MAE 158: Aricraft Performance
Recommended Homework #4

Jordi Ventura Siches

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- **11.1** A twin turbofan transport airplane is cruising at 31,000 ft pressure altitude at a Mach number of 0.78. Outside air temperature is -60ºF. The airplane gross weight is 98,000 lb. The airplane has unsealed aerodynamically balanced control surfaces. Following are the airplane dimensional data:

Wing

Span	= 93.2 ft
Planform area	= 1000 ft^2
Average t/c	= 0.106
Sweepback angle	= 24.5 deg
Taper ratio	= 0.2
Root chord	= 17.8 ft
Wing area covered by fuselage	= 17%

Horizontal Tail

Exposed planform area	= 261 ft^2
t/c	= 0.09
Sweepback	= 31.6 deg
Taper ratio	= 0.35
Root chord	= 11.1 ft

Pylons

Total wetted area	= 117 ft^2
t/c	= 0.06
Sweepback	= 0 deg
Taper ratio	= 1.0
Chord	= 16.2 ft

Fuselage

Length	= 107 ft
Diameter	= 11.5 ft
Wetted area	= 3280 ft^2

Vertical Tail

Exposed planform area	= 161 ft^2
t/c	= 0.09
Sweepback	= 43.5 deg
Taper ratio	= 0.80
Root chord	= 15.5 ft

Nacelles

Total wetted area	= 455 ft^2
Effective fineness ratio	= 5.0
Length	= 16.8 ft

Flap Hinge Fairings

Δf	= 0.15 ft^2

Determine

(a) Incompressible parasite drag coefficient and equivalent flat-plate area.
From table A.2, we get the pressure $p_0 = 601.61$ lb/ft^2, and by the equation of state we get the density (with $T_0 = -60ºF = 400º$R)

$$\rho_0 = \frac{p_0}{RT_0} = \frac{601.61\,\text{lb/ft}^2}{1718\,\text{lb ft/(slug ºR)} \cdot 400\text{ºR}} = 0.00087545\,\text{slug/ft}^3$$

The speed of sound and the airspeed are

$$a_0 = \sqrt{\gamma R T_0} = \sqrt{1.4 \cdot 1718\,\text{lb ft/(slug ºR)} \cdot 400\text{ºR}} = 980.86\,\text{ft/s}$$
$$V_0 = M_0 \cdot a_0 = 0.78 \cdot 980.86\,\text{ft/s} = 765.07\,\text{ft/s}$$

Viscosity is, according to Fig. 10.14 $\mu_0 = 3.04 \cdot 10^{-7}$ lb s/ft^2, and according to equation 10.7:

$$10^{10}\mu = 0.3170\,\mathrm{T}(^{\circ}\mathrm{R})^{3/2}\left(\frac{734.7}{T(^{\circ}\mathrm{R})+216}\right)$$

it has a value of $\mu_0 = 3.025 \cdot 10^{-7}$ lb s/ft^2. The Reynolds number over length is

$$\mathrm{Re}/L = \frac{\rho_0 V_0}{\mu_0} = \frac{0.00087545\,\mathrm{slug/ft}^3 \cdot 765.07\,\mathrm{ft/s}}{3.04 \cdot 10^{-7}\,\mathrm{lb\ s/ft}} = 2203225\,\mathrm{ft}^{-1}$$

where L is the characteristic length. The dynamic pressure is found as

$$q_0 = \frac{1}{2}\rho_0 V_0^2 = \frac{\gamma}{2}p_0 M_0^2 = 0.7 \cdot 601.61\,\mathrm{lb/ft}^2 \cdot 0.78^2 = 256.21\,\mathrm{lb/ft}^2$$

We will compute the total parasite drag coefficient summing each contribution:

$$C_{D_P} = \frac{\sum_i K_i C_{f_i} S_{\mathrm{wet}}}{S_{\mathrm{ref}}}$$

where K_i is the form factor and C_{f_i} is the flat plate skin friction of the i-th component. We have the following components:

- **Wing**: We will use the following: taper ratio $\sigma = 0.2$, root chord $C_R = 17.8\,\mathrm{ft}$, fuselage diameter $D_{\mathrm{fus}} = 11.5\,\mathrm{ft}$, from which, we have fuselage radius $R_{\mathrm{fus}} = 5.75\,\mathrm{ft}$ (see figure 1

