HW3, Problem 3

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CSE 2321

$$\sum_{k=1}^{N} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Base Case:

$k=1 \quad 1^2 = 1$

$n=1 \quad \frac{1(1+1)(2(1)+1)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1$

Inductive Hypothesis: $N=n \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

Inductive Step: $N = n+1 \quad \sum_{k=1}^{n+1} k^2$

$$\sum_{k=1}^{n} k^2 + (n+1)^2 \rightarrow \frac{n(n+1)(2n+1)}{6} + (n+1)^2 \rightarrow \frac{n(n+1)(2n+2)}{6} + \frac{6(n+1)^2}{6}$$

$$\rightarrow \frac{n(n+1)(2n+1)+6(n+1)^2}{6} \rightarrow \frac{(n^2+n)(2n+1)+6(n+1)^2}{6} \rightarrow \frac{2n^3+n^2+2n^2+n+6(n+1)^2}{6}$$

$$\rightarrow \frac{2n^3+3n^2+n+6n^2+12n+6}{6} \rightarrow \frac{2n^3+9n^2+13n+6}{6}$$

$$\rightarrow \frac{(n+1)(n+2)(2n+3)}{6}$$

They are <u>not</u> equal.