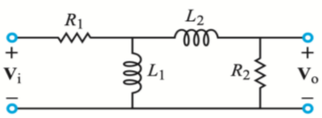
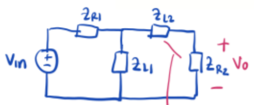
THE OHIO STATE UNIVERSITY

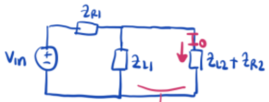
Extra Problem 1

Determine the transfer function $H = Vo/Vi$





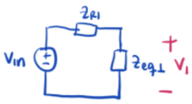
series, same current



$V_o = I_o Z_{R2}$

$Z_{eq1} = Z_{L1} \| (Z_{L2}+Z_{R2}) = \dfrac{Z_{L1}(Z_{L2}+Z_{R2})}{Z_{L1}+Z_{L2}+Z_{R2}}$

parallel, same voltage



$V_1 = V_{series}$ $\quad I_o = \dfrac{V_{series}}{Z_{L2}+Z_{R2}}$

voltage division

$V_1 = V_{in} \cdot \dfrac{Z_{eq1}}{Z_{R1}+Z_{eq1}}$

$$= V_{in}\,\frac{\dfrac{Z_{L1}(Z_{L2}+Z_{R2})}{Z_{L1}+Z_{L2}+Z_{R2}}}{Z_{R1}+\dfrac{Z_{L1}(Z_{L2}+Z_{R2})}{Z_{L1}+Z_{L2}+Z_{R2}}}$$

$$= V_{in}\,\frac{Z_{L1}(Z_{L2}+Z_{R2})}{Z_{R1}(Z_{L1}+Z_{L2}+Z_{R2})+Z_{L1}(Z_{L2}+Z_{R2})}$$

$I_o = \dfrac{V_{series}}{Z_{L2}+Z_{R2}}$

$$= \frac{V_{in}\,\dfrac{Z_{L1}\cancel{(Z_{L2}+Z_{R2})}}{Z_{R1}(Z_{L1}+Z_{L2}+Z_{R2})+Z_{L1}(Z_{L2}+Z_{R2})}}{\cancel{(Z_{L2}+Z_{R2})}}$$

$$= V_{in}\,\frac{Z_{L1}}{Z_{R1}(Z_{L1}+Z_{L2}+Z_{R2})+Z_{L1}(Z_{L2}+Z_{R2})}$$

$V_o = I_o Z_{R2}$

$$V_o = V_{in}\,\frac{Z_{L1}Z_{R2}}{Z_{R1}(Z_{L1}+Z_{L2}+Z_{R2})+Z_{L1}(Z_{L2}+Z_{R2})}$$

$$\frac{V_o}{V_{in}} = \frac{Z_{L1}Z_{R2}}{Z_{R1}(Z_{L1}+Z_{L2}+Z_{R2})+Z_{L1}(Z_{L2}+Z_{R2})}$$

$Z_{R2} = R_2$
$Z_{R1} = R_1$
$Z_{L1} = j\omega L_1$
$Z_{L2} = j\omega L_2$

$$= \frac{j\omega L_1 R_2}{R_1(j\omega L_1 + j\omega L_2 + R_2) + j\omega L_1(j\omega L_2 + R_2)}$$

$$= \frac{j\omega L_1 R_2}{R_1R_2 + R_1 j\omega(L_1+L_2) + j\omega L_1 R_2 - \omega^2 L_1 L_2}$$

THE OHIO STATE UNIVERSITY

Extra Problem 2

Convert the following voltage ratios to dB:

$H(\omega) = \left|\dfrac{V_{out}}{V_{in}}\right|$

$20\log_{10}\left(\dfrac{V_{out}}{V_{in}}\right) = dB$

1. 3×10^2 $\rightarrow$ $20\log_{10}(300) = 49.54\text{dB}$
2. 0.510^{-2} $\rightarrow$ $20\log_{10}(0.005) = -46.02\text{dB}$
3. $\sqrt{2000}$ $\rightarrow$ $20\log_{10}(\sqrt{2000})$
   $= 20\log_{10}(2000)^{1/2}$
   $= 10\log_{10}(2000) = 33.01\text{dB}$
4. $(360)^{1/4}$
   $\downarrow$
   $20\log_{10}(360^{1/4}) = 5\log_{10}(360) = 12.78\text{dB}$

THE OHIO STATE UNIVERSITY

Extra Problem 3

Convert the following dB values to voltage ratios:

$20\log_{10}\left(\dfrac{V_{out}}{V_{in}}\right) = dB$

$\log_{10}\left(\dfrac{V_{out}}{V_{in}}\right) = \dfrac{dB}{20}$

$\dfrac{V_{out}}{V_{in}} = 10^{dB/20}$

1. 36 dB $\rightarrow 10^{36/20} = 63.09$ V/V
2. 0.6 dB $\rightarrow 10^{0.6/20} = 1.07$ V/V
3. −2 dB $\rightarrow 10^{-2/20} = 0.79$ V/V
4. −60 dB $\rightarrow 10^{-60/20} = 10^{-3} = 0.001$

THE OHIO STATE UNIVERSITY

Extra Problem 4

Generate Bode magnitude and phase plots (straight-line approximations) for the following voltage transfer function.

$$H(j\omega) = \frac{j\omega}{10+j\omega} \times \frac{1/10}{1/10}$$

look lik TF for

so High pass filter

$$= \frac{j\omega/10}{1+j\omega/10}$$

$1+j\omega$ $\quad \omega = \dfrac{1}{\omega_c}$ $\quad \dfrac{1}{10} = \dfrac{1}{\omega_c}$ $\quad \omega_c = 10$

$$M(\omega) = \frac{\omega/10}{\sqrt{1+\omega^2/100}}$$

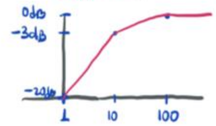
$$\phi(\omega) = 90 - \tan^{-1}\left(\frac{\omega/10}{1}\right)$$

$\lim_{\omega\to 0} H(\omega) = \dfrac{0}{1+0} = 0$ $\qquad \lim_{\omega\to\infty} H(\omega) = \dfrac{\infty}{1+\infty}$

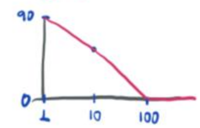
$$= \lim_{\omega\to\infty} \frac{1/10}{\sqrt{1/\omega^2 + 1/100}}$$

$$= \frac{1/10}{\sqrt{0+1/100}} = 1$$

Magnitude



Phase



THE OHIO STATE UNIVERSITY

Extra Practice 5

Generate Bode magnitude and phase plots (straight-line approximations) for the following voltage transfer function

$$H(s) = \frac{30(j\omega+10)}{(2j\omega+200)(2j\omega+1000)}$$

$j\omega/\omega_c + 1$

ω_c

$$= 30\cdot(j\omega+10)\cdot\frac{1}{2j\omega+200}\cdot\frac{1}{2j\omega+1000}$$

$$= 30\,(j\omega/10+1)\,10\cdot\frac{1}{\left(\frac{2j\omega}{200}+1\right)200}\cdot\frac{1}{\left(\frac{2j\omega}{1000}+1\right)1000}$$

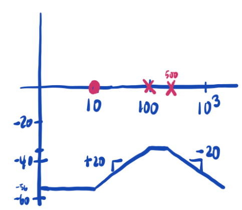
$$= \underset{①}{\frac{3}{2\times10^3}}\,\underset{②}{(j\omega/10+1)}\cdot\underset{③}{\frac{1}{\left(\frac{2j\omega}{200}+1\right)}}\cdot\underset{④}{\frac{1}{\left(\frac{2j\omega}{1000}+1\right)}}$$

zeros $\rightarrow$ 10

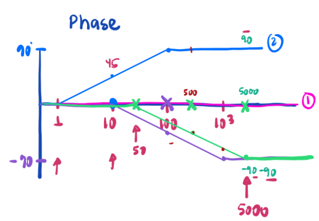
poles $\rightarrow$ 100 , 500

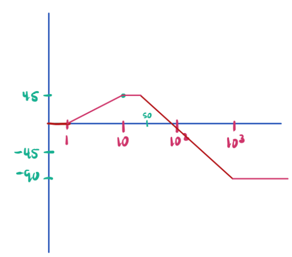
$K = \dfrac{3}{2\times10^3} \rightarrow 20\log_{10}(1.5m) = -56.48$

Magnitude



Phase



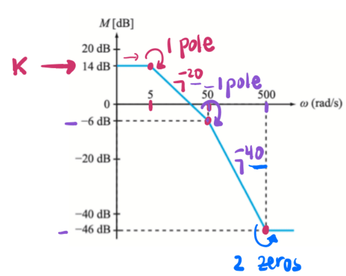


THE OHIO STATE UNIVERSITY

Extra Practice 6

Determine the voltage transfer function H(jω) corresponding to the Bode magnitude plot shown

K



$(1+j\omega/\omega_c)$

$$H(j\omega) = \frac{5.012\,(1+j\omega/500)^2}{(1+j\omega/5)(1+j\omega/50)}$$

$20\log_{10}(K) = 14$

$10^{14/20} = K$

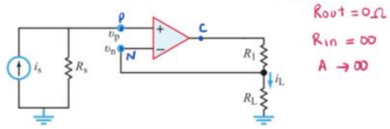
$K = 5.012$

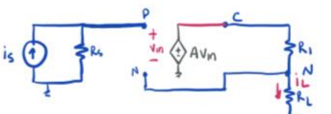
THE OHIO STATE UNIVERSITY

Extra Problem 1

For the op-amp circuit below, use the behavioral model to find an expression for the current gain $G_i = i_L/i_s$.

