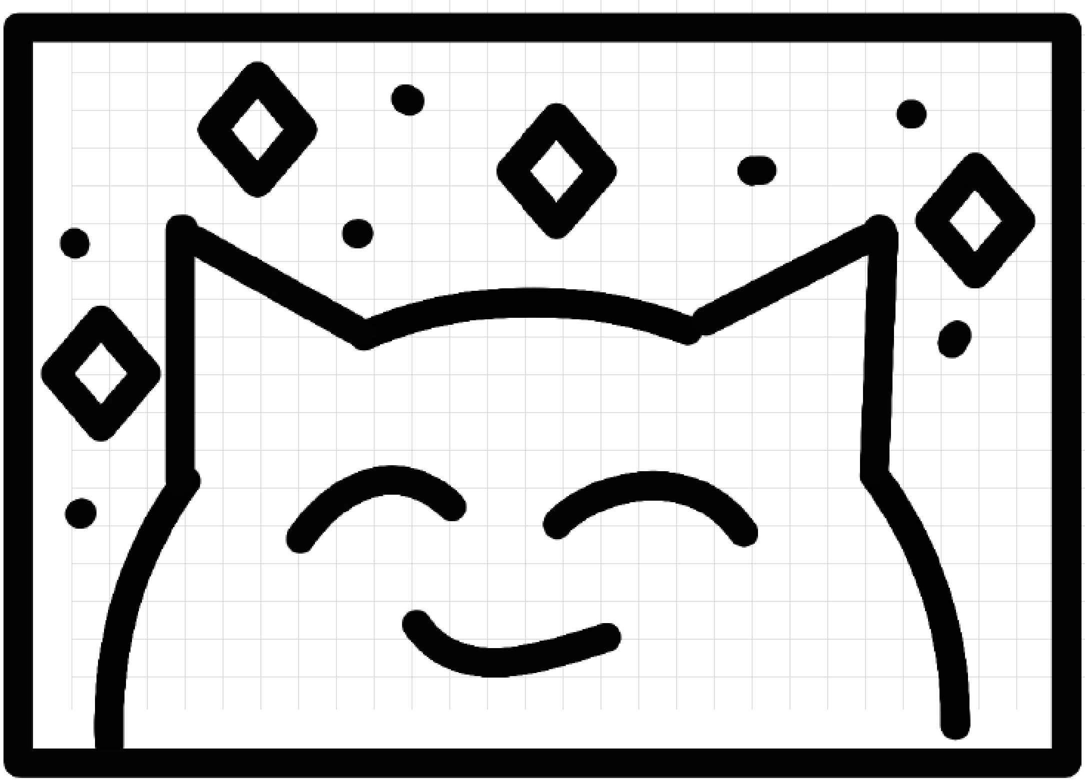
MAE 112

HW 5

TRIET



1. Consider one-stage of a compressor with an 8% static pressure rise across the rotor followed by another 9% pressure rise across the stator (compounded to be 17.7%). The incoming flow has a velocity of 75 ft/sec in the axial direction, a temperature of 560°R and a pressure of 2.0 atmospheres. $\gamma = 1.4$; $c_p = 0.24$ Btu/1bm°R; polytropic efficiency = 0.95 for the compressor stage. (a) What is the power per unit mass flow of the compressor? (b) If the rotor blade speed averages 1000 ft/sec, what is the tangential component of velocity exiting the rotor?

(a)

$$\frac{P_2}{P_1} = \frac{117.7\,\%}{100\,\%} = 1.177$$

$$\eta = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_2}{T_1} - 1} \Rightarrow 0.95 = \frac{1.177^{\frac{1.4-1}{1.4}} - 1}{\frac{T_2}{560} - 1}$$

