Final Review
Lectures 12 through 19

See Midterm Exam Review for earlier material.

It is particularly important that students clearly understand that the unauthorized sale or commercial distribution of course notes or recordings is a violation of campus policy. Specifically, Section 102.23 of Policies Applying to Campus Activities, Organizations, and Students states that the following activities constitute grounds for discipline:

Selling, preparing, or distributing for any commercial purpose course lecture notes or video or audio recordings of any course unless authorized by the University in advance and explicitly permitted by the course instructor in writing. The unauthorized sale or commercial distribution of course notes or recordings by a student is a violation of these Policies whether or not it was the student or someone else who prepared the notes or recordings.

Matching of Turbine and Compressor

(Select operating speed N (rpm) and turbine inlet temperature $T_4^\circ \approx T_4$

$$\mu \equiv \frac{\dot{m}_{comp} - \dot{m}_{bleed}}{\dot{m}_f} = \frac{\dot{m}_{air-into-combustor}}{\dot{m}_{fuel}}$$

$$H_c = h_3^\circ - h_2^\circ = c_p T_2^\circ \left(\frac{T_3^\circ}{T_2^\circ} - 1\right) = c_p T_2^\circ \left[\left(\frac{P_3^\circ}{P_2^\circ}\right)^{\frac{\gamma_d - 1}{\gamma_d e_c}} - 1\right]$$

$$\dot{m}_{comp} H_c = \left(\dot{m}_{comp} - \dot{m}_{bleed} + \dot{m}_{fuel}\right) H_t$$
$$= \dot{m}_f (1 + \mu) H_t$$

$$H_t = \frac{\mu + \dot{m}_{bleed}/\dot{m}_f}{\mu + 1} H_c = c_p T_4^\circ \left[1 - \left(\frac{P_5^\circ}{P_4^\circ}\right)^{\frac{(\gamma_n - 1)e_t}{\gamma_n}}\right]$$

So now, with the knowledge of $H_c, T_4^\circ, \mu, \dot{m}_{bleed}/\dot{m}_f, e_t$, we obtain temperature and pressure at station 5

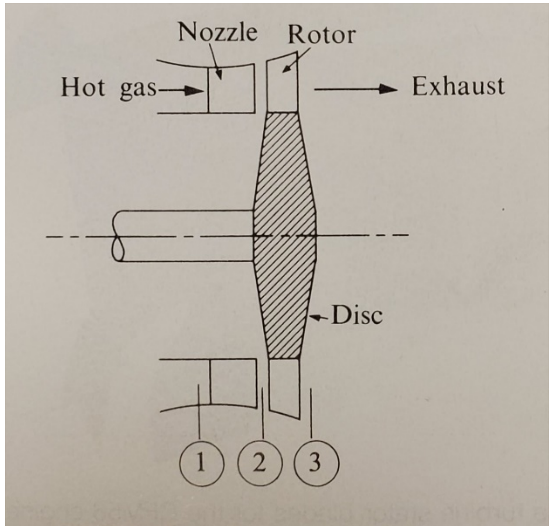


FIGURE 8.4 An axial turbine stage.

$$\frac{\dot{m}_{turbine}\sqrt{T_4^\circ}}{P_4^\circ} = \Gamma(\gamma_t) A^* / \sqrt{R}$$

For a given mass flow, this can determine A^* for stator nozzle!

$$K = c_{2_a}[\tan(\beta) + \tan(|\alpha|)]$$

$$\frac{T_5^\circ}{T_4^\circ} = 1 + \frac{1}{c_p}\frac{K}{\sqrt{T_4^\circ}}\frac{U}{\sqrt{T_4^\circ}} - \left(\frac{U}{\sqrt{T_4^\circ}}\right)^2 \frac{1}{c_p}$$

Turbojet engines

V

M_1

$P_1 = P_a$

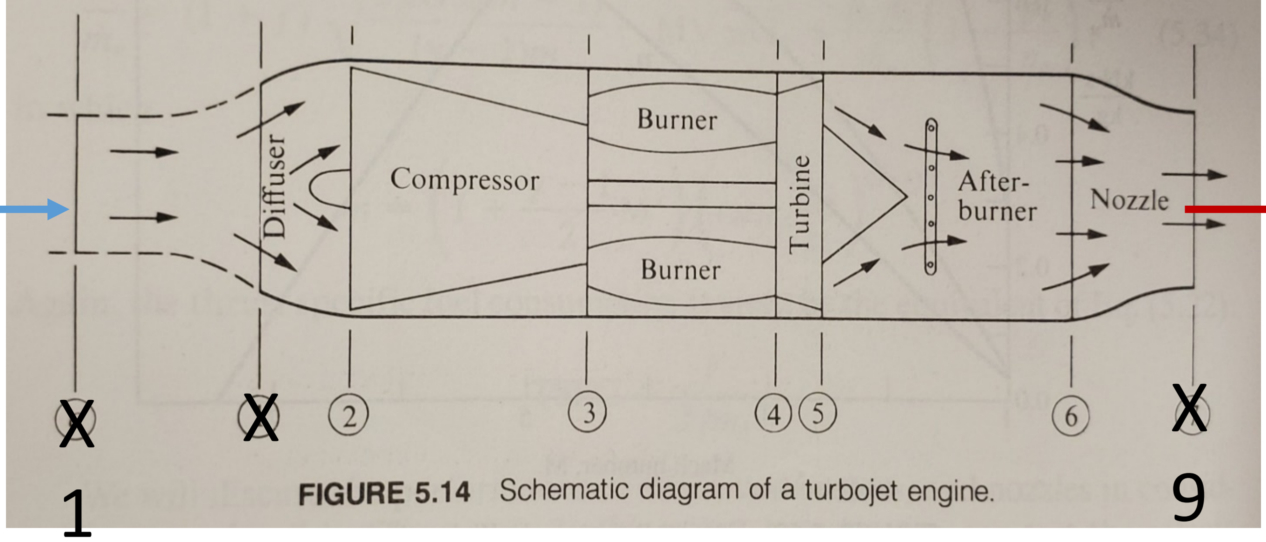


FIGURE 5.14 Schematic diagram of a turbojet engine.

