Lecture 1 Outline

Reminder to self: turn on lecture recording to Cloud!

- Last Lecture
  - Reviewed Syllabus
  - Course Goals
- Today's Lecture
  - Overview
  - Number Systems and Conversion

Handouts and Announcements

- Announcements
  - Homework Problems 1-2, 1-3, 1-4
    - Will be posted on Carmen this evening or tomorrow
    - Due in Carmen 11:59pm, Tuesday 1/17
  - Homework Problem 1-1 reminder
    - Assigned on Carmen on 1/10
    - Due in Carmen 11:59pm, Thursday 1/12
  - Read for Friday: Pages 12-21

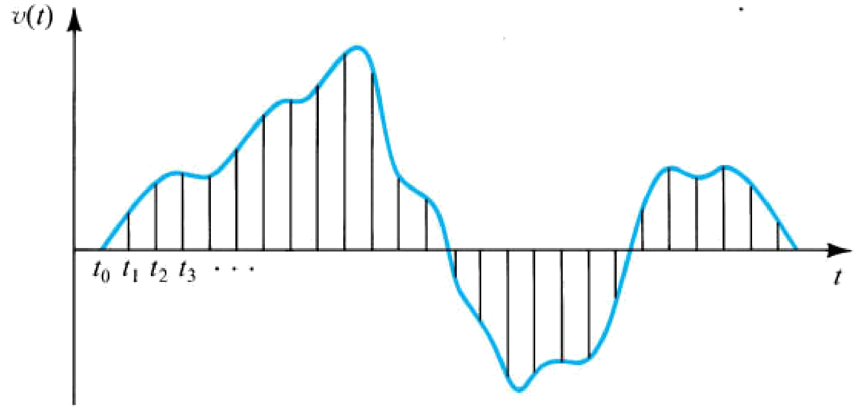
Overview

Analog vs. Digital Systems

- All “real” signals are analog
  - An analog signal can have any value, and
  - Is continuous in time
- Digital signals are discrete in time and have a limited set of values
  - 2 values → binary (e.g. 1/0, high/low, true/false)
  - In principle any integer >1 could be used
- Majority of circuits are digital
  - Binary signals → Circuits simpler to design than analog
  - Fewer different kind of blocks (inverter, memory, etc.)
  - But large numbers of those blocks needed for complex functions
- Physical world is Analog (temperature, speed, …)
  - Digital processing of signals is pervasive
  - Need both! → Mixed-signal or mixed-mode design
  - Analog → Digital → Analog

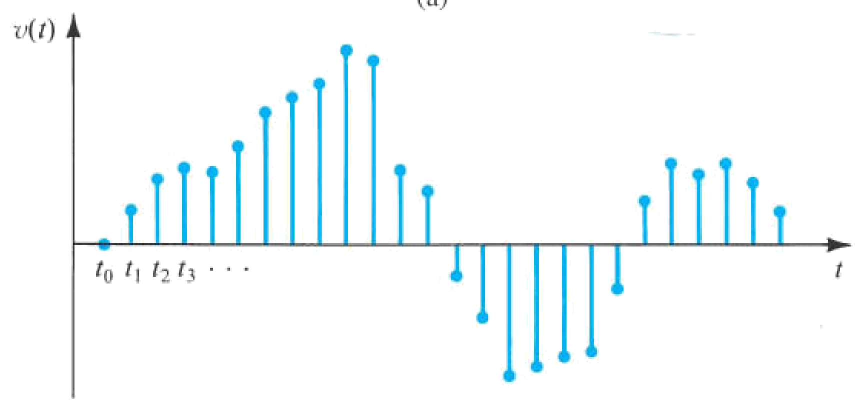
Sampling a continuous-time analog signal

continuous-time analog signal

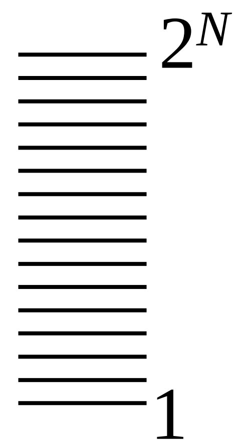


(a)

discrete-time signal



(b)

