

Haskell Tutorial

A guide to Haskell basics

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1 Introduction

Haskell is a purely functional programming language. The following sections will give a brief overview of Haskell, and how to install it.

1.1 Language Overview

In Haskell, everything is a *pure* function - that is, they abide by the Mathematical definition of a function; they map inputs to a unique output.

Data is immutable, meaning that our data types cannot be changed in-place. Combined, this means that there a re few or no side-effects from functions, which make programming more simple.

Haskell is declarative, meaning that the program defines what the issue is, rather than simply giving an algorithm to solve a problem.

Functional programs are easier to verify as we can use maths to verify an algorithm.

1.2 Installation

Link: https://www.haskell.org/ghcup/

I used GHCup to install several components of the Haskell toolchain.

1.2.1 The Haskell Toolchain

The Haskell Toolchain consists of several useful tools for Haskell compilatio and development:

- GHC the Glasgow Haskell Compiler;
- cabal-install Cabal installation tool for managing Haskell software;
- Stack a cross-platform proram for developing Haskell projects;
 - Msys2 provides a UNIX shell and environment which is necessary for executing configuration scripts.
- haskell-language-server a language server which may be integrated into an IDE;

1.2.2 Install Command

The command to use on Windows (in a normal Powershell instance) is

```
Set-ExecutionPolicy Bypass -Scope Process -Force; [System.Net.
    ServicePointManager]::SecurityProtocol = [System.Net.ServicePointManager]::
    SecurityProtocol -bor 3072; try { Invoke-Command -ScriptBlock ([ScriptBlock ]::Create((Invoke-WebRequest https://www.haskell.org/ghcup/sh/bootstrap-haskell.ps1 -UseBasicParsing))) -ArgumentList $true } catch {
    Write-Error $_ }
```

2 Variables

Variables are name-bounded values. Variables in Haskell are immutable - they cannot be changed.

```
name = <value>
```

Variable types are *inferred*. To explicitly assign variables a type,

```
name :: Type
```

In GHCI, the type of a symbol may be retriesed by :t name.

2.1 Local Name Binding

Two methods are provided to bind a symbol to a value in a local scope: let and where. Each follow a "main" expression.

2.1.1 let

Syntax:

For example, re-define ${\tt in_range}$ as follows:

```
in_range x min max =

let

in_lb = min <= x

in_ub = max > x

in

in_lb && in_ub
```

2.1.2 where

Syntax:

For example, re-define in_range as follows:

```
in_range x min max = in_lb && in_ub

where
in_lb = min <= x
in_ub = max > x
```

3 Functions

Functions are defined to map a value from an input set - the *domain* - into an output set - the co-domain. Every output of the function is contained in a subset of the co-domain, called the *range*. **Pure** mathematical functions must be able to map every value from the domain, and each input value must map to only one output value.

In Haskell, occurences of functions are expanded into their RHS.

3.1 Function Definition

Functions are defined by providing its name, a list of arguments, and setting it equal to an expression.

```
name arg1 arg2 ...argn = <expr>
```

The arguments may be set to constants, or may be given a name to accept a variable value.

The type of a function is specified by an arrow (->) separated list of its argument types and its return type:

```
func :: type<sub>1</sub> -> type<sub>2</sub> -> ...-> type<sub>n</sub> -> type<sub>return</sub>
```

For an example, take a function which returns the sum of the elements of an array:

```
sum :: [a] -> Int -- Takes an array of arbitrary type and returns an integer
sum [] = 0 -- Define the sum of an empty array to be zero
sum (h:t) = h + sum t -- Define the sum of an array to be the head plus the
sum of the tail
```

3.1.1 Infix Functions

A good example of infix functions are operators such as +. Functions which take two arguments may be writen between the arguments instead.

For example, say we had add a b = a + b. Then add 5 7 and 5 `add` 7 are equivalent.

To reference the function defined by an operator i.e. +, surround it with parenthesis i.e. (+).

The associativity and precedence of infix operators may be changed via infixr/infixl $\operatorname{prec} \operatorname{sfunction}$ name> where $0 < \operatorname{prec} \le 9$.

3.1.2 Pattern Matching

Parameters of functions may be matched against a pattern. Note that order matters: more specific patterns should be defined first, with general cases last.