U

$$U^2 = 2c_{p_n}T_6^\circ\left[1 - \left(\frac{P_9}{P_6^\circ}\right)^{\frac{\gamma_n}{\gamma_n-1}\eta_n}\right]$$

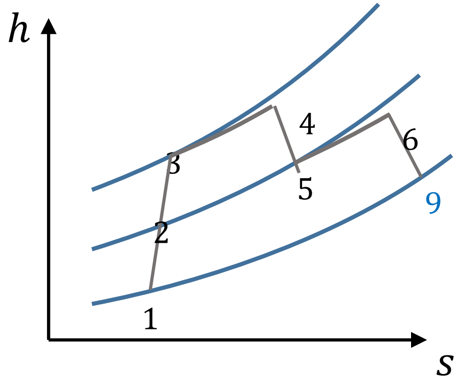
$$U^2 = 2c_{p_n}T_6^\circ\left[1 - \left(\frac{P_9}{P_1}\frac{P_1}{P_2^\circ}\frac{P_2^\circ}{P_3^\circ}\frac{P_3^\circ}{P_4^\circ}\frac{P_4^\circ}{P_5^\circ}\frac{P_5^\circ}{P_6^\circ}\right)^{\frac{\gamma_n}{\gamma_n-1}\eta_n}\right]$$

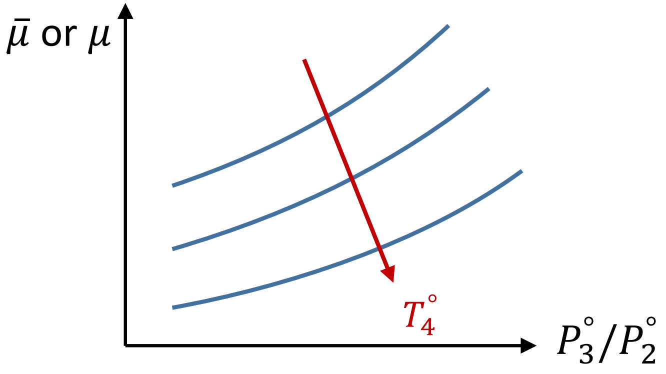
$$U^2 = 2c_{p_n}T_6^\circ\left[1 - \left(1 + \frac{\gamma_d - 1}{2}M_1^2\right)^{-\delta\eta_n\eta_d} \underline{* (1 - CM_c^2)^{-\frac{\gamma_n-1}{\gamma_n}\eta_n}} * \left(1 + \frac{H_c}{c_{p_d}T_2^\circ}\right)^{-\eta_n\eta_c\delta} * \left(1 - \frac{H_t}{c_{p_t}T_4^\circ}\right)^{-\frac{\delta'\eta_n}{\eta_t}} \underline{* (1 - C'(M_c')^2)^{-\frac{\gamma_n-1}{\gamma_n}\eta_n}}\right]$$

$\frac{P_9}{P_1} = 1$ above

Approximate by 1 if $M_c^2 << 1$.

Remove if there is no afterburner





Turboprop engines

Turboprop engine – Secondary flow gives same advantage as ducted fan but the propeller has variable pitch (more thrust at takeoff). Thrust occurs due to propeller and due to jet!

The propeller could be in the rear of the propulsion unit [propfan] when exhaust velocity is low.

$$(\dot{m}_a - \dot{m}_{bleed} + \dot{m}_f)H_t = \dot{m}_a H_c + (\eta_{prop}\eta_{gear})^{-1} T_{prop} V$$

$$T = T_{prop} + (\dot{m}_a - \dot{m}_{bleed} + \dot{m}_f)U - \dot{m}_a V + (P_e - P_a)A_e$$

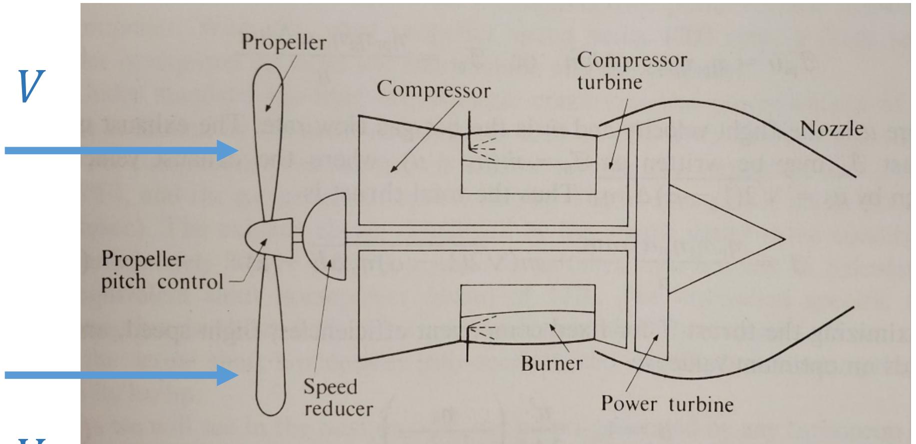
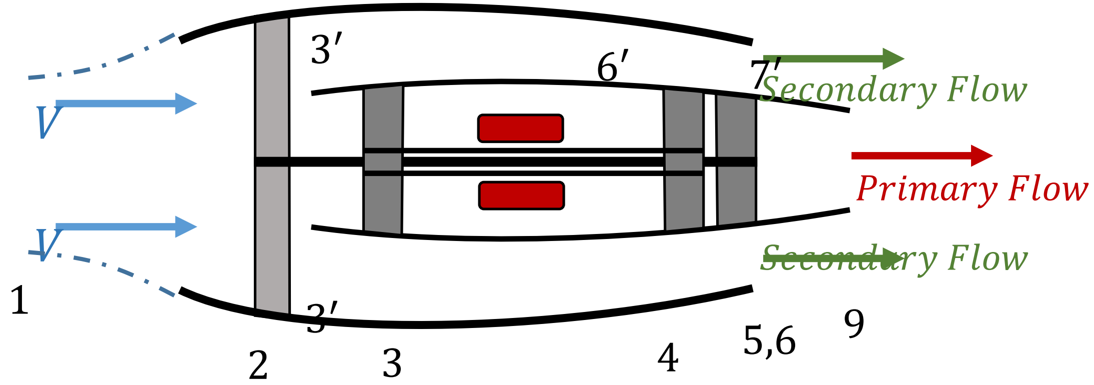


