MAE 158 Lecture 7
Fall 2024

Announcements: • **Week 3 Quiz**

Fri 12am – Mon 11:59pm

topics: Lecture 4 - 7 (induced drag, compressibility drag)

+ Recommended HW 3 + 4

Today's Objectives: Finish Compressibility Drag

+ Drag Buildup Ch. 11 Shevell

Last Time: $D_{total} = D_P + D_i + \Delta D_c$ civil transport aircraft

$\Delta C_{D,C}$ ~ compressibility drag increment

What about Super sonic cases?

$M_\infty > M_{Div} \rightarrow$ compressibility drag ↑↑

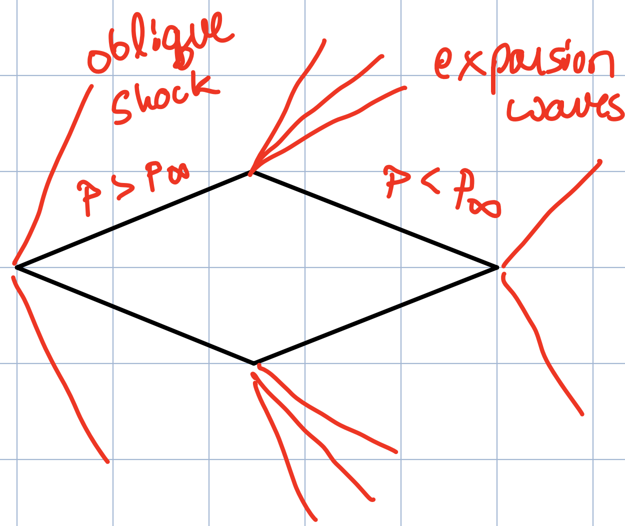
↳ Wave Drag → drag caused by shocks for supersonic flow

↳ has both lift & thickness components

thickness component

$M_\infty > 1$

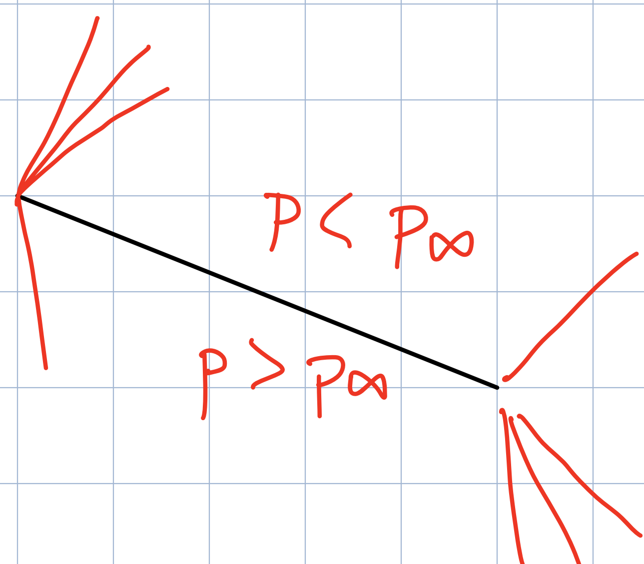
P_∞



$p\uparrow$ front, $P\downarrow$ back

→ Drag

lift component



→ increased drag

from thin airfoil theory,

Supersonic conditions : $C_L = \frac{4}{\sqrt{M_\infty^2 - 1}} \cdot \alpha$

$\alpha \equiv$ angle of attack in Radians

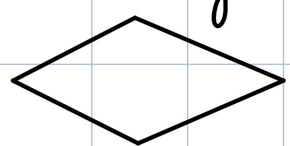
Supersonic lift drag:

$$C_{D_{\text{wave, lift}}} = \frac{4 \cdot \alpha^2}{\sqrt{M_\infty^2 - 1}}$$

thickness drag:

