MAE 158 Lecture 12
Nov. 7th 2024

Announcements: **Week 6 Quiz**

Fri 12am – Mon 11:59pm

topics: Lecture 9 – 11 (power Required, climbing/descending flight)

+ Recommended HW 6

Today's Objectives: Range
Endurance

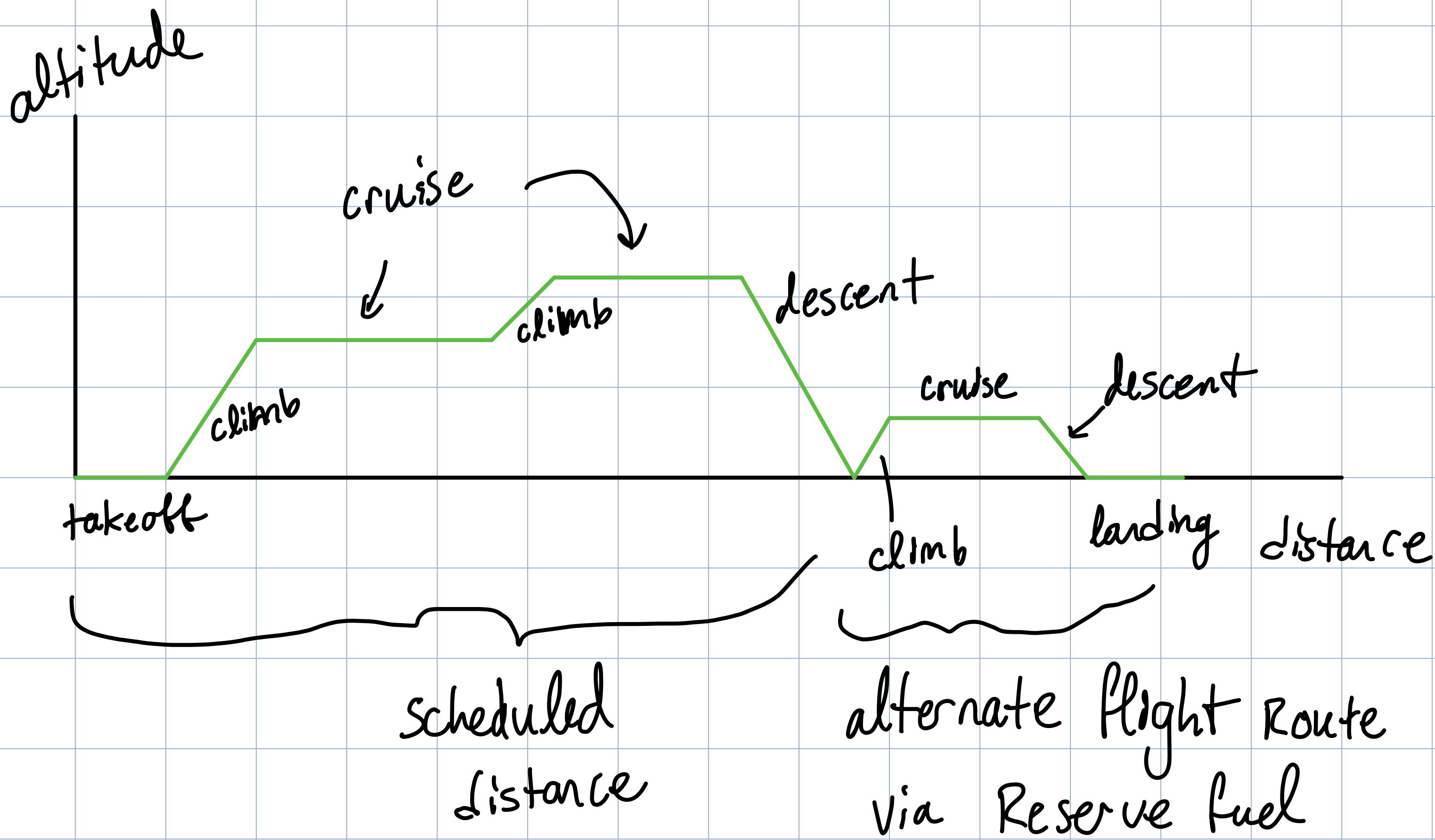
<u>Range</u>: distance to fly from one location to another on one unit of energy

