Lecture 2
Thrust

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Impulse =

$$T' = \dot{m}c = \dot{m}\,U + (P_e - P_a)A_e = \dot{m}\,[\,\mathrm{U} + (P_e - P_a)A_e/\ \dot{m}\,]$$

So, $c = U + (P_e - P_a)A_e/\ \dot{m}$, an effective exhaust velocity ; ***Weight flow:*** $\dot{w} = \dot{m}g$

$$\frac{Thrust\ Force}{Weight\ Flow} = Specific\ Impulse = I_{sp}$$

$$I_{sp} = \frac{T'}{\dot{w}} = \frac{T'}{\dot{m}g} = \frac{c}{g}\ \text{(units of seconds)}$$

Specific impulse is a function of fuel-oxidizer types, mixture ratio, and nozzle design!
This is a good unit to compare small engines to larger engines and engines of different types.

3600 converts hours into seconds because $\dot{m}_f$ (mass/sec)

$$Specific\ Fuel\ Consumption = \frac{lbm\ of\ fuel/hour}{thrust}$$

$$S = \frac{\dot{m}_f g \cdot 3600}{T'}$$

In the case of a rocket, $\dot{m} = \dot{m}_f = \dot{m}_p$

$$\boxed{S = \frac{3600}{I_{sp}}}$$

Efficiency

For one mole of fuel (or propellants), $n_p \cdot H\big(T_{fuel}, products\big) - n_r \cdot H(T_{matched}, reactants)$ is released. Divide by molecular mass of fuel (or propellants) to get heat released per unit mass of fuel (or propellants). $\Delta h_c = \frac{n_p \cdot H\left(T_{fuel}, products\right) - n_r \cdot H(T_{matched}, reactants)}{m_F}$

$$Efficiency = \frac{Power\ Delivered}{Rate\ of\ Chemical\ Energy\ Converted}$$

$$\eta = \frac{T'V}{\dot{m}_f \Delta h_c}$$

Heat per unit mass released during combustion

In the one-dimensional case: $\eta = \frac{T'Vg}{\dot{m}_f \mathrm{g} \Delta h_c} = \frac{I_{sp}Vg}{\Delta h_c} = \frac{cV}{\Delta h_c}$

Again, I_{sp} and c are directly related to efficiency and useful in comparing engines of different types and sizes.

Delta V

Consider a simple rocket case:

$$T' = T = m\frac{dV}{dt}$$

$$\dot{m}c = m\frac{dV}{dt}$$

Zero gravity, no drag:

$$T' = T$$

$$\dot{m} = -\frac{dm}{dt}$$

$$\frac{dV}{dt} = -\frac{c}{m}\frac{dm}{dt}$$

or

$$dV = -c\frac{dm}{m}$$

$$-\int_{V_i}^{V_f} dV = c\int_{m_i}^{m_f} d(\log m)$$

In this course, “log” is the natural logarithm.

$$\boxed{V_f - V_i = c\log\left(\frac{m_i}{m_f}\right) = c\log\left(\frac{m_f + m_{propellants}}{m_f}\right)}$$

Delta V

So the change in velocity depends on c (or I_{sp}) and the fraction of mass exhausted.

$$\Delta V = I_{sp} g \log\left(1 + \frac{m_{propellants}}{m_f}\right) = I_{sp} g \log\left(\frac{m_i}{m_i \; - m_{propellants}}\right)$$

$$\Delta V = c \log\left(1 + \frac{m_{propellants}}{m_f}\right) = c \; \log\left(\frac{m_i}{m_i \; - m_{propellants}}\right)$$

$$\boxed{\Delta V = c \log\left(\frac{1}{1 - {m_{propellants}}/{m_i}}\right)}$$

This again shows the usefulness of c (or I_{sp})!