MAE 158: Aricraft Performance
Recommended Homework #7

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The Bede BD-5J is a very small single-seat home-built jet airplane which became available in the early 1970s. The data for the BD-5J are as follows

- Wing span: 17 ft
- Wing planform area: 37.8 ft^2
- Gross weight at takeoff: 960 lb
- Fuel capacity: 55 gal
- Power plant: one French-built Microturbo TRS 18 turbojet engine with maximum thrust at sea level of 202 lb and a specific fuel consumption of 1.3 lb/(lb · h)

We will approximate the drag polar for this airplane by

$$C_D = 0.02 + 0.062 C_L^2$$

- **5.11 (Anderson)** Consider the BD-5J flying at 10,000 ft. Assume a sudden and total loss of engine thrust. Calculate (a) the minimum glide path angle, (b) the maximum range covered over the ground during the glide, and (c) the corresponding equilibrium glide velocities at 10,000 ft and at sea level.

  (a) The maximum lift to drag ratio is obtained when the parasite drag equals the induced drag and has a value of

  $$\left(\frac{L}{D}\right)_{\max} = \frac{1}{\sqrt{4 C_{D_P} k}} = \frac{1}{\sqrt{4 \cdot 0.02 \cdot 0.062}} = 14.2$$

  The minimum gliding angle will thus be

  $$\theta_{\min} = \arctan\left(\frac{1}{(L/D)_{\max}}\right) = \arctan(1/14.2) = 4.03^{\circ} \tag{1}$$

  (b) The maximum range will be

  $$R_{\max} = \frac{h}{\tan\theta_{\min}} = \frac{10000\,\mathrm{ft}}{\tan 4.03^{\circ}} = 141938\,\mathrm{ft} = 26.88\,\mathrm{mi} = 23.36\,\mathrm{NM}$$

  (c) Density at 10000 ft is $\rho_{10000\,\mathrm{ft}} = 1.7556 \cdot 10^{-3}\,\mathrm{slug/ft^3}$. while at sea level, we have $\rho_{\mathrm{SL}} = 2.3769 \cdot 10^{-3}\,\mathrm{slug/ft^3}$. Thus, we will have

  $$V_{(L/D)_{\max},\,10000\,\mathrm{ft}} = \sqrt{\frac{2}{\rho_{10000\,\mathrm{ft}}}\sqrt{\frac{k}{C_{D_P}}}\frac{W}{S}} = $$
  $$= \sqrt{\frac{2}{1.7556 \cdot 10^{-3}\,\mathrm{slug/ft^3}}\sqrt{\frac{0.062}{0.02}}\frac{960\,\mathrm{lb}}{37.8\,\mathrm{ft}^2}} = 225.7\,\mathrm{ft/s}$$

and

$$V_{(L/D)_{\max},\mathrm{SL}} = \sqrt{\frac{2}{\rho_{\mathrm{SL}}}\sqrt{\frac{k}{C_{D_P}}}\frac{W}{S}} =$$

$$= \sqrt{\frac{2}{2.3769\cdot 10^{-3}\,\mathrm{slug/ft^3}}\sqrt{\frac{0.062}{0.02}}\frac{960\,\mathrm{lb}}{37.8\,\mathrm{ft^2}}} = 194\,\mathrm{ft/s}$$

- **15.3 (Shevell)** A four-engine 747 with a capacity of 365 passengers is cruising at a Mach number of 0.82 at a pressure altitude of 37,000 ft. Outside air temperature is -50ºF. The initial cruise weight was 630,000 lb. According to the pilot's flight plan, he will start his descent at a weight of 488,000 lb. The 747 may be assumed to have a C_{D_P} of 0.0145, an e of 0.86, a wing area of 5500 ft^2, and a wing span of 195.7 ft. The compressibility drag coefficient, ΔC_{D_C}, at the average cruise weight is 0.0010. The JT9D-7 high bypass ratio turbofans have an installed specific fuel consumption at cruise of 0.65 lb/lb-h. Determine:

  (a) Distance covered at cruise altitude (assume conditions at average cruise weight can be considered as the average for the flight)

  (b) Required engine thrust, per engine, at the average cruise weight.

