Exercise - 7

1. a) We know, 2-Points forward difference formula,

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$$

$t = 0$

$h = \frac{0.5-0}{1} = 0.5$

$$V(0) = S'(0) = \frac{S(x_0+h) - S(x_0)}{h}$$

$$= \frac{S(0+0.5) - S(0)}{0.5}$$

$$= \frac{S(0.5) - S(0)}{0.5}$$

$$= \frac{3.65 - 0}{0.5}$$

$$= 7.3\ ms^{-1}$$

Again,

we know, 2-points backward difference formula,

$$f'(x_0) = \frac{f(x_0) - f(x_0-h)}{h}$$

$t = 2$

$h = \frac{2-1.5}{1} = 0.5$

$$V(2) = S'(2) = \frac{S(x_0) - S(x_0-h)}{h}$$

$$= \frac{S(2) - S(2-0.5)}{0.5}$$

$$= \frac{S(2) - S(1.5)}{0.5}$$

$$= \frac{12.15 - 9.9}{0.5} = 4.5\ ms^{-1}$$

(Ans:)

b) We know, three points central difference formula for first derivative,

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$t = 0.5$

$h = \frac{1-0}{2} = 0.5$

$$v(0.5) = s'(0.5) = \frac{s(x_0+h) - s(x_0-h)}{2h}$$

$$= \frac{s(0.5+0.5) - s(0.5-0.5)}{2\times 0.5}$$

$$= \frac{s(1) - s(0)}{1}$$

$$= \frac{6.8 - 0}{1}$$

$$= 6.8\ ms^{-1}$$

at, $t = 1.25$

$$h = \frac{1.5-1}{2} = 0.25$$

$$v(1.25) = s'(1.25) = \frac{s(x_0+h) - s(x_0-h)}{2h}$$

$$= \frac{s(1.25+0.25) - s(1.25-0.25)}{2\times 0.25}$$

$$= \frac{s(1.5) - s(1)}{0.5}$$

$$= \frac{9.9 - 6.8}{0.5}$$

$$= 6.2\ ms^{-1}$$

(Ans)

c) We know, three points central ~~derivative~~ difference formula for second derivative,

$$f''(x_0) = \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}$$

$t = 1$

$h = \frac{1.5-0.5}{2}$

$= 0.5$

$$a(1) = s''(1) = \frac{s(x_0-h) - 2s(x_0) + s(x_0+h)}{h^2}$$

$$= \frac{s(1-0.5) - 2s(1) + s(1+0.5)}{(0.5)^2}$$

$$= \frac{s(0.5) - 2s(1) + s(1.5)}{0.25}$$

$$= \frac{3.65 - (2\times 6.8) + 9.9}{0.25}$$

$$= -0.2\ ms^{-2}$$ (Ans)

$t = 1.5,\ h = \frac{2-1}{2} = 0.5$

$$f' = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$$= \frac{f(1.5+0.5) - f(1.5-0.5)}{2h}$$

$$= \frac{f(2) - f(1)}{2h}$$

$$= \frac{s(5) - s(3)}{2h}$$

$$f'' = \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}$$

$$= \frac{f(1.5-0.5) - 2f(1.5) + f(1.5+0.5)}{h^2}$$

$$= \frac{f(1) - 2f(1.5) + f(2)}{h^2}$$

$$= \frac{s(3) - 2s(4) + s(5)}{h^2}$$

d)

```
>> clear
>> t = [0  0.5  1  1.5  2];
>> s = [0  3.65  6.8  9.9  12.15];
>> h = t(4) - t(3);
>> D1 = (s(5) - s(3)) / (2*h)
>> D2 = (s(3) - 2*s(4) + s(5)) / (h^2)
```

(Ans)