- Accept any value by using a symbol e.g. f x = ...
- Accept a certain value e.g. $f \ 0 = \ldots, g \ [] = \ldots$
- List splicing using (x:xs) where x is the head, xs is the tail.
- Tuple unpacking e.g. (Int, Int) could be unpacked using (x,y).
- Accept and extract a custom types. E.g., say we had type Pos = (Int, Int), we would define getX (x, y) = x.
- Accept and extract a custom datatype. E.g., say we had data Num = Zero | Succ Num. We could then define f Zero = ... and f (Succ n) =

Pattern matching may also be done using case ... of ... construct.

3.2 Function Application

A function is applied (called) to some arguments as follows:

```
name arg1 arg2 ...argn
```

For example, consider the function in range x min max = x >= min && x < max, an implementation of $x \in [min, max)$.

Then, in range 3 0 5 would evaluate to True, but in range 5 0 5 would evaluate to False.

3.3 Recursion

Recursion is the process of a function calling itself. Recursion requires a *base case* to stop the function recursing indefinitely.

There are many ways to implement recursion, which will be demonstrated using the factorial, defined as

$$n! = n \cdot (n-1) \cdot \ldots \cdot 1 = \prod_{k=1}^{n} k$$

3.3.1 Defined Base Case

We can hard-code the case where the function is called with the base case:

```
1 fac 1 = 1
2 fac n = n * fib (n-1)
```

3.3.2 If-Else Expression

We can use the if-else expression:

```
if <expr> then <ifTrue> else <ifFalse>
```

For example,

```
1 fac n = if n \le 1 then 1 else n * fac (n-1)
```

3.3.3 Guards

Guards are similar to piece-wise functions.

Where <expr> is a boolean expression. If <expr> is matches, then <value> will be returned. If none is matched, the otherwise is returned.

For example,

3.3.4 Accumulators

In this example, we define an auxiliary function aux inside fac to calculate the the factorial

```
fac n = aux n 1
where
aux n acc
l n <= 1 = acc
l otherwise = aux (n-1) (n*acc)</pre>
```

This is called *tail recursion*. This is because the final result of <code>aux</code> is the result we want, meaning that it is much more memory efficient. A good compiler could even unwind this into a non-recursive imperative approach using a loop. (For more insight, see https://www.youtube.com/watch?v=_JtPhF8MshA.) Normal recursion (using an above definition of <code>fac</code>):

```
fac 4
= 4 * fac 3
= 4 * (3 * fac 2)
= 4 * (3 * (2 * fac 1))
= 4 * (3 * (2 * 1))
= 4 * (3 * 2)
= 4 * 6
= 24
```

Tail recursion (using the definition in this sub-section):

```
fac 4
= aux 3 4
= aux 2 12
= aux 1 24
= 24
```

3.4 Lambdas

Syntax:

Some examples:

- ($\x -> x+1$) 2 returns 3.
- (\x y z -> x+y+z) 1 2 3 returns 6.

Lambdas may be bound to names.

3.5 Higher Order Functions

Higher order functions are functions that take other functions as arguments. For example, a function which takes another function and applies it to an argument:

```
1 app :: (a -> b) -> a -> b
2 app f x = f x
```

A synonym of such a function is the dollar (\$) operator: (\$) :: (a -> b) -> a -> b.

3.5.1 Useful Higher Order Functions

• map :: (a -> b) -> [a] -> [b] applies a function to every element on an array.

$$L' = \{ f(x) : x \in L \}$$

Example: map ($x \rightarrow x^2$) [1,2,3] returns [1,4,9].

• filter :: (a -> Bool) -> [a] -> [a] filters the list on a predicate.

$$L' = \{x \in L : P(x)\}$$

Example: filter ($x \rightarrow mod x 2 == 0$) [1,2,3,4,5] returns [2,4].

• fold :: (a -> b -> b) -> b -> [a] -> b processes a list with some function to produce a single value, starting at a given value.

In Haskell, fold doesn't exist, but rather foldr and foldl which start folding at either end of the list respectively.

foldr
$$(op)$$
 $a [x_1, x_2, ..., x_n] = a (op) x_n (op) ... (op) x_2 (op) x_1$
foldl (op) $a [x_1, x_2, ..., x_n] = a (op) x_n (op) x_{n-1} (op) ... (op) x_1$

Example: foldr (+) 0 [1,2,3,4,5] returns 15.

