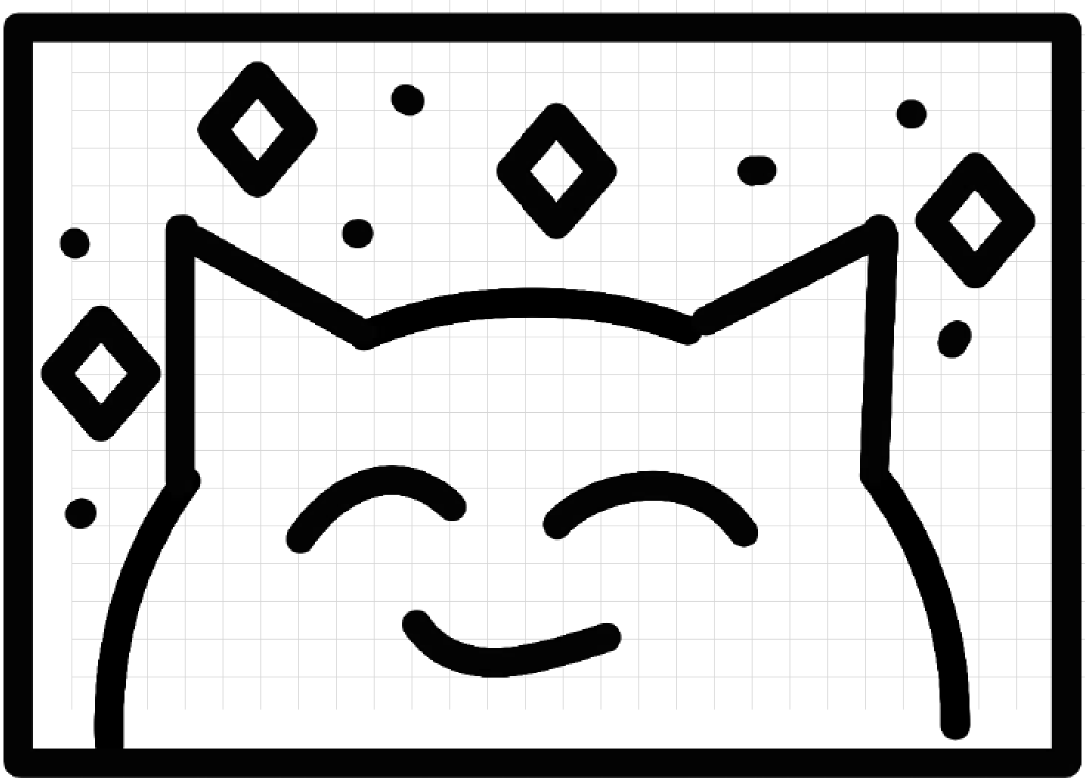
MAE 112

HW 8

TRIET



1. Consider a crossed-field two-dimensional electromagnetic accelerator. The propellant flow begins with negligible velocity and is accelerated to 25,000 meters per second at the exit of the thruster. The exit plane pressure is 0.01 atmosphere while the ambient pressure is zero. Xenon gas with molecular weight of 54 and $\gamma = 5/3$ is the propellant. The gas temperature (T = 3,000 K) and the cross-sectional flow area are constant with downstream distance. The electric field is directed transverse to the flow with B = 1.8 webers per square meter. The conductivity σ = 1800 per ohm-meter. Neglect Hall current. Determine

a) the electric field E in units of volts per meter at the exit;

b) the current density j_y at the exit in units of coulombs per meter squared per second;

c) the Lorentz force in units of newtons per cubic meter.

Note that one weber = Newton-second-meter per coulomb; one ohm = volt-sec per coulomb; and one joule = Newton-meter = volt-coulomb.

(a) Gas constant:

$$R = \frac{\bar{R}}{W} = \frac{8.314}{0.13129} \frac{J}{K \cdot \cancel{mol}} \frac{\cancel{mol}}{kg} = 63.325 \left[\frac{J}{K\,kg}\right]$$

Sound velocity:

$$C = \sqrt{\gamma R T} = \sqrt{\frac{5}{3} \times 63.325 \times 3000} = 562.7 \,[m/s]$$

Mach number:

$$M = \frac{V}{C} = \frac{25000}{562.7} = 44.43 \quad [1]$$

Convert unit for B:

$$B = 1.8 \frac{\cancel{Wb}}{m^2} \frac{1 \cancel{N} s \cancel{m}}{1 \cancel{Wb} \cancel{C}} \frac{V \cancel{C}}{1 \cancel{N} \cancel{m}} = 1.8 \frac{Vs}{m^2}$$