3.a) We know, three points central difference formula for first derivative,

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$t = 1$

$h = \frac{1.5-0.5}{2}$

$= 0.5$

$$a(1) = v'(1) = \frac{v(x_0+h) - v(x_0-h)}{2h}$$

$$= \frac{v(1+0.5) - v(1-0.5)}{2\times 0.5}$$

$$= \frac{v(1.5) - v(0.5)}{1}$$

$$= \frac{41.075 - 11.860}{1}$$

$$= 29.215 \ ms^{-2}$$

at $t = 1.75$

$$h = \frac{2-1.5}{2} = 0.25$$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$$a(1.75) = v'(1.75) = \frac{v(x_0+h) - v(x_0-h)}{2h}$$

$$= \frac{v(1.75+0.25) - v(1.75-0.25)}{2\times 0.25}$$

$$= \frac{v(2) - v(1.5)}{0.5}$$

$$= \frac{59.05 - 41.075}{0.5}$$

$$= 35.95 \ ms^{-2}$$

(Ans)

b) $t = 2$

$h_1 = 2 - 1.5 = 0.5$

$h_2 = 2 - 1 = 1$

Now,

$$a(t, h_1) = V'(t, h_1) = \frac{V(t) - V(t - h_1)}{h_1}$$

$$a(2, 0.5) = \frac{V(2) - V(2 - 0.5)}{0.5}$$

$$= \frac{59.05 - V(1.5)}{0.5}$$

$$= \frac{59.05 - 41.075}{0.5}$$

$$= 35.95 \text{ ms}^{-2}$$

Again,

$$a(t, h_2) = V'(t, h_2) = \frac{V(t) - V(t - h_2)}{h_2}$$

$$a(2, 1) = \frac{V(2) - V(2 - 1)}{1}$$

$$= \frac{59.05 - V(1)}{1}$$

$$= 59.05 - 26.335$$

$$= 32.715 \text{ ms}^{-2}$$

Richardson extrapolation,

$$r = \frac{h_2}{h_1} = \frac{1}{0.5} = 2$$

$$n = 1$$

Now,

$$a_R(2) = a(2, 0.5) + \frac{a(2, 0.5) - a(2, 1)}{r^n - 1}$$

$$= 35.95 + \frac{35.95 - 32.715}{2^1 - 1}$$

$$= 39.185 \text{ ms}^{-2}$$

(Ans:)

c) $t = 1$

$h_1 = 1 - 0.5 = 0.5$

$h_2 = 1 - 0 = 1$

Now,

$$a(t, h_1) = v'(t, h_1) = \frac{v(t + h_1) - v(t - h_1)}{2h_1}$$

$$a(1, 0.5) = \frac{v(1 + 0.5) - v(1 - 0.5)}{2 \times 0.5}$$

$$= \frac{v(1.5) - v(0.5)}{1}$$

$$= \frac{41.075 - 11.860}{1}$$

$$= 29.215 \text{ ms}^{-2}$$

again,

$$a(t, h_2) = v'(t, h_2) = \frac{v(t+h_2) - v(t-h_2)}{2h_2}$$

$$a(1,1) = \frac{v(1+1) - v(1-1)}{2\times1}$$

$$= \frac{v(2) - v(0)}{2}$$

$$= \frac{59.05 - 0}{2}$$

$$= 29.525 \ ms^{-2}$$

extrapolation:

$$r = \frac{h_2}{h_1} = \frac{1}{0.5} = 2$$

$$n = 2$$

now,

$$a_R(1) = a(1, 0.5) + \frac{a(1,0.5) - a(1,1)}{r^n - 1}$$

$$= 29.215 + \frac{29.215 - 29.525}{2^2 - 1}$$

$$= 29.11 \ ms^{-2}$$

(Ans:)

d)

```
>> clear all
>> x = [0  0.5  1  1.5  2];
>> y = [0  11.860  26.335  41.075  59.05];
>> V = spline (x,y)
>> a = fnder (V, 1)
>> x1 = [0.5  1.25  2];
>> aval = fnval (a, x1)
```

(Ans 2)

