MAE 158 Lecture 6
Fall 2024

Announcements:
- Matlab + Solidworks guides For drag project posted under "Resources"
- Midterm Thursday Week 5

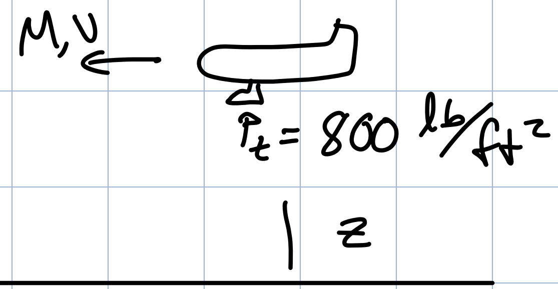
Today's Objectives: Compressibility Drag
Ch. 12

Last Time: Mach # $= M = V/a$
& $a = \sqrt{\gamma R T}$

$M > 0.3$
$\rightarrow$ compressible

$$M_1 = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P_T}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

Ex: Determine the true airspeed in compressible conditions



→ pitot tube measures $P_t = 800$ lb/ft^2

- atmospheric conditions are such that "pressure altitude" = $h_p = 35{,}000$ ft

& temperature is 10 °F above standard

- "pressure altitude" → it is the altitude at the pressure that is the same as the local pressure @ that altitude in standard conditions

TABLE A.2 CHARACTERISTICS OF THE STANDARD ATMOSPHERE (ENGLISH UNITS)

Altitude, ft	Temperature, T, °R	Pressure, p, lb/ft^2	Density ρ, lb s^2/ft^4 (slugs/ft^3)	Speed of sound, ft/s	Kinematic viscosity, ft^2/s
34,000	397.64	523.47	7.6696	977.52	3.9348
35,000	394.08	499.34	7.3820	973.14	4.0575
36,000	390.53	476.12	7.1028^{-4}	968.75	4.1852^{-4}
37,000	389.99	453.86	6.7800	968.08	4.3794

$$P_1 = 499.34 \text{ lb/ft}^2$$

$$T = 394.08\,°R + 10\,°F = 404.08\,°R$$

→ if you need ρ

→ $P = \rho R T$

get true airspeed

$$M_1 = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_T}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

γ: 1.4

P_T: 800 lb/ft²

P_1: 499.34 lb/ft²

$= 0.85$

$$V = M\sqrt{\gamma R T}$$

γ: 1.4

R: $\left(1718 \frac{ft\,lb}{slug\,R}\right)$

T: 404 °R

$= 840$ ft/s

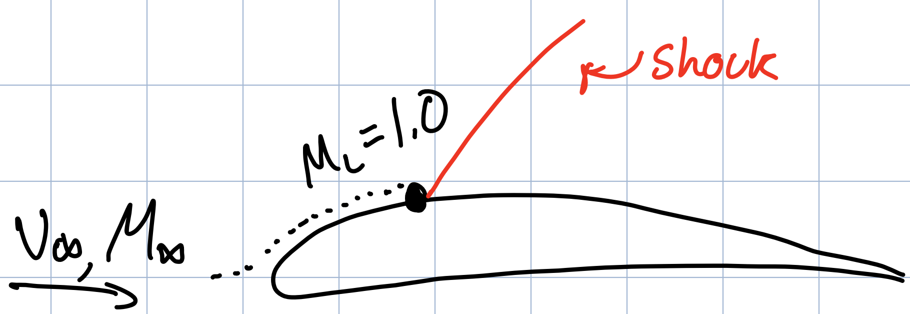
Compressibility Drag Ch. 12

@ low speeds ($M < 0.3$)

aircraft drag vs. C_L is invariant with Mach #

as Mach # ↑ → drag increases due to compressibility

$\Delta C_{D,c}$ → incremental drag increase due to compressibility



$M_\infty < 1.0$ → get energy loss behind shock → early separation etc. . . .