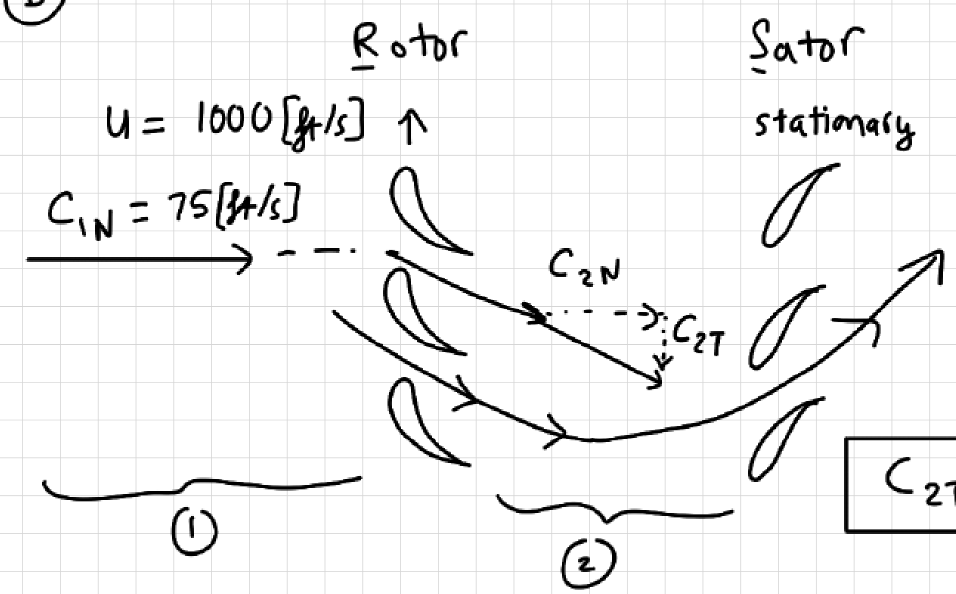
$$\Rightarrow T_2 = 588.0964 \; [°R]$$

$$\frac{P}{\dot{m}} = c_p(T_2 - T_1) = 0.24\left[\frac{Btu}{lbm\,\cancel{°R}}\right](588.0964 - 560)[\cancel{°R}]$$

$$= 6.743136\left[\frac{\cancel{Btu}}{\cancel{lbm}}\right]\frac{778\;ft\cdot\cancel{lbf}}{1\;\cancel{Btu}} \times \frac{1\,\cancel{lbm} \times 32.2\,\frac{ft}{s^2}}{1\,\cancel{lbf}}$$

$$\boxed{\frac{P}{\dot{m}} = 168926.3458\left[\frac{ft^2}{s^2}\right]}$$

(b)



$$\frac{P}{\dot{m}} = U(C_{2T} - \cancel{C_{1T}})$$

$$C_{2T} = \frac{P}{\dot{m}} \div U$$

$$= \frac{168926}{1000}$$

$$\boxed{C_{2T} = 168.93\;[ft/s]}$$

2. Suppose a particular compressor has a compression ratio $P_2/P_1 = 15$ and the incoming air temperature is 300K. If the adiabatic efficiency is .95, what is (a) the final temperature, (b) the average polytropic efficiency, and (c) the entropy change? (d) What is the power required, if 25 kgm per sec. flow through the compressor?

(a)

$$\eta = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{1.4-1}{1.4}} - 1}{\frac{T_2}{T_1} - 1} \qquad 0.95 = \frac{15^{\frac{1.4-1}{1.4}} - 1}{\frac{T_2}{300} - 1}$$

$$\Rightarrow \boxed{T_2 = 668.7898\ [K]}$$

(b)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}}$$

$$\frac{668.7898}{300} = 15^{\frac{1}{e} \times \frac{1.4-1}{1.4}}$$

$$\boxed{e = 96.51\ \%}$$

(c)

$$C_p = 1004.5\ J/kg-k$$

$$\Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$= 1004.5 \ln\left(\frac{668.7898}{300}\right) - 287 \ln(15)$$

