Prelab 2

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Math 1008
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Ans to the Q. No. 1(a)

For a step input of amplitude V_a, input in Laplace domain: $v(t) = V_a u(t)$

$$\Rightarrow V(s) = \frac{V_a}{s}$$

output in Laplace domain: $\Omega(s) = G_o(s)V(s) = \frac{A}{s+1} \cdot \frac{V_a}{s}$

Using the final value theorem,

$$\omega_{ss} = \lim_{t\to\infty} \omega(t) = \lim_{s\to 0} s\Omega(s) = \lim_{s\to 0} s\left(\frac{AV_a}{s(s+1)}\right) = \lim_{s\to 0} \frac{AV_a}{s+1} = AV_a$$

Ans to the Q. No. 1(b)

For a step input $v(t) = V_a u(t)$ and initial condition,

$\omega(0) = \omega_0$, the output is:

$$\omega(t) = AV_a\left(1 - e^{-\frac{t}{\tau}}\right) + \omega_0 e^{-\frac{t}{\tau}}, \; t \geq 0$$

$$= AV_a - AV_a e^{-\frac{t}{\tau}} + \omega_0 e^{-\frac{t}{\tau}}$$

$$= AV_a + (\omega_0 - AV_a)e^{-\frac{t}{\tau}}$$

Ans to the Q. No. 1(c)

From 1(b), $\omega(t) = AV_a + (\omega_0 - AV_a)e^{-\frac{t}{\tau}}$

Since $\lim_{t\to\infty} e^{-\frac{t}{\tau}} = 0$,

$$\lim_{t\to\infty} \omega(t) = \lim_{t\to\infty}\left(AV_a + (\omega_0 - AV_a)e^{-\frac{t}{\tau}}\right)$$

$$= \lim_{t\to\infty} AV_a + \lim_{t\to\infty}(\omega_0 - AV_a)e^{-\frac{t}{\tau}}$$

$$= AV_a + 0$$

$$= AV_a$$

In 1(b), using the final value theorem the steady-state value for a step input of amplitude V_a is: $\omega(\infty) = AV_a$.

So, the result from 1(b) matches exactly with the steady-state value found in 1(a).

Ans. to the Q. No. 1(d)

From 1(b), for a step at time $t \geq 0$ from input V_a with initial condition $\omega(0) = \omega_0$, the solution was:

$$\omega(t) = AV_a + (\omega_0 - AV_a)e^{-\frac{t}{\tau}}$$

Now, we make the motor with constant input V_{min} for a long time ($t_1 >> \tau$), so the system reaches steady state:

$$\omega_s = \lim_{t\to\infty} \omega(t) = AV_{min} \ldots (i)$$

At $t = t_1$, the input changes from V_{min} to V_{max}. The new steady state (as it was after the change) is: $\omega_{ss} = AV_{max} \ldots (ii)$

Given,

$$\Delta V = V_{max} - V_{min},$$

$$\Delta\omega = \omega_{ss} - \omega_s$$

<from eq (i) and (ii)>

$$\Delta\omega = AV_{max} - AV_{min}$$

$$\Delta\omega = A(V_{max} - V_{min})$$

$$\Delta\omega = A\Delta V$$

$$\therefore A = \frac{\Delta\omega}{\Delta V} \text{ [Proved]}$$

Ans to the Q. No. 2

The formula derived in question 1 is: $A = \frac{\Delta\omega}{\Delta V}$

The input step change, $\Delta V = V_{max} - V_{min}$

" output " " , $\Delta\omega = \omega_{ss} - \omega_s$

From the graph,

$$V_{min} = 1\ V$$

$$V_{max} = 5\ V$$

$$\omega_s = 25\ rad/s$$

$$\omega_s = 5\ rad/s$$

$$\therefore \Delta V = V_{max} - V_{min} = (5 - 1)\ V = 4\ V$$

$$\Delta\omega = \omega_{ss} - \omega_s = (25 - 5)\ rad/s = 20\ rad/s$$

$$\therefore A = \frac{\Delta\omega}{\Delta V} = \frac{20}{4} = 5\ \frac{rad/s}{V}$$

