Lecture 12
Turbines

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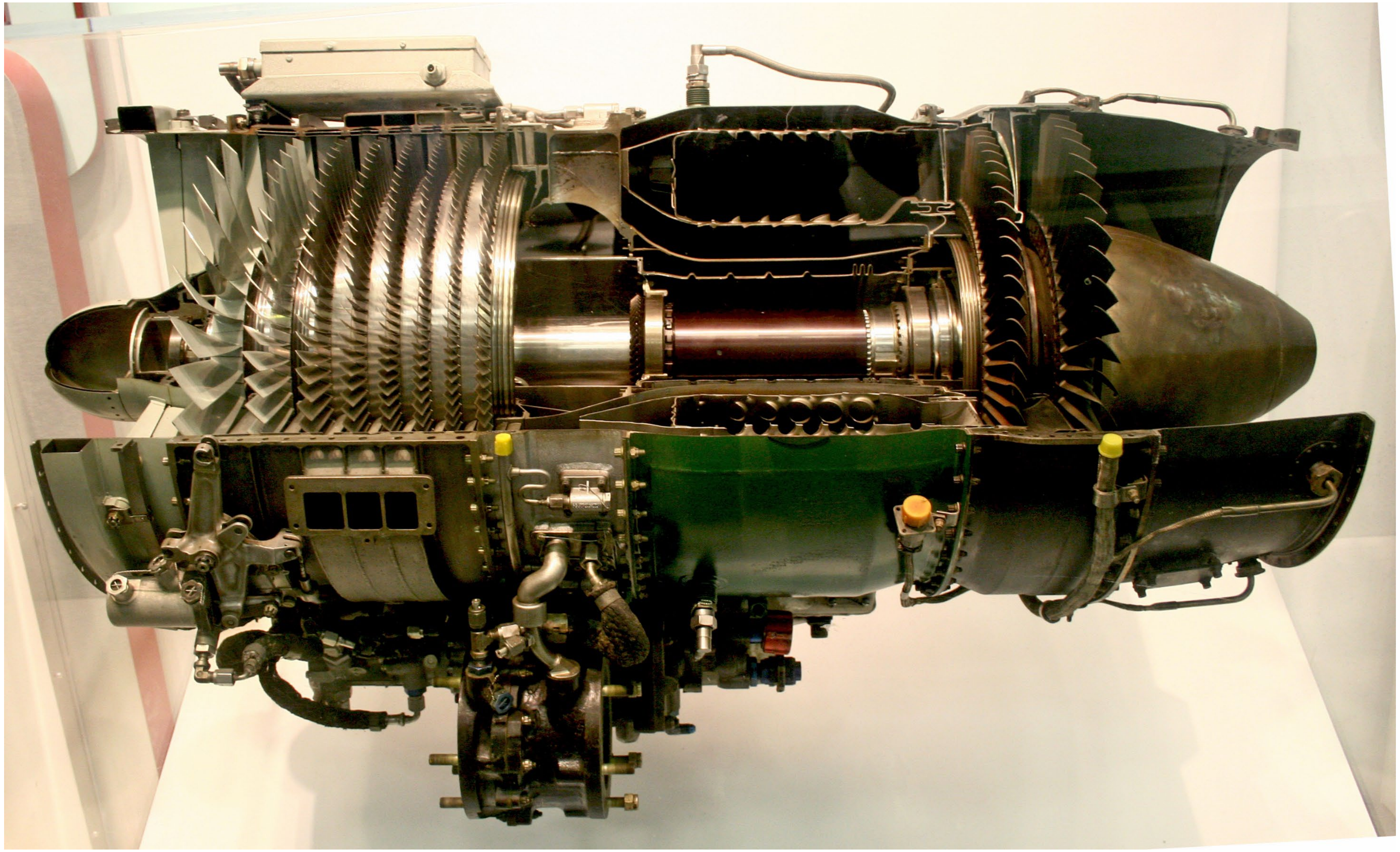
GE J85 Turbojet



