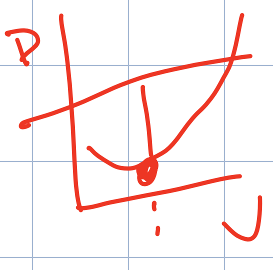
MAE 158 Lecture 11
Nov. 5th 2024

Announcements: Drag project posted

Recommended HW 6 posted

Last Time: R/C_{max} & γ_{max}

- prop: $\underline{R/C} = \dfrac{P_A - P_R}{W} = \left(\dfrac{T}{W} - \dfrac{D}{W}\right)V$



because $P_A \approx$ ¢ w/ V

then R/C_{max} @ condition

where $\underline{C_L^{3/2}/C_D}$ max

$\sim P\eta/V$

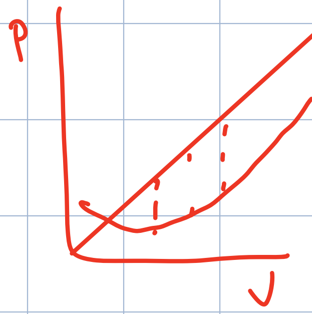
estimate $\gamma_{max} = \dfrac{T}{W} - \dfrac{D}{W}$

where solve for T & D

using $V_{\gamma max} \sim \dfrac{4(W/S)K}{\underline{\rho\,\eta\, P/W}}$

- Jet: $R/C = \left(\dfrac{T}{W} - \dfrac{D}{W}\right)V$

$\gamma = T/W - 1/(L/D)$

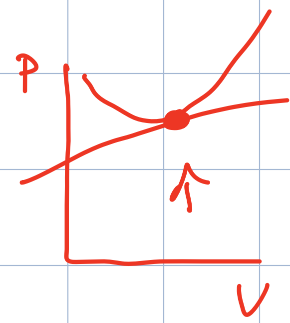


because $T_A \approx$ & w/ V

γ_{max} obtained @ L/D_{max}

R/C_{max} must be obtained via subtraction of P_A & P_R curves

1. Ceilings → how high you can fly



2. Time to climb: $t = \int_{h_1}^{h_2} \frac{dh}{R/C}$

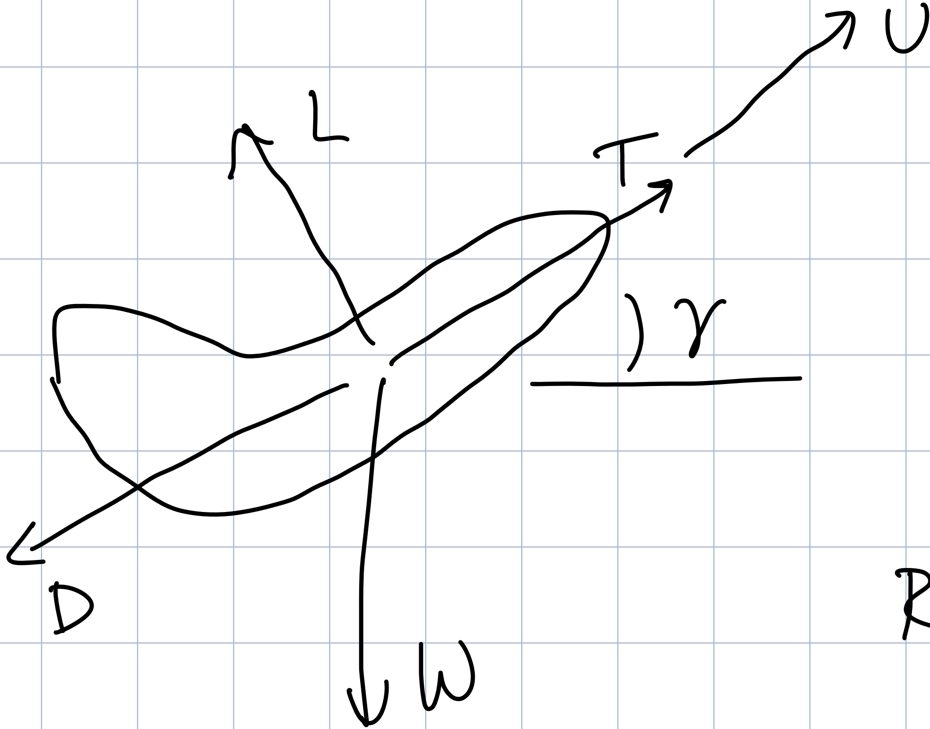
h (ft)	$(R/C)_{max}$ (ft/s)	$V_{(R/C)_{max}}$ (ft/s)
0	179.9	747.4
10,000	156.6	798.0
20,000	133.8	858.3
30,000	111.0	931.9
40,000	85.9	1,033.4
50,000	58.2	1,176.6
60,000	30.1	1,358.7

