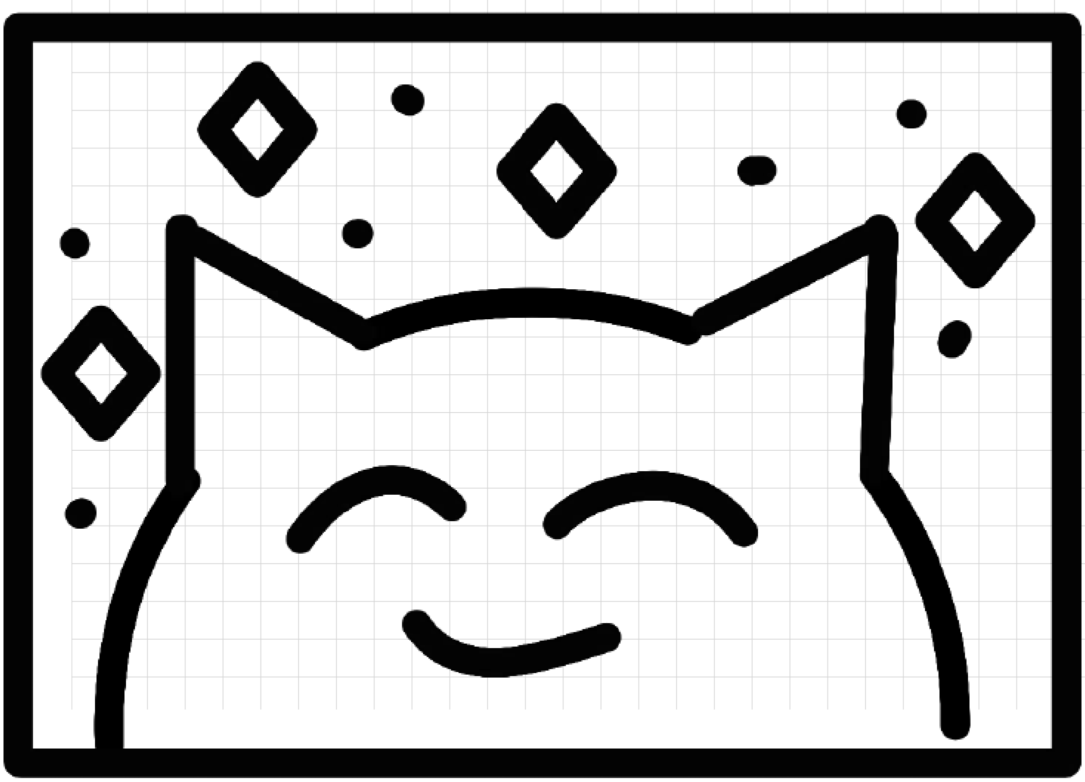
MAE 112

HW 6

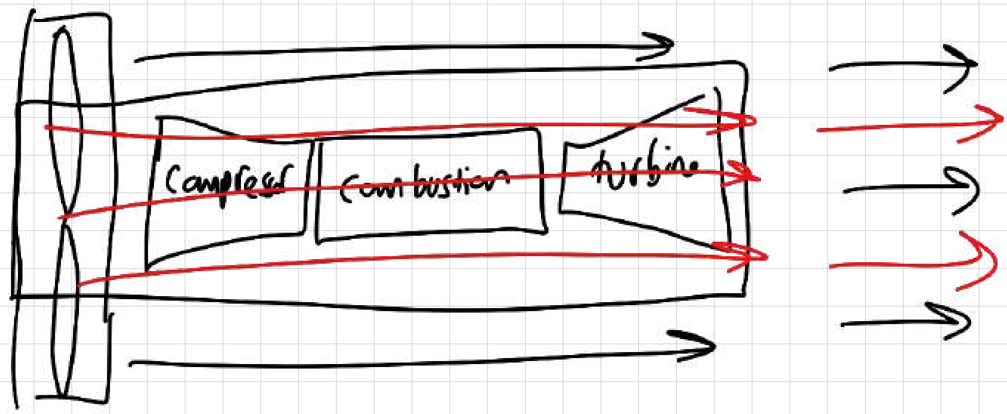
TRIET



1. Suppose we had two parallel flows mimicking the primary and secondary exhaust flows from a turbofan engine. One flow has a stagnation temperature of 1100°R and the other at 2700°R. Each flow had a stagnation pressure of five atmospheres, a value of $\gamma = 1.4$, and a value of $c_p = .33$ Btu/1bm°R. The colder flow has five times the mass flow of the hotter flow. Calculate the ratio of total thrust to total mass flow from nozzle expansion in each of the following two cases.

