LESSON 7

BOOK CHAPTER 25

CAPACITANCE

A Spherical Capacitor:

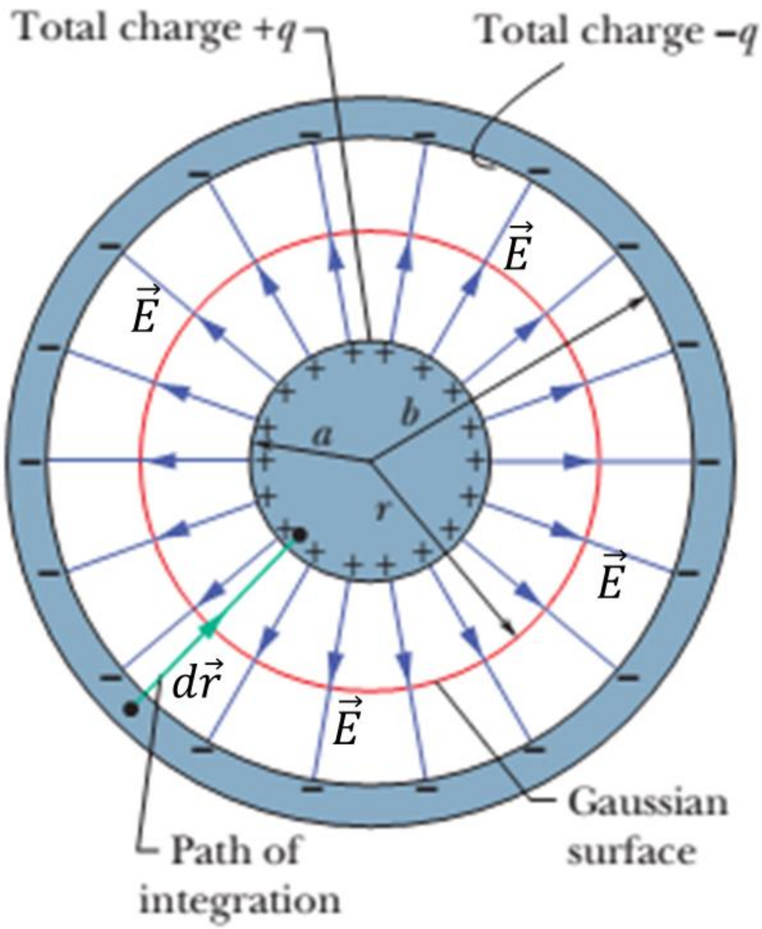


Figure : A central cross section of a capacitor that consists of two concentric spherical shells, of radii a and *b.*

The adjacent figure shows a central cross section of a spherical capacitor that consists of two concentric spherical shells, of radii *a* and *b*..

As a Gaussian surface we draw a sphere of radius *r* concentric with the two shells. Then

Applying Gauss' law $\varepsilon_0 EA = q$

$$\varepsilon_0 E(4\pi r^2) = q$$

Where , $A = 4\pi r^2$ is the area of the spherical Gaussian surface.

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

We know

$$V = \int_{-}^{+} E\, ds = -\int_{-}^{+} E\, dr \qquad \text{[since } ds = -dr\text{]}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{r=b}^{r=a}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\left|\frac{1}{r}\right|_{r=b}^{r=a}\right)$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q(b-a)}{4\pi\varepsilon_0\, ab}$$

$$\boxed{C = \frac{q}{V} = \frac{4\pi\varepsilon_0\, ab}{b-a}}$$

An isolated sphere:

We can assign a capacitance to a *single* isolated spherical conductor of radius *R* by assuming that the "missing plate" is a conducting sphere of infinite radius.

