Lecture 4
Combustion Chamber

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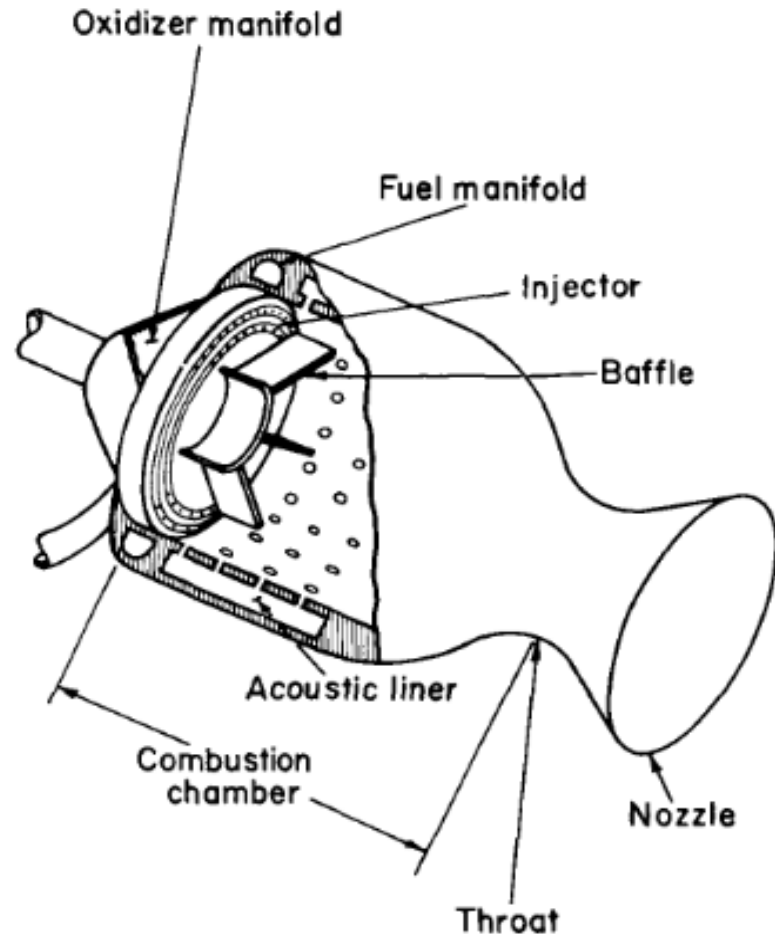
Sample Rocket Engine and Combustor