a) The flows are mixed adiabatically at constant pressure and then expanded to ambient pressure of 0.7 atmosphere in one nozzle.

b) Each flow is expanded to the 0.7 atmosphere ambient pressure through a separate nozzle.



Unit analysis:

$$C_p = 0.33 \frac{\cancel{Btu}}{\cancel{lbm}°R} \times \frac{778 \text{ ft } \cancel{lbf}}{1 \cancel{Btu}} \times \frac{1 \cancel{lbm}}{1 \cancel{lbf}} \frac{32.2 \text{ ft}}{s^2} = 8267.028 \left[\frac{ft^2}{s^2 \cdot R}\right]$$

Gas constant:

$$R = c_p \frac{\gamma - 1}{\gamma} = 8267.028 \times \frac{1.4-1}{1.4} = 2360.1 \left[\frac{ft^2}{s^2 \, °R}\right]$$

(a) Mixed temperature:

$$\text{Temp} = \frac{5 \times 1100 + 1 \times 2700}{6} = 1366.667 [°R]$$

Total thrust to mass flow ratio:

$$\frac{\text{Thrust}}{\dot{m}} = \sqrt{\frac{2 \gamma R \,\text{Temp}}{\gamma - 1}\left[1 - \left(\frac{P_e}{P_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \sqrt{\frac{2 \times 1.4 \times 2360.1 \times 1366.667}{1.4 - 1}\left[1 - \left(\frac{0.7}{5}\right)^{\frac{1.4-1}{1.4}}\right]}$$

$$\boxed{\frac{\text{Thrust}}{\dot{m}} = 3115.1 \quad [ft/s]}$$

(b)

Cold flow thrust to mass flow ratio

$$\frac{T_{cold}}{\dot{m}_{cold}} = \sqrt{\frac{2 \gamma R T_{emp}}{\gamma - 1}\left[1 - \left(\frac{P_e}{P_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \sqrt{\frac{2 \times 1.4 \times 2360.1 \times 1100}{1.4 - 1}\left[1 - \left(\frac{0.7}{5}\right)^{\frac{1.4-1}{1.4}}\right]}$$

$$= 2794.7152$$

$$T_{cold} = 2794.7152 \ \dot{m}_{cold}$$

Hot flow thrust to mass flow ratio

$$\frac{T_{hot}}{\dot{m}_{hot}} = \sqrt{\frac{2 \gamma R T_{emp}}{\gamma - 1}\left[1 - \left(\frac{P_e}{P_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$= \sqrt{\frac{2 \times 1.4 \times 2360.1 \times 2700}{1.4 - 1}\left[1 - \left(\frac{0.7}{5}\right)^{\frac{1.4-1}{1.4}}\right]}$$

$$= 4378.477$$

$$T_{hot} = 4378.477 \ \dot{m}_{hot}$$

We know $\dot{m}_{cold} = 5\,\dot{m}_{hot}$ $(*)$

Total thrust:

$$T_{total} = T_{cold} + T_{hot}$$

$$= 2794.7152\ \dot{m}_{cold} + 4378.477\ \dot{m}_{hot}$$

$$(*) \Rightarrow T_{total} = 2794.7152\,(5\,\dot{m}_{hot}) + 4378.477\ \dot{m}_{hot}$$

$$= 18352.05\ \dot{m}_{hot}$$

Total mass flow rate:

$$\dot{m}_{total} = \dot{m}_{cold} + \dot{m}_{hot}$$

$$(*) \Rightarrow \dot{m}_{total} = 5\,\dot{m}_{hot} + \dot{m}_{hot} = 6\,\dot{m}_{hot}$$

