Lecture 7
Air Intakes

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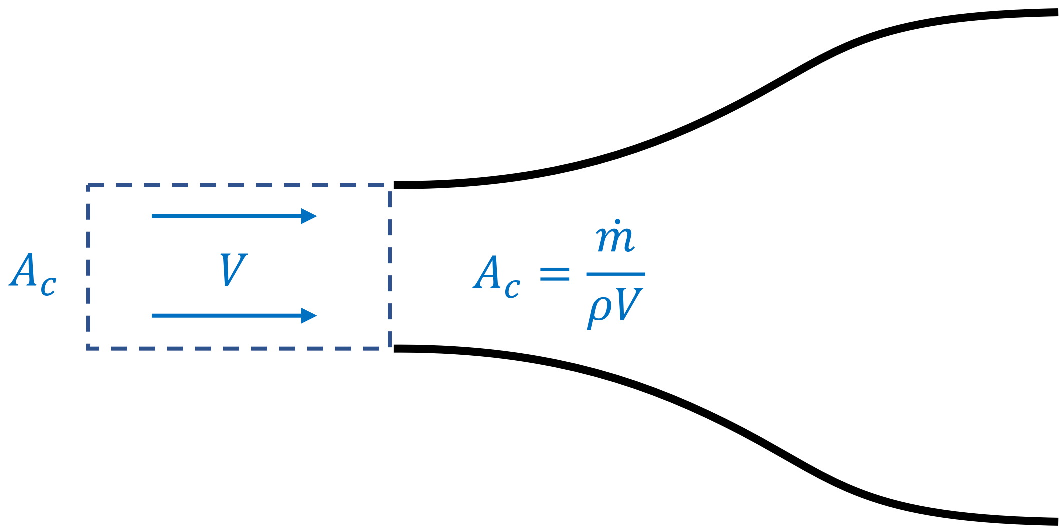
Subsonic Air Intakes & Diffusers

Entrance area equal to the capture area is one possibility (at the design point only)!



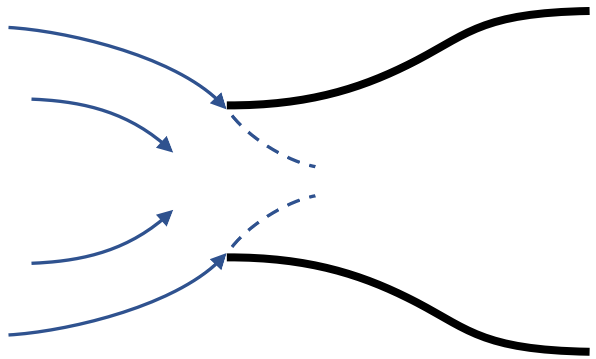
Internal diffusion only

There are inefficiencies at an angle of attack since separation can occur! Separation occurs in fluid mechanical situations where the flow is required to turn too sharply!

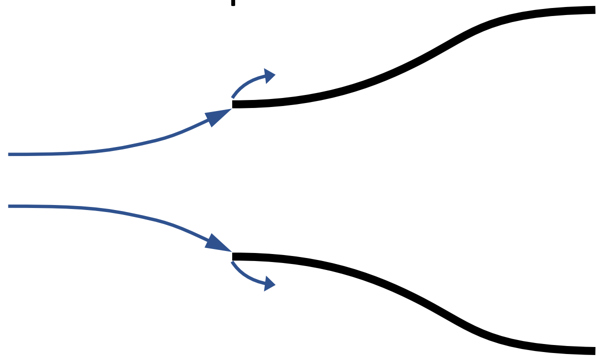
Subsonic Air Intakes & Diffusers

Separation can occur at zero angle of attack in off-design conditions!

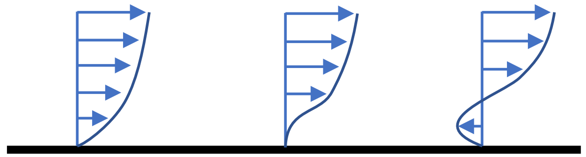
High mass flow, Low speed (Takeoff)
Large capture area

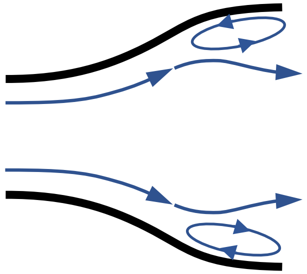


Low mass flow, High speed (Cruise)
Smaller capture area



Separation may also occur internally due to an adverse pressure gradient in the boundary layer!



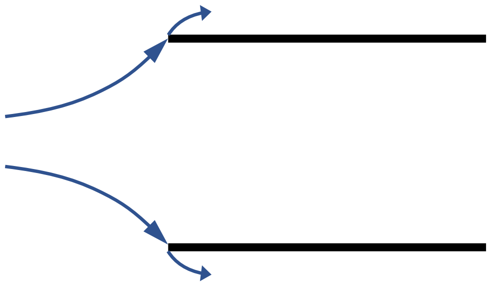


This results in less diffusion because the effective area is reduced by the separation zone!

Subsonic Air Intakes & Diffusers

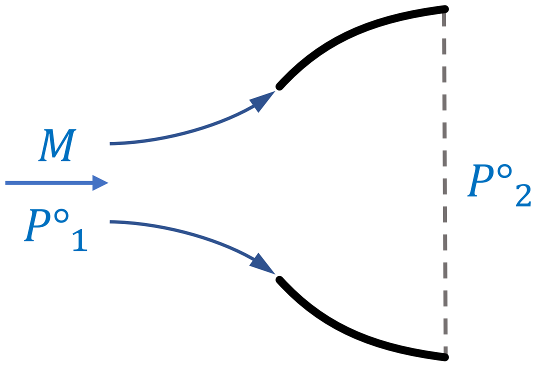
External diffusion helps here since there is no boundary layer that occurs externally!

The increase in area occurs externally here



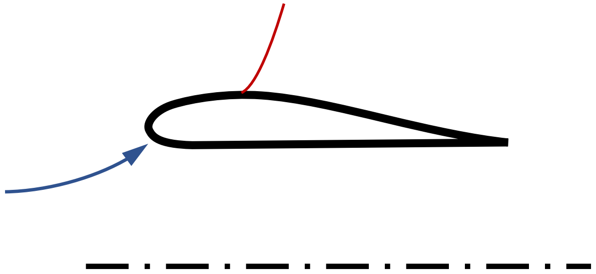
There is still a risk of separation externally, at the entrance here so a combination is more desirable!

