MAE 158 Lecture 19
Dec. 5 2024

Announcements: Drag project due Friday 11:59pm

- note - if late, submission will be marked down -25% for every 24 hours late

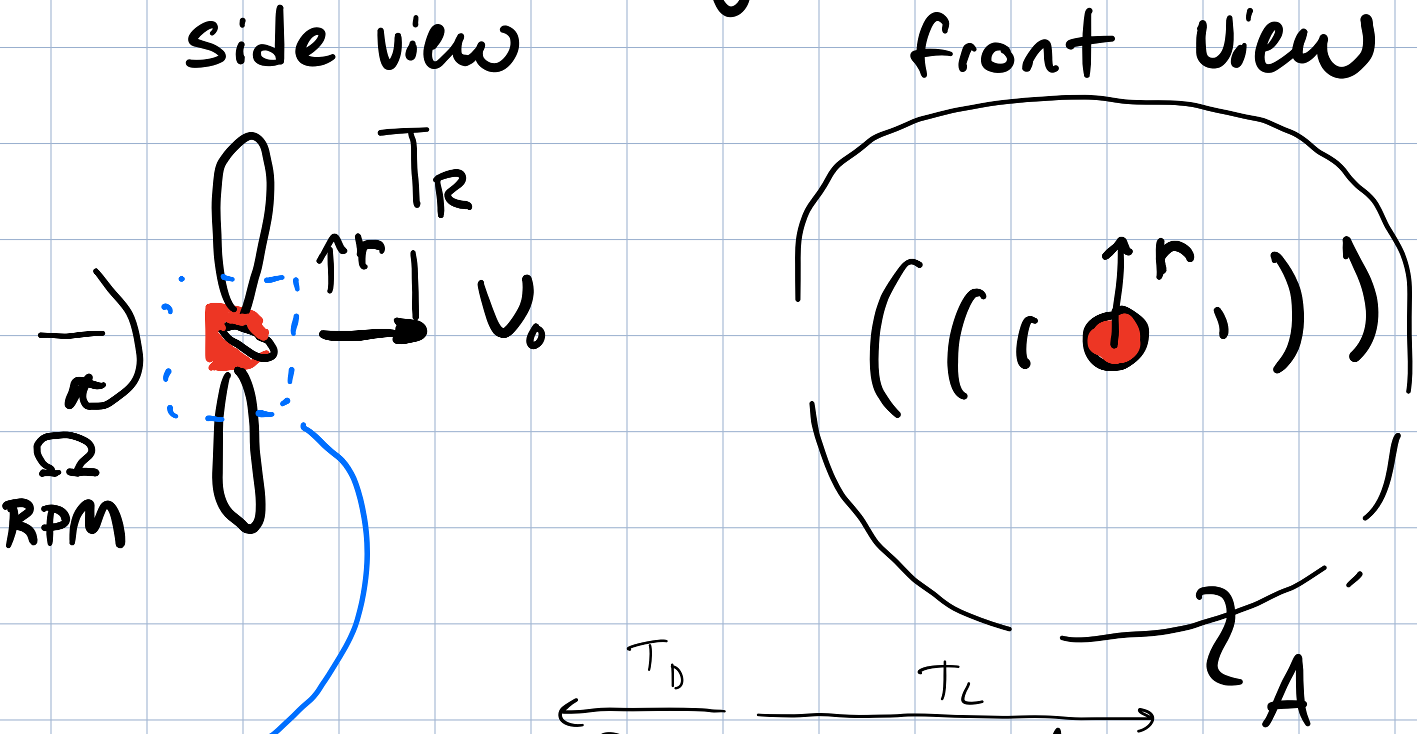
- Extra credit practice quiz
  Fri 12am - Mon 11:59pm
  Topics: Stability & propulsion

