Lecture 9
Efficiencies

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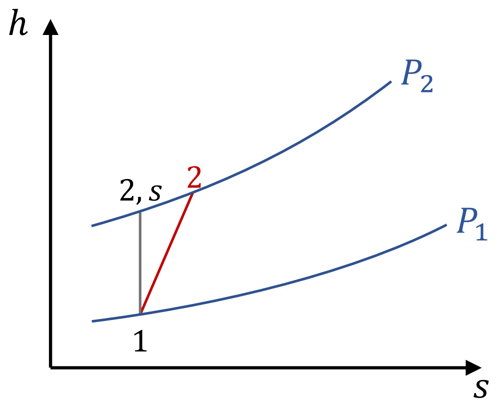
Efficiencies for Compression

h_s is enthalpy for the isentropic case.



$$\eta_{ad} = \frac{h_{2,s} - h_1}{h_2 - h_1} = \frac{\int \frac{dP}{\rho}}{c_p(T_2 - T_1)}$$

$$Tds = dh - dP/\rho$$

So $dh_s = dP/\rho$

$$\Delta h_s = \int \frac{dP}{\rho}$$

$$\frac{\rho}{\rho_1} = \left(\frac{P}{P_1}\right)^{1/\gamma}$$ For an isentropic process

$$\int_{P_1}^{P_2} \frac{dP}{\rho} = \frac{P_1^{1/\gamma}}{\rho_1} \int_{P_1}^{P_2} \frac{dP}{P^{1/\gamma}} = \frac{P_1^{1/\gamma}}{\rho_1} \frac{\gamma}{\gamma - 1} \left[P_2^{(\gamma-1)/\gamma} - P_1^{(\gamma-1)/\gamma}\right]$$

$$c_p = c_v + R$$

$$\frac{c_p}{R} = \frac{c_v}{R} + 1$$

$$\frac{c_p}{R} = \frac{\gamma - 1}{\gamma}$$

Used: $\frac{P_1}{R\rho_1 T_1} = 1$

$$\eta_{ad} = \frac{\frac{\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]}{\frac{c_p}{R}\left[\frac{T_2}{T_1} - 1\right]} = \frac{\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1}{\frac{T_2}{T_1} - 1}$$

Efficiencies for Compression

$$e = \frac{dh_s}{dh} = \frac{dP/\rho}{dh} = \frac{\left(\frac{RT}{P}\right)dP}{c_p dT}$$

Polytropic or Elementary Efficiency

$$e = \frac{\gamma - 1}{\gamma}\frac{dP/P}{dT/T}$$

If e were constant: $$\ln\frac{T_2}{T_1} = \frac{\gamma - 1}{\gamma e}\ln\frac{P_2}{P_1}$$

Or: $$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma e}$$

Shockwave Compression Relations

For compression process:

$$\frac{dT}{T} = \frac{\gamma - 1}{\gamma e}\frac{dP}{P} \quad \rightarrow \quad \ln\frac{T_2}{T_1} = \frac{\gamma - 1}{\gamma e}\ln\frac{P_2}{P_1}$$

$$\Delta \ln T = \frac{\gamma - 1}{\gamma e}\Delta \ln P$$

$$\Delta \ln T = \Delta \ln P - \Delta \ln \rho$$

$$\boxed{\begin{aligned} &e = \eta_{el} \\ &\frac{dT}{T} = \frac{R}{c_p e}\frac{dP}{P} = \frac{\gamma - 1}{\gamma e}\frac{dP}{P} \\ &d\sigma = c_p\frac{dT}{T} = \frac{R}{e}\frac{dP}{P} \end{aligned}}$$

