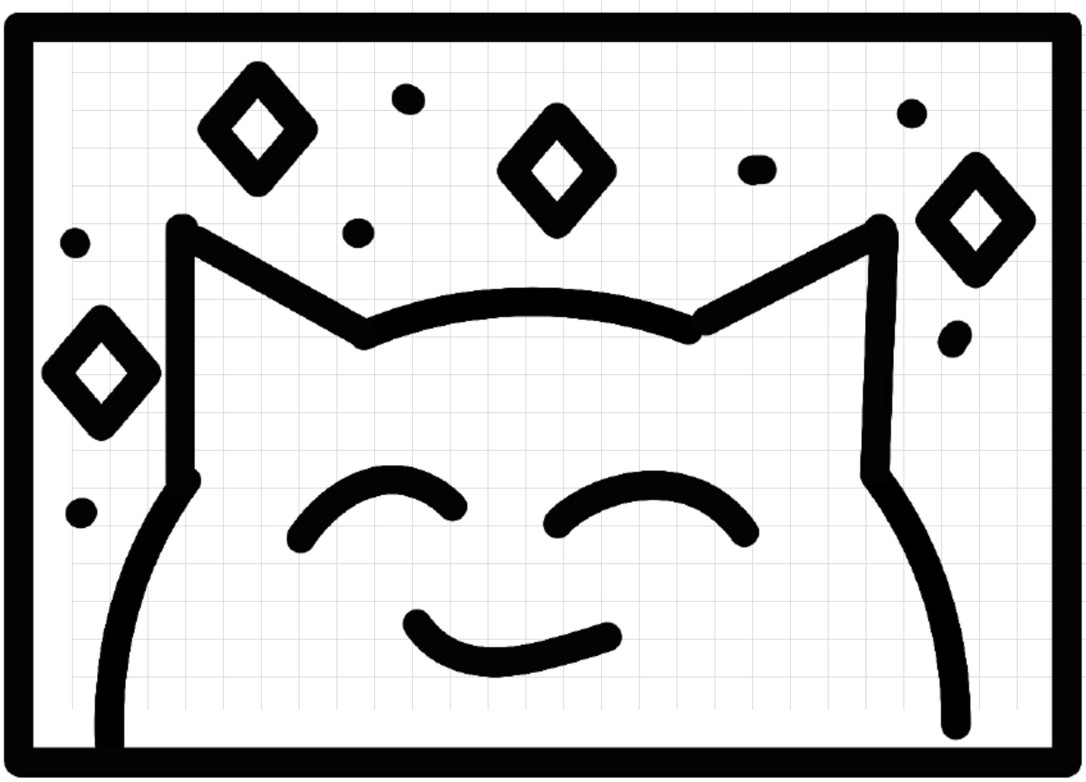
MAE 112

HW 1

TRIET



1. A rocket uses propellants with a specific impulse equal to 270 seconds. It has an empty weight (i.e., without propellants) of 200 lbf (measured at sea level). The desire is to accelerate it from rest to a velocity of 8,000 feet per second. Assume that drag forces are negligible and gravity acts normal to the flight direction.

(a) What is the required mass of propellant?

(b) If the rocket had an initial horizontal velocity of 1000 feet per second (e.g., with a launch from an aircraft), how much propellant mass is required to achieve the 8,000 feet per second speed?

(c) If the specific impulse were increased to the new value of 290 seconds, what would be the answer to part (a)?

$$I_{sp} = 270 \ [s]$$
$$m_{dry} = 200 \ [lb]$$
$$m_p + m_{dry} = m_{total}$$

(a)

$$\Delta V = I_{sp} \times g \times \ln\left(1 + \frac{m_p}{m_{dry}}\right)$$

$$8000 \left[\frac{\cancel{ft}}{s}\right] = 270 \ [\cancel{s}] \times 32.2 \left[\frac{\cancel{ft}}{\cancel{s^2}}\right] \times \ln\left(1 + \frac{m_p}{200}\right)$$

$$\boxed{m_p = 301.9458 \ [lb]}$$

(b)

$$\Delta V = I_{sp} \times g \times \ln\left(1 + \frac{m_p}{m_{dry}}\right)$$

$$7000 \left[\frac{\cancel{ft}}{s}\right] = 270 \ [\cancel{s}] \times 32.2 \left[\frac{\cancel{ft}}{\cancel{s^2}}\right] \times \ln\left(1 + \frac{m_p}{200}\right)$$

$$\boxed{m_p = 247.4077 \ [lb]}$$

(c)

$$\Delta V = I_{sp} \times g \times \ln\left(1 + \frac{m_p}{m_{dry}}\right)$$

$$8000 \left[\frac{\cancel{ft}}{s}\right] = 290 \ [\cancel{s}] \times 32.2 \left[\frac{\cancel{ft}}{\cancel{s^2}}\right] \times \ln\left(1 + \frac{m_p}{200}\right)$$

$$\boxed{m_p = 271.0819 \ [lb]}$$

2. Consider a rocket at takeoff from Earth at sea level. The mass flux of propellants is 160 kg/s. The hot gas exits the nozzle at a velocity of 1200 m/sec and a pressure of 0.85 atmospheres through an exit area of one square meter. Ambient pressure is one atmosphere. What is the thrust magnitude? If the initial acceleration is 50 m/sec^2, what is the initial mass of the vehicle including propellants?

$$\frac{dm}{dt} = \dot{m} = 160 \ [kg/s]$$

$$v_e = 1200 \ [m/s]$$

$$P_e = 0.85 \ [atm] = 86126.25 \ [Pa]$$

$$A_e = 1 \ [m^2]$$

$$P_a = 1 \ [atm] = 101325 \ [Pa]$$

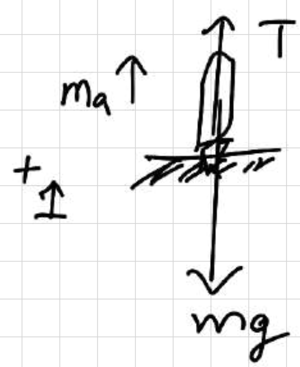
$$a = 50 \ [m/s^2]$$

Thrust equation:

$$T = \dot{m} v_e + (P_e - P_a) A_e$$

$$= 160 \times 1200 + (86126.25 - 101325) 1$$

$$\boxed{T = 176801.25 \ [lb]}$$



Newton 2nd law in y direction:

$$T - mg = ma$$

$$T = ma + mg = m(a + g)$$

$$m = \frac{T}{a+g} = \frac{176801.25}{50 + 9.81}$$

$$\boxed{m = 2956.04832 \ [kg]}$$

3. Consider an air-breathing jet engine which is flying at a velocity of 600 feet per second. For every lbm/second of air mass flow, a 0.030 lbm/sec mass flow of fuel is injected into the engine. The thrust force is 5000 lbf.; the entrance pressure equals the ambient pressure at the altitude (0.5 atm) and the exhaust pressure is 0.75 times the ambient atmospheric pressure. The incoming air temperature is 525°R. The entrance area and exhaust areas are both ten square feet. Determine:

(a) exhaust velocity

(b) specific fuel consumption

$V_\infty = 600$ [ft/s]

$\dot{m}_{fuel} = 0.03\ \dot{m}_{air}$

$f = \dfrac{\dot{m}_{fuel}}{\dot{m}_{air}} = 0.03$

$T = 5000$ [lb]

$P_{in} = P_a = 0.5 \times 2116.2 = 1058.1$ [lb/ft²]

$P_{out} = 0.75\ P_a = 793.575$ [lb/ft²]

$T_{in} = 525$ [°R]

$A = 10$ [ft²]

(a) $\rho = \dfrac{P_{in}}{RT_{in}} = \dfrac{1058.1}{1718 \times 525} = 0.001173$ [lb/ft³]

$\dot{m}_{air} = \rho A V_\infty = 0.001173 \times 10 \times 600 = 7.038$ [slug/s]

$\dot{m}_{fuel} = f\ \dot{m}_{air} = 0.03 \times 7.038 = 0.21114$ [slug/s]

Thrust equation:

$$T = (\dot{m}_{air} + \dot{m}_{fuel}) V_{out} - \dot{m}_{air} \times V_{in} + (P_{out} - P_a) A - (P_{in} - P_a) A$$

0

$$V_{out} = \frac{T + \dot{m}_{air} V_{in} - (P_{out} - P_a) A}{\dot{m}_{air} + \dot{m}_{fuel}}$$

$$= \frac{5000 + 7.038 \times 600 - (793.575 - 1058.1)\,10}{7.038 + 0.21114}$$

$$\boxed{V_{out} = 1637.1666 \text{ [ft/s]}}$$

(b)

$$1 \; slug \frac{ft}{s^2} = 1 \; lbf$$

$$1 \; lbm \times 32.2 \frac{ft}{s^2} = 1 \; lbf$$

$$1 \; slug \cancel{\frac{ft}{s^2}} = 32.2 \; lbm \cancel{\frac{ft}{s^2}}$$

$$\dot{m}_f = 0.21114 \left[\frac{\cancel{slug}}{\cancel{s}}\right] \times \frac{32.2 \, [lbm]}{1 \, [\cancel{slug}]} \times \frac{3600 \, [\cancel{s}]}{1 \, [hr]} = 24475.3488 \left[\frac{lbm}{hr}\right]$$

Thrust - specific fuel consumption

$$TSFC = \frac{\dot{m}_f \leftarrow [lbm/hr]}{T \leftarrow [lbf]}$$

$$= \frac{24475.3488}{5000}$$

$$\boxed{TSFC = 4.8951} \left[\frac{lbm/hr}{lbf}\right]$$

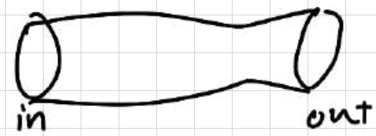
mass flow rate of fuel

Thrust

4. For an air-breathing jet engine, specific thrust is defined as thrust divided by air mass flow rate. Consider an engine that is flying at a velocity equal to 250 meters per second. For every kilogram/second of air mass flow, 0.040 kgm/sec mass flow of fuel is injected into the engine. The exit pressure and the entrance pressure both equal the ambient pressure, which is 0.7 atm. What must be the value of the exhaust velocity u_e if a specific thrust equal to 400m/s is desired?