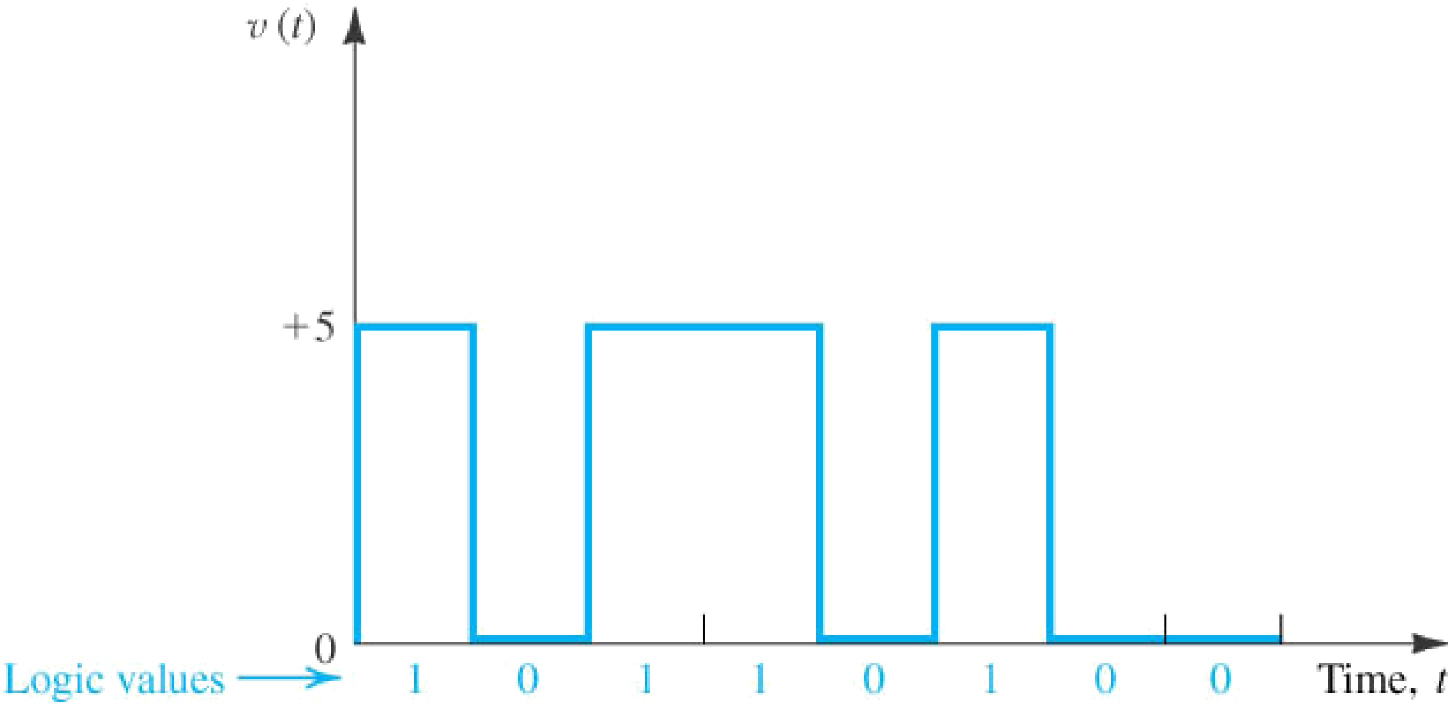


N is the number of binary "bits"
N small $\rightarrow$
N large $\rightarrow$

Quantized, discretized or digitized signal

Sampling the continuous-time analog signal in **(a)** results in the discrete-time signal in **(b).**

Binary signals: 2 values



Variation of a particular binary digital signal with time.

