MAE 158 Lecture 13
Nov. 12th 2024

Announcements: Week 7 HW posted

Today's Objectives: Takeoff (accelerated flight)

Last time: $E_{prop} = \frac{\eta}{c_p}(2\rho S)^{1/2}\frac{C_L^{3/2}}{C_D}\left(\frac{1}{W_1^{1/2}} - \frac{1}{W_0^{1/2}}\right)$

from week 7 HW:

$$E_{prop} = 37.9\,\frac{\eta}{c_p}\frac{C_L^{3/2}}{C_D}\sqrt{\frac{\sigma S}{W_0}}\left(\sqrt{\frac{W_0}{W_1}} - 1\right)$$

$\sigma = \frac{\rho}{\rho_{SL}}$

move $\frac{1}{W_0^{1/2}}$ outside

$$E_{prop} = \frac{\eta}{c_p}\left(\frac{2\rho S}{W_0}\right)^{1/2}\frac{C_L^{3/2}}{C_D}\left(\left(\frac{W_0}{W_1}\right)^{1/2} - 1\right)$$

$$\sigma \cdot \rho_{SL} = \rho$$

Note $\quad 550 \, \frac{\text{ft-lb}}{\text{s}} = 1 \text{ hp}$

$$= \underbrace{2^{1/2} \rho_{SL}^{1/2}}_{} \frac{\eta}{C_P} \left(\frac{\sigma S}{W_0}\right)^{1/2} \frac{C_L^{3/2}}{C_D} \left(\left(\frac{W_0}{W_1}\right)^{1/2} - 1 \right)$$

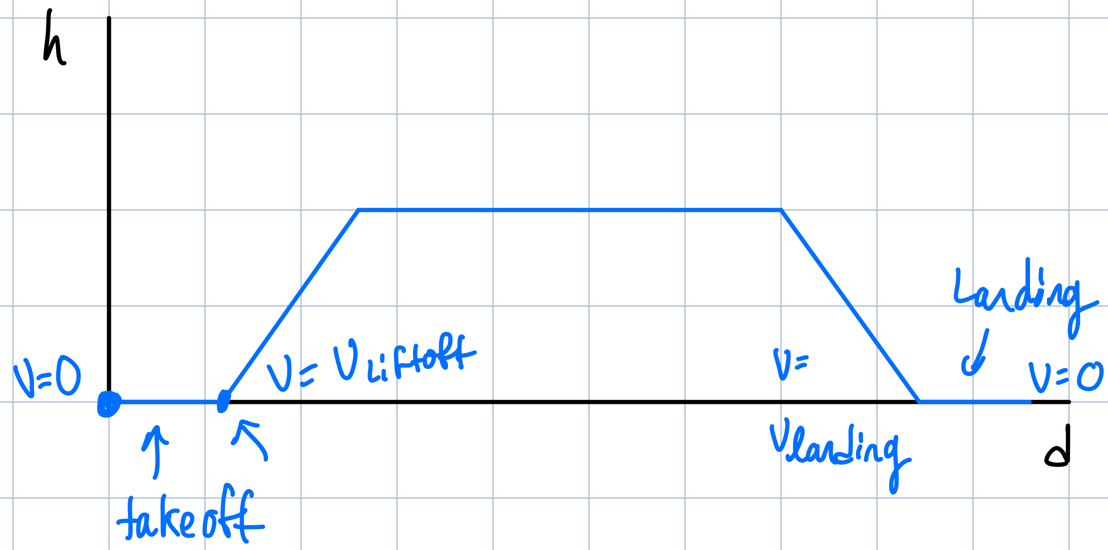
$\frac{\text{lb}}{\text{BhP-hr}}$, multiply by $550 \, \frac{\text{ft-lb}}{\text{s}}$ to convert from BHP to english units $\frac{\text{ft-lb}}{\text{s}}$