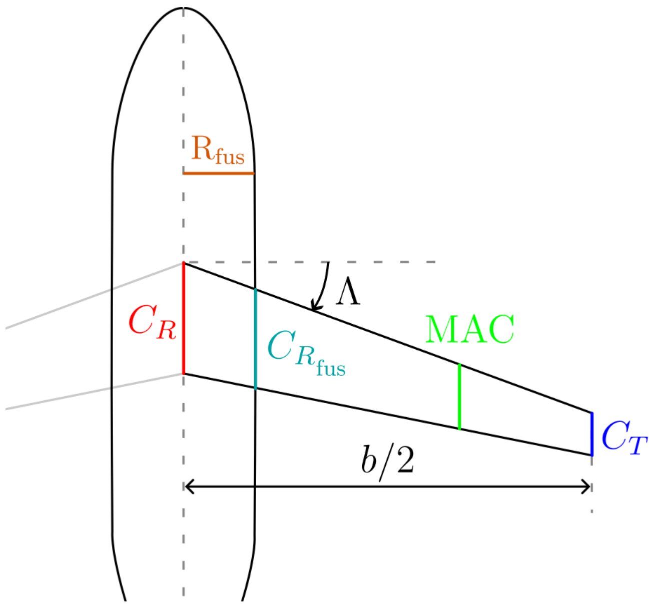

Figure 1: Relevant distances in the wing

We can find the tip chord by

$$C_T = \sigma C_R = 0.2 \cdot 17.8\,\mathrm{ft} = 3.56\,\mathrm{ft}$$

The mean aerodynamic chord (m.a.c.) can be computed the following two ways

$$\mathrm{m.a.c.} = \frac{2}{3}\left(C_R + C_T - \frac{C_R C_T}{C_R + C_T}\right) = \frac{2}{3}C_R\left(1+\sigma-\frac{\sigma}{1+\sigma}\right) = 11.12\,\mathrm{ft}$$

The corresponding Reynolds number is

$$\text{Re} = 2203225 \cdot \text{m.a.c.} = 2203225 \cdot 11.12 = 2.25 \cdot 10^7$$

We use Figure 11.2 from the book, and with this Reynolds number we obtain $C_f = 0.00295$ (see figure 2). From figure 11.3 from the book, and using $t/c = 0.106$ and $\Lambda = 24.5^{\text{o}}$, we get $K = 1.197$ (we can also use the formula, see figure 7 for an application of the chart in another problem).