FIGURE 5.35 Schematic diagram of a typical free-turbine turboprop engine.



Turbofan engines – primary (core) flow

More turbine work is required on account of the fan so that T_5° and the primary flow exhaust velocity are less than for a turbojet engine. Extra energy is in the secondary flow. We sacrifice some kinetic energy in the exhaust of the core engine to gain more mass flow through the bypass flow.

$$\bar{\mu} \equiv \mu - \frac{\dot{m}_{bleed}}{\dot{m}_{fuel}}$$

Bypass Ratio: $\beta = \dfrac{Secondary\ mass\ flow\ rate}{Primary\ mass\ flow\ rate}$

As before for a turbojet and assuming small Mach number in the combustor:

$$(\bar{\mu}+1)H_t = \mu H_c + \beta\mu c_{p_d}\left(T_{3'}^{\circ} - T_2^{\circ}\right) = (\bar{\mu}+1)c_{p_t}\left(T_4^{\circ} - T_5^{\circ}\right)$$

U_p is primary exhaust velocity →

$$U_p^2 = 2\left(c_{p_n}T_4^{\circ} - H_t\right)\left[1 - \left(1+\frac{\gamma_d-1}{2}M_1^2\right)^{-\delta\eta_n\eta_d} * \left(1+\frac{H_c}{c_{p_d}T_2^{\circ}}\right)^{-\eta_n\eta_c\delta} * \left(1-\frac{H_t}{c_{p_n}T_4^{\circ}}\right)^{-\frac{\delta'\eta_n}{\eta_t}}\right]$$

* Relation for H_t is different:

$$H_t = \left(\frac{\mu}{\bar{\mu}+1}\right)c_{p_d}T_2^{\circ}\left[\left(\frac{P_3^{\circ}}{P_2^{\circ}}\right)^{\frac{\gamma_d-1}{\gamma_d\eta_c}} - 1\right] + \left(\frac{\beta\mu}{\bar{\mu}+1}\right)c_{p_d}T_2^{\circ}\left[\left(\frac{P_{3'}^{\circ}}{P_2^{\circ}}\right)^{\frac{\gamma_d-1}{\gamma_d\eta_f}} - 1\right]$$

$$\bar{\mu} = \frac{\eta_b Q - h(T_4^{\circ}, products)}{h(T_4^{\circ}, products) - h(T_3^{\circ}, air)}$$

$$\boxed{\begin{aligned}\delta &\equiv \frac{\gamma_d}{\gamma_d-1}\frac{\gamma_n-1}{\gamma_n}\\ \delta' &\equiv \frac{\gamma_t}{\gamma_t-1}\frac{\gamma_n-1}{\gamma_n}\end{aligned}}$$

Turbofan engines – secondary (bypass) flow

Now, we must determine the secondary flow: $U_{secondary\ flow} = U_s$

$$U_s^2 = 2c_{p_d} T^{\circ}_{3'} \left[1 - \left(\frac{P_{7'}}{P^{\circ}_{3'}} \right)^{\frac{\gamma_d - 1}{\gamma_d}\eta_{ns}} \right]$$

η_{ns} is polytropic efficiency for the secondary (bypass) nozzle

Assume perfect expansion across the secondary nozzle:

Then: $P_{7'} = P_1$

$$U_s^2 = 2c_{p_d} T^{\circ}_{2} \left(\frac{P^{\circ}_{3'}}{P^{\circ}_{2}} \right)^{\frac{\gamma_d - 1}{\gamma_d \eta_f}} \left[1 - \left(\frac{P_1}{P^{\circ}_{2}} \right)^{\frac{\gamma_d - 1}{\gamma_d}\eta_{ns}} \left(\frac{P^{\circ}_{3'}}{P^{\circ}_{2}} \right)^{-\frac{\gamma_d - 1}{\gamma_d}\eta_{ns}} \right]$$

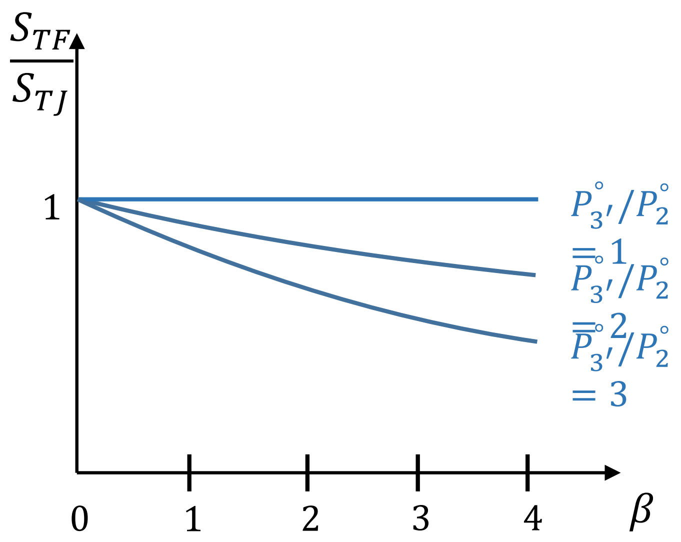
$$U_p = U_p \left(c_{p_n}, c_{p_d}, \gamma_n, \gamma_d, \eta_d, \eta_c, \eta_f, \eta_t, \eta_n, \eta_b Q, fuel\ section, \beta, M_1, T_1, T^{\circ}_4, P^{\circ}_3/P^{\circ}_2, P^{\circ}_{3'}/P^{\circ}_2 \right)$$

