Lecture 13
Turbojet with Afterburner

By W. A. Sirignano
Prepared by Colin Sledge

Turbojet engines

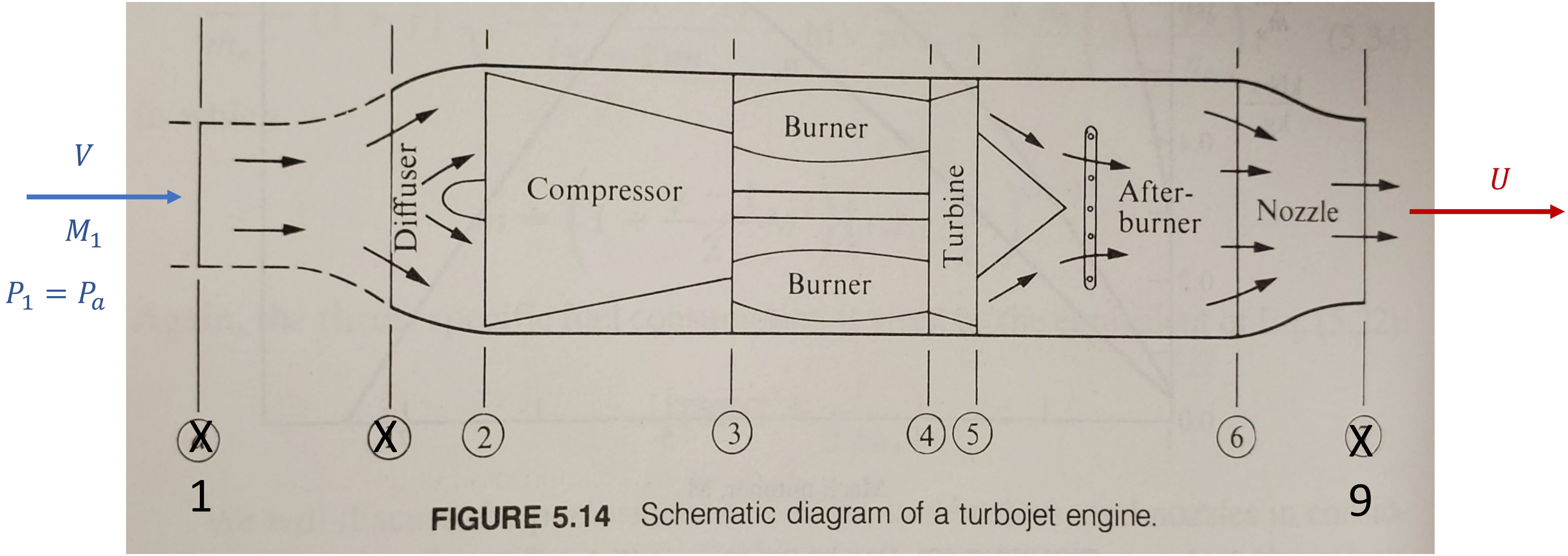


FIGURE 5.14 Schematic diagram of a turbojet engine.

Turbojet engine from Hill & Peterson [1]

Turbojet engines

Subscripts: *n* for nozzle, *d* for diffuser, *c* for compressor, *t* for turbine.

Same nozzle equations as before:

$$U^2 = 2c_{p_n}T_6^\circ\left[1-\left(\frac{P_9}{P_6^\circ}\right)^{\frac{\gamma_n}{\gamma_n-1}e_n}\right]$$

$$U^2 = 2c_{p_n}T_6^\circ\left[1-\left(\frac{P_9}{P_1}\frac{P_1}{P_2^\circ}\frac{P_2^\circ}{P_3^\circ}\frac{P_3^\circ}{P_4^\circ}\frac{P_4^\circ}{P_5^\circ}\frac{P_5^\circ}{P_6^\circ}\right)^{\frac{\gamma_n}{\gamma_n-1}e_n}\right]$$

For perfect expansion: $\frac{P_9}{P_1} = 1$

Consider now the contribution to pressure ratio from each component

$$\frac{P_1}{P_2^\circ} = \left(1+\frac{\gamma_d-1}{2}M_1^2\right)^{-\frac{\gamma_d}{\gamma_d-1}e_d}$$

Turbojet engines

Define: $\delta \equiv \frac{\gamma_d}{\gamma_d - 1} \frac{\gamma_n - 1}{\gamma_n}$ Typically, $\delta < 1$ since $\gamma_n < \gamma_d$

$$\left(\frac{P_1}{P_2^\circ}\right)^{\frac{\gamma_n - 1}{\gamma_n} e_n} = \left(1 + \frac{\gamma_d - 1}{2} M_1^2\right)^{-e_n e_d \delta}$$