e.g. one bit of the previous discretized signal

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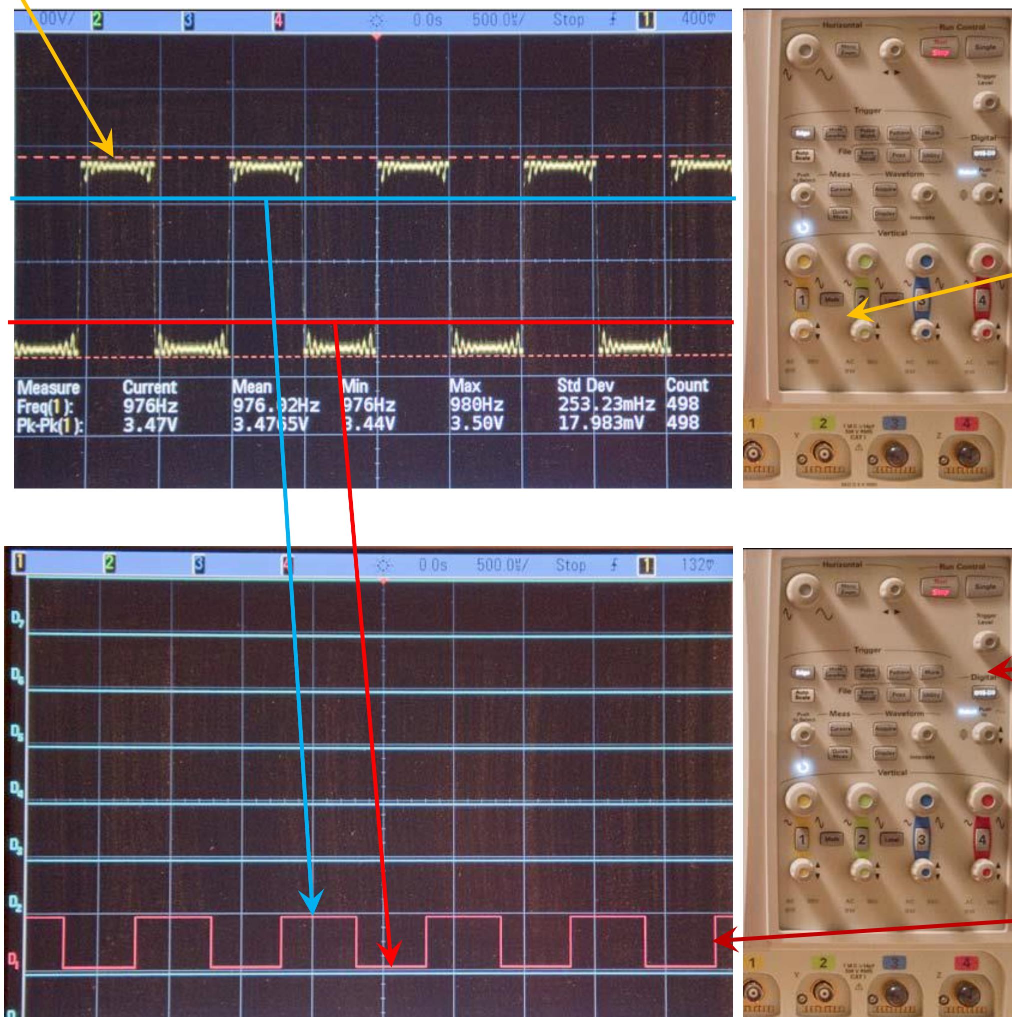
Binary signals: 2 values

Analog (voltage) Signal

Mixed Signal Oscilloscope

V_H

V_L



Analog

Digital signal is Hi or 1 if analog signal > blue voltage

Digital signal is Lo or 0 if analog signal < red voltage

Digital

Hi, 1

Lo, 0

1 Bit (Binary Digit) Digital Signal

Logical Values, not actual Voltages

Overview

This course is about Digital Systems → specifically Binary Systems

- Digital: Data processing, controls, communications
  - An analog signal can have any value, and
  - Is continuous in time
- Digital Design:
  1) System Design
     a) What does it do?

     Types & numbers of subsystems (e.g. memory, arithmetic units, I/O devices, etc.; how interconnected & controlled)
  2) Logic Design
     a) What logic function describes 1)

     Interconnections of things like gates and flip-flops to implement logic function
  3) Circuit Design
     a) Take logic function and implement it with circuits Transistor Level
     b) Computer aided design (CAD) of integrated circuits

Main focus of the class

Overview

Logic Circuits – "switching circuits": All inputs will be "discrete" values

- Combinational logic:
  - Output depends on only the current input(s)
- Sequential logic:
  - Output depends on the current inputs and on prior inputs (memory)
- Synchronous logic:
  - Has a common timing signal, called a "clock"
- Asynchronous logic:
  - No clock

Number Systems and Conversion

- When we write decimal (base 10) numbers, we use a positional notation
- Each digit multiplied by appropriate power of 10 depending on position in number

$$469.14_{10} = 4 \times 10^2 + 6 \times 10^1 + 9 \times 10^0 + 1 \times 10^{-1} + 4 \times 10^{-2}$$

- When we write binary (base 2) numbers, each digit is multiplied by an appropriate power of 2 on its position in the number

$$1101.11 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$
$$= 8 + 4 + 0 + 1 + \frac{1}{2} + \frac{1}{4} = 13.75_{10}$$

Number Systems and Conversion

- $R\,(R > 1) \equiv$ "Radix" or "Base" of number system (positive integer) **Carmen Quiz** Decimal point
- In general, a number written in positional notation can be expanded as a power series in R. For example:

$$N = (a_4 a_3 a_2 a_1 a_0 \,.\, a_{-1} a_{-2} a_{-3})_R$$
$$= a_4 \times R^4 + a_3 \times R^3 + a_2 \times R^2 + a_1 \times R^1 + a_0 \times R^0$$
$$+ a_{-1} R^{-1} + a_{-2} R^{-2} + a_{-3} R^{-3}$$