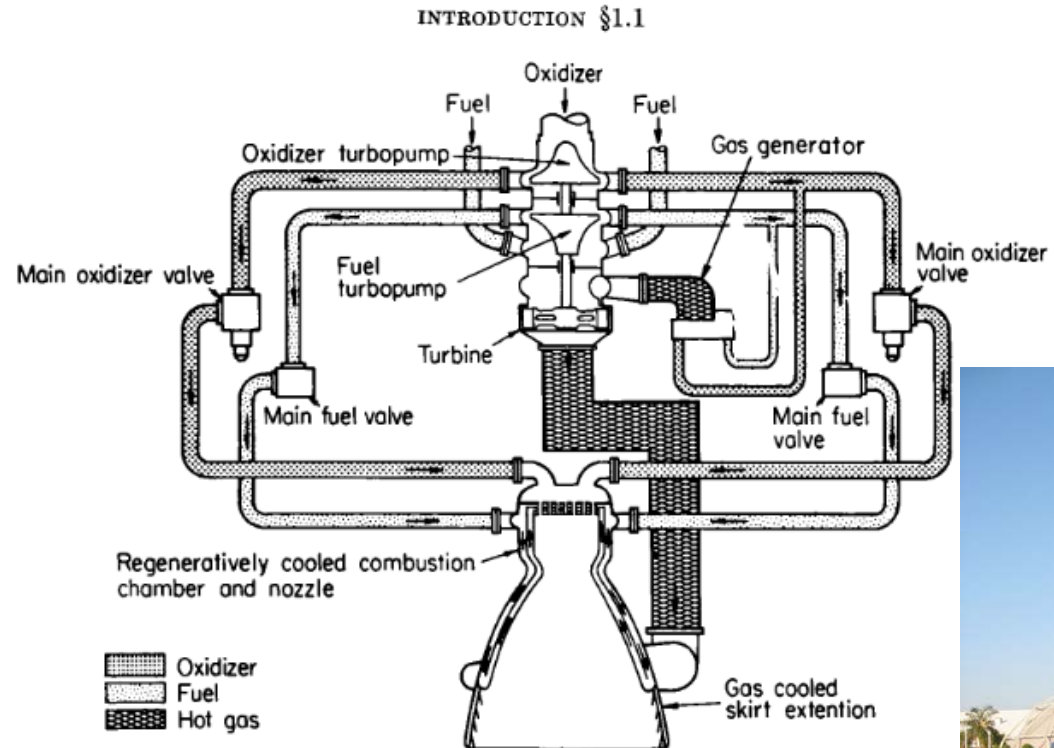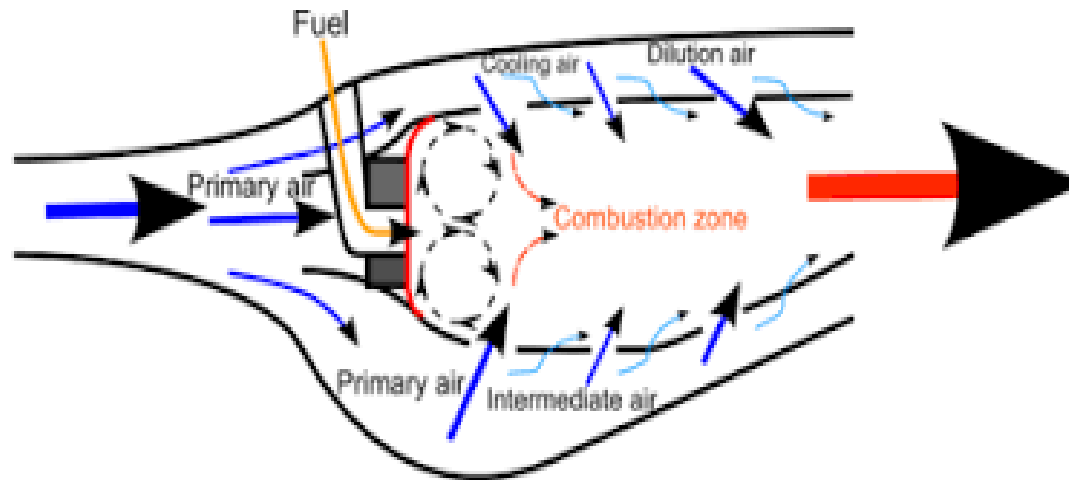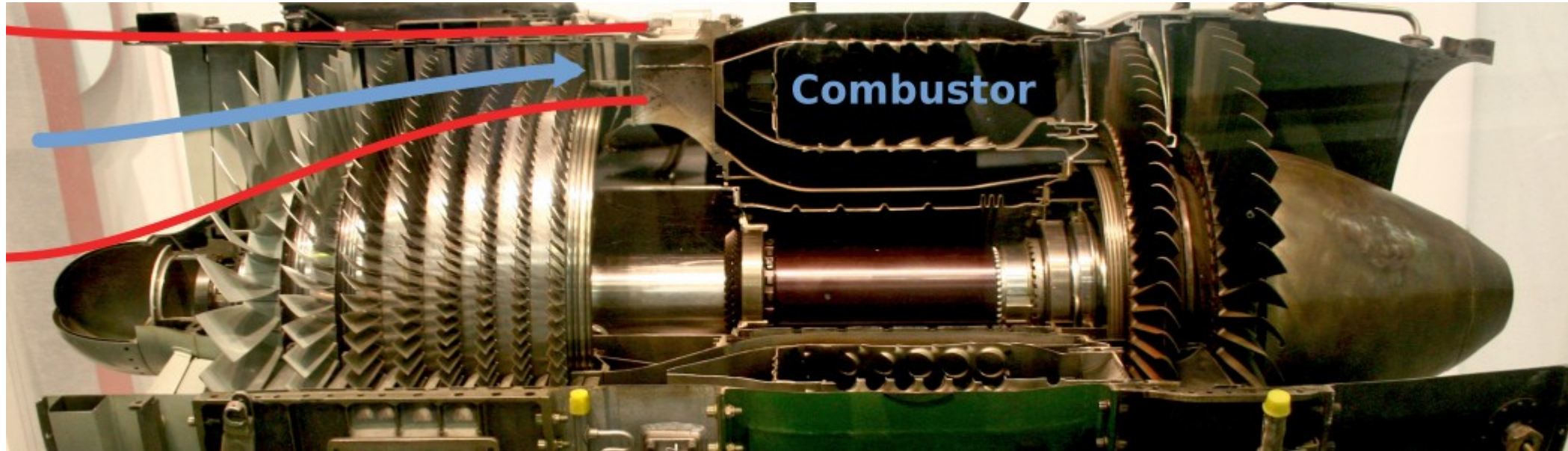FIGURE 1.1.1a.—Typical components of a thrust chamber assembly.

