AMERICAN INTERNATIONAL UNIVERSITY-BANGLADESH

Faculty of Science and Technology

AIUB

DEPARTMENT OF COMPUTER SCIENCE

CSC3113: THEORY OF COMPUTATION

Lecture: # 4 Week: # 2 Semester: Spring 2022-2023

DETERMINISTIC FINITE AUTOMATON (DFA)
REGULAR LANGUAGE

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Lecture Outline

- Regular Language
  - Problem solving applying regular operation, Union.
  - Design Issues.

LEARNING OBJECTIVE

- Build one machine from multiple machines using closure under union.

Learning Outcome

All outcome are represented with examples

- Analyze and Design new machine model from one or more machine model(s) using closure rule of regular operation (example: Union).
- In doing so, understand that, there are certain cases where DFA might not give a desired machine model.

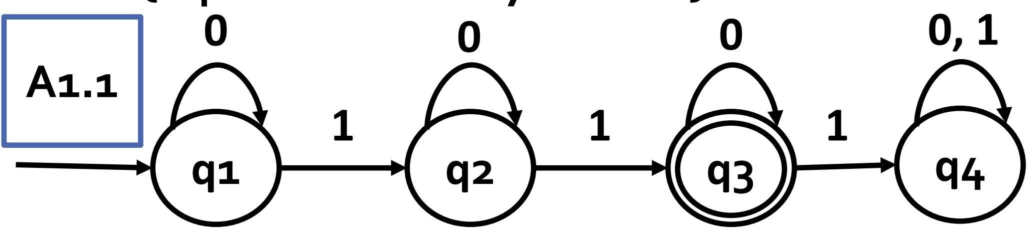
Regular Language closed under Union

- We will prove it by construction.
- Let M_1 recognize A_1, where $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, and M_2 recognize A_2, where $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$.
- Construct M to recognize $A_1 \cup A_2$, where $M = (Q, \Sigma, \delta, q_0, F)$.
  - $Q = \{(r_1, r_2) : r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$.
    - $Q = Q_1 \times Q_2$. (All combination of states of machine M_1 and M_2).
  - $\Sigma = \Sigma_1 \cup \Sigma_2$.
    - But here, for simplicity, we have considered $\Sigma_1 = \Sigma_2$ to be same.
  - For each $(r_1,r_2) \in Q$ and each $a \in \Sigma$, let $\delta((r_1,r_2), a) = (\delta(r_1, a), \delta(r_2, a))$.
    - Hence δ gets a state of M (which actually is a pair of states from M_1 and M_2), together with an input symbol, and returns M's next state.
  - q_0 is the pair (q_1, q_2).
  - $F = \{ (r_1, r_2) : r_1 \in F_1 \text{ or } r_2 \in F_2 \}$
    - $F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$

A1={w| w has exactly two 1s or at least two 0s} for ∑ = {0,1}.

Let us consider them separately

A1.1={w| w has exactly two 1s}

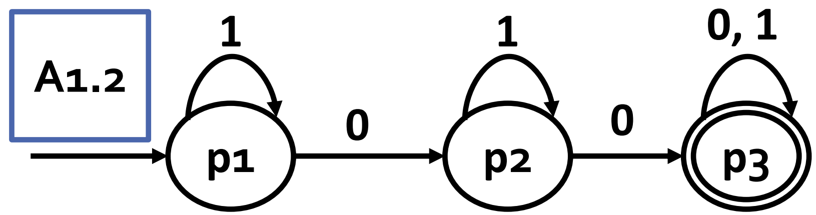


$M_{1.1}= (Q_{1.1}, \Sigma_{1.1}, \delta_{1.1}, q_{0.1.1}, F_{1.1})$,
where –

$Q_{1.1} = \{q_1,q_2,q_3 ,q_4\}$,
$\Sigma_{1.1} = \{0, 1\}$
$q_{0.1.1} = q_1$,
$F_{1.1} = \{q_3\}$,
$\delta_{1.1}$

	0	1
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_4
q_4	q_4	q_4

A1.2={w| w has at least two 0s}



$M_{1.2}= (Q_{1.2}, \Sigma_{1.2}, \delta_{1.2}, q_{0.1.2}, F_{1.2})$,
where –

$Q_{1.2} = \{p_1,p_2,p_3\}$,
$\Sigma_{1.2} = \{0, 1\}$
$q_{0.1.2} = p_1$,
$F_{1.2} = \{p_3\}$,
$\delta_{1.2}$

	0	1
p_1	p_2	p_1
p_2	p_3	p_2
p_3	p_3	p_3

A1={w| w has exactly two *1*s or at least two *0*s} for ∑ = {0,1}.