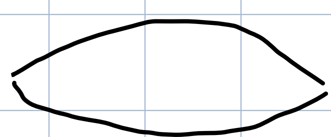
flat plate $C_{D_{\text{wave thickness}}} = 0$

double-wedge airfoil

t/c 

$$C_{D_{\text{wave thickness}}} = \frac{4}{\sqrt{M_\infty^2 - 1}} \cdot \left(\frac{t}{c}\right)^2$$

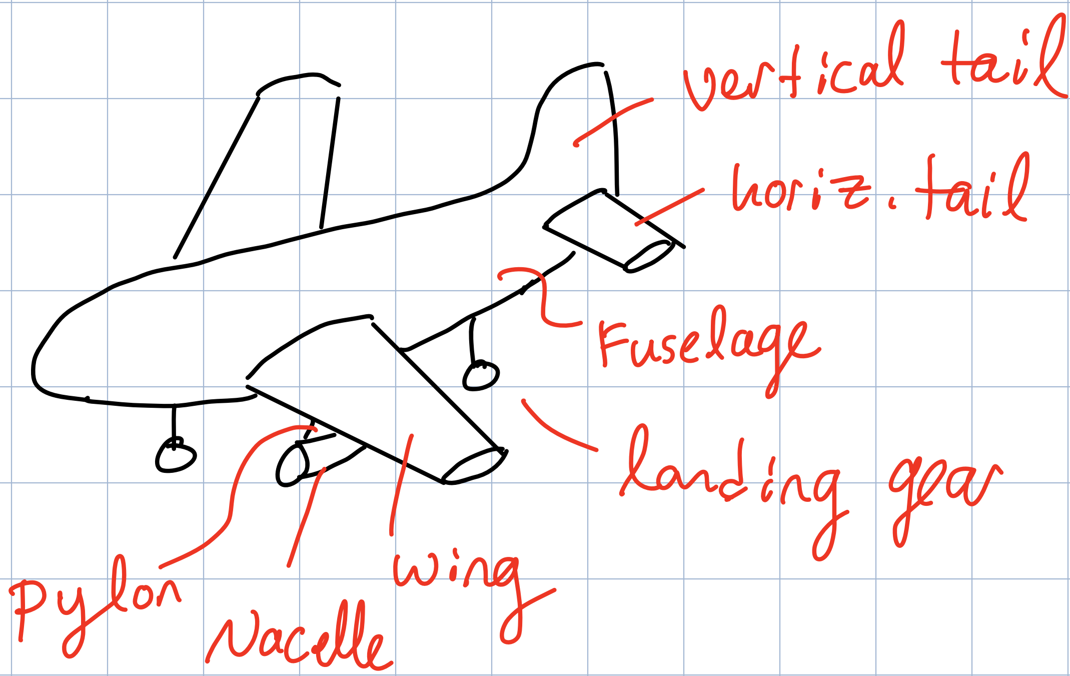
circular-arc airfoil

t/c 

$$C_{D_{\text{wave thickness}}} = \frac{16}{3\sqrt{M_\infty^2 - 1}} \cdot \left(t/c\right)^2$$

↑ quiz

Total Drag Buildup



forms of drag

- Induced drag ~ lift producing bodies
- profile drag — skin friction (Re) + pressure drag