to climb to 30,000 ft, what is min time? → R/C_{max}

$$t = \int_0^{30000\,ft} \frac{dh}{R/C} \approx \sum_{i=1}^{n} \left(\frac{\Delta h}{R/C}\right)_i$$

$$t = \frac{10k\,ft}{\frac{1}{2}(179.9+156.6)\frac{ft}{s}} + \frac{10k\,ft}{\frac{1}{2}(156.6+133.8)\frac{ft}{s}} + \frac{10k\,ft}{\frac{1}{2}(111.0+133.8)\frac{ft}{s}}$$

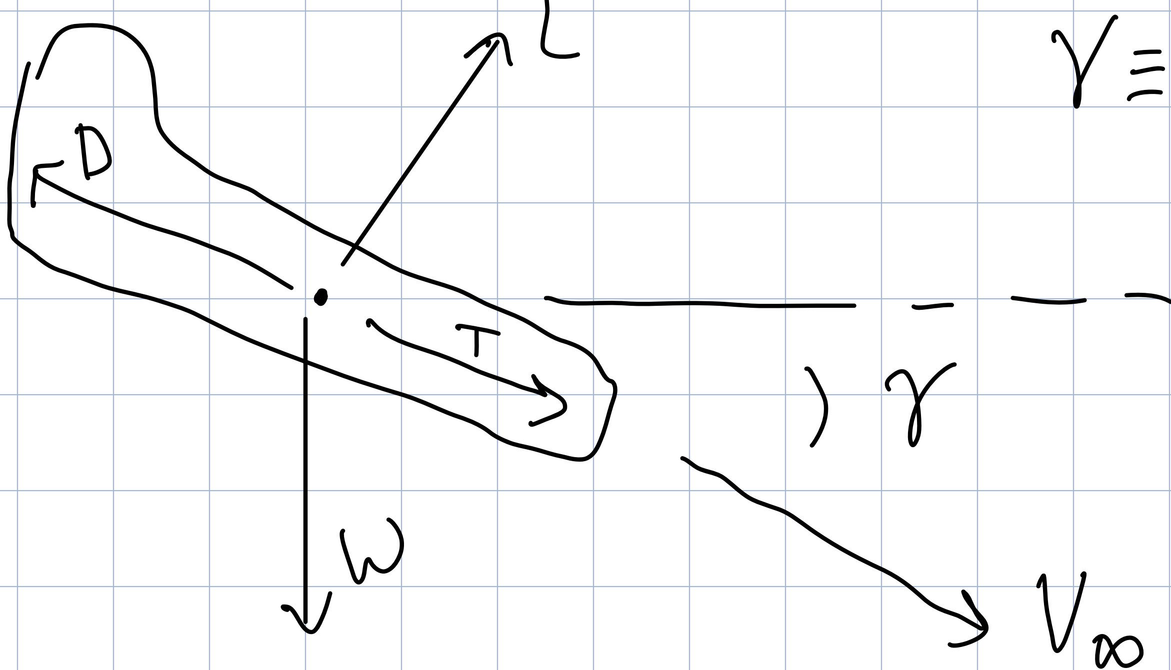
$\approx$ 210s



$$R/C = \frac{P_A - P_R}{W}$$

What happens if $P_R > P_A$?

descending flight!



