MAE 158: Aricraft Performance
Recommended Homework #3

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January 23, 2023

- **8.2** An airplane is cruising at a density altitude of 38,000 ft at a speed of 500 mph. The ambient temperature is 409ºR.

  (a) At a point on the wing the local velocity is 14% higher than the freestream velocity. What is the pressure coefficient at this point? If the C_p is equal to the average upper surface C_p, and if the upper surface provides three-quarters of the lift, what is the wing lift coefficient? Note that at this Mach number the compressible fluid equations must be used.

  We will learn these equation in future lectures. For this homework you may assume the pressure on the upper surface is 404 lb/ft^2.
  The pressure coefficient of the upper surface is defined as

  $$C_{P_u} = \frac{p_u - p_\infty}{\frac{1}{2}\rho V_\infty^2}$$

  where subindex ∞ refers to a point far enough from the airfoil. We find the density from Table A.2 in the appendix of the book to be

  $$\rho_{38{,}000\,\mathrm{ft}} = 6.4629 \cdot 10^{-4}\,\mathrm{slug/ft^3}$$

  We compute p_∞ using the equation of state

  $$p_\infty = \rho R T_\infty = 6.4629 \cdot 10^{-4}\,\mathrm{slug/ft^3} \cdot 1718\,\frac{\mathrm{ft\ lb}}{\mathrm{slug\ °R}} \cdot 409\ \ \mathrm{°R} = 454.1\,\frac{\mathrm{lb}}{\mathrm{ft^2}}$$

  Using $p_u = 404$ lb/ft^2, as stated, we can now find C_{P_u} ($V_\infty = 500\,\mathrm{mph} = 733.3\,\mathrm{ft/s}$)

  $$C_{p_u} = \frac{p_u - p_\infty}{\frac{1}{2}\rho V_\infty^2} = \frac{404\,\mathrm{lb/ft^2} - 454.1\,\mathrm{lb/ft^2}}{\frac{1}{2}\,6.4629 \cdot 10^{-4}\,\mathrm{slug/ft^3}(733.3\,\mathrm{ft/s})^2} = -0.2878$$

  If the C_p is equal to the average in the upper surface, then lift coefficient integral is reduced to

  $$C_L = \frac{1}{c}\int_{TE}^{LE} (C_{p_\ell} - C_{p_u})\,dx = C_{p_\ell} - C_{p_u}$$

  Also, if the upper surface provides 3/4 of the lift, we have that $C_L = -4C_{p_u}/3$ (or that $C_{p_\ell} = -C_{p_u}/3$) and so

  $$C_L = -4C_{p_u}/3 = -4 \cdot (-0.2878)/3 = 0.3837.$$

- **9.1** An airplane with a wing platform area of 650 ft^2 and a span of 80 ft is flying at a density altitude of 38,000 ft at a speed of 500 mph. The ambient pressure is 409ºR. The gross weight is 52,000 lb. Determine:

  (a) Lift coefficient
  From table A.2, we get that the density at this altitude is $\rho_{38{,}000\,\mathrm{ft}} = 6.4629 \cdot 10^{-4}$ lb s^2/ft^4. Velocity is $500\,\mathrm{mph} = 733.3$ ft/s. As we are in the cruise phase, $L = W$ and so we can find the lift coefficient C_L as

  $$C_L = \frac{L}{\frac{1}{2}\rho V^2 S_{ref}} = \frac{52000\,\mathrm{lbf}}{\frac{1}{2}\,6.4629 \cdot 10^{-4}\,\mathrm{lbf\ s^2/ft^4} \cdot (733.3\,\mathrm{ft/s})^2 \cdot 650\mathrm{ft}^2} = 0.4604$$

(b) Induced drag in pounds, assuming that the drag due to lift differs from that for the ideal elliptical distribution only by having $u = 0.99$.
The coefficient of induced drag is

$$C_{D_i} = C_L\,\varepsilon = \frac{C_L^2}{\pi\,\mathrm{AR}\,u}$$

where $\varepsilon = w/V_\infty$, AR=b^2/S is the aspect ratio and u (sometimes denoted by e) is Oswald's efficiency factor. We will use the second expression, for which we have C_L, the aspect ratio

$$\mathrm{AR} = \frac{b^2}{S} = \frac{(80\,\mathrm{ft})^2}{650\,\mathrm{ft}^2} = 9.85$$

and $u = 0.99$. Thus

$$C_{D_i} = \frac{C_L^2}{\pi\,\mathrm{AR}\,u} = \frac{0.4604^2}{\pi \cdot 9.85 \cdot 0.99} = 6.92 \cdot 10^{-3}$$

