MAE 158: Aricraft Performance
Recommended Homework #9

Jordi Ventura Siches

March 15, 2023

- **16.1** A large turboprop transport has the following characteristics:

$$\text{wing area, } S_W = 3520\,\text{ft}^2$$
$$\text{wing span, } b_W = 155\,\text{ft}$$
$$\text{mean aerodynamic chord, } \bar{c} = 25.9\,\text{ft}$$
$$\text{horizontal tail aspect ratio} = 4.2$$
$$\text{tail length, } \ell'_H = 83.0\,\text{ft}$$

The airplane has a T-tail, the horizontal being mounted on top of the vertical tail. The wing is unswept. The fuselage and nacelle contribution to stability about the quarter-chord is unstable; that is,

$$\left(\frac{dC_M}{dC_L}\right)_{\text{fus + nac}} = 0.06; \qquad \text{also} \qquad \frac{d\epsilon_H}{d\alpha}(\text{at the tail}) = 0.50$$

Assume that the wing and tail have elliptical lift distributions so that $dC_L/d\alpha = a_0/(1 + 57.3a_0/(\pi AR))$ (per degree). The most aft allowable center of gravity position, at which stability, dC_M/dC_L, is at least -0.10, is 35% of the m.a.c. (Note that $dC_M/dC_L = -0.10$ corresponds to the c.g. being 10% of the mean aerodynamic chord ahead of the 'neutral' point.) Determine the horizontal tail area.

We first find the following

$$\text{AR} = \frac{b^2}{S} = \frac{(155\,\text{ft})^2}{3520\,\text{ft}^2} = 6.83$$
$$a_0 = \frac{2\pi\eta}{57.3} = \frac{2\pi 0.95}{57.3} = 0.104\,\text{deg}^{-1}$$

where we hav asseumed $\eta = 0.95$. Now, from the formula

$$\left(\frac{dC_M}{dC_L}\right)_{CG} = \frac{x}{\bar{c}} - \left[\frac{\left(\frac{dC_L}{d\alpha}\right)_H}{\left(\frac{dC_L}{d\alpha}\right)_W}\left(1 - \frac{d\epsilon}{d\alpha}\right)\frac{S_H \ell'_H}{S_W \bar{c}}\eta_H - \left(\frac{dC_M}{dC_L}\right)_{F,N}\right]\left[\frac{1}{1 + \frac{(dC_L/d\alpha)_H}{(dC_L/d\alpha)_W}\left(1 - \frac{d\epsilon_H}{d\alpha}\right)\frac{S_H}{S_W}\eta_H}\right]$$

we have

- $\frac{dC_M}{dC_L} = -0.1$
- $\frac{x}{\bar{c}} = 0.35 - 0.25 = 0.1$
- $\left(\frac{dC_L}{d\alpha}\right)_H = \frac{0.104}{1+(57.3\cdot 0.104/\pi 4.2)} = 0.0716$ (using the AR of the horizontal tail $\text{AR}_H = 4.2$).
- $\left(\frac{dC_L}{d\alpha}\right)_W = \frac{0.104}{1+(57.3\cdot 0.104/\pi 6.83)} = 0.0814$
- $\frac{d\epsilon_H}{d\alpha} = 0.5$
- $\ell'_H = 83.0\,\text{ft}$
- $S_W = 3520\,\text{ft}^2$
- $\bar{c} = 25.9\,\text{ft}$

- $\eta_H = 1.0$, since T-tail
- $\left(\frac{dC_M}{dC_L}\right)_{\text{F,N}} = 0.06$

and we want to find S_H. By substituting above, we get

$$
\begin{aligned}
-0.1 &= 0.1 - \frac{0.000415\, S_H - 0.06}{1 + 0.000125\, S_H} \\
-0.2 - 0.000025\, S_H &= -0.000415\, S_H + 0.06 \\
S_H &= 666.7\,\text{ft}^2
\end{aligned}
$$

