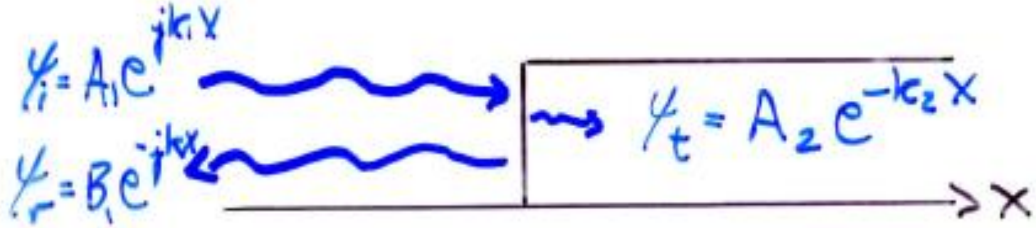
Solve for B_1 and A_2 in terms of A_1

$$B_1 = \frac{-(k_2^2 + 2jk_1k_2 - k_1^2)A_1}{(k_2^2 + k_1^2)}$$

$$A_2 = \frac{2k_1(k_1 - jk_2)A_1}{(k_2^2 + k_1^2)}$$



note: "particle" bounces back. (Almost) duplicates classical phenomena.

Part of the wave incident on the barrier is reflected but part is transmitted!

Purely a QM effect!

Note: For infinite barrier, $K_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} \longrightarrow \infty$

so $A_2 \longrightarrow 0$ and $A_1 = -B_1$ (classical)

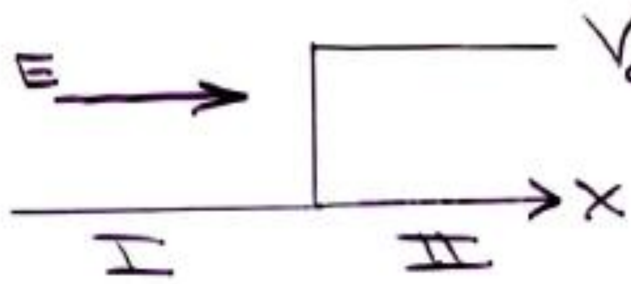
For V finite, $\int A_2^* A_2 \, dx = \int \psi^* \psi \, dx$ is finite

= Transmitted Wave Probability

$$R = \frac{v_r}{v_i} \frac{|B_1|^2}{|A_1|^2}$$ reflection coefficient (definition)

$$T = \frac{v_t}{v_i} \frac{|A_2|^2}{|A_1|^2}$$ transmission coefficient (definition)

$$v = \frac{\hbar k}{m}$$



$E < V_0$

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\kappa_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

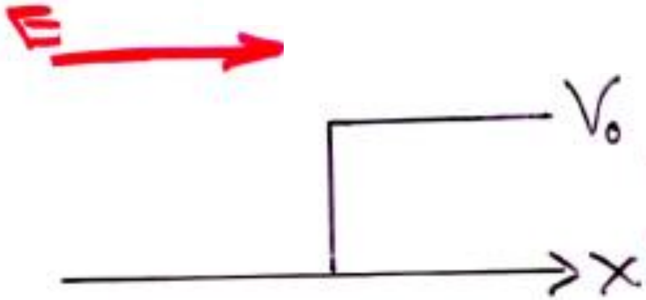
$$A_1 + B_1 = A_2$$

$$jk_1A_1 - jk_1B_1 = -\kappa_2 A_2 = -\kappa_2(A_1 + B_1)$$

$$A_1(jk_1 + \kappa_2) = B_1(-\kappa_2 + jk_1)$$

$$B_1 = \frac{A_1(jk_1 + \kappa_2)}{-\kappa_2 + jk_1} = \frac{A_1(jk_1 + \kappa_2)(jk_1 + \kappa_2)}{(jk_1 - \kappa_2)(jk_1 + \kappa_2)}$$

$$\frac{B_1}{A_1} = \frac{-k_1^2 + 2jk_1\kappa_2 + \kappa_2^2}{-k_1^2 - \kappa_2^2} = -\frac{(\kappa_2^2 + 2jk_1\kappa_2 - k_1^2)}{(\kappa_2^2 + k_1^2)}$$



$E > V_0$

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

$$A_1 + B_1 = A_2$$

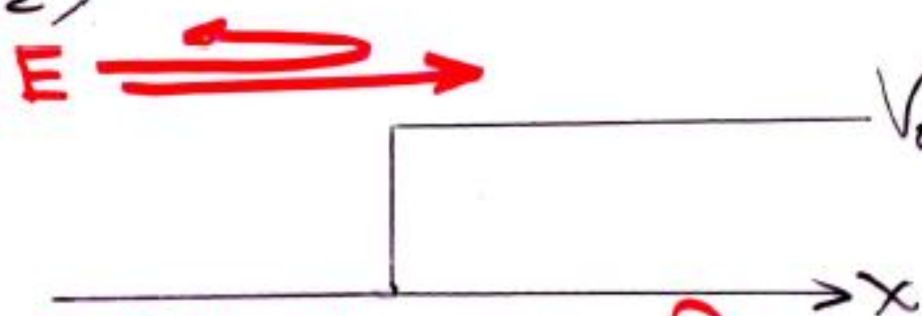
$$jk_1 A_1 - jk_1 B_1 = jk_2 A_2 = jk_2(A_1 + B_1)$$