$M_{1.1} = (Q_{1.1}, \Sigma_{1.1}, \delta_{1.1}, q_{0.1.1}, F_{1.1})$,
where –
$Q_{1.1} = \{q_1, q_2, q_3, q_4\}$,
$\Sigma_{1.1} = \{0, 1\}$
$q_{0.1.1} = q_1$,
$F_{1.1} = \{q_3\}$,

$\delta_{1.1}$	*0*	*1*
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_4
q_4	q_4	q_4

$M_{1.2} = (Q_{1.2}, \Sigma_{1.2}, \delta_{1.2}, q_{0.1.2}, F_{1.2})$,
where –
$Q_{1.2} = \{p_1, p_2, p_3\}$,
$\Sigma_{1.2} = \{0, 1\}$
$q_{0.1.2} = p_1$,
$F_{1.2} = \{p_3\}$,

$\delta_{1.2}$	*0*	*1*
p_1	p_2	p_1
p_2	p_3	p_2
p_3	p_3	p_3

$\Sigma = \Sigma_{1.1} \cup \Sigma_{1.2}$

δ	*0*	*1*
(q_1, p_1)	(,)	(,)
(q_1, p_2)	(,)	(,)
(q_1, p_3)	(,)	(,)
(q_2, p_1)	(,)	(,)
(q_2, p_2)	(,)	(,)
(q_2, p_3)	(,)	(,)
(q_3, p_1)	(,)	(,)
(q_3, p_2)	(,)	(,)
(q_3, p_3)	(,)	(,)
(q_4, p_1)	(,)	(,)
(q_4, p_2)	(,)	(,)
(q_4, p_3)	(,)	(,)

$Q = Q_{1.1} \times Q_{1.2}$
(Tuples)

A1={w| w has exactly two *1*s or at least two *0*s} for ∑ = {0,1}.

$M_{1.1} = (Q_{1.1}, \Sigma_{1.1}, \delta_{1.1}, q_{0.1.1}, F_{1.1})$,
where –
$Q_{1.1} = \{q_1, q_2, q_3, q_4\}$,
$\Sigma_{1.1} = \{0, 1\}$
$q_{0.1.1} = q_1$,
$F_{1.1} = \{q_3\}$,

$\delta_{1.1}$	*0*	*1*
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_4
q_4	q_4	q_4

$M_{1.2} = (Q_{1.2}, \Sigma_{1.2}, \delta_{1.2}, q_{0.1.2}, F_{1.2})$,
where –
$Q_{1.2} = \{p_1, p_2, p_3\}$,
$\Sigma_{1.2} = \{0, 1\}$
$q_{0.1.2} = p_1$,
$F_{1.2} = \{p_3\}$,

$\delta_{1.2}$	*0*	*1*
p_1	p_2	p_1
p_2	p_3	p_2
p_3	p_3	p_3

$\Sigma = \Sigma_{1.1} \cup \Sigma_{1.2}$

δ	*0*	*1*
(q_1, p_1)	$(q_1, \quad)$	$(q_2, \quad)$
(q_1, p_2)	$(q_1, \quad)$	$(q_2, \quad)$
(q_1, p_3)	$(q_1, \quad)$	$(q_2, \quad)$
(q_2, p_1)	$(q_2, \quad)$	$(q_3, \quad)$
(q_2, p_2)	$(q_2, \quad)$	$(q_3, \quad)$
(q_2, p_3)	$(q_2, \quad)$	$(q_3, \quad)$
(q_3, p_1)	$(q_3, \quad)$	$(q_4, \quad)$
(q_3, p_2)	$(q_3, \quad)$	$(q_4, \quad)$
(q_3, p_3)	$(q_3, \quad)$	$(q_4, \quad)$
(q_4, p_1)	$(q_4, \quad)$	$(q_4, \quad)$
(q_4, p_2)	$(q_4, \quad)$	$(q_4, \quad)$
(q_4, p_3)	$(q_4, \quad)$	$(q_4, \quad)$

$Q = Q_{1.1} \times Q_{1.2}$

Fill up the tuples in the table from the transition tables $\delta_{1.1}$ of machine $\boldsymbol{M_{1.1}}$.