- compressibility drag
  $\hookrightarrow M > 0.3$

Review Profile drag

$$D_P = D_f + D_B$$

D_f: skin friction drag

D_B: pressure / bluff body drag

get D_f using C_f

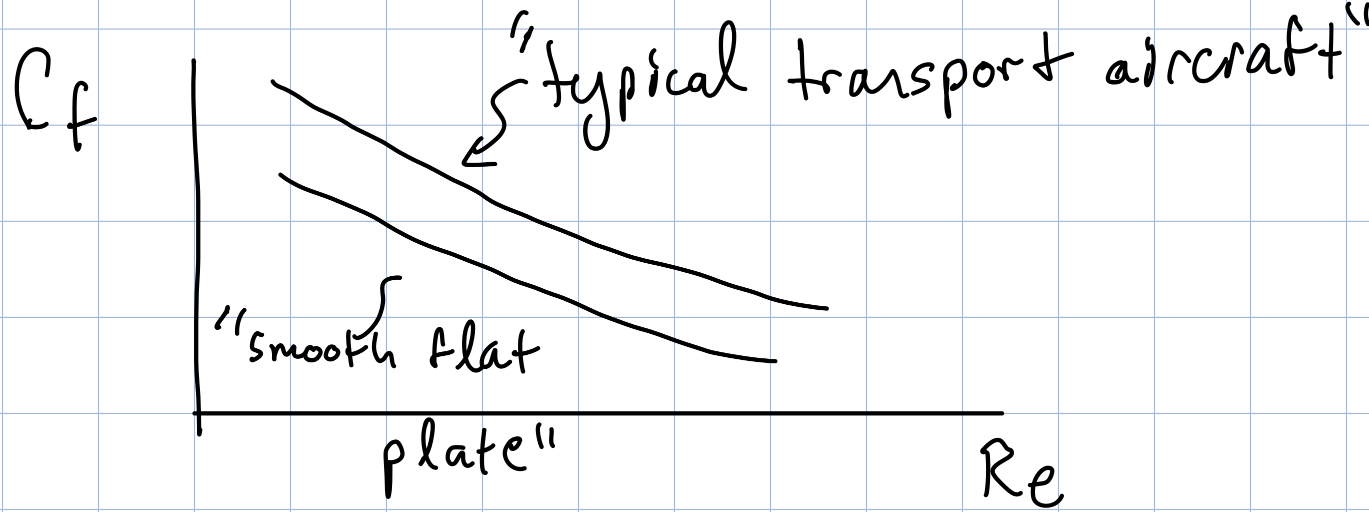
C_f depends on flow Regime
AKA Re
must determine if you
are laminar or turbulent

