CSE 2321 Homework 5 Template

1a

$f_1(n) = \Theta(\lg(n))$
$\lg(n^3) + \lg(n^2) = 5\lg(n)$
$5\lg(n) \leq 6\lg(n)$ for $n \geq 1$
$5\lg(n) \geq \lg(n)$ for $n \geq 1$
$\lg(n) \leq 5\lg(n) \leq 6\lg(n)$ for $n \geq 1$

$f_2(n) = \Theta(3^n)$
3^{n+1} is the dominant term.
$3^{n+1} \geq \frac{2}{3} \cdot 3^{n+1}$ for all $n \geq 1$
$3^{n+1} \leq \frac{4}{3} \cdot 3^{n+1}$ for all $n \geq 1$
$\frac{2}{3} \cdot 3^{n+1} \leq 3^{n+1} \leq \frac{2}{3} \cdot 3^{n+1}$ for all $n \geq 1$

$f_3(n) = \Theta(n^{0.8})$
$n^{0.8}$ is the dominant term.
$\lim_{n\to\infty} \frac{3n^{0.2}+3n^{0.8}}{n^{0.8}} = \lim_{n\to\infty} \left(\frac{3n^{0.2}}{n^{0.8}} + \frac{3n^{0.8}}{n^{0.8}}\right) = \lim_{n\to\infty}(3n^{-0.6} + 3) = 0 + 3 = 3$

$f_4(n) = \Theta(1)$
$2^{16} \leq 2^{17}$ for all $n \geq 1$
$2^{16} \geq 1$ for all $n \geq 1$
$1 \leq 2^{16} \leq 2^{17}$ for all $n \geq 1$

$f_5(n) = \Theta(n^{0.7})$
$n^{0.7}$ is the dominant term.
$\lim_{n\to\infty} \frac{4\lg(n^2+2n)+6n^{0.7}}{n^{0.7}} = \lim_{n\to\infty} \left(\frac{4\lg(n^2+2n)}{n^{0.7}} + \frac{6n^{0.7}}{n^{0.7}}\right) = 0 + 6 = 6$

$f_6(n) = \Theta(2^n)$
2^{n-2} is the dominant term.
$2^{n-2} \leq 2^n$ for all $n \geq 1$
$2^{n-2} \geq 2^{-2} \cdot 2^n$ for all $n \geq 1$
$2^{-2} \cdot 2^n \leq 2^{n-2} \leq 2^n$

$f_7(n) = \Theta(n^2)$

The dominant term is n^2.
$\lim_{n\to\infty} \frac{3n\lg(n^2+2)+n^2}{n^2} = \lim_{n\to\infty}\left(\frac{3n\lg(n^2+2)}{n^2}+\frac{n^2}{n^2}\right) = \lim_{n\to\infty}(0+1) = 1$

$f_8(n) = \Theta(n^{1.5})$
$\sqrt{(9n^2)(6n)(121)} = \sqrt{6534}\cdot n^{1.5}$
$\lim_{n\to\infty} \frac{\sqrt{6534}\cdot n^{1.5}}{n^{1.5}} = \sqrt{6534}$

$f_9(n) = \Theta(n^{\frac{1}{2}})$
$2+\sqrt{5}n^{\frac{1}{2}}+\frac{1}{2}\lg(n)$
The dominant term is $n^{1/2}$.
$(5n)^{\frac{1}{2}} \leq (4+5n+\lg(n))^{1.2} \leq (10n)^{\frac{1}{2}}$

$f_{10}(n) = \Theta(nlg(n))$
The dominant term is $nlg(n)$.
$\lim_{n\to\infty} \frac{\lg(2^n)+n\lg(n^3)+500n}{n\lg(n)} = \lim_{n\to\infty}(\frac{n\lg(2)}{n\lg(n)}+\frac{3n\lg(n)}{n\lg(n)}+\frac{500n}{n\lg(n)}) = 0+3+0 = 3$

$f_{11}(n) = \Theta(n^{16/5})$
$\sqrt[5]{(5n^5)(6n)(121n^{10})}+n\lg(100n) = \sqrt[5]{Cn^{16}}+100n\lg(n)$
The dominant term is $n^{16/5}$.
$\lim_{n\to\infty} \frac{Cn^{16/5}}{n^{16/5}}+\frac{100n\lg(n)}{n^{16/5}} = C+0 = C$

$f_{12}(n) = \Theta(n^n)$
$n^{n-2} \leq n^{n-1} \leq n^n$

$f_{13}(n) = \Theta(8^n)$
$8^n-1 \leq 8n^{\frac{1}{2}} \leq 8^n$

$f_{14}(n) = \Theta(n^2)$
The dominant term is n^2.
$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{1}{2}n^2+\frac{1}{2}n$
$\lim_{n\to\infty} \frac{\frac{1}{2n^2}+\frac{1}{2n}}{n^2} = \lim_{n\to\infty}\frac{\frac{1}{2n^2}}{n^2}+\lim_{n\to\infty}\frac{\frac{1}{2n}}{n^2} = \frac{1}{2}+0 = \frac{1}{2}$

1b

$$f_4, f_9, f_5, f_1, f_{10}, f_3, f_7 = f_{14}, f_{11} = f_8, f_6, f_2, f_{13}, f_{12}$$

2a

$T(n) = \Theta(n^4)$
$\sum_{a=1}^{n^2} \sum_{b=1}^{n^2} 1 = n^2 \cdot n^2 = n^4$

$n^4 \leq 2n^4$ for all $n \geq 1$
$n^4 \geq \frac{1}{2}n^4$ for all $n \geq 1$
$\frac{1}{2}n^4 \leq n^4 \leq 2n^4$

2b

$T(n) = \Theta(n^6)$
$\sum_{a=1}^{n^2} \left(\sum_{b=1}^{a^2} 1\right)$
$\sum_{a=1}^{n^2} a^2 = \frac{n^2(n^2+1)(2n^2+1)}{6}$
Dominant term is n^6. Discard other terms (polynomials with power of less than 6).
$n^6 \leq 2n^6$ for all $n \geq 1$
$n^6 \geq \frac{1}{2}n^6$ for all $n \geq 1$
$\frac{1}{2}n^6 \leq n^6 \leq 2n^6$