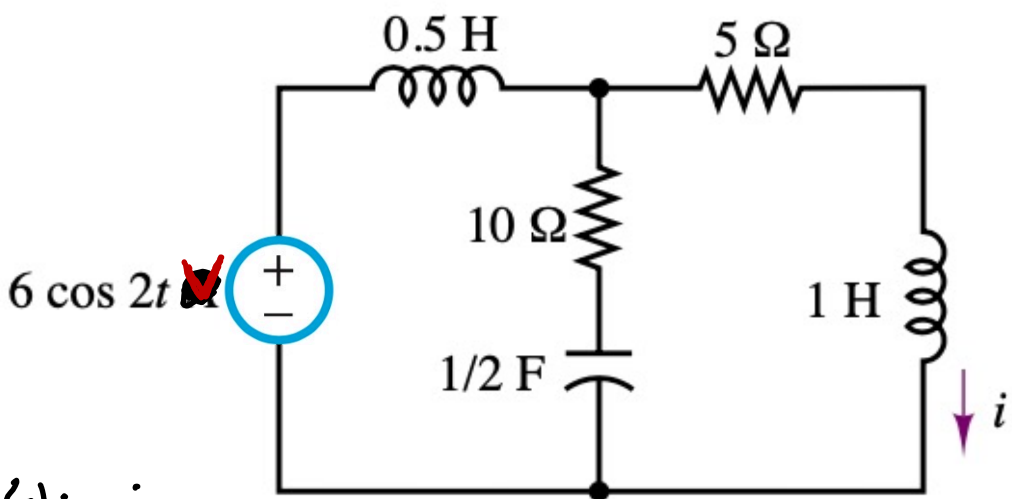
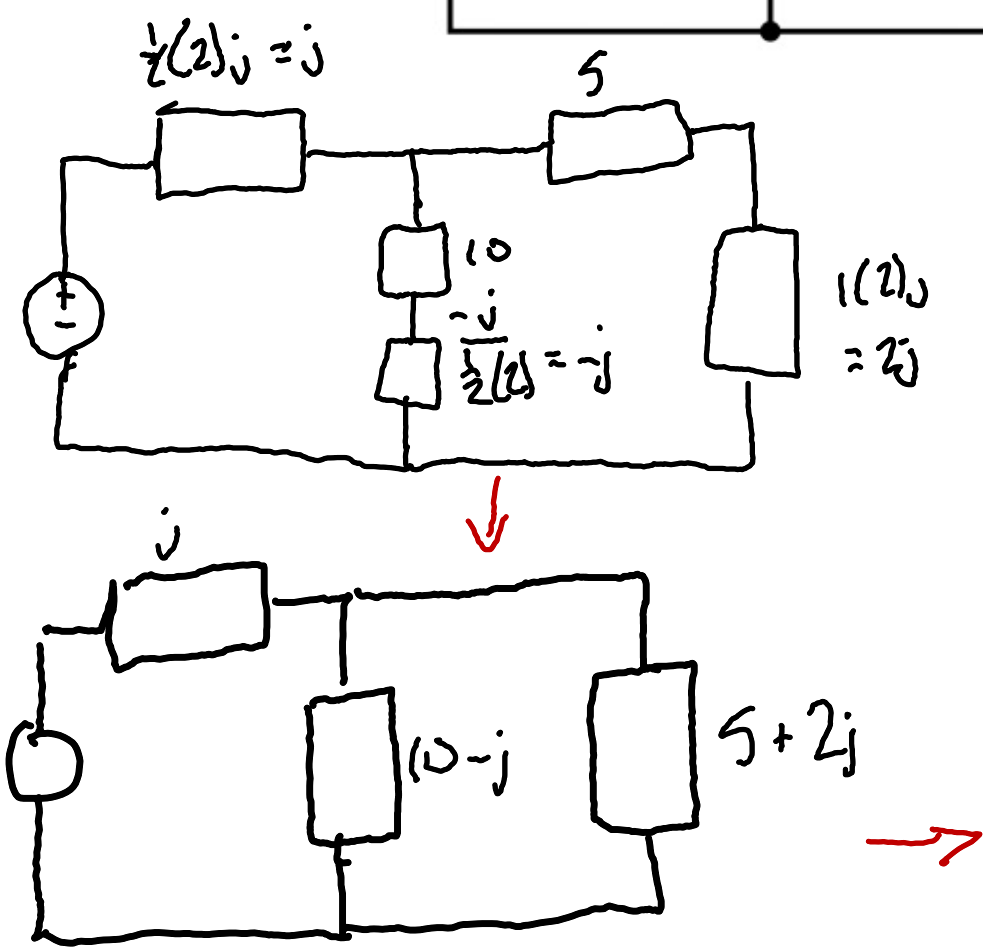
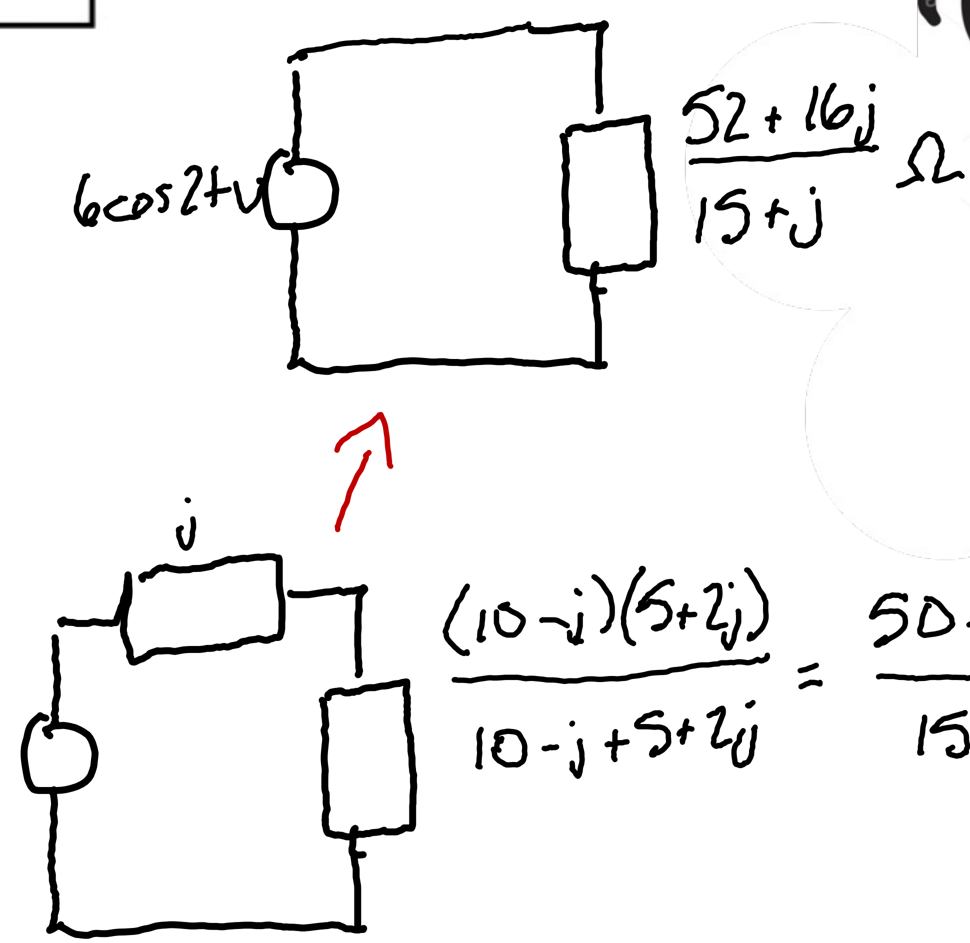
Last Class…

