# MAE 158 Lecture 9
## Fall 2024

Announcements: Week 4 Quiz

Fri 12am – Mon 11:59pm

topics: Lecture 7-9 (total drag + airfoils)

+ Recommended HW 5

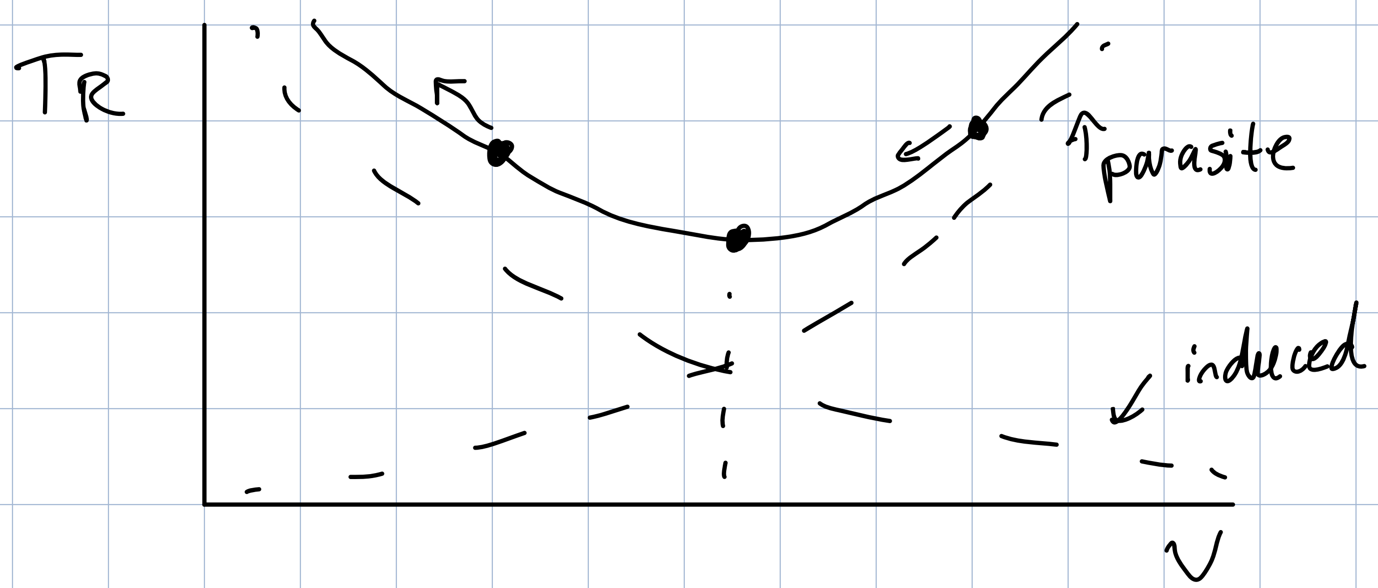
Today's Objectives: Thrust Required

Power Required

last time:

$C_L = \frac{2W}{\rho V^2 S}$

$$T_R = \tfrac{1}{2}\rho V^2 S \left( \overset{V^2}{C_{D_P}} + \overset{1/V^2}{K C_L^2} \right)$$



- there is a point where $T_R$ is min

- there is a side of the curve that is more "stable" than the other

- Right side of the curve is more stable because a momentary decrease in $V$ will $\downarrow T_R$

- left side is also known as "back side" of power curve

Analytically, in SLF

$$T = D \qquad \frac{W}{W} D = T = \frac{W}{L} D$$
$$W = L$$

$$\underline{T_{Req} = \frac{W}{(L/D)}} \qquad \frac{L}{D} = \text{lift to Drag Ratio}$$

$L/D \sim$ aerodynamic efficiency of the aircraft

$T_{R\,min}$ will occur @ $V_{L/D\,max}$

$$\frac{L}{D} \sim \frac{C_L \cancel{\frac{1}{2}\rho V^2 S_{REF}} = L}{C_D \cancel{\frac{1}{2}\rho V^2 S_{REF}} = D}$$