To find the capacitance of the conductor, we first rewrite $C = \frac{4\pi\varepsilon_0\, ab}{b-a}$ as

$$C = \frac{4\pi\varepsilon_0\, a}{1 - \frac{a}{b}}$$

[Dividing both numerator and denominator by b]

If we then let $b \rightarrow \infty$ (infinity), we get

$$C = \frac{4\pi\varepsilon_0\, a}{1 - 0} = 4\pi\varepsilon_0 a$$

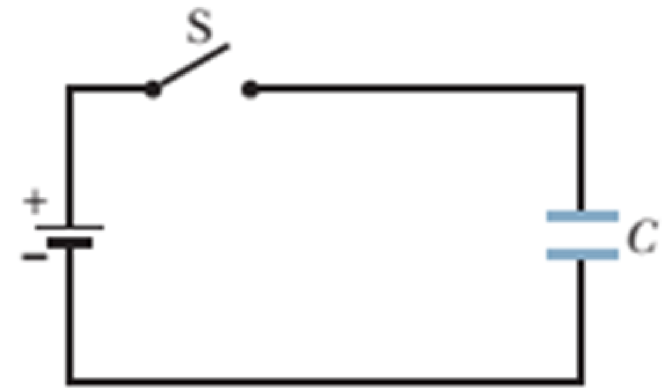
By substituting R for *a*, we get

$$\boxed{C = 4\pi\varepsilon_0 \mathrm{R}}$$

Which is the capacitance for isolated sphere.

Problem 2 (Book chapter 25):

The capacitor in the adjacent Fig. has a capacitance of $25\,\mu F$ and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?



Answer:

We know

$$q = CV$$

$$q = 25 \times 10^{-6} \times 120 = 3 \times 10^{-3}\ \text{C}$$

Given

$$C = 25\,\mu F = 25 \times 10^{-6} F$$

$$V = 120\,V \qquad q = ?$$

Problem 3 (Book chapter 25):

A parallel-plate capacitor has circular plates of 8.20 cm radius and 1.30 mm separation. (a) Calculate the capacitance. (b) Find the charge for a potential difference of 120 V.

Answer:

(a) We know

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 (\pi r^2)}{d}$$

Given

$$r = 8.20\ cm = 0.082\ m$$

$$d = 1.30\ mm = 1.3 \times 10^{-3}\ m$$

$$V = 120\ V$$

$$C = \frac{8.854 \times 10^{-12} \times 3.1416 \times (0.082)^2}{1.3 \times 10^{-3}} = 143.87 \times 10^{-12}\ F$$

(b) We know

$$q = CV = 143.87 \times 10^{-12} \times 120 = 17.26 \times 10^{-9}\ C$$

Problem 4 (Book chapter 25):

The plates of a spherical capacitor have radii 38.0 mm and 40.0 mm. (a) Calculate the capacitance. (b) What must be the plate area of a parallel-plate capacitor with the same plate separation and capacitance?

Answer:

(a) We know

Given

$$a = 38\,mm = 38 \times 10^{-3}\,m$$

$$b = 40\,mm = 40 \times 10^{-3}\,m$$

$$b - a = 2\,mm = 2 \times 10^{-3}\,m$$

$$C = \frac{4\pi\varepsilon_0\,ab}{b-a} = \frac{1}{9 \times 10^9}\,\frac{38 \times 40 \times 10^{-6}}{2 \times 10^{-3}}$$

$$C = 84.44 \times 10^{-12}\,F$$

(b) For a parallel plate capacitor, we know

$$C = \frac{\varepsilon_0 A}{d}$$

Here

$$d = b - a = 2\,mm = 2 \times 10^{-3}\,m$$

$$A = ?$$

$$A = \frac{Cd}{\varepsilon_0} = \frac{84.44 \times 10^{-12} \times 2 \times 10^{-3}}{8.854 \times 10^{-12}}$$

$$A = 19.074 \times 10^{-3}\,m^2$$

Problem 6 (Book chapter 25):

You have two flat metal plates, each of area 1.00 m^2, with which to construct a parallel-plate capacitor. (a) If the capacitance of the device is to be 1.00 F, what must be the separation between the plates? (b) Could this capacitor actually be constructed?

