MAE 158: Aricraft Performance
Recommended Homework #4

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- **7.1** An airfoil is moving through the air at 540 mph at 15,000-ft pressure altitude. The temperature is 45ºF. At a point on the airfoil surface, the local velocity is 630 mph. What is the pressure at that point? If this is the average pressure on the surface, how much lift per square foot is being provided by the top surface? If the average speed on the lower surface is 195 mph, what is the pressure and average lift per square foot on the lower surface? What is the total lift per square foot of wing area?

  (a) Solve using *incompressible* equations.
  We take the pressure from table A.2 $p_0 = 1194.8$ lb/ft^2. With $T = 45$ºF$= 504.69$ºR, density is obtained using the equation of state

  $$\rho_0 = \frac{p_0}{RT} = \frac{1194.8\,\text{lb/ft}^2}{1718\,\text{ft lb/slug ºR} \cdot 504.69\text{ ºR}} = 1.378 \cdot 10^{-3}\,\text{slug/ft}^3$$

  We then apply Bernoulli's equation with $V_0 = 540\,\text{mph} = 792$ ft/s and $V_u = 630\,\text{mph} = 924$ ft/s:

  $$\begin{aligned} \tfrac{1}{2}\,\rho_0 V_0^2 + p_0 &= \tfrac{1}{2}\,\rho_0 V_u^2 + p_u \\ p_u = p_0 + \tfrac{1}{2}\,\rho_0 \left(V_0^2 - V_u^2\right) &= \\ &= 1194.8\,\text{lb/ft}^2 + \tfrac{1}{2}\,1.378 \cdot 10^{-3}\,\text{slug/ft}^3 (792^2 - 924^2)\,\text{ft}^2/\text{s}^2 \\ &= 1038.7\,\text{lb/ft}^2 \end{aligned}$$

  The lift over square foot that is being provided by the top surface is the difference in pressures

  $$\ell_u = p_0 - p_u = 1194.8 - 1038.7 = 156.1\,\text{lb/ft}^2$$

  In the lower surface, we do the same with $V_\ell = 195\,\text{mph} = 286$ ft/s

  $$\begin{aligned} p_\ell = p_0 + \tfrac{1}{2}\,\rho_0 (V_0^2 - V_\ell^2) &= \\ &= 1194.8\,\text{lb/ft}^2 + \tfrac{1}{2}\,1.378 \cdot 10^{-3}\,\text{slug/ft}^3 (792^2 - 726^2)\,\text{ft}^2/\text{s}^2 \\ &= 1263.8\,\text{lb/ft}^2 \end{aligned}$$

  and so $\ell_\ell = p_\ell - p_0 = 1263.8 - 1194.8 = 69$ lb/ft^2. The total wing lift is the sum $\ell = \ell_u + \ell_\ell = 225.1$ lb/ft^2.

  (b) Solve using *compressible* equations.
  In this case, what is constant along a streamline is the energy, since the flow is adiabatic. We have

  $$c_p T_0 + \tfrac{1}{2}\,V_0^2 = c_p T_u + \tfrac{1}{2}\,V_u^2$$

  with $c_p = 6006$ ft lb/slug ºR the specific heat at constant pressure. Thus

  $$\begin{aligned} T_u = T_0 + \tfrac{1}{2c_p}\left(V_0^2 - V_u^2\right) &= 504.69\text{ºR} + \frac{1}{2 \cdot 6006\,\text{ft lb/slug ºR}} \left(792^2 - 924^2\right)\,\text{ft}^2/\text{s}^2 \\ &= 485.83\,\text{ºR} \end{aligned}$$

Using the isentropic gas law (since the process is both adiabatic and reversible) we can get the pressure by equation (7.2):

$$p_u = p_0 \left(\frac{T_u}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = 1194.8\,\text{lb/ft}^2 \left(\frac{485.83\,^{\text{o}}\text{R}}{504.69\,^{\text{o}}\text{R}}\right)^{3.5} = 1045.7\,\text{lb/ft}^2$$

where γ is the adiabatic index or heat capacity ratio and equals 1.4 for air. The lift per square foot is then

$$\ell_u = p_0 - p_u = 1194.8 - 1045.7 = 149.1\,\text{lb/ft}^2$$

We do the same for the lower surface, obtaining $T_\ell = 513^{\text{o}}$R, $p_\ell = 1265.2$ lb/ft^2 and $\ell_\ell = 70.55$ lb/ft^2. Total lift per surface is then $\ell = 219.7$ lb/ft^2.

(c) What are the percentage differences in the lift value determined by parts (a) and (b)?
We have that the percentage differences are 4.6% for ℓ_u, 2.2% for ℓ_u and 2.5% for ℓ.

