ECE 2050 Autumn 2023 Homework 8
Due Friday November 3, 5:00 pm
upload as single PDF to Carmen

BC:8.1 A periodic signal with period $T_o = 125$ ns is known to have the form

$$f(t) = \sum_{k=-\infty}^{\infty} \alpha_k \exp\left(j\frac{2\pi}{T_o}kt\right)$$

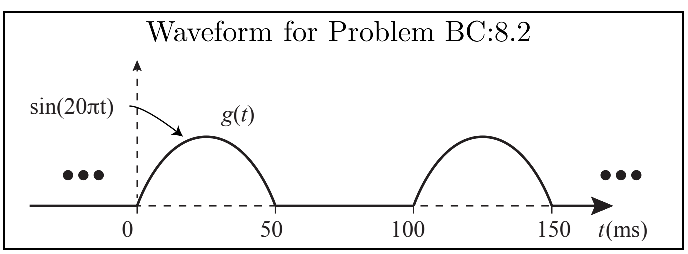
where $\alpha_0 = -15$, $\alpha_1 = 12.5\angle\text{-}13^o$, $\alpha_{-1} = 12.5\angle 13^o$, $\alpha_2 = 3\angle 7^o$, $\alpha_{-2} = 3\angle\text{-}7^o$, and all other $\alpha_k = 0$.

a.) What is the fundamental frequency (in Hz) for the periodic signal.

b.) Use the Euler formula to express $f(t)$ as the sum of cosines plus a constant.

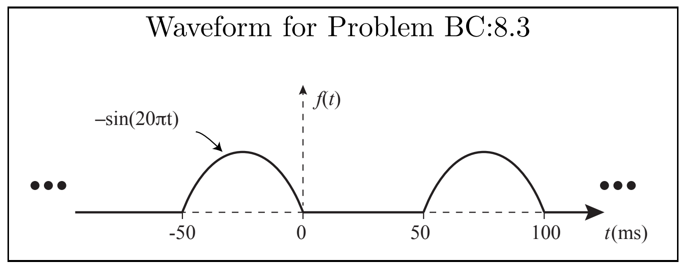
BC:8.2 The periodic waveform, $g(t)$, shown below is a *halfwave rectified* sine where only the positive parts of the sine are nonzero, and the signal is zero whenever the sine would be negative. Note the time units on the graph are milliseconds.

$$g(t) = \sum_{k=-\infty}^{\infty} \alpha_k \exp\left(j\frac{2\pi}{T_o}kt\right)$$



Determine a formula for the coefficients, α_k using the definition integral. (Hint: it is recommended to treat α_0 as a special case, and it is useful to expand the sine using the Euler identity for other values of α_k. If any α_k is 0/0 use L'Hôpital's rule to evaluate for that value of k.) Evaluate the coefficients α_0, α_1, α_{-1}, α_2 and α_{-2}. If the values are complex, express them in polar form with the angle in degrees.

BC:8.3 The periodic waveform, $f(t)$ shown below is related to the waveform $g(t)$ from Problem 8.2 by a time shift.



where the periodic signal, $f(t)$, can be expressed as a Fourier series:

$$f(t) = \sum_{k=-\infty}^{\infty} \beta_k \exp\left(j\frac{2\pi}{T_o}kt\right)$$

a.) Show a formula relating g(t) to f(t) that expresses the time shift, including the amount of the time shift in seconds.

b.) Use the properties of the Fourier series to express the relationship between the α_k coefficients from problem 8.2 and the β_k coefficients for $f(t)$.

c.) Evaluate the coefficients β_0, β_1, β_{-1}, β_2 and β_{-2}. If the values are complex, express them in polar form with the angle in degrees.

d.) A *full wave rectifed* sine is defined as the absolute value of a sine, and can be represented as

$$x(t) = g(t) + f(t) = \sum_{k=-\infty}^{\infty} \sigma_k \exp\left(j\frac{2\pi}{T_o}kt\right)$$

Use the properties of the Fourier series to express the relationship between the α_k coefficients from problem 8.2 and the σ_k coefficients for $x(t)$.

BC:8.4 The periodic waveform, $y(t)$, is related to $g(t)$ from Problem 8.2 by the relationship

$$y(t) = \frac{d}{dt}g(t)$$

where the periodic signal, $y(t)$, can be expressed as a Fourier series:

$$y(t) = \sum_{k=-\infty}^{\infty} \gamma_k \exp\left(j\frac{2\pi}{T_o}kt\right)$$

a.) Sketch (by hand) $y(t)$ through two complete cycles and label landmarks where $y(t)$ is minimum, maximum and where it crosses zero. In your labels, include the timepoint of the landmark and the value of $y(t)$ at that point in time.

b.) Use the properties of the Fourier series to express the relationship between the α_k coefficients from problem 8.2 and the γ_k coefficients for $x(t)$.

c.) Evaluate the coefficients γ_0, γ_1, γ_{-1}, γ_2 and γ_{-2}. If the values are complex, express them in polar form with the angle in degrees.

BC:8.5 For the periodic time signal

$$g(t) = 61\cos(1753.5\pi t - 17^o) + 29\cos(3006\pi t + 39^o)$$

$$+17\cos(3757.5\pi t - 90^o)$$

where

$$g(t) = \sum_{k=-\infty}^{\infty} \alpha_k \exp\left(j\frac{2\pi}{T_o}kt\right)$$

a.) Find the period, T_o, of $g(t)$ and the fundamental frequency, f_o (in Hz).

b.) For the complex Fourier series expansion, specify which values of k represent harmonics at frequencies $f_k = kf_o$ that have nonzero coefficients?

c.) Express all nonzero values of the complex Fourier coefficients, α_k, as complex numbers in polar form with the angle in degrees.