Turbines – Axial Turbines

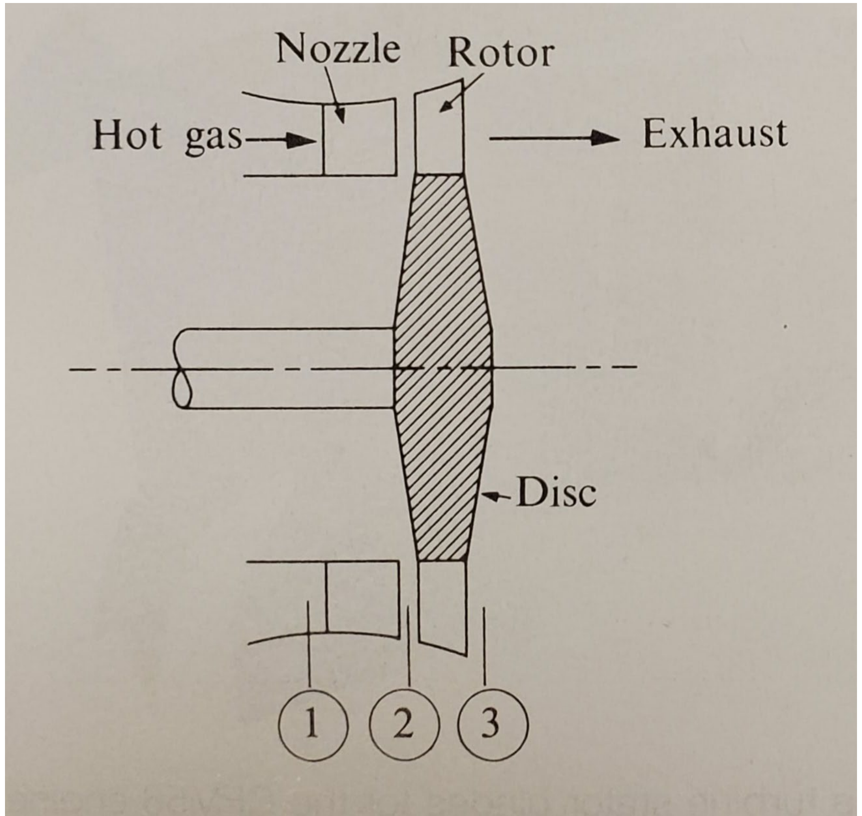


FIGURE 8.4 An axial turbine stage.

Axial turbine stage from Hill & Peterson [1]

Turbine stages have favorable pressure gradients allowing greater pressure change per stage. So, there are fewer turbine stages than compressor stages.

Shrouds at tips prevent leakage and minimize vibrations. At the high temperatures, we are more concerned about mechanical stress and vibration!

Single Turbine Stage

Minimal cross-sectional area at throat – flow from subsonic to supersonic

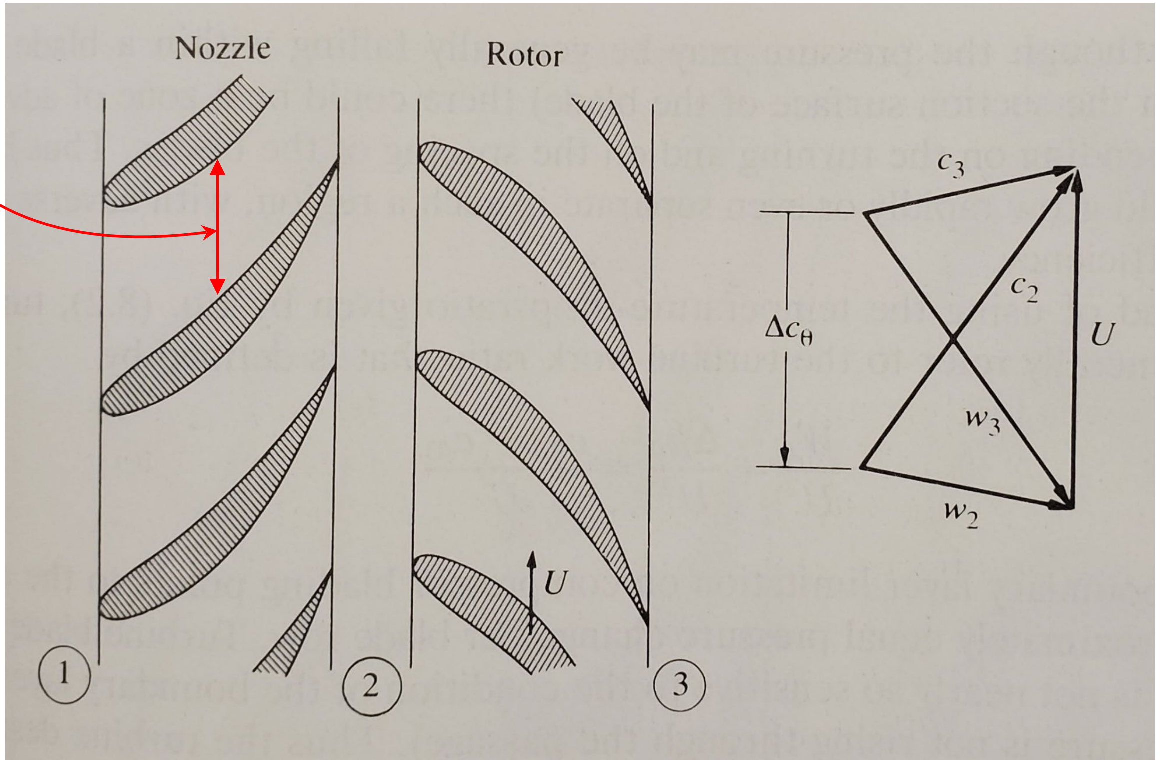


FIGURE 8.5 Turbine blading and velocity triangles.