coefficient in front

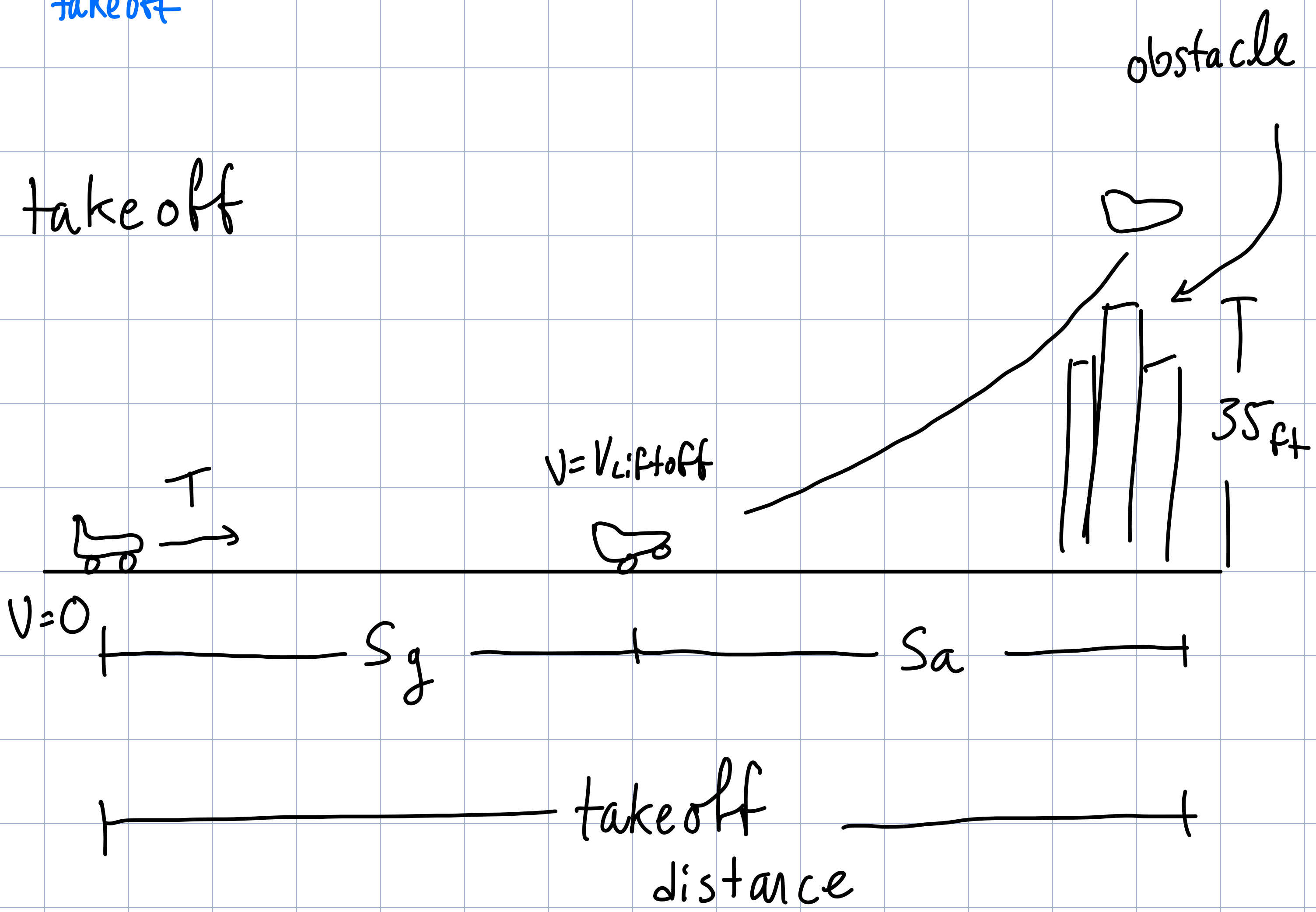
$$\Rightarrow \quad 2^{1/2} \cdot \rho_{SL}^{1/2} \cdot 550$$