INTRODUCTION §1.1

FIGURE 1.1.1c.—F-1 engine propellant schematic.

Sample Turbojet Combustor

Momentum Equation

1D, Steady, Inviscid, and No Gravity: $\rho u du + dp = 0$

$u \longrightarrow$

$$p = \rho RT$$

$$\dot{m} = \rho uA = \frac{puA}{RT}$$

$$\rho uAdu + Adp = 0$$

$$\frac{puA}{RT} du + Adp = 0$$

Increase $T°$ via combustion.
a is speed of sound. M is Mach number.

Here, R is the
specific gas constant.

$$\frac{dp}{p} + \frac{udu}{RT} = 0$$

$$a^2 = \gamma RT \quad ; \quad M_c = u/a \quad ; \quad \frac{u^2}{RT} = \gamma M_c^2$$

$$\frac{dp}{p} + \gamma M_c^2 \frac{du}{u} = 0$$
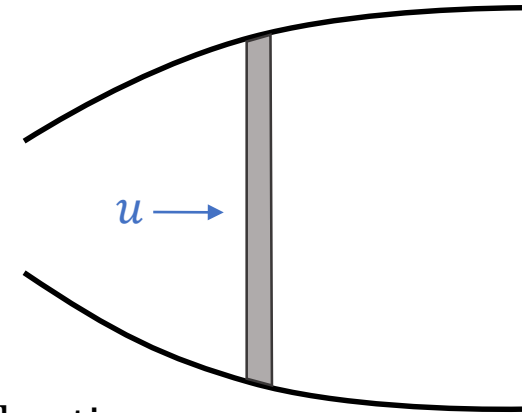
Energy

$$\frac{u^2}{2} = c_p(T^\circ - T); \quad \gamma = \frac{c_p}{c_v} \quad ; \quad R = c_p - c_v = c_p\left(1 - 1/\gamma\right)$$

$$(T^\circ - T) = \frac{u^2}{2c_p} = \frac{u^2}{2\gamma R}(\gamma - 1) = \frac{\gamma - 1}{2} M_c^2 \, T$$

$$\frac{dT^\circ}{T} = \frac{dT}{T} + \frac{(\gamma - 1)u\,du}{\gamma RT} = \frac{dT}{T} + \frac{(\gamma - 1)M_c^2 \, du}{u} = \left(1 + \frac{\gamma - 1}{2} M_c^2\right)\frac{dT^\circ}{T^o} \qquad \boxed{\text{Since } \frac{T^\circ}{T} = 1 + \frac{\gamma - 1}{2} M_c^2}$$

$$\frac{dT}{T} = \left(1 + \frac{\gamma - 1}{2} M_c^2\right)\frac{dT^\circ}{T^o} - (\gamma - 1)M_c^2 \frac{du}{u}$$

$$\boxed{\textbf{\textit{Isentropic:}} \left(\frac{p^\circ}{p}\right) = \left(\frac{T^\circ}{T}\right)^{\frac{\gamma}{\gamma - 1}}} \longrightarrow \quad \ln p^\circ - \ln p = \frac{\gamma}{\gamma - 1}(\ln T^\circ - \ln T)$$

$$\frac{dp^\circ}{p^o} - \frac{dp}{p} = \frac{\gamma}{\gamma - 1}\left[\frac{dT^\circ}{T^o} - \frac{dT}{T}\right]$$

Energy

$$\frac{dp°}{p°} = \frac{dp}{p} + \frac{\gamma}{\gamma - 1}\left[\frac{dT°}{T°} - \left\{\left(1 + \frac{\gamma - 1}{2}M_c^2\right)\frac{dT°}{T°} - (\gamma - 1)M_c^2\frac{du}{u}\right\}\right]$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{\textcolor{red}{\frac{dT}{T}}}$$

$$\frac{dp°}{p°} = \frac{dp}{p} + \frac{\gamma}{\gamma - 1}\left[1 - 1 - \frac{\gamma - 1}{2}M_c^2\right]\frac{dT°}{T°} + \frac{\gamma}{\gamma - 1}\left[(\gamma - 1)M_c^2\frac{du}{u}\right]$$

$$\frac{dp°}{p°} = \frac{dp}{p} + \gamma M_c^2\frac{du}{u} - \frac{\gamma}{2}M_c^2\frac{dT°}{T°}$$

$$\underbrace{\qquad\qquad\qquad}_{\textcolor{red}{0}}$$

Decrease in $p°$ as $T°$ increases is proportional to M_c^2

$$\boxed{\frac{dp°}{p°} = -\frac{\gamma}{2}M_c^2\frac{dT°}{T°}}$$

This result is independent whether we have constant pressure or constant area!

