LESSON 2

BOOK CHAPTER 4

Projectile Motion

Projectile Motion:

A particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the freefall acceleration $\vec{g}$, which is downward. Such a particle is called a **projectile** (meaning that it is projected or launched), and its motion is called **projectile motion**.

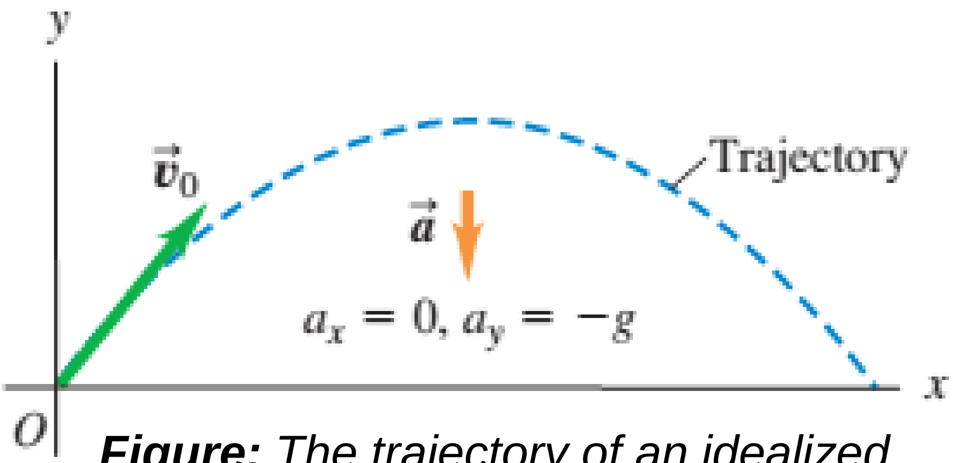
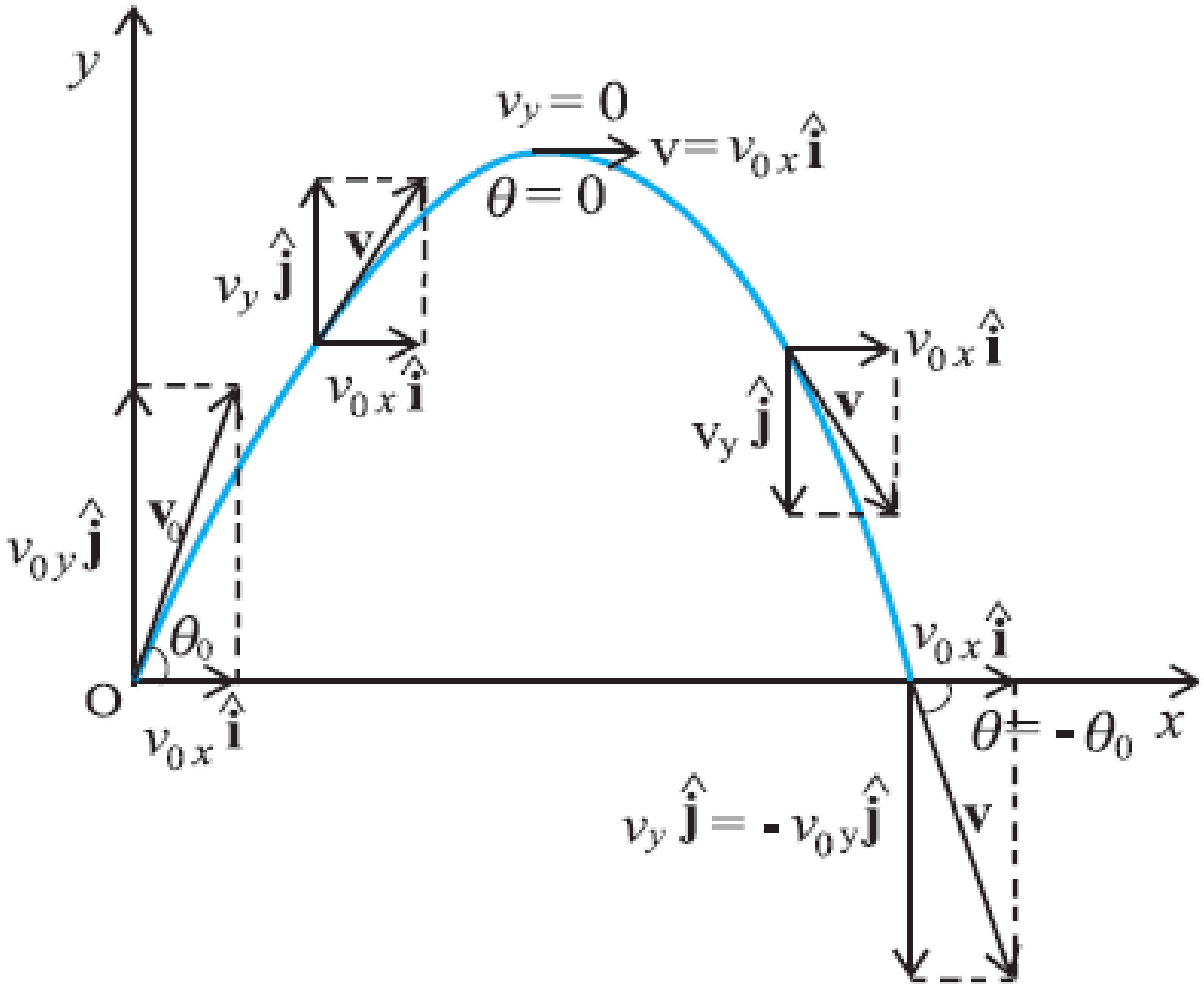