A1={w| w has exactly two *1*s or at least two *0*s} for ∑ = {0,1}.

$M_{1.1}= (Q_{1.1}, \Sigma_{1.1}, \delta_{1.1}, q_{0.1.1}, F_{1.1})$,
where –
$Q_{1.1} = \{q_1,q_2,q_3 ,q_4\}$,
$\Sigma_{1.1} = \{0, 1\}$
$q_{0.1.1} = q_1$,
$F_{1.1} = \{q_3\}$,

$\delta_{1.1}$	0	1
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_4
q_4	q_4	q_4

$M_{1.2}= (Q_{1.2}, \Sigma_{1.2}, \delta_{1.2}, q_{0.1.2}, F_{1.2})$,
where –
$Q_{1.2} = \{p_1,p_2,p_3\}$,
$\Sigma_{1.2} = \{0, 1\}$
$q_{0.1.2} = p_1$,
$F_{1.2} = \{p_3\}$,

$\delta_{1.2}$	0	1
p_1	p_2	p_1
p_2	p_3	p_2
p_3	p_3	p_3

$\Sigma = \Sigma_{1.1} \cup \Sigma_{1.2}$

δ	0	1
(q_1,p_1)	(q_1,p_2)	(q_2,p_1)
(q_1,p_2)	(q_1,p_3)	(q_2,p_2)
(q_1,p_3)	(q_1,p_3)	(q_2,p_3)
(q_2,p_1)	(q_2,p_2)	(q_3,p_1)
(q_2,p_2)	(q_2,p_3)	(q_3,p_2)
(q_2,p_3)	(q_2,p_3)	(q_3,p_3)
(q_3,p_1)	(q_3,p_2)	(q_4,p_1)
(q_3,p_2)	(q_3,p_3)	(q_4,p_2)
(q_3,p_3)	(q_3,p_3)	(q_4,p_3)
(q_4,p_1)	(q_4,p_2)	(q_4,p_1)
(q_4,p_2)	(q_4,p_3)	(q_4,p_2)
(q_4,p_3)	(q_4,p_3)	(q_4,p_3)

$Q = Q_{1.1} \times Q_{1.2}$

Fill up the tuples in the table from the transition tables $\delta_{1.2}$ of machine $M_{1.2}$.

A1={w| w has exactly two *1*s or at least two *0*s} for $\Sigma = \{0,1\}$.

$M_{1.1} = (Q_{1.1}, \Sigma_{1.1}, \delta_{1.1}, q_{0.1.1}, F_{1.1})$,
where –

$Q_{1.1} = \{q_1, q_2, q_3, q_4\}$,
$\Sigma_{1.1} = \{0, 1\}$
$q_{0.1.1} = q_1$,
$F_{1.1} = \{q_3\}$,

$\delta_{1.1}$	0	1
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_4
q_4	q_4	q_4

$M_{1.2} = (Q_{1.2}, \Sigma_{1.2}, \delta_{1.2}, q_{0.1.2}, F_{1.2})$,
where –

$Q_{1.2} = \{p_1, p_2, p_3\}$,
$\Sigma_{1.2} = \{0, 1\}$
$q_{0.1.2} = p_1$,
$F_{1.2} = \{p_3\}$,