ex: $$C_{f_{turb}} = \frac{0.455}{\left[\log_{10}(Re)\right]^{2.58}}$$

considering Real transport aircraft,

C_f often higher than a pure smooth flat plate

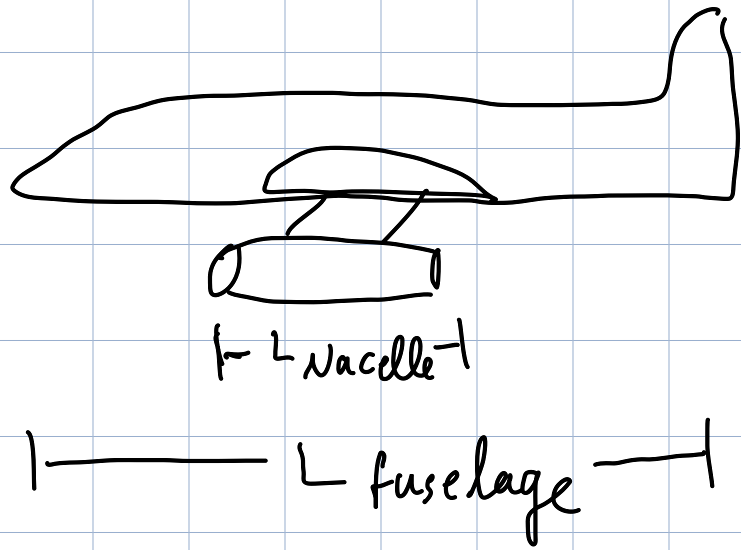
ex: Shevell Ch. 11 Fig 2



to get C_f, need Re

$$Re = \frac{\rho V L}{\mu}$$ $L \equiv$ characteristic length

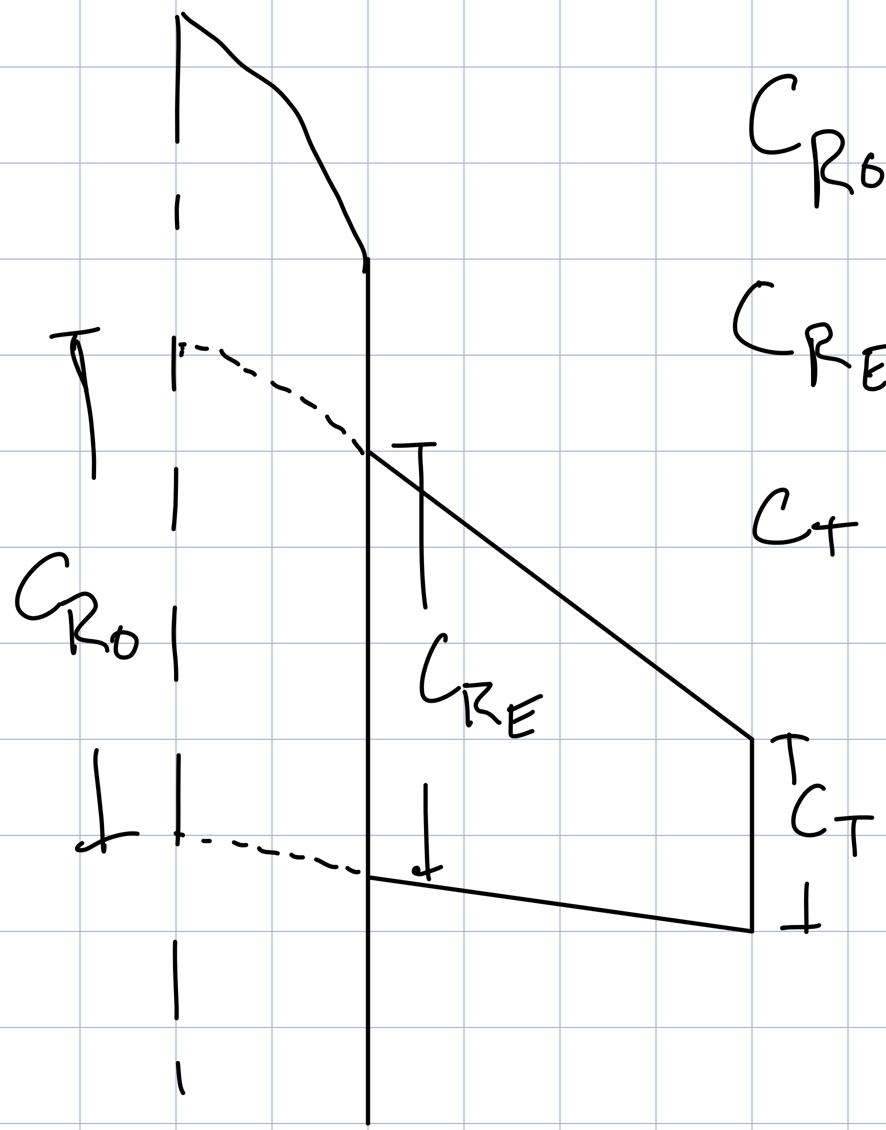
Nacelles / fuselages ?



Wings / tails / Pylons

$L \equiv$ mean aerodynamic chord
MAC, $\bar{c}$

$\equiv$ chord of an imaginary wing with a constant chord with same aero characteristics as actual wing