U_p decreases as β increases, while U_s increases with β

Turbofan engines –Thrust and Performance

$$\frac{T}{\dot{m}_f} = (1+\bar{\mu})U_p + \beta\mu U_s - \mu(1+\beta)V + \frac{\left(P_{e_p} - P_a\right)A_{e_p}}{\dot{m}_f} + \frac{\left(P_{e_s} - P_a\right)A_{e_s}}{\dot{m}_f}$$

$$S = \frac{3600g}{(1+\bar{\mu})U_p + \beta\mu U_s - \mu(1+\beta)V + \dfrac{\left(P_{e_p} - P_a\right)A_{e_p} + \left(P_{e_s} - P_a\right)A_{e_s}}{\dot{m}_f}}$$



$$\frac{mg}{T} = \frac{L}{D} \quad \text{or} \quad T = \frac{mg}{L/D}$$

The propulsive efficiency is: $\eta = \frac{TV}{\dot{m}_f Q} = \frac{mg}{L/D}\frac{V}{\dot{m}_f Q}$

$$\dot{m}_f = -\frac{dm}{dt} = \frac{mgV}{\eta(L/D)Q}$$

Brequet range formula

$$\Delta s = \frac{\eta(L/D)Q}{g}\ln\left(1+\frac{M_F}{m_2}\right) = \frac{(L/D)V}{S}\ln\left(1+\frac{M_F}{m_2}\right)$$

Electrothermal propulsion –Arcjet (Plasmajet)

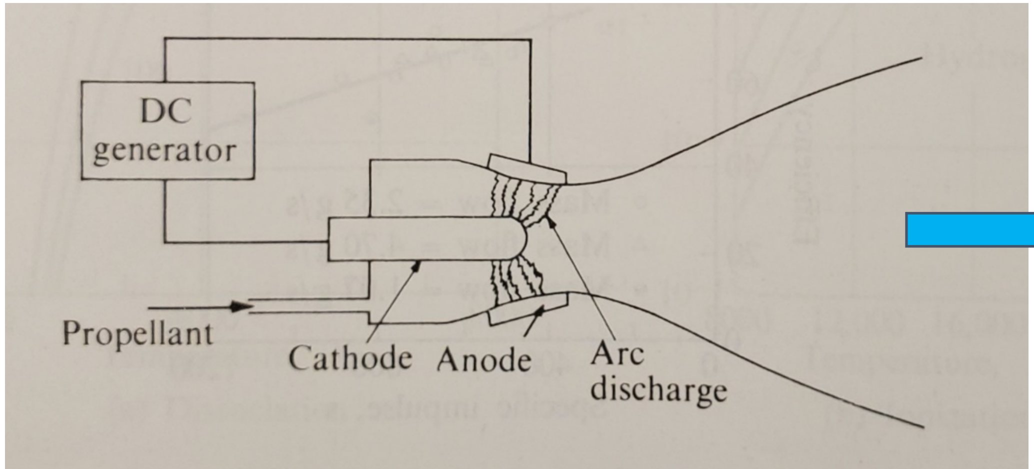


FIGURE 14.19 Schematic diagram of an arcjet.

$$u_e = \sqrt{\frac{2\gamma}{\gamma - 1} \frac{RT^{\circ}}{mw} \left[1 - \left(\frac{P_e}{P^{\circ}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}}$$

$$P^{\circ} = \frac{\dot{m}c^*}{A^*} \quad \text{Where:} \quad c^* = \sqrt{\frac{RT^{\circ}}{mw} \frac{1}{\Gamma(\gamma)}}$$

$$n_e^2 = \left(\frac{2\pi m_e}{h^2}\right)^{3/2} (kT)^{3/2} e^{-\epsilon_i/kT} n_n$$

$m_e = electron\ mass\ (9.11 \times 10^{-31}\ kg)$

$h = Planck's\ constant\ (6.62377 \times 10^{-34}\ joule - sec)$

$\epsilon_i = ionization\ potential\ (24.48\ for\ helium, 1ev = 1.602 \times 10^{-19}\ joules)$

$k = Boltzmann's\ constant\ (1.38062 \times 10^{-23} joule/{}^{o}K)$

Arcjet schematic from Hill & Peterson [1]