Electric field:

$$\frac{B}{E} = \frac{1}{V} \times \frac{\gamma M^2 - 1}{\gamma M^2}$$

$$\frac{1.8}{E} = \frac{1}{25000} \times \frac{\frac{5}{3} \times 44.43^2 - 1}{\frac{5}{3} \times 44.43^2}$$

$$\boxed{E = 45013.6818 \left[\frac{V}{m}\right]}$$

(b) Current density:

$$j = \sigma\,(E - vB)$$

$$= 1800\,\frac{1}{\Omega m}\left(45013.6818\,\frac{V}{m} - 25000\,\frac{\cancel{m}}{\cancel{s}} \times 1.8\,\frac{V\cancel{s}}{m^{\cancel{2}}}\right)$$

$$= 24627.24\;\frac{\cancel{V}}{\cancel{\Omega}m^2}\;\frac{1\,\cancel{\Omega}}{\frac{1\,\cancel{V}s}{C}}$$

$$\boxed{j = 24627.24\left[\frac{C}{m^2 s}\right]}$$

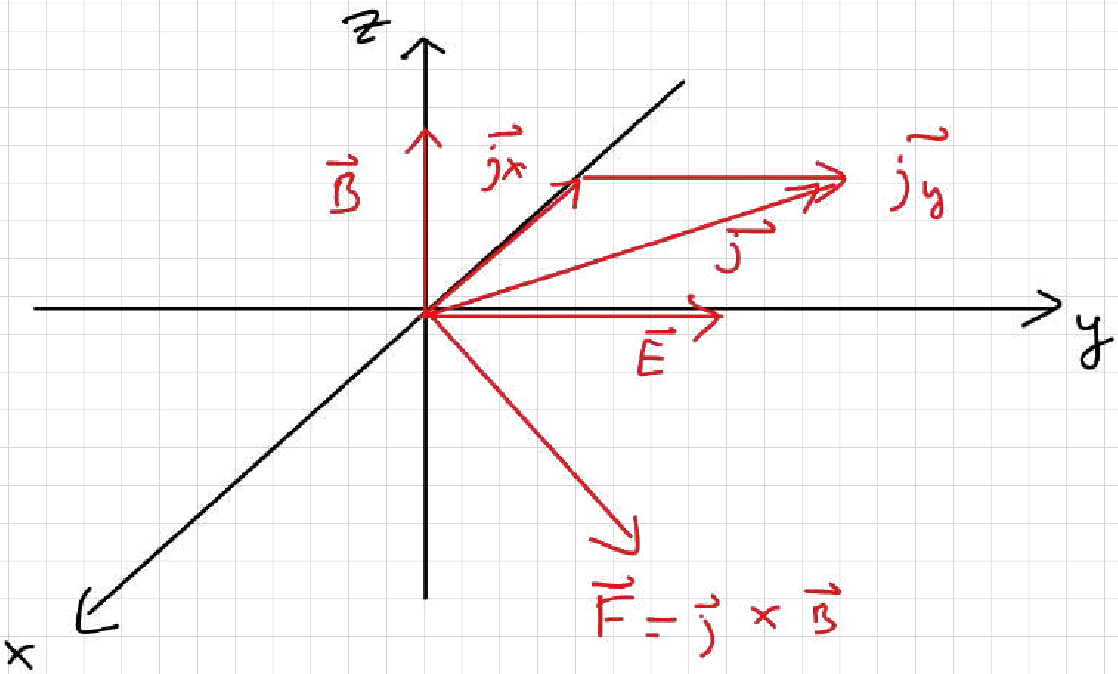
(c)

$$\vec{F} = \vec{j} \times \vec{B} = 24627.24\left[\frac{C}{m^2 s}\right] \times 1.8\left[\frac{Vs}{m^2}\right]$$

$$= 44329.032\;\frac{\cancel{s}\;\cancel{Vc}}{m^{\cancel{4}\,3}\cancel{s}}\qquad\frac{N\cancel{m}}{\cancel{Vc}}$$

$$\boxed{|\vec{F}| = 44329.032\left[\frac{N}{m^3}\right]}$$

2. We have a crossed E and B field with the electric field vector pointing in the positive y-direction and the magnetic field vector pointing in the positive z-direction. Ohm's Law tells us that the current density has two components:
$j_y = \sigma(E\text{-}u\,B)/(1+\Omega^2)$; and $j_x = -\,\sigma(E\text{-}u\,B)\,\Omega\,/(1+\Omega^2)$. Furthermore, the velocity $u = 0$ and $\Omega = \omega_e / \nu$ where the gyro frequency $\omega_e = q\,B/\,m = 1.76 \times 10^{11}$ (webers sec)$^{-1}$ B; $\nu = 10^8$ collisions per second. $\sigma = 2000$ per ohm-meter. *Note that one weber = Newton-second per meter-coulomb; one ohm = volt-sec per coulomb; and one joule = Newton-meter = volt-coulomb.* Design a Hall thruster that produces a Lorentz force in the positive y-direction of magnitude one kilo-Newton per cubic meter; i.e., choose the values of E and B in a consistent manner. Determine the consequential Lorentz force in the x-direction.



