University of California, Irvine
Department of Mechanical and Aerospace Engineering

MAE 112 - Homework
Fall 2024

Prepared by: Wes Hellwig

1. Suppose we had two parallel flows mimicking the primary and secondary exhaust flows from a turbofan engine. One flow has a stagnation temperature of 1100°R and the other at 2700°R. Each flow had a stagnation pressure of five atmospheres, a value of $\gamma = 1.4$, and a value of $c_p = 0.33$ Btu/lbm°R. The colder flow has five times the mass flow of the hotter flow. Calculate the ratio of total thrust to total mass flow from nozzle expansion in each of the following two cases.
a) The flows are mixed adiabatically at constant pressure and then expanded to ambient pressure of 0.7 atmosphere in one nozzle.
b) Each flow is expanded to the 0.7 atmosphere ambient pressure through a separate nozzle.

Solution:
(a) First we convert the units of specific heat and calculate the specific gas constant.

$$c_p = 0.33\,\frac{Btu}{lbm^\circ R}\left(778\,\frac{ft\cdot lbf}{Btu}\right)\left(32.174\,\frac{lbm\cdot ft}{lbf}\right) = 8260.35\,\frac{ft^2}{s^2\cdot^\circ R} \tag{1}$$

$$R = c_p\frac{\gamma-1}{\gamma} = 2360.1\,\frac{ft^2}{s^2\cdot^\circ R} \tag{2}$$

If the flows are mixed the total temperature is:

$$T_0 = \frac{1100^\circ R\times 5 + 2700^\circ R}{6} = 1366.6667^\circ R \tag{3}$$

$$\frac{T_{tot\ thrust}}{\dot{m}_{tot}} = u_e = \sqrt{\frac{2\gamma R T_0}{\gamma-1}\left[1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} = 3115.1\ ft/s \tag{4}$$

(b) If the flows are expanded individually, we calculate $T/\dot{m}$ for each and then combine them based on mass flow.

$$\frac{T_{in}}{\dot{m}_{in}} = 4378.48\ ft/s \tag{5}$$

$$\frac{T_{out}}{\dot{m}_{out}} = 2794.72\ ft/s \tag{6}$$

$$T_{in} = 4378.48\dot{m}_{in} \tag{7}$$

$$T_{out} = 2794.72\dot{m}_{out} = 13973.56\dot{m}_{in} \tag{8}$$

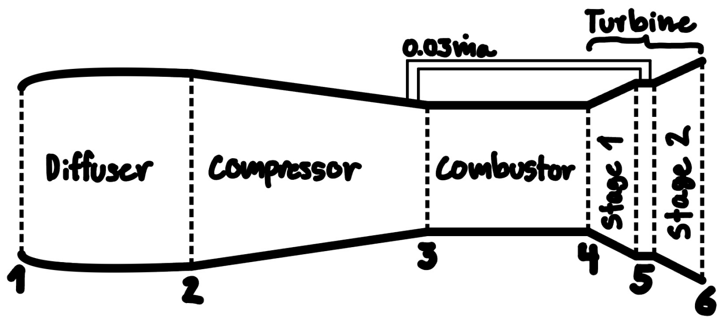
$$T_{tot\ thrust} = 18352.06\dot{m}_{in} \tag{9}$$

$$\dot{m}_{tot} = 6\dot{m}_{in} \tag{10}$$

$$\frac{T_{tot\ thrust}}{\dot{m}_{tot}} = 3058.68\ ft/s \tag{11}$$

2. Consider a turbine engine that powers helicopter flight with ambient conditions at 298 K and 1.0 atmospheres of pressure. There are two turbine stages with different rotational speeds; the first one drives the compressor while the other drives the main propeller of the helicopter. The turbine exit pressure is practically 1.0 atmospheres. Consider a combustor exit temperature of 1600 K and a mass mixture ratio $\mu = 30$.