↪ drag

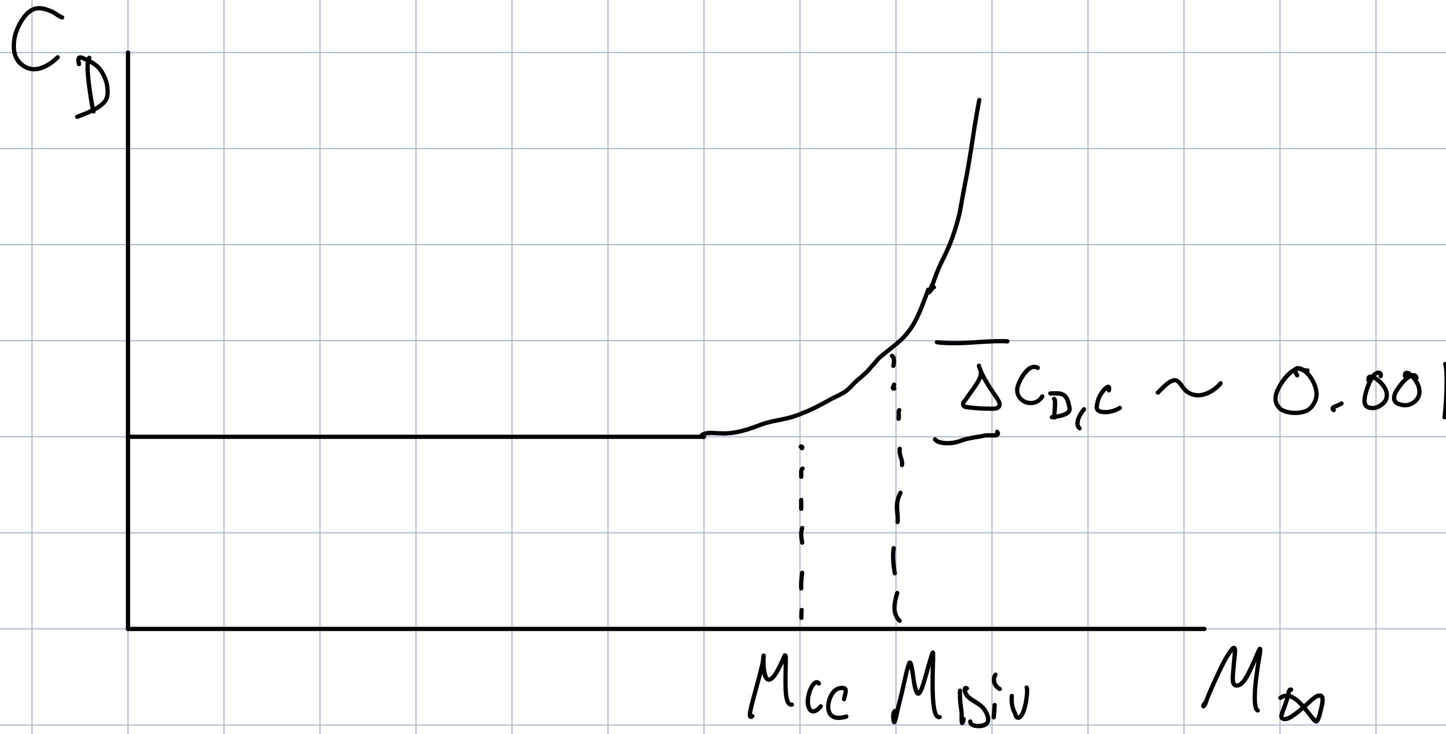
- estimate $\Delta C_{D,c}$

M_{cc} ≡ crest critical Mach # → freestream Mach # where $M_L = 1.0$ @ crest of airfoil

M_{DIV} ≡ Drag divergence Mach #

M_{DIV} is the freestream Mach # where an abrupt drag rise occurs

↳ M_{DIV} where the drag coefficient of the whole airplane increases ~ 0.0010



$M_{cc} \sim$ 2–4% less than M_{Div}

- how to estimate M_{cc} for civil transport aircraft
  ↳ estimate $C_{p,crest}$ such that local flow conditions → $M = 1.0$ @ the crest → M_{cc}

$$C_p = \frac{P - P_\infty}{q_\infty} \Rightarrow \frac{P_\infty}{q_\infty}\left(\frac{P}{P_\infty} - 1\right)$$