$C_L$ & $C_D$ are functions of $V, \alpha$

Solve for $\frac{L}{D}_{max}$

$$\frac{C_L}{C_D}_{max} \rightarrow \text{same as } \frac{C_D}{C_L}_{min}$$

$$\frac{C_D}{C_L} = \frac{C_{Dp} + \frac{C_L^2}{\pi A\!R e}}{C_L} = \frac{C_{Dp}}{C_L} + \frac{C_L}{\pi A\!R e}$$

$$\frac{d\left(\frac{C_D}{C_L}\right)}{dC_L} = -\frac{C_{Dp}}{C_L^2} + \frac{1}{\pi A\!R e} = 0 \text{ @ min}$$

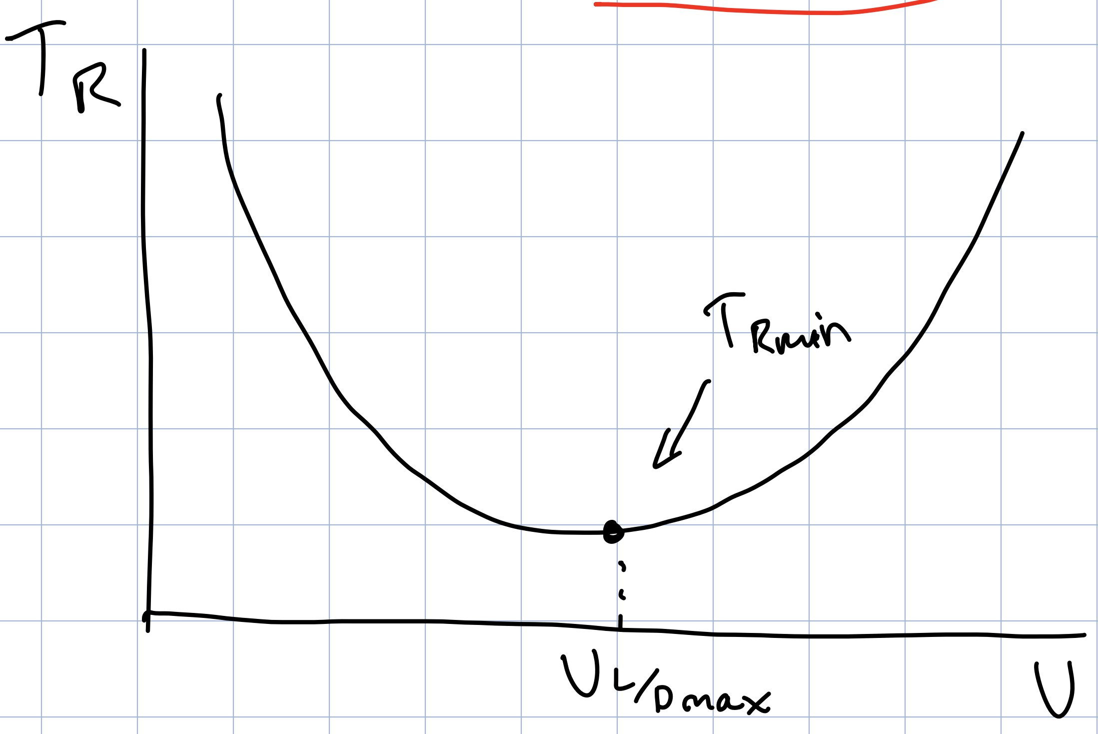
$\hookrightarrow \boxed{C_{L_{L/D\max}} = \sqrt{C_{D_p} \pi A\!\!\!/ Re} = \sqrt{\dfrac{C_{Dp}}{K}}}$

