Math 5 (F): Lecture 2

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3. (a) → Homework

Hint: $t \rightarrow$

$v(t) \rightarrow$

$$a(t) = v'(t) = \frac{v(t+h) - v(t-h)}{2h}, \quad h = *$$

Richardson extrapolation:

$O(h_1^n)$ & $O(h_2^n)$

$$M_R = M(h_1) + \frac{M(h_1) - M(h_2)}{r^n - 1}$$

where, $r = \frac{h_2}{h_1}$, & n is order of error.

$v(t,h) \Rightarrow v(t)$

3(b)

Time t (s)	0	0.5	1	1.5	2
Speed v (m/s)	0	11.860	26.335	41.075	59.05

Use two point backward difference formula and Richardson extrapolation to estimate the acceleration of the rocket at time $t = 2$ s.

We have, $h_1 = 2 - 1.5 = 0.5$

and, $h_2 = 2 - 1 = 1$

$$a(t, h_1) = v'(t, h_1) = \frac{v(t) - v(t - h_1)}{h_1}$$

and, $$a(t, h_2) = v'(t, h_2) = \frac{v(t) - v(t - h_2)}{h_2}$$

At time $t = 2$ s,

$$a(2, 0.5) = \frac{v(2) - v(1.5)}{0.5} = \frac{59.05 - 41.075}{0.5}$$

$$= 35.95 \text{ ms}^{-2}$$

$$a(2, 1) = \frac{v(2) - v(1)}{1} = \frac{59.05 - 26.335}{1}$$

$= 32.715 \ ms^{-2}$

Richardson extrapolation,

$$r = \frac{h_2}{h_1} = \frac{1}{0.5} = 2$$

and $n = 1$.

$$a_R(2) = a(2, 0.5) + \frac{a(2, 0.5) - a(2, 1)}{2^1 - 1}$$

$$= 35.95 + \frac{35.95 - 32.715}{2 - 1}$$

$$= 35.95 + 3.235$$

$$= 39.185 \ ms^{-2}.$$

(c)

t	0	0.5	1	1.5	2
v	0	11.860	26.335	41.075	59.05

Here, $t=1s$, $h_1 = 0.5$ and $h_2 = 1$

$$a(t, h_1) = v'(t, h_1) = \frac{v(t+h_1) - v(t-h_1)}{2h_1}$$

and

$$a(t, h_2) = v'(t, h_2) = \frac{v(t+h_2) - v(t-h_2)}{2h_2}$$

at time $t = 1s$,

$$a(1, 0.5) = \frac{v(1+0.5) - v(1-0.5)}{2 \times 0.5}$$

$$= \frac{v(1.5) - v(0.5)}{1}$$

$$= 41.075 - 11.860$$

$$= 29.215 \ ms^{-2}$$

$$a(1,1) = \frac{v(1+1) - v(1-1)}{2 \times 1}$$

$$= \frac{v(2) - v(0)}{2}$$

$$= \frac{59.05 - 0}{2}$$

$$= 29.525 \ ms^{-2}$$

Extrapolation: $r = \frac{1}{0.5} = 2, \quad n = 2$

$$a_R(1) = a(1,0.5) + \frac{a(1,0.5) - a(1,1)}{2^2 - 1}$$

$$= 29.215 + \frac{29.215 - 29.525}{4-1}$$

$$= 29.215 - 0.1033$$

$$= 29.11 \ ms^{-2}.$$

(d) >> clear all

>> x = [0 0.5 1 1.5 2];

>> y = [0 11.860 26.335 41.075 59.05];

>> V = spline(x, y)

>> a = fnder(V, 1)

>> x1 = [0.5 1.25 2];

>> aval = fnval(a, x1)