  (c) Average cruise fuel flow in gallons per hour (kerosene fuel weights 6.7 lb/gal).

  (d) Seat-miles produced per gallon.

  (e) Compare the 747 seat-miles/gal with a five passenger automobile having a fuel consumption of 25 mi/gal.

  (a) From the pressure altitude, we get $p_0 = 153.86$ lb/ft^2. Using $T_0 = -50$ºF and $M = 0.82$, we have $q = \frac{\gamma}{2}pM^2 = 213.62$ lb/ft^2. The aspect ratio is $\mathrm{AR} = b^2/S = 195.7^2/5500 = 6.963$.
  To find the lift coefficient, we will use the average weight $W_{\mathrm{av}} = 0.5\,(W_i + W_f) = 0.5\,(488000 + 630000) = 559000\,\mathrm{lb}$:

$$C_L = \frac{W_{\mathrm{av}}}{qS_{\mathrm{ref}}} = \frac{559000\,\mathrm{lb}}{213.62\,\mathrm{lb/ft^2}\cdot 5500\,\mathrm{ft^2}} = 0.476$$

  The induced drag coefficient is

$$C_{D_i} = \frac{C_L^2}{\pi\,\mathrm{AR}\,e} = \frac{0.476^2}{\pi\,6.963\cdot 0.86} = 0.012$$

  The total drag is

$$C_D = C_{D_P} + C_{D_i} + \Delta C_{D_C} = 0.0145 + 0.012 + 0.001 = 0.0275$$

  The ratio L/D is

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{0.476}{0.0275} = 17.31$$

  and the velocity is

$$V = M\sqrt{\gamma RT} = 0.82\sqrt{1.4\cdot 1718\,\mathrm{lb\ ft/slug\ ºR}\cdot 410\,\mathrm{ºR}}\,\frac{1\,\mathrm{kt}}{1.69\,\mathrm{ft/s}} = 481.8\,\mathrm{kt}$$

  All in all, for the jet

$$R = \frac{V}{c}\frac{L}{D}\ln\left(\frac{W_i}{W_f}\right) = \frac{481.8\,\mathrm{kt}}{0.65\,\mathrm{lb/lb\ h}}\,17.31\,\ln\left(\frac{630000\,\mathrm{lb}}{488000\,\mathrm{lb}}\right) = 3275\,\mathrm{NM}$$

(b) We have

$$T = D = C_D q S = 0.0275 \cdot 213.62\,\text{lb/ft}^2 \cdot 5500\,\text{ft}^2 = 32310\,\text{lb}$$

The thrust per engine is then

$$T/e = \frac{32310\,\text{lb}}{4} = 8078\,\text{lb/engine}$$

(c) The fuel flow is

$$\text{Fuel flow} = Tc = 32310\,lb \cdot 0.65\,\text{lb/lb h} = 21002\,\text{lb/h}$$

and so

$$\frac{21002\,\text{lb/h}}{6.7\,\text{lb/gal}} = 3135\,\text{gal/h}$$

(d) The seat miles produced per gallon are

$$\text{seat mi/gallon} = \frac{\text{seats} \cdot V}{\text{gal/h}} = \frac{365\,\text{seats} \cdot 481.8\,\text{NM/h}}{3135\,\text{gal/h}} = 56.09\,\text{NM/gal} = 64.62\,\text{statute mi/gal}$$

(e) For the automobile, we have

$$\text{seat miles/gallon} = \text{seats} \cdot \text{mi/gal} = 5 \cdot 25 = 125\,\text{statute mi/gal}$$

And so, the ratio is

$$\frac{\text{Auto}}{747} = \frac{125\,\text{statute mi/gal}}{64.62\,\text{statute mi/gal}} = 1.93$$

A two-place airplane is flying at a pressure altitude of 4000 ft at a speed of 120 mph. Outside air temperature is 50ºF. The gross weight is 2000 lb. The rectangular wing has an area of 170 ft^2 with a span of 33.25 ft. Wing thickness is 14%. Wing parasite drag is 39% of the total parasite drag; 88% of the wing is exposed. Assuming a propeller (or propulsive) efficiency of 0.84.