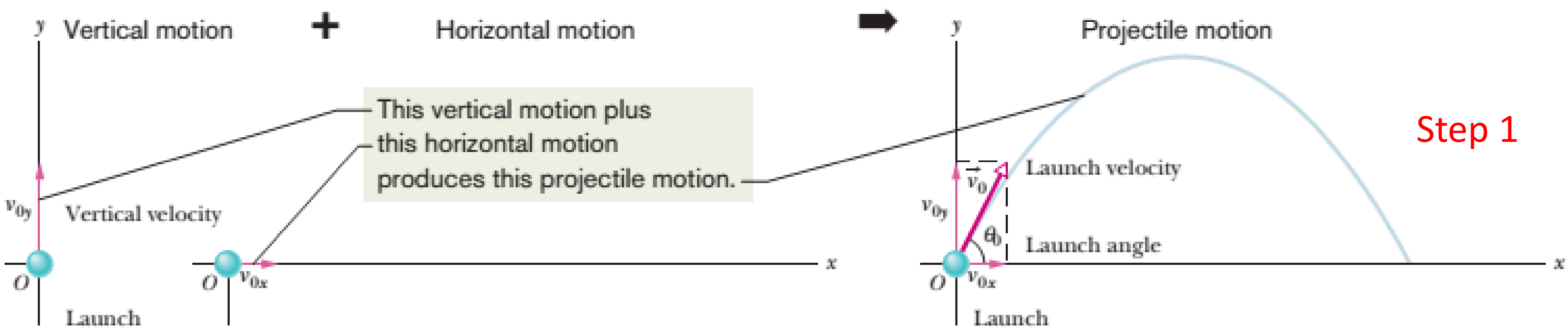
Figure: *The trajectory of an idealized projectile.*

Examples: A batted baseball, a thrown football, a package dropped from an airplane, and a bullet shot from a rifle are all projectiles.

