MAE 158: Aricraft Performance
Recommended Homework #6

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- A two-place airplane is flying at a pressure altitude of 4000 ft at a speed of 120 mph. Outside air temperature is 50ºF. The gross weight is 2000 lb. The rectangular wing has an area of 170 ft^2 with a span of 33.25 ft. Wing thickness is 14%. Wing parasite drag is 39% of the total parasite drag; 88% of the wing is exposed. Assuming a propeller (or propulsive) efficienct of 0.84, determine the required cruising brake horsepower. Note Brake Horsepower = BHP = $\frac{\text{Thrust}\cdot\text{Velocity}}{550\eta}$.

Velocity is $V = 176\,\text{ft/s}$ and pressure is obtained from the table to be $p = 1827.7\,\text{lb/ft}^2$. From the equation of state, we can find the density

$$\rho = \frac{p}{RT} = \frac{1827.7\,\text{lb/ft}^2}{1718\,\text{lb ft/slug ºR}\cdot 510\,\text{ºR}} = 2.086\cdot 10^{-3}\,\text{slug/ft}^3$$

and dynamic pressure

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2}2.086\cdot 10^{-3}\,\text{slug/ft}^3\,(176\,\text{ft/s})^2 = 32.31\,\text{lb/ft}^2$$

We obtain the dynamic viscosity from equation 10.7

$$10^{10}\mu = 0.317\cdot T(\text{ºR})^{3/2}\left(\frac{734.7}{T(\text{ºR})+216}\right)$$

which is $\mu = 3.693\cdot 10^{-7}$ lb s/ft^2. The Reynolds number, based on the mean chord

$$\bar{c} = \frac{S}{b} = \frac{170\,\text{ft}^2}{33.25\,\text{ft}} = 5.11\,\text{ft}$$

is

$$\text{Re}_{\bar{c}} = \frac{\rho V_0 L}{\mu} = \frac{2.086\cdot 10^{-3}\,\text{slug/ft}^3\cdot 176\,\text{ft/s}\cdot 5.11\,\text{ft}}{3.693\cdot 10^{-7}\,\text{lb s/ft}^2} = 5.076\cdot 10^6$$

From figure 11.2, using this Reynolds number, we obtain $C_f = 0.0038$. The wetted surface is

$$S_{\text{wet}} = 2.04\cdot S_{\text{ref}}\cdot \%_{\text{exp}} = 2.04\cdot 170\,\text{ft}^2\cdot 0.88 = 305.2\,\text{ft}^2$$

To find the form factor K, we will use the aspect ratio $AR = \frac{b^2}{S} = \frac{33.25\,\text{ft}^2}{170\,\text{ft}^2} = 6.50$. From figure 11.3, using $\Lambda = 0^{\text{º}}$ and $t/c = 0.14$, we get $K = 1.32$. The wing parasite drag coefficient is

$$C_{D_{P,\text{wing}}} = \frac{C_f K S_{\text{wet}}}{S_{\text{ref}}} = \frac{0.0038\cdot 305.2\,\text{ft}^2\cdot 1.32}{170\,\text{ft}^2} = 0.00901$$

Since the wing parasite drag accounts for 39% of the total parasite drag, the total parasite drag coefficient is

$$C_{D_P} = \frac{C_{D_{P,\text{wing}}}}{0.39} = \frac{0.00901}{0.39} = 0.02309$$

For the induced drag we will need the efficiency factor and the lift coefficient. Using the value of C_{D_P}, and $AR = 6.5$, from figure 11.8, we get $e = 0.82$. Also,

$$C_L = \frac{W}{qS_{\text{ref}}} = \frac{2000\,\text{lb}}{32.31\,\text{lb/ft}^2 \cdot 170\,\text{ft}^2} = 0.364$$

The total drag coefficient is

$$C_D = C_{D_P} + C_{D_i} = C_{D_P} + \frac{C_L^2}{\pi\,\text{AR}\,e} = 0.02309 + \frac{0.364^2}{\pi \cdot 6.5 \cdot 0.82} = 0.02309 + 0.0791 = 0.031$$

and so the total drag force is

$$D = q\,C_D\,S_{\text{ref}} = 32.31\,\text{lb/ft}^2 \cdot 0.031 \cdot 170\,\text{ft}^2 = 170.27\,\text{lb}$$

