MAE 158 Lecture 10
Fall 2024

Announcements: Midterm Thursday Oct 31st
2-3:20pm

* Exam coverage Weeks 1-5 (thrust Required)
* Exam is closed book, closed notes, closed online Resources. You
→ may use a 1-sided 8.5"x11" sheet of Notes & a calculator

- Drag Project posted Friday Nov 1st (Due Friday week 10, 11:59pm)
  - plot digitization & solidworks videos have been uploaded on canvas on the modules page under "Resources"

Today's objectives: climbing/descending Flight

Last time:

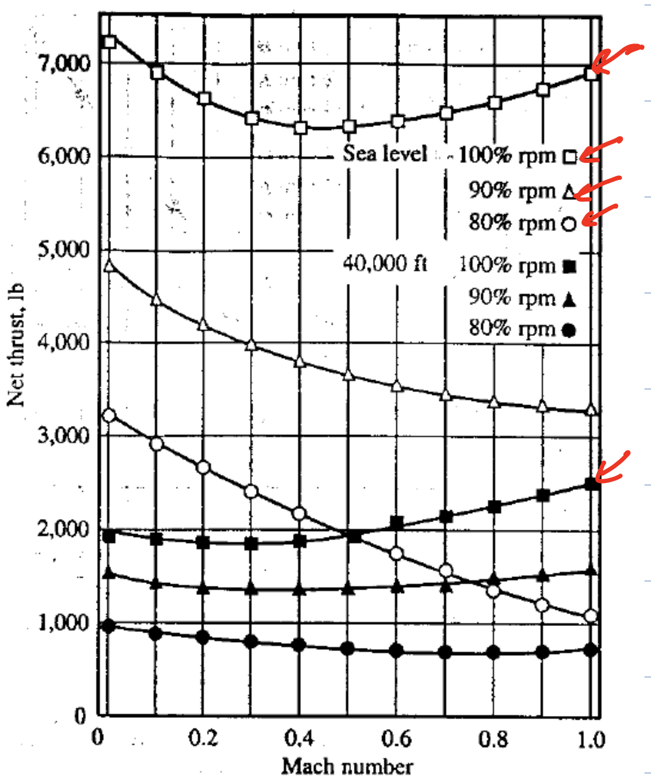
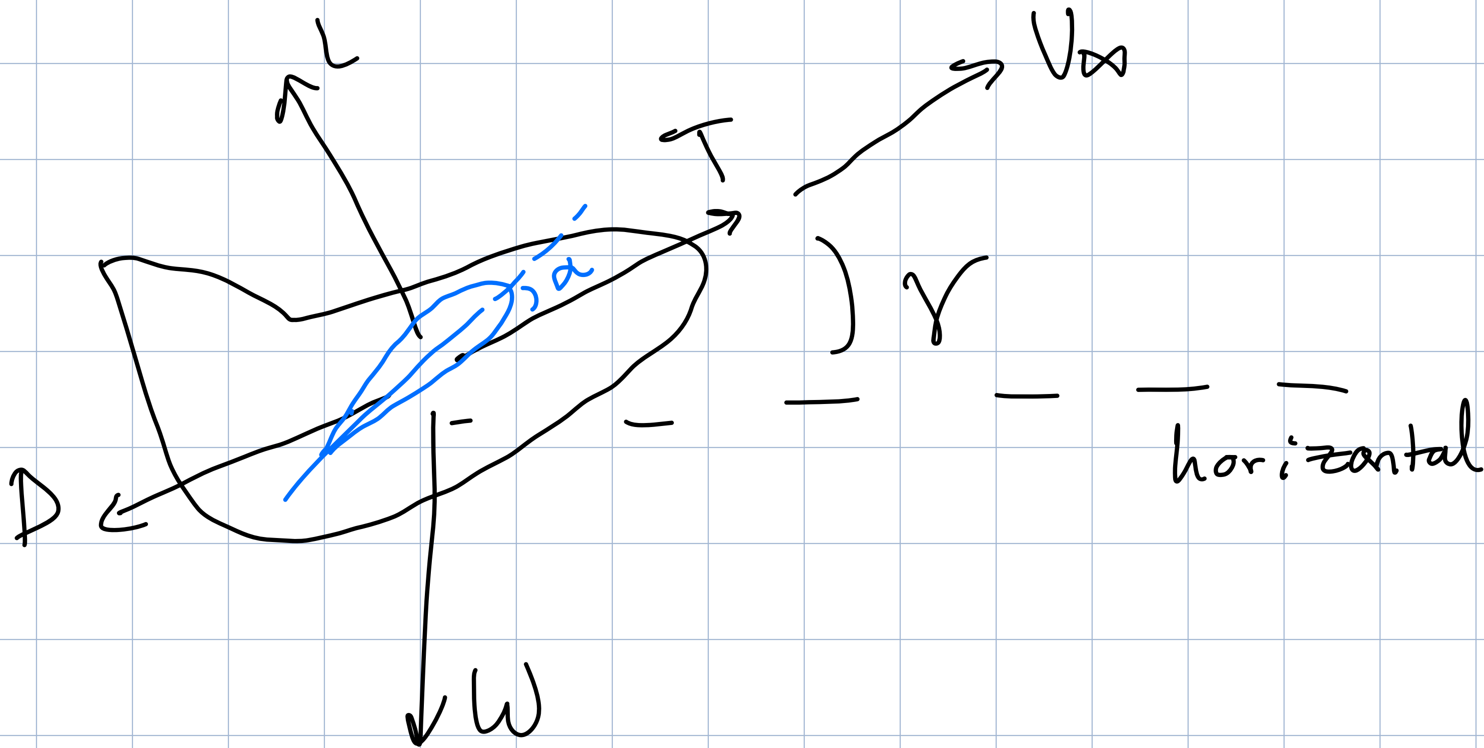