- **16.4** A DC-8-50, the original turbofan-powered version of the DC-8, has the following characteristics:

$$
\begin{gathered}
\text{wing area, } S_W = 2883\,\text{ft}^2 \\
\text{wing span, } b_W = 148.4\,\text{ft} \\
\text{mean aerodynamic chord } \bar{c} = 22.98\,\text{ft} \\
\text{horizontal tail area, } S_H = 559.1\,\text{ft}^2 \\
\text{horizontal tail span, } b_H = 47.5\,\text{ft} \\
\text{tail length, } \ell'_H = 68.4\,\text{ft.}
\end{gathered}
$$

The total of the wing, fuselage, and nacelle contributions to stability is lightly stable about the $\bar{c}/4$ point; that is,

$$\left(\frac{dC_M}{dC_L}\right)_{\text{wing + fus + nac}} = -0.04$$

At the tail,

$$\frac{d\epsilon_H}{d\alpha} = 0.45$$

Assuming that the wing and tail have elliptical lift distributions so that $dC_L/d\alpha = a_0/(1 + 57.3\, a_0/(\pi\, AR))$ (per degree), what is the most aft allowable center of gravity position at which stability, dC_M/dC_L, is at least -0.10?

We first compute

$$
\begin{aligned}
\text{AR}_W &= \frac{b_W^2}{S_W} = \frac{(148.4\,\text{ft})^2}{2883\,\text{ft}^2} = 7.64 \\
\text{AR}_H &= \frac{b_H^2}{S_H} = \frac{(47.5\,\text{ft})^2}{539.1\,\text{ft}^2} = 4.04
\end{aligned}
$$

From

$$\left(\frac{dC_M}{dC_L}\right)_{CG} = \frac{x}{\bar{c}} - \left[\frac{\left(\frac{dC_L}{d\alpha}\right)_H}{\left(\frac{dC_L}{d\alpha}\right)_W}\left(1 - \frac{d\epsilon}{d\alpha}\right)\frac{S_H \ell'_H}{S_W \bar{c}}\,\eta_H - \left(\frac{dC_M}{dC_L}\right)_{F,N}\right]\left[\frac{1}{1 + \frac{(dC_L/d\alpha)_H}{(dC_L/d\alpha)_W}\left(1 - \frac{d\epsilon_H}{d\alpha}\right)\frac{S_H}{S_W}\,\eta_H}\right]$$

we have

- $\frac{dC_M}{dC_L} = -0.1$
- $\left(\frac{dC_L}{d\alpha}\right)_H = \frac{0.104}{1+(57.3 \cdot 0.104/\pi 4.04)} = 0.071$ (using $\text{AR}_H = 4.04$).
- $\left(\frac{dC_L}{d\alpha}\right)_W = \frac{0.104}{1+(57.3 \cdot 0.104/\pi 7.64)} = 0.0836$ (using $\text{AR}_W = 7.64$)
- $\frac{d\epsilon_H}{d\alpha} = 0.45$

- $\ell'_H = 68.4\,\text{ft}$
- $S_W = 2883\,\text{ft}^2$
- $S_H = 559.1\,\text{ft}^2$
- $\bar{c} = 22.98\,\text{ft}$
- Assume $\eta_H = 0.9$
- $\left(\frac{dC_M}{dC_L}\right)_{\text{F,N,W}} = -0.04$

Substituting we get $\frac{x}{\bar{c}} = 0.1614$ and so the aft c.g. point is found at $0.25+0.1614 = 0.4114$, that is 41.14% of the chord.

- **17.1** A proposed twin-engine turboprop transport airplane will cruise 28,000 ft at a true speed of 400 mph. Air temperature is 430ºR. The airplane will be powered by engines with a takeoff rating of 1,500 shp and a maximum cruise rating at 28,000 ft of 800 shp. At cruise, propeller speed is 1200 revolutions per minute.

  (a) Assuming four-bladed propellers, with a blade activity factor of 135, so that the propeller chart in Figure 17.20 is directly applicable, find the variation of propulsive efficiency with propeller diameter. What propeller diameter would you choose to obtain the highest cruise efficiency at maximum cruise power? (Assume several propeller diameters and determine their efficiencies. Plot η versus diameter. Check additional values of diameter as necessary to locate the optimum.)

