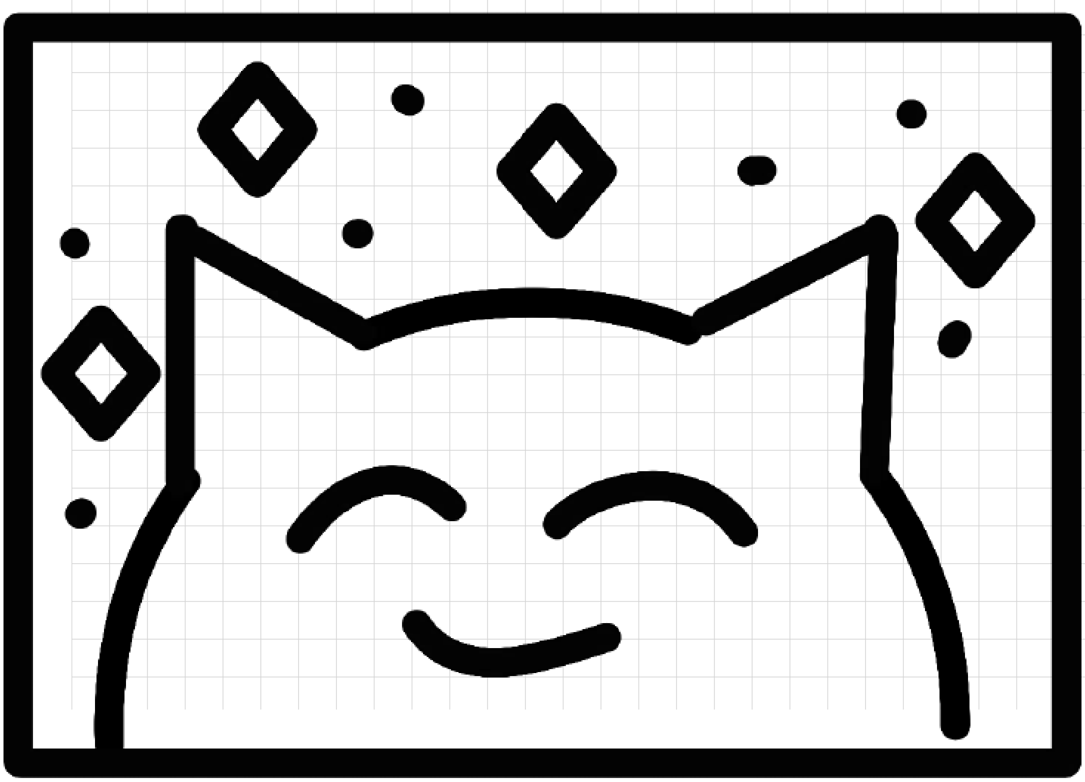
MAE 112

HW 1

TRIET



1. A rocket uses propellants with a specific impulse equal to 270 seconds. It has an empty weight (i.e., without propellants) of 200 lbf (measured at sea level). The desire is to accelerate it from rest to a velocity of 8,000 feet per second. Assume that drag forces are negligible and gravity acts normal to the flight direction.

(a) What is the required mass of propellant?

(b) If the rocket had an initial horizontal velocity of 1000 feet per second (e.g., with a launch from an aircraft), how much propellant mass is required to achieve the 8,000 feet per second speed?

(c) If the specific impulse were increased to the new value of 290 seconds, what would be the answer to part (a)?

$$I_{sp} = 270 \text{ [s]}$$
$$m_{dry} = 200 \text{ [lb]}$$
$$m_p + m_{dry} = m_{total}$$

