Lecture 6
Rocket Engines

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Liquid Rocket Engine

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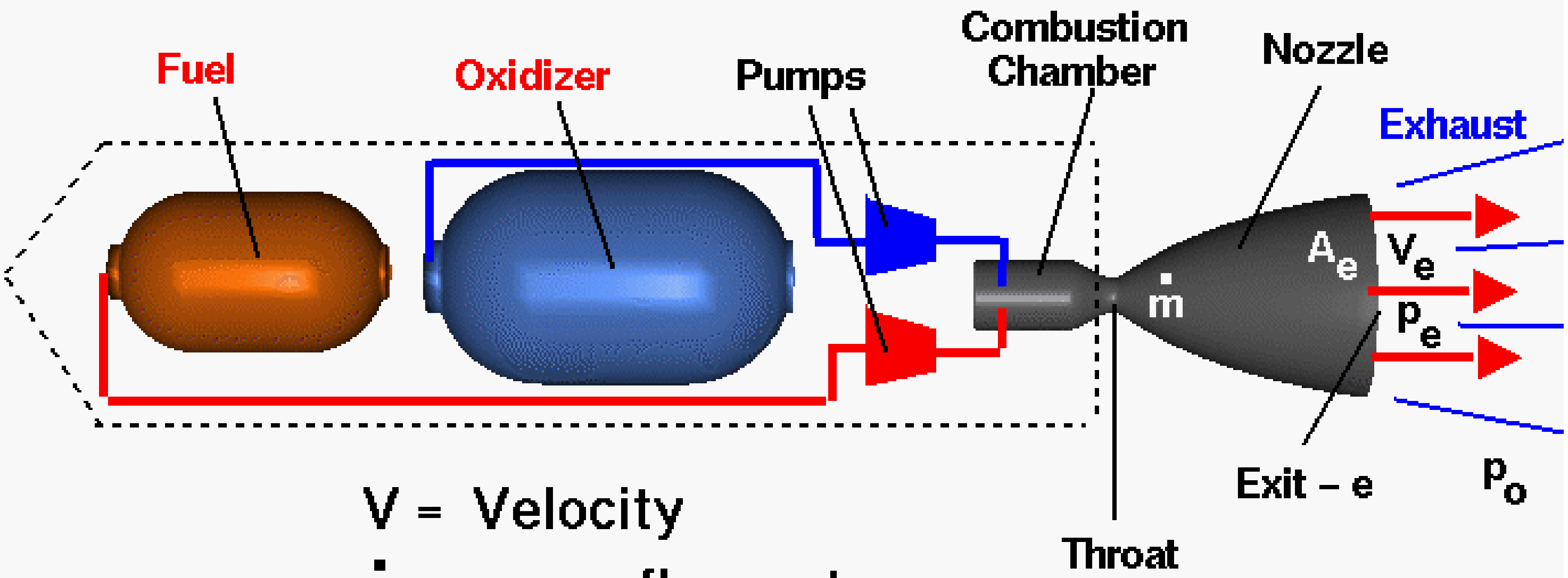


