

MAE 112 Fall 2022
Homework Assignment #7
Due: 11:59 pm, Sunday, December 1, 2024
Follow submission instructions from TAs.

1) Helium has the ionization potential of $\varepsilon = 24.48$ electron volts. (1 eV = 1.602×10^{-19} joules.) The Saha equation, adjusted for degeneracy in the helium states, gives

$$\frac{n_{electron} n_{ion}}{n_{neutral}} = 4 \frac{(2\pi m_e kT)^{3/2}}{h^3} e^{-\varepsilon/kT}$$

m_e , k , and h are respectively the electron mass, Boltzmann constant, and Planck constant. (use class notes or search for values.) We start with two moles of helium at standard atmospheric conditions. This initially cool helium gas with initially negligible ionization is heated at constant volume to 2900 K. At equilibrium, what is the free electron number density? What is the number density of positive ions?

2) Consider an ion rocket with a voltage drop in potential between the accelerating electrodes placed 100 centimeters apart. Cesium of atomic weight 55 is the propellant. One volt equals one newton-meter per coulomb. The cross-sectional area is 40 cm^2 . $\epsilon_0 = 8.85 \times 10^{-12} \text{ (coulomb)}^2/\text{newton-meter}$. It is desired to obtain a specific impulse of 2500 seconds.

- a) Determine the required voltage drop.
- b) Determine the final jet (beam) velocity.
- c) Determine the maximum thrust which is achievable here.
- d) Determine the mass flow for operation with a current equal to 90% of the maximum beam current.

3) Consider an ion rocket with a 600-volt drop in potential between the accelerating electrodes placed 60 centimeters apart. Xenon of atomic weight 54 is the propellant. The charge-to-mass ratio is 7.34×10^5 coulombs per kilogram. One volt equals one newton-meter per coulomb. The cross-sectional area is 100 cm^2 . $\epsilon_0 = 8.85 \times 10^{-12} \text{ (coulomb)}^2/\text{newton-meter}$. Determine the

- (a) exit velocity,
- (b) specific impulse,
- (c) maximum beam current,
- (d) thrust at maximum beam current, and
- (e) mass flow at maximum beam current.
- (f) What are the thrust and the mass flow if the rocket operates on one-half of the maximum current?

4) Design an arc jet that produces 40 lbf of thrust. Give maximum chamber temperature and pressure. Describe propellant choice. Give mass flow of propellant, throat area, exit velocity, and exit pressure.

END