Lecture 15
Electrothermal & Electrostatic Rockets

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Electric propulsion -Overview

Electric propulsion devices give low mass flow and low thrust, but high specific impulses compared to chemical propulsion.
Would not make good boosters but would make good systems for long space travel.

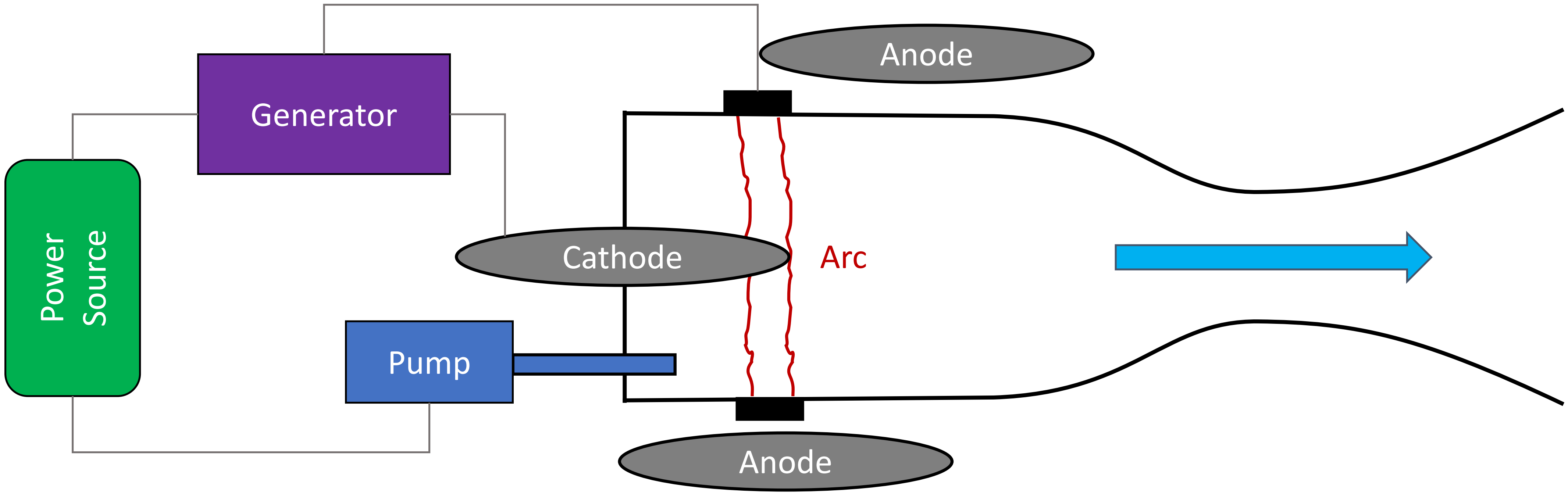
Clarifications:

1. Electrothermal uses electric energy to heat propellants which are expanded through exhaust nozzle – specific impulse is still temperature limited
2. Electrostatic uses electric field to accelerate electrically-charged propellants
3. Electromagnetic uses electric and magnetic fields for propellant acceleration.
4. DC electrical generation could originate with isotope nuclear power, solar energy, or fuel cell (batteries are typically too heavy).