3.6 Currying

The principle behind currying is that given

We could re-write this as

For example, one could define a function add in multiple ways:

```
1 add x y = x + y
2 add x = (\y -> x + y)
3 add = (\x -> (\y -> x + y))
```

3.7 Currying & Uncurrying

Let's illustrate these terms by defining functions,

3.7.1 Partial Function Application

Using the last definition of add, consider the result of add 1. This would be a new function; add 1:: Int -> Int. This is known as a section.

A good example would be using map.

```
doubleList = map (\xspace x = 2)
```

3.8 Function Composition

Function composition is a way to combine functions. For this, we use the dot (.) operator.

(.) ::
$$(b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow (a \rightarrow c)$$

Then, (f.g) is equivalent to $(\x -> f (g x))$.

For example, all three definitions of descSort are equivalent:

```
descSort = reverse . sort
descSort = (\x -> reverse (sort x))
descSort = reverse (sort x)
```

4 Types

Every expression in Haskell has a type. Types are inferred, even when given explicitly.

Types always begin with an UPPERCASE letter.

The cardinality of a type is how many *states* the data type could hold. Polymorphic types have no cardinality.

4.1 Variable Types

```
var :: type
```

4.1.1 Lists

To define a list of a type, one would write [type]. This may be nested.

4.2 Function Types

```
func :: type1 -> ... -> typeN -> ret_type
```

Where the function func takes n arguments of types type1, ..., typeN and returns ret_type.

4.2.1 Type Variables

Type variables may be used where any type would be permissable and must be lowercase. For example,

This is called "parametric polymorphism".

4.3 Type Aliasing

This doesn't define a new datatype, but rather an alias for another type.

A common example is using a tuple e.g. type Pos = (Int, Int). The type name need not be stated in pattern matching.

4.4 Type Classes

Type classes may be used to restrict the types a polymorphic function may take. This is useful if we would like to use features in a polymorphic function that may only be available to certain types. For a type to be a member of a type class, it must implement all of the required methods.

To impose a constraint on variable a in a function f: f :: (<TypeClass> a, ...) => This is called "ad-hoc polymorphism".

4.4.1 Definition

A type class definition begins with

class <Name> <var> where

Below is a list of function declarations. For a type to be a member of <Name>, it must implement all of these functions.

4.4.2 Implementation

To define a new type which belongs to a type class: instance <TypeClass> <TypeName> where Where below this is a list of function definitions.

4.4.3 Common Type Classes

Common type classes include:

- Eq types which may be compared i.e. (==) is defined;
- Num numeric types, gives us access to standard mathematical operations i.e. (+), (-), (*), abs, ... are defined;
- Ord types which may be ordered, imposes a total ordering i.e. (<), (>), (<=) are defined;
- Read types which may be converted from a string i.e. read is defined;
- Show types which may be converted to a string i.e. show is defined;
- Integral types which are integer-like i.e. div, ... is defined;
- Floating types which are float-like i.e. (/), ... is defined;
- Enum types which may be enumerated i.e. succ, pred, ... are defined;

4.4.4 Instances

Intances allow us to write functions which make use of type classes. Syntax:

instance (<constraints>) => <typeClass> <value> where followed by a list of function definitions.

4.4.5 Derivation

The deriving keyword can be used to automatically generate implementations for the given type class(es).

```
Syntax: data <Name> = ...deriving (<Class1>, ...) 
 Example: data Shape = Circle Int | Rect Int Int deriving Show. Then, print (Circle 5) \rightarrow Circle 5.
```

4.5 Defining Datatypes

The data keyword is used to define a new datatype; unlike the above, these are entirely custom. This may be done using the data keyword:

```
data Name = Constructor1 [<args>] | ...
```

where ${\tt args>}$ are the types of each argument, not literals.

Constructors are either plain values, or functions which take args and return the datatype.

Data consturctors can include polymorphism by including type variables after <Name> e.g. data Maybe a = Nothing | Just a.

