CSE 2321 Foundations I Spring 2024 Dr. Estill

Homework 6 Due: Friday, March 8

Write a recurrence relation describing the worst case running time of each the following algorithms and make a guess about the asymptotic complexity of the functions defined by the recurrence relation using either expanding into a series or a recursion tree.

$A[i \dots j]$ represents an array of $n = j - i + 1$ numbers starting at index i and ending at index j and $A[k]$ represents the value at index k.

(20 points each)

1.) `FUNCTION` $F_1(A[1 \dots n])$
```
    IF n ≤ 5 THEN RETURN(A[n])
    x ← 0
    FOR i ← 1 TO 5 DO
      BEGIN (FOR i)
        FOR j ← 1 TO n − 4 DO
          A[i] ← A[j] + A[j + 2]
        x ← x + F1(A[1 ... ⌊n/2⌋])
      END
    RETURN(x)
```

2.) `FUNCTION` $F_2(A[i \dots j])$
```
    n ← j − i + 1
    IF n ≤ 1 THEN RETURN(A[i])
    x ← F2(A[i ... ⌊i + 2n/3⌋])
    FOR i ← ⌊n/4⌋ TO ⌊n/4⌋ + 12 DO
      x ← x + A[i]
    RETURN(x)
```

3.) `FUNCTION` $F_3(A[1 \dots n])$
```
    IF n ≤ 1 THEN RETURN(A[1])
    FOR i ← 1 TO n DO
      FOR j ← 1 TO ⌊n/3⌋ DO
        A[i] ← A[i] − A[j]
    x ← F3(A[1..⌊3n/5⌋])
    RETURN(x)
```

For each of the following, prove that the given recursive relation defines a function in the given Θ-set using the substitution method (i.e. induction).

(20 points each)

4.) $T_4(n) = 4T_4(n/5) + cn^2$, with a base case of $n = 1$
Guess: $T_4(n) \in \Theta(n^2)$

5.) $T_5(n) = 5T_5(n/5) + c\sqrt{n}$, with a base case of $n = 1$
Guess: $T_5(n) \in \Theta(n)$