

# MAE 158 Lecture 16

Nov. 21 2024

Announcements: Week 8 quiz  
landing + turn/pullup/  
pull down maneuver

Prof Huynh's OH week 9  
tues @ 1pm in EG 4212

TA Reza's OH week 9  
Mon @ 9am in EG 2146

Week 8 discussions →  
additional time to  
Review Midterm  
as desired

Today's Objectives: turning (fn)  
stability

last time:

determine "theoretical min  
turn Radius"

$$W/S = 76.84 \text{ lb/ft}^2 \quad \underline{C_{L \max} = 1.2}$$

$$K = 0.08$$

Sea level conditions

$$\rightarrow \rho = 0.00238 \text{ slug/ft}^3$$

$$T/W = 0.3795$$

$$C_{D,P} = 0.015$$

$$r_{\min} = \frac{V_{r_{\min}}^2}{g \sqrt{n_{r_{\min}}^2 - 1}} = \frac{4K(W/S)}{g \rho \left(\frac{T}{W}\right) \sqrt{1 - 4K \frac{C_{D,P}}{(T/W)^2}}}$$

$$= 4 \cdot 0.08 (76.84 \text{ lb/ft}^2)$$

$$(32.2 \frac{\text{ft}}{\text{s}^2})(0.00238 \frac{\text{slug}}{\text{ft}^3})(0.3795).$$

$$\sqrt{1 - 4(0.08) \frac{0.015}{(0.3795)^2}}$$

$$= \underline{861 \text{ ft}} \quad \underline{\text{"theoretical min"}}$$

Is this feasible?  $\rightarrow$  is it safe?  
 $\rightarrow$  is the load factor reasonable

$\rightarrow$  is this  $V_{rmin}$  feasible?  
 $\rightarrow$  is it too low or too high?

$$n_{rmin} = \sqrt{2 - \frac{4K C_{Dp}}{(T/w)^2}} = \sqrt{2 - \frac{4(0.08)(0.015)}{(0.3795)^2}} = \underline{1.4}$$

$$V_{\min} = \sqrt{\frac{4K(W/S)}{\rho(T/W)}} = \sqrt{\frac{4(0.08)(76.84 \frac{lb}{ft^2})}{0.00238 \frac{slug}{ft^3} (0.3795)}} \\ = \underline{165 ft/s} \quad \leftarrow$$

compare load factor to structural load factor

$n_{\text{struct}} \sim 2$  thus safe for  $n_{\text{struct}} \checkmark$

165 ft/s because  $n_{\min} < n_{\text{struct}}$

$$n_{\text{thrust}} = \left\{ \frac{\frac{1}{2} \rho V^2}{k(W/S)} \left[ \left( \frac{T}{W} \right) - \frac{\frac{1}{2} \rho V^2 C_{DP}}{W/S} \right] \right\}^{1/2}$$

$\nearrow$  0.08       $\uparrow$  76.84  $\frac{lb}{ft^2}$        $\uparrow$  usually want max  $T/W$

$\nwarrow$  0.3795       $\downarrow$  0.015

$$= \underline{1.4}$$

using  $T/\omega = 0.3795$

because  $T_{A \max}$  is higher,

then ✓

$n_{CL \max}$

check  $C_L = \frac{2 n \omega}{\rho V^2 S}$

$$= \frac{2 \cdot 1.4 \cdot 76.84 \frac{\text{lb}}{\text{ft}^2}}{0.00238 \frac{\text{slug}}{\text{ft}^3} \cdot 165 \frac{\text{ft}}{\text{s}}}$$

$$= \underline{\underline{3.32}}$$

~~X~~

given a velocity  $\rightarrow$  What is  
the min safe  
turn Radius?