The increase in area occurs externally in addition to internally here



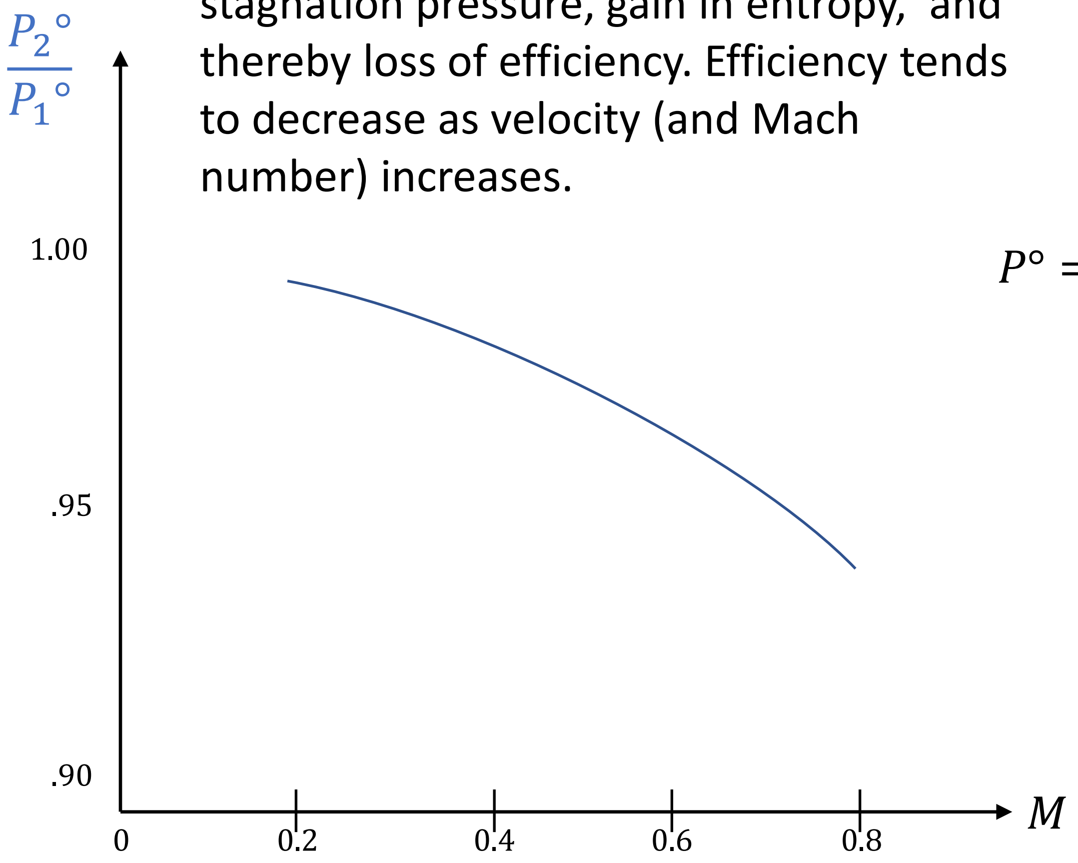
Highest Efficiency!

If the entrance is faired externally (contoured lip) but the internal area is constant, shocks can form causing additional drag!



Subsonic Air Intakes & Diffusers

Non-isentropric flow results in loss of stagnation pressure, gain in entropy, and thereby loss of efficiency. Efficiency tends to decrease as velocity (and Mach number) increases.



For isentropic flow and $c_p = Const = \frac{\gamma R}{\gamma - 1}$

$$T^\circ = Const = T_1\left[1 + \frac{\gamma - 1}{2}M_1^2\right] = T_2\left[1 + \frac{\gamma - 1}{2}M_2^2\right]$$

$$P^\circ = Const = P_1\left[1 + \frac{\gamma - 1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}} = P_2\left[1 + \frac{\gamma - 1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}$$

Combined First and Second Law of Thermodynamics

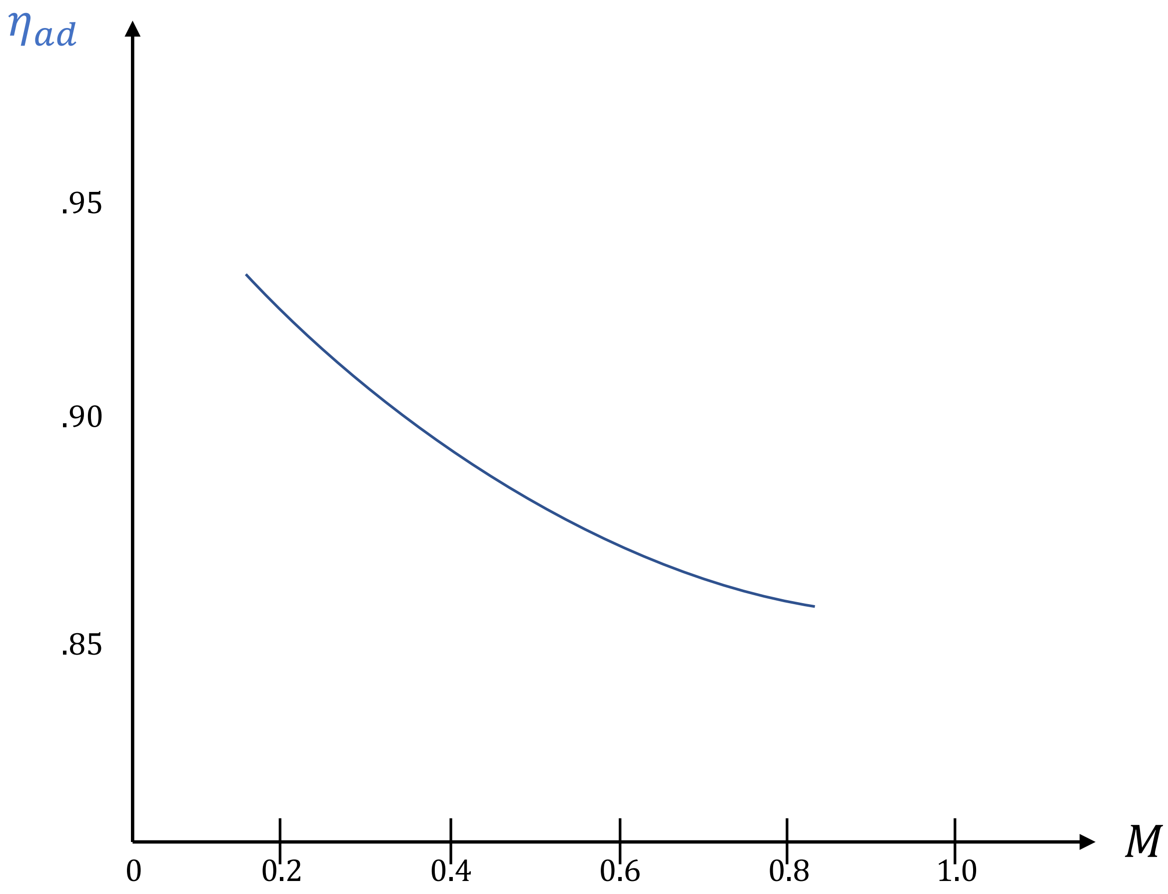
$$T\,ds = dh - (1/\rho)\ dP = c_p\,dT - (RT/P)\,dP$$

$$ds = c_p\,d(\log T) - R\,d(\log P)$$

$$\Delta s = s_2 - s_1 = c_p \log(T_2/T_1) - R\log(P_2/P_1)$$

$$\Delta s = s_2 - s_1 = c_p \log(T_2{}^o/T_1{}^o) - R\log(P_2{}^o/P_1{}^o)$$

Subsonic Air Intakes & Diffusers

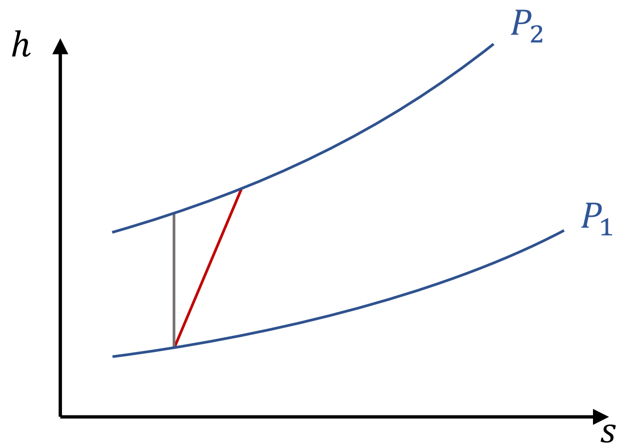


h° = h + q² /2 ; q² = u² + v² + w²

$$\eta = \frac{h_{2,s} - h_1}{h_2 - h_1} = \frac{q_{1,s}^2 - q_2^2}{q_1^2 - q_2^2} \approx \frac{q_{1,s}^2}{q_1^2}$$

