MAE 158 Lecture 4
Fall 2024

Announcements: Week 2 Quiz
available Fri 12 AM - Mon 11:59 pm
- 1 hour to answer questions & upload any written work
- topics: Lect 1-4 (up to effects of viscosity) & HWs 1,2, HW3 problem 8.2

No in-class lecture thursday Oct 10th (will post lecture video instead)

Today's Objectives:
- Conclude effects of viscosity
- Nature of lift (ch 8 Shevelle)

last time:

$$\text{Drag}_{total} = \tfrac{1}{2}\rho V^2 C_{D_{total}} \cdot S_{REF}$$

$C_{D_{total}}$: Drag coefficient

$$D_f\,(\text{lbs}, N) = \underbrace{\tfrac{1}{2}\rho V^2}_{q} \cdot C_f \cdot S_{wet}$$

C_f: depends on flow Regime

$$\left\{ \; \frac{\text{Drag}_{total}}{q\,S_{REF}} = \frac{D_p}{q\,S_{REF}} + \frac{D_i}{q\,S_{REF}} + \frac{D_{compressibility}}{q\,S_{REF}} \right.$$

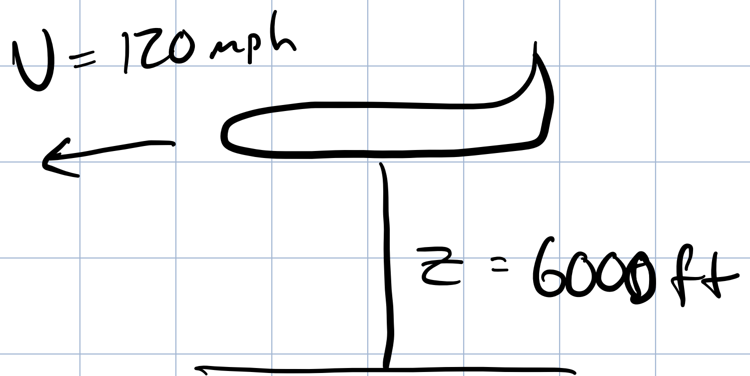
D_p: D_f, D_B

$q\,S_{REF}$

$$C_{D_{total}} = \underbrace{C_{D_f} + C_{D_B}}_{C_{D_p}} + C_{D_i} + C_{D,c} \quad *$$

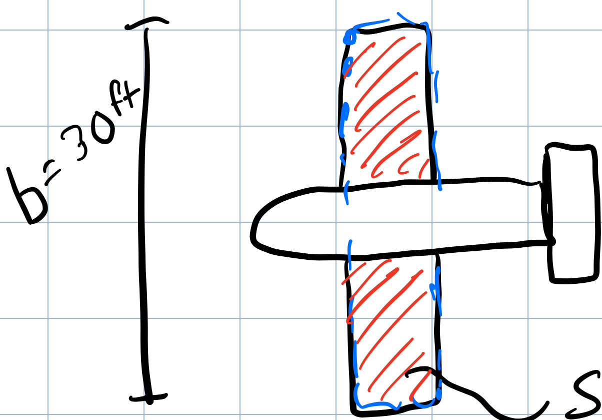
$$C_{D_f} = \frac{S_{wet}}{S_{REF}} \cdot C_f \rightarrow \frac{D_f}{\frac{1}{2}\rho V^2 S_{REF}}$$

Last time



true airspeed
$V = 120$ mph

assume turbulent flow everywhere



$S_{REF} = 160\ ft^2$

assume wing is 85% exposed

$S_{wet} = 277\ ft^2$

$C_{f_{turb}} = 0.00335$

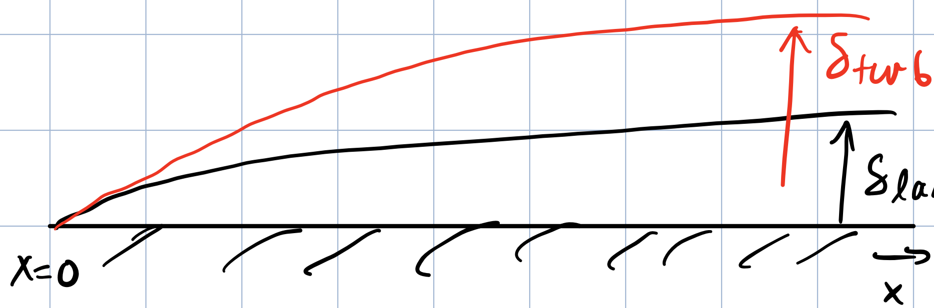
$\underline{D_f = 28.6\ lbs}$

$C_{D_f} = ?$

$$= \frac{S_{wet}}{S_{REF}} \cdot C_f = \frac{277\,ft^2}{160\,ft^2} \cdot 0.00335$$

$$= 0.00580$$

Final thoughts with Boundary layers



$$\delta_{lam} = \frac{5.2x}{(Re_x)^{0.5}} \qquad \delta_{turb} = \frac{0.37x}{(Re_x)^{0.2}}$$

