Complex Variable, Laplace & Z-Transformation

Lecture 04

- Inverse Laplace Transformation
- This Lecture Covers
- Important Formulae
- Examples Using Direct Formula
- Exercise Using Direct Formula
- First Translation Property & Example
- Examples Using First Shifting Property
- Exercises Using First Shifting Property
- Learning Outcomes

Inverse Laplace Transformation

This Lecture Covers

1. Formula of Inverse Laplace Transformation.

2. Examples & Exercise of Inverse Laplace Transformation Using Direct Formula.

3. First Shifting Property of Inverse Laplace Transformation.

4. Examples & Exercises of Inverse Laplace Transformation Using First Shifting Property.

Important Formulae

Examples Using Direct Formula

Exercise Using Direct Formula

First Translation Property & Example

Examples Using First Shifting Property

Exercises Using First Shifting Property

Learning Outcomes

1. $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1,$

2. $\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!},$

3. $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at},$

4. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at,$

5. $\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at,$

6. $\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at,$

7. $\mathcal{L}^{-1}\left\{\frac{a}{s^2-a^2}\right\} = \sinh at.$

Learning Outcomes

Exercises Using First Shifting Property

Examples Using First Shifting Property

First Translation Property & Example

Exercise Using Direct Formula

Examples Using Direct Formula

1. $\mathcal{L}^{-1}\left\{\frac{s^2+1}{s^3}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{1}{s^3}\right\}$$

$$= 1 + \frac{t^2}{2!} = 1 + \frac{t^2}{2}.$$

2. $\mathcal{L}^{-1}\left\{\frac{1}{2s-5}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{2(s-\frac{5}{2})}\right\}$$

$$= \frac{1}{2}e^{\frac{5}{2}t}$$

3. $\mathcal{L}^{-1}\left\{\frac{2s}{s^2-9}\right\}$

$$= 2\mathcal{L}^{-1}\left\{\frac{s}{s^2-3^2}\right\}$$

$$= 2\cosh 3t$$

4. $\mathcal{L}^{-1}\left\{\frac{5}{s} - \frac{3s}{s^2+16} + \frac{2}{s^2+4}\right\}$

$$= 5\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 3\mathcal{L}^{-1}\left\{\frac{s}{s^2+4^2}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{s^2+2^2}\right\}$$

$$= 5 - 3\cos 4t + \sin 2t.$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!},$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at,$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at,$$

$$\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at,$$

Find The Inverse Laplace Transformation of the following functions:

1. $F(s) = \frac{1}{s-5}$,

2. $F(s) = \frac{1}{s^5}$,

3. $F(s) = \frac{s^3-5s^2+6}{s^4}$,

4. $F(s) = \frac{2+4s}{s^2+25}$,

5. $F(s) = \frac{3}{s^2+4}$,

6. $F(s) = \frac{3}{s^2-4}$.

Learning Outcomes

Exercises Using First Shifting Property

Examples Using First Shifting Property

First Translation Property & Example

First translation property

If $\mathcal{L}^{-1}\{F(s)\} = f(t)$ then

$$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}\mathcal{L}^{-1}\{F(s)\}.$$

Example: 01

$$\mathcal{L}^{-1}\left\{\frac{10}{(s+3)^4}\right\}$$
$$= 10\mathcal{L}^{-1}\left\{\frac{1}{(s-(-3))^4}\right\}$$
$$= 10e^{-3t}\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\}$$
$$= 10e^{-3t}\frac{t^3}{3!} = \frac{10}{6}e^{-3t}t^3.$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\}$$
$$= \frac{t^n}{n!}$$

Example: 02

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2+1}\right\}$$
$$= e^{2t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$
$$= e^{2t}\sin t\,.$$

$$\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\}$$
$$= \sin at,$$

Exercise Using Direct Formula

Examples Using Direct Formula

Important Formulae

This Lecture Covers

Inverse Laplace Transformation

Example 3.

$$\mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+4s+13}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+2)-3}{(s+2)^2-4+13}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+2)-3}{(s+2)^2+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+2)}{(s+2)^2+3^2}-\frac{3}{(s+2)^2+3^2}\right\}$$

$$= 2e^{-2t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\} - e^{-2t}\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\}$$

$$= 2e^{-2t}\cos 3t - e^{-2t}\sin 3t.$$

$$\mathcal{L}^{-1}\left\{\frac{a}{s^2+a^2}\right\} = \sin at,$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at,$$

Learning Outcomes

Exercises Using First Shifting Property

$$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}\mathcal{L}^{-1}\{F(s)\}.$$

Find Inverse Laplace of the following functions:

1. $F(s) = \frac{1}{(s-3)^4}$
2. $F(s) = \frac{3}{(s+2)^2+9}$
3. $F(s) = \frac{s-2}{(s-2)^2-16}$
4. $F(s) = \frac{s}{s^2+4s-9}$
5. $F(s) = \frac{5s-7}{s^2-6s+25}$
6. $F(s) = \frac{s}{s^2-6s+10}$

Examples Using First Shifting Property

First Translation Property & Example

Exercise Using Direct Formula

Examples Using Direct Formula

Important Formulae

This Lecture Covers

Inverse Laplace Transformation

Learning Outcomes

After completing this chapter you can easily

- evaluate the inverse Laplace transformation of function
  - using direct formula
  - also using property.

THE END

Sample MCQ

1. $\mathcal{L}^{-1}\left\{\dfrac{s^2+1}{s^3}\right\} = ?$

(a) $1+\frac{t}{2}$ (b) $1+\frac{t^2}{2}$ (c) $1-\frac{t^2}{2}$ (d) $\frac{t^2}{2}$

2. $\mathcal{L}^{-1}\left\{\frac{4}{s-2} - \frac{s}{s^2-16} + \frac{4}{s^2-4}\right\} = ?$

(a) $e^{2t} - \cosh 4t + 2\sinh 2t$ (b) $4e^{2t} + \cosh 4t + 2\sinh 2t$

(c) $4e^{2t} - \cosh 4t + 2\sinh 2t$ (d) $4e^{2t} - \cosh 4t$

3. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+13}\right\} = ?$

(a) $e^{-2t}\cos 3t - \frac{2}{3}e^{-2t}\sin 3t$ (b) $e^{-2t}\cos 3t - \frac{2}{3}e^{-2t}\sin 3t$

(c) $e^{-2t}\cos 3t - \frac{2}{3}e^{-2t}\sin 3t$ (d) $e^{-2t}\cos 3t - \frac{2}{3}e^{-2t}\sin 3t$

4. $\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2-16}\right\} = ?$

(a) $\frac{e^{2t}}{2} + \frac{e^{6t}}{2}$ (b) $\frac{e^{-2t}}{2} + \frac{e^{-6t}}{2}$ (c) $\frac{e^{-2t}}{2} + \frac{e^{4t}}{2}$ (d) $\frac{e^{-2t}}{2} + \frac{e^{6t}}{2}$