↳ energy can come from

liquid fuel ←

Batteries

hybrid - - - -



Cruise phase: typically longest phase where most fuel burn occurs

Range cruise

- 1. Jet Specific Range $= \dfrac{\frac{1}{hr}\ \text{nautical miles}}{\frac{1}{hr}\ \text{weight fuel}}$

$$= \frac{V}{T \cdot C_T} = \frac{V}{D \cdot C_T}$$

SLF

$$D = D \cdot \frac{W}{L}$$

$C_T \equiv$ thrust specific fuel consumption

↳ property of an engine

units: $\left[\frac{lbs}{lb\text{-}hr}\right] = \frac{lbs\ fuel}{hr} \frac{1}{lbs\ Thrust}$

for a jet

Specific Range: $\frac{V}{C_T} \frac{L}{D} \cdot \frac{1}{W}$

property of Engine

aerodynamic efficiency

weight

weight will vary in cruise

to get Range, integrate specific Range

with weight

$$R_{jet} = \int_{W_0}^{W_1} \frac{V}{C_T}\frac{L}{D}\frac{dW}{W} = \underbrace{\overset{\frac{nmi}{hr}}{\frac{V}{C_T}} \cdot \frac{L}{D} \cdot \ln\left(\frac{W_0}{W_1}\right)}$$

$\frac{lb}{lb\,hr} = \frac{1}{hr}$

$W_0 \equiv$ starting weight

$W_1 \equiv$ ending weight

W_0 is bigger than W_1

Breguet Range Equation

how do I increase R_{jet}?

⇝ increase V & $\frac{L}{D}$ together

⇝ decrease C_T

⇝ increase $W_0 - W_1$ carry a

lot of fuel

V & L/D are Related

thus maximize $V \cdot L/D$

Recal $\frac{L}{D} V = \sqrt{\frac{2W}{\rho S C_L}} \cdot \frac{L}{D} \approx \frac{C_L}{C_D}$

$$V \cdot \frac{L}{D} = \sqrt{\frac{2W}{\rho S}} \cdot \frac{C_L^{1/2}}{C_D}$$

$\therefore$ $V \frac{L}{D}$ is max if $\frac{C_L^{1/2}}{C_D}$ is max

What is C_L such that $\frac{C_L^{1/2}}{C_D}$ is max

$$\frac{d \frac{C_D}{C_L^{1/2}}}{d C_L} = \frac{d\left(\frac{C_{Dp}}{C_L^{1/2}} + \frac{K C_L^2}{C_L^{1/2}}\right)}{d C_L}$$

$$= -\frac{1}{2}\frac{C_{Dp}}{C_L^{3/2}} + \frac{3}{2} C_L^{1/2} \cdot K = 0$$

$$\hookrightarrow \quad C_{Dp} = 3 K C_L^2$$

$$C_L = \sqrt{\frac{C_{Dp}}{3K}}$$

C_L where $C_L^{1/2}/C_D$ is max

then $$V_{\frac{C_L^{1/2}}{C_D}\max} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{3K}{C_{Dp}}}}$$

$$R_{jet} = \int_{W_0}^{W_1} \frac{1}{C_T} \cdot \left(\frac{2W}{\rho S}\right)^{1/2} \frac{C_L^{1/2}}{C_D} \frac{dW}{W}$$

$$= \frac{2}{C_T}\sqrt{\frac{2}{\rho S}} \cdot \frac{C_L^{1/2}}{C_D}\left(W_0^{1/2} - W_1^{1/2}\right)$$

- for R_{jet}, $\rho \downarrow$, $R \uparrow$ (flying higher is useful for R_{jet})

· Prop A/C

Not pressure coefficient!

instead of C_T, define C_p

$C_p \equiv$ specific fuel consumption Referenced to horse power

(lbs of fuel per hp-hr)

unit: $\left[\dfrac{\text{lbs fuel}}{\text{BHP - hr}}\right]$

Brake horsepower, outputted by propulsor shaft

Specific Range $\dfrac{\text{distance}/\text{hr}}{\text{lb fuel}/\text{hr}}$

thus: $\dfrac{V}{C_p \cdot BHP}$ V [knots] compare to jet $\quad BHP = \dfrac{T \cdot V}{550\,\eta}$