$n_{struct}$   
solve  $n_{thrust}$   
solve  $n_{clmax}$  } whichever is  
Smallest  
 $\rightarrow n_{safe}$   
 $\hookrightarrow$  min  
safe Radius  
@ given  $V$

quiz  $\uparrow$

---

Stability & control topics **shewell**  
**Ch. 16**

1. Stability: ability of Aircraft  
to Return to  
equilibrium condition  
after a disturbance

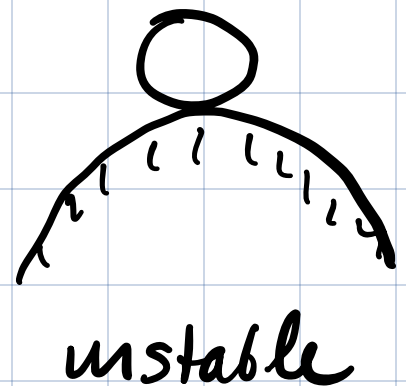
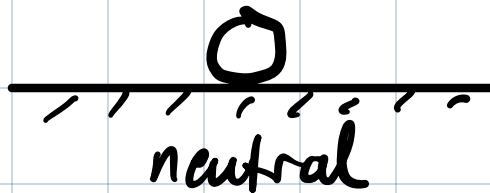
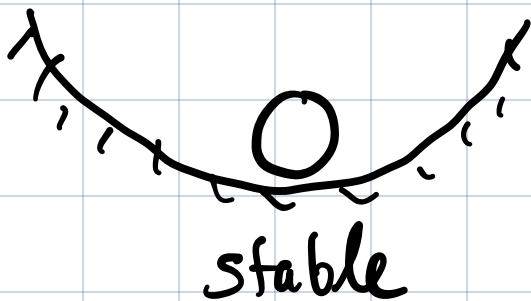
equilibrium :

$$\sum F = 0$$

$$\sum M = 0$$

on the body

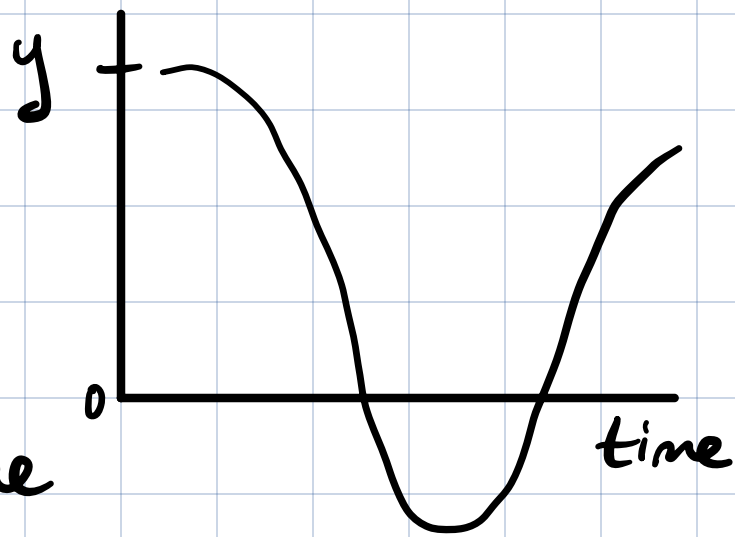
Static Stability : when forces  
& moments on a body  
Return body to equilibrium  
after a disturbance



Dynamic Stability - after a disturbance  
the body Returns to equilibrium  
after some time period



