Lecture 10
Ramjet Analysis

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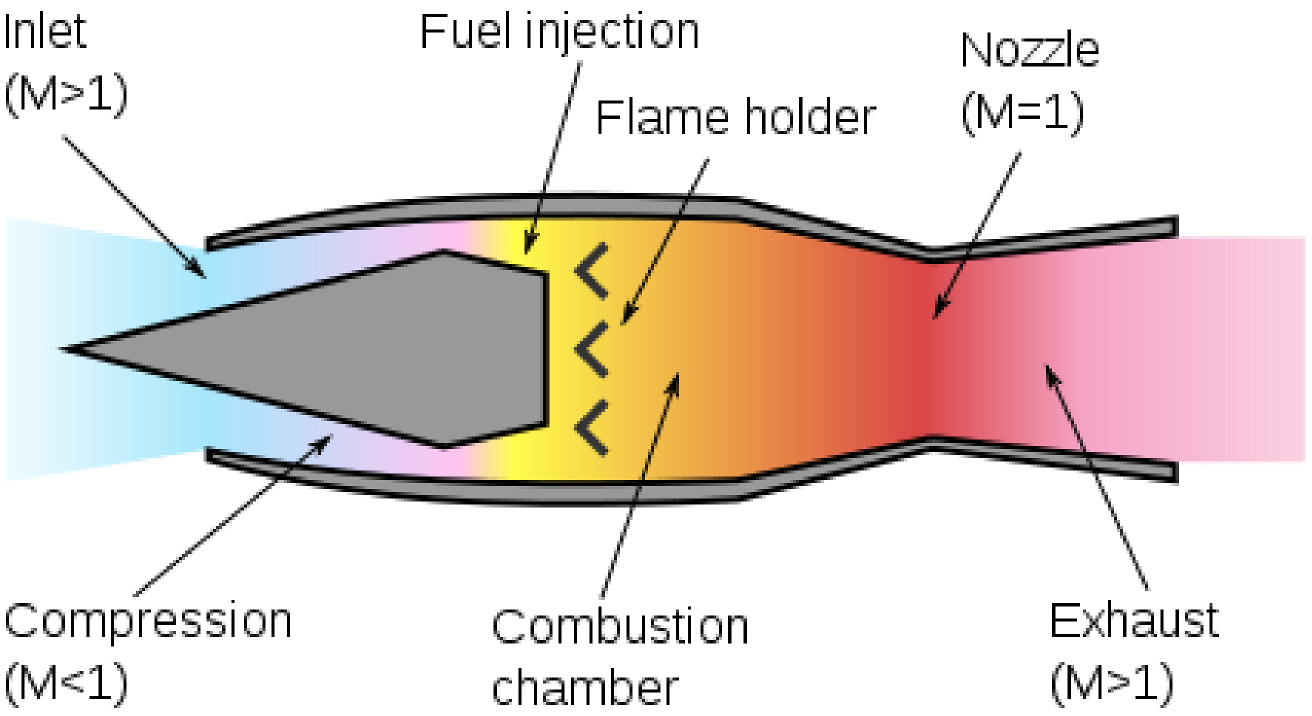
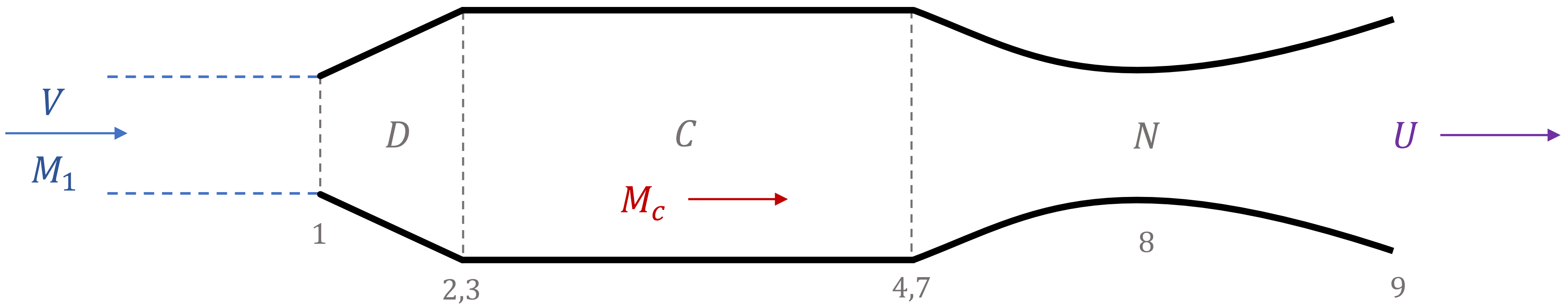
Ramjet