Turbine stage from Hill & Peterson [1]

The net force on the rotor is in the direction of rotation; thus, the flow does work on the rotor. The diagram has a cylindrical surface rolled out on a plane. Vertical is the tangential direction of rotation. Horizontal is axial in the turbine, i.e., the main flow direction.

Power output of turbine

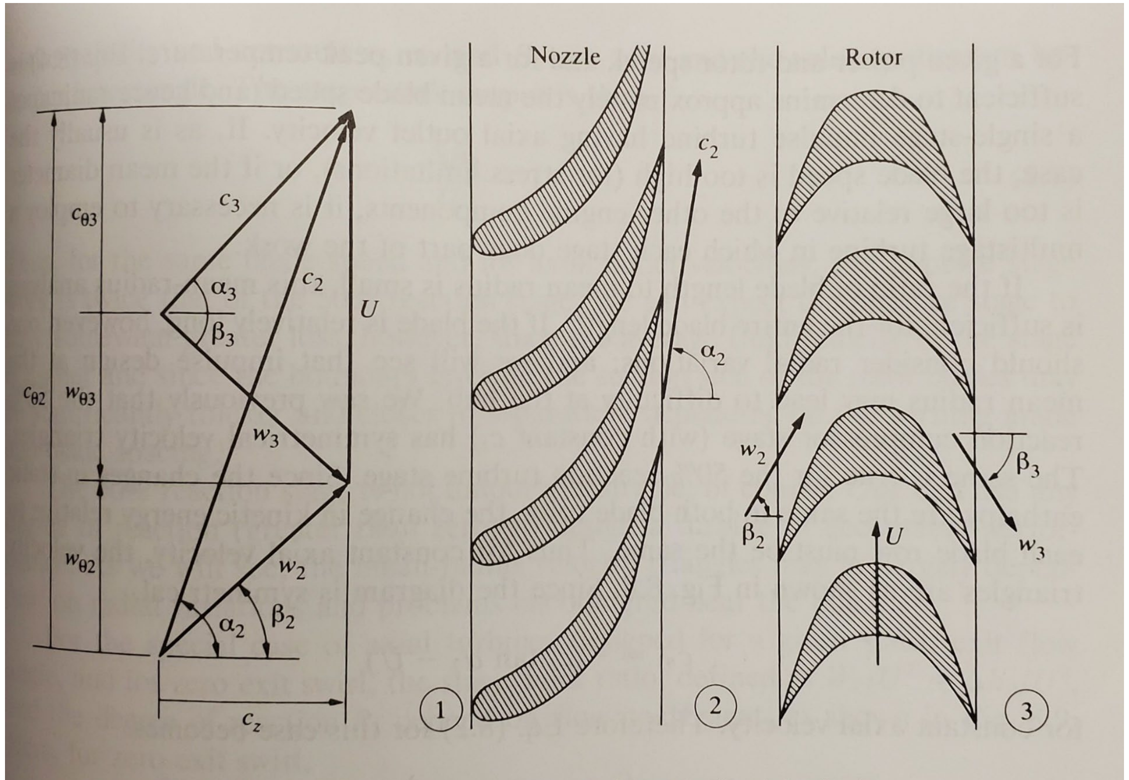


FIGURE 8.7 Impulse turbine stage and constant axial velocity.

Impulse turbine stage from Hill & Peterson [1]

$$Power = Force \times Velocity$$

$$Power = \frac{Rate\ of\ change\ of}{tangential\ momentum} \times Tangential\ velocity$$

$$P = \dot{m}U\left(c_{t_2} - c_{t_3}\right) = \dot{m}\left(h_2^{\circ} - h_3^{\circ}\right)$$

Stagnation enthalpy is measured in non-rotating frame.

Impulse stage: The entire pressure drop & enthalpy drop are in the nozzle (stator). The rotor maintains pressure & static enthalpy but turns the flow.

Power output of zero-reaction turbine

$$h_2 + \frac{(w_2)^2}{2} = h_3 + \frac{(w_3)^2}{2}$$

Kinetic energy increases through the nozzle but remains constant in the rotor. Angular momentum changes in the rotor.

