

Lecture 9

Efficiencies

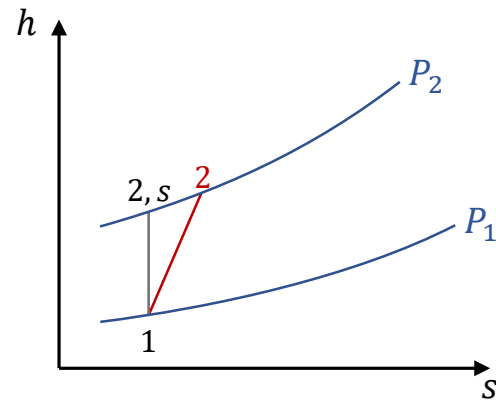
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It is particularly important that students clearly understand that the unauthorized sale or commercial distribution of course notes or recordings is a violation of campus policy. Specifically, Section 102.23 of [Policies Applying to Campus Activities, Organizations, and Students](#) states that the following activities constitute grounds for discipline:

Selling, preparing, or distributing for any commercial purpose course lecture notes or video or audio recordings of any course unless authorized by the University in advance and explicitly permitted by the course instructor in writing. The unauthorized sale or commercial distribution of course notes or recordings by a student is a violation of these Policies whether or not it was the student or someone else who prepared the notes or recordings.

Efficiencies for Compression

h_s is enthalpy for the isentropic case.



$$\eta_{ad} = \frac{h_{2,s} - h_1}{h_2 - h_1} = \frac{\int \frac{dP}{\rho}}{c_p(T_2 - T_1)}$$

$$\frac{\rho}{\rho_1} = \left(\frac{P}{P_1}\right)^{1/\gamma} \quad \text{For an isentropic process}$$

$$Tds = dh - dP/\rho$$

$$\text{So } dh_s = dP/\rho$$

$$\Delta h_s = \int \frac{dP}{\rho}$$

$$\int_{P_1}^{P_2} \frac{dP}{\rho} = \frac{P_1^{1/\gamma}}{\rho_1} \int_{P_1}^{P_2} \frac{dP}{P^{1/\gamma}} = \frac{P_1^{1/\gamma}}{\rho_1} \frac{\gamma}{\gamma - 1} \left[P_2^{(\gamma-1)/\gamma} - P_1^{(\gamma-1)/\gamma} \right]$$

$$\eta_{ad} = \frac{\frac{\gamma}{\gamma - 1} \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1 \right]}{\frac{c_p}{R} \left[\frac{T_2}{T_1} - 1 \right]} = \frac{\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1}{\frac{T_2}{T_1} - 1}$$

Used: $\frac{P_1}{R\rho_1 T_1} = 1$

$$c_p = c_v + R$$

$$\frac{c_p}{R} = \frac{c_v}{R} + 1$$

$$\frac{c_p}{R} = \frac{\gamma - 1}{\gamma}$$

Efficiencies for Compression

$$e = \frac{dh_s}{dh} = \frac{dP/\rho}{dh} = \frac{\left(\frac{RT}{P}\right) dP}{c_p dT}$$

Polytropic or
Elementary Efficiency

$$e = \frac{\gamma - 1}{\gamma} \frac{dP/P}{dT/T}$$

If e were constant: $\ln \frac{T_2}{T_1} = \frac{\gamma - 1}{\gamma e} \ln \frac{P_2}{P_1}$

Or: $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma e}$

Shockwave Compression Relations

For compression process:

$$\frac{dT}{T} = \frac{\gamma - 1}{\gamma e} \frac{dP}{P} \quad \rightarrow \quad \ln \frac{T_2}{T_1} = \frac{\gamma - 1}{\gamma e} \ln \frac{P_2}{P_1}$$

$$\Delta \ln T = \frac{\gamma - 1}{\gamma e} \Delta \ln P$$

$$\Delta \ln T = \Delta \ln P - \Delta \ln \rho$$

$$\begin{aligned} e &= \eta_{el} \\ \frac{dT}{T} &= \frac{R}{c_p e} \frac{dP}{P} = \frac{\gamma - 1}{\gamma e} \frac{dP}{P} \\ d\sigma &= c_p \frac{dT}{T} = \frac{R}{e} \frac{dP}{P} \end{aligned}$$