$C_{R_0} \equiv$ Root chord actual

C_{R_E} = Root chord Exposed

C_T = tip chord

$$\sigma = \text{taper Ratio} = \frac{C_T}{C_R}$$

$$\sigma_E = \frac{C_T}{C_{R_E}} \qquad \sigma_0 = \frac{C_T}{C_{R_0}}$$

$$MAC = \bar{C} = \frac{2}{3}\left(C_R + C_T - \frac{C_R \cdot C_T}{C_R + C_T}\right)$$

$$= \frac{2}{3} \cdot C_R \left(1 + \sigma - \frac{\sigma}{1+\sigma} \right)$$

$MAC_E \rightsquigarrow C_{RE}, \sigma_E \to$ use to get "L" for drag

$MAC_0 \rightsquigarrow C_{R0}, \sigma_0 \to$ aircraft stability

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- D_B (pressure drag)
  - ↳ BL separation pressure $\rightsquigarrow$ scales with fineness Ratio

fineness Ratio $\quad l/d$

length diameter

for fuselages / Nacelles

but for wings / tails / pylons

t/c

from fineness Ratio, thickness to Chord Ratio

↳ get form factor, k

$$D_P = C_{DP} \cdot q \cdot S_{REF}$$

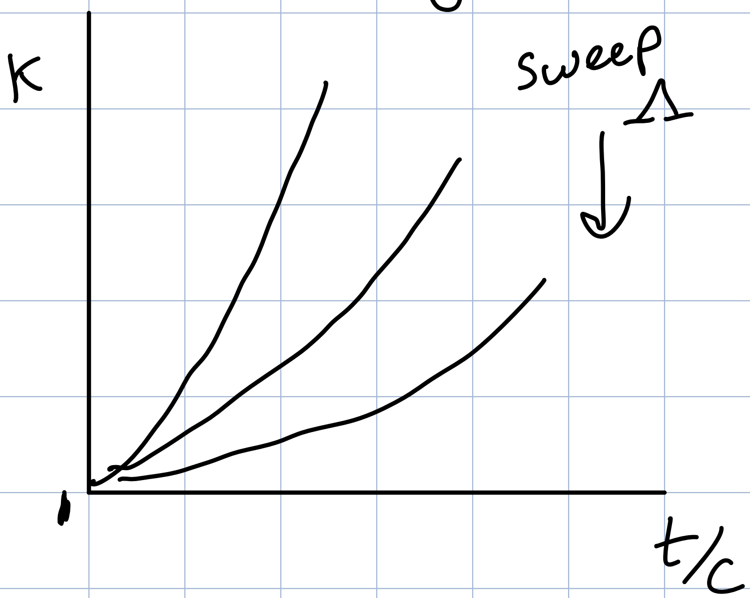
$$C_{DP} = \sum_i \cdot \frac{K_i \cdot C_{f_i} \cdot S_{wet\,i}}{S_{REF}}$$

$(\,)_i$ → "i^{th}" component
(wing, tails, fuselage, Pylons . . .)