$\rho_{SL} = 0.00238 \, \frac{\text{slugs}}{\text{ft}^3} \rightsquigarrow 37.9$

takeoff Procedures



takeoff

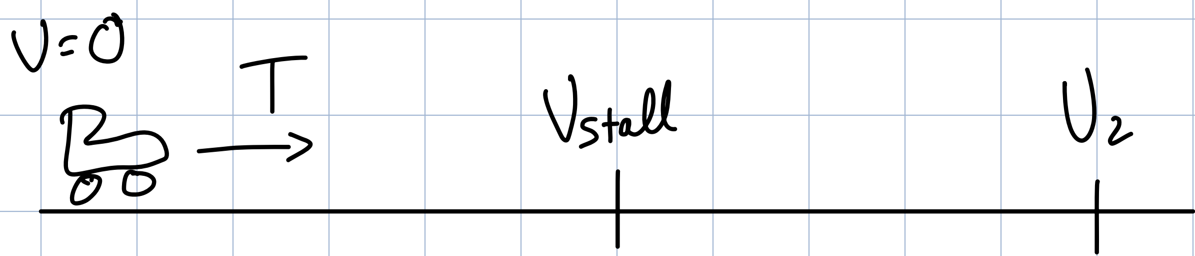


$S_g \equiv$ ground Roll distance between brake Release & liftoff

$S_a \equiv$ air borne distance b/t Liftoff & clearance of obstacle

total TO distance $S_g + S_a$

looking specifically @ S_g



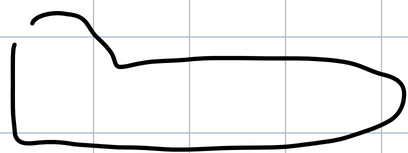
- start to accellerate to V_{stall}
- accelerate to $V_2 \equiv$ "take off safety speed"