High flight Mach, no need of compressor and turbine, light, simple,
But need assist for taking off

Analysis of Ramjet Flow

Subscript n for nozzle ;
Subscript d for diffuser or air intake



$$c_p T^\circ = c_p T + \frac{1}{2} u^2 = Constant \quad \rightarrow \quad U^2 = 2 c_{p_n} T_4^\circ \left[1 - \left(\frac{P_9}{P_7^\circ} \right)^{\frac{\gamma_n - 1}{\gamma_n} e_n} \right]$$

$$\frac{P_4^\circ}{P_3^\circ} = 1 - \frac{\gamma_c}{2} M_c^2 \frac{\Delta T^\circ}{T_{34}^\circ} = 1 - C M_c^2$$

Where: $C = \frac{\gamma_c}{2} \frac{\Delta T^\circ}{T_{34}^\circ}$

$$\frac{P_9}{P_7^\circ} = \frac{P_9}{P_4^\circ} = \frac{P_9}{P_3^\circ} \frac{P_3^\circ}{P_4^\circ} = \frac{P_9}{P_3^\circ} (1 - C M_c^2)^{-1} \quad = \quad \frac{P_1}{P_3^\circ} \frac{P_9}{P_1} (1 - C M_c^2)^{-1}$$

Analysis of Ramjet Flow

Subscripts: d for diffuser, n for nozzle, M_c is Mach number through the combustor. The energy equation (given below in two forms) is fundamental to the analysis here.

$$c_p T^\circ = c_p T + \frac{1}{2}u^2 = Constant$$

$$\frac{T^\circ}{T} = 1 + \frac{\gamma - 1}{2}M^2$$

$$U^2 = 2c_{p_n}T_4^\circ \left[1 - \left(\frac{P_1}{P_3^\circ}\right)^{\frac{\gamma_n}{\gamma_n - 1}e_n} \left(\frac{P_9}{P_1}\right)^{\frac{\gamma_n}{\gamma_n - 1}e_n} (1 - CM_c^2)^{-\frac{\gamma_n}{\gamma_n - 1}e_n}\right]$$

$$\frac{P_1}{P_3^\circ} = \frac{P_1}{P_2}\frac{P_3}{P_3^\circ} = \frac{P_1}{P_2}\left[\frac{1}{1 + \frac{\gamma_d - 1}{2}M_c^2}\right]^{\frac{\gamma_d}{\gamma_d - 1}}$$

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma_d}{\gamma_d - 1}e_d} = \left[\frac{1 + \frac{\gamma_d - 1}{2}M_c^2}{1 + \frac{\gamma_d - 1}{2}M_1^2}\right]^{\frac{\gamma_d}{\gamma_d - 1}e_d}$$

So:

$$\frac{P_1}{P_3^\circ} = \frac{\left[1 + \frac{\gamma_d - 1}{2}M_c^2\right]^{\frac{\gamma_d}{\gamma_d - 1}e_d}}{\left[1 + \frac{\gamma_d - 1}{2}M_1^2\right]^{\frac{\gamma_d}{\gamma_d - 1}e_d}\left[1 + \frac{\gamma_d - 1}{2}M_c^2\right]^{\frac{\gamma_d}{\gamma_d - 1}}}$$