(a)

$$\Delta V = I_{sp} \times g \times \ln\left(1 + \frac{m_p}{m_{dry}}\right)$$

$$8000 \left[\frac{\cancel{ft}}{s}\right] = 270 [\cancel{s}] \times 32.2 \left[\frac{\cancel{ft}}{s^2}\right] \times \ln\left(1 + \frac{m_p}{200}\right)$$

$$\boxed{m_p = 301.9458 \text{ [lb]}}$$

(b)

$$\Delta V = I_{sp} \times g \times \ln\left(1 + \frac{m_p}{m_{dry}}\right)$$

$$7000 \left[\frac{\cancel{ft}}{s}\right] = 270 [\cancel{s}] \times 32.2 \left[\frac{\cancel{ft}}{s^2}\right] \times \ln\left(1 + \frac{m_p}{200}\right)$$

$$\boxed{m_p = 247.4077 \text{ [lb]}}$$

(c)

$$\Delta V = I_{sp} \times g \times \ln\left(1 + \frac{m_p}{m_{dry}}\right)$$

$$8000 \left[\frac{\cancel{ft}}{s}\right] = 290 [\cancel{s}] \times 32.2 \left[\frac{\cancel{ft}}{s^2}\right] \times \ln\left(1 + \frac{m_p}{200}\right)$$

$$\boxed{m_p = 271.0819 \text{ [lb]}}$$

2. Consider a rocket at takeoff from Earth at sea level. The mass flux of propellants is 160 kg/s. The hot gas exits the nozzle at a velocity of 1200 m/sec and a pressure of 0.85 atmospheres through an exit area of one square meter. Ambient pressure is one atmosphere. What is the thrust magnitude? If the initial acceleration is 50 m/sec^2, what is the initial mass of the vehicle including propellants?

$$\frac{dm}{dt} = \dot{m} = 160 \ [kg/s]$$

$$V_e = 1200 \ [m/s]$$

$$P_e = 0.85 \ [atm] = 86126.25 \ [Pa]$$

$$A_e = 1 \ [m^2]$$

$$P_a = 1 \ [atm] = 101325 \ [Pa]$$

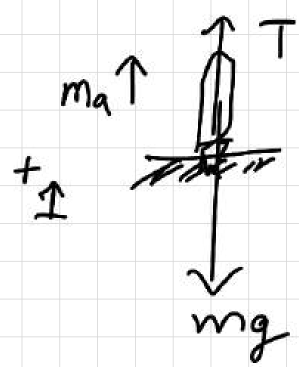
$$a = 50 \ [m/s^2]$$

Thrust equation:

$$T = \dot{m} V_e + (P_e - P_a) A_e$$

$$= 160 \times 1200 + (86126.25 - 101325) \ 1$$

$$\boxed{T = 176801.25 \ [lb]}$$



Newton 2nd law in y direction:

$$T - mg = ma$$

$$T = ma + mg = m(a+g)$$

$$m = \frac{T}{a+g} = \frac{176801.25}{50 + 9.81}$$

$$\boxed{m = 2956.04832 \ [kg]}$$

3. Consider an air-breathing jet engine which is flying at a velocity of 600 feet per second. For every lbm/second of air mass flow, a 0.030 lbm/sec mass flow of fuel is injected into the engine. The thrust force is 5000 lbf.; the entrance pressure equals the ambient pressure at the altitude (0.5 atm) and the exhaust pressure is 0.75 times the ambient atmospheric pressure. The incoming air temperature is 525°R. The entrance area and exhaust areas are both ten square feet. Determine:

(a) exhaust velocity

(b) specific fuel consumption

$$V_\infty = 600 \ [ft/s]$$

$$\dot{m}_{fuel} = 0.03 \ \dot{m}_{air}$$

$$f = \frac{\dot{m}_{fuel}}{\dot{m}_{air}} = 0.03$$

$$T = 5000 \ [lb]$$

$$P_{in} = P_a = 0.5 \times 2116.2 = 1058.1 \ [lb/ft^2]$$

$$P_{out} = 0.75 \ P_a = 793.575 \ [lb/ft^2]$$

$$T_{in} = 525 \ [°R]$$

$$A = 10 \ [ft^2]$$

ⓐ $$\rho = \frac{P_{in}}{RT_{in}} = \frac{1058.1}{1718 \times 525} = 0.001173 \ [lb/ft^3]$$

