CSE 2321 Homework 4 Template

The align environment is good for when you have a sequence of equations:

$$\begin{aligned} x^2 &= -2x - 1 \\ x^2 + 2x + 1 &= 0 \\ (x+1)(x+1) &= 0 \end{aligned}$$

1

The product of two odd numbers is odd.

Proof.
Proof Let n, m be an odd integer.
Then there $\exists j, k \in \mathbb{Z}$ such that
n = 2j + 1 and m = 2k + 1

Therefore,
n * m = (2j + 1)(2k + 1)
= 4jk + 2j + 2k + 1
= 2(jk + j + k) + 1

Since $\exists j, k \in \mathbb{Z}$, we have that 2(jk + j + k) + 1 $\in \mathbb{Z}$

All multiples of 2 are even, and the sum of an even number plus one is always odd.
So 2(jk + j + k) + 1 is odd.
So n * m is odd.
Therefore, the products of 2 odd integers is also odd.

□

2

Let $G = (V, E)$ be an undirected graph.
G is bipartite $\Rightarrow$ G contains no cycles of odd length.

Proof By Contradiction.

Assume that G is bipartite and G contains a cycle of odd length.
So we know G has a bipartition, A and B.
So G contains a cycle of odd length called C.

Let C = $V_0, V_1, V_2, ..., V_k, V_0$
So let $V_0 \in A$ and $V_1 \in B$
All odd-numbered vertices $V_e \in A$ and even numbered vertices $V_o \in B$.

So, if the cycle starts at $V_0 \in A$, it must end at V_0.
Because the graph has a bipartition, it must go to $V_1 \in B$.
Then the next vertex must be $V_2 \in A$.

So, regardless of cycle length, for the graph to start and end at a vertice $V_0 \in A$, it must travel to and back from a vertex $V_k \in B$.
Which increases the cycle length k by 2.

To increase the cycle length by 1 and obtain a cycle with an odd length, the cycle must start from an even vertex $V_e \in A$ and end at an odd vertex $V_o \in B$.

Contradiction: $V_0 \in B \wedge V_0 \in A$

Conclusion: G is bipartite and only contains cycles of even length.

□

3

Let $a, b \in \mathbb{R}$. If $b > 0$ then $(n + a)^b = \Theta(n^b)$.

Proof. Your proof goes here. □