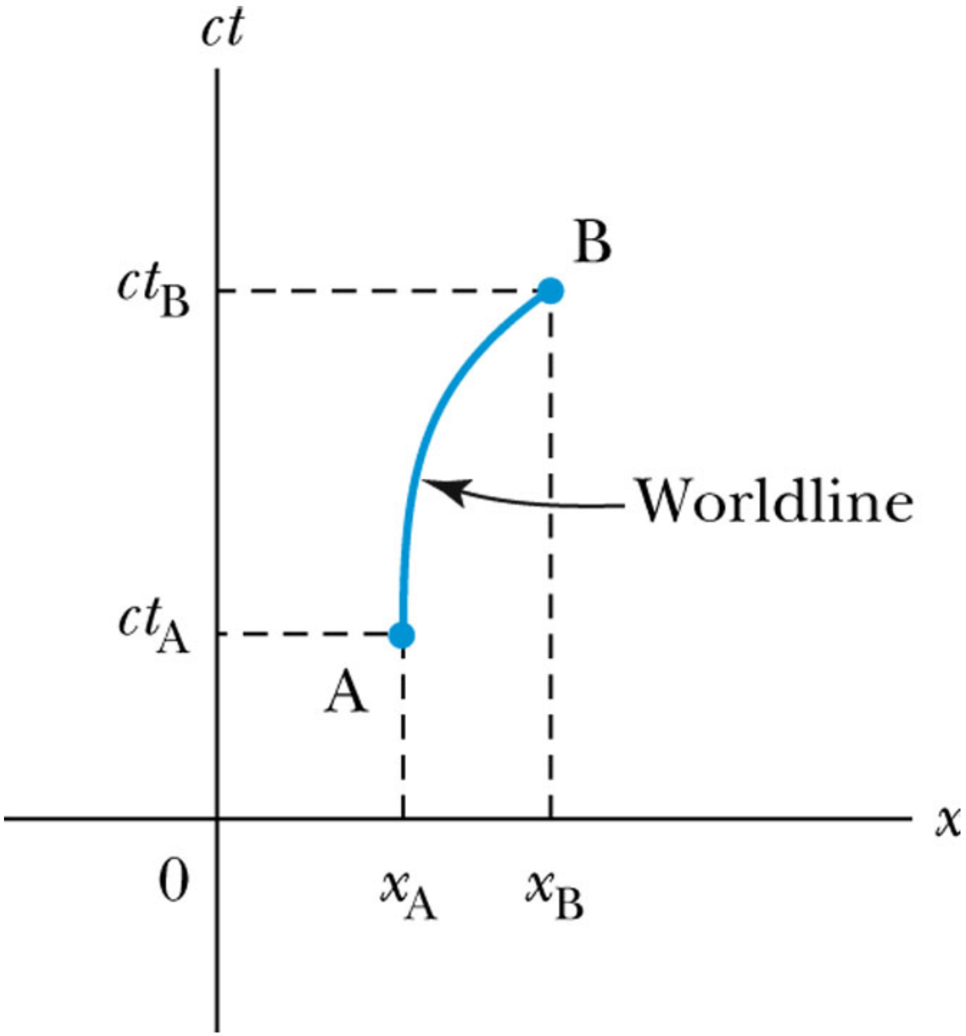
Spacetime

- When describing events in relativity, it is convenient to represent events on a **spacetime** diagram.