Combustors

When combustion occurs at a fixed pressure (continuous combustion is one example):

$\dot{q}$ is energy per unit time converted from chemical form to thermal form

$\dot{m} = \dot{m}_{air} + \dot{m}_{fuel}$ is total mass flow (mass/time)
For a rocket, $\dot{m} = \dot{m}_{propellants}$

Then: $\dot{q} = \dot{m}[h(T_{final}) - h(T_{initial})]$

> This is an approximation. Use stagnation enthalpy for higher velocity flows

So heat released causes an increase in temperature!

Here, $h(T)$ is determined for the products in units of energy per unit mass

$H(T)$ is energy per mole; $h = \dfrac{H}{mW}$

There is some confusion in the textbook about the enthalpy H. Sometimes, it is given in units of energy per mole, other times it is given as simply energy. We always mean here energy per mole.

Combustors

For a given compound: $h(T) = \int_{T_{ref}}^{T} c_p(T')dT' + h_f(T_{ref})$

$$H(T) = \int_{T_{ref}}^{T} C_p(T')dT' + H_f(T_{ref})$$

Heat of formation at T_{ref}

(Sensible enthalpy from the elements in their natural form)
Positive when formation is endothermic!

$\boxed{T_{ref} = 298.15K \text{ (typically)}}$

For CO_2, formation is given by: $C + O_2 \rightarrow CO_2$

For H_2O, formation is given by: $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$

Both processes are exothermic so h_f is negative!

Products have lower heat of formation than reactants

We must specify the phase of the reactants and the products to determine the enthalpy at a given temperature since heat of melting and heat of vaporization affects results!

$$\int_{T_{initial}}^{T_{final}} \sum n_j \cdot C_{p_j} \cdot dT' = n_{reactants} H_{T_{initial}}(reactants, T_{initial}) - n_{products} H_{T_{initial}}(products, T_{initial})$$

$$\int_{T_{initial}}^{T_{final}} \sum n_j \cdot C_{p_j} \cdot dT' = \int_{T_{ref}}^{T_{initial}} \left[\sum n_i \cdot C_{p_i} - \sum n_j \cdot C_{p_j} \right] dT' + \sum n_i \cdot H_{f_i} - \sum n_j \cdot H_{f_j}$$

Chemical Heat Release

The chemical heat release for reaction at T_{ref} is:

$$n_p Q(T_{ref}) = \sum_{reactants} n_i H_{f\,i}(T_{ref}) - \sum_{products} n_j H_{f\,j}(T_{ref})$$

Sensible enthalpy is zero at T_{ref}

The chemical heat release for reaction at $T_{initial}$ is:

$$n_p Q(T_{initial}) = \int_{T_{ref}}^{T_{initial}} \left[\sum_{reactants} n_i C_{p_i} - \sum_{products} n_j C_{p_j} \right] dT' + \sum_{reactants} n_i H_{f\,i} - \sum_{products} n_j H_{f\,j}$$

$$n_p Q(T_{initial}) = n_p Q(T_{ref}) + \int_{T_{ref}}^{T_{initial}} \left[\sum_{reactants} n_i C_{p_i} - \sum_{products} n_j C_{p_j} \right] dT'$$

So:

$$n_p H(products, T_{final}) - n_p H(products, T_{initial}) = n_p Q(T_{initial}) \approx n_p Q(T_{ref})$$

Enthalpy – Explanatory Statement

For the reactants, $n_r = \sum n_i$

For the products, $n_p = \sum n_j$

1. $n_r H(reactants, T_{initial}) = \int_{T_{ref}}^{T_{initial}} \left(\sum n_i C_{p_i}\right) dT' + \sum n_i H_{f_i}(T_{ref})$

2. $n_p H(products, T_{final}) = \int_{T_{ref}}^{T_{final}} \left(\sum n_j C_{p_j}\right) dT' + \sum n_j H_{f_j}(T_{ref})$

For an adiabatic, constant pressure process:

3. $\Delta(nH) = 0$ or $n_r H(reactants, T_{initial}) = n_p H(products, T_{final})$

4. $\int_{T_{ref}}^{T_{final}} \sum n_j C_{p_j} dT' = \int_{T_{initial}}^{T_{final}} \left(\sum n_j C_{p_j}\right) dT' + \int_{T_{ref}}^{T_{initial}} \left(\sum n_j C_{p_j}\right) dT'$

Enthalpy – Explanatory Statement

Substitute 4 into 2 to get:

5. $n_p H(products, T_{final}) = \int_{T_{initial}}^{T_{final}} \left(\sum n_j C_{p_j} \right) dT' + n_p H(products, T_{initial})$

Now use 3 and 5 to obtain:

6. $n_p Q(T_{initial}) \equiv \int_{T_{initial}}^{T_{final}} \left(\sum n_j C_{p_j} \right) dT' = n_r H(reactants, T_{initial}) - n_p H(products, T_{initial})$

$$= \int_{T_{ref}}^{T_{initial}} \left[\sum n_i C_{p_i} - \sum n_j C_{p_j} \right] dT' + \sum n_i H_{f_i} - \sum n_j H_{f_j}$$

7. $\boxed{\int_{T_{initial}}^{T_{final}} \sum n_j C_{p_j} dT' \approx \sum n_i H_{f_i} - \sum n_j H_{f_j} \equiv n_p Q(T_{ref})}$

Sketch of Thermal Paths
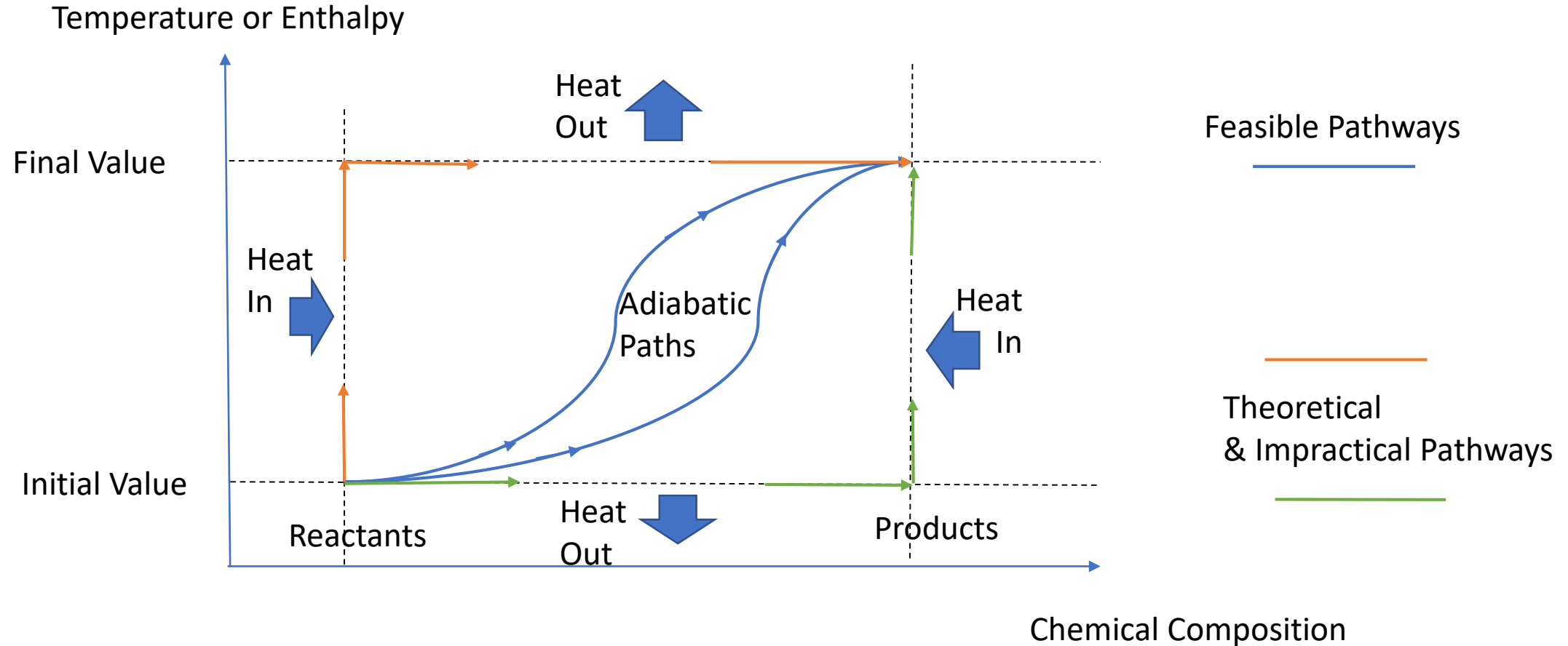
Chemical Heat Release