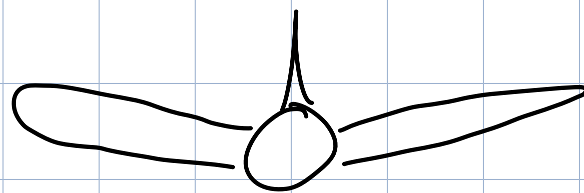
for typical civil transport A/C

$$V_2 = 1.2 \sqrt{\frac{2W}{\rho S C_{L_{max}}}}$$

take off configuration

"clean" configuration

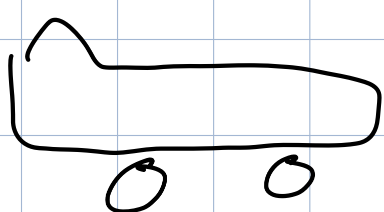


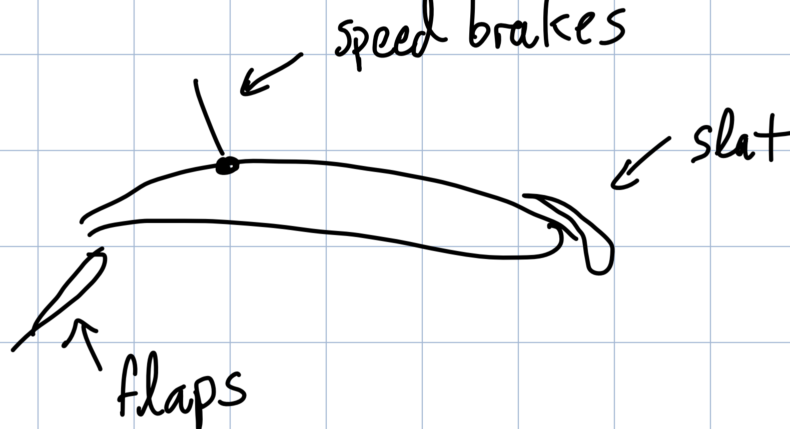


gear, flaps, slats are all Retracted

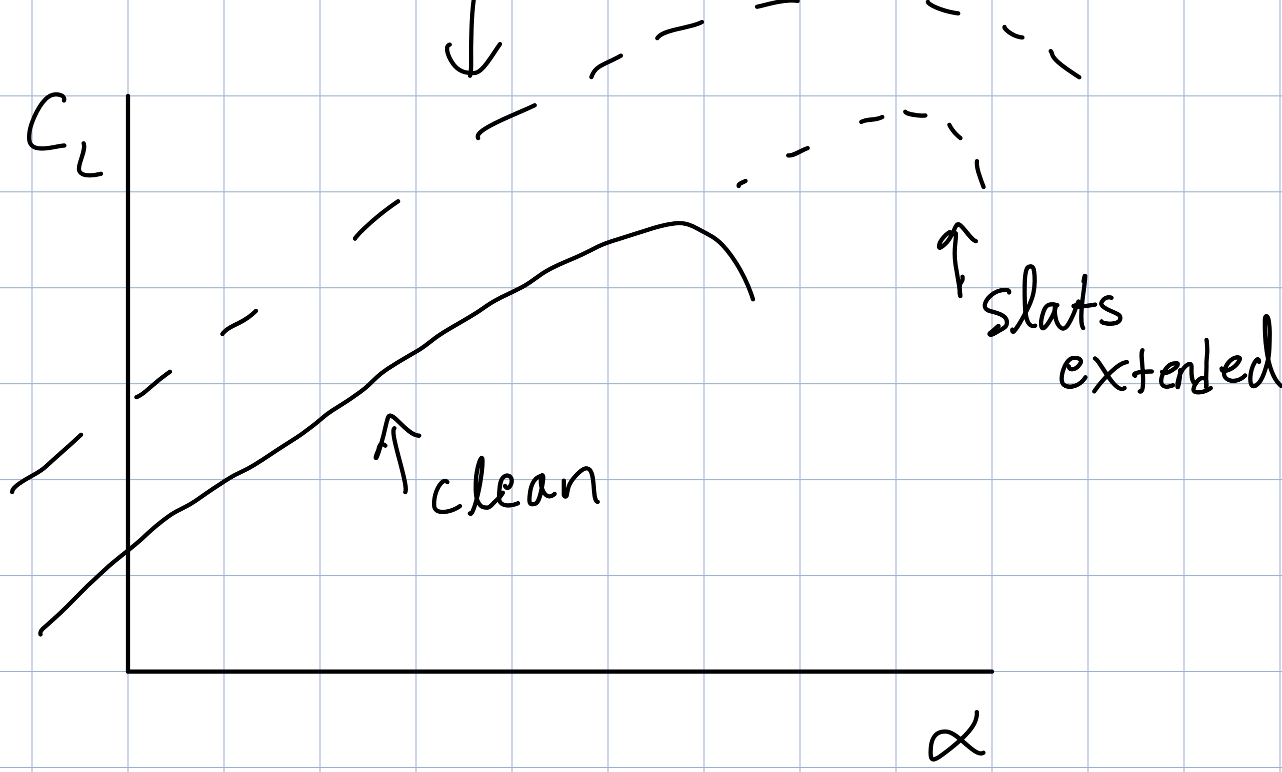
Cruise, climb, descent

Takeoff Configuration

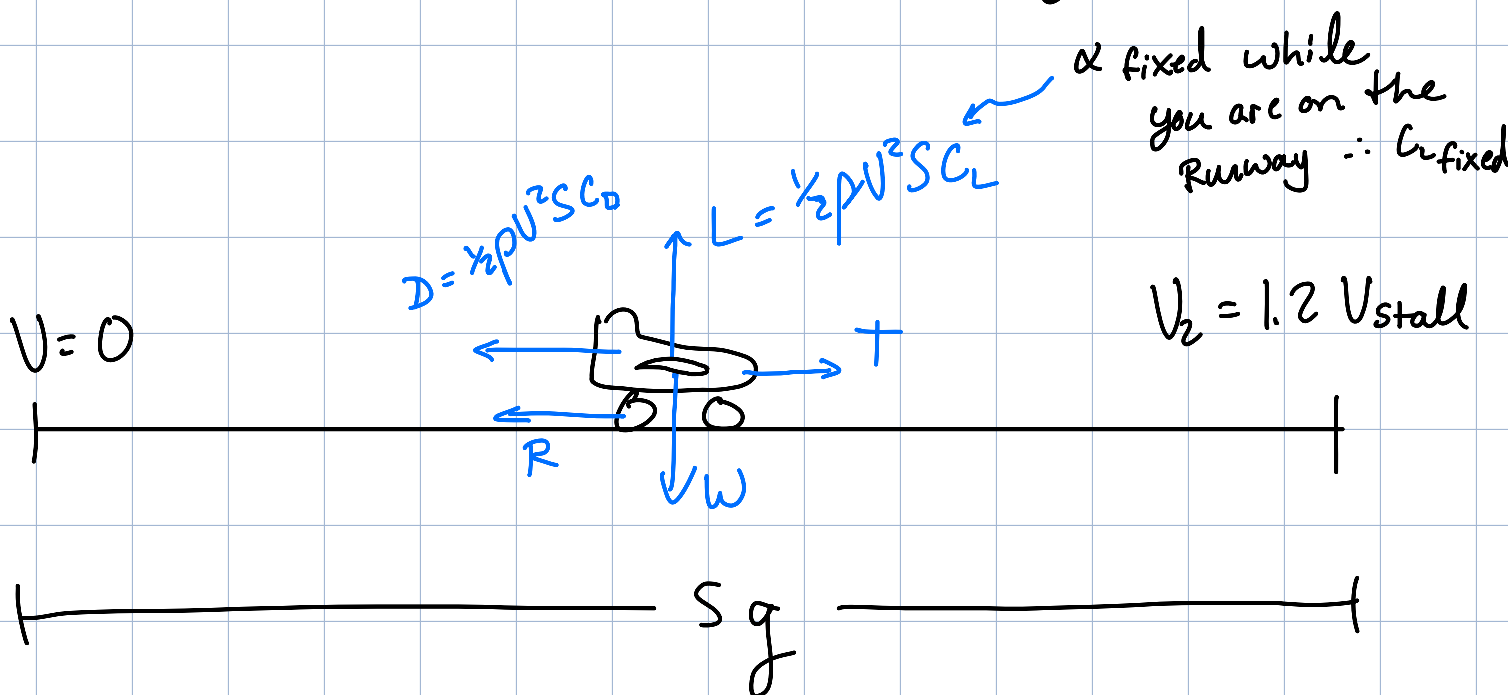




flaps & slats extended



Shevell Fig 14.15 gives C_L for various configurations



Rolling Resistance $= R = \mu_r (W - L)$

$\mu_r \equiv$ Rolling friction coefficient