Figure 3.13 Typical results for the variation of thrust with subsonic Mach number for a turbojet.

$$T_A \sim T_{SL} \left(\frac{\rho}{\rho_{SL}} \right)^{m}$$

m from data for the specific engine

Unaccelerated Climbing Flight



$\gamma \equiv$ flight path angle

angle between V_∞

& horizontal

ROC

$R/C = V_\infty \sin\gamma$

$$V\left(T - D - W\sin\gamma\right) = (0)\,V$$

since $R/C = V_\infty \sin\gamma$

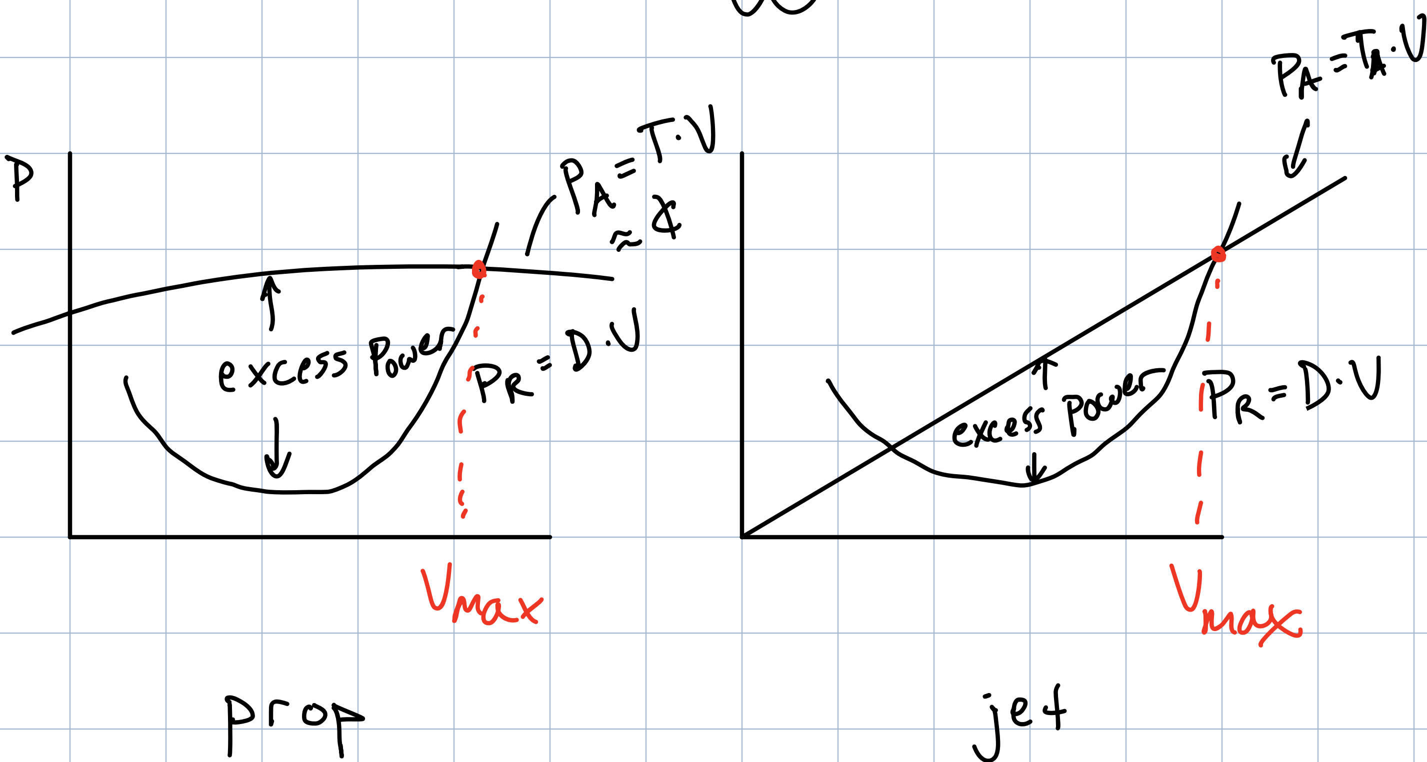
$$R/C = \left(\frac{T \cdot V - D \cdot V}{W}\right)$$

