Lecture 14
Turboprops and Turbofans

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Turboprop engines

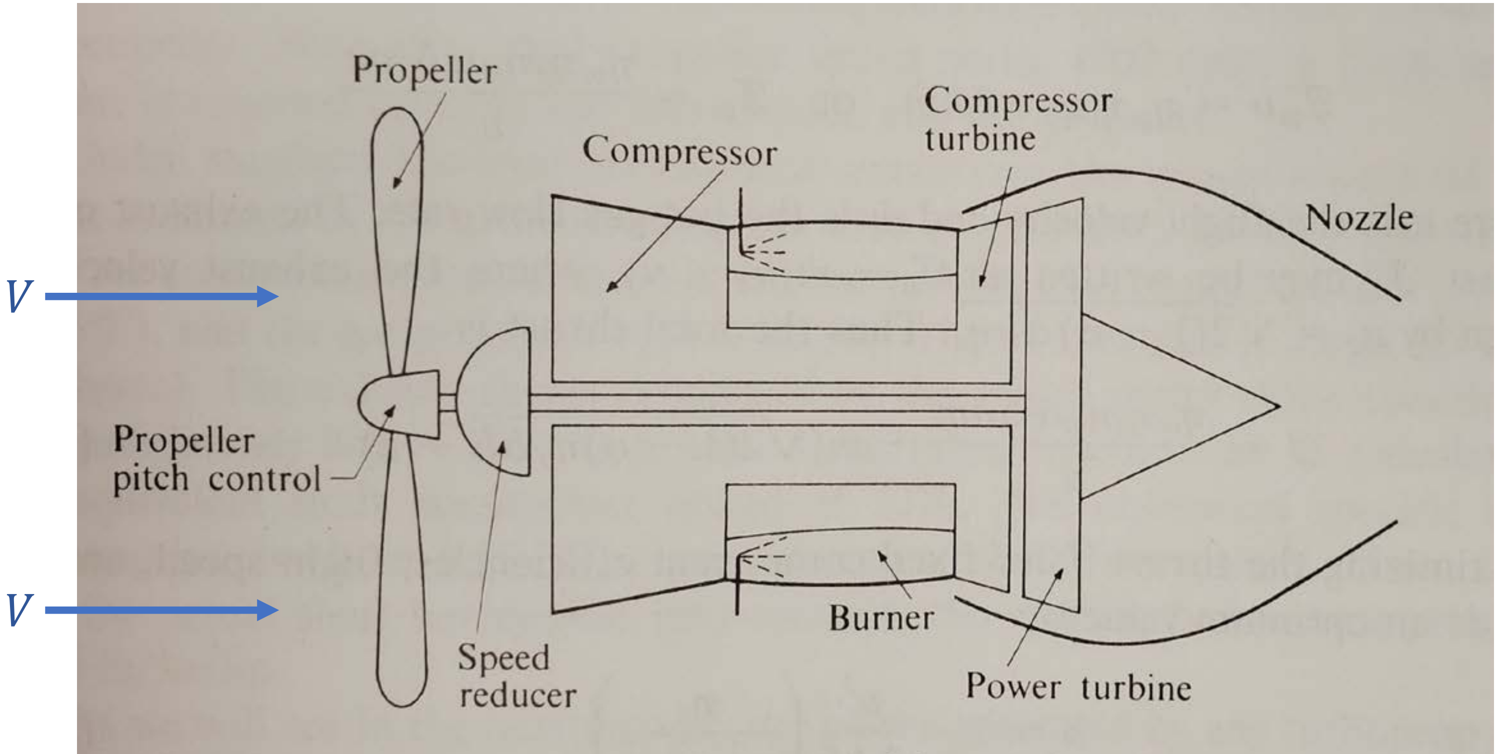
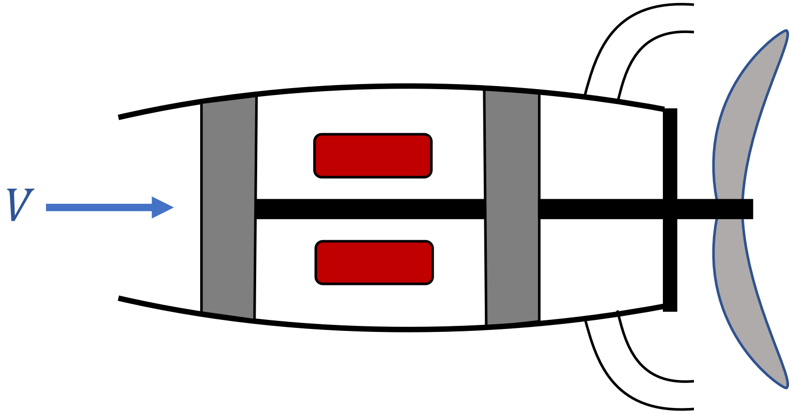