And so, the induced drag is

$$\begin{aligned} D_i &= \tfrac{1}{2}\,C_{D_i}\rho V^2 S = \\ &= \tfrac{1}{2}\,6.92 \cdot 10^{-3} \cdot 6.4629 \cdot 10^{-4}\mathrm{lb\ s^2/ft^4} \cdot (733\,\mathrm{ft/s})^2 \cdot 650\,\mathrm{ft}^2 = 781.6\,lb \end{aligned}$$

(c) What is the wing angle of attack (above zero lift). Assume $\eta = 0.95$.
We first compute the Mach number to check in which regime we are (incompressible if $C_L < 0.3$)

$$M = \frac{V}{\sqrt{\gamma R T}} = \frac{733.3\,\mathrm{ft/s}}{\sqrt{1.4 \cdot 1718\,\mathrm{lb\ ft/(slug\ ^{o}R)} \cdot 409^{o}\mathrm{R}}} = 0.7393.$$

where $\gamma_{\mathrm{air}} = 1.4$ is the adiabatic index or heat capacity ratio. We are in the compressible regime but we can use the Prandtl-Glauert relation

$$C_{L,\mathrm{in}} = C_{L,\mathrm{com}}\sqrt{1 - M^2} = 0.4604 \cdot \sqrt{1 - 0.7393^2} = 0.31$$

where "in" and "com" stand for incompressible and compressible respectively. We will find the aircraft's angle of attack using $C_{L,\mathrm{in}} = \mathrm{a} \cdot \alpha$, where

$$\mathrm{a} = \frac{\mathrm{a}_0(\mathrm{rad}^{-1})}{1 + \frac{\mathrm{a}_0(\mathrm{rad}^{-1})}{\pi\,\mathrm{AR}}} = \frac{\mathrm{a}_0(\mathrm{deg}^{-1})}{1 + \frac{57.3\,\mathrm{a}_0(\mathrm{deg}^{-1})}{\pi\,\mathrm{AR}}}$$

where a_0 is the 2D lift curve slope

$$\mathrm{a}_0 = 2\pi\eta = 2\pi \cdot 0.95 = 5.969\,\mathrm{rad}^{-1}$$

for which we get

$$\mathrm{a} = \frac{5.969}{1 + \frac{5.969}{\pi 9.85}} = 5.003\,\mathrm{rad}^{-1} = 0.08733\,\mathrm{deg}^{-1}$$

and so

$$\alpha = \frac{C_{L,\mathrm{in}}}{\mathrm{a}} = \frac{0.31}{0.08733} = 3.55\,\mathrm{deg}$$

- **9.2** An airplane with a wing area of 350 ft^2 and a span of 56 ft is flying at a pressure altitude of 10,000 ft at a speed of 320 mph. The ambient temperature is 495ºR. The wing has a geometric angle of attack of 4 degrees, above the angle for zero lift. Assume the two-dimensional lift curve slope is 95% of the theoretical value. Determine

(a) Lift in pounds
Aspect ratio is AR $= b^2/S = 56^2/350 = 8.96$. We get the pressure from table A.2 ($p = 1455.6$ lb/ft^2). Density is found using equation of state

$$\rho = \frac{p}{RT} = \frac{1455.6\,\text{lb/ft}}{1918\,\text{lb ft/slug }^{\text{o}}\text{R} \cdot 495\,^{\text{o}}\text{ R}} = 0.001712\,\text{slug/ft}^3$$

The Mach number is, using $V = 320\,\text{mph} = 469.33$ ft/s

$$M = \frac{V}{\sqrt{\gamma RT}} = \frac{469.33\,\text{ft/s}}{\sqrt{1.4 \cdot 1718\,\text{lb ft/(slug }^{\text{o}}\text{R)} \cdot 495^{\text{o}}\text{R}}} = 0.4301.$$

The 2D lift curve slope a_0 is

$$\text{a}_0 = 2\pi\eta = 2\pi \cdot 0.95 = 5.969\,\text{rad}^{-1} = 0.1042\,\text{deg}^{-1}$$

and so, for the 3D

$$\text{a} = \left(\frac{dC_L}{d\alpha}\right) = \frac{\text{a}_0(\text{deg}^{-1})}{1 + \frac{57.3\,\text{a}_0(\text{deg}^{-1})}{\pi\text{AR}}} = \frac{0.1042}{1 + \frac{57.3 \cdot 0.1042}{8.96\pi}} = 0.08597\,\text{deg}^{-1}$$

The incompressible lift coefficient is

$$C_{L,\text{in}} = \text{a} \cdot \alpha = 0.08597\,\text{deg}^{-1} \cdot 4\,\text{deg} = 0.3439$$

Applying Prandtl-Glauert relation, we get the compressible lift coefficient

$$C_{L,\text{com}} = \frac{C_{L,\text{in}}}{\sqrt{1 - M^2}} = \frac{0.3439}{\sqrt{1 - 0.4301^2}} = 0.381$$

and finally,

$$L = \tfrac{1}{2}\,C_L \rho V^2 S = \tfrac{1}{2}\,0.381 \cdot 0.001712\,\text{slug/ft}^3 \cdot (469.33\,\text{ft/s})^2 \cdot 350\,\text{ft}^2 = 25143\,\text{lb}$$