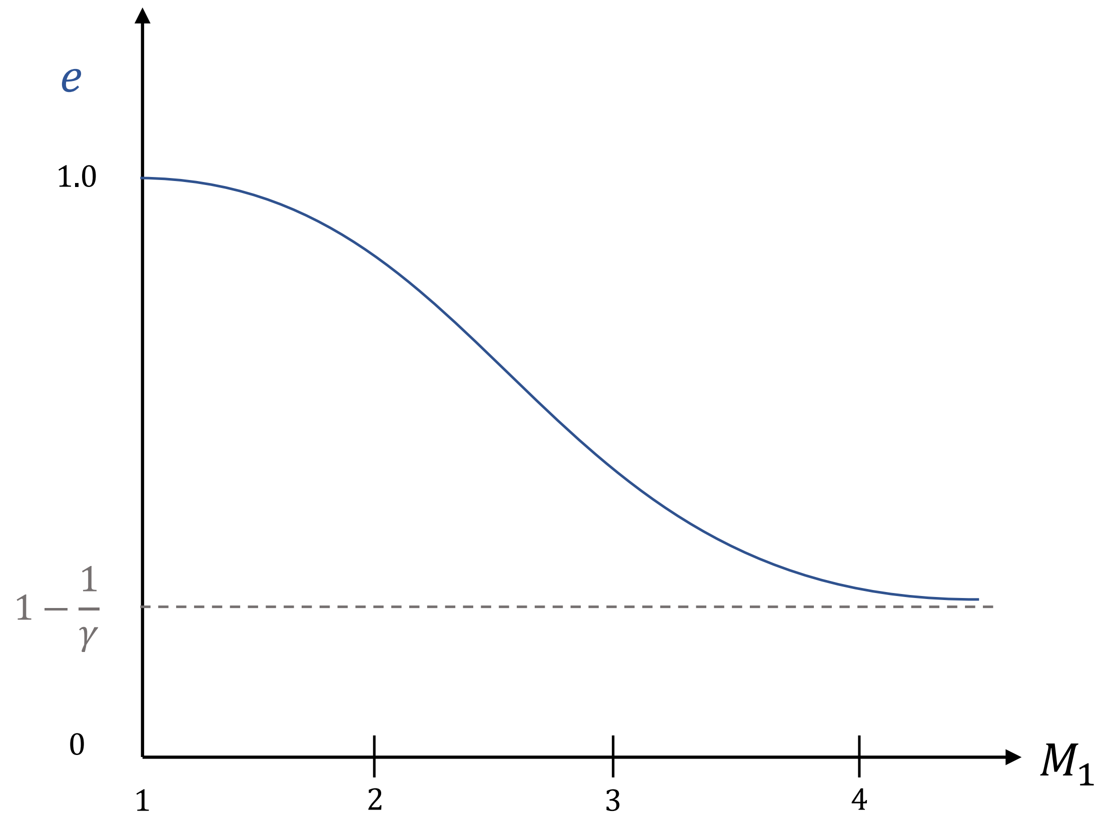
Shockwave Compression Relations

$$\frac{\gamma - 1}{\gamma e} = 1 - \frac{\Delta \ln \rho}{\Delta \ln P}$$

$$e = \frac{\dfrac{\gamma - 1}{\gamma}}{1 - \dfrac{\Delta \ln \rho}{\Delta \ln P}} = \frac{1 - \dfrac{1}{\gamma}}{1 - \dfrac{\Delta \ln \rho}{\Delta \ln P}} \quad \dashrightarrow$$

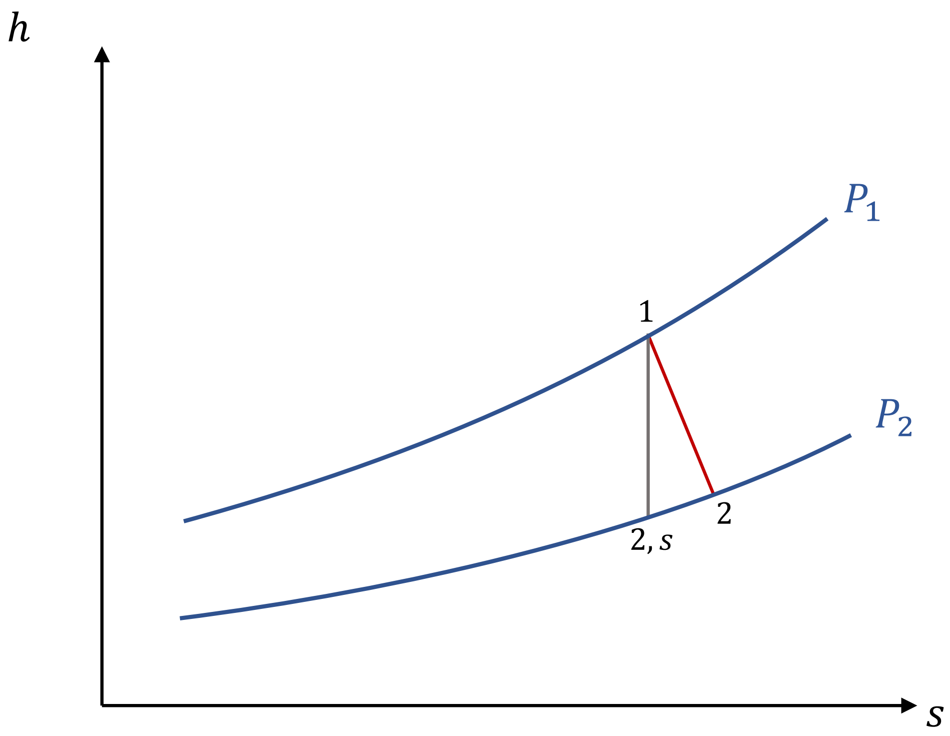
$1 \quad$ as $\dfrac{P_2}{P_1} \to 1, \;\; M_1 \to 1$