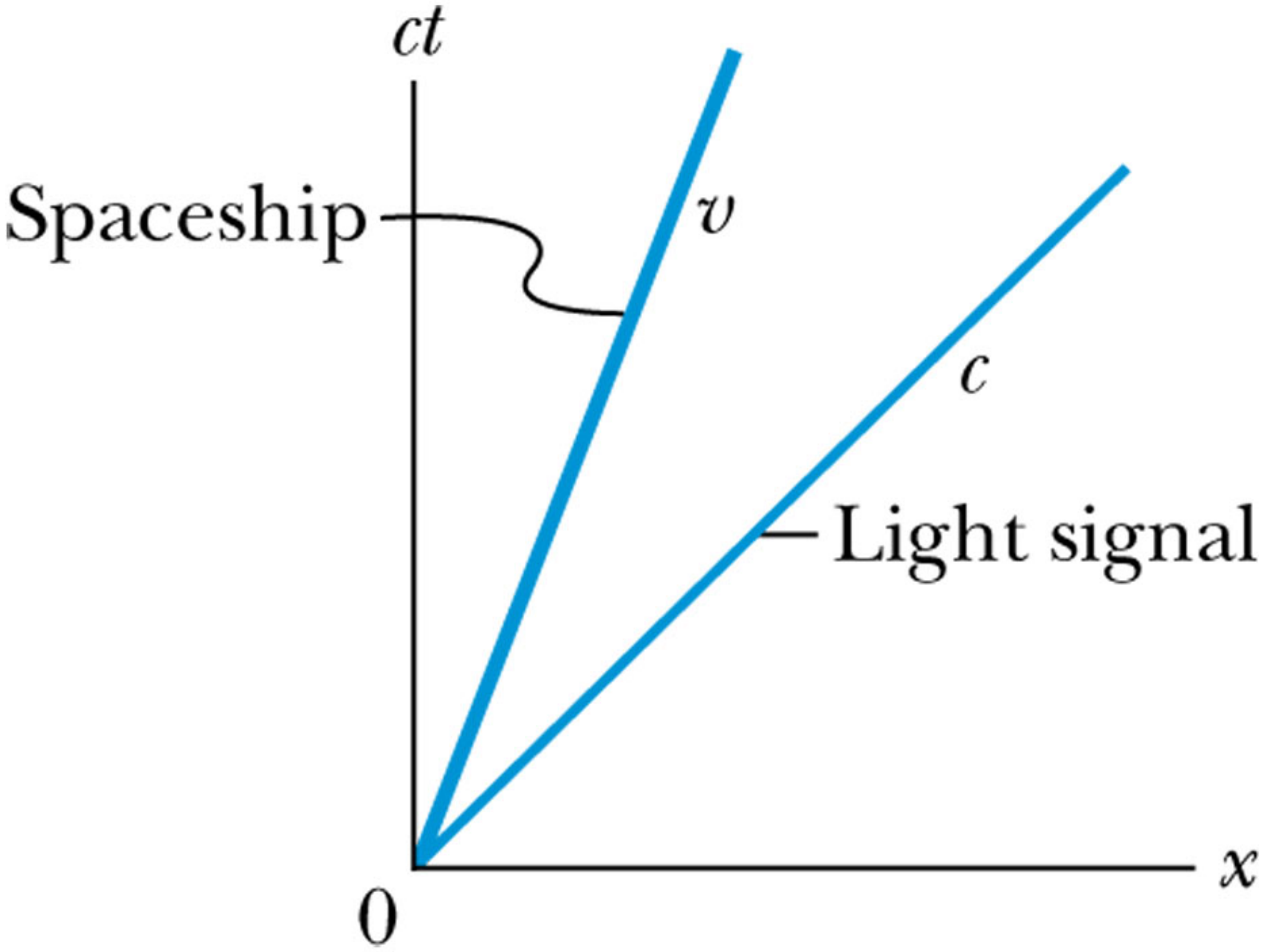
- In this diagram one spatial coordinate *x,* to specify position, is used and instead of time *t*, *ct* is used as the other coordinate so that both coordinates will have dimensions of length.

- Spacetime diagrams were first used by H. Minkowski in 1908 and are often called **Minkowski diagrams**. Paths in Minkowski spacetime are called **worldlines**.