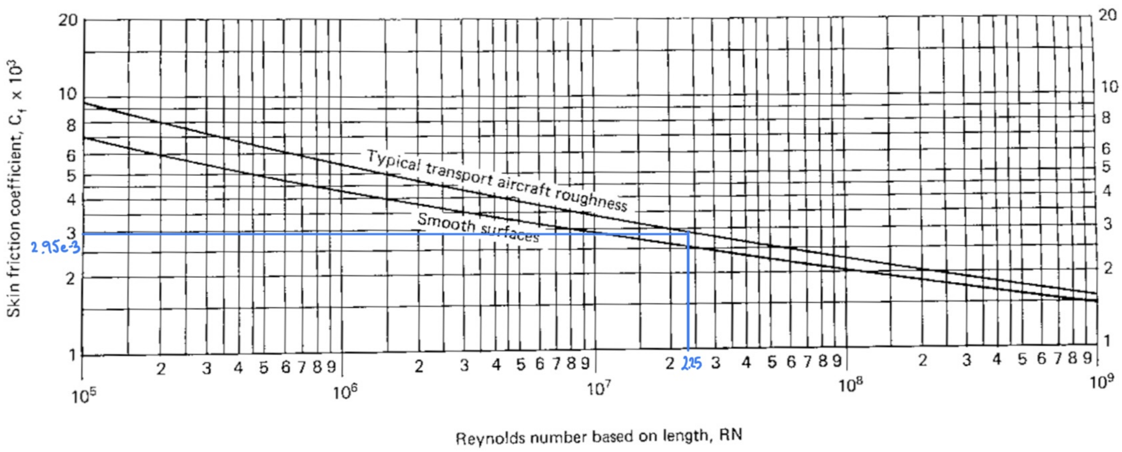

Figure 2: Use of figure 11.2 for the wing

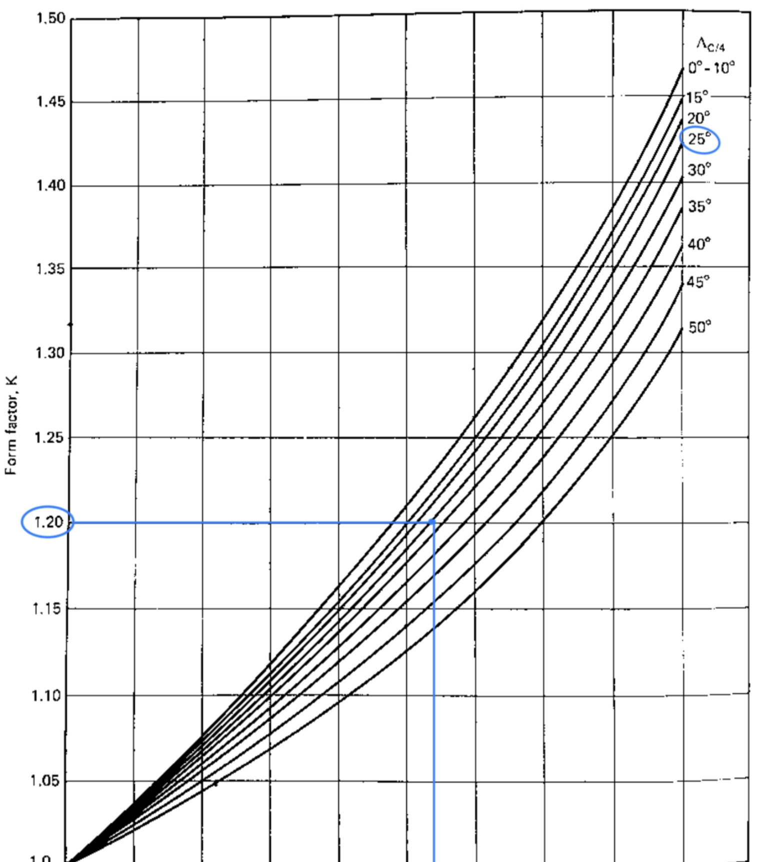

Figure 3: Use of figure 11.3 for the wing

The wetted surface of the wing is

$$S_{\text{wet}} = S_{\text{ref}} \cdot \%_{\text{exp}} \cdot 2 \cdot 1.02 = 1693.2\,\text{ft}^2$$

The equivalent profile drag area of the wings is

$$f_{\text{wing}} = KC_f S_{\text{wet}} = 1.197 \cdot 0.00295 \cdot 1693.2\,\text{ft}^2 = 5.98\,\text{ft}^2$$

The parasite drag coefficient due to the wing is

$$\Delta C_{D_{P,\text{wing}}} = \frac{f_{\text{wing}}}{S_{\text{ref}}} = \frac{5.98}{1000} = 0.00598$$

– **Fuselage**. We will use the $S_{\text{wet}} = 3280\,\text{ft}^2$, and the length and diameter to compute the ratio

$$\frac{\text{Length}}{\text{Diameter}} = \frac{107\,\text{ft}}{11.5\,\text{ft}} = 9.3$$

From table 11.4 (see figure 4), we obtain $K = 1.11$.

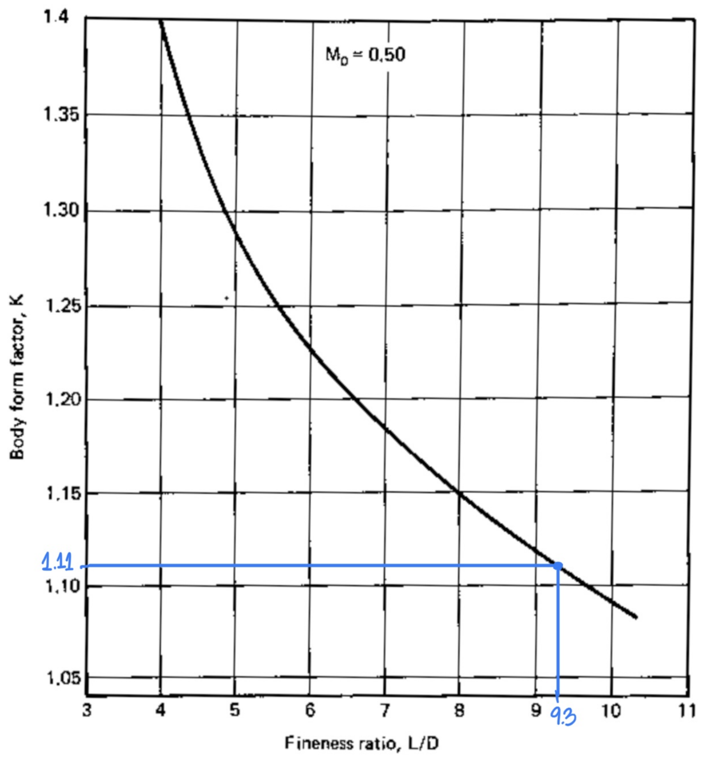

Figure 4: Use of table 11.4

The Reynolds number based on the length of the fuselage is

$$\text{Re} = 2203225 \cdot 107\,\text{ft} = 2.357 \cdot 10^7$$

From figure 11.2, we obtain the skin friction coefficient $C_f = 0.002$, using this Reynolds number. Thus we have

$$f_{\text{fus}} = K \cdot C_f \cdot S_{\text{wet}} = 1.11 \cdot 0.002 \cdot 3280\,\text{ft}^2 = 7.28\,\text{ft}^2$$

giving a coefficient of

$$\Delta C_{D_{P,\text{wing}}} = \frac{f_{\text{wing}}}{S_{\text{ref}}} = \frac{7.28\,\text{ft}^2}{1000\,\text{ft}^2} = 0.00728.$$