depends on surface

$= 0.05$ for grass

$= 0.015$ for asfault

$$a = \frac{\Sigma F}{m} = \frac{T - D - R}{W/g} = \frac{T - q_\infty S C_D - \mu(W - q_\infty S C_L)}{W/g}$$

$T(V)$, $D(V)$, $R(V)$

$\rightsquigarrow$ function of V

not t!

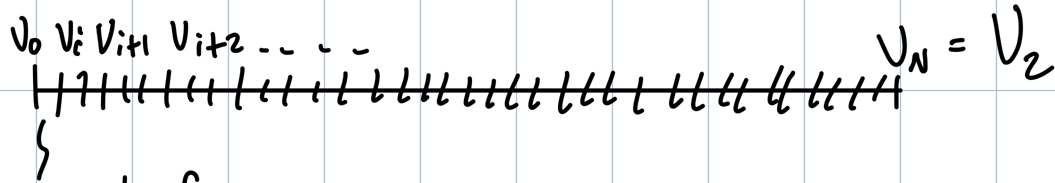
suppose $a \simeq$ const

then $S_g = \dfrac{V_2^2 - \cancel{V_0^2}}{2a}$ $= 0$

from kinematics

only works if a is constant

① Reimann sum



$a \approx$ ¢ for each segment

↳ assume $D \approx$ ¢ using V @ start of the segment
$R \approx$ ¢
$T \approx$ ¢ to compute forces

