

# Lecture 2

## Thrust

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# Impulse =

$$T' = \dot{m}c = \dot{m} U + (P_e - P_a)A_e = \dot{m} [ U + (P_e - P_a)A_e / \dot{m} ]$$

So,  $c = U + (P_e - P_a)A_e / \dot{m}$ , an effective exhaust velocity ; **Weight flow:**  $\dot{w} = \dot{m}g$

$$\frac{\text{Thrust Force}}{\text{Weight Flow}} = \text{Specific Impulse} = I_{sp}$$

$$I_{sp} = \frac{T'}{\dot{w}} = \frac{T'}{\dot{m}g} = \frac{c}{g} \text{ (units of seconds)}$$

Specific impulse is a function of fuel-oxidizer types, mixture ratio, and nozzle design!  
This is a good unit to compare small engines to larger engines and engines of different types.

3600 converts hours  
into seconds because  
 $\dot{m}_f$  (mass/sec)

$$\text{Specific Fuel Consumption} = \frac{\text{lbm of fuel/hour}}{\text{thrust}}$$

$$S = \frac{\dot{m}_f g \cdot 3600}{T'}$$

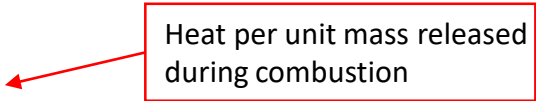
In the case of a rocket,  
 $\dot{m} = \dot{m}_f = \dot{m}_p$

$$S = \frac{3600}{I_{sp}}$$

# Efficiency

For one mole of fuel (or propellants),  $n_p \cdot H(T_{fuel, products}) - n_r \cdot H(T_{matched, reactants})$  is released. Divide by molecular mass of fuel (or propellants) to get heat released per unit mass of fuel (or propellants).  $\Delta h_c = \frac{n_p \cdot H(T_{fuel, products}) - n_r \cdot H(T_{matched, reactants})}{m_F}$

$$Efficiency = \frac{Power\ Delivered}{Rate\ of\ Chemical\ Energy\ Converted}$$

$$\eta = \frac{T'V}{\dot{m}_f \Delta h_c}$$


Heat per unit mass released during combustion

In the one-dimensional case:  $\eta = \frac{T'Vg}{\dot{m}_f g \Delta h_c} = \frac{I_{sp} V g}{\Delta h_c} = \frac{cV}{\Delta h_c}$

Again,  $I_{sp}$  and  $c$  are directly related to efficiency and useful in comparing engines of different types and sizes.

# Delta $V$

Consider a simple rocket case:

$$T' = T = m \frac{dV}{dt}$$

$$\dot{m}c = m \frac{dV}{dt}$$

$$\frac{dV}{dt} = -\frac{c}{m} \frac{dm}{dt} \quad \text{or} \quad dV = -c \frac{dm}{m}$$

$$-\int_{V_i}^{V_f} dV = c \int_{m_i}^{m_f} d(\log m)$$

In this course, “log” is the natural logarithm.

$$V_f - V_i = c \log \left( \frac{m_i}{m_f} \right) = c \log \left( \frac{m_f + m_{\text{propellants}}}{m_f} \right)$$

# Delta $V$

So the change in velocity depends on  $c$  ( or  $I_{sp}$ ) and the fraction of mass exhausted.

$$\Delta V = I_{sp}g \log \left( 1 + \frac{m_{propellants}}{m_f} \right) = I_{sp}g \log \left( \frac{m_i}{m_i - m_{propellants}} \right)$$

$$\Delta V = c \log \left( 1 + \frac{m_{propellants}}{m_f} \right) = c \log \left( \frac{m_i}{m_i - m_{propellants}} \right)$$

$$\Delta V = c \log \left( \frac{1}{1 - m_{propellants}/m_i} \right)$$

This again shows the usefulness of  $c$  ( or  $I_{sp}$ )!