– **Horizontal tail**. The wetted area is

$$S_{\text{wet}} = S_{\text{exp}} \cdot 2 \cdot 1.02 = 261\,\text{ft}^2 \cdot 2.04 = 532.44\,\text{ft}^2$$

Now, using figure 11.3, with $t/c = 0.09$ and $\Lambda = 31.6^{\text{o}}$, we get $K = 1.155$. To find the m.a.c., we have $C_R = 11.1\,\text{ft}$ and need $C_T = \sigma C_R = 0.35 \cdot 11.1\,\text{ft} = 3.89\,\text{ft}$. Thus

$$\text{m.a.c.} = \frac{2}{3}C_R\left(1+\sigma-\frac{\sigma}{1+\sigma}\right) = \frac{2}{3}11.1\cdot\left(1+0.35-\frac{0.35}{1+0.35}\right) = 8.07\,\text{ft}$$

The Reynolds number based on the chord of the horizontal tail is

$$\text{Re} = 2203225 \cdot 8.07\,\text{ft} = 1.779 \cdot 10^7$$

and so, from figure 11.2, we obtain $C_f = 0.00305$, giving

$$f_{\text{ht}} = K \cdot C_f \cdot S_{\text{wet}} = 1.155 \cdot 0.00305 \cdot 532.44\,\text{ft}^2 = 1.88\,\text{ft}^2$$

and so

$$\Delta C_{D_{P,\text{ht}}} = \frac{f_{\text{ht}}}{S_{\text{ref}}} = \frac{1.88\,\text{ft}^2}{1000\,\text{ft}} = 0.00188$$

– **Vertical tail**. Similarly, $S_{\text{wet}} = S_{\text{exp}} \cdot 2 \cdot 1.02 = 161\,\text{ft} \cdot 2.04 = 328.44\,\text{ft}^2$. Using figure 11.3, with $t/c = 0.09$ and $\Lambda = 43.5^{\text{o}}$, we get $K = 1.127$. We find the tip chord as $C_T = \sigma C_R = 0.8 \cdot 15.5\,\text{ft} = 12.4\,\text{ft}$. The mean chord is

$$\text{m.a.c.} = \frac{2}{3}C_R\left(1+\sigma-\frac{\sigma}{1+\sigma}\right) = \frac{2}{3}15.5\cdot\left(1+0.8-\frac{0.8}{1+0.8}\right) = 14.01\,\text{ft}$$

and so, the Reynolds number is $\text{Re} = 2203225 \cdot 14.01 = 3.107 \cdot 10^7$ and from figure 11.2 we get $C_f = 0.0028$. All in al

$$f_{\text{vt}} = K \cdot C_f \cdot S_{\text{wet}} = 1.127 \cdot 0.0028 \cdot 328.44\,\text{ft}^2 = 1.04\,\text{ft}^2$$

and so

$$\Delta C_{D_{P,\text{vt}}} = \frac{f_{\text{vt}}}{S_{\text{ref}}} = \frac{1.04\,\text{ft}^2}{1000\,\text{ft}^2} = 0.00104$$

– **Pylons**. We have $S_{\text{wet}} = 117\,\text{ft}^2$. From $t/c = 0.06$ and $\Lambda = 0^{\text{o}}$, we use figure 11.3 and get $K = 1.12$. From m.a.c.=16.2 ft we get the Reynolds number $\text{Re} = 3.569 \cdot 10^7$ and from figure 11.2, we obtain $C_f = 0.0027$. Thus

$$f_{\text{pyl}} = K \cdot C_f \cdot S_{\text{wet}} = 1.12 \cdot 0.0027 \cdot 117\,\text{ft} = 0.354\,\text{ft}^2$$

and

$$\Delta C_{D_{P,\text{pyl}}} = \frac{f_{\text{pyl}}}{S_{\text{ref}}} = \frac{0.354\,\text{ft}^2}{1000\,\text{ft}^2} = 0.000354$$

– **Nacelles**. We have $S_{\text{wet}} = 455\,\text{ft}^2$, the effective fineness ratio $L/D = 5.0$ and the length 16.8 ft. From Figure 11.4, using $L/D = 5$, we get the form factor $K = 1.29$. While for the C_f, we use the Reynolds number based on the length of the nacelles:

$$\text{Re} = 2203225 \cdot 16.8\,\text{ft} = 3.7 \cdot 10^7$$

in Figure 11.2, getting a skin friction coefficient of $C_f = 0.0027$. And so

$$f_{\text{nac}} = K \cdot C_f \cdot S_{\text{wet}} = 1.29 \cdot 0.0027 \cdot 455\,\text{ft}^2 = 1.585\,\text{ft}^2$$

and

$$\Delta C_{D_{P,\text{nac}}} = \frac{f_{\text{nac}}}{S_{\text{ref}}} = \frac{1.585\,\text{ft}^2}{1000\,\text{ft}^2} = 0.001585$$

– **Flap hinges**: simply $\Delta f = 0.15\,\text{ft}^2$.

