MAE 158 Fall 2024

Lecture 2

Announcements: Discussions start this week

Today's Objectives: Fluid Mechanics Review (Shevell Ch. 6)
Atmosphere (Ch. 5)
Nature of Aerodynamic Forces (Ch. 3)

Last time:

Conservation of Mass (continuity): $\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$

if incompressible: $A_1 V_1 = A_2 V_2$

Conservation of Momentum: $P_2 + \rho \frac{V_2^2}{2} = P_1 + \rho \frac{V_1^2}{2}$

↳ Bernoulli's equation for incompressible, inviscid, & along a streamline

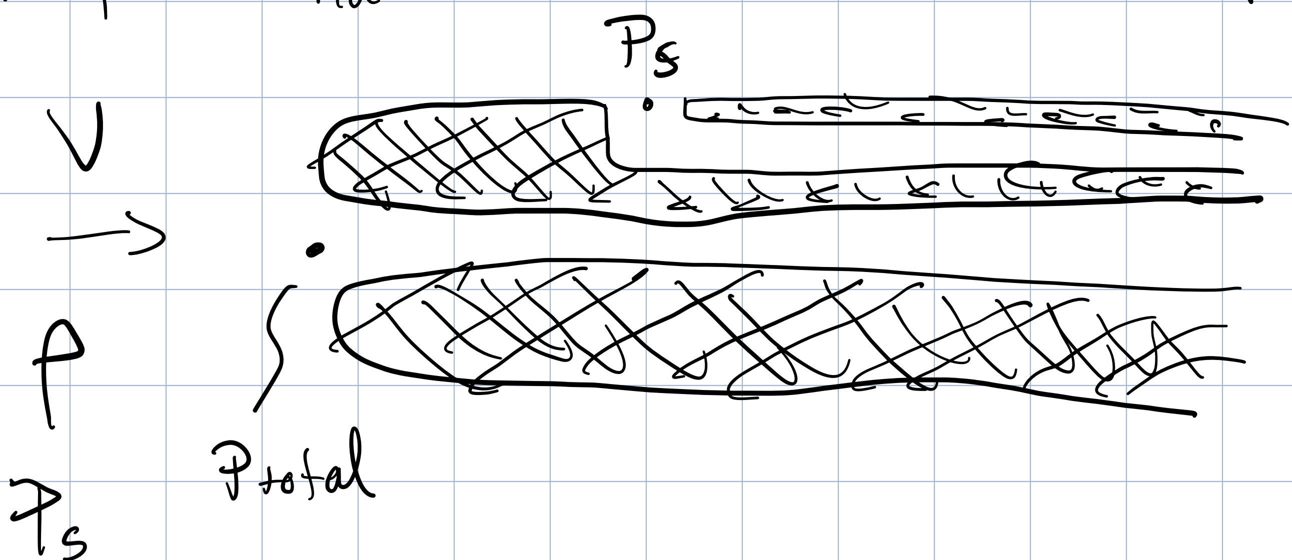
also $\boxed{P + \frac{1}{2}\rho V^2 = C}$

↳ Relation between P, V

Finally, equation of state,

$$\boxed{P = \rho R T} \quad \text{@ some point}$$

Airspeed Indicators Pitot Static tube.



$$P_{total} = P_s + \tfrac{1}{2}\rho V^2$$

$$\underbrace{\tfrac{1}{2}\rho V^2} = \underbrace{P_{total} - P_s}$$

$$\hookrightarrow \quad V = \sqrt{\frac{2}{\rho}(P_{total} - P_s)}$$

$$L = \underbrace{\tfrac{1}{2}\rho V^2} S_{REF}\, C_L$$

$\frac{1}{2}\rho V^2 \rightarrow \rho$ @ altitude

$\hookrightarrow V_{\text{true airspeed}}$

true velocity of airplane WRT the air it is flying through

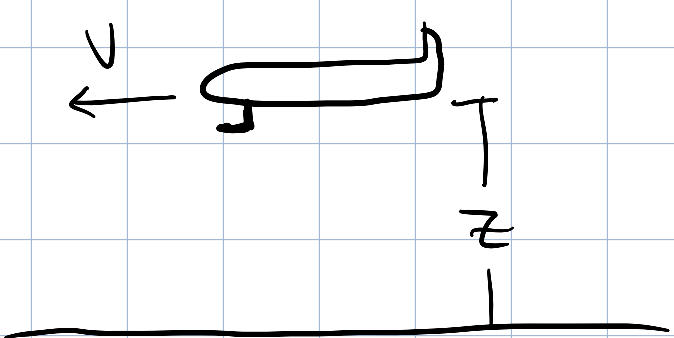
$V_{\text{Reading}} \Rightarrow V_{\text{equivalent airspeed}} = V_{\text{indicated}}$ (incompressible flow)