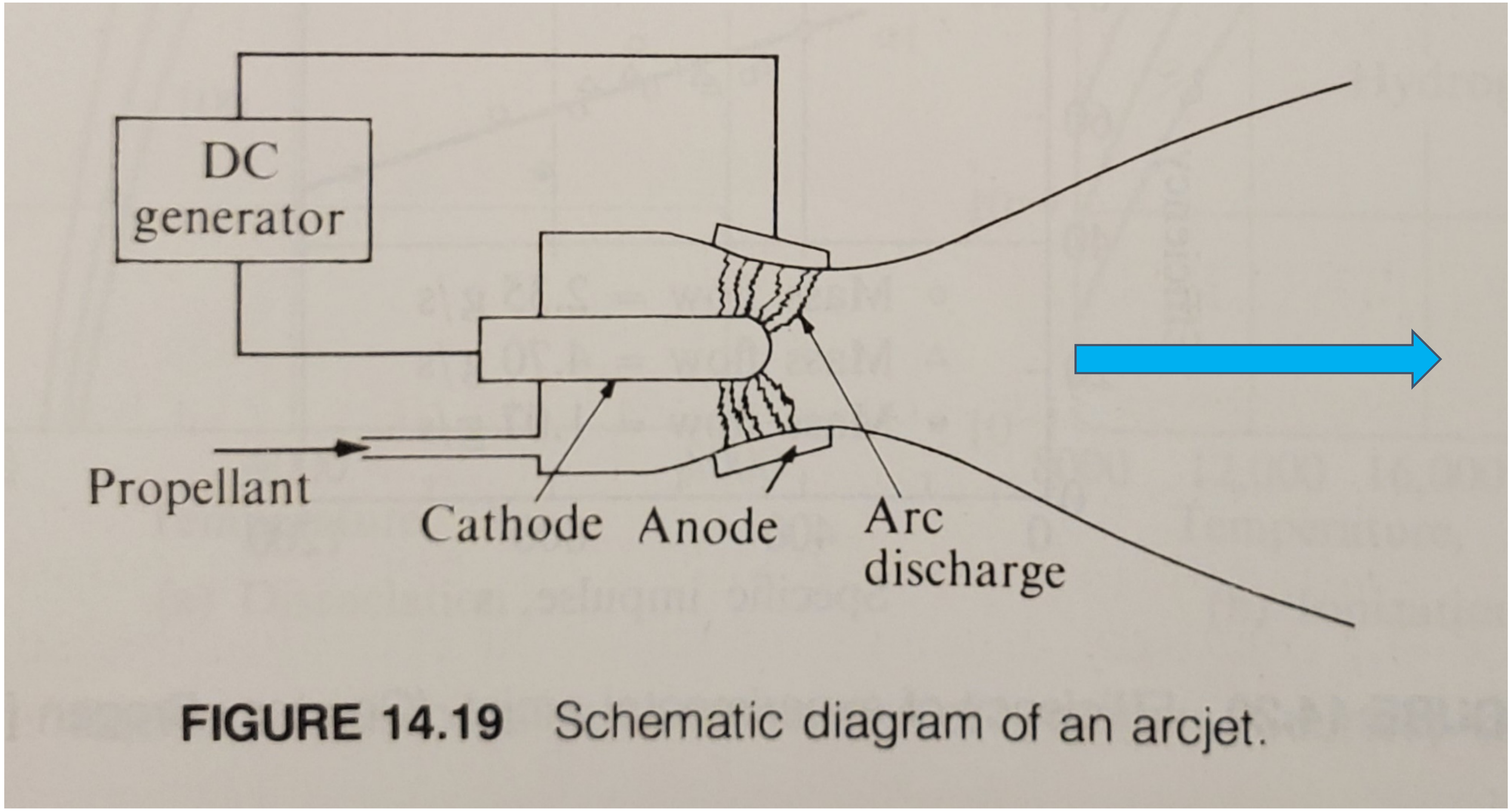
Consider first an example of electrothermal – Arcjet or Plasmajet

An arc occurs across the propellant stream in a chamber, heating the propellant. Nuclear power yields heat to drive a turbine that drives the generator plus drives the pump for the propellants.

Electric propulsion – Arcjet (Plasmajet)



Electric propulsion -Arcjet



Arcjet schematic from Hill & Peterson [1]

Electric propulsion - Arcjet

Suppose T° and P° are the stagnation temperature and pressure in the chamber. The stagnation temperature will depend upon heat added to propellant. Then, with choked flow in the nozzle, P° depends upon $\dot{m}$, T°, A^{*}, and molecular weight.

The exhaust velocity and stagnation pressure are given by:

$$u_e = \sqrt{\frac{2\gamma}{\gamma - 1}\frac{RT^{\circ}}{mw}\left[1 - \left(\frac{P_e}{P^{\circ}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}}$$

$$P^{\circ} = \frac{\dot{m}c^{*}}{A^{*}} \quad \text{Where:} \quad c^{*} = \sqrt{\frac{RT^{\circ}}{mw}}\frac{1}{\Gamma(\gamma)}$$

The choice of exit cross-sectional area determines P_e

Electric propulsion - Arcjet

Hydrogen and helium are prime candidates for propellants because of their low molecular weights.

Hydrogen has the advantage of lower molecular weight but some disadvantages:

1. It is chemically active while helium is inert. This leads to reaction with metal or graphite compounds.
2. It is diatomic while helium is monatomic so that some energy goes into dissociation.
3. Helium has higher ionization potential which results in more joule heating.

Most of the voltage drop that occurs is in the gas, but some voltage drop and joule heating occurs within anode and cathode leading to inefficiencies!

The high temperature lead to ionization of the gases. The fraction ionized can be estimated by assuming equilibrium conditions similar to chemical equilibrium.

Electric propulsion – some physics

$n \rightleftarrows e + I$ Neutral $\rightleftarrows$ electron + ion

P_e is partial pressure of the electron

P_I is partial pressure of the ion

P_n is partial pressure of the neutral atom

$$\frac{P_e P_I}{P_n} = K_p(T)$$

Here: T is T° in the chamber

Let n represent number density, then for neutral plasma: $n_e = n_I$

The Saha equilibrium equation can be derived:

$$n_e^2 = \left(\frac{2\pi m_e}{h^2}\right)^{3/2} (kT)^{3/2} e^{-\epsilon_i/kT} n_n$$

Where:

$m_e = electron\ mass\ (9.11 \times 10^{-31}\ kg)$

$h = Planck's\ constant\ (6.62377 \times 10^{-34}\ joule - sec)$

$\epsilon_i = ionization\ potential\ (24.48\ for\ helium, 1ev = 1.602 \times 10^{-19}\ joules)$

$k = Boltzmann's\ constant\ (1.38062 \times 10^{-23} joule/{}^{o}K)$

Clearly, n_e increases with temperature increase!