Analysis of Ramjet Flow

$$\left(\frac{P_1}{P_3^\circ}\right)^{\frac{\gamma_n-1}{\gamma_n}e_n} = \frac{\left[1+\frac{\gamma_d-1}{2}M_c^2\right]^{\left(\frac{\gamma_d}{\gamma_d-1}\right)\left(\frac{\gamma_n-1}{\gamma_n}\right)e_n(e_d-1)}}{\left[1+\frac{\gamma_d-1}{2}M_1^2\right]^{\left(\frac{\gamma_d}{\gamma_d-1}\right)\left(\frac{\gamma_n-1}{\gamma_n}\right)e_n e_d}}$$

Note for $M_c \ll 1$: $$\left(\frac{P_1}{P_3^\circ}\right)^{\frac{\gamma_n-1}{\gamma_n}e_n} \approx \left[1+\frac{\gamma_d-1}{2}M_1^2\right]^{-\left(\frac{\gamma_d}{\gamma_d-1}\right)\left(\frac{\gamma_n-1}{\gamma_n}\right)e_n e_d}$$

$$U^2 = 2c_{p_n}T_4^\circ\left[1-\frac{\left[1+\frac{\gamma_d-1}{2}M_c^2\right]^{\left(\frac{\gamma_d}{\gamma_d-1}\right)\left(\frac{\gamma_n-1}{\gamma_n}\right)e_n(e_d-1)}}{\left[1+\frac{\gamma_d-1}{2}M_1^2\right]^{\left(\frac{\gamma_d}{\gamma_d-1}\right)\left(\frac{\gamma_n-1}{\gamma_n}\right)e_n e_d}}\left(\frac{P_9}{P_1}\right)^{\frac{\gamma_n}{\gamma_n-1}e_n}\left(1-CM_c^2\right)^{-\frac{\gamma_n}{\gamma_n-1}e_n}\right]$$

Analysis of Ramjet Flow

For $M_c^2 \ll 1$: $$\boxed{U^2 \approx 2c_{p_n} T_4^{\circ} \left[1 - \left(1 + \frac{\gamma_d - 1}{2} M_1^2\right)^{-\left(\frac{\gamma_d}{\gamma_d - 1}\right)\left(\frac{\gamma_n - 1}{\gamma_n}\right) e_n e_d}\right]}$$

u increases with T_4° and increases with M_1^2!

Analysis of Ramjet Flow

Note that the ramjet cannot start from zero velocity. It must be launched from rocket or an aircraft!

Consider thrust: $T = (\dot{m}_a + \dot{m}_f)U - \dot{m}_a V + (p_e - p_a)A_e = \dot{m}_4 U - \dot{m}_1 V + (p_9 - p_a)A_e$

For perfect expansion: $T = \dot{m}_4 U - \dot{m}_1 V$

Consider mass mixture ratio: $\mu = \dfrac{\dot{m}_a}{\dot{m}_f} = \dfrac{\dot{m}_1}{\dot{m}_f} = A/F$

$\dot{m}_4 = (1+\mu)\dot{m}_f$ When $\mu \gg 1$: $\dot{m}_4 \approx \mu \dot{m}_f$

$$\frac{T}{\dot{m}_f} = (1+\mu)U - \mu V$$

$$I = \frac{T}{\dot{m}_f g} = \frac{(1+\mu)U - \mu V}{g}$$

I is specific impulse.