↑ pressure coefficient

$$q_\infty = \tfrac{1}{2}\rho_\infty V_\infty^2 = \tfrac{\gamma}{2} P_\infty M_\infty^2$$

$(\ )_\infty \equiv$ free stream properties
$(\ ) \equiv$ local properties on the airfoil

$$\frac{P_t}{P_\infty} = \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\gamma/\gamma-1} \qquad \text{(lecture 5)}$$

$$\frac{P_t}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1}$$

$$\hookrightarrow \quad \frac{P}{P_\infty} = \frac{\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\gamma/\gamma-1}}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1}}$$

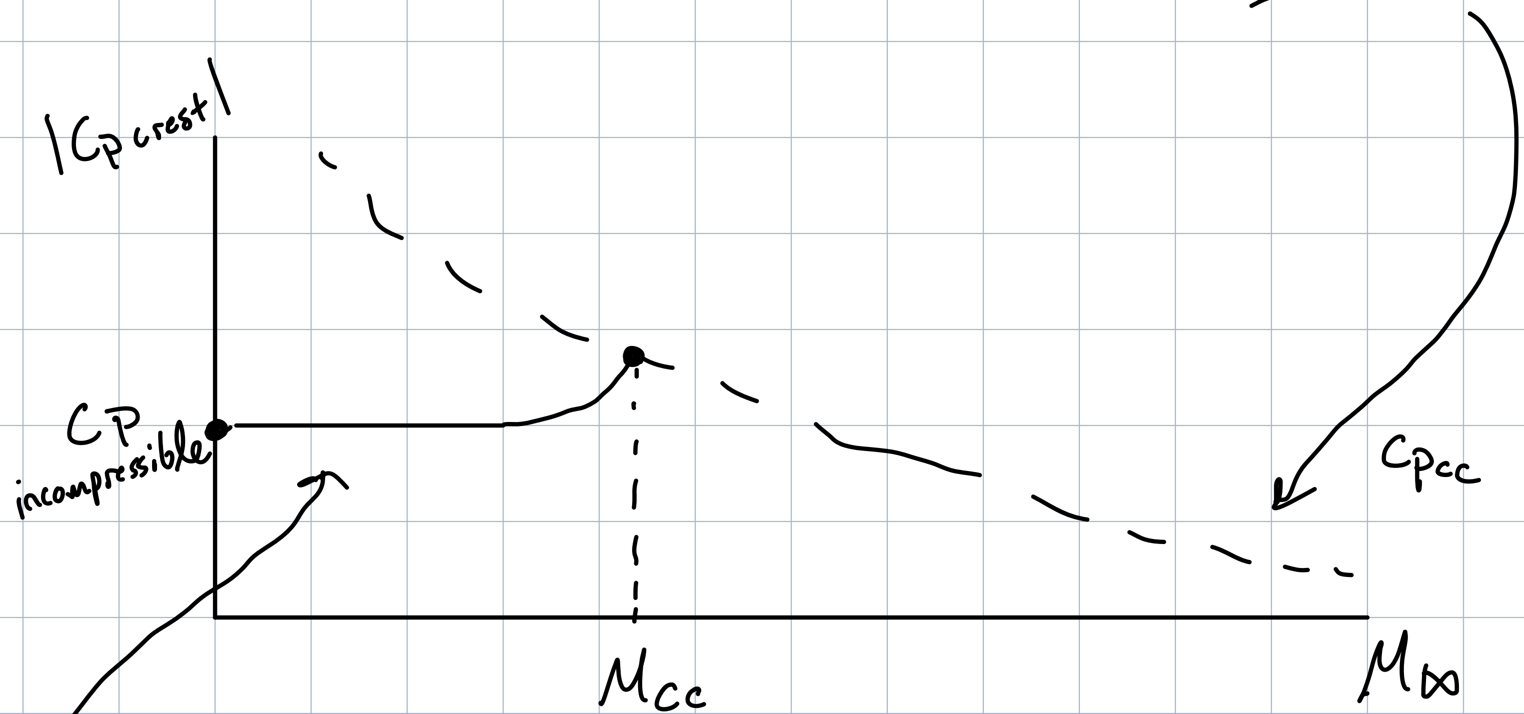
can plug into C_p equation

$$C_p = \frac{P_\infty}{q_\infty}\left(\frac{P}{P_\infty} - 1\right) = \frac{P_\infty}{\frac{\gamma}{2} P_\infty M_\infty^2}\left(\qquad -1\right)$$

