University of California, Irvine
Department of Mechanical and Aerospace Engineering

MAE 112 - Homework 1
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1 A rocket uses propellants with a specific impulse equal to 270 seconds. It has an empty weight (i.e., without propellants) of 200 lbf (measured at sea level). The desire is to accelerate it from rest to a velocity of 8,000 feet per second. Assume that drag forces are negligible and gravity acts normal to the flight direction.

- (a) What is the required mass of propellant?
- (b) If the rocket had an initial horizontal velocity of 1000 feet per second (e.g., with a launch from an aircraft), how much propellant mass is required to achieve the 8,000 feet per second speed?
- (c) If the specific impulse were increased to the new value of 290 seconds, what would be the answer to part (a)?

Solution:
(a) Directly apply the equation to calculate the required mass of propellant.

$$\Delta V = I_{sp} g ln \left[1 + \frac{m_{propellant}}{m_{final}} \right] \tag{1}$$

Solving this equation for the given inputs gives $m_{propellant} = 301.95\ lbm$.

*Note:
$1\ lbf = 1\ lbm \times 32.2\ ft/s^2$
$1\ lbf = 1\ slug \times 1\ ft/s^2$

(b) If the initial velocity is 1000 ft/s, then $\Delta V = 8000 - 1000 = 7000\ ft/s$. Solving the equation above again with the new ΔV gives $m_{propellant} = 247.41\ lbm$.

(c) Solve again using the new I_{sp} value gives $m_{propellant} = 271.08\ lbm$.

2 Consider a rocket at takeoff from Earth at sea level. The mass flux of propellants is 160 kg/s. The hot gas exits the nozzle at a velocity of 1200 m/sec and a pressure of 0.85 atmospheres through an exit area of one square meter. Ambient pressure is one atmosphere. What is the thrust magnitude? If the initial acceleration is 50 m/sec2 , what is the initial mass of the vehicle including propellants?

Solution:
Taking a control volume around the rocket and applying the integral momentum equation, we get the thrust generated by the rocket.

$$T = \dot{m} u_e + (p_e - p_a) A_e \tag{2}$$

Solving this equation for the given inputs gives $T = 176801.25\ N$. To calculate the total mass of the rocket, apply Newton's second law to the system and solve for m.

$$\sum F = T - mg = ma \tag{3}$$

Solving this equation for the given inputs gives $m = 2956.05\ kg$.

3 Consider an air-breathing jet engine which is flying at a velocity of 600 feet per second. For every lbm/second of air mass flow, a 0.030 lbm/sec mass flow of fuel is injected into the engine. The thrust force is 5000 lbf; the entrance pressure equals the ambient pressure at the altitude (0.5 atm) and the exhaust pressure is 0.75 times the ambient atmospheric pressure. The incoming air temperature is 525°R. The entrance area and exhaust areas are both 10 ft^2. Determine:

- (a) exhaust velocity
- (b) specific fuel consumption

Solution:
(a) Taking a control volume around the engine and applying the integral momentum equation, we get the thrust generated by the engine.

$$T = (\dot{m}_a + \dot{m}_f)\, u_e - \dot{m}_a u_i + (p_e - p_a)\, A_e - (p_i - p_a)\, A_i \tag{4}$$

Since the inlet pressure is equal to the ambient pressure the last term is zero. Divide Eq. 4 by $\dot{m}_a$ where $f = \frac{\dot{m}_f}{\dot{m}_a} = 0.030\ lbm/s$ is the given fuel/air mass flow rate. This equation can now be solved for u_e once $\dot{m}_a$ is known.

$$u_e = \frac{1}{1+f}\left[\frac{T - (P_e - P_a)A_e}{\dot{m}_a} + u_i\right] \tag{5}$$

Using the ideal gas law $P = \rho R_{sp} T$, the mass flow rate of air can be calculated.

$$\dot{m}_a = \rho u_i A_i = \frac{P_a}{R_a T_a} u_i A_i = \frac{1058.11(lbf/ft^2)}{53.3523(ft\text{-}lbf/lbm^\circ R)525(^\circ R)}(600ft/s)(10ft^2) = 226.66\ lbm/s \tag{6}$$

With $\dot{m}_a$ known, $u_e = 615.27\ ft/s$.

(b) Thrust-specific fuel consumption is defined as (units are per hr):

$$TSFC = \frac{\dot{m}_f}{T} = \frac{f}{T/\dot{m}_a} = \frac{0.030}{5000(lbf)/226.66(lbm/s)}\frac{3600(s)}{1(hr)} = 4.896(\frac{lbm/hr}{lbf}) \tag{7}$$

4 For an air-breathing jet engine, specific thrust is defined as thrust divided by air mass flow rate. Consider an engine that is flying at a velocity equal to 250 meters per second. For every kilogram/second of air mass flow, 0.040 kgm/sec mass flow of fuel is injected into the engine. The exit pressure and the entrance pressure both equal the ambient pressure, which is 0.7 atm. What must be the value of the exhaust velocity ue if a specific thrust equal to 400m/s is desired?

Solution:
Using the jet thrust equation, we can zero-out both pressure terms since $p_e = p_i = p_a$.

$$T = (\dot{m}_a + \dot{m}_f)\, u_e - \dot{m}_a u_i + \cancel{(p_e - p_a)}A_e - \cancel{(p_i - p_a)}A_i \tag{8}$$

Dividing this equation by $\dot{m}_a$ yields specific thrust.

$$\frac{T}{\dot{m}_a} = (1+f)\, u_e - u_i \tag{9}$$

This equation can then be solved for u_e using the known parameters. $u_e = 625\ m/s$.