Lecture 16
Hall Thrusters

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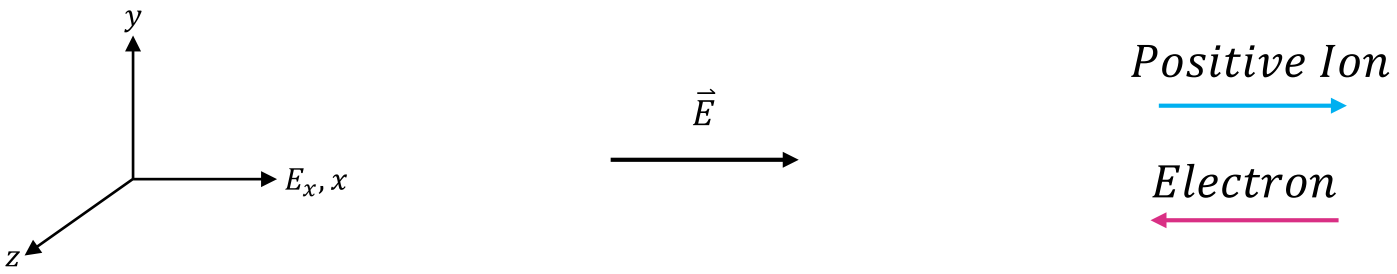
Hall Current and Electromagnetic Fields

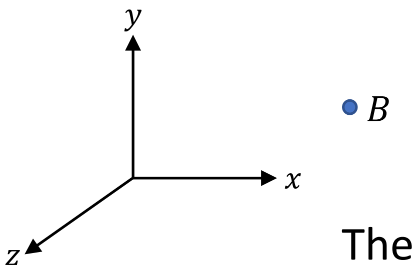
Effects on a charged particle at low densities – neglect collisions with other particles for the first examination. Consider no forced flow without pressure gradients in this first examination!

The force on a charged particle is: $\vec{F} = q\left(\vec{E} + \vec{V} \times \vec{B}\right)$

1. With only an electric field, linear motion results:



2. With only a magnetic field, $\vec{B}$, B_z, rotational motion in *y-x* plane results:

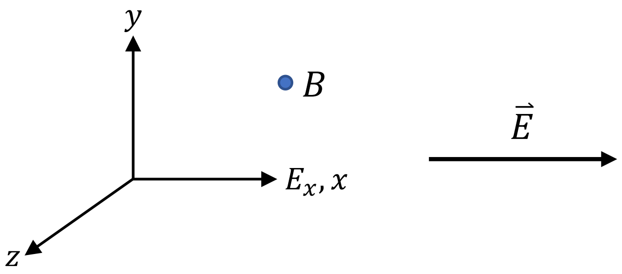


$$\frac{mV_\theta^2}{r} = qBV_\theta \quad \Rightarrow \quad \frac{V_\theta}{r} = \omega = \frac{q}{m}B$$

The electron with lower mass has a higher angular frequency!

Hall Current and Electromagnetic Fields

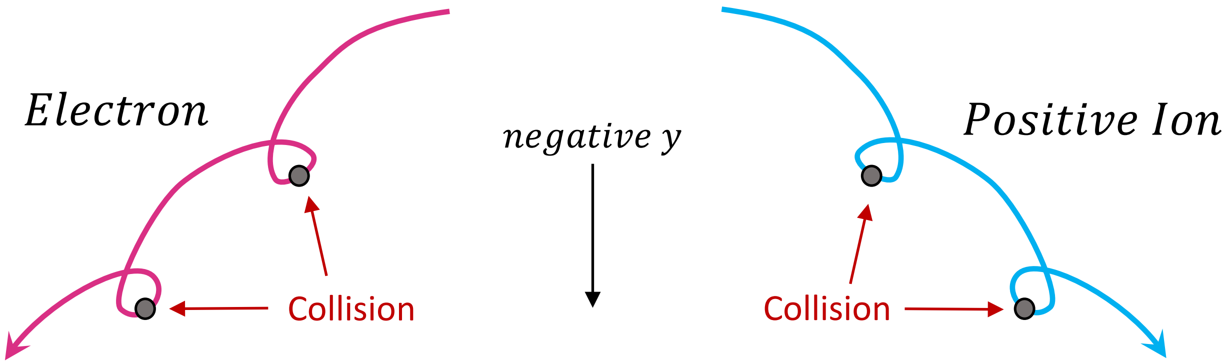
3. With both electric and magnetic fields crossed (i.e., orthogonal) linear motion with rotation results:



$$\frac{mV_\theta^2}{r} = qBV_\theta \quad \Rightarrow \quad \frac{V_\theta}{r} = \omega = \frac{q}{m}B$$

As the electric field, $\vec{E}$ changes the velocity, it also changes the radius of curvature for the motion

$$r = \left(\frac{q}{m}\frac{B}{V_\theta}\right)^{-1}$$



Both the electron and positive ion drift in the negative-y direction

Lower velocity or less mass produces lower radius!

Hall Current and Electromagnetic Fields

The drift velocity in the y-direction interacts with the magnetic field to create force and acceleration in the *x*-direction.

$$\vec{F} = q[\vec{E} + \vec{V} \times \vec{B}]$$

If: $\vec{V} = \vec{V}_{drift} + \vec{V}_{other} = \vec{e}_y V_{drift} + \vec{V}_{other}$

Then: $\vec{V} \times \vec{B} = \vec{V}_{other} \times \vec{B} + \vec{e}_x |B| V_{drift}$

Taking x-components:

This only fluctuates with zero mean value

$$F_x = q\left[E + |B|V_{drift} + V_{other,y}|B|\right]$$

$|B|V_{drift}$ and *E* each act in opposing directions for either the ion or electron. As the F_x force is applied, acceleration occurs in x and y towards a situation where no force is applied to the electron on average in the *x*-direction.
A stable drift velocity is the result.

$$E+|B|V_{drift} = 0 \qquad \text{or} \qquad V_{drift} = -\frac{E}{|B|}$$

Hall Current and Electromagnetic Fields

So, the positive ions + electrons tend to move towards this drift velocity. Positive ions have more inertia and do not respond as quickly; so, on balance, negative charge flows in the negative y-direction which amounts to a positive j_y.

Note: $\vec{j}_x$ will still exist due to $\vec{E}$

This is the Hall effect and j_y is the Hall current.

The Lorentz force, $\vec{j}_y \times \vec{B}$, due to the Hall current, is in the positive x-direction. This force is on the fluid, a mixture of electrons, ions, and neutral atoms.

Although the electrogmagnetic force acts directly only on charged particles, collisions between neutral particles and charged particles will cause the momentum change (i.e., acceleration) to be shared.

So there will be a flow with velocity, u, in the x-direction which results in thrust due to the Hall effect!

Hall Current and Electromagnetic Fields

Now, consider collision effects:

$$\omega_{electron} = \frac{q}{m_{electron}} B \gg \omega_{ion} = \frac{q}{m_{ion}} B$$

Assume $\omega_{ion} \approx 0$; then, with $\vec{E} = \vec{e}_x E$, the positive ion moves only in the *x*-direction

The electric field tends to move electrons in the negative x-direction. Combined with the B field, the "gyro" effect at high ω_e causes drift in the negative y-direction.

Collisions with the positive ions and neutral particles (both much heavier particles) will change the momentum of the electrons causing more motion towards the positive direction and reducing electron momentum in the y-direction.

So, at high densities with many collisions, the Hall effect is substantially reduced.

ν is the collision frequency

$$\Omega = \frac{\omega_e}{\nu} \quad \text{is the Hall parameter}$$

Hall thrusters work at high Ω, $\omega_e/\nu \gg 1$, or $\omega_e \ggg \nu$. That is, at low densities and high B values.

Distinction between force on charged particle and force on fluid

Consider the number density (number per unit volume of ions and electrons to be equal in the thruster. We started with ionizing a neutral atoms.