Don't confuse the subscripts here with the subscripts we need for compressor and turbine stages!

$$T_2^\circ = T_1^\circ = T_1 \left(1 + \frac{\gamma_d - 1}{2} M_1^2\right)$$

For the compressor:

$$\frac{P_3^\circ}{P_2^\circ} = \left(\frac{T_3^\circ}{T_2^\circ}\right)^{\frac{\gamma_d}{\gamma_d - 1} e_c} = \left[1 + \frac{c_p\left(T_3^\circ - T_2^\circ\right)}{c_p T_2^\circ}\right]^{\frac{\gamma_d}{\gamma_d - 1} e_c}$$

$$\frac{P_3^\circ}{P_2^\circ} = \left(1 + \frac{H_c}{c_p T_2^\circ}\right)^{\frac{\gamma_d}{\gamma_d - 1} e_c}$$

Where: $H_c = h_3^\circ - h_2^\circ$
Is compressor work per unit mass

$$\left(\frac{P_3^\circ}{P_2^\circ}\right)^{\frac{\gamma_n - 1}{\gamma_n} e_n} = \left(1 + \frac{H_c}{c_p T_2^\circ}\right)^{e_n e_c \delta}$$

Turbojet engines

In the combustor:

$$\frac{P_4^\circ}{P_3^\circ} = 1 - CM_c^2 \qquad \text{Where:} \quad C \equiv \frac{\gamma_c}{2}\frac{\Delta T^\circ}{T_3^\circ}$$

In the turbine:

$$\frac{P_5^\circ}{P_4^\circ} = \left(\frac{T_5^\circ}{T_4^\circ}\right)^{\frac{\gamma_t}{(\gamma_t-1)e_t}} = \left[1 - \frac{H_t}{c_{p_t}T_4^\circ}\right]^{\frac{\gamma_t}{(\gamma_t-1)e_t}}$$

Where: $H_t = -h_5^\circ + h_4^\circ$
Is turbine work per unit mass

$$\left(\frac{P_5^\circ}{P_4^\circ}\right)^{\frac{\gamma_n-1}{\gamma_n}e_n} = \left(1 - \frac{H_t}{c_{p_t}T_4^\circ}\right)^{\frac{\delta' e_n}{e_t}} \qquad \text{where:} \quad \delta' \equiv \frac{\gamma_t}{\gamma_t - 1}\,\frac{\gamma_n - 1}{\gamma_n}$$

Turbojet engines without Afterburner

Look first at the case without an afterburner:

$$P_5^\circ = P_6^\circ \,; T_6^\circ = T_5^\circ = T_4^\circ - H_t/c_{p_n} \,; c_{p_t} = c_{p_n} \,; \gamma_t = \gamma_n \,; \delta' = 1 \qquad \frac{P_9}{P_1} = 1$$

$$U^2 = 2\left(c_{p_n}T_4^\circ - H_t\right)\left[1 - \left(1 + \frac{\gamma_d - 1}{2}M_1^2\right)^{-\delta e_n e_d} * (1 - CM_c^2)^{-\frac{\gamma_n - 1}{\gamma_n}e_n} * \left(1 + \frac{H_c}{c_{p_d}T_2^\circ}\right)^{-e_n e_c \delta} * \left(1 - \frac{H_t}{c_{p_t}T_4^\circ}\right)^{-\frac{\delta' e_n}{e_t}}\right]$$

In case when $H_c = H_t = 0$, we recover the ramjet formula!

For a turbojet: $$\underbrace{\left(\mu + 1 - \frac{\dot{m}_{bleed}}{\dot{m}_{fuel}}\right)}_{(\bar{\mu}+1)} H_t = \mu H_c$$