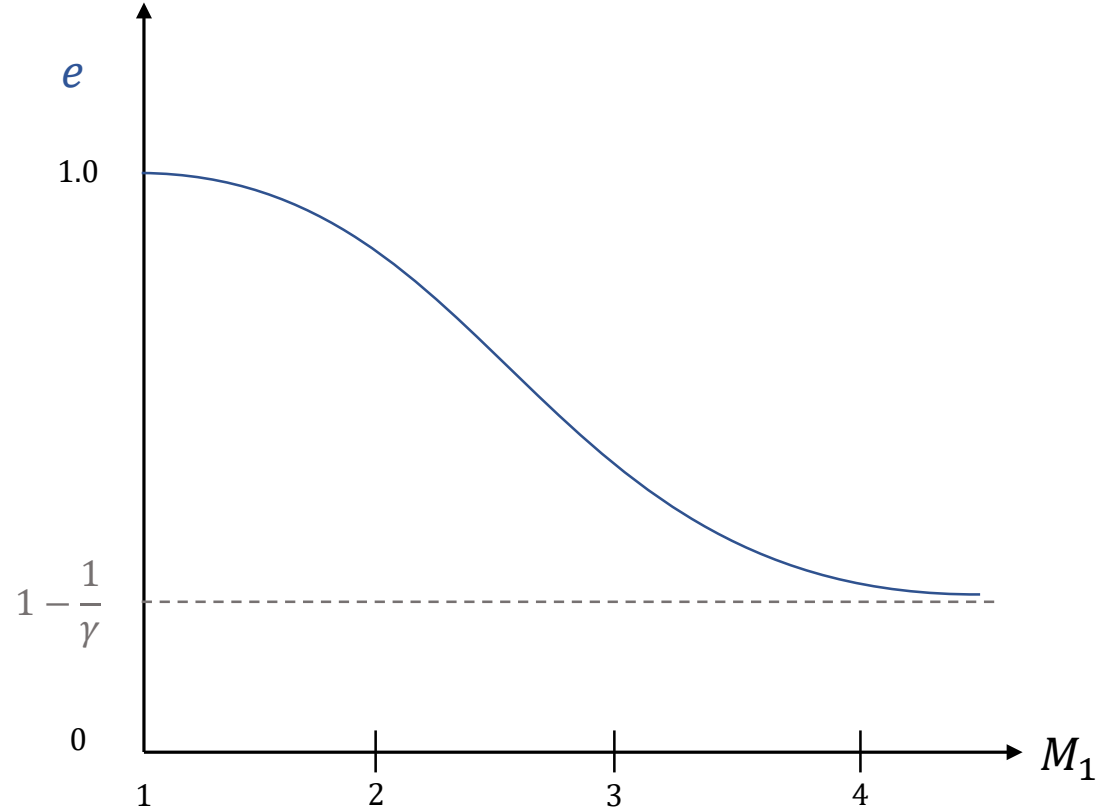
Shockwave Compression Relations

$$\frac{\gamma - 1}{\gamma e} = 1 - \frac{\Delta \ln \rho}{\Delta \ln P}$$

$$e = \frac{\frac{\gamma - 1}{\gamma}}{1 - \frac{\Delta \ln \rho}{\Delta \ln P}} = \frac{1 - \frac{1}{\gamma}}{1 - \frac{\Delta \ln \rho}{\Delta \ln P}} \quad \text{----->}$$

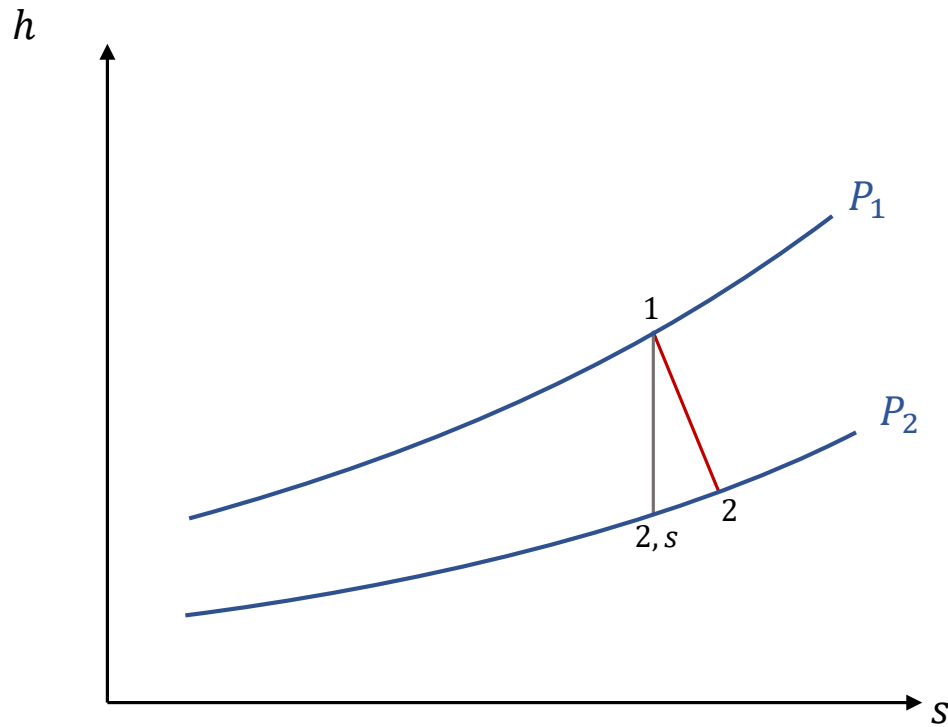
$$1 \quad \text{as} \quad \frac{P_2}{P_1} \rightarrow 1, \quad M_1 \rightarrow 1$$

