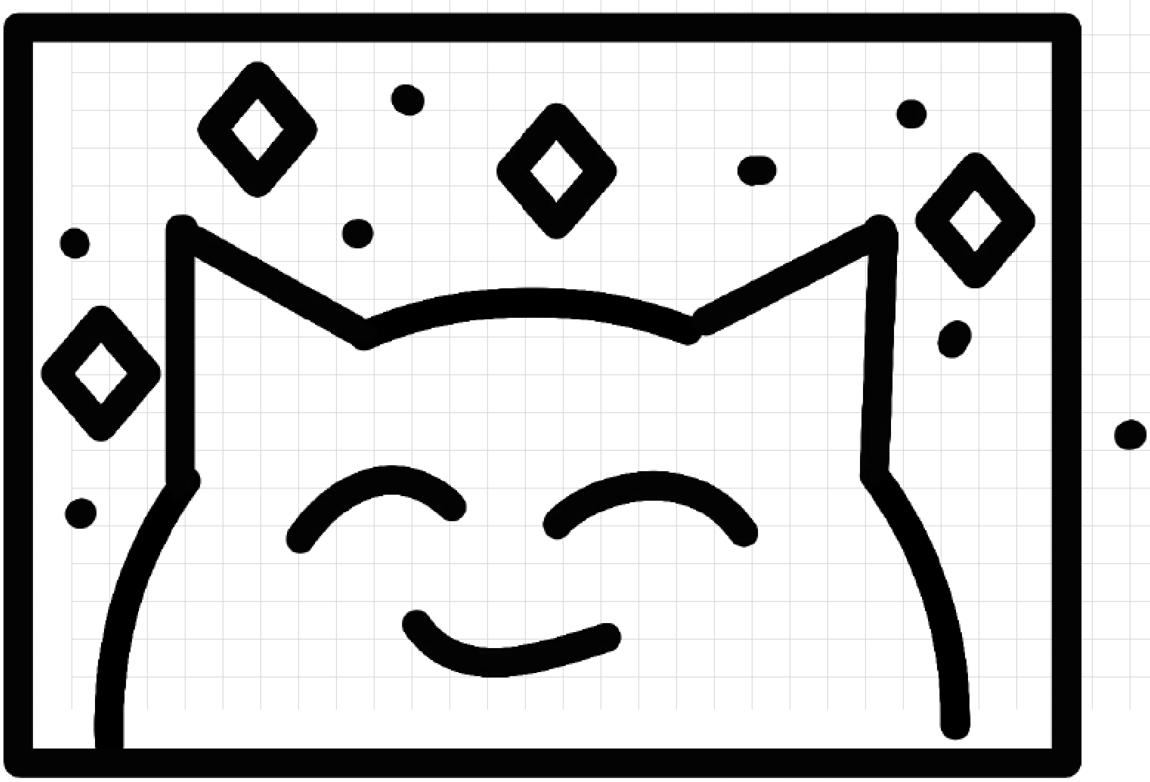
MAE 112

HW 4

TRIET



1. . Compare a normal shock with an oblique shock. Suppose the inflowing velocity of the air had a Mach number of 2.0 at a temperature of 250 K and an ambient pressure of 0.70 atm.

(a) With the normal shock, determine the pressure, stagnation pressure, temperature, velocity, and Mach number behind (downstream of) the shock.

(b) Suppose we aim for a downstream stagnation pressure that is 15% higher than the value found in part (a). What is the angle of oblique shock here to the incoming velocity vector? Use the charts from Chapter 3, making the best interpolations you can.

(c) Determine the downstream values for the temperature, Mach number, velocity component normal to the oblique shock, and velocity component parallel to the oblique shock.

(a)

Stagnation pressure before the shock:

$$\frac{P_{01}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{P_{01}}{0.7} = 7.8244$$

$$\Rightarrow P_{01} = 5.47708 \text{ [atm]}$$

Stagnation temperature before the shock:

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{T_{01}}{250} = 1 + \frac{1.4 - 1}{2} 2^2$$

$$T_{01} = 450 \text{ [K]}$$

Right after the shock.