$\delta_{1.2}$	0	1
p_1	p_2	p_1
p_2	p_3	p_2
p_3	p_3	p_3

δ	0	1
(q_1, p_1)	(q_1, p_2)	(q_2, p_1)
(q_1, p_2)	(q_1, p_3)	(q_2, p_2)
(q_1, p_3)	(q_1, p_3)	(q_2, p_3)
(q_2, p_1)	(q_2, p_2)	(q_3, p_1)
(q_2, p_2)	(q_2, p_3)	(q_3, p_2)
(q_2, p_3)	(q_2, p_3)	(q_3, p_3)
(q_3, p_1)	(q_3, p_2)	(q_4, p_1)
(q_3, p_2)	(q_3, p_3)	(q_4, p_2)
(q_3, p_3)	(q_3, p_3)	(q_4, p_3)
(q_4, p_1)	(q_4, p_2)	(q_4, p_1)
(q_4, p_2)	(q_4, p_3)	(q_4, p_2)
(q_4, p_3)	(q_4, p_3)	(q_4, p_3)

Σ

Q

Mark the start state from the start states $\boldsymbol{q_{0.1.1}}$ and $\boldsymbol{q_{0.1.2}}$ of the two machines $\boldsymbol{M_{1.1}}$ and $\boldsymbol{M_{1.2}}$.
Mark the final states from the set of final states $\boldsymbol{F_{1.1}}$ and $\boldsymbol{F_{1.2}}$ of the two machines $\boldsymbol{M_{1.1}}$ and $\boldsymbol{M_{1.2}}$.

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$\Sigma_{1.1} = \{0, 1\}$
$q_{0.1.1} = q_1$,
$F_{1.1} = \{q_3\}$,

$\delta_{1.1}$	0	1
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_4
q_4	q_4	q_4

$M_{1.2} = (Q_{1.2}, \Sigma_{1.2}, \delta_{1.2}, q_{0.1.2}, F_{1.2})$,
where –
$Q_{1.2} = \{p_1, p_2, p_3\}$,
$\Sigma_{1.2} = \{0, 1\}$
$q_{0.1.2} = p_1$,
$F_{1.2} = \{p_3\}$,

$\delta_{1.2}$	0	1
p_1	p_2	p_1
p_2	p_3	p_2
p_3	p_3	p_3

δ	0	1
$\rightarrow r_1\ (q_1, p_1)$	$r_2\ (q_1, p_2)$	$r_4\ (q_2, p_1)$
$r_2\ (q_1, p_2)$	$r_3\ (q_1, p_3)$	$r_5\ (q_2, p_2)$
$r_3\ (q_1, p_3)$	$r_3\ (q_1, p_3)$	$r_6\ (q_2, p_3)$
$r_4\ (q_2, p_1)$	$r_5\ (q_2, p_2)$	$r_7\ (q_3, p_1)$
$r_5\ (q_2, p_2)$	$r_6\ (q_2, p_3)$	$r_8\ (q_3, p_2)$
$r_6\ (q_2, p_3)$	$r_6\ (q_2, p_3)$	$r_9\ (q_3, p_3)$
$r_7\ (q_3, p_1)$	$r_8\ (q_3, p_2)$	$r_{10}\ (q_4, p_1)$
$r_8\ (q_3, p_2)$	$r_9\ (q_3, p_3)$	$r_{11}\ (q_4, p_2)$
$r_9\ (q_3, p_3)$	$r_9\ (q_3, p_3)$	$r_{12}\ (q_4, p_3)$
$r_{10}\ (q_4, p_1)$	$r_{11}\ (q_4, p_2)$	$r_{10}\ (q_4, p_1)$
$r_{11}\ (q_4, p_2)$	$r_{12}\ (q_4, p_3)$	$r_{11}\ (q_4, p_2)$
$r_{12}\ (q_4, p_3)$	$r_{12}\ (q_4, p_3)$	$r_{12}\ (q_4, p_3)$

Σ

Q

[*optional*]
Rename each state (tuples) for the 1st column
Put these names against each state for each input column according to the 1st column

A1={w| w has exactly two 1s or at least two 0s} for $\Sigma = \{0,1\}$.

1. Draw state diagram by level, *starting with start state*.

2. *Mark* all the ***current level*** states (**green**) in the *first column* & the ***transitions*** states (**orange**) in *2nd & 3rd column*.

3. Get the *next level* states (**orange**) from *2nd & 3rd column* & **draw**. *Mark* (**orange**) all next level states in the *whole table*.

4. Draw the **transitions.**

5. Now, ***current level complete*** (**green** to **red**) and next level becomes current level (**orange** to **green**).