Total thrust to total mass flow rate ratio:

$$\frac{T_{total}}{\dot{m}_{total}} = \frac{18352.05\ \cancel{\dot{m}_{hot}}}{6\ \cancel{\dot{m}_{hot}}} = \boxed{3058.675\left[\frac{ft}{s}\right]}$$

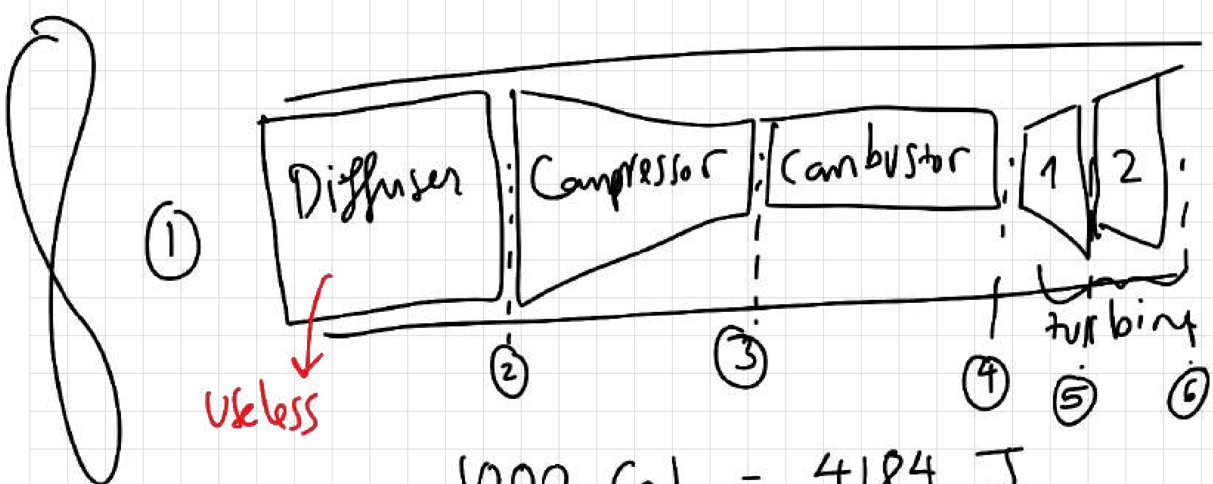
2. Consider a turbine engine that powers helicopter flight with ambient conditions at 298 K and 1.0 atmospheres of pressure. There are two turbine stages with different rotational speeds; the first one drives the compressor while the other drives the main propeller of the helicopter. The turbine exit pressure is practically 1.0 atmospheres.
Consider a combustor exit temperature of 1600 K and a mass mixture ratio $\mu = 30$. Consider that 3% of the air is taken from the flow downstream of the compressor but before entry to the combustor to be used for cooling of the turbine. The specific heat of combustion products equals 0.30 cal/gm°K with γ=1.3, and the specific heat of air is 0.24 cal/gm°K with γ=1.4. Assume that flight velocity has negligible effect on the intake flow pressure.

a.) What is the pressure entering the compressor? Assume a polytropic diffuser efficiency equal to 0. 95.

b.) Assume a pressure ratio of 20 across the compressor. If the polytropic compressor efficiency is 0.95, what is the power requirement of the compressor (per unit mass flux)?

c.) Neglect pressure change across the combustor. The first turbine stage drives the compressor and has a polytropic efficiency of 0.92. Determine the pressure change and temperature change across that stage.

d.) All of the power from the second stage drives the propeller. The polytropic efficiency of this stage is 0.95. The mechanical efficiency for power transfer to the propeller is 0.9. What is the final exhaust-gas temperature? What is the extracted power (per unit mass flux) delivered to the propeller?



