1.) Find **numerical values** for each of the following:

For parts (a) and (b) convert each complex number to polar form (express angle in degrees) :

a) (3 pts): $-50 + j200$

b) (3 pts): $-4 - j3$

For parts (c–e) convert each complex number to cartesian form:

c) (3pts): $2.5\exp(-j\pi/9)$ (note angle is in radians)

d) (3 pts): $(100\angle 180^o) + (200\angle - 145^o)$

e) (3 pts): $(2 + \frac{7}{j14}) \cdot 200\exp(j35^o)$

f) (5 pts): Use phasors to find A and θ_A for

$$A\cos(120\pi t + \theta_A) = 50\cos(120\pi t + 140^o) - 250\sin(120\pi t - 55^o)$$

a) $206.155 \angle 104.036$

b) $5 \angle -143.13$

c) $2.35 - 0.86j$

d) $-263.83 - 114\ 715j$

e)

f)

$$50\cos(120\pi t + 140°) \to \underline{50e^{j140°}}$$

$$-250\sin(120\pi t - 55°) = -250\cos(120\pi t - 55° - 90°)$$

$$= +250\cos(120\pi t - 144° + 180°) \to \underline{250e^{j35}}$$

$$50e^{j140°} + 250e^{j35} = 241.9e^{46.5°j}$$

$A = 241.9$

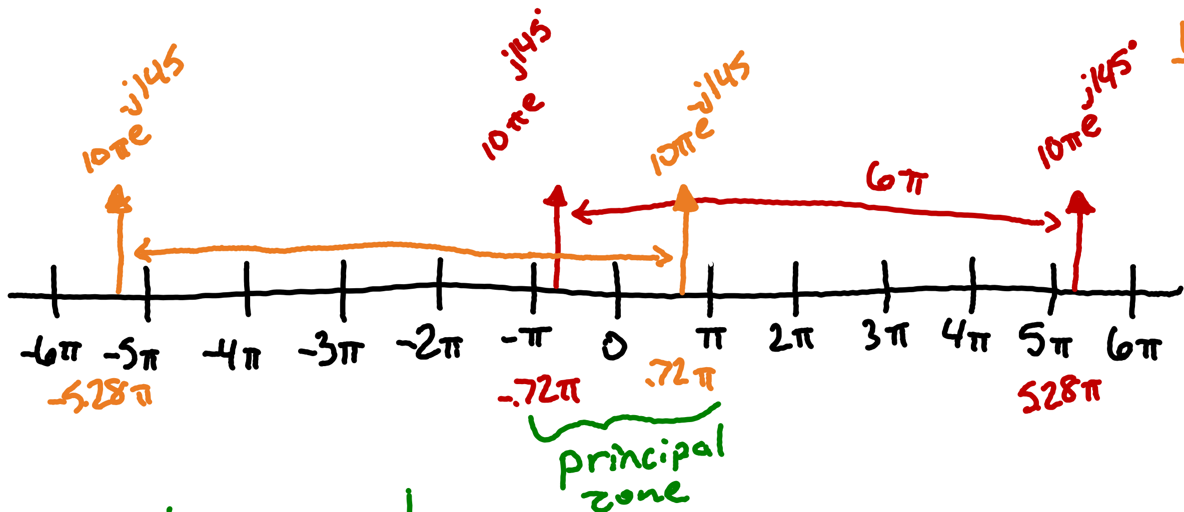
$\Theta = 46.5°$

2.) (20 pts): A continuous time signal, $f(t) = 10\cos(240\pi t + 145^o)$ is sampled at $T_s = \Delta t = 0.022$ sec so that the sampled signal is $f[n] = 10\cos(\hat{\omega}_o n + \theta_o)$ where $\hat{\omega}_o$ is the *normalized radial frequency* for the **principal zone (principal alias)** description of the sampled signal. Find $\hat{\omega}_o$ and θ_o for the sampled signal. Identify whether the signal is oversampled or undersampled and justify why.

