Lecture 5 (5F)

20.11.2023

4. A river is 50 m wide. The depth 'd' in meters at distance x m from one bank is given by the following table. Calculate the area of the cross-section of the river using Trapezoidal rule.

x	0	10	20	30	40	50
d	0	4	7	9	12	15

[* The area of the cross-section is the product of river's width multiplied by depth]

$\boxed{\text{say } d = f(x)}$

Area,

$$A = \int_0^{50} f(x)\,dx$$

$$= \frac{h}{2}\left[f(0) + f(50) + 2\{f(10) + f(20) + f(30) + f(40)\}\right]$$

$$= \frac{10}{2}\left[0 + 15 + 2(4 + 7 + 9 + 12)\right]$$

$$= 5\left[15 + 2 \times 32\right]$$

$$= 395$$

Homework : 5

Romberg Integration:

$$I^{(1)}\left(2n, \frac{h}{2}\right) = I^{(0)}\left(2n, \frac{h}{2}\right) + \frac{I^{(0)}\left(2n, \frac{h}{2}\right) - I^{(0)}(n, h)}{2^2 - 1};$$

$O(h^4)$

$$I^{(2)}\left(4n, \frac{h}{4}\right) = I^{(1)}\left(4n, \frac{h}{4}\right) + \frac{I^{(1)}\left(4n, \frac{h}{4}\right) - I^{(1)}\left(2n, \frac{h}{2}\right)}{2^4 - 1}$$

$O(h^6)$

Exercise

7.

x	1	1.2	1.4	1.6	1.8
$f(x)$	1.831	2.592	3.515	4.643	5.926

Evaluate $\int_1^{1.8} f(x)\,dx$ using Trapezoidal rule with 1, 2 and 4 subintervals. Improve your results using Romberg integration.

Using Trapezoidal rule, we have

Note:
$$h = \frac{b-a}{n}$$

for subinterval, $n=1$,

$$h = \frac{1.8 - 1.0}{1} = 0.8$$

So, $(1.0, 1.8)$.

Note
$$I^{(0)}(n,h) = \frac{h}{2}[f(x_0) + f(x_n)]$$

$$I^{(0)}(1, 0.8) = \frac{0.8}{2}[f(1) + f(1.8)]$$

$$= 0.4[1.831 + 5.926]$$

$$= 3.1028$$

For $n=2$, $h = \frac{1.8-1.0}{2} = \frac{0.8}{2} = 0.4$

data points $(1, 1.4)$, $(1.4, 1.8)$.

$$I^{(0)}(2, 0.4) = \frac{0.4}{2}[f(1) + f(1.8) + 2f(1.4)]$$

$$= 0.2[1.831 + 5.926 + 2 \times 3.515]$$

$I^{(0)}(n,h)$

$$= 2.9574$$

For $n=4$, $h = \frac{1.8-1.0}{4} = \frac{0.8}{4} = 0.2$

subintervals: $(1, 1.2), (1.2, 1.4), (1.4, 1.6), (1.6, 1.8)$

$$I^{(0)}(4, 0.2) = \frac{0.2}{2}\left[f(1) + f(1.8) + 2\left\{f(1.2) + f(1.4) + f(1.6)\right\}\right]$$

$$= 0.1\left[1.831 + 5.926 + 2(2.592 + 3.515 + 4.643)\right]$$

$$= 2.9257.$$

First order extrapolated values are

$$I^{(1)}\left(2n, \frac{h}{2}\right) = I^{(0)}\left(2n, \frac{h}{2}\right) + \frac{I^{(0)}\left(2n, \frac{h}{2}\right) - I^{(0)}(n, h)}{2^2 - 1}$$

For $n=1$,

$$I^{(1)}(2, 0.4) = I^{(0)}(2, 0.4) + \frac{I^{(0)}(2, 0.4) - I^{(0)}(1, 0.8)}{3}$$

$\boxed{\text{For } n=1}$

$$= 2.9574 + \frac{2.9574 - 3.1028}{3}$$

$$= 2.9089.$$

For $n=2$,

$$I^{(1)}(4,0\cdot 2) = I^{(0)}(4,0\cdot 2) + \frac{I^{(0)}(4,0\cdot 2) - I^{(0)}(2,0\cdot 4)}{2^2-1}$$

$$= 2\cdot 9257 + \frac{2\cdot 9257 - 2\cdot 9574}{3}$$

$$= 2\cdot 9151$$

Second order extrapolated value

$$I^{(2)}\left(4n, \frac{h}{4}\right) = I^{(1)}\left(4n, \frac{h}{4}\right) + \frac{I^{(1)}\left(4n, \frac{h}{4}\right) - I^{(1)}\left(2n, \frac{h}{2}\right)}{2^4-1}$$

For $n=1$,

$$I^{(2)}(4,0\cdot 2) = I^{(1)}(4,0\cdot 2) + \frac{I^{(1)}(4,0\cdot 2) - I^{(1)}(2,0\cdot 4)}{15}$$

$$= 2\cdot 9151 + \frac{2\cdot 9151 - 2\cdot 9089}{15}$$

$$= 2\cdot 9155$$

n	h	$O(h^2)$	$O(h^4)$	$O(h^6)$
1	0.8	3.1028		
2	0.4	2.9574	2.9089	
4	0.2	2.9257	2.9151	2.9155

; $O(h^6)$

$$\boxed{\text{Homework : 8}}$$

Numerical Evaluation of Double Integrals:

$$\iint_R f(x,y)\,dA = \int_a^b \left(\int_{f_1(x)}^{f_2(x)} f(x,y)\,dy \right) dx$$

or,
$$\iint_R f(x,y)\,dA = \int_c^d \left(\int_{f_1(y)}^{f_2(y)} f(x,y)\,dx \right) dy$$

Exercise

9. (b) $\int_{1.4}^{2.0}\int_{1.0}^{1.5} \ln(2x+y)\,dx\,dy$

Use Simpson's rule with two subintervals evaluate the above double integral.

We have, the integral is over the rectangular region bounded by $1.0 \le x \le 1.5$ and $1.4 \le y \le 2.0$.

Using 2-subintervals,

$$h = \frac{1.5-1.0}{2} = 0.25$$

$$K = \frac{2.0-1.4}{2} = \frac{0.6}{2} = 0.3$$

$\longrightarrow$ Remaining we will do on next class

<u>Quiz</u> : Lecture 1 to Lecture 4

on Friday.