Electric propulsion

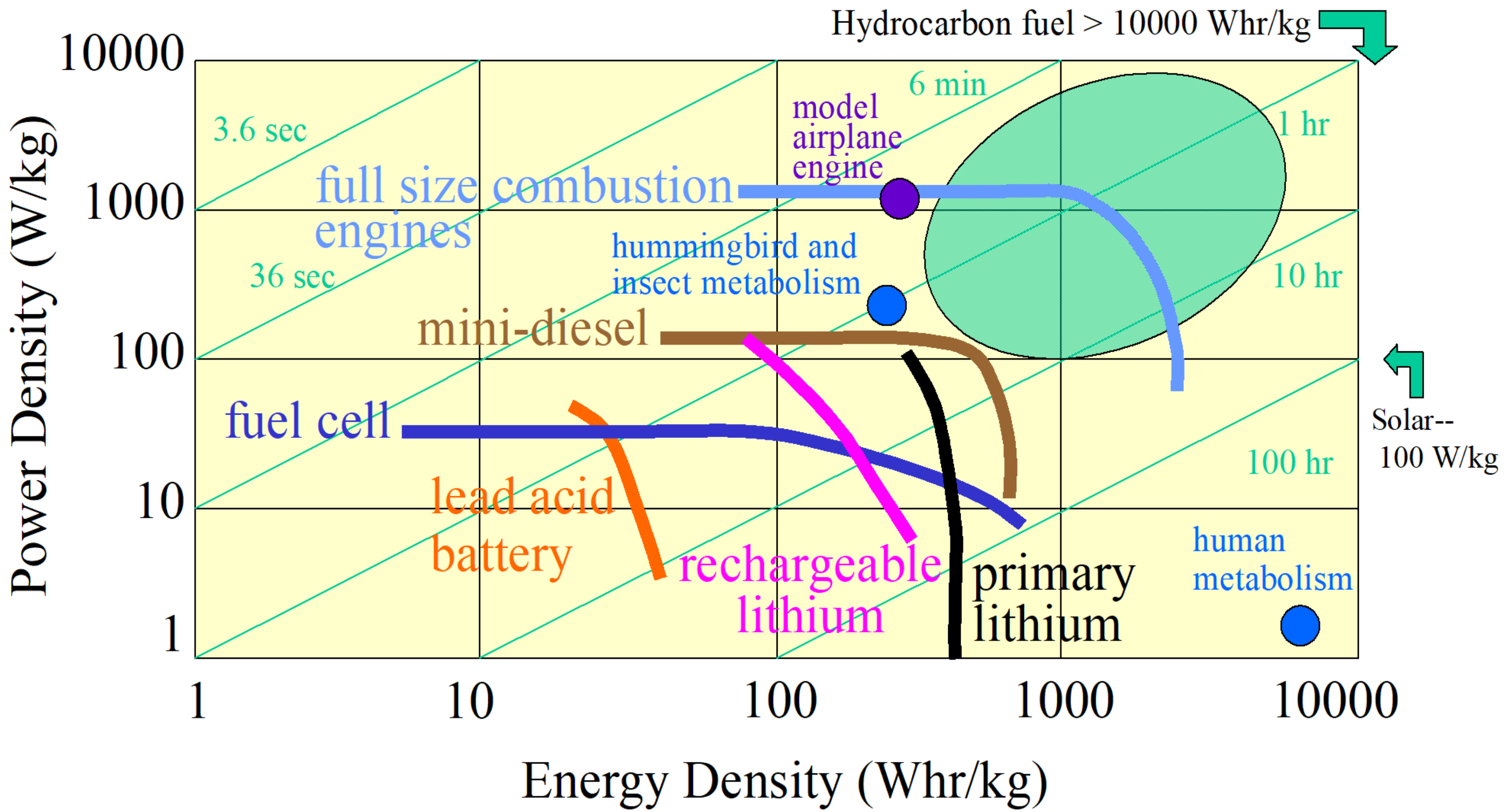
Ions are accelerated toward the cathode while electrons are accelerated toward the anode. Collisions with neutral atoms cause joule heating of the gas. Some ions can collide into the cathode resulting in possible sputtering or loss of material!

As temperature increases, the gas becomes a better electrical conductor resulting in less resistance and less joule heating at higher temperatures.

Helium has higher ionization potential than hydrogen so that a smaller fraction of helium ionizes at a given temperature. This is desirable because:

1. A smaller amount of energy goes into ionization energy and a larger fraction goes into thermal energy and eventually kinetic energy of exhaust.
2. Joule heating is greater for helium.

