Lecture No. 6 (5F) 22.11.2023

9. (b) $\int_{1.4}^{2} \int_{1}^{1.5} \ln(2x+y)\,dx\,dy$

(See previous Lecture 05)

step

$x \rightarrow 1 \le x \le 1.5 \rightarrow h$

$y \rightarrow 1.4 \le y \le 2 \rightarrow k$

Using 2 subintervals in each direction, we have,

step size on x direction,

$$h = \frac{1.5-1}{2} = 0.25$$

step size on y direction,

$$k = \frac{2-1.4}{2} = 0.3$$

Note

$$\text{step size, } h = \frac{b-a}{n}$$

x_n = Terminal point

x_0 = initial point

number of subintervals

So, x : 1 1.25 1.5

y : 1.4 1.7 2

Let, $f(x,y) = \ln(2x+y)$

Integration using fixed values of $y = 1.4, 1.7$ and 2, are

$$I(1.4) = \int_{1}^{1.5} f(x, 1.4)\, dx$$

$$= \frac{0.25}{3}\left[f(1, 1.4) + 4f(1.25, 1.4) + f(1.5, 1.4) \right]$$

$$= \frac{0.25}{3}\left[1.223 + 5.443 + 1.48 \right]$$

$$= 0.678$$

$$I(1.7) = \int_1^{1.5} f(x, 1.7)\,dx$$

$$= \frac{0.25}{3}\left[f(1, 1.7) + 4f(1.25, 1.7) + f(1.5, 1.7)\right]$$

$$= \frac{0.25}{3}\left[1.308 + 5.74 + 1.547\right]$$

$$= 0.716$$

$$I(2) = \int_1^{1.5} f(x, 2)\,dx$$

$$= \frac{0.25}{3}\left[f(1, 2) + 4f(1.25, 2) + f(1.5, 2)\right]$$

$$= \frac{0.25}{3}\left[1.386 + 6.016 + 1.609\right]$$

$$= 0.751$$

Finally, combining the integral for y,

$$I(\text{simp}) = \int_{1.4}^{2}\int_{1}^{1.5} f(x,y)\,dx\,dy$$

$$= \frac{0.3}{3}\left[I(1.4) + 4I(1.7) + I(2)\right]$$

$$= 0.1\left[0.678 + 4\times 0.716 + 0.751\right]$$

$$= 0.429$$

Ans

Homework : 9 (a, c, d, e, f).

8.7. ** Using Simpson's rule with 2-subintervals evaluate the double integral,

$$\int_{1}^{1.4}\int_{x}^{1+x^2} (1+xy)\, dy\, dx$$

Soln: Region of integration:

$$1 \le x \le 1.4$$

$$x \le y \le 1+x^2$$

Let, $f(x,y) = 1+xy$

step size for x, $h = \dfrac{1.4-1}{2} = 0.2$

The x data points: 1, 1.2 and 1.4.

The length of y depends on x, as

For $x=1$, $1 \le y \le 2$

step size for y, $k = \dfrac{2-1}{2} = 0.5$

y data points: 1, 1.5, 2.

$f(x,y) = 1+xy$

$$I(1) = \int_1^2 f(1,y)\,dy$$

$$= \frac{0.5}{3}\left[f(1,1) + 4f(1,1.5) + f(1,2)\right]$$

$$= 0.166\left[2 + 4\times 2.5 + 3\right]$$

$$= 2.49$$

For $x = 1.2$,

$$1.2 \le y \le 1+(1.2)^2$$

or, $1.2 \le y \le 2.44$; $k = \frac{2.44-1.2}{2} = 0.62$

$y : 1.2,\ 1.82,\ 2.44$

$$I(1.2) = \int_{1.2}^{2.44} f(1.2, y)\,dy$$

$$= \frac{0.62}{3}\left[f(1.2,1.2) + 4f(1.2,1.82) + f(1.2,2.44)\right]$$

$$= 0.206\left[2.44 + 12.736 + 3.928\right]$$

$$= 3.935$$

For $x = 1.4$, $\quad 1.4 \le y \le 1 + (1.4)^2$

or, $1.4 \le y \le 2.96$

step size, $k = \dfrac{2.96 - 1.4}{2} = 0.78$

y : $1.4, \ 2.18, \ 2.96$

$$I(1.4) = \int_{1.4}^{2.96} f(1.4, y)\, dy$$

$$= \frac{0.78}{3}\Big[f(1.4, 1.4) + 4 f(1.4, 2.18) + f(1.4, 2.96) \Big]$$

$$= 6.3211$$

Combining the integrals for x, we have

$$\int_1^{1.4} \int_x^{1+x^2} f(x,y)\, dy\, dx = \frac{0.2}{3}\Big[I(1) + 4 I(1.2) + I(1.4) \Big]$$

$$= \frac{0.2}{3}\left[2.49 + 4 \times 3.935 + 6.3211\right]$$

$$= 1.636.$$ Ans

MATLAB.

Command:

For single integral: integral(function, xmin, xmax)

For double integral:

integral2(function, xmin, xmax, ymin, ymax)

For triple integral,

integral3(function, xmin, xmax, ymin, ymax, zmin, zmax).

Exercise 8 (MATLAB part):

(a) $$\int_0^1 \sqrt{1+2x^3}\, dx$$

MATLAB Code:

```
>> clear all
>> F = @(x) sqrt(1+2*x^3);
>> I = integral(F, 0, 1)
```

Exercise 9 (MATLAB)

(a) $$\int_1^{1.5}\int_0^1 \left(x^2+2\sqrt{y}\right) dy\, dx$$

MATLAB Code:

```
>> clear all
>> F = @(x,y) x^2+2*sqrt(y);
>> I = integral2(F, 1, 1.5, 0, 1).
```

Example 8.8

$$\int_0^2 \int_{-x}^{\sqrt{x}} \frac{1}{\sqrt{x^2+y^2}} \, dy \, dx$$

MATLAB code

```
>> clear all
>> F = @(x,y) 1/sqrt(x^2+y^2);
>> ymin = @(x) -x;
>> ymax = @(x) sqrt(x);
>> I = integral2(F, 0, 2, ymin,
                 ymax)
```