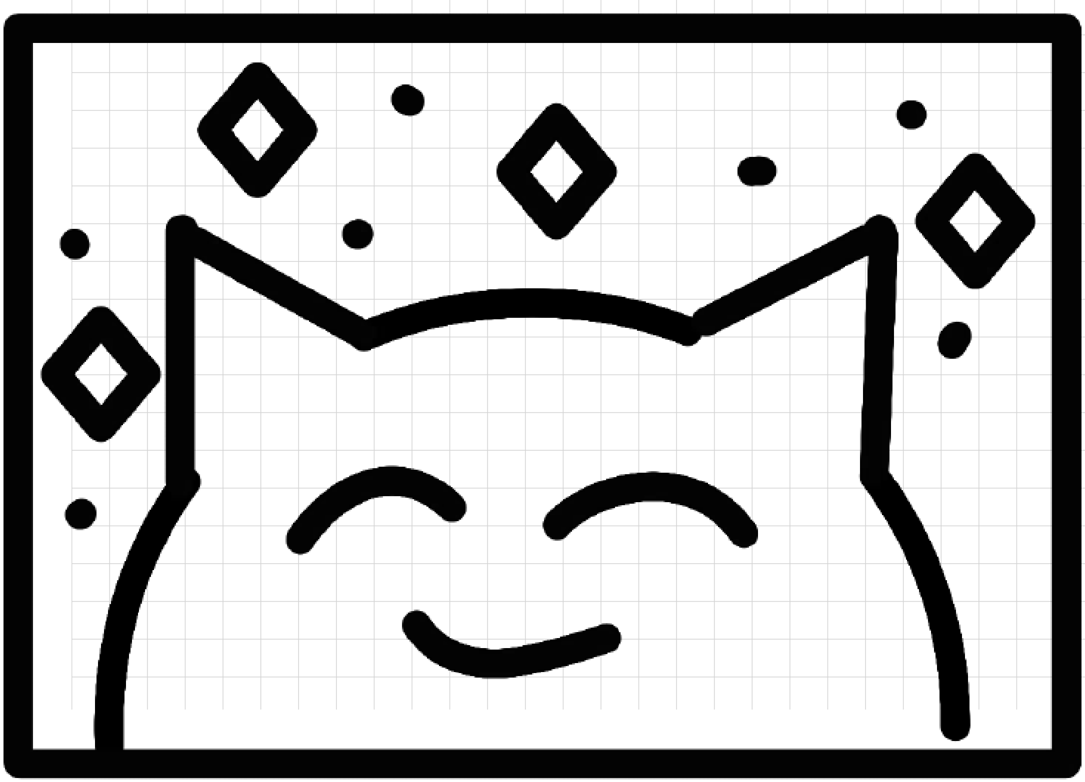
MAE 112

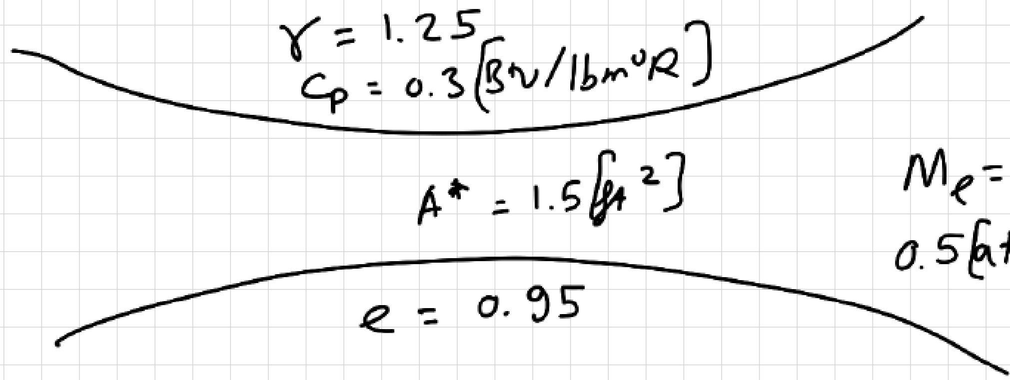
HW 3

TRIET



1. A rocket nozzle has initial pressure and temperature of fifty atmospheres and 5000°R with $\gamma = 1.25$; $c_p = 0.30$ Btu/1bm °R; and $A^* = 1.5\ ft^2$. The flow is slightly over-expanded to a Mach number $M_e = 3.5$ at the nozzle exit with the ambient pressure at 0.50 atmosphere. Assume 95% for nozzle polytropic efficiency. Calculate: (a) the characteristic velocity c*; (b) the mass flow; (c) nozzle exit pressure and cross-sectional area (*beware of tables and graphs constructed for air flow*); (d) nozzle exit velocity U; and (e) effective exhaust velocity c.

50 [atm]
5000 [°R]



$M_e = 3.5$
0.5 [atm]

ⓐ

$$1\,\text{Btu} = 778\ \text{ft}\cdot\text{lbf}$$

$$1\,\text{lbf} = 1\,\text{lbm} \times 32.2\ \frac{\text{ft}}{\text{s}^2}$$

$$c_p = 0.3\ \frac{\cancel{\text{Btu}}}{\cancel{\text{lbm}}\,°R} \times \frac{778\ \text{ft}\cdot\cancel{\text{lbf}}}{1\ \cancel{\text{Btu}}} \times \frac{1\ \cancel{\text{lbm}}\ 32.2\ \frac{\text{ft}}{\text{s}^2}}{1\ \cancel{\text{lbf}}}$$

$$= 7515.48 \left[\frac{\text{ft}^2}{\text{s}^2\,°R}\right]$$

$$R = c_p\,\frac{\gamma - 1}{\gamma} = 7515.48\ \frac{1.25 - 1}{1.25} = 1503 \left[\frac{\text{ft}^2}{\text{s}^2\ °R}\right]$$