The electric field forces are equal but opposite for ions and electrons. Thus, the net force locally Due to the electric on a fluid mixture is zero.

The magnetic force is equal and in the same direction for electrons and ions. Thus, it can create a net force on the fluid.

The magnetic field does create a net force on the fluid because there is a electric current, i.e., $j > 0$ because the lighter electrons move faster than the ions. The net force is the Lorentz force.

Hall Effect Thruster

Hall current is caused by a drift velocity with crossed electric and magnetic fields. While the two fields must have a non-zero cross product, they need not be perfectly orthogonal.

Force on charged particles $\vec{F} = q\left[\vec{E} + \vec{V} \times \vec{B}\right]$

$E_\perp$ is perpendicular to $\vec{B}$

$E_\parallel$ is parallel to $\vec{B}$

$\overrightarrow{E_\parallel}$, $\overrightarrow{E_\perp}$, and $\vec{B}$ are all in the x-y plane

Let $\vec{V} = \vec{V}' + \vec{V_z}$ where $\vec{V_z}$ is perpendicular to both $\vec{E}$ and $\vec{B}$

$\left|\vec{V}_\perp\right| = E_\perp/|B|$; $\overrightarrow{\ V'}$ might still have a component outside of the $E - B$ plane .

$$\vec{F} = q\left[\overrightarrow{E_\parallel} + \overrightarrow{E_\perp} + \vec{V}' \times \vec{B} + \vec{V_z} \times \vec{B}\right]$$

$\overrightarrow{E_\perp} + \vec{V_z} \times \vec{B}$ produces no net force in the perpendicular direction and no acceleration to the charged particles.

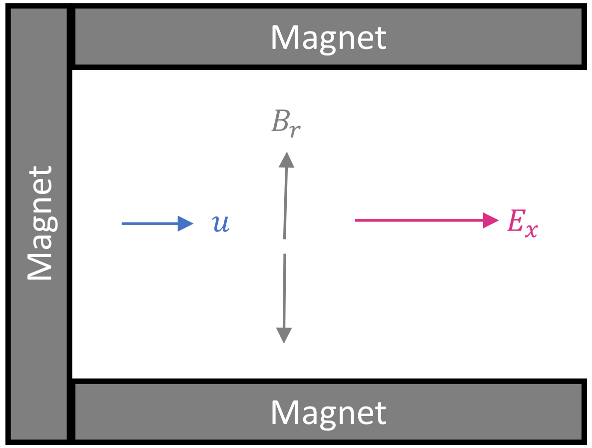
So: $\vec{V_z} = -\dfrac{E_\perp}{|B|}\vec{e}_z$

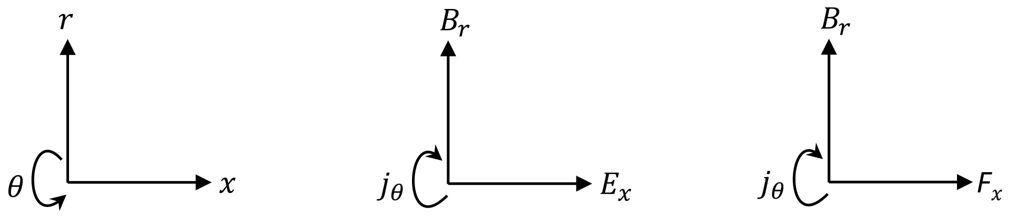
Hall Effect Thruster in Cylindrical Geometry

$q\overrightarrow{E_{\parallel}}$ produces linear acceleration and $\vec{V}' \times \vec{B}$ produces a spiraling motion.

The drift velocity causes the Hall current. Here, it will be in the azimuthal (tangential) direction. The drift velocity is independent of particle mass. The lighter electrons, however, will spiral more than heavier positive ions.

The Hall current can now interact with the $\vec{B}$ field to produce a $\vec{j} \times \vec{B}$ acceleration





j_θ is Hall current