$\gamma \equiv$ descent angle

"descending @ $3°$"

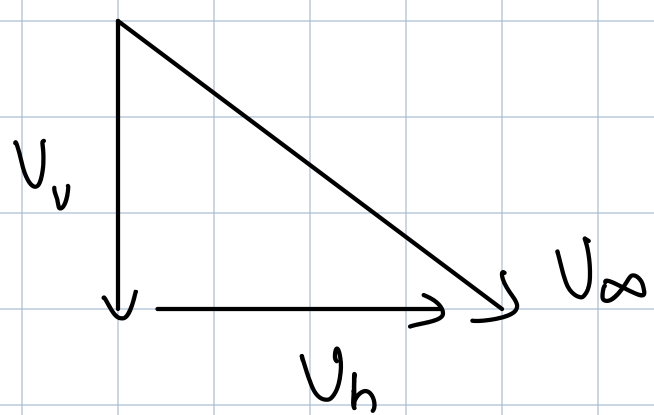
"descending @ $-3°$"

T can be zero

if $T = 0$ ⇝ idle thrust

or Engine is out

-or- don't have engines

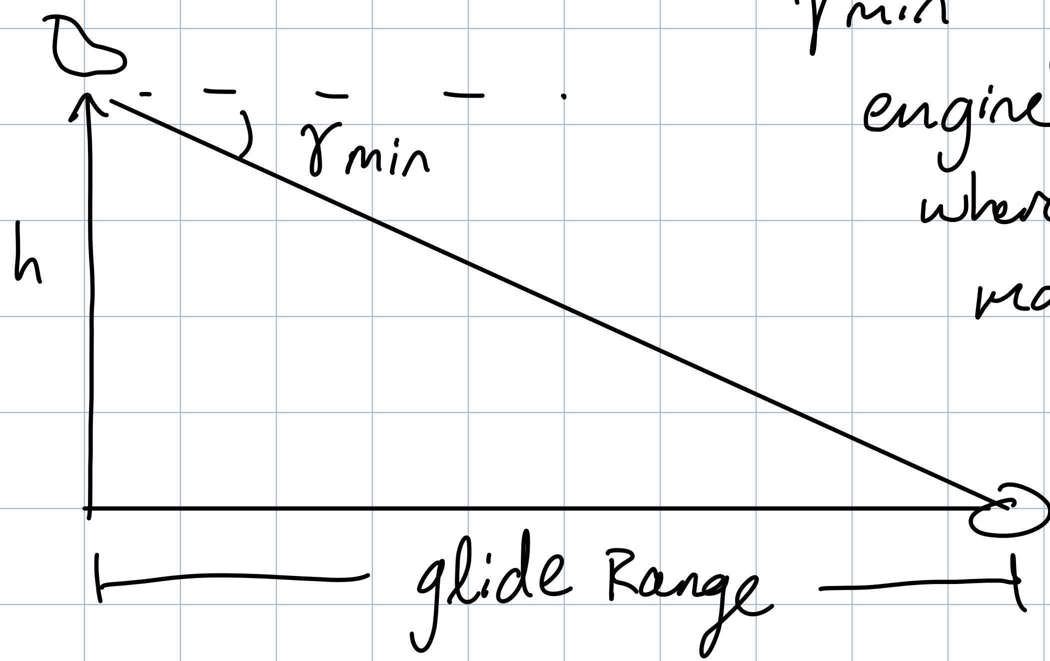


$V_v \equiv$ descent Rate (sink Rate)

for case of $T = 0$

$$\left.\begin{array}{l} L = W\cos\gamma \\ D = W\sin\gamma \end{array}\right\} \tan\gamma = \frac{1}{L/D}$$

γ_{min} occurs @ L/D_{max}



γ_{min} good for engine out cases where you want max glide Range

$\rightarrow$ $V_{L/D_{max}}$ $\rightarrow$ must fly @ this velocity to get to L/D_{max}

What is equation of L/D_{max} a function of ?

$\hookrightarrow$ C_{D_p} & K

$K \rightsquigarrow$ not change w/ altitude

$C_{Dp} \rightsquigarrow$ function of Re $\rightsquigarrow$ small

if neglecting Re effects $\sim$ ¢

γ_{min} function of L/D_{max} only

& $L/D_{max} \simeq$ const w/ altitude

γ_{min} doesn't vary w/ altitude

however - if you want to maintain L/D_{max}, must fly @ $V_{L/D_{max}}$ $\rightsquigarrow$ does depend on altitude

define equilibrium glide velocity

$$L = \tfrac{1}{2}\rho U^2 S C_L = W\cos\gamma$$

$$V = \sqrt{\frac{2\cos\gamma}{\rho C_L}\frac{W}{S}}$$

$\uparrow$ @ L/D_{max}

$\cos\gamma \approx 1$ since γ is small

thus $$V_{L/D_{max}} = \sqrt{\frac{2}{\rho}\left(\sqrt{\frac{K}{C_{D_P}}}\right)\frac{W}{S}}$$ $\rightsquigarrow$ velocity @ γ_{min}

$$= V_{\gamma_{min}}$$

$V_{\gamma_{min}}$ $\to$ does depend on altitude b/c ρ is directly in the equation

Ex: Say A/C that looses all Engine thrust

what is γ_{min}, max glide Range in descent?

