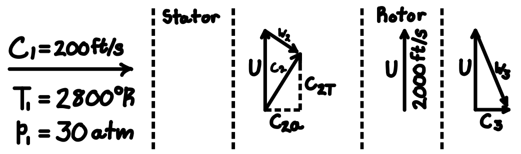
1) 30 Pts



$e = 0.95$

$P_3 = 20\,atm$

a) 7.5

$$\frac{T_3}{T_1} = \frac{T_2}{T_1}\frac{T_3}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{e\gamma}}\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{e\gamma}} = \left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{e\gamma}} = 0.9062 \longrightarrow T_3 = 2537.36°R$$

$$\Delta T_{1,3} = T_1 - T_3 = 262.64\,°R \qquad c_p = \frac{0.3\,\cancel{Btu}}{lbm\,°R}\left|\frac{778\,ft\cdot\cancel{lbf}}{\cancel{Btu}}\right|\frac{32.174\,lbm\cdot ft/s^2}{\cancel{lbf}} = 7509.41\,\frac{ft^2}{s^2\,°R}$$

$$\Delta h_{1,3} = c_p \Delta T_{1,3} = 78.79\,\frac{Btu}{lbm} = 1{,}972{,}239\,\frac{ft^2}{s^2}$$

$$\Delta T_{1,3} = 262.64\,°R \qquad \boxed{\Delta T_{1,3} = 183.85\,°R}$$

$$\Delta h_{1,3} = 78.79\,\frac{Btu}{lbm} \qquad \boxed{\Delta h_{1,3} = 55.153\,\frac{Btu}{lbm}}$$

b) 7.5

$$\Delta h^\circ_{1,3} = c_p \Delta T^\circ_{1,3} = c_p\left(\Delta T_{1,3} + \left[\frac{1}{2c_p}\cancel{(c_1^2 - c_3^2)}\right]\right) \quad c_1 = c_3$$

$$\Delta h^\circ_{1,3} = \Delta h_{1,3}$$

$$U(c_{2T} - \cancel{c_{1T}}^{\,0}) = \Delta h_{3,1}$$

$$c_{2T} = \frac{\Delta h_{3,1}}{U} = \frac{-1{,}972{,}239\,ft^2/s^2}{2000\,ft/s} = -986.12\,\frac{ft}{s}$$

$$\boxed{c_{2T} = -986.12\,\frac{ft}{s}}$$

c) 30% of $\Delta h_{1,3}$ occurs across rotor

7.5

$$\Delta h_{2,3} = 23.64\,\frac{Btu}{lbm} \longrightarrow \Delta T_{2,3} = 78.79\,°R \quad \therefore \quad T_2 = 2616.15\,°R$$

$$P_2 = P_3\left(\frac{T_3}{T_2}\right)^{-\frac{e\gamma}{\gamma-1}} = 22.68\,atm$$

$$\boxed{\begin{aligned}\Delta h_{2,3} &= 23.64\,\frac{Btu}{lbm}\\ \Delta T_{2,3} &= 78.79\,°R\\ \Delta P_{2,3} &= 2.683\,atm\end{aligned}}$$

d) Flow through stator is adiabatic and no work is done

7.5

$$\frac{P}{\dot m} = h_{o_3} - h_{o_2} = U(\cancel{c_{3T}}^{\,0} - c_{2T})$$

$$\boxed{\frac{P}{\dot m} = 78.79\,\frac{Btu}{lbm}}$$

$$c_1 = c_{2a} = c_3 \qquad c_2 = \sqrt{c_{2T}^2 + c_{2a}^2}$$

$$c_2 = 769.45\,ft/s$$

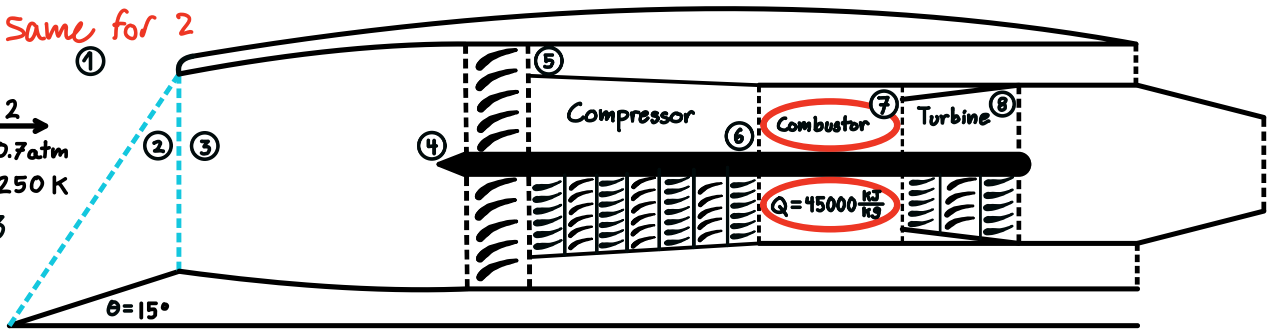
2) Same for 2

$M_\infty = 2$

$P_\infty = 0.7\,atm$

$T_\infty = 250\,K$

$\beta = 3$



$\frac{P_{06}}{P_{04}} = 25 \quad e_{cF} = 0.97 \qquad T_7 = 1500\,K \qquad \eta_B = 0.99 \qquad C_{P_{air}} = 0.24\,\frac{cal}{gK} \qquad \gamma_{air} = 1.4$