4.a) $t = 1.2$

$h_1 = 1.2 - 1.1 = 0.1$

$h_2 = 1.2 - 1.0 = 0.2$

$$\frac{di}{dt}(t, h_1) = i'(t, h_1) = \frac{i(t+h_1) - i(t-h_1)}{2h_1}$$

$$\frac{di}{dt}(1.2, 0.1) = \frac{i(1.2+0.1) - i(1.2-0.1)}{2 \times 0.1}$$

$$= \frac{i(1.3) - i(1.1)}{0.2}$$

$$= \frac{4.5260 - 7.2428}{0.2}$$

$$= -13.584$$

Again, $\frac{di}{dt}(t, h_2) = i'(t, h_2) = \frac{i(t+h_2) - i(t-h_2)}{2h_2}$

$$\frac{di}{dt}(1.2, 0.2) = \frac{i(1.2+0.2) - i(1.2-0.2)}{2 \times 0.2}$$

$$= \frac{i(1.4) - i(1)}{0.4}$$

$$= \frac{2.9122 - 8.2277}{0.4}$$

$$= -13.28875$$

extrapolation:

$$\frac{di}{dt}(1.2) = \frac{di}{dt}(1.2, 0.1) + \frac{\frac{di}{dt}(1.2, 0.1) - \frac{di}{dt}(1.2, 0.2)}{r^n - 1}$$

$r = \frac{h_2}{h_1} = \frac{0.2}{0.1} = 2$

$n = 2$

$$= -13.584 + \frac{(-13.584) + 13.28875}{2^2 - 1}$$

$$= -13.682$$

Now,

$L = 0.05$

$R = 2$

$$E(1.2) = L\frac{di}{dt}(1.2) + Ri(1.2)$$

$$= 0.05 \times (-13.682) + 2 \times 5.9908$$

$$= 11.2975$$

(Ans.)

b) given, $i(t) = 10\, e^{(-t/10)} \sin 2t$

$$\frac{di}{dt}(t) = 10\left[e^{-\frac{t}{10}} \sin 2t\right]$$

$$= 10\left[e^{-\frac{t}{10}} \cdot 2\cos 2t + \sin 2t \left(-\frac{1}{10} e^{-\frac{t}{10}}\right)\right]$$

$$= 20e^{-\frac{t}{10}} \cos 2t - e^{-\frac{t}{10}} \sin 2t$$

$$\therefore \frac{di}{dt}(1.2) = 20e^{-\frac{1.2}{10}} \cos(2\times 1.2) - e^{-\frac{1.2}{10}} \sin(2\times 1.2)$$

$$= -13.679$$

$$\text{Error} = \left|-13.679 - (-13.682)\right|$$

$$= 0.003 \quad \text{(Ans:)}$$

c)

```
>> clear
>> x = [1.0  1.1  1.2  1.3  1.4];
>> y = [8.2277  7.2428  5.9908  4.5260  2.9122];
>> I = spline (x,y)
>> DI = fnder (I,x)
>> R = 2;
>> L = 0.05;
>> Et = L*DI + R*I
```

(Ans)

5.a) (i) We know, three points central difference formula for first derivative.

