MAE 185: Aricraft Performance
Recommended Homework #2

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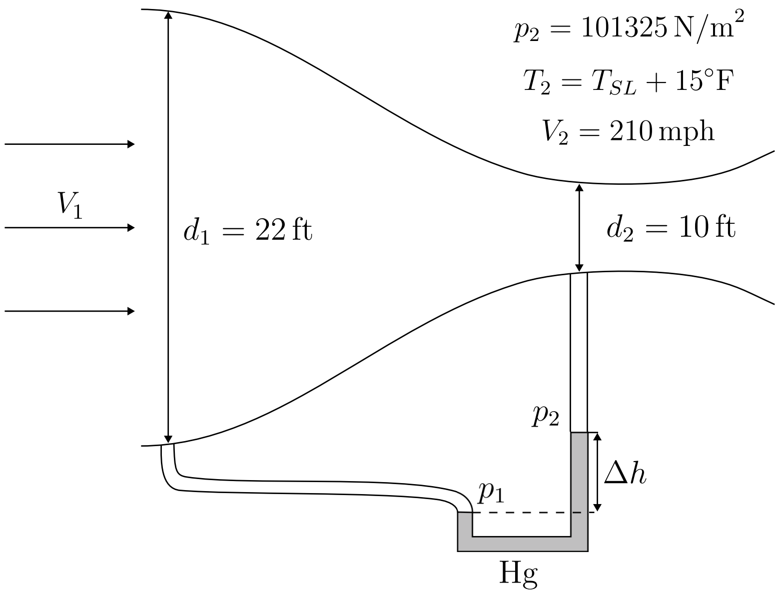
- 6.1 Consider a sea-level, low-speed wind tunnel of circular cross section with a diameter upstream of the contraction of 22 ft and a working section diameter of 10 ft. The working section is vented to atmosphere. Temperature is 15ºF above standard. If the working section velocity is 210 mph, what is



(a) the upstream (22 ft section) velocity?
We use the continuity equation for incompressible fluids

$$A_1 V_1 = A_2 V_2$$

to determine the upstream velocity V_1:

$$V_1 = \frac{A_2}{A_1} V_2 = \frac{\pi 5^2 \mathrm{ft}^2}{\pi 11^2 \mathrm{ft}^2} 210\mathrm{mph} = 43.4\,\mathrm{mph}$$

(b) the upstream pressure?
In order to apply the incompressible Bernoulli equation

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

we first need to find ρ. The temperature at the tunnel is

$$T_2 = T_{SL} + 15º\mathrm{F} = 288.15\mathrm{K} + 8.33\mathrm{K} = 296.5\mathrm{K}$$

and using the equation of state, we get

$$\rho = \frac{p_2}{RT_2} = \frac{101325\,\mathrm{N/m^2}}{287\,\mathrm{J/(kg\ K)} \cdot 296.5\,\mathrm{K}} = 1.191\,\mathrm{kg/m^3}$$

Now, we can apply Bernoulli and get (velocities are conveniently expressed as $V_1 = 19.4$ m/s and $V_2 = 93.88$ m/s):

$$p_1 = p_2 + \tfrac{1}{2}\,\rho\left(V_2^2 - V_1^2\right) = 101325\,\mathrm{N/m^2} + \tfrac{1}{2}\,1.191\,\mathrm{kg/m^3}\left(93.88^2 - 19.4^2\right)\mathrm{m^2/s^2} =$$
$$= 106350\,\mathrm{N/m^2} \qquad \left(= 2221\,\mathrm{lb/ft^2}\right)$$

(c) the height of a mercury column being used to regulate the tunnel speed by measuring the pressure difference between the upstream and working section pressure? Mercury weighs 0.49 lb/in^3.

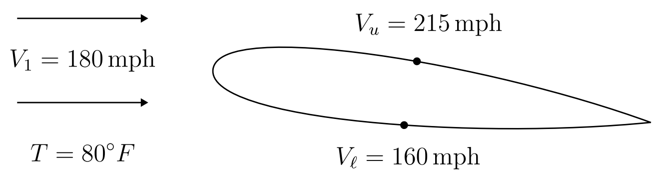
The change in pressure is

$$\Delta p = p_1 - p_2 = 106350\mathrm{N/m^2} - 101325\mathrm{N/m^2} = 5025\mathrm{N/m^2} = 105\,\mathrm{lb/ft^2}$$

and using formula (6.17) from the book (note that $W_{Hg} = 0.49$ lb/in^3 = 846.7 lb/ft^3)

$$\Delta p = w\Delta h; \qquad \Delta h = \frac{\Delta p}{W_{Hg}} = \frac{105\,\mathrm{lb/ft^2}}{846.7\,\mathrm{lb/ft^3}} = 0.124\,\mathrm{ft}$$

- 6.2 An airfoil is moving through the air at 180 mph at 5000 ft pressure altitude. The temperature is 80ºF. At a point on the airfoil upper surface the local velocity is 215 mph. What is the pressure at that point? If this is the average pressure on the surface, how much lift per square foot is being provided by the top surface? If the average speed on the lower surface is 160 mph, what is the pressure and average lift per square foot on the lower surface? What is the total lift per square foot of wing area?