- **15.6 (Shevell)** The airplane of Problem 15.2 has an initial cruise weight of 2250 lb and will start its descent after consuming 430 lb of fuel in cruise. Specific fuel consumption is 0.48 lb/bhp · h. Determine the cruise range and endurance.

The range is defined as

$$R = 325\,\frac{\eta}{c}\frac{L}{D}\,\ln\frac{W_i}{W_f}$$

We have the efficiency $\eta = 0.84$ and the consumption $c = 0.48$ lb/bhp · h. Besides, the initial cruise weight is $W_i = 2250$ lb and its final weight is $W_f = W_i - \Delta W = 2250 - 430 = 1820$ lb. We need the lift coefficient to find the drag foce. We will approximate it with the average weight, that is $W_{\text{av}} = 0.5(2250 + 1820) = 2035$ lb:

$$C_L = \frac{W_{AV}}{qS} = \frac{2035\,\text{lb}}{32.31\,\text{lb/ft}^2 \cdot 170\,\text{ft}^2} = 0.3705$$

and so (from problem 15.2, that we did last week)

$$D = \left(C_{D_P} + \frac{C_L^2}{\pi\,\text{AR}\,e}\right) qS = \left(0.02309 + \frac{0.3705^2}{\pi \cdot 6.5 \cdot 0.82}\right) 32.31\,\text{lb/ft}^2 \cdot 170\,\text{ft}^2 = 171.85\,\text{lb}$$

and so, finally (using $L = W_{\mathrm{av}}$)

$$R = 325 \left(\frac{0.84}{0.48\,\mathrm{lb/bhp \cdot h}} \right) \left(\frac{2035\,\mathrm{lb}}{171.85\,\mathrm{lb}} \right) \ln \left(\frac{2250\,\mathrm{lb}}{1820\,\mathrm{lb}} \right) = 1428\,\mathrm{NM}$$

The endurance time is found to be

$$\begin{aligned} t_e &= 37.9\,\frac{\eta}{c}\frac{C_L^{3/2}}{C_D}\sqrt{\frac{\sigma S}{W_i}}\left(\sqrt{\frac{W_i}{W_f}}-1\right) = \\ &= 325\left(\frac{0.84}{0.48\,\mathrm{lb/bhp\cdot h}}\right)\frac{0.3705^{3/2}}{0.031167}\sqrt{\frac{0.8779\cdot 170\,\mathrm{ft}^2}{2250\,\mathrm{lb}}}\left(\sqrt{\frac{2250\,\mathrm{lb}}{1820\,\mathrm{lb}}}-1\right) = 13.8\,h \end{aligned}$$

where $C_D = C_{D_P} + C_L^2/(\pi\,\mathrm{AR}\,e)$ and the density ratio is

$$\sigma = \frac{\rho_h}{\rho_{\mathrm{SL}}} = \frac{0.002086\,\mathrm{slug/ft}^3}{0.002377\,\mathrm{slug/ft}^3} = 0.8779$$

where ρ_h has been taken from problem 15.2 from last week.

