University of California, Irvine
Department of Mechanical and Aerospace Engineering

MAE 112 - Homework 5
Fall 2024

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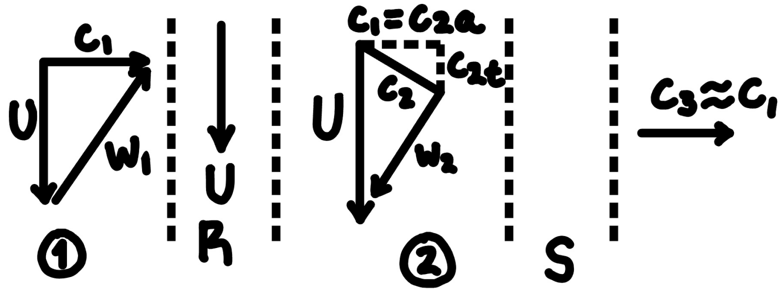
1. Consider one-stage of a compressor with an 8% static pressure rise across the rotor followed by another 9% pressure rise across the stator (compounded to be 17.7%). The incoming flow has a velocity of 75 ft/sec in the axial direction, a temperature of 560°R and a pressure of 2.0 atmospheres. $\gamma = 1.4$; $c_p = 0.24 Btu/lbm -^{\circ} R$; polytropic efficiency = 0.95 for the compressor stage. (a) What is the power per unit mass flow of the compressor? (b) If the rotor blade speed averages 1000 ft/sec, what is the tangential component of velocity exiting the rotor?



Solution:
(a) Power required is $P = \dot{m}(h_{03} - h_{01})$. Consider the total enthalpy $h_0 = h + \frac{1}{2}c^2$ where $h = c_pT$ is the static enthalpy.

$$h_{03} - h_{01} = c_p(T_{03} - T_{01}) = c_p(T_3 - T_1) + \frac{1}{2}(c_3^2 - c_1^2) \tag{1}$$

$$T_{03} - T_{01} = T_3 - T_1 + \frac{1}{2c_p}(c_3^2 - c_1^2) \tag{2}$$

With the low Mach number assumption $c_3 \approx c_1$, therefore, $T_{03} - T_{01} \approx T_3 - T_1$. The polytropic relation is used to get the exit temperature. The pressure ratio is the compounded increase in pressure, i.e., 1.177.

$$\frac{T_3}{T_1} = \left(\frac{P_3}{P_1}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}} = 1.05023 \tag{3}$$

$$\frac{P}{\dot{m}} = c_p(T_3 - T_1) = 6.75\ Btu/lbm \tag{4}$$

(b) Power can also be written as $P = \dot{m}U(c_{2t} - c_{1t}) = \dot{m}U(w_{2t} - w_{1t})$. We are looking for the tangential velocity in the frame of reference of the engine, i.e. c_{2t}. First, convert the units of the answer from part (a) so $P/\dot{m} = 168999\ ft^2/s^2$. Also, $c_{1t} = 0$ as implied by the problem statement (says axial direction).

$$c_{2t} = \frac{P}{\dot{m}U} = 168.999\ ft/s \tag{5}$$

2. Suppose a particular compressor has a compression ratio P 2 /P1 = 15 and the incoming air temperature is 300K. If the adiabatic efficiency is .95, what is (a) the final temperature, (b) the average polytropic efficiency, and (c) the entropy change? (d) What is the power required, if 25 kgm per sec. flow through the compressor?

Solution:
(a)

$$T_2 = T_1 \left(1 + \frac{1}{\eta_{ad}} \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right) = 668.79\ K \tag{6}$$

(b) Solve the polytropic relation for e.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}} \rightarrow e = \frac{\gamma-1}{\gamma} \left[\frac{ln(p_2/p_1)}{ln(T_2/T_1)} \right] = 0.965 \tag{7}$$

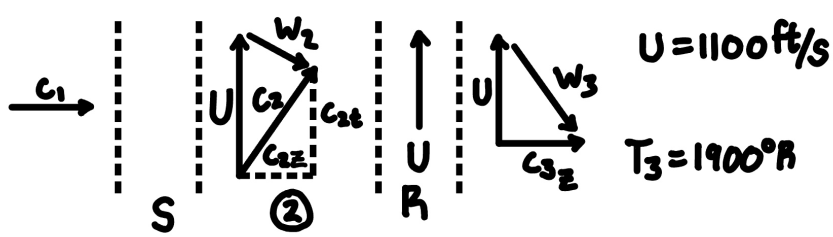
(c) Using $c_p = 1004.5\ J/kg - K$,

$$\Delta s = c_p ln\left(\frac{T_2}{T_1}\right) - Rln\left(\frac{p_2}{p_1}\right) = 28.08\ J/kg - K \tag{8}$$