The pressure at 5000 ft (1512.5 m) is computed using the standard atmosphere model:

$$p_1 = p_0\left(\frac{T_0 - a(h-h_0)}{T_0}\right)^{-g_0/aR} = 101325\,\mathrm{Pa}\left(\frac{288.16\,\mathrm{K} - 0.0065\,\mathrm{K/m}\cdot 1512.5\,\mathrm{m}}{288.16\,\mathrm{K}}\right)^{5.25} =$$
$$= 84435\,\mathrm{Pa} \qquad \left(= 1763.5\,\mathrm{lb/ft^2}\right)$$

(Remember that $1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$). Density is found by means of the equation of state, using $T = 80$ºF = 300 K,

$$\rho = \frac{p_1}{RT_1} = \frac{84435\,\mathrm{N/m^2}}{287\,\mathrm{J/(kg\cdot K)}\cdot 300\,\mathrm{K}} = 0.9807\,\mathrm{kg/m^3}$$

As we are in the incompressible regime, we can apply Bernoulli's principle to find the upper pressure p_u using a point far enough from the airfoil, for which $p_1 = p$, just found and $V_1 = 180$ mph = 80.46 m/s:

$$p_1 + \tfrac{1}{2}\,\rho V_1^2 = p_u + \tfrac{1}{2}\,\rho V_u^2 \qquad \rightarrow \qquad p_u = p_1 + \tfrac{1}{2}\,\rho\left(V_1^2 - V_u^2\right)$$
$$= 84435\,\mathrm{Pa} + \frac{0.9807\,\mathrm{kg/m^3}}{2}\left(80.46^2 - 96.2^2\right)\mathrm{m^2/s^2}$$
$$= 83072\,\mathrm{Pa} \qquad \left(= 1735\,\mathrm{lb/ft^2}\right)$$

The lift per surface generated by the upper surface is the difference in pressures:

$$\ell_u = p - p_u = 84435 - 83072 = 1363\,\mathrm{Pa} \qquad \left(= 28.4\,\mathrm{lb/ft^2}\right)$$

We do the same to find the pressure in the lower surface p_ℓ, using $V_\ell = 160$ mph $= 71.5$ m/s,

$$\ell_\ell = 84435\,\mathrm{Pa} + \frac{0.9807\,\mathrm{kg/m^3}}{2}\left(80.46^2 - 71.5^2\right)\mathrm{m^2/s^2} = 85102\,\mathrm{Pa} \qquad \left(= 1777\,\mathrm{lb/ft^2}\right)$$

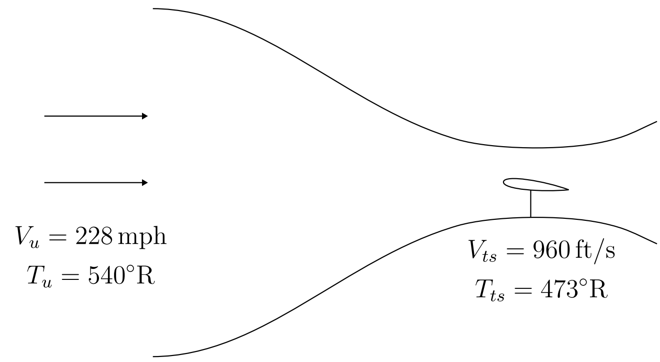
Then we get

$$\ell_\ell = p_\ell - p = 85102 - 84435 = 667\,\mathrm{Pa} \qquad \left(= 13.9\,\mathrm{lb/ft^2}\right)$$

And so, the total lift is

$$\ell = \ell_u + \ell_\ell = 1363 + 667 = 2030\,\mathrm{Pa} \qquad \left(= 42.4\,\mathrm{lb/ft^2}\right)$$

- 10.1 Consider a high speed subsonic wind tunnel. The conditions in the large-diameter section upstream of the test section are $V = 228$ mph and $T = 540^{\mathrm{o}}$R. At the test section, $T = 473^{\mathrm{o}}$R and the pressure is two standard sea-level atmospheres. Assume velocity 960 ft/s at the test section.



