Lecture 8
Oblique Shockwaves

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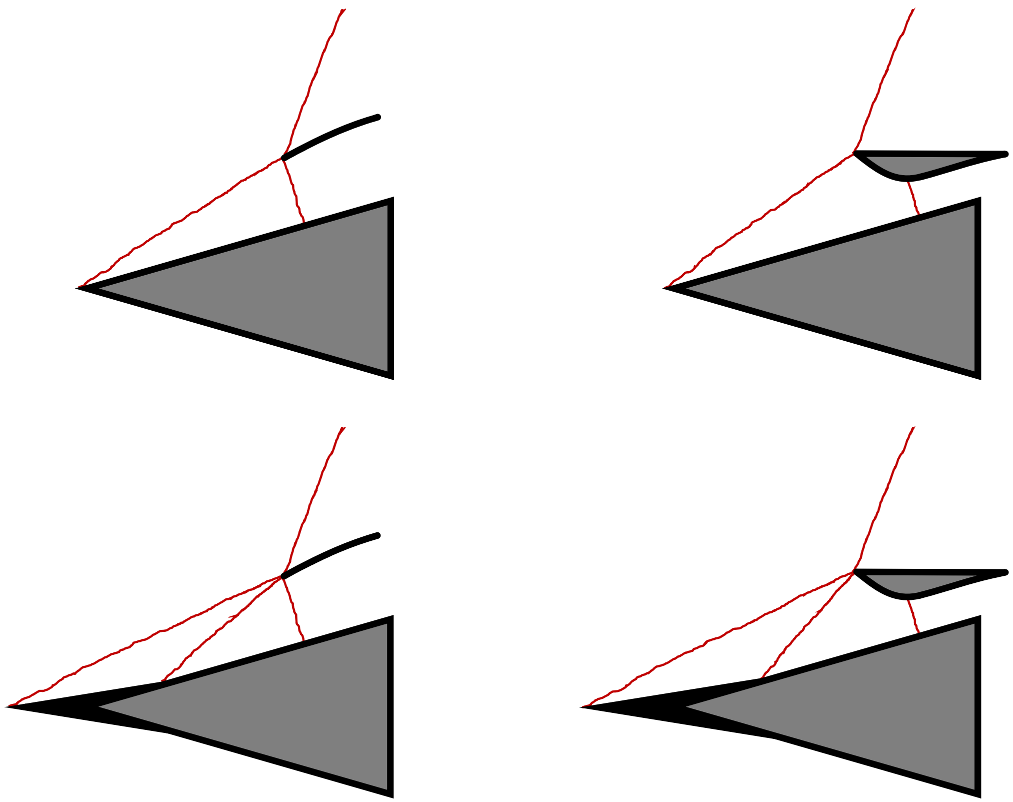
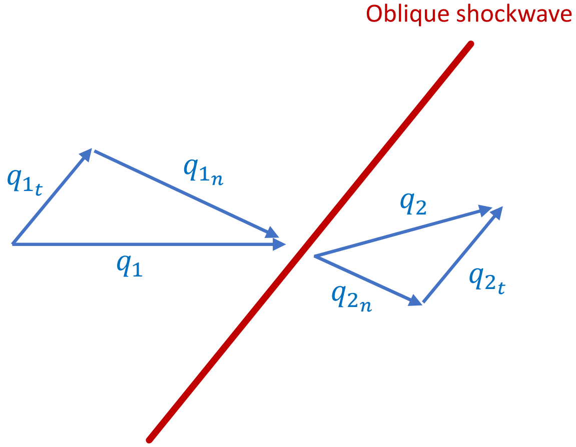
External Compression with Oblique Shocks



Flow goes through the external oblique shock resulting in a lower Mach number supersonic flow behind the shock. The flow then goes through either a normal shock (with subsonic flow behind the shock) with a divergent diffuser or Kantrowitz-Donaldson diffuser!

At higher flight Mach numbers, inlet ramps with multiple angles might be used to promote more oblique shocks followed by the terminating normal shock!