Sketch of the path taken in projectile motion:



Sketch of the path taken in projectile motion (Step-by-Step):



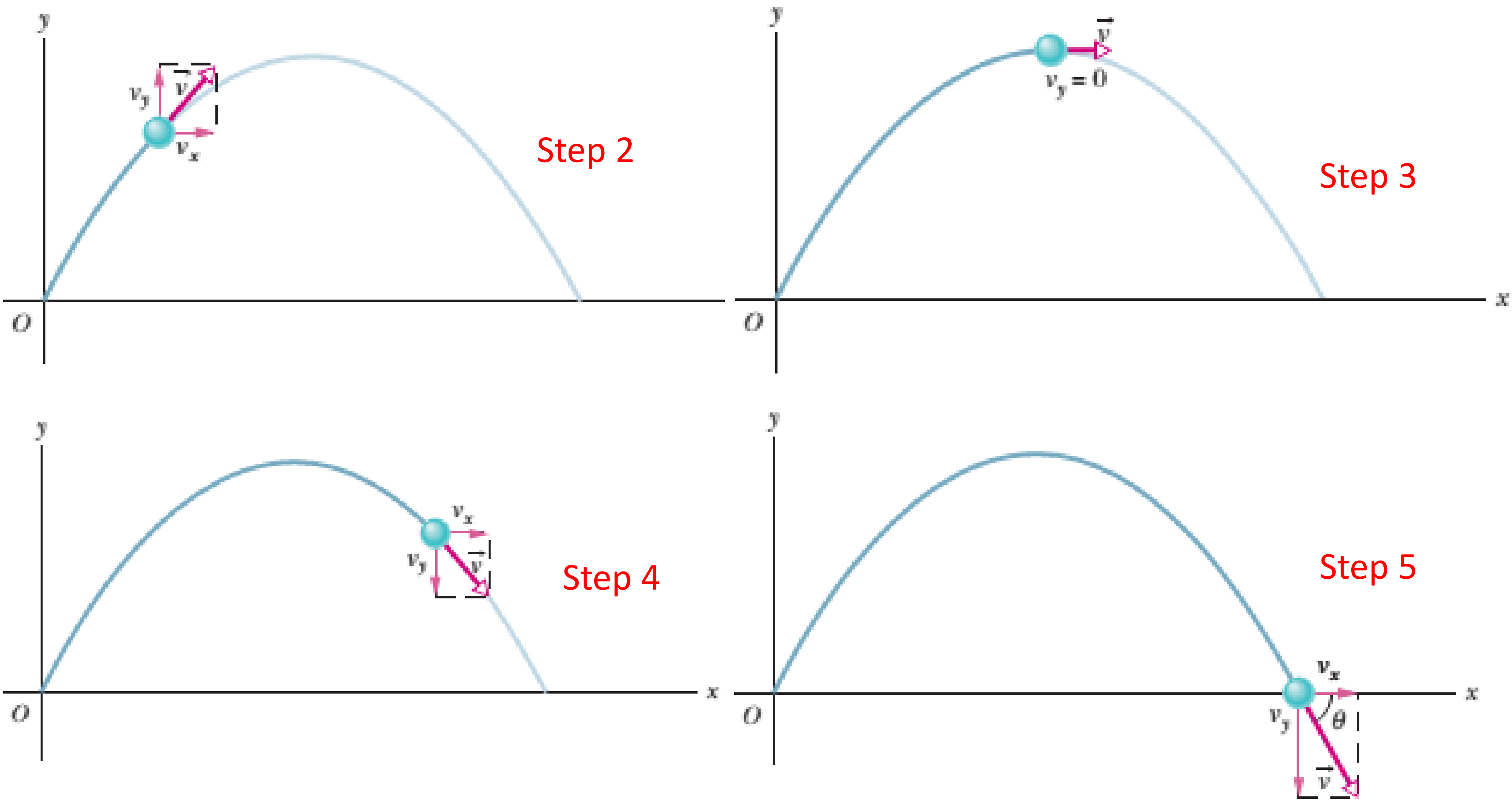
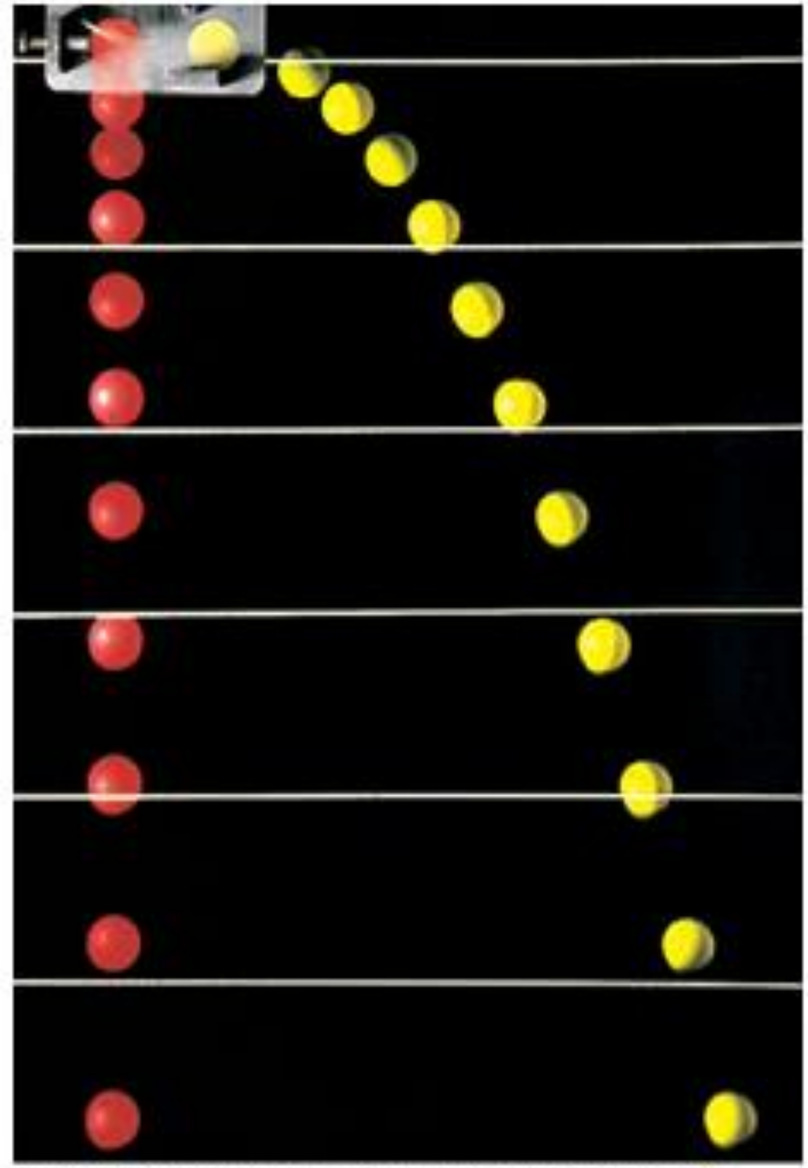