K_i = form factor of "i^{th}" component

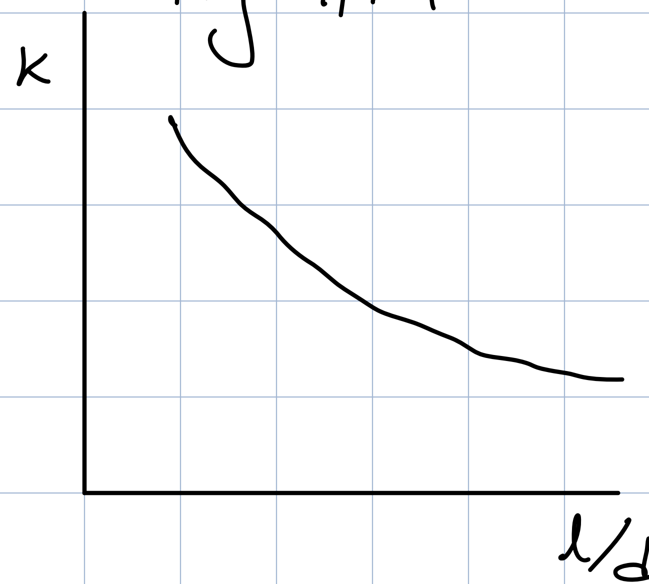
↳ pressure drag correction factor

Shevell Fig 11.3



Wings / tails
pylons

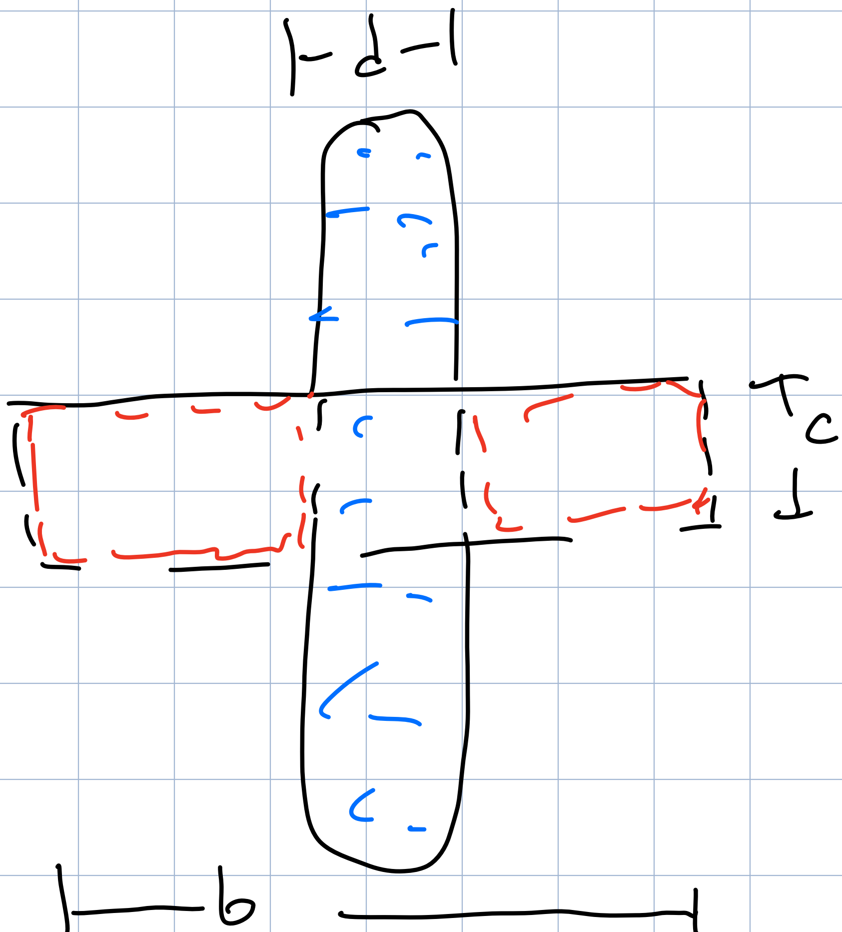
Fig 11.4



fuselages
nacelles

$C_{f_i} \equiv$ flat plate skin friction coefficient $(Re_{L,i})$

S_{wet_i} = area of Exposed portion of component



$$S_{wet_{wing}} = 2 \cdot 1.02 \cdot S_{REF_{Exposed}}$$

$$S_{REF\,Exposed} = b \cdot C - d \cdot c$$

$S_{wet_{fuselage}} = \pi d \cdot l$

$S_{REF} \equiv$ Reference area of wing

- Note $f_i = K_i \cdot C_{f_i} \cdot S_{wet_i}$

$f_i \equiv$ "equivalent profile drag area"

area of a theoretical flat plate $\perp$ to the flow & has $C_D = 1.0$ with same drag force as i^{th} component

$$f_{total} = \sum f_i = f_{wing} + f_{tail} + f_{fuselage} + \ldots$$

$$C_{D_P} = \frac{f_{total}}{S_{REF}}$$

$$D_P = \frac{f_{total}}{S_{REF}} \cdot q \cdot S_{REF}$$

$$= f q$$

Drag due to lift

3-D C_L

- $$C_{Di} = \frac{C_L^2}{\pi \, AR \, e}$$

convert C_l to C_L if needed

$$L = \tfrac{1}{2}\rho V^2 S_{REF} \cdot C_L$$

for C_{Di} (most rigourous definition)