We add up all the terms, multiply by a 1.1 factor accounting for surface roughness, and obtain

$$f = \sum_i f_i = (5.98 + 7.28 + 1.88 + 1.04 + 0.354 + 1.585 + 0.15) \cdot 1.1 = 20.1\,\text{ft}^2$$

and so,

$$C_{D_P} = \frac{f}{S_{\text{ref}}} = \frac{20.1\,\text{ft}^2}{1000\,\text{ft}^2} = 0.0201$$

(b) Induced drag coefficient.
We will use

$$C_{D_i} = \frac{C_L^2}{\pi\,\text{AR}\,e}$$

where

$$C_L = \frac{W}{q_0 S_{\text{ref}}} = \frac{98000\,\text{lb}}{256.21\,\text{lb/ft}^2 \cdot 1000\,\text{ft}^2} = 0.383$$

aspect ratio is $\text{AR} = 8.69$. Using $C_{D_P} = 0.0201$, $\text{AR} = 8.69$ and $\Lambda = 24.5^{\text{o}}$, from figure 11.8, we get $e = 0.805 \cdot 0.98 = 0.789$ (see figure 5 below) and so

$$C_{D_i} = \frac{0.383^2}{\pi \cdot 8.69 \cdot 0.789} = 0.00681$$

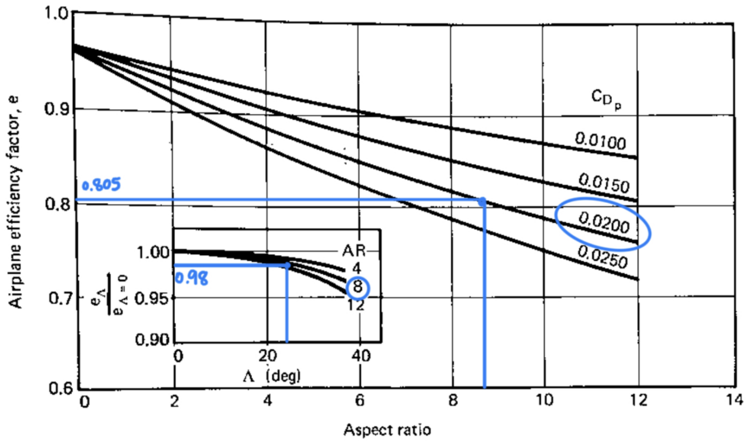

Figure 5: Use of figure 11.8

(c) Total incompressible drag coefficient.
The total incompressible drag coefficient is the sum of the parasite and induced drag coefficients

$$C_D = C_{D_P} + C_{D_i} = 0.0201 + 0.0068 = 0.0269$$

(d) Total incompressible drag in pounds.
The total drag is

$$D = C_D q_0 S_{\text{ref}} = 0.0269 \cdot 256.21\,\text{lb/ft}^2 \cdot 1000\,\text{ft}^2 = 6892\,\text{lb}$$

(e) Ratio of lift to drag, neglecting compressibility.
The lift-to-drag ratio is computed using the fact that in cruise, $L = W$,

$$\frac{L}{D} = \frac{98000}{6892} = 14.22$$

- **11.2** At speed where compressibility may be ignored,

$$C_D = C_{D_P} + \frac{C_L^2}{\pi \text{AR} e}$$

(a) Determine the C_L for best C_L/C_D (Try for minimum C_D/C_L).
We want to minimize

$$\frac{C_D}{C_L} = \frac{C_{D_P}}{C_L} + \frac{C_L}{\pi\,\text{AR}\,e}$$

And so we impose $\frac{d(C_D/C_L)}{dC_L} = 0$, that is

$$\frac{d(C_D/C_L)}{dC_L} = -\frac{C_{D_P}}{C_L^2} + \frac{1}{\pi\,\text{AR}\,e} = 0 \qquad \rightarrow \qquad C_{D_P} = \frac{C_L^2}{\pi\,\text{AR}\,e}$$

and so, $C_L = \sqrt{C_{D_P}\,\pi\,\text{AR}\,e}$.

(b) What is the maximum ratio of lift per drag (C_L to C_D) in terms of C_{D_P}, AR and e?
Applying the condition above to L/D, we get

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D_P} + \frac{C_L^2}{\pi\,\text{AR}\,e}} = \frac{\sqrt{C_{D_P}\,\pi\,\text{AR}\,e}}{C_{D_P} + \frac{C_{D_P}\,\pi\,\text{AR}\,e}{\pi\,\text{AR}\,e}} =$$

$$= \frac{\sqrt{C_{D_P}\,\pi\,\text{AR}\,e}}{2C_{D_P}} = \frac{\sqrt{\pi}}{2}\sqrt{\frac{\text{AR}\,e}{C_{D_P}}} = 0.886\,\frac{b\sqrt{e}}{\sqrt{f}}$$

where b is the wingspan and $f = C_{D_P} S$ is the equivalent profile drag area.