FIGURE 5.35 Schematic diagram of a typical free-turbine turboprop engine.

Turboprop engine from Hill & Peterson [1]

Turboprop engine – Secondary flow gives same advantage as ducted fan but the propeller has variable pitch (more thrust at takeoff). Thrust occurs due to propeller and due to jet!

Note: The propeller could be in the rear of the propulsion unit [propfan] when exhaust velocity is low.



Turboprop engines

$$\left(\dot{m}_a - \dot{m}_{bleed} + \dot{m}_f\right)H_t = \dot{m}_a H_c + \left(\eta_{prop}\eta_{gear}\right)^{-1} T_{prop} V$$

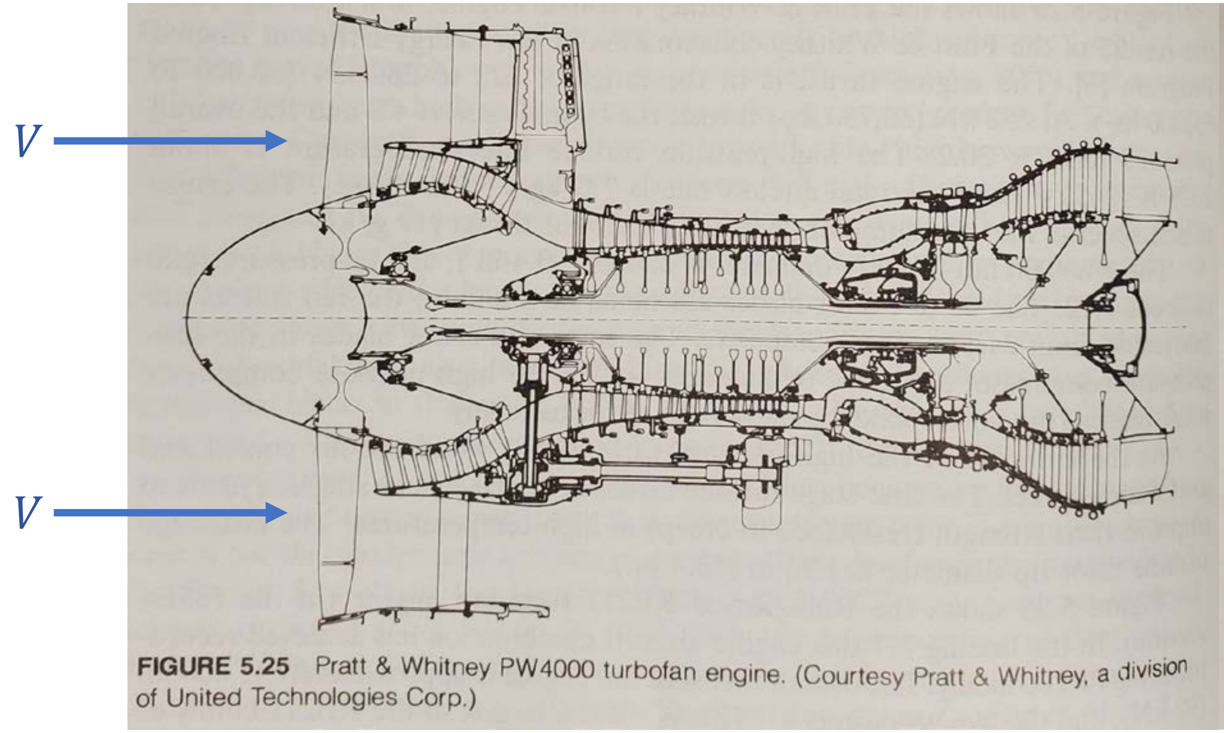
η_{prop} is the propeller efficiency, η_{gear} is the gearbox efficiency (gearing from turbine to propeller) T_{prop} is the propeller thrust, V is the flight velocity. Thrust can be produced by two methods of momentum transfer: reaction to propeller force on the external air and reaction to the increase of gas velocity and pressure difference through the engine core.