where $h°_1 = h°_2$

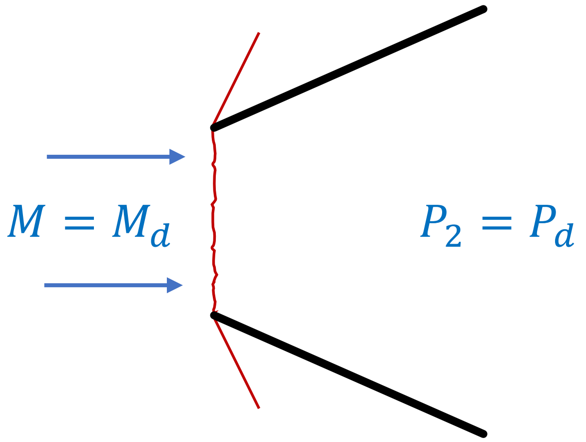
P_1, P_2 are given



Supersonic Air Intakes & Diffusers

It is still desirable to have subsonic flow leave the diffuser and enter the compressor or combustor. Otherwise, there will be severe pressure losses due to shocks occurring in the blade passages! Supersonic combustors for hypersonic ramjets are becoming a reality – still an area of research and development!

Consider a divergent diffuser with a normal shock wave at the entrance at the design condition:

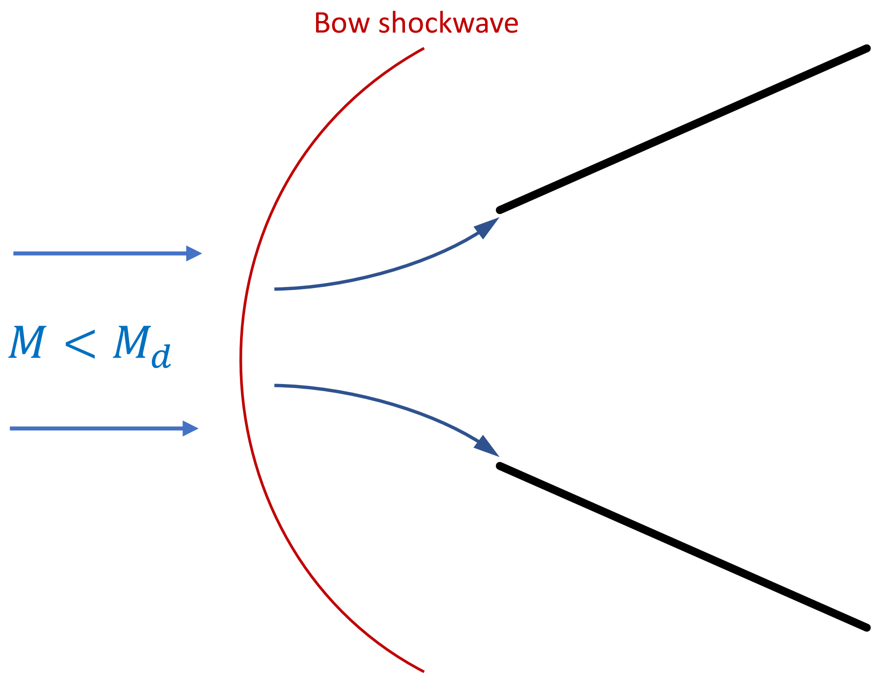


The [static] pressure increases across the shock, then subsonic diffusion occurs!

Stagnation pressure decreases across the shock causing entropy to increase.

Supersonic Air Intakes & Diffusers

Off-design operation

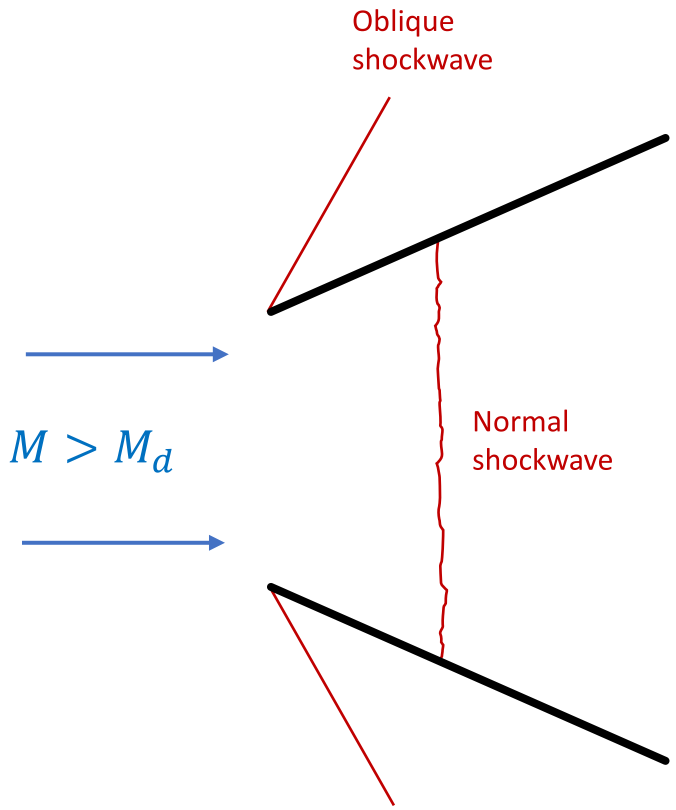


1a. $M < M_d, P_2 = P_d$ OR 1b. $M = M_d, P_2 > P_d$

Bow shock occurs with external diffusion and $\dot{m} < \dot{m}_d$ since the capture area is less!

Supersonic Air Intakes & Diffusers

Off-design operation

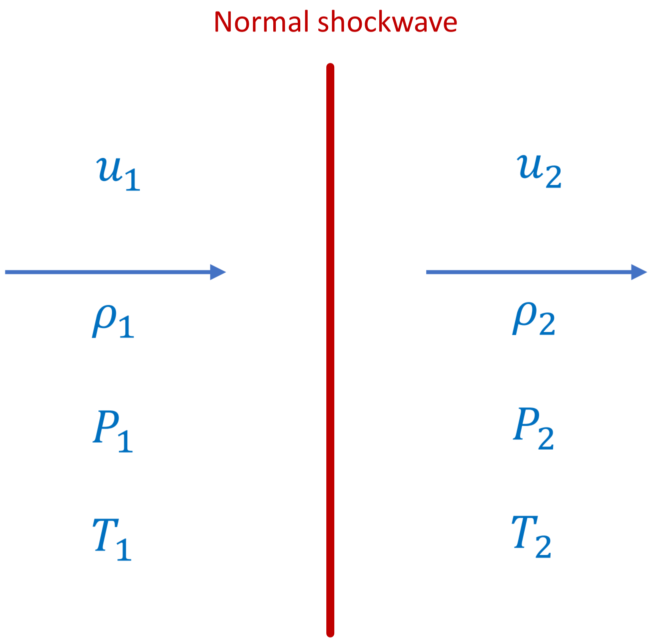


2a. $M > M_d$, $P_2 = P_d$ OR 2b. $M = M_d$, $P_2 < P_d$

Shock moves into duct and efficiency decreases, but $\dot{m} = \dot{m}_d$ for case 2b.

