MAE 158 Lecture 17
Nov. 26 2024

Announcements: No quiz this week

No HW this week
(last HW to be posted
Monday week 10)

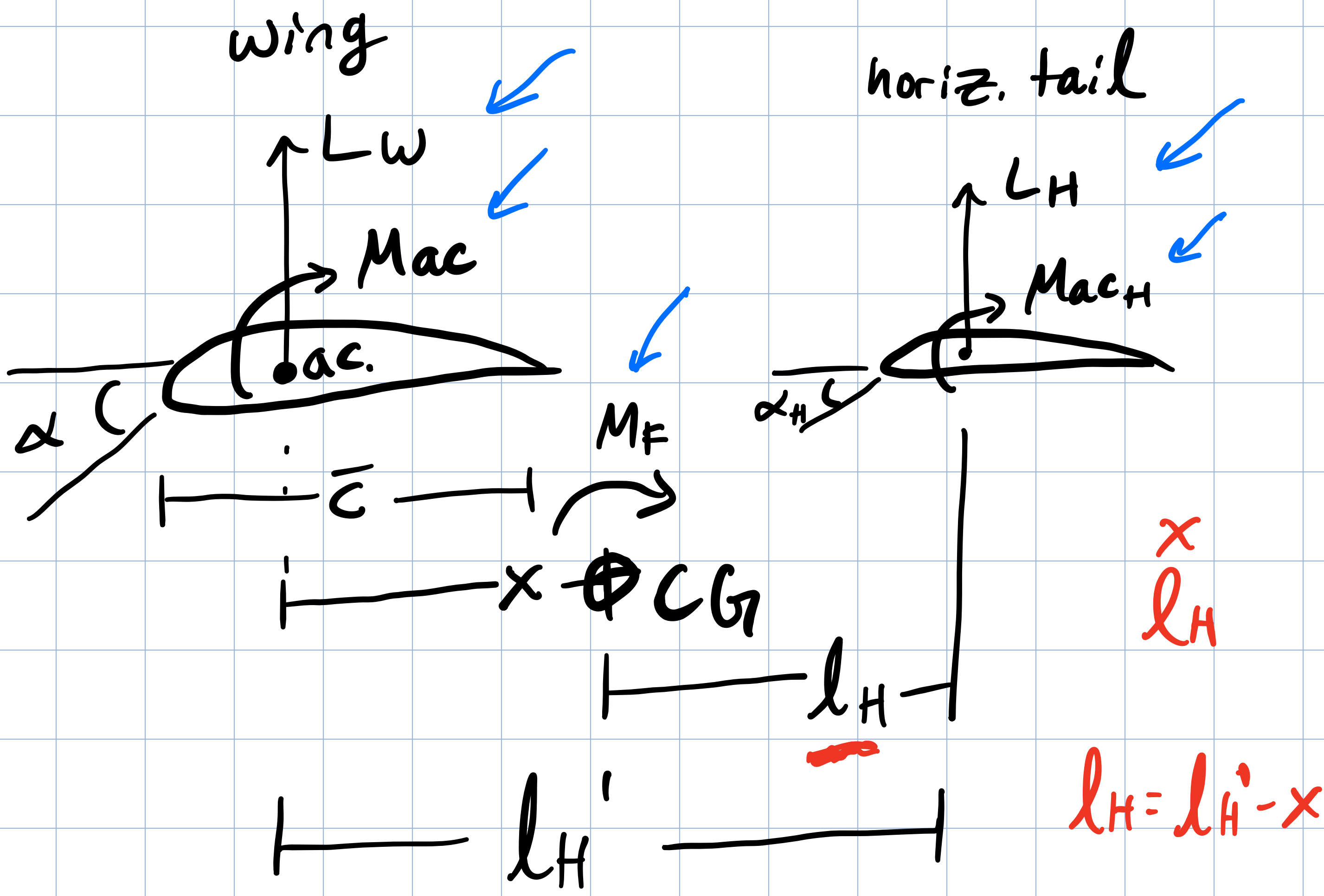
Today's Objectives: Stability (cont.)

last time:

longitudinal Stability when

$$\frac{dC_M}{dC_L} \leq 0$$

Solve location of CG
that leads to Stability



$M_{ac.H}$ → negligible for horizontal tail

M_F → Moments due to fuselage nacelles etc.

