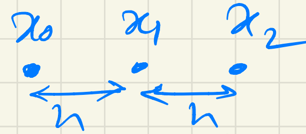
Lecture 04. 5(F) 15.11.2023

⊞ Simpson's 1/3 rule:



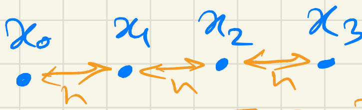
$[x_0, x_1], [x_1, x_2]$

For $n=2$, we have

$$\int_{x_0}^{x_2} f(x)\,dx \simeq \frac{h}{3}\left(f(x_0) + 4f(x_1) + f(x_2)\right).$$

where $h = \dfrac{x_2 - x_0}{2}$; $O(h^4)$

⊞ Simpson's 3/8 rule:



$[x_0, x_1], [x_1, x_2], [x_2, x_3]$

For $n=3$,

$$\int_{x_0}^{x_3} f(x)\,dx \simeq \frac{3h}{8}\left(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right)$$

where, $h = \dfrac{x_3 - x_0}{3}$; $O(h^4)$.

Example 8.3 The table below shows the values of $f(x)$ at different values of x:

0.1   0.1   0.2   0.2

x	0.4	0.5	0.6	0.8	1.0
$f(x)$	1.083	1.133	1.197	1.377	1.649

Evaluate $\int_{0.4}^{1.0} f(x)\,dx$ using Simpson's rule.

Sol:

$$\int_{0.4}^{1.0} f(x)\,dx = \int_{0.4}^{0.6} f(x)\,dx$$

0.1

$$+ \int_{0.6}^{1.0} f(x)\,dx$$

0.2

For $[0.4, 0.5]$ and $[0.5, 0.6]$,

$h_1 = 0.1$

and for $[0.6, 0.8]$, $[0.8, 1]$,

$h_2 = 0.2$

Now, $$\int_{0.4}^{1.0} f(x)\,dx = \underbrace{\int_{0.4}^{0.6} f(x)\,dx} + \int_{0.6}^{1.0} f(x)\,dx$$

$$= \frac{0.1}{3}\left[f(0.4) + 4f(0.5) + f(0.6)\right]$$

$$+ \frac{0.2}{3}\left[f(0.6) + 4f(0.8) + f(1)\right]$$

$$= \frac{0.1}{3}\left[1.083 + 4 \times 1.133 + 1.197\right]$$

$$+ \frac{0.2}{3}\left[1.197 + 4 \times 1.377 + 1.649\right]$$

$$= 0.033 \times 6.812 + 0.066 \times 8.354$$

$$= 0.776$$ Ans

☐ Composite Quadrature Rule:

* Composite Trapezoidal Rule:

we have given n subintervals $[x_{r-1}, x_r]$, step size $h = \frac{b-a}{n}$,

$x_0 = a$, $x_n = b$ and $x_r = x_0 + rh$,

$r = 1, 2, 3, \dots, n.$

**

$$\int_a^b f(x)\,dx = \frac{h}{2}\Big[f(x_0) + f(x_n) + 2\big(f(x_1) + f(x_2) + \dots + f(x_{n-1})\big)\Big]$$

$O(h^2)$

☐ Composite Simpson's Rule:

$$\int_a^b f(x)\,dx = \frac{h}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big]$$

$O(h^4)$

Exercise

2.

x	1.2	1.4	1.6	1.8	2
$f(x)$	3.728	4.124	4.525	5.123	5.626

a, b, x_0, x_1, x_2, x_3, x_4, 0.2, 0.2, 0.2, 0.2

(i) Use Trapezoidal rule and Richardson's extrapolation to estime $\int_{1.2}^{2} f(x)\,dx$.

Here, number of intervals $= 4$

$$\therefore\ h_1 = \frac{2-1.2}{4} = \frac{0.8}{4} = 0.2$$

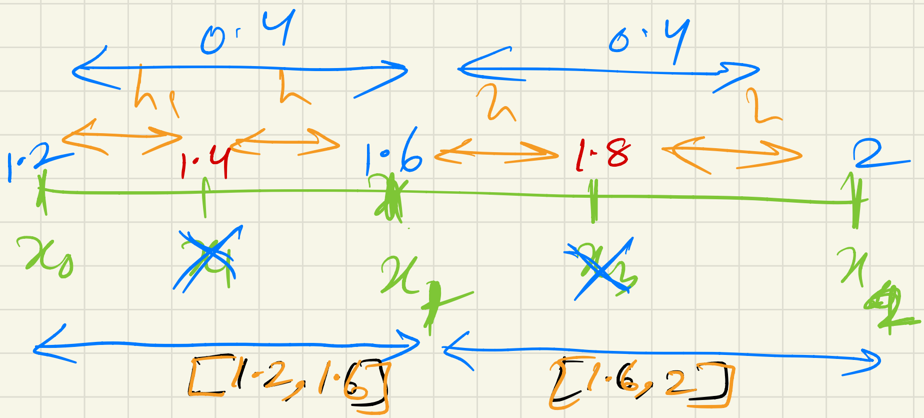
$$I_1 = \int_{1.2}^{2} f(x)\,dx = \frac{h_1}{2}\Big[f(x_0)+f(x_4) + 2\big(f(x_1)+f(x_2)+f(x_3)\big)\Big]$$

