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SFWR ENG 3DX4 Tutorial Quiz 3 L03: Mechanical Systems

1. **Linear Mechanical Systems Models** (10 marks)

Consider the system show below where the two carts have negligible rolling friction and b_1 and b_2 are coefficients of viscous friction:

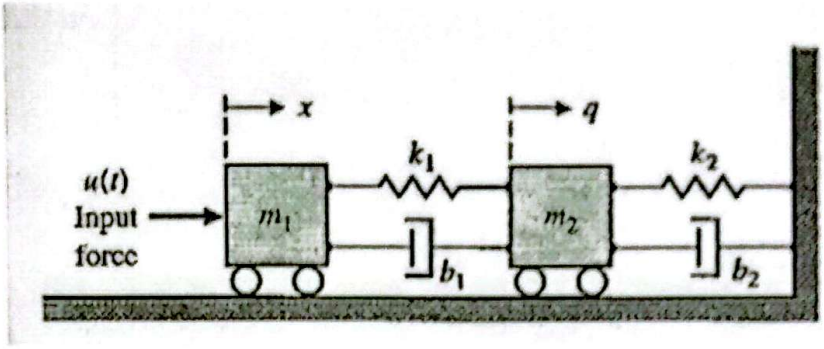

Figure 1: Mechanical System

(a) (5 marks) Write down the equations of motion for the system. Do not forget your free-body diagrams!

(b) (5 marks) What is the transfer function $\frac{Q(s)}{U(s)}$?

1(a) Free-body diagrams are below. → + (positive direction)

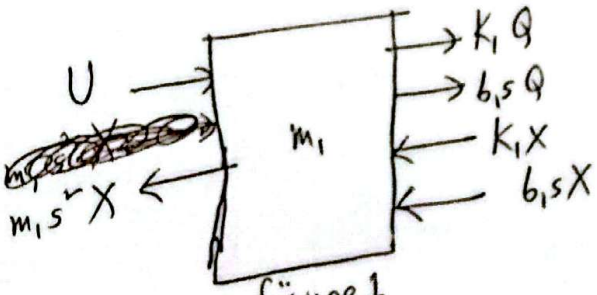

figure 1

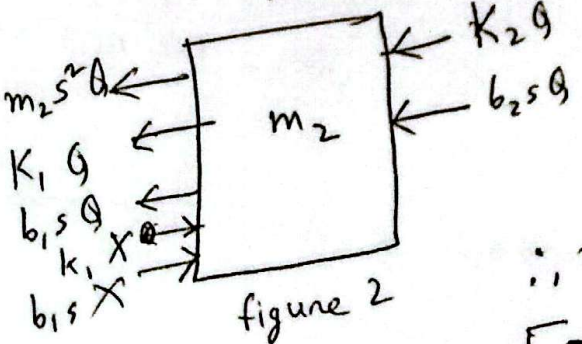

figure 2

From fig. 1,

$(-m_1s^2 - K_1 - b_1s)X + (K_1 + b_1s)Q = -U \quad \ldots (i)$

From fig. 2,

$(K_1 + b_1s)X + (-m_2s^2 - K_1 - b_1s - K_2 - b_1s)Q = 0 \quad \ldots (ii)$

∴ The eqn in matrix form is

$$\begin{bmatrix} -m_1s^2 - K_1 - b_1s & K_1 + b_1s \\ K_1 + b_1s & -m_2s^2 - K_1 - b_1s - K_2 - b_1s \end{bmatrix} \begin{bmatrix} X \\ Q \end{bmatrix} = \begin{bmatrix} -U \\ 0 \end{bmatrix}$$

(b) To get transfer function, first get $Q(s)$, which can be found using Cramer's rule.

$$\therefore Q(s) = \frac{\det(A_2)}{\det(A)}$$

$$\det(A_2) = \begin{vmatrix} -m_1 s^2 - K_1 - b_1 s & -U \\ K_1 + b_1 s & 0 \end{vmatrix}$$

$$= 0 - (-U)(K_1 + b_1 s)$$

$$= U(K_1 + b_1 s)$$

$$\det(A) = \begin{vmatrix} -m_1 s^2 - K_1 - b_1 s & K_1 + b_1 s \\ K_1 + b_1 s & -m_2 s^2 - K_1 - b_1 s - K_2 - b_2 s \end{vmatrix}$$

$$= (-m_1 s^2 - K_1 - b_1 s)(-m_2 s^2 - K_1 - b_1 s - K_2 - b_2 s) - (K_1 + b_1 s)(K_1 + b_1 s)$$

$$= m_1 m_2 s^4 + m_1 K_1 s^2 + m_1 b_1 s^3 + m_1 K_2 s^2 + m_1 b_2 s^3 + K_1 m_2 s^2 + K_1^2 + K_1 b_1 s + K_1 K_2 + K_1 b_2 s + m_2 b_1 s^3 + b_1 K_1 s + b_1^2 s^2 + b_1 K_2 s + b_1 b_2 s^2 - K_1^2 - K_1 b_1 s - K_1 b_1 s - b_1^2 s^2$$

$$= m_1 m_2 s^4 + s^3(m_1 b_1 + m_1 b_2 + m_2 b_1) + s^2(m_1 K_1 + m_1 K_2 + K_1 m_2 + b_1 b_2) + (K_1 b_2 + b_1 K_2)s + K_1 K_2$$

$$Q(s) = \frac{\det(A_2)}{\det(A)} = \frac{U(K_1 + b_1 s)}{m_1 m_2 s^4 + (m_1 b_1 + m_1 b_2 + m_2 b_1)s^3 + (m_1 K_1 + m_1 K_2 + K_1 m_2 + b_1 b_2)s^2 + (K_1 b_2 + b_1 K_2)s + K_1 K_2}$$

$$\therefore \frac{Q(s)}{U(s)} = \frac{K_1 + b_1 s}{m_1 m_2 s^4 + (m_1 b_1 + m_1 b_2 + m_2 b_1)s^3 + (m_1 K_1 + m_1 K_2 + K_1 m_2 + b_1 b_2)s^2 + (K_1 b_2 + b_1 K_2)s + K_1 K_2}$$

↳ transfer function