Where: $\bar{\mu} \equiv \mu - \dfrac{\dot{m}_{bleed}}{\dot{m}_{fuel}}$

Turbojet engines with Afterburner

Now consider the case WITH an afterburner:

$$P_6^\circ / P_5^\circ = 1 - C'(M_c')^2 \quad \text{Where:} \quad C' \equiv \frac{\gamma_{ab}}{2} \frac{\Delta T_{ab}^\circ}{T_5^\circ}$$

$$\frac{P_9}{P_1} = 1$$

$$\left(\frac{P_5^\circ}{P_6^\circ}\right)^{\frac{\gamma_n - 1}{\gamma_n} e_n} = \left(1 - C'(M_c')^2\right)^{-\frac{\gamma_n - 1}{\gamma_n} e_n}$$

$$U^2 = 2 c_{p_n} T_6^\circ \left[1 - \left(1 + \frac{\gamma_d - 1}{2} M_1^2\right)^{-\delta e_n e_d} * \left(1 - C M_c^2\right)^{-\frac{\gamma_n - 1}{\gamma_n} e_n} * \left(1 + \frac{H_c}{c_{p_d} T_2^\circ}\right)^{-e_n e_c \delta} * \left(1 - \frac{H_t}{c_{p_t} T_4^\circ}\right)^{-\frac{\delta' e_n}{e_t}} * \left(1 - C'(M_c')^2\right)^{-\frac{\gamma_n - 1}{\gamma_n} e_n} \right]$$

Turbojet engines

Neglecting $\mathcal{O}(M_c^2)$ and $\mathcal{O}\big((M_c')^2\big)$ we have:

$$U^2 = 2c_{p_n}T_6^\circ\left[1-\left(1+\frac{\gamma_d-1}{2}M_1^2\right)^{-\delta e_n e_d} * \left(1+\frac{H_c}{c_{p_d}T_2^\circ}\right)^{-e_n e_c \delta} * \left(1-\frac{H_t}{c_{p_t}T_4^\circ}\right)^{-\frac{\delta' e_n}{e_t}}\right]$$

This implies that: $P_5^\circ \approx P_6^\circ$ and $P_4^\circ \approx P_3^\circ$

Determine the relation between mixture ratio in afterburner and temperature T_6°

Define: $\alpha \equiv \dfrac{mass\ flux\ of\ fuel\ in\ afterburner}{mass\ flux\ of\ fuel\ in\ main\ burner}$

$$\bar{\mu} = \frac{\dot{m}_{air}-\dot{m}_{bleed}}{\dot{m}_{fuel}} = \mu - \frac{\dot{m}_{bleed}}{\dot{m}_{fuel}}$$

This considers only the fuel in the main combustor.

$$\bar{\mu}-\mu_{st} = \frac{\dot{m}_{air}-\dot{m}_{bleed}-\dot{m}_{air\,st}}{\dot{m}_{fuel}} = \frac{mass\ flux\ of\ air\ in\ afterburner}{mass\ flux\ \ of\ fuel\ in\ main\ burner}$$

Turbojet engines

$$\mu_{ab} \equiv \frac{\bar{\mu} - \mu_{st}}{\alpha} = \left.\frac{mass\ of\ air}{mass\ of\ fuel}\right|_{ab}$$

This is mixture ratio for afterburner.
Afterburner also has inflow of products from main combustor.

Now, write the energy balance:

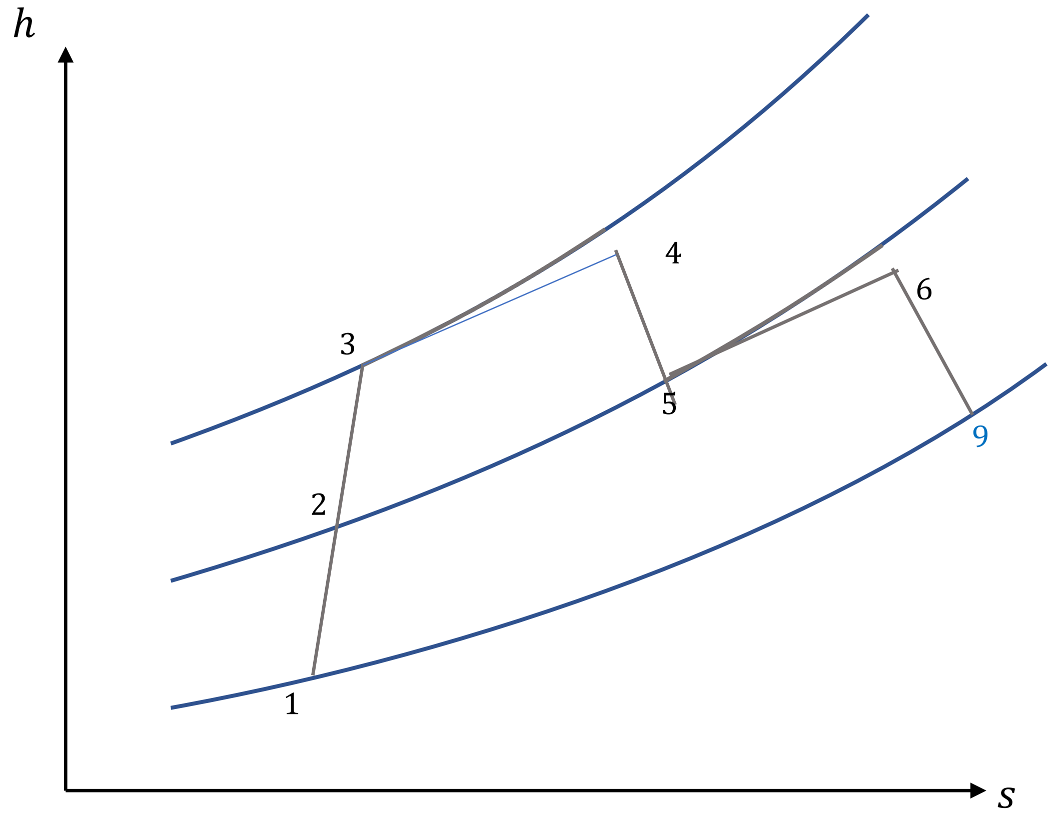
$$\left[\dot{m}_a - \dot{m}_{bleed} + \dot{m}_f\ \right] h_5^{\circ}\left(T_5^{\circ}, main\ burner\ products\right) + \alpha \dot{m}_f \eta_{ab} Q = \left[\dot{m}_a - \dot{m}_{bleed} + (1+\alpha)\dot{m}_f\right] h_6^{\circ}\left(T_6^{\circ}, afterburner\ products\right)$$