6. Repeat from step 2 until all states are **red**.

δ	0	1
$\rightarrow r_1\ (q_1,p_1)$	$r_2\ (q_1,p_2)$	$r_4\ (q_2,p_1)$
$r_2\ (q_1,p_2)$	$r_3\ (q_1,p_3)$	$r_5\ (q_2,p_2)$
$r_3\ (q_1,p_3)$	$r_3\ (q_1,p_3)$	$r_6\ (q_2,p_3)$
$r_4\ (q_2,p_1)$	$r_5\ (q_2,p_2)$	$r_7\ (q_3,p_1)$
$r_5\ (q_2,p_2)$	$r_6\ (q_2,p_3)$	$r_8\ (q_3,p_2)$
$r_6\ (q_2,p_3)$	$r_6\ (q_2,p_3)$	$r_9\ (q_3,p_3)$
$r_7\ (q_3,p_1)$	$r_8\ (q_3,p_2)$	$r_{10}\ (q_4,p_1)$
$r_8\ (q_3,p_2)$	$r_9\ (q_3,p_3)$	$r_{11}\ (q_4,p_2)$
$r_9\ (q_3,p_3)$	$r_9\ (q_3,p_3)$	$r_{12}\ (q_4,p_3)$
$r_{10}\ (q_4,p_1)$	$r_{11}\ (q_4,p_2)$	$r_{10}\ (q_4,p_1)$
$r_{11}\ (q_4,p_2)$	$r_{12}\ (q_4,p_3)$	$r_{11}\ (q_4,p_2)$
$r_{12}\ (q_4,p_3)$	$r_{12}\ (q_4,p_3)$	$r_{12}\ (q_4,p_3)$

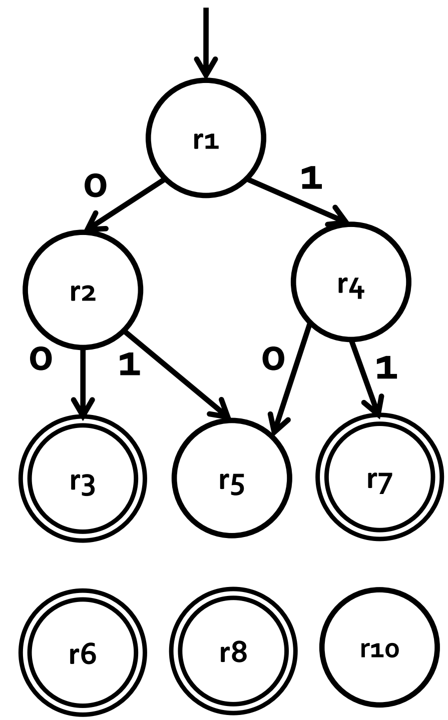
GREEN: State Processing

$r_3\ r_5\ r_7$

ORANGE: State Appeared

RED: State Processing Done

$r_1\ r_2\ r_4$



A1={w| w has exactly two 1s or at least two 0s} for Σ = {0,1}.

1. Draw state diagram by level, *starting with start state*.

2. *Mark* all the ***current level*** states (**green**) in the *first column* & the ***transitions*** states (**orange**) in *2nd & 3rd column*.

3. Get the *next level* states (**orange**) from *2nd & 3rd column* & **draw**. *Mark* (**orange**) all next level states in the *whole table*.

4. Draw the **transitions.**

5. Now, ***current level complete*** (**green** to **red**) and next level becomes current level (**orange** to **green**).