$M_1 = 2 \qquad \gamma = 1.4$

$$\boxed{M_2 = 0.5774}$$

$$\frac{P_2}{P_1} = 4.5$$

$$\boxed{P_2 = 4.5\, P_1 = 4.5 \times 0.7 = 3.15 \; [atm]}$$

$$\frac{P_{02}}{P_{01}} = 0.7209 \Rightarrow P_{02} = 0.7209\, P_{01} = 0.7209 \times 5.47708$$

$$\boxed{P_{02} = 3.9484 \; [atm]}$$

$$\frac{T_2}{T_1} = 1.6875 \Rightarrow T_2 = 1.6875\, T_1 = 1.6875 \times 250$$

$$\boxed{T_2 = 421.875 \; [K]}$$

$$V_2 = M_2 \sqrt{\gamma R T_2} = 0.5774 \sqrt{1.4 \times 287 \times 421.875}$$

$$\boxed{V_2 = 237.7243 \; [m/s]}$$

(b)

Stagnation pressure after shock wave:

$$P_{02} = 115\%\, P_{02\,(a)} = 115\% \times 3.9484$$

$$P_{02} = 4.54066 \; [atm]$$

Pressure ratio

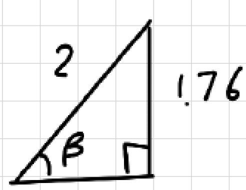
$$\frac{P_{02}}{P_{01}} = \frac{4.54066}{5.47708} = 0.83$$

Mach number before shock way for normal component

$$\gamma = 1.4 \qquad \frac{P_{02}}{P_{01}} = 0.83$$

$$M_{1n} = 1.76$$

$$M_1 = 2$$



$$\beta = \sin^{-1}\left(\frac{M_{1n}}{M_1}\right)$$

$$= \sin^{-1}\left(\frac{1.76}{2}\right)$$

$$\boxed{\beta = 61.64^\circ} \qquad \boxed{\delta = 23^\circ}$$

Ⓒ $M_{1n} = 1.76 \qquad \gamma = 1.4$

Temperature after the oblique shock:

$$\frac{T_2}{T_1} = 1.5019 \Rightarrow T_2 = 1.5019\, T_1 = 1.5019 \times 250$$

$$\boxed{T_2 = 375.475\ [K]}$$

Normal Component of Mach number after shock way:

$$M_{2n} = 0.6257$$

Mach number after oblique shock wave:

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = \frac{0.6257}{\sin(61.64 - 23)} = \boxed{1.0620418}$$

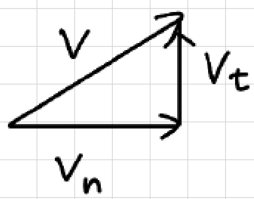
Velocity:

$$V_2 = M_2 \sqrt{\gamma R T_2} = 1\sqrt{1.4 \times 287 \times 375.475}$$

$$= 388.41454\ [m/s]$$

$$V_{2n} = M_{2n} \sqrt{\gamma R T_2} = 0.6257 \sqrt{1.4 \times 287 \times 375.475}$$

