MAE 185: Aricraft Performance
Recommended Homework #1

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Chapter 3

- 3.1 The gross weight of a two-place Piper Cherokee is 2000 lb and its wing area is 160 ft^2. What is its wing loading?
  The wing loading is the ratio weight over reference surface, thus

$$\text{Wing loading} = \frac{\text{Weight}}{S_{\text{ref}}} = \frac{2000\,\text{lb}}{160\,\text{ft}^2} = 12.5\,\text{lb}/\text{ft}^2$$

- 3.2 A Boeing 747B has a wing loading of 123 lb/ft^2 at takeoff and a wing area of 5500 ft^2. What is its takeoff weight?
  Using the same expression from the previous exercise,

$$\text{Weight} = \text{Wing loading} \cdot S_{\text{ref}} = 123\,\text{lb}/\text{ft}^2 \cdot 5500\,\text{ft}^2 = 676500\,\text{lb}$$

- 3.3 A 747 with a gross (total) weight of 650,000 lb is criusing at 35,000 ft at a speed of 465 knots. The 747 has a wing area of 5500 ft^2. The density at 35,000 ft is 0.0007382 slug/ft^3. What is the value of the cruise lift coefficient, C_L, and the wing loading? (When using equations 3.9, V must be in feet per second in the English system.)
  For the lift coefficient, we use

$$C_L = \frac{\text{Lift}}{\frac{1}{2}\rho V^2 S}$$

  Noting that $L = W = mg$ as we are on the cruise phase, and converting

$$V = 465\,\text{knot} \cdot \frac{1.69\,\text{ft/s}}{1\,\text{knot}} = 785.9\,\text{ft/s}$$

  and with $g = 32.17\,\text{ft/s}^2$. Thus,

$$C_L = \frac{2 \cdot 650000\,\text{lb} \cdot 32.17\,\text{ft/s}^2}{0.0007382\,\frac{\text{slug}}{\text{ft}^3}\,\frac{32.17\,\text{lb}}{1\,\text{slug}} \cdot \left(785.9\,\frac{\text{ft}}{\text{s}}\right)^2 \cdot 5500\text{ft}^2} = 0.518$$

  On the other hand, the wing loading is computed as before

$$\text{Wing loading} = \frac{\text{Weight}}{S_{\text{ref}}} = \frac{650000\,\text{lb}}{5500\,\text{ft}^2} = 118.18\,\text{lb}/\text{ft}^2$$

- 3.4 If the 747 in Problem 3.3 has a drag of 40,000 lb, what is the drag coefficient?
  Similarly, we have

$$C_D = \frac{\text{Drag}}{\frac{1}{2}\rho V^2 S} = \frac{2 \cdot 40000\,\text{lb} \cdot 32.17\,\text{ft/s}^2}{0.0007382\,\frac{\text{slug}}{\text{ft}^3}\,\frac{32.17\,\text{lb}}{1\,\text{slug}} \cdot \left(785.9\,\frac{\text{ft}}{\text{s}}\right)^2 \cdot 5500\text{ft}^2} = 0.0319$$

- 3.5 A DC-9 is carrying 100 passengers at 30,000 ft at a C_L of 0.35. The density at 20,000 ft is 0.0008907 slug/ft^3. The DC-9 wing area is 1000 ft^2 and its weight is 100,000 lb. What is the cruise speed in feet per second? in miles per hour? in knots?
  Now, we have, as $L = W$

$$V = \sqrt{\frac{W}{\frac{1}{2}\rho C_L S}} = \sqrt{\frac{2 \cdot 100000\,\text{lb} \cdot 32.17\text{ft/s}^2}{0.0008907\,\frac{\text{slug}}{\text{ft}^3} \cdot \frac{32.17\,\text{lb}}{1\,\text{slug}} \cdot 0.35 \cdot 1000\,\text{ft}^2}} = 801\,\text{ft/s}$$

  In miles per hour

$$V = 801\,\text{ft/s}\,\tfrac{0.3025\,\text{m}}{1\,\text{ft}}\,\tfrac{1\,\text{mile}}{1609\,\text{m}}\,\tfrac{3600\,\text{s}}{1\,\text{h}} = 546\,\text{mph}$$

  And in knots:

$$V = 546\,\text{mph}\,\tfrac{0.869\,\text{knot}}{1\,\text{mph}} = 474\,\text{knot}$$