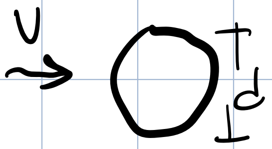
$$\delta_{lam} < \delta_{turb}$$

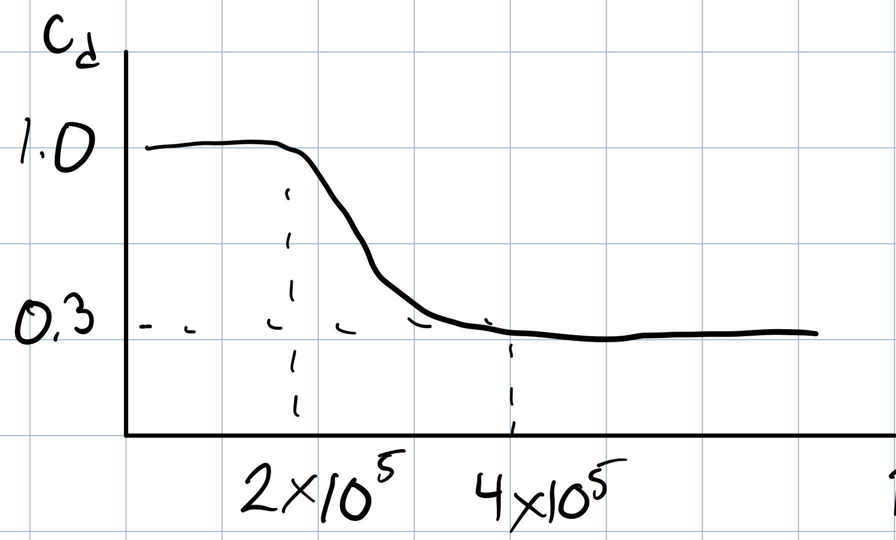
$\delta_{turb}, \delta_{lam} \uparrow$ with x

profile drag of cylinders or spheres ect...