$$\boxed{V_{2n} = 243.031\ [m/s]}$$



$$V_{2t} = \sqrt{V_2^2 - V_{2n}^2} = \sqrt{388.4^2 - 243.031^2}$$

$$\boxed{V_{2t} = 303\ [m/s]}$$

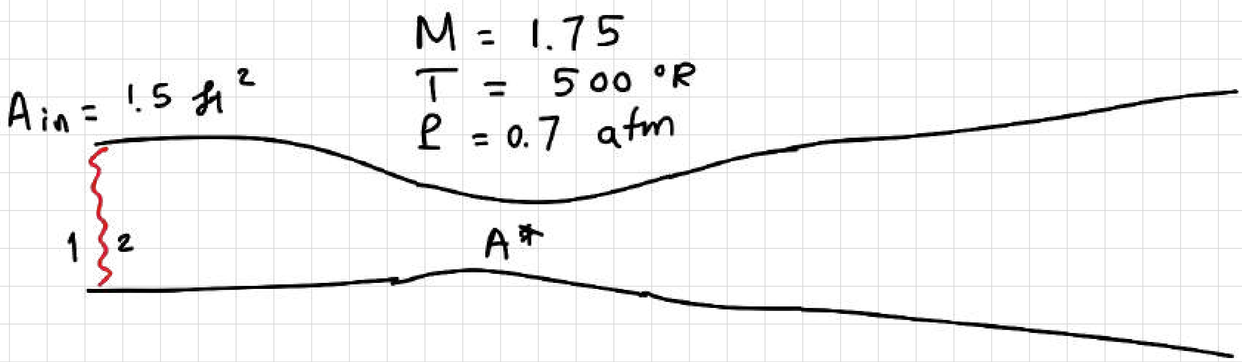
2. Consider a Kantrowitz-Donaldson diffuser designed for a flight Mach number of 1.75. The entrance area equals 1.5 ft^2 and the ambient air temperature and pressure are 500°R and 0.7 atmosphere. The flow is isentropic everywhere except across the normal shockwave. Determine:

(a) the minimum cross-sectional area of the throat such that a normal shock may be stabilized at the entrance,

(b) the maximum mass flow, and

(c) the maximum stagnation pressure possible at the end of the diffuser (with subsonic flow only in the divergent portion).

In each of these optimizations, consider the flight Mach number fixed at the design value while the final pressure (at the end of the diffuser) is allowed to adjust.



(a) $M_1 = 1.75 \Rightarrow M_2 = 0.6281$

$$\frac{A_2}{A^*} = 1.1571$$

$$\Rightarrow A^* = \frac{A_2}{1.1571} = \frac{1.5}{1.1571}$$

$$\boxed{A^* = 1.2963 \ [ft^2]}$$

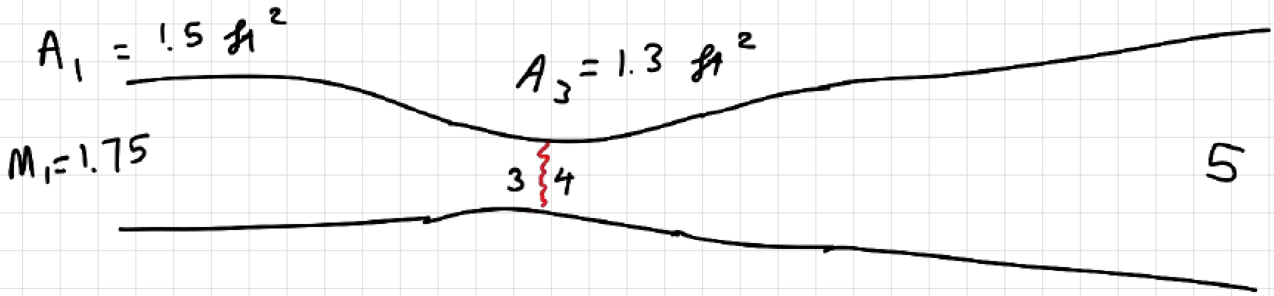
(b)

$$\dot{m} = \rho_1 V_1 A_1 = \frac{P_1}{RT_1}\left(M_1\sqrt{\gamma R T_1}\right) A_1 = \frac{0.7 \times 2116.2}{1718 \times 500}\left(1.75\sqrt{1.4 \times 1718 \times 500}\right) 1.5$$

$$\boxed{\dot{m} = 4.9642 \ [slug/s]}$$

$$\dot{m} = 4.9642 \left[\frac{\cancel{slug}}{s}\right]\left[\frac{32.2 \ lbm}{1 \ \cancel{slug}}\right] = 159.848 \left[\frac{lbm}{s}\right]$$