then compute a for an incremental distance along segment

if $V_N = V_2$, that is TO distance

② assume an average value for a along entire TO distance

use a obtained when calculating $T - D - R$ @ $V = 0.7\,V_2$

in other words, compute T, D, R @ $V = 0.7\,V_2$

plug into $a = \frac{\Sigma F}{m}$

first calculate $V_{stall} = \sqrt{\frac{2W}{\rho S C_{L_{max}}}}$

$\uparrow$ to config

calculate $V_2 = 1.2 \cdot V_{stall}$

calculate $0.7 V_2$

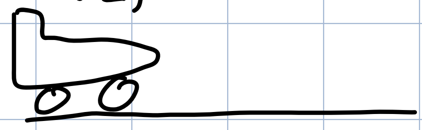
calculate forces $\rightarrow a$

$$S_g = \frac{V_2^2}{2 a_{0.7V_2}}$$

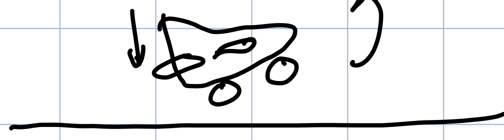
$$= \frac{1.44 W^2}{g \rho S C_{L_{max}} (T - D - R)_{V=0.7V_2}}$$

Note when Taking off,

V_2, C_L not be $C_{L\,max}$



$V_2 = 1.2\,V_{stall}$



when taking off