6. Repeat from step 2 until all states are **red**.

δ	0	1
$\rightarrow r_1\ (q_1,p_1)$	$r_2\ (q_1,p_2)$	$r_4\ (q_2,p_1)$
$r_2\ (q_1,p_2)$	$r_3\ (q_1,p_3)$	$r_5\ (q_2,p_2)$
$r_3\ (q_1,p_3)$	$r_3\ (q_1,p_3)$	$r_6\ (q_2,p_3)$
$r_4\ (q_2,p_1)$	$r_5\ (q_2,p_2)$	$r_7\ (q_3,p_1)$
$r_5\ (q_2,p_2)$	$r_6\ (q_2,p_3)$	$r_8\ (q_3,p_2)$
$r_6\ (q_2,p_3)$	$r_6\ (q_2,p_3)$	$r_9\ (q_3,p_3)$
$r_7\ (q_3,p_1)$	$r_8\ (q_3,p_2)$	$r_{10}\ (q_4,p_1)$
$r_8\ (q_3,p_2)$	$r_9\ (q_3,p_3)$	$r_{11}\ (q_4,p_2)$
$r_9\ (q_3,p_3)$	$r_9\ (q_3,p_3)$	$r_{12}\ (q_4,p_3)$
$r_{10}\ (q_4,p_1)$	$r_{11}\ (q_4,p_2)$	$r_{10}\ (q_4,p_1)$
$r_{11}\ (q_4,p_2)$	$r_{12}\ (q_4,p_3)$	$r_{11}\ (q_4,p_2)$
$r_{12}\ (q_4,p_3)$	$r_{12}\ (q_4,p_3)$	$r_{12}\ (q_4,p_3)$

GREEN: State Processing

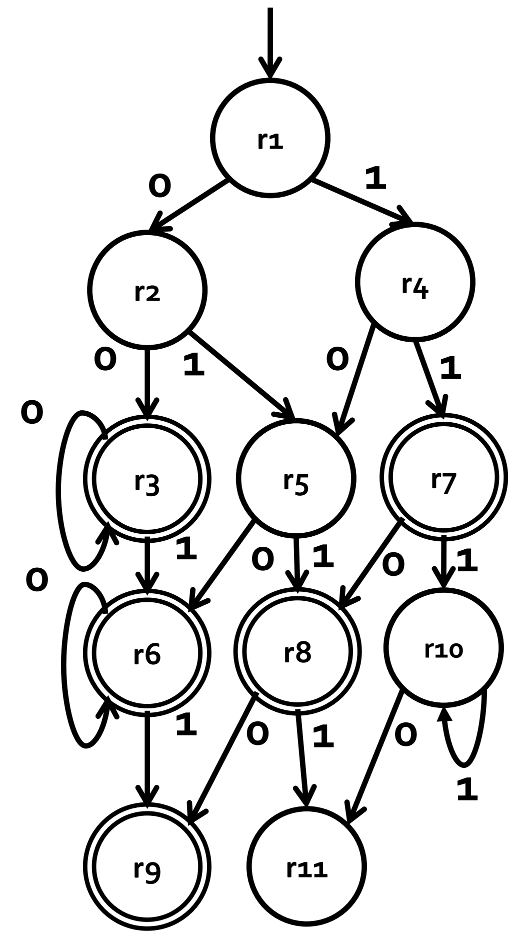
$r_6\ r_8\ r_{10}$

ORANGE: State Appeared

$r_9\ r_{11}$

RED: State Processing Done

$r_1\ r_2\ r_4$



A1={w| w has exactly two 1s or at least two 0s} for $\Sigma = \{0,1\}$.

1. Draw state diagram by level, *starting with start state*.

2. *Mark* all the ***current level*** states (**green**) in the *first column* & the ***transitions*** states (**orange**) in *2nd & 3rd column*.

3. Get the *next level* states (**orange**) from *2nd & 3rd column* & **draw**. *Mark* (**orange**) all next level states in the *whole table*.

4. Draw the **transitions.**

5. Now, ***current level complete*** (**green** to **red**) and next level becomes current level (**orange** to **green**).