- **12.1** An airplane with an unswept, 12% thick wing, a wing planform area of 450 ft^2, a span of 60 ft, and a mean aerodynamic chord (m.a.c) of 8 ft is flying at a density altitude of 28,000 ft at a speed of 400 mph. The ambient temperature is 430ºR. The gross weight is 30,000 lb. The exposed wing area is 80% of the total wing area. The wing parasite drag is 35% of the total parasite drag. The airfoil is a conventional peaky type. Determine (You can use $\mu_0 = 3.25 \cdot 10^{-7}$ lb-s/ft^2 from Fig 10.14 (homework 2))

(b) Total parasite drag in pounds.
Density is obtained from Table A.2 $\rho_0 = 0.0095801$ slug/ft^3, velocity is $V_0 = 400\,\text{mph} = 585.67\,\text{ft/s}$. The dynamic pressure is then

$$q_0 = \frac{1}{2}\rho_0 V_0^2 = \frac{1}{2}0.0095801\,\text{slug/ft}^3 \cdot (585.67\,\text{ft/s})^2 = 164.87\,\text{lb/ft}^2$$

The Reynolds number, based on the mean aerodynamic chord is

$$\text{Re} = \frac{\rho_0 V_0 c}{\mu_0} = \frac{0.0095801\,\text{slug/ft}^3 \cdot 585.67\,\text{ft/s} \cdot 8\,\text{ft}}{3.25 \cdot 10^{-7}\,\text{lb s/ft}^2} = 1.3835 \cdot 10^7$$

To find the skin friction coefficient, we use figure 11.2 from the book, taking into account that both axes are logarithmic (see figure 6). To find the form factor K, we

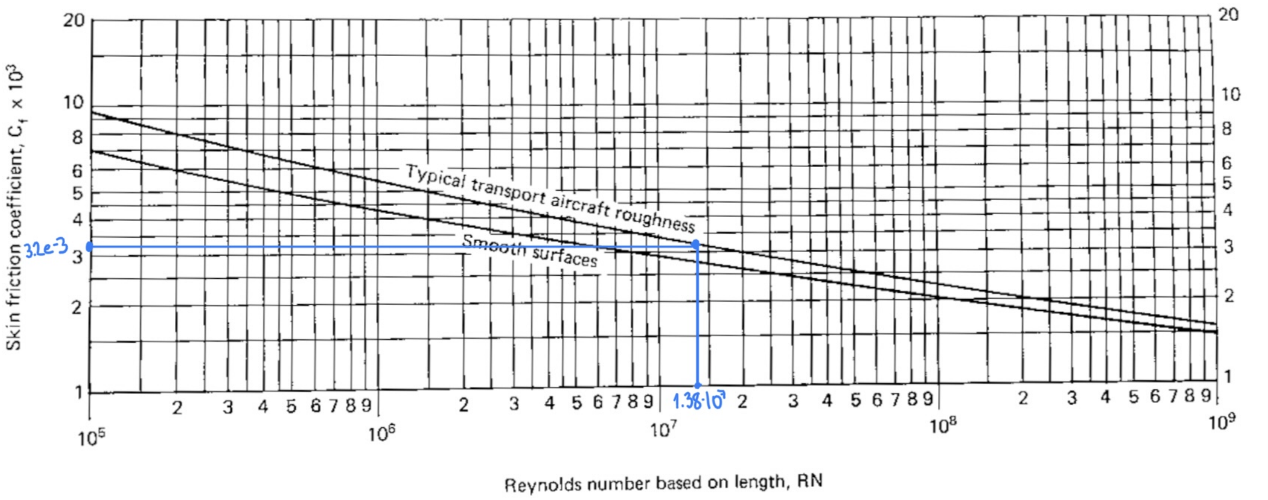

Figure 6: Use of figure 11.2 (Flat-plate skin friction coefficient ; turbulent boundary layer; $M = 0.50$)

can either use the formula

$$K = (1 + Z \cdot (t/c) + 100(t/c)^4)$$

with

$$Z = \frac{(2 - M_0^2)\cos\Lambda_{c/4}}{\sqrt{1 - M_0^2\cos^2\Lambda_{c/4}}} = 2.0207$$

since $M_0 = 0.5$ and $\Lambda_{c/4} = 0^{\text{o}}$. Then we get $K = 1.263$. Alternatively, we can use figure 11.3 from the book taking $\Lambda_{c/4} = 0^{\text{o}}$ and $t/c = 0.12$, from which we get $K = 1.26$ (see figure 7).