(d) Find the local Mach number on the upper surface of the wing.
We will use the temperature in the upper surface

$$M_u = \frac{V_u}{\sqrt{\gamma R T_u}} = \frac{924\,\text{ft/s}}{\sqrt{1.4 \cdot 1718\,\text{lb ft/slug}\,^{\text{o}}\text{R} \cdot 485.83\,^{\text{o}}\text{R}}} = \frac{924\,\text{ft/s}}{1080.9\,\text{ft/s}} = 0.855$$

As it is greater than 0.3, the fluid is compressible.

- **7.2** A Boeing 727 is cruising at its assigned altitude at a Mach number of 0.82. The outside air temperature is 227 K. At a given point on the upper surface of the wing, the pressure is measured at 19,000 N/m^2. The temperature at this point is 216 K. How much lift per square meter (referred to ambient pressure) is provided by the upper surface at this point? What is the assigned pressure altitude? What is the density altitude? What is the true speed of the airplane? (Use SI units, except also give the upper surface lift in lb/ft^2).
To find p_0, we apply the isentropic relation

$$\frac{p_u}{p_0} = \left(\frac{T_u}{T_0}\right)^{\gamma/(\gamma-1)} \qquad \rightarrow \qquad p_0 = p_u \left(\frac{T_0}{T_u}\right)^{3.5} = 19 \cdot 10^3\,\text{N/m}^2 \left(\frac{227}{216}\right)^{3.5} = 22608\,\text{Pa}$$

The lift per square meter from the upper surface is, thus,

$$\ell_u = p_0 - p_u = 22608 - 19000 = 3608\,\text{N/m}^2 = 75.36\,\text{lb/ft}^2$$

To find the assigned pressure altitude that corresponds to 22608 Pa, we interpolate from the data in table A.1: $h_1 = 10800$ m $\leftrightarrow p_1 = 23422$ Pa and $h_2 = 11100$ m $\leftrightarrow p_2 = 22346$ Pa:

$$h_{p_0} = h_1 + (p_0 - p_1) \cdot \frac{h_2 - h_1}{p_2 - p_1} = 10800 + (22608 - 23422) \cdot \frac{11100 - 10800}{22346 - 23422} = 11027\,\text{m}$$

To obtain the density altitude, we first compute the density from the equation of state

$$\rho_0 = \frac{p_0}{RT_0} = \frac{22608\,\text{Pa}}{287.05\,\text{J/(kg K)} \cdot 227\,\text{K}} = 0.3470\,\text{kg/m}^3$$

Again we interpolate using the table ($h_1 = 11100$ m $\leftrightarrow \rho_1 = 0.3593$ kg/m^3, $h_2 = 11400$ m $\leftrightarrow \rho_2 = 0.3428$ kg/m^3)

$$h_{\rho_0} = h_1 + (\rho_0 - \rho_1) \cdot \frac{h_2 - h_1}{\rho_2 - \rho_1} = 11100 + (0.347 - 0.3593) \cdot \frac{11400 - 11100}{0.3428 - 0.3593} = 11324\,\text{m}$$

Finally, we will compute the true airspeed using the Mach number and the speed of sound:

$$\text{TAS} = M_0 \cdot a_0 = M_0\sqrt{\gamma R T_0} = 0.82\sqrt{1.4 \cdot 287\,\text{J}/(\text{kg K}) \cdot 227\,\text{K}} = 247.65\,\text{m/s}$$

- **12.1** An airplane with an unswept, 12% thick wing, a wing planform area of 450 ft^2, a span of 60 ft, and a mean aerodynamic chord (m.a.c) of 8 ft is flying at a density altitude of 28,000 ft at a speed of 400 mph. The ambient temperature is 430ºR. The gross weight is 30,000 lb. The exposed wing area is 80% of the total wing area. The wing parasite drag is 35% of the total parasite drag. The airfoil is a conventional peaky type. Determine

  (a) Lift coefficient.
  The lift force equals the weight and the density is obtained from table A.2 ($\rho_0 =$ 0.00095801 slug/ft^3). Using $V_0 = 400$ mph$= 586.7$ ft/s, we get

$$W = \tfrac{1}{2}\rho_0 V_0^2 C_L S \quad \rightarrow \quad C_L = \frac{W}{\frac{1}{2}\rho_0 V_0^2 S} = \frac{30000\,\text{lb}}{\frac{1}{2}0.00095801\,\text{slug/ft}^3(586.7\,\text{ft/s})^2 \cdot 450\,\text{ft}^2} = 0.404$$

  (e) Crest critical Mach number, M_{cc}
  We go to figure 12.7 (page 194 in Shevell 1st edition, page 199 in 2nd edition), with a thickness ratio of $t/c = 0.12$, we get $M = 0.70$ (see figure 1).