Ch. 10 Shevelle

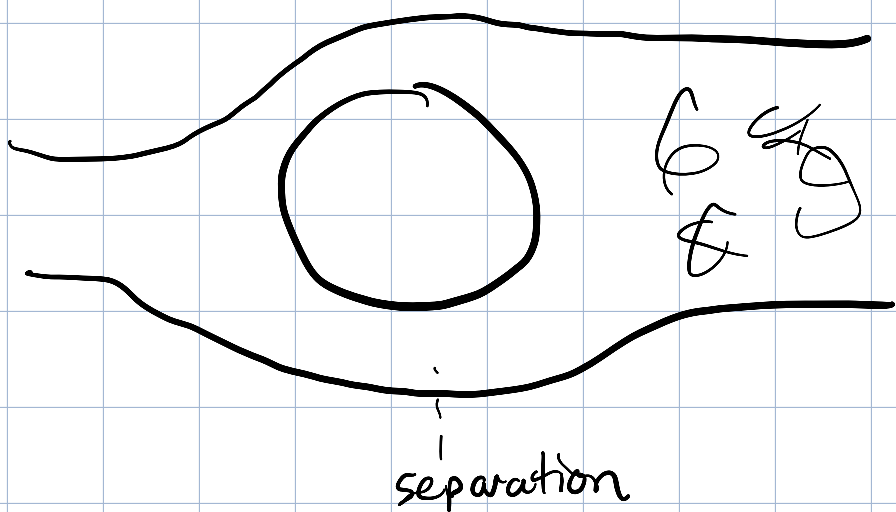
Cylinder



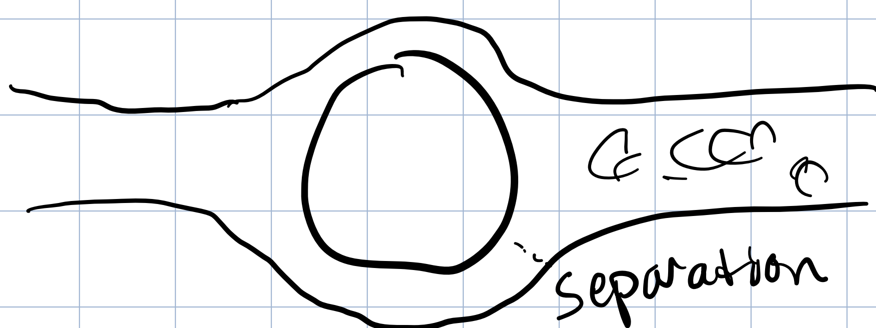


$Re = \frac{\rho V d}{\mu}$

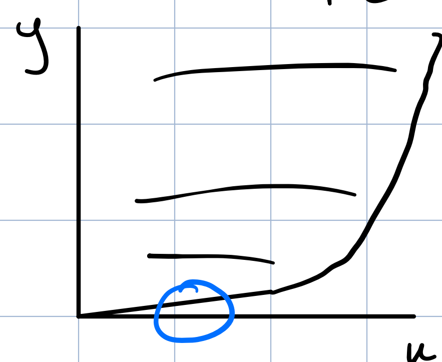
Bluff Bodies → pressure drag

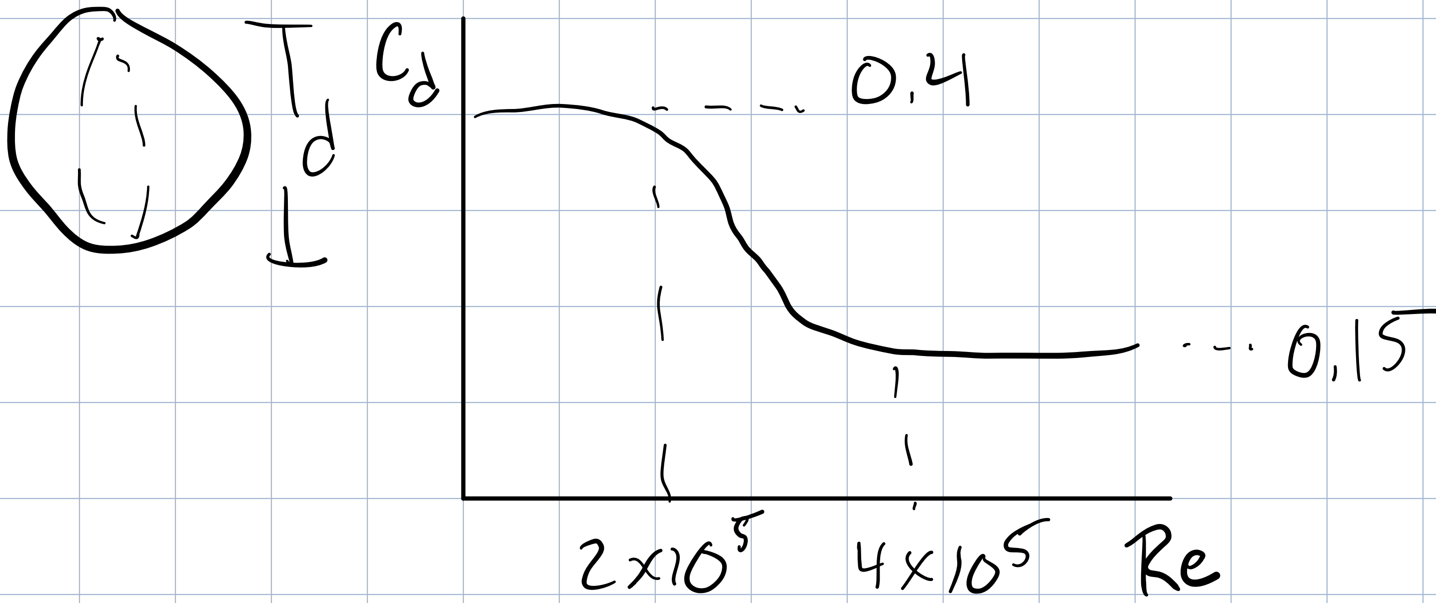


@ laminar Re



@ turbulent Re

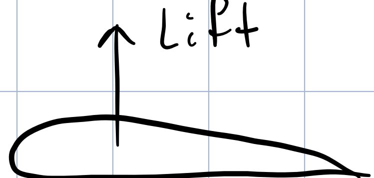




$$D_{\text{sphere/cylinder}} = \tfrac{1}{2}\rho V^2 \underbrace{S}_{\pi (d/2)^2} \cdot C_d$$

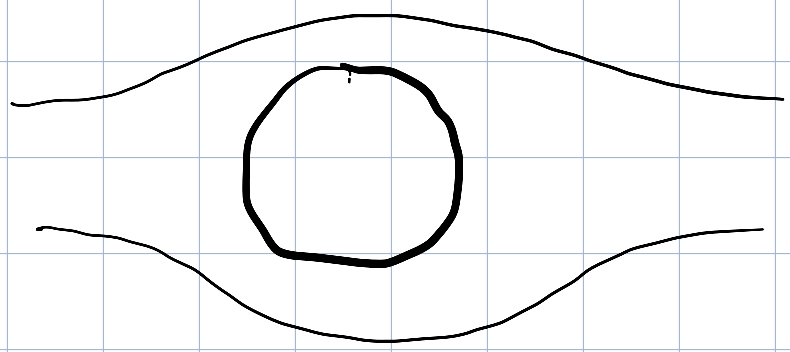
$$D_{total} = D_p + D_i + D_c$$

Induced drag ≡ Drag due to lift



↳ need circulation to get to lift forces

flow around cylinder

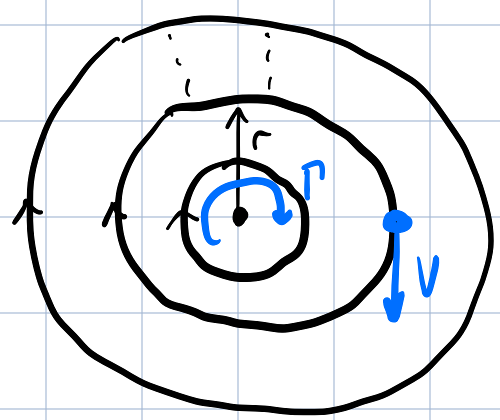


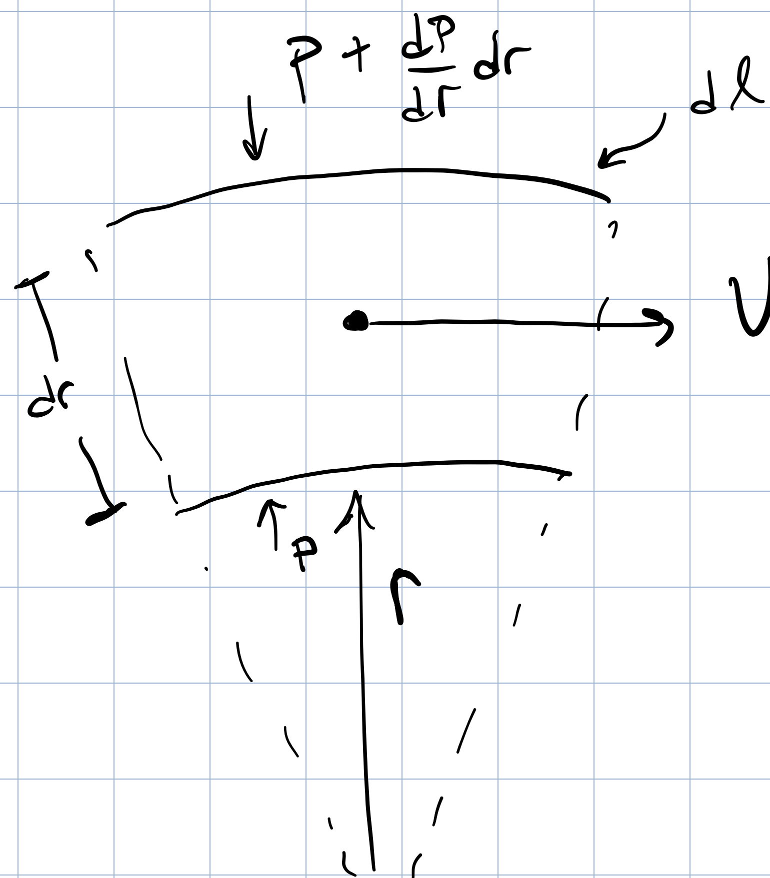
$L = \emptyset$

$D = \emptyset$

inviscid, irrotational

Vortex flow





Net of the forces, noting that pressure forces must balance the centrifugal forces