Power and Energy Density

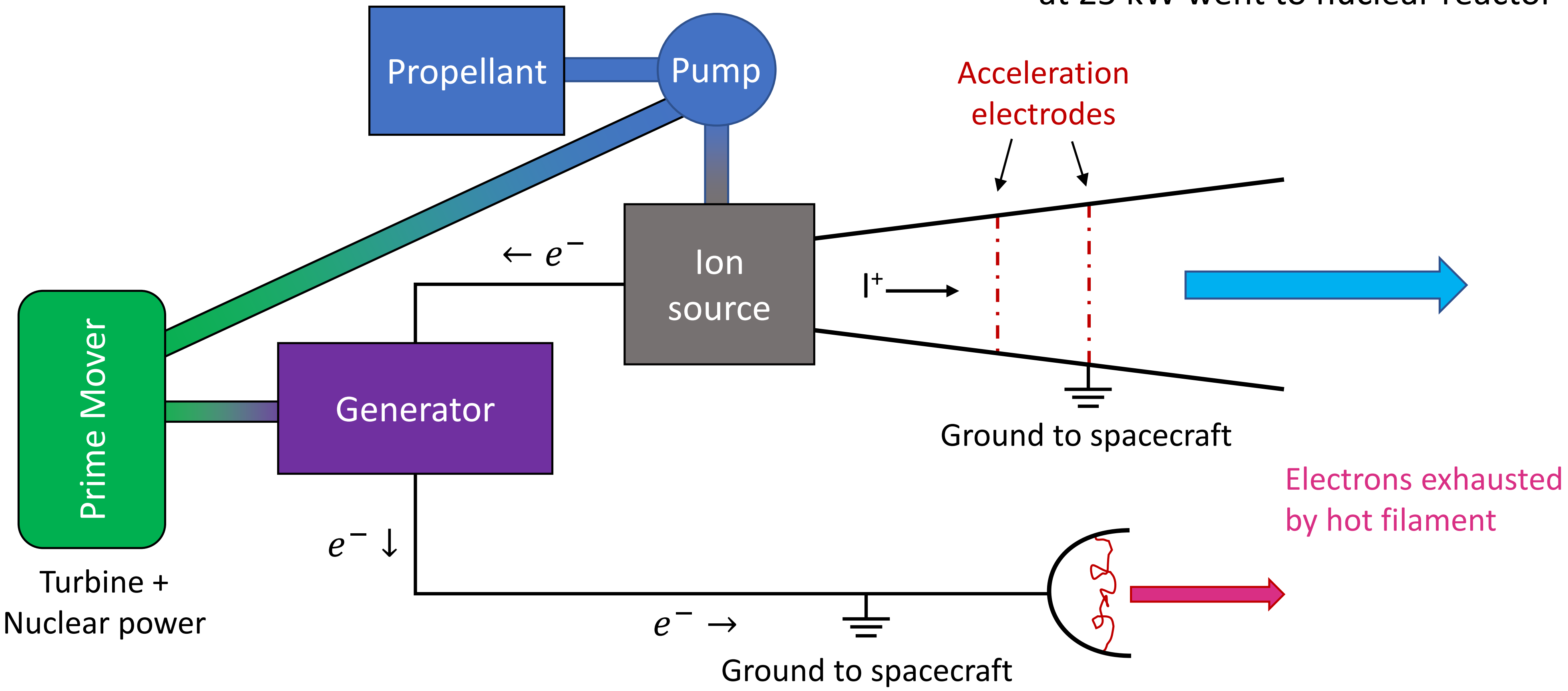


Courtesy of Prof. D. Dunn-Rankin

Electric propulsion - Ion Rocket

Consider an example of electrostatic rocket (Ion rocket)

JIMO (Jupiter mission) – Several engines at 25 kW went to nuclear reactor



Electric propulsion - Ion Rocket

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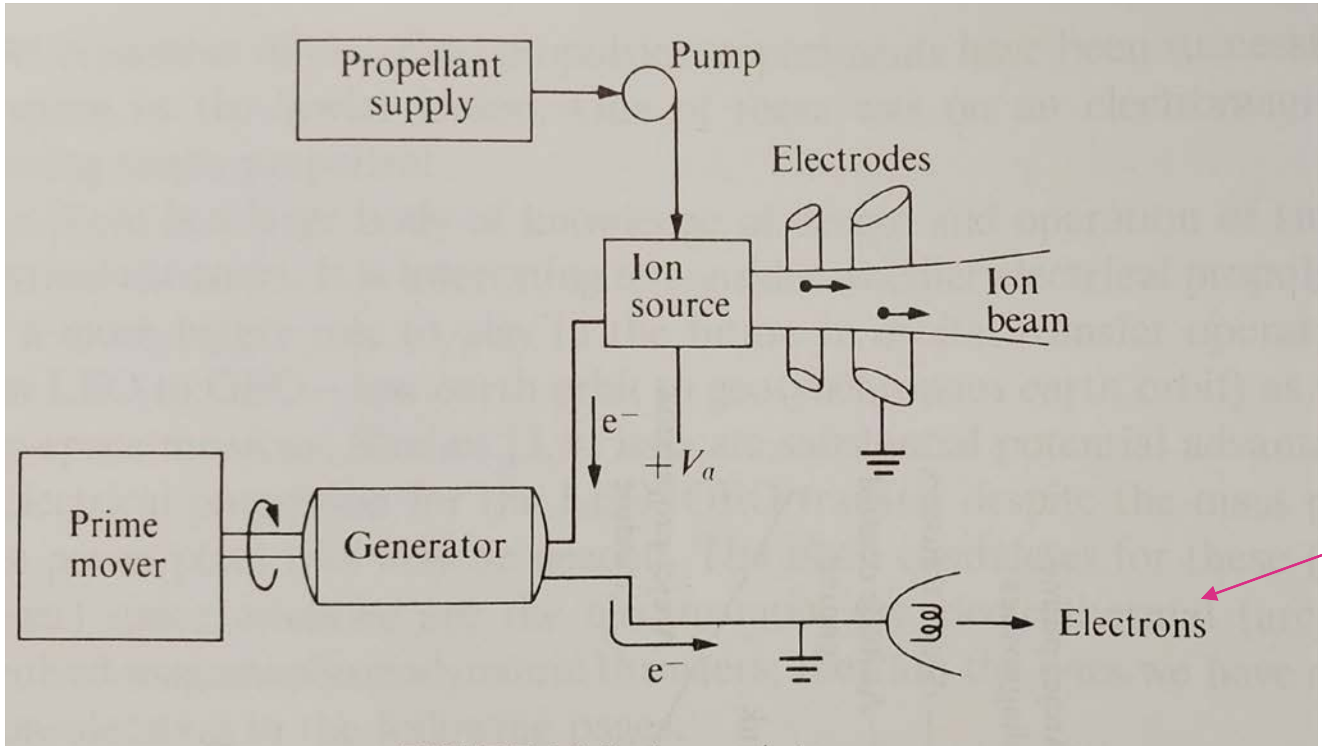