$$A_1(jk_1 - jk_2) = B_1(jk_1 + jk_2)$$

$$A_1 = (k_1 - k_2) = B_1(k_1 + k_2)$$

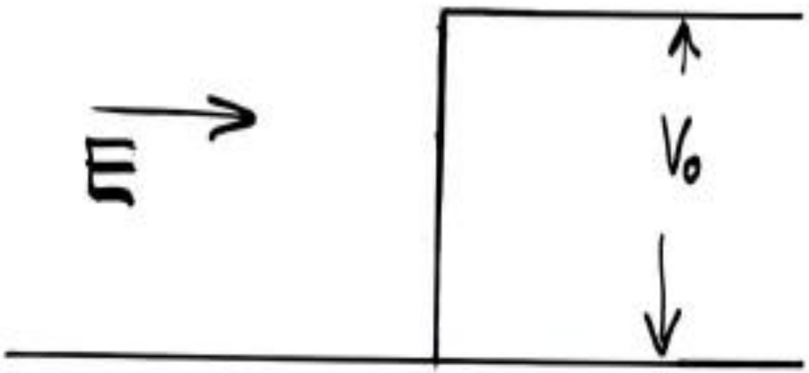
$$B_1 = \frac{(k_1 - k_2)}{(k_1 + k_2)} A_1$$

$$\frac{|B_1|^2}{|A_1|^2} = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2$$



Reflection even for $E > V_0$!

Wave functions for Potential Barrier Problems



In problems with "incident particles and different potential regions

If $E > V$, then Traveling wave, so $\psi \propto e^{jkx}$

If $E < V$, then Decaying wave, so $\psi \propto e^{-kx}$

~#

Probability of Finding a Particle Inside A Barrier ($E<V$): ~~Start~~ with General Case

in General (No integral since probability at just one point in space)

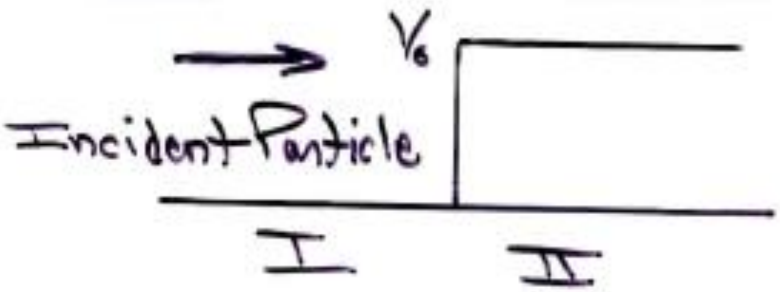
so for $E<V$, then $\psi \propto e^{-kx}$

and $P \propto e^{-2kx}$

Relative Probability just means normalizing to some reference.

For example, $\dfrac{P(x)}{P(0)} = \dfrac{\psi^*(x)\psi(x)}{\psi^*(0)\psi(0)} = \dfrac{A^*A e^{-kx} e^{-kx}}{A^*A e^0 e^0} = e^{-2kx}$

Example: Potential Barrier Problem



Incident $v = 1 \times 10^{5}$ m/s

$m = 9.11 \times 10^{-31}$ kg

$E = \frac{1}{2}mv^2 + V_0 = \frac{1}{2}mv^2$ in region I

$= \frac{1}{2}(9.11 \times 10^{-31}\,\text{kg})(10^{5}\,\text{m/s})^2 = 4.56 \times 10^{-21}\,\text{J}$

Assume $V_0 = 2E$ at boundary.