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$t = 9$

$h = \frac{10-8}{2} = 1$

$$V(9) = S'(9) = \frac{S(t_0+h) - S(t_0-h)}{2h}$$

$$= \frac{S(9+1) - S(9-1)}{2\times 1}$$

$$= \frac{S(10) - S(8)}{2}$$

$$= \frac{25.752 - 17.453}{2}$$

$$= 4.1495 \ ms^{-1}$$

(ii) $t = 10.5$

$h = \frac{11-10}{2} = 0.5$

$$V(10.5) = S'(10.5) = \frac{S(10.5+0.5) - S(10.5-0.5)}{2\times 0.5}$$

$$= \frac{S(11) - S(10)}{1}$$

$$= \frac{30.302 - 25.752}{1}$$

$$= 4.55 \ ms^{-1}$$

b) (i) For, $t = 8$,

Two points forward difference formula,

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$$

$$h = \frac{9-8}{1} = 1$$

$$V(8) = s'(8) = \frac{s(8+1) - s(8)}{1}$$

$$= \frac{s(9) - s(8)}{1}$$

$$= \frac{21.460 - 17.453}{1}$$

$$= 4.007 \ ms^{-1}$$

(ii) For, $t = 12$

Two points backward difference formula,

$$f'(x_0) = \frac{f(x_0) - f(x_0-h)}{h}$$

$$h = \frac{12-11}{1} = 1$$

$$V(12) = s'(12) = \frac{s(12) - s(12-1)}{1}$$

$$= \frac{s(12) - s(11)}{1}$$

$$= 35.084 - 30.302$$

$$= 4.782 \ ms^{-1}$$

(Ans)

c) $t = 10$

$h_1 = 10 - 9 = 1$

$h_2 = 10 - 8 = 2$

$$v(t, h_1) = s'(t, h_1) = \frac{s(t + h_1) - s(t - h_1)}{2h_1}$$

$$v(10, 1) = \frac{s(10 + 1) - s(10 - 1)}{2 \times 1}$$

$$= \frac{s(11) - s(9)}{2}$$

$$= \frac{30.302 - 21.460}{2}$$

$$= 4.421 \ ms^{-1}$$

Again, $$v(t, h_2) = s'(t, h_2) = \frac{s(t + h_2) - s(t - h_2)}{2h_2}$$

$$v(10, 2) = \frac{s(10+2) - s(10-2)}{2 \times 2}$$

$$= \frac{s(12) - s(8)}{4}$$

$$= \frac{35.084 - 17.453}{4}$$

$$= 4.408$$

extrapolation:

$$r = \frac{h_2}{h_1} = \frac{2}{1} = 2$$

$$n = 2$$

$$V_R(1) = V(10,1) + \frac{V(10,1) - V(10,2)}{r^n - 1}$$

$$= 4.421 + \frac{4.421 - 4.408}{2^2 - 1}$$

$$= 4.425 \ ms^{-1}$$ (Ans)

(ii)

d) for $t = 10$

$$h = \frac{11-9}{2} = 1$$

we know

$$f''(x_0) = \frac{f(x_0 - h) - 2f(x_0) + f(x_0 + h)}{h^2}$$

$$a(10) = s'(10) = \frac{s(10-1) - 2s(10) + s(10+1)}{1^2}$$

$$= \frac{s(9) - 2s(10) + s(11)}{1}$$

$$= \frac{21.460 - 2 \times 25.752 + 30.302}{1}$$

$$= 0.258 \ ms^{-2}$$

(ii) for, $t = 8$

$h = \frac{10-8}{2} = 1$

$$f''(x_0) = \frac{f(x_0) - 2f(x_0+h) + f(x_0+2h)}{h^2}$$

$$a(8) = s'(8) = \frac{s(8) - 2s(8+1) + s(8+2)}{1^2}$$

$$= \frac{s(8) - 2s(9) + s(10)}{1}$$

$$= 17.453 - 2 \times 21.460 + 25.752$$

$$= 0.285 \ ms^{-2}$$

(iii) for, $t = 12$

$h = \frac{12-10}{2} = 1$

$$f''(x_0) = \frac{f(x_0) - 2f(x_0-h) + f(x_0-2h)}{h^2}$$

$$a(12) = s'(12) = \frac{s(12) - 2s(12-1) + s(12-2)}{1^2}$$

$$= \frac{s(12) - 2s(11) + s(10)}{1}$$

$$= 35.084 - 2 \times 30.302 + 25.752$$

$$= 0.232 \ ms^{-2}$$

(Ans:)

e)

```
>> clear all
>> x = [8 9 10 11 12];
>> y = [17.453 21.460 25.752 30.302 35.084];
>> v = spline(x,y)
>> a = fnder(v,1)
>> x1 = [8.5 10.5 11.2];
>> aval = fnval(a,x1)
```