Oblique Shockwave Relations



Supersonic flow can occur behind an oblique shock. Only the normal component must be subsonic behind the shockwave. Behind an oblique shock, the total velocity and Mach number are reduced.

$$\rho_1 q_{1_n} = \rho_2 q_{2_n}$$

$$\rho_1 q_{1_t} q_{1_n} = \rho_2 q_{2_t} q_{2_n} \rightarrow q_{1_t} = q_{2_t}$$

$$P_1 + \rho_1 q_{1_n}^2 = P_2 + \rho_2 q_{2_n}^2$$

$$h^\circ{}_1 = h^\circ{}_2 = h_1 + q_1^2/2 = h_2 + q_2^2/2$$

$$h_1 + q_{1_n}^2\big/2 = h_2 + q_{2_n}^2\big/2$$

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1} + q_{1_n}^2\big/2 = \frac{\gamma}{\gamma-1}\frac{P_2}{\rho_2} + q_{2_n}^2\big/2$$

$$\frac{a_1^2}{\gamma-1} + q_{1_n}^2\big/2 = \frac{a_2^2}{\gamma-1} + q_{2_n}^2\big/2 = \frac{(\gamma+1)}{2(\gamma-1)}a_*^2$$

Value of a when $q = a$

a_* is the critical sonic velocity for given h°

Oblique Shockwave Relations

$$\rho_1 q_{1_n} = \rho_2 q_{2_n} \qquad P_1 + \rho_1 q_{1_n}^2 = P_2 + \underbrace{\rho_2 q_{2_n}^2}_{\rho_1 q_{1_n} q_{2_n}} \qquad \frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1} + \frac{q_{1_n}^2}{2} = \frac{\gamma}{\gamma-1}\frac{P_2}{\rho_2} + \frac{q_{2_n}^2}{2} = \frac{(\gamma+1)}{2(\gamma-1)} a_*^2$$

$$P_2 - P_1 = \rho_1 q_{1_n}\left[q_{1_n} - q_{2_n}\right]$$

$$\frac{P_2}{\rho_2} = \left[\frac{(\gamma+1)}{2(\gamma-1)} a_*^2 - \frac{q_{2_n}^2}{2}\right]\frac{\gamma-1}{\gamma}$$

$$\frac{P_1}{\rho_1} = \left[\frac{(\gamma+1)}{2(\gamma-1)} a_*^2 - \frac{q_{1_n}^2}{2}\right]\frac{\gamma-1}{\gamma}$$

Combining the relations

$$\frac{P_2}{\rho_2 q_{2_n}} - \frac{P_1}{\rho_1 q_{1_n}} = q_{1_n} - q_{2_n}$$

$$\boxed{a_*^2 = q_{1_n} q_{2_n}}$$

Prandtl Relation

Oblique Shockwave Relations

Solutions: $q_{1_n} = q_{2_n}$, $P_1 = P_2$, etc. (Trivial Solution!)

Prandtl Relation: $a_*^2 = q_{1_n} q_{2_n}$

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1}\left(\frac{q_{1_n}}{a_1}\right)^2 - \frac{\gamma-1}{\gamma+1} = \frac{1}{\frac{2\gamma}{\gamma+1}\left(\frac{q_{2_n}}{a_2}\right)^2 - \frac{\gamma-1}{\gamma+1}}$$

$$= \frac{2\gamma}{\gamma+1} {M_{1n}}^2 - \frac{\gamma-1}{\gamma+1} = \frac{1}{\frac{2\gamma}{\gamma+1} {M_{2n}}^2 - \frac{\gamma-1}{\gamma+1}}$$

$$\frac{q_{1_n}}{a_1} = M_{1n} \quad ; \frac{q_{2_n}}{a_2} = M_{2n}$$

$$\frac{\rho_2}{\rho_1} = \frac{\gamma-1}{\gamma+1} + \frac{1}{\gamma-1}\left(\frac{q_{1_n}}{a_1}\right)^2 = \frac{1}{\frac{\gamma-1}{\gamma+1} + \frac{1}{\gamma-1}\left(\frac{q_{2_n}}{a_2}\right)^2}$$

$$= \frac{\gamma-1}{\gamma+1} + \frac{1}{\gamma-1} {M_{1n}}^2 = \frac{1}{\frac{\gamma-1}{\gamma+1} + \frac{1}{\gamma-1} {M_{2n}}^2}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_2}{\rho_1}$$

Oblique Shockwave Relations

$$\frac{P_2}{P_1} = \frac{\dfrac{\rho_2}{\rho_1} - \dfrac{\gamma - 1}{\gamma + 1}}{1 - \dfrac{\gamma - 1}{\gamma + 1}\dfrac{\rho_2}{\rho_1}}$$ Rankine-Hugoniot Relation