already know $\psi_2(x) = A_2 e^{-k_2 x}$

where $k_2 = \sqrt{2m(V_0 - E)/\hbar^2}$

"Penetration depth" = $\frac{1}{e}$ depth or $k_2 d = 1$

$$1 = d\sqrt{\frac{2m(2E-E)}{\hbar^2}} = d\sqrt{\frac{2mE}{\hbar^2}}$$

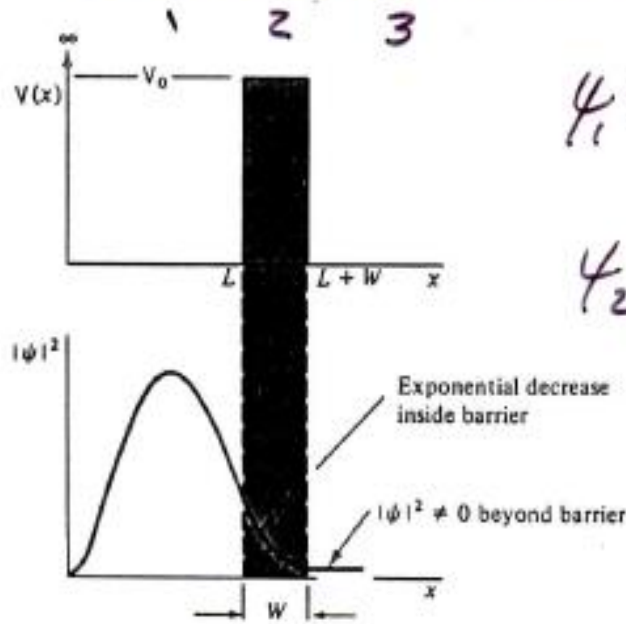
$$d = \frac{\hbar}{\sqrt{2mE}} = \frac{1.054\times 10^{-34}\ \text{J-sec}}{\left[2(9.11\times 10^{-31}\,\text{kg})(4.56\times 10^{-21}\,\text{J})\right]^{1/2}}$$

$$=$$

$$=$$

equivalent to 2 lattice constants of Si, for example.

If step has finite height ($V_0 < \infty$) and finite width, then Tunnel Barrier



$$\psi_1(x) = A_1 e^{jk_1 x} + B_1 e^{-jk_1 x} \qquad k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi_2(x) = A_2 e^{k_2 x} + B_2 e^{-k_2 x} \qquad k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

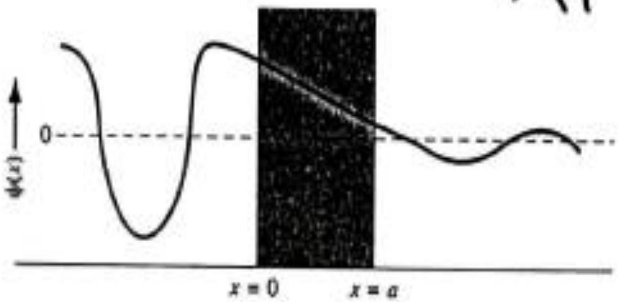
$$\psi_3(x) = A_3 e^{jk_1 x} + B_3 e^{-jk_1 x} \qquad (k_3 = k_1)$$

$B_3 = 0$ since no reflected wave in region 3.

so now 4 boundary conditions

→ Get B_1, A_2, B_2, and A_3 in terms of A_1

$$T = \frac{v_t}{v_i} \cdot \frac{A_3^* A_3}{A_1^* A_1} = \frac{A_3^* A_3}{A_1^* A_1}$$



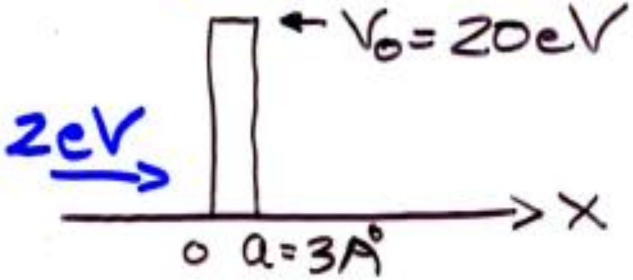
For $E \ll V_0$

$T \approx$

Particle does not go over barrier.
Instead, it "tunnels" through!

If barrier w [blank] or
height V_0 [blank] or
energy E [blank] then T decreases.

Tunnel Barrier Example



Probability of a 2eV electron tunneling through a 20eV high, 3Å thick barrier.

Use $T \cong$

as the tunneling probability. (okay since $E \ll V_0$)

$$K_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \sqrt{\frac{2(9.11 \times 10^{-31} kg)(20eV - 2eV)(1.602 \times 10^{-19} J/eV)}{(1.054 \times 10^{-34} J\text{-}s)^2}}$$

(Note: $(kg)(eV)(J/eV)/J^2\text{-}s^2 = kg/J\text{-}s^2$

$= kg/kg\,m^2 s^{-2} s^2 = m^{-2}$ so $K_2 \propto \sqrt{m^{-2}} \propto m^{-1}$

okay!)

$K_2 = 21.75 \times 10^{9}\,m^{-1} = 2.175 \times 10^{10}\,m^{-1}$

Then $T \cong 16\left(\frac{2}{20}\right)\left(1-\frac{2}{20}\right) \exp\left[-2\left(2.175\times10^{+10}\,m^{-1}\right)\left(3\times10^{-10}\,m\right)\right]$

$= 16(0.1)(0.9)\exp[-13.048]$

$=$

(note: handout didn't carry significant digits: 2.175, not 2.17).

small but not zero.

Since a large number of particles impinge on the barrier, a significant number can get through.

Example: $1\mu A = 10^{-6}\,\frac{\text{Coulomb}}{\text{sec}} \,/\, (1.6\times10^{-19}\,\text{Coulomb/electron})$

$= 6.25\times10^{12}$ electrons/sec

$T \cdot J = 3.1\times10^{-6} \cdot 6.25\times10^{12} = 1.9\times10^{7}$ electrons/sec