$\rightarrow$ Relationship between C_p & M

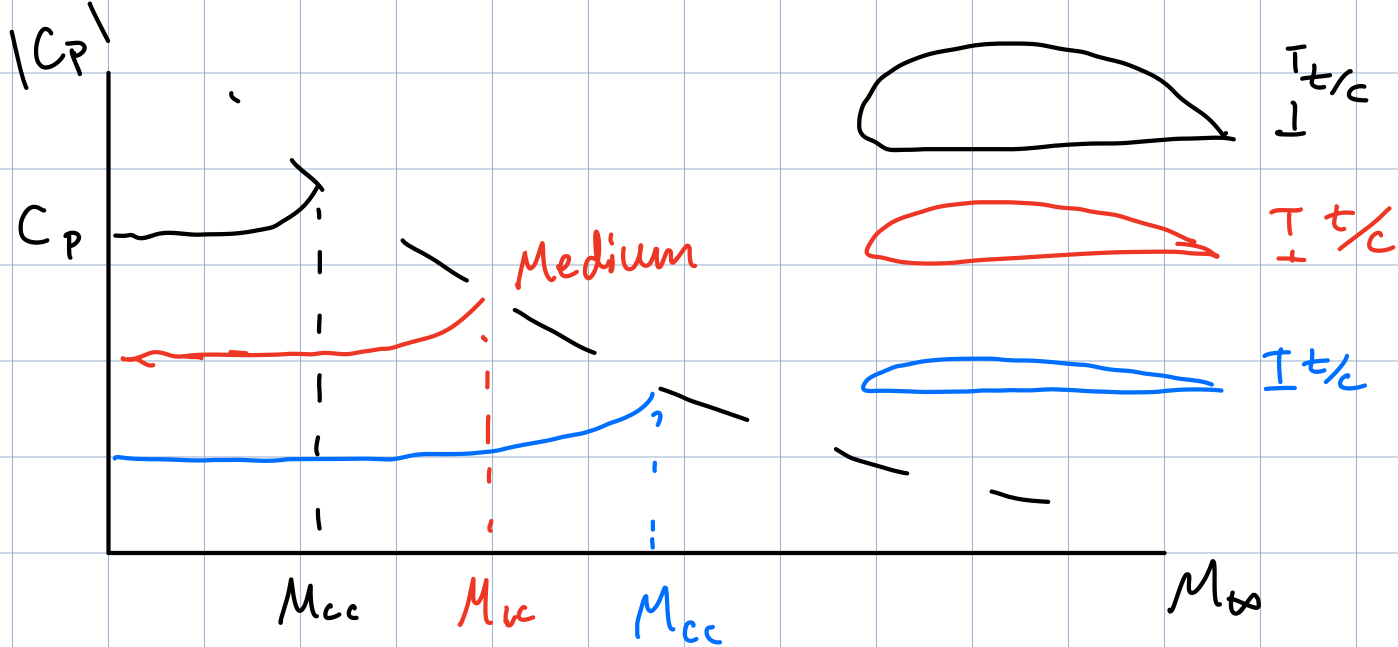
if you want C_p on crest such that $M = 1.0$ $\rightarrow$ plug in $M = 1.0$ into this equation

$$C_{p_{cc}} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right)^{\gamma/\gamma-1} - 1\right] \ast$$