Consider that 3% of the air is taken from the flow downstream of the compressor but before entry to the combustor to be used for cooling of the turbine. The specific heat of combustion products equals $c_{p,p} = 0.30$ cal/gm-K with $\gamma_p = 1.3$, and the specific heat of air is $c_{p,a} = 0.24$ cal/gm-K with $\gamma = 1.4$. Assume that flight velocity has negligible effect on the intake flow pressure.



Solution:

(a) First lets get the units of specific heat in a more usable form. $c_{p,a} = 1004.16\ J/kg \cdot K$ and $c_{p,p} = 1255.2\ J/kg \cdot K$. Since the flight velocity has negligible effect on the intake flow $(M << 1)$ $p_{01} \approx p_1 \approx p_{02} \approx p_2$.

(b) The power required for the compressor is based on the temperature change so we need the temperature before, and at the end of, the compressor. Since the Mach number is low $T_2 = 298\ K$ by the same logic as part (a).

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{1}{e}\frac{\gamma-1}{\gamma}} = 2.462 \rightarrow T_3 = 733.67\ K \tag{12}$$

$$\frac{P_{comp}}{\dot{m}} = c_{p,a}(T_3 - T_2) = 437,487\ J/kg \tag{13}$$

(c) To find the temperature and pressure change across the first turbine stage we do a power balance between the first turbine and the compressor. Since some air is taken out of the compressor to cool the turbine, we define $\dot{m}_a' = 0.97\dot{m}_a$ as the mass flow rate of air passing through the turbine.

$$\dot{m}_a' + \dot{m}_f = \dot{m}_a' + \frac{\dot{m}_a'}{\mu} = 0.97\dot{m}_a + \frac{0.97\dot{m}_a}{30} = 1.002333\dot{m}_a \tag{14}$$

$$\frac{P_t}{\dot{m}_a} = 1.002333c_{p,p}(T_4 - T_5) = \frac{P_c}{\dot{m}_a} \rightarrow T_5 = 1252.27\ K \tag{15}$$

$$\frac{p_5}{p_4} = \left(\frac{T_5}{T_4}\right)^{\frac{\gamma_p}{e(\gamma_p-1)}} = 0.3153 \rightarrow p_5 = 6.306\ atm \tag{16}$$

So the temperature and pressure change across the turbine are $\Delta T = -347.73\ K$ and $\Delta p = -13.69\ atm$, respectively. These values are negative because both quantities are decreasing.

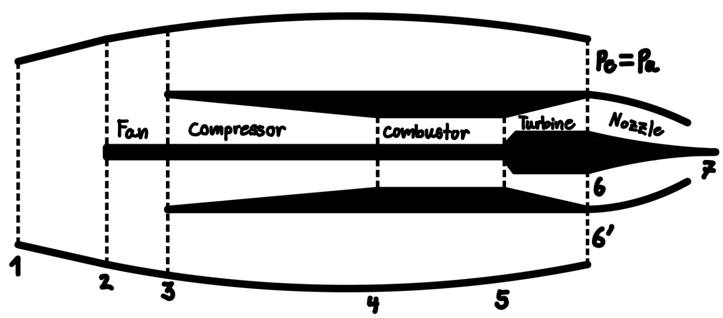
(d) To do the power balance, we need the turbine exit temperature.

$$\frac{T_6}{T_5} = \left(\frac{p_6}{p_5}\right)^{e\frac{\gamma_p-1}{\gamma_p}} = 0.667835 \rightarrow T_3 = 836.31\ K \tag{17}$$

$$\frac{P_{t2}}{\dot{m}_a} = 1.00233c_{p,p}(T_5 - T_6) = 523,331.23\ J/kg \tag{18}$$

$$\frac{P_{prop}}{\dot{m}_a} = 0.9\frac{P_{t2}}{\dot{m}_a} = 470,998.11\ J/kg \tag{19}$$