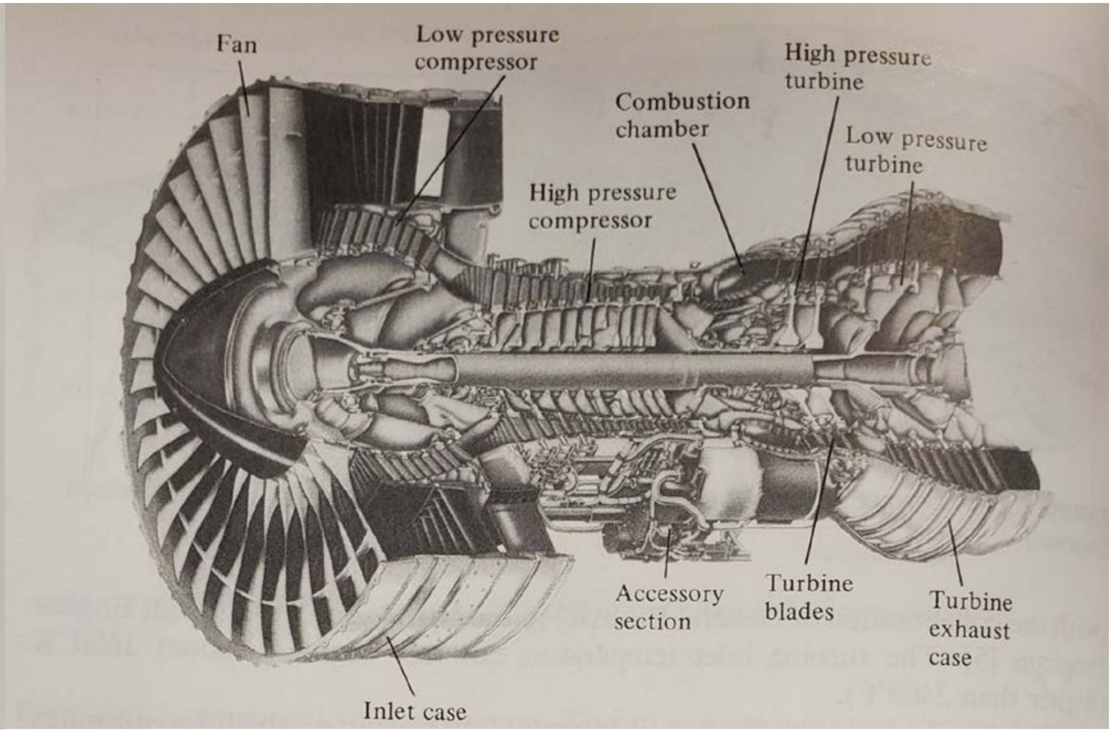
Now: $$T = T_{prop} + \left(\dot{m}_a - \dot{m}_{bleed} + \dot{m}_f\right)U - \dot{m}_a V + (P_e - P_a)A_e$$

At high flight velocity, the propeller tips can move at supersonic speed due to the vector combination of rotational speed and flight speed. This will produce shock waves and large losses in efficiency.

Note: If we shroud the turboprop, we have a turbofan engine. The diffuser then slows down the flow before the propeller blades and decreases losses!