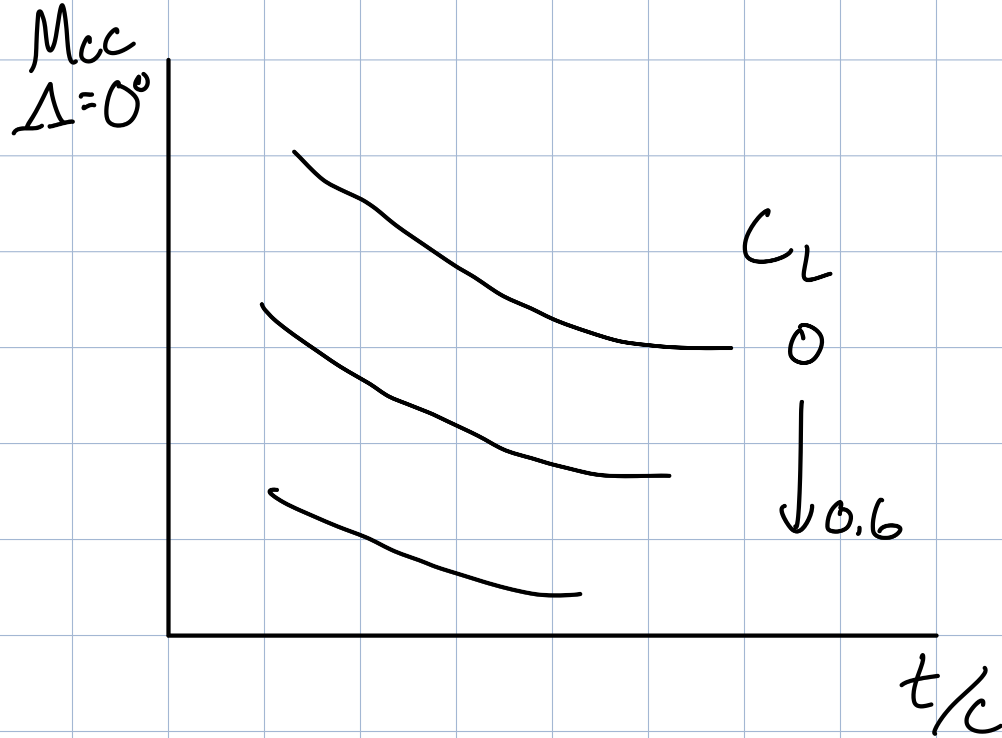
prandtl - glauert Relation

$$C_{p\,compressible} = \frac{C_{p\,incompressible}}{\sqrt{1 - M_\infty^2}}$$

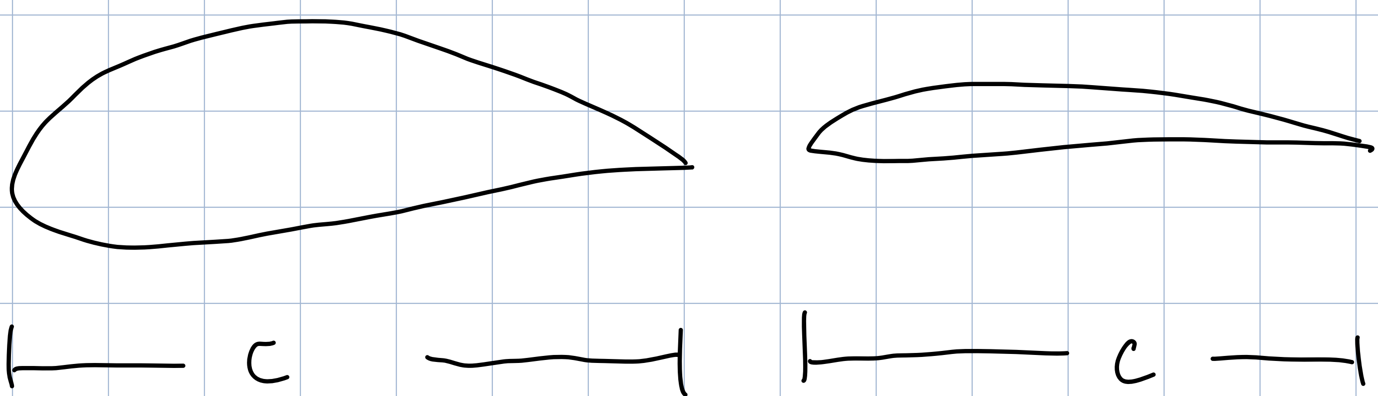


$t/c \rightarrow$ thickness to chord Ratio of the airfoil

Shevell Fig 12.7

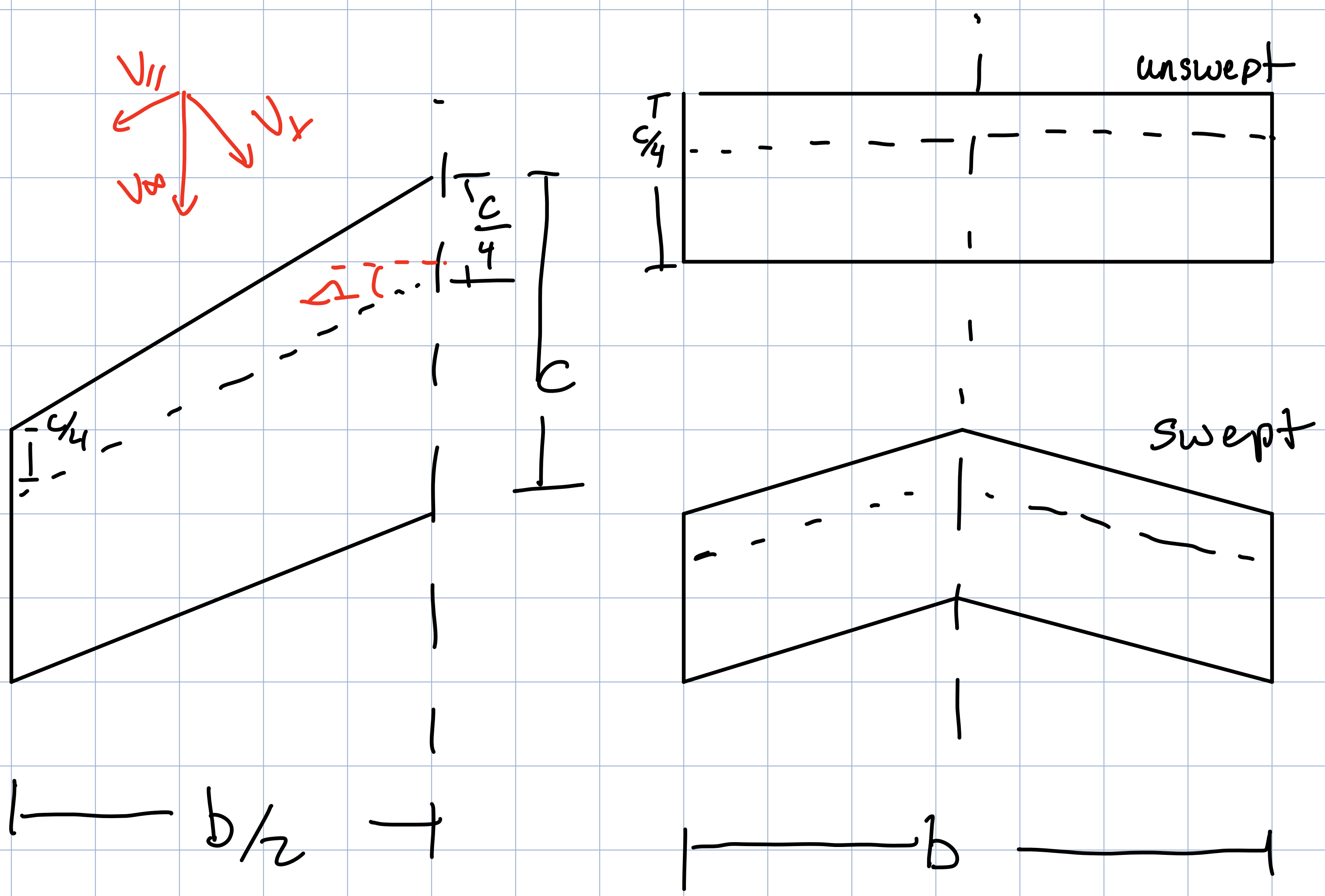


Which airfoil has a higher M_{cc} for a given C_L?



increase M_{cc} with Sweep

Sweep theory : Λ = sweep angle



wing "sees" effective values

$$V_{\infty\,\text{effective}} = V_\infty \cos\Lambda$$

M_∞ effective $= M_\infty \cos\Lambda$

$$M_{cc_\Lambda} = \frac{M_{cc_{\Lambda=0°}}}{\cos\Lambda}$$

theoretical case

because q effective $< q_\infty$

↳ Need higher C_L, C_D

↳ effectively lowers M_{cc_Λ} by a bit...