$K = \dfrac{1}{\pi A\!\!\!/ Re}$

$$L/D_{\max} = \frac{C_L}{C_{D\max}} = \frac{C_{L_{L/D\max}}}{C_{D_p} + K\left(C_{L_{L/D\max}}\right)^2}$$

$$= \frac{\sqrt{\frac{C_{D_p}}{K}}}{C_{D_p} + K\left(C_{Dp}/K\right)}$$

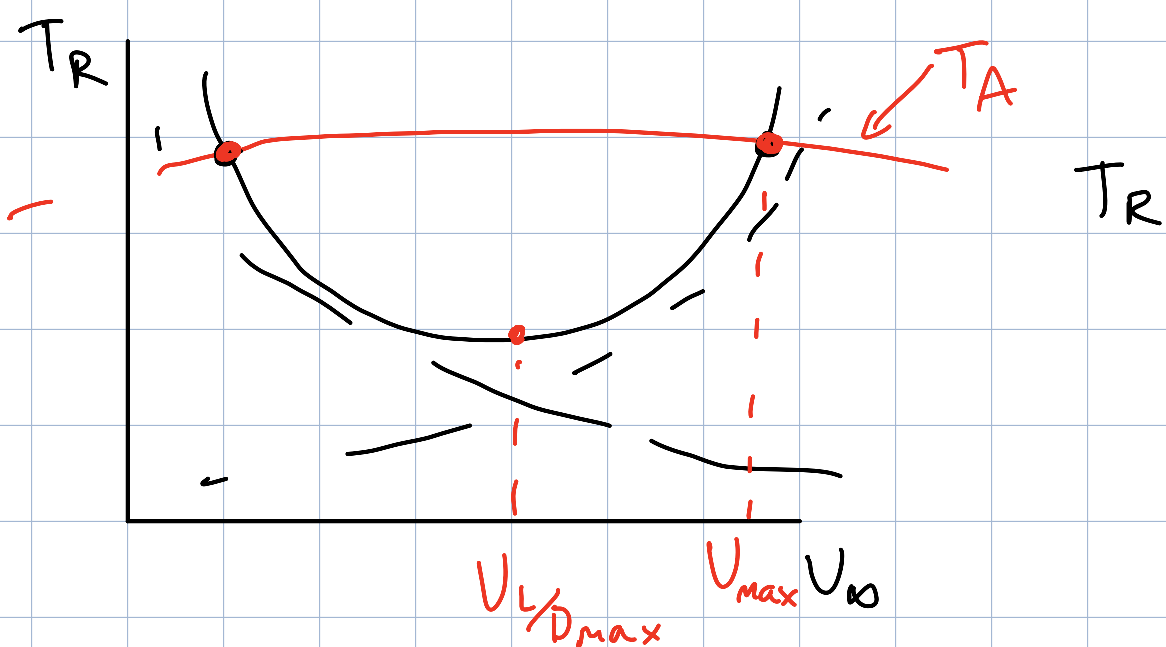
$$= \sqrt{\frac{1}{4KC_{D_p}}}$$



determine $V_{L/D_{max}}$ by taking $C_{L_{L/D_{max}}}$, plugging into Lift equation

$$V_{L/D_{max}} = \sqrt{\frac{2W}{\rho S C_{L_{L/D_{max}}}}} = \sqrt{\frac{2W}{\rho S \left(\sqrt{C_{D_p} \pi ARe}\right)}}$$