6. Repeat from step 2 until all states are **red**.

	δ	0	1
→	$r_1\ (q_1,p_1)$	$r_2\ (q_1,p_2)$	$r_4\ (q_2,p_1)$
	$r_2\ (q_1,p_2)$	$r_3\ (q_1,p_3)$	$r_5\ (q_2,p_2)$
◎	$r_3\ (q_1,p_3)$	$r_3\ (q_1,p_3)$	$r_6\ (q_2,p_3)$
	$r_4\ (q_2,p_1)$	$r_5\ (q_2,p_2)$	$r_7\ (q_3,p_1)$
	$r_5\ (q_2,p_2)$	$r_6\ (q_2,p_3)$	$r_8\ (q_3,p_2)$
◎	$r_6\ (q_2,p_3)$	$r_6\ (q_2,p_3)$	$r_9\ (q_3,p_3)$
◎	$r_7\ (q_3,p_1)$	$r_8\ (q_3,p_2)$	$r_{10}\ (q_4,p_1)$
◎	$r_8\ (q_3,p_2)$	$r_9\ (q_3,p_3)$	$r_{11}\ (q_4,p_2)$
◎	$r_9\ (q_3,p_3)$	$r_9\ (q_3,p_3)$	$r_{12}\ (q_4,p_3)$
	$r_{10}\ (q_4,p_1)$	$r_{11}\ (q_4,p_2)$	$r_{10}\ (q_4,p_1)$
	$r_{11}\ (q_4,p_2)$	$r_{12}\ (q_4,p_3)$	$r_{11}\ (q_4,p_2)$
◎	$r_{12}\ (q_4,p_3)$	$r_{12}\ (q_4,p_3)$	$r_{12}\ (q_4,p_3)$

GREEN: State Processing

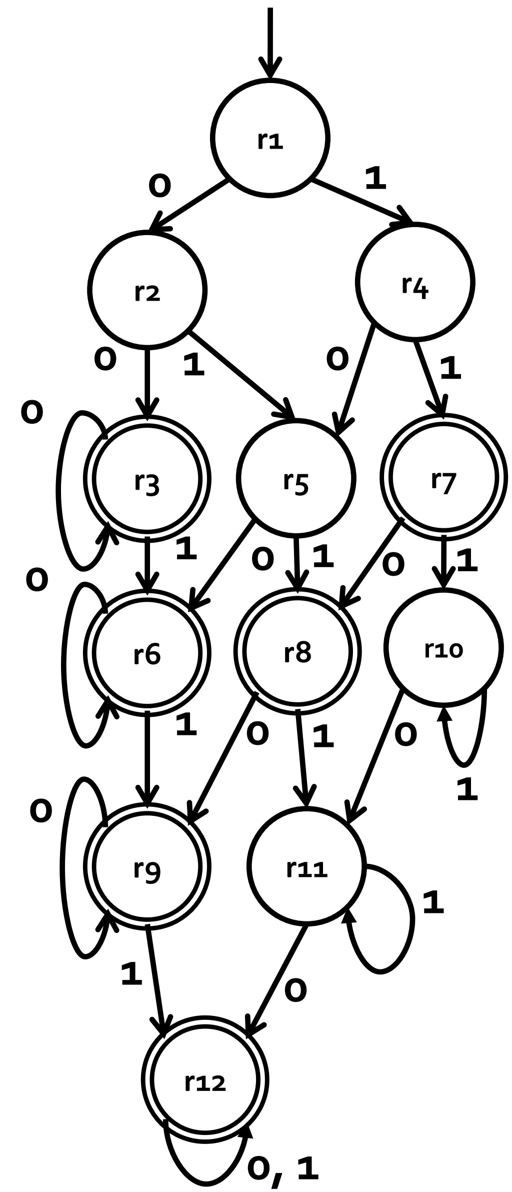
r_{12}

ORANGE: State Appeared

r_{12}

RED: State Processing Done

$r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7\ r_8\ r_9\ r_{10}\ r_{11}\ r_{12}$



A1={w| w has exactly two *1*s or at least two *0*s} for $\sum = \{0,1\}$.

$M_{1.1} = (Q_{1.1}, \Sigma_{1.1}, \delta_{1.1}, q_{0.1.1}, F_{1.1})$,
where –

$Q_{1.1} = \{q_1, q_2, q_3, q_4\}$,
$\Sigma_{1.1} = \{0, 1\}$
$q_{0.1.1} = q_1$,
$F_{1.1} = \{q_3\}$,

$\delta_{1.1}$	0	1
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_4
q_4	q_4	q_4