$T \cdot V$ = power available

$D \cdot V$ = power Required

$$R/C = \frac{(\text{Power available} - \text{power Required})}{W}$$

$$= \frac{\text{excess power}}{W}$$



R/C by taking the distance between P_A & P_R curves, & dividing them by weight

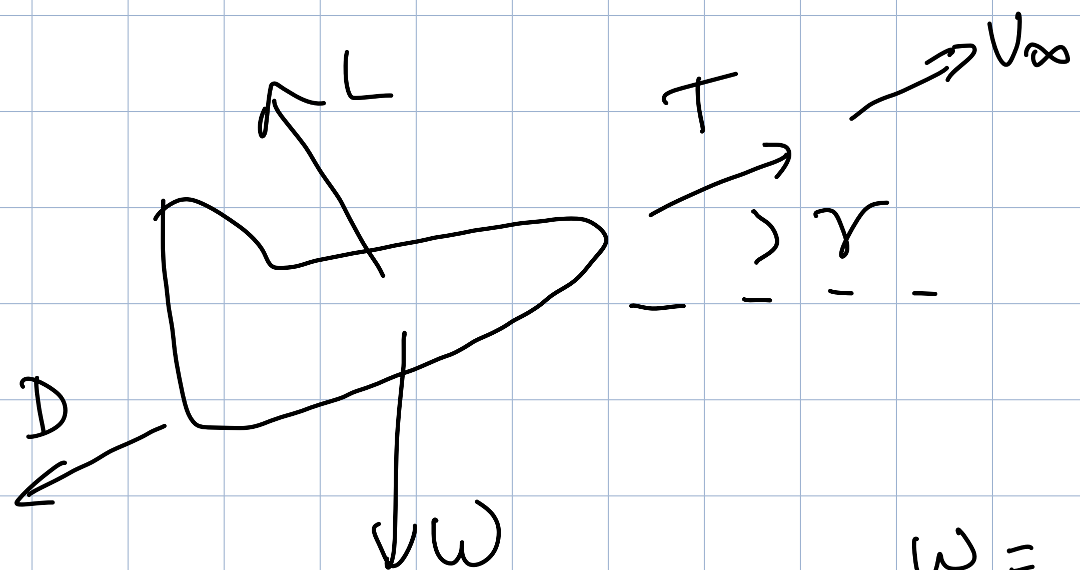
↳ check units

R/C is ϕ @ V_{max} because no excess power

max R/C is @ the velocity where distance between P_A & P_R is maximum

γ_{max} ⇝ max angle of climb

R/C_{max} ⇝ max verticle Rate



$W = L/\cos\gamma$

$$\sin\gamma = \frac{T}{W} - \frac{D}{W}$$

$$= \frac{T}{W} - \frac{\cos\gamma}{L/D}$$ assume γ small

$$\approx \qquad \gamma = \frac{T}{W} - \frac{1}{(L/D)}$$

1. Jet powered A/C $\quad T(V) \sim ¢$

$$\gamma_{max} \approx \frac{T}{W} - \frac{1}{L/D_{max}}$$

$$= \frac{T}{W} - \sqrt{4KC_{D_P}}$$

Velocity that corresponds to γ_{max}

recall the C_L @ L/D_{max}

$$C_L = \sqrt{\frac{C_{DP}}{K}} \ @ \ L/D_{max}$$

$$L = W\cos\gamma$$

$$W \cos\gamma = \tfrac{1}{2}\rho V^2 S \sqrt{\frac{C_{D_0}}{K}}$$

Rearrange to get $V_{\gamma max}$

$$R/C_{\gamma max} = V_{\gamma max} \cdot \sin(\gamma_{max})$$

↳ not R/C_{max}

2. Propeller Powered A/C

$$T = \frac{P \cdot \eta}{V}$$