Shockwaves

Consider the normal shock relations:



- Continuity: $\rho_1 u_1 = \rho_2 u_2$
- Momentum: $P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$
- Energy: $c_p T_1 + {u_1^2}/{2} = c_p T_2 + {u_2^2}/{2}$
- Ideal Gas: $P_1 = \rho_1 R T_1$, $P_1 = \rho_1 R T_1$

Shockwaves

The following relations may be derived from there if $a = \sqrt{\gamma R T}$

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}$$

Where: $\begin{cases} M_1 \equiv \dfrac{u_1}{a_1} \\ M_2 \equiv \dfrac{u_2}{a_2} \end{cases}$

$$\frac{P°_2}{P°_1} = \left[\frac{\frac{\gamma + 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_1^2}\right]^{\frac{\gamma}{\gamma - 1}} \frac{1}{\left[\frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}\right]}$$

$$\sigma = \int \frac{dh}{T} = \int c_p \frac{dT}{T}$$

For constant c_p, $\sigma = c_p \ln T$

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

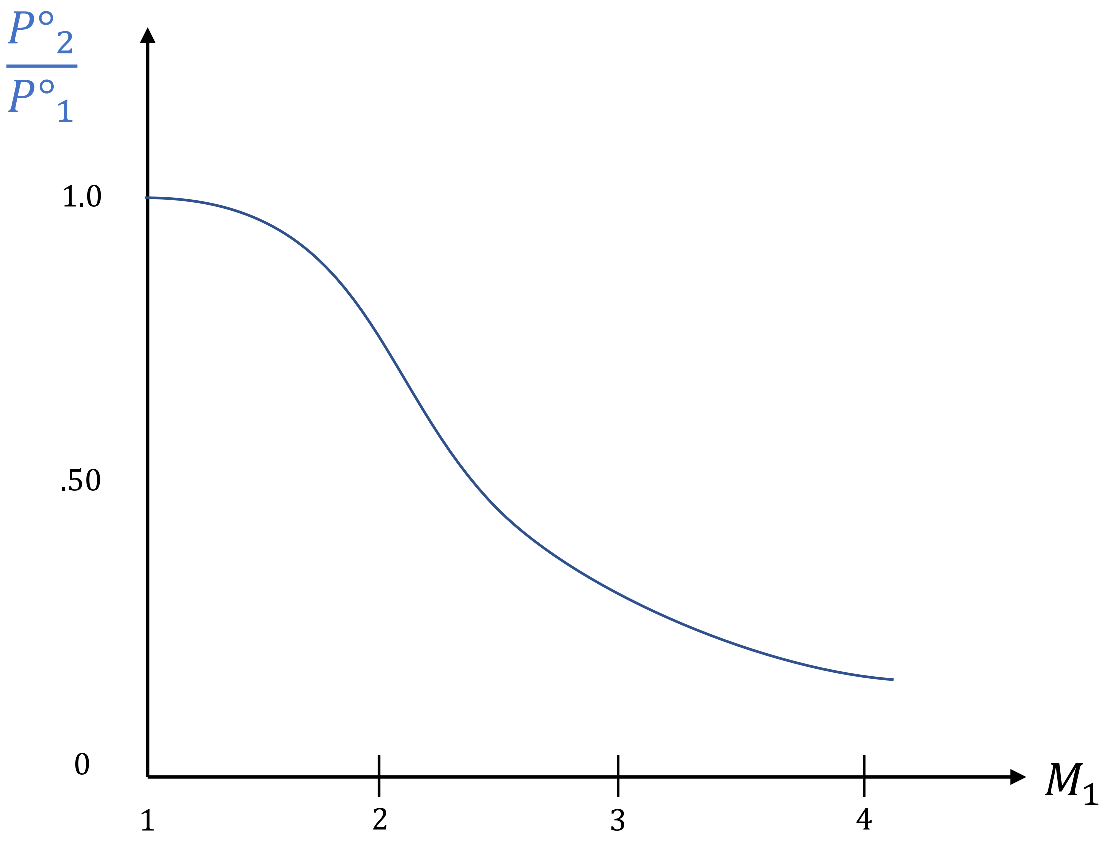
$$\Delta s = \Delta\sigma(T°) - R \ln \frac{P°_2}{P°_1}$$

$$\Delta s = R \ln \frac{P°_1}{P°_2}$$

Since $\Delta\sigma = \sigma(T°_2) - \sigma(T°_1) = 0$

Shockwaves

Entropy increases across the shockwave and results in loss of efficiency!



$$\frac{P°_2}{P°_1} = \left[\frac{\frac{\gamma+1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \frac{1}{\left[\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right]}$$

Obviously, this type of diffuser can only be acceptable for low M_1, $M_1 \leq 1.5$ typically!

Isentropic Flow

Consider isentropic flow (no shockwave) with variable area – diffuser or nozzle

Continuity: $\dot{m} = \rho u A = Constant$

$$\dot{m} = PuA \ / \ RT$$

Energy: $c_p T^\circ = c_p T + \frac{1}{2}u^2 = Constant$

Entropy: $s = Constant$, but $T^\circ = Constant$

This implies $P^\circ = Constant, \rho^\circ = Constant$

$$\frac{T^\circ}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{P^\circ}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho^\circ}{\rho} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{1}{\gamma - 1}}$$

Isentropic Flow

It follows that: $u = M\sqrt{\gamma RT} = M\sqrt{\dfrac{\gamma RTo}{1+\dfrac{\gamma-1}{2}M^2}}$