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{(2-e)\gamma + e}{2e(\gamma-1)}}$$

$$= \sqrt{1.25}\left(\frac{2}{1.25+1}\right)^{\frac{(2-0.95)1.25+0.95}{2\times 0.95(1.25-1)}}$$

$$\Gamma = 0.638$$

$$c^* = \frac{\sqrt{RT_o}}{\Gamma} = \frac{\sqrt{1503 \times 5000}}{0.638} = 4296.79 \left[\frac{\text{ft}}{\text{s}}\right]$$

(b) $P_o = 50 \text{ atm} = 105811 \left[\text{lb}/\text{ft}^2\right]$

$$\dot{m} = \frac{P_o A^*}{C^*} = \frac{105811\left[\frac{\text{lb}}{\text{ft}^2}\right] \times 1.5\left[\text{ft}^2\right]}{4296.79\left[\frac{\text{ft}}{\text{s}}\right]} = 36.9384\left[\frac{\text{lb}}{\frac{\text{ft}}{\text{s}}}\right]$$

$$1 \text{ lbf} = 1 \text{lbm} \times 32.2 \ \frac{\text{ft}}{\text{s}^2}$$

$$1 \text{ lbf} = 1 \text{ slug} \times \frac{\text{ft}}{\text{s}^2}$$

$$\dot{m} = 36.9384 \frac{\cancel{\text{lbf}}}{\frac{\cancel{\text{ft}}}{\cancel{\text{s}}}} \times \frac{1 \text{lbm} \times 32.2 \ \frac{\cancel{\text{ft}}}{\text{s}^{\cancel{2}}}}{1 \ \cancel{\text{lbf}}} = 1189.4\left[\frac{\text{lbm}}{\text{s}}\right]$$

$$\dot{m} = 36.9384 \frac{\cancel{\text{lbf}}}{\frac{\cancel{\text{ft}}}{\cancel{\text{s}}}} \times \frac{1 \text{slug} \times \frac{\cancel{\text{ft}}}{\text{s}^{\cancel{2}}}}{1 \ \cancel{\text{lbf}}} = 36.9384\left[\frac{\text{slug}}{\text{s}}\right]$$