V = Velocity

$\dot{m}$ = mass flow rate

p = pressure

Thrust = $F = \dot{m}\, V_e + (p_e - p_0)\, A_e$

Liquid Propellant Rocket Engine

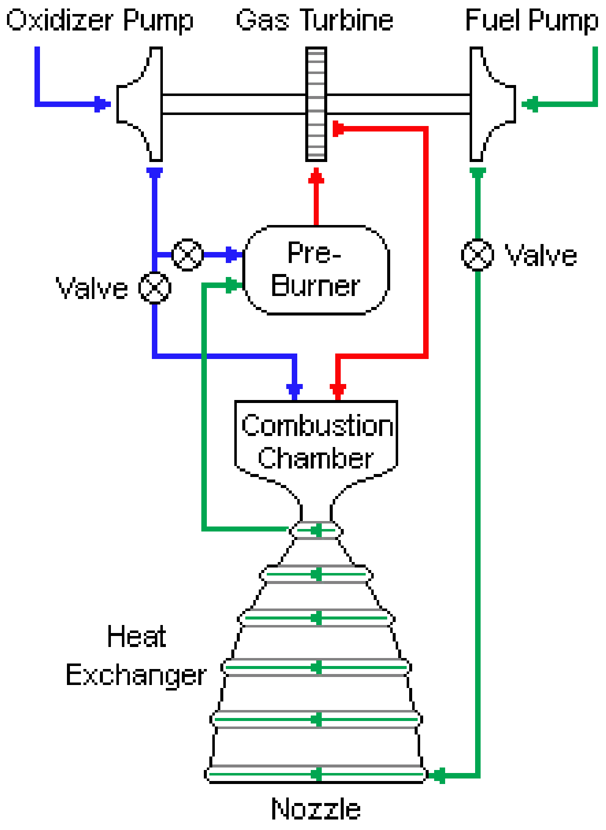
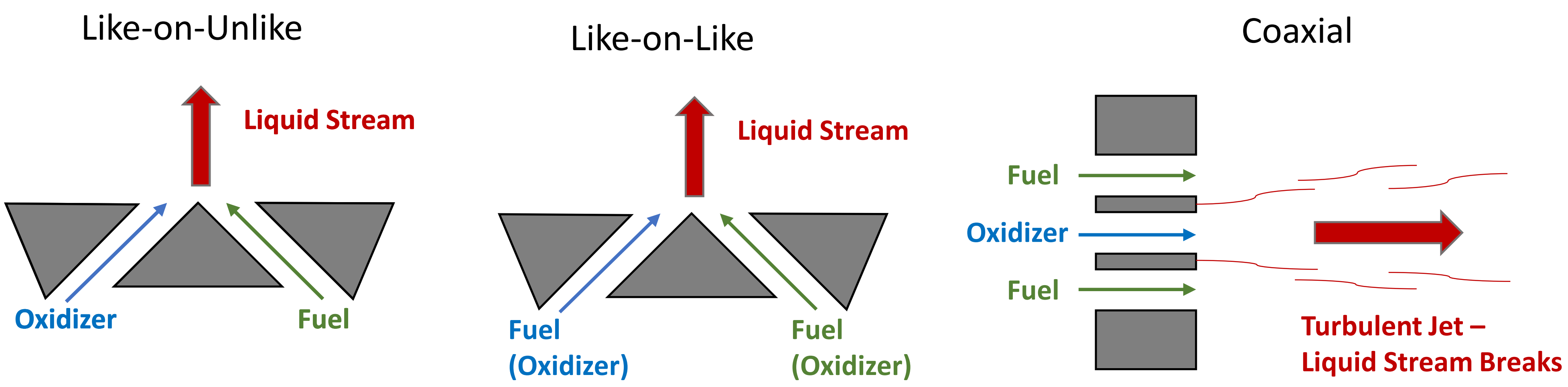


Fig. 1.9 - STAGED COMBUSTION

Liquid propellant rocket engine from Braeunig [1]

Liquid Propellant Rocket Engine

Different kinds of injectors



Formation of a spray maximizes surface area and thereby maximizes vaporization rate

$\dot{m}_d =$ the mass vaporization rate per droplet $\sim \quad D \to D^{3/2}$ Droplet Diameter

$$N \sim \frac{1}{D^3} \qquad \dot{m} = N\dot{m}_d \sim \frac{1}{D^2} \; to \; \frac{1}{D^{3/2}}$$

Solid Rocket Engine

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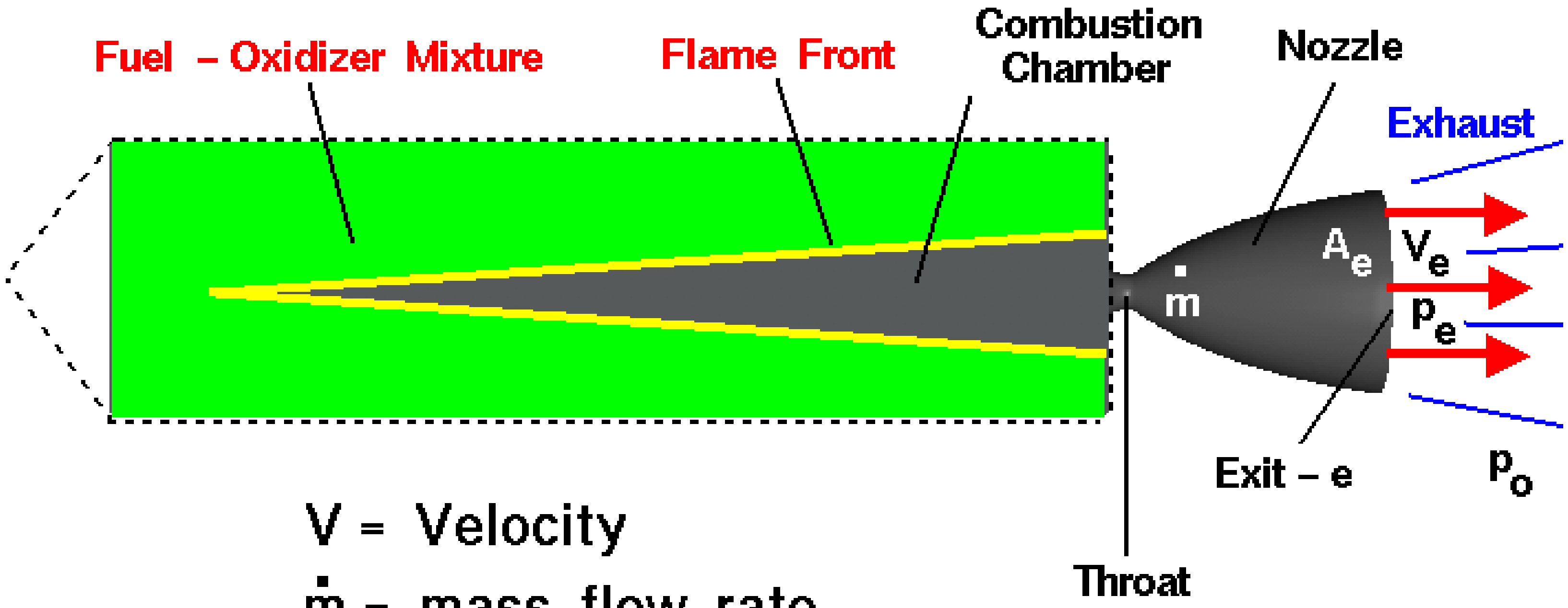