(c)



$$M_1 = 1.75 \Rightarrow \frac{A_1}{A^*} = 1.3865 \Rightarrow \frac{1.5}{A^*} = 1.3865$$

$$\Rightarrow A^* = 1.08186\ [ft^2]$$

$$\frac{A_3}{A^*} = \frac{1.3}{1.08186} = 1.20163 \Rightarrow M_3 = 1.535$$

Stagnation pressure:

$$\frac{P_{01}}{P_1} = 5.3241 \Rightarrow P_{01} = 5.3241\ P_1 = 5.3241 \times 0.7$$

$$P_{03} = P_{01} = 3.72687\ [atm]$$

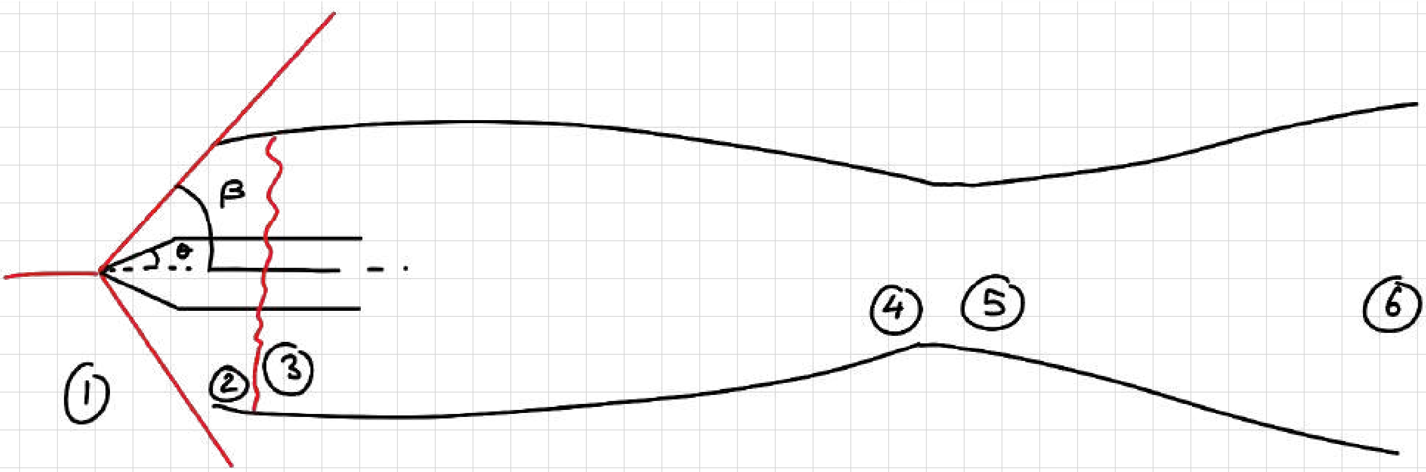
$$M_2 = 1.535 \Rightarrow \frac{P_{04}}{P_{03}} = 0.9183$$

$$\Rightarrow P_{04} = 0.9183 \times 3.72687$$

$$\boxed{P_{05} = P_{04} = 3.4224\ [atm]}$$

3 Consider a ramjet in flight at a Mach number of 2.75 with ambient conditions at 298 K and 0.9 atmosphere of pressure. The air capture area is 0.70 square meters. The inlet design involves first a wedge that deflects the stream by an angle of 15 degrees followed by a Kantrowitz-Donaldson (K-D) diffuser. Operation is at design conditions except for part (h).

(a) What is the mass flow through the ramjet?
(b) What is the stagnation temperature for that flow through the inlet / diffuser?
(c) What are the stagnation-pressure values ahead of and immediately behind the first shock?
(d) What is the flow Mach number immediately behind the first shock? What is the flow Mach number at the entrance to the K-D diffuser?
(e) What is the Mach number at the diffuser throat?
(f) What is the final stagnation pressure?
(g) Determine the value of the polytropic efficiency for this inlet design.
(h) Determine the polytropic efficiency value for a shock at the entrance of the K- D diffuser.