$$\boxed{\Delta s = 28.0845\ [J/kg \cdot k]}$$

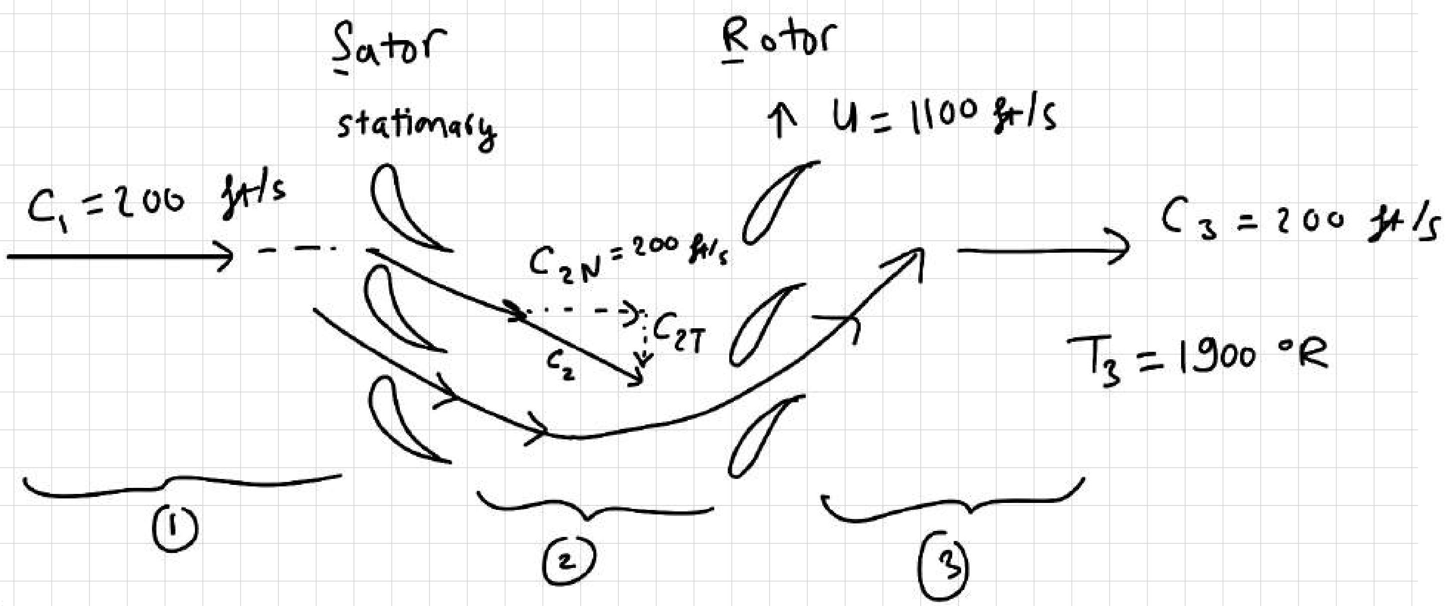
(d)

$$P = \dot{m} c_p (T_2 - T_1) = 25 \times 1004.5 \times (668.7898 - 300)$$

$$\boxed{P = 9261233.853\ [W]}$$

3. Consider a turbine stage that has a polytropic efficiency of 0.95 for the stator (nozzle) and for the rotor flow. 30% of the total static enthalpy drop through the stage occurs in the rotor portion. The initial and final velocities for the stage are axial and have no swirl (tangential component). Assume that only the tangential component of velocity changes through the stator (nozzle) portion. The flow has $\gamma = 1.3$ and $c_p = 0.30$ Btu/1bm°R; the incoming flow has a static temperature of 2400°R, a static pressure of 25 atmospheres, and a velocity of 200 ft/sec. The average rotational velocity of the rotor blade is 1100 ft/sec. The flow exiting the stage has a temperature of 1900°R.

(a) What are the enthalpy drop and pressure drop across the stator?
(b) What is the tangential velocity at the position between the stator and the rotor measured in a frame of reference fixed to the stator?
(c) What is the Mach number of the flow at the position between the stator and rotor measured in a frame of reference fixed to the rotor?
(d) What are the enthalpy drop and the pressure drop across the rotor?
(e) What is the power output per unit mass flux?



ⓐ Total enthalpy drop:

$$h_1 - h_3 = c_p (T_1 - T_3) = 0.3 \frac{\text{Btu}}{\text{lbm}^\circ\text{R}} (2400 - 1900)\ ^\circ\text{R}$$

$$h_1 - h_3 = 150 \left[\frac{\text{Btu}}{\text{lbm}}\right]$$

Sator enthalpy drop:

$$\boxed{h_1 - h_2 = 70\% (h_1 - h_3) = 70\% \times 150 = 105 \left[\frac{\text{Btu}}{\text{lbm}}\right]}$$

Temperature at 2

$$h_1 - h_2 = c_p (T_1 - T_2)$$

$$105 = 0.3 \ (2400 - T_2)$$

$$T_2 = 2050 \, [°R]$$

Pressure at 2

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{e} \frac{\gamma}{\gamma - 1}} \Rightarrow \frac{P_2}{25} = \left(\frac{2050}{2400}\right)^{\frac{1}{0.95} \cdot \frac{1.3}{1.3-1}}$$

$$P_2 = 12.18 \, [atm]$$

Pressure difference across stator:

$$\boxed{P_1 - P_2 = 25 - 12.18 = 12.82 \, [atm]}$$

(b) Unit analysis for c_p:

$$C_p = 0.3 \ \frac{\cancel{Btu}}{\cancel{lbm} °R} \times \frac{778 \ ft \ \cancel{lbf}}{1 \ \cancel{Btu}} \times \frac{1 \ \cancel{lbm} \ 32.2 \ \frac{ft}{s^2}}{1 \ \cancel{lbf}} = 7515.48 \left[\frac{ft^2}{s^2 \, °R}\right]$$