$$V_{in} = 250 \; [m/s]$$

$$1 \, \frac{kg}{s} \text{ air} \; : \; 0.04 \, \frac{kg}{s} \text{ fuel}$$



$$\dot{m}_{fuel} = 0.04 \, \dot{m}_{air}$$

$$P_{in} = P_{out} = P_a = 0.7 \; [atm]$$

$$V_{out} = \, ?$$

$$\frac{T}{\dot{m}_{air}} = 400 \; [m/s]$$

Thrust equation:

$$T = \underbrace{(\dot{m}_{air} + \dot{m}_{fuel})} V_{out} - \dot{m}_{air} V_{in} + \cancelto{0}{(P_{out} - P_a)} A_{out} - \cancelto{0}{(P_{in} - P_a)} A_{in}$$

$$T = 1.04 \, \dot{m}_{air} V_{out} - \dot{m}_{air} V_{in}$$

$$\frac{T}{\dot{m}_{air}} = 1.04 \, V_{out} - V_{in}$$

$$V_{out} = \left(\frac{T}{\dot{m}_{air}} + V_{in} \right) \div 1.04$$

$$= (400 + 250) \div 1.04$$

$$\boxed{V_{out} = 625 \; [m/s]}$$

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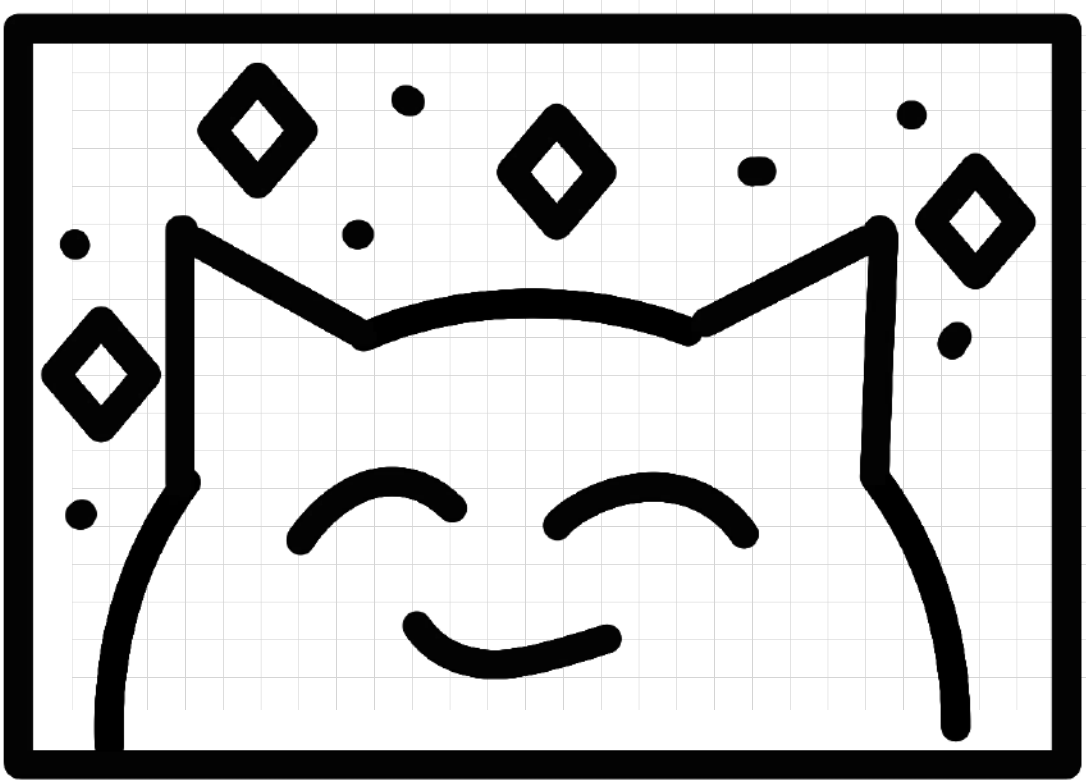
HW 2

TRIET



CH_4

1. Calculate theoretical (ideal) flame temperature for methane in stoichiometric ratio with enriched air (50% O_2, 50% N_2 by volume). Pressure is constant at 15 atm and the initial temperature is 298 K.

Stoichiometric reaction:

$$a\,CH_4 + b(O_2 + N_2) \longrightarrow c\,CO_2 + d\,H_2O + e\,N_2$$

$$1\,CH_4 + 2(O_2 + N_2) \longrightarrow 1\,CO_2 + 2\,H_2O + 2\,N_2$$

Mole number (n), heat of formation ($h_{f,m}$) and specific heats ($c_{p,m}$)

	n	$h_{f,m}$ [J/mol]	$C_{p,m}$ [J/ mol-k]
CH_4	1	−74 831	N/A
O_2	2	0	N/A
N_2	2	0	34.805
CO_2	1	−393 546	58.292
H_2O	2	−241 845	47.103
N_2	2	0	34.805

Energy balance:

$$\sum_{\text{reactants}} n\left(h_{f,m} + \int_{T_{ref}}^{T_i} C_{p,m}\,dT\right) = \sum_{\text{Products}} n\left(h_{f,m} + \int_{T_{ref}}^{T_{final}} C_{p,m}\,dT\right)$$

$$n_{CH4}(h_{f,CH_4} + 0) + 0 + 0 = n_{CO_2}(h_{f,CO_2} + C_{p,CO_2}\Delta T) + n_{H_2O}(h_{f,H_2O} + C_{p,H_2O}\Delta T)$$
$$+ n_{N_2}(0 + C_{p,N_2}\Delta T)$$

$$n_{CH4} \times h_{f,CH_4} = n_{CO_2}\,h_{f,CO_2} + n_{H_2O}\,h_{f,H_2O} + \Delta T\,(n_{O_2}C_{p,O_2} + n_{H_2O}C_{p,H_2O} + n_{N2}C_{p,N_2})$$

$$\Delta T = \frac{n_{CH4} \times h_{f,CH_4} - (n_{CO_2}\,h_{f,CO_2} + n_{H_2O}\,h_{f,H_2O})}{n_{O_2}C_{p,O_2} + n_{H_2O}C_{p,H_2O} + n_{N2}C_{p,N_2}}$$

$$T_f - 298 = \frac{1 \times (-74.831) - [1 \times (-393546) + 2(-241845)]}{1 \times 58.292 + 2 \times 47.103 + 2 \times 34.805}$$

$$\boxed{T_f = 3910.68 \quad [K]}$$

3. (a) Calculate AF at stoichiometric condition (AF_{st}) for ethyl alcohol C_2H_5OH (aka ethanol) initially at 550 º R burning in air at 20 atmospheres of pressure. AF is the ratio of mass flow of air to mass flow of fuel. Also, calculate FA = 1/ AF for the same condition.
(b) Calculate AF and $\Phi = AF_{st} / AF = FA / FA_{st}$ for ethyl alcohol and 50% excess air at the same conditions.

Molecular weight of single element:

$$W_C = 12.011 \left[\frac{g}{mol}\right] \quad W_H = 1 \left[\frac{g}{mol}\right] \quad W_O = 16 \left[\frac{g}{mol}\right] \quad W_N = 14 \left[\frac{g}{mol}\right]$$

$$a\, C_2H_5OH + b(O_2 + 3.76\, N_2) \rightarrow c\, CO_2 + d\, H_2O + e\, N_2$$

$$1\, C_2H_5OH + 3(O_2 + 3.76\, N_2) \rightarrow 2\, CO_2 + 3\, H_2O + 11.28\, N_2$$

(a) Air to fuel ratio:

$$A/F = \frac{m_{Air}}{m_{Fuel}} = \frac{3 \times (2 \times 16 + 3.76 \times 2 \times 14)}{1 \times (2 \times 12.011 + 5 + 16 + 1)} = \boxed{8.9488}$$

Fuel to air ratio:

$$F/A = 1 \div A/F = 1 \div 8.9488 = \boxed{0.1117}$$

(b) Air mole: $1.5\, n_{air} = 1.5 \times 3 = 4.5$

$$1\, C_2H_5OH + 4.5(O_2 + 3.76\, N_2) \rightarrow \ldots$$

Air to fuel ratio:

$$A/F = \frac{m_{Air}}{m_{Fuel}} = \frac{4.5 \times (2 \times 16 + 3.76 \times 2 \times 14)}{1 \times (2 \times 12.011 + 5 + 16 + 1)} = \boxed{13.4231}$$

Fuel to air ratio:

$$F/A = 1 \div A/F = 1 \div 13.4231 = \boxed{0.0745}$$

Equivalent ratio:

$$\Phi = \frac{A/F_{\text{stoichimometry}}}{A/F_{\text{at 50\% excess air}}} = \frac{8.9488}{13.4231} = \boxed{0.6667}$$

4. (a) Establish the equations which can be employed for the calculation of the equilibrium composition and the flame temperature when one mole of propane C_3H_8 burns adiabatically at a constant pressure of ten atmospheres. The mixture is lean with 75% excess air. Both air and fuel enter at a temperature of 800°R. Consider the products to be CO_2, CO, H_2O, H_2, O_2, and N_2 only. Write all the required equations with known quantities and parameters substituted into the equation. Identify the unknowns. Propane is gaseous at room temperature. Explain what would be different in the analysis if propane entered at a lower temperature in liquid form.
(b) Use the computer software to calculate the final flame temperature and concentrations of the products with the gaseous propane fuel.
(c) For the adiabatic situation with gaseous fuel described in Part a, establish the equations to solve for the theoretical (ideal) temperature and composition. What are the products in this case? Again, write the necessary equations, identify the known quantities, and identify the unknowns. Solve the equations for the final temperature and composition. Which of the two temperatures from 2b and 2c is larger? Why?