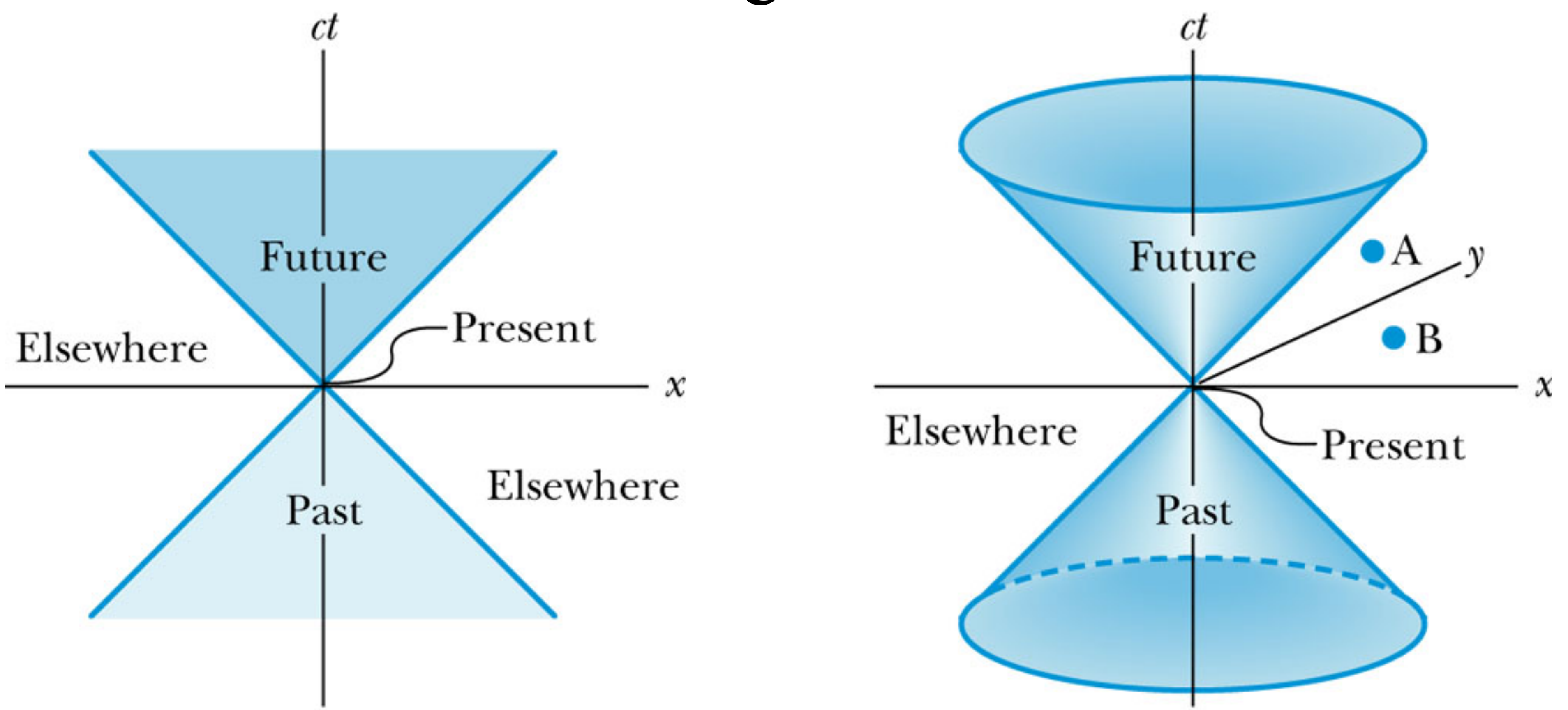
Spacetime Diagram



Particular Worldlines



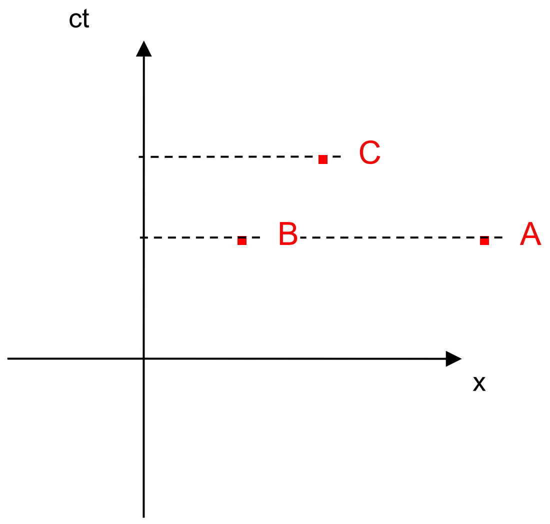
The Light Cone



(a) (b)

Spacetime Diagrams and Events

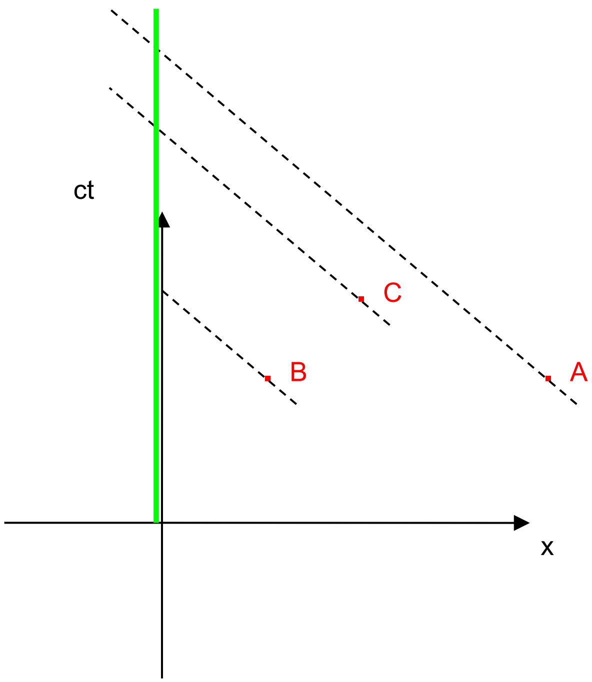
Events on spacetime diagrams are represented by points typically. We may see when the event occurs in a given reference frame by drawing a line parallel to the x-axis and seeing where it intersects the ct-axis:



Events A and B occur simultaneously, while event C occurs at a later time in the rest frame of the observer.

Spacetime Diagrams and Events

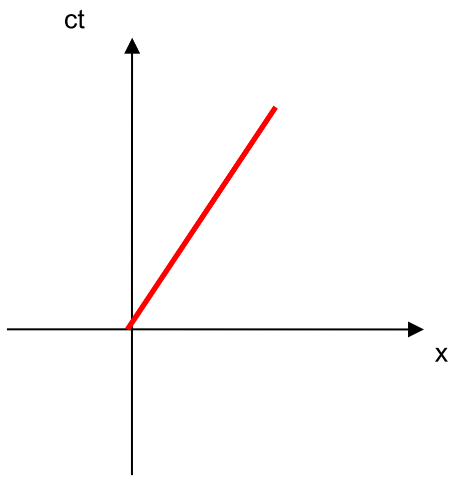
Note that the observer will not actually see these events happening at the times they occur in the frame…to find when the observer sees the events happen, one would draw a world line for light traveling from each event to the observer's world line. Where they intersect would be when the observer sees the event: (In this case, the observer sees event B first, then C, then A)…be very careful not to confuse when the observer sees an event with when an event occurs in the observer's frame.



Spacetime Diagrams and Events

An object moving relative to the observer will have a world line as well, which will no longer be along the time axis. To find the direction of the world line of the object we may look at the slope of any such graph:

$$slope = \frac{d(ct)}{dx} = \frac{cdt}{dx\frac{dt}{dt}} = \frac{cdt}{dt\frac{dx}{dt}} = \frac{c}{v}$$



The slope is the speed of light divided by the velocity relative to the reference frame.

