1 (i) $A = \{w \mid$ each "x" followed by atleast two "y" $\}$

$= L = \{xyy, yxyy, yyyxyyyxyy, \ldots\}$

$S \to YxyyYT$

$Y \to yY \mid \varepsilon$

$T \to xyyYT$

(ii) $A = \{w \mid$ starts and ends with same symbol $\}$

$L = \{a, xyyx, xx, x, y, yxy, \ldots\}$

$S \to xTx \mid yTy \mid x \mid y$

$T \to xT \mid yT \mid \varepsilon$

(iii) $A = \{w \mid w$ contains 'xyxyxy' substring$\}$

$= L = \{yyxyxyxyx, xyxyxy, \ldots\}$

$S \to TxyxyxyT$

$T \to xT \mid yT \mid \varepsilon$

2 $D \to xDx \mid yE \mid \varepsilon$

$E \to yEy \mid xF \mid \varepsilon$

$F \to xD \mid \varepsilon$

Adding S_0, start variable (i)

$S_0 \to D$

$D \to xDx \mid yE \mid \varepsilon$

$E \to yEy \mid xF \mid \varepsilon$

$F \to xD \mid \varepsilon$

Removing $F \to \varepsilon$ (ii)

$S_0 \to D$

$D \to xDx \mid yE \mid \varepsilon$

$E \to yEy \mid xF \mid \varepsilon \mid x$

$F \to xD$

Removing $E \to \varepsilon$ (iii)

$S_0 \to D$

$D \to xDx \mid yE \mid \varepsilon \mid y$

$E \to yEy \mid xF \mid x \mid y$ [yy equivalent to y]

$F \to xD$

Removing $D \to \varepsilon$ (iv)

$S_0 \to D \mid \varepsilon$

$D \to xDx \mid yE \mid y \mid x$ [xx equivalent to x]

$E \to yEy \mid xF \mid x \mid y$

$F \to xD \mid x$

Substitute,

$S_0 \to xDx \mid yE \mid y \mid x \mid \varepsilon$

$D \to xDx \mid yE \mid y \mid x$

$E \to yEy \mid xF \mid x \mid y$

$F \to xD \mid x$

Converting to new rules,

$S_0 \to XDX \mid YE \mid y \mid x \mid \varepsilon$

$D \to XDX \mid YE \mid y \mid x$

$E \to YEY \mid XF \mid x \mid y$

$F \to XD \mid x$

$X \to x$

$Y \to y$

Finally, $S_0 \to PX \mid YE \mid y \mid x \mid \varepsilon$

$D \to PX \mid YE \mid y \mid x$

$E \to YR \mid XF \mid x \mid y$

$F \to PX \mid x$

$X \to x$

$Y \to y$

$P \to XD$

$R \to EY$

(Ans)