FIGURE 14.4 Ion rocket.

Electrostatic rocket schematic from Hill & Peterson [1]

The pump starts propellant flowing at low velocity . The ionized propellant is accelerated through the electrodes to produce thrust.

Electrons exhausted by hot filament will neutralize ion beam downstream and prevent reversal of the beam

Electric propulsion - Ion Rocket

If the potential difference of V_a is applied across the accelerator, a positive ion at negligible initial velocity and of mass, M, and charge, q, will achieve the final velocity of:

$$\frac{Mu_e^2}{2} = qV_a \leftrightarrow V_a = \frac{Mu_e^2}{2q}$$

From a balance of energy: $I_{sp} = \frac{u_e}{g} = \frac{1}{g}\sqrt{\frac{2qV_a}{M}}$

$$T = gI_{sp}\dot{m}$$

Also note: $Mu_e \sim \sqrt{M}$

This shows that more thrust is obtained when larger atoms are used

Total beam current: $I = \frac{q}{M}\dot{m}$

Ideally, beam power: $P = IV_a = \left(\frac{q}{M}\dot{m}\right)\left(\frac{Mu_e^2}{2}\frac{1}{q}\right) = \dot{m}\frac{u_e^2}{2} = exhaust\ kinetic\ energy\ flux$

Electric propulsion - Ion Rocket

Not all power goes into beam power. Radiation accounts for some losses. Also, the ion source generator requires power to operate. Collectively, these are called ‘charging power’. Low Ionization potential implies low charging power. Alkali metals make good ion sources. Cesium has lower ionization potential than others. Cesium also has high atomic weight. Alkali metals have one electron in the outer shell that is easier to “strip” electron or ionize!

$$\boxed{1\ ev = 1.60 \times 10^{-19}\ joules}$$

	Element	***Atomic #***	***1st Ionization potential***	***2nd Ionization potential***
Alkali	*Li*	*3*	*5.36 ev*	*75.3 ev*
	Na	*11*	*5.12 ev*	*47.1 ev*
	K	*19*	*4.32 ev*	*31.7 ev*
	Rb	*37*	*4.16 ev*	*27.4 ev*
	Cs	*55*	*3.87 ev*	*23.4 ev*
Inert	*He*	*2*	*24.5 ev*	*54.1 ev*
	Ne	*10*	*21.5 ev*	*40.9 ev*
	Ar	*18*	*15.7 ev*	*27.8 ev*
	Xe	*54*	*12.13 ev*	*21.21 ev*
	Hg	*80*	*10.44 ev*	*18.75 ev*

Ionization process

1. Electron bombardment: Direct collision between energetic electrons and atoms produces more electrons plus positive ions
2. Contact ionization: Atom makes contact with a particular solid surface