Lorentz force:

$$\vec{F} = \vec{j} \times \vec{B}$$

$$F_y = j_x B$$

$$F_y = \sigma(E - \cancel{u}^{\,0} B)\frac{\Omega}{1+\Omega^2}\,B$$

$$F_y = \sigma E \frac{\Omega}{1+\Omega^2}\,B$$

We know:

$$\omega_e = 1.76 \times 10^{11}\ B$$
$$\nu = 10^8$$

$$\Rightarrow \Omega = \frac{\omega_e}{V} = 1.76 \times 10^{3}\ B$$

Plug back in:

$$F_y = \sigma E \frac{1760\,B}{1+3097600B^2}\ B$$

$$1000 = 2000 \times E \times \frac{1760\ B^2}{1+3097600B^2}$$

One degree of freedom

Choose $\boxed{B = 2\ \left[\frac{Wb}{m^2}\right]}$

Then $$1000 = 2000 \times E \times \frac{1760 \times 2^2}{1+3097600 \times 2^2}$$

$$\boxed{E = 880\left[\frac{V}{m}\right]}$$

Lorentz force in x direction:

$$F_x = j_y B = \sigma\left(E - \cancelto{0}{uB}\right)\frac{1}{1+\Omega^2}\ B$$

$$= \sigma E \frac{1}{1+(1.76\times 10^{3}\ B)^2} \times B$$

$$= 2000 \times 880 \times \frac{1}{1+(1.76\times 10^{3} \times 2)^2} \times 2$$

$$\boxed{F_x = 0.284\ \frac{N}{m^3}}$$

3. Consider a solid-core nuclear rocket where the propellant is helium gas. Suppose the gas temperature and the gas pressure for the flow entering the nozzle are 4000 K and 35 atmospheres, respectively. Assume that the Mach number at the nozzle entrance is sufficiently low so that we need not distinguish between static and stagnation values at that location. The helium mass flow rate is 100 kgm / second. Assume that $\gamma = 5/3$.

a) Assuming frozen isentropic flow of a perfect gas through the nozzle, determine the nozzle-throat area required.

b) Determine the thrust with an expansion through the nozzle to an exit pressure equal to 0.60 atmospheres with the ambient pressure of 0 atmospheres.

Gas constant for helium

(a)

$$R = \frac{\bar{R}}{w} = \frac{8.314}{0.004002\text{6}} \frac{J}{\cancel{mol} \cdot k} \times \frac{\cancel{mol}}{kg} = 2077.15 \left[\frac{J}{kg \cdot K}\right]$$

At throat:

Characteristic velocity:

$$\Gamma = \sqrt{\gamma} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$= \sqrt{\frac{5}{3}} \left(\frac{2}{\frac{5}{3} + 1}\right)^{\frac{\frac{5}{3}+1}{2(\frac{5}{3}-1)}}$$

$$= 0.7262$$

$$c^* = \frac{\sqrt{RT_0}}{\Gamma} = \frac{\sqrt{2077.15 \times 4000}}{0.7262}$$

$$c^* = 3969.2428 \ [m/s]$$

Mass flow rate formula:

$$\dot{m} = \frac{P_0 A^*}{c^*}$$

$$100 = \frac{35 \times 101300 \times A^*}{3969.2428}$$

$$\boxed{A^* = 0.1119515668 \ [m^2]}$$

(b) At exit:

Mach number:

$$\frac{P_0}{P_e} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{35}{0.6} = \left(1 + \frac{\frac{5}{3} - 1}{2} M^2\right)^{\frac{\frac{5}{3}}{\frac{5}{3} - 1}}$$

$$M = 3.501 \quad [1]$$

Exit area:

$$\frac{A_e}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A_e}{0.11195} = \frac{1}{3.501}\left[\frac{2}{\frac{5}{3}+1}\left(1 + \frac{\frac{5}{3} - 1}{2} \times 3.501^2\right)\right]^{\frac{\frac{5}{3}+1}{2(\frac{5}{3}-1)}}$$

$$A_e = 0.465 \, [m^2]$$

Temperature:

$$\frac{T_0}{T_e} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{4000}{T_e} = 1 + \frac{\frac{5}{3} - 1}{2} \times 3.501^2$$

$$T_e = 786.52 \, [k]$$

Density:

$$\rho_e = \frac{P_e}{RT_e} = \frac{0.6 \times 101300}{2077.15 \times 786.52} = 0.0372 \left[\frac{kg}{m^3}\right]$$