Electrostatic propulsion - Ion Rocket

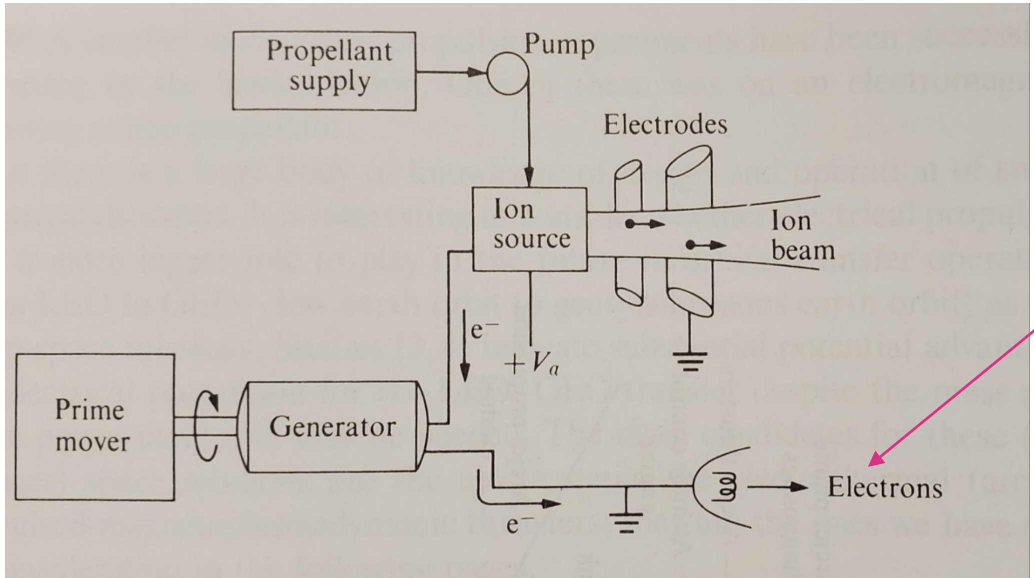


FIGURE 14.4 Ion rocket.

The pump starts propellant flowing at low velocity . The ionized propellant is accelerated through the electrodes to produce thrust.

Electrons exhausted by hot filament will neutralize ion beam downstream and prevent reversal of the beam

From a balance of energy:

$$\frac{Mu_e^2}{2} = qV_a \leftrightarrow V_a = \frac{Mu_e^2}{2q} \qquad I_{sp} = \frac{u_e}{g} = \frac{1}{g}\sqrt{\frac{2qV_a}{M}}$$

$$T = gI_{sp}\dot{m}$$

Magnetic field helps bombardment and ionization of neutral atoms by electrons but only the electric field accelerates the ions.

Total beam current: $I = \dfrac{q}{M}\dot{m}$

Ideally, beam power: $P = IV_a = \left(\dfrac{q}{M}\dot{m}\right)\left(\dfrac{Mu_e^2}{2}\dfrac{1}{q}\right) = \dot{m}\dfrac{u_e^2}{2}$

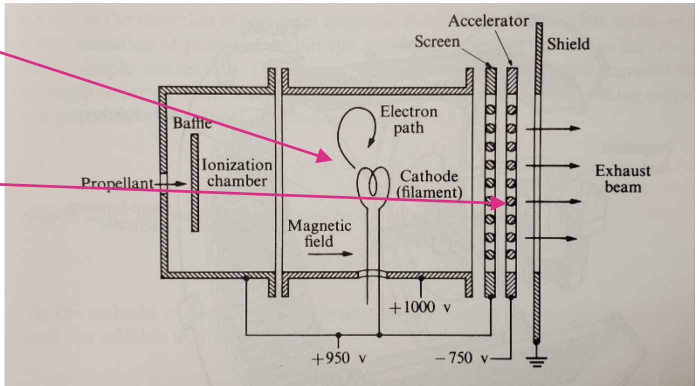


FIGURE 14.8 Schematic of low-density ion engine of Kaufman, showing typical operating voltages.

Optimization of Space Charge and Current for Ion rocket

At: $x = L,\ V_* = V_a$ and $\sqrt{\alpha}L = \frac{2}{3}\left(C + V_a^{1/2}\right)^{3/2} - 2C\left(C + V_a^{1/2}\right)^{1/2} + \frac{4}{3}C^{3/2} = f(V_a, C)$

We can maximize α and therefore j by maximizing $f(V_a, C)$ through the selection of the value of C!

$\frac{df}{dC} < 0$ so that α and j are maximized at $C = 0$; $\text{i.e., zero value for voltage gradient at } x = 0$

Thereby: $\sqrt{\alpha_{max}}L = \frac{2}{3}V_a^{3/4}$ and $j_{max} = \frac{4}{9}\epsilon_0\sqrt{\frac{2q}{m}}\frac{V_a^{3/2}}{L^2}$

The thrust T per unit area A is given by $\frac{T}{A} = \frac{\dot{m}u_e}{A} = \frac{\rho_e M u_e}{q}u_e = \frac{j}{q/M}u_e = \frac{j}{q/M}\sqrt{\frac{2q}{M}V_a}$

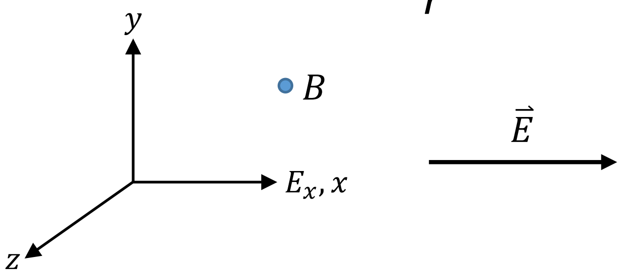
Therefore, the maximum possible thrust is given by: $\frac{T_{max}}{A} = \sqrt{\frac{2M}{q}}j_{max}V_a^{1/2} = \frac{8}{9}\epsilon_0\left(\frac{V_a}{L}\right)^2$

This maximum value is independent of M and depends on the average electric field intensity V_a/L, i.e., and average derivative dV/dx between $x = 0$ and $x = L$.