$$F_r = \frac{mV^2}{r} = \rho\, dr\, dl \frac{V^2}{r}$$

Net pressure forces

$$-p\, dl$$

$$\left(p + \frac{dp}{dr} dr\right) dl$$

Net $\frac{dp}{dr}\, dr\, dl$

equate : $\frac{dp}{dr} dr\, dl = \rho\, dr\, dl \frac{V^2}{r}$

$$dp = \rho V^2 \frac{dr}{r}$$

want to relate V @ position r
to the circulation strength Γ

from momentum equation (lecture 1) $dp = -\rho V dV$

incompressible

then $\cancel{\rho} V^2 \frac{dr}{r} = -\cancel{\rho} V dV$

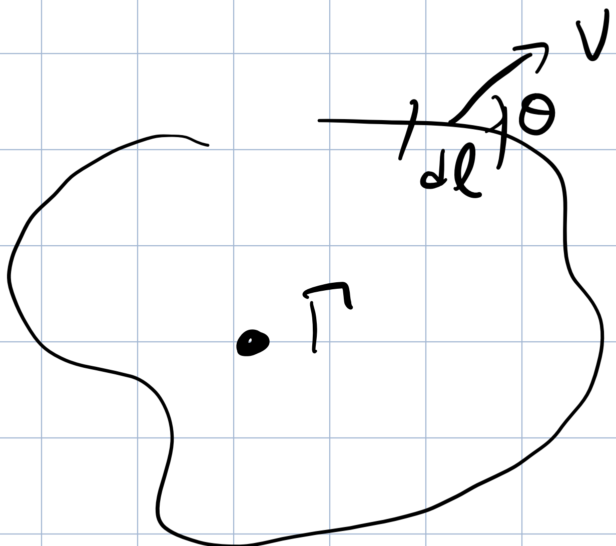
$$-\int \frac{dV}{V} = \int \frac{dr}{r}$$

$$\ln(V) = -\ln(r) + C$$

$\hookrightarrow$ $V = \frac{\text{Const}}{r}$ → velocity for a vortex @ r

as $r \to 0$
$V \to \infty$

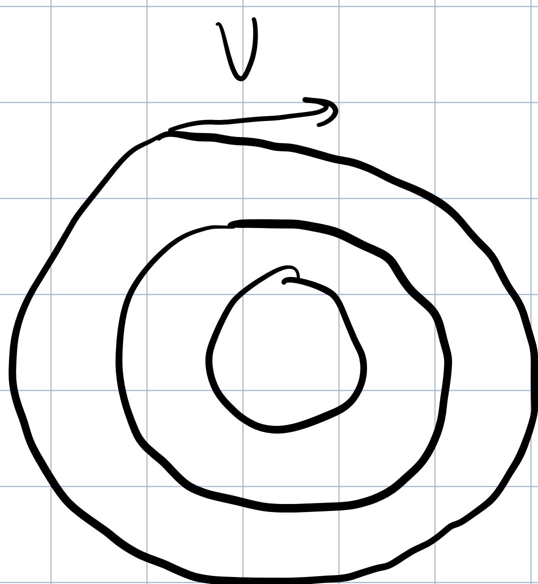
$\Gamma \equiv$ Circulation Strength



mathmatically define Γ for a contour of fluid

$$\Gamma = \oint V \cos\theta \, dl$$

θ is angle between V & dl



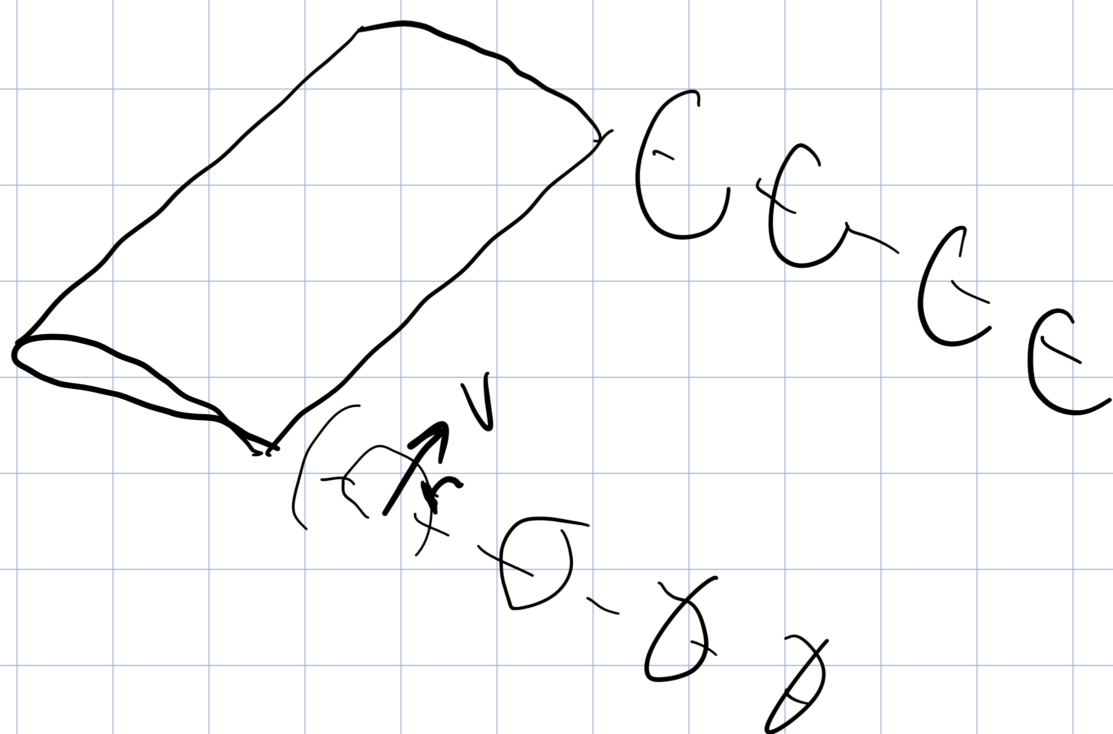
$$\underset{\frac{\text{const}}{r}}{V} \cdot \underbrace{\cos\theta}_{1}$$