Ideal values
$R_{out} = 0\Omega$
$R_{in} = \infty$
$A \to \infty$





$V_P = i_s \cdot R_s$ (1)

$V_{in} = V_P - V_N$ (2)

voltage division:

(3) $V_N = V_{RL} = AV_{in} \cdot \frac{R_L}{R_L + R_I}$

(2) in (3)

$$V_N = A(V_P - V_N)\cdot \frac{R_L}{R_L+R_I}$$

(1) $\nearrow$

$$V_N = A(i_sR_s - V_N)\cdot \frac{R_L}{R_L+R_I}$$

$$V_N\left[1 + \frac{AR_L}{R_L+R_I}\right] = \frac{A i_s R_s R_L}{R_L+R_I}$$

$$V_N = \frac{\frac{A i_s R_s R_L}{R_L+R_I}}{1 + \frac{AR_L}{R_L+R_I}}$$

$$V_N = \frac{A i_s R_s R_L}{R_L + R_I + AR_L}$$

$V_N = V_{RL} = R_L \cdot i_L$

$$\frac{A i_s R_s R_L}{R_L + R_I + AR_L} = R_L i_L$$

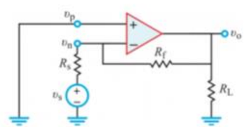
$$\frac{i_L}{i_s} = \frac{AR_sR_L}{R_L^2 + R_IR_L + AR_L^2}$$

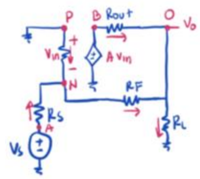
$$\lim_{A\to\infty} \frac{i_L}{i_s} = \frac{R_sR_L}{R_L^2} = \frac{R_s}{R_L} \qquad \boxed{\frac{i_L}{i_s} = \frac{R_s}{R_L}}$$

THE OHIO STATE UNIVERSITY

Extra Problem 2

- Use the behavioral circuit model to obtain an expression for the closed-loop gain $G = v_o/v_i$, in terms of R_S, R_i, R_o, R_L, R_F, and A.
- Determine the value of G for $R_S = 10\Omega$, $R_i = 10M\Omega$, $R_o = 50\Omega$, $R_L = 1k\Omega$, $R_F = 1k\Omega$, and $A = 10^6$.
- Determine the value of G by letting $A \to \infty$, $R_i \to \infty$, $R_o \to 0$ (ideal op-amp model).





$V_o = V_{RL}$

$V_P = 0\,V$

$V_A = V_s$

$V_B = AV_{in} = -AV_N$

$V_{in} = \cancel{V_P}^{0} - V_N = -V_N$

KCL @ N:

$i_{Rs} + i_{in} = i_{RF}$

$$\frac{V_{Rs}}{R_s} + \frac{V_{in}}{R_{in}} = \frac{V_{RF}}{R_F}$$

$$V_N = \frac{R_{in}R_F V_s + R_sR_{in}V_o}{R_sR_{in} + R_{in}R_F + R_sR_F} \quad (1)$$

KCL @ O:

$i_{RF} + i_{out} = R_L$

$$\frac{V_{RF}}{R_F} + \frac{V_{out}}{R_{out}} = \frac{V_{RL}}{R_L}$$

$$\left[\frac{V_N - V_o}{R_F} + \frac{-AV_N - V_o}{R_{out}} = \frac{V_o - 0}{R_L}\right] R_F R_{out} R_L$$

$$R_{out}R_LV_N - R_{out}R_LV_o - AR_LR_FV_N - R_FR_LV_o = R_FR_{out}V_o$$

$$(R_{out}R_L - AR_FR_L)V_N = (R_FR_{out} + R_FR_L + R_{out}R_L)V_o$$

$$V_N = \frac{(R_FR_{out} + R_FR_L + R_{out}R_L)V_o}{(R_{out}R_L - AR_FR_L)} \quad (2)$$