- **15.7 (Shevell)** At takeoff from San Francisco, a twin-engine DC-9-30 required a takeoff runway length of 5700 ft. The wing area is 100 ft^2. The airport runways may be assumed to be at sea-level pressure altitude. The temperature was 72°F. The engines have a static ($V = 0$) rating of 14500 lb of thrust each, but lose 14% in thrust at the effective average takeoff speed ($0.7V_{LO}$) due to bleed and power extraction losses and the engine ram drag due to forward speed. For the takeoff, the flap angle is 20 degrees, and the slats are extended. (You may assyme a $C_{L,\max} = 2.53$)

  (a) Determine the takeoff weight

  (b) What is the lift-off speed at $1.2V_S$

  (a) For a takeoff runway length of 5700 ft, we have, from figure 15.29 in the book, that $\frac{W^2}{\sigma S C_{L,\max} T} = 152$ (see figure 1). At sea level, we have $p = 2116\,\mathrm{lb/ft}^2$ and we get a density of $\rho_0 = 0.002315\,\mathrm{slug/ft}^3$ using the equation of state.

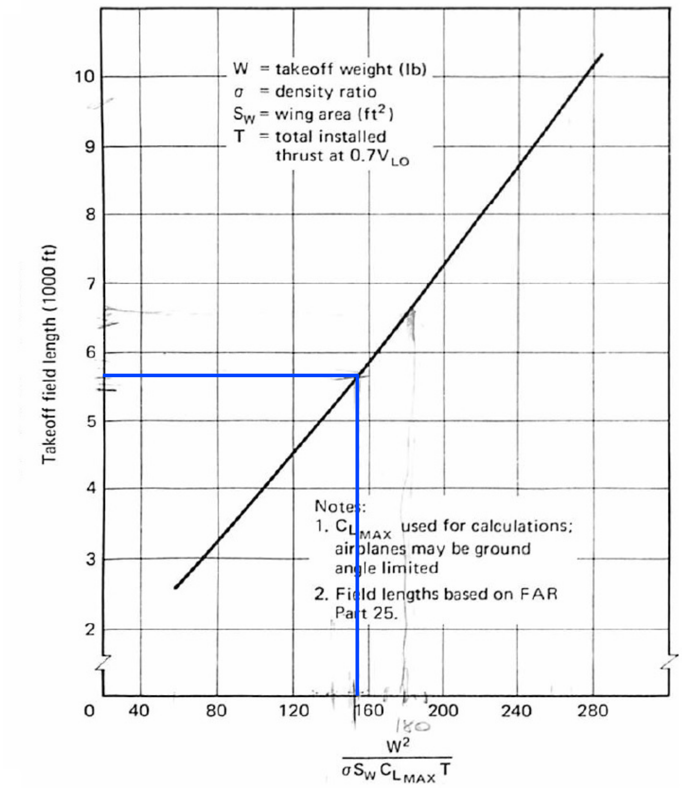

Figure 1: Use of figure 15.29

The density ratio is

$$\sigma = \frac{\rho_0}{\rho_{SL}} = \frac{0.002315\,\mathrm{slug/ft^3}}{0.002377\,\mathrm{slug/ft^3}} = 0.974$$

The maximum lift coefficient is given $C_{L,\max} = 2.53$ but it could just as well be found at figure 14.15 using the 20º flap deflection angle and the fact that the slats are extended. The total thrust is $T_t = 0.86 \cdot 2T = 0.86 \cdot 2 \cdot 14500 = 24940\,\mathrm{lb}$. Finally,

$$W = \sqrt{152\sigma S C_{L,\max} T} = \sqrt{152 \cdot 0.974 \cdot 1000\,\mathrm{ft}^2 \cdot 2.53 \cdot 24940\,\mathrm{lb}} = 96652\,\mathrm{lb}$$

(b) The lift-off speped is

$$V_{TO} = 1.2\sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\frac{1}{C_{L,\max}}} = 1.2\sqrt{\frac{2}{0.002315\,\mathrm{slug/ft^3}}\frac{96652\,\mathrm{lb}}{1000\,\mathrm{ft}^2}\frac{1}{2.53}} = 218\,\mathrm{ft}/s = 129\,\mathrm{kt}$$