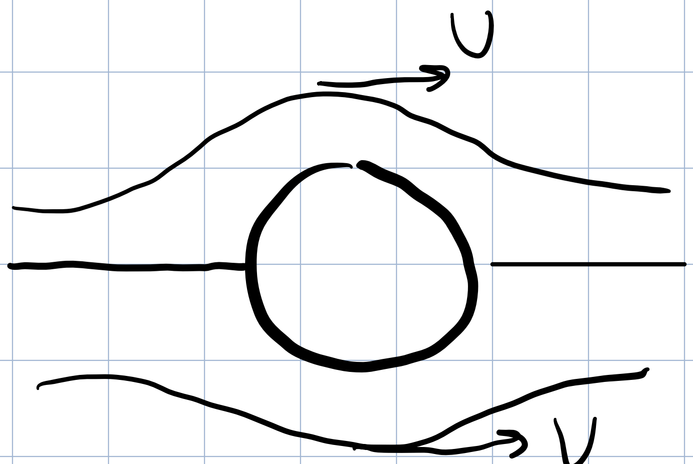
$$\Gamma = \oint \frac{\text{const}}{r} \underbrace{r\,d\theta}_{dl}$$

$$= 2\pi \,\text{const}$$

thus, $\text{const} = \dfrac{\Gamma}{2\pi}$

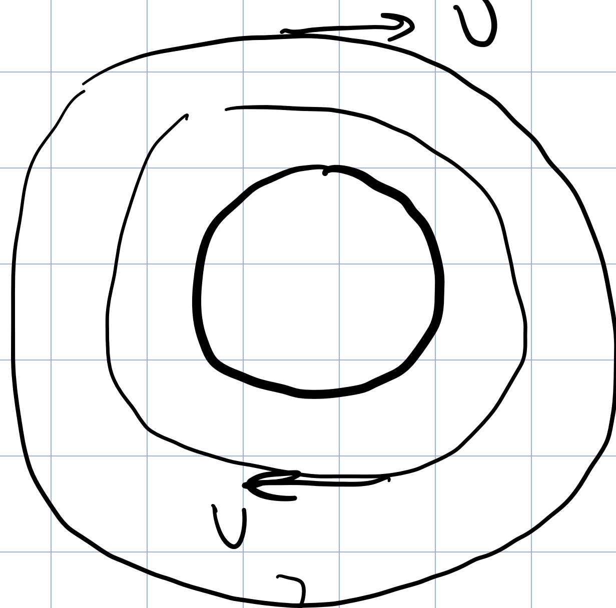
$$V = \frac{\Gamma}{2\pi r}$$ for a circular vortex flow





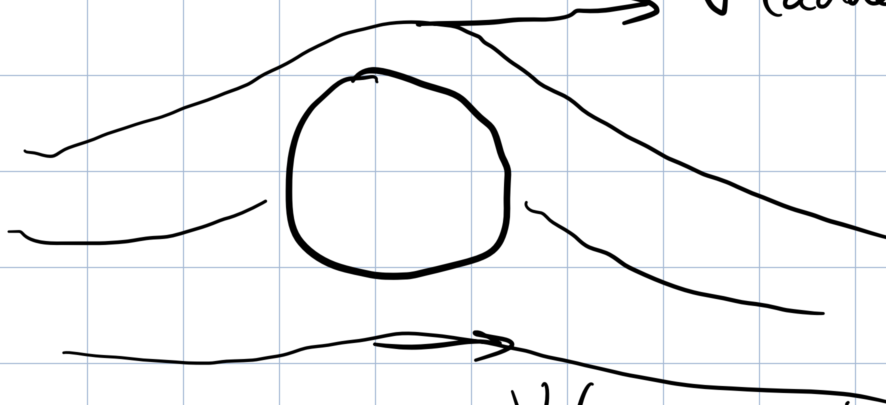
uniform

Drag, Lift = 0



$V = \frac{const}{r}$

add



$L \neq 0$, Drag $= 0$

- Kutta Joukowski:
  - if circulation exists about a body in uniform flow, a force L' that is $\perp$ to the uniform flow is produced

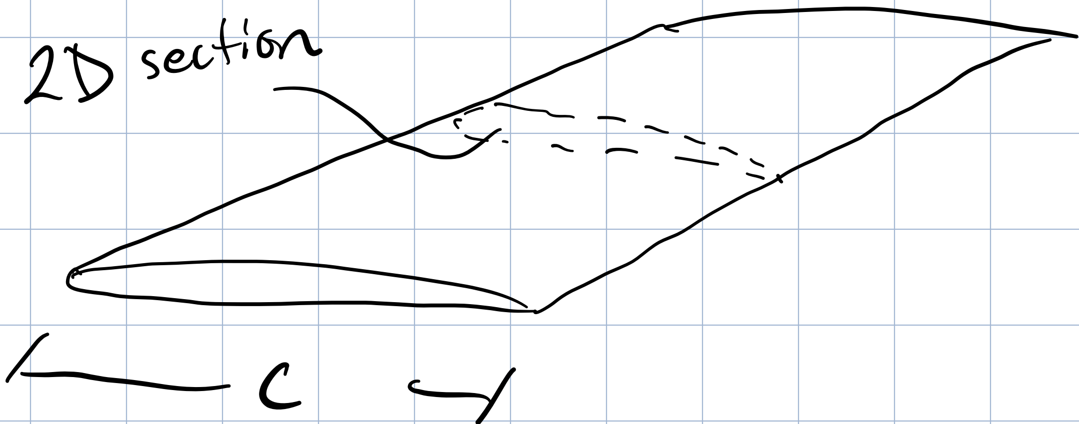
$\hookrightarrow \quad L' = \rho V \Gamma$