(c) If a wind tunnel model, placed in the test section, has a wing chord of 12 in., what is the test Reynolds number based on that chord?
We have

$$V_u = 102\,\mathrm{m/s}, \qquad T_u = 300\,\mathrm{K}, \qquad V_{ts} = 290.4\,\mathrm{m/s}, \qquad T_{ts} = 262.8\,\mathrm{K}$$

where u stands for upstream and ts for test section. Also

$$P_{ts} = 2 \cdot 1.01325 \cdot 10^5\,\mathrm{N/m^2} = 2.0265 \cdot 10^5\,\mathrm{N/m^2} \qquad \left(= 4.232 \cdot 10^3\,\mathrm{lb/ft^2}\right)$$

We compute the density at the test section using the equation of state

$$\rho_{ts} = \frac{p_{ts}}{RT_{ts}} = \frac{2.0265 \cdot 10^5\,\mathrm{N/m^2}}{287\,\mathrm{J/K \cdot kg} \cdot 262.8\,\mathrm{K}} = 2.687\,\mathrm{kg/m^3} \qquad \left(= 0.005213\,\mathrm{slug/ft^3}\right)$$

From figure 10.14 from the book, we can compute the viscosity μ, using $T_{ts}/T_0 = 0.909$ ($T_0 = 288.16$ K is the standard sea level temperature) and obtain $\mu/\mu_0 = 0.93$ thus, we have

$$\mu = 0.92 \cdot \rho_0 = 0.93 \cdot 1.7894 \cdot 10^{-5}\,\mathrm{kg/m\ s} = 1.664 \cdot 10^{-5}\,\mathrm{kg/m\ s} \qquad \left(= 3.48 \cdot 10^{-7}\,\mathrm{lb\text{-}s/ft^2}\right)$$

Note that lb-s/ft^2 = slug/(ft $\cdot$ s). Finally, the Reynolds number is computed using $L = 12\,\mathrm{in} = 1\,\mathrm{ft} = 0.305$ m:

$$\mathrm{Re} = \frac{\rho_{ts} V_{ts} L}{\mu} = \frac{2.687\,\mathrm{kg/m^3} \cdot 290.4\,\mathrm{m/s} \cdot 0.305\,\mathrm{m}}{1.664 \cdot 10^{-5}\,\mathrm{kg/m\ s}} = 14.3 \cdot 10^6$$

(d) What is the overall smooth flat-plate skin friction coefficient of the model wing if the boundary layer is turbulent? What is the C_{D_P}?
Note that we are in the turbulent regime ($\mathrm{Re} > 5 \cdot 10^5$). Using Schlichting empirical formula for turbulent boundary layers (eq 10.6b), we can compute the total smooth flat-plate skin friction drag coefficient C_f:

$$C_f = \frac{0.455}{(\log_{10} \mathrm{Re})^{2.58}} = 2.838 \cdot 10^{-3}$$

We have

$$S_{wet} = 2 \cdot 1.02 \cdot S_{ref}$$

The skin friction drag D_f is

$$D_f = C_f q S_{wet}$$

And C_{D_P} is defined as

$$C_{D_P} = \frac{D_f}{qS_{ref}} = \frac{C_f q S_{wet}}{qS_{ref}} = C_f \frac{S_{wet}}{S_{ref}} = 2.04\, C_f = 5.79 \cdot 10^{-3}$$

(e) Determine the boundary layer thickness at the trailing edge of the model wing.
Again, since flow is turbulent, we use

$$\delta = \frac{0.37\,\ell}{(\mathrm{Re}_\ell)^{0.2}} = \frac{0.37 \cdot 0.305\,\mathrm{m}}{(14.3 \cdot 10^6)^{0.2}} = 4.183 \cdot 10^{-3}\,\mathrm{m} \qquad (= 0.1647\,\mathrm{in})$$

(f) Calculate the skin friction drag in pounds. Model span is 3 ft.

The reference surface is the span ($3\,\mathrm{ft} = 0.914$ m) times the chord

$$S_{ref} = c \cdot b = 0.305\,\mathrm{m} \cdot 0.914\,\mathrm{m} = 0.279\,\mathrm{m}^2 \qquad \left(3\,\mathrm{ft}^2\right)$$

The skin friction drag uses the reference surface and C_{D_P}:

$$D_f = C_{D_P}\,\frac{1}{2}\,\rho_{ts} V_{ts}^2 S_{ref} = 5.79 \cdot 10^{-3} \cdot \frac{1}{2} 2.687\,\mathrm{kg/m^3} \cdot (290.4)^2 \mathrm{m^2/s^2} \cdot 0.279\,\mathrm{m}^2 = 183\,\mathrm{N} \qquad (= 41.14\,\mathrm{lb})$$

- 10.3 A Piper Cherokee is flying at 4000 ft altitude at 120 mph on a standard day. The gross weight is 1850 lb. The wing has an area of 160 ft^2, 85% of which is exposed, is of rectangular platform, and has a span of 30 ft. Assume completely turbulent boundary layer flow. Pressure drag and surface roughness add 25% to wing parasite drag (above pure skin friction). Wing parasite drag is 38% of the total airplane parasite drag.