$$= \frac{0.2}{2}\Big[f(1.2)+f(2)+2\big(f(1.4)+f(1.6)+f(1.8)\big)\Big]$$

$$= 0.1\Big[3.728+5.626+2\big(4.124+4.525+5.123\big)\Big]$$

$= 0.1 \times 36.898$

$= 3.689 \quad = I_1(0.2)$



When considering Extrapolation

We have, $h_2 = \dfrac{2 - 1.2}{2}$

$= \dfrac{0.8}{2} = 0.4$

$$I(0.4) = \int_{1.2}^{2} f(x)\, dx$$

$$= \frac{h}{2}\left[f(x_0) + f(x_2) + 2f(x_1)\right]$$

$$= \frac{h}{2}\left[f(1.2) + f(2) + 2f(1.6)\right]$$

$$= \frac{0.4}{2}\left[3.728 + 5.626 + 2 \times 4.525\right]$$

$$I(0.4) = 0.2 \times 18.404$$

$$= 3.6808$$

<u>Extrapolation:</u> $r = \frac{h_2}{h_1} = \frac{0.4}{0.2} = 2,\ n = 2$

$$I = \int_{1.2}^{2} f(x)\,dx = I(0.2) + \frac{I(0.2) - I(0.4)}{r^n - 1}$$

$$= 3.689 + \frac{3.689 - 3.6808}{2^2 - 1}$$

$$= 3.689 + 0.0027$$

$$= 3.691.$$

2. (ii)

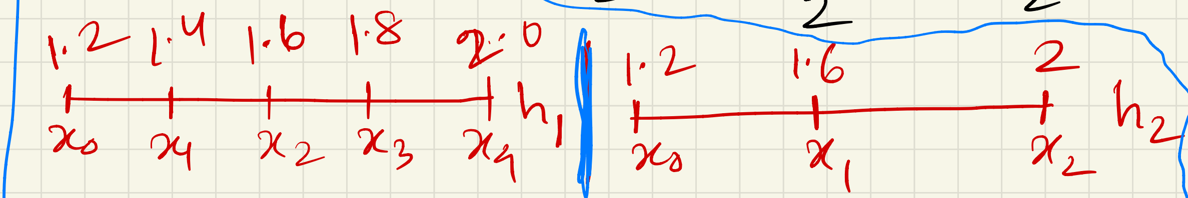
x	1.2	1.4	1.6	1.8	2
$f(x)$	3.728	4.124	4.525	5.123	5.626

Use Simpson's rule and Richardson's extrapolation to estimate $\int_{1.2}^{2} f(x)\,dx$.

we have, $n = 4$, $h_1 = \frac{2-1.2}{4}$

$= \frac{0.8}{4} = 0.2$

and, for $n = 2$, $h_2 = \frac{2-1.2}{2} = \frac{0.8}{2} = 0.4$



$$I(h_1) = \int_{x_0}^{x_4} f(x)\,dx = \frac{h}{3}\Big[f(x_0) + f(x_4) + 4f(x_1) + 2f(x_2) + 4f(x_3)\Big]$$

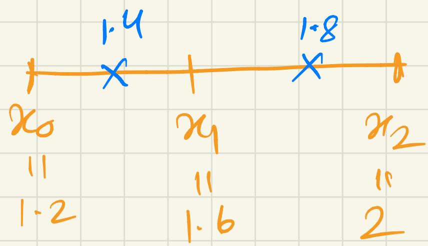
$$= \frac{0.2}{3}\Big[f(1.2) + f(2) + 4f(1.4) + 2f(1.6)$$

$$+ 4f(1.8)\Big]$$

$$= 0.066\Big[3.728 + 5.626 + 4 \times 4.124 + 2 \times 4.525 + 4 \times 5.123\Big]$$

$$= 0.066 \times 55.392$$

$$I(0.2) = 3.656.$$



$h = 0.4$

$[x_0, x_1]$, $[x_1, x_2]$

$n = 2$

$$I(h_2) = \int_{x_0}^{x_2} f(x)\,dx$$

$$= \frac{h_2}{3}\Big[f(x_0) + 4f(x_1) + f(x_2)\Big]$$

$$I(0.4) = \int_{1.2}^{2} f(x)\,dx = \frac{0.4}{3}\Big[f(1.2) + 4f(1.6) + f(2)\Big]$$

$$= 0.133\big(3.728 + 4 \times 4.525 + 5.626\big)$$

$= 0.133 \times 27.454$

$= 3.651$

<u>Extrapolation</u>: $r = \frac{h_2}{h_1} = \frac{0.4}{0.2} = 2, \quad n = 4$

$$I = \int_{1.2}^{2} f(x)\,dx = I(0.2) + \frac{I(0.2) - I(0.4)}{r^n - 1}$$

$$= 3.656 + \frac{3.656 - 3.651}{2^4 - 1}$$

$$= 3.656 + 0.00033$$

$$= 3.65633.$$

Homework – 3, 6