(a)

$$\dot{m} = \rho_1 V_1 A_1 = \frac{P_1}{RT_1} M_1 \sqrt{\gamma R T_1}\, A_1$$

$$= \frac{0.9 \times 101325}{287 \times 298}\, 2.75 \sqrt{1.4 \times 287 \times 298} \times 0.7$$

$$\boxed{\dot{m} = 710.2395 \ [kg/s]}$$

(b)

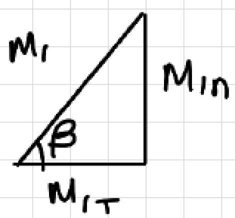
$$M_1 = 2.75 \Rightarrow \frac{T_{01}}{T_1} = 2.5125$$

$T_{01} = 2.5125\, T_1 = 2.5125 \times 298 = \boxed{748.725\,[K]}$

(c)

$$M_1 = 2.75 \Rightarrow \frac{P_{01}}{P_1} = 25.14$$

$$\Rightarrow P_{01} = 25.14\, P_1 = 25.14 \times 0.9 = \boxed{22.626\,[atm]}$$



$M_1 = 2.75 \quad \theta = 15° \Rightarrow \beta \approx 34°$

$M_{1N} = M_1 \sin\beta = 2.75 \times \sin(34°)$

$M_{1N} = 1.53778$

$$\frac{P_{02}}{P_{01}} = 0.9174 \Rightarrow P_{02} = P_{01}\, 0.9174 = 22.626 \times 0.9174$$

$$\boxed{P_{02} = 20.7516\,[atm]}$$

(d)

$$M_{1N} = 1.53778 \Rightarrow M_{2N} = 0.6881$$

$$M_2 = \frac{M_{2N}}{\sin(\beta - \theta)} = \frac{0.6881}{\sin(34° - 15°)} = \boxed{2.1135}$$

(e) Shock wave at entrance:

$$M_{1N} = 1.53778 \Rightarrow \frac{P_2}{P_1} = 2.5922, \quad \frac{T_2}{T_1} = 1.3455$$

$$P_2 = 2.5922\, P_1 = 2.5922 \times 0.9 = 2.333\,[atm] = 2.3639 \times 10^5\,[Pa]$$

$$T_2 = 1.3455 \times T_1 = 1.3455 \times 298 = 401\,[K]$$

$$A_2 = \frac{\dot{m}}{\frac{P_2}{RT_2} M_2 \sqrt{\gamma R T_2}} = \frac{710.2395}{\frac{2.3639 \times 10^5}{287 \times 401} \times 2.1135 \sqrt{1.4 \times 287 \times 401}}$$

$$A_2 = 0.4076\,[m^2]$$

$$M_2 = 2.1135 \Rightarrow M_3 = 0.56 \Rightarrow \frac{A_3}{A^*} = 1.24$$

$$\Rightarrow A^* = \frac{A_3}{1.24} = \frac{A_2}{1.24} = \frac{0.4076}{1.24} = 0.3287\ [m^2]$$

$$A_4 = A^* = 0.3287\ [m^2]$$

Move shockwave to throat:

$M_2 = 2.1135$ $\quad A_2 = 0.4076$ $\quad A_4 = 0.3287$ $\quad M_4 = ?$

$\Downarrow$

$$\frac{A_2}{A^*} = 1.8585 \Rightarrow A^* = \frac{A_2}{1.8585} = \frac{0.4076}{1.8585} = 0.2193\ [m^2]$$

$$\frac{A_4}{A^*} = \frac{0.3287}{0.2193} = 1.5 \Rightarrow \boxed{M_4 = 1.854}$$