(c)

$$\frac{P_o}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{(\gamma-1)e}}$$

$$\frac{50\,[\text{atm}]}{P_e} = \left[1 + \frac{1.25 - 1}{2} \times 3.5^2\right]^{\frac{1.25}{(1.25-1)0.95}}$$

$$\boxed{P_e = 0.3768 \ [\text{atm}]}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{\frac{(2-e)\gamma + e}{2e(\gamma-1)}}$$

$$\frac{A_e}{1.5\,[\text{ft}^2]} = \frac{1}{3.5}\left[\frac{2}{1.25+1}\left(1 + \frac{1.25-1}{2} 3.5^2\right)\right]^{\frac{(2-0.95)1.25+0.95}{2\times0.95(1.25-1)}}$$

$$\boxed{A_e = 26.395 \ [\text{ft}^2]}$$

(d)

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{5000\,[°R]}{T_e} = 1 + \frac{1.25 - 1}{2} \times 3.5^2$$

$$T_e = 1975.3\ [°R]$$

$$\rho_e = \frac{P_e}{R\,T_e} = \frac{0.3768 \times 2116.2}{1503 \times 1975.3} = 2.6858 \times 10^{-4}\ \left[\frac{slug}{ft^3}\right]$$

$$\dot{m} = \rho_e A_e V_e$$

$$V_e = \frac{\dot{m}}{\rho_e A_e} = \frac{36.9384\left[\frac{\cancel{slug}}{s}\right]}{2.6858 \times 10^{-4}\left[\frac{\cancel{slug}}{ft^{\cancel{3}}}\right] 20.395\ [\cancel{ft^2}]}$$

$$\boxed{V_e = 6743.4\ [ft/s]}$$

$$V_e = \sqrt{2 c_p (T_0 - T_e)}$$

$$= \sqrt{2 \times 7515.48 \times (5000 - 1975.3)}$$

$$V_e = 6742.71\ [ft/s]$$

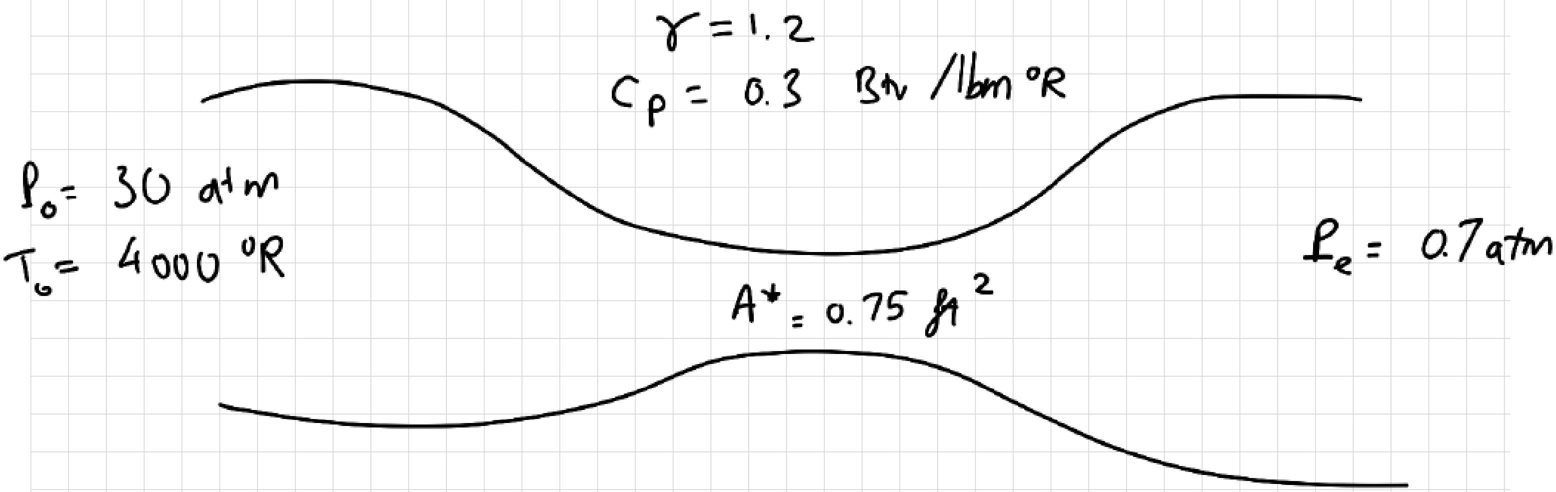
(e)