$$\rho_{\text{sealevel}} \rightarrow \Delta P = \frac{1}{2}\rho V^2$$

$$\frac{\rho_{\text{sealevel}} \cdot V_{ind}^2}{2} = \Delta P = \frac{\rho_{@\text{altitude}} \cdot V_{true}^2}{2}$$

$$V_{true} = V_{ind.} \left(\frac{\rho_{\text{sea level}}}{\rho_{@\text{atitude}}} \right)^{1/2}$$

ex:



@ z,

$\rho = 0.00175 \ \frac{\text{slugs}}{\text{ft}^3}$

~ 1.69 ft/s / knot

$V = 180$ KIAS

indicated Airspeed (knots)

$= 300$ ft/s

True airspeed in knots : KTAS

what is the Drag (lbs)

Assume $S_{REF} = 1000\ ft^2$
$C_D = 0.01$

$$D = \underbrace{\tfrac{1}{2}\rho}\, V^2 S_{REF} \cdot C_D$$

$\rho_{SL} = 0.00238 \frac{slugs}{ft^3}$

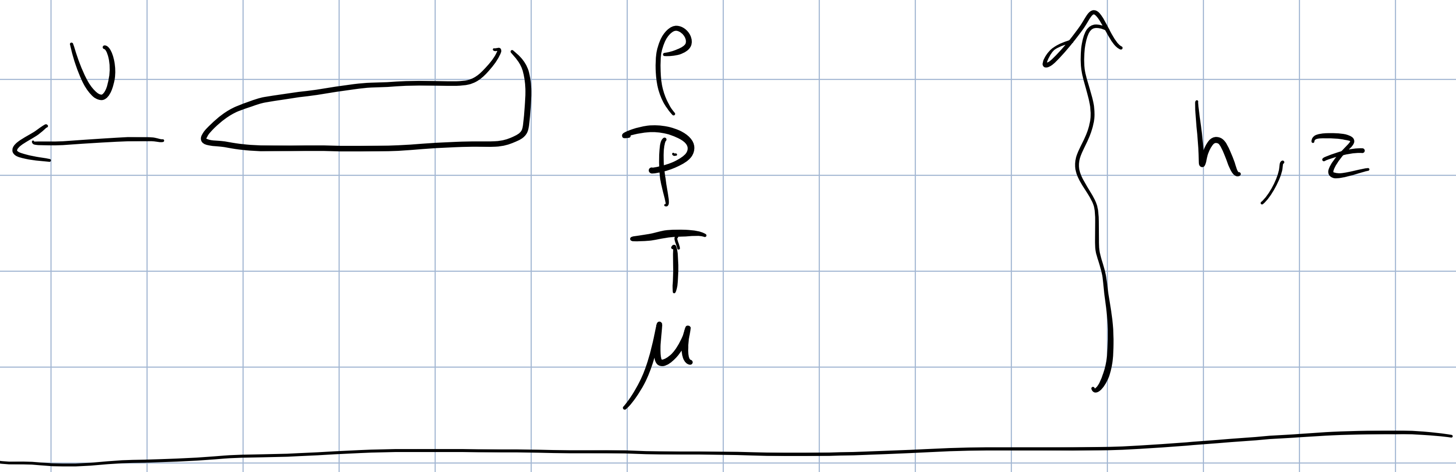
$$D = \tfrac{1}{2}\rho V^2 S_{REF} \cdot C_D = \tfrac{1}{2}\rho_{@alt} \cdot V_{true}^2\, S_{REF} \cdot C_D$$

$$= \tfrac{1}{2}\rho_{SL} \cdot V_{ind}^2\, S_{REF}\, C_D$$

$$= \tfrac{1}{2}\, 0.00238 \tfrac{slug}{ft^3} \cdot (300\ ft/s)^2 \cdot 1000\ ft^2 \cdot 0.01$$