Note: $\eta_{ab} \equiv$ afterburner efficiency ; $\dot{m}_f$ is fuel mass flux in main burner; same fuel and Q in both burners.

$$[\bar{\mu}+1]h_5^{\circ} + \alpha \eta_{ab} Q = [\bar{\mu}+1+\alpha]h_6^{\circ}$$

So: $$h_6^{\circ} = \frac{[\bar{\mu}+1]h_5^{\circ} + \alpha \eta_{ab} Q}{[\bar{\mu}+1+\alpha]}$$

Turbojet engines

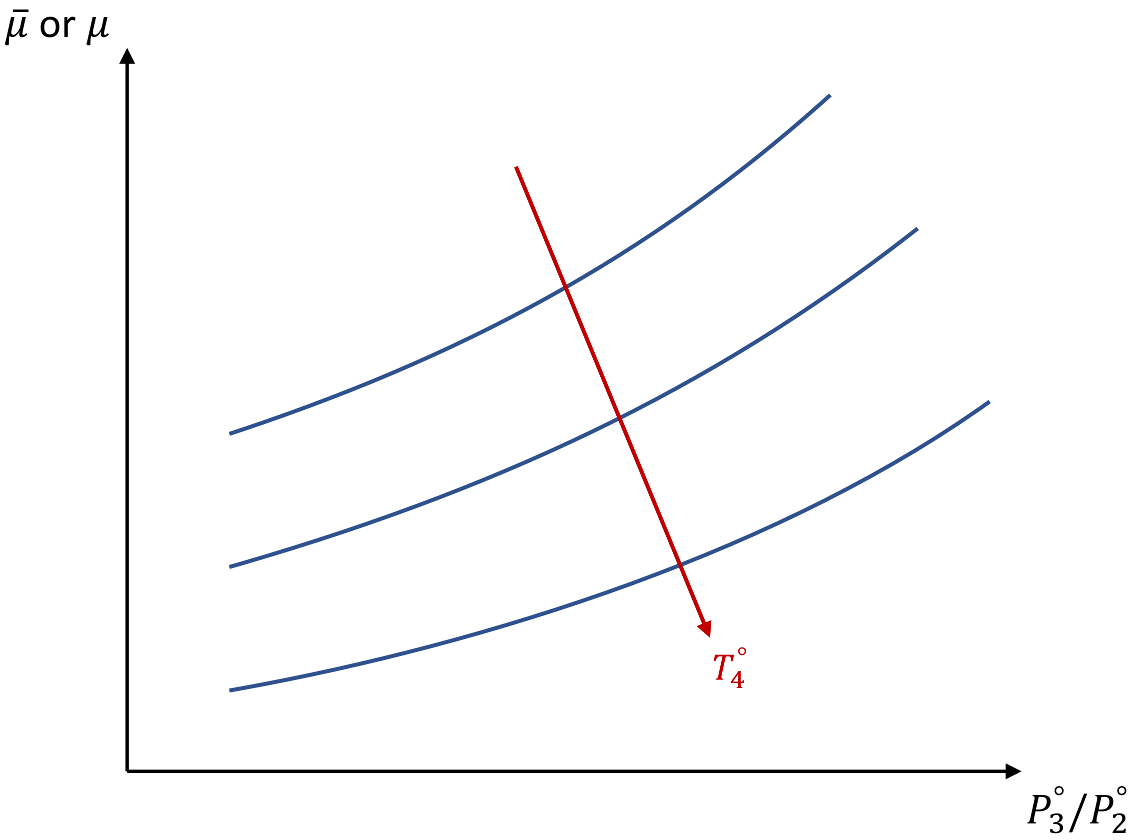


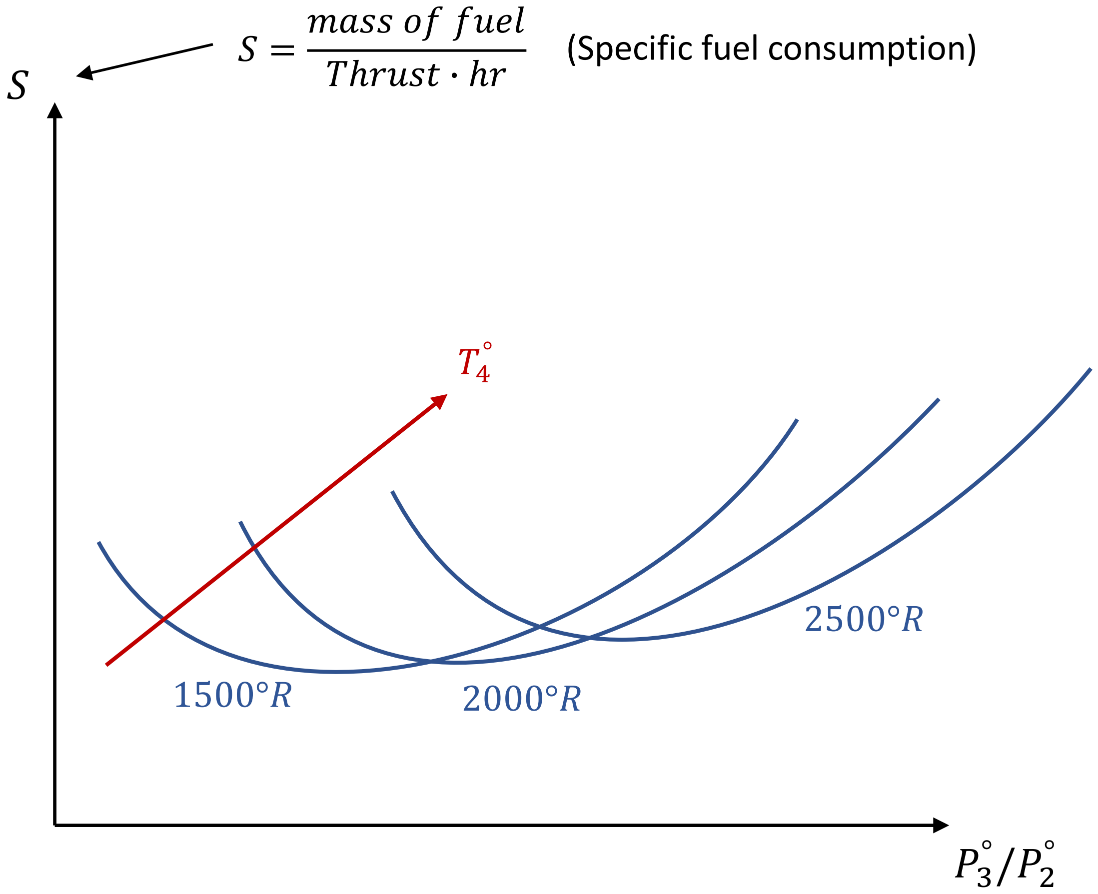
Determine the relationship between h_6° (or T_6°) and α

Note: $h_5^\circ = h_4^\circ - H_t$

The relationship between h_4° and μ remain the same as for the ramjet!

Turbojet engines





An optimum compression ratio exists. Use of afterburner will increase specific fuel consumption. Note that ramjet S is still higher. Units are different from the book (with a factor of g)!

References

[1] Hill, Philip G., and Carl R. Peterson. *Mechanics and Thermodynamics of Propulsion*. Reading, Mass: Addison-Wesley Longman, 1992.