$$\dot{m}_{air} = \rho A V_\infty = 0.001173 \times 10 \times 600 = 7.038 \ [slug/s]$$

$$\dot{m}_{fuel} = f \ \dot{m}_{air} = 0.03 \times 7.038 = 0.21114 \ [slug/s]$$

Thrust equation:

$$T = (\dot{m}_{air} + \dot{m}_{fuel}) V_{out} - \dot{m}_{air} \times V_{in} + (P_{out} - P_a)A - \cancelto{0}{(P_{in} - P_a)A}$$

$$V_{out} = \frac{T + \dot{m}_{air} V_{in} - (P_{out} - P_a)A}{\dot{m}_{air} + \dot{m}_{fuel}}$$

$$= \frac{5000 + 7.038 \times 600 - (793.575 - 1058.1)\, 10}{7.038 + 0.21114}$$

$$\boxed{V_{out} = 1637.1666 \ [ft/s]}$$

(b)

$$1 \text{ slug } \frac{ft}{s^2} = 1 \text{ lbf}$$

$$1 \text{ lbm} \times 32.2 \frac{ft}{s^2} = 1 \text{ lbf}$$

$$1 \text{ slug } \cancel{\frac{ft}{s^2}} = 32.2 \text{ lbm } \cancel{\frac{ft}{s^2}}$$

$$\dot{m}_f = 0.21114 \left[\frac{\cancel{slug}}{\cancel{s}}\right] \times \frac{32.2\,[lbm]}{1\,[\cancel{slug}]} \times \frac{3600\,[\cancel{s}]}{1\,[hr]} = 24475.3488 \left[\frac{lbm}{hr}\right]$$

Thrust-specific fuel consumption

$$TSFC = \frac{\dot{m}_f}{T} \quad \begin{matrix} [lbm/hr] \\ [lbf] \end{matrix}$$

$$= \frac{24475.3488}{5000}$$

$$\boxed{TSFC = 4.8951} \left[\frac{lbm/hr}{lbf}\right]$$

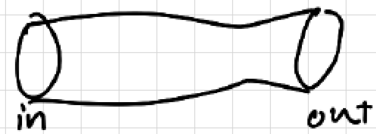
mass flow rate of fuel

Thrust

4. For an air-breathing jet engine, specific thrust is defined as thrust divided by air mass flow rate. Consider an engine that is flying at a velocity equal to 250 meters per second. For every kilogram/second of air mass flow, 0.040 kgm/sec mass flow of fuel is injected into the engine. The exit pressure and the entrance pressure both equal the ambient pressure, which is 0.7 atm. What must be the value of the exhaust velocity u_e if a specific thrust equal to 400m/s is desired?

$$V_{in} = 250 \ [m/s]$$

$$1 \frac{kg}{s} \text{ air} : 0.04 \frac{kg}{s} \text{ fuel}$$



$$\dot{m}_{fuel} = 0.04 \ \dot{m}_{air}$$

$$P_{in} = P_{out} = P_a = 0.7 \ [atm]$$

$$V_{out} = ?$$

$$\frac{T}{\dot{m}_{air}} = 400 \ [m/s]$$

Thrust equation:

$$T = \underbrace{(\dot{m}_{air} + \dot{m}_{fuel})}_{} V_{out} - \dot{m}_{air} V_{in} + \cancel{(P_{out} - P_a)}^{0} A_{out} - \cancel{(P_{in} - P_a)}^{0} A_{in}$$

$$T = 1.04 \ \dot{m}_{air} V_{out} - \dot{m}_{air} V_{in}$$

$$\frac{T}{\dot{m}_{air}} = 1.04 \ V_{out} - V_{in}$$

$$V_{out} = \left(\frac{T}{\dot{m}_{air}} + V_{in} \right) \div 1.04$$

$$= (400 + 250) \div 1.04$$

$$\boxed{V_{out} = 625 \ [m/s]}$$