The degree of reaction is defined as the fraction of enthalpy drop in the rotor.

Consider the turbine stage results in a final axial velocity only.

$$c_{3_t} = 0, \;\; w_{3_t} = -U$$

(1) Impulse stage – this is a zero-reaction machine

$$w_{2_t} = -w_{3_t} = U, \;\; c_{2_t} = 2U$$

$$w_{2_a} = w_{3_a}$$

$$w_{2_t} = -w_{3_t}$$

$$w_2^2 = w_3^2$$

$$h^o{}_1 = h^o{}_2$$

$$h_2 = h_3$$

Power output of zero-reaction turbine

Across the stator nozzle: $$h_2 + \frac{(2U)^2}{2} + \frac{\left(c_{2a}\right)^2}{2} = h_1 + \frac{\left(c_{1a}\right)^2}{2}$$

With: $c_{2a} \approx c_{1a}$, $$h_1 - h_2 = 2U^2 + \frac{\left(c_{2a}\right)^2 - \left(c_{1a}\right)^2}{2}$$

$h_3 = h_2$ So: $$h_1 - h_3 = 2U^2 + \frac{\left(c_{2a}\right)^2 - \left(c_{1a}\right)^2}{2}$$

In the non-rotating frame: $$h_1^\circ = h_1 + \frac{\left(c_{1a}\right)^2}{2}, \quad h_3^\circ = h_3 + \frac{\left(c_{3a}\right)^2}{2}$$ $\longleftarrow$ $c_{3a} \approx c_{1a}$

$$h_3^\circ - h_1^\circ \approx h_3 - h_1 = -2U^2 = h_3^\circ - h_2^\circ$$

$$\frac{h_3^\circ - h_2^\circ}{h_2^\circ} = \frac{-2U^2}{h_2^\circ}$$

Power output of 50% reaction turbine

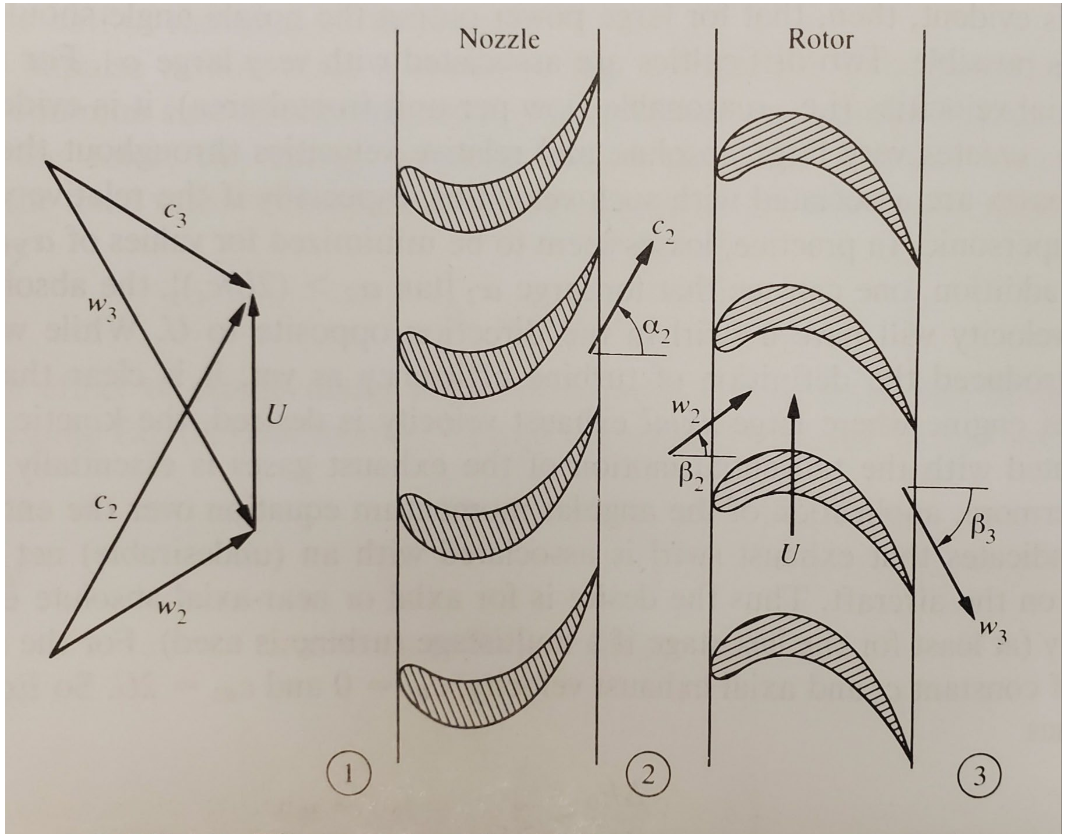


FIGURE 8.8 Fifty-percent reaction blading and constant axial velocity.