Hall Current and Electromagnetic Fields

With both electric and magnetic fields crossed (i.e., orthogonal) linear motion with rotation results:

$$\vec{F} = q[\vec{E} + \vec{V} \times \vec{B}] \qquad \frac{mV_\theta^2}{r} = qBV_\theta \quad \Rightarrow \quad \frac{V_\theta}{r} = \omega = \frac{q}{m}B$$



$$\vec{V} = \vec{V}_{drift} + \vec{V}_{other} = \vec{e}_y V_{drift} + \vec{V}_{other}$$

$$F_x = q\left[E + |B|V_{drift} + V_{other,y}|B|\right]$$

Acceleration occurs towards a situation where no average force is applied on the electron in the *x*-direction. A stable drift velocity results.

$$E + |B|V_{drift} = 0 \quad \text{or} \quad V_{drift} = -\frac{E}{|B|}$$

$$\omega_{electron} = \frac{q}{m_{electron}}B \gg \omega_{ion} = \frac{q}{m_{ion}}B$$

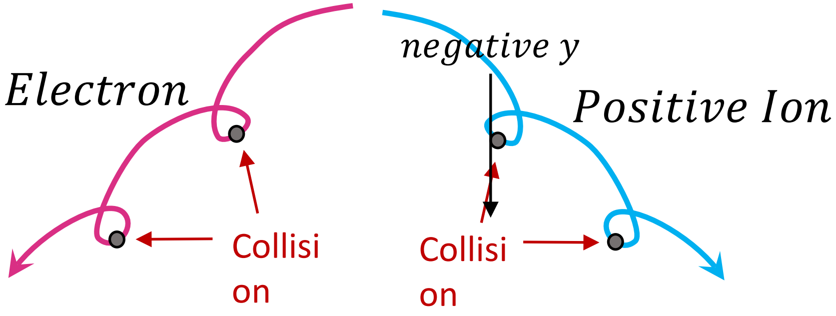
$\Omega = \dfrac{\omega_e}{\nu}$ is the Hall parameter.

As the electric field, $\vec{E}$ changes the velocity, it also changes the radius of curvature for the motion

$$r = \left(\frac{q}{m}\frac{B}{V_\theta}\right)^{-1}$$

Lower velocity or less mass produces lower radius!

$$r = \left(\frac{q}{m}\frac{B}{V_\theta}\right)^{-1}$$



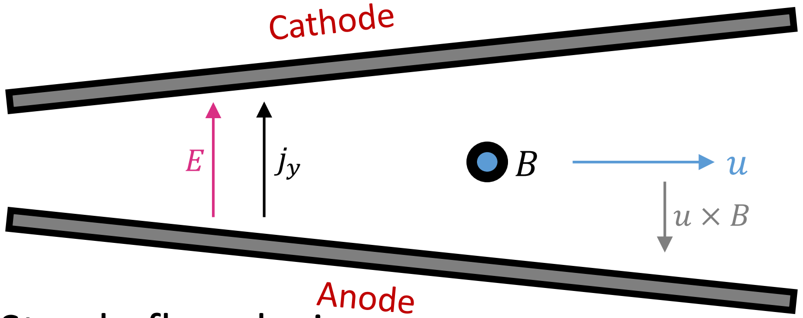
Both the electron and positive ion drift in the negative-y direction

The Hall current interacts with the $\vec{B}$ field to produce a $\vec{j} \times \vec{B}$ acceleration.

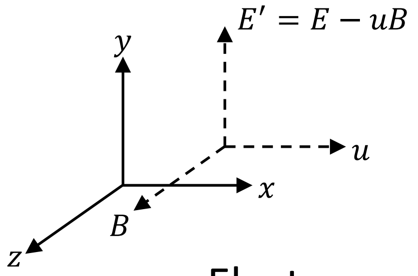
Hall thrusters work at high Ω, $\omega_e/\nu \gg 1$, or $\omega_e \ggg \nu$.
That is, at low densities and high B values.

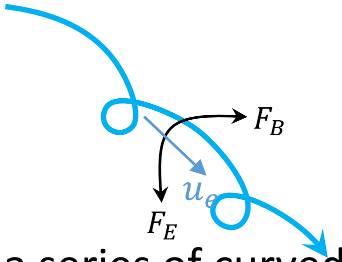
Electromagnetic crossed-fields thrusters

One-dimensional representation.



Steady-flow device





Electron path is a series of curved lines between collisions

1. $j_y = \sigma(E - uB)$ — Ohm's Law; σ is Plasma conductivity

2. $p = \rho RT$ — Equation of state

3′. $\dfrac{d\rho}{\rho} + \dfrac{du}{u} = 0$ or $\rho u = \dfrac{\dot{m}}{A}$ — Continuity

4. $\dfrac{dp}{dx} + \rho u \dfrac{du}{dx} = j_y B$ — Momentum

5′. $\rho u^2 \dfrac{du}{dx} = j_y E - \rho u c_p \dfrac{dT}{dx}$ — Energy

$\vec{j} \times \vec{B}$ is the Lorentz force, the main cause of acceleration

$$j_y E = -j_y \frac{dV}{dy}$$

Consider constant area, constant temperature case

Electromagnetic thrusters

Typically alkali metal is used to seed gas. An arc discharge is employed to create plasma $\vec{j} \times \vec{B}$ is primary acceleration force – Lorentz force!