$$= \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{C_{D_p}}}}$$



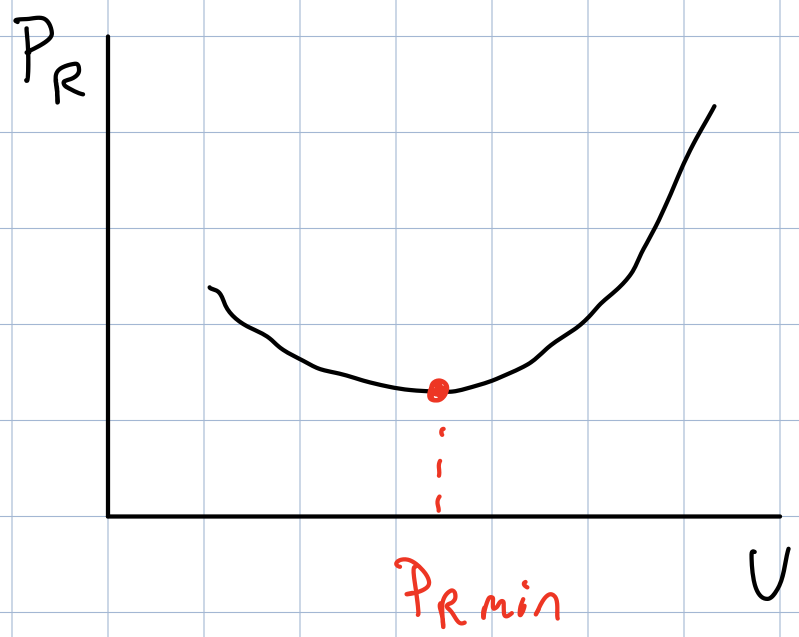
# Power Required

$$\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{\text{Force} \cdot \text{Distance}}{\text{time}}$$

