MAE 158 Lecture 15
Nov 18 2024

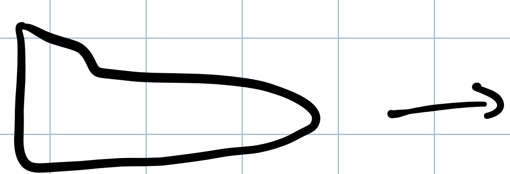
Announcements: Week 8 HW posted

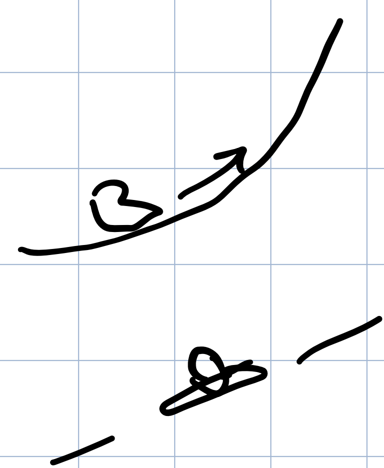
Today's Objectives: Turning performance

turning Performance:

- pilots use controls in order to manipulate flight path

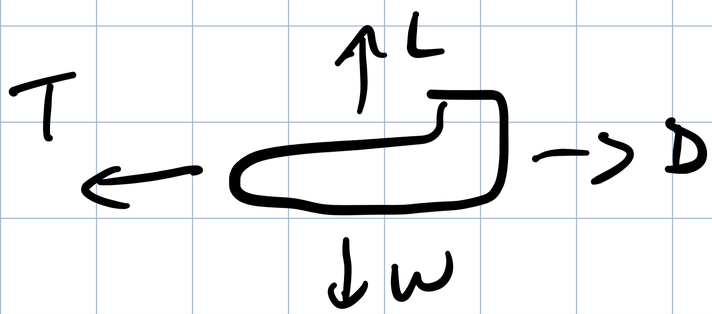
  2o Rudders, elevators, ailerons, differential thrust etc...





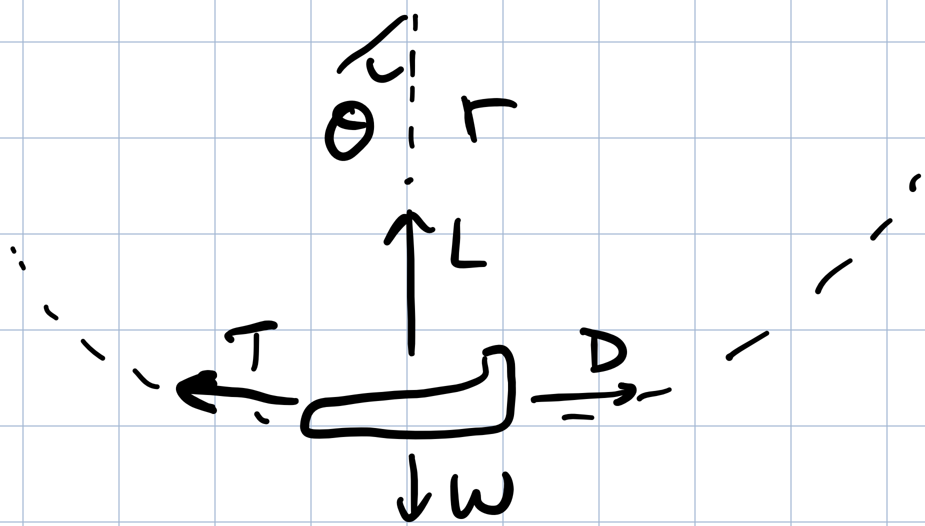
consider a few turn procedures

compared to SLF



$W = L$
$T = D$

pullup maneuver (flare maneuver)