$\uparrow$ lift/span

$$L = \tfrac{1}{2}\rho V^2 S_{REF} \cdot C_L$$ ← 3D Lift coefficient

$$C_l = \frac{L'}{\tfrac{1}{2}\rho V^2 c}$$ $c \equiv$ chord length

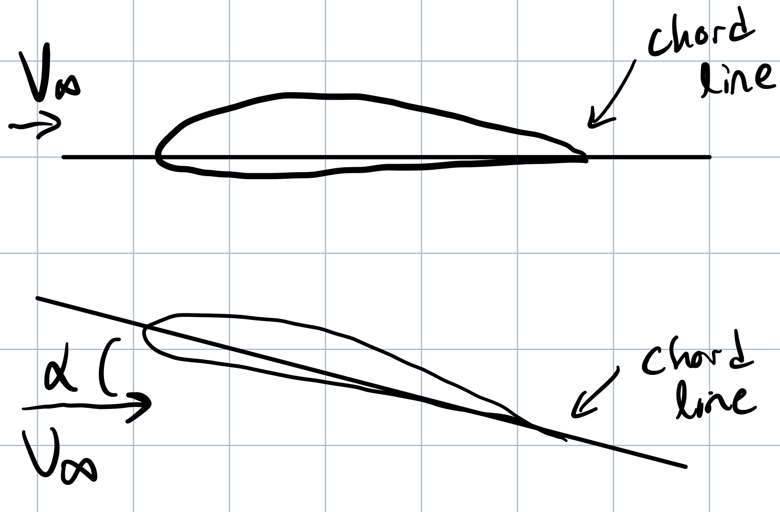
2D section

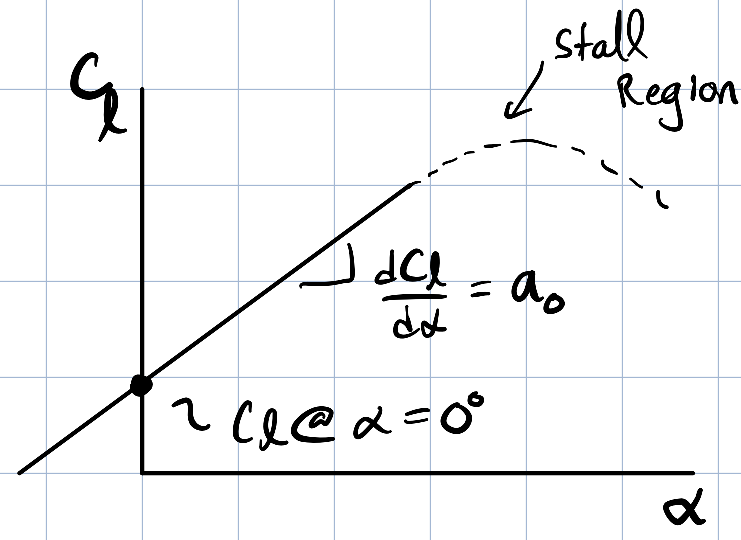


$$L' = c_l \cdot \tfrac{1}{2}\rho V^2 \cdot c = \rho V \Gamma$$

$$\hookrightarrow \quad \frac{c_l \cdot V \cdot c}{2} = \Gamma$$

2D C_l for airfoil sections





$\alpha \equiv$ angle of attack

$a_o \equiv$ lift curve slope

$C_l = C_l @ \alpha = 0° + a_o \cdot \alpha$, applies if airfoil hasn't stalled

$a_o = 2\pi$ for thin airfoils (if α in radians)

in Reality $a_o = 2\pi\eta$

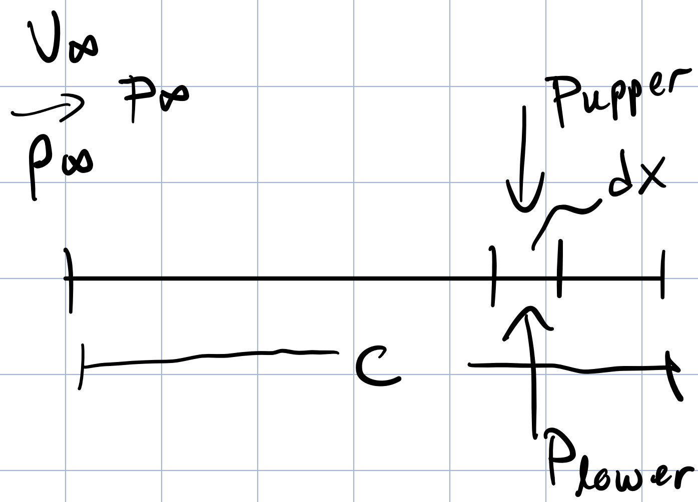
$\eta \sim 0.95$ for various airfoils

Consider lift by looking at pressure above & below surfaces

↳ Coefficient of pressure $\quad C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$

