1)

a) $\dot{m}_{O_2} = 1\ kg/s = 1000\ g/s$

$\dot{N}_{O_2} = \frac{\dot{m}_{O_2}}{MW_{O_2}} = 31.25\ mol/s$

$\dot{N}_{CH_4} = \frac{\dot{N}_{O_2}}{2} = 15.625\ mol/s$

$\dot{m}_{CH_4} = \dot{N}_{CH_4} MW_{CH_4} = 250\ g/s = 0.25\ kg/s$

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

$\dot{N}_{CO_2} = \dot{N}_{CH_4} \quad \dot{N}_{H_2O} = 2\dot{N}_{CH_4}$

$\dot{m}_{CO_2} = \dot{N}_{CO_2} MW_{CO_2} = 0.6875\ kg/s$

$\dot{m}_{H_2O} = \dot{N}_{H_2O} MW_{H_2O} = 0.5625\ kg/s$

b) $T_{i_{CH_4}} = 111\ K \quad T_{i_{O_2}} = 90K \quad N_{tot} = 3 \quad X_{CO_2} = \frac{1}{3} \quad X_{H_2O} = \frac{2}{3}$

$$n_{CH_4}\left(h_{f_{CH_4}} + \int_{T_{ref}}^{T_i} c_{p_{CH_4}} dT\right) + n_{O_2}\left(h_{f_{O_2}} + \int_{T_{ref}}^{T_i} c_{p_{O_2}} dT\right) = n_{CO_2}\left(h_{f_{CO_2}} + \int_{T_{ref}}^{T_{ad}} c_{p_{CO_2}} dT\right) + n_{H_2O}\left(h_{f_{H_2O}} + \int_{T_{ref}}^{T_i} c_{p_{H_2O}} dT\right)$$

$$h_{f_{CH_4}} + c_{p_{CH_4}}(T_{i_{CH_4}} - T_{ref}) + n_{O_2} c_{p_{O_2}}(T_{i_{O_2}} - T_{ref}) = n_{CO_2}\left(h_{f_{CO_2}} + c_{p_{CO_2}} \Delta T\right) + n_{H_2O}\left(h_{f_{H_2O}} + c_{p_{H_2O}} \Delta T\right)$$

$$h_{f_{CH_4}} + c_{p_{CH_4}}(T_{i_{CH_4}} - T_{ref}) + n_{O_2} c_{p_{O_2}}(T_{i_{O_2}} - T_{ref}) = n_{CO_2} h_{f_{CO_2}} + n_{CO_2} c_{p_{CO_2}} \Delta T + n_{H_2O} h_{f_{H_2O}} + n_{H_2O} c_{p_{H_2O}} \Delta T$$

$$h_{f_{CH_4}} + c_{p_{CH_4}}(T_{i_{CH_4}} - T_{ref}) + n_{O_2} c_{p_{O_2}}(T_{i_{O_2}} - T_{ref}) - n_{CO_2} h_{f_{CO_2}} - n_{H_2O} h_{f_{H_2O}} = \Delta T\left(n_{CO_2} c_{p_{CO_2}} + n_{H_2O} c_{p_{H_2O}}\right)$$

$$T_{ad} = T_{ref} + \frac{h_{f_{CH_4}} + c_{p_{CH_4}}(T_{i_{CH_4}} - T_{ref}) + n_{O_2} c_{p_{O_2}}(T_{i_{O_2}} - T_{ref}) - n_{CO_2} h_{f_{CO_2}} - n_{H_2O} h_{f_{H_2O}}}{n_{CO_2} c_{p_{CO_2}} + n_{H_2O} c_{p_{H_2O}}}$$

$$T_{ad} = 298 + \frac{-74,831 + 35.8(-187) + 2(29.315)(-208) - 1(-393,546) - 2(-241,845)}{1(58.836) + 2(48.035)}$$