Use mesh current analysis to solve for i(t) in the circuit below.

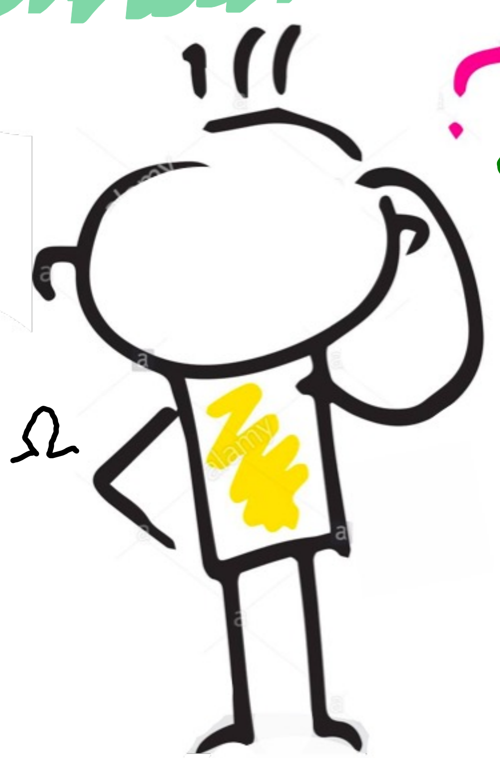






$$i(t) = \frac{6\cos 2t}{\frac{52+16j}{15+j}} = (6\cos 2t)\left(\frac{15+j}{52+16j}\right)$$

I don't know how to do this stuff on my calculator



$$\frac{(10-j)(5+2j)}{10-j+5+2j} = \frac{50+20j-5j+2}{15+j} = \frac{52+15j}{15+j}$$

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AC Power

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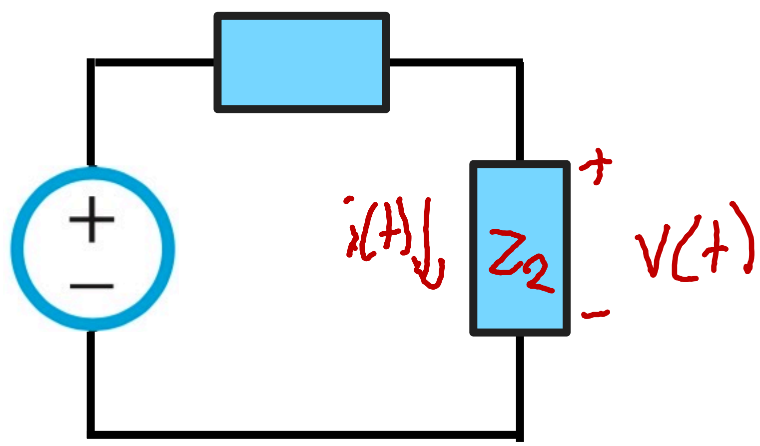
This Lecture…

- Determine the complex power, average real power, and reactive power for any complex load with known input voltage or current.