4.5.1 Examples

- rock-paper-scissors.hs A basic example revolving around Rock-Paper-Scissors;
- expr.hs A program to build and evaluate expressions;
- tree.hs A representation of a tree structure;
- nat-num.hs A definition of natural numbers using the successor function;

4.6 Records

Records allow data to be stored with an associated name. Syntax:

```
data <Name> = <Name> { <field> :: <type>, ...}
```

This will automatically generate functions <field> :: <Name> -> <type> to extract said properties. This has the side-effect that field names must be globally unique.

4.6.1 Multiple Constructors

```
Note that records may also have mulliple constructors, data Point = D2 { x :: Int, y :: Int } | D3 { x :: Int, y :: Int } Duplicate field names in this context is OK.
```

This will generate functions x, y and z all with the signature x/y/z :: Point -> Int. Both x and y will work on either D2 or D3, but applying z to D2 will throw an exception. For an example, see code/vector.hs.

5 Useful Types

This section will list some common, useful types which should be known.

5.1 Maybe

The Maybe type is incredibly useful, as it can be used to represent the *absence* of a value. This is useful when our function is passed invalid data, for example.

If it defined as: data Maybe a = Nothing | Just a

Functions Found inside Data. Maybe.

- isJust :: Maybe a -> Bool returns if the passed Maybe is a Just value;
- fromMaybe :: a -> Maybe a -> a returns the Just value if a value is present, else returns the default value;
- fromJust :: Maybe a -> a returns the Just value, or throws an exception if recieved Nothing;
- catMaybes :: [Maybe a] -> [a] returns a list containing all the Just values.
- mapMaybe :: (a -> Maybe b) -> [a] -> [b] maps a function over a list of Maybes, discarding any Nothings;
- maybe :: b -> (a -> b) -> Maybe a -> b takes a maybe. If Just, applies a function and returns. Else, returns a default;

5.2 Either

The Either type can be used to represent a union of types – a value which is either one type, or another. It is defined as data Either a $b = Left a \mid Right b$

Functions Found inside Data. Either.

- lefts :: [Either a b] -> [a] returns an array of left-hand side values;
- rights :: [Either a b] -> [b] returns an array of right-hand side values;
- isLeft :: Either a b -> Bool returns whether the provided value is a left-hand side value;
- isRight :: Either a b -> Bool returns whether the provided value is a right-hand side value;
- fromLeft :: a -> Either a b -> a returns the left-hand side value if a Left is provided, else returns a default;
- fromRight :: n -> Either a b -> n returns the right-hand side value if a Right is provided, else returns a default;
- either :: (a -> c) -> (b -> c) -> Either a b -> c processes the left- or right-hand value in the Either as per the given functions;
- partitionEithers :: [Either a b] -> ([a],[b]) traverses the list, placing and Left values in one list and any Right values in another;

Use - Error Handling Either can be used in place of Maybe for error handling. Left can represent a correct input/output with Right holding the incorrect value.

6 Kinds

The types of types and type constructors are called kinds.

A kind signature is denoted by :::. The number of kinds correspond to the number of type parameters a type constructor takes.

• Types which take no type parameters are of kind * ("type").

```
- Bool :: *
- String :: *
- Maybe :: * -> *
```

• When we apply a type parameter, one of the parameters is consumed in the kind

```
- Maybe Int :: *
```

7 Collections

Haskell has two collections: lists, and tuples.

7.1 Lists

A mutable collection of elements of the same type. Every elements has an ordinal. A list of type type has the given type signature

```
name :: [type]
```

7.1.1 Construction

A list may be greated by the following constructor:

- Using square brackets: $[x_1, x_2, \ldots, x_n]$
- Using the prepend opeator: $x_1: x_2: \ldots : x_n: []$. With the syntax of x:list, it prepends x to the list list.

7.1.2 Pre-defined functions

Many pre-defined functions for lists are defined in the Data.List module.

General Functions These functions work on a list of any type, namely [a].

- head t>. This function returns the head (x_1) of the list. Example: head [1,2,3] returns 1.
- tail tail This function returns the tail of the list. Example: tail [1,2,3] returns [2,3].
- length t>. This function returns the length of the list. Example: length [1,2,3] returns 3.
- init init init (1,2,3] returns [1,2].
- null null This function returns whether the list is empty. Example: null [1,2,3] returns False.
- take n take 1,2]. This function returns a list of the first n elements of the list. Example: take 2 [1,2,3,4,5] returns [1,2].
- drop <n> 1ist>. This function returns a list excluding the first n elements of the list. Example:
 drop 2 [1,2,3,4,5] returns [3,4,5].
- ++ ++ 1; this function "append" returns a concatenation of both lists. Example: [1,2] ++ [3,4] returns [1,2,3,4].