stable

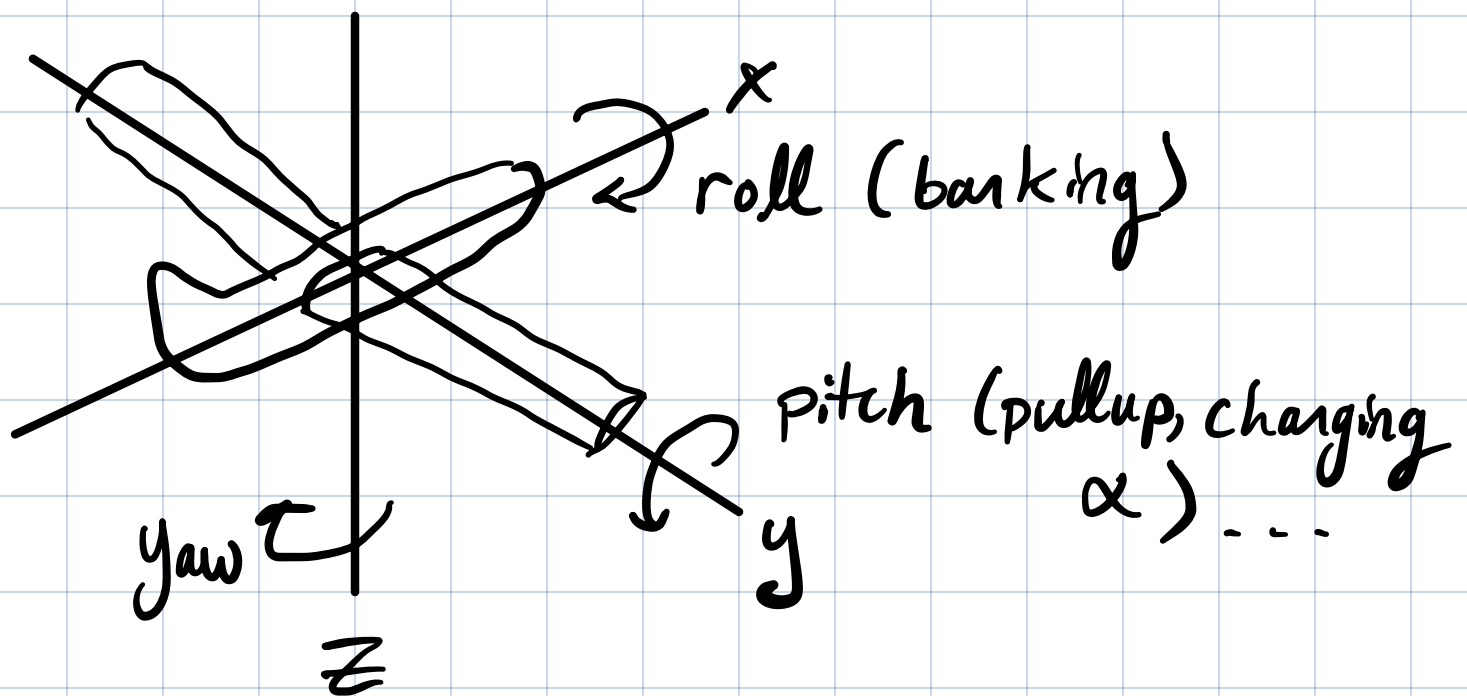


unstable

2. Control: ability of pilots to  
Produce forces & moments  
on A/C using ailerons,  
elevators, rudder, differential  
thrust etc... to change  
A/C direction

We will assume A/C is  
a Rigid body





6 degrees of freedom

- motion along  $x, y, z$  axis
- Rotate around:

$x$  axis (Roll)

$y$  axis (pitch)

$z$  axis (yaw)

• + moment defined as:

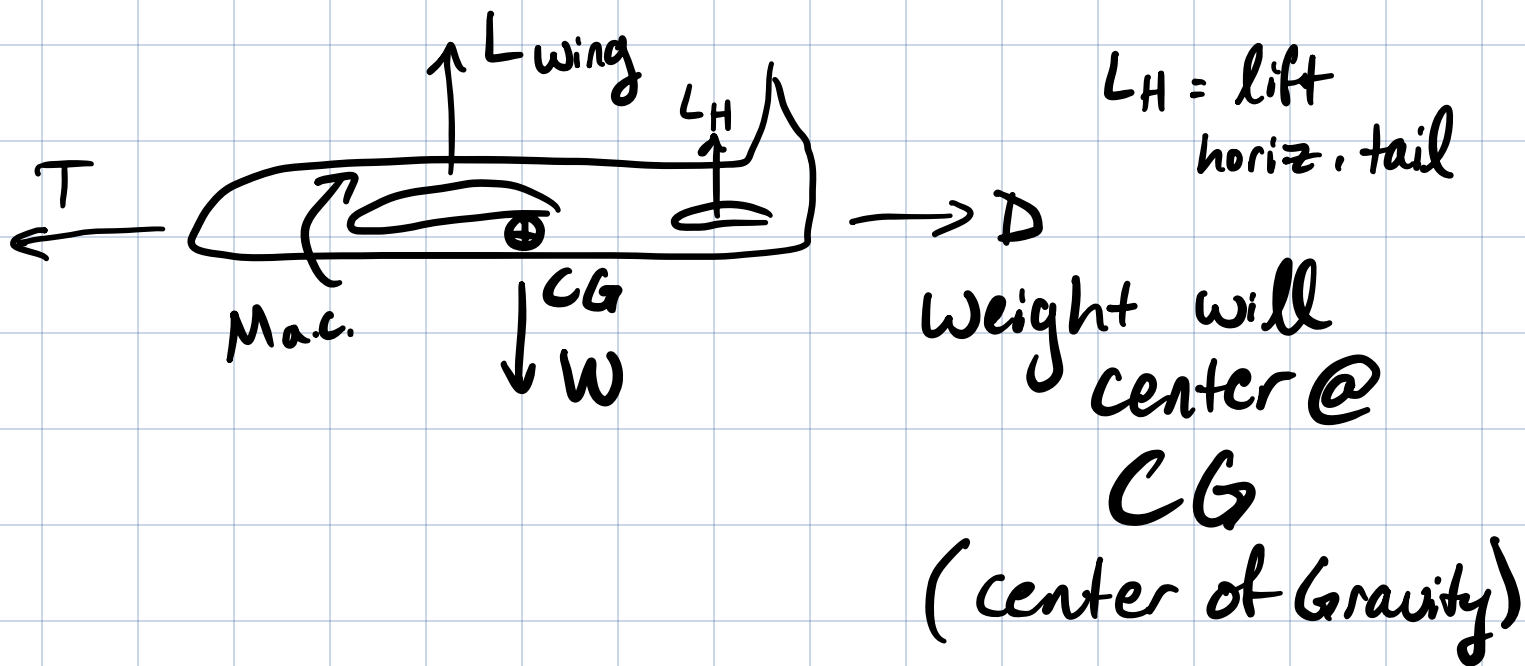
- Rolling Right
- Pitching up
- yawing nose goes to Right

## longitudinal Static Stability

Recall  $C_M = \frac{M}{\frac{1}{2} \rho V^2 S \bar{c}}$

↑  
aerodynamic mac. when  
dealing with stability  
& control

↳  $C_{R_0} \rightarrow \text{mac.}$



Conditions for equilibrium ( $C_{M_{CG}} = 0$ )  
 $\rightarrow$  trim point

Few other points:

@ aerodynamic center:

$C_M$  const w/  $\alpha$   
 const w/  $C_L$

if  $C_L$  is 0  $\rightarrow$  condition where no force vectors from wing / tails

contributing to moment  
assume  $T, D$  contributions  
to moment are  
negligible