pilot pull back on control stick, & A/C move in a curved flight path,
$L > W$

define $n \equiv$ load factor $= \frac{L}{W}$

if $n = 1$ $(L = W)$ means

A/c experiences 1 "g" of load

if $n = 3 \rightarrow 3$ "g"s

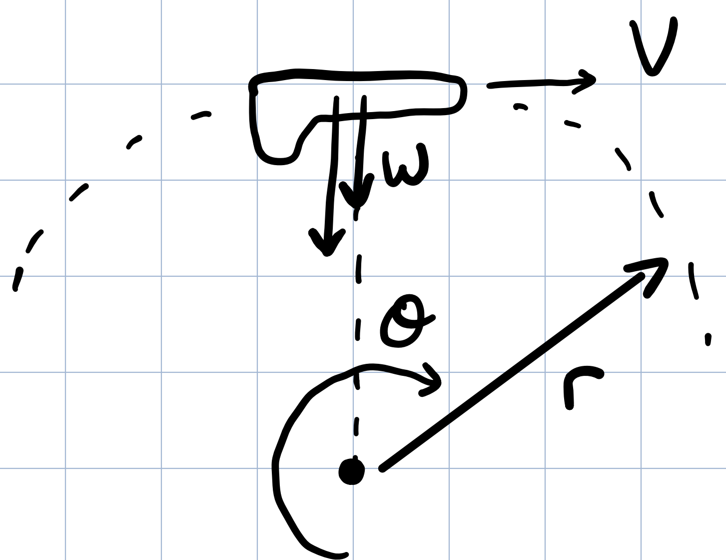
$$r = \frac{V^2}{g(n-1)} \qquad \frac{d\theta}{dt} = \omega = \frac{V}{r} = \frac{g(n-1)}{V}$$

omega

turn Radius

turn Rate

pull-down maneuver:



turn Rate & r?
ω

forces ⊥ to flight path

$$\frac{mV^2}{r} = L - W\cos\theta$$

$\theta = 180°$

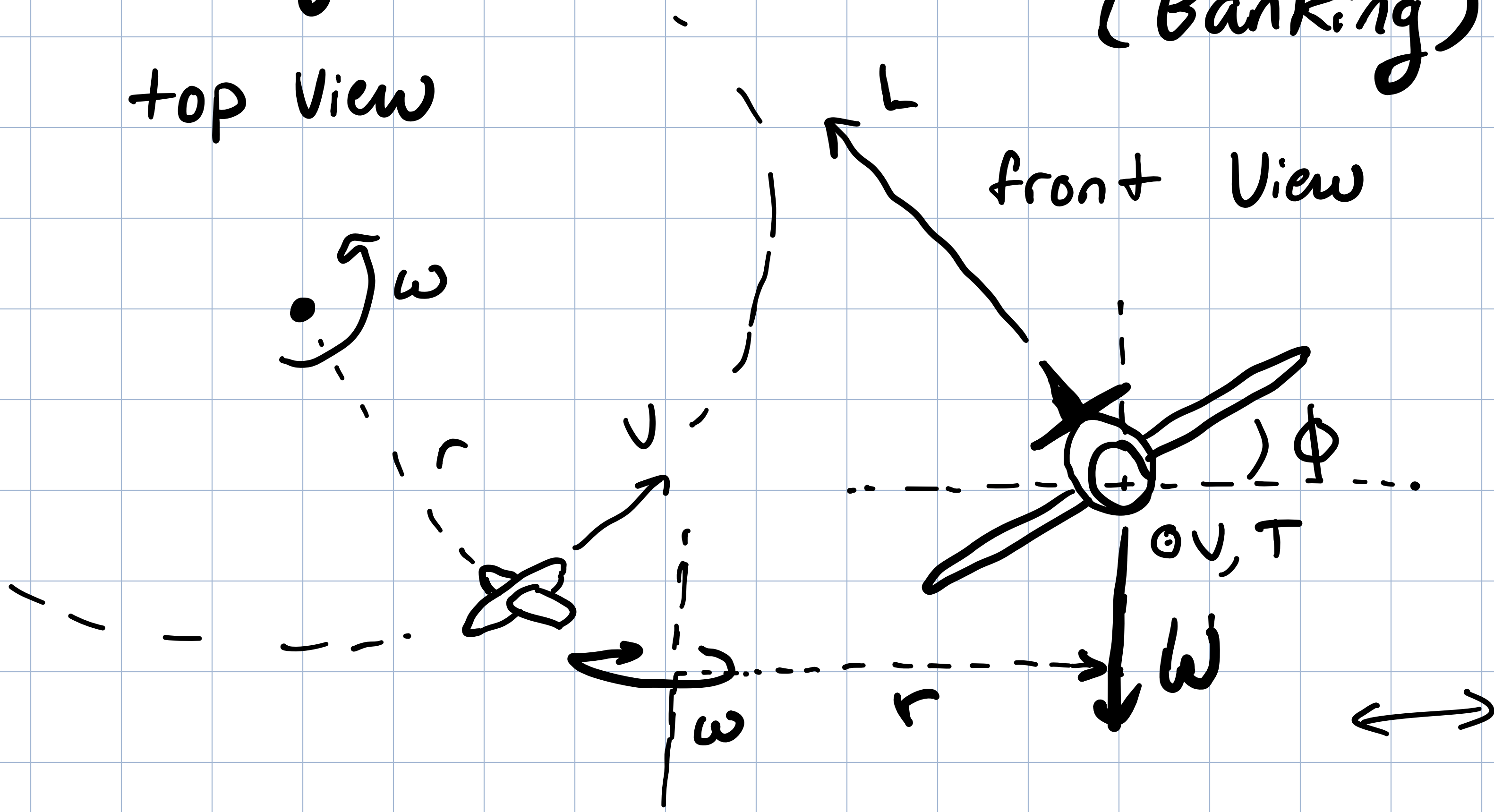
$$\rightarrow \frac{mV^2}{r} = L + W$$

$$r = \frac{mV^2}{(L+W)} = \frac{1}{g}\frac{V^2}{(n+1)}$$

$$\omega = g\frac{(n+1)}{V}$$

pull down maneuver

turning in a level plane (Banking)



- Lift vector turn inwards
- Wing @ ϕ WRT the horizontal

$$\therefore \quad L\cos\phi = W \qquad \phi \equiv \text{Bank angle}$$

lateral forces

$$L \sin\phi = \frac{mV^2}{r}$$

also $\cos\phi = \frac{W}{L} = \frac{1}{n}$

bank angle corresponds dicrectly to load factor

$$r = \frac{mV^2}{L\sin\phi} = \frac{1}{n}\frac{V^2}{g\sin\phi}$$

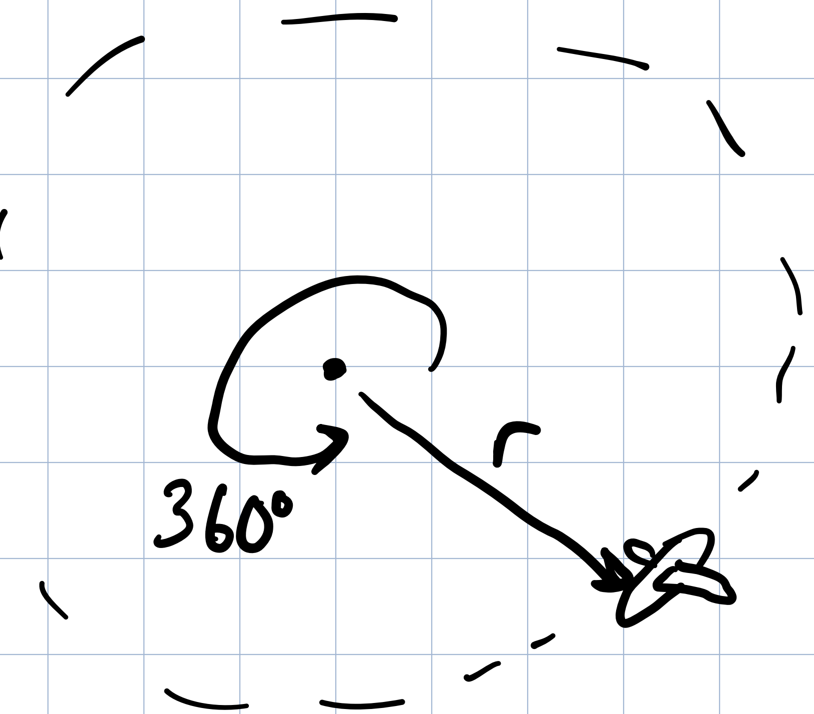
Recall $\cos^2\phi + \sin^2\phi = 1$

thus $\left(\frac{1}{n}\right)^2 + \sin^2\phi = 1$

then $r = \frac{V^2}{g\sqrt{n^2-1}}$ $\quad \omega = \frac{V}{r} = \frac{g\sqrt{n^2-1}}{V}$

turn Radius turn Rate

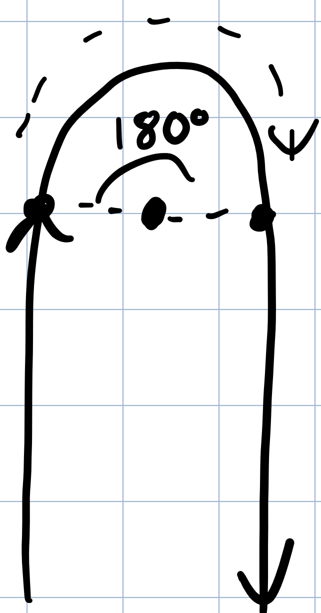
total distance traveled in a turn on level plane?



total distance

$2\pi r$ if travel 360°

can get time from ω



if I want to minimize r & maximize ω

- low V
- high n

} what are these

limits?