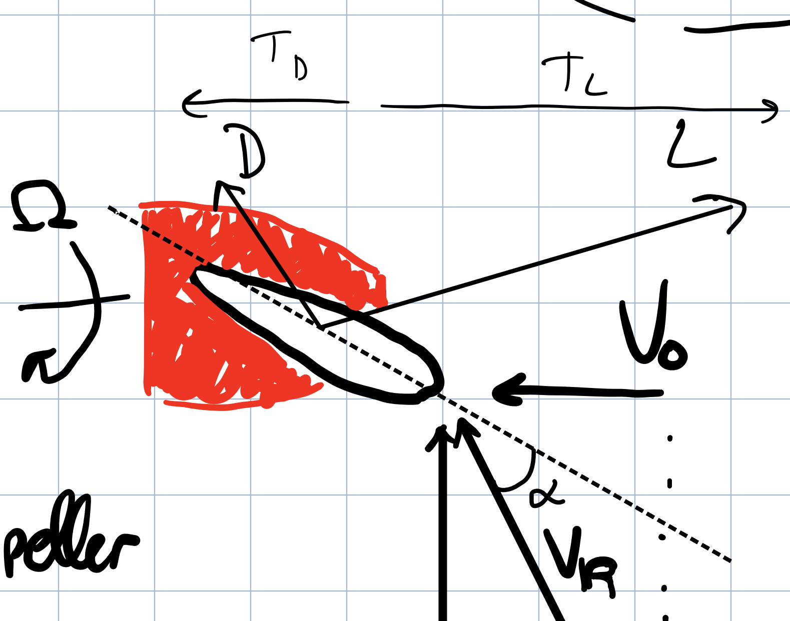
- Final Exam Thurs Dec 12th 1:30pm - 3:30pm
  - Cumulative but heavy focus on second ½ of course content
  - 4 problems, each w/ multiple parts, conceptual + solve probs
  - allowed 1 two-sided 8.5" x 11" cheatsheet

Today's objectives: Propulsion (finish)
propeller + Electrified A/C

$$\eta_P = \frac{2}{V_j/V_0 + 1} \quad \text{for jet aircraft}$$

propeller (not gas turbine)

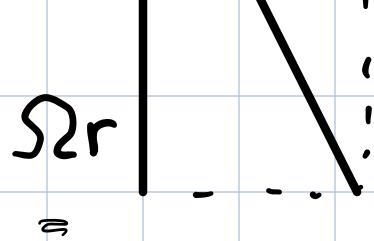




slice of propeller

V_R = Resultant

blade @ radius r from center of Propeller



velocity vector

Thrust component due to lift

+ Thrust component due to drag

= Net thrust produced by local blade section

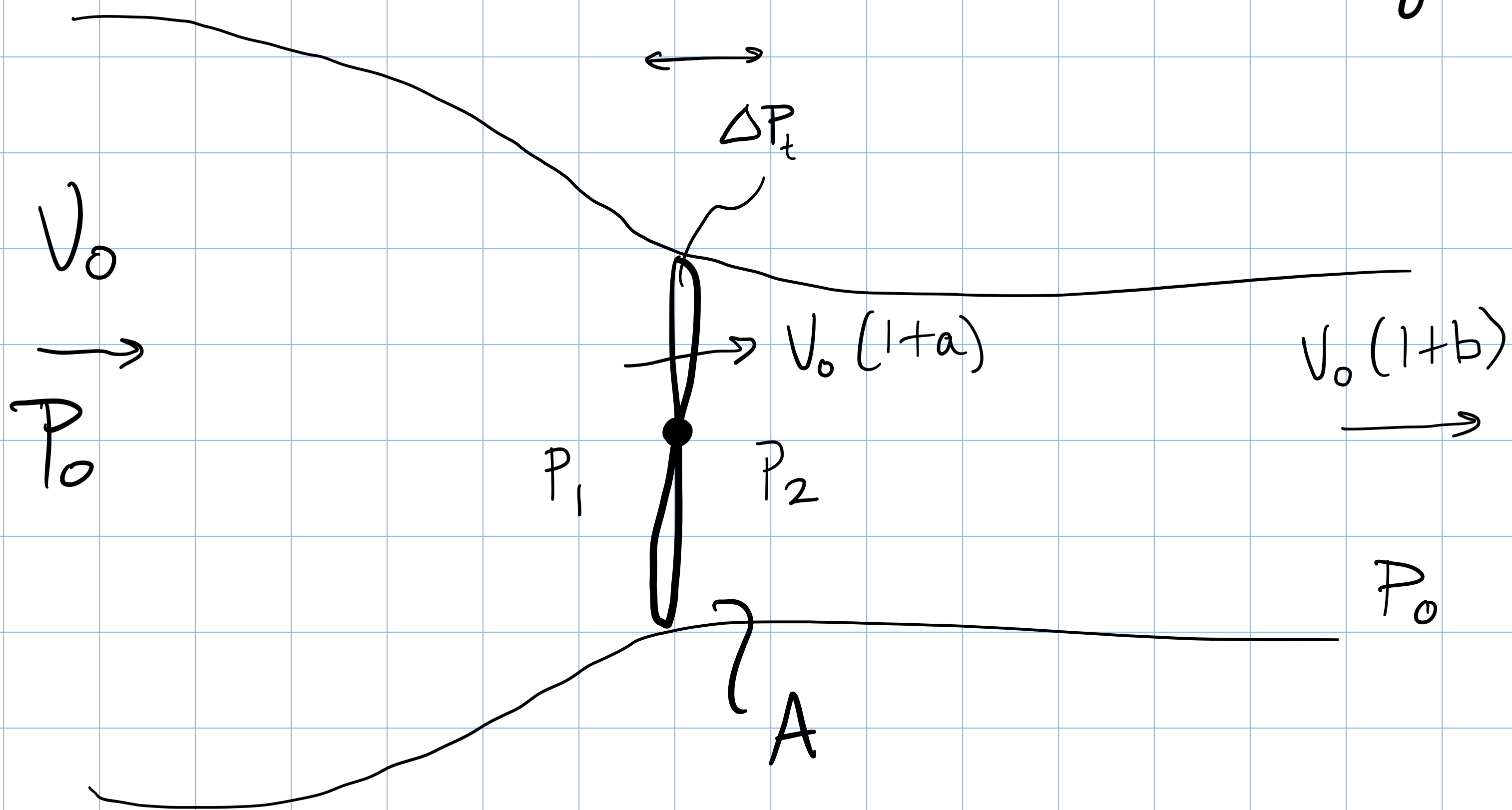
because Ωr changes along the blade length, analytical solutions for the total thrust difficult to solve $\rightarrow$ integral summation along the blade