Electron bombardment

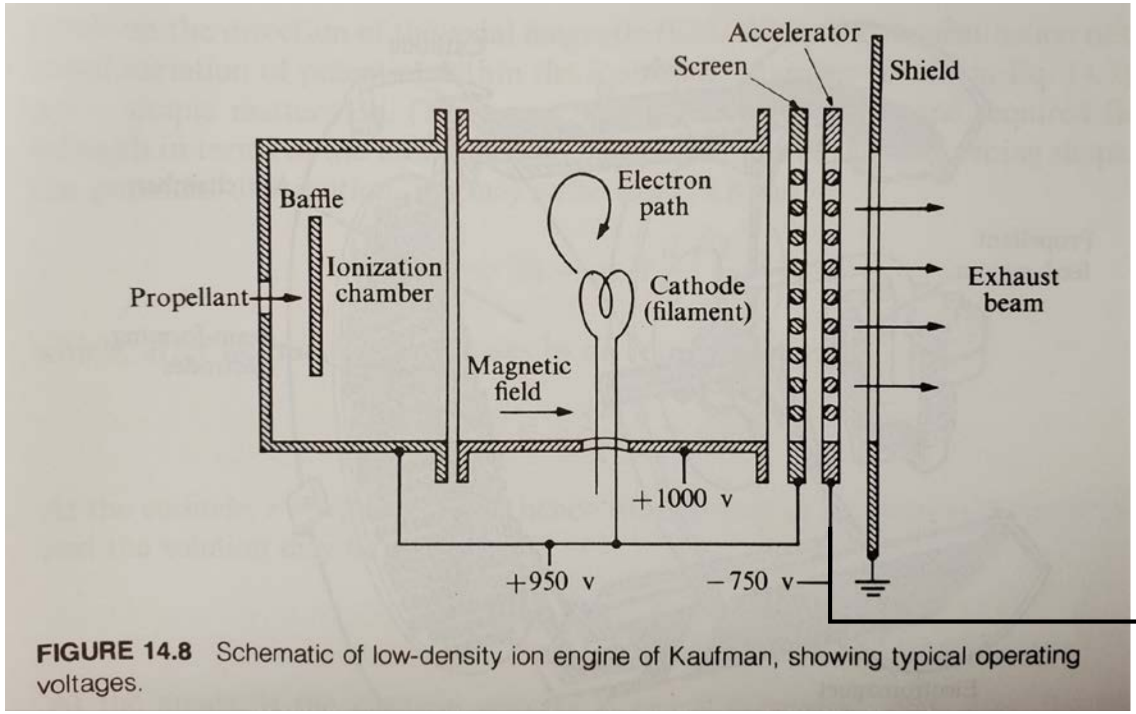


FIGURE 14.8 Schematic of low-density ion engine of Kaufman, showing typical operating voltages.

Electron bombardment engine schematic from Hill & Peterson [1]

$$\vec{F} = q\vec{V} \times \vec{B}$$

Causes spiraling motion of electrons increasing collisions

Neutralization keeps the positive ion from returning

Electrons for neutralization exhausted by hot filament

Ionization process

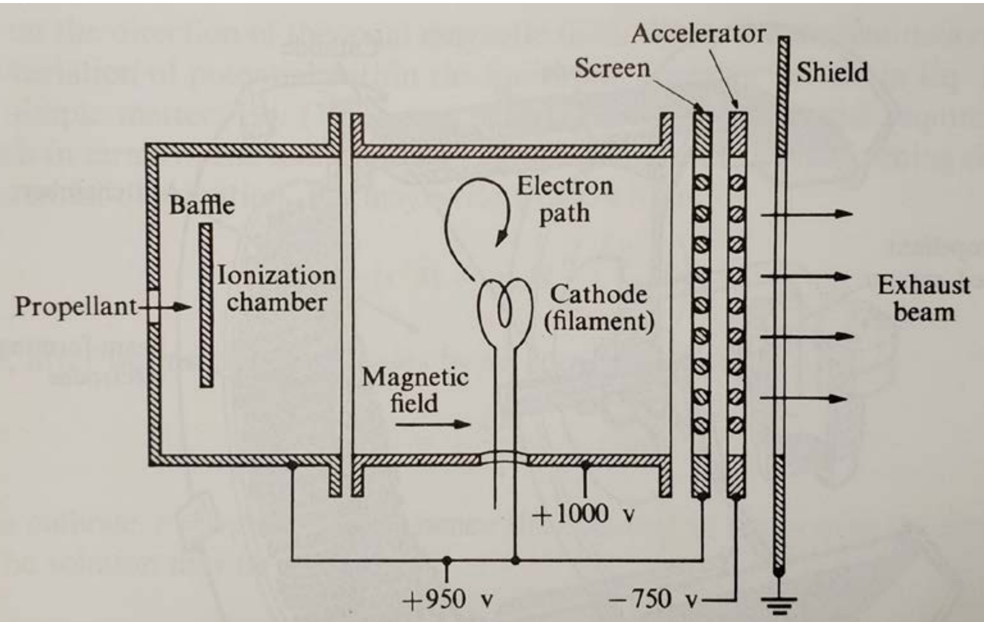


FIGURE 14.8 Schematic of low-density ion engine of Kaufman, showing typical operating voltages.