3. Consider a turbofan engine flying at a Mach number of 0.85 with ambient pressure of 0.6 atmosphere and ambient temperature of 430°R. The intake has 0.95 polytropic efficiency. The air divides with a bypass ratio of 9 into a primary and a secondary (or bypass) flow at the air intake exit. The primary flow passes through a fan and compressor with an overall pressure ratio of 22 and polytropic efficiency of 0.92 while the secondary flow passes through a fan with pressure ratio of four and efficiency of 0.95. The primary flow burns at constant pressure with liquid kerosene fuel (heating value = 20,000 Btu/lbm) with a final combustor temperature of 2500°R and then expands through a turbine and ultimately through the primary nozzle to ambient pressure. Burner efficiency is 0.98 while the polytropic efficiencies of the turbine and the nozzle are each 0.95. All work taken from the turbine flow is employed to drive the fan and compressor. The secondary flow expands isentropically to ambient pressure through a secondary nozzle. $c_p = 0.24$ Btu/lbm°R for air; $c_p = 0.31$ Btu/lbm°R for products; $\gamma = 1.4$ for air; $\gamma_p = 1.25$ for products.



Solution:
(a) Since the Mach number is low through the inlet and fan, the static temperature and pressure at these locations is approximately equal to the stagnation temperature. Furthermore, $T_{01} = T_{02}$ because the process is isentropic.

$$T_{01} = T_1\left(1 + \frac{\gamma - 1}{2}M_1^2\right) = 492.135°R = T_{02} \tag{20}$$

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{1}{e_f}\frac{\gamma-1}{\gamma}} = 1.5173 \rightarrow T_3 = 746.75°R \approx T_{03} \tag{21}$$

(b) $p_{02} \approx p_2$, $p_{03} \approx p_3$, $p_{04} \approx p_5$ because $M << 1$.

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{e_i\gamma}{\gamma-1}} = 1.566 \rightarrow p_2 = 0.94\ atm \rightarrow p_3 = 3.76\ atm \rightarrow p_4 = 22p_2 = 20.68\ atm \tag{22}$$

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{1}{e_c}\frac{\gamma-1}{\gamma}} = 2.612 \rightarrow T_4 = 1285.24°R \approx T_{04} \tag{23}$$

(c) To find the turbine exit temperature, we do a power balance between the turbine, compressor, and fan. However, first, we need the mixture ratio. This is found from an enthalpy balance before and after the combustor. In the equation below, η_b is the burner efficiency and Q is the heating value, and $f = \dot{m}_f/\dot{m}_a$.

$$\dot{m}_f\eta_b Q + \dot{m}_a c_{p,a}T_4 = (\dot{m}_a + \dot{m}_f)c_{p,p}T_5 \tag{24}$$

$$f = \frac{c_{p,p}T_5 - c_{p,a}T_4}{\eta_b Q - c_{p,p}T_5} = 0.0249 \tag{25}$$

Now we can match the power captured by the turbine to the power required by the fan and the compressor.

$$(\dot{m}_a + \dot{m}_f)c_{p,p}(T_5 - T_6) = \dot{m}_a c_{p,a}(T_4 - T_3) + (\dot{m}_a + \dot{m}_s)c_{p,a}(T_3 - T_2) \tag{26}$$

$$T_6 = T_5 - \frac{c_{p,a}(T_4 - T_3) + (\beta + 1)c_{p,a}(T_3 - T_2)}{(1 + f)c_{p,p}} = 1516.31°R \tag{27}$$

(d) The flow leaving the nozzle is perfectly expanded so $p_7 = 0.6\ atm$. Since we know the temperature change across the turbine we can determine the total pressure in the nozzle. $p_4 = 20.68\ atm \approx p_{04} = p_{05}$

$$\frac{p_6}{p_5} = \left(\frac{T_6}{T_5}\right)^{\frac{\gamma_p}{e_t(\gamma_p-1)}} = 0.072 \rightarrow p_6 = 1.488\ atm \tag{28}$$

$$u_{e,p} = \sqrt{2c_{p,p}T_{06}\left[1-\left(\frac{p_7}{p_{06}}\right)^{\frac{e_N(\gamma_p-1)}{\gamma_p}}\right]} = 1931.29\ ft/s \tag{29}$$