Analysis of Ramjet Flow

$$I = \frac{(1+\mu)}{g}\sqrt{2c_{p_n}T_4^{\circ}\left\{1-\left(1+\frac{\gamma_d-1}{2}M_1^2\right)^{-\left(\frac{\gamma_n-1}{\gamma_n}\right)\left(\frac{\gamma_d}{\gamma_d-1}\right)e_n e_d}\right\}^{1/2}} - \frac{\mu}{g}M_1\sqrt{\gamma R T_1}$$

Specific Fuel Consumption: $S = \dfrac{3600g}{(1+\mu)U-\mu V} \approx \dfrac{3600g}{\mu(U-V)}$

Propulsive Efficiency: $\eta = \dfrac{TV}{\dot{m}_f Q} = \dfrac{(1+\mu)UV-\mu V^2}{Q} \approx \dfrac{\mu V(U-V)}{Q}$

Analysis of Ramjet Flow

Relation between mixture ratio and combustion chamber temperature

Energy equation: $\left(\dot{m}_a + \dot{m}_f\right)h_4^\circ = \dot{m}_a h_2^\circ + \dot{m}_f Q \eta_b$ η_b is burner efficiency or fraction burned

$$(1+\mu)h_4^\circ = \mu h_2^\circ + \eta_b Q \qquad \mu = \dot{m}_a / \dot{m}_f$$

Neglecting kinetic energy (low Mach number) ; $h_4^\circ \approx h_4$; $h_2^\circ \approx h_2$

$$(1+\mu)h\left(T_{4,products}\right) \approx \mu h\left(T_{2,air}\right) + \eta_b Q$$

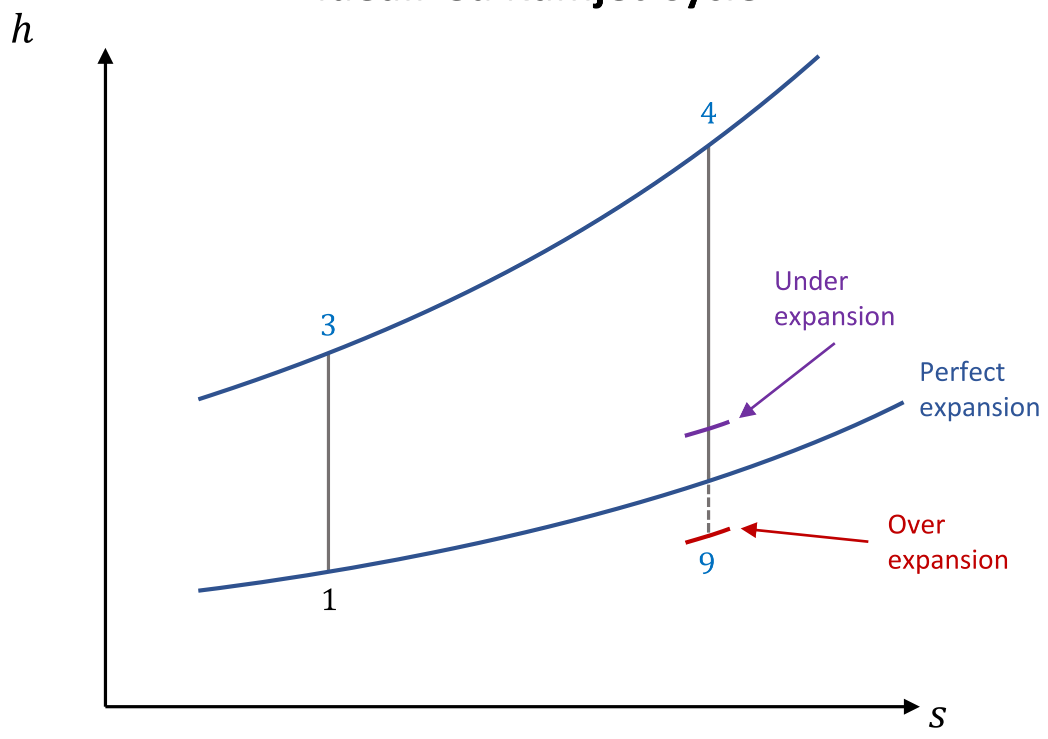
$$\mu\left[h\left(T_{4,products}\right) - h\left(T_{2,air}\right)\right] = \eta_b Q - h\left(T_{4,products}\right)$$

$$\mu = \frac{\eta_b Q - h\left(T_{4,products}\right)}{\left[h\left(T_{4,products}\right) - h\left(T_{2,air}\right)\right]} \qquad T_2 \approx T_2^\circ = T_1^\circ = T_1\left[1 + \frac{\gamma_d - 1}{2} M_1^2\right]$$