plug into $\gamma = \frac{T}{W} - \frac{D}{W}$

solve by differentiating with V setting $= 0$

↳ (see full derivation in Anderson text)

$$V_{\gamma max} \sim \frac{4\left(\frac{W}{S}\right)\cdot K}{\left(\rho\, \eta\, \frac{P}{W}\right)}$$

$P \equiv$ shaft power

to get γ_{max}, plug $V_{\gamma max}$ into equations for T & D

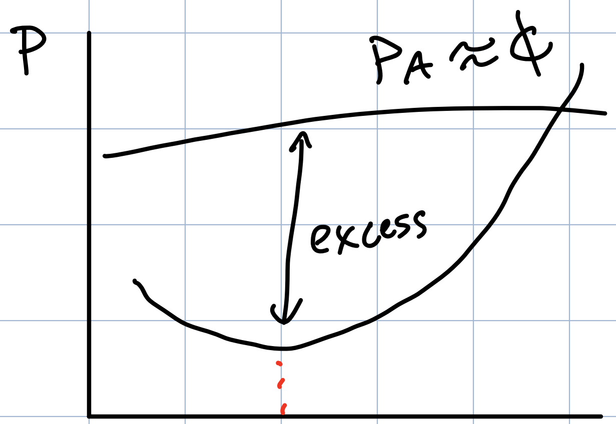
$$\hookrightarrow \quad \gamma_{max} = \frac{T}{W} - \frac{D}{W}$$

for prop A/C

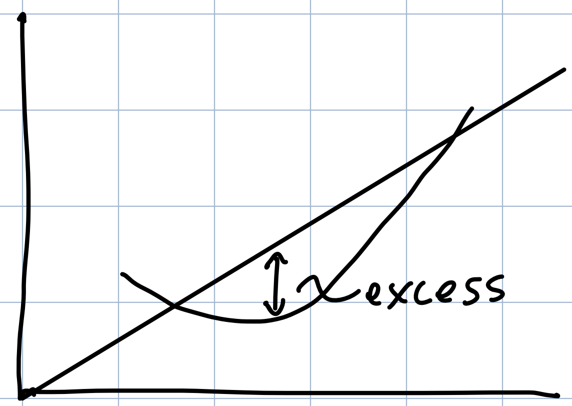
$$R/C_{\gamma max} = V_{\gamma max} \cdot \sin \gamma_{max}$$

$$R/C_{max} \neq R/C_{\gamma max}$$

$$R/C_{max} = \frac{\text{max excess power}}{W}$$



Prop



Jet

R/C$_{max}$ prop ⤳ occurs @ $P_{R\,min}$

AKA $\left(\frac{C_L^{3/2}}{C_D}\right)_{max}$

R/C$_{max}$ jet ⤳ take PA & PR curves subtract them, find the Max

- prop : $R/C = \frac{P_A - P_R}{W} = \left(\frac{T}{W} - \frac{D}{W}\right)V$

because $P_A \approx$ ¢ with V.

then R/C$_{max}$ occurs where

$- \dfrac{C_L^{3/2}}{C_D}$ is maximum

estimate $\gamma_{max} = \dfrac{T}{W} - \dfrac{D}{W}$

$\dfrac{T}{W} \rightsquigarrow \dfrac{P\eta}{V}$

solve for T & D using

$$V_{\gamma max} \sim \frac{4\left(\frac{W}{S}\right) \cdot K}{\rho\, \eta\, P/W}$$