  (a) Determine the smooth flat-plate skin friction drag of the wing in both coefficient and force terms.
  First off, from table A.2 in the book's appendix, we find the values of ρ and μ at $h = 4000$ ft in standard conditions:

$$\rho_{4000\,\mathrm{ft}} = 2.111 \cdot 10^{-3}\,\mathrm{slug/ft^3} = 1.088\,\mathrm{kg/m^3}$$
$$\nu_{4000\,\mathrm{ft}} = 1.7324 \cdot 10^{-4}\,\mathrm{ft^2/s} = 1.609 \cdot 10^{-5}\,\mathrm{m^2/s}$$

To find the skin friction drag, we first need the Reynolds number, for which we have to compute the chord

$$c = \frac{S_{ref}}{b} = \frac{160\,\mathrm{ft}^2}{30\,\mathrm{ft}} = 5.33\,\mathrm{ft} = 1.625\,\mathrm{m}$$

The Reynolds number is then, with $V = 120\,\mathrm{mph} = 53.65\,\mathrm{m/s}$

$$\mathrm{Re}_c = \frac{\rho V c}{\mu} = \frac{V c}{\nu} = \frac{53.65\,\mathrm{m/s} \cdot 1.625\,\mathrm{m}}{1.609 \cdot 10^{-5}\,\mathrm{m^2/s}} = 5.418 \cdot 10^6$$

Next, we compute C_f as

$$C_f = \frac{0.455}{\left(\log_{10}(\mathrm{Re})\right)^{2.58}} = \frac{0.455}{\left(\log_{10}(5.418 \cdot 10^6)\right)^{2.58}} = 3.32 \cdot 10^{-3}$$

Now to compute C_{D_f} we first find S_{wet}, using $\%_{exp} = 0.85$. Note that C_{D_f} is not still the same as C_{D_P}, as it does not account for the surface roughness extra drag (C_{D_P} will be computed in section (b))

$$S_{wet} = 2 \cdot 1.02 \cdot S_{ref} \cdot \%_{exp} = 1.734\, S_{ref}$$

and finally

$$C_{D_f} = C_f \frac{S_{wet}}{S_{ref}} = 3.32 \cdot 10^{-3} \frac{1.734\, S_{ref}}{S_{ref}} = 5.76 \cdot 10^{-3}$$

and the wing skin friction drag is

$$\begin{aligned} D_f = C_{D_f} \tfrac{1}{2} \rho V^2 S_{ref} &= 5.76 \cdot 10^{-3} \cdot \tfrac{1}{2}\, 1.088\,\mathrm{kg/m^3} \cdot (53.65\,\mathrm{m/s})^2 \cdot 14.86\,\mathrm{m}^2 \\ &= 134\, N \qquad (= 30.12\,\mathrm{lbf}) \end{aligned}$$

(b) Determine the total parasite drag of the wing and the airplane in both coefficient and force terms.

We must take into account the extra drag due to pressure drag and surface roughness to compute $C_{D_{P_{wing}}}$

$$C_{D_{P_{wing}}} = 1.25 \cdot C_{D_{f_{wing}}} = 1.25 \cdot 5.76 \cdot 10^{-3} = 7.2 \cdot 10^{-3}$$

and so

$$\begin{aligned} D_{wing} = C_{D_{P_{wing}}} \tfrac{1}{2} \rho V^2 S_{ref} &= 7.2 \cdot 10^{-3} \cdot \tfrac{1}{2}\, 1.088\,\mathrm{kg/m^3} \cdot (53.65\,\mathrm{m/s})^2 \cdot 14.86\,\mathrm{m}^2 \\ &= 167.5\, N \qquad (= 37.65\,\mathrm{lbf}) \end{aligned}$$

Now, for the whole airplane, we use the fact that parasite drag is 38% of the total airplane parasite drag:

$$C_{D_P} = \frac{C_{D_{P_{wing}}}}{0.38} = 0.01895$$

and so

$$\begin{aligned} D = C_{D_P} \tfrac{1}{2} \rho V^2 S_{ref} &= 18.95 \cdot 10^{-3} \cdot \tfrac{1}{2}\, 1.088\,\mathrm{kg/m^3} \cdot (53.65\,\mathrm{m/s})^2 \cdot 14.86\,\mathrm{m}^2 \\ &= 440.9\, N \qquad (= 99.12\,\mathrm{lbf}) \end{aligned}$$

(d) What is the boundary layer thickness of the wing trailing edge?

We simply use the boundary layer thickness for turbulent flows

$$\delta = \frac{0.37\ell}{\mathrm{Re}_\ell^{0.2}} = \frac{0.37 \cdot 1.625\,\mathrm{m}}{(5.418 \cdot 10^6)^{0.2}} = 0.0271\,\mathrm{m} \qquad (= 1.067\,\mathrm{in})$$