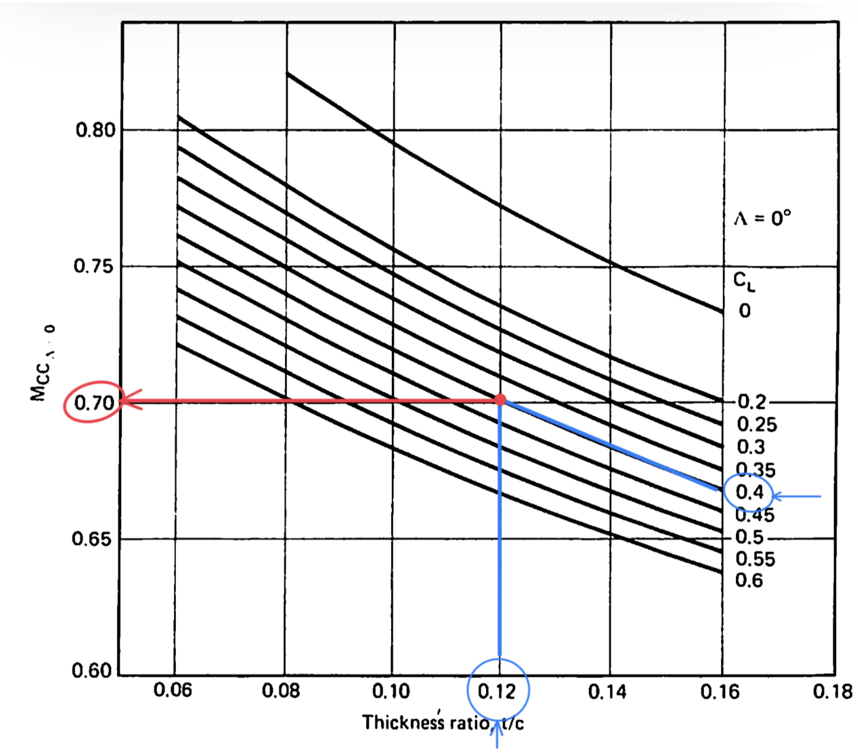

Figure 1: Use of figure 12.7 to find crest critical Mach number

  (f) Compressibility drag
  We first compute the Mach number

$$M_0 = \frac{V_0}{\sqrt{\gamma R T_0}} = \frac{586.7\text{ft/s}}{\sqrt{1.4 \cdot 1718\,\text{lb ft/slug ºR} \cdot 430\,\text{ºR}}} = 0.577$$

which gives a ratio $M_0/M_{cc} = 0.577/0.7 = 0.824$. From figure 12.13 in the book, we find that $\Delta C_{D_C}/\cos^3(\Lambda) = 2{\cdot}10^{-4}$ (see figure 2 below). Since the airplane is unswept, the sweep angle $\Lambda = 0^{\text{o}}$, and so the compressibility drag is simply $\Delta C_{D_C} = 2 \cdot 10^{-4}$