  (b) What is the ideal efficiency of the propeller you selected?

  (a) At this altitude we have $p = 688.96\,\text{lb/ft}^2$ and with temperature 430ºR and using the equation of state, we get a density of $\rho = 0.000933$ slug/ft^3. Velocity is $V_{\text{true}} = 400\,\text{mph} = 586.7\,\text{ft/s}$. The power coefficient, as a function of the diameter is

$$CP = \frac{\text{BHP} \cdot 550}{\rho \eta^3 D^5} = \frac{800\,\text{shp} \cdot 550}{0.000933\,\text{slug/ft}^3\,(20\,rps)^3\,D^5} = \frac{58949}{D^5}$$

  where η is the revolutions per second of the propeller. The advance ratio is

$$J = \frac{V_{\text{true}}}{\eta\,D} = \frac{586.7\,\text{ft/s}}{20\,\text{rps}\,D} = \frac{29.3}{D}$$

  We will consider different values of D, ranging from 1 to 13. To obtain the corresponding η value in %, we use Figure 17.20 (see figue 1). Note that for $D < 8$ ft, CP values are not available in the chart, so we won't be able to find their efficiency. For the rest, we will store and plot these values (see figure 2). We see that the diameter for which we have the greatest efficiency is $D = 11\,\text{ft}$, for which we obtain $\eta = 0.838$.

