**MAE 112 Fall 2024**
**Homework Assignment #8**

Due: 11:59 pm, Sunday, December 8, 2024
Follow submission instructions from TAs

1. Consider a crossed-field two-dimensional electromagnetic accelerator. The propellant flow begins with negligible velocity and is accelerated to 25,000 meters per second at the exit of the thruster. The exit plane pressure is 0.01 atmosphere while the ambient pressure is zero. Xenon gas with molecular weight of 54 and $\gamma = 5/3$ is the propellant. The gas temperature (T = 3,000 K) and the cross-sectional flow area are constant with downstream distance. The electric field is directed transverse to the flow with B = 1.8 webers per square meter. The conductivity $\sigma$ = 1800 per ohm-meter. Neglect Hall current. Determine

   a) the electric field E in units of volts per meter at the exit;
   b) the current density $j_y$ at the exit in units of coulombs per meter squared per second;
   c) the Lorentz force in units of newtons per cubic meter.

   *Note that one weber = Newton-second-meter per coulomb; one ohm = volt-sec per coulomb; and one joule = Newton-meter = volt-coulomb.*

2. We have a crossed E and B field with the electric field vector pointing in the positive y-direction and the magnetic field vector pointing in the positive z-direction. Ohm's Law tells us that the current density has two components:
$j_y = \sigma(E\text{-}u B)/(1+\Omega^2)$ ; and $j_x = -\sigma(E\text{-}u B)\,\Omega/(1+\Omega^2)$ . Furthermore, the velocity $u = 0$ and $\Omega = \omega_e / \nu$ where the gyro frequency $\omega_e = q B/ m = 1.76 \times 10^{11}$ (webers sec)$^{-1}$ B ; $\nu = 10^8$ collisions per second. $\sigma$ = 2000 per ohm-meter. *Note that one weber = Newton-second per meter-coulomb; one ohm = volt-sec per coulomb; and one joule = Newton-meter = volt-coulomb.* Design a Hall thruster that produces a Lorentz force in the positive y-direction of magnitude one kilo-Newton per cubic meter; i.e., choose the values of E and B in a consistent manner. Determine the consequential Lorentz force in the x-direction.

3. Consider a solid-core nuclear rocket where the propellant is helium gas. Suppose the gas temperature and the gas pressure for the flow entering the nozzle are 4000 K and 35 atmospheres, respectively. Assume that the Mach number at the nozzle entrance is sufficiently low so that we need not distinguish between static and stagnation values at that location. The helium mass flow rate is 100 kgm / second. Assume that $\gamma = 5/3$.

   a) Assuming frozen isentropic flow of a perfect gas through the nozzle, determine the nozzle-throat area required.
   b) Determine the thrust with an expansion through the nozzle to an exit pressure equal to 0.60 atmospheres with the ambient pressure of 0 atmospheres.

4. Consider a solid-core nuclear rocket where the propellant is hydrogen gas. Suppose the gas temperature and the gas pressure for the flow entering the nozzle are 3500 K and 40 atmospheres, respectively. The hydrogen mass flow rate is 2.0 kgm / second. Assume that $\gamma = 1.4$ for the

mixture, which is dominated by the diatomic species, and $c_p$ is constant for each species through the nozzle flow.

a) Determine the fraction of hydrogen mass in diatomic form ($H_2$) and the fraction in monatomic (H) form at the nozzle entrance.

b) Determine the total static enthalpy of the inflow to the nozzle; i.e., the sum of the sensible enthalpy and the chemical energy associated with the dissociated species.

c) Assuming frozen isentropic flow of a perfect gas through the nozzle, determine the nozzle-throat area required.

d) Determine the thrust for an exit pressure of 0.2 atmospheres and 0 atmospheres for ambient pressure.

e) Determine the temperature and the static sensible enthalpy at the exit. Also, determine the kinetic energy per unit mass at that exit.

**END.**