$= 1100$ lbs $\sim$

Atmosphere



"Standard Atmosphere" → to get Representative conditions

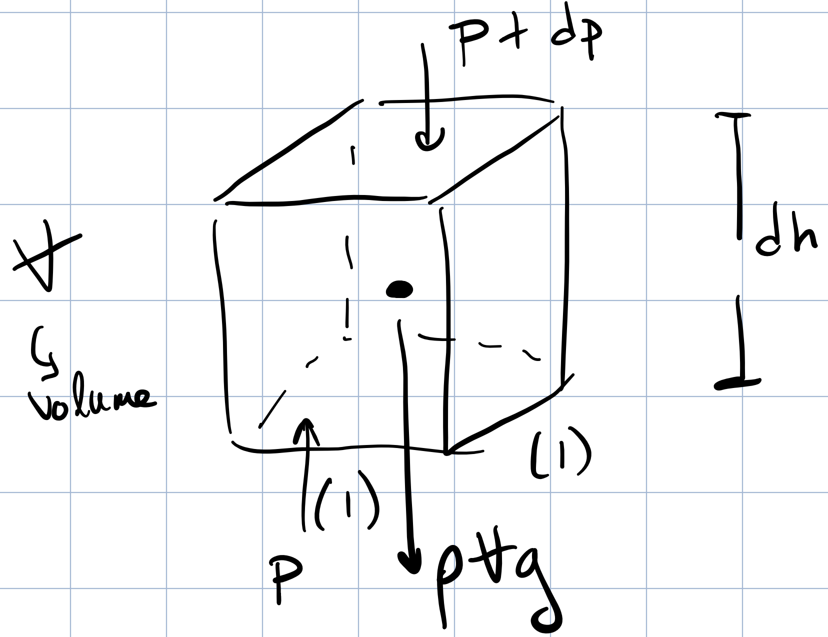
<u>sea level standard</u> conditions

- $\rho_{SL} = 0.00238 \ \frac{slug}{ft^3}$
  $= 1.22 \ kg/m^3$

- $p_{SL} = 2116.2 \ lb/ft^2$
  $= 1.01325 \times 10^5 \ N/m^2$

- $T = 518.69 \ R$
  $= 288.15 \ K$

- hydrostatics (fluid element @ REST)



unit length & width

Forces acting:

Bottom: $p(1)(1)$

top: $(p+dp)(1)(1)$

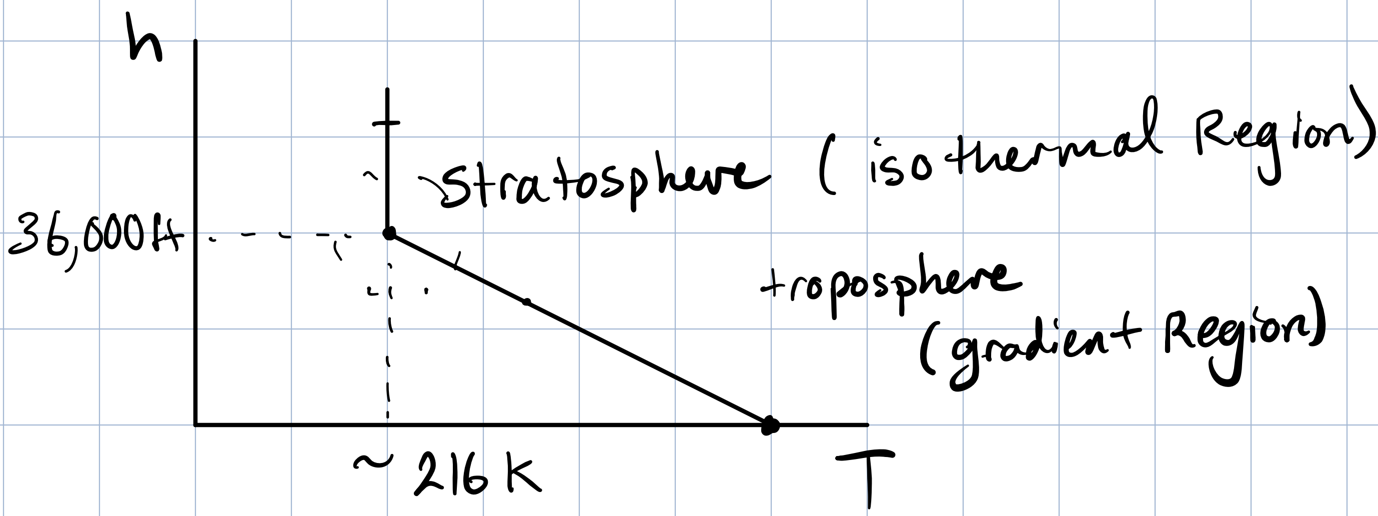
weight: $\rho \forall g = \rho g (1)(1)(dh)$