$$= \text{Thrust} \cdot V$$

$$P_R = T_R \cdot V$$

graphically




$P_R$ also has a minimum

analytically

$$P_R = T_R \cdot V$$

$$= \frac{W}{C_L/C_D} \cdot V$$

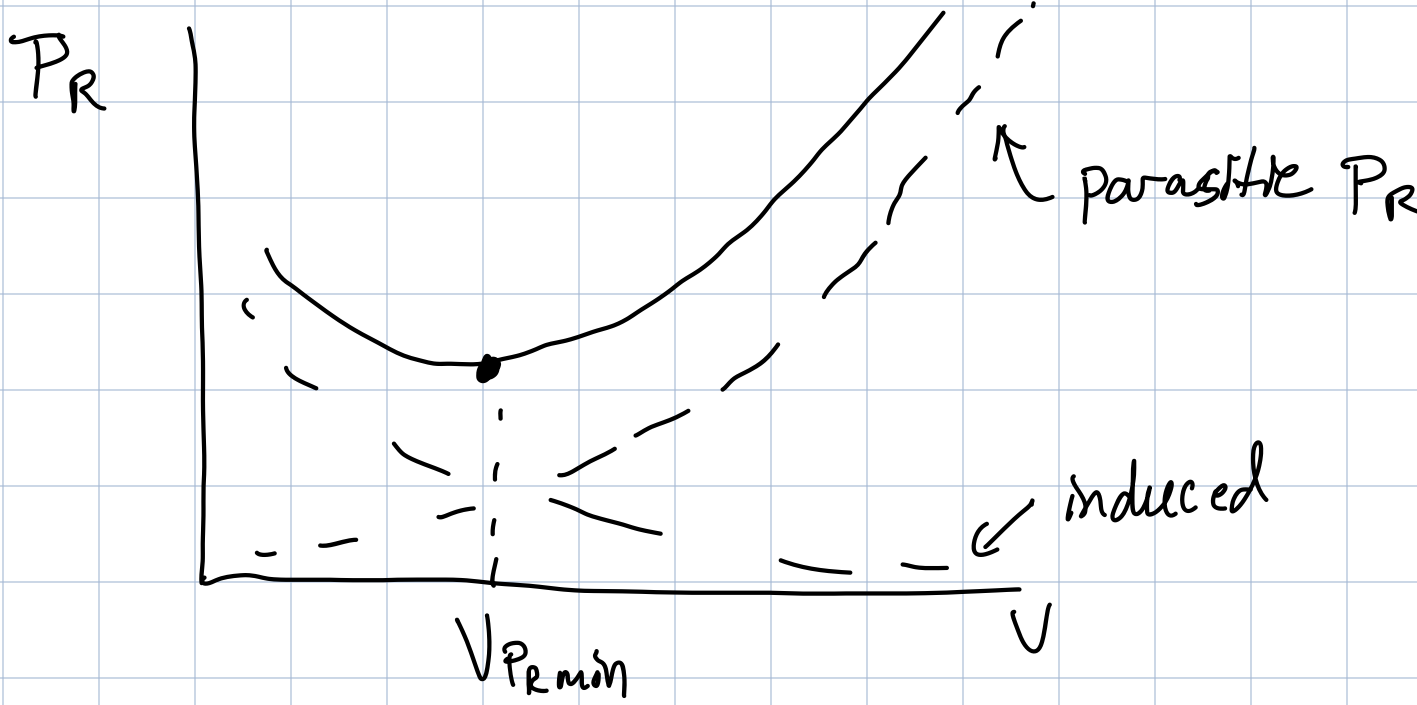
$$V = \sqrt{\frac{2W}{\rho S C_L}}$$

$$= \sqrt{\frac{2W^3}{\rho S} \cdot \frac{1}{[C_L^{3/2}/C_D]}}$$

$P_R$ is min when $C_L^{3/2}/C_D$ max

$$P_R = T_R \cdot V = \underset{\frac{1}{2}\rho V^2}{q} S V \left(C_{Dp} + K C_L^2\right)$$

$$P_R = \frac{1}{2}\rho V^3 S C_{Dp} + \frac{1}{2}\rho V^3 S K \left(\frac{W}{\frac{1}{2}\rho V^2 S}\right)^2$$



(solving for V @ $P_{R\min}$)

$$\frac{dP_r}{dV} = \frac{3}{2}\rho V^3 S C_{Dp} - \frac{W^2}{\frac{1}{2}\rho V^2 S} k = 0$$

$$\hookrightarrow \frac{3}{2}\rho V^2 S \left(C_{Dp} - \frac{1}{3} K C_L^2\right) = 0$$

note

$C_{Di} = K C_L^2$

@ $P_{R\min}$

$$C_{Dp} = \frac{1}{3} C_{Di}$$

$$\boxed{C_{L\,P_{R\min}} = \sqrt{3\pi C_{Dp} A\!R e}}$$

$$\boxed{V_{P_{R\min}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{K}{3C_{Dp}}}}}$$

$V_{P_{R\min}}$ & $V_{L/D\max}$

$\sqrt{\sqrt{1/3}}$

$V_{PR_{min}} = 0.76\, V_{L/D_{max}}$

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$T_A$ & $P_A$ for different engine types

$(\ )_R$ → Required → what you need to overcome the drag / power

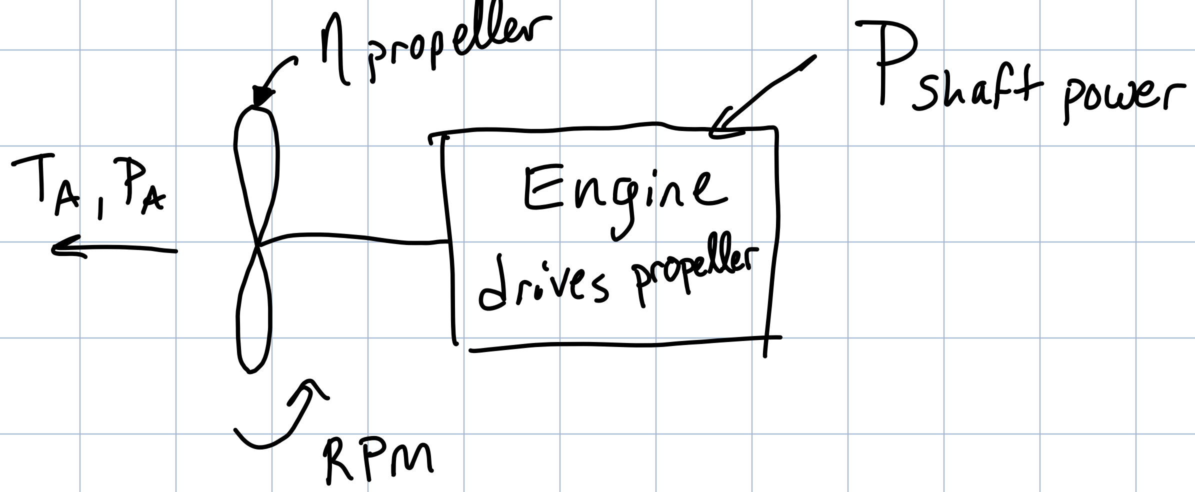
→ function of W, S . . . .