$\hookrightarrow$ Blade element theory

XROTOR

for initial thrust predictions

use actuator <u>Disk</u> theory



Bernoulli's equations to solve for

ΔP_t

ahead of disk

$$P_{t_1} = P_1 + \rho/2 \,(V_0 + aV_0)^2$$

$$= P_0 + \rho/2 \,(V_0)^2$$

behind disk

$$P_{t_2} = P_2 + \rho/2 \,(V_0 + aV_0)^2$$

$$= P_0 + \rho/2 \,(V_0 + bV_0)^2$$

$$\Delta P_t = P_{t_2} - P_{t_1} = \frac{\rho}{2}\left(2bV_0^2 + b^2V_0^2\right)$$

$$= \rho V_0^2 \left(1 + \frac{b}{2}\right) b$$

Recall thrust is Rate

of change of Momentum

$$T - \rho\underbrace{(V_0 + aV_0)}_{\text{Velocity through the propeller}} A\,(\underbrace{V_0 + bV_0}_{\text{aft}} \; \underbrace{-V_0}_{\text{fwd}})$$

$$= \rho\,(V_0 + aV_0)\,A\,b\,V_0$$

$$T = \Delta P_t \cdot A = A\rho\, V_0^2 (1 + b/2)\, b$$

thus $\rho(V_0 + aV_0)\,b\,V_0 = \rho\, V_0^2 (1 + b/2)\, b$

$\hookrightarrow \quad a = b/2$

$$\text{thrust} = 2 A \rho V_0^2 (1+a) a$$

What is the KE through propeller?

$$\Delta KE = \Delta \frac{\dot{m} V^2}{2}$$

$$= \dot{m} \left(\frac{V_0 (1+b)}{2} \right)^2 - \dot{m} \frac{V_0^2}{2}$$

$$= \frac{\rho A V_0 (1+a)}{2} \left([V_0 (1+b)]^2 - V_0^2 \right)$$

↳ Recall $a = b/2$

$$= 2 A \rho V_0^3 (1+a)^2 a$$

$$\Delta KE = T \cdot V_0 (1+a)$$

I can use this to get propeller efficiency (ideal efficiency)

$$\eta_{propeller} = \frac{\text{output work}}{\text{input Energy}} = \frac{\cancel{T} \cdot \cancel{V_0}}{\cancel{T} \cdot \cancel{V_0} \cdot (1+a)}$$

$$= \frac{1}{(1+a)} \rightarrow \text{ideal propeller efficiency}$$

for a given Thrust, can solve for a $\longrightarrow$ use to get $\eta_{propeller}$ efficiency

Ex: Thrust = 1750 lb

What is ideal Efficiency for a propeller?