- a_i is the integer coefficient of R^i and $0 \le a_i \le R - 1$

Carmen Quiz

a_i of base 4
cannot be > 3

Number Systems and Conversion

Example:

Using the previous slide, the following conversion can be made from base 6 to base 10.

$$235.3_6 = 2 \times 6^2 + 3 \times 6^1 + 5 \times 6^0 + 3 \times 6^{-1} = 72 + 18 + 5 + \frac{3}{6}$$

$$= 95.5_{10}$$

$i = 2$
$a_2 = 2$

$i = 1$
$a_1 = 3$

$i = 0$
$a_0 = 5$

$i = -1$
$a_{-1} = 3$

Conversion to other bases:

The power series expansion can be used to convert to any base. For example, converting 147_{10} to base 3 would be written as

$$147_{10} = \overset{1}{1} \times (\overset{10}{101})^2 + (\overset{4}{11}) \times (\overset{10}{101})^1 + (\overset{7}{21}) \times (\overset{10}{101})^0$$

where all the numbers on the right-hand side are base 3 numbers. (*Note:* In base 3, 10 is 101, 7 is 21, etc.) To complete the conversion, base 3 arithmetic would be used.

Table relating (1 through $10)_{10}$ to base 3:

Decimal	Base 3
0	0
1	1
2	2
3	10
4	11
5	12
6	20
7	21
8	22
9	100
10	101

Number Systems and Conversion

Bases greater than 10:

- For bases greater than 10, more than 10 symbols are needed to represent the digits
- Letters are typically used to represent digits >9
- For example, in hexadecimal (base 16)
  - $A_{16} = 10_{10} = 1010_2$
  - $B_{16} = 11_{10} = 1011_2$
  - $C_{16} = 12_{10} = 1100_2$
  - $D_{16} = 13_{10} = 1101_2$
  - $E_{16} = 14_{10} = 1110_2$
  - $F_{16} = 15_{10} = 1111_2$

$$B3E_{16} = 11 \times 16^2 + 3 \times 16^1 + 14 \times 16^0 = 2816 + 48 + 14 = 2878$$

Number Systems and Conversion

Conversion of decimal integer to base R using division method:

- The base R equivalent of a decimal integer N is as follows (from previous slides):

$$N = (a_n a_{n-1} \cdots a_2 a_1 a_0)_R = a_n R^n + a_{n-1} R^{n-1} + \cdots + a_2 R^2 + a_1 R^1 + a_o$$

- If we divide N by R, the remainder is a_0

$$\frac{N}{R} = a_n R^{n-1} + a_{n-1} R^{n-2} + \cdots + a_2 R^1 + a_1 = Q_1 \text{, remainder } a_0$$

- Then divide quotient Q_1 by R

$$\frac{Q_1}{R} = a_n R^{n-2} + a_{n-1} R^{n-3} + \cdots + a_3 R^1 + a_2 = Q_2 \text{, remainder } a_1$$

Conversion of decimal integer to base R using division method (continued):

- Next divide Q_2 by R

$$\frac{Q_2}{R} = a_n R^{n-3} + a_{n-1} R^{n-4} + \cdots + a_3 = Q_3, \text{ remainder } a_2$$

- Repeat the process until we finally obtain a_n
- Note: remainder obtained at each step is one of the desired digits, and the least significant digit is obtained first

Emphasis: **<u>Least</u>** significant digit is obtained first

Example 1: Conversion of decimal integer to binary

Example Convert 53_{10} to binary.

$2\,\underline{/53}$	
$2\,\underline{/26}$	rem. $= 1 = a_0$
$2\,\underline{/13}$	rem. $= 0 = a_1$
$2\,\underline{/6}$	rem. $= 1 = a_2$
$2\,\underline{/3}$	rem. $= 0 = a_3$
$2\,\underline{/1}$	rem. $= 1 = a_4$
0	rem. $= 1 = a_5$