(b) Induced drag in pounds.
The coefficient of induced drag C_{D_i} is

$$C_{D_i} = \frac{C_L^2}{\pi\,\text{AR}\,u} = \frac{0.381^2}{\pi \cdot 8.96 \cdot 0.99} = 0.00521$$

and so, the induced drag is

$$D_i = \tfrac{1}{2}\,C_{D_i}\,\rho\,V^2\,S = 343.16\,\text{lb}$$

- **9.6** An airplane with a wing area of 450 ft^2 and a span of 70 ft is flying at a pressure altitude of 8000 ft at a speed of 280 mph. The ambient temperature is 500ºR. The wing has a geometric angle of attack of 4 degrees above the angle for zero lift. Assume the two-dimensional lift curve slope is 95% of the theoretical value. Determine:

(a) Lift in pounds.
Aspect ratio is AR $= b^2/S = 70^2/450 = 10.89$. Pressure from table A.2 is $p = 1572.1$ lb/ft^2. Density is obtained using the equation of state

$$\rho = \frac{p}{RT} = \frac{1572.1\text{lb/ft}^2}{1718\,\text{lb ft/slug }^{\text{o}}\text{R} \cdot 500\,^{\text{o}}\text{R}} = 0.00183\,\text{slug/ft}^3$$

Mach number is, using $V = 280\,\text{mph} = 410.67$ ft/s

$$M = \frac{V}{\sqrt{\gamma RT}} = \frac{410.67\,\text{ft/s}}{\sqrt{1.4 \cdot 1718\,\text{lb ft/(slug ºR)} \cdot 500\text{ºR}}} = 0.3745.$$

which is within the compressible regime. Thus we will need to account for it. The 2D lift curve slope a_0 is

$$\mathrm{a}_0 = 2\pi\eta = 2\pi \cdot 0.95 = 5.969\,\text{rad}^{-1} = 0.1042\,\text{deg}^{-1}$$

and

$$\mathrm{a} = \frac{\mathrm{a}_0(\text{deg}^{-1})}{1 + \frac{57.3\,\mathrm{a}_0(\text{deg}^{-1})}{\pi \text{AR}}} = \frac{0.1042}{1 + \frac{57.3 \cdot 0.1042}{10.89\pi}} = 0.08872\,\text{deg}^{-1}$$

and thus

$$C_{L,\text{in}} = \mathrm{a} \cdot \alpha = 0.08872\,\text{deg}^{-1} \cdot 4\,\text{deg} = 0.3549$$

And the compressible lift coefficient is

$$C_{L,\text{com}} = \frac{C_{L,\text{in}}}{\sqrt{1 - M^2}} = \frac{0.3549}{\sqrt{1 - 0.3745^2}} = 0.3828$$

The lift force, is then

$$L = \tfrac{1}{2}\,C_L \rho V^2 S = \tfrac{1}{2}\,0.3828 \cdot 0.00183\,\text{slug/ft}^3 \cdot (410.67\,\text{ft/s})^2 \cdot 450\,\text{ft}^2 = 26582\,\text{lb}$$

(b) Parasite drag (lb) if the value of C_{D_P} is 0.0190.
Simply:

$$D_P = \tfrac{1}{2}\,C_{D_P} \rho V^2 S = \tfrac{1}{2}\,0.019 \cdot 0.00183\,\text{slug/ft}^3 \cdot (410.67\,\text{ft/s})^2 \cdot 450\,\text{ft}^2 = 1319.4\,\text{lb}$$

(c) The induced drag coefficient.
It is a function of the lift coefficient:

$$C_{D_i} = \frac{C_L^2}{\pi\,\text{AR}\,u} = \frac{0.3828^2}{\pi \cdot 10.89 \cdot 0.99} = 0.00433$$

(d) The induced drag in pounds.
Using the induced drag coefficient, computed above, we get

$$D_i = \tfrac{1}{2}\,C_{D_i} \rho V^2 S = \tfrac{1}{2}\,0.0433 \cdot 0.00183\,\text{slug/ft}^3 \cdot (410.67\,\text{ft/s})^2 \cdot 450\,\text{ft}^2 = 300.7\,\text{lb}$$

(e) The total drag in pounds.
It is the sum of the parasite and the induced drag

$$D = D_P + D_i = 1319.4 + 300.7 = 1620.1\,\text{lb}$$

(f) The ratio of lift to drag.
The Lift-to-Drag ratio is

$$\frac{L}{D} = \frac{26582\,\text{lb}}{1320.1\,\text{lb}} = 16.41.$$