Tautologies, Contingencies and Contradictions

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A compound proposition that is sometimes true and sometimes false is called a contingency.

An Example ... of One of Those

Consider the example

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•

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this is a tautology

C

A	В	$A \wedge B$	$\neg (A \land B)$	$\neg A$	$\neg B$	$(\neg A) \lor (\neg B)$	$(C) \Leftrightarrow (D)$
0	0	0				1	
0	1	0		1	0		
1	0	0		0	(1
1	1		0	0	D	0	1

When we considered $P\Rightarrow Q$ and its contrapositive $(\neg Q)\Rightarrow (\neg P)$, we came to the conclusion that they mean the same thing.

$$(P \Rightarrow Q) \Longleftrightarrow ((\neg Q) \Rightarrow (\neg P))$$
is a tautology

When we considered $P \Rightarrow Q$ and its contrapositive $(\neg Q) \Rightarrow (\neg P)$, we came to the conclusion that they mean the same thing. We can be precise about what we mean by that by saying that two propositions are logically equivalent if they have the same truth values for any choice of truth values of their combined atomic propositions.

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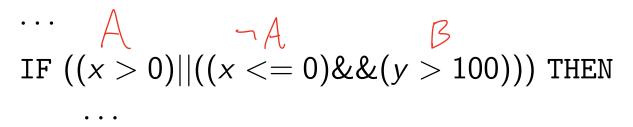
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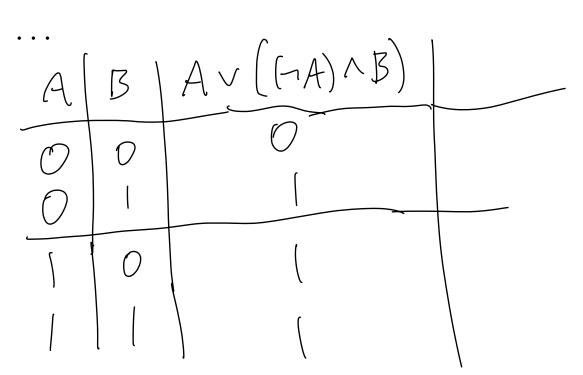
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Notice that we just showed that $\neg(P \land Q)$ is logically equivalent to $(\neg P) \lor (\neg Q)$. We can see from that that \Leftrightarrow is essentially the symbol for logical equivalence. Although, sometimes, if we want to talk about the logical equivalence being "about the logic instead of *in* the logic" we'll use \equiv .

An Example from Programming

As another example consider the following code:





An Example from Programming

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. . .

IF
$$((x > 0)||((x <= 0)\&\&(y > 100)))$$
 THEN

. . .

If we name x > 0 "A" and name y > 100 "B" than the following truth table shows that $A \lor (\neg A \land B)$ is logically equivalent to just $A \lor B$.

A	В	$A \lor (\neg A \land B)$	$A \lor B$
0	0	0	0
0	1	1	1
1	0	1	1
	1	1	1

Just as in arithmetic, we can reduce our use of braces by deciding on an order of operations for the common logical connectives. The one in force is:

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By the way, \wedge is also called a conjunction, and \vee is called a disjunction.

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The argument that will be familiar to most of you is the famous syllogism:

Socrates is a man

All men are mortal

∴ Socrates is mortal

An Example of an Argument

Let us determine if the following argument is valid

$$P \Rightarrow (Q \lor (\neg R))$$

$$Q \Rightarrow (P \land R)$$

$$\therefore P \Rightarrow R$$

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We need to make sure that the conclusion $(P \Rightarrow R)$ is true whenever all the premises are true.

$$((P\Rightarrow (QV-P))\wedge (Q\Rightarrow (P\wedge P))\Rightarrow (P\Rightarrow P)$$

The Truth Table Work

					Premise	premise2	the	argumin
P	Q	R	$Q \lor \neg R$	$P \wedge R$	$P \Rightarrow (Q \lor \neg R)$	$Q \Rightarrow (P \wedge R)$	$P \Rightarrow R$	is valid
0	0	0	1	0	Ĺ			I'M Vallo
0	0	1	0	0		1	ĺ	
0	1	0		0	ſ	0	X	
0	1	1	1	\bigcirc		0	X	
1	0	0	1	Ö	(1	0	
1	0	1	0		0	1	\times	
1	1	0		0	1	0	X	
1	1	1	1					

This row can't make the argument invalid

This vov tells us

Logical Rules

First there are two standard basic arguments:

modus ponens

("mode that affirms by affirming")

$$P \Rightarrow Q$$
 P
 $\therefore Q$

and

modus tollens

("mode that denies by denying")

$$P \Rightarrow Q$$
 $\neg Q$
 $\therefore \neg P$