(Ans:)

6.a) for, $f'(1)$

$$h = \frac{1.2-0.8}{2} = 0.2$$

$$f'(1) = \frac{f(1.2) - f(0.8)}{2h} = \frac{2.623 - 0.954}{0.4} = 4.1725$$

for, $f'(1.3)$

$$h = \frac{1.4-1.2}{2} = 0.1$$

$$f'(1.3) = \frac{f(1.4) - f(1.2)}{2h} = \frac{3.947 - 2.623}{0.2}$$

$$= 6.62$$

(Ans)

b) $\cancel{x} = 0.8$

$h_1 = 1 - 0.8 = 0.2$

$h_2 = 1.2 - 0.8 = 0.4$

$$f'(\cancel{x}, h_1) = \frac{f(x + h_1) - f(x)}{h_1}$$

$$f'(0.8, 0.2) = \frac{f(0.8 + 0.2) - f(0.8)}{0.2}$$

$$= \frac{f(1) - f(0.8)}{0.2}$$

$$= \frac{1.648 - 0.954}{0.2}$$

$$= 3.47$$

$$f'(x, h_2) = \frac{f(x + h_2) - f(x)}{h_2}$$

$$f'(0.8, 0.4) = \frac{f(0.8 + 0.4) - f(0.8)}{0.4}$$

$$= \frac{f(1.2) - f(0.8)}{0.4}$$

$$= \frac{2.623 - 0.954}{0.4}$$

$$= 4.1725$$

extrapolation:

$$r = \frac{h_2}{h_1} = \frac{0.4}{0.2} = 2$$

$$n = 1$$

$$f'(0.8) = 3.47 + \frac{3.47 - 4.1725}{2^1 - 1}$$

$$= 2.7675 \quad \text{(Ans)}$$

e) $x = 1.2$

$$h_1 = 1.2 - 1.0 = 0.2$$

$$h_2 = 1.2 - 0.8 = 0.4$$

$$f'(x, h_1) = \frac{f(x_0 + h_1) - f(x - h_1)}{2h_1}$$

$$f'(1.2, 0.2) = \frac{f(1.2 + 0.2) - f(1.2 - 0.2)}{2 \times 0.2}$$

$$= \frac{f(1.4) - f(1)}{0.4}$$

$$= \frac{3.947 - 1.648}{0.4}$$

$$= 5.7475$$

$$f'(x, h_2) = \frac{f(x+h_2) - f(x-h_2)}{2h_2}$$

$$f'(1.2,\ 0.4) = \frac{f(1.2+0.4) - f(1.2-0.4)}{2 \times 0.4}$$

$$= \frac{f(1.6) - f(0.8)}{0.8}$$

$$= \frac{5.697 - 0.954}{0.8}$$

$$= 5.92875$$

extrapolation:

$$p = \frac{h_2}{h_1} = \frac{0.4}{0.2} = 2$$

$$n = 2$$

$$f'(1.2) = 5.7475 + \frac{5.7475 - 5.92875}{2^2 - 1}$$

$$= 5.687$$

(Ans:)

d) $f''(x_0) = \dfrac{f(x_0) - 2f(x_0 + h) + f(x_0 - 2h)}{h^2}$

$x = 1.6$

$h = \dfrac{1.6 - 1.2}{2} = 0.2$

$f''(1.6) = \dfrac{f(1.6) - 2f(1.6 - 0.2) + f(1.6 - 0.4)}{(0.2)^2}$

$= \dfrac{5.697 - 2 \times 3.947 + 2.623}{0.04}$

$= 10.65$ (Ans)

e)

```
>>clear
>> x = [0.8   1.0   1.2   1.4   1.6];
>>y = [0.954   1.648   2.623   3.947   5.697];
>> sp = spline(x,y)
>>a = fnder (sp,
>> x0 = [0.9   1.1   1.42];
>> aval = fnval (a, x0)
```

(Ans)