(a) Stoichiometric reaction:

$$1\ C_3H_8 + 5(O_2 + 3.76 N_2) \rightarrow 3CO_2 + 4H_2O + 18.8 N_2$$

75% excess air:

$$C_3H_8 + 8.75(O_2 + 3.76 N_2) \rightarrow a\,CO_2 + b\,H_2O + 32.9\,N_2 + c\,CO + d\,H_2 + e\,O_2$$

C: $3 = a + c$

H: $8 = 2b + 2d$

O: $17.5 = 2a + b + c + 2e$

N: Aldready balanced

Two dissociation:

$$CO_2 \longleftrightarrow CO + \frac{1}{2} O_2$$

$$K_1 = \frac{X_{CO} X_{O_2}^{1/2}}{X_{CO_2}} = P^{-1/2} K_{P1}(T_f)$$

$$H_2O \longleftrightarrow H_2 + \frac{1}{2} O_2$$

$$K_2 = \frac{X_{H_2} X_{O_2}^{1/2}}{X_{H_2O}} = P^{-1/2} K_{P2}(T_f)$$

Energy conservation:

$$\sum_{reactants} n_m \left(h_{f,m} + \int_{T_{ref}}^{T_i} c_{p,m}\, dT \right) = \sum_{products} n_m \left(h_{f,m} + \int_{T_{ref}}^{T_f} c_{p,m}\, dT \right)$$

© Stoichiometric reaction:

$$1\ C_3H_8 + 5\,(O_2 + 3.76\,N_2) \rightarrow 3\,CO_2 + 4\,H_2O + 18.8\,N_2$$

75 % excess air:

$$1\,C_3H_8 + 8.75(O_2 + 3.76\,N_2) \rightarrow 3\,CO_2 + 4\,H_2O + 32.9\,N_2 + 3.75\,O_2$$

From textbook / internet:

	$h_{f,m}$ [J/mol]	$c_{p,m}$ [J/mol-K]
C_3H_8	-103847	73.920
O_2	0	34.88
N_2	0	32.676
H_2O	-241845	41.103
CO_2	-393546	54.299
CO	-110500	28.56
H_2	0	14.5
O_2	0	34.88
N_2	0	32.676

Energy conservation:

$$\sum_{reactants} n_m \left(h_{f,m} + \int_{T_{ref}}^{T_i} c_{p,m}\, dT \right) = \sum_{products} n_m \left(h_{f,m} + \int_{T_{ref}}^{T_f} c_{p,m}\, dT \right)$$

$T_i = 800\,[°R] = 444\,[K] \qquad T_{ref} = 298.15\,[K]$

$\Rightarrow T_f = 1769.7475\,[K]$

Less than part ⓑ $T_f = 1765.7\,[K]$

$\Rightarrow$ Excess conditions decrease T_f

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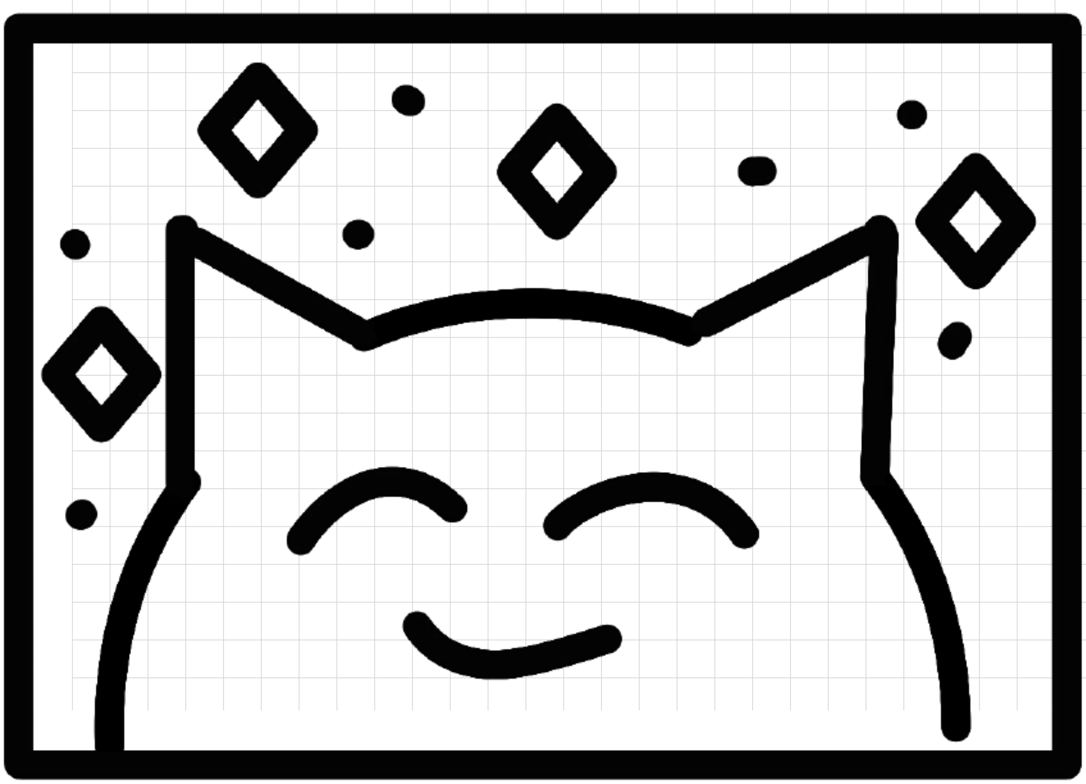
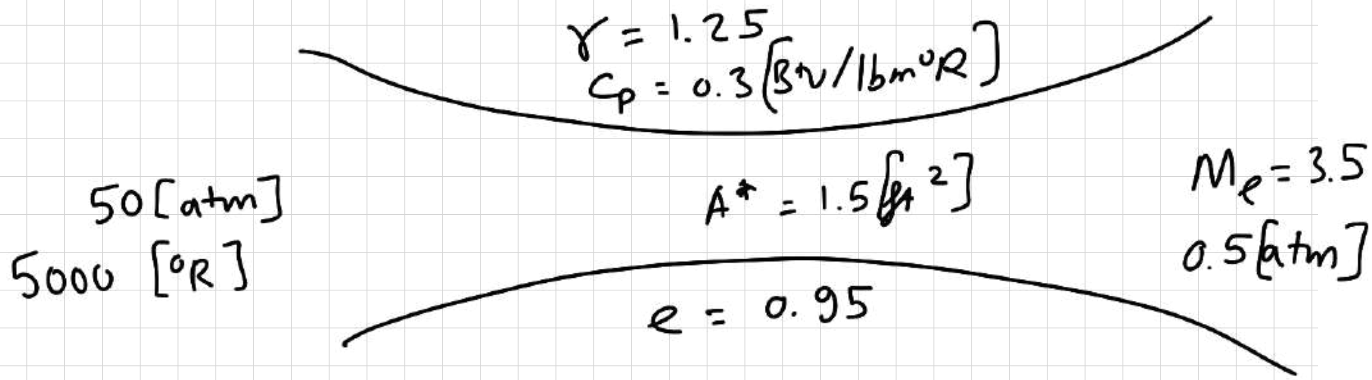
HW 3

TRIET



1. A rocket nozzle has initial pressure and temperature of fifty atmospheres and 5000°R with $\gamma = 1.25$; $c_p = 0.30$ Btu/1bm °R; and $A^* = 1.5$ ft^2. The flow is slightly over-expanded to a Mach number $M_e = 3.5$ at the nozzle exit with the ambient pressure at 0.50 atmosphere. Assume 95% for nozzle polytropic efficiency. Calculate: (a) the characteristic velocity c*; (b) the mass flow; (c) nozzle exit pressure and cross-sectional area (*beware of tables and graphs constructed for air flow*); (d) nozzle exit velocity U; and (e) effective exhaust velocity c.



ⓐ

$$1\,\text{Btu} = 778\ \text{ft}\cdot\text{lbf}$$

$$1\,\text{lbf} = 1\,\text{lbm} \times 32.2\ \frac{\text{ft}}{\text{s}^2}$$

$$c_p = 0.3\ \frac{\cancel{\text{Btu}}}{\cancel{\text{lbm}}\,{}^\circ\text{R}} \times \frac{778\ \text{ft}\cdot\cancel{\text{lbf}}}{1\ \cancel{\text{Btu}}} \times \frac{1\ \cancel{\text{lbm}}\ 32.2\ \frac{\text{ft}}{\text{s}^2}}{1\ \cancel{\text{lbf}}}$$

$$= 7515.48 \left[\frac{\text{ft}^2}{\text{s}^2\,{}^\circ\text{R}}\right]$$

$$R = c_p\frac{\gamma-1}{\gamma} = 7515.48\,\frac{1.25-1}{1.25} = 1503 \left[\frac{\text{ft}^2}{\text{s}^2\ {}^\circ\text{R}}\right]$$

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{(2-e)\gamma+e}{2e(\gamma-1)}}$$

$$= \sqrt{1.25}\left(\frac{2}{1.25+1}\right)^{\frac{(2-0.95)1.25+0.95}{2\times0.95(1.25-1)}}$$

$$\Gamma = 0.638$$

$$c^* = \frac{\sqrt{RT_0}}{\Gamma} = \frac{\sqrt{1503\times5000}}{0.638} = 4296.79 \left[\frac{\text{ft}}{\text{s}}\right]$$

(b) $P_0 = 50 \text{ atm} = 105811 \, [lb/ft^2]$

$$\dot{m} = \frac{P_0 A^*}{C^*} = \frac{105811 \left[\frac{lb}{ft^2}\right] \times 1.5 \, [ft^2]}{4296.79 \left[\frac{ft}{s}\right]} = 36.9384 \left[\frac{lb}{\frac{ft}{s}}\right]$$

$$1 \, lbf = 1 lbm \times 32.2 \, \frac{ft}{s^2}$$

$$1 \, lbf = 1 slug \times \frac{ft}{s^2}$$

$$\dot{m} = 36.9384 \, \frac{\cancel{lbf}}{\frac{\cancel{ft}}{\cancel{s}}} \times \frac{1 lbm \times 32.2 \, \frac{\cancel{ft}}{s^{\cancel{2}}}}{1 \, \cancel{lbf}} = 1189.4 \left[\frac{lbm}{s}\right]$$

$$\dot{m} = 36.9384 \, \frac{\cancel{lbf}}{\frac{\cancel{ft}}{\cancel{s}}} \times \frac{1 slug \times \frac{\cancel{ft}}{s^{\cancel{2}}}}{1 \, \cancel{lbf}} = 36.9384 \left[\frac{slug}{s}\right]$$