50% reaction turbine stage from Hill & Peterson [1]

(2) Examine 50% reaction core

$$h_2 - h_1 = h_3 - h_2$$

$$\frac{(c_{1_t})^2 - (c_{2_t})^2}{2} = \frac{(w_{2_t})^2 - (w_{3_t})^2}{2} = \frac{(c_{2_t} - U)^2 - U^2}{2}$$

Consider: $c_{1_t} = 0$

Then: $-(c_{2_t})^2 = (c_{2_t} - U)^2 - U^2 = (c_{2_t})^2 - 2Uc_{2_t}$

$$2(c_{2_t})^2 = 2Uc_{2_t} \Rightarrow \quad c_{2_t} = U, \;\; w_{2_t} = 0$$

Power output of 50% reaction turbine

$$\frac{\Delta h^{\circ}}{h_2^{\circ}} = \frac{h_3^{\circ} - h_2^{\circ}}{h_2^{\circ}} = \left[h_3 + \underbrace{\frac{(c_{3_t})^2}{2}}_{=0} + \cancel{\frac{(c_{3_a})^2}{2}} - \left(h_2 + \frac{(c_{2_t})^2}{2} + \cancel{\frac{(c_{2_a})^2}{2}}\right)\right]/h_2^{\circ} \quad = \left[h_3 - h_2 - \frac{U^2}{2}\right]/h_2^{\circ}$$

Also: $\Delta h^{\circ} = Blade\ velocity \times Change\ in\ tangential\ flow\ velocity$

$$U(-U) = -U^2$$

$$h_3 - h_2 - \frac{U^2}{2} = -U^2 \quad \Rightarrow \quad h_3 - h_2 = -\frac{U^2}{2} \quad \Rightarrow \quad h_2 - h_1 = h_3 - h_2 = -\frac{U^2}{2}$$

$$h_3 - h_1 = h_2 - h_1 + h_3 - h_2 = -U^2$$ Which is energy (max) removed

So, the impulse stage does twice as much work as the 50% reaction stage, however, the higher velocities of the impulse stage can result in greater losses!

Matching of Turbine and Compressor

(1) Select operating speed N (rpm)

(2) Select turbine inlet temperature $T_4^\circ \approx T_4$

(3) Select pressure ratio for compressor P_3° / P_2°

(4) Determine turbine and compressor efficiencies

Polytropic efficiencies e_t and e_c

(5) Calculate compressor work per unit mass

$$H_c = h_3^\circ - h_2^\circ = c_p T_2^\circ \left(\frac{T_3^\circ}{T_2^\circ} - 1 \right) = c_p T_2^\circ \left[\left(\frac{P_3^\circ}{P_2^\circ} \right)^{\frac{\gamma_d - 1}{\gamma_d e_c}} - 1 \right]$$

(6) Calculate T_5° and turbine pressure ratio that are required

$$\dot{m}_{comp} H_c = \left(\dot{m}_{comp} - \dot{m}_{bleed} + \dot{m}_{fuel} \right) H_t = \dot{m}_f (1 + \mu) H_t$$

Where: $H_t = h_4^\circ - h_5^\circ = c_p T_4^\circ \left(1 - \frac{T_5^\circ}{T_4^\circ} \right)$

Matching of Turbine and Compressor

$$\mu \equiv \frac{\dot{m}_{comp} - \dot{m}_{bleed}}{\dot{m}_f} = \frac{\dot{m}_{air-into-combustor}}{\dot{m}_{fuel}}$$

$$H_t = \frac{\mu + \dot{m}_{bleed}/\dot{m}_f}{\mu + 1} H_c = c_p T_4^{\circ} \left[1 - \left(\frac{P_5^{\circ}}{P_4^{\circ}} \right)^{\frac{(\gamma_n - 1) e_t}{\gamma_n}} \right]$$

If we have an afterburner or mixing with bypass air, $\gamma_t \neq \gamma_n$

$$\left(\frac{P_5^{\circ}}{P_4^{\circ}} \right)^{\frac{(\gamma_n - 1) e_t}{\gamma_n}} = \frac{T_5^{\circ}}{T_4^{\circ}}$$

So now, with the knowledge of $H_c, T_4^{\circ}, \mu, \dot{m}_{bleed}/\dot{m}_f, e_t$, we obtain temperature and pressure at station 5

