CSE 2321 Homework 7

Problem 1

$$
\begin{aligned}
T(n) &= \sum_{k=0}^{n-1} ar^k && \text{Geometric series is really useful!} \\
&= a\left(\frac{1-r^n}{1-r}\right) && \text{If } r \neq 1
\end{aligned}
$$

1A

Recurrence relation:

$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$

Running time:

$$T(n) = \Theta(n)$$

Generic formula after k substitutions:

$$T(n) = 2^{k+1}T\left(\frac{n}{2^{k+1}}\right) + \sum_{i=0}^{k} 2^i \tag{1}$$

$$\text{Stop when } \frac{n}{2^{k+1}} = 1$$

$$k = \log_2(n) - 1$$

$$T(n) = 2^{(\log_2(n)-1+1)}T\left(\frac{n}{2^{(\log_2(n)-1+1)}}\right) + \sum_{i=0}^{(\log_2(n)-1)} 2^i \tag{2}$$

$$n \cdot 1 + n - 1$$

$$T(n) = \Theta(n)$$

1B

Recurrence relation:

$$T(n) = 3T\left(\frac{n}{3}\right) + 1$$

Running time:

$$T(n) = \Theta(n)$$

Generic formula after k substitutions:

$$T(n) = 3^{k+1}T\left(\frac{n}{3^{k+1}}\right) + \sum_{i=0}^{k} 3^i \tag{3}$$

$$\text{Stop when } \frac{n}{3^{k+1}} = 1$$

$$k = \log_3(n) - 1$$

$$T(n) = 3^{(\log_3(n)-1+1)}T\left(\frac{n}{3^{(\log_3(n)-1+1)}}\right) + \sum_{i=0}^{(\log_3(n)-1)} 3^i \tag{4}$$

$$n \cdot 1 + \frac{n-1}{2}$$

$$T(n) = \Theta(n)$$

1C

Recurrence relation:

$$T(n) = T(n-1) + 1$$

Running time:

$$T(n) = \Theta(n)$$

Generic formula after k substitutions:

$$T(n) = T(n-(k+1)) + (k+1) \tag{5}$$

$$\text{Stop when } n-(k+1) = 1$$

$$k = n-2$$

$$T(n-((n-2)+1)) + ((n-2)+1) \tag{6}$$

$$1 + n - 1 = n$$

$$T(n) = \Theta(n)$$

1D

Recurrence relation:

$$T(n) = T(n-2) + 1$$

Running time:

$$T(n) = \Theta(n)$$

Generic formula after k substitutions:

$$T(n) = T(n - 2(k+1)) + (k+1) \tag{7}$$

$$\text{Stop when } n - 2(k+1)) = 1$$

$$k = \frac{n-3}{2}$$

$$T(n) = T\left(n - 2\left(\frac{n-3}{2} + 1\right)\right) + \left(\frac{n-3}{2} + 1\right)$$

$$1 + \frac{n-3}{2} + 1$$

$$T(n) = \Theta(n)$$

1E

Recurrence relation:

$$T(n) = 3T\left(\frac{n}{2}\right) + 1$$

Running time:

$$T(n) = \Theta(n^{\frac{\log_3(n)}{\log_2(n)}})$$

Generic formula after k substitutions:

$$T(n) = 3^{k+1}T\left(\frac{n}{2^{k+1}}\right) + \sum_{i=0}^{k} 3^i \tag{8}$$

$$\text{Stop when } \frac{n}{2^{k+1}} = 1$$

$$k = \log_2(n) - 1$$

$$T(n) = 3^{(\log_2(n)-1+1)} T\left(\frac{n}{2^{(\log_2(n)-1+1)}}\right) + \sum_{i=0}^{(\log_2(n)-1)} 3^i \tag{9}$$

$$3^{\log_2(n)} \cdot 1 + \frac{1-3^{\log_2(n)}}{-2}$$

$$T(n) = \Theta(n^{\frac{\log_3(n)}{\log_2(n)}})$$

Problem 3

```
int BinarySearch(A, i, j, k)
    if i>j
        index = -1
    else
        midpt1 = (i+j)/3
        midpt2 = midpt1
        if k = A[midpt1]
            index = midpt1
        else if k < A[midpt1]
            index = BinarySearch(A, i, midpt1 - 1, k)
        else if k = A[midpt2]
            index = midpt2
        else if k > A[midpt2]
            index = BinarySearch(A, midpt2 + 1, j, k)
        else
            index = BinarySearch(A, midpt1 + 1, midpt2 - 1, k)
    return index
```

TrinarySearch:
Recurrence relation:

$$T(n) = T\left(\frac{n}{3}\right) + C$$

Running time:

$$T(n) = \Theta(log(n))$$

Generic formula after k substitutions:

$$T(n) = T\left(\frac{n}{3^{k+1}}\right) + (k+1) \tag{10}$$

$$\text{Stop when } \frac{n}{3^{k+1}} = 1$$

$$k = \log_3(n) - 1$$

$$T(n) = T\left(\frac{n}{3^{(\log_3(n)-1+1)}}\right) + (\log_3(n) - 1 + 1) \tag{11}$$

$$1 + log_3(n)$$

$$T(n) = \Theta(log(n))$$

BinarySearch (from class notes):

Recurrence relation:

$$T(n) = T\left(\frac{n}{2}\right) + C$$

Running time:

$$T(n) = \Theta(log(n))$$

TrinarySearch has same running time as BinarySearch.