(e)

$$u_{e,s} = \sqrt{2c_{p,a}T_{03}\left[1-\left(\frac{p_7}{p_{03}}\right)^{\frac{\gamma-1}{\gamma}}\right]} = 1849.83\ ft/s \tag{30}$$

(f) Since we don't know the mass flow rates we will solve for $T/\dot{m}_a$.

$$T = (\dot{m}_a + \dot{m}_s)u_{e,p} + \dot{m}_s u_{e,s} - (\dot{m}_a + \dot{m}_s)u_1 \tag{31}$$

$$u_1 = M_1\sqrt{\gamma R T_1} = 1016.51\ ft/s \tag{32}$$

$$\frac{T}{\dot{m}_a} = \beta u_{e,s} + (1+f)u_{e,p} - (1+\beta)u_1 = 2629.51\ ft/s = 81.73\ \frac{lbf}{lbm/s} \tag{33}$$

$$TSFC = \frac{\dot{m}_s}{T} = \frac{f}{T/\dot{m}_a} = 3.0467\times10^{-4}\ \frac{lbf}{lbm/s} = 1.097\ \frac{lbm/hr}{lbf} \tag{34}$$

4. Design a turbofan engine that cruises at a Mach number of 0.85 with an ambient temperature of 490°R. It must produce 60,000 lbf of thrust. Use a bypass ratio of six. Give the mass flows of fuel and two air streams, pressure ratios across each component, cross-sectional areas of capture, nozzle throat, and nozzle exit.

Solution:
Here we assume that we are using the same fuel as in problem 3 where the heating value is 20,000 Btu/lbm.

Step 1
We need to know the ambient pressure and temperature. An good estimate for cruising altitude is around 40,000 ft where $p_\infty = 0.184\ atm$.

Step 2
If nozzle expansion is assumed to be perfectly expanded and $u_\infty = M_\infty\sqrt{\gamma R T_\infty}$, we have our thrust as:

$$T = (\dot{m}_a + \dot{m}_f)u_{e,p} + \dot{m}_s u_{e,s} - (\dot{m}_a + \dot{m}_s)u_\infty \tag{35}$$

Step 3
For the gas properties, we take those from problem 3.

$$c_{p,a} = 0.24\ \frac{Btu}{lbm^\circ R} \quad c_{p,a} = 0.31\ \frac{Btu}{lbm^\circ R} \quad \gamma = 1.4 \quad \gamma_p = 1.25 \tag{36}$$

We also take the polytropic efficiency from problem 3 into consideration in this problem.

$$e_d = 0.98 \quad e_c = 0.96 \quad e_f = 0.98 \quad e_t = 0.97 \quad e_n = 0.99 \tag{37}$$

The efficiency for the secondary flow nozzle is still 100%. Of course, you could use a different value to see how these parameters effect the overall performance as long as your number is realistic.

Step 4
Here we need to pick some design parameters, later we may need to adjust them to make the design satisfy the requirements.

The design parameters you need to give are: compressor pressure ratio r_c, fan pressure ratio r_f, and the temperature at the exit of the combustor T_{05} (the temperature is limited by the material of the turbine so the turbine blades are not damaged. Estimate $2200°R < T_{05} < 3600°R$).

Step 5
After steps 1-4 are completed, the determination of values at each location directly parallels that of problem 3. Remember, after the diffuser, the Mach number at station 2, 3, 4, and 5 is low enough that the static property there is approximately equal to the total property. At station 6 and beyond, the velocity is high enough that static and total quantities can no longer be approximated as equal. Throughout this problem when referring to stations, I am referring to the drawing in problem 3.

Without changing the pressure or temperature values across various stations, the thrust can be adjusted just by changing the inlet/exit areas. Some degree of iteration may be necessary with the area and/or the pressure and temperature ratios to meet the design requirements.