(c)

$$\frac{P_0}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{(\gamma-1)e}}$$

$$\frac{50 \, [atm]}{P_e} = \left[1 + \frac{1.25 - 1}{2} \times 3.5^2\right]^{\frac{1.25}{(1.25-1)0.95}}$$

$$\boxed{P_e = 0.3768 \, [atm]}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{\frac{(2-e)\gamma + e}{2e(\gamma-1)}}$$

$$\frac{A_e}{1.5 \, [ft^2]} = \frac{1}{3.5}\left[\frac{2}{1.25+1}\left(1 + \frac{1.25-1}{2} 3.5^2\right)\right]^{\frac{(2-0.95)1.25 + 0.95}{2 \times 0.95(1.25-1)}}$$

$$\boxed{A_e = 26.395 \, [ft^2]}$$

(d)

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\frac{5000[°R]}{T_e} = 1 + \frac{1.25-1}{2} \times 3.5^2$$

$$T_e = 1975.3\ [°R]$$

$$\rho_e = \frac{P_e}{R\,T_e} = \frac{0.3768 \times 2116.2}{1503 \times 1975.3} = 2.6858 \times 10^{-4} \left[\frac{slug}{ft^3}\right]$$

$$\dot{m} = \rho_e A_e V_e$$

$$V_e = \frac{\dot{m}}{\rho_e A_e} = \frac{36.9384 \left[\frac{\cancel{slug}}{s}\right]}{2.6858 \times 10^{-4} \left[\frac{\cancel{slug}}{ft^{\cancel{3}}}\right] 20.395\ [\cancel{ft^2}]}$$

$$\boxed{V_e = 6743.4\ [ft/s]}$$

$$V_e = \sqrt{2 c_p (T_0 - T_e)}$$

$$= \sqrt{2 \times 7515.48 \times (5000 - 1975.3)}$$

$$V_e = 6742.71\ [ft/s]$$

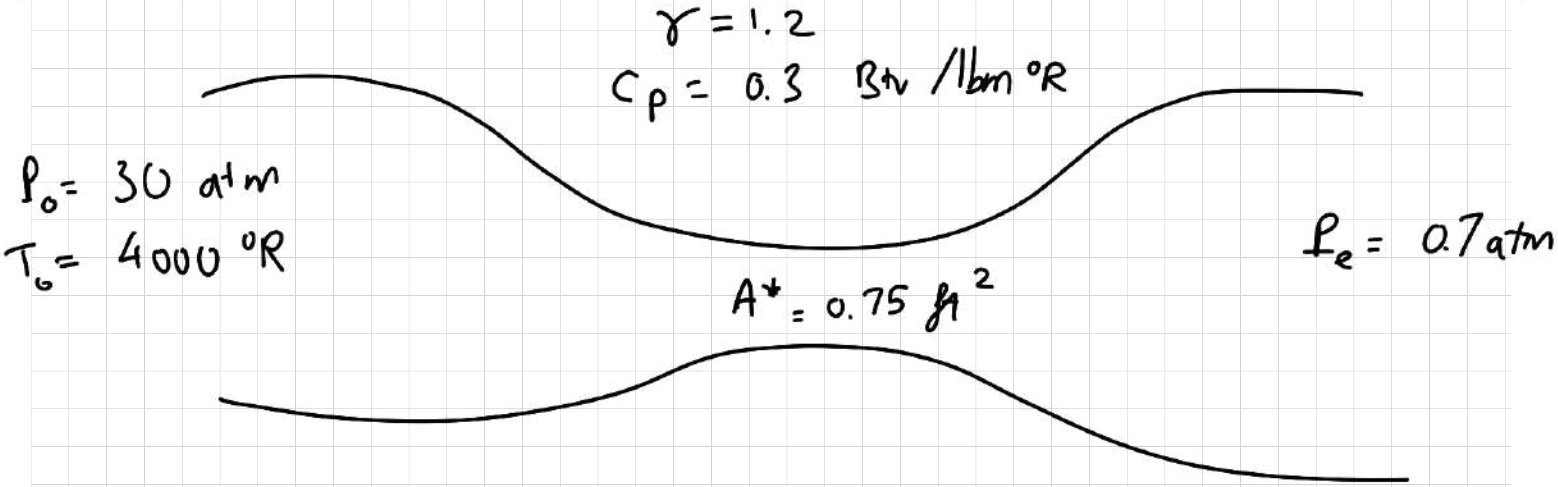
(e)

$$C = \frac{T}{\dot{m}} = \frac{\dot{m} V_e + (P_e - P_a) A_e}{\dot{m}}$$

$$= \frac{36.9384 \times 6743.4 + (0.3768 \times 2116.2 - 0.5 \times 2116.2)\,20.395}{36.9384}$$

$$C = 6600\ [ft/s]$$

2. Consider a nozzle with initial upstream entry pressure and temperature of thirty atmospheres and 4000°R. The value of $\gamma = 1.2$ and the value of $c_p = .30$ Btu/1bm °R. The throat area is 0.75 ft^2. The flow is perfectly expanded to the ambient pressure of 0.70 atmospheres. Calculate: (a) the mass flow, (b) the exhaust velocity, (c) the exit area, and (d) the thrust coefficient.



(a)

$$C_p = 0.3 \frac{\cancel{Btu}}{\cancel{lbm}\,°R} \times \frac{778\ ft\cdot \cancel{lbf}}{1\ \cancel{Btu}} \times \frac{1\ \cancel{lbm} \times 32.2\ \frac{ft}{s^2}}{1\ \cancel{lbf}}$$

$$C_p = 7515.48 \left[\frac{ft^2}{s^2\ °R}\right]$$

$$R = C_p \frac{\gamma - 1}{\gamma} = 7515.48 \frac{1.2 - 1}{1.2} = 1252.58 \left[\frac{ft^2}{s^2\ °R}\right]$$

$$\dot{m} = \frac{P_o A^*}{\sqrt{R T_o}} \sqrt{\gamma} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\dot{m} = \frac{30 \times 2116.2 \times 0.75}{\sqrt{1252.58 \times 4000}} \sqrt{1.2} \left(\frac{2}{1.2 + 1}\right)^{\frac{1.2+1}{2(1.2-1)}}$$

$$\boxed{\dot{m} = 13.7955 \left[\frac{slug}{s}\right]} = 444.215 \left[\frac{lbm}{s}\right]$$

(b)

At the exit:

$$\frac{P_0}{P} = \left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{30\ \cancel{atm}}{0.7\ \cancel{atm}} = \left[1+\frac{1.2-1}{2}M^2\right]^{\frac{1.2}{1.2-1}}$$

$$\Rightarrow M = 2.95$$

$$\frac{T_0}{T} = 1+\frac{\gamma-1}{2}M^2$$

$$\frac{4000}{T} = 1+\frac{1.2-1}{2}\times 2.95^2$$

$$\Rightarrow T = 2138.7515\ [°R]$$

$$M = \frac{V}{\sqrt{\gamma RT}} \Rightarrow V = M\sqrt{\gamma RT}$$

$$= 2.95\sqrt{1.2\times 1252.58\times 2138.7515}$$

$$\boxed{V = 5289.27\ [ft/s]}$$

(c) At the exit:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A}{0.75} = \frac{1}{2.95}\left[\frac{2}{1.2+1}\left(1+\frac{1.2-1}{2}2.95^2\right)\right]^{\frac{1.2+1}{2(1.2-1)}}$$

$$\boxed{A = 4.71\ [ft^2]}$$

(d)

$$C_F = \frac{T}{P_0 A^*} = \frac{\dot{m} V_e}{P_0 A^*} = \frac{13.7955\left[\cancel{\frac{slug}{s}}\right]5289.27\left[\cancel{\frac{ft}{s}}\right]}{30\times 2116.2\left[\cancel{\frac{lb}{ft^2}}\right]0.75\left[\cancel{ft^2}\right]} = \boxed{1.53}$$

3. Consider a rocket engine that uses liquid oxygen and liquid ethanol (C_2H_5OH) fuel aka ethyl alcohol. The oxygen mass-flow rate is 2.0 times greater than the fuel mass-flow rate. Ethanol is stored at 298K while the oxygen is stored at 80K just slightly below its boiling point. Oxygen has a heat of vaporization of 6.81kJ/mole while the value for ethanol is 38.6 kJ/mole. The heat of formation of liquid ethanol is -277.0 kJ/mole. The specific heat at constant pressure for gaseous oxygen is 30.77 joules/mole °K. The liquids are sprayed into the combustion chamber.

(a) How much energy per mole is required to vaporize and heat a mole of oxygen to the temperature of 298K.