$$1000 \text{ Cal} = 4184 \text{ J}$$

$$1000 \text{ gm} = 1 \text{ kg}$$

Specific heat of combustion products:

$$C_{p,P} = 0.3 \frac{\cancel{\text{Cal}}}{\cancel{\text{gm}}\,{}^\circ\text{K}} \times \frac{4184 \text{ J}}{1000 \cancel{\text{Cal}}} \times \frac{1000 \cancel{\text{gm}}}{1 \text{ Kg}} = 1255.2 \frac{\text{J}}{\text{Kg} \cdot \text{K}}$$

Specific heat of air:

$$C_{p,a} = 0.24 \frac{\cancel{cal}}{\cancel{gm}\,°K} \times \frac{4184\,J}{1000\,\cancel{cal}} \times \frac{1000\,\cancel{gm}}{1\,kg} = 1004.16\ \frac{J}{kg\cdot K}$$

(a) Helicopter $\rightarrow$ slow $\rightarrow$ $M << 1 \Rightarrow P_1 = \boxed{P_2 = 1\ [atm]}$

(b) Temperature after the compressor:

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_3}{298} = (20)^{\frac{1}{0.95}\times\frac{1.4-1}{1.4}}$$

$$T_3 = 733.67\ [K]$$

Power requirement of the compressor:

$$\frac{P}{\dot{m}_a} = C_{p,a}(T_3 - T_2) = 1004.16\ \frac{J}{kg\cdot\cancel{K}}(733.67 - 298)\ \cancel{K}$$

$$\boxed{\frac{P}{\dot{m}_a} = 437482.3872\ [J/kg]}$$

(c)

Air mass flow rate before turbine:

$$\dot{m}_a' = 97\%\ \dot{m}_a$$

Fuel mass flow rate before turbine:

$$\mu = \frac{\dot{m}_a'}{\dot{m}_f} \Rightarrow \dot{m}_f = \frac{\dot{m}_a'}{\mu} = \frac{\dot{m}_a'}{30} = \frac{97\%\ \dot{m}_a}{30}$$

Total mass flow rate before turbine:

$$\dot{m} = \dot{m}_a' + \dot{m}_f = 97\%\ \dot{m}_a + \frac{97\%\ \dot{m}_a}{30}$$

$$\dot{m} = 1.0023333\ \dot{m}_a$$

Power for 1st stage turbine:

$$P_{T1} = \dot{m}\, c_{p,p}(T_4 - T_5) = 1.0023333\ \dot{m}_a c_{p,p}(T_4 - T_5)$$

$$\frac{P_{T_1}}{\dot{m}_a} = 1.0023333\ c_{p,p}(T_4 - T_5)$$

If 1st stage of turbine drives compressor, powers are same:

$$\frac{P_{T_1}}{\dot{m}_a} = \frac{P_c}{\dot{m}_a}$$

$$1.0023333\ c_{p,p}(T_4 - T_5) = 437482.3872$$

$$1.0023333 \times 1255.2 \times (1600 - T_5) = 437482.3872$$

$$T_5 = 1252.27\ [K]$$

Temperature change:

$$\Delta T = T_5 - T_4 = 1252.27 - 1600$$

$$\boxed{\Delta T = -347.72\ [K]}$$

Pressure before turbine:

$$\frac{P_3}{P_2} = 20 \Rightarrow P_3 = 20\, P_2 = 20 P_1 = 20\ [atm]$$

$$P_4 = P_3 = 20\ [atm]$$

Pressure after stage 1 turbine:

$$\frac{P_5}{P_4} = \left(\frac{T_5}{T_4}\right)^{\frac{\gamma}{e(\gamma-1)}} \Rightarrow \frac{P_5}{20} = \left(\frac{1252.27}{1600}\right)^{\frac{1.3}{0.92(1.3-1)}}$$

$$P_5 = 6.3062\ [atm]$$

Pressure difference:

$$\Delta P = P_5 - P_4 = 6.3 - 20 = \boxed{-13.7\ [atm]}$$

(d) Final exhaust gas temperature:

$$\frac{T_6}{T_5} = \left(\frac{P_6}{P_5}\right)^{\frac{\gamma-1}{\gamma}\cdot e} \Rightarrow \frac{T_6}{1252.27} = \left(\frac{1}{6.3}\right)^{\frac{1.3-1}{1.3}\cdot 0.95}$$

$$\boxed{T_6 = 836.48\ [K]}$$

Power of stage 2 turbine:

$$P_{T2} = \dot{m}\ c_{p,p}(T_5 - T_6)$$

$$= 1.0023333\ \dot{m}_a\ c_{p,p}(T_5 - T_6)$$

$$\frac{P_{T_2}}{\dot{m}_a} = 1.0023333\ c_{p,p}(T_5 - T_6)$$

$$= 1.0023333 \times 1255.2 \times (1252.27 - 836.48)$$

$$= 523\ 117.3564\ [J/kg]$$

Extracted power delivered to the propeller:

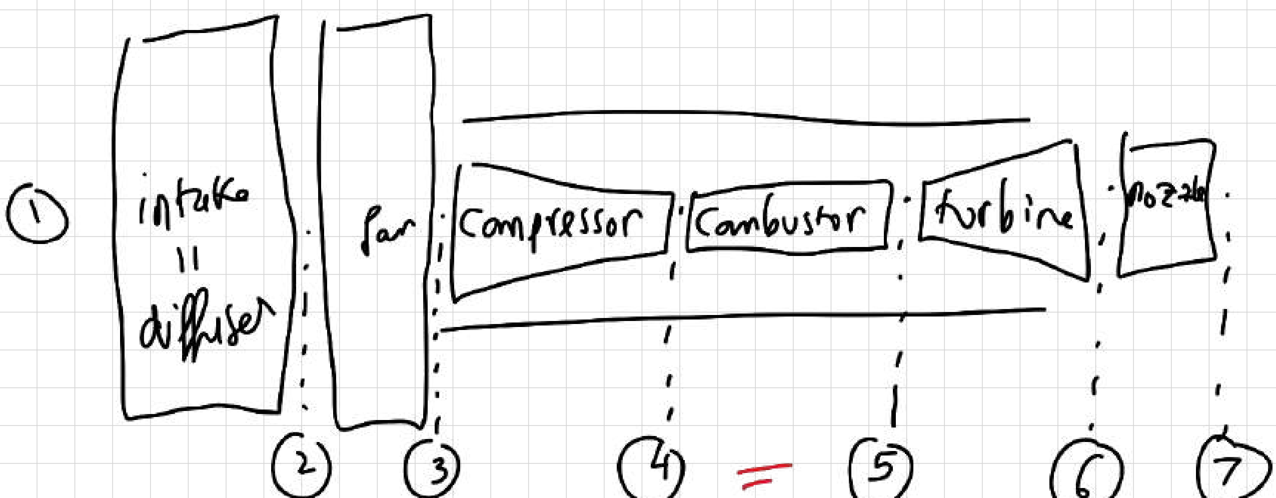
$$\frac{P_p}{\dot{m}_a} = 90\%\ \frac{P_{T_2}}{\dot{m}_a} = 90\% \times 523\ 117.3564$$

$$\boxed{\frac{P_p}{\dot{m}_a} = 470\ 805.62\ J/kg}$$

3. Consider a turbofan engine flying at a Mach number of 0.85 with ambient pressure of 0.6 atmosphere and ambient temperature of 430°R. The intake has .95 polytropic efficiency. The air divides with a bypass ratio of 2 into a primary and a secondary (or bypass) flow at the air intake exit. The primary flow passes through a fan and compressor with an overall pressure ratio of 22 and polytropic efficiency of .92 while the secondary flow passes through a fan with pressure ratio of four and efficiency of .95.