AC Power

- AC Circuit:



- On DC: $p = vi$

- Instantaneous power: $p(t) = v(t)\,i(t)$

- Average Power:
  - Power delivery (utilities).
  - Electronics (laptops, mobile phones, etc.).
  - Logic circuits.

Average Power

The average amount of work done or energy transferred per unit time.

in phasor domain

$$P = \frac{1}{T_0}\int_0^{T_0} \overbrace{v(t)i(t)}^{p(t)}\,dt = \frac{V_m I_m}{2}\cos(\emptyset_V - \emptyset_I) = \frac{V_m I_m}{2}\cos(\theta_z)$$

$v(t) = V_m \cos(\omega t + \emptyset_V)$

magnitude phase

$i(t) = I_m \cos(\omega t + \emptyset_I)$

Polar form

$Z = \frac{V}{I}$

$Z_m \angle \theta_z = \frac{V_m \angle \theta_V}{I_m \angle \theta_I}$

$Z_m \angle \theta_z = \frac{V_m}{I_m} \angle \theta_V - \theta_I$

$Z_m = \frac{V_m}{I_m}$ $\theta_z = \theta_V - \theta_I$

$Z_R = Re^{0j}$

$Z_C = \frac{1}{\omega C} e^{-90j}$

$Z_L = \omega L e^{90j}$

$P_R = \frac{V_m I_m}{2}$

$P_C = 0$

$P_L = 0$

Avg Power

Reactive Power makes more sense

Complex Power

$x = a + bj$

$x^* = a - bj$

$y = m\angle\theta$

$y^* = m\angle -\theta$

Define **S** as complex power

$$\boldsymbol{S} = \frac{\boldsymbol{VI}^*}{2}$$

Average

$\boldsymbol{S}$ = "Complex Power" = Real Power + Reactive Power

$$\boldsymbol{S} = P + jQ$$

- * indicates the complex conjugate
- Real part is the real power
- Imaginary part is the reactive power
  - The dissipated power resulting from inductive and capacitive loads measured in volt-amperes reactive (VAR).
  - A purely reactive load can store power and then release it, but the net average power it absorbs is zero

Complex Power

$$\frac{\boldsymbol{VI}^*}{2}$$

$\boldsymbol{S}$ = "Complex Power" = Real Power + Reactive Power

$V = V_m \angle \theta_V \qquad I = I_m \angle \theta_I$

From polar to rectangular:

$$S = \frac{(V_m \angle \theta_V)(I_m \angle -\theta_I)}{2} = \frac{V_m I_m}{2} \angle \theta_V - \theta_I$$

$$S = \underbrace{\frac{V_m I_m}{2} \cos(\theta_V - \theta_I)}_{\text{Real}} + \underbrace{\frac{V_m I_m}{2} \sin(\theta_V - \theta_I) j}_{\text{Imaginary}}$$

Avg Power + Reactive Power

Time Domain / Phasor Domain

$$v(t) = V_m \cos(\omega t + \phi_v) \quad \longleftrightarrow \quad \mathbf{V} = V_m e^{j\phi_v}$$

$$i(t) = I_m \cos(\omega t + \phi_i) \quad \longleftrightarrow \quad \mathbf{I} = I_m e^{j\phi_i}$$

Complex Power

$$\mathbf{S} = \tfrac{1}{2}\,\mathbf{V}\mathbf{I}^* = P_{av} + jQ$$

Real Average Power

$$P_{av} = \mathfrak{Re}\,[\mathbf{S}] = \frac{V_m I_m}{2} \cos(\phi_v - \phi_i)$$

Reactive Power

$$Q = \mathfrak{Im}\,[\mathbf{S}] = \frac{V_m I_m}{2} \sin(\phi_v - \phi_i)$$

Apparent Power

$$S = |\mathbf{S}| = \sqrt{P_{av}^2 + Q^2} = \frac{V_m I_m}{2}$$

$$\mathbf{S} = S e^{j\phi_s}$$

$$\phi_s = \phi_v - \phi_i = \phi_z$$

Power Factor

$$pf = \frac{P_{av}}{S} = \frac{P_{av}}{|S|} = \cos(\phi_v - \phi_i) = \cos\phi_z$$

Power Factor Significance

From the perspective of an energy supplier:

- The amount of power the company has to supply is S, but it can charge for only Pav, because Pav is the only real power consumed by the load.
- The company appears to supply S—hence, the name apparent power—but it gets paid for a fraction of that, and the power factor is that fraction.