$$c = \frac{T}{\dot{m}} = \frac{\dot{m} V_e + (P_e - P_a) A_e}{\dot{m}}$$

$$= \frac{36.9384 \times 6743.4 + (0.3768 \times 2116.2 - 0.5 \times 2116.2)\,20.395}{36.9384}$$

$$c = 6600\ [ft/s]$$

2. Consider a nozzle with initial upstream entry pressure and temperature of thirty atmospheres and 4000°R. The value of $\gamma = 1.2$ and the value of $c_p = .30$ Btu/1bm °R. The throat area is 0.75 ft^2. The flow is perfectly expanded to the ambient pressure of 0.70 atmospheres. Calculate: (a) the mass flow, (b) the exhaust velocity, (c) the exit area, and (d) the thrust coefficient.



(a)

$$C_p = 0.3 \frac{\cancel{Btu}}{\cancel{lbm}\,°R} \times \frac{778\ ft \cdot \cancel{lbf}}{1\ \cancel{Btu}} \times \frac{1\ \cancel{lbm} \times 32.2\ \frac{ft}{s^2}}{1\ \cancel{lbf}}$$

$$C_p = 7515.48 \left[\frac{ft^2}{s^2\ °R}\right]$$

$$R = C_p \frac{\gamma - 1}{\gamma} = 7515.48 \frac{1.2 - 1}{1.2} = 1252.58 \left[\frac{ft^2}{s^2\ °R}\right]$$

$$\dot{m} = \frac{P_0 A^*}{\sqrt{R T_0}} \sqrt{\gamma} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\dot{m} = \frac{30 \times 2116.2 \times 0.75}{\sqrt{1252.58 \times 4000}} \sqrt{1.2} \left(\frac{2}{1.2+1}\right)^{\frac{1.2+1}{2(1.2-1)}}$$

$$\boxed{\dot{m} = 13.7955 \left[\frac{slug}{s}\right]} = 444.215 \left[\frac{lbm}{s}\right]$$

(b)

At the exit:

$$\frac{P_0}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{30 \, \cancel{atm}}{0.7 \, \cancel{atm}} = \left[1 + \frac{1.2 - 1}{2} M^2\right]^{\frac{1.2}{1.2 - 1}}$$

$$\Rightarrow M = 2.95$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{4000}{T} = 1 + \frac{1.2 - 1}{2} \times 2.95^2$$

$$\Rightarrow T = 2138.7515 \, [°R]$$

$$M = \frac{V}{\sqrt{\gamma R T}} \Rightarrow V = M \sqrt{\gamma R T}$$

$$= 2.95 \sqrt{1.2 \times 1252.58 \times 2138.7515}$$

$$\boxed{V = 5289.27 \, [ft/s]}$$

(c) At the exit:

$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

$$\frac{A}{0.75} = \frac{1}{2.95} \left[\frac{2}{1.2 + 1} \left(1 + \frac{1.2 - 1}{2} 2.95^2\right)\right]^{\frac{1.2 + 1}{2(1.2 - 1)}}$$

$$\boxed{A = 4.71 \, [ft^2]}$$

(d)

$$C_F = \frac{T}{P_0 A^*} = \frac{\dot{m} V_e}{P_0 A^*} = \frac{13.7955 \left[\cancel{\frac{slug}{s}}\right] 5289.27 \left[\cancel{\frac{ft}{s}}\right]}{30 \times 2116.2 \left[\cancel{\frac{lb}{ft^2}}\right] 0.75 \left[\cancel{ft^2}\right]} = \boxed{1.53}$$

3. Consider a rocket engine that uses liquid oxygen and liquid ethanol (C_2H_5OH) fuel aka ethyl alcohol. The oxygen mass-flow rate is 2.0 times greater than the fuel mass-flow rate. Ethanol is stored at 298K while the oxygen is stored at 80K just slightly below its boiling point. Oxygen has a heat of vaporization of 6.81kJ/mole while the value for ethanol is 38.6 kJ/mole. The heat of formation of liquid ethanol is -277.0 kJ/mole. The specific heat at constant pressure for gaseous oxygen is 30.77 joules/mole °K. The liquids are sprayed into the combustion chamber.

(a) How much energy per mole is required to vaporize and heat a mole of oxygen to the temperature of 298K.

(b) What is the expected ideal flame temperature? The ideal flame temperature aka theoretical flame temperature is the value with no dissociation. Assume that the products are H_2O, CO_2, and CO. You should make this calculation without using the online computer code. A key step is to determine what fraction of the carbon will appear in CO_2 and what fraction will appear in CO.

$$\dot{m}_{O_2} = 2\dot{m}_{C_2H_5OH} \Rightarrow m_{O_2} = 2\, m_{C_2H_5OH}$$

$$T_i = 298\ [K]$$

$$h_{vap} = 6810\ [J/mol]$$

$$c_{p,O_2} = 30.77\ [J/mol\,K]$$

(a)

$$Q = h_{vap} + \int_{80}^{298} C_{p,O_2}\, dT = 6810 + 30.77(298-80)$$

$$\boxed{Q = 13517.86\ [J/mol]}$$

(b) Stoichiometric reaction:

$$a\, C_2H_5OH + b\, O_2 \longrightarrow c\, CO_2 + d\, H_2O + e\, CO$$

$$m_{O_2} = 2\, m_{C_2H_5OH}$$

$$b \times W_{O_2} = 2 \times a \times W_{C_2H_5OH}$$

$$b \times 32 = 2 \times a \times (2\times 12 + 5 + 16 + 1)$$

$$b = 2.875\, a$$

$$1\, C_2H_5OH + 2.875\, O_2 \longrightarrow 1.75\, CO_2 + 3\, H_2O + 0.25\, CO$$

Energy conservation:

$$\sum_{reactants} n_m \left(h_{f,m} + \int_{T_{ref}}^{T_i} c_{p,m} dT \right) = \sum_{products} n_m \left(h_{f,m} + \int_{T_{ref}}^{T_f} c_{p,m} dT \right)$$

$$n_{C_2H_5OH} \left[h_{f,C_2H_5OH} + \cancel{c_{p,C_2H_5OH} (T_i - T_{ref})} \right] + n_{O_2} (-Q)$$
$$= n_{CO_2} \left[h_{f,CO_2} + c_{p,CO_2} (T_f - T_{ref}) \right] + n_{H_2O} \left[h_{f,H_2O} + c_{p,H_2O} (T_f - T_{ref}) \right] +$$
$$n_{CO} \left[h_{f,CO} + c_{p,CO} (T_f - T_{ref}) \right]$$

	n	$h_{f,m}$ [J/mol]	$c_{p,m}$ [J/mol-K]
C_2H_5OH	1	-277000	N/A
O_2	2.875	0	30.77
CO_2	1.75	-393500	37.14
H_2O	3	-241800	34.74
CO	0.25	-110500	28.56