Exit velocity:

$$\dot{m} = \rho_e V_e A_e \Rightarrow V_e = \frac{\dot{m}}{\rho_e A_e} = \frac{100}{0.0372 \times 0.465}$$

$$\Rightarrow V_e = 5781.015 \, [m/s]$$

Thrust:

$$T = \dot{m} V_e + (P_e - P_a) A_e$$

$$= 100 \times 5781.015 + (0.6 \times 101300 - 0)\, 0.465$$

$$\boxed{T = 606364.2 \, [N]}$$

4. Consider a solid-core nuclear rocket where the propellant is hydrogen gas. Suppose the gas temperature and the gas pressure for the flow entering the nozzle are 3500 K and 40 atmospheres, respectively. The hydrogen mass flow rate is 2.0 kgm / second. Assume that $\gamma = 1.4$ for the mixture, which is dominated by the diatomic species, and c_p is constant for each species through the nozzle flow.

a) Determine the fraction of hydrogen mass in diatomic form (H_2) and the fraction in monatomic (H) form at the nozzle entrance.

b) Determine the total static enthalpy of the inflow to the nozzle; i.e., the sum of the sensible enthalpy and the chemical energy associated with the dissociated species.

c) Assuming frozen isentropic flow of a perfect gas through the nozzle, determine the nozzle-throat area required.

d) Determine the thrust for an exit pressure of 0.2 atmospheres and 0 atmospheres for ambient pressure.

e) Determine the temperature and the static sensible enthalpy at the exit. Also, determine the kinetic energy per unit mass at that exit.

(a) $1 H_2 \longrightarrow H + H_2$

Starting temperature : 3500 K

pressure : 40 atm

Use online Calculator :

$Y_{H_2} = 0.9536$

$Y_H = 0.0464$

(b) Enthalpy $= 6.3972 \times 10^{11}$ [J/g]

(c) Specific heat of mixture

$$C_p = Y_{H_2} C_{p,H_2} + Y_H C_{p,H}$$

$$= 0.9536 \times 14556 + 0.0464 \times 20790$$

$$= 14839.536 \left[\frac{J}{kg\,K}\right]$$

Gas constant of mixture :

$$R = C_p \frac{\gamma - 1}{\gamma} = 14839.536 \times \frac{1.4 - 1}{1.4}$$

$$R = 4239.867 \left[\frac{J}{kg \cdot k}\right]$$

Charateristic velocity:

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = \sqrt{1.4}\left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{2(1.4-1)}}$$

$$= 0.6847$$

$$C^* = \frac{\sqrt{R T_0}}{\Gamma} = \frac{\sqrt{4239.867 \times 3500}}{0.6847} = 5626.13 \left[m/s\right]$$

Mass flow rate formula:

$$\dot{m} = \frac{P_0 A^*}{C^*}$$

$$2 = \frac{40 \times 101300 \times A^*}{5626.13}$$

$$\boxed{A^* = 0.00278 \left[m^2\right]}$$

d) At exit:

Mach number:

$$\frac{P_0}{P_e} = \frac{40}{0.2} = 200 \Rightarrow \frac{P_e}{P_0} = 0.005 \Rightarrow M = 4.258$$

Exit temperature:

$$M = 4.258 \Rightarrow \frac{T_e}{T_0} = 0.2174 \Rightarrow T_e = 0.2174 \times 3500$$

$$T_e = 760.9 \left[K\right]$$

Exit velocity:

$$V_e = M\sqrt{\gamma R T_e} = 4.258\sqrt{1.4 \times 4239.867 \times 760.9} = 9649.2 \left[m/s\right]$$

Exit density

$$\rho_e = \frac{P_e}{RT_e} = \frac{0.2 \times 101366}{4239.867 \times 760.9} = 0.00628 \ [kg/m^3]$$

Exit area:

$$A_e = \frac{\dot{m}}{\rho_e V_e} = \frac{2}{0.00628 \times 9049.2} = 0.0352 \ [m^2]$$

Thrust:

$$T = \dot{m} V_e + (P_e - P_a) A_e$$

$$= 2 \times 9049.2 + (0.2 \times 101300 - 0)\, 0.0352$$

$$\boxed{T = 18811.552 \ [N]}$$

ⓔ Exit temperature : $\boxed{T_e = 760.9 \ [K]}$

Static sensible enthalpy:

$$h = c_p T_e = 14839.536 \times 760.9 = \boxed{11291402.94 \left[\frac{J}{kg}\right]}$$

Kinetic energy formula:

$$KE = \frac{1}{2} m v^2$$

$$\frac{KE}{m} = \frac{1}{2} v^2 = \frac{1}{2} \times 9049.2^2 = \boxed{4.0944 \times 10^7 \left[\frac{J}{kg}\right]}$$