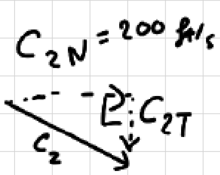
Enthalpy conservation:

$$c_p T_1 + \frac{1}{2} C_1^2 = C_p T_2 + \frac{1}{2} C_2^2$$

$$7515.48 \times 2400 + \frac{1}{2} \times 200^2 = 7515.48 \times 2050 + \frac{1}{2} C_2^2$$

$$C_2 = 2302.3544 \, [ft/s]$$

Tagential component of C_2:



$$C_{2T} = \sqrt{C_2^2 - C_{2N}^2} = \sqrt{2302.35^2 - 200^2}$$

$$\boxed{C_{2T} = 2293.65 \, [ft/s]}$$

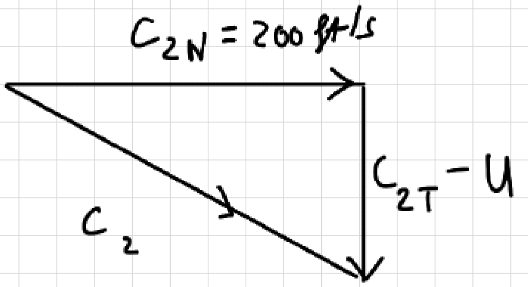
(c) Gas constant:

$$R = C_p \frac{\gamma - 1}{\gamma} = 7515.48 \left[\frac{ft^2}{s^2\,{}^\circ R}\right] \frac{1.3-1}{1.3} = 1734.34 \left[\frac{ft^2}{s^2\,{}^\circ R}\right]$$

Sound velocity:

$$c = \sqrt{\gamma R T_2} = \sqrt{1.3 \times 1734.34 \times 2050} = 2150 \ [ft/s]$$

Flow velocity at (2):



$$C_2 = \sqrt{C_{2N}^2 + (C_{2T} - u)^2}$$
$$= \sqrt{200^2 + (2293.65 - 1100)^2}$$
$$= 1210.29 \ [ft/s]$$

Mach number: $\boxed{M = \frac{c_2}{c} = \frac{1210.29}{2150} = 0.563 \ [1]}$

(d) Enthalpy drop:

$$\boxed{h_2 - h_3} = 30\% (h_1 - h_3) = 30\% \times 150 = \boxed{45 \left[\frac{Btu}{lbm}\right]}$$

Pressure at (3)

$$\frac{P_3}{P_1} = \left(\frac{T_3}{T_1}\right)^{\frac{1}{e}\frac{\gamma}{\gamma-1}} \Rightarrow \frac{P_3}{25} = \left(\frac{1900}{2400}\right)^{\frac{1}{0.95}\cdot\frac{1.3}{1.3-1}}$$

$$P_3 = 8.613 \ [atm]$$

Pressure difference across stator:

$$\boxed{P_2 - P_3 = 12.18 - 8.613 = 3.567 \ [atm]}$$

(e) Power per unit mass flux:

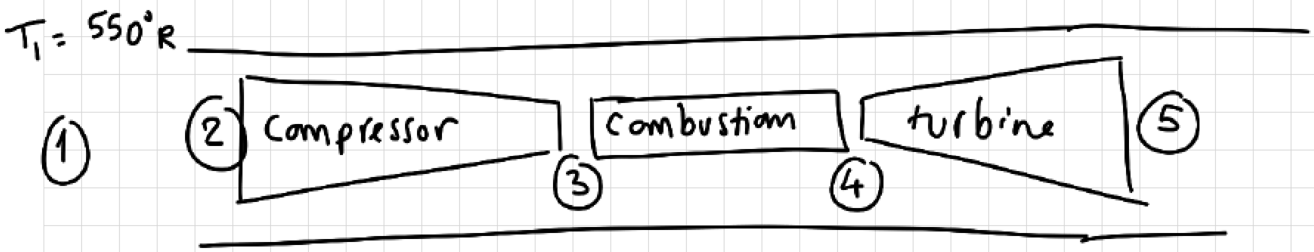
$$\frac{P}{\dot{m}} = u\left(C_{2t} - \cancel{C_{3t}}\right)$$