Turbofan engines

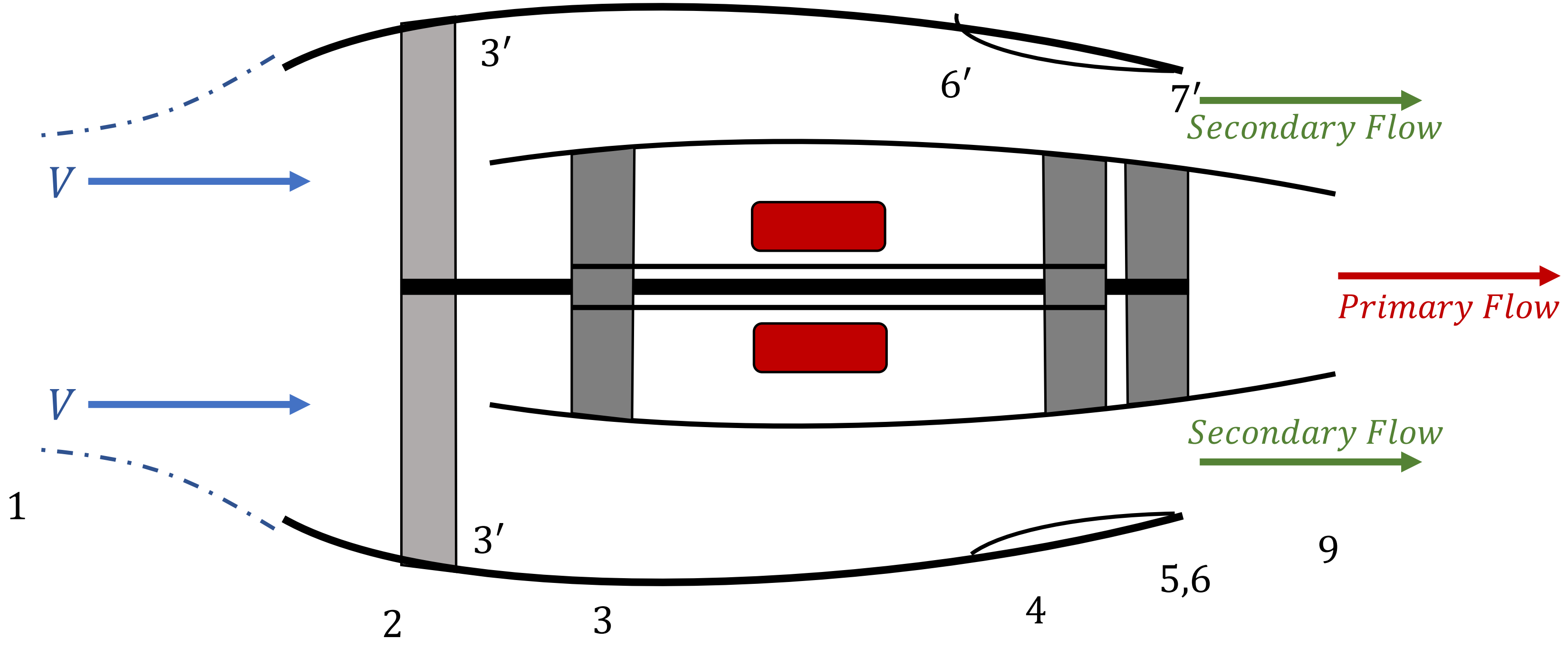




Turbofan engine from Hill & Peterson [1]

First, there is no combustion in the secondary (bypass) flow and two flows are not mixed. There is also no afterburner in this case.

Turbofan engines



Turbine power is used for both fan and compressor. Both primary and secondary flows go through fan while only the primary flow goes through the compressor. Generally, fan and compressor are identical in rotation.
Note: prime superscripts are used for positions in secondary (i.e., bypass) flow.

Bypass Ratio: $\beta = \dfrac{Secondary\ mass\ flow\ rate}{Primary\ mass\ flow\ rate}$

Turbofan engines

Recall $\bar{\mu} \equiv \mu - \frac{\dot{m}_{bleed}}{\dot{m}_{fuel}}$

$$(\bar{\mu}+1)H_t = \mu H_c + \beta\mu c_{p_d}\left(T_{3'}^{\circ} - T_2^{\circ}\right) = (\bar{\mu}+1)c_{p_t}\left(T_4^{\circ} - T_5^{\circ}\right)$$

Note the * equation

More turbine work is required on account of the fan so that T_5° and the primary flow exhaust velocity are less than for a turbojet engine. Extra energy is in the secondary flow. We sacrifice some kinetic energy in the exhaust of the core engine to gain more mass flow through the bypass flow.