$(\ )_A$ → available → actual force/ power that the propulsor can produce

→ depends on propulsor

Propulsors

1. Propeller A/c




Aerodynamic forces generated @ prop drives A/C forward

$T_A$ is highest @ $V = 0$ (static thrust)

Propeller attached to shaft getting power from engine

$P_A = P_{shaft} \cdot \eta_{prop}$ $\equiv$ Power available to drive the A/c

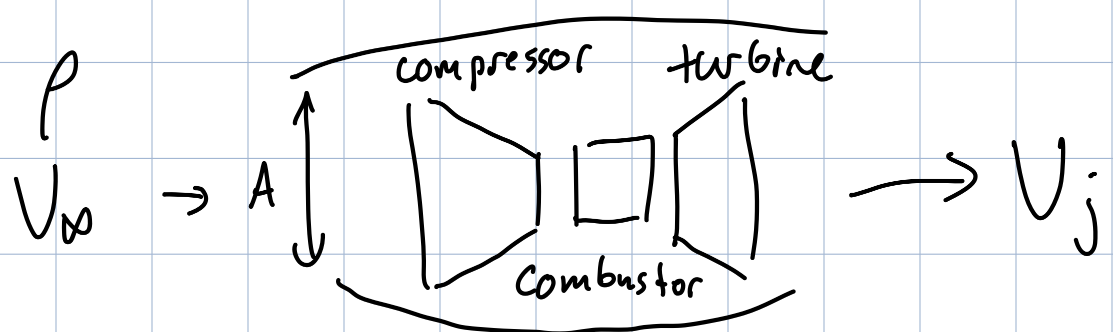
$$T_A = \frac{P_{shaft} \cdot \eta_{prop}}{V}$$