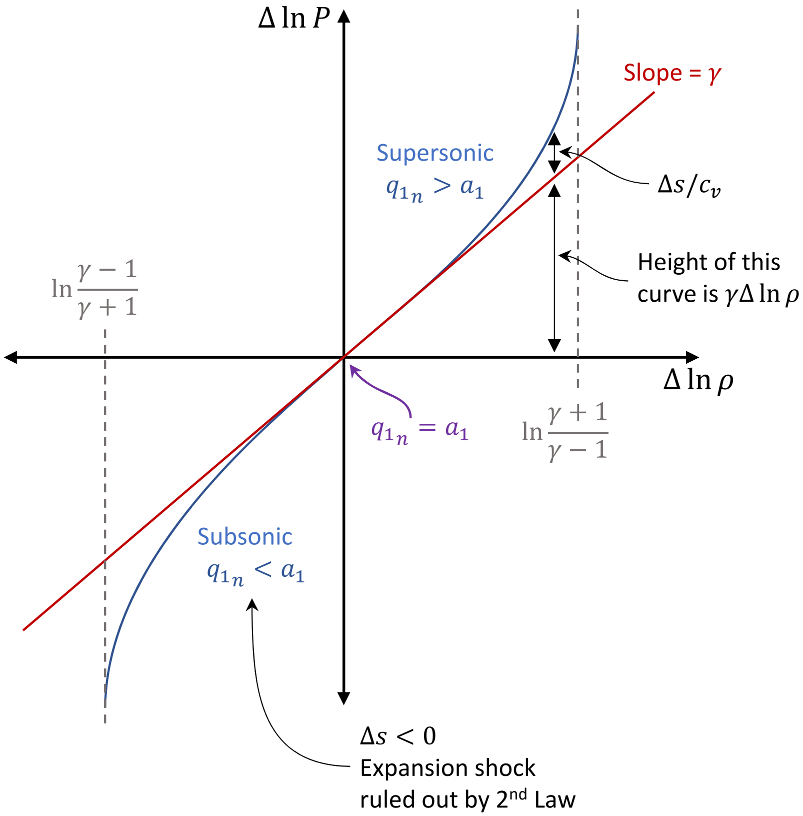
$$\Delta \ln P = \ln P_2/P_1 \qquad \Delta \ln \rho = \ln \rho_2/\rho_1$$

$$\frac{\Delta s}{c_v} = \Delta \ln P - \gamma \Delta \ln \rho$$

Isentropic: $\dfrac{\Delta \ln P}{\Delta \ln \rho} = \gamma$

Non-Isentropic: $\dfrac{\Delta \ln P}{\Delta \ln \rho} > \gamma$ Since $\Delta s > 0$ for adiabatic

Oblique Shockwave Relations



The non-linearity of the curve implies that it is better to achieve pressure rise (compression) through multiple [oblique] shocks

$$\Delta \ln \rho = \ln \frac{\rho_3}{\rho_1} = \ln \frac{\rho_2}{\rho_1} + \ln \frac{\rho_3}{\rho_2}$$

There is less entropy increase with more shocks

$$Tds = dh - dP/\rho$$

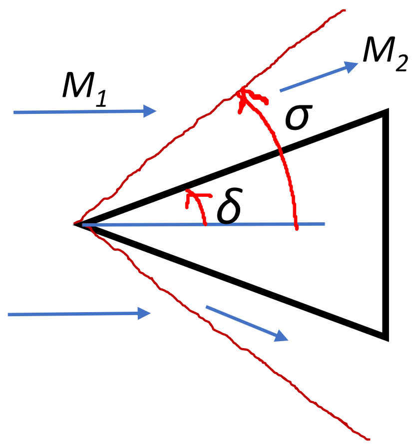
$$ds = c_p \frac{dT}{T} - \frac{dP}{\rho T} = c_p \left(\frac{dP}{\rho} - \frac{d\rho}{\rho} \right) - R \frac{dP}{\rho}$$