n constraints

- structural limits of aircraft
  1. limit load factor: permanent damage occurs
  2. ultimate load factor: structural failure occurs (AKA, something Breaks)

$n_{max, structural}$ ~ 2 commercial
~ 7-8 military

- thrust constraint:

- in a level turn, lift will increase by a ratio of $1/\cos\phi$

- if you increase L, induced drag $\uparrow$

@ a given V, $T_R \uparrow$ in the turn as a funtion of $1/\cos\phi$ in the induced drag

ϕ limited by thrust available

$n_{\text{thrust constraint}}$, Drag = Thrust

$$= \tfrac{1}{2}\rho V^2 S \left(C_{D_P} + K C_L^2\right)$$

$$C_L = \frac{n \cdot 2W}{\rho V^2 S}, \quad K = \frac{1}{\pi A\!R e}$$

$$\underset{T_{A,max}}{\text{Thrust}} = \tfrac{1}{2}\rho V^2 S\left(C_{D_P} + K\left(\frac{\underset{n_{\text{thrust constraint}}}{n}\, 2W}{\rho V^2 S}\right)^2\right)$$

$n_{\text{thrust constraint}} = \left[\left(\frac{\frac{1}{2}\rho V^2}{K \, W/S} \right) \left(\frac{T_{A\max}}{W} - \frac{1}{2}\rho V^2 \frac{C_{D_0}}{W/S} \right) \right]^{1/2}$

$n_{C_{L\max}, \text{constraint}}$

@ a given V in the turn you need corresponding C_L to get necessary lift @ desired ϕ

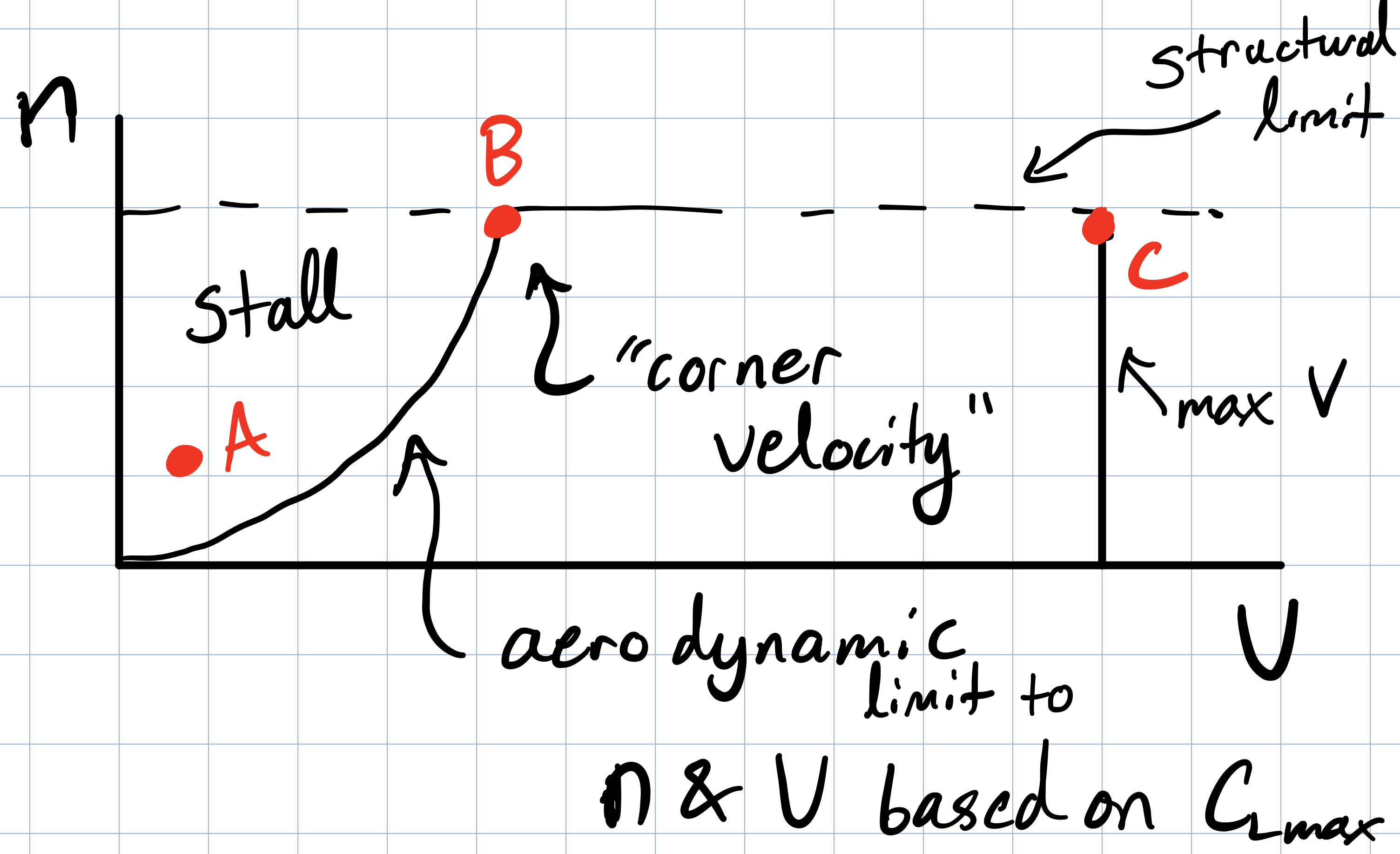
$$n = \frac{L}{W} = \frac{\frac{1}{2}\rho V^2 S C_L}{W}$$