Note we think of Power as fundamental characteristic of propeller engines

2. Jet engines

↳ tend to Rate in terms of thrust instead of power

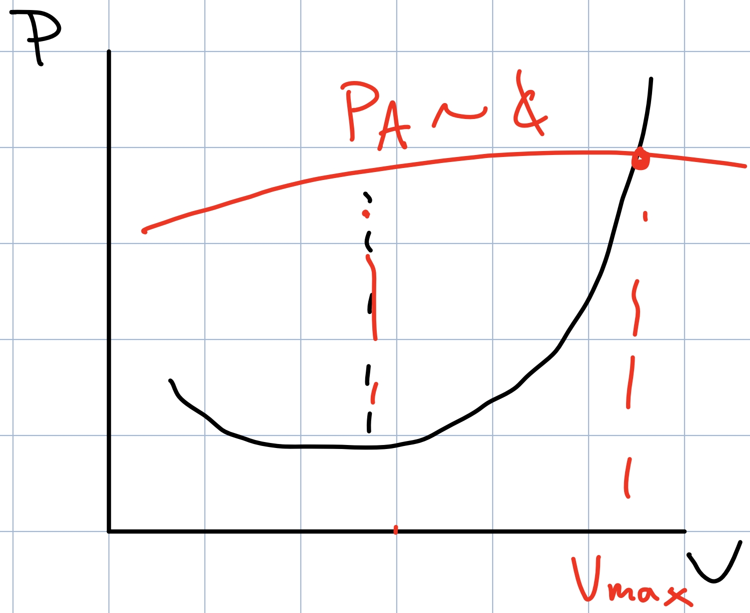
$T_A \sim$ ¢ with velocity



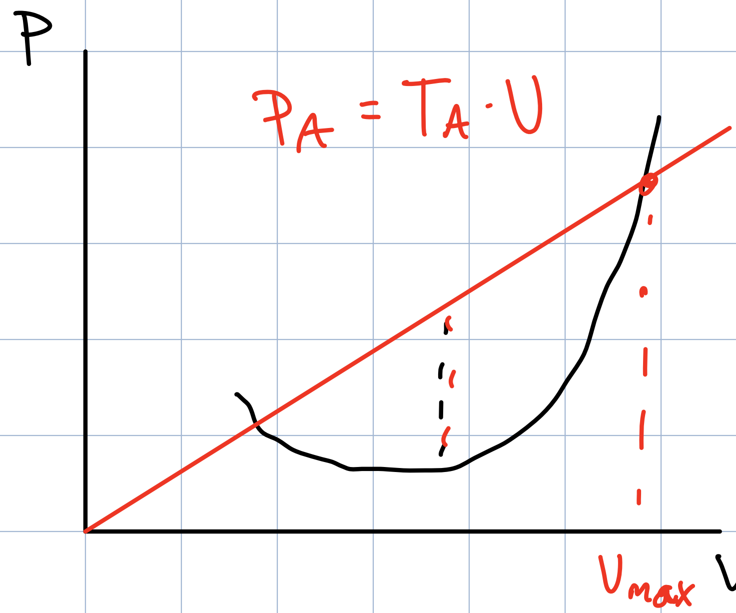

$$T_A = \rho A V_\infty (V_j - V_\infty)$$

$\underbrace{\phantom{xxx}}_{\text{mass}}$

flow Rate




Prop Aircraft




Jet Aircraft

can determine max difference between Available & the Required

## Altitude effects

$$V = \sqrt{\frac{2W}{\rho S C_L}} \qquad P_R = \sqrt{\frac{2W^3}{\rho S}\frac{C_D^{\,2}}{C_L^{\,3}}}$$

has $\rho^{-1/2}$ Relation

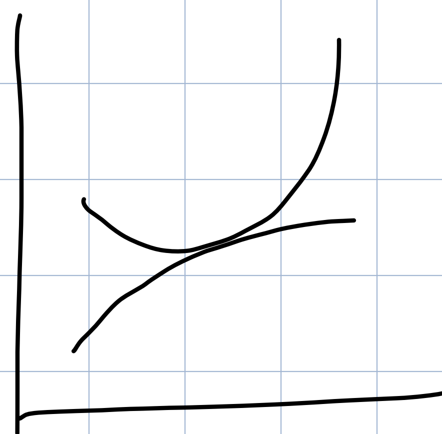
$$P_{A_{alt}} = P_{A_{SL}} \left( \frac{\rho_{alt}}{\rho_{SL}} \right)^m$$