(d)

$$P = \dot{m} c_p (T_2 - T_1) = 9261.24\ W \tag{9}$$

3. Consider a turbine stage that has a polytropic efficiency of 0.95 for the stator (nozzle) and for the rotor flow. 30% of the total static enthalpy drop through the stage occurs in the rotor portion. The initial and final velocities for the stage are axial and have no swirl (tangential component). Assume that only the tangential component of velocity changes through the stator (nozzle) portion. The flow has $\gamma = 1.3$ and $c_p = 0.30\ Btu/lbm°R$; the incoming flow has a static temperature of $2400°R$, a static pressure of 25 atmospheres, and a velocity of 200 ft/sec. The average rotational velocity of the rotor blade is 1100 ft/sec. The flow exiting the stage has a temperature of $1900°R$.
(a) What are the enthalpy drop and pressure drop across the stator?
(b) What is the tangential velocity at the position between the stator and the rotor measured in a frame of reference fixed to the stator?
(c) What is the Mach number of the flow at the position between the stator and rotor measured in a frame of reference fixed to the rotor?
(d) What are the enthalpy drop and the pressure drop across the rotor?
(e) What is the power output per unit mass flux?



Solution:
(a) The total enthalpy drop is $h_1 - h_3 = c_p(T_1 - T_3) = 150\ Btu/lbm$. Since 70% of it occurs in the stator $h_1 - h_2 = 105\ Btu/lbm$. By the same logic, $T_1 - T_2 = 350\ °R \rightarrow T_2 = 2050\ °R$.

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{e}\frac{\gamma}{\gamma-1}} = 0.4872 \rightarrow p_2 = 12.18\ atm \rightarrow p_1 - p_2 = 12.82\ atm \tag{10}$$

(b) The total enthalpy is constant across the stator since no work is extracted, thus $h_{01} = h_{02}$ which may also be written as $c_pT_1 + \frac{1}{2}c_1^2 = c_pT_2 + \frac{1}{2}c_2^2$. Since the inflow and outflow have no swirl (tangential component) we also know $c_1 = c_{1z} = c_{2z} = c_{3z} = 200\ ft/s$. By Pythagorean theorem $c_2 = \sqrt{c_{2t}^2 + c_{2z}^2}$. Combining these three relations gives $c_2 = 2301.68\ ft/s$ and $c_{2t} = 2292.98\ ft/s$.

(c) The gas constant is $R = \frac{\gamma-1}{\gamma}c_p = 1733.32\ ft/sft^2/s^{2\circ}R$.

$$w_2 = \sqrt{c_{2t}^2 + (c_{2t} - U)^2} = 1209.33\ ft/s \tag{11}$$

$$M_{2R} = \frac{w_2}{\sqrt{\gamma RT_2}} = 0.563 \tag{12}$$

(d) $h_3 - h_2 = 45\ Btu/lbm$

(e)

$$\frac{P}{\dot{m}} = h_{01} - h_{03} = h_{02} - h_{03} = c_pT_2 + \frac{1}{2}c_2^2 - c_pT_3 + \frac{1}{2}c_3^2 = 3,755,524.411\ ft^2/s^2 \tag{13}$$

4. Suppose we had a gas turbine engine driving a propeller. Consider takeoff only where flight velocity is 120 ft/sec. Consider the product of gearbox efficiency and propeller efficiency to be 0.8. The pressure ratio across the compressor is ten and the pressure ratio across the turbine is ten. The pressure drops across the combustor and the nozzle are negligible. The fuel heating value is $Q = 10,000\ Btu/lbm$ and the mixture ratio is $1/f = 28$. For air or products, consider $\gamma = 1.4$ and $c_p = 0.24 Btu/lbm^\circ R$. Ambient temperature is $550^\circ R$. (a) What is the propeller power per unit mass flow of air? (b) What is the propeller thrust per unit mass flow of air? Assume isentropic compression and expansion.

Solution:
(a) To determine the propeller power per unit mass flow of air we need to set up a power balance between the various engine components. The turbine supplies the power for the propeller and the compressor thus by knowing how much power is produced by the turbine and how much is taken by the compressor, we will know the remainder sent to the propeller. First, it is necessary to determine the temperature and pressure across each component. To begin, we check the flight Mach number. It says "consider takeoff only" so the Mach number should be very low, and indeed it is.

$$c_p = 0.24\ \frac{Btu}{lbm^\circ R} = 6008 \frac{ft^2}{s^{2\circ}R} \qquad R = \frac{\gamma - 1}{\gamma}c_p = 1717\ \frac{ft^2}{s^{2\circ}R} \tag{14}$$

$$M_1 = \frac{u_1}{\sqrt{\gamma RT_1}} = 0.104 \tag{15}$$

Because the Mach number is very low, the static quantities are approximately equal to the total quantities and these quantities do not change very much between station 1 and 2. $T_{01} \approx T_1 \approx T_{02} \approx T_2 = 550\ ^\circ R$.