diameter = 14 ft

Velocity = 304 Knots = V_0

$\rho = 0.00126 \text{ slug/ft}^3$

$$T = 2A\rho V_0^2(1+a)a$$

$\frac{\pi}{4}(14 \text{ ft})^2$ 304 knots × 1.69

$$= 103000\,(a + a^2) = 1750$$

→ move over to one side

$$a^2 + a - 0.017 = 0$$

quadratic equation

$$a = \frac{-1 \pm \sqrt{1 - 4(1)(-0.017)}}{2}$$

$$= \frac{-1 \pm 1.03}{2}$$

$\rightsquigarrow$ positive solution

~ 0.017

$= a$

$$\eta = \frac{1}{1+a} = 0.98$$

Range for fuel burning Aircraft

$$R = \frac{V}{C_T} \frac{L}{D} \ln\left(\frac{W_0}{W_1}\right)$$

$$= \eta_0 \, h_f \, \frac{L}{D} \ln\left(\frac{W_0}{W_1}\right)$$

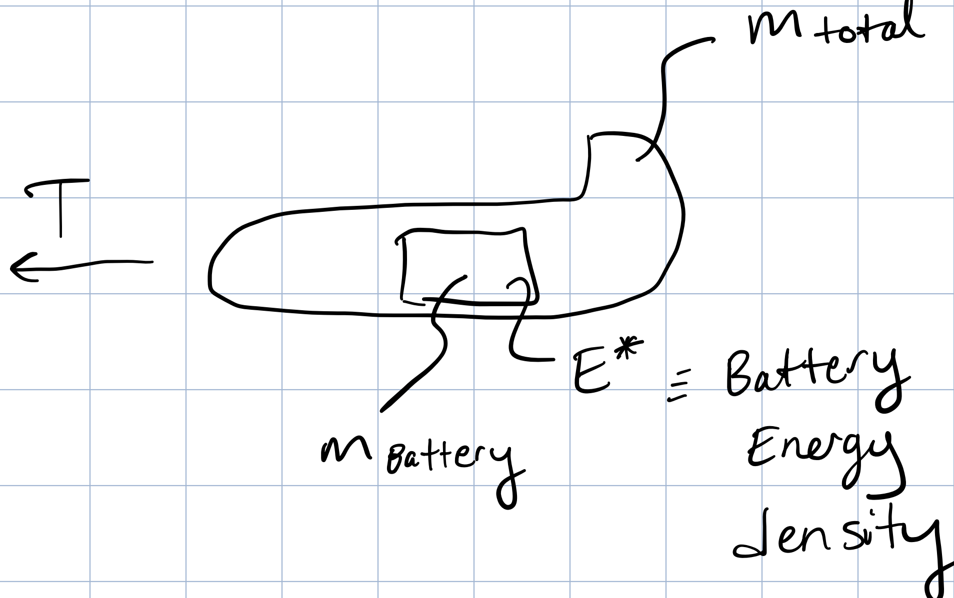
η_0: engine

h_f: fuel type

$\ln\left(\frac{W_0}{W_1}\right)$: Change weight from burning fuel

What if I have energy on board, but I don't burn fuel?

↳ Batteries $\rightarrow \frac{W_0}{W_1} = 1$



used to produce work

$$E = T \cdot d$$

total energy available

$$E = m_{Battery} \cdot E^* \cdot \eta_o$$

What is T?

Steady level flight $T = \dfrac{W}{(L/D)}$

$$T \cdot d = m_{Battery} \cdot E^* \cdot \eta_o$$

$$d = \frac{E^* \cdot m_{Battery} \cdot \eta_o}{W/(L/D)}$$

engine efficiency

$$= E^{*} \cdot \frac{M_{Battery}}{M_{total}} \frac{1}{g} \frac{L}{D} \cdot \eta_0$$

energy source

ratio of weights

aero efficiency

= Range for Electric Aircraft

to maximize R. of electric A/C

L/D max

$\frac{M_{Batt}}{M_{total}}$ high as possible

let's compare Range of a "Electrified" B747 to traditional jet fuel burning A/C

assume $\frac{m_{fuel}}{m_{total}} = 0.44 = \frac{m_{Batt}}{m_{total}}$

$L/D = 17$

$\eta_o \approx 0.33$ for the traditional jet engine

$\eta_o \simeq 0.8$ for electric

$W_o = 735\,000 \text{ lb}$

$$R_{jet} = h_f \cdot \eta_o \frac{L}{D} \ln\left(\frac{W_0}{W_1}\right)$$

$$h_f \approx 14.3 \text{ million } \frac{\text{ft-lb}}{\text{lb}}$$

$$= (0.33)(17)(14.3 \text{ mill}) \cdot \ln\left(\frac{735000 \text{ lb}}{735000(1-0.44)}\right)$$

$$= \underline{7700 \text{ nmi}} \sim \underline{14500 \text{ km}}$$

$$R_{elec} = E^*\left(\frac{m_{Batt}}{m_{tot}}\right)\frac{1}{g}\frac{L}{D}\eta_o$$

$$E^* \approx 300 \frac{\text{Wh}}{\text{kg}} = 1.08 \frac{\text{MJ}}{\text{kg}}$$

$$= (1.08 \times 10^6)(0.44)\frac{1}{9.8} 17 \cdot 0.8$$

$$= \underline{660 \text{ km}}$$