- 3.6 A Cessna 150 is cruising at 115 mph at 7000 ft. The airplane weights 1500 lb and has a wing area of 157 ft^2. What is the lift coefficient? If the drag coefficient is 0.0300, what is the drag in pounds? The air density, ρ, at 7000 ft is 0.001927 slug/ft^3.
  We first convert the velocity

$$115\,\text{mph} \cdot \frac{1\,\text{h}}{3600\,\text{s}} \cdot \frac{5280\,\text{ft}}{1\,\text{mile}} = 168.6\,\text{ft/s}$$

  Again

$$C_L = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{1500\,\text{lb} \cdot 32.17\,\text{ft/s}^2}{\frac{1}{2}\,0.001927\,\text{slug/ft}^3 \cdot \frac{32.17\,\text{lb}}{1\,\text{slug}}(168.6\,\text{ft/s})^2 \cdot 157\,\text{ft}^2} = 0.349$$

  The drag can be computed as

$$D = \tfrac{1}{2}\rho V^2 S C_D = \tfrac{1}{2} \cdot 0.001927\,\text{slug/ft}^3 (168.6\,\text{ft/s})^2 \cdot 157\,\text{ft}^2 \cdot 0.03 = 129\,\text{lb}$$

Chapter 5

- 5.2 Calculate the values of pressure, pressure ration, density, density ration, and temperature for the standard atmosphere at geometric altitudes of 5000, 10,000, 11,019 and 15,000 m. Show results in SI units. The sea-level values are pressure = 1.01325×10^5 N/m^2, density = $1.2250\,\text{kg/m}^3$, and temperature = 288.16 K. The temperature lapse rate $a = -0.0065$ K/m; above 11,019 m, $a = 0$.
  For temperatures, we will use

$$T(h) = T_0 + a(h - h_0)$$

  with $T_0 = 288.16$ K, $a = -0.0065$ K/m below $h = 11019$ m, $a = 0$ above, and $h_0 = 0$. Thus, we have

$$T_1 = 255.66\,\text{K}; \qquad T_2 = 223.16\,\text{K}; \qquad T_3 = 216.5\,\text{K}; \qquad T_4 = 216.5\,\text{K}$$

  As after $h = 11019$ m, the temperature keeps constant. In the gradient region, the pressure is

$$p(h) = p_0 \left(\frac{T(h)}{T_0}\right)^{-g_0/aR}$$

  where the exponent is approximately 5.253. Thus, we get

$$p_1 = 54040\,\text{N/m}^2; \qquad p_2 = 26457\,\text{N/m}^2; \qquad p_3 = 22568\,\text{N/m}^2$$

In the isothermal region, we use

$$p(h) = p_0 e^{-(g_0/RT)(h-h_0)}$$

h_0 being 11019 m and $T = 216.5$ K ($p_0 = 22568$ N/m^2 is the pressure at 11019 m). Thus, we get $p_4 = 12046$ N/m^2. The pressure ratia are

$$\frac{p_1}{p_0} = 0.533; \qquad \frac{p_2}{p_0} = 0.261; \qquad \frac{p_3}{p_0} = 0.223; \qquad \frac{p_4}{p_0} = 0.119.$$

Densities can be obtained using the similar expressions

$$\rho(h) = \rho_0 \left(\frac{T(h)}{T_0}\right)^{-g_0/(aR)-1}$$

in the gradient region and

$$\rho(h) = \rho_0 e^{-g_0/(RT)(h-h_0)}$$

in the isothermal. Also, we can find the density with the equation of state $\rho = p/RT$. Values are

$$\rho_1 = 0.7366\,\mathrm{kg/m^3}; \qquad \rho_2 = 0.4133\,\mathrm{kg/m^3}; \qquad \rho_3 = 0.3631\,\mathrm{kg/m^3}; \qquad \rho_4 = 0.1938\,\mathrm{kg/m^3}.$$

Density ratia are

$$\frac{\rho_1}{\rho_0} = 0.6013; \qquad \frac{\rho_2}{\rho_0} = 0.3374; \qquad \frac{\rho_3}{\rho_0} = 0.2964; \qquad \frac{\rho_4}{\rho_0} = 0.1582.$$

Note that the equation of state, $p = \rho RT$ can also be used, giving the same results.