Lab 1 prelab

Monday, January 19, 2026 12:31 AM

Name: Tridib Banik
St#: 400514461
MacID: banikt
Section: L03
Group: 7

Ans to the Q. No. 1

Given the open-loop transfer function for angular velocity:

$$G_\omega(s) = \frac{\Omega(s)}{R(s)} = \frac{A}{\tau s + 1}$$

where $\Omega(s) = \mathcal{L}\{\omega(t)\}$

$R(s) =$ Laplace transform of the input $r(t)$

$A, \tau > 0$

Relationship between angular position, $\theta(t)$ and angular velocity $\omega(t)$:

$$\omega(t) = \frac{d}{dt}\left(\theta(t)\right)$$

Taking Laplace Transforms (assuming zero initial conditions):

$$\Omega(s) = s\,\theta(s)$$

So, $\theta(s) = \frac{\Omega(s)}{s}$

<substitute $\Omega(s) = G_\omega(s)R(s)$>

$$\theta(s) = \frac{G_\omega(s)R(s)}{s}$$

So, the transfer function from input $R(s)$ to angular position $\theta(s)$ is:

$$G_\theta(s) = \frac{\theta(s)}{R(s)} = \frac{G_\omega(s)}{s} = \left(\frac{1}{s}\right)\left(\frac{A}{\tau s + 1}\right).$$

Ans. to Q. No. 2

Input: $r(t) = U_0\, u(t) = \begin{cases} U_0, & t \geq 0 \\ 0, & t < 0 \end{cases}$

<take Laplace Transform>

$$R(s) = \mathcal{L}\{U_0 u(t)\} = \frac{U_0}{s}$$

The output angular velocity is Laplace domain is:

$$\Omega(s) = G_\omega(s)R(s) = \left(\frac{A}{\tau s + 1}\right)\left(\frac{U_0}{s}\right) = \frac{A U_0}{s(\tau s + 1)}$$

To find steady-state value $\omega_{ss} = \lim_{t\to\infty} \omega(t)$, use Final Value Theorem: $\omega_{ss} = \lim_{t\to\infty}\omega(t) = \lim_{s\to 0} s\,\Omega(s)$

<substitute $\Omega(s)$>

$$\omega_{ss} = \lim_{s\to 0}(s)\left(\frac{A U_0}{s(\tau s + 1)}\right) = \lim_{s\to 0}\frac{A U_0}{\tau s + 1}$$

Evaluating the limit as $s \to 0$: $\omega_{ss} = \frac{A U_0}{\tau(0) + 1} = A U_0$

So, the steady-state angular velocity is $A U_0$.