$\frac{P_{05}}{P_{04}} = 3 \quad e_F = 0.98 \qquad e_T = e_N = 0.97 \qquad C_{P_P} = 0.31\,\frac{cal}{gK} \qquad \gamma_P = 1.25$

a) $\theta = 15° \longrightarrow \beta = 45°$

$M_{1n} = M_1 \sin\beta = \sqrt{2} \longrightarrow \frac{P_{02}}{P_{01}} = 0.9531$ & $M_{2n} = 0.7314 \qquad M_2 = \frac{M_{2n}}{\sin(\beta-\theta)} = 1.463 \longrightarrow \frac{P_{03}}{P_{02}} = 0.942$

$P_{01} = P_\infty(1+0.2\cdot 4)^{\frac{\gamma}{\gamma-1}} = 5.477\,atm \qquad P_{02} = 5.22\,atm \qquad \boxed{P_{03} = 4.92}$

b) $P_{04} = P_{03} \qquad P_{05} = \frac{P_{05}}{P_{04}} P_{04} = 3(4.92) = 14.75\,atm \qquad P_{05} = 14.75\,atm$

$T_{01} = T_1(1+0.8) = 450\,K$

$T_{01} = T_{02} = T_{03} = T_{04} = 450\,K$

after 4 static quantities are approximately equal to total quantities

$$T_{05} \approx T_5 = T_4 \left(\frac{P_{05}}{P_{04}}\right)^{\frac{\gamma-1}{e_F \gamma}} = 619.8\,K$$

$\boxed{T_{05} = 619.8\,K}$

c) $P_{04} = P_{03} \qquad P_{06} = \frac{P_{06}}{P_{04}} P_{04} = 25(4.92) = 123\,atm \qquad P_{06} = 123\,atm$

$$T_{06} \approx T_6 = T_4 \left(\frac{P_{06}}{P_{04}}\right)^{\frac{\gamma-1}{e_{cF} \gamma}} = 1161.4\,K$$

$\boxed{T_{06} = 1161.4\,K}$

d) $\dot m_f \eta_b Q_R + \dot m_a C_{P_{air}} T_{06} = (\dot m_a + \dot m_f) C_{P_P} T_{07}$

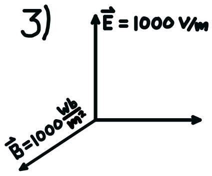
$C_{P_{air}} = 0.24\,\frac{cal}{gK}\Big|\frac{4.184\,J}{cal}\Big|\frac{1000\,g}{Kg} = 1004.16\,\frac{J}{Kg\cdot K} \qquad f = \frac{C_{P_P} T_{07} - C_{P_a} T_{06}}{\eta_b Q_R - C_{P_P} T_{07}} = 0.0183$

$C_{P_P} = 0.31\,\frac{cal}{gK}\Big|\frac{4.184\,J}{cal}\Big|\frac{1000\,g}{Kg} = 1297.04\,\frac{J}{Kg\cdot K}$

$\dot m_p (1+f) C_{P_P}(T_{07} - T_{08}) = \dot m_p C_{P_a}(T_{06} - T_{04}) + \dot m_s C_{P_a}(T_{05} - T_{04})$

$$T_{08} = T_{07} - \frac{C_{P_a}(T_{06} - T_{04}) + \beta C_{P_a}(T_{05} - T_{04})}{(1+f) C_{P_P}}$$

$\boxed{T_{08} = 571.85\,K}$

3) 

$\frac{q}{m} = 1.76\times10^{11}\,\frac{C}{Kg}$ $\nu = 10^{8}\,\frac{\text{collisions}}{s}$ $Wb = \frac{N\cdot s\cdot m}{C}$ $\Omega = \frac{V\cdot s}{C}$ $J = N\cdot m = V\cdot C$