4. $$E(t) = L\frac{di}{dt} + Ri$$

$L = 0.05$, $R = 2$, $i(t) \longrightarrow$ table

t	1	1.1	1.2	1.3	1.4
$i(t)$	8.22^{77}	7.24^{28}	5.9908	4.52^{60}	2.9122

We have, $h_1 = 1.2 - 1.1 = 0.1$

and, $h_2 = 1.2 - 1 = 0.2$

$$\frac{di}{dt}(t, h_1) = \frac{i(t+h_1) - i(t-h_1)}{2h_1}$$

and $$\frac{di}{dt}(t, h_2) = \frac{i(t+h_2) - i(t-h_2)}{2h_2}$$

At time $t = 1.2$,

$$\frac{di}{dt}(1.2, 0.1) = \frac{i(1.2+0.1) - i(1.2-0.1)}{2\times 0.1}$$

$$\frac{di}{dt}(1.2, 0.1) = \frac{i(1.3) - i(1.1)}{0.2}$$

$$= \frac{4.5260 - 7.2428}{0.2}$$

$$= -13.584$$

$$\frac{di}{dt}(1.2, 0.2) = \frac{i(1.4) - i(1)}{2 \times 0.2}$$

$$= \frac{2.9199 - 8.2277}{0.4}$$

$$= -13.2695$$

Extrapolation: $r = \frac{0.2}{0.1} = 2, n = 2$

$$\frac{di}{dt}(1.2) = \frac{di}{dt}(1.2, 0.1) + \frac{\frac{di}{dt}(1.2, 0.1) - \frac{di}{dt}(1.2, 0.2)}{2^2 - 1}$$

$$= -13.584 + \frac{-13.584 + 13.2695}{4 - 1}$$

$$\frac{di}{dt}(1.2) = -13.688$$

Now,

$$E(1.2) = L\frac{di}{dt}(1.2) + R\,i(1.2)$$

$$= 0.05 \times (-13.688) + 2 \times 5.9908$$

$$= 11.297$$

(b) Compare your result with the exact solution $i(t) = 10e^{-t/10}\sin(2t)$.

$$\frac{di}{dt} = 10\left[e^{-t/10}\cdot 2\cos(2t) + \sin(2t)\cdot\left(-\frac{1}{10}e^{-t/10}\right)\right]$$

$$= 20e^{-t/10}\cos(2t) - e^{-t/10}\sin 2t$$

$$\therefore \frac{di}{dt}(1.2) = 20e^{-\frac{1.2}{10}}\cos(2\times 1.2)$$

$$- e^{-\frac{1.2}{10}}\sin(2\times 1.2)$$

$$= 20\times 0.886\times(-0.737)$$

$$-0.886\times(0.675)$$

$$= -13.069 - 0.598$$

$$= -13.667$$

$$\text{Error} = |-13.667 - (-13.688)|$$

$$= 0.021$$

(c)

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>> clear
>> x = [1  1.1  1.2  1.3  1.4]
>> y = [8.2277  7.2428  5.9908
        4.5260  2.9122];
>> I = spline(x, y)
>> DI = fnder(I, 1)
>> R = 2;
>> L = 0.05;
>> Et = L * DI + R * I
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6.

x	0.8	1.0	1.2	1.3	1.4	1.6
$f(x)$	0.954	1.648	2.623		3.947	5.697

(a) $h = \frac{1.2-0.8}{2} = 0.2$

$$f'(1) = \frac{f(1.2)-f(0.8)}{2\times 0.2} = \frac{2.623-0.954}{0.4}$$

$$= 4.1725$$

at $x = 1.3$, $h = \frac{1.4-1.2}{2} = 0.1$

$$f'(1.3) = \frac{f(1.4)-f(1.2)}{2\times 0.1}$$

$$= \frac{3.947-2.623}{0.2}$$

$$= 6.62$$

please do in details

(b) $x \rightarrow$ 0.8 1 1.2 1.4 1.6

$y \rightarrow$

h_1 h_2

at $x = 0.8$, $h_1 = 1 - 0.8 = 0.2$

and $h_2 = 1.2 - 0.8$

$= 0.4$

$$f'(x, h_1) = \frac{f(x+h_1) - f(x)}{h_1}$$

$$f'(x, h_2) = \frac{f(x+h_2) - f(x)}{h_2}$$

$$f'(0.8, 0.2) = \frac{f(1) - f(0.8)}{0.2} = \frac{1.648 - 0.954}{0.2}$$

$= 3.47$

$$f'(0.8, 0.4) = \frac{f(1.2) - f(0.8)}{0.4} = \frac{2.623 - 0.954}{0.4}$$

$= 4.1725$

Extrapolation, $r = \frac{h_2}{h_1} = \frac{0.4}{0.2} = 2$, $n = 1$

$$f'(0.8) = f'(0.8, 0.2) + \frac{f'(0.8, 0.2) - f'(0.8, 0.4)}{2^1 - 1}$$

$$= 3.47 + \frac{3.47 - 4.1725}{1}$$

$$= 2.767$$