$$p(1)(1) - (p+dp)(1)(1) - \rho g (1)(1) dh = 0$$

$$\rightarrow \quad dp = -\rho g \, dh$$

↳ hydrostatics equation

$\hookrightarrow \quad g \approx$ ¢ $= 32.2\ \mathrm{ft/s^2}\ ,\ 9.8\ \mathrm{\frac{m}{s^2}}$



divide the hydrostatics equation by equation of state

$$\frac{dp}{p} = \frac{-\cancel{\rho} g\, dh}{\cancel{\rho} RT} = \frac{-g}{RT} dh$$

Iso thermal Region $\quad T =$ ¢

$$\int \frac{dP}{P} = \frac{-g}{RT} \int dh$$

$$\rightarrow \quad \ln\left(\frac{P}{P_1}\right) = \frac{-g}{RT}(h - h_1)$$

$$\hookrightarrow \quad \frac{P}{P_1} = e^{-g/RT\,(h-h_1)}$$

$$\frac{P}{P_1} = \frac{\rho \cancel{RT}}{\rho_1 \cancel{RT}} \rightarrow \frac{P}{P_1} = \frac{\rho}{\rho_1}$$

iso thermal Region

$$\boxed{\frac{P}{P_1} = e^{-g/RT(h-h_1)} = \frac{\rho}{\rho_1}}$$

gradient Region → T is linear

$$T = T_1 + a(h-h_1)$$

↑ lapse Rate

$$= \frac{dT}{dh} \rightarrow dh = \frac{1}{a}dT$$

$$a \simeq -0.00356\ R/ft$$
$$= -0.0065\ K/m$$

plug in hydrostatics equation, divide by equation of state

$$\frac{dP}{P} = \frac{-g}{R \cdot a}\frac{1}{T}dT$$

thus $\int \frac{dP}{P} = -\frac{g}{Ra}\int \frac{dT}{T}$

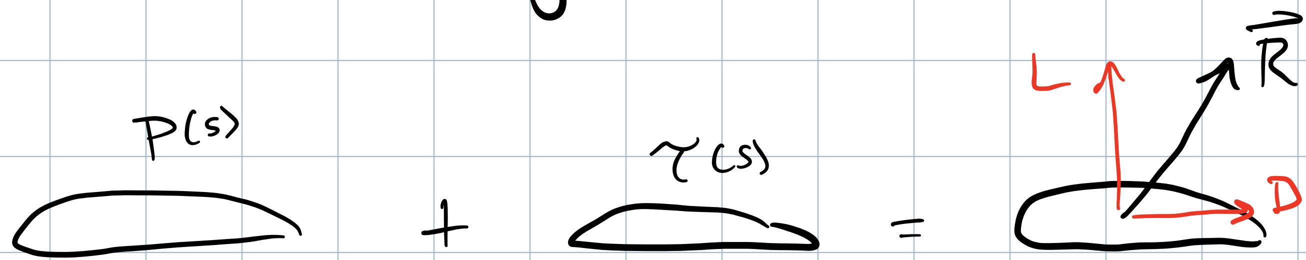
$$\hookrightarrow \boxed{\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-g/aR}}$$

gradient Region

$$\frac{P}{P_1} = \frac{\rho \cancel{R} T}{\rho_1 \cancel{R} T_1} = \frac{\rho T}{\rho_1 T_1}$$

thus, $\boxed{\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-(g/aR+1)}}$

Nature of Aerodynamic forces (ch 3)



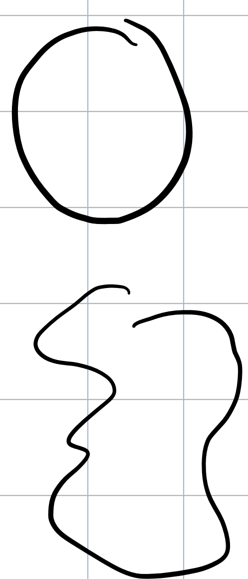
$\rightarrow$ $\vdash l \dashv$

U

ρ

μ

"shape factor"