Finally, the brake horsepower, with $\eta = 0.84$:

$$\text{BHP} = \frac{T \cdot V}{550\eta} = \frac{170.27\,\text{lb} \cdot 176\,\text{ft/s}}{550 \cdot 0.84} = 64.9\,\text{lb ft/s}$$

The Bede BD-5J is a very small single-seat home-built jet airplane which became available in the early 1970s. The data for the BD-5J are as follows

- Wing span: 17 ft
- Wing planform area: 37.8 ft^2
- Gross weight at takeoff: 960 lb
- Fuel capacity: 55 gal
- Power plant: one French-built Microturbo TRS 18 turbojet engine with maximum thrust at sea level of 202 lb and a specific fuel consumption of 1.3 lb/(lb · h)

We will approximate the drag polar for this airplane by

$$C_D = 0.02 + 0.062C_L^2$$

- For the BD-5J calculate *analytically* (directly) (a) the maximum velocity at sea level and (b) the maximum velocity at 10,000 ft.

(a) For the maximum velocity at sea level, we will have that $T_A = D$, and so

$$T_A = D = C_D\,q\,S = \left(0.02 + 0.062C_L^2\right) q\,S = \left(0.02 + 0.062\,\frac{W^2}{q^2S^2}\right) q\,S$$

Multiplying at both sides by q, we get

$$0.02Sq^2 - T_A\,q + 0.062\,\frac{W^2}{S} = 0$$

a quadratic equation with coefficients

$$\begin{aligned} a &= 0.02\,S = 0.02\,37.8\,\text{ft}^2 = 0.756\,\text{ft}^2 \\ b &= -T_A = -202\,\text{lb} \qquad \text{and} \\ c &= 0.062\,\frac{W^2}{S} = 0.062\,\frac{960^2\,\text{lb}^2}{37.8\,\text{ft}^2} = 1511.62\,\text{lb}^2/\text{ft}^2 \end{aligned}$$

that gives, as solutions $q_1 = 259.49\,\text{lb/ft}^2$ (corresponding to $V_{\text{max, SL}}$) and $q_2 = 7.7055\,\text{lb/ft}^2$. Then, using the density at sea level $\rho_{\text{SL}} = 2.377 \cdot 10^{-3}\,\text{slug/ft}^3$, we get

$$V_{\text{max, SL}} = \sqrt{\frac{q}{\frac{1}{2}\rho_{\text{SL}}}} = \sqrt{\frac{259.49\,\text{lb/ft}^2}{0.5 \cdot 2.377 \cdot 10^{-3}\,\text{slug/ft}^3}} = 467.3\,\text{ft/s}$$

(b) We first find the density at $h = 10,000$ ft, which is $\rho = 1.756 \cdot 10^{-3}\,\text{slug/ft}^3$. Using the following relation, we can find the thrust at this altitude:

$$\frac{T_h}{T_{\text{SL}}} = \frac{\rho_h}{\rho_{\text{SL}}} \qquad \rightarrow \qquad T_h = T_{\text{SL}}\left(\frac{\rho_h}{\rho_{\text{SL}}}\right) = 202\,\text{lbs}\left(\frac{1.756 \cdot 10^{-3}\,\text{slug/ft}^3}{2.377 \cdot 10^{-3}\,\text{slug/ft}^3}\right) = 149.2\,\text{lbs}$$

Note that there is usually an exponent n in the density ratio in the above expression. Since the value of n has not been given, we will take $n = 1$. We use the same quadratic equation as before, now with coefficients:

$$a = 0.756\,\text{ft}^2; \qquad b = -149.2\,\text{lb}; \qquad c = 1511.62\,\text{lb}^2/\text{ft}^2.$$

From the solutions $q_1 = 186.6\,\text{lb/ft}^2$ and $q_2 = 10.71\,\text{lb/ft}^2$, we take the first and get the velocity

$$V_{\text{max, h}} = \sqrt{\frac{186.6\,\text{lb/ft}^2}{0.5 \cdot 1.756 \cdot 10^{-3}\,\text{slug/ft}^3}} = 461.1\,\text{ft/s}$$

- For the BD-5J, plot the power required and power available curves at sea level. From these curves, estimate the maximum rate of climb at sea level.