$$ds = c_v \frac{dP}{P} - c_p \frac{d\rho}{\rho}$$

$$\frac{\Delta s}{c_v} = d(\ln P) \ - \gamma d(\ln \rho)$$

$$\frac{\Delta s}{c_v} = \Delta \ln P - \gamma \Delta \ln \rho$$

Wave Angle σ versus Mach Number M_1 and Turning Angle δ for Air, $\gamma = 1.4$



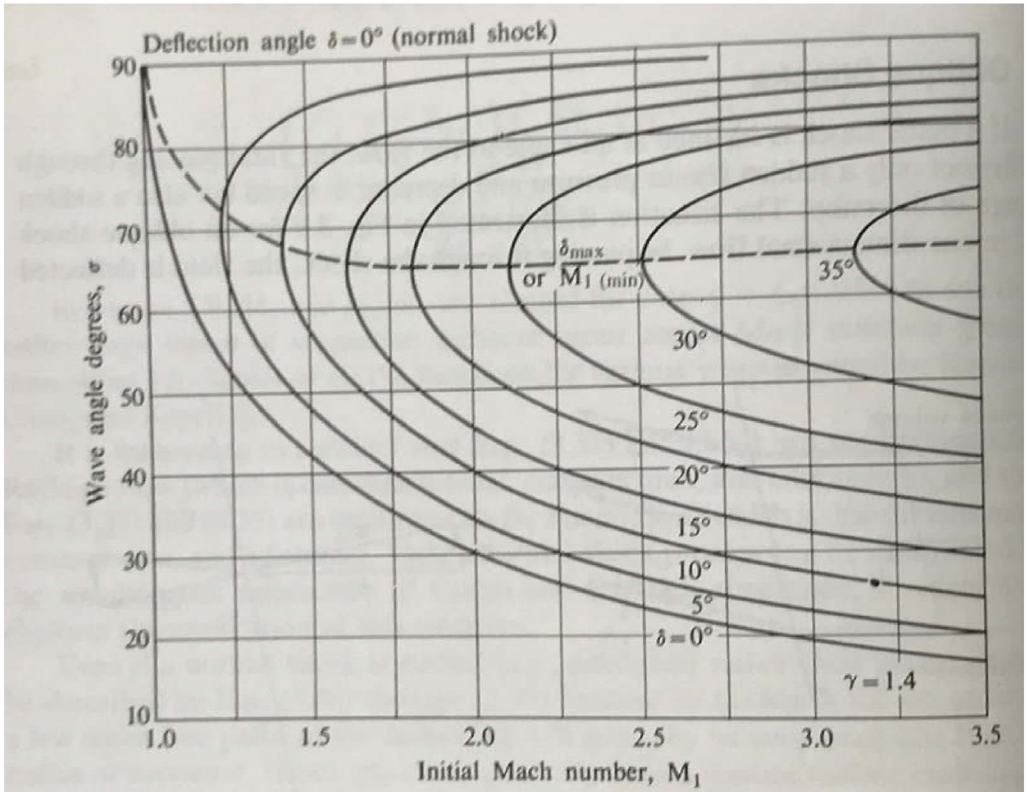
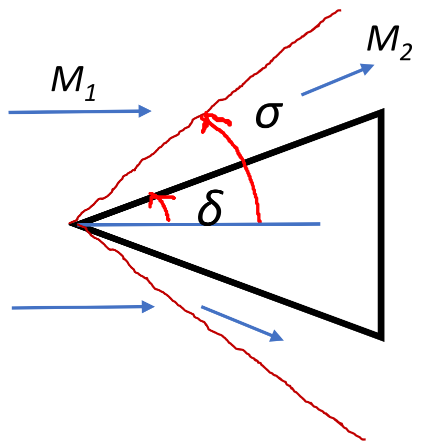


FIGURE 3.10 Shock angle versus inlet Mach number and turning angle [1]. Curves above dashed line hold for $M_2 < 1$, and curves below hold for $M_2 > 1$. (From Shapiro [1].)

Stagnation Pressure Ratio versus Mach Number M_1 and Turning Angle δ for Air, $\gamma = 1.4$