$$n_{C_2H_5OH} \, h_{f,C_2H_5OH} + n_{O_2} (-Q) = n_{CO_2} \left[h_{f,CO_2} + c_{p,CO_2} (T_f - T_{ref}) \right]$$
$$+ n_{H_2O} \left[h_{f,H_2O} + c_{p,H_2O} (T_f - T_{ref}) \right] + n_{CO} \left[h_{f,CO} + c_{p,CO} (T_f - T_{ref}) \right]$$

$$1 \times (-277000) + 2.875 (-13517.86) = 1.75 [-393500 + 37.14 (T_f - 298)]$$
$$+ 3 [-241800 + 34.74 (T_f - 298)] + 0.25 [-110500 + 28.56 (T_f - 298)]$$

$$\boxed{T_f = 6681.6361 [K]}$$

4. Suppose we have a rocket combustor that has hot products produced at the following conditions:

T = temperature = 4600 ° R $= 2555.\bar{5}$ K

P = pressure = 75 atmospheres $= 7.599 \times 10^6$ [Pa]

γ = ratio of specific heats = 1.25

MW = average molecular weight = 27 $\left[\frac{g}{mol}\right]$

(a) Design a nozzle that will produce 75,000 pounds of thrust with an ambient pressure of one atmosphere. In particular, determine the following quantities: mass flow rate, exit pressure, exit or exhaust velocity, effective exhaust velocity, thrust coefficient, throat cross-sectional area, and exit cross-sectional area.

(b) Design a nozzle that produces 100,000 pounds of thrust with an ambient pressure at vacuum conditions. Limit the nozzle exit cross-sectional area to no more than thirty times the throat cross-sectional area. Determine the same quantities as described in Part (a).

(a)

$$\bar{R} = 8.3144598 \left[\frac{J}{mol \cdot K}\right]$$

$$R = \frac{\bar{R}}{MW} = 8.3144598 \left[\frac{J}{\cancel{mol} \cdot K}\right] \frac{1}{27} \left[\frac{\cancel{mol}}{\cancel{g}}\right] \times \frac{1000 [\cancel{g}]}{1 [kg]}$$

Gas constant:

$$R = 307.943 \left[\frac{m^2}{s^2 \cdot K}\right]$$

Pressure ratio:

$$\boxed{P = 1 [atm]}$$

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{75}{1} = \left(1 + \frac{1.25 - 1}{2} \times M^2\right)^{\frac{1.25}{1.25-1}}$$

$$M = 3.3123$$

Temperature ratio:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{2555.\bar{5}}{T} = 1 + \frac{1.25 - 1}{2} \times 3.3123^2$$

$$T = 1077.65\ [K]$$

Velocity at exit:

$$V = M\sqrt{\gamma R T}$$

$$= 3.3123\sqrt{1.25 \times 367.943 \times 1077.65}$$

$$\boxed{V = 2133.28\ [m/s]}$$

Thrust equation:

$$T = 75\,000[lb] = 333616.62\ [N]$$

$$T = \dot{m} V_e + \cancel{(P_e - P_a)A_e}$$

$$\boxed{\dot{m} = \frac{T}{V_e} = \frac{333616.62}{2133.28} = 156.4\ [kg/s]}$$

At the throat:

$$M = 1$$

$$\frac{P_0}{P^*} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{7.599 \times 10^6}{P^*} = \left(1 + \frac{1.25-1}{2} \times 1^2\right)^{\frac{1.25}{1.25-1}}$$

$$P^* = 4.217 \times 10^6\ [Pa]$$

$$\frac{T_0}{T^*} = 1 + \frac{\gamma-1}{2} M^2$$

$$\frac{2555.5}{T^*} = 1 + \frac{1.25-1}{2} \times 1^2$$

$$\Rightarrow T^* = 2271.6 \ [K]$$

$$\rho^* = \frac{P^*}{R T^*} = \frac{4.217 \times 10^6}{307.943 \times 2271.6} = 6.028 \left[\frac{kg}{m^3}\right]$$

$$V^* = M \sqrt{\gamma R T^*} = 1 \sqrt{1.25 \times 307.943 \times 2271.6} = 935.096 \left[\frac{m}{s}\right]$$

$$\boxed{A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{156.4}{6.028 \times 935.096} = 0.02775 \ [m^2]}$$