$$\omega_A = 240\pi \rightarrow \hat{\omega}_A = 240\pi(\underbrace{.022}_{\Delta t}) = 528\pi$$

not principal zone

Undersampled



$\hat{\omega}_o = |6\pi - 5.28\pi| = 0.72\pi$

$\theta_o = -145^\circ$

3.) (30 points): In each discrete time system below, the input signal is $x[n]$ and the output signal is $y[n]$. For each system determine if the system is linear or nonlinear and if the system is time-invariant or not time-invariant. Briefly justify your answers:

a.) $y[n] = 2e^{j\pi n/2} \cdot x[n]$

$x_1 = f \rightarrow y_1 = 2e^{j\frac{\pi n}{2}} \cdot f$

$x_2 = g \rightarrow y_2 = 2e^{j\frac{\pi n}{2}} \cdot g$

$x_3 = af + bg$

$y_3 = 2e^{j\frac{\pi n}{2}}(af + bg)$

$y_3 = ay_1 + by_2$

Linear

$x_1 = f[n] \rightarrow y_1 = 2e^{j\frac{\pi n}{2}} \cdot f[n]$

$x_2 = f[n-q] \rightarrow y_2 = 2e^{j\frac{\pi n}{2}} \cdot f[n-q]$

$y_1[n-q] = 2e^{j\frac{\pi[n-q]}{2}} \neq y_2$

Not Time-Invariant

b.) $y[n] = 100x[n] - 40x[n-10]$

$x_1 = f \rightarrow y_1 = 100f - 40f[n-10]$

$x_2 = g \rightarrow y_2 = 100g - 40g(n-10]$

$x_3 = af + bg$

$y_3 = 100(af - bg) + 40(af(n-10) - bg(n-10))$

$y_3 = ay_1 + by_2$

Linear

$x_1 = f[n] \rightarrow y_1 = 100f(n) - 40f(n-10)$

$x_2 =$

Time-Invariant

c.) $y[n] = 100\cos(2\pi x[n]/100)$

$x_1 = f \rightarrow y_1 = 100\cos(2\pi f/100)$

$x_2 = g \rightarrow y_2 = 100\cos(2\pi g/100)$

$x_3 = af + bg$

$y_3 = 100\cos\left(\frac{2\pi}{100}(af + bg)\right)$

Non Linear

Time invariant

4.) (30 points): A LTI system starts at rest (no stored values) and has an impulse response

$$h[n] = 1.25\delta[n] - 0.5\delta[n-1] + \delta[n-3]$$

and an input signal

$$x[n] = 10\cos(0.25\pi n + 15^o)\,(u[n-1] - u[n-3])$$

Find a closed form (analytic expression) for the output of the system, $y[n]$.

$x[n] = 10\cos(0.25\pi n + 15^\circ)(\delta[n-1] + \delta[n-2])$

$x[n] = 5\delta[n-1] - 2.588\,\delta[n-2]$

$y[n] = h[n] * x[n] = x[n] * h[n]$

$y[n] = \sum_{q=-\infty}^{\infty} h[n-q]\,x[q] = \sum (1.25\,\delta[n-q] - 0.5\delta[n-q-1] + \delta[n-q-3]) \cdot (5\delta[q-1] - 2.588\delta[q-2])$

$y[n] = 1.25\,\delta[n-1](5) - 0.5\delta[n-2](5) + \delta[n-4]5 - 1.25\,\delta[n-2](2.588) + 0.5\,\delta[n-3](2.588)$

Different Way

$y[n] = (1.25\,\delta[n] - 0.5\,\delta[n-1] + \delta[n-3]) * 10\cos(0.25\pi n + 15^\circ)(u[n-1] - u[n-3])$

$$\boxed{\begin{aligned} y[n] = {} & 12.5\cos(0.25\pi n + 15^\circ)(u[n-1] - u[n-3]) \\ & - 5\cos(0.25\pi[n-1] + 15^\circ)(u[n-2] - u[n-4]) \\ & + 10\cos(0.25\pi[n-3] + 15^\circ)(u[n-4] - u[n-6]) \end{aligned}}$$