A/C pitches up Right at Liftoff

on ground

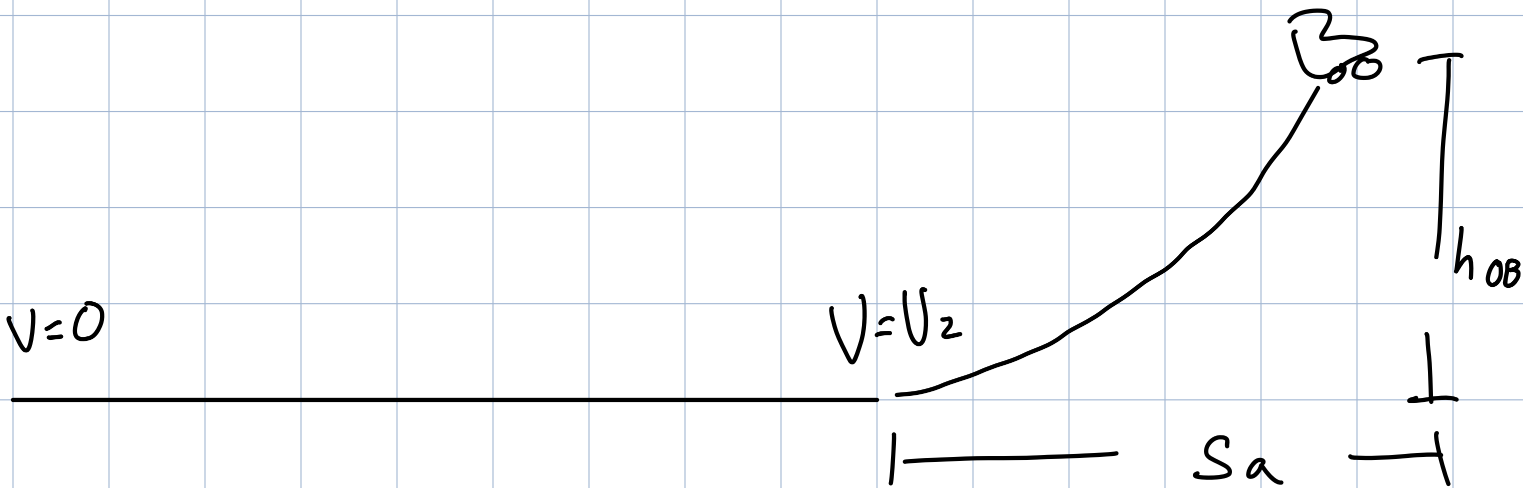
$$C_D = C_{D_p} + KC_L^2$$

$$C_{D_{p0}} + \Delta C_{D_{p\,configuration}}$$

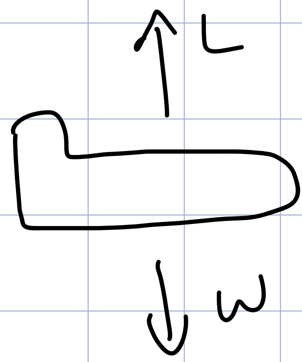
"clean" C_{D_p}

gear, flaps, slats etc....

use C_L on ground (Not $C_{L\,max}$!!)



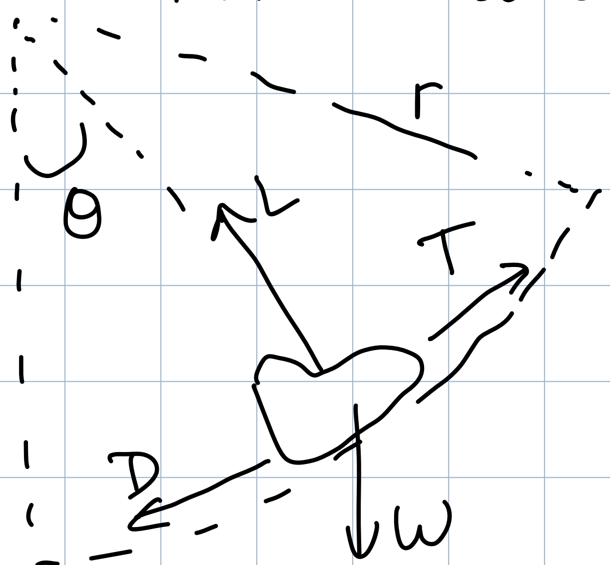
in order to get Sa, pull-up maneuver



SLF $L = W$

when pitching A/C for Liftoff, typically Need α higher than needed for SLF, sudden increase in L such that $L > W$

A/C archs upward



$r \equiv$ Radius of curved arch

$\frac{d\theta}{dt} \equiv$ Rate of change of turn angle θ

looking @ instantaneous moment where pullup is initiated

$\theta = 0°$

in verticle direction

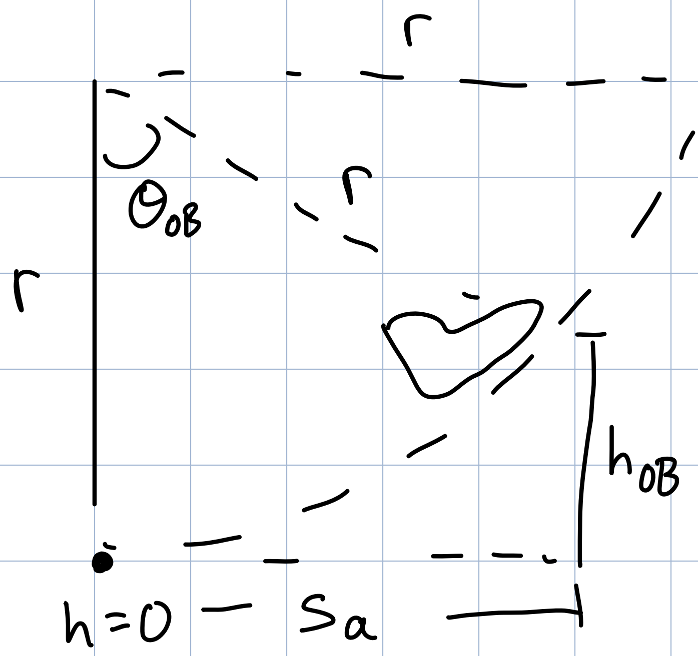
$$L - W = \frac{mV^2}{r}$$ ← centripetal force