As before for a turbojet and assuming small Mach number in the combustor:

$$T_2^{\circ} = T_1^{\circ} = T_1\left(1 + \frac{\gamma_d - 1}{2}M_1^2\right)$$

U_p is primary exhaust velocity

$$U_p^2 = 2\left(c_{p_n}T_4^{\circ} - H_t\right)\left[1 - \left(1 + \frac{\gamma_d - 1}{2}M_1^2\right)^{-\delta\eta_n\eta_d} * \left(1 + \frac{H_c}{c_{p_d}T_2^{\circ}}\right)^{-\eta_n\eta_c\delta} * \left(1 - \frac{H_t}{c_{p_n}T_4^{\circ}}\right)^{-\frac{\delta'\eta_n}{\eta_t}}\right]$$

* Relation for H_t is different:

$$H_t = \left(\frac{\mu}{\bar{\mu}+1}\right)c_{p_d}T_2^{\circ}\left[\left(\frac{P_3^{\circ}}{P_2^{\circ}}\right)^{\frac{\gamma_d-1}{\gamma_d\eta_c}} - 1\right] + \left(\frac{\beta\mu}{\bar{\mu}+1}\right)c_{p_d}T_2^{\circ}\left[\left(\frac{P_{3'}^{\circ}}{P_2^{\circ}}\right)^{\frac{\gamma_d-1}{\gamma_d\eta_f}} - 1\right]$$

Turbofan engines

The relation for $\bar{\mu}$ is the same as for the ramjet and turbojet:

$$\bar{\mu} = \frac{\eta_b Q - h\left(T_4^\circ, products\right)}{h\left(T_4^\circ, products\right) - h\left(T_3^\circ, air\right)}$$

$$\boxed{\begin{aligned} \delta &\equiv \frac{\gamma_d}{\gamma_d - 1}\frac{\gamma_n - 1}{\gamma_n} \\ \delta' &\equiv \frac{\gamma_t}{\gamma_t - 1}\frac{\gamma_n - 1}{\gamma_n} \end{aligned}}$$

From above, we determine:

$$U_p = U_p\left(c_{p_n}, c_{p_d}, \gamma_n, \gamma_d, \eta_d, \eta_c, \eta_f, \eta_t, \eta_n, \eta_b Q, fuel\ section, \beta, M_1, T_1, T_4^\circ, P_3^\circ/P_2^\circ, P_{3'}^\circ/P_2^\circ\right)$$

Now, we must determine the secondary flow:

$$U_{secondary\ flow} = U_s$$

$$U_s^2 = 2c_{p_d}T_{3'}^\circ\left[1 - \left(\frac{P_{7'}}{P_{3'}^\circ}\right)^{\frac{\gamma_d - 1}{\gamma_d}\eta_{ns}}\right]$$

η_{ns} is polytropic efficiency for the secondary (bypass) nozzle

Turbofan engines

Assume perfect expansion across the secondary nozzle:

Then: $P_{7'} = P_1$

$$U_s^2 = 2c_{p_d} T_2^\circ \left(\frac{P_{3'}^\circ}{P_2^\circ}\right)^{\frac{\gamma_d - 1}{\gamma_d \eta_f}} \left[1 - \left(\frac{P_1}{P_2^\circ}\right)^{\frac{\gamma_d - 1}{\gamma_d}\eta_{ns}} \left(\frac{P_{3'}^\circ}{P_2^\circ}\right)^{-\frac{\gamma_d - 1}{\gamma_d}\eta_{ns}}\right]$$

$$U_s^2 = 2c_{p_d} T_1 \left(1 + \frac{\gamma_d - 1}{2} M_1^2\right) \left(\frac{P_{3'}^\circ}{P_2^\circ}\right)^{\frac{\gamma_d - 1}{\gamma_d \eta_f}} \left[1 - T_1 \left(1 + \frac{\gamma_d - 1}{2} M_1^2\right)^{-\eta_{ns}\eta_d} \left(\frac{P_{3'}^\circ}{P_2^\circ}\right)^{-\frac{\gamma_d - 1}{\gamma_d}\eta_{ns}}\right]$$

So U_s is readily determined. U_s is also less than U_p.