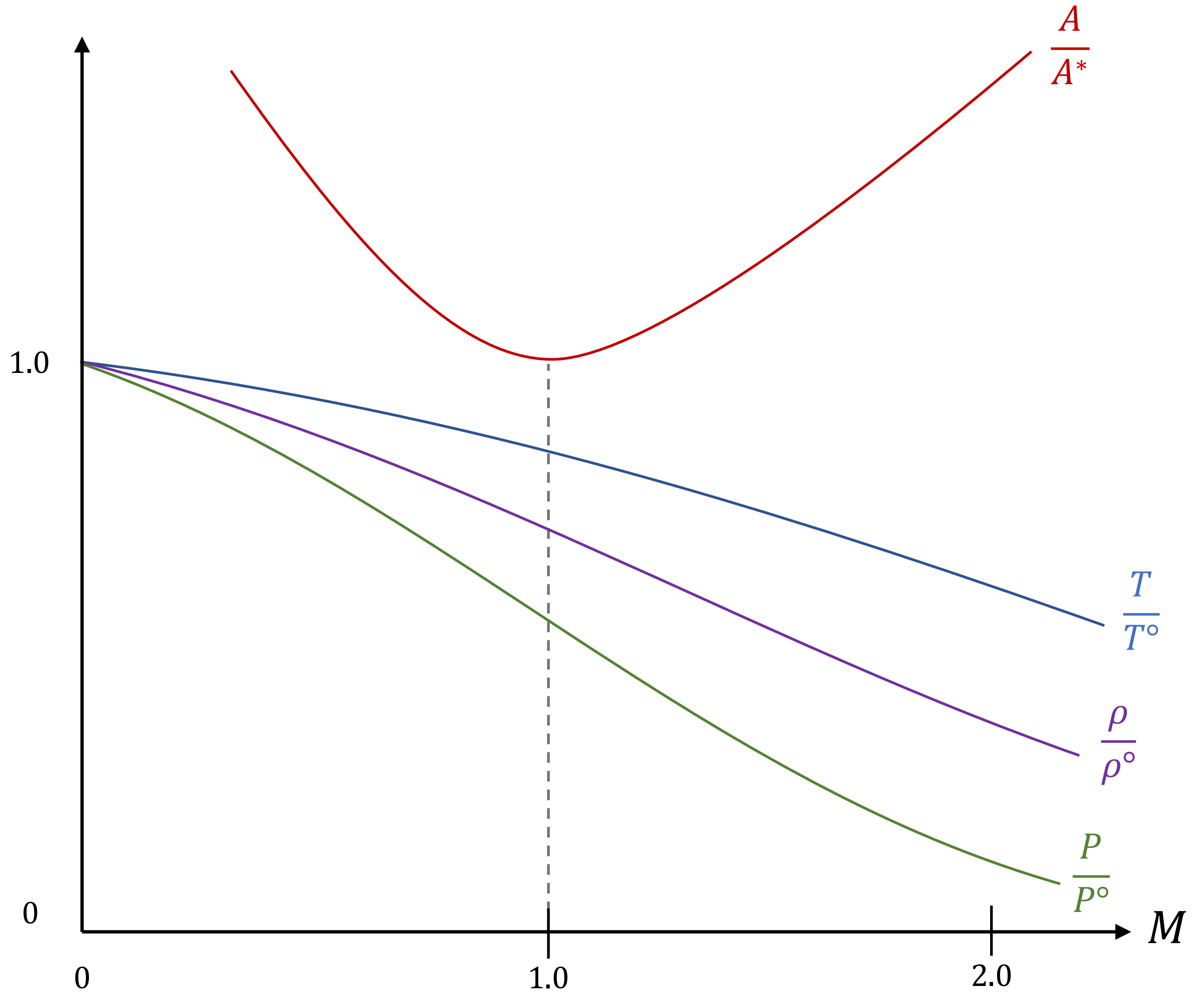
And therefore: $\dfrac{\dot{m}}{A} = \dfrac{P^\circ\sqrt{\gamma}}{\sqrt{RT^\circ}}M\left[\dfrac{1}{1+\dfrac{\gamma-1}{2}M^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$

At $M = 1$, $A = A^*$, So That: $\dfrac{\dot{m}}{A^*} = \dfrac{P^\circ\sqrt{\gamma}}{\sqrt{RT^\circ}}\left[\dfrac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$

Dividing

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

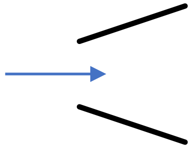
Isentropic Flow



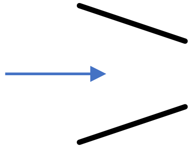
$\gamma = 1.4$ (Air)

Isentropic Flow

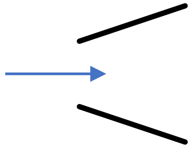
Supersonic flow with INCREASING area			
$M \uparrow$	$T \downarrow$	$P \downarrow$	$\rho \downarrow$



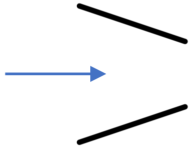
Supersonic flow with DECREASING area			
$M \downarrow$	$T \uparrow$	$P \uparrow$	$\rho \uparrow$



Subsonic flow with INCREASING area			
$M \downarrow$	$T \uparrow$	$P \uparrow$	$\rho \uparrow$



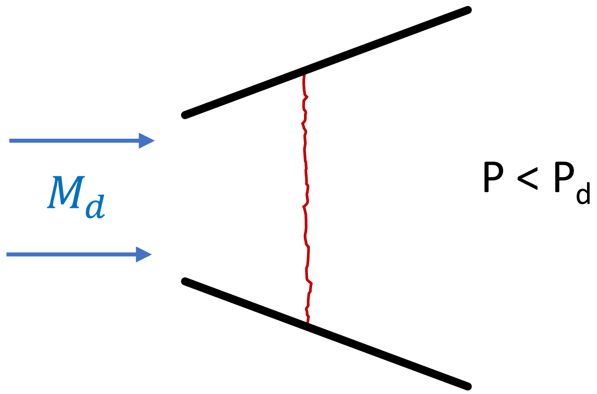
Subsonic flow with DECREASING area			
$M \uparrow$	$T \downarrow$	$P \downarrow$	$\rho \downarrow$

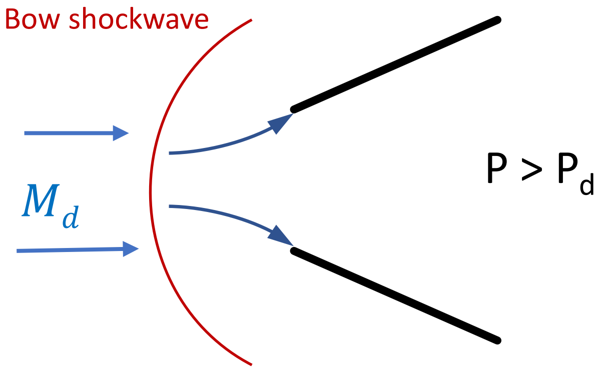


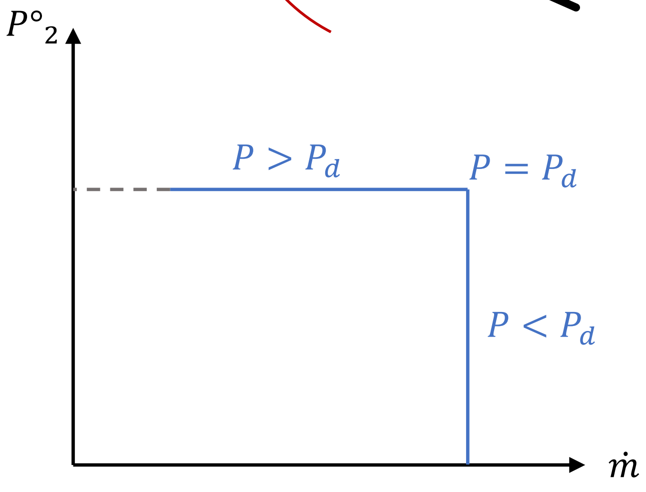
Piecewise Isentropic Flow

Isentropic flow upstream of the shock; entropy gain through the shock ; isentropic flow downstream of the shock.