Spacetime Diagram of a Moving System

$$x' = \gamma(x - ut)$$

$$t' = \gamma(t - \frac{u}{c^2}x)$$

We would like to see how the time (ct') and position (x') axes of a second reference frame (O') would appear on our current spacetime diagram.

To begin with, set x'=x=0 at ct'=ct=0 and let S' move with velocity u in the positive x-direction.

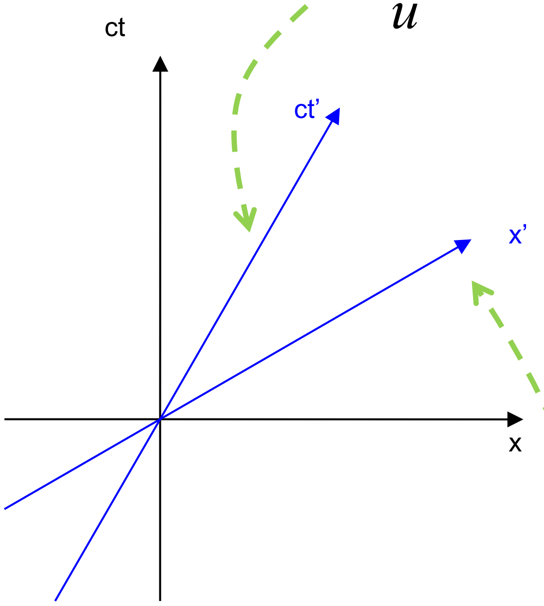
Along the ct' axis, x'=0 so we can see that the slope of the ct' axis must be the slope of a worldline of an object moving with velocity +u (slope = c/u).

Now how does the x'-axis look? It is not perpendicular to ct' in the observer's reference frame. All along the x'-axis we know ct'=0 and may look at the Lorentz transformation to find the slope.

The slope of u/c is just the inverse of the slope of the ct'-axis:

$$x' = \gamma(x - ut) = 0$$

$$x = ut \quad \Rightarrow \quad ct = \frac{cx}{u}$$

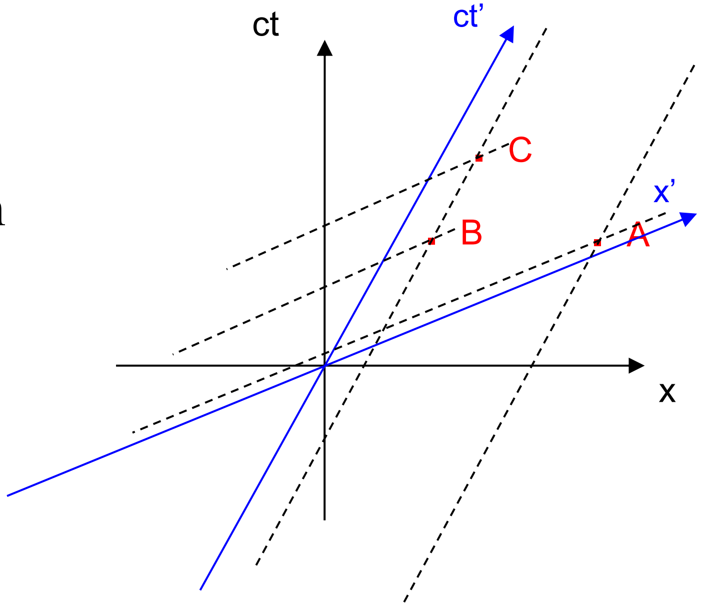


$$ct' = c\gamma(t - \frac{ux}{c^2}) = 0$$

$$ct = \frac{u}{c}x$$

Spacetime Diagrams and Events

Similar to finding when events occur in the observer's reference frame, we may draw lines from events parallel to the x'-axis and see when they intersect the ct'-axis:



Here you can simply see that in O' frame, event A occurs first, followed by B and then C.

In the O frame A and B occurred simultaneously followed by event C.

Spacetime Diagrams and Events