Now, considering both exhausts, thrust can be calculated!

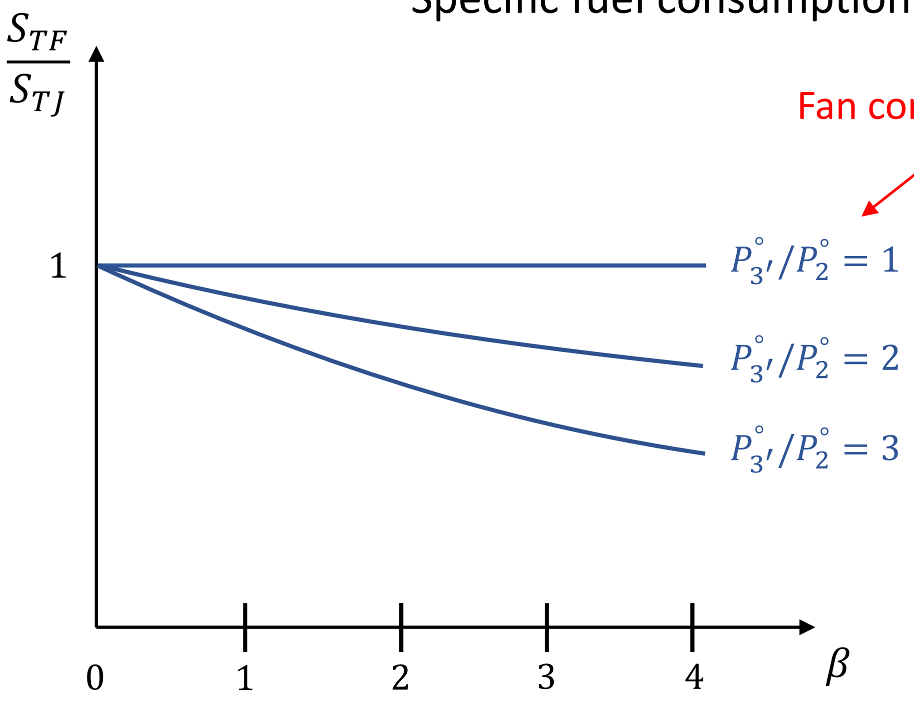
Turbofan engines

$$\frac{T}{\dot{m}_f} = (1+\bar{\mu})U_p + \beta\mu U_s - \mu(1+\beta)V + \frac{\left(P_{e_p} - P_a\right)A_{e_p}}{\dot{m}_f} + \frac{\left(P_{e_s} - P_a\right)A_{e_s}}{\dot{m}_f}$$

$$S = \frac{3600g}{(1+\bar{\mu})U_p + \beta\mu U_s - \mu(1+\beta)V + \frac{\left(P_{e_p} - P_a\right)A_{e_p} + \left(P_{e_s} - P_a\right)A_{e_s}}{\dot{m}_f}}$$

Turbofan engines

Specific fuel consumption compared to turbojet with same core flow



Approximately 30% decrease for $\beta = 4,\ \ P^\circ_{3'}/P^\circ_2 = 3$

U_p decreases as β increases, while U_s increases with β

Note that jet noise power goes as $(exhaust\ velocity)^8$ so placing energy into the larger mass flow results in less jet noise! Compressor noise still remains.

Drag and weight increase as β increases. Drag increases as engine size increases.
Weight ∝ volume ∝ diameter squared to diameter cubed!

Fuel can be added to secondary stream and burned, increasing the stagnation temperature and secondary stream exhaust velocity [afterburning]!