@ a given Velocity

constrained by $C_{L\max}$

$$n_{C_{L\max}\text{ constraint}} = \frac{\frac{1}{2}\rho V^2 S C_{L\max}}{W}$$

V-n diagram → "flight envelope"



Which point leads to "Best turn performance?"

A = stalling

B = high n & low V

→ best turn performance

C = high V & high n

for a given problem,

If asked to determine lowest r or fastest ω

→ check n_{struct}, n_{thrust}, $n_{C_L max}$ } whichever is lowest is constraint

→ @ a given V

because there is a tradeoff with n & V, how do we get theoretical r_{min} & ω_{max}?

- r_{min} doesn't necessarily happen @ n_{max} because of dependence on V

$$\frac{dr}{dV} = 0 = \frac{d}{dV}\left(\frac{V^2}{g\sqrt{n^2-1}}\right)$$

n also depends on V

full mathematical derivation
Anderson Ch. 6

$$\Rightarrow \quad V_{r_{min}} = \sqrt{\frac{4K\left(W/S\right)}{\rho\left(T/W\right)}}$$

this is V @ theoretical minimum turn Radius

also:

$$n_{r_{min}} = \sqrt{2 - \left[\frac{4KC_{D0}}{\left(T/W\right)^2}\right]}$$

$$r_{min} = \frac{V_{r_{min}}^2}{g\sqrt{n_{r_{min}}^2 - 1}} = \frac{4K\left(W/S\right)}{g\rho\left(\frac{T}{W}\right)\sqrt{1 - 4K\frac{C_{D0}}{\left(T/W\right)^2}}}$$

$\uparrow$ min turn Radius conditions @ a given T/W ratio

-but- ~~iff~~ $V_{r_{min}}$ is below stall velocity, then have to adjust V & n & turn @ a higher radius

for max ω:

$$\frac{d\omega}{dV} = 0 = \frac{d}{dV}\left(g\frac{\sqrt{n^2-1}}{V}\right)$$

Anderson ch. 6

$$V_{\omega max} = \left[\frac{2\left(W/S\right)}{\rho}\right]^{1/2}\left(\frac{K}{C_{D_P}}\right)^{1/4}$$

$$n_{\omega max} = \left(\frac{T/W}{\sqrt{K C_{D_P}}} - 1\right)^{1/2}$$

$$\omega_{max} = g \cdot \sqrt{\frac{\rho}{W/S}\left[\frac{T/W}{2 \cdot K} - \left(\frac{C_{D_P}}{K}\right)^{1/2}\right]}$$

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determine "theoretical min turn Radius"

$W/S = 76.84\ lb/ft^2$

$K = 0.08$

Sea level conditions

$\rightarrow \rho = 0.00238 \ slug/ft^3$

$T/W = 0.3795$

$C_{D,P} = 0.015$

$$r_{min} = \frac{V_{r_{min}}^2}{g\sqrt{n_{r_{min}}^2 - 1}} = \frac{4K(W/S)}{g\rho\left(\frac{T}{W}\right)\sqrt{1 - 4K\frac{C_{DP}}{(T/W)^2}}}$$

$$= \frac{4 \cdot 0.08\,(76.84 \ lb/ft^2)}{(32.2 \ \frac{ft}{s^2})(0.00238 \ \frac{slug}{ft^3})(0.3795) \cdot \sqrt{1 - 4(0.08)\frac{0.015}{(0.3795)^2}}}$$

$= 861 \ ft$ "theoretical min"

Sahaj singh