& γ_{min} (instantaneous)

A/C: $W = 73{,}000$ lb

$S = 950$ ft^2

$z = 30{,}000$ ft $\rightsquigarrow \rho = 8.9\times10^{-4}$ slug/ft^3

$$C_D = \underbrace{0.015}_{C_{D_p}} + \underbrace{0.08}_{K = \frac{1}{\pi AR e}} C_L^2$$

$e = 0.9$

$$L/D_{max} = \frac{1}{\sqrt{4 C_{D_p} K}}$$

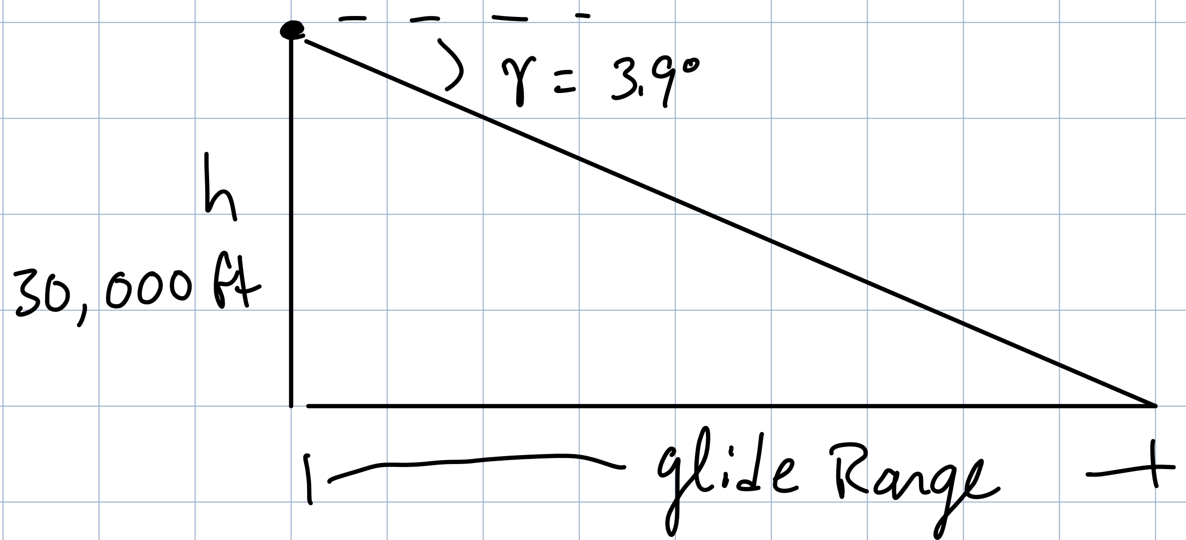
$C_{D_p} = 0.015$, $K = 0.08$

$0.08 = \frac{1}{\pi AR e}$, $e = 0.9$, AR

$$= \frac{1}{\sqrt{4(0.015)(0.08)}} = \underline{14.4}$$

$$\gamma_{min} = \tan^{-1}\left(\frac{1}{14.4}\right) = 3.9°$$

Glide
2. Range max



$$\text{glide Range}_{max} = \frac{h}{\tan(\gamma_{min})} = \underline{432{,}900 \text{ ft}}$$

3. $V_{\gamma min}$ @ altitude of 30,000 ft

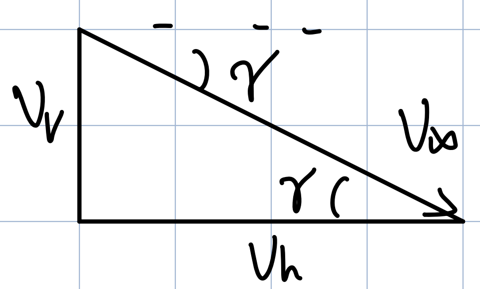
$$V_{\gamma min} = V_{L/D max} = \left(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_P}}}\,\frac{W}{S}\right)^{1/2}$$

$$= \left(\frac{2}{8.9\times10^{-4}\,\frac{\text{slug}}{\text{ft}^3}}\sqrt{\frac{0.08}{0.015}}\,\frac{73000\text{ lb}}{950\text{ ft}^2}\right)^{1/2}$$

$$= 630 \text{ ft/s}$$ $\rightarrow$ equilibrium glide Velocity

$\rightarrow$ V_∞ along the flight path (Not sink Rate!)