4] String: a a b b a a #

q_0: a a b b a a #

q_1: # a b b a a #

q_2: # a x b a a #

q_2: # a x b a a #

q_3: # a x b x a #

q_4: # a x b x a #

q_4: # a x b x a #

q_4: # a x b x a #

q_4: # a x b x a #

q_4: # a x b x a #

q_5: # a x b x a #

q_1: # x x b x a #

q_1: # x x b x a #

q_2: # x x x x a #

q_2: # x x x x a #

q_3: # x x x x x #

q_6: # x x x x x #

q_6: # x x x x x #

q_6: # x x x x x #

q_6: # x x x x x #

q_6: # x x x x x #

q_6: # x x x x x #

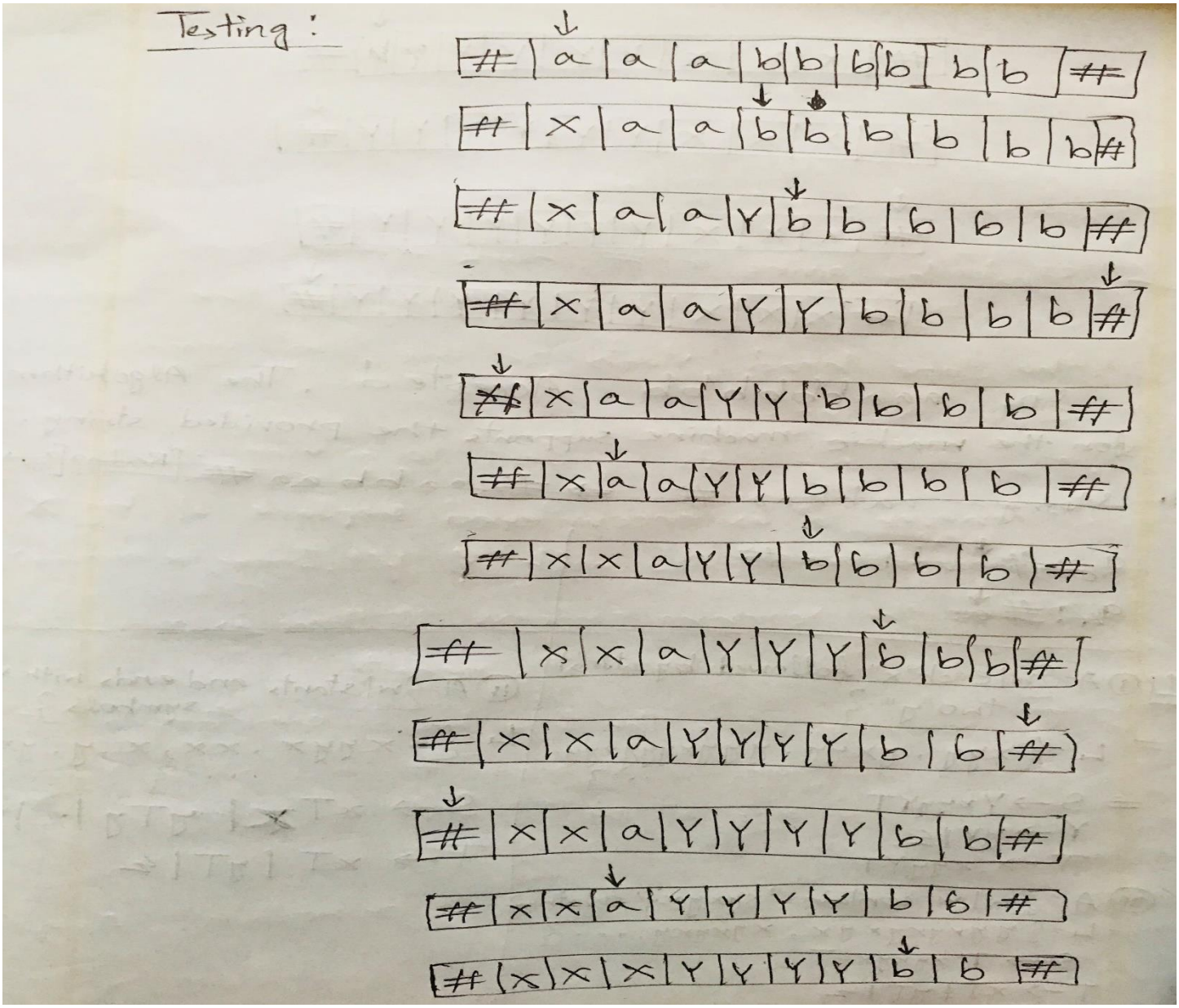
q_{accept}

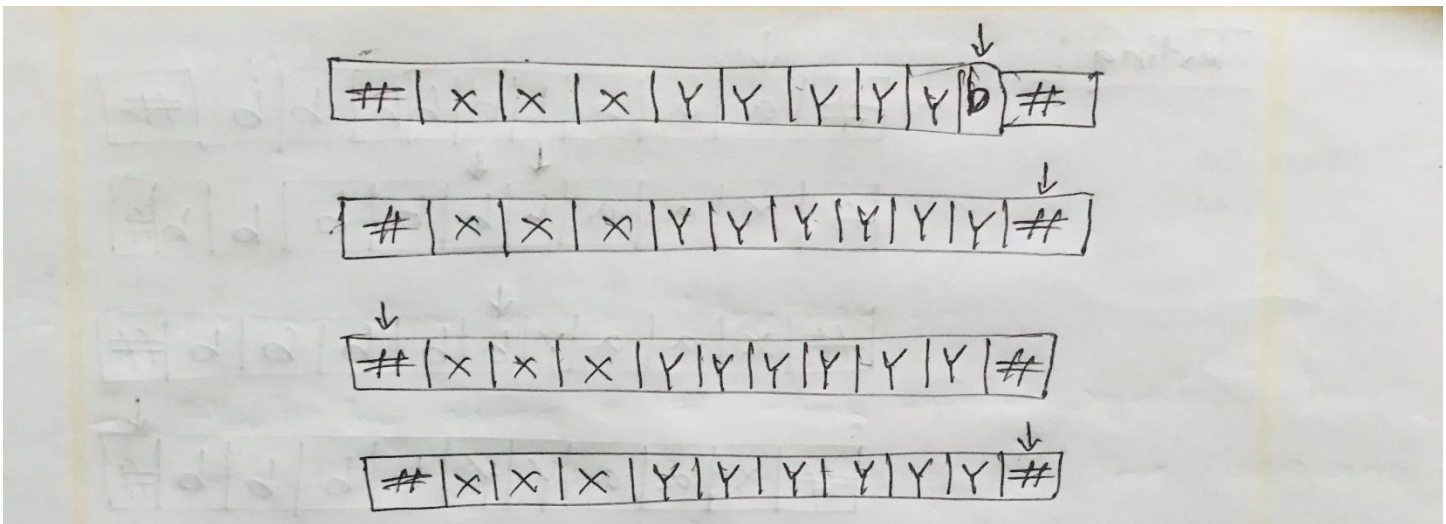
∴ a a b b a a # accepted

7] Implementation level Strategy:

1. Start by giving a "#" blank symbol before putting in the input string on the tape and the tape head starts on the first symbol of the input string.
2. Scan the input string left to right.
3. Whenever 'a' is encountered, replace it by 'X' and after "a" for 'b' replace with 'Y'. Ignore the encounter X, Y,
4. During the scanning process, if there is more than two "b"s for a single "a", reject.
5. By scanning, when the tape head reaches a "#", it goes to left untill it finds "#" and starts scan again to right if any "a" or "b" left scanning and repeating from steps 2.
6. if all the "a" and "b" marks on replaced, Accept.

Testing :





So, aaabbbbbb Accepted. The Algorithm for the ~~machie~~ machine supports the provided string.