(8)

$$P_{04} = P_{02} = 20.7516\ [atm]$$

$$M_4 = 1.854 \Rightarrow \frac{P_{05}}{P_{04}} = 0.7884 \Rightarrow P_{05} = 0.7884 P_{04} = 0.7884 \times 20.7516$$

$$P_{05} = 16.36\ [atm]$$

$$\boxed{P_{06} = P_{05} = 16.36\ [atm]}$$

(9) $M_4 = 1.854 \Rightarrow M_5 = 0.6049$

$$M_5 = 0.6049 \Rightarrow \frac{P_{05}}{P_5} = 1.2804 \Rightarrow P_5 = \frac{P_{05}}{1.2804} = \frac{16.36}{1.2804}$$

$$P_5 = 12.777\ [atm]$$

$$T_2 = 401\ [K]$$

$$M_2 = 2.1135 \Rightarrow \frac{T_{02}}{T_2} = 1.8934 \Rightarrow T_{02} = 1.8934\ T_2$$

$$T_{02} = 1.8934 \times 401 = 759.2534\ [K]$$

$$T_{05} = T_{04} = T_{02} = 759.2534\ [K]$$

$$M_5 = 0.6049 \Rightarrow \frac{T_{05}}{T_5} = 1.0732 \Rightarrow T_5 = \frac{T_{05}}{1.0732}$$

$$T_5 = \frac{759.2534}{1.6732} = 707.467 \; [K]$$

Polytropic effiency:

$$\frac{T_{final}}{T_{initial}} = \left(\frac{P_{final}}{P_{initial}}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_5}{T_1} = \left(\frac{P_5}{P_1}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}}$$

$$\frac{707.467}{298} = \left(\frac{12.777}{0.7}\right)^{\frac{1}{e}\times\frac{1.4-1}{1.4}} \Rightarrow e = 0.95976$$

$$\boxed{e = 95.976\ \%}$$

h

$$M_2 = 2.1135 \Rightarrow \frac{T_3}{T_2} = 1.7819 \qquad \frac{P_3}{P_2} = 5.0447$$

$$\Rightarrow T_3 = 1.7819\, T_2 = 1.7819 \times 401 = 714.542 \; [K]$$

$$\Rightarrow P_3 = 5.0447 P_2 = 5.0447 \times 2.333 = 11.77 \; [atm]$$

Polytropic effiency:

$$\frac{T_{final}}{T_{initial}} = \left(\frac{P_{final}}{P_{initial}}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}}$$

$$\frac{T_3}{T_1} = \left(\frac{P_3}{P_1}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}}$$

$$\frac{714.542}{298} = \left(\frac{11.77}{0.7}\right)^{\frac{1}{e}\times\frac{1.4-1}{1.4}}$$

$$e = 0.92202$$

$$\boxed{e = 92.202\ \%}$$

4. Suppose a particular compressor has a compression ratio $P_3/P_2 = 25$; the incoming air temperature is 300 K and its pressure is 1.2 atm. 20 kgm per sec. of air flows through the compressor.

(a) If the adiabatic efficiency is 90%, what is the final temperature?

(b) What is the power required?

(c) What is the minimum number of stages (pairs of rotor and stator sections) required to protect against separation due to adverse pressure gradients?

(a)

$$\eta = \frac{\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_2}{T_1} - 1}$$

$$90\% = \frac{25^{\frac{1.4-1}{1.4}} - 1}{\frac{T_2}{300} - 1}$$

$$\boxed{T_2 = 802.8282\ [K]}$$

(b)

$$C_p = 1004.5\ J/kg\text{-}K$$

$$P_r = \dot{m}\, c_p (T_2 - T_1)$$

$$= 20 \times 1004.5 \times (802.8282 - 300)$$

$$\boxed{P_r = 10101818.54\ [W]}$$

(c)

$$\frac{P_{after}}{P_{before}} \leq 1.6$$

$$\frac{P_3/P_2}{1.6} = \frac{25}{1.6} = 15.625$$

$$\boxed{16}$$