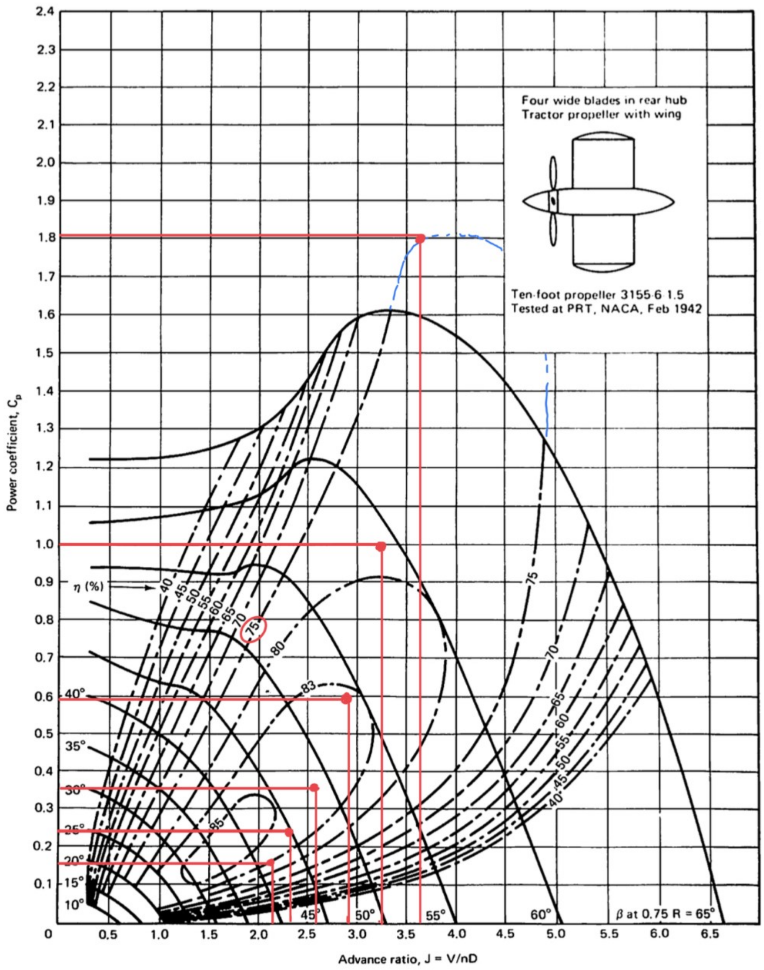

Figure 1: Use of figure 17.20

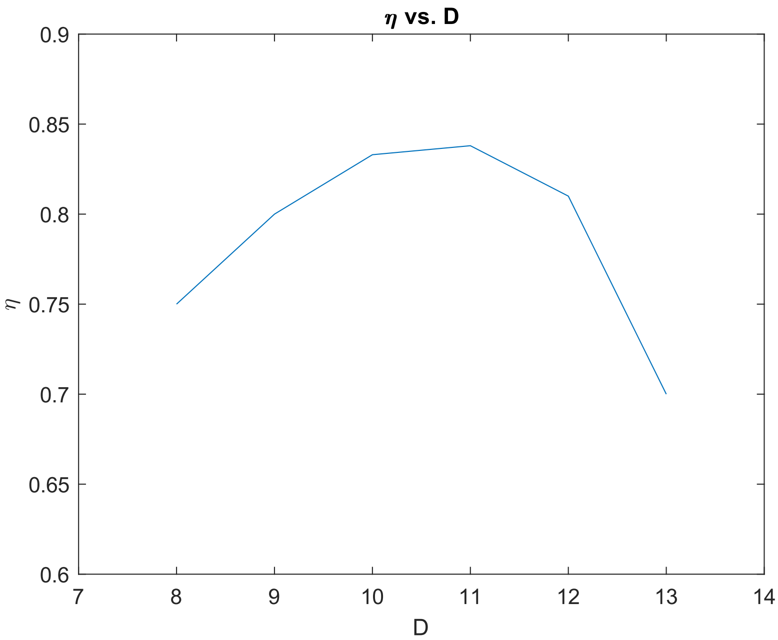

Figure 2: Efficiency vs. propeller diameter

(b) Thrust is given by

$$T = \frac{\text{BHP}\,550}{V_0}\eta = \frac{800 \cdot 550}{586.67\,\text{ft/s}}\,0.838 = 629\,\text{lbs}$$

where we have used the cruise efficiency η obtained in the previous section. From disk actuator theory, we have

$$T = 2A\rho V_0^2(1+a)a$$

and so, we can solve for a, by substituting $A = \pi(D/2)^2 = \pi(11\,\text{ft}/2)^2 = 95\,\text{ft}^2$:

$$629\,\text{lbs} = 2 \cdot 95\,\text{ft}^2 \cdot 0.000933\,\text{slug/ft}^3(586.7\,\text{ft/s})^2(1+a)a$$

that reduces to

$$a^2 + a - 0.01031 = 0$$

with solutions

$$a = -0.5 \pm 0.5102$$

from which we take the positive solution, which is the only physically meaningful, that is $a = 0.0102$. We compute the ideal efficiency as

$$\eta_{\text{ideal}} = \frac{1}{1+a} = \frac{1}{1.0102} = 0.9899$$

The actual efficiency is less than the ideal.

- **17.3** A turboprop-powered Navy P3V is flying at 350 mph at 20,000 ft on a standard day. Each of the four 14-ft-diameter propellers is delivering 1750 lb of thrust. What is the *ideal* efficiency of this propeller?
Velocity is $V = 350\,\text{mph} = 513.3\,\text{ft/s}$. At $h = 20000\,\text{ft}$ we have in standard conditions $\rho = 1.2673 \cdot 10^{-3}\,\text{slug/ft}^3$. The area is $A = \pi(7\,\text{ft})^2 = 153.94\,\text{ft}^2$ Thrust is

$$\begin{aligned} T &= 2A\rho V_0^2(1+a)a \\ a(1+a) &= \frac{T}{2A\rho V_0^2} = \frac{1750\,\text{lb}}{2 \cdot 153.94\,\text{ft}^2 \cdot 1.2673 \cdot 10^{-3}\,\text{slug/ft}^3 \cdot (513.3\,\text{ft/s})^2} = 0.0017 \end{aligned}$$

We solve the quadratic equation

$$a^2 + a - 0.017 = 0$$

and obtain

$$a = -0.5 \pm 0.5167$$

from which, we take the positive solution $a = 0.0167$ and express the ideal efficiency as

$$\eta = \frac{1}{1+a} = \frac{1}{1.0167} = 0.9835$$