V = Velocity

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Solid Propellant Rocket Engine

Consider now a solid propellant rocket

Typically regression rate r of burning surface is given by: $r = aP^n$

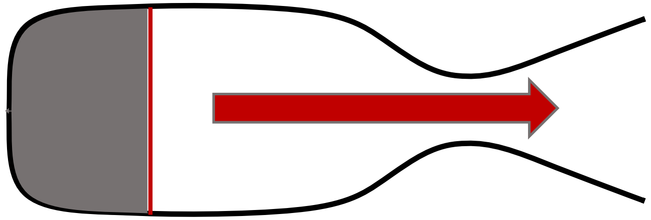
- P is pressure
- r is velocity
- a, n are constants

Mass burning rate: $\dot{m}_p = \rho_p r A_b$

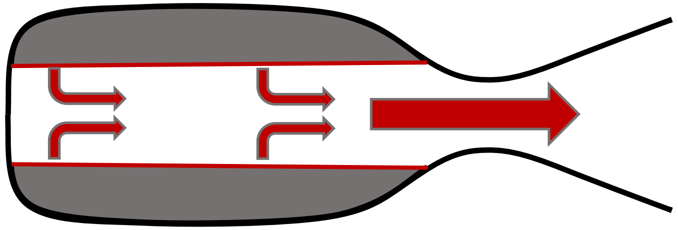
- ρ_p is propellant density (solid)
- A_b burn surface area

A_b can depend upon time

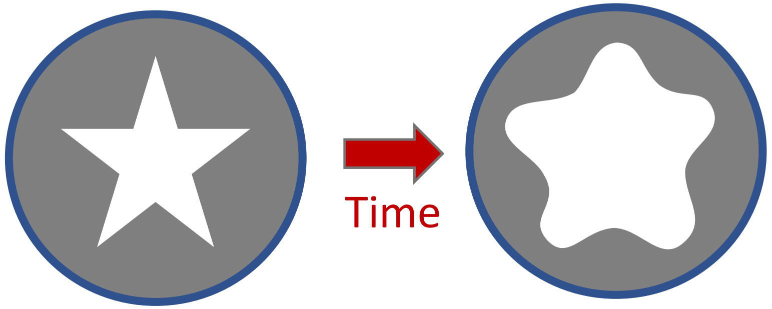
$$Mass\ burn\ rate = Mass\ flow\ thru\ the\ nozzle$$



End-Burning



Cylindrical Grain



Minuteman [ICBM missiles] had star-shaped channels for more uniform burning!
Many channels many nozzles

Attempt to keep A_b nearly constant with time!

Rocket Engine Example

Ammonium Perchlorate
NH_4ClO_4

$$R = \frac{\mathfrak{R}}{mw}$$ unit gas constant

$$\dot{m}_p = \dot{m} = P^\circ \frac{A^*}{c^*}$$

Where $c^* = \frac{\sqrt{RT^o}}{\Gamma(\gamma)} \approx \frac{\sqrt{RT}}{\Gamma(\gamma)}$

And $1/mw = \sum_i (1/mw_i)(\rho_i/\rho)$; $\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$

Since the velocity in the combustion chamber is very low, in particular $M \ll 1$:

$T^\circ \approx T$ and $P^\circ \approx P$

For a solid rocket:

$$\rho_p A_b a P^n \approx P \frac{A^*}{c^*}$$

$$P^{1-n} = \left[a \rho_p c^* \frac{A_b}{A^*}\right]$$

$$P = \left[a \rho_p c^* \frac{A_b}{A^*}\right]^{\frac{1}{1-n}}$$

If $n > 1$, an unstable situation arises!

Rocket Engine Example

$$\frac{d\dot{m}_p}{\dot{m}_p} = n\frac{dP}{P} \quad \text{While} \quad \frac{d\dot{m}}{\dot{m}} = \frac{dP}{P}$$

$\boxed{\dot{m}_p \sim P^n}$

For gas generation

Since $\dot{m} \sim P$ for nozzle flow

An increase in pressure causes $\dot{m}_p$ to rise faster than $\dot{m}$, which cause accumulation of mass in the combustion chamber and further pressure rise. This eventually causes engine failure!

References

[1] Braeunig, R., 2020. *Basics Of Space Flight: Rocket Propulsion*. [online] Braeunig.us. Available at: <http://www.braeunig.us/space/propuls.htm?fbclid=IwAR2W5i0Ai7J7apvhk4WV0jysDbGte6hJC2nK4SH31mHlCnPBzatYiZYksp8> [Accessed 20 July 2020].