Compressor

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = 1.9307 \rightarrow T_3 = 1061.88\ ^\circ R \approx T_{03} \tag{16}$$

Combustor
The combustion parameters are given in terms of the heating value Q which is the amount of energy released per unit mass of fuel. This simplifies the energy balance significantly as compared to the flame temperature calculation performed in prior homeworks. The energy balance is below with the burning efficiency η_b included for reference. In this problem, $\eta_b = 1$.

$$H_{04} - H_{03} = \eta_b \dot{m}_f Q \tag{17}$$

$$(\dot{m}_a + \dot{m}_f)c_pT_4 - \dot{m}_a c_pT_3 = \eta_b \dot{m}_f Q \tag{18}$$

This equation can be used to solve for T_4 where $f = \dot{m}_f/\dot{m}_a = 1/mixtureratio$.

$$T_4 = \frac{\eta_b f Q + c_p T_3}{(1+f)c_p} = 2462.04\ ^\circ R \tag{19}$$

Turbine

$$\frac{T_5}{T_4} = \left(\frac{p_5}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = 0.518 \rightarrow T_5 = 1275.23\ ^\circ R \tag{20}$$

Here it is interesting to notice that since the pressure ratios across the compressor and turbine are the same but inverted, at the exit of the turbine we are recovering $p_5 = p_1$. Thus, the nozzle is unnecessary unless we want an overexpanded flow for some reason. We cannot really expand the flow anymore, thus the thrust is only produced by the propeller and not the ejected air. This is the typical configuration of turboprop engines.

Power Balance

Now we can balance the power to see how much if left over for the propeller. $\eta_{gp} = 0.8$ is the combined efficiency of the gearbox and the propeller.

$$\frac{1}{\eta_{gp}} P_{propeller} + P_{compressor} = P_{turbine} \tag{21}$$

$$\frac{1}{\eta_{gp}} P_{propeller} + \dot{m}_a c_p (T_3 - T_2) = (\dot{m}_a + \dot{m}_f) c_p (T_4 - T_5) \tag{22}$$

$$\frac{P_{propeller}}{\dot{m}_a} = -\eta_{gp} c_p (T_3 - T_2) + \eta_{gp}(1+f) c_p (T_4 - T_5) = 3,447,845.507\ \frac{ft^2}{s^2} = 137.71\ \frac{Btu}{lbm} \tag{23}$$

(b) The thrust can be calculated from the power with the equation below where T_P is propeller thrust.

$$P_{propeller} = T_P u_1 \tag{24}$$

$$\frac{T_P}{\dot{m}_a} = \frac{P_{propeller}}{\dot{m}_a} \frac{1}{u_1} = 28732.05\ ft/s \tag{25}$$

5. Do a preliminary design on a turbojet engine which produces 250,000 newtons of thrust. Size constraints limit the intake cross-sectional capture area to 0.3 square meters. The engine is intended to cruise at a Mach number of 2.5. Assume ambient air conditions are one atmosphere of pressure and 270 K. Indicate clearly your choice of fuel, temperature at entrance to turbine, diffuser type, and extent of expansion in nozzle. Also indicate whether you elect to have an afterburner. All choices must be rational and defensible, of course. Indicate mass flows of air and fuel, mixture ratio(s), thrust specific fuel consumption, exhaust velocity, work done by compressor, work done on turbine, stagnation pressure ratios across each component, throat area, nozzle exit area, and nozzle exit pressure.

Solution:

The solution to this design problem follows the same steps as Problem 5 of Homework 4. However, here, we add a compressor and a turbine. Because of this, we only provide some guidelines about each component of the turbojet engine.

Diffuser

We have to slow down the air flow from the flight conditions to a negligible Mach number before entering the compressor. The flight Mach number is 2.5. Thus, a divergent diffuser is out of the picture since the total pressure loss would be too high. A better choice is a K-D diffuser and perhaps it is needed to deflect the flow with a wedge in order to generate an oblique shock before the diffuser. This is exactly the same as HW4 P5. Expect the K-D diffuser with wedge to produce a pressure ratio (between the entrance of the compressor and the freestream) of around 10 (it could be less than 10 perhaps 6-7 and it could be more than 10, perhaps 14-15 as in HW4 P5).

Compressor
Keep in mind that we want to target a combustion chamber pressure of around 30-40 atm in a jet engine. Only rocket engines go up to 200 atm. So, finding the compression ratio across the diffuser can guide you to the necessary pressure ratio across the compressor which in turn, tells you the power needed by the compressor (to be produced by the turbine).

Combustion Chamber
You can either use the online software approach or the simplified energy balance as given in the previous exercise. Aim for a realistic flame temperature ($\sim 1850K$). Since we have a rotating turbine after the combustion chamber, we don't want flame temperature exceeding 1800-1900 K otherwise the turbine blades will be damaged.

Turbine
Design the turbine to power the compressor. You will know the compressor power once you get to this stages of the problem since you know the pressure and temperature ratios this far. Thus, determine the temperature exiting the turbine such that the power produced by the turbine matched that required by the compressor.

Nozzle
Lastly, expand the flow exiting the turbine through a convergent-divergent nozzle with a reasonable exit pressure condition (easiest would be perfectly expanded $p_e = p_a$).

Note
Keep in mind we want to satisfy the thrust requirement given the size constraints. Since your solution involves assumptions, such as compressor pressure ratio, mixture ratio, etc... your solution may not be perfect on the first try. That is okay as real design involves iterations to hone in on the desired parameters. You do not need to iterate this problem though. Once you have done the first full iteration you may stop, but think to yourself how you might alter the design in the next iteration to better satisfy the requirements.