So, we can conceptualize a two-step process:
1. An exothermic reaction at temperature $T_{initial}$
2. All the released energy is used to heat the products from $T_{initial}$ to T_{final}

Actually, both steps occur simultaneously, but thermodynamics depends only on the end state, not the path!
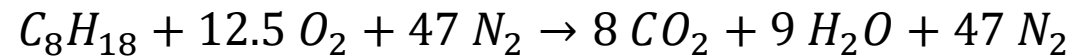
This presents the energy equation!

Composition of Products

First, the idealized case:
- No dissociation
- Mass balance determines the number of moles
- Assume complete oxidation
- Example: ***Octane (C_8H_{18}) and Air***

Stoichiometric: Air has 1 mole O_2 and 3.76 moles N_2

$$C_8H_{18} + 12.5\ O_2 + 47\ N_2 \rightarrow 8\ CO_2 + 9\ H_2O + 47\ N_2$$

$$n_{C_8H_{18}} = 1; n_{O_2} = 12.5; n_{N_2} = 47; n_{CO_2} = 8; n_{H_2O} = 9$$

Consider 20% excess air: $C_8H_{18} + 15\ O_2 + 56.4\ N_2 \rightarrow 8\ CO_2 + 9\ H_2O + 2.5\ O_2 + 56.4\ N_2$ ← | Combustors often operate lean to keep temperature lower! |

In a rocket that burns pure O_2 without N_2, much higher temperatures are obtained.

So in this idealized case, we can immediately determine the number of moles of each product if we know the reactants!

Composition of Products

Second case:
- Dissociation
- Example: **Octane (C_8H_{18}) and Air**

At high temperatures, the vibrational energy in atoms causes dissociation

Dissociation:
- $H_2O \leftrightarrow H + OH$
- $CO_2 \leftrightarrow CO + O$
- $O_2 \leftrightarrow O + O$
- $H_2 \leftrightarrow H + H$

The arrows are in both directions because some recombination occurs!

So some fraction of the product molecules will be dissociated. It takes some energy to break the bonds so the final temperature will be a little lower than the idealized value!

Composition of Products

Consider 20% excess air with dissociation: $C_8H_{18} + 15\ O_2 + 56.4\ N_2 \rightarrow$

- $a\ CO_2 + b\ H_2O$
- $+\ c\ O_2 + d\ H_2$
- $+\ e\ C + f\ CO$
- $+\ g\ H + i\ O$
- $+\ j\ OH + k\ N_2$
- $+\ l\ NO + m\ N$

l and m are typically very small and we can assume $k = 56.4$, no N_2 dissociation for the purpose of calculating flame temperature. Small amounts can be important for pollution purposes. Also, small amounts of C will exist allowing $e \approx 0$.

Carbon mass balance: $a + f = 8$

Hydrogen mass balance: $2b + 2d + g + j = 18$

Oxygen mass balance: $2a + b + 2c + f + i + j = 30$

Nitrogen mass balance: $k = 56.4$

We have four relations for nine unknowns. Five more relations are needed.

Composition of Products

The products will be in equilibrium which give certain mass action formulas. The equilibrium balance for example can be:

- $CO_2 \leftrightarrow CO + \frac{1}{2}O_2$

- $H_2O \leftrightarrow H_2 + \frac{1}{2}O_2$

- $H_2 \leftrightarrow H + H$

- $O_2 \leftrightarrow O + O$

- $H_2O \leftrightarrow H + OH$

Five more equations can result from this.
Note that the equations above must be independent equations.
For example: $CO_2 \leftrightarrow CO + O$ is not independent!

Thermal Cycle -- Effect of Combustor Pressure

Three stages:
1-2 Compression
2-3 Combustion
3-4 Expansion

Pressure increases
in direction of arrow

Constant pressure –isobars
These curves diverge

High-pressure engine
1- 2b- 3b – 4b

Low-pressure engine
1- 2a- 3a – 4a

$Q = h_{3b} - h_{2b}$

$Q = h_{3a} - h_{2a}$

h

s

3b

2b

3a

2a

4a

4b

1

The higher the pressure in the combustor, the smaller
the gain in entropy. This means greater efficiency.