$T_{ad} = 5356\ K$

$MW_{mix} = \sum X_m MW_m = \frac{1}{3}(44) + \frac{2}{3}(18) = 26.\overline{66}\ \frac{g}{mol} = 0.02\overline{66}\ \frac{kg}{mol}$

c) $\dot{m} = \dot{m}_o + \dot{m}_f = 1.25\ kg/s$

$\dot{m} = \rho^* u^* A^* \qquad M^* = 1 = \frac{u^*}{\sqrt{\gamma R T^*}} \qquad R = \frac{\bar{R}}{MW_{mix}} = 311.79375\ \frac{J}{kg \cdot K}$

$\dot{m} = \frac{P^*}{RT^*}\sqrt{\gamma R T^*}\, A^*$

$\dot{m} = P^* A^* \sqrt{\frac{\gamma}{RT^*}}$

$A^* = \frac{\dot{m}}{P^*}\sqrt{\frac{RT^*}{\gamma}}$

$P_o = P^*\left(1 + \frac{\gamma-1}{2}M^{*2}\right)^{\frac{\gamma}{\gamma-1}} \rightarrow P^* = P_o\left(1 + \frac{\gamma-1}{2}M^{*2}\right)^{-\frac{\gamma}{\gamma-1}} = P_o\left(1 + \frac{\gamma-1}{2}\right)^{-\frac{\gamma}{\gamma-1}} = 11.098\ atm$

$T_o = T^*\left(1 + \frac{\gamma-1}{2}M^*\right) \longrightarrow T^* = T_o\left(1 + \frac{\gamma-1}{2}M^{*2}\right)^{-1} = T_o\left(1 + \frac{\gamma-1}{2}\right)^{-1} = 4760.\overline{58}\ K$

$$A^* = \dot{m}\sqrt{\frac{R}{\gamma}}\frac{\left(T_o\left(1 + \frac{\gamma-1}{2}M^{*2}\right)^{-1}\right)^{1/2}}{P_o\left(1 + \frac{\gamma-1}{2}M^{*2}\right)^{-\frac{\gamma}{\gamma-1}}} = \frac{\dot{m}}{P_o}\sqrt{\frac{RT_o}{\gamma}}\frac{\left(1 + \frac{\gamma-1}{2}M^{*2}\right)^{\frac{\gamma}{\gamma-1}}}{\left(1 + \frac{\gamma-1}{2}M^{*2}\right)^{1/2}} = \frac{\dot{m}}{P_o}\sqrt{\frac{RT_o}{\gamma}}\left(1 + \frac{\gamma-1}{2}M^{*2}\right)^{\frac{\gamma}{\gamma-1} - \frac{1}{2}}$$

$$A^* = \frac{\dot{m}}{P_o}\sqrt{\frac{RT_o}{\gamma}}\left(1 + \frac{\gamma-1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$\frac{2\gamma}{2\gamma-2} - \frac{\gamma-1}{2\gamma-2} = \frac{2\gamma-\gamma+1}{2(\gamma-1)} = \frac{\gamma+1}{2(\gamma-1)}$

$A^* = 0.001211\ m^2$

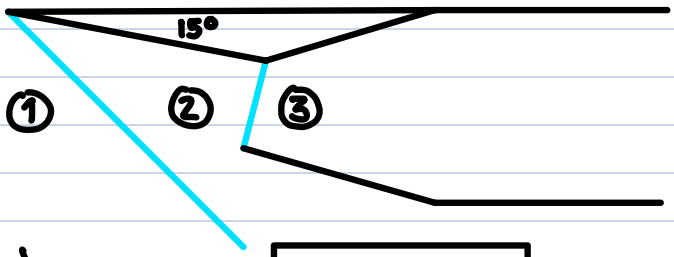
d) $T = \dot{m} u_e + (P_e - P_a) A_e$ (the $(P_e - P_a)A_e$ term $\rightarrow 0$)