$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A}{0.02775} = \frac{1}{3.3123} \left[\frac{2}{1.25+1} \left(1 + \frac{1.25-1}{2} \times 3.3123^2 \right) \right]^{\frac{1.25+1}{2(1.25-1)}}$$

$$\boxed{A = 0.24015 \ [m^2]}$$

Effective exhaust velocity:

$$C = \frac{T}{\dot{m}} = \frac{333616.62}{156.4} = \boxed{2133.0986 \left[\frac{m}{s}\right]}$$

Thrust coefficient:

$$C_F = \frac{\dot{m} C}{P_o A^*} = \frac{156.4 \times 2133.0986}{7.599 \times 10^6 \times 0.02775}$$

$$\boxed{C_F = 1.58208}$$

ⓑ Area ratio:

$$\frac{A_e}{A^*} = 30 = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$30 = \frac{1}{M}\left(\frac{2}{1.25+1}\left(1+\frac{1.25-1}{2}\times M^2\right)\right]^{\frac{1.25+1}{2(1.25-1)}}$$

$$M_e = 4.3015$$

Temperature ratio at the exit:

$$\frac{T_0}{T_e} = 1 + \frac{\gamma-1}{2}M^2$$

$$\frac{2555.\bar{5}}{T_e} = 1 + \frac{1.25-1}{2} \times 4.3015^2$$

$$T_e = 771.4 \ [K]$$

Pressure ratio at exit:

$$\frac{P_0}{P_e} = \left(\frac{T_0}{T_e}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{7.599\times10^6}{P_e} = \left(1 + \frac{1.25-1}{2} \times 4.3015^2\right)^{\frac{1.25}{1.25-1}}$$

$$\boxed{P_e = 19043.1683 \ [Pa]}$$

Velocity at exit:

$$V_e = M_e\sqrt{\gamma R T_e} = 4.3015\sqrt{1.25\times367.943\times771.4}$$

$$= 2343.96 \ [m/s]$$

Area at exit:

Thrust: $T = \rho_e A_e \left(\frac{V_e^2}{R T_e} + 1 \right)$

$$444\ 822.16 = 19043.1683 \times A_e \left(\frac{2343.96^2}{307.943 \times 771.4} + 1 \right)$$

$$\boxed{A_e = 0.968\ [m^2]}$$

Area at throat:

$$\frac{A_e}{A^*} = 30 \Rightarrow \frac{0.968}{A^*} = 30 \Rightarrow A^* = \boxed{0.03227\ [m^2]}$$

Mass flow rate:

$$\dot{m} = \frac{\rho_e}{R T_e} A_e V_e = \frac{19043.1683}{307.943 \times 771.4} \times 0.968 \times 2343.96$$

$$\boxed{\dot{m} = 181.8925\ [kg/s]}$$

Effective exhaust velocity:

$$c = \frac{T}{\dot{m}} = \frac{444\ 822.16}{181.8925}$$

$$\boxed{c = 2445.5223\ [m/s]}$$

Thrust coefficient:

$$C_F = \frac{T}{\rho_o A^*} = \frac{444\ 822.16}{7.599 \times 10^6 \times 0.03227}$$

$$\boxed{C_F = 1.814}$$

5. Consider a jet engine flying at a Mach number of 1.4. A normal shock sits at the entrance of the divergent diffuser. The diffuser entrance cross-sectional area is 2.5 ft^2. The ambient conditions are 500°R for temperature and 0.8 atmosphere for pressure.