Boolean Functions These functions are of the type fn :: [Bool] -> Bool

- and t>. This functions returns True if every elements in t> is True.
- or t>. This functions returns True if at least one element in t> is True.

7.1.3 List Comprehension

List comprehension can be used to transform one or more lists according to a predicate. Syntax:

```
[ <gen> | <elem> <- <li>! <guard>, ...]
```

Where

• <elem> <- st> is called a generator - it binds each value from to <elem> in turn so that they may be used. There may be multiple generators, in which case they will be worked through left-to-right.

• <guard> is a statement which returns a Bool. If false, the current bound value(s) will not be output.

Examples:

- [2*x | x <- [1,2,3]] generates [2,4,6]
- [x^2 | $x \leftarrow [1,2,3], x > 1$] generates [4,9]
- [$(x,y) \mid x \leftarrow [1,2,3], y \leftarrow ['a','b']$] generates [(1,'a'),(1,'b'),(2,'a'),(2,'b'),(3,'a'),(3,'b')]

7.1.4 Ranges

Generate ranges (arithmetic sequences) in Haskell using the ellipse ..:

Where

- step is optional, and default to 1 e.g. [1..5] = [1,2,3,4,5], [1,3..5] = [1,3,5]
- <end> may be omitted to generate an infinite list e.g. [1 ..] = [1,2,3,...].

7.2 Tuples

An immutable collection of elements of $different\ types$.

A tuple has the signature

$$(x,y,\ldots)$$
 :: $(type_x, type_y, \ldots)$

8 Modules

In Haskell, each file is a module. Hence, each file (other than the entry file) must begin with a module declaration:

```
module filename [(n1, n2, ...)] where
```

By default, every symbol is exported. If (n1, n2, ...) is included, only symbols n1, n2 etc. are exported.

8.1 Importing

To import a module, use an import statement:

```
import module [(n1, n2, ...)]
```

By default, every symbol exported by module is imported. If (n1, n2, ...) is included, only symbols n1, n2 etc. are imported.

To import Animals.hs one would write import Animals. To import Farm/Tractor.hs one would write import Farm.Tractor.

Once imported, symbols may be used freely. For example, if the function double is imported, to reference it we would write double. However, if two different definitions for double are imported, we must use its full name e.g. Module.double.

8.1.1 Qualified Imports

Syntax:

```
import qualified module [(...)]
```

This forces the module name to precede any symbols imported. In the example above, Module.double must be used.

8.1.2 Aliased Imports

Syntax:

```
import module [(...)] as alias
```

Using the above example, now, instead of writing Module.double one would now write Alias.double.

8.1.3 Hiding Imports

The hiding keyword can be used to omit imports. E.g. import Prelude hiding (map) would import every function from Prelude but omit map.

8.2 Importing Datatypes

Let's say data DataType = A | B | C is defined inside module Module.

- To import every constructor, we'd write import Module (DataType(..))
- To import only constructor A, we'd write import Module (DataType(A))
- To import constructors A and B, we'd write import Module (DataType(A,B))

9 I/O

I/O produces an issue with Haskell as I/O functions aren't pure.

9.1 The IO Type

All I/O functions in Haskell have the following type: IO <value>.

This special type holds a given I/O action. When the IO value is used, the stored action will be carried out, and IO <value> is returned.

```
For example, in GHCI
> hi = putStrLn "Hello, World!"
> hw
```

Notice how nothing was outputted until the IO value was used. Note that hw may be used mutliple times.

9.2 Input

Hello, World!

• getLine :: IO String - retrieves a line of input from STDIN;

9.3 Output

```
• putStr :: String -> IO () - puts the given string to STDOUT;
```

• putStrLn :: String -> 10 () - puts the given string to STDOUT on a new line;

9.4 Extracting <value>

```
IO is a monad, and should be extracted as such.
greet :: IO ()
greet = do
  putStrLn "What's your name? "
  name <- getLine
  putStrLn $ "Hello, " ++ name ++ "!"
You can only extract values from IO inside of another IO action.</pre>
```

For a more complex example, see code/IO.hs.