Answer:

Given

$$A = 1.00\, m^2$$

$$C = 1.00\, F$$

(a) $d = ?$

(b) Could this capacitor actually be constructed?

(a) We know

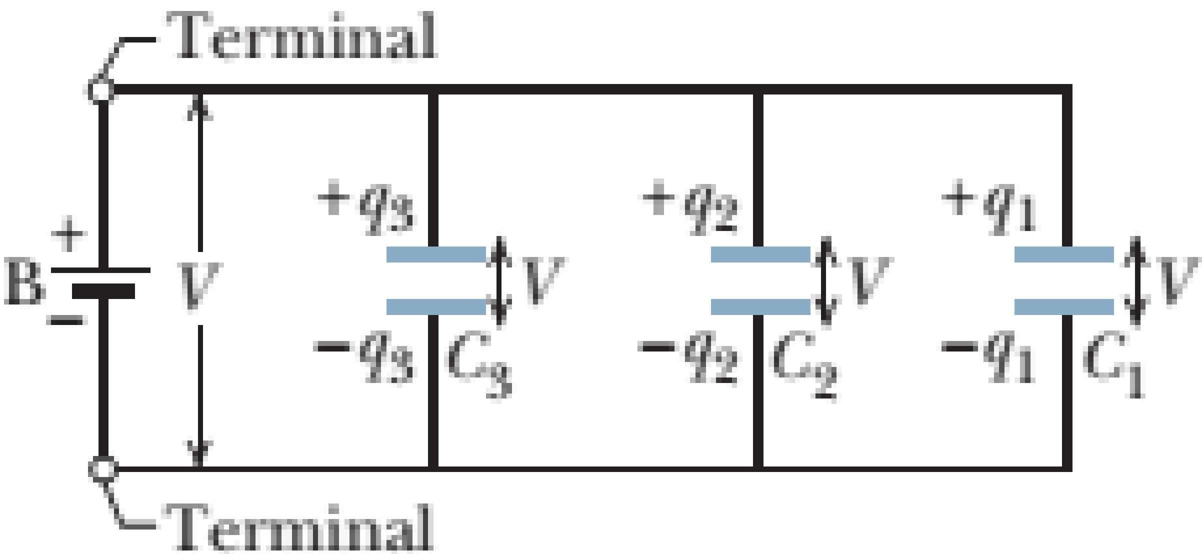
$$C = \frac{\varepsilon_0 A}{d}$$

$$d = \frac{\varepsilon_0 A}{C} = \frac{8.854 \times 10^{-12} \times 1}{1} = 8.854 \times 10^{-12}\, m$$

(b)

It is not possible to construct a capacitor by the separation distance, $d = 8.854 \times 10^{-12}\, m$, because d value is less than the minimum size of an atom.

Capacitors in parallel combination:



Charge on each capacitor:

$$q_1 = C_1 V$$

$$q_2 = C_2 V$$

$$q_3 = C_3 V$$

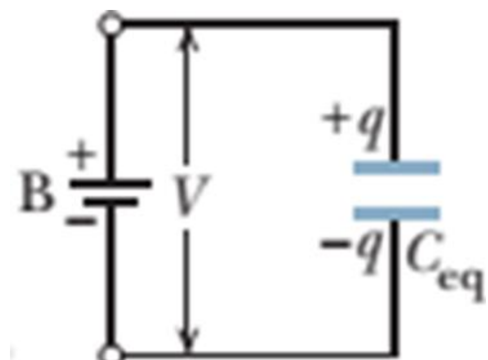
The total charge on the parallel combination is then

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

The equivalent capacitance, with the same total charge *q* and applied potential difference V as the combination, is then

$$C_{eq} = \frac{q}{V} = \frac{(C_1 + C_2 + C_3)V}{V} = C_1 + C_2 + C_3$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$



Thank You