$$C_{Di} = K C_L^2 + \frac{C_L^2}{\pi AR u}$$

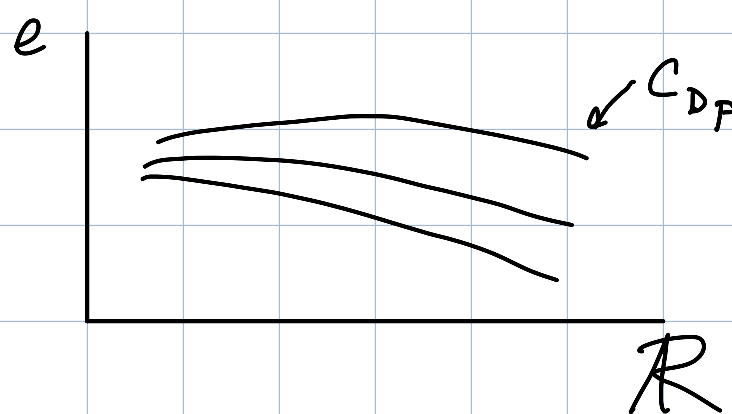
profile drag sometimes varies with lift....

$$= \left(K + \frac{1}{\pi AR u} \right) C_L^2$$

$$\approx \frac{C_L^2}{\pi AR e}$$

oswald efficiency factor

shevell Fig 11.8



Incompressible drag

$$\text{Drag (lbs)} = C_{D_P}\, q\, S_{REF} + \underset{\substack{\uparrow \\ \frac{C_L^2}{\pi AR e}}}{C_{D_i}}\, q\, S_{REF}$$

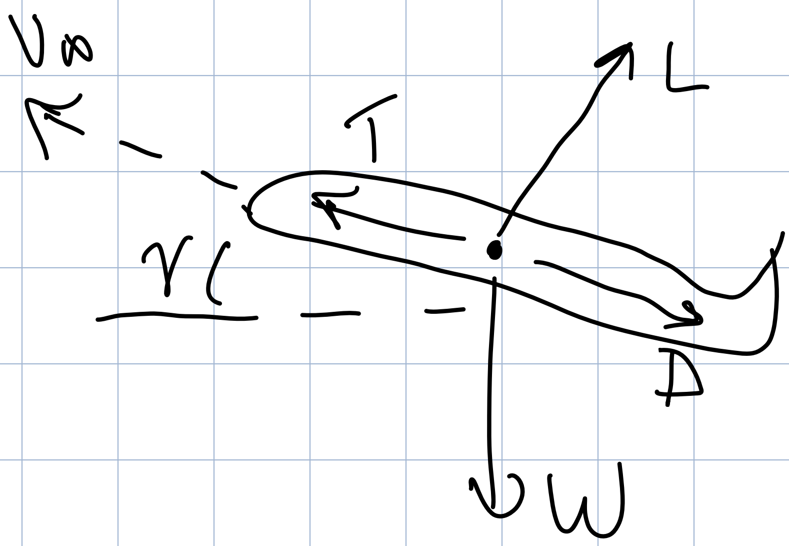
$$+ \quad \Delta C_{D,C} \cdot q \cdot S_{REF}$$

if compressibility drag is Relevant

Can also write

$$\text{Drag} = \underbrace{f \cdot q}_{\text{profile}} + \underbrace{\left(\frac{L}{b}\right)^2 \frac{1}{\pi q e}}_{\text{induced}} + \text{compressibility}$$

Why does this Matter?



if you are flying steady & level, $\gamma = 0°$
flight path angle

& acceleration $= 0$

↳ $L = \text{Weight}$
$D = T$ ⟶ predict Required thrust