(1) = (2)

$$\frac{R_{in}R_F V_s + R_sR_{in}V_o}{R_sR_{in} + R_{in}R_F + R_sR_F} = \frac{(R_FR_{out} + R_FR_L + R_{out}R_L)V_o}{(R_{out}R_L - AR_FR_L)}$$

$$R_{in}R_FV_s(R_{out}R_L - AR_FR_L) + V_oR_sR_{in}(R_{out}R_L - AR_FR_L) = (R_sR_{in} + R_{in}R_F + R_sR_F)(R_FR_{out} + R_FR_L + R_{out}R_L)V_o$$

$$R_{in}R_FV_s(R_{out}R_L - AR_FR_L) = V_o\left[R_sR_{in}(AR_FR_L - R_{out}R_L) + (R_sR_{in} + R_{in}R_F + R_sR_F)(R_FR_{out} + R_FR_L + R_{out}R_L)\right]$$

$$\frac{V_o}{V_s} = \frac{R_{in}R_F(R_{out}R_L - AR_FR_L)}{R_sR_{in}(AR_FR_L - R_{out}R_L) + (R_sR_{in} + R_{in}R_F + R_sR_F)(R_FR_{out} + R_FR_L + R_{out}R_L)}$$

B) $\frac{V_o}{V_s} = -99.98$

C)
$$\lim_{\substack{A\to\infty \\ A_{out}\to 0 \\ R_{in}\to\infty}} \frac{R_{in}R_F(R_{out}R_L - AR_FR_L)}{R_sR_{in}(AR_FR_L - R_{out}R_L) + (R_sR_{in} + R_{in}R_F + R_sR_F)(R_FR_{out} + R_FR_L + R_{out}R_L)}$$

$$\lim_{\substack{A\to\infty \\ A_{out}\to 0 \\ R_{in}\to\infty}} \frac{-AR_FR_LR_{in}R_F}{R_sR_{in}(AR_FR_L) + (R_sR_{in} + R_{in}R_F + R_sR_F)R_FR_L}$$

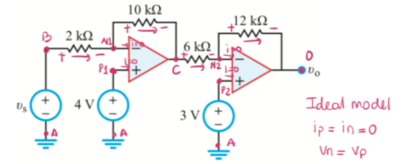
$$\frac{-\cancel{AR_FR_L}\cancel{R_{in}}R_F}{\cancel{AR_FR_L}R_s\cancel{R_{in}}} = -\frac{R_F}{R_s}$$

Inverting amplifier.

THE OHIO STATE UNIVERSITY

Extra problem 1

- Solve for v_o in terms of v_s.



Ideal model
$i_p = i_n = 0$
$V_n = V_p$

Node voltage analysis $V_A = 0V$ $V_B = V_s$ $V_{N1} = V_{P1} = 4V$

$V_{P1} = 4V$ $V_{P2} = 3V$ $V_{N2} = V_{P2} = 3V$

Do not do KCL @ C and O

KCL @ N1:

$i_{2K} = i_{10K}$

$$\frac{V_{2K}}{2K} = \frac{V_{10K}}{10K}$$

$$\frac{V_s - V_{N1}}{2K} = \frac{V_{N1} - V_c}{10K}$$

$10K(V_s - 4) = 2K(4 - V_c)$

$10KV_s - 40K = 8K - 2KV_c$

$5V_s - 20 = 4 - V_c$

$5V_s - 24 = -V_c$ (1)

KCL @ N2:

$i_{6K} = i_{12K}$

$$\frac{V_{6K}}{6K} = \frac{V_{12K}}{12K}$$

$$\frac{V_c - V_{N2}}{6K} = \frac{V_{N2} - V_o}{12K}$$

$2V_c - 2(3) = 3 - V_o$

$2V_c - 6 = 3 - V_o$

$2V_c - 9 = -V_o$

(2) $V_c = \frac{9 - V_o}{2}$

(1) = (2) $-5V_s + 24 = \frac{9 - V_o}{2}$

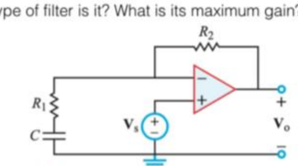
$-10V_s + 48 = 9 - V_o$

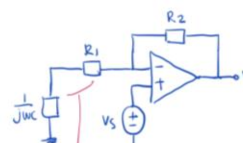
$V_o = 10V_s - 39$

THE OHIO STATE UNIVERSITY

For the op-amp circuit below:

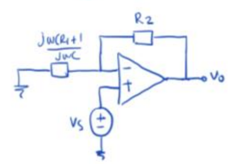
- Obtain an expression for H(ω) = Vo/Vs in standard form.
- Generate spectral plots for the magnitude and phase of H(ω), given that R1 =1k Ω, R2 =4k Ω, and C=1μF.
- What type of filter is it? What is its maximum gain?