The primary flow burns at constant pressure with liquid kerosene fuel (heating value = 20,000 Btu/ lbm) with a final combustor temperature of 2500°R and then expands through a turbine and ultimately through the primary nozzle to ambient pressure. Burner efficiency is .98 while the polytropic efficiencies of the turbine and the nozzle are each .95. All work taken from the turbine flow is employed to drive the fan and compressor. The secondary flow expands isentropically to ambient pressure through a secondary nozzle. cp = 0.24 Btu/1bm°R for air; cp = 0.31 Btu/1bm°R for products; $\gamma = 1.4$ for air; $\gamma = 1.25$ for products.

a) Calculate the stagnation temperature of the secondary flow at the fan exit.
b) Calculate the stagnation temperature of the primary flow at the compressor exit.
c) What is the temperature at the turbine exit?
d) Calculate the exit jet velocity for the primary flow.
e) Calculate the bypass exit jet velocity.
f) Calculate the thrust and the thrust specific fuel consumption.



(a) Stagnation temperature at (2):

$$T_{02} = T_{01} = T_1\left(1 + \frac{\gamma - 1}{2} M^2\right) = 430\left(1 + \frac{1.4 - 1}{2} \times 0.85^2\right)$$

$$T_{02} = 492.135 \ [°R]$$

After diffuser, $M << 1 \Rightarrow T \simeq T_0$, $P \approx P_0$

Temperature at ②: $T_2 = T_{02} = 492.135\,[°R]$

Temperature at ③:

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{1}{e} \times \frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_3}{492.135} = (4)^{\frac{1}{0.95} \times \frac{1.4-1}{1.4}}$$

$$T_3 = 746.715\,[°R]$$

$$\boxed{T_{03} = T_3 = 746.715\,[°R]}$$

ⓑ Pressure at ②:

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{e \times \frac{\gamma}{\gamma-1}} \Rightarrow \frac{P_2}{0.6} = \left(\frac{492.135}{430}\right)^{0.95\,\frac{1.4}{1.4-1}}$$

$$P_2 = 0.9398\,[atm]$$

Pressure at ③

$$\frac{P_3}{P_2} = \left(\frac{T_3}{T_2}\right)^{e \times \frac{\gamma}{\gamma-1}} \Rightarrow \frac{P_3}{0.9398} = \left(\frac{746.715}{492.135}\right)^{0.95\,\frac{1.4}{1.4-1}}$$

$$P_3 = 3.76\,[atm]$$

Pressure at ④:

$$P_4 = 22\,P_2 = 22 \times 0.9398 = 20.68\,[atm]$$

Temperature at ④:

$$\frac{T_4}{T_2} = \left(\frac{P_4}{P_2}\right)^{\frac{1}{e}\,\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_4}{492.135} = \left(\frac{20.68}{0.9398}\right)^{\frac{1}{0.92}\,\frac{1.4-1}{1.4}}$$

$$\boxed{T_4 = T_{04} = 1285.33\,[°R]}$$

(c) Enthalpy balance across combustion chamber

$$\dot{m}_f \eta_b Q + \dot{m}_a C_{p,a} T_4 = (\dot{m}_a + \dot{m}_f) C_{p,p} T_5$$

$$\dot{m}_f\, 0.98 \times 20\,000 + \dot{m}_a \times 0.24 \times 1285.33 = (\dot{m}_a + \dot{m}_f)\, 0.31 \times 2500$$

$$19600\, \dot{m}_f + 308.4792 = 775\, \dot{m}_a + 775\, \dot{m}_f$$

$$18825\, \dot{m}_f = 466.5208\, \dot{m}_a$$

$$\dot{m}_f = \frac{466.5208}{18825} \dot{m}_a = 0.0248\, \dot{m}_a$$

Bypass is 2 $\Rightarrow$ $\dot{m}_s = 2\, \dot{m}_a$

Power balance:

$$P_{turbine} = P_{fan} + P_{compressor}$$

$$(\dot{m}_a + \dot{m}_f) C_{p,p} (T_5 - T_6) = (\dot{m}_a + \dot{m}_s) C_{p,a} (T_3 - T_2) + \dot{m}_a C_{p,a} (T_4 - T_3)$$

$$(\cancel{\dot{m}_a} + 0.0248\, \cancel{\dot{m}_a})\, 0.31 (2500 - T_6) = 3 \cancel{\dot{m}_a} \times 0.24 (746.715 - 492.135) + \cancel{\dot{m}_a} \times 0.24 (1285.33 - 746.715)$$