Matching of Turbine and Compressor

(7) Match the allowable mass flow through the combustor and turbine i.e. determine throat area in turbine stator because of choked flow in turbine stator nozzle.

$$\frac{\dot{m}_{turbine}\sqrt{T_4^\circ}}{P_4^\circ} = \Gamma(\gamma_t) A^* / \sqrt{R}$$

Sum of all throat area in all stator nozzles

$$\frac{\dot{m}_{turbine}}{\dot{m}_{comp}} = \frac{\dot{m}_{comp} - \dot{m}_{bleed} + \dot{m}_f}{\dot{m}_{air} + \dot{m}_{bleed}} = \frac{\mu + 1}{\mu + \frac{\dot{m}_{bleed}}{\dot{m}_{fuel}}}$$

So: $$\frac{\dot{m}_{comp}\sqrt{T_4^\circ}}{P_4^\circ} \frac{\mu + 1}{\mu + \frac{\dot{m}_{bleed}}{\dot{m}_f}} = \Gamma(\gamma_t) \frac{A^*}{\sqrt{R}}$$

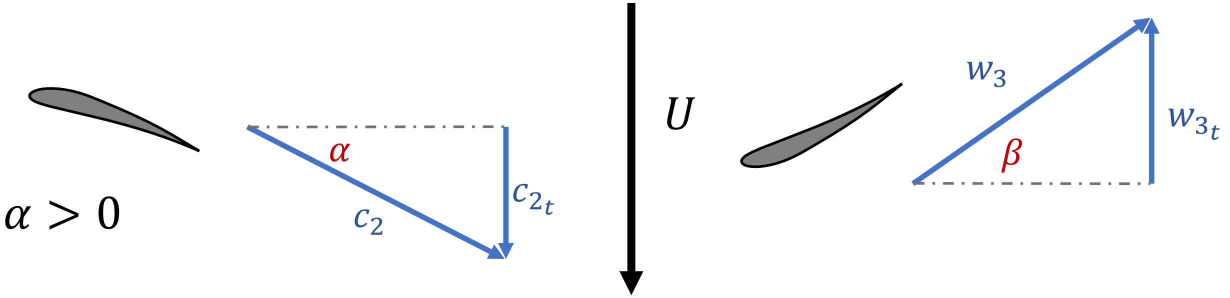
For a given mass flow, this can determine A^* for stator nozzle!

Matching of Turbine and Compressor

More on determination of T_5°

$$c_p\left(T_5^\circ - T_4^\circ\right) = U\left(c_{3_t} - c_{2_t}\right) \; < 0$$

Stator ***Rotor***



$\alpha > 0$

Assume: $c_{2_a} = c_{3_a}$

$$c_{2_t} = c_{2_a}\tan(\alpha) = -c_{2_a}\tan(|\alpha|)$$

$$c_{2_a} = c_{3_a} = w_a$$

$$w_{3_t} = w_{3_a}\tan(\beta) = c_{3_a}\tan(\beta) = c_{2_a}\tan(\beta)$$

$$c_{3_t} = w_{3_t} - U = c_{2_a}\tan(\beta) - U$$

$$\Delta c_t = c_{3_t} - c_{2_t} = c_{2_a}\tan(\beta) - U + c_{2_a}\tan(|\alpha|)$$

$$\Delta c_t = \underbrace{c_{2_a}[\tan(\beta) + \tan(|\alpha|)]}_{\text{Constant}} - U = K - U$$

$K > 0$

Matching of Turbine and Compressor

$$\frac{c_p\left(T_5^\circ - T_4^\circ\right)}{T_4^\circ} = \frac{-U(K-U)}{T_4^\circ}$$

$$c_p\left(\frac{T_5^\circ}{T_4^\circ} - 1\right) = \frac{-U}{\sqrt{T_4^\circ}}\left(\frac{K}{\sqrt{T_4^\circ}} - \frac{U}{\sqrt{T_4^\circ}}\right)$$

$$\frac{T_5^\circ}{T_4^\circ} = 1 - \frac{1}{c_p}\frac{K}{\sqrt{T_4^\circ}}\frac{U}{\sqrt{T_4^\circ}} + \left(\frac{U}{\sqrt{T_4^\circ}}\right)^2\frac{1}{c_p} \qquad U = \ constant \ \ x\,N(rpm)$$

So: $\dfrac{T_5^\circ}{T_4^\circ}$ Is determined by $\dfrac{N}{\sqrt{T_4^\circ}}$ and $\dfrac{c_{2a}}{\sqrt{T_4^\circ}}$ which have been chosen.