When the shock moves into the duct:
$M_{shock} > M_{entrance} = M_d$



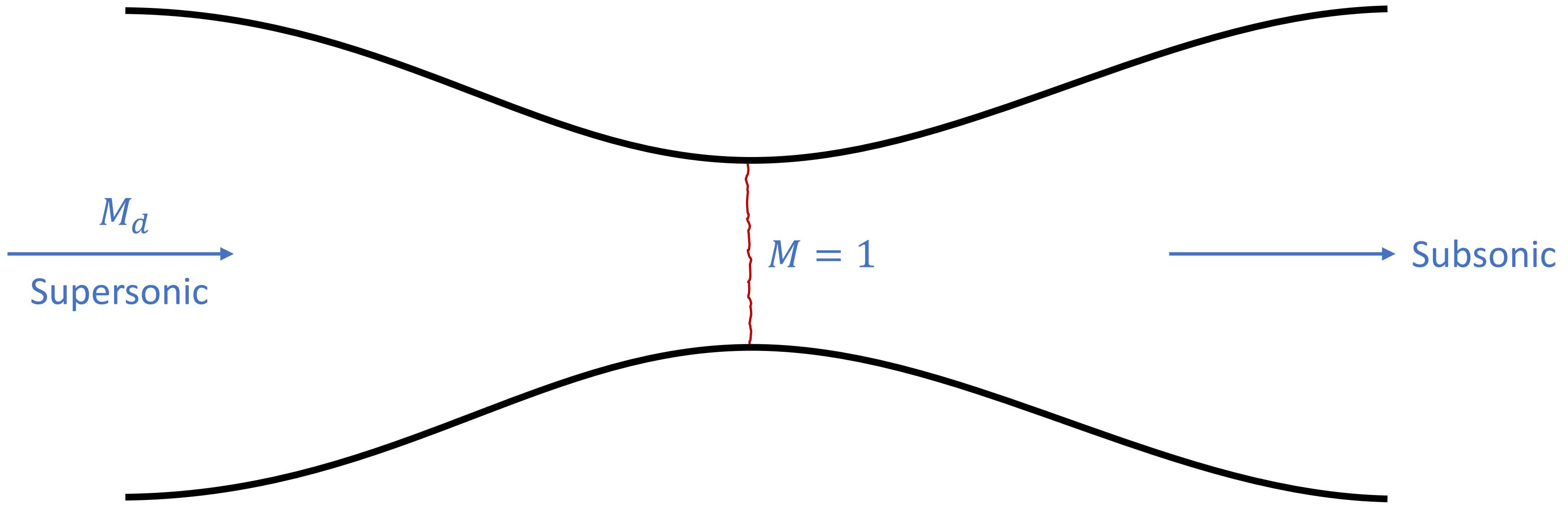




From previous results, the stagnation pressure loss for shocks is greater at higher Mach number, so this is less efficient!

Continuous Supersonic Diffuser

Inverse of a nozzle



Designed to give sonic conditions at the throat when $M = M_d$

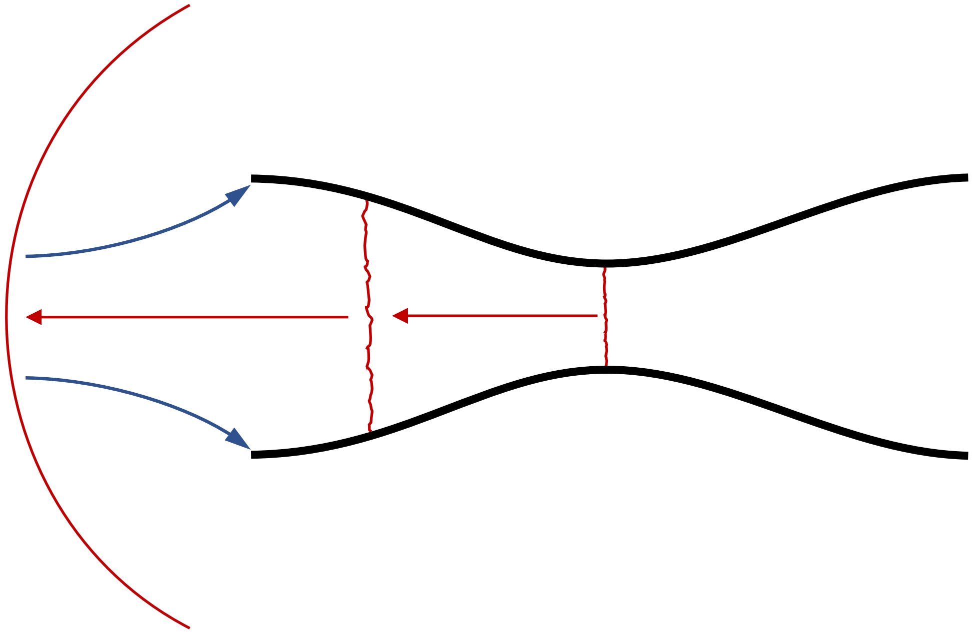
M_d determines the geometry, $A_{entrance} \,/\, A_{throat}$

Continuous Supersonic Diffuser

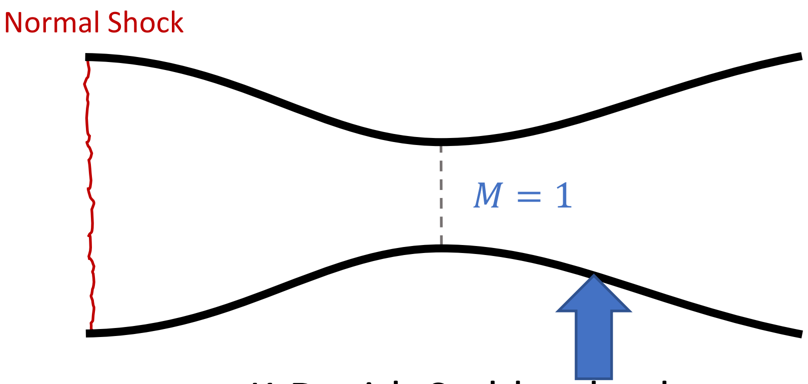
Stability of this operation:

$$\dot{m}_{max} = \frac{\sqrt{\gamma} P^{\circ} A_t}{\sqrt{R T^{\circ}}} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

Suppose a small disturbance propagates upstream past the throat. Does the disturbance grow? Consider a small shock – stagnation pressure decreases across the shock. In this condition there is less mass flow through the throat, but the same mass flow through the entrance. This causes the shock to grow and move forward, causing the mass flow through the throat to continue decreasing. This disturbance continues to grow until the shock moves out of the entrance and forms a bow shock. Decrease in the mass flow is dangerous to the operation of the system and causes loss of thrust.