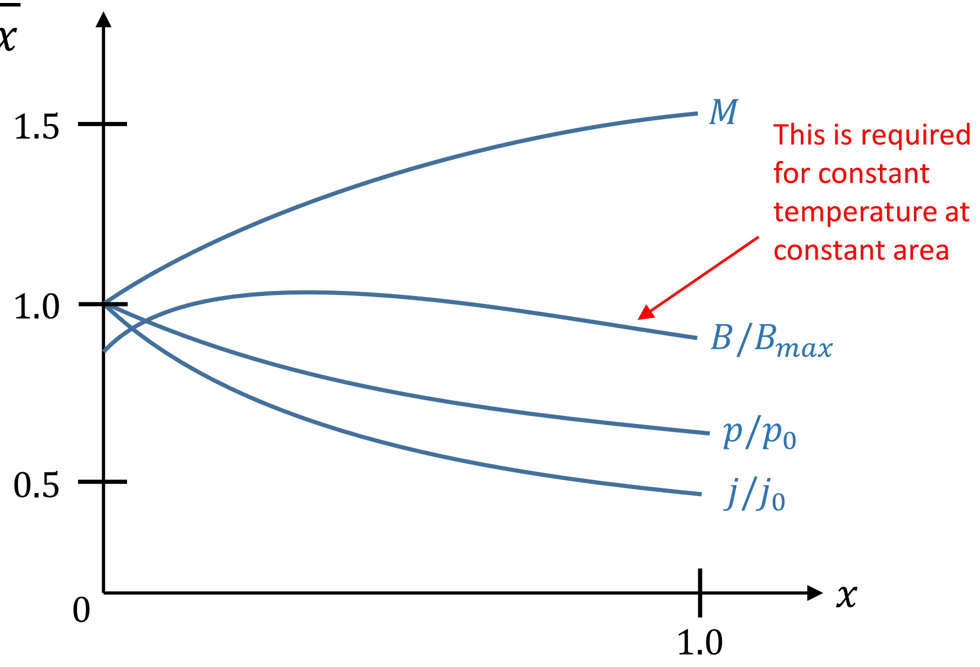
$$u\frac{du}{dx} = \frac{j_y E}{\rho u} - c_p\frac{dT}{dx} = \frac{j_y B}{\rho} - \frac{1}{\rho}\frac{dp}{dx}$$

$$\frac{j_y B}{\rho} = u\frac{du}{dx}\left[1 - \frac{RT}{u^2}\right] + R\frac{dT}{dx} = \left(\frac{j_y E}{\rho u} - c_p\frac{dT}{dx}\right)\left[1 - \frac{a^2}{\gamma u^2}\right] + R\frac{dT}{dx}$$

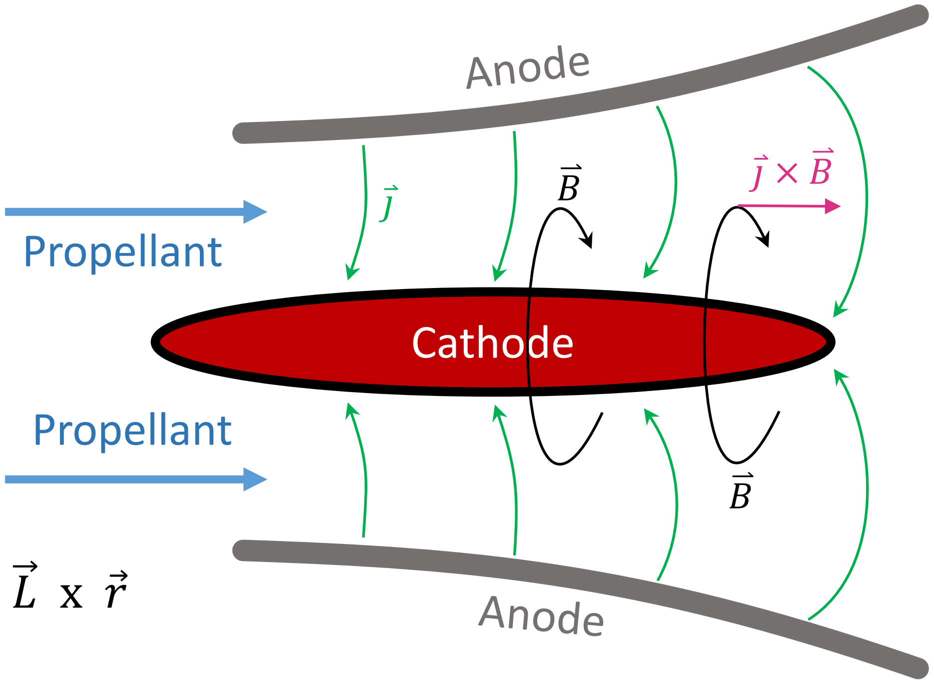
$$\frac{B}{E} = \frac{1}{u}\frac{\gamma M^2 - 1}{\gamma M^2}$$

Note: Constant temperature model has weakness that at $M < 1$, $j_y B < 0$ so, pressure gradient drives flow!

$$\rho u = \frac{\dot{m}}{A}$$



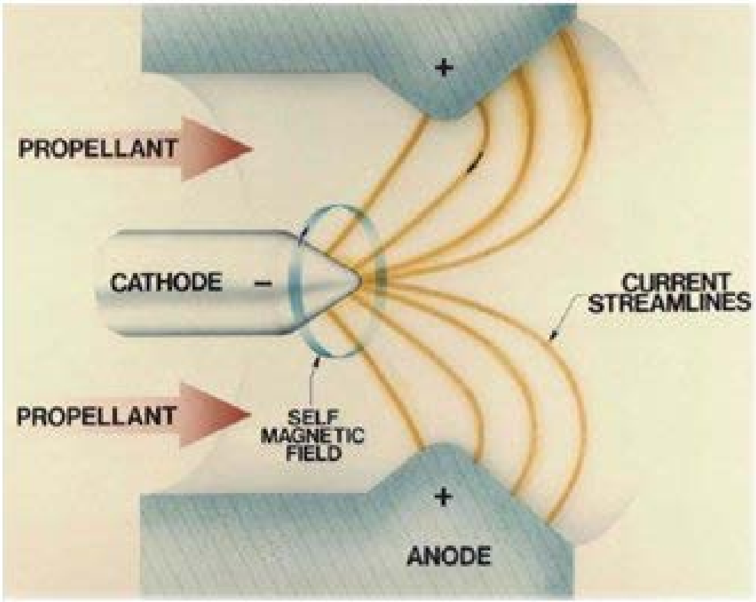
Electromagnetic thrusters



Typically alkali metal is used to seed gas. An arc discharge is employed to create plasma $\vec{j} \times \vec{B}$ is primary acceleration force – Lorentz force!