overall effect is sweep ↑ M_{cc}

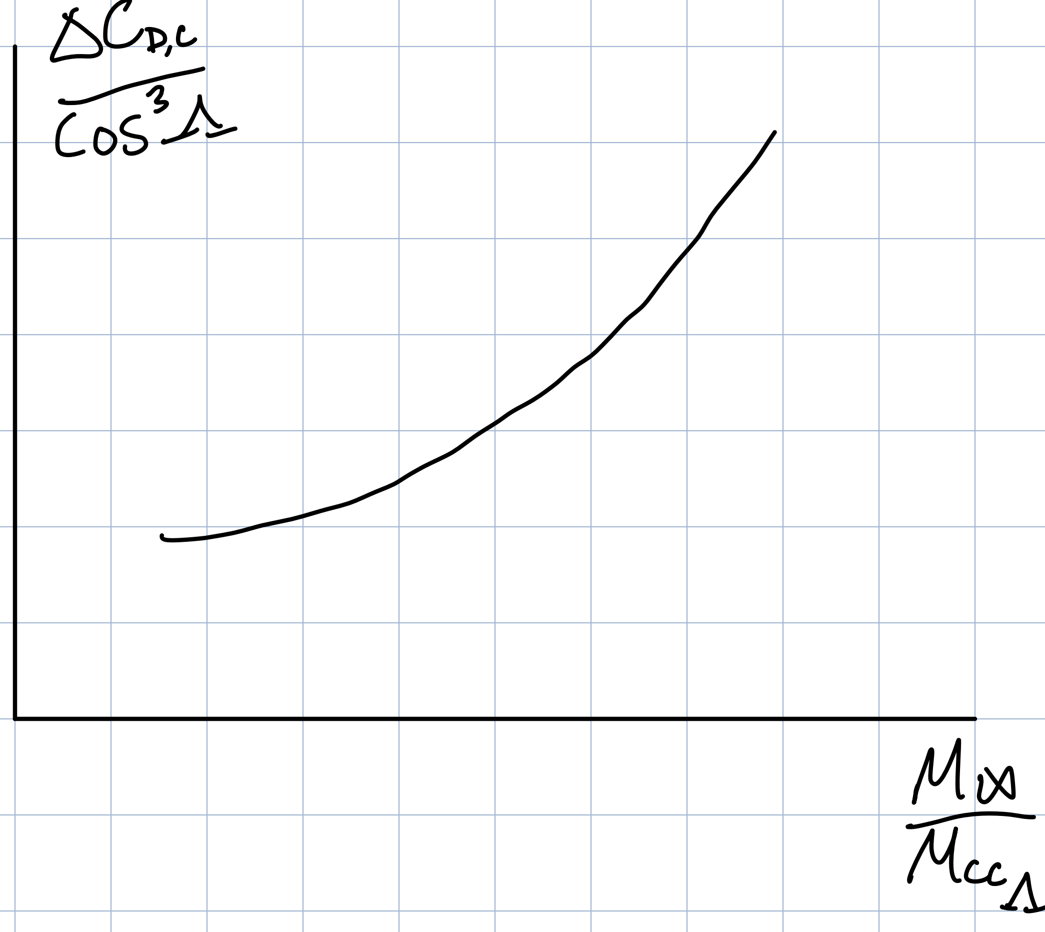
$$M_{cc_\Lambda} = \frac{M_{cc_{\Lambda=0°}}}{\cos^m\Lambda}$$

account for lift effects

Shevell
Fig 12.9

use data to estimate ΔC_{DC}
given M_{cc_Λ} & Λ

Fig
12.13
Shevell



$$C_{D_{total}} = \underbrace{C_{D,p} + C_{D_i}}_{\text{incompressible}} + \underbrace{\Delta C_{D,C}}_{\text{compressible}}$$

$\rightarrow M_\infty \sim> 0.8 \sim 0.85$

commercial aircraft

subsonic

What about Supersonic cases?

Wave drag

as $M_\infty > M_{DIV}$

compressibility drag ↑↑