Lecture 17
Electromagnetic Thrusters

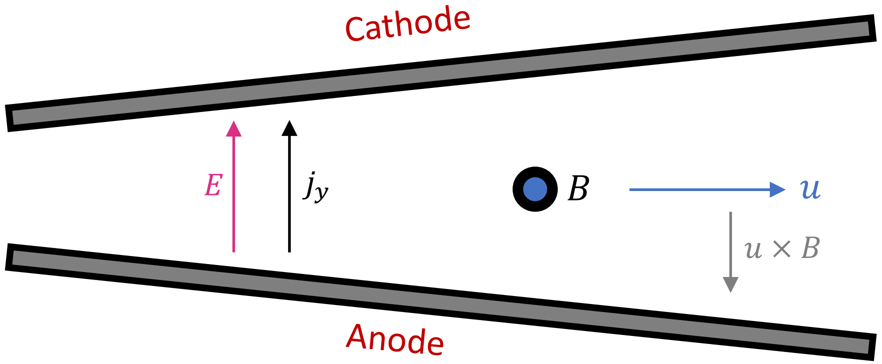
By W. A. Sirignano

Prepared by Colin Sledge

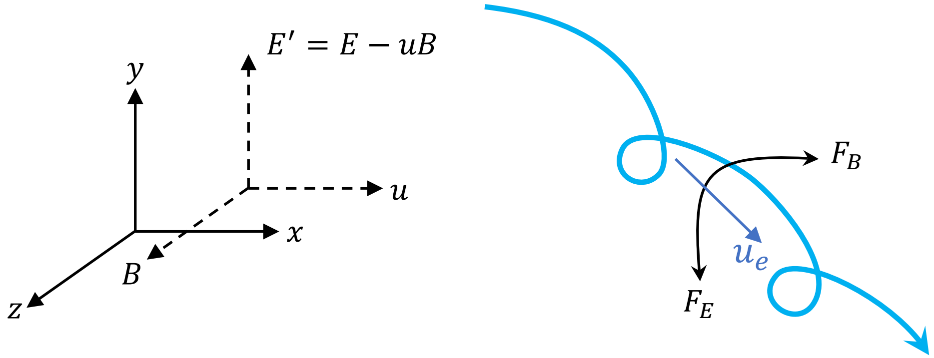
Electromagnetic thrusters

Consider now a crossed-field accelerator:

One-dimensional representation



Steady-flow device



Electron path is a series of curved lines between collisions

Note: The x-component of current or the Hall current is neglected

Electromagnetic thrusters

1. $j_y = \sigma(E - uB)$ Ohm's Law σ is Plasma conductivity

2. $p = \rho RT$ Equation of state

3. $\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$ Continuity

Lorentz force is key!

4. $\frac{dp}{dx} + \rho u \frac{du}{dx} = j_y B$ Momentum

$\vec{j} \times \vec{B}$ is the Lorentz force that is the main cause of acceleration

5. $\rho u c_p \frac{dT}{dx} + \rho u^2 \frac{du}{dx} = j_y E$ Energy

$j_y E = -j_y \frac{dV}{dy}$ This is the joule heating, energy per unit time per unit volume.

3'. $\frac{d\rho}{\rho} + \frac{du}{u} = 0$ or $\rho u = constant = \frac{\dot{m}}{A}$

5'. $\rho u^2 \frac{du}{dx} = j_y E - \rho u c_p \frac{dT}{dx}$

Consider constant area, constant temperature case

Electromagnetic thrusters

Given E and B, we have five equations for j_y, u, p, ρ, and T

For constant temperature and constant E, we have five equations for j_y, B, u, p, and ρ

From equations 4 and 5':

$$u\frac{du}{dx} = \frac{j_y E}{\rho u} - c_p\frac{dT}{dx} = \frac{j_y B}{\rho} - \frac{1}{\rho}\frac{dp}{dx}$$

We prefer to put energy into kinetic energy rather than thermal energy

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = \frac{dp}{p} - \frac{dT}{T} + \frac{du}{u} + \frac{dA}{A} = 0$$

Electromagnetic thrusters

For constant area case:

$$\boxed{\frac{dp}{p}+\frac{du}{u}-\frac{dT}{T}=0}$$

$$\frac{j_yB}{\rho}=u\frac{du}{dx}+\frac{1}{\rho}\frac{dp}{dx}=u\frac{du}{dx}+\frac{RT}{p}\frac{dp}{dx}=u\frac{du}{dx}-\frac{RT}{u}\frac{du}{dx}+\frac{RT}{T}\frac{dT}{dx}$$

$$\frac{j_yB}{\rho}=u\frac{du}{dx}\left[1-\frac{RT}{u^2}\right]+R\frac{dT}{dx}=\left(\frac{j_yE}{\rho u}-c_p\frac{dT}{dx}\right)\left[1-\frac{a^2}{\gamma u^2}\right]+R\frac{dT}{dx}$$

For constant area and constant temperature case:

6’. $\dfrac{B}{E}=\dfrac{1}{u}\dfrac{\gamma M^2-1}{\gamma M^2}$

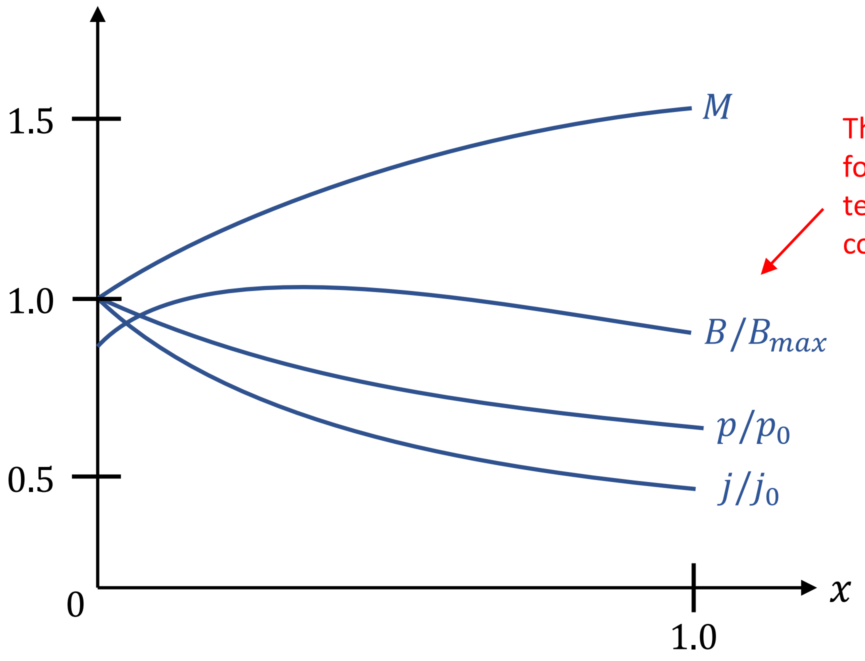
Note: Constant temperature model has weakness that at $M<1$, $j_yB<0$ so, pressure gradient drives flow!

7’. $\rho u=\dfrac{\dot{m}}{A}$

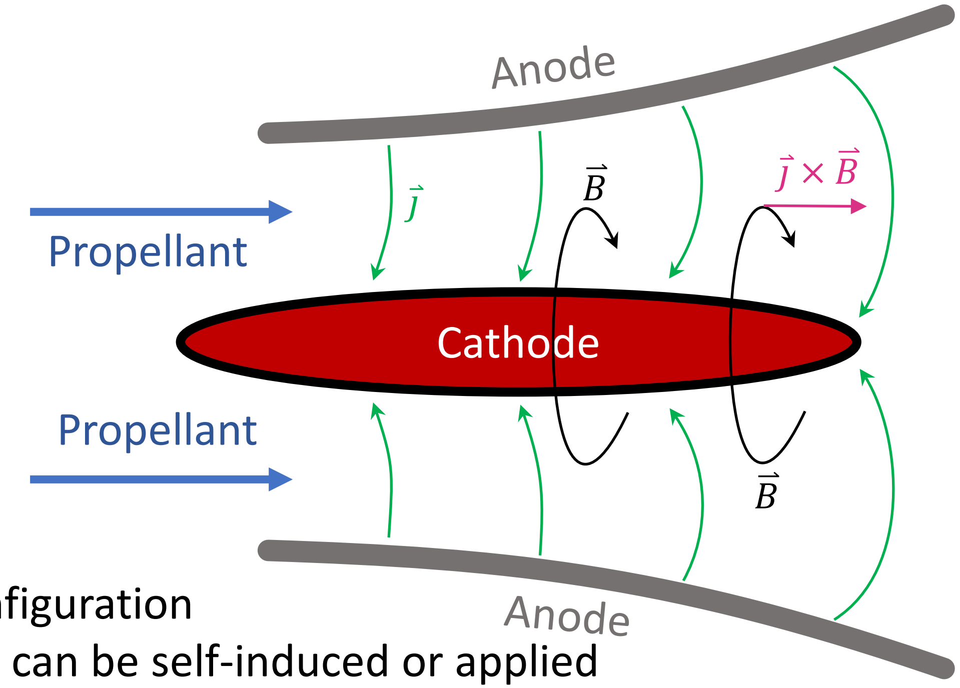
Equations 1, 2, 5’, 6’, and 7’ can be used to determine behavior

For constant temperature, joule heating goes entirely into kinetic energy (no change in thermal energy). At subsonic speeds, not much change in kinetic energy occurs; so, constant T model is inaccurate.

Electromagnetic thrusters

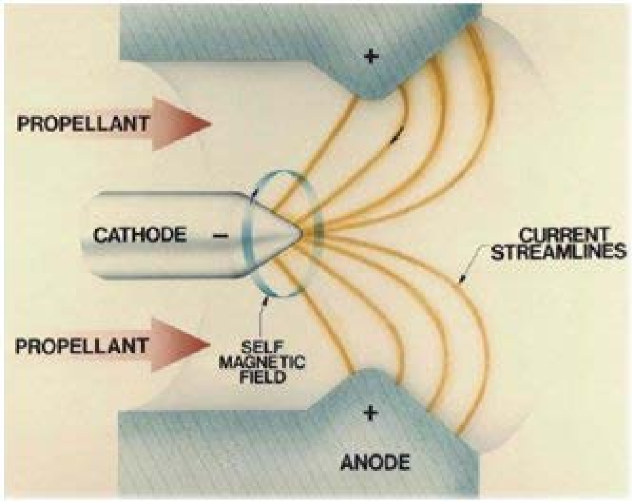


This is required for constant temperature at constant area



Axisymmetric configuration

- Magnetic field can be self-induced or applied
- Current ionizes the propellant



$$d\vec{B} = \frac{\mu \ I}{4 \ \pi r^2} d \vec{L} \ x \ \vec{r}$$

Typically alkali metal is used to seed gas. An arc discharge is employed to create plasma $\vec{j} \times \vec{B}$ is primary acceleration force – Lorentz force!