Axisymmetric configuration

- Current ionizes the propellant
- Magnetic field can be applied or self-induced: $d\vec{B} = \frac{\mu \ I}{4\ \pi r2} d\vec{L} \ x \ \vec{r}$



Momentum

$$\frac{dp}{dx} + \rho u \frac{du}{dx} = j_y B$$

$\vec{j} \times \vec{B}$ is the Lorentz force that is the main cause of acceleration

Energy

$$\rho u c_p \frac{dT}{dx} + \rho u^2 \frac{du}{dx} = j_y E$$

$$j_y E = -j_y \frac{dV}{dy}$$

This is the joule heating, energy per unit time per unit volume.

Solar sails & solar panels

Consider solar photovoltaic process to convert solar energy to electrical energy

Solar → Photovoltaic cells → Electric rocket (versus Solar sails)

For ion rocket: beam power = kinetic energy flux

Suppose 1/10 of solar power is converted to beam energy

Average radius of the earth's orbit around sun

$$\frac{\dot{E}}{A} = 1366\left[\frac{W}{m^2}\right]\left(\frac{R}{r}\right)^2$$

Distance from the sun

Consider a $1\ m^2$ panel with 10% efficiency :

$$P = 137\ watts = \frac{1}{2}\dot{m}UU = T\frac{U}{2} = \frac{TgI_{sp}}{2}$$

$$\frac{T}{meter^2} = \frac{9.14\times10^{-3}\ newton}{meter^2}$$

Now, consider using pressure from photon momentum collision with sail surface.

The momentum flux per area of the photons is: $\frac{\dot{E}}{Ac} = \frac{F}{A} = P$

$$\frac{1366\ joules/sec}{m^2\ 3\times10^8 m/sec} = \frac{0.457\times10^{-5}}{m^2}\left[\frac{newton\ meter}{m}\right] = 0.457\times10^{-5}\mathit{newton/m^2}$$

This is 1000 to 2000 times lower than the thrust with solar panel !

Nuclear Rockets

Type	Energy $\left[\frac{joules}{kgm}\right]$
Heat of Fusion - Water	$334{,}000 \approx 93\ Wh/kgm$
Batteries, Fuel Cells	$10^5\ - 10^6$
Melted matter, $LiOH, LiH, LiF$	$> 10^6$
Combustion – oxidation chemistry	$> 10^7$
Binding energy (dissociation - recombination	$> 10^8$
Fission	8×10^{13}
Fusion	4×10^{14}
Antimatter annihilation $E/m = c^2$	9×10^{16}

$10 - 100\ Wh/kgm$ for batteries

$30{,}000\ Wh/kgm$ for H_2

Chemical

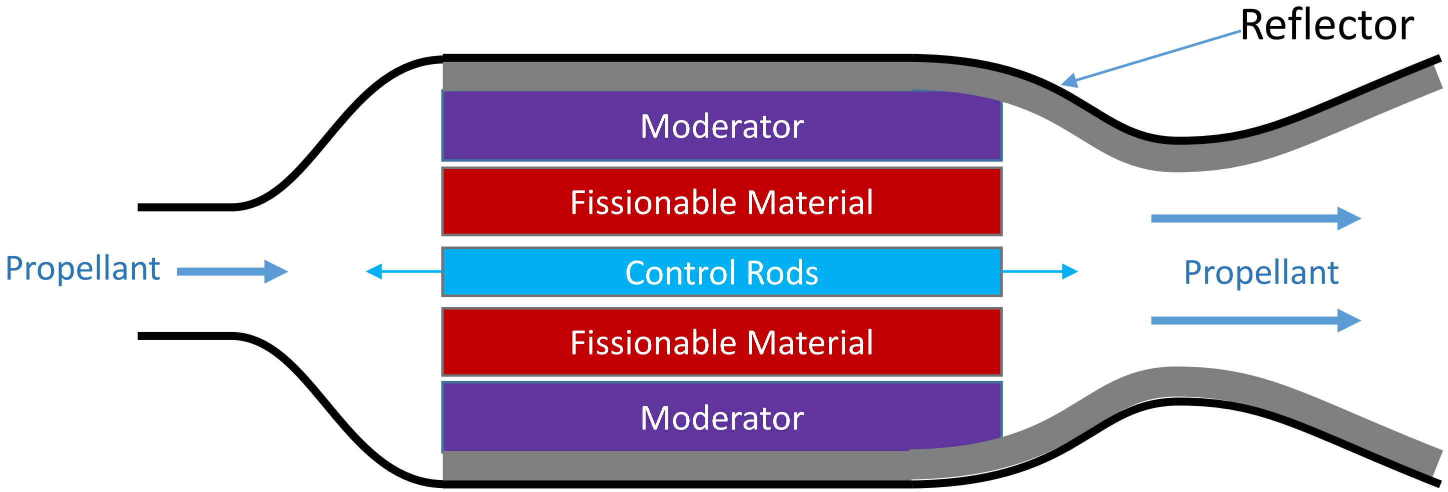
Nuclear binding energy

Nuclear Rockets

In the solid-core and gaseous-core rockets,
the fissionable material passes energy to the propellant.

Fissionable fragments and ORION concepts
use the fissionable material as propellants.

Solid-core nuclear rocket design



Reflector – keeps neutrons from escaping
Control Rod – absorbs neutrons at sufficient rate to control reaction and prevent explosion
Moderator – absorbs energy from neutrons and heats up. The transfers heat to the propellant flow

Graphite is a good moderator
1. Sublimes at high temperature $3620\ K$
2. Doesn't crack under thermal shock
3. Low molecular weight so it takes more energy in collision with neutrons

Hydrogen is a good propellant because of low molecular weight

Uranium carbide is a typical fissionable material

The boiling point of the moderator is the limitation!

The choked nozzle flow behaves exactly the same as a chemical-rocket nozzle flow!