$\sigma = 2000\,\frac{1}{\Omega m}$

a) $\omega_e = \frac{qB}{m} = 1000\,Wb\left(1.76\times10^{11}\,\frac{C}{Kg}\right) = \frac{1.76\times10^{14}\,Wb\cdot C}{Kg\cdot m^2}\left|\frac{N\cdot s\cdot m\cdot C}{C\cdot Kg\cdot m^2}\right|\frac{Kg\cdot \cancel{m}\cdot s\cdot \cancel{m}\cdot \cancel{C}}{s^2\cdot \cancel{C}\cdot \cancel{Kg}\cdot \cancel{m^2}} = 1.76\times10^{14}\,Hz$

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$\boxed{\omega_e = 1.76\times10^{14}\,Hz}$

For effective Hall thruster, $\Omega = \frac{\omega_e}{\nu} \gg 1$

$\Omega = 1{,}760{,}000 \gg 1$ ✓

b) $V_{drift} = -\frac{E}{|B|} = -\frac{1000\,V/m}{|1000\,Wb|} = -1\,\frac{V\cdot m^2}{Wb\cdot m}\left|\frac{C\cdot V\cdot \cancel{m^2}}{N\cdot s\cdot \cancel{m}\cdot \cancel{m}}\right|\frac{N\cdot m}{N\cdot s} = -1\,\frac{m}{s}$

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$\boxed{V_{drift} = -1\,\frac{m}{s}}$

c) $j_y = -\sigma(E - \cancel{u}B)\frac{1}{1+\Omega^2} = -\sigma E\frac{1}{1+\Omega^2}$

0 because propellant is accelerated in y direction

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$j_y = -2000\,\frac{1}{\Omega m}(1000\,V/m)\frac{1}{1+1{,}760{,}000^2}$

$j_y = -6.457\times10^{-23}\,\frac{V}{\Omega\cdot m^2}\left|\frac{A\cdot\Omega}{\Omega\cdot m^2}\right.$

$\boxed{j_y = -6.457\times10^{-23}\,\frac{A}{m^2}}$

d) $F_x = j_y B = -6.457\times10^{-20}\,\frac{A\cdot Wb}{m^4}\left|\frac{V\cdot N\cdot s\cdot m}{\Omega\cdot C\cdot m^4}\right|\frac{\cancel{C}\cdot \cancel{V}\cdot N\cdot \cancel{s}\cdot \cancel{m}}{\cancel{V}\cdot \cancel{s}\cdot \cancel{C}\cdot m^3} = -6.457\times10^{-20}\,\frac{N}{m^3}$

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$\boxed{F_x = -6.457\times10^{-20}\,\frac{N}{m^3}}$

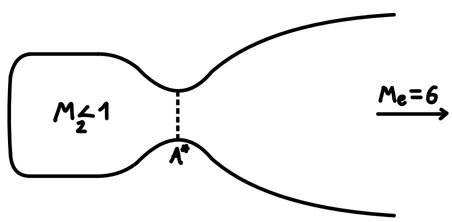
4) $P_o = 75$ atm

$T_o = 3200K$

$\gamma_c = 1.25$

For diatomic gases $\gamma = 7/5$

For monoatomic gases $\gamma = 5/3$



a)

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Chemical Rocket	Nuclear Rocket H_2	Nuclear Rocket He
$T_e = T_o\left(1+\frac{\gamma_c-1}{2}M_e^2\right)^{-1}$	$T_e = T_o\left(1+\frac{\gamma_{H_2}-1}{2}M_e^2\right)^{-1}$	$T_e = T_o\left(1+\frac{\gamma_{He}-1}{2}M_e^2\right)^{-1}$
$T_e = 581.81$ K	$T_e = 390.24$K	$T_e = 246.15$ K

b) $\frac{A}{A^*} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$

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Chemical Rocket	Nuclear Rocket H_2	Nuclear Rocket He
$\frac{A}{A^*} = 210.52$	$\frac{A}{A^*} = 53.180$	$\frac{A}{A^*} = 15.842$

c) $u_e = \sqrt{\frac{2\gamma R T_0}{\gamma - 1}}\left(1 - \frac{T_e}{T_0}\right)^{1/2}$ $\qquad F_T = \dot{m} u_e \qquad I_{sp} = \frac{F_T}{\dot{m} g} = \frac{u_e}{g} \qquad R = \frac{8.314\ J/mol \cdot K}{MW}$

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$2H_2 + O_2 \rightarrow 2H_2O \qquad MW = (2 \cdot 1\,\frac{g}{mol} + 16\,\frac{g}{mol})/1000\,\frac{g}{kg} = 0.018\,\frac{kg}{mol}$

<u>Chemical Rocket</u>	<u>Nuclear Rocket H_2</u>	<u>Nuclear Rocket He</u>
$R = 461.89$ J/kg·K	$R = 4157$ J/kg·K	$R = 2078.5$ J/kg·K
$\boxed{I_{sp} = 354.49\ s}$	$\boxed{I_{sp} = 921.73\ s}$	$\boxed{I_{sp} = 564.78\ s}$