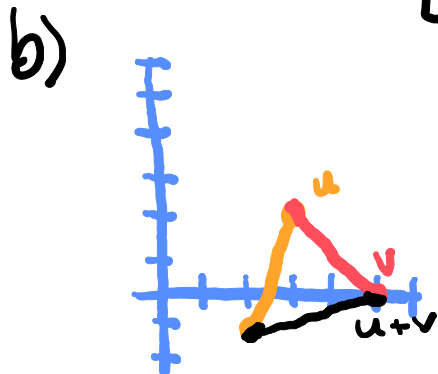
19) $u = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ $A = (2, -1)$

$B = (3, 2)$

$C = (5, 0)$

a) $B = u + A = (1+2, 3+(-1)) = (3, 2)$

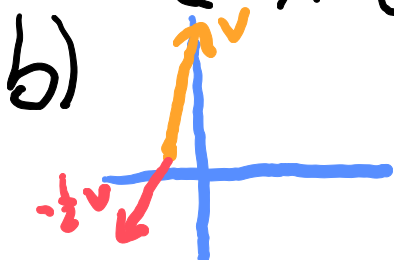
$C = (u+v) + A = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + A = (3+2, 1-1) = (5, 0)$



26) $v = 2i + 6j = [2, 6]$ $A = (-2, 1)$

a) $B = (-2+2, 1+6) = (0, 7)$

$C = A + (-\frac{1}{2}v) = (-2+(-1), 1+(-3)) = (-3, -2)$



$$35) \quad v \parallel [1, 3] \quad A = (3, 1) \quad B \text{ on } y = 7$$

$$B = A + 2[1, 3] = (3+2, 1+6) = \boxed{(5, 7)}$$

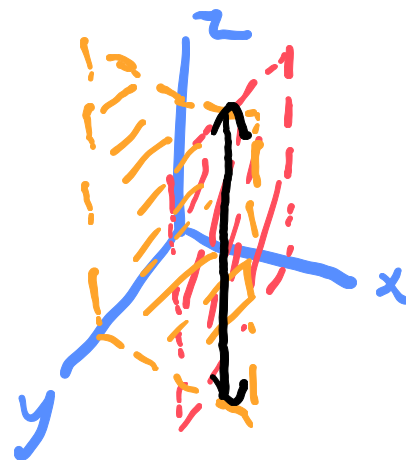
Section 2.2

$$5) \quad P = (2, 3, 1) \quad Q = (0, 5, 7)$$

$$M = \left(\frac{2}{2}, \frac{3+5}{2}, \frac{1+7}{2} \right) = (1, 4, 4)$$

$$d(M, 0) = \sqrt{1^2 + 4^2 + 4^2} = \boxed{\sqrt{33}}$$

$$12) \quad (5, 0, 0) \text{ \& } (0, 5, 0)$$



It's a line

$$22) \quad v = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} \quad B = (4, 3, 2)$$

$$A = v + B = (0+4, 3+3, 2+2) = \boxed{(4, 6, 4)}$$

$$28) u = [9, -3, 2] \quad v = [1, 0, 1]$$

$$a) u + 2v = [9+2, -3+0, 2+2] = 11i - 3j + 4k$$

$$b) \|u - v\| = \|[9-1, -3, 2-1]\| = \sqrt{8^2 + 3^2 + 1^2} = \sqrt{74}$$

$$c) w = \frac{v - u}{2} = \frac{[-8, 3, -1]}{2} = \left[-4, \frac{3}{2}, -\frac{1}{2}\right]$$

$$33) v = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \quad \|u\| = 5$$

$$\|v\| = \sqrt{1^2 + 2^2 + 2^2} = 3$$

$$u = -5\left(\frac{1}{3}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}\right) = -5\begin{bmatrix} -1/3 \\ 2/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 5/3 \\ -10/3 \\ -10/3 \end{bmatrix}$$

Section 2.3

$$4) u = [4, 2, -3] \quad v = [-2, 1, -2]$$

$$u \cdot v = 4(-2) + 2(1) - 3(-2) = 0$$

$$8) u = [2, -3, 1] \quad v = [1, -2, 3]$$

$$\|u\| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$$

$$\|v\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$\theta = \tan^{-1} = 45^\circ$$

$$\cos 45 = 0.707$$

$$16) u \cdot i = 12 \quad u \cdot k = 3 \quad \|u\| = 13$$

$$u = [12, 4, 3] \text{ or } [12, -4, 3]$$

$$23) u = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \quad q = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \|q\| = \sqrt{2}$$

$$u_1 = \text{proj}_q u = \frac{5\sqrt{2}}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$u_2 = u - u_1 = \begin{bmatrix} 7 \\ 3 \end{bmatrix} - \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$48) u = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$u \cdot (v \times w) = ?$$

$$v \times w = (-2, -2, 4)$$

$$u \cdot (-2, -2, 4) = 0 \quad \text{They are coplanar}$$