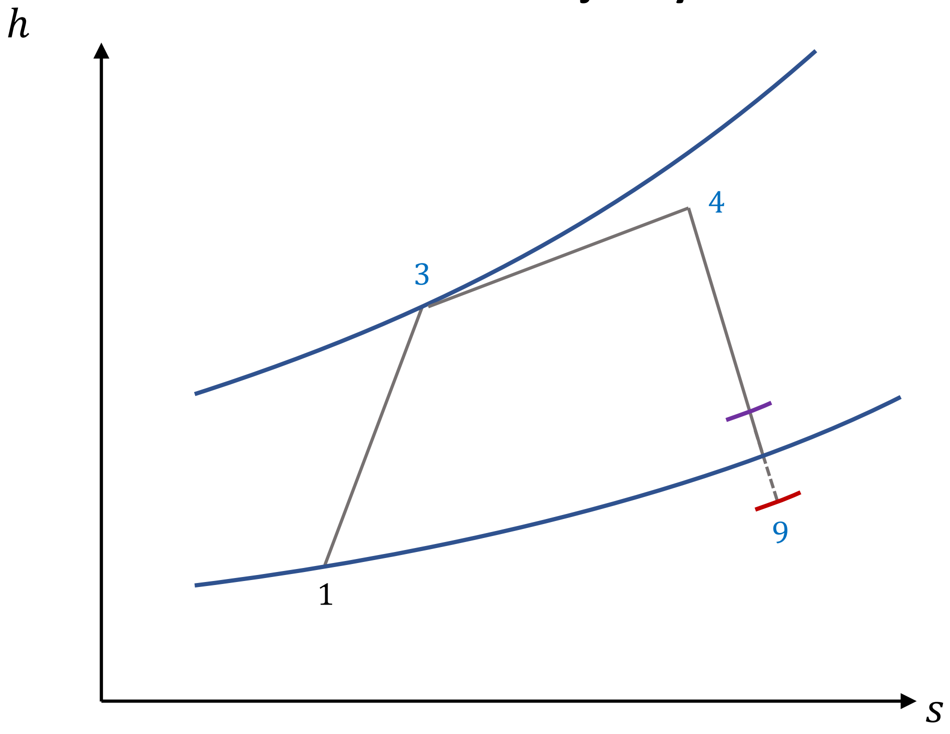
Analysis of Ramjet Flow

As $T_4 \uparrow, \mu \downarrow$ or as $\mu \uparrow, T_4 \downarrow$

Idealized Ramjet Cycle

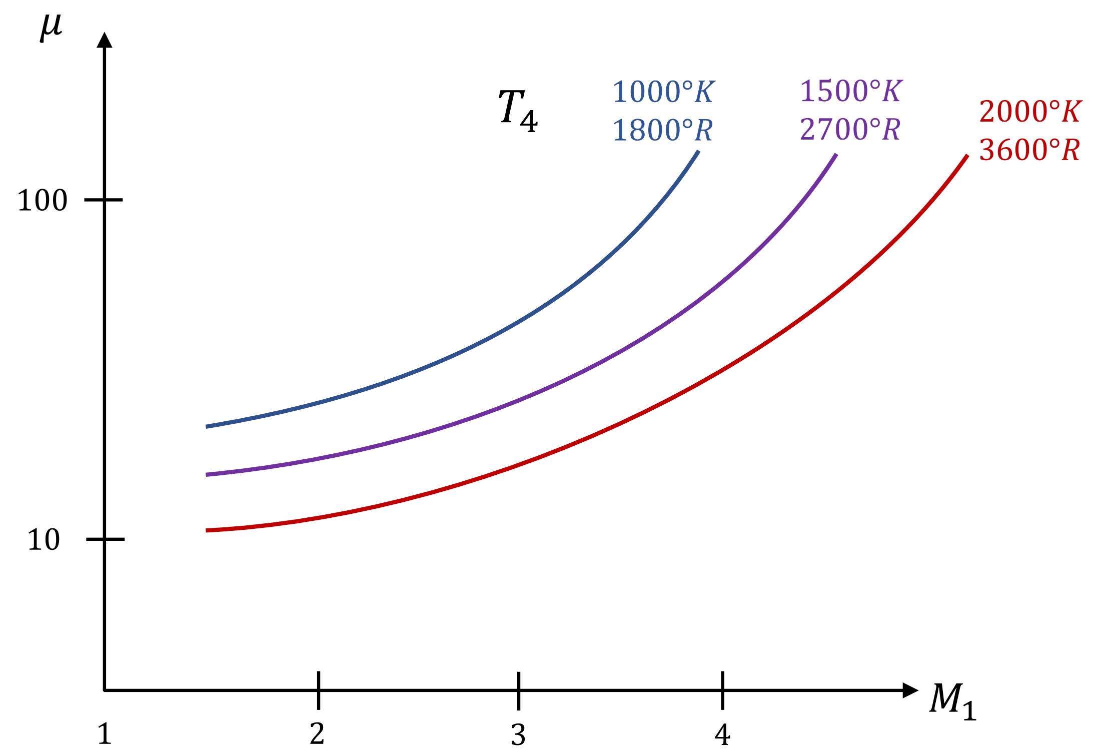


Practical Ramjet Cycle

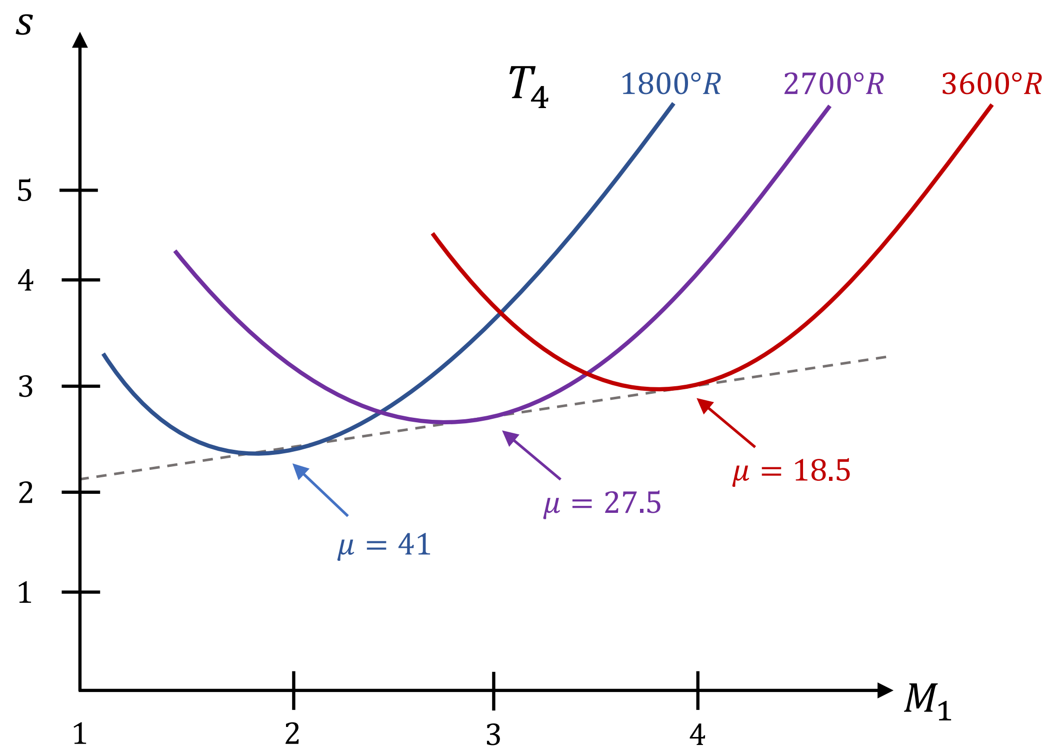


Analysis of Ramjet Flow

Performance characteristics of a Ramjet

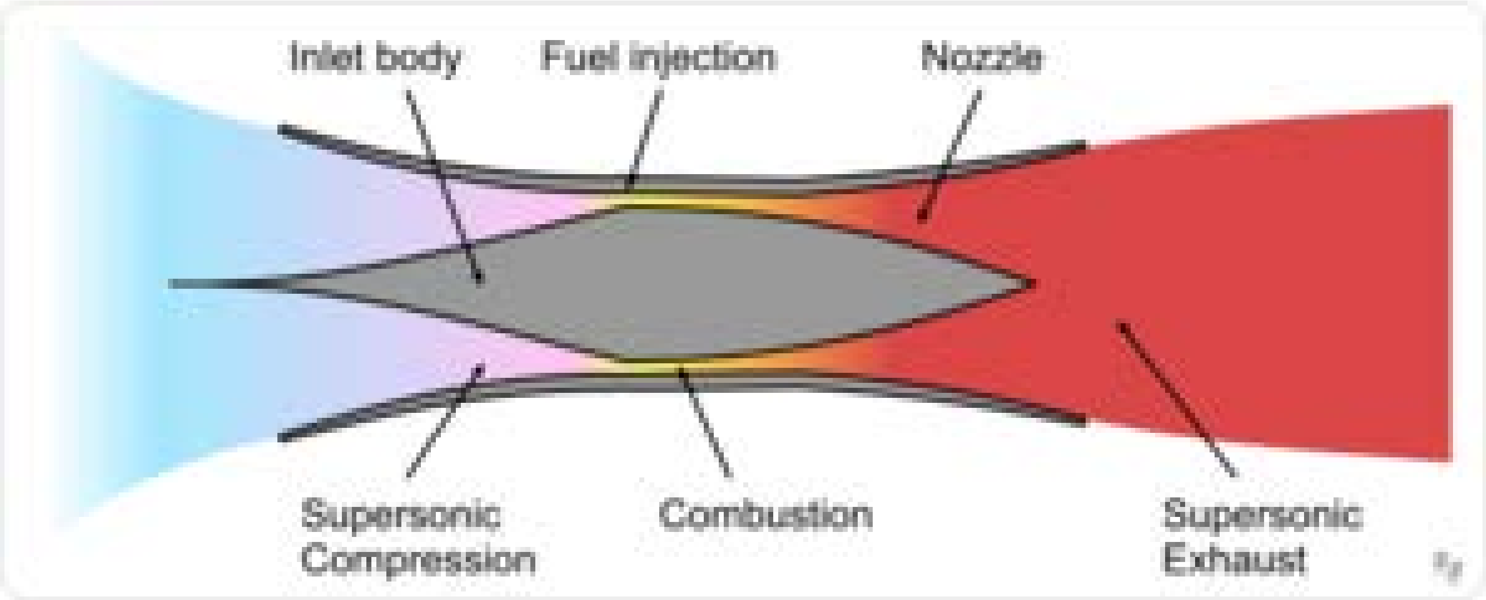


2-shock diffuser $T_1 = -56.5°C$



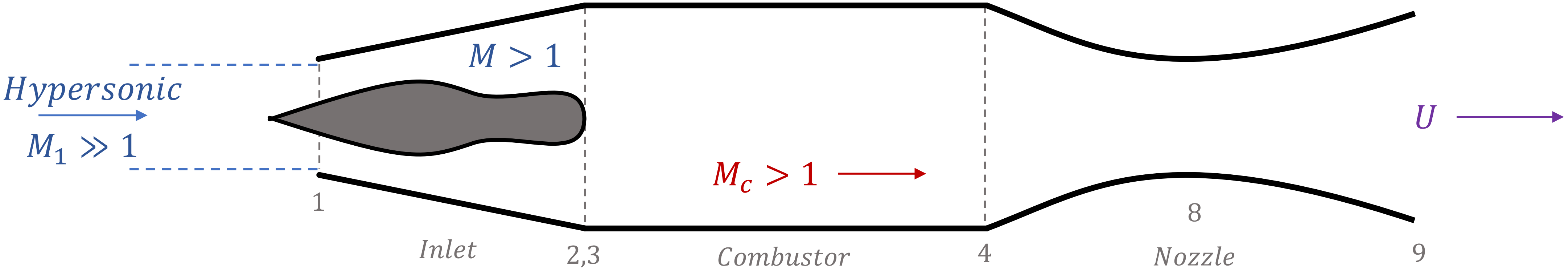
Low temperature operation

Scramjet (Supersonic-Combustion Ramjet)



Still Experimental and unmanned: e.g. X43A, X51, M>5
