Lecture 9 (5F) 04.12.2023

5(d) Use MATLAB function
"sol = bvp4c(odefun, bcfun, solinit)"
to solve the BVP:
(i) Estimate the values of y at
$x = 1/3$ and $2/3$.

(ii) plot the solution curve $y(x)$.

From (c):

$y_1' = y_2$

$y_2' = 2y_2 - 2y_1 + x$

$y_1(0) = 1$

$y_1(1) = 2$

Soln: MATLAB Code:

```
>> clear all
>> F = @(x,y) [y(2); 2*y(2)
                -2*y(1)+x];
>> bc = @(ya,yb) [ ya(1)-1 ; yb(1)-2];
>> yinit = @(x) [0;1] ;
>> solinit = bvpinit(linspace(0,0.1, 1) );
>> sol = bvp4c(F, bc, solinit)
```

$\boxed{\text{i}}$

```
>> xint = [0  1/3  2/3  1]
>> Exint = deval(sol, xint)
```

(ii)

```
>> xval = linspace(0,1)
>> yval = deval(sol, xval);
>> plot(xval, yval(1,:))
```

Exercise

1. Given the initial value problem

$$y' = 2x^2 - y + 3y^2 \text{ with } y(2) = 0.5.$$

(a) solved. (b) solved (c) solved.

(d) (i) Use three point central difference formula for derivative to derive a recurrence relation for the above IVP.

Soln: We have, 3-point central difference formula:

$$(y')_n = (2x^2 - y + 3y^2)_n$$

or, $$\frac{y_{n+1} - y_{n-1}}{2h} = 2x_n^2 - y_n + 3y_n^2$$

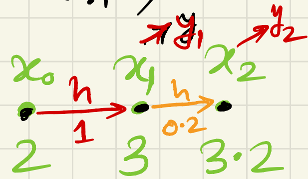
or, $$y_{n+1} - y_{n-1} = 2h\left(2x_n^2 - y_n + 3y_n^2\right)$$

(ii) Estimate the values of y at $x=3$ and 3.2 using the above recurrence relation.

Soln: Here, $f(x,y) = 2x^2 - y + 3y^2$

we have, the recurrence relation

$$y_{n+1} - y_{n-1} = 2h\left(2x_n^2 - y_n + 3y_n^2\right)$$



~~we~~ and, $x_0 = 2$, $x_1 = 3$, $x_2 = 3.2$ and

$y_0 = 0.5$, $h_1 = 1$ and $h_2 = 0.2$

For $n = 1$,

$$y_2 - y_0 = 2h\left(2x_1^2 - y_1 + 3y_1^2\right) \quad \text{(1)}$$

By Euler's method,

$$y_1 = y_0 + h_1 f(x_0, y_0)$$

$$= 0.5 + 1 f(2, 0.5)$$

$\therefore \; y(3) = 8.75$

Now, from eqn ①,

$$y_2 = y_0 + 2h_2\left(2x_1^2 - y_1 + 3y_1^2\right)$$

$$= 0.5 + 2\times 0.2\left(2(3)^2 - 8.75 + 3(8.75)^2\right)$$

$$y(3.2) = 96.075$$

(e) Use MATLAB function

"[x1,y1] = ode23(f,[x0,xn],y0)"

(i) to estimate the values of y in $1 \le x \le 4$ using $h = 0.2$.

(ii) to plot the solution curve in $[1,4]$

soln:

(i)

```
>> clear all
>> F = @(x,y) 2*x^2 - y + 3*y^2;
>> [x1,y1] = ode23(F,[1:0.2:4],0.5)
>> solution = [x1,y1]
```

(11)

```
>> xin = 1:0.2:4;
>> [x2, y2] = ode23(F, xin, 0.5)
>> plot (x2, y2)
```

2. Given the initial value problem

$$y' = 2x + x^2 y \quad \text{with } y(0) = -1$$

(a) Estimate the values of $y(0.2)$ using Runge-Kutta method of order 2.

Soln: Here, $f(x,y) = 2x + x^2 y$

$x_0 = 0$, $y_0 = -1$ and let, $h = 0.2$

$$K_1 = h f(x_0, y_0)$$
$$= 0.2\, f(0, -1)$$
$$= 0.2 \times 0 = 0$$

$$K_2 = h f(x_0 + h,\ y_0 + K_1)$$
$$= 0.2\, f(0.2, -1) = 0.072$$

$$\therefore y(0.2) = y_1 = y_0 + \frac{1}{2}[k_1 + k_2]$$

$$= -1 + \frac{1}{2}[0 + 0.072]$$

$$= -0.964$$

2(b). Estimate ~~the~~ value of $y(0.2)$ using Runge-Kutta method of order 4.