$$= u\left(C_{2t} + u\right)$$

$$= 1100\left(2293.65 + 1100\right)$$

$$\boxed{\frac{P}{\dot{m}} = 3733015 \left[\frac{m^2}{s^2}\right]}$$

4. Suppose we had a gas turbine engine driving a propeller. Consider takeoff only where flight velocity is 120 ft/sec. Consider the product of gearbox efficiency and propeller efficiency to be 0.8. The pressure ratio across the compressor is ten and the pressure ratio across the turbine is ten. The pressure drops across the combustor and the nozzle are negligible. The fuel heating value is 10,000 Btu/1bm and the mixture ratio is 28. For air or products, consider $\gamma = 1.4$ and $c_p = 0.24$ Btu/1bm°R. Ambient temperature is 550°R. (a) What is the propeller power per unit mass flow of air? (b) What is the propeller thrust per unit mass flow of air? Assume isentropic compression and expansion.



(a)

Temperature at (2): $T_2 = T_1 = 550\,[°R]$

Temperature at (3):

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_3}{550} = (10)^{\frac{1.4-1}{1.4}}$$

$$\Rightarrow T_3 = 1061.88\,[°R]$$

Temperature at (4):

Mixture ratio:

$$\frac{\dot m_{air}}{\dot m_{fuel}} = 28 \Rightarrow \dot m_a = 28\,\dot m_f \quad (*)$$

Energy conservation:

$$(\dot m_a + \dot m_f)\,c_p T_4 - \dot m_a c_p T_3 = \overset{1}{\cancel{\eta_b}}\,\dot m_f Q$$

$$(*) \Rightarrow (28\cancel{\dot m_f} + \cancel{\dot m_f})\,c_p T_4 - 28\cancel{\dot m_f}\,c_p T_3 = \cancel{\dot m_f}\,Q$$

$$29\, C_p T_4 - 28\, C_p T_3 = Q$$

$$29 \times 0.24 \times T_4 - 28 \times 0.24 \times 1061.88 = 10\,000$$

$$\Rightarrow T_4 = 2462.045 \; [°R]$$

Temperature at ⑤

$$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_5}{2462.045} = \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}}$$

$$\Rightarrow T_5 = 1275.21 \; [°R]$$

Propeller power:

$$P_{propeller} + P_{compressor} = P_{turbine}$$

$$P_p + \dot{m}_a C_p (T_3 - T_2) = (\dot{m}_a + \dot{m}_f) C_p (T_4 - T_5)$$

$$P_p + \dot{m}_a\, 0.24\,(1061.88 - 550) = \left(\dot{m}_a + \frac{1}{28}\dot{m}_a\right) 0.24\,(2462.045 - 1275.21)$$

$$P_p + 122.8512\, \dot{m}_a = 295.013\, \dot{m}_a$$

$$P_p = 172.162\, \dot{m}_a$$

$$\frac{P_p}{\dot{m}_a} = 172.162$$

$$\frac{P_{propeller}}{\dot{m}_a} = \frac{P_p}{\dot{m}_a} \times 0.8 = 172.162 \times 0.8$$

$$\boxed{\frac{P_{propeller}}{\dot{m}_a} = 137.73 \quad \left[\frac{Btu}{lbm}\right]}$$

(b)

$$\frac{P_{propeller}}{\dot{m}_a} = \frac{T}{\dot{m}_a} \times u$$

$$137.73 \left[\frac{Btu}{lbm}\right] = \frac{T}{\dot{m}_a} \times 120 \left[\frac{ft}{s}\right]$$

$$\frac{T}{\dot{m}_a} = \frac{137.73}{120}$$

$$= 1.14775 \left[\frac{Btu}{lbm} \times \frac{s}{ft}\right]$$

Unit analysis:

$$\frac{T}{\dot{m}_a} = 1.14775 \ \frac{\cancel{Btu}}{\cancel{lbm}} \cdot \frac{\cancel{s}}{\cancel{ft}} \cdot \frac{778 \ \cancel{ft} \ \cancel{lbf}}{1 \ \cancel{Btu}} \ \frac{32.2 \ \cancel{lbm} \ \frac{ft}{s^{\cancel{2}}}}{1 \ \cancel{lbf}}$$

$$\boxed{\frac{T}{\dot{m}_a} = 28752.9739 \left[\frac{ft}{s}\right]}$$