$$\Rightarrow \boxed{T_6 = 1516.1253\ [{}^\circ R]}$$

(d) Pressure at (6)

$$\frac{P_6}{P_5} = \left(\frac{T_6}{T_5}\right)^{\frac{\gamma}{e(\gamma - 1)}} \Rightarrow \frac{P_6}{20.68} = \left(\frac{1516.125}{2500}\right)^{\frac{1.25}{0.95(1.25-1)}}$$

$$P_6 = 1.4872\ [atm]$$

$$C_{p,a} = 0.24\ [Btu/lbm\,{}^\circ R] \times 778 \times 32.2 = 6012.384\ \left[\frac{ft^2}{s^2 \cdot R}\right]$$

$$C_{p,p} = 0.31\ [Btu/lbm\,{}^\circ R] \times 778 \times 32.2 = 7766\ \left[\frac{ft^2}{s^2 \cdot R}\right]$$

Exit velocity of primary flow:

$$V_{e,p} = \sqrt{2 c_{p,p} T_{06} \left[1 - \left(\frac{P_7}{P_{06}}\right)^{e \frac{\gamma - 1}{\gamma}}\right]}$$

$$= \sqrt{2 \times 7766 \times 1516.13 \times \left[1 - \left(\frac{0.6}{1.4872}\right)^{0.95 \times \frac{1.25-1}{1.25}}\right]}$$

$$\boxed{V_{e,p} = 1931.43 \text{ [ft/s]}}$$

(e) Exit velocity of bypass:

$$V_{e,s} = \sqrt{2 c_{p,a} T_{03} \left[1 - \left(\frac{P_7}{P_{03}}\right)^{e \frac{\gamma - 1}{\gamma}}\right]}$$

$$= \sqrt{2 \times 6012.38 \times 746.715 \times \left[1 - \left(\frac{0.6}{3.76}\right)^{1 \times \frac{1.4-1}{1.4}}\right]}$$

$$\boxed{V_{e,s} = 1913.652 \text{ [ft/s]}}$$

(f) Initial velocity at (1):

$$V_1 = M\sqrt{\gamma R T} = M\sqrt{\cancel{\gamma} c_{p,a} \frac{\gamma - 1}{\cancel{\gamma}} T}$$

$$= 0.85 \sqrt{6012.384 \times (1.4-1) \times 430}$$

$$V_1 = 864.38 \text{ [ft/s]}$$

Thrust:

$$T = (\dot{m}_a + \dot{m}_f) V_{e,p} + \dot{m}_s V_{e,s} - (\dot{m}_a + \dot{m}_s) V_1$$

$$= 1.0248\,\dot{m}_a V_{e,p} + 2\dot{m}_a V_{e,s} - 3\dot{m}_a V_1$$

$$\frac{T}{\dot{m}_a} = 1.0248\,V_{e,p} + 2\,V_{e,s} - 3\,V_1$$

$$= 1.0248 \times 1931.43 + 2 \times 1913.652 - 3 \times 864.38$$

$$\boxed{\frac{T}{\dot{m}a} = 3213.5\ \frac{ft}{s}}$$

$$\frac{T}{\dot{m}a} = 3213.5\ \frac{\cancel{ft}}{\cancel{s}} \times \frac{1\ lbf \times s^{\cancel{2}}}{1\,lbm\ 32.2\ \cancel{ft}} \times \frac{1\,hr}{3600\ \cancel{s}} = 0.02772\left[\frac{lbf}{lbm/hr}\right]$$

Thrust specific fuel consumption:

$$TSFC = \frac{\dot{m}_f}{T} = \frac{\dot{m}_f/\dot{m}_a}{T/\dot{m}_a} = \frac{0.0248}{0.02772}\left[\frac{lbm/hr}{lbf}\right]$$

$$\boxed{TSFC = 0.8946\left[\frac{lbm/hr}{lbf}\right]}$$