$$1 - \frac{1}{\gamma} \quad \text{as} \quad \frac{P_2}{P_1} \rightarrow \infty, \quad M_1 \rightarrow \infty$$



So, oblique shock is preferred at high Mach numbers!

Efficiencies of Expansion



$$\eta_{ad} = \frac{h_2 - h_1}{h_{2,s} - h_1}$$

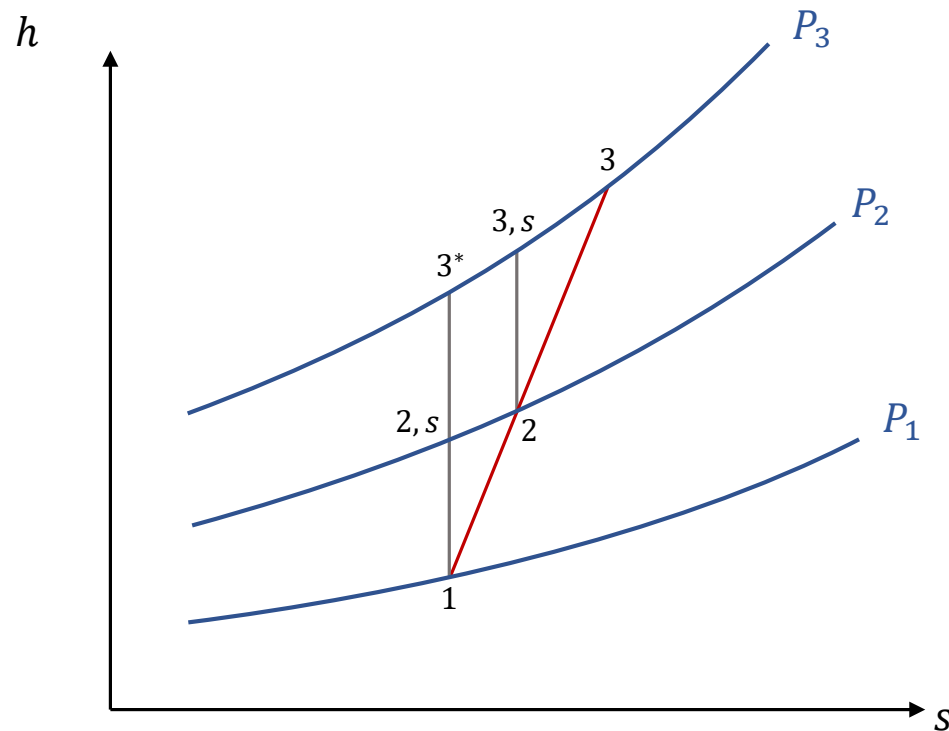
$$e = \frac{dh}{dh_s}$$

$$e = \frac{c_p dT}{dP/\rho} = \frac{c_p}{R} \frac{dT/T}{dP/P} = \frac{\gamma}{\gamma - 1} \frac{dT/T}{dP/P}$$

$$\text{For constant } e : \ln \frac{T_2}{T_1} = \frac{(\gamma - 1)e}{\gamma} \ln \frac{P_2}{P_1}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(\gamma - 1)e/\gamma}$$