Drag → force impeding forward motion

Lift → force serving to lift body upward

$$F = f(\rho, \mu, V, l, \text{shape})$$

dimensional analysis, π-theorem, can get an algebraic expression for this Relation

$$F = \rho^a \mu^b V^c l^d \cdot C$$

↑ shape factor

dimensions: M = mass
L = length
T = time

$$F = ma \Rightarrow \frac{ML}{T^2}$$

$$\rho = \text{mass/volume} \Rightarrow M/L^3$$

$V = \text{length}/\text{time} \Rightarrow L/T$

$l = \text{length} \Rightarrow L$

$\tau = \mu \frac{du}{dy}$

$\mu = \tau / \frac{du}{dy} \Rightarrow \frac{MLT^{-2}L^{-2}}{LT^{-1}L^{-1}}$

$\Rightarrow \frac{M}{LT}$

dimensions F = dimensions $\rho^a \mu^b V^c l^d$

$$\frac{ML}{T^2} = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{LT}\right)^b \left(\frac{L}{T}\right)^c L^d$$

$M: \quad 1 = a + b$

$L: \quad 1 = c + d - 3a - b$

$T: \quad -2 = -b - c$

solve in terms of b:

$$\left.\begin{aligned} a &= 1 - b \\ c &= 2 - b \\ d &= 2 - b \end{aligned}\right\} \text{plug into } F$$

$$F = \rho^{1-b}\mu^{b}V^{2-b}\ell^{2-b}\cdot C$$

$$= \rho V^2 \ell^2 \left(\frac{\mu}{\rho V \ell}\right)^{b}\cdot C$$

$$\frac{\rho V \ell}{\mu} \equiv \text{Reynolds Number} = Re$$

RN

call $C\cdot\left(\frac{\mu}{\rho V \ell}\right)^{b} \rightarrow$ some coefficient that is a function of shape, Re

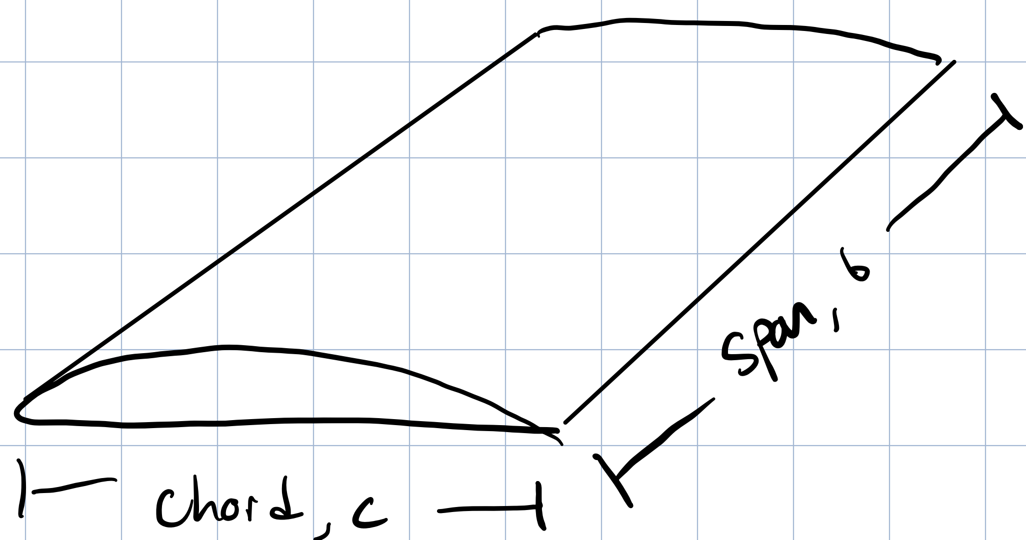
$C_F \equiv$ force Coefficient

$$2\cdot C\left(\frac{\mu}{\rho V \ell}\right)^{b}$$

$$F = \frac{1}{2}\rho V^2\cdot \ell^2\cdot C_F$$

$\ell^2 \to S_{REF}$

in general for an aircraft wing S_{REF} is planform area



if wing is Rectangular, no fuse

$$S_{REF} = c \cdot b$$