Soln: $f(x,y) = 2x + x^2 y$

$x_0 = 0, \quad y_0 = -1, \quad h = 0.2$

$$k_1 = hf(x_0, y_0) = 0.2 f(0, -1) = 0$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2 f(0.1, -1)$$
$$= 0.038$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.2 f(0.1, -0.981)$$
$$= 0.038038$$

$$k_4 = hf(x_0 + h, y_0 + k_3) = 0.2 f(0.2, -0.9619)$$
$$= 0.0723$$

$\therefore y(0.2) = y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= -1 + \frac{1}{6}[0 + 2 \times 0.038 + 2 \times 0.038038 + 0.0723]$

$= -0.9626$

2(c). (i) Use 3-point central difference formula for derivative to derive a recurrence relation for the above IVP.

Soln: Here, the IVP: $y' = 2x + x^2 y$

By 3-point central difference formula;

$$\frac{y_{n+1} - y_{n-1}}{2h} = 2x_n + x_n^2 y_n$$

$$\therefore y_{n+1} - y_{n-1} = 2h(2x_n + x_n^2 y_n)$$

(ii) Estimate the values of y at $x = 0.4$ and 0.6 using the above recurrence relation.

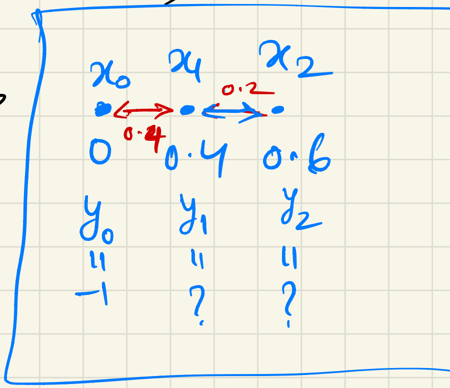
Soln: we have,

$$y_{n+1} - y_{n-1} = 2h(2x_n + x_n^2 y_n)$$

and, $x_0 = 0$, $x_1 = 0.4$, $x_2 = 0.6$

$y_0 = -1$ and $h_1 = 0.4$

and $h_2 = 0.2$



For $n = 1$,

$$y_2 - y_0 = 2h(2x_1 + x_1^2 y_1) \quad \text{——} \ (1)$$

By Euler's method,

$$y_1 = y(0.4) = y_0 + h_1 f(x_0, y_0)$$

$$= -1 + 0.4 f(0, -1)$$

$$= -1 + 0$$

$$= -1$$

From eq^n (I):

$$y_2 = y_0 + 2h_2(2x_1 + x_1^2 y_1)$$

$$= -1 + 2 \times 0.2\,(2 \times 0.4 + (0.4)^2 \times (-1))$$

$$= -1 + 0.256$$

$$y(0.6) = -0.744$$

3. Given the system
$\frac{dy}{dx} = y^2 + xz + x^2$ and $\frac{dz}{dx} = y + 2x + yz$
with initial conditions $y(1) = -1$ and $z(1) = 1$.

(a) Estimate $y(1.1)$ and $z(1.1)$ using Runge-Kutta method of order 2.

Soln: Here, $f_1(x, y, z) = y^2 + xz + x^2$

$$f_2(x, y, z) = y + 2x + yz$$

and, $x_0 = 1,\ y_0 = -1,\ z_0 = 1.$

Let, $h=0.1$

$$k_1 = hf_1(x_0, y_0, z_0)$$
$$= 0.1\, f_1(1, -1, 1)$$
$$= 0.1\,(1+1+1)$$
$$= 0.3$$

$$m_1 = hf_2(x_0, y_0, z_0)$$
$$= 0.1\, f_2(1, -1, 1)$$
$$= 0.1\,(-1+2-1)$$
$$= 0$$

$$k_2 = hf_1(x_0+h,\ y_0+k_1 + z_0+m_1)$$
$$= 0.1\, f_1(1.1,\ -0.7,\ 1)$$
$$= 0.1\,(0.49+1.1+1.21)$$
$$= 0.28$$

$$m_2 = 0.1\, f_2(1.1, -0.7, 1)$$
$$= 0.08$$

$$y(1.1) = y_1 = y_0 + \frac{1}{2}[k_1 + k_2]$$

$$= -1 + \frac{1}{2}[0.3 + 0.28]$$

$$= -1 + 0.29$$

$$= -0.71$$

$$z(1.1) = z_1 = z_0 + \frac{1}{2}[m_1 + m_2]$$

$$= 1 + \frac{1}{2}[0 + 0.08]$$

$$= 1.04$$