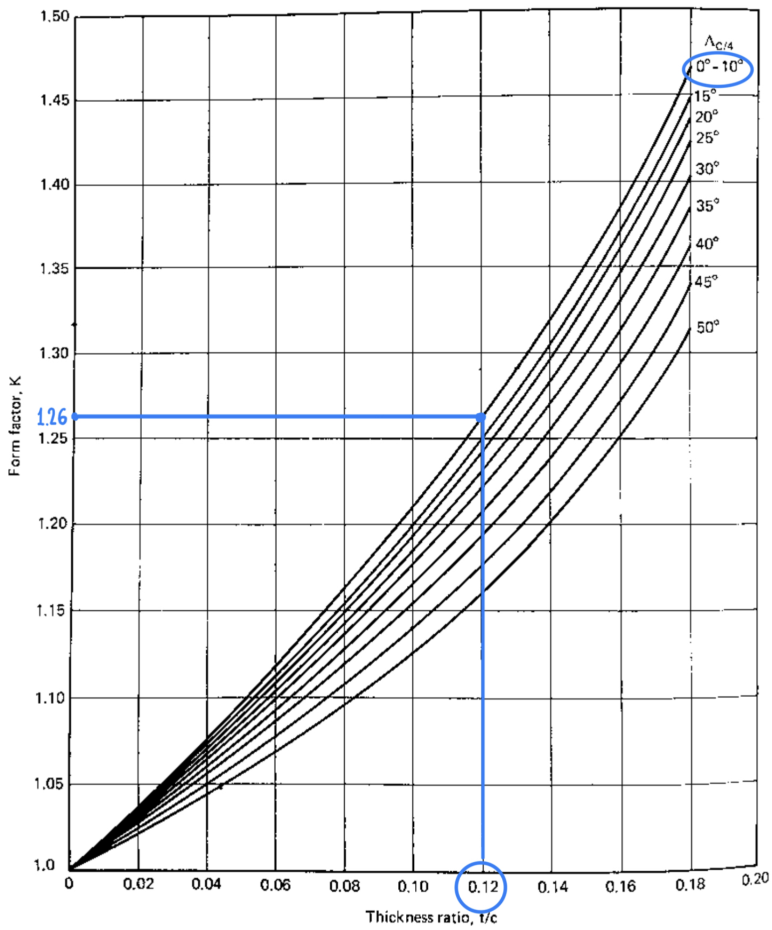

Figure 7: Use of figure 11.3 (Aerodynamic surface form factor)

The exposed surface is

$$S_{\text{exp}} = \%_{\text{exp}} \cdot S_{\text{ref}} = 0.8 \cdot 450\,\text{ft}^2 = 360\,\text{ft}^2$$

and the wetted surface (the surface in contact with air) is

$$S_{\text{wet}} = 2 \cdot 1.02 \cdot S_{\text{exp}} = 2.04 \cdot 360\,\text{ft}^2 = 734.4\,\text{ft}^2$$

The parasite drag coefficient due to the wings is

$$\Delta C_{D_{P,\text{ wing}}} = \frac{K\,C_f\,S_{\text{wet}}}{S_{\text{ref}}} = \frac{1.263 \cdot 3.2 \cdot 10^{-3} \cdot 734.4\,\text{ft}^2}{450\,\text{ft}^2} = 0.0066$$

Since the wing parasite drag accounts for 35% of the total parasite drag, we can find it by

$$\Delta C_{D_{P,\text{total}}} = \frac{\Delta C_{D_{P,\text{ wing}}}}{0.35} = 0.01886$$

and so, the parasite drag is

$$D_P = C_{D_{P,\text{total}}} \cdot q_0 \cdot S_{\text{ref}} = 0.01886 \cdot 164.87\,\text{lb/ft}^2 \cdot 450\,\text{ft}^2 = 1399\,\text{lb}$$

(c) Boundary layer thickness at the trailing edge of the m.a.c., assuming the flat-plate turbulent boundary layer equation.
The boundary layer thickness is, for turbulent flows,

$$\delta = \frac{0.37\,c}{\text{Re}^{0.2}} = \frac{0.37 \cdot 8\,\text{ft}}{(1.3835 \cdot 10^7)^{0.2}} = 0.1101\,\text{ft}$$

(d) Induced drag in pounds.
From section (a), we have that $C_L = 0.404$, the Aspect Ratio is

$$\mathrm{AR} = \frac{b^2}{S_{\mathrm{ref}}} = \frac{(60\,\mathrm{ft})^2}{450\,\mathrm{ft}^2} = 8$$

and from figure 11.8, we obtain the airplane efficiency factor $e = 0.82$, using AR=8 and $C_{D_P} \simeq 0.02$ (see figure 8).