$$\sum M_{CG} = \underbrace{M_W}_{M_{ac_W} + L_W \cdot x} + \underbrace{M_H}_{- L_H l_H} + M_F$$

$$C_{M_{CG}} = \frac{M_{CG}}{q S_w \bar{C}} = C_{M_{AC}} + \frac{C_{L_W} \cdot X}{\bar{c}} - C_{L_H}\left[\frac{S_H \cdot l_H}{S_w \bar{C}}\right] \cdot \eta_H + C_{M_F}$$

horiz. tail efficiency

Sref wing

mean aerodynamic chord wing (Aerodynamic MAC, -Not- exposed MAC)

Refer total drag lecture

get stable condition if $\frac{dC_m}{dC_L} \leq 0$

limiting condition @ $\frac{dC_m}{dC_L} = 0$

↳ When X positioned such that this is true → neutral stability

- CG forward of this position
  → stability

- CG aft of this position
  → unstable

Note we want $\frac{dC_M}{dC_L}$

total Aircraft C_L

Not wing C_L!

$$L_{total} = L_W + L_H \cdot \eta_H$$

divide by $q S_W$

$$C_L = C_{L_W} + \eta_H C_{L_H} \cdot \frac{S_H}{S_W}$$

Start by solving $\frac{dC_M}{dC_{L_W}}$

$\frac{d}{dC_{LW}}$ Recall $\frac{dC_{mac}}{dC_L} = 0$

$$\left(C_{m_{CG}} = \frac{M_{CG}}{q S_W \bar{c}} = \cancel{C_{mac}} + C_{L_W} \cdot \frac{X}{\bar{c}} - C_{L_H}\left[\frac{S_H \cdot l_H}{S_W \bar{c}}\right] \cdot \eta_H + C_{m_F} \right)$$

$$\frac{dC_{m_{CG}}}{dC_{L_W}} = \frac{X}{\bar{c}} - \frac{dC_{L_H}}{dC_{L_W}}\left[\frac{S_H l_H}{S_W \bar{c}}\right]\eta_H + \frac{dC_{m_F}}{dC_{L_W}} \quad ①$$

by definition, $\frac{dC_{mac}}{dC_{L_W}} = 0$

because M_{ac} is invariant with C_L by definition

↳ airfoil lecture

What is $\dfrac{dC_{LH}}{dC_{LW}}$?

$C_{L_{wing}} = \dfrac{dC_{Lw}}{d\alpha} \cdot \alpha$

$C_{LH} = \underset{\substack{\uparrow \\ C_L \text{ curve horizontal tail}}}{\dfrac{dC_{LH}}{d\alpha}} \cdot \alpha_H$

↑ from C_L curve

in general, $\alpha_H = \alpha - \varepsilon$

Angle of attack @ tail

downwash due to wing (induced drag lecture)

$$\ast \quad \frac{dC_{LH}}{d\alpha} = \frac{dC_{LH}}{d\alpha_H} \cdot \frac{d\alpha_H}{d\alpha} = \frac{dC_{LH}}{d\alpha_H} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$dC_{LH}/d\alpha$

$$\text{thus, } \frac{dC_{LH}}{dC_{Lw}} = \frac{\overbrace{dC_{LH}/d\alpha_H \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)}^{dC_{LH}/d\alpha}}{dC_{Lw}/d\alpha}$$

plug into ①

$$\frac{dC_{m_{CG}}}{dC_{L_W}} = \frac{X}{\bar{c}} - \frac{(dC_{L_H}/d\alpha_H)}{(dC_{L_W}/d\alpha)}\left(1-\frac{d\varepsilon}{d\alpha}\right)\cdot\eta_H\cdot\frac{S_H l_H}{S_W \bar{c}} + \frac{dC_{m_F}}{dC_{L_W}}$$

almost there, -but- $\frac{dC_m}{dC_L}$

& $C_L = C_{L_W} + \eta_H C_{L_H} \frac{S_H}{S_W}$

$$\frac{dC_{m_{CG}}}{dC_L} = \left(\frac{dC_m}{dC_{L_W}}\right)\frac{\left(\frac{dC_{L_W}}{d\alpha}\right)}{\left(\frac{dC_L}{d\alpha}\right)}$$ ← lift curve

$$= \left(\frac{dC_m}{dC_{L_W}}\right)\cdot\frac{\left(\frac{dC_{L_W}}{d\alpha}\right)}{\left(\frac{dC_{L_W}}{d\alpha}\right)+\frac{dC_{L_H}}{d\alpha_H}\left(1-\frac{d\varepsilon}{d\alpha}\right)\frac{S_H}{S_W}\cdot\eta_H}$$ I

$$= \frac{dC_m}{dC_{L_w}} \cdot \frac{1}{1 + \frac{dC_{L_H}/d\alpha_H}{dC_{L_W}/d\alpha}\left(1-\frac{d\varepsilon}{d\alpha}\right)\frac{S_H}{S_W}\cdot\eta_H} \quad ②$$