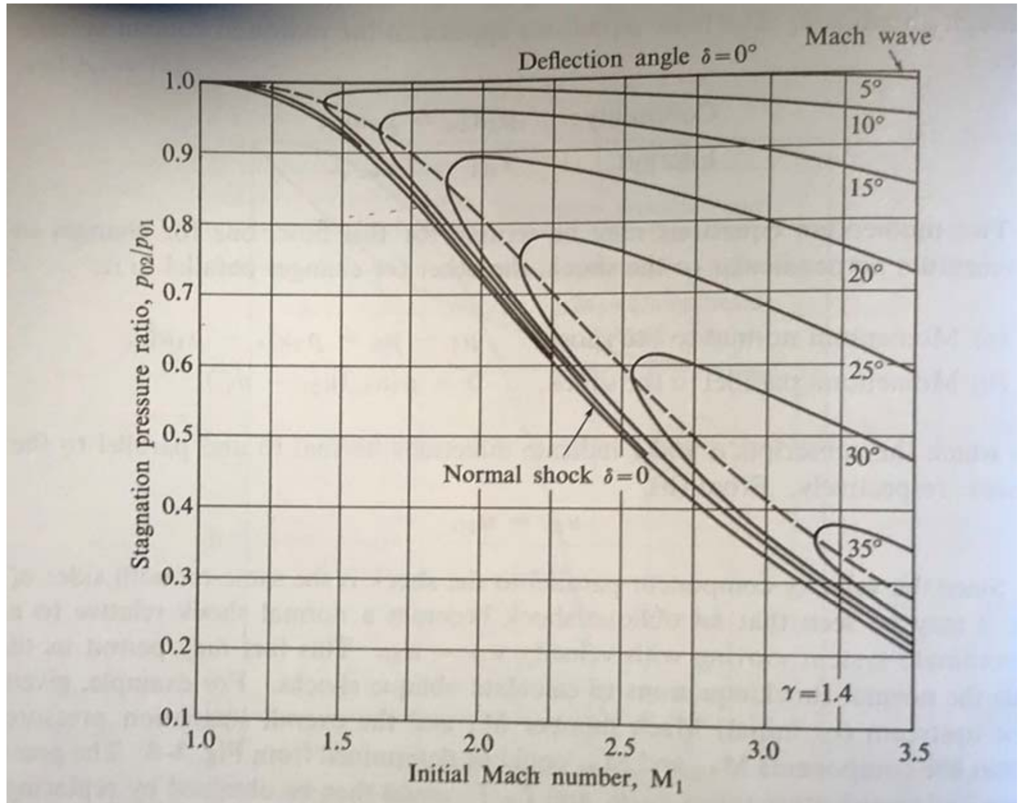
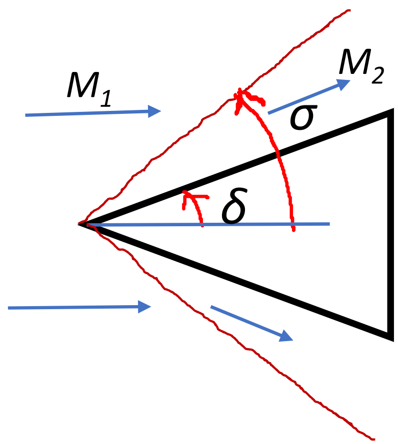


FIG. 3–11. Stagnation pressure ratio versus inlet Mach number, with turning angle parameter [1]. Curves above dashed line hold for $M_2 > 1$ and curves below hold f $M_2 < 1$. (After Shapiro [1].)

Final Mach Number M_2 versus Mach Number M_1 and Turning Angle δ for Air, $\gamma = 1.4$



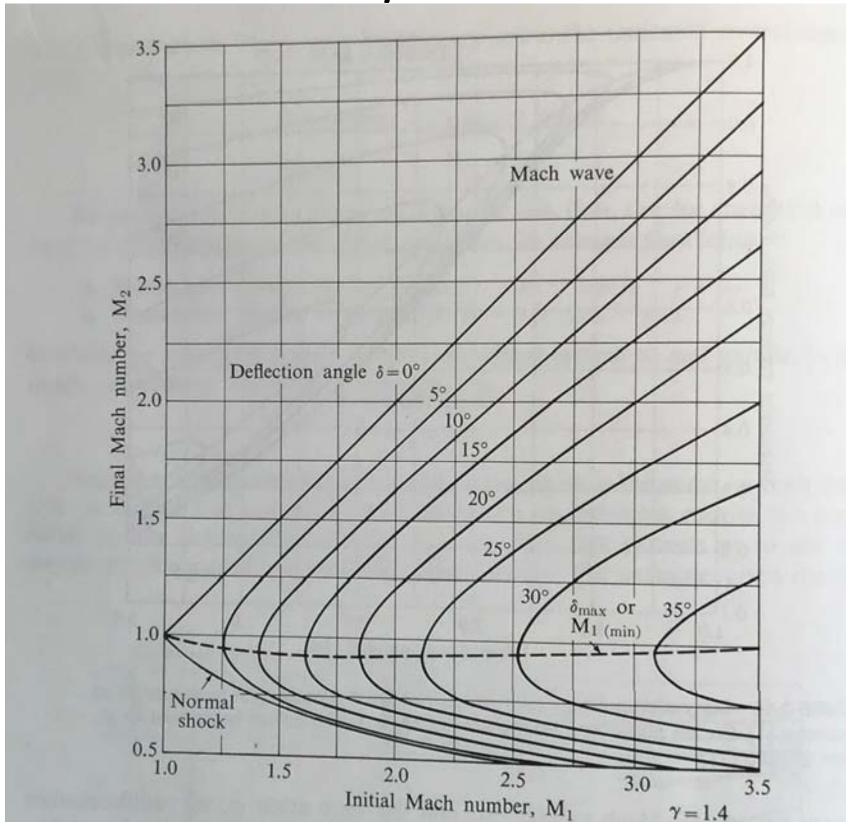


FIGURE 3.12 Exit Mach number versus inlet Mach number, with turning angle as parameter [1]. Curves above dashed line correspond to small σ, and curves below correspond to large σ. (From Shapiro [1].)