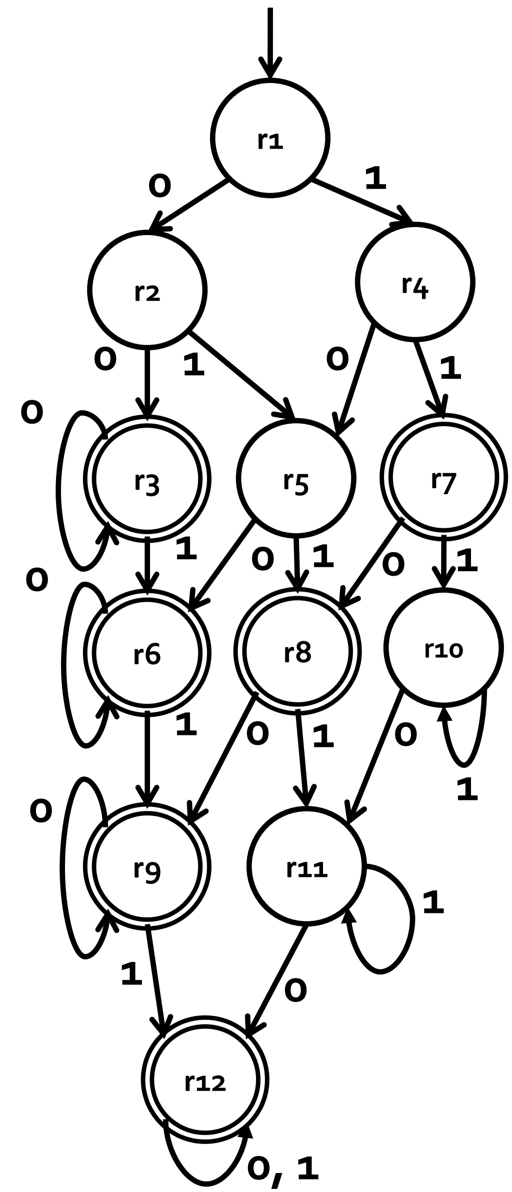
$M_{1.2} = (Q_{1.2}, \Sigma_{1.2}, \delta_{1.2}, q_{0.1.2}, F_{1.2})$,
where –

$Q_{1.2} = \{p_1, p_2, p_3\}$,
$\Sigma_{1.2} = \{0, 1\}$
$q_{0.1.2} = p_1$,
$F_{1.2} = \{p_3\}$,

$\delta_{1.2}$	0	1
p_1	p_2	p_1
p_2	p_3	p_2
p_3	p_3	p_3

	δ	0	1
→	$r_1\ (q_1, p_1)$	$r_2\ (q_1, p_2)$	$r_4\ (q_2, p_1)$
	$r_2\ (q_1, p_2)$	$r_3\ (q_1, p_3)$	$r_5\ (q_2, p_2)$
◎	$r_3\ (q_1, p_3)$	$r_3\ (q_1, p_3)$	$r_6\ (q_2, p_3)$
	$r_4\ (q_2, p_1)$	$r_5\ (q_2, p_2)$	$r_7\ (q_3, p_1)$
	$r_5\ (q_2, p_2)$	$r_6\ (q_2, p_3)$	$r_8\ (q_3, p_2)$
◎	$r_6\ (q_2, p_3)$	$r_6\ (q_2, p_3)$	$r_9\ (q_3, p_3)$
◎	$r_7\ (q_3, p_1)$	$r_8\ (q_3, p_2)$	$r_{10}\ (q_4, p_1)$
◎	$r_8\ (q_3, p_2)$	$r_9\ (q_3, p_3)$	$r_{11}\ (q_4, p_2)$
◎	$r_9\ (q_3, p_3)$	$r_9\ (q_3, p_3)$	$r_{12}\ (q_4, p_3)$
	$r_{10}\ (q_4, p_1)$	$r_{11}\ (q_4, p_2)$	$r_{10}\ (q_4, p_1)$
	$r_{11}\ (q_4, p_2)$	$r_{12}\ (q_4, p_3)$	$r_{11}\ (q_4, p_2)$
◎	$r_{12}\ (q_4, p_3)$	$r_{12}\ (q_4, p_3)$	$r_{12}\ (q_4, p_3)$

Final Answer
→ Transition table with start and final states marked
→ State Diagram



References

- Introduction to Theory of Computation, Sipser, (3rd ed), [Regular Language](#).