[ft/s]

[hp]

prop efficiency

550 is conversion for hp

AKA $\quad 550\,\dfrac{ft\,lb}{s} = 1\,hp$

plug BHP into Specific Range equation...

$$\frac{V\,[\text{knots}]}{C_p \cdot BHP} = \frac{V\,[\text{knots}] \cdot 550 \cdot \eta}{C_p \cdot V\,[ft/s] \cdot T}$$

$T = D \cdot \dfrac{W}{L}$

Note $\quad \dfrac{V\,[\text{knots}]}{V\,[ft/s]} = \dfrac{1}{1.69}$

property of propeller

$\therefore$ Specific Range for prop $= 325 \cdot \dfrac{\eta}{C_p} \cdot \dfrac{L}{D} \cdot \dfrac{1}{W}$

property of Engine

integrate $R_{prop} = \int_{W_0}^{W_1} 325 \frac{\eta}{c_p} \cdot \frac{L}{D} \cdot \frac{1}{W} \, dW$

$$= 325 \cdot \frac{\eta}{c_p} \cdot \frac{L}{D} \cdot \ln\left(\frac{W_0}{W_1}\right)$$ [nmi]

want to increase R_{prop}, what do I do?

- increase L/D (or max $\frac{L}{D}$ for max R_{prop})
- decrease c_p
- increase η
- carry more fuel

Endurance (E): time to fly on 1 unit of Energy

time on station

Specific endurance : $\dfrac{\text{hrs}}{\text{lb fuel}}$

jet Endurance :

Specific Endurance : $\dfrac{1}{T \cdot C_T}$

$T = D \cdot \dfrac{W}{L}$

then $= \dfrac{1}{C_T} \cdot \dfrac{L}{D} \cdot \dfrac{1}{W}$

$$E_{jet} = \int_{W_0}^{W_1} \frac{1}{C_T} \cdot \frac{L}{D} \cdot \frac{dW}{W}$$

$$= \underbrace{\frac{1}{C_T} \cdot \frac{L}{D} \cdot \ln}_{\text{Jet}} \left(\frac{W_0}{W_1} \right)$$

$\therefore$ thus maximize E_{jet} if $\underline{\dfrac{L}{D}_{max}}$

· Prop Endurance:

$$\text{Specific Endurace} = \frac{1}{BHP \cdot C_P}$$

relate to T

$$= \frac{\eta}{C_P} \cdot \frac{L}{D} \cdot \frac{1}{V} \cdot \frac{1}{W}$$

$$V = \sqrt{\frac{2W}{\rho S C_L}}$$

$$E_{prop} = \int_{W_0}^{W_1} \frac{\eta}{C_P} \cdot \left(\frac{\rho S}{2}\right)^{1/2} \frac{C_L^{3/2}}{C_D} \cdot \frac{1}{W^{3/2}} \, dW$$

$$= \frac{\eta}{C_P} (2 \rho S)^{1/2} \frac{C_L^{3/2}}{C_D} \cdot \left(\frac{1}{W_1^{1/2}} - \frac{1}{W_0^{1/2}}\right)$$

Want to maximize prop Endurance

· $C_L^{3/2}/C_D$ max

· $\rho \uparrow$ fly @ $\downarrow h$

- $\eta \uparrow$
- $C_p \downarrow$
- carry a lot of fuel

for Ex, if I want to loiter for 1 day, what is the min fuel I need to carry?