Ans to the Q. No. 3

From Fig 1, $\Delta\omega = \omega_{ss} - \omega_s (25 - 5)\ rad/s = 20\ rad/s \ldots (i)$

A first-order system reaches 63.2% of the total change at one time constant:

$$\omega(t_1) = \omega_s + \underbrace{0.632\,\Delta\omega}_{\text{<from eq (i) and Fig 1>}}$$

$$\omega(t_1) = 5 + 0.632(20)$$

$$= 17.64\ rad/s$$

This is the output value at exactly one time constant after the step.

From the definition, $\tau = t_1 - t_0$. So, time constant is the horizontal distance between t_0 (the moment the input step from V_{min} to V_{max}) and t_1 (the moment the output reaches 17.64 rad/s).

From Fig 1, $t_0 = 0.5s$, $t_1 = 1s$ respectively.

$\therefore \tau = t_1 - t_0 = 1s - 0.5s = 0.5s$.

$\therefore$ The time constant for the system in Fig 1 is $0.5s$.

Ans to the Q. No. 4

Since the motor is modelled as a first-order system, the transfer function always takes the form:

$$G_o(s) = \frac{\Omega(s)}{V(s)} = \frac{A}{\tau s + 1} = \frac{5}{0.5s + 1} \quad \text{[from Q.2, } A = 5 \text{ and from Q.3, } \tau = 0.5s\text{]}$$

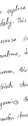
$$= \frac{(100)(s+5)}{0.5s(s+20)}$$

$$= \frac{100}{s + 20}$$

$$\therefore G_o(s) \text{ in terms of } s \text{ is: } \frac{100}{s+20}$$

Ans to the Q. No. 5

The figure for problem 2.3(a) in Chapter 4 in 3rd ed.:



The figure is the response for a unit step input (amplitude = 2), a first-order response. Final (steady state) value = 2.

For a unit step input, $\omega_s = K \cdot 1 = K$.

From the graph, the steady state value is about $2 \Rightarrow K = 2$.

For a first-order system: $\omega(t) = K(1 - e^{-\frac{t}{\tau}})$

At one time constant: $\omega(\tau) = 0.632K$

Since $K = 2$, $\omega(\tau) = (0.632)(2) = 1.264$.

The response reaches 1.264 at roughly $\tau = 0.625s$.

$\therefore \tau = 0.625s$.

A third-order transfer function is: $G(s) = \frac{K}{\tau s + 1}$

$$\langle K = 2, \tau = 0.625 \rangle$$

$$= \frac{2}{0.625s + 1}$$

$$= \frac{(2)(1.6s + 1.25)}{0.625(1.6s + 1.25)}$$

$$= \frac{3.2}{1.6s + 1.25}$$

$$= \frac{32}{16s + 12.5}$$

$$\therefore \text{The transfer function of the system is } \frac{32}{16s + 12.5}$$

Ans to the Q. No. 6

It is preferable to derive a transfer function experimentally when analyzing directly with the physical plant would be risky, expensive, or impractical. For example, if the system is dangerous, such as a small DC motor with limited thermal capacity, repeated step tests or aggressive calibration testing in the hardware could overheat or damage the motor. Instead, one can measure the response experimentally and fit a simple model (like a first-order system) from the data, allowing the system experimentally once, building a reliable transfer function, and then performing all calibration, design and tuning in simulation. Moreover, engineers often rely this approach also speeds up development because iterations can be run repeatedly without damaging the hardware. In short, experimental transfer functions, damage, or likely consequences.

However, this approach is not preferable when the plant exhibits strong nonlinearities, time-varying behaviour, or operating ranges that vary significantly from the identified transfer function. In such cases, a single experimental transfer function would be misleading, and the system would need to be repeatedly tested and remodelled for different operating points, making the approach time-consuming and impractical. In contrast, when the system is experimentally tested, linear model, is cost saving because the calculation directly, or the physical system is necessary to replace the linear behaviour and accurate stability and performance under real operating conditions.