$1 - \dfrac{1}{\gamma} \quad$ as $\dfrac{P_2}{P_1} \to \infty, \;\; M_1 \to \infty$



So, oblique shock is preferred at high Mach numbers!

Efficiencies of Expansion



$$\eta_{ad} = \frac{h_2 - h_1}{h_{2,s} - h_1}$$

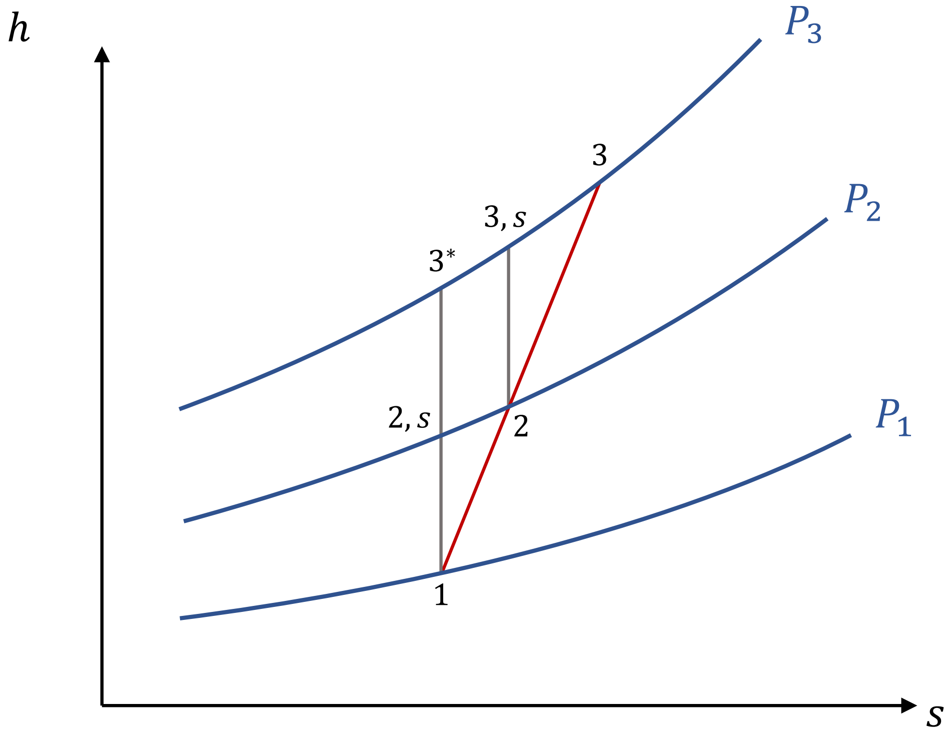
$$e = \frac{dh}{dh_s}$$

$$e = \frac{c_p dT}{dP/\rho} = \frac{c_p}{R}\frac{dT/T}{dP/P} = \frac{\gamma}{\gamma - 1}\frac{dT/T}{dP/P}$$

For constant e : $\ln\frac{T_2}{T_1} = \frac{(\gamma - 1)e}{\gamma}\ln\frac{P_2}{P_1}$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)e/\gamma}$$