(b) What is the expected ideal flame temperature? The ideal flame temperature aka theoretical flame temperature is the value with no dissociation. Assume that the products are H_2O, CO_2, and CO. You should make this calculation without using the online computer code. A key step is to determine what fraction of the carbon will appear in CO_2 and what fraction will appear in CO.

$$\dot{m}_{O_2} = 2\dot{m}_{C_2H_5OH} \Rightarrow m_{O_2} = 2\, m_{C_2H_5OH}$$

$$T_i = 298\ [K]$$

$$h_{vap} = 6810\ [J/mol]$$

$$c_{p,O_2} = 30.77\ [J/mol\,K]$$

(a)

$$Q = h_{vap} + \int_{80}^{298} c_{p,O_2}\, dT = 6810 + 30.77(298-80)$$

$$\boxed{Q = 13517.86\ [J/mol]}$$

(b) Stoichiometric reaction:

$$a\,C_2H_5OH + b\,O_2 \longrightarrow c\,CO_2 + d\,H_2O + e\,CO$$

$$m_{O_2} = 2\, m_{C_2H_5OH}$$

$$b \times W_{O_2} = 2 \times a \times W_{C_2H_5OH}$$

$$b \times 32 = 2 \times a \times (2\times 12 + 5 + 16 + 1)$$

$$b = 2.875\, a$$

$$1\, C_2H_5OH + 2.875\, O_2 \longrightarrow 1.75\, CO_2 + 3\, H_2O + 0.25\, CO$$

Energy conservation:

$$\sum_{\text{reactants}} n_m \left(h_{f,m} + \int_{T_{ref}}^{T_i} c_{p,m} dT \right) = \sum_{\text{products}} n_m \left(h_{f,m} + \int_{T_{ref}}^{T_f} c_{p,m} dT \right)$$

$$n_{C_2H_5OH} \left[h_{f,C_2H_5OH} + \cancel{c_{p,C_2H_5OH}(T_i - T_{ref})} \right] + n_{O_2}(-Q)$$

$$= n_{CO_2}\left[h_{f,CO_2} + c_{p,CO_2}(T_f - T_{ref})\right] + n_{H_2O}\left[h_{f,H_2O} + c_{p,H_2O}(T_f - T_{ref})\right] +$$

$$n_{CO}\left[h_{f,CO} + c_{p,CO}(T_f - T_{ref})\right]$$

	n	$h_{f,m}$ [J/mol]	$c_{p,m}$ [J/mol-K]
C_2H_5OH	1	−277000	N/A
O_2	2.875	0	30.77
CO_2	1.75	−393500	37.14
H_2O	3	−241800	34.74
CO	0.25	−110500	28.56

$$n_{C_2H_5OH}\, h_{f,C_2H_5OH} + n_{O_2}(-Q) = n_{CO_2}\left[h_{f,CO_2} + c_{p,CO_2}(T_f - T_{ref})\right]$$

$$+ n_{H_2O}\left[h_{f,H_2O} + c_{p,H_2O}(T_f - T_{ref})\right] + n_{CO}\left[h_{f,CO} + c_{p,CO}(T_f - T_{ref})\right]$$

$$1 \times (-277000) + 2.875(-13517.86) = 1.75\left[-393500 + 37.14(T_f - 298)\right]$$

$$+ 3\left[-241800 + 34.74(T_f - 298)\right] + 0.25\left[-110500 + 28.56(T_f - 298)\right]$$

$$\boxed{T_f = 6681.6361 \,[K]}$$

4. Suppose we have a rocket combustor that has hot products produced at the following conditions:

T = temperature = 4600 º R $= 2555.\bar{5}$ K

P = pressure = 75 atmospheres $= 7.599 \times 10^{6}$ [Pa]

γ = ratio of specific heats = 1.25

MW = average molecular weight = 27 $\left[\frac{g}{mol}\right]$

(a) Design a nozzle that will produce 75,000 pounds of thrust with an ambient pressure of one atmosphere. In particular, determine the following quantities: mass flow rate, exit pressure, exit or exhaust velocity, effective exhaust velocity, thrust coefficient, throat cross-sectional area, and exit cross-sectional area.

(b) Design a nozzle that produces 100,000 pounds of thrust with an ambient pressure at vacuum conditions. Limit the nozzle exit cross-sectional area to no more than thirty times the throat cross-sectional area. Determine the same quantities as described in Part (a).

(a)

$$\bar{R} = 8.3144598 \left[\frac{J}{mol \cdot K}\right]$$

$$R = \frac{\bar{R}}{MW} = 8.3144598 \left[\frac{J}{\cancel{mol} \cdot K}\right] \frac{1}{27} \left[\frac{\cancel{mol}}{\cancel{g}}\right] \times \frac{1000 \, [\cancel{g}]}{1 \, [kg]}$$

Gas constant:

$$R = 307.943 \left[\frac{m^2}{s^2 \cdot K}\right]$$

Pressure ratio:

$$\boxed{P = 1 \, [atm]}$$

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{75}{1} = \left(1 + \frac{1.25 - 1}{2} \times M^2\right)^{\frac{1.25}{1.25 - 1}}$$

$$M = 3.3123$$

Temperature ratio:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{2555.\bar{5}}{T} = 1 + \frac{1.25 - 1}{2} \times 3.3123^2$$

$$T = 1077.65 \ [K]$$

Velocity at exit:

$$V = M\sqrt{\gamma R T}$$

$$= 3.3123\sqrt{1.25 \times 307.943 \times 1077.65}$$

$$\boxed{V = 2133.28 \ [m/s]}$$

Thrust equation:

$$T = 75\,000 [lb] = 333616.62 \ [N]$$

$$T = \dot{m} V_e + \cancel{(P_e - P_a) A_e}$$

$$\boxed{\dot{m} = \frac{T}{V_e} = \frac{333616.62}{2133.28} = 156.4 \ [kg/s]}$$

At the throat:

$$M = 1$$

$$\frac{P_0}{P^*} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{7.599 \times 10^6}{P^*} = \left(1 + \frac{1.25 - 1}{2} \times 1^2\right)^{\frac{1.25}{1.25 - 1}}$$

$$P^* = 4.217 \times 10^6 \ [Pa]$$

$$\frac{T_0}{T^*} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{2555.5}{T^*} = 1 + \frac{1.25 - 1}{2} \times 1^2$$

$$\Rightarrow T^* = 2271.6\ [K]$$

$$\rho^* = \frac{P^*}{R T^*} = \frac{4.217\times10^6}{307.943\times2271.6} = 6.028\ \left[\frac{kg}{m^3}\right]$$

$$V^* = M\sqrt{\gamma R T^*} = 1\sqrt{1.25\times307.943\times2271.6} = 935.096\left[\frac{m}{s}\right]$$

$$\boxed{A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{156.4}{6.028\times935.096} = 0.02775\ [m^2]}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A}{0.02775} = \frac{1}{3.3123}\left[\frac{2}{1.25+1}\left(1+\frac{1.25-1}{2}\times3.3123^2\right)\right]^{\frac{1.25+1}{2(1.25-1)}}$$

$$\boxed{A = 0.24015\ [m^2]}$$

Effective exhaust velocity:

$$C = \frac{T}{\dot{m}} = \frac{333616.62}{156.4} = \boxed{2133.0986\left[\frac{m}{s}\right]}$$

Thrust coefficient:

$$C_F = \frac{\dot{m}\,C}{P_0 A^*} = \frac{156.4\times2133.0986}{7.599\times10^6\times0.02775}$$

$$\boxed{C_F = 1.58208}$$

(b) Area ratio:

$$\frac{A_e}{A^*} = 30 = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$30 = \frac{1}{M}\left(\frac{2}{1.25+1}\left(1+\frac{1.25-1}{2}\times M^2\right)\right]^{\frac{1.25+1}{2(1.25-1)}}$$

$$M_e = 4.3015$$

Temperature ratio at the exit:

$$\frac{T_0}{T_e} = 1 + \frac{\gamma-1}{2}M^2$$

$$\frac{2555.\bar{5}}{T_e} = 1 + \frac{1.25-1}{2} \times 4.3015^2$$

$$T_e = 771.4 \text{ [K]}$$

Pressure ratio at exit:

$$\frac{P_0}{P_e} = \left(\frac{T_0}{T_e}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{7.599\times10^6}{P_e} = \left(1 + \frac{1.25-1}{2} \times 4.3015^2\right)^{\frac{1.25}{1.25-1}}$$

$$\boxed{P_e = 19043.1683 \text{ [Pa]}}$$

Velocity at exit:

$$V_e = M_e\sqrt{\gamma R T_e} = 4.3015\sqrt{1.25\times367.943\times771.4}$$

$$= 2343.96 \text{ [m/s]}$$

Area at exit:

Thrust: $T = P_e A_e \left(\frac{V_e^2}{R T_e} + 1 \right)$

$$444\ 822.16 = 19043.1683 \times A_e \left(\frac{2343.96^2}{307.943 \times 771.4} + 1 \right)$$

$$\boxed{A_e = 0.968\ [m^2]}$$

Area at throat:

$$\frac{A_e}{A^*} = 30 \Rightarrow \frac{0.968}{A^*} = 30 \Rightarrow A^* = \boxed{0.03227\ [m^2]}$$

Mass flow rate:

$$\dot{m} = \frac{P_e}{R T_e} A_e V_e = \frac{19043.1683}{307.943 \times 771.4} \times 0.968 \times 2343.96$$

$$\boxed{\dot{m} = 181.8925\ [kg/s]}$$

Effective exhaust velocity:

$$c = \frac{T}{\dot{m}} = \frac{444\ 822.16}{181.8925}$$

$$\boxed{c = 2445.5223\ [m/s]}$$

Thrust coefficient:

$$C_F = \frac{T}{P_0 A^*} = \frac{444\ 822.16}{7.599 \times 10^6 \times 0.03227}$$

$$\boxed{C_F = 1.814}$$

5. Consider a jet engine flying at a Mach number of 1.4. A normal shock sits at the entrance of the divergent diffuser. The diffuser entrance cross-sectional area is 2.5 ft^2. The ambient conditions are 500°R for temperature and 0.8 atmosphere for pressure.

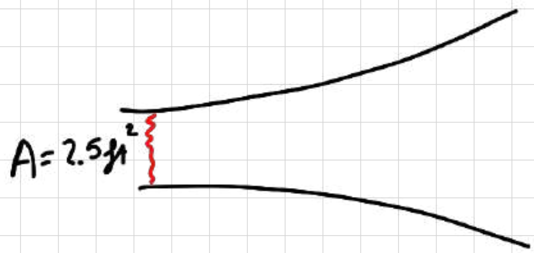
(a) What is the stagnation pressure immediately in front (upstream) of the shock? What is the stagnation pressure immediately behind (downstream) the shock? What is the Mach number immediately behind the shock? What is the mass flow through the diffuser?

(b) What is the minimum cross-sectional area required at the downstream end of the diffuser in order to assure that the Mach number of the flow there does not exceed 0.10?