Matching of Turbine and Compressor

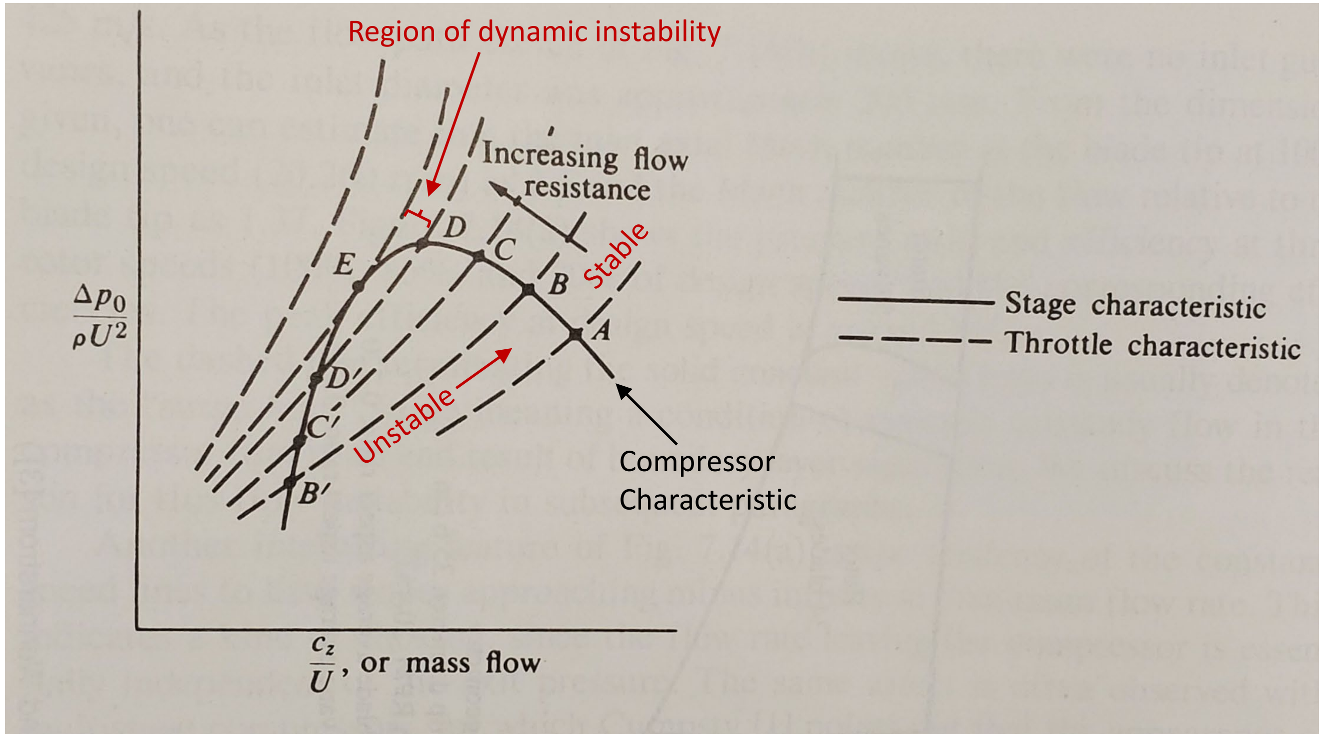


FIGURE 7.16 Pressure-rise characteristic compared with throttle characteristic of downstream resistance.

Pressure-rise and throttle characteristic from Hill & Peterson [1]

Each dashed line is for a fixed throttle setting

There is an optimum mass flow for pressure rise

Increasing the mass flow causes increase in velocity and affects the angle of attack of the blades.