Efficiencies for Multi-stage Compression



Suppose: $\eta_{ad_{12}} = \eta_{ad_{23}}$

$$\frac{h_{2,s} - h_1}{h_2 - h_1} = \frac{h_{3,s} - h_2}{h_3 - h_2}$$

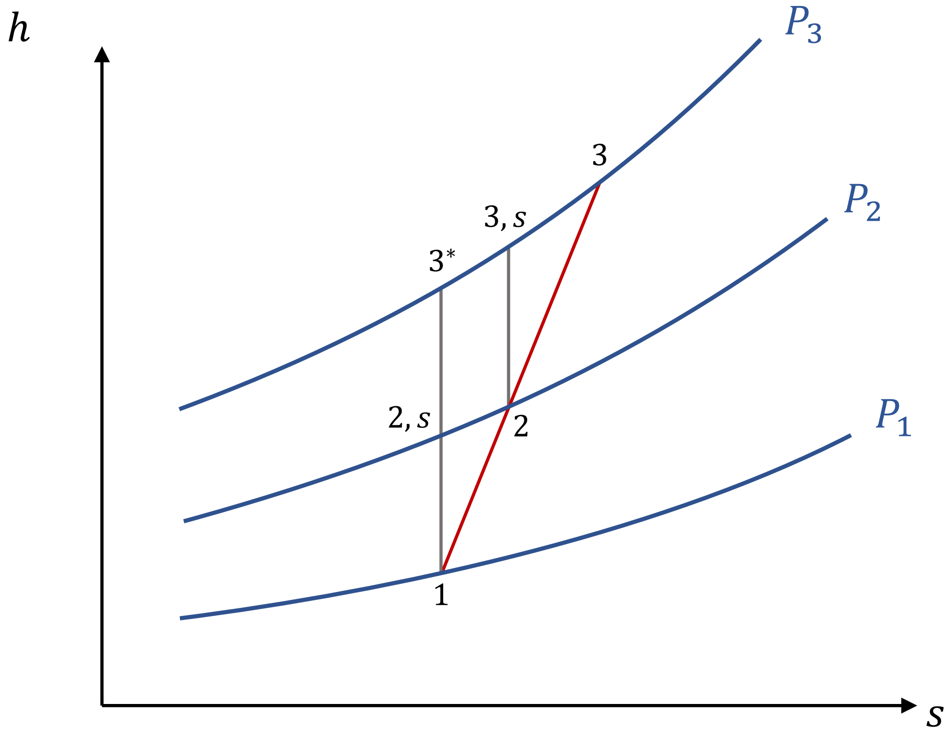
$$h_{2,s} - h_1 + h_{3,s} - h_2 = \eta_{ad_{12}}[h_2 - h_1 + h_3 - h_2]$$

$$h_{3,s} - h_1 - (h_2 - h_{2,s}) = \eta_{ad_{12}}[h_3 - h_1]$$

For the overall process:

$$\eta_{ad_{13}} = \frac{h_{3^*} - h_1}{h_3 - h_1} \quad \text{So} \quad h_{3^*} - h_1 = \eta_{ad_{13}}[h_3 - h_1]$$

Efficiencies for Multi-stage Compression



$$h_{3^*} - h_1 < h_{3,s} - \left(h_2 - h_{2,s}\right) - h_1$$

Because of diverging pressure curves, $\underbrace{h_{3,s} - h_{3^*} > h_2 - h_{2,s}}$

$$h_{3,s} - (h_2 - h_{2,s}) > h_{3^*}$$

So: $h_{3^*} - h_1 < h_{3,s} - \left(h_2 - h_{2,s}\right) - h_1$

And: $\eta_{ad_{13}} < \eta_{ad_{12}} = \eta_{ad_{23}}$

Efficiency overall $\neq$ Efficiency of stages

Polytropic efficiency is superior

Approximate Integration

$$\int_{P^\circ_1}^{P^\circ_2} \frac{dP^\circ}{P^\circ} \approx \frac{1}{P^\circ_{12}} \int_{P^\circ_1}^{P^\circ_2} dP^\circ \approx \frac{\Delta P^\circ}{P^\circ_{12}}$$

If P°_{12} is an average

$$\int_{T^\circ_1}^{T^\circ_2} \frac{dT^\circ}{T^\circ} \approx \frac{\Delta T^\circ}{T^\circ_{12}}$$

Where T°_{12} is an average

$$\frac{\Delta P^\circ}{P^\circ_{12}} \approx -\frac{\gamma}{2} M_c^2 \frac{\Delta T^\circ}{T^\circ_{12}}$$

Where M_c is approximately constant in the combustion chamber!