$$T_{A_{alt}} = T_{A_{SL}} \left( \frac{\rho_{alt}}{\rho_{SL}} \right)^n$$

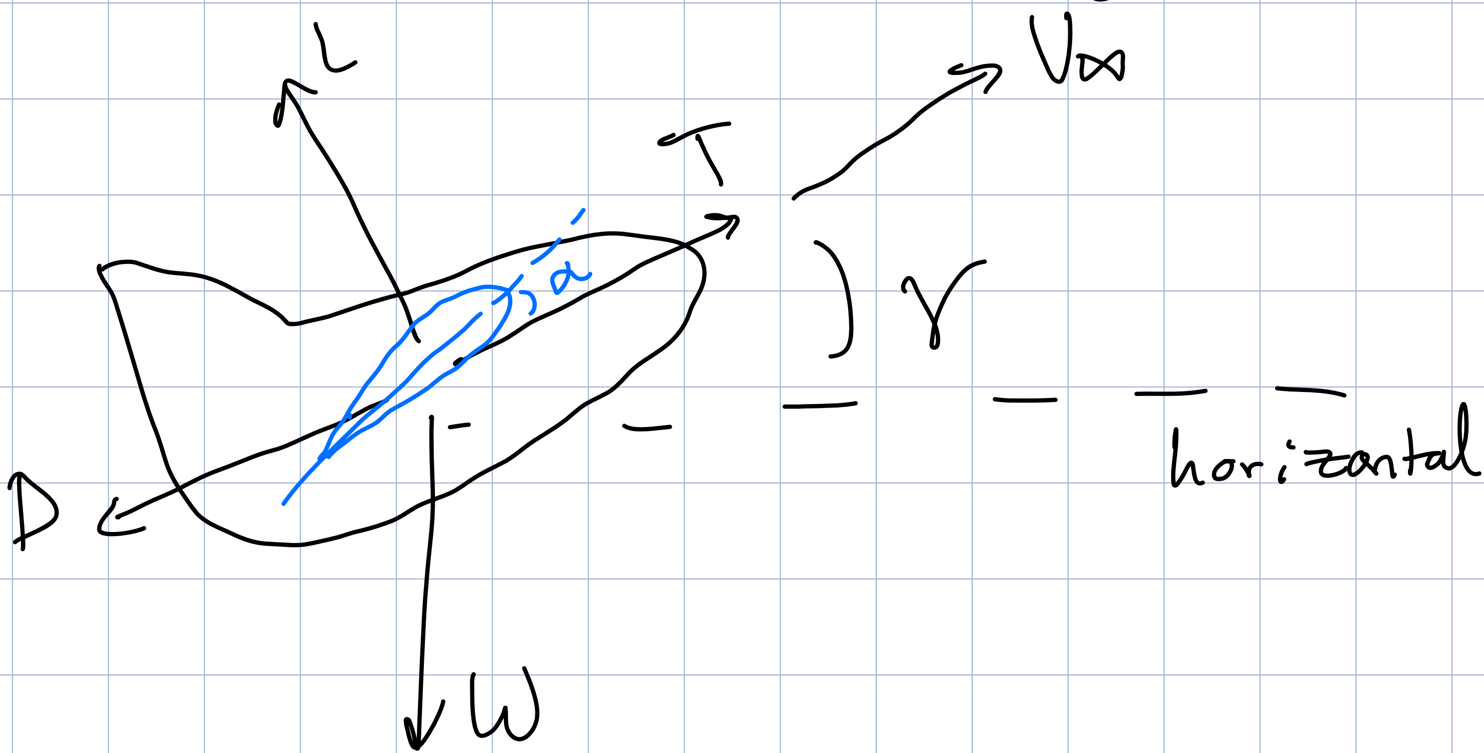
m & n are specific to the engine

we assume $(\ )_A$ follow a relation vs. $\rho$ → change with altitude

→



# Unaccelerated Climbing Flight




$\gamma \equiv$ flight path angle

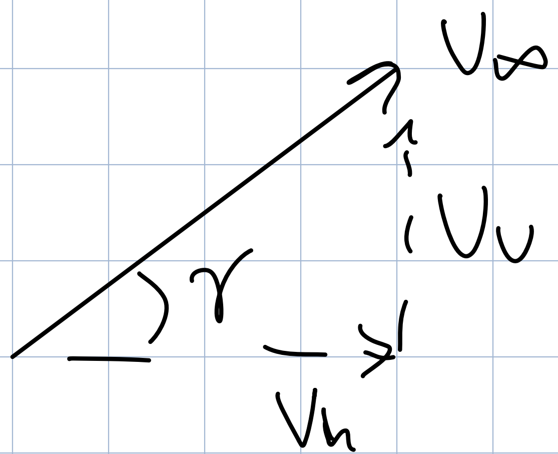
angle between $V_\infty$ & horizontal

Sum forces

① $//$ to flight direction $\quad T - D - W\sin\gamma = 0$

② $\perp$ to flight direction $\quad L - W\cos\gamma = 0$

Break $V_\infty$ into components




$V_v$ = Rate of Climb = R/C

$= V_\infty \sin\gamma$

Velocity of A/C in verticle direction

take ①, multiply by V

$$V(T - D - W\sin\gamma) = 0 \cdot V$$

$$T \cdot V = D \cdot V + VW\sin\gamma$$

Since $R/C = V\sin\gamma$

$$= \left(\frac{T \cdot V - D \cdot V}{W}\right)$$

$D \cdot V$ = Power Required

$T \cdot V$ = Power available

$$R/C = \frac{\text{Power available} - \text{Power Required}}{W}$$

$$= \frac{\text{excess power}}{W}$$