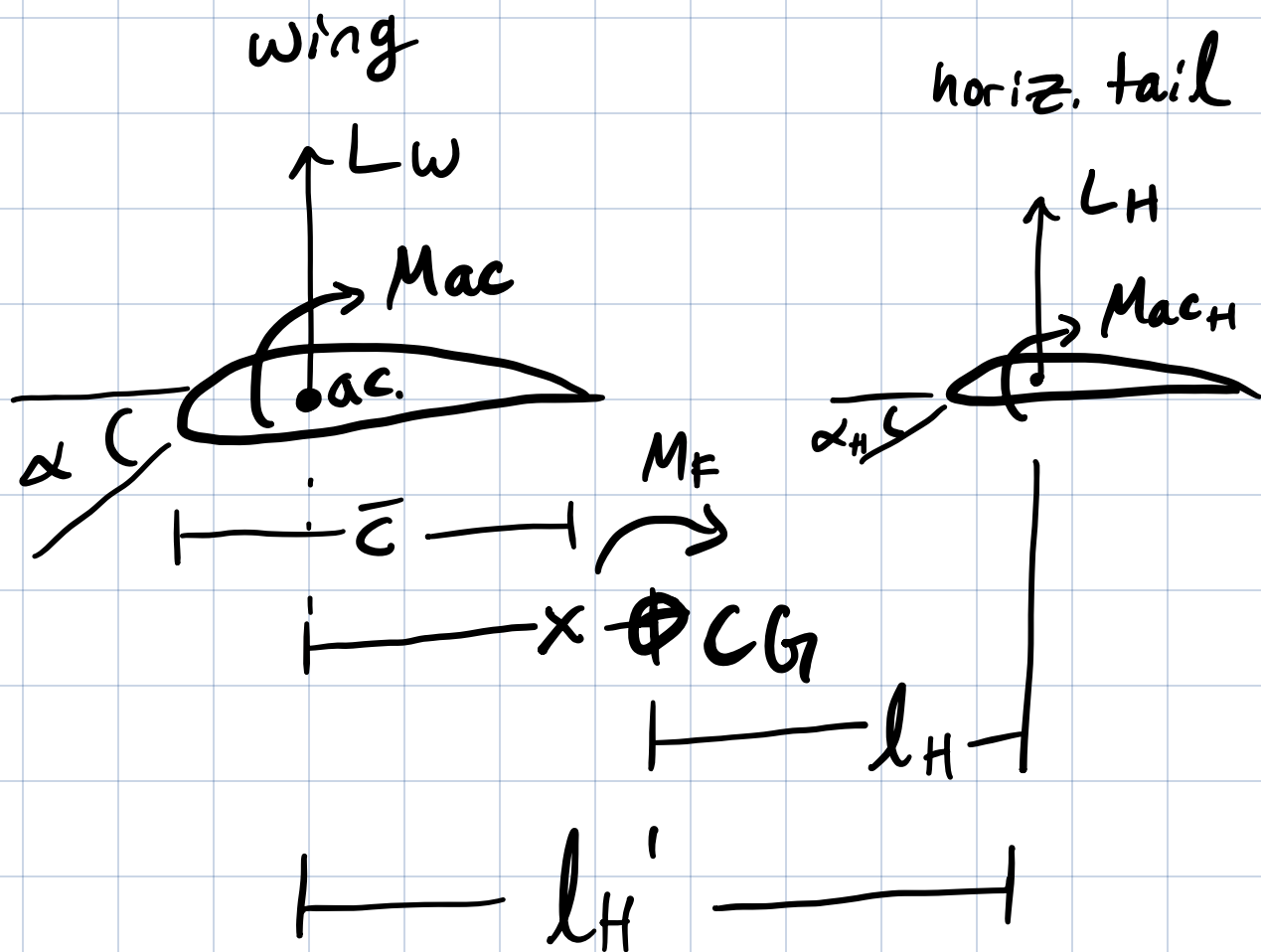
conditions for longitudinal static  
stability?

Stable if  $C_m$  becomes  
more negative w/  $\alpha$  or  $C_L$   
increases  $\rightarrow$  restores

the pitching moment  $\rightarrow$   
prevent stalling

- $\frac{dC_m}{dC_L} < 0 \quad \leftarrow$

Solve location of CG  
that leads to Stability



$\bar{c}$  = M.A.C aerodynamic

$x$  = distance between a.c. of wing & CG

$l_H$  = distance between a.c. of

tail & CG

$l_H'$  = distance between ac. of wing & a.c. tail

$M_F$  = moment contribution due to fuselage

$M_{ac_H} \sim$  small,  
negligible

$$\begin{aligned}\sum M_{CG} &= M_w + M_H + M_F \\ &= M_{ac_w} + L_w \cdot X - L_H \cdot l_H + M_F\end{aligned}$$

$$\begin{aligned}C_{M_{CG}} &= \frac{M}{q S \bar{c}} = C_{mac} + \frac{C_{L_w} X}{\bar{c}} - C_{L_H} \left[ \frac{S_H \cdot l_H}{S_w \cdot \bar{c}} \right] \eta_H \\ &\quad + C_{m_F}\end{aligned}$$

$\eta_H \equiv$  tail efficiency factor due  
to wing wake influence