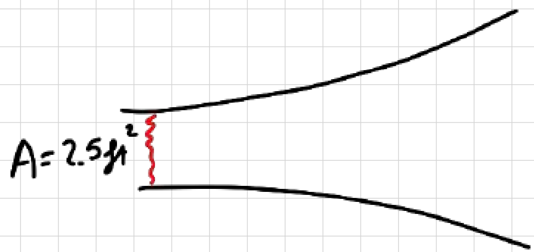
(a) What is the stagnation pressure immediately in front (upstream) of the shock? What is the stagnation pressure immediately behind (downstream) the shock? What is the Mach number immediately behind the shock? What is the mass flow through the diffuser?

(b) What is the minimum cross-sectional area required at the downstream end of the diffuser in order to assure that the Mach number of the flow there does not exceed 0.10?

$M = 1.4$

$T = 500\,°R$

$P = 0.8$ atm



(a)

At the entrance, before normal shock (upstream)

Pressure ratio:

$$\frac{P_{01}}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{01}}{0.8} = \left(1 + \frac{1.4-1}{2} \times 1.4^2\right)^{\frac{1.4}{1.4-1}}$$

$$\boxed{P_{01} = 2.5458 \ [\text{atm}]}$$

Stagnation pressure downstream:

$$\frac{P_{02}}{P_{01}} = \left[\frac{(\gamma+1) M^2}{(\gamma-1)M^2 + 2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

$$\frac{P_{02}}{2.5458} = \left[\frac{(1.4+1)\,1.4^2}{(1.4-1)\,1.4^2 + 2}\right]^{\frac{1.4}{0.4}} \left[\frac{1.4+1}{2 \times 1.4 \times 1.4^2 - (1.4-1)}\right]^{\frac{1}{0.4}}$$

$$\boxed{P_{02} = 2.4394 \ [\text{atm}]}$$

Down stream Mach number:

$$M_2 = \sqrt{\frac{(\gamma-1)M^2+2}{2\gamma M^2-(\gamma-1)}} = \sqrt{\frac{(1.4-1)1.4^2+2}{2\times1.4\times1.4^2-(1.4-1)}}$$

$$\boxed{M_2 = 0.7397}$$

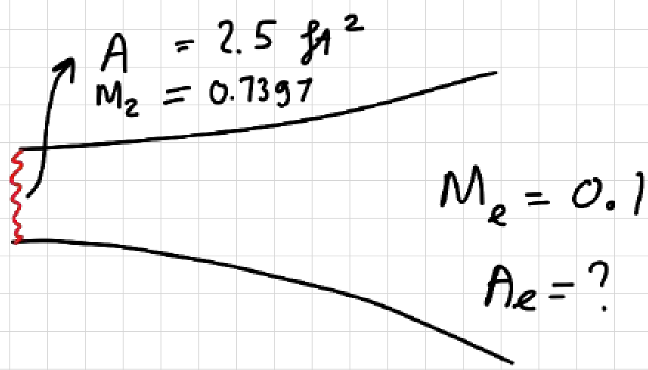
Mass flow rate:

$$\dot{m} = \rho A V = \frac{P}{RT} A M\sqrt{\gamma RT}$$

$$= \frac{0.8\times2116.2}{1718\times500} \times 2.5\times1.4\sqrt{1.4\times1718\times500}$$

$$\boxed{\dot{m} = 7.5645\ [slug/s]}$$

(b)



Right after the shock way

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{2.5}{A^*} = \frac{1}{0.74}\left[\frac{2}{1.4+1}\left(1+\frac{1.4-1}{2}0.74^2\right)\right]^{\frac{1.4+1}{2(1.4-1)}}$$

$$A^* = 2.34\ [ft^2]$$

At the end of the diffuser:

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A_e}{2.34} = \frac{1}{0.1}\left[\frac{2}{1.4+1}\left(1+\frac{1.4-1}{2}0.1^2\right)\right]^{\frac{1.4+1}{2(1.4-1)}}$$

$$\boxed{A_e = 13.623\ [ft^2]}$$