$M = 1.4$

$T = 500°R$

$P = 0.8$ atm



(a)

At the entrance, before normal shock (upstream)

Pressure ratio:

$$\frac{P_{01}}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{01}}{0.8} = \left(1 + \frac{1.4-1}{2} \times 1.4^2\right)^{\frac{1.4}{1.4-1}}$$

$$\boxed{P_{01} = 2.5458 \text{ [atm]}}$$

Stagnation pressure downstream :

$$\frac{P_{02}}{P_{01}} = \left[\frac{(\gamma+1)M^2}{(\gamma-1)M^2 + 2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

$$\frac{P_{02}}{2.5458} = \left[\frac{(1.4+1)\,1.4^2}{(1.4-1)\,1.4^2 + 2}\right]^{\frac{1.4}{0.4}} \left[\frac{1.4+1}{2 \times 1.4 \times 1.4^2 - (1.4-1)}\right]^{\frac{1}{0.4}}$$

$$\boxed{P_{02} = 2.4394 \text{ [atm]}}$$

Down stream Mach number:

$$M_2 = \sqrt{\frac{(\gamma-1)M^2+2}{2\gamma M^2-(\gamma-1)}} = \sqrt{\frac{(1.4-1)1.4^2+2}{2\times1.4\times1.4^2-(1.4-1)}}$$

$$\boxed{M_2 = 0.7397}$$

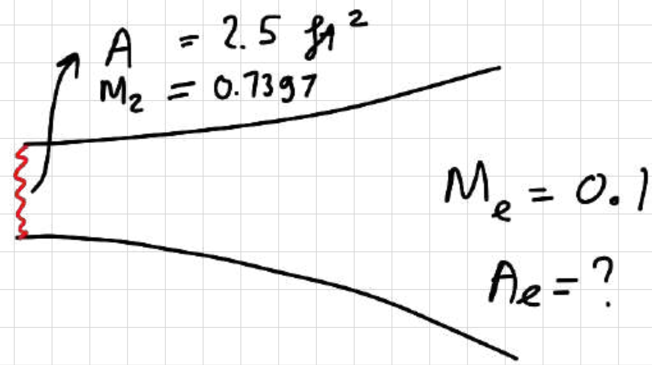
Mass flow rate:

$$\dot{m} = \rho A V = \frac{P}{RT} A M\sqrt{\gamma RT}$$

$$= \frac{0.8\times2116.2}{1718\times500} \times 2.5\times1.4\sqrt{1.4\times1718\times500}$$

$$\boxed{\dot{m} = 7.5645 \text{ [slug/s]}}$$

(b)



Right after the shock way

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{2.5}{A^*} = \frac{1}{0.74}\left[\frac{2}{1.4+1}\left(1+\frac{1.4-1}{2}0.74^2\right)\right]^{\frac{1.4+1}{2(1.4-1)}}$$

$$A^* = 2.34 \text{ [ft}^2\text{]}$$

At the end of the diffuser:

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

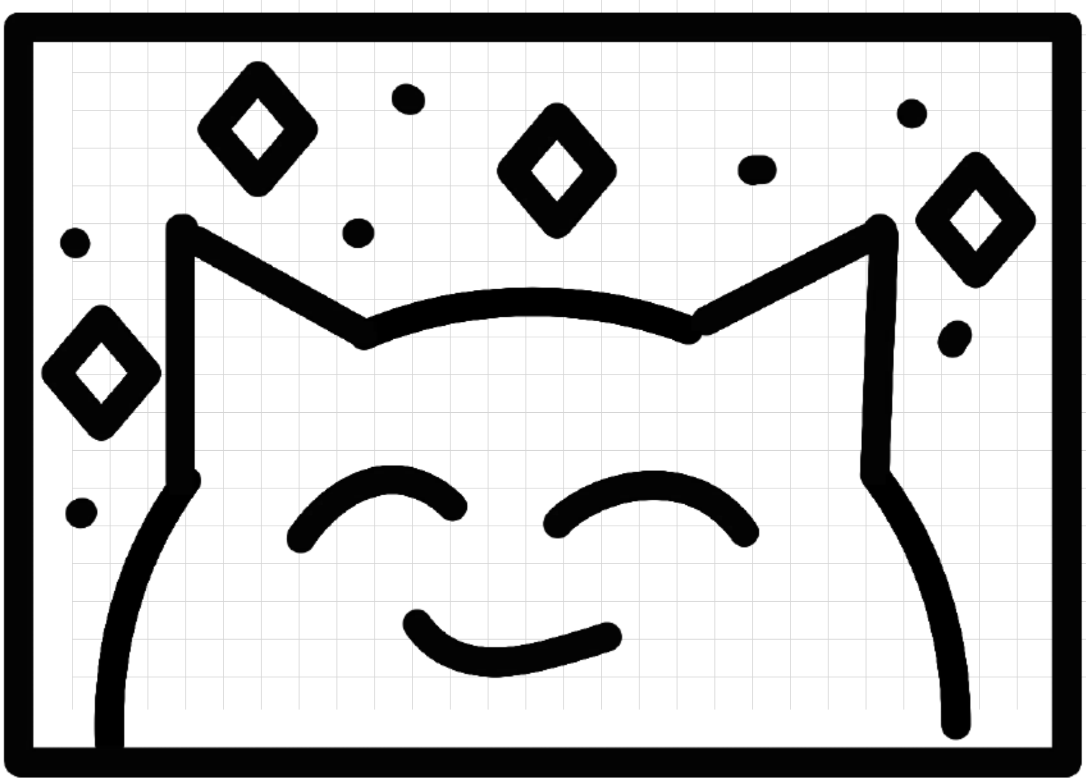
$$\frac{A_e}{2.34} = \frac{1}{0.1}\left[\frac{2}{1.4+1}\left(1+\frac{1.4-1}{2}0.1^2\right)\right]^{\frac{1.4+1}{2(1.4-1)}}$$

$$\boxed{A_e = 13.623\ [ft^2]}$$

MAE 112

HW 4

TRIET



1. . Compare a normal shock with an oblique shock. Suppose the inflowing velocity of the air had a Mach number of 2.0 at a temperature of 250 K and an ambient pressure of 0.70 atm.

(a) With the normal shock, determine the pressure, stagnation pressure, temperature, velocity, and Mach number behind (downstream of) the shock.

(b) Suppose we aim for a downstream stagnation pressure that is 15% higher than the value found in part (a). What is the angle of oblique shock here to the incoming velocity vector? Use the charts from Chapter 3, making the best interpolations you can.

(c) Determine the downstream values for the temperature, Mach number, velocity component normal to the oblique shock, and velocity component parallel to the oblique shock.

(a)

Stagnation pressure before the shock:

$$\frac{P_{01}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{P_{01}}{0.7} = 7.8244$$

$$\Rightarrow P_{01} = 5.47708 \text{ [atm]}$$

Stagnation temperature before the shock:

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{T_{01}}{250} = 1 + \frac{1.4 - 1}{2} 2^2$$

$$T_{01} = 450 \text{ [K]}$$

Right after the shock.

$$M_1 = 2 \qquad \gamma = 1.4$$

$$\boxed{M_2 = 0.5774}$$

$$\frac{P_2}{P_1} = 4.5$$

$$\boxed{P_2 = 4.5\,P_1 = 4.5 \times 0.7 = 3.15\ [\text{atm}]}$$

$$\frac{P_{02}}{P_{01}} = 0.7209 \Rightarrow P_{02} = 0.7209\,P_{01} = 0.7209 \times 5.47708$$

$$\boxed{P_{02} = 3.9484\ [\text{atm}]}$$

$$\frac{T_2}{T_1} = 1.6875 \Rightarrow T_2 = 1.6875\,T_1 = 1.6875 \times 250$$

$$\boxed{T_2 = 421.875\ [\text{K}]}$$

$$V_2 = M_2\sqrt{\gamma R T_2} = 0.5774\sqrt{1.4 \times 287 \times 421.875}$$

$$\boxed{V_2 = 237.7243\ [\text{m/s}]}$$

(b)

Stagnation pressure after shock wave:

$$P_{02} = 115\%\ P_{02\,(a)} = 115\% \times 3.9484$$

$$P_{02} = 4.54066\ [\text{atm}]$$

Pressure ratio

$$\frac{P_{02}}{P_{01}} = \frac{4.54066}{5.47708} = 0.83$$

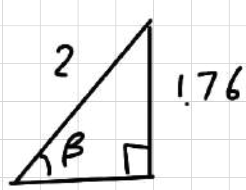
Mach number before shock way for normal component

$$\gamma = 1.4 \qquad \frac{P_{02}}{P_{01}} = 0.83$$

$$M_{1n} = 1.76$$

$$M_1 = 2$$

$$\beta = \sin^{-1}\left(\frac{M_{1n}}{M_1}\right)$$



$$= \sin^{-1}\left(\frac{1.76}{2}\right)$$

$$\boxed{\beta = 61.64°} \qquad \boxed{\delta = 23°}$$

(c)

$M_{1n} = 1.76 \qquad \gamma = 1.4$

Temperature after the oblique shock:

$$\frac{T_2}{T_1} = 1.5019 \Rightarrow T_2 = 1.5019\, T_1 = 1.5019 \times 250$$

$$\boxed{T_2 = 375.475\ [K]}$$

Normal Component of Mach number after shock way:

$$M_{2n} = 0.6257$$

Mach number after oblique shock wave:

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = \frac{0.6257}{\sin(61.64 - 23)} = \boxed{1.0620418}$$

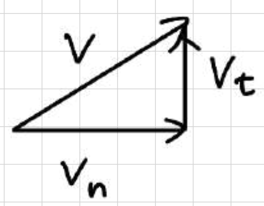
Velocity:

$$V_2 = M_2 \sqrt{\gamma R T_2} = 1\sqrt{1.4 \times 287 \times 375.475}$$

$$= 388.41454\ [m/s]$$

$$V_{2n} = M_{2n} \sqrt{\gamma R T_2} = 0.6257 \sqrt{1.4 \times 287 \times 375.475}$$

$$\boxed{V_{2n} = 243.031\ [m/s]}$$



$$V_{2t} = \sqrt{V_2^2 - V_{2n}^2} = \sqrt{388.4^2 - 243.031^2}$$

$$\boxed{V_{2t} = 303\ [m/s]}$$

2. Consider a Kantrowitz-Donaldson diffuser designed for a flight Mach number of 1.75. The entrance area equals 1.5 ft^2 and the ambient air temperature and pressure are 500°R and 0.7 atmosphere. The flow is isentropic everywhere except across the normal shockwave. Determine:

(a) the minimum cross-sectional area of the throat such that a normal shock may be stabilized at the entrance,

(b) the maximum mass flow, and

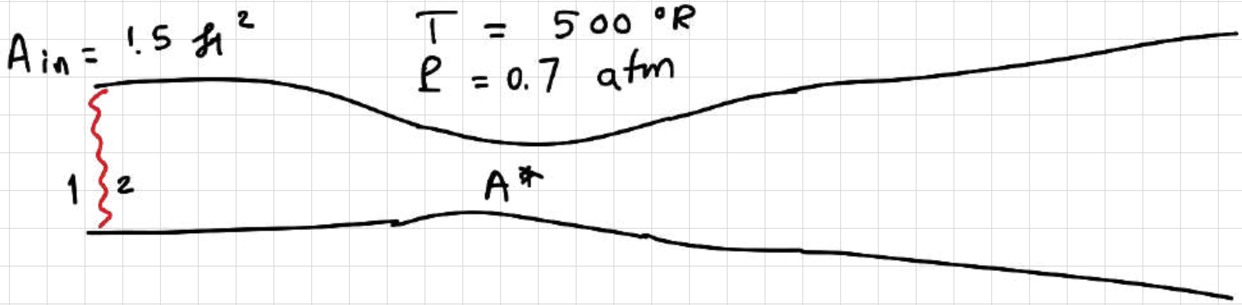
(c) the maximum stagnation pressure possible at the end of the diffuser (with subsonic flow only in the divergent portion).