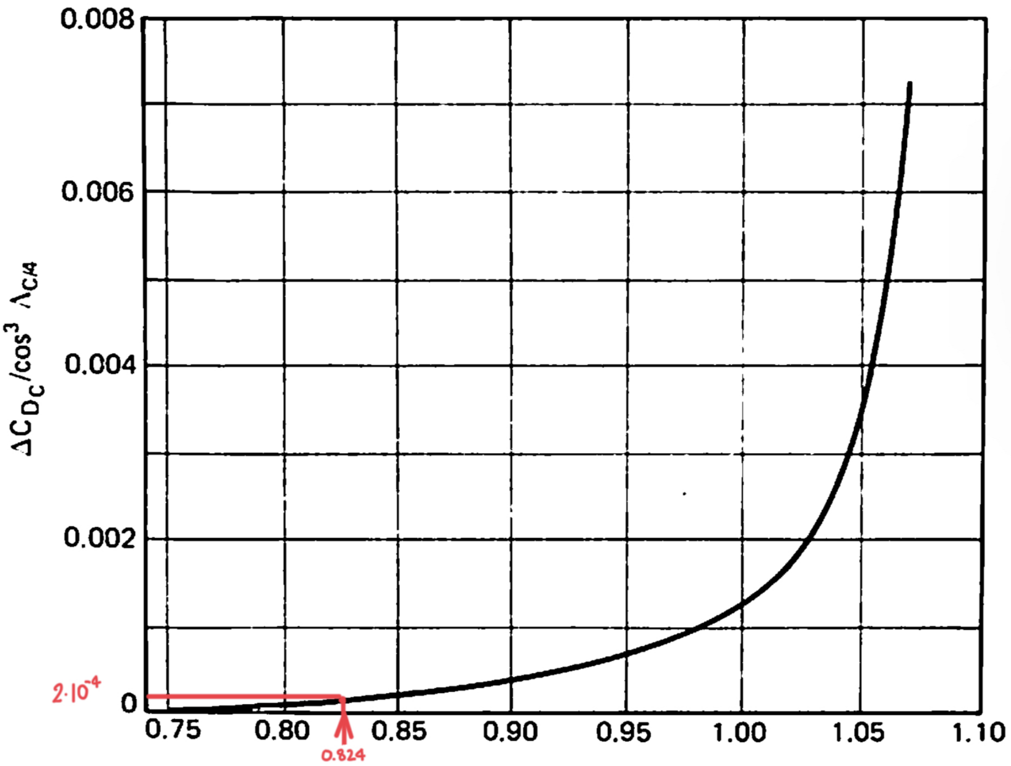

Figure 2: Use of table 12.13 to find $\Delta C_{D_C}/\cos^3 \Lambda$

Finally, we compute the compressibility drag

$$D_{\text{comp}} = \tfrac{1}{2}\,\rho_0 V_0^2 C_{D_C} S = \tfrac{1}{2}\,0.00095801\,\text{slug/ft}^3 \cdot (586.7\,\text{ft/s})^2 \cdot 2 \cdot 10^{-4} \cdot 450\,\text{ft}^2 = 14.84\,\text{lb}$$

- **12.3** A straight wing supersonic fighter with a circular arc airfoil is flying at $M = 2.0$ at 45,000 ft on a standard day. It has a wing area of 320 ft^2 and a span of 36 ft. The total weight is 21,000 lb. Eighty percent of the wing area is exposed. Assume only the exposed wing carries lift supersonically. Neglect tip effects. The total wing wave drag due to thickness and lift is 1630 lb. What is the wing thickness ratio?
  The exposed surface is $S_{\text{exp}} = 320 \cdot 0.8 = 256$ ft^2. We find the pressure from table A.2 to be $p_0 = 309.45$ lb/ft^2. Then, the dynamic pressure will be found as

  $$q_0 = \tfrac{1}{2}\,\rho_0 V_0^2 = \tfrac{1}{2}\,\frac{p_0}{RT_0}\,M^2 \gamma R T_0 = 0.7\,p_0 M^2 = 0.7 \cdot 309.45\,\text{lb/ft} \cdot 2^2 = 866.5\,\text{lb/ft}^2$$

  We compute the lift coefficient

  $$C_L = \frac{W}{q_0 S_{\text{exp}}} = \frac{21000\,\text{lb}}{866.5\,\text{lb/ft}^2 \cdot 256\,\text{ft}^2} = 0.0947$$

  which also equals (eq. 12.12 in the book)

  $$C_L = \frac{4}{\sqrt{M_0^2 - 1}}\,\alpha, \qquad \rightarrow \qquad \alpha = \frac{C_L\sqrt{M_0^2 - 1}}{4} = \frac{\sqrt{3} \cdot 0.0947}{4} = 0.041\,\text{rad}$$

The wave drag coefficient due to lift is (eq. 12.13)

$$C_{D_{\text{wave, lift}}} = \frac{4\alpha^2}{\sqrt{M_0^2 - 1}} = \frac{4 \cdot 0.041^2}{\sqrt{3}} = 0.00388$$

and so the wave drag due to lift is

$$D_{\text{wave, lift}} = qC_{D_{\text{wave, lift}}} S_{\text{exp}} = 866.5\,\text{lb/ft}^2 \cdot 0.00388 \cdot 256\,\text{ft}^2 = 861.14\,\text{lb}$$

The total wave drag, that is the sum of the wave drag due to lift plus the wave drag due to wing thickness, equals 1630 lb, thus the wave drag due to thickness is $D_{\text{wave, t/c}} = 1630 - 861.14 = 768.86$ lb. And so, we can get the wave drag coefficient due to thickness as

$$C_{D_{\text{wave, t/c}}} = \frac{D_{\text{wave, t/c}}}{qS_{\text{exp}}} = \frac{768.86\,\text{lb}}{866.5\,\text{lb/ft}^2 \cdot 256\,\text{ft}^2} = 0.00347$$

which, since it is a circular arc airfoil, equals (eq. 12.15)

$$C_{D_{\text{wave, t/c}}} = \frac{16}{3\sqrt{M_0^2 - 1}} \left(\frac{t}{c}\right)^2$$

and so, the wing thickness ratio is

$$\left(\frac{t}{c}\right) = \sqrt{\frac{C_{D_{\text{wave, t/c}}} \cdot 3\sqrt{M_0^2 - 1}}{16}} = \sqrt{\frac{0.00347 \cdot 3\sqrt{2^2 - 1}}{16}} = 0.0336$$