Technical difficulties for engine: mixing, ignition, combustion efficiency, particularly with conventional liquid fuel.
Technical difficulty with manned flight : cooling of aircraft

Analysis of a Scramjet



$$U^2 = 2c_{p_n} T_4^\circ \left[1 - \left(\frac{P_9}{P_4^\circ} \right)^{\frac{\gamma_n - 1}{\gamma_n} e_n} \right]$$

$$U^2 = 2c_{p_n} T_4^\circ \left[1 - \left\{ \left(\frac{P_9}{P_1} \right) \left(\frac{P_1}{P_2^\circ} \right) \left(\frac{P_2^\circ}{P_4^\circ} \right) \right\}^{\frac{\gamma_n - 1}{\gamma_n} e_n} \right]$$

Analysis of a Scramjet

Suppose $P_9 = P_1$ (perfect expansion):

$$\frac{P_1}{P_2^\circ} = \frac{1}{\left(1 + \frac{\gamma_d - 1}{2} M_1^2\right)^{\frac{\gamma_d}{\gamma_d - 1} e_d}}$$

$\frac{P_2^\circ}{P_4^\circ}$ Can be large due to shock losses.
Fuel is injected into high-speed stream causing shockwaves

$$\frac{P_4^\circ}{P_2^\circ} = 1 - \mathcal{O}(M_c^2)$$

Large!

$$T_2^\circ = T_2 \left(1 + \frac{\gamma_d - 1}{2} M_2^2\right) \quad \text{or} \quad T_2 = \frac{T_2^\circ}{\left(1 + \frac{\gamma_d - 1}{2} M_2^2\right)}$$

This approximation assumes $c_p = constant$ which is not exact; c_p increase due to vibrational excitation!

Analysis of a Scramjet

Energy balance: $(1+\mu)h_4^{\circ}\left(T_{4,products}^{\circ}\right) = \mu h_2^{\circ}\left(T_{2,air}^{\circ}\right) + \eta_b Q$

There will be substantial dissociation, but it is T_4 not T_4° that determines the amount of dissociation

Hypersonic ramjets could provide a re-useable launch vehicle. Rockets or a gas turbine engine would be required to gain motion as first, then the ramjet could be used to go to the outer reaches of the atmosphere. Rockets could then be used to move the vehicle beyond the atmosphere and for maneuvering. At this point, the lower stage Scramjet could be released from the vehicle and recovered. This would be less expensive since less oxidizer has to be carried onboard. Also, recovering the Scramjet would further reduce costs!