In each of these optimizations, consider the flight Mach number fixed at the design value while the final pressure (at the end of the diffuser) is allowed to adjust.

$M = 1.75$

$T = 500\ °R$

$P = 0.7$ atm



ⓐ $M_1 = 1.75 \Rightarrow M_2 = 0.6281$

$$\frac{A_2}{A^*} = 1.1571$$

$$\Rightarrow A^* = \frac{A_2}{1.1571} = \frac{1.5}{1.1571}$$

$$\boxed{A^* = 1.2963\ [ft^2]}$$

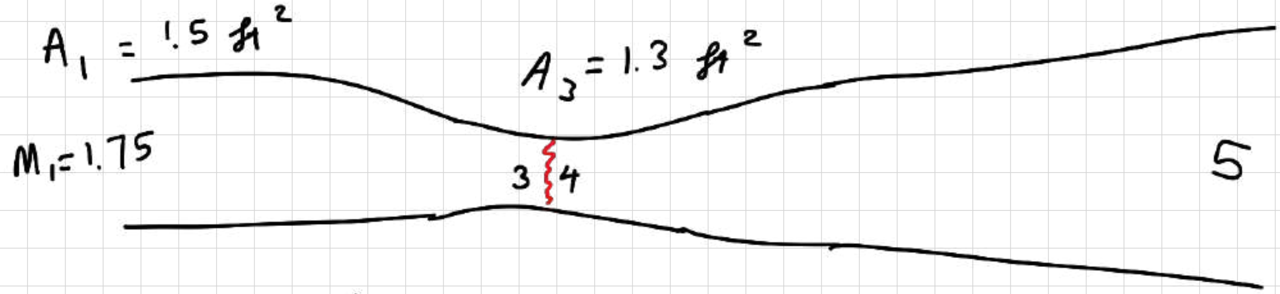
ⓑ

$$\dot{m} = \rho_1 V_1 A_1 = \frac{P_1}{RT_1}\left(M_1\sqrt{\gamma R T_1}\right) A_1 = \frac{0.7 \times 2116.2}{1718 \times 500}\left(1.75\sqrt{1.4 \times 1718 \times 500}\right) 1.5$$

$$\boxed{\dot{m} = 4.9642\ [slug/s]}$$

$$\dot{m} = 4.9642\left[\frac{\cancel{slug}}{s}\right]\left[\frac{32.2\ lbm}{1\ \cancel{slug}}\right] = 159.848\left[\frac{lbm}{s}\right]$$

(c)



$$M_1 = 1.75 \Rightarrow \frac{A_1}{A^*} = 1.3865 \Rightarrow \frac{1.5}{A^*} = 1.3865$$

$$\Rightarrow A^* = 1.08186\ [ft^2]$$

$$\frac{A_3}{A^*} = \frac{1.3}{1.08186} = 1.20163 \Rightarrow M_3 = 1.535$$

Stagnation pressure:

$$\frac{P_{01}}{P_1} = 5.3241 \Rightarrow P_{01} = 5.3241\ P_1 = 5.3241 \times 0.7$$

$$P_{03} = P_{01} = 3.72687\ [atm]$$

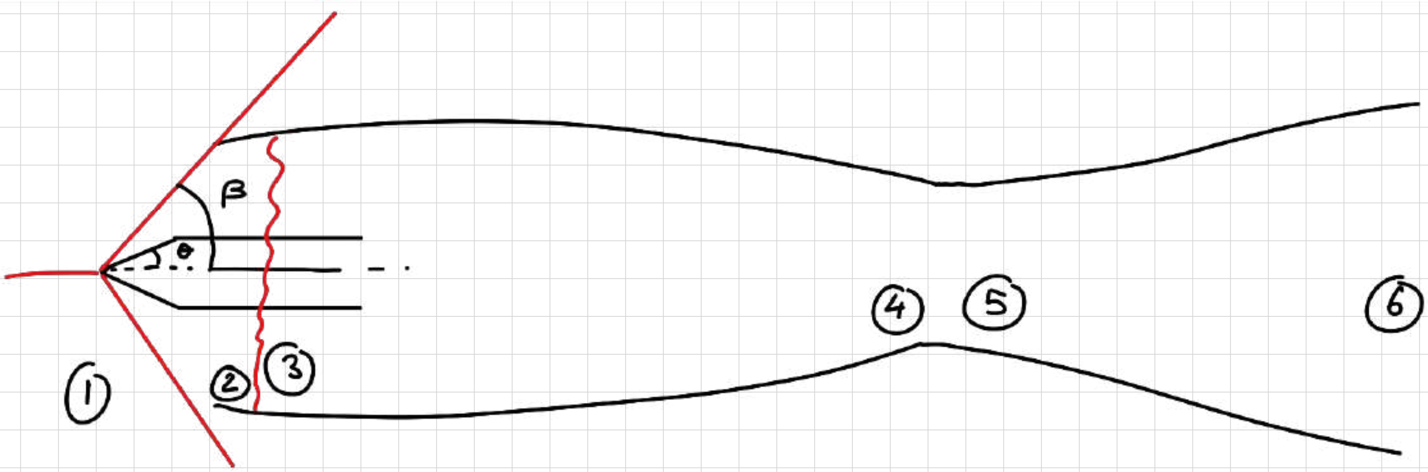
$$M_2 = 1.535 \Rightarrow \frac{P_{04}}{P_{03}} = 0.9183$$

$$\Rightarrow P_{04} = 0.9183 \times 3.72687$$

$$\boxed{P_{05} = P_{04} = 3.4224\ [atm]}$$

3 Consider a ramjet in flight at a Mach number of 2.75 with ambient conditions at 298 K and 0.9 atmosphere of pressure. The air capture area is 0.70 square meters. The inlet design involves first a wedge that deflects the stream by an angle of 15 degrees followed by a Kantrowitz-Donaldson (K-D) diffuser. Operation is at design conditions except for part (h).

(a) What is the mass flow through the ramjet?
(b) What is the stagnation temperature for that flow through the inlet / diffuser?
(c) What are the stagnation-pressure values ahead of and immediately behind the first shock?
(d) What is the flow Mach number immediately behind the first shock? What is the flow Mach number at the entrance to the K-D diffuser?
(e) What is the Mach number at the diffuser throat?
(f) What is the final stagnation pressure?
(g) Determine the value of the polytropic efficiency for this inlet design.
(h) Determine the polytropic efficiency value for a shock at the entrance of the K- D diffuser.



(a)

$$\dot{m} = \rho_1 V_1 A_1 = \frac{P_1}{RT_1} M_1 \sqrt{\gamma R T_1}\, A_1$$

$$= \frac{0.9 \times 101325}{287 \times 298}\, 2.75 \sqrt{1.4 \times 287 \times 298} \times 0.7$$

$$\boxed{\dot{m} = 710.2395 \ [kg/s]}$$

(b)

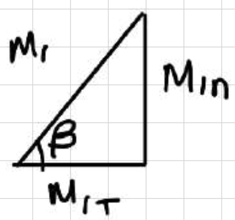
$$M_1 = 2.75 \Rightarrow \frac{T_{01}}{T_1} = 2.5125$$

$T_{01} = 2.5125\, T_1 = 2.5125 \times 298 = \boxed{748.725\,[K]}$

(c)

$$M_1 = 2.75 \Rightarrow \frac{P_{01}}{P_1} = 25.14$$

$$\Rightarrow P_{01} = 25.14\, P_1 = 25.14 \times 0.9 = \boxed{22.626\,[atm]}$$



$M_1 = 2.75 \quad \theta = 15° \Rightarrow \beta \simeq 34°$

$M_{1N} = M_1 \sin\beta = 2.75 \times \sin(34°)$

$M_{1N} = 1.53778$

$$\frac{P_{02}}{P_{01}} = 0.9174 \Rightarrow P_{02} = P_{01}\, 0.9174 = 22.626 \times 0.9174$$

$$\boxed{P_{02} = 20.7516\,[atm]}$$

(d)

$$M_{1N} = 1.53778 \Rightarrow M_{2N} = 0.6881$$

$$M_2 = \frac{M_{2N}}{\sin(\beta - \theta)} = \frac{0.6881}{\sin(34° - 15°)} = \boxed{2.1135}$$

(e) Shockwave at entrance:

$$M_{1N} = 1.53778 \Rightarrow \frac{P_2}{P_1} = 2.5922, \quad \frac{T_2}{T_1} = 1.3455$$

$$P_2 = 2.5922\, P_1 = 2.5922 \times 0.9 = 2.333\,[atm] = 2.3639 \times 10^5\,[Pa]$$

$$T_2 = 1.3455 \times T_1 = 1.3455 \times 298 = 401\,[K]$$

$$A_2 = \frac{\dot{m}}{\frac{P_2}{RT_2} M_2 \sqrt{\gamma R T_2}} = \frac{710.2395}{\frac{2.3639 \times 10^5}{287 \times 401} \times 2.1135 \sqrt{1.4 \times 287 \times 401}}$$

$$A_2 = 0.4076\,[m^2]$$

$$M_2 = 2.1135 \Rightarrow M_3 = 0.56 \Rightarrow \frac{A_3}{A^*} = 1.24$$

$$\Rightarrow A^* = \frac{A_3}{1.24} = \frac{A_2}{1.24} = \frac{0.4076}{1.24} = 0.3287 \ [m^2]$$

$$A_4 = A^* = 0.3287 \ [m^2]$$

Move Shockwave to throat:

$M_2 = 2.1135 \quad A_2 = 0.4076 \quad A_4 = 0.3287 \quad M_4 = ?$

$$\Downarrow$$

$$\frac{A_2}{A^*} = 1.8585 \Rightarrow A^* = \frac{A_2}{1.8585} = \frac{0.4076}{1.8585} = 0.2193 \ [m^2]$$

$$\frac{A_4}{A^*} = \frac{0.3287}{0.2193} = 1.5 \Rightarrow \boxed{M_4 = 1.854}$$

(8) $P_{04} = P_{02} = 20.7516 \ [atm]$

$$M_4 = 1.854 \Rightarrow \frac{P_{05}}{P_{04}} = 0.7884 \Rightarrow P_{05} = 0.7884 P_{04} = 0.7884 \times 20.7516$$

$$P_{05} = 16.36 \ [atm]$$

$$\boxed{P_{06} = P_{05} = 16.36 \ [atm]}$$

(9) $M_4 = 1.854 \Rightarrow M_5 = 0.6049$

$$M_5 = 0.6049 \Rightarrow \frac{P_{05}}{P_5} = 1.2804 \Rightarrow P_5 = \frac{P_{05}}{1.2804} = \frac{16.36}{1.2804}$$

$$P_5 = 12.777 \ [atm]$$

$$T_2 = 401 \ [K]$$

$$M_2 = 2.1135 \Rightarrow \frac{T_{02}}{T_2} = 1.8934 \Rightarrow T_{02} = 1.8934 \ T_2$$

$$T_{02} = 1.8934 \times 401 = 759.2534 \ [K]$$

$$T_{05} = T_{04} = T_{02} = 759.2534 \ [K]$$

$$M_5 = 0.6049 \Rightarrow \frac{T_{05}}{T_5} = 1.0732 \Rightarrow T_5 = \frac{T_{05}}{1.0732}$$

$$T_5 = \frac{759.2534}{1.6732} = 707.467 \; [K]$$

Polytropic effiency:

$$\frac{T_{final}}{T_{initial}} = \left(\frac{P_{final}}{P_{initial}}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_5}{T_1} = \left(\frac{P_5}{P_1}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}}$$

$$\frac{707.467}{298} = \left(\frac{12.777}{0.7}\right)^{\frac{1}{e}\times\frac{1.4-1}{1.4}} \Rightarrow e = 0.95976$$

$$\boxed{e = 95.976 \%}$$

(h)

$$M_2 = 2.1135 \Rightarrow \frac{T_3}{T_2} = 1.7819 \qquad \frac{P_3}{P_2} = 5.0447$$

$$\Rightarrow T_3 = 1.7819\, T_2 = 1.7819 \times 401 = 714.542 \; [K]$$

$$\Rightarrow P_3 = 5.0447 P_2 = 5.0447 \times 2.333 = 11.77 \; [atm]$$

Polytropic effiency:

$$\frac{T_{final}}{T_{initial}} = \left(\frac{P_{final}}{P_{initial}}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}}$$

$$\frac{T_3}{T_1} = \left(\frac{P_3}{P_1}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}}$$

$$\frac{714.542}{298} = \left(\frac{11.77}{0.7}\right)^{\frac{1}{e}\times\frac{1.4-1}{1.4}}$$

$$e = 0.92202$$

$$\boxed{e = 92.202 \%}$$

4. Suppose a particular compressor has a compression ratio $P_3/P_2 = 25$; the incoming air temperature is 300 K and its pressure is 1.2 atm. 20 kgm per sec. of air flows through the compressor.

(a) If the adiabatic efficiency is 90%, what is the final temperature?

(b) What is the power required?

(c) What is the minimum number of stages (pairs of rotor and stator sections) required to protect against separation due to adverse pressure gradients?

(a)

$$\eta = \frac{\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_2}{T_1} - 1}$$

$$90\% = \frac{25^{\frac{1.4-1}{1.4}} - 1}{\frac{T_2}{300} - 1}$$

$$\boxed{T_2 = 802.8282 \ [K]}$$

(b)

$$C_p = 1004.5 \ J/kg\text{-}K$$

$$P_r = \dot{m} \, C_p (T_2 - T_1)$$

$$= 20 \times 1004.5 \times (802.8282 - 300)$$

$$\boxed{P_r = 10101818.54 \ [W]}$$

(c)

$$\frac{P_{after}}{P_{before}} \leq 1.6$$

$$\frac{P_3/P_2}{1.6} = \frac{25}{1.6} = 15.625$$

$$\boxed{16}$$

1. Consider one-stage of a compressor with an 8% static pressure rise across the rotor followed by another 9% pressure rise across the stator (compounded to be 17.7%). The incoming flow has a velocity of 75 ft/sec in the axial direction, a temperature of 560°R and a pressure of 2.0 atmospheres. $\gamma = 1.4$; $c_p = 0.24$ Btu/1bm°R; polytropic efficiency = 0.95 for the compressor stage. (a) What is the power per unit mass flow of the compressor? (b) If the rotor blade speed averages 1000 ft/sec, what is the tangential component of velocity exiting the rotor?

(a)

$$\frac{P_2}{P_1} = \frac{117.7\,\%}{100\,\%} = 1.177$$

$$\eta = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_2}{T_1} - 1} \Rightarrow 0.95 = \frac{1.177^{\frac{1.4-1}{1.4}} - 1}{\frac{T_2}{560} - 1}$$

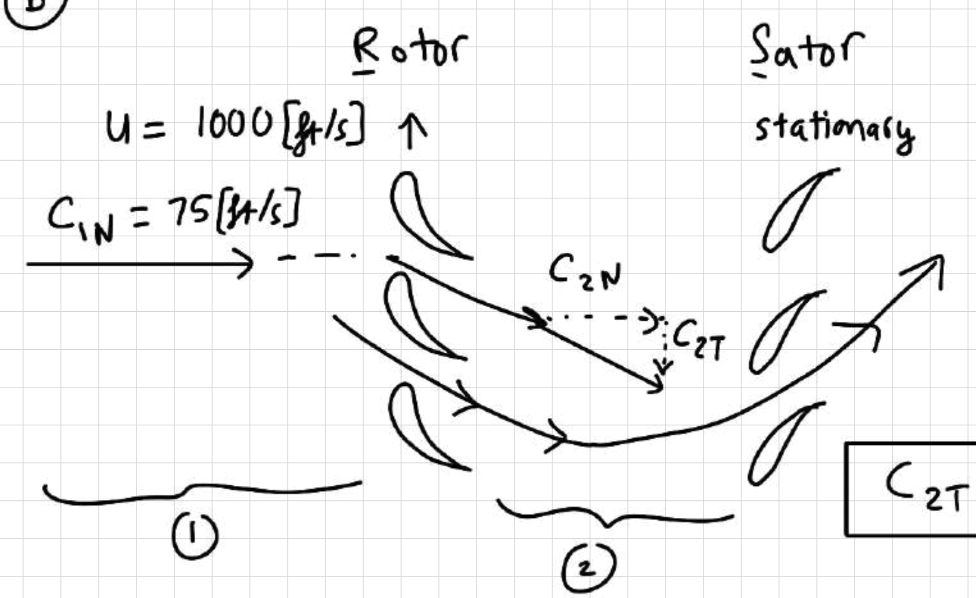
$$\Rightarrow T_2 = 588.0964\ [°R]$$

$$\frac{P}{\dot{m}} = C_p(T_2 - T_1) = 0.24\left[\frac{Btu}{lbm\,\cancel{°R}}\right](588.0964 - 560)[\cancel{°R}]$$

$$= 6.743136\left[\frac{\cancel{Btu}}{\cancel{lbm}}\right]\frac{778\ ft\cdot \cancel{lbf}}{1\ \cancel{Btu}} \times \frac{1\,\cancel{lbm} \times 32.2\ \frac{ft}{s^2}}{1\ \cancel{lbf}}$$

$$\boxed{\frac{P}{\dot{m}} = 168926.3458\left[\frac{ft^2}{s^2}\right]}$$

(b)



$$\frac{P}{\dot{m}} = U(C_{2T} - \cancel{C_{1T}})$$

$$C_{2T} = \frac{P}{\dot{m}} \div U$$

$$= \frac{168926}{1000}$$

$$\boxed{C_{2T} = 168.93\ [ft/s]}$$

2. Suppose a particular compressor has a compression ratio $P_2/P_1 = 15$ and the incoming air temperature is 300K. If the adiabatic efficiency is .95, what is (a) the final temperature, (b) the average polytropic efficiency, and (c) the entropy change? (d) What is the power required, if 25 kgm per sec. flow through the compressor?

(a)

$$\eta = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{1.4-1}{1.4}} - 1}{\frac{T_2}{T_1} - 1} \qquad 0.95 = \frac{15^{\frac{1.4-1}{1.4}} - 1}{\frac{T_2}{300} - 1}$$

$$\Rightarrow \boxed{T_2 = 668.7898\ [K]}$$

(b)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}}$$

$$\frac{668.7898}{300} = 15^{\frac{1}{e} \times \frac{1.4-1}{1.4}}$$

$$\boxed{e = 96.51\%}$$

(c)

$$C_p = 1004.5\ J/kg\text{-}k$$

$$\Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$= 1004.5 \ln\left(\frac{668.7898}{300}\right) - 287 \ln(15)$$

$$\boxed{\Delta s = 28.0845\ [J/kg\text{-}k]}$$

(d)

$$P = \dot{m} C_p (T_2 - T_1) = 25 \times 1004.5 \times (668.7898 - 300)$$

$$\boxed{P = 9261233.853\ [W]}$$