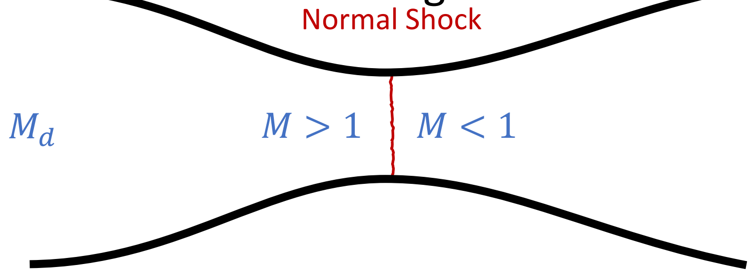


Kantrowitz-Donaldson Diffuser



K-D with Stable shock at entrance

K-D On-design



Isentropic flow behind the shock through the convergent and divergent portions

Isentropic flow in front of the shock through the convergent portion and isentropic flow after the shock through the divergent portion

$\dot{m}_{max} \approx P^{\circ} A_t$

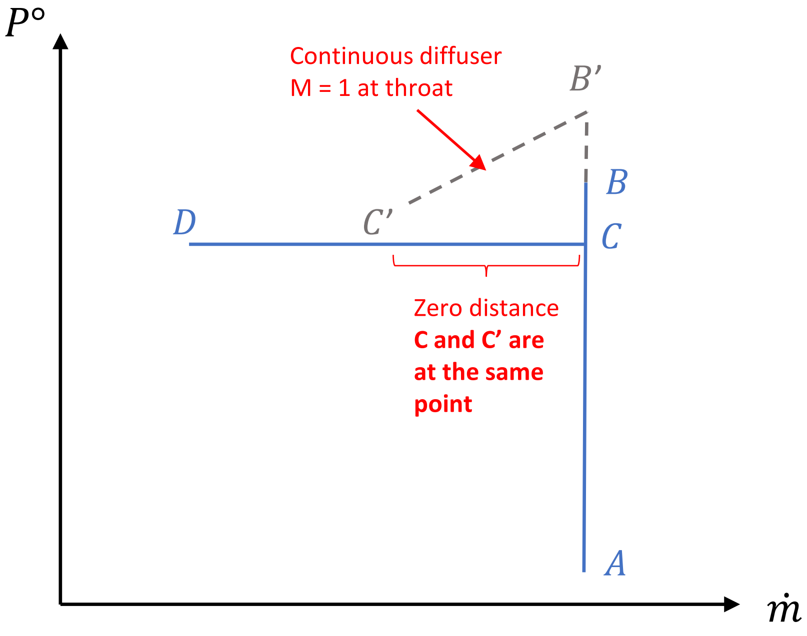
Lower P° means higher A_t for the same mass flux

Continuous flow would require a smaller throat area at M_d

Desire to operate with the shock just downstream of the throat for stability C purposes!

Kantrowitz-Donaldson (K-D) Diffuser

In this system, when the shock jumps to the inlet, this causes a decrease in stagnation pressure, but no change in the mass flow.



The aim is to have a stable shock at the throat (B). However, the throat is enlarged compared to continuous diffuser in order to achieve stability.

Back pressure increases in direction: A, C, B, D

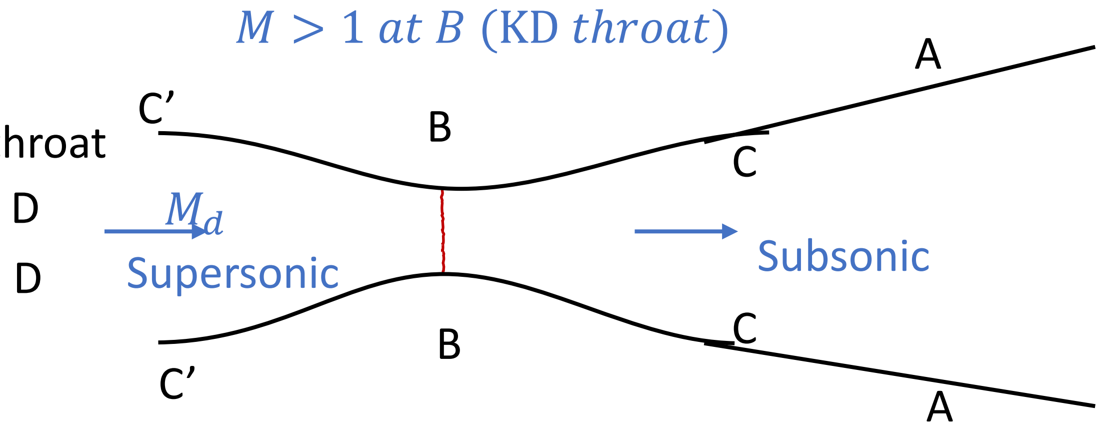
A: Shock forms downstream in divergent section

C: Shock in divergent section at location where $area = inlet\ area$

C': Shock at inlet

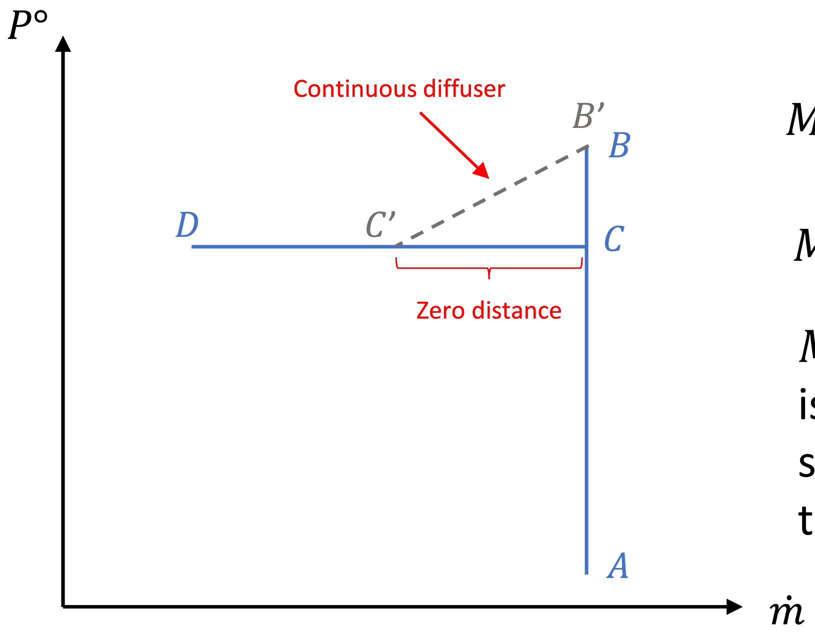
D: Bow shock

B: Shock at K-D throat



Kantrowitz-Donaldson Diffuser

On-design and off-design operation – Consider back pressure is constant and Mach number increase:

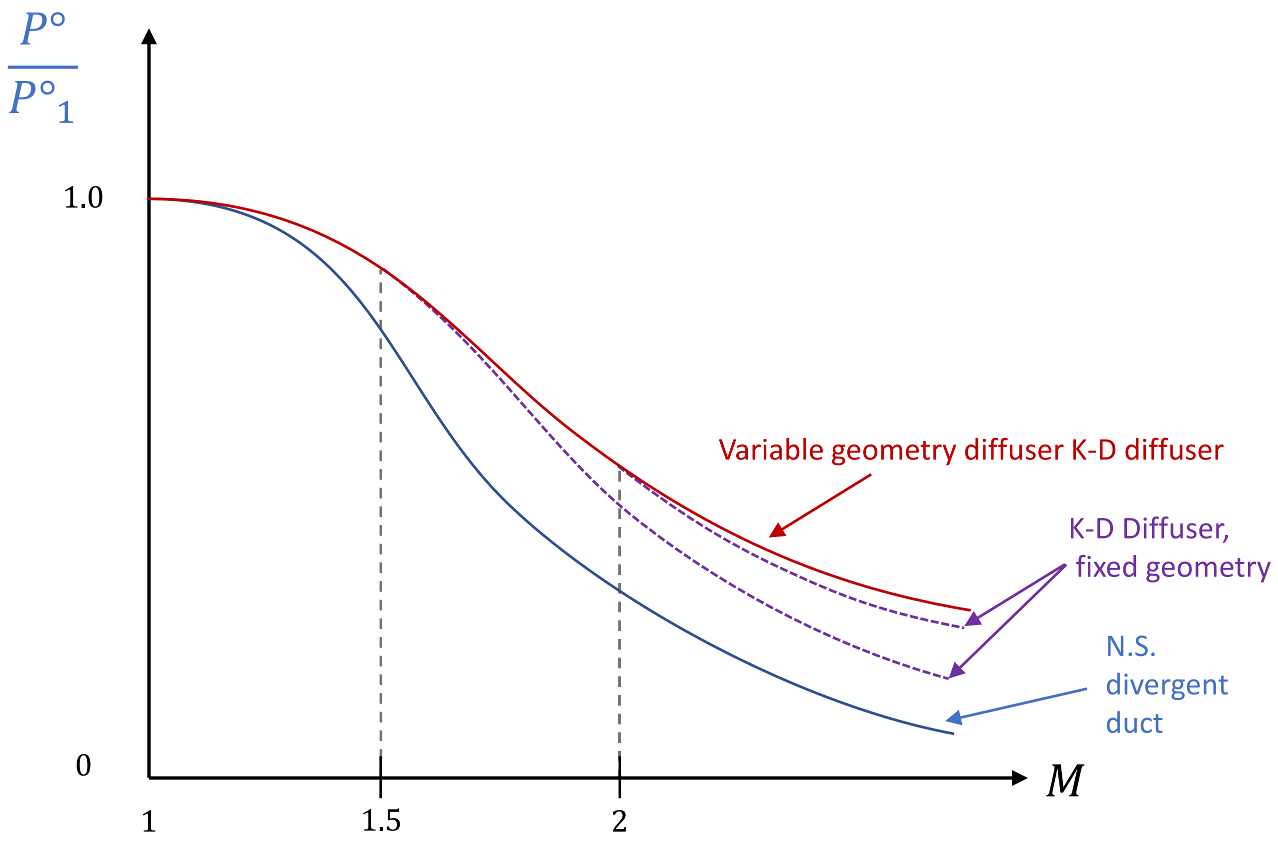


$M < M_d$: Detached shock, inflow is less than design

$M = M_d$: Shock sits at the entrance and is swallowed

$M > M_d$: Shock swallowed, $\dot{m}$ increases – stagnation pressure is determined by the back pressure since the portion of the shock is so determined. If the back pressure remains constant, then the shock moves downstream as $M \uparrow$

Kantrowitz-Donaldson Diffuser



Not much improvement over the N.S. divergent duct

Also for stability, the diffuser cannot be operated at optimum condition – shock somewhat downstream of the throat

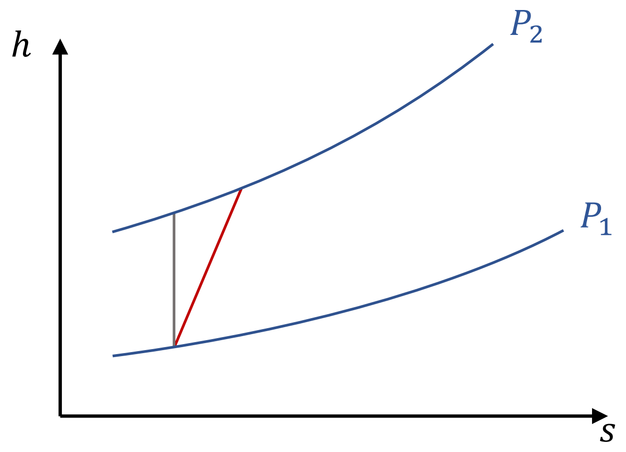
Other devices, such as oblique shocks, can be used at higher Mach numbers

Efficiencies for Compression – Air Intakes and Compressors
Kinetic energy or work converts to thermal energy

$$\eta_{ad} = \frac{h_{2,s} - h_1}{h_2 - h_1}$$ ← Adiabatic Efficiency

Polytropic or Elementary Efficiency → $$e = \frac{dh_s}{dh} = \frac{dP/\rho}{dh} = \frac{\left(\frac{RT}{P}\right) dP}{c_p dT}$$

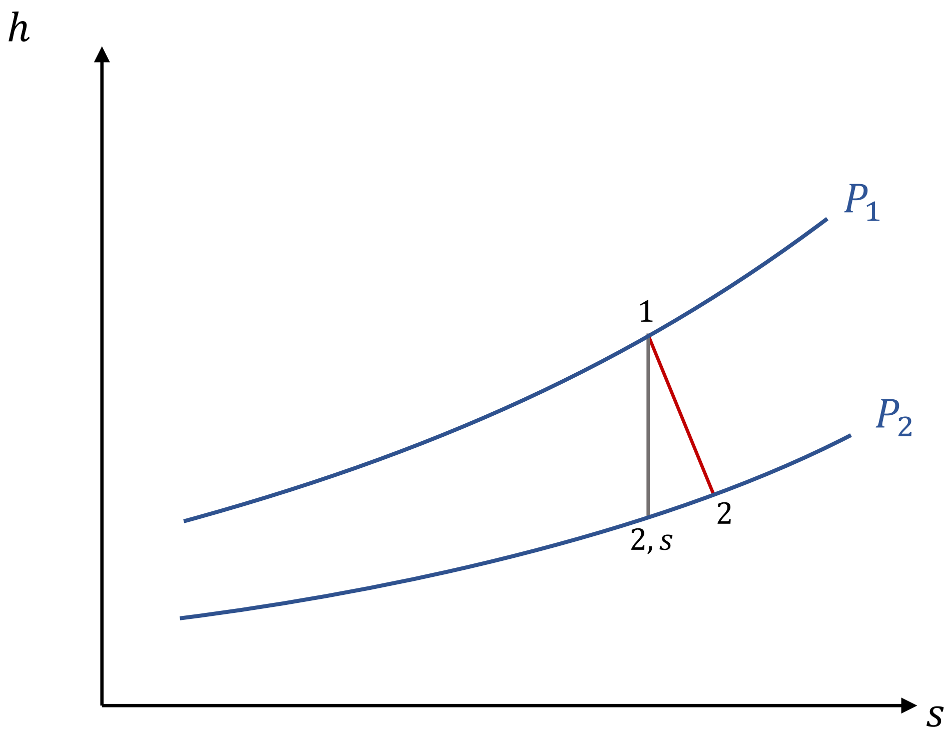
$$e = \frac{\gamma - 1}{\gamma} \frac{dP/P}{dT/T}$$



If e were constant: $$\ln \frac{T_2}{T_1} = \frac{\gamma - 1}{\gamma e} \ln \frac{P_2}{P_1}$$

Or: $$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma e}$$

Efficiencies of Expansion in Nozzles and Turbines
Thermal energy converts to kinetic energy or work



$$\eta_{ad} = \frac{h_2 - h_1}{h_{2,s} - h_1}$$

$$e = \frac{dh}{dh_s}$$

$$e = \frac{c_p dT}{dP/\rho} = \frac{c_p}{R}\frac{dT/T}{dP/P} = \frac{\gamma}{\gamma - 1}\frac{dT/T}{dP/P}$$

For constant e : $\ln\frac{T_2}{T_1} = \frac{(\gamma - 1)e}{\gamma}\ln\frac{P_2}{P_1}$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)e/\gamma}$$