Sink Rate:



$$V_v \equiv \text{Sink Rate}$$
$$= V_\infty \sin\gamma$$

for example, if you want sink Rate @ γ_{min}, then:

$$V_{\infty\,\gamma_{min}} \cdot \sin\gamma_{min} = V_{v\,\gamma_{min}}$$

distinguish from min sink Rate

↳ want to maximize the time it takes to get down, different from $V_{v\,\gamma_{min}}$

in general $\quad V_v = V_\infty \sin\gamma \equiv$ sink Rate

note $\quad D = W \sin\gamma \quad$ if $T = 0$

multiply both sides by V_∞

$$V D = V \cdot W \sin\gamma$$

$$= W V_v$$

$$V_v = \frac{D}{W} \cdot V$$ analytically

if small angles, $D \cdot V$ is just Power Req.

$V_{v\,min} \approx$ occurs @ velocity is that needed for $P_{R\,min}$

$$V_{v\,min} = V_\infty \sin\gamma$$

$$= \left(\frac{2}{\rho}\sqrt{\frac{K}{3C_{Dp}}}\,\frac{W}{S}\right)^{1/2} \cdot \underbrace{\sin\gamma}_{1}$$

Note from $\tan\gamma = \frac{1}{L/D}$

then $\sin\gamma = \frac{D}{L}\cos\gamma$

$\frac{D}{L} \leftarrow C_D/C_L$

$\cos\gamma \simeq 1$ thus,

$$V_{v\,min} = \left(\frac{2}{\rho}\sqrt{\frac{K}{3C_{Dp}}}\,\frac{W}{S}\right)^{1/2} \cdot \frac{C_D}{C_L}$$

Note @ min power Required

$$C_{Dp} = \tfrac{1}{3} C_{Di}$$

$\hookrightarrow C_L^{3/2}/C_D$

$$V_{v_{min}} = \sqrt{\frac{2}{\rho}\frac{W}{S}\left(\frac{C_L^{3/2}}{C_D}\right)^{-2}}$$

occurs @ $\frac{C_L^{3/2}}{C_D}_{max}$

Ex: same A/C

$$C_D = 0.015 + 0.08\,C_L^2$$

1. min sink Rate?

2. sink Rate @ γ_{min}?

$$\hookrightarrow \quad V_{v_{\gamma min}} = V_{\infty_{\gamma min}} \cdot \sin\gamma_{min}$$

$$= 630\,\text{ft/s} \cdot \sin(3.9°)$$

$$= 43.6\,\text{ft/s}$$

min sink Rate?

$$V_{min} = \sqrt{\frac{2}{\rho}\frac{W}{S}\left(\frac{C_L^{3/2}}{C_D}\right)^{-2}}$$

$\uparrow$ max

what is $\dfrac{C_L^{3/2}}{C_D}$ max?

power Required lecture

for $\dfrac{C_L^{3/2}}{C_D}$ max $\rightsquigarrow$ $C_{Dp} = \frac{1}{3} C_{Di}$

$$= \frac{1}{3} C_L^2 \cdot K$$

$$C_L @ \frac{C_L^{3/2}}{C_D}\text{max} = \sqrt{\frac{3C_{Dp}}{K}}$$

$$\frac{C_L^{3/2}}{C_D}\text{max} = \frac{\left(\sqrt{\frac{3C_{Dp}}{K}}\right)^{3/2}}{\underset{\uparrow\ 0.015}{C_{Dp}} + \underset{\uparrow\ 0.08}{K}\left(\sqrt{\frac{3C_{Dp}}{K}}\right)^2}$$

$$= 10.8$$

$$V_{v\,min} = \sqrt{\frac{2}{8.9\times10^{-4}\ \frac{slug}{ft^3}} \cdot \frac{73000\,lb}{950\,ft^2}\,(10.8)^{-2}}$$

$$= 38.6\ ft/s$$

Range → distance to fly from one location to another

↳ expending Energy