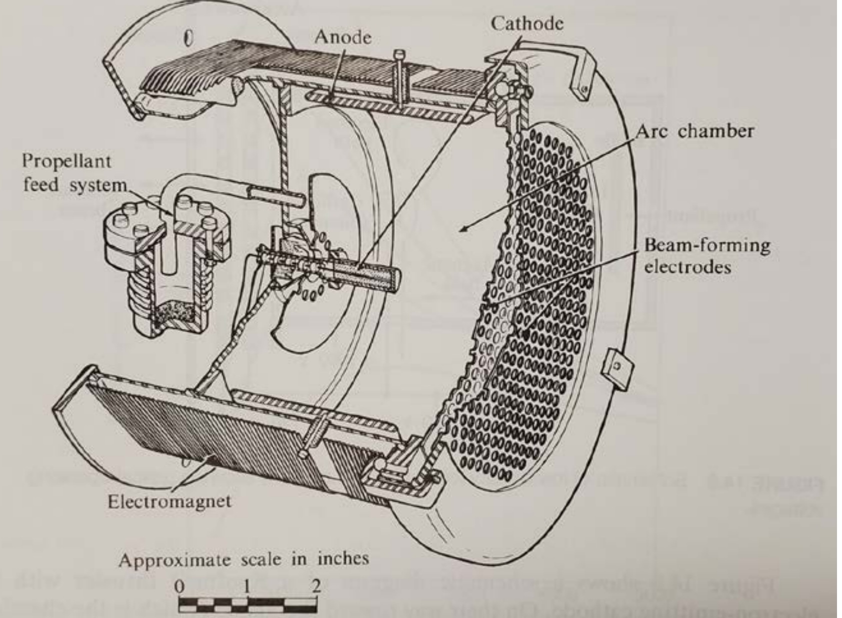


FIGURE 14.9 Low-current-density ion engine. (Courtesy Ion Physics Corporation. Reprinted with permission.)

Electron bombardment engine schematic and engine assembly from Hill & Peterson [1]

Ionization process

Electron bombardment

Electrons are confined in the ionization chamber by electric fields. They are emitted from the cathode and move toward the anode. The magnetic field causes spiraling. Without the magnetic field, electrons would not travel as long of a distance and move as fast in the ionization chamber before reaching the anode, reducing the chance of collisions with neutral atoms which is needed to cause ionization.

Kinetic energy of bombarding electron should be greater than the ionization potential in order to cause ionization. A factor of three to five times the ionization potential is ideal!

Contact Ionization

Work function is the amount of energy required to move an electron from the surface of the material to an infinite distance. For example, tungsten is a material with a very high work function, while cesium has a low work function. If a cesium atom were in the vicinity of a tungsten surface, its outer electron would experience a strong attraction to the tungsten. A porous tungsten plug is used with cesium vapor flowing through it. Current density increases as the surface temperature increases.

Ion Beam Acceleration

Acceleration of positive ions between electrodes in x-direction

For positive ion: $\rho_e > 0$

$V = V(x)$

$$\vec{E} = -\nabla V$$

$$E_x = -\frac{dV}{dx}; \quad \frac{d^2V}{dx^2} = -\frac{\rho_e}{\epsilon_0}$$

This is a one-dimensional representation

ϵ_0 is electric permeability of free space (vacuum)

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{(coulomb)^2}{newton\text{−}meter}$$

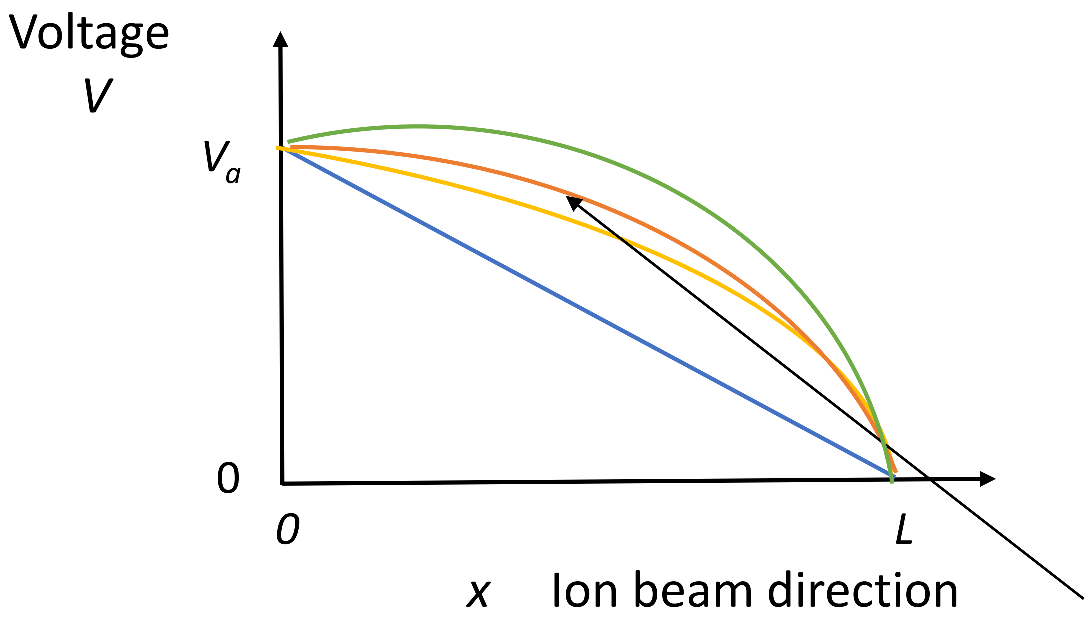
ρ_e is the electric charge density

$j = \rho_e u$ $\quad$ j is current density for ion beam (constant with x in steady state),
u*(x)* is charge velocity,

$$qV_a = qV + \frac{mu^2}{2}$$

This assumes positive charges have negligible initial velocity!

Ion Beam Acceleration – Space Charge



Blue line – no space charge , linear voltage between electrodes, second derivative is zero.

Yellow curve – some space charge, ion mass flux has increased but ions adds to voltage, decrease rate of acceleration near electrode at x = 0.