Efficiencies for Multi-stage Compression



Suppose: $\eta_{ad12} = \eta_{ad23}$

$$\frac{h_{2,s} - h_1}{h_2 - h_1} = \frac{h_{3,s} - h_2}{h_3 - h_2}$$

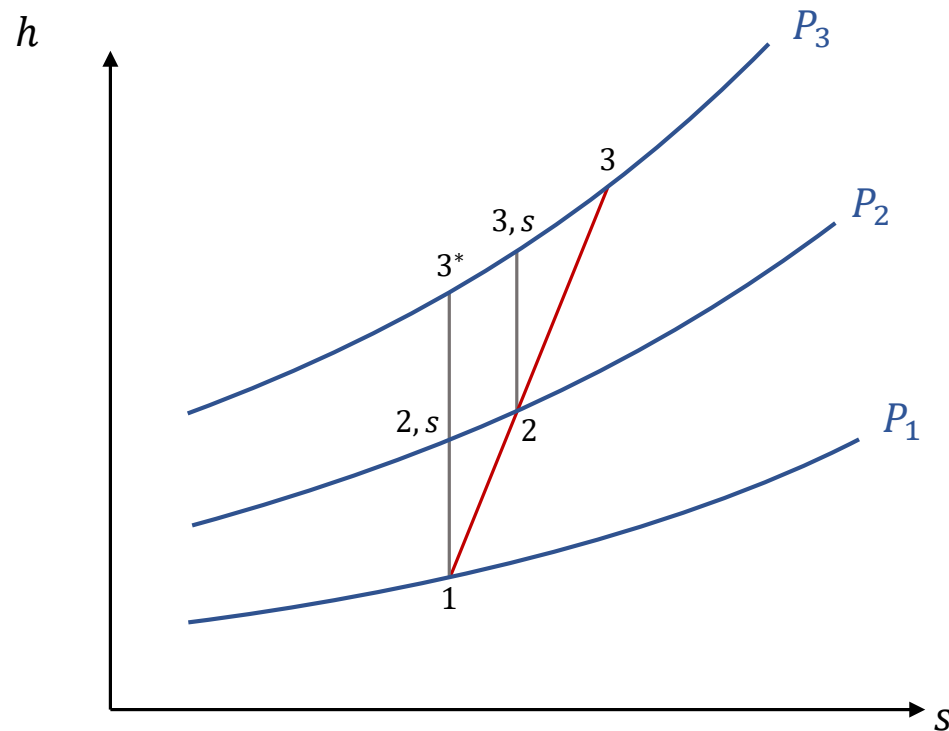
$$h_{2,s} - h_1 + h_{3,s} - h_2 = \eta_{ad12} [h_2 - h_1 + h_3 - h_2]$$

$$h_{3,s} - h_1 - (h_2 - h_{2,s}) = \eta_{ad12} [h_3 - h_1]$$

For the overall process:

$$\eta_{ad13} = \frac{h_{3^*} - h_1}{h_3 - h_1} \quad \text{So} \quad h_{3^*} - h_1 = \eta_{ad13} [h_3 - h_1]$$

Efficiencies for Multi-stage Compression



$$h_{3^*} - h_1 < h_{3,s} - (h_2 - h_{2,s}) - h_1$$

Because of diverging pressure curves, $\underbrace{h_{3,s} - h_{3^*} > h_2 - h_{2,s}}$

$$h_{3,s} - (h_2 - h_{2,s}) > h_{3^*}$$

So:
$$h_{3^*} - h_1 < h_{3,s} - (h_2 - h_{2,s}) - h_1$$

And:
$$\eta_{ad_{13}} < \eta_{ad_{12}} = \eta_{ad_{23}}$$

Efficiency overall \neq Efficiency of stages
 Polytropic efficiency is superior

Approximate Integration

$$\int_{P^\circ_1}^{P^\circ_2} \frac{dP^\circ}{P^\circ} \approx \frac{1}{P^\circ_{12}} \int_{P^\circ_1}^{P^\circ_2} dP^\circ \approx \frac{\Delta P^\circ}{P^\circ_{12}} \quad \text{If } P^\circ_{12} \text{ is an average}$$

$$\int_{T^\circ_1}^{T^\circ_2} \frac{dT^\circ}{T^\circ} \approx \frac{\Delta T^\circ}{T^\circ_{12}} \quad \text{Where } T^\circ_{12} \text{ is an average}$$

$$\frac{\Delta P^\circ}{P^\circ_{12}} \approx -\frac{\gamma}{2} M_c^2 \frac{\Delta T^\circ}{T^\circ_{12}}$$

Where M_c is approximately constant in the combustion chamber!