CSE 2321 Foundations I Spring, 2024 Dr. Estill

Homework 4 Due: Friday, February 23

1.) (20 points each) Give the approximate running times of each of the following algorithms (a.k.a. functions) using Θ-notation. Note that you can immediately drop any floor or ceiling functions and that if you forget the "+1" in something like $\sum_{k=a(n)}^{b(n)} C = C(b(n) - a(n) + 1)$ it won't affect your answers. (If either summation limit contains a explicit number it is better to include it as it won't complicate the work.)

(a) FUNCTION $f_1(n)$

```
s ← 0
FOR i ← n TO ⌊n^(3/2)⌋ DO
    FOR j ← 5i TO 2i^2 DO
        s ← s + i − j
RETURN(s)
```

(b) FUNCTION $f_2(n)$

```
s ← 0
FOR i ← ⌊n/5⌋ TO n DO
    FOR j ← i TO n DO
        s ← s + i − j
RETURN(s)
```

(c) FUNCTION $f_3(n)$

```
s ← 0
FOR i ← n TO 2n^2 DO
    FOR j ← 5i TO 5i + 15 DO
        s ← s + i − j
RETURN(s)
```

(d) FUNCTION $f_4(n)$

```
s ← 0
FOR i ← 5n TO 6n^3 DO
    FOR j ← 5 TO i DO
        FOR k ← j TO i DO
            s ← s + i − j
RETURN(s)
```

2.) (20 points total) Either by showing exact inequalities or using precise limits, find the asymptotic complexity of each of the following functions in the simplest terms that you can and then order them by "asymptotic dominance", i.e. produce an ordering $f_1(n) \ll f_2(n) \ll f_3(n) \ll \cdots$ where $g(n) \ll h(n)$ means that $g(n) \in O(h(n))$. So, for example, $1 \ll \log(n) \ll n \ll n^2$. Your ordering doesn't need justification.

(a) $f_a(n) = \sum_{k=1}^{n^2} \left(\frac{3}{4}\right)^k$
(b) $f_b(n) = 2\log_4(4n + 17)$
(c) $f_c(n) = \sum_{j=1}^{2n}(4j + 1)$
(d) $f_d(n) = 6^{13}$
(e) $f_e(n) = 5n^{0.6} + 3n^{0.7}$
(f) $f_f(n) = \sqrt{3n^3 - 2n^2 + 7n}$
(g) $f_g(n) = 2n\log_3(2n^3 + 17n + 1)$