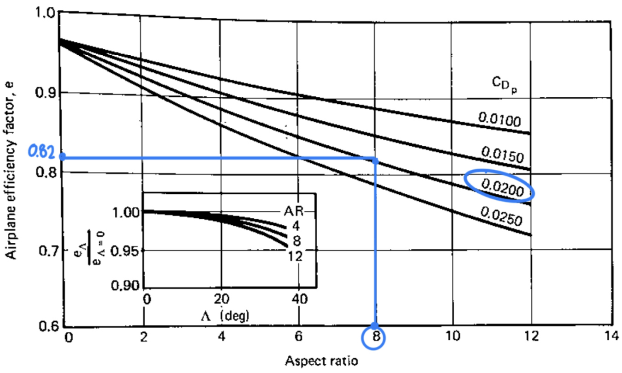

Figure 8: Use of figure 11.8 (Airplane efficiency factor, e)

The induced drag coefficient is

$$C_{D_i} = \frac{C_L^2}{\pi\,\mathrm{AR}\,e} = \frac{0.404^2}{\pi \cdot 8 \cdot 0.82} = 0.00792$$

and so,

$$D_i = C_{D_i} \cdot q_0 \cdot S_{\mathrm{ref}} = 0.00792 \cdot 164.87\,\mathrm{lb/ft}^2 \cdot 450\,\mathrm{ft}^2 = 587.6\,\mathrm{lb}$$

(f) Compressibility drag and total drag in pounds and the ratio of lift to drag.
(Done last week).

- **2.4** For the NACA 2412 airfoil, with the measured data in Fig. 2.6 calculate the location of the aerodynamic center.
The location of the aerodynamic center x_{AC}/c is located at

$$\frac{x_{AC}}{c} = -\frac{m_0}{\mathrm{a}_0} + \frac{1}{4}$$

where

$$\mathrm{a}_0 = \frac{dC_L}{d\alpha} \qquad \text{and} \qquad m_0 = \frac{dC_m}{d\alpha}$$

and so, we take data points within the linear region of the curves, as it is shown in figure 9 and get:

$$\mathrm{a}_0 = \frac{\Delta C_L}{\Delta\alpha} = \frac{1.1 - (-0.6)}{8 - (-8)} = \frac{1.7}{16\,\mathrm{deg}} = 0.1062\,\mathrm{deg}^{-1}$$

and similarly

$$m_0 = \frac{\Delta C_m}{\Delta\alpha} = \frac{-0.1 - (-0.2)}{14 - (-10)} = \frac{0.1}{24\,\mathrm{deg}} = 4.167 \cdot 10^{-3}\,\mathrm{deg}^{-1}$$

and so

$$\frac{x_{AC}}{c} = -\frac{4.167 \cdot 10^{-3}\,\mathrm{deg}^{-1}}{0.1062\,\mathrm{deg}^{-1}} + \frac{1}{4} = 0.2108$$

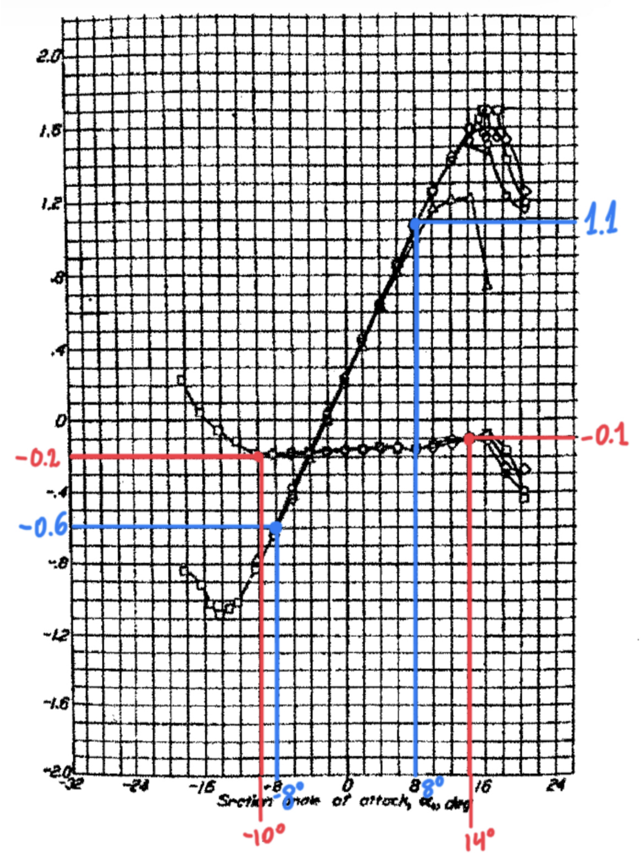

Figure 9: Data points used to find a_0 (blue) and m_0 (red)