  We have used the following relations:

$$P_R = D\,V, \qquad P_A = T_A\,V, \qquad EP = P_A - P_R.$$

Listing 1: Code used

```
S=37.8;
W=960;
rhoSL=2.377e-3;

V=100:10:500;
q=0.5*rhoSL*V.^2;
CL=W./(q*S);
CD=0.02+0.062*CL.^2;
D=CD.*q*S;
PR=D.*V;
PA=202*V;
EP=PA-PR;
ROC=60*EP/W;
figure
plot(V,PR)
hold on
plot(V,PA)
hold on
plot(V,EP)
legend('PR','PA','EP','Location','Best')
xlabel('V (ft/s)')
ylabel('P (ft lb/s)')
```

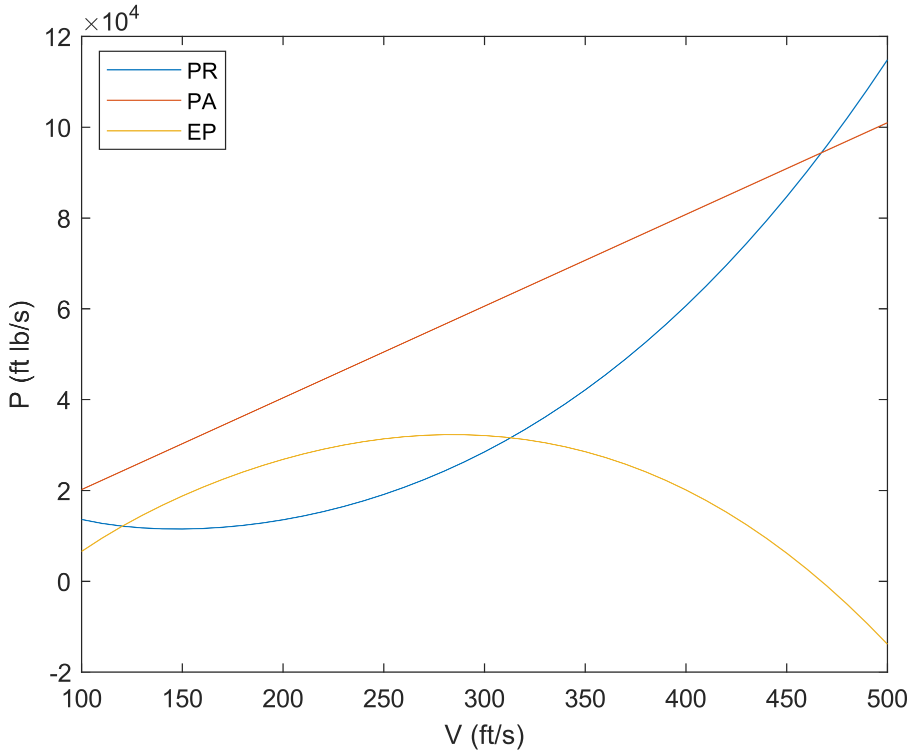

Figure 1: Required, available and excess power vs. velocity.

We have used the code listed above and obtained the plot in figure 1. The maximum ROC, defined as

$$ROC = \frac{60\,EP}{W}$$

is approximated to be when the derivative of the excessive power is zero, which happens approximately at $V = 280\,\mathrm{ft/s}$.

- For the BD-5J use the analytical results to calculate directly (b) Maximum climb angle at sea level and the velocity at which it occurs.
  We use the relation

$$\sin\theta_{\max} = \frac{T}{W} - \sqrt{4\,C_{D_P}\,k}$$

where $k = \frac{1}{\pi\,\mathrm{AR}\,e}$. Thus, we have

$$\theta_{\max} = \arcsin\left(\frac{202}{960} - \sqrt{4\cdot 0.02\cdot 0.062}\right) = 8.05^{\text{o}}$$

Finally

$$V_{\theta_{\max}} = \sqrt{\frac{2}{\rho}\left(\frac{k}{C_{D_P}}\right)^{1/2}\frac{W}{S}\,\cos\theta_{\max}} =$$

$$= \sqrt{\frac{2}{2.377\cdot 10^{-3}\,\mathrm{slug/ft^3}}\left(\frac{0.062}{0.02}\right)^{1/2}\frac{960\,\mathrm{lb}}{37.8\,\mathrm{ft^2}}\,\cos(8.05)} = 193\,\mathrm{ft/s}$$