$$r = \frac{mV^2}{L - W} = \frac{V^2}{g\left(\frac{L}{W} - 1\right)}$$

$m = W/g$

$L/W \equiv$ load factor $= n$

$$r = \frac{V^2}{g(n-1)}$$



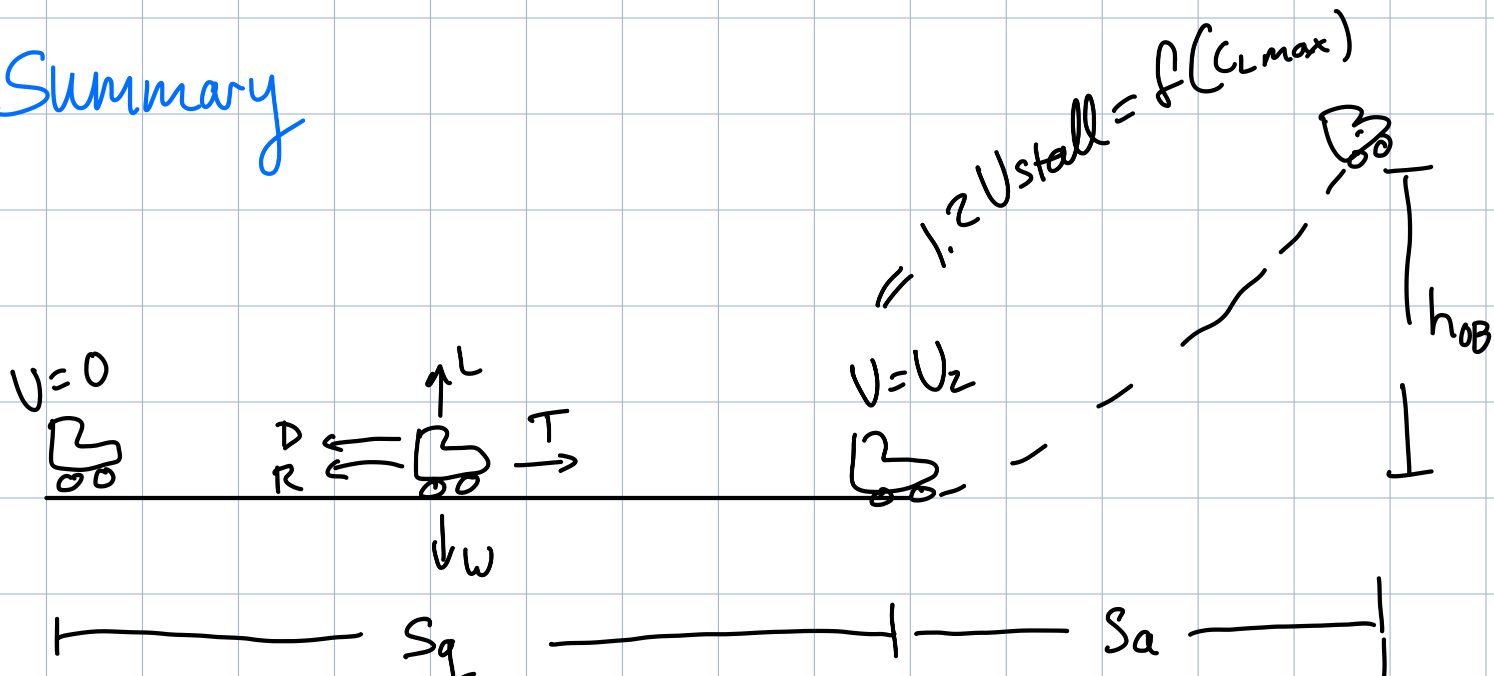
$$\cos\theta_{OB} = \frac{r - h_{OB}}{r}$$

$$\theta_{OB} = \cos^{-1}\left(1 - \frac{h_{OB}}{r}\right)$$

$$\underline{S_a = r\sin\theta_{OB}}$$

need to know n, & $V_{Liftoff}$

Summary



$$S_g = \frac{V^2}{2a_{0.7V_2}}$$

from r & geometry