series

$$R_1 + \frac{1}{j\omega C} = \frac{j\omega CR_1 + 1}{j\omega C}$$



Non-inverting amplifier.

$$\frac{V_o}{V_s} = \frac{R_2 + Z_{series}}{Z_{series}}$$

$$= \frac{R_2 + \frac{j\omega CR_1 + 1}{j\omega C}}{\frac{j\omega CR_1 + 1}{j\omega C}}$$

$$= \frac{j\omega CR_2 + j\omega CR_1 + 1}{1 + j\omega CR_1}$$

$1 + j\omega/\omega_c \longrightarrow$

$$= \frac{j\omega C(R_1 + R_2) + 1}{1 + j\omega CR_1}$$

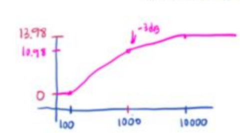
$$\frac{V_o}{V_s} = \frac{j\omega(1\times10^{-6})5K + 1}{1 + j\omega(1\times10^{-6})1K} = \frac{1 + j\omega 5\times10^{-3}}{1 + j\omega 1\times10^{-3}}$$

$$H(j\omega) = \frac{\sqrt{1 + (5\times10^{-3})^2\omega^2}}{\sqrt{1 + (1\times10^{-3})^2\omega^2}}$$

$\omega \to 0$ $H = 1$

$\omega \to \infty$ $H = 5$

HPF



$H = 1$ $20\log(1) = 0$ dB

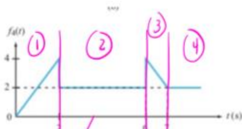
$H = 5$ $20\log(5) = 13.98$dB

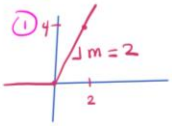
$1 + j\omega(1\times10^{-3}) = 1 + j\frac{\omega}{\omega_c}$

$$\omega_c = \frac{1}{1\times10^{-3}} = 1000$$

Extra Problem 1

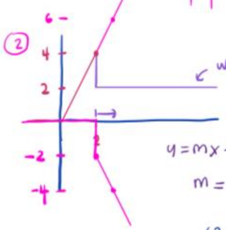
- Express the waveforms in terms of step functions and then determine its Laplace transform. [Recall that the ramp function is given by r(t − T) = (t − T) u(t − T)]. Assume that all waveforms are zero for t < 0.





$f(t) = 2r(t)$

$= 2t\,u(t)$



$t = 2$ $\quad 4 - 2 = 2$

$t = 3$ $\quad 6 - 4 = 2$

$y = mx + b = -2x + b$

$m = \frac{-2-(-4)}{2-3} = \frac{2}{-1} = -2$

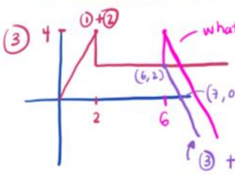
$(2,-2)$ $\quad -2 = -2(2) + b$

$-2 + 4 = b$

$b = 2$

$(-2t+2)u(t-2)$

$f(t) = 2t\,u(t) - 2(t-1)u(t-2)$ (1) (2)



$y = mx + b = -2x + b$

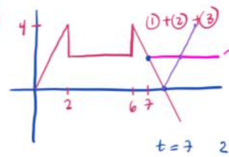
$m = \frac{2-0}{6-7} = \frac{2}{-1} = -2$

$(7,0)$ $\quad 0 = -2(7) + b$

$b = 14$

$(-2t+14)u(t-6)$

$f(t) = 2tu(t) - 2(t-1)u(t-2) - 2(t-7)u(t-6)$



purple line

$(2t - 14)u(t-7)$

$2(7) - 14 = 0$

$2(8) - 14 = 2$

$2(9) - 14 = 4$

$t = 7$ $\quad 2 + 0 = 2$

$t = 8$ $\quad 0 + 2 = 2$

$t = 9$ $\quad -2 + 4 = 2$

$f(t) = 2tu(t) - 2(t-1)u(t-2) - 2(t-7)u(t-6) + 2(t-7)u(t-7)$
(1) (6)

$2(t-1-1+1)$ $\quad 2(t-7+1-1)$

$-[2(t-2)+2]u(t-2)$ $\quad -[2(t-6)-2]u(t-6)$
(2) (3) (4) (5)

$F(s) = \frac{2}{s^2} - \frac{2}{s^2}e^{-2s} - \frac{2}{s}e^{-2s} - \frac{2}{s^2}e^{-6s} + \frac{2}{s}e^{-6s} + \frac{2}{s^2}e^{-7s}$
(1) (2) (3) (4) (5) (6)

Extra Problem 2

Find the Laplace transform of the following signals:

1. $h_1(t) = 4te^{-2t}u(t)$
2. $h_2(t) = 10\cos(12t + 60°)u(t)$
3. $h_3(t) = 12e^{-3(t-4)}\,u(t-4)$

Laplace Transform Pairs

	$f(t)$		$F(s) = \mathcal{L}[f(t)]$
1	$\delta(t)$	⟷	1
1a	$\delta(t-T)$	⟷	e^{-Ts}
2	1 or $u(t)$	⟷	$\frac{1}{s}$
2a	$u(t-T)$	⟷	$\frac{e^{-Ts}}{s}$
3	$e^{-at}\,u(t)$	⟷	$\frac{1}{s+a}$
3a	$e^{-a(t-T)}\,u(t-T)$	⟷	$\frac{e^{-Ts}}{s+a}$
4	$t\,u(t)$	⟷	$\frac{1}{s^2}$
4a	$(t-T)\,u(t-T)$	⟷	$\frac{e^{-Ts}}{s^2}$
5	$t^2\,u(t)$	⟷	$\frac{2}{s^3}$
6	$te^{-at}\,u(t)$	⟷	$\frac{1}{(s+a)^2}$
7	$t^2e^{-at}\,u(t)$	⟷	$\frac{2}{(s+a)^3}$
8	$t^{n-1}e^{-at}\,u(t)$	⟷	$\frac{(n-1)!}{(s+a)^n}$
9	$\sin\omega t\,u(t)$	⟷	$\frac{\omega}{s^2+\omega^2}$
10	$\sin(\omega t+\theta)\,u(t)$	⟷	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
11	$\cos\omega t\,u(t)$	⟷	$\frac{s}{s^2+\omega^2}$
12	$\cos(\omega t+\theta)\,u(t)$	⟷	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
13	$e^{-at}\sin\omega t\,u(t)$	⟷	$\frac{\omega}{(s+a)^2+\omega^2}$
14	$e^{-at}\cos\omega t\,u(t)$	⟷	$\frac{s+a}{(s+a)^2+\omega^2}$
15	$2e^{-at}\cos(bt-\theta)\,u(t)$	⟷	$\frac{e^{j\theta}}{s+a+jb} + \frac{e^{-j\theta}}{s+a-jb}$
16	$\frac{2t^{n-1}}{(n-1)!}e^{-at}\cos(bt-\theta)\,u(t)$	⟷	$\frac{e^{j\theta}}{(s+a+jb)^n} + \frac{e^{-j\theta}}{(s+a-jb)^n}$

$H_1(s) = \frac{4}{(s+2)^2}$

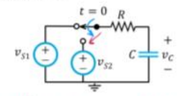
$H_2(s) = 10 \cdot \frac{s\cos(60) - 12\sin(60)}{s^2 + 12^2} = \frac{5s - 10.392}{s^2 + 144}$

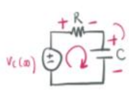
$H_3(s) = 12 \cdot \frac{e^{-4s}}{s+3}$

Extra Problem 4

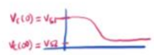
Module 7

Determine $v_C(t)$ for $t \geq 0$.





series, same current



$V_R(t) = i_R(t) \cdot R$

$i_C(t) = C \cdot \frac{dV_C(t)}{dt} = i_R(t)$

KVL $\quad V_C(\infty) = V_R(t) + V_C(t)$

$V_C(\infty) = R \cdot i_R(t) + V_C(t)$

differential equation → $V_C(\infty) = RC \cdot \frac{dV_C(t)}{dt} + V_C(t)$

Laplace Transform

$\frac{V_C(\infty)}{s} = RC \cdot (sV_C(s) - V_C(0)) + V_C(s)$

$K \leftrightarrow \frac{K}{s}$
$\frac{df(t)}{dt} \leftrightarrow s \cdot F(s) - f(0)$

initial conditions

$\frac{V_C(\infty)}{s} + RC\,V_C(0) = V_C(s)[RCs + 1]$

$V_C(s) = \frac{\frac{V_C(\infty)}{s} + RC\,V_C(0)}{RCs + 1}$

$= \frac{V_C(\infty) + RC\,s\,V_C(0)}{s(RCs+1)}$ $\quad$ 1/RC, 1/RC

$= \frac{V_C(\infty)/RC + sV_C(0)}{s(s + 1/RC)}$

Partial Fraction Expansion

$\frac{1}{RC} = 1/RC$

$V_C(s) = \frac{A}{s} + \frac{B}{s + 1/RC}$

$A = V_C(s) \cdot s \,\big|_{s=0}$

$= \frac{V_C(\infty)/RC + 0\,V_C(0)}{0 + 1/RC} = V_C(\infty)$

$B = V_C(s) \cdot (s + 1/RC)\,\big|_{s=-1/RC}$

$= \frac{V_C(\infty)/RC - 1/RC\,V_C(0)}{-1/RC} = -V_C(\infty) + V_C(0)$

$V_C(s) = \frac{V_C(\infty)}{s} + \frac{V_C(0) - V_C(\infty)}{s + 1/RC}$

↳

$v_C(t) = \left[V_C(\infty) + (V_C(0) - V_C(\infty))\,e^{-1/RC\,t}\right]u(t)$

Extra Problem 3

Use partial fraction expansion to find $f(t)$:

$$F(s) = \frac{3s^3 + 36s^2 + 131s + 144}{s(s+4)(s^2+6s+9)}$$

↳ $(s+3)^2$

$= \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+3} + \frac{D}{(s+3)^2}$

$A = F(s) \cdot s \,\big|_{s=0}$

$= \frac{3s^3 + 36s^2 + 131s + 144}{(s+4)(s+3)^2}\Big|_{s=0} = \frac{144}{(4)(3^2)} = \frac{144}{36}$

$\boxed{A = 4}$

$B = F(s)(s+4)\,\big|_{s=-4}$

$= \frac{3s^3 + 36s^2 + 131s + 144}{(s)(s+3)^2}\Big|_{s=-4}$

$= \frac{3(-4)^3 + 36(-4)^2 + 131(-4) + 144}{-4(-4+3)^2}$

$= \frac{-3(64) + 36(16) - 131(4) + 144}{-4} = -1$ $\quad \boxed{B = -1}$

$D = F(s) \cdot (s+3)^2\,\big|_{s=-3}$

$= \frac{3s^3 + 36s^2 + 131s + 144}{(s)(s+4)}\Big|_{s=-3}$

$= \frac{3(-3)^3 + 36(-3)^2 + 131(-3) + 144}{-3(-3+4)}$

$= \frac{-81 + 36(9) + 131(-3) + 144}{-3} = 2$ $\quad \boxed{D = 2}$

$F(s) = \frac{4}{s} + \frac{-1}{s+4} + \frac{C}{s+3} + \frac{2}{(s+3)^2}$

$N(s) = 4(s+3)^2(s+4) - s(s+3)^2 + C(s+4)(s+3)s + 2(s+4)s$

$= 4(s^2+6s+9)(s+4) - s^3 - 6s^2 - 9s + C(s^3 + 7s^2 + 12s) + 2s^2 + 8s$

$= 4s^3 + 40s^2 + 132s + 144 - s^3 - 6s^2 - 9s + C(s^3 + 7s^2 + 12s) + 2s^2 + 8s$

$= s^3(4 - 1 + C) + s^2(40 - 6 + 7C + 2) + s(132 - 9 + 12C + 8) + 144$

$\quad\quad \| \quad\quad \| \quad\quad \| \quad\quad \|$

$\quad\quad 3 \quad\quad 36 \quad\quad 131 \quad\quad 144$

$3 + C = 3$

$C = 0$

$F(s) = \frac{4}{s} - \frac{1}{s+4} + \frac{2}{(s+3)^2}$

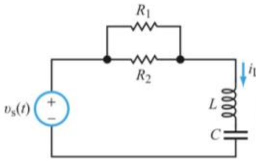
↳

$f(t) = 4u(t) - e^{-4t}u(t) + 2te^{-3t}u(t)$

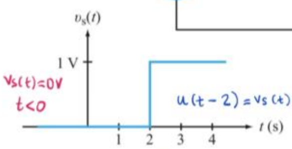
$= [4 - e^{-4t} + 2te^{-3t}]\,u(t)$

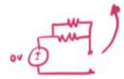
Extra Problem 1

- $v_s(t)$ is given by the waveform displayed. Determine $i_L(t)$, given that $R1 = R2 = 4\Omega$, $L = 2H$, and $C = 0.5F$.



initial conditions
$i_L(0) = 0A$
$v_C(0) = 0V$

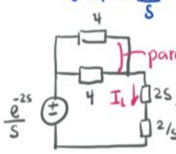




$V_s(s) = \frac{e^{-2s}}{s}$

$Z_L = sL = 2s$

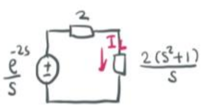
$Z_C = \frac{1}{sC} = \frac{2}{s}$



parallel $4 \| 4 = 2$

series, same current

$2s + \frac{2}{s} = \frac{2s^2+2}{s}$



$I_L = \frac{V_s}{Z_{eq}} = \frac{e^{-2s}/\cancel{s}}{\frac{2(s^2+s+1)}{\cancel{s}}} = \frac{e^{-2s}}{2(s^2+s+1)} = \frac{0.5\,e^{-2s}}{s^2+s+1}$

coefficient $= 1$

$Z_{eq} = \frac{2(s^2+1)}{s} + 2$

$= \frac{2(s^2+s+1)}{s}$

$I_L = \left[\frac{0.5}{\underbrace{s^2+s+1}_{\text{poles}}}\right]\underbrace{e^{-2s}}_{u(t-2)}$ time-shift