Matching of Turbine and Compressor

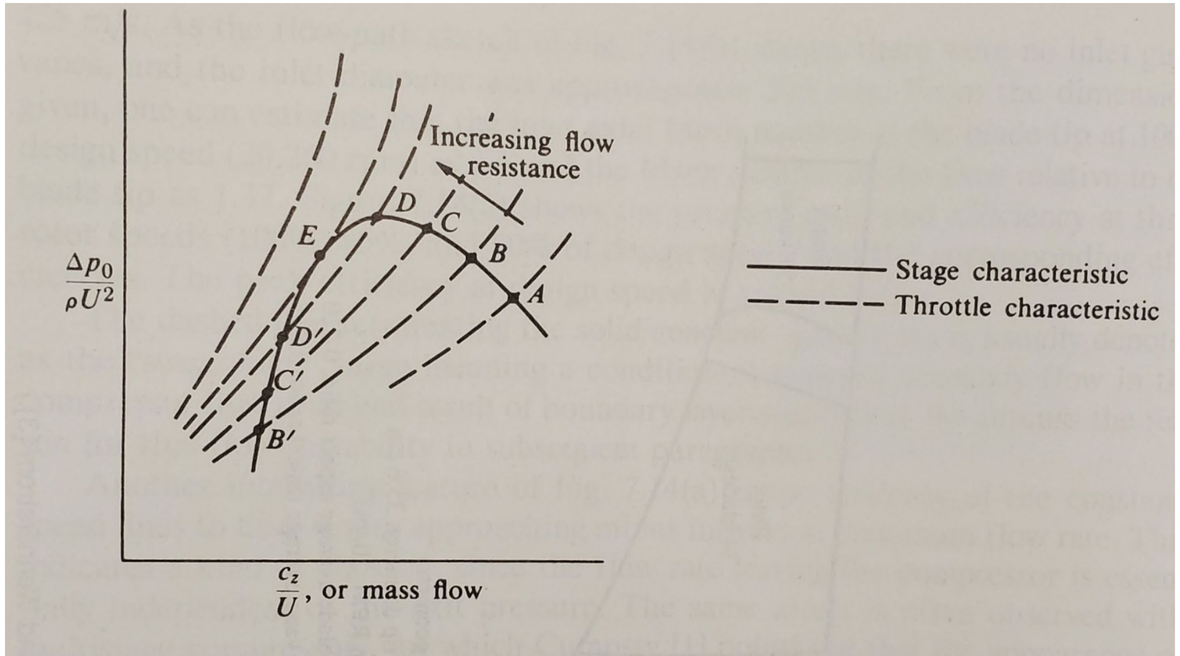


FIGURE 7.16 Pressure-rise characteristic compared with throttle characteristic of downstream resistance.

Stable at point B, C, or D – increase in mass flow for fixed throttle setting (dashed line) requires pressure rise; however, compressor characteristic results in drop in pressure so mass flow decreases – returns to the original point.

Unstable at point B′, C ‘ or D′– increase in mass flow for fixed throttle setting requires pressure rise; the compressor produces even greater pressure rise causing further mass flow increase until stable point at B, C, or D is reached. It could also go the other way towards zero with decrease in mass flow at point B. Point E is neutrally stable.

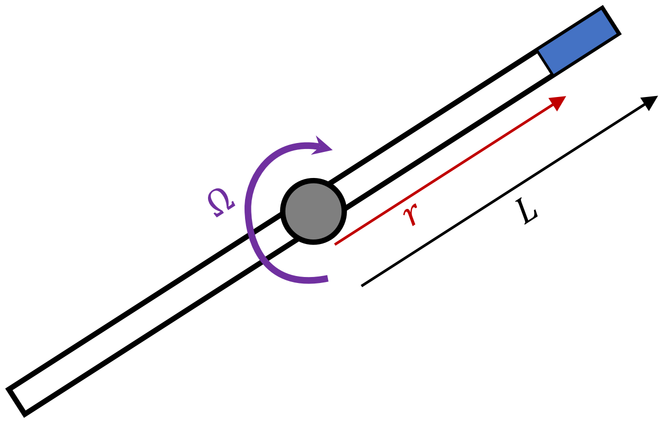
If the throttle is shut down beyond tangent line, the mass flow can only be maintained with a larger pressure rise than the compressor can provide. Therefore, mass flow drops drastically and sometimes flow even reverses. Then the compressor starts again and cyclic behavior occurs.

Blade speed limitations due to stresses

Types of stresses:
- Centrifugal – dominant
- Thermal – especially present in turbine
- Bending – due to aerodynamic forces

Bending stresses due to aerodynamic forces can be compensated by tilting blade out of the plane to produce a centrifugal force to return blade to the plane!

Generally, the centrifugal forces are dominate!



$$F = m\frac{v^2}{R} = m\Omega^2 R$$

m = mass of rotating section between r and L

Tip speed $U = \Omega L$ $\quad R = \frac{L+r}{2}$ $\quad m = \rho A(L-r)$

$$\sigma = \frac{F}{A} = \frac{m}{A}\Omega^2\left(\frac{L+r}{2}\right) \qquad \sigma = \frac{\rho A \Omega^2}{2A}(L^2 - r^2) = \frac{\rho U_{tip}^2}{2}\left(1 - \frac{r^2}{L^2}\right)$$

Stress σ increases as the square of tip velocity – This limit the tip velocity and rpm

References

[1] Hill, Philip G., and Carl R. Peterson. *Mechanics and Thermodynamics of Propulsion*. Reading, Mass: Addison-Wesley Longman, 1992.