$P \equiv$ pressure @ some point

$(\ )_\infty \equiv$ values in the freestream



Negative C_p on upper surface
positive C_p on lower surface

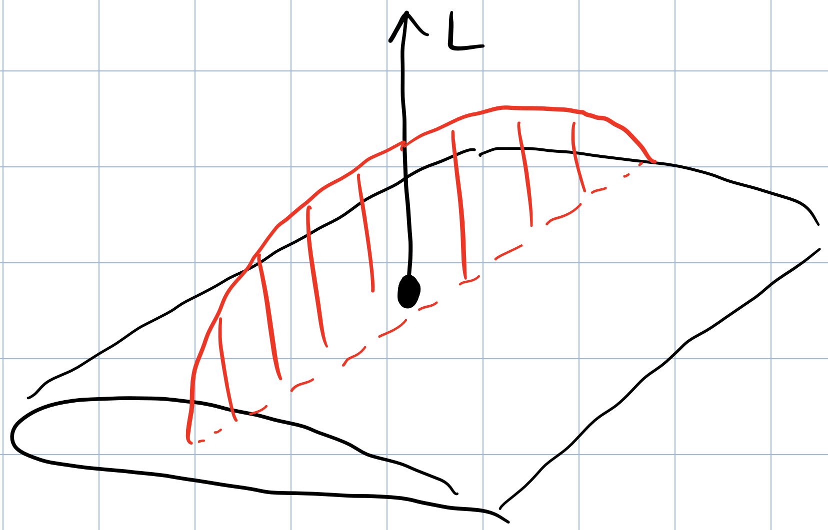
lift

$$L' = \int P_{lower} - p_\infty \, dx - \int P_{upper} - P_\infty \, dx$$

qc

$$c_l = \frac{1}{c}\int C_{p\,lower} - C_{p\,upper} \, dx$$

Real 3D wing



$$L = \tfrac{1}{2}\rho U^2 C_L \cdot S_{REF}$$

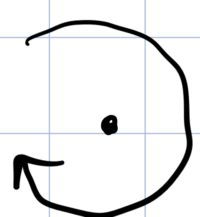
Lift is NOT constant per unit span

if you know $c_\ell \rightarrow C_L$ knowing wing geometry

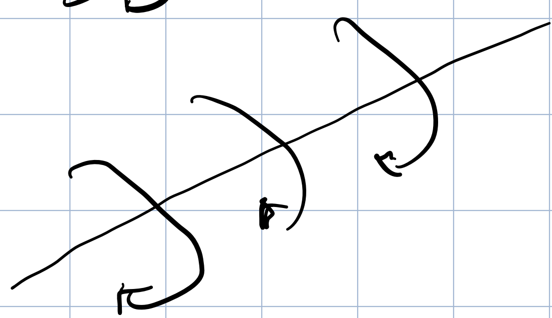
Induced drag $\rightarrow$ Drag due to Lift

must look @ 3D vortex filaments

2D



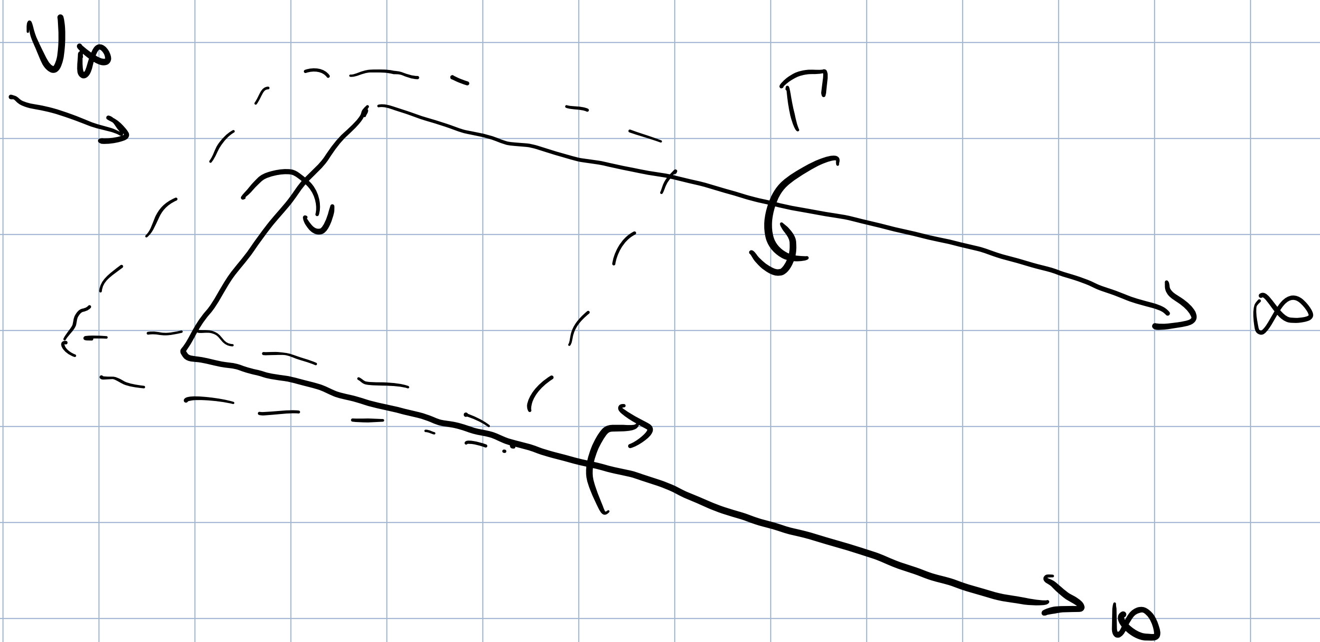
3D:

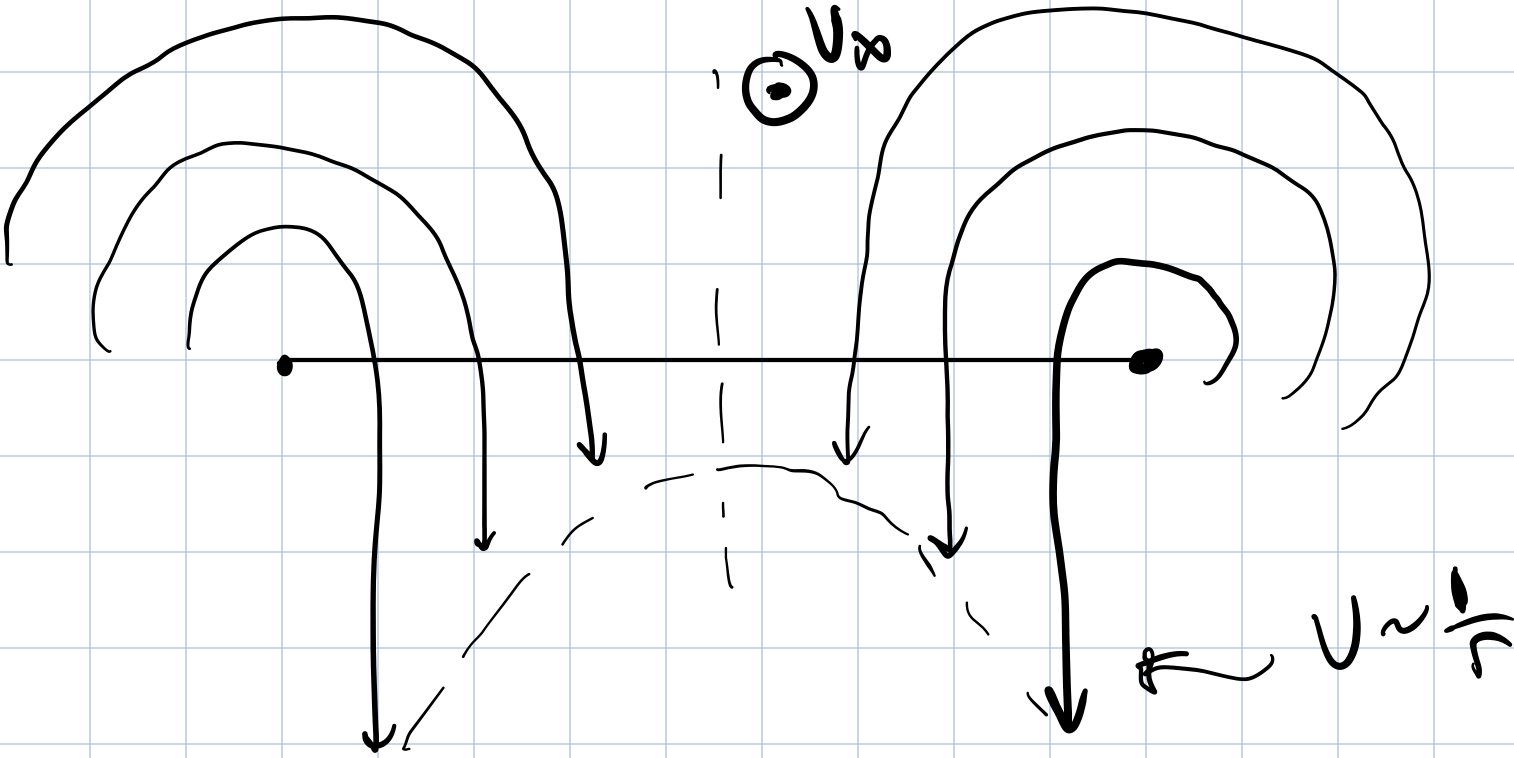


helmholtz Vortex theorems

1. Vortex filament cannot end in a fluid
   ↳ extends to ∞ or forms a loop
2. Strength of filament is $\not c$ along its path

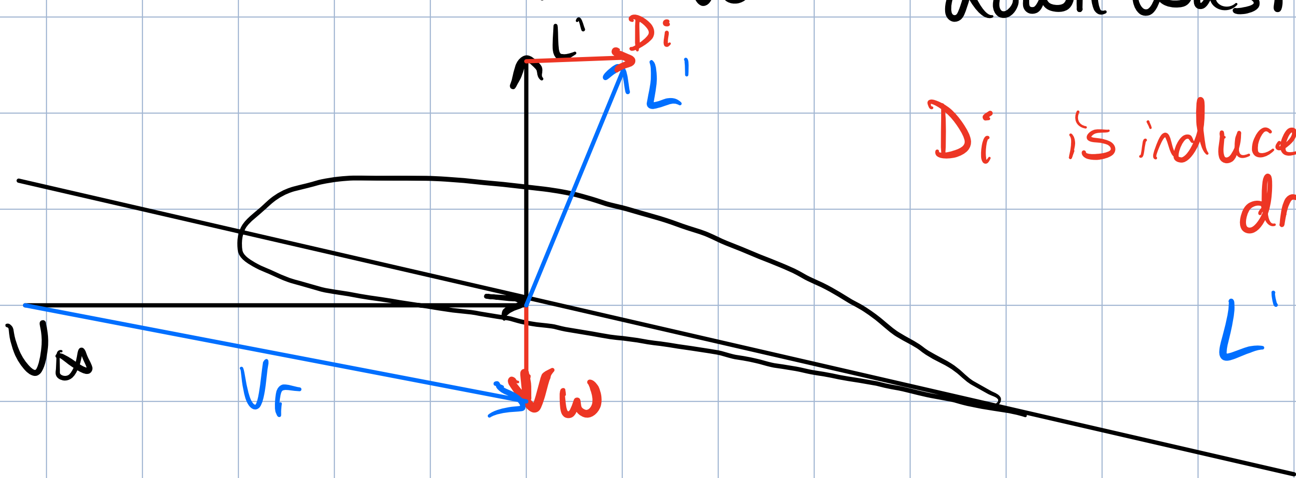
mathematically Replace 3D wing with a vortex filament





velocity distribution behind wing

↳ w → downwash



D_i is induced drag

L' is $\perp$ to V_R