9.5 Environment

The following functions are defined in System. Environment.

9.5.1 Command-Line Arguments

```
Command-line arguments are arguments passed to the executable e.g. ./prog.exe arg1 arg2 ...

These can be accessed via getArgs :: IO [String]

Note, the program name "prog.exe" is ommited; this can be access via getProgName :: IO String
```

9.5.2 Environment Variables

The function getEnvironment :: IO [(String, String)] gets a list of all environment variables in name-value pairs.

To get only one variable, the function lookupEnv :: String -> IO (Maybe String) returns the value of an environment variable.

The function withArgs :: [String] -> IO a -> IO a loads sets the environment variables inside an IO action.

9.5.3 Error Handling

Defined in System.Exit

- exitWith :: ExitCode -> IO a exits the program with the provided exit code (ExitCode = ExitSuccess | ExitFailure Int);
- exitSuccess :: IO a exits the program with exit code of success;
- exitFailure :: IO a exits the program with exit code of failure (1);
- die :: String -> IO a prints the given message, then exits the program with exit code of failure (1);

10 Functors

Function is a type class which requires fmap to be implemented.

```
fmap :: Functor f => (a -> b) -> f a -> f b
```

10.1 Laws

- \bullet The mapping must preserve the structure of the arguments;
- Mapping over the identity changes nothing i.e. $fmap id \dots == id \dots;$
- \bullet Distributivity of mapping over composition i.e. fmap (f . g) == fmap f . fmap g;

11 Monads

With the signature of Monad m => m a, a monad is simply a wrapper around another type. A monad implements the following functions (only (>>=) is required): bind ((<<=)); then ((>>)); return; and fail.

11.1 Bind

```
(>>=) :: Monad m => m a -> (a -> m b) -> m b
```

This function takes a monad and a function which takes a raw value and returns a new monad, and returns another new monad.

When implemented, then, we may vary the action taken depending on the value of the provided monad, such as returning a default value – this is what (>>=) does with Maybe, as shown below:

11.1.1 Example with Maybe

```
add :: Num a => Maybe a -> Maybe a a add mx my = mx >>= (\x -> my >>= (\y -> Just (x + y)))
```

Then addition works as expected:

> add (Just 1) (Just 2)

Just 3

But if either mx or my is Nothing, (>>=) skips the function and returns a default of Nothing, propagating the "error".

> add Nothing (Just 2)
Nothing

11.2 Then

```
(>>) :: Monad m => m a -> m b -> m a
```

This function discards the second monad given to it. m >> n is equivalent to $m >>= \setminus_- -> n$. "Then" can be though of wanting to carry out an action but not caring what the result is.

11.3 Return

```
return :: Monad m => a -> m a
```

Return wraps a monad around a raw value.

Using the example from the Bind section, we could substitute the explicit Just with the more general return. Now, this would theoretically work with any appropriately-defined monad.

```
add mx my = mx >>= (\x -> my >>= (\y -> return (x + y)))
```

11.4 Fail

```
fail :: Monad m => String -> m a
```

Fail is intended to be called when something goes wrong. The default implementation is to call error (i.e. error out of the program), but it may be implemented so that certain errors may be handled and return an appropriate monad as a response.

11.5 "do" Syntax

Chaining together applications of (>>=), (>>) and lambda functions can get tedious; that's where the syntactic sugar "do" expression comes in.

The "statements" inside of do are executed in order, and if one "statement" fails this will be propagated through.

11.5.1 Bind

We can substitute the "bind" construct

```
m >>= (\x -> ...)
with

do
    x <- m
    ...
```

11.5.2 Then

We can substitute the "then" construct

```
m >> ...
with

do
    m
    ...
```

11.5.3 Example

Translating the above add function into do-syntax:

```
add mx my = do

x <- mx

y <- yx

return (x + y)
```

11.6 Monad Laws

Defined monads have the following properties. Any monad implemented must abide by these also.

- Left identity return a >>= $k \equiv k$ a where a is any value, k is a function;
- Right identity m >>= return ≡ m where m is a monad;
- Associativity m >>= (\x -> k x >>= h) \equiv (m >>= k) >>= h where m is a monad, k,m are functions.