Jet: $R/C = \dfrac{T}{W} - \dfrac{D}{W}$

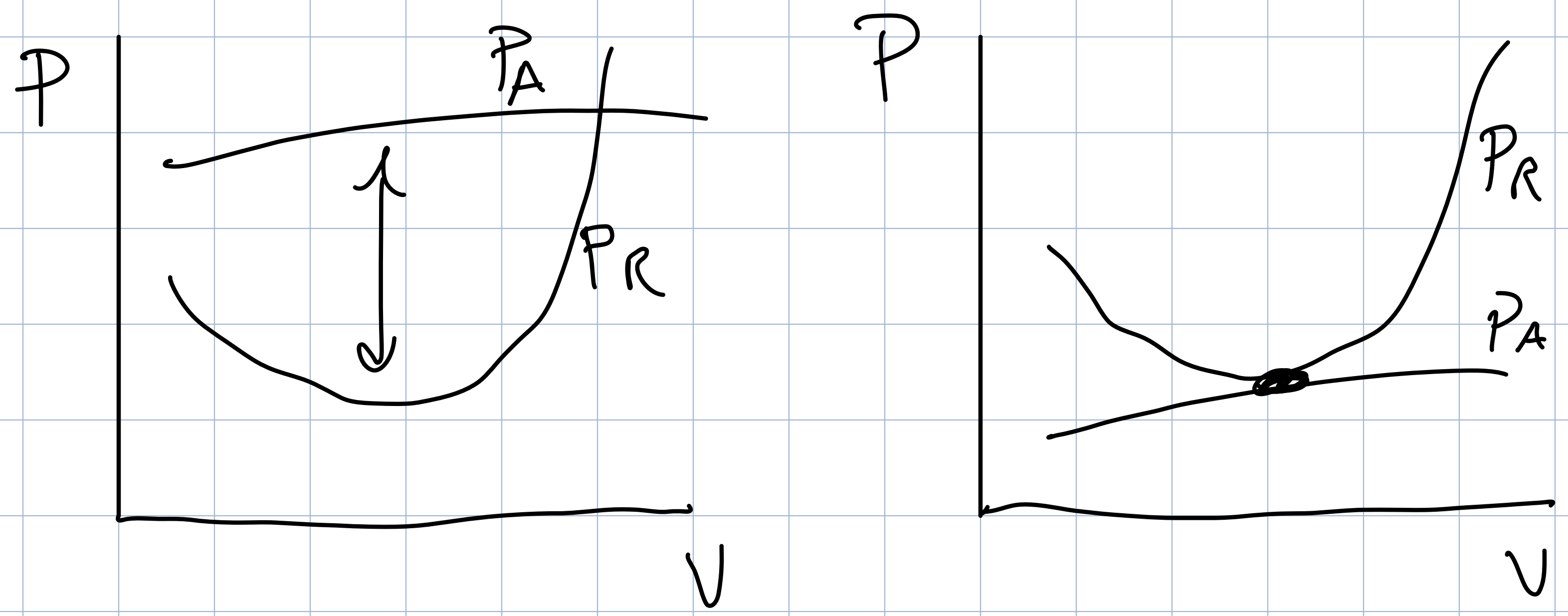
$$\gamma = \frac{T}{W} - \frac{D}{W} \approx \frac{T}{W} - \frac{1}{L/D}$$

because $T_A \simeq ¢$

γ_{max} obtained @ L/D_{max}

R/C_{max} must be obtained via subtraction of P_A & P_R curves a few more R/C concepts

1. ceilings: how high you can fly in SLF



- altitude where $R/C_{max} \to 0$

absolute ceiling

$$P_A = P_{R_{min}}$$

- Service ceiling ⇝ altitude where $R/C_{max} \approx 100 \frac{ft}{min}$

2. Time to climb

- time to Reach altitude

$$R/C = \frac{dh}{dt}$$

$$dt = \frac{dh}{R/C}$$

time to climb : $\int_{h_1}^{h_2} \frac{dh}{R/C}$

h (ft)	$(R/C)_{max}$ (ft/s)	$V_{(R/C)_{max}}$ (ft/s)
0	179.9	747.4
10,000	156.6	798.0
20,000	133.8	858.3
30,000	111.0	931.9
40,000	85.9	1,033.4
50,000	58.2	1,176.6
60,000	30.1	1,358.7

Estimate absolute ceiling