Note $l_H' = l_H + x$, $l_H = l_H' - x$

plug in ②, & $l_H = l_H' - x$

re-write just in terms of x

given

$$\left.\frac{dC_m}{dC_L}\right|_{CG} = \frac{x}{\bar{c}} - \overbrace{\left[\frac{dC_{L_H}/d\alpha_H}{dC_{L_W}/d\alpha}\left(1-\frac{d\varepsilon}{d\alpha}\right)\cdot\frac{S_H l_H'}{S_W \bar{c}}\cdot\eta_H - \left.\frac{dC_m}{dC_L}\right|_F\right]}^{②} \cdot \left[\frac{1}{1 + \frac{dC_{L_H}/d\alpha_H}{dC_{L_W}/d\alpha}\left(1-\frac{d\varepsilon}{d\alpha}\right)\frac{S_H}{S_W}\cdot\eta_H}\right]$$

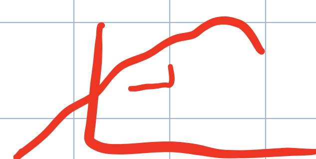
ε induced drag lecture

lift curves

given

$\frac{dC_{L_W}}{d\alpha}$

wing



tail

$\frac{dC_{L_H}}{d\alpha_H}$

solve for neutral stability

@ $\frac{dC_{mCG}}{dC_L} = 0 \rightarrow \frac{x}{\bar{c}}$

@ this location, we call it

the <u>Neutral point</u>

$$\text{Static Margin} = \frac{x_{np}}{\bar{c}} - \frac{x_{CG}}{\bar{c}}$$

$\frac{dC_m}{dC_L} < 0$

$\frac{dC_m}{dC_L} = 0$

position of CG

Ex: Jet AC which wing & horizontal tail geometry is the following

- $S_w = 2927 \text{ ft}^2$
  $b_w = 148.4 \text{ ft}$ $\Big\}$ $AR = \frac{b^2}{S}$ $= 7.52$

  $\bar{c} = 22.73 \text{ ft}$

- $S_H = 559.1 \text{ ft}^2$
  $b_H = 47.5 \text{ ft}$ $\Big\}$ $AR_H = 4.04$

  l_H' = distance between wings, horiz tail

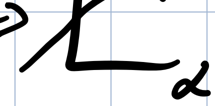
  $= 71.2 \text{ ft}$

$\eta_H = 0.9$

assume $d\varepsilon / d\alpha = 0.43$ $\quad \frac{dC_m}{dC_L}_F = -0.016$

what is $\frac{x}{\bar{c}}$ such that $\frac{dC_m}{dC_L}$ is @ least -0.10 ?

$2\pi\eta \rightarrow$



Need $\frac{dC_{Lw}}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi AR}}$

lets say $\eta_{wing} = 0.95$

$\frac{dC_{Lw}}{d\alpha} = \underline{0.0831}$ $2\pi\eta = a_0$

span efficiency

lets say $\eta_H = 0.95$

$\frac{dC_{LH}}{d\alpha_H} = \underline{0.0708}$

$$\frac{dC_{m\,CG}}{dC_L} = \frac{X}{\bar{C}} - \left[\frac{dC_{LH}/d\alpha_H}{dC_{Lw}/d\alpha}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot \frac{S_H l_H'}{S_W \bar{C}} \cdot \eta_H - \frac{dC_m}{dC_L}_F\right]$$

$$\cdot \left[\frac{1}{1 + \frac{dC_{LH}/d\alpha_H}{dC_{LW}/d\alpha}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\frac{S_H}{S_W} \cdot \eta_H}\right]$$

$$= \frac{x}{c} - \left[\frac{0.0708}{0.0831}\left(1-0.43\right)\cdot 0.9 \cdot \frac{559.1}{2927}\left(\frac{71.2}{22.7}\right) - \left(-0.016\right)\right] \cdot$$

$$\left[\frac{1}{1 + \frac{0.0708}{0.0831}\left(1-0.43\right)\frac{559}{2927}(.9)}\right]$$

$$= -0.10$$

$\uparrow$

$\frac{dC_m}{dC_L}$ that I want

$$\underline{\underline{-0.1}} = \frac{x}{c} - 0.2561$$

$$\frac{x}{\bar{c}} = \underline{0.1561}$$