Prop. assume engine properties are given, Airframe & altitude given, W_1

$\rightarrow$ max $C_L^{3/2}/C_D$

Summary

	Jet	Prop
Range :	$C_L^{1/2}/C_D$	C_L/C_D

Endurance: C_L/C_D $\quad C_L^{3/2}/C_D$

ROC

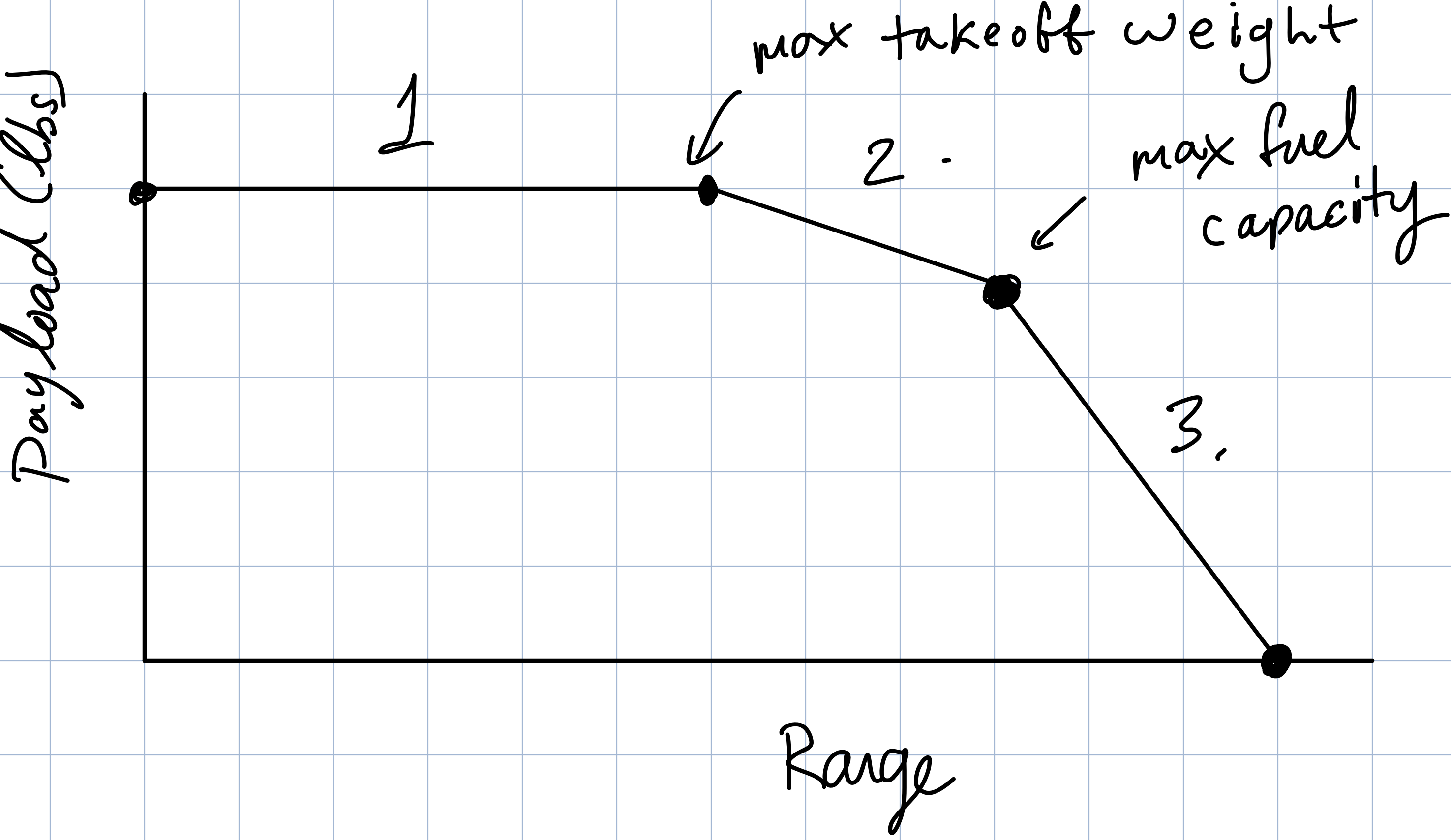
γ_{max} climb

descent

⋮

there is a tradeoff between
carrying fuel & carrying payload

examine this w/ payload
Range chart



1. Carry max payload, increase Range by carrying more fuel
2. @ Max takeoff weight, but you can extend Range furthur by dropping payload
3. @ max fuel capacity, decrease payload to exend the Range furthur ↳ decrease W_1