110101_2

Number Systems and Conversion

Example: Convert 62_{10} to binary

$2\,\underline{|62}$

$2\,\underline{|31}$ 0

$2\,\underline{|15}$ 1

$2\,\underline{|7}$ 1

$2\,\underline{|3}$ 1

$2\,\underline{|1}$ 1

0 1

111110_2

Number Systems and Conversion

Conversion of a decimal fraction F to base R using successive multiplications:

$$F = (.a_{-1}a_{-2}a_{-3}\cdots a_{-m})_R = a_{-1}R^{-1} + a_{-2}R^{-2} + a_{-3}R^{-3} + \cdots + a_{-m}R^{-m}$$

radix point

- Multiplying by R yields

$$FR = a_{-1} + a_{-2}R^{-1} + a_{-3}R^{-2} + \cdots + a_{-m}R^{-m+1} = a_{-1} + F_1$$

where F_1 represents the rational part of the result, and a_{-1} is the integer part

- Multiplying F_1 by R yields

$$F_1R = a_{-2} + a_{-3}R^{-1} + \cdots + a_{-m}R^{-m+2} = a_{-2} + F_2$$

Number Systems and Conversion

Conversion of a decimal fraction F to base R using successive multiplications (continued):

- Next, Multiply F_2 by R

$$F_2R = a_{-3} + \cdots + a_{-m}R^{-m+3} = a_{-3} + F_3$$

- Repeat the process until we obtain a sufficient number of digits
- Note: The integer part obtained at each step is one of the desired digits, and the most significant digit is obtained first

Emphasis: **Most** significant digit is obtained first

Number Systems and Conversion

Example: Conversion of decimal fraction F to base R using successive multiplications

Example Convert 0.625_{10} to binary.

$F = .625$	$F_1 = .250$	$F_2 = .500$	
$\times\ 2$	$\times\ 2$	$\times\ 2$	$.625_{10} = .101_2$
$\underline{1.250}$	$\underline{0.500}$	$\underline{1.000}$	
$(a_{-1} = 1)$	$(a_{-2} = 0)$	$(a_{-3} = 1)$	

Number Systems and Conversion

Example: Conversion of decimal fraction F to base R using successive multiplications

Example Convert 0.7_{10} to binary.

$$
\begin{array}{r}
.7 \\
\underline{\quad 2} \\
(1).4 \\
\underline{\quad 2} \\
(0).8 \\
\underline{\quad 2} \\
(1).6 \\
\underline{\quad 2} \\
(1).2 \\
\underline{\quad 2} \\
(0).4 \\
\underline{\quad 2} \\
(0).8
\end{array}
$$

$\longleftarrow$ process starts repeating here because 0.4 was previously obtained

$0.7_{10} = 0.1\underline{0110}\,\underline{0110}\,\underline{0110}\ldots_2$

inf.
repeat

Example: Conversion between two bases other than decimal

- In principal could be done directly
- Generally easier: 1st base → base 10 → 2nd base

Convert base 4 to decimal:
- Power series in base 4
- Base 10 arithmetic

Example Convert 231.3_4 to base 7.

$$231.3_4 = 2 \times 16 + 3 \times 4 + 1 + \tfrac{3}{4} = 45.75_{10}$$

Decimal to base 7

$7\,\overline{)45}$			$.75$
$7\,\overline{)\ 6}$	rem. 3		$\times 7$
0	rem. 6		$(5).25$
			$\times 7$
			$(1).75$
			$\times 7$
			$(5).25$
			$\times 7$
			$(1).75$

Division for int. part

Multiplication for decimal part

$$45.75_{10} = 63.5151\ldots_7$$

Number Systems and Conversion

Example: Convert $2AB.13_{13}$ to base 5

$2 \times 13^2 + 10 \times 13 + 11 + \frac{1}{13} + \frac{3}{13^2} = 479.0947_{10}$

$5 \underline{\big) 479}$

- I rounded to 4 digits base 10
- Extra precision at intermediate step to not introduce rounding error into final result

Check: $3 \times 5^3 + 4 \times 5^2 + 4 + \frac{2}{25} + \frac{1}{125} + \frac{4}{5^4} = 479.0944$

Number Systems and Conversion

Conversion from binary to hexadecimal and binary to octal:

- Conversion from binary to hexadecimal (and conversely) can be done by inspection because each hexadecimal digit corresponds to exactly four binary digits (bits).

$$1001101.010111_2 = \underbrace{0100}_{4}\ \underbrace{1101}_{D} . \underbrace{0101}_{5}\ \underbrace{1100}_{C} = 4D.5C_{16}$$

- A similar conversion can be done from binary to octal, base 8 (and conversely), except each octal digit corresponds to three binary digits, instead of four.

$100101101011010_2 = 100 \quad 101 \quad 101 \quad 011 \quad 010 = 45532_8$

- If the number of bits is not a multiple of 4 (for hex) or of 3 (for octal)
  - For the integer part, add leading 0s
  - For the fractional part, add trailing 0s