$T = 3690N$

$$u_e = \sqrt{\frac{2R\gamma T_o}{\gamma - 1}\left[1 - \left(\frac{P_e}{P_o}\right)^{\frac{\gamma-1}{\gamma}}\right]} = \sqrt{\frac{2(311.8)(1.25)(5356)}{0.25}\left[1 - \left(\frac{0.5}{2.0}\right)^{0.2}\right]} = 2952\ m/s$$

2)

$M_1 = 2.25$
$T_1 = 298K$
$P_1 = 1\,atm$
$A_1 = 1m^2$



a) $T_0 = T_1\left(1+\frac{\gamma-1}{2}M_1^2\right) = 599.725$ $\boxed{T_0 = 599.725}$

b) $\theta = 15° \rightarrow \beta-\theta \rightarrow \beta = 40.5°$

$M_{1n} = 2.25\sin(40.5°) = 1.46 \rightarrow$ TA.2 $\quad \frac{P_2}{P_1} = 2.32 \quad \frac{P_{02}}{P_{01}} = 0.942 \quad \frac{T_2}{T_1} = 1.294 \quad M_{2n} = 0.7157$

$M_2 = \frac{M_{2n}}{\sin(\beta-\theta)} = \frac{0.7157}{\sin(25.5)} = 1.6624$

$P_{01} = P_1\left(1+\frac{\gamma-1}{2}M_1^2\right)^{3.5} = 11.563\,atm$

$P_{02} = \frac{P_{02}}{P_{01}}P_{01} = 10.892\,atm$

$\boxed{P_{01} = 11.563\,atm \quad P_{02} = 10.892\,atm}$

c) $\boxed{M_2 = 1.6624}$

$M_2 = 1.6642 \rightarrow$ TA.2 $\rightarrow \frac{1.6642-1.66}{1.68-1.66} = \frac{x-0.6512}{0.6458-0.6512} \quad M_3 = 0.65$

d) $\boxed{M_3 = 0.65}$

3) $P_1 = 1.5\,atm \quad T_1 = 550°R \quad \frac{P_e}{P_1} = 12 \quad c_p = 0.24\frac{Btu}{lbm°R} \quad e = 0.96$

a) $\frac{T_e}{T_1} = \left(\frac{P_e}{P_1}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}} \quad T_e = 550(12)^{\frac{1}{0.96}\frac{0.4}{1.4}} = 1152.25°R$ $\boxed{T_e = 1152.25°R}$

b) $P = \dot{m}c_p(T_e-T_i)$

$c_p = 0.24\frac{Btu}{lbm°R}\left|\frac{778\,ft\cdot lbf}{Btu}\right|\frac{32.174\,lbm\cdot ft/s^2}{lbf} = 6007.53\frac{ft^2}{s^2°R}$

$c_p = 0.24\frac{Btu}{lbm°R}\left|\frac{778\,ft\cdot lbf}{Btu}\right. = 186.72\frac{ft\cdot lbf}{lbm°R}$

$R = c_p\frac{\gamma-1}{\gamma} = 1716.4\frac{ft^2}{s^2°R} = 0.06857\frac{Btu}{lbm°R} = 53.35\frac{ft\cdot lbf}{lbm°R}$

$\frac{P}{\dot{m}} = 3{,}618{,}034.509\frac{ft^2}{s^2}$

$\frac{P}{\dot{m}} = 144.54\frac{Btu}{lbm}$

$\frac{P}{\dot{m}} = 112{,}452\frac{ft\cdot lbf}{lbm}$

c) $\Delta s = c_p\ln\left(\frac{T_e}{T_i}\right) - R\ln\left(\frac{P_e}{P_i}\right) = 177.8\frac{ft^2}{s^2°R} = 0.0071\frac{Btu}{lbm°R} = 5.52\frac{ft\cdot lbf}{lbm°R}$

$\boxed{\Delta s = 177.8\frac{ft^2}{s^2°R} \quad \Delta s = 0.0071\frac{Btu}{lbm°R} \quad \Delta s = 5.52\frac{ft\cdot lbf}{lbm°R}}$