Figure: The *projectile motion* of an object launched into the air at the origin of a coordinate system and with launch velocity $\vec{v}_0$ at angle θ_0. The motion is a combination of vertical motion (constant acceleration) and horizontal motion (constant velocity), as shown by the velocity components.

The adjacent figure is a stroboscopic photograph of two golf balls. One ball is released from rest and the other ball is shot horizontally at the same instant. The golf balls have the same vertical motion, both falling through the same vertical distance in the same interval of time. *The fact that one ball is moving horizontally while it is falling has no effect on its vertical motion;* that is, the horizontal and vertical motions are independent of each other.

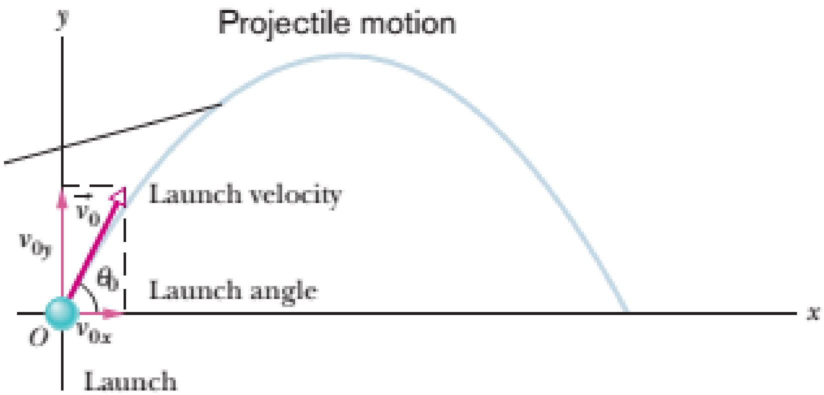


Richard Megna/Fundamental Photographs

The Horizontal Motion:

At any time *t*, the projectile's horizontal displacement $x - x_0$ from an initial position x_0 is given by

$$x - x_0 = v_{0x}\, t + \frac{1}{2} a_x t^2$$



Where $acceleration\ along\ x - axis, a_x = 0$

Using $v_{0x} = v_0 \cos\theta_0$ we can write

$$x - x_0 = (v_0 \cos\theta_0)\ t \qquad \text{.......... (1)}$$

At any time *t*, the projectile's horizontal velocity $v_{0x} = v_x$

The Vertical Motion:

At any time *t*, the projectile's vertical displacement $y - y_0$ from an initial position y_0 is given by

$$y - y_0 = v_{0y}\, t - \frac{1}{2} g t^2 \qquad [\text{ where, } a_y = -g]$$

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2} g t^2 \qquad [\text{ where, } v_{0y} = v_0 \sin\theta_0]$$

.... (2)

At any time *t*, the projectile's vertical velocity

$$v_y = v_0 \sin\theta_0 - gt$$

And we can express $v_y^2\ as$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2\,g(y - y_0)$$

- ❑ Show that the path of a projectile is a parabola.

From equation (1) we can write

$$t = \frac{x - x_0}{v_0 \cos\theta_0}$$

Using the value of t in equation (2), we get

$$y - y_0 = v_0 \sin\theta_0 \frac{x - x_0}{v_0 \cos\theta_0} - \frac{1}{2} g \left(\frac{x - x_0}{v_0 \cos\theta_0}\right)^2$$

For simplicity, we let $x_0 = 0 \; and \; y_0 = 0$.

Therefore, the equation becomes

$$y = (\tan\theta_0)x - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta_0}\right)^2 \qquad \text{.....................} \; (3)$$

Where $\theta_0, g \; and \; v_0 \; are \; constants$.

Equation (3) is of the form $y = ax \mp bx^2 \; , where \; a \; and \; b \; are \; constants$.

This is the equation of a parabola, so the path is *parabolic*.

❑ Equations for the horizontal range and the maximum horizontal range of a projectile:

The ***horizontal range R*** of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height (the height at which it is launched). That is $x - x_0 = R \; when \; y - y_0 = 0$.

Using $x - x_0 = R \; in \; equation \; (1) and \; y - y_0 = 0 \; in \; equation \; (2)$, we get

$$R = (v_0 \cos\theta_0)\, t$$ [From equation (1)]

And $$0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2$$ [From equation (2)]

$$or \quad (v_0 \sin\theta_0)\, t = \frac{1}{2} g t^2 \qquad or \quad t = \frac{2 v_0 \sin\theta_0}{g}$$

Therefore, $$R = (v_0 \cos\theta_0)\, \frac{2 v_0 \sin\theta_0}{g} = \frac{v_0^2\, (2 \sin\theta_0 \cos\theta_0)}{g}$$

$$\boxed{R = \frac{v_0^2 \sin 2\theta_0}{g}} \quad \ldots\ldots(3)$$

***Caution:** This equation does not give the horizontal distance traveled by a projectile when the final height is not the launch height.*