Module 10 video 2

property #4 of first table = time shift

$S_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = -0.5 \pm 0.87j$

$F(s) = \frac{A}{s+0.5+0.87j} + \frac{B}{s+0.5-0.87j}$

$A = F(s)\cdot(s+0.5+0.87j)\Big|_{s=-0.5-0.87j}$

$= \frac{0.5}{-0.5-0.87j+0.5-0.87j} = 0.29j = 0.29\,e^{90j}$

$B = F(s)\cdot(s+0.5-0.87j)\Big|_{s=-0.5+0.87j}$

$= \frac{0.5}{-0.5+0.87j+0.5+0.87j} = -0.29j = 0.29\,e^{-90j}$

$I_L(s) = \left[\frac{0.29\,e^{90j}}{s+0.5+0.87j} + \frac{0.29\,e^{-90j}}{s+0.5-0.87j}\right]e^{-2s}$

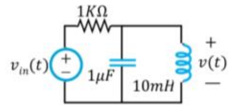
second table

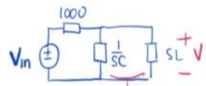
$i_L(t) = [2(0.29)\,e^{-0.5t-2}\cos(0.87t-90)]\,u(t-2)$ line 15

Extra Problem 2

For the circuit below, assume all initial conditions are $\mathbf{0}$.

a. Find the s-domain transfer function $\mathbf{V}(s)/V_{in}(s)$.

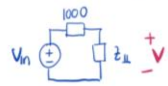




$Z_C = \frac{1}{sC} = \frac{10^6}{s}$

$Z_L = sL = 0.01s$

parallel, same voltage



$Z_{\|} = \frac{\left(\frac{10^6}{\cancel{s}}\cdot 0.01\cancel{s}\right)}{\frac{10^6}{s} + 0.01s}$

$= \frac{10^4 s}{0.01s^2 + 10^6}$

voltage division

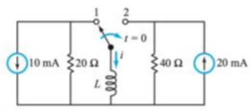
$V(s) = V_{in}(s)\cdot\frac{Z_{\|}}{Z_{\|}+1000}$

$\frac{V(s)}{V_{in}(s)} = \frac{\frac{10^4 s}{0.01s^2+10^6}}{1000 + \frac{10^4 s}{0.01s^2+10^6}} = \frac{10^4 s}{10s^2 + 10^9 + 10^4 s}\ \frac{1/10}{1/10}$

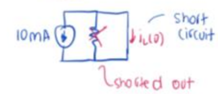
$= \frac{10^3 s}{s^2 + 10^3 s + 10^8}$

Extra Problem 3

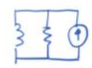
The switch in the below was moved from position 1 to position 2 at t = 0, after it had been in position 1 for a long time. Determine i(t) for t ≥ 0, if L = 80mH,



I.C.

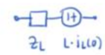


$i_L(0) = -10mA$



$I_s(s) = \frac{20m}{s}$

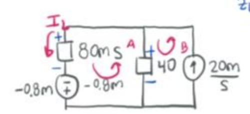
$Z_R = 40$



$Z_L = s\cdot L = 80ms$

$= 80m(-10m)$

$= -800\mu = -0.8m$



$I_B = \frac{20m}{s}$

KVL @ A:

$V_S + V_R = V_{Z_L}$

$-0.8m + 40(I_B - I_A) = 80ms\,I_A$

$-0.8m + \frac{0.8}{s} = (80ms + 40)\,I_A$

$I_A = \frac{0.8 - 0.8ms}{s(80ms+40)}\ \frac{1/0.08}{1/0.08}$

$= \frac{10 - 0.01s}{s(s+500)}$

$I_A = I_L = \frac{A}{s} + \frac{B}{s+500}$

$A = F(s)\,s\Big|_{s=0} = \frac{10 - 0.01(0)}{0+500} = 0.02$

$B = F(s)(s+500)\Big|_{s=-500} = \frac{10 - 0.01(-500)}{-500}$

$= -0.01$

$I_L = \frac{0.02}{s} - \frac{0.01}{s+500}$

$i_L(t) = [0.02 - 0.01\,e^{-500t}]\,u(t)$ A

$= [20 - 10\,e^{-500t}]\,u(t)$ mA