AMERICAN INTERNATIONAL UNIVERSITY-BANGLADESH

Faculty of Science & Technology

Department of Mathematics

MAT1102: Differential Calculus and Coordinate Geometry (Sections: All)

Total Marks: 40 Time: 2 hours.

Coordinators:

Instruction: Answer all the questions.

1. Answer any **ONE** of the following questions: $(\mathbf{04 \times 1 = 04})$

(a) Given, $f(x) = \begin{cases} x^2; x < 1 \\ x; x \geq 1. \end{cases}$

Sketch the function and check the continuity at $x = 1$.

(b) Find $\frac{d}{dx}(\sin 2x)$ by first principle of differentiation.

2. Find $\frac{dy}{dx}$ (any **FOUR**): $(\mathbf{03 \times 4 = 12})$

(a) $y = e^{3x} \arctan(2x)$, **(b)** $y = \frac{\cos(x^2)}{x^3}$,

(c) $y = x^x$, **(d)** $y = \ln(\sin(\cos(x^2)))$,

(e) $x = 3\cos t, y = 3\sin t$, **(f)** $x^2y + 3xy^2 + 5x = 6$,

(g) $y = \text{arcsec}\,(x)$, **(h)** $y = \frac{(2x+3)\,(3x+2)}{(x-2)}$.

3. Evaluate the limit using L'Hôpital rule (if possible) (any **ONE**): $(\mathbf{04 \times 1 = 04})$

(a) $\lim_{x \to 1} \frac{x^4 - 4x^3 + 6x^2 - 4x + 1}{x^3 - 3x^2 + 3x - 1}$,

(b) $\lim_{x \to 0^+} x^{2x}$.

4. Given $f(x) = x^3 - 3x^2 + 1$. $(\mathbf{06 \times 1 = 06})$

(a) Find the stationary point(s) and find the maximum and minimum value of $f(x)$.

(b) Find the point(s) of inflection.

(c) Sketch the graph of $f(x)$.

(d) Find the interval(s) where the function is concave up and concave down.

5. Answer any **TWO** of the following questions: $(\mathbf{04 \times 2 = 08})$

(a) Given $u(x,y) = \arcsin\left(\frac{x^2+y^2}{x+y}\right)$. Using Euler's theorem prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u.$$

(b) Show that $u = x^3 - 3xy^2$ and $v = 3x^2y - y^3$ satisfy Cauchy-Riemann equations ($u_x = v_y$ and $u_y = -v_x$) in rectangular form.

(c) If $f(x,y,z) = x^3y^3z^3$, prove that $f_{xyz} = f_{zxy}$.

(d) Find local maximum and minimum values, and saddle points (if any) of the following function

$f(x,y) = x^4 + y^4 - 4xy + 1$.

6. Answer any **ONE** of the following questions: $(\mathbf{03 \times 1 = 03})$

(a) Find first **three** non-zero terms in the expansion of Taylor series of $f(x) = \cos x$ at $x = \frac{\pi}{4}$.

(b) Find first **three** non-zero terms in the expansion of Maclaurin series of $f(x) = e^{5x}$.

7. Answer any **ONE** of the following questions: $(\mathbf{03 \times 1 = 03})$

(a) Find the equations of the tangent and normal line of $y = 3x^2 - 5x + 4$ at $(1,2)$.

(b) Find the equations of the tangent and normal line of $2x^2 + 3xy + y^2 = 0$ at $(1,-2)$.