The value of R is maximum in equation (3) when $\sin 2\theta_0 = 1$

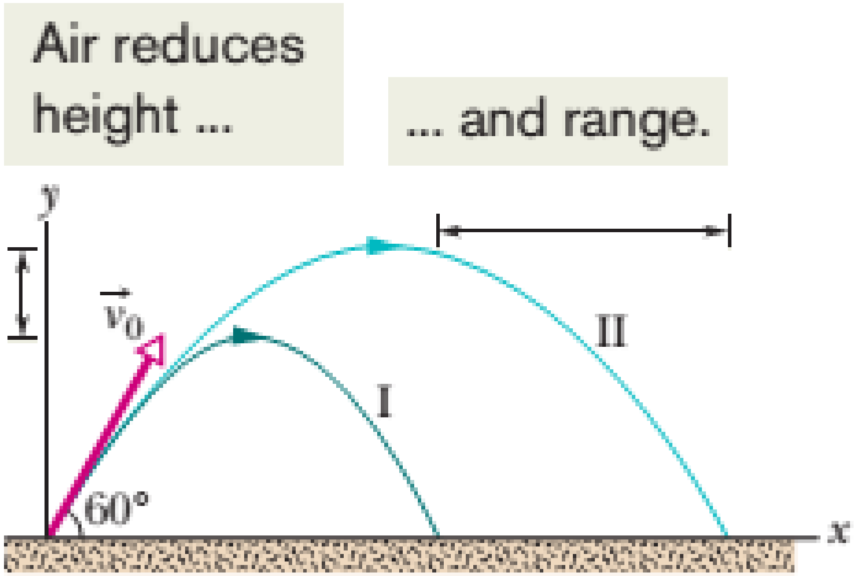
$$or\ \ 2\theta_0 = \sin^{-1} 1$$

$$or\ 2\theta_0 = 90^0 \qquad [since\ \ \sin^{-1} 1 = 90^0]$$

$$\boxed{\theta_0 = 45^0}$$

The Effects of the Air (in the projectile motion):

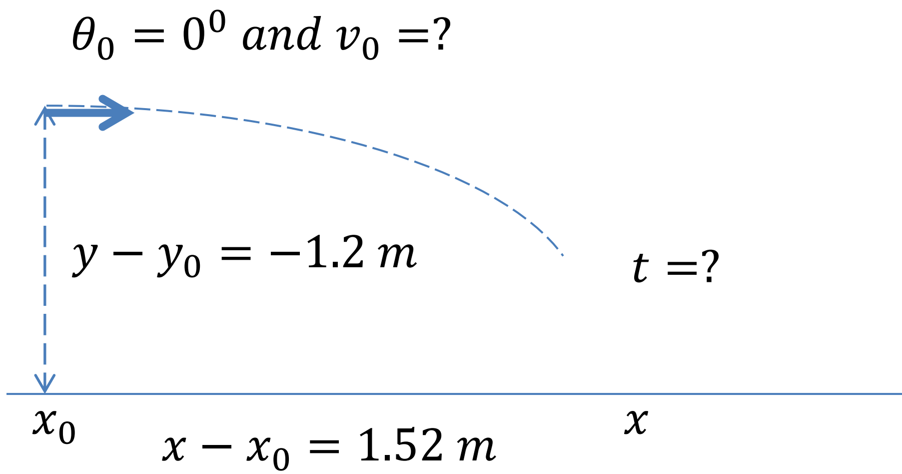
	Path I (Air)	Path II (Vacuum)
Range	98.5 m	177 m
Maximum height	53.0 m	76.8 m
Time of flight	6.6 s	7.9 s



The launch angle is 60° and the launch speed is 44.7 m/s.

Problem 22 (Book chapter 4):

A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. (a) How long is the ball in the air? (b) What is its speed at the instant it leaves the table?



Answer:

(a) We know

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2} g t^2$$

$$-1.20 = (v_0\ sin\ 0^0)\ t - 4.9t^2$$

$$-1.20 = 0 - 4.9t^2$$

$$t = \sqrt{\frac{1.2}{4.9}} = 0.495\ s$$

(b) We know

$$x - x_0 = (v_0 \cos\theta_0)\ t$$

$$1.52 = (v_0 \cos 0^0)(\ 0.495)$$

$$1.52 = (v_0 \cos 0^0)(\ 0.495)$$

$$1.52 = (v_0)(1)(0.495)$$

$$\boxed{v_0 = \frac{1.52}{0.495} = 3.07\ m/s}$$

Thank You