Turbofan engines

With burning of the secondary stream:

$$U_s^2 = 2c_{p_{ns}}T^{\circ}_{6'}\left[1 - \left(\frac{P_{7'}}{P^{\circ}_{6'}}\right)^{\frac{\gamma_n - 1}{\gamma_n}\eta_{ns}}\right]$$

$$\frac{P_{7'}}{P^{\circ}_{6'}} = \frac{P_{7'}}{P_1}\left(\frac{P_1}{P^{\circ}_2}\right)\frac{P^{\circ}_2}{P^{\circ}_{3'}}\frac{P^{\circ}_{3'}}{P^{\circ}_{6'}}$$

Diffuser

Fan

Combustor ratio.
Without burning in bypass flow $P^{\circ}_{3'} = P^{\circ}_{6'}$

$$\frac{P_{7'}}{P_1} = 1$$ For perfect expansion

Turbofan engines

So: $$U_s^2 = 2c_{p_{ns}} T^\circ_{7'} \left[1 - \left(1 - \frac{\gamma_d - 1}{2} M_1^2\right)^{-\delta_s \eta_{ns} \eta_d} \left(\frac{P_{7'}}{P^\circ_2}\right)^{-\frac{\gamma_n - 1}{\gamma_n}\eta_{ns}} \left(1 - C_s M_s^2\right)^{-\frac{\gamma_n - 1}{\gamma_n \eta_{ns}}}\right]$$

C_s, M_s

For secondary stream combustor

η_{ns} Polytropic efficiency for secondary flow

$$\delta_s \equiv \frac{\gamma_d}{\gamma_d - 1} \frac{\gamma_{ns} - 1}{\gamma_{ns}} = 1$$ For no combustor

$$\mu_s = \frac{\eta_{bs} Q - h(T_{6'}, products)}{h(T_{6'}, products) - h(T_{3'}, air)}$$

Same relationship for secondary stream mixture ratio

Coupling of engine to aircraft performance

Aircraft range:

The aircraft is in a cruise configuration for most of the flight – no acceleration or deceleration [trimmed flight]

$$Thrust = Drag \quad \text{and} \quad Lift \approx Weight = mg$$

L/D is a characteristic of the aircraft $\qquad \dfrac{mg}{T} = \dfrac{L}{D}$ or $T = \dfrac{mg}{L/D}$

Thrust power is: $T \cdot V$

The propulsive efficiency is: $\eta = \dfrac{TV}{\dot{m}_f Q} = \dfrac{mg}{L/D}\dfrac{V}{\dot{m}_f Q}$

$$\dot{m}_f = -\frac{dm}{dt} = \frac{mgV}{\eta(L/D)Q}$$

Coupling of engine to aircraft performance

$$\frac{1}{mV}\frac{dm}{dt} = -\frac{g}{\eta(L/D)Q}$$

$\boxed{\text{Distance:} \quad ds = Vdt}$

$$\frac{d(\ln m)}{ds} = -\frac{g}{\eta(L/D)Q} = constant$$

Theoretically, g should be taken at altitude which provides a correction in the third significant digit.

$$s_2 - s_1 = \Delta s = \frac{\eta(L/D)Q}{g}\ln\left(\frac{m_1}{m_2}\right)$$

Brequet range formula

$$Range = \Delta s = \frac{\eta(L/D)Q}{g}\ln\left(1 + \frac{M_F}{m_2}\right)$$

$$m_1 = m_2 + M_F$$

Dry weight

Total fuel consumed

Coupling of engine to aircraft performance

We can replace ηQ by $\dfrac{TV}{\dot{m}_f}$

$$Range = \frac{(L/D)}{g}\left[\ln\left(1 + \frac{M_F}{m_2}\right)\right]\frac{TV}{\dot{m}_f}$$

$$S = specific\ fuel\ consumption = \frac{\dot{m}_f g}{T}$$

Brequet range formula

So: $$Range = \frac{(L/D)V}{S}\ln\left(1 + \frac{M_F}{m_2}\right)$$

Therefore, to maximize range, we wish to maximize $(L/D)V$, minimize S, and minimize dry weight m_2!

References

[1] Hill, Philip G., and Carl R. Peterson. *Mechanics and Thermodynamics of Propulsion*. Reading, Mass: Addison-Wesley Longman, 1992.