Red curve – more space charge but **optimal mass flux,** zero slope at x = 0.

Green curve – positive slope at x = 0 will decelerate ion beam. This cannot work.

Optimization of Space Charge and Current for Ion rocket

Solving the differential equation:

$$\frac{d^2V}{dx^2} = -\frac{\rho_e}{\epsilon_0} = -\frac{j}{u\epsilon_0} = -\frac{j}{\epsilon_0\sqrt{\frac{2q}{m}(V_a - V)}}$$

For: $0 \le x \le L$ with $V(0) = V_a$ and $V(L) = 0$

With the definitions: $\alpha \equiv \frac{j}{\epsilon_0}\frac{1}{\sqrt{2q/m}}$; $V_* \equiv V_a - V(x)$; $C \equiv \frac{1}{4\alpha}\left(\frac{dV_*}{dx}(0)\right)^2 \ge 0$

$$\frac{d^2V_*}{dx^2} = \frac{\alpha}{\sqrt{V_*}} \ ; \qquad V_*(0) = 0$$

We obtain: $\sqrt{\alpha}x = \frac{2}{3}\left(C + V_*^{1/2}\right)^{3/2} - 2C\left(C + V_*^{1/2}\right)^{1/2} + \frac{4}{3}C^{3/2}$ See note slide for solution details.

We have C and $\frac{dV_*}{dx}(0)$ as constants for any particular configuration, but now we aim to optimize C to maximize current, j, and therefore maximize thrust

Notes on Differential Equation Solution

Multiply $\frac{dV_*}{dx}$ to obtain $\frac{dV_*}{dx}\frac{d^2V_*}{dx^2} = \frac{\alpha}{\sqrt{V_*}}\frac{dV_*}{dx}$; $\quad V_*(0) = 0$

Integrate once to obtain $\frac{1}{2}\left(\frac{dV_*}{dx}(x)\right)^2 = 2\alpha\sqrt{V_*(x)} + C'$ where $C' = 2\alpha C$ is the constant of integration. Rearrange to get $\frac{1}{2\sqrt{\alpha}}\frac{dV_*}{dx}(x) = (V_*^{1/2} + C)^{1/2}$

Rearrange again to get

$$2\sqrt{\alpha}dx = (V_*^{1/2} + C)^{-1/2}\, dV_* = [\left(C + V_*^{1/2} - C\right)/V_*^{1/2}](V_*^{1/2} + C)^{-1/2}\, dV_*$$

where $[\ ....] = 1$. Finally, integrate again using $V_*(0) = 0$ to obtain

$$\sqrt{\alpha}x = \frac{2}{3}\left(C + V_*^{1/2}\right)^{3/2} - 2C\left(C + V_*^{1/2}\right)^{1/2} + \frac{4}{3}C^{3/2}$$

Optimization of Space Charge and Current for Ion rocket

At: $x = L,\ V_* = V_a$ and $\sqrt{\alpha}L = \frac{2}{3}\left(C + V_a^{1/2}\right)^{3/2} - 2C\left(C + V_a^{1/2}\right)^{1/2} + \frac{4}{3}C^{3/2} = f(V_a, C)$

We can maximize α and therefore j by maximizing $f(V_a, C)$ through the selection of the value of C!

$$\frac{df}{dC} = \left[C^{1/2} - \left(C + V_a^{1/2}\right)^{1/2}\right] + \left[C^{1/2} - C^{1/2}\frac{C^{1/2}}{\left(C + V_a^{1/2}\right)^{1/2}}\right] = C^{1/2}\left\{[1-p] + \left[1 - \frac{1}{p}\right]\right\} = -C^{1/2}\frac{(p-1)^2}{p}$$

Where: $p \equiv \left(1 + \frac{V_a^{1/2}}{C}\right)^{1/2} > 0$

Clearly, $\frac{df}{dC} < 0$ so that α and j are maximized at $C = 0$; $\text{i.e., zero value for voltage gradient at } x = 0$

Thereby: $\sqrt{\alpha_{max}}L = \frac{2}{3}V_a^{3/4}$ and $j_{max} = \frac{4}{9}\epsilon_0\sqrt{\frac{2q}{m}}\frac{V_a^{3/2}}{L^2}$

Optimization of Space Charge and Current for Ion rocket

The thrust T per unit area A is given by:

$$\frac{T}{A} = \frac{\dot{m}u_e}{A} = \frac{\rho_e M u_e}{q} u_e = \frac{j}{q/M} u_e = \frac{j}{q/M} \sqrt{\frac{2q}{M} V_a}$$

Therefore, the maximum possible thrust is given by:

$$\frac{T_{max}}{A} = \sqrt{\frac{2M}{q}} j_{max} V_a^{1/2} = \frac{8}{9} \epsilon_0 \left(\frac{V_a}{L} \right)^2$$

Note that this maximum value is independent of M and depends on the average electric field intensity V_a/L, i.e., and average derivative dV/dx between $x = 0$ and $x = L$.

References

[1] Hill, Philip G., and Carl R. Peterson. *Mechanics and Thermodynamics of Propulsion*. Reading, Mass: Addison-Wesley Longman, 1992.