Solar Radiation Calculation

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Solar Constant

Solar Constant is the intensity of the solar radiation hitting one square meter of the Earth

Or it is the intensity of radiation from the spherical black body, whose temperature is 5785^oK and diameter is $696 \cdot 10^6$ m, per square meter on a spherical surface whose radius is 150.10^9m and with the Sun placed at its centre.

The Solar Constant

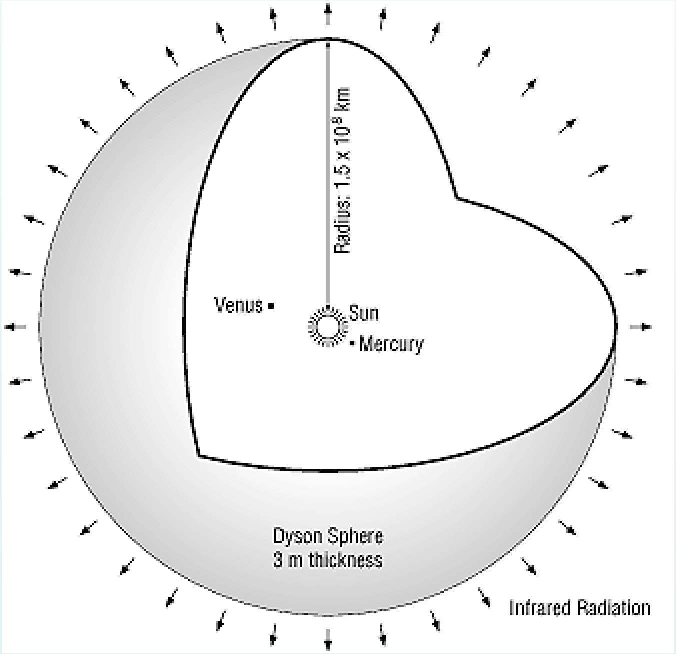
$$G_{sc} = \sigma . T^4 . \left(\frac{4\pi . R}{4\pi . D} \right)^2 = 1367\, W/m^2$$

Where

σ= 5.67·10^{-8} $W/m^2.K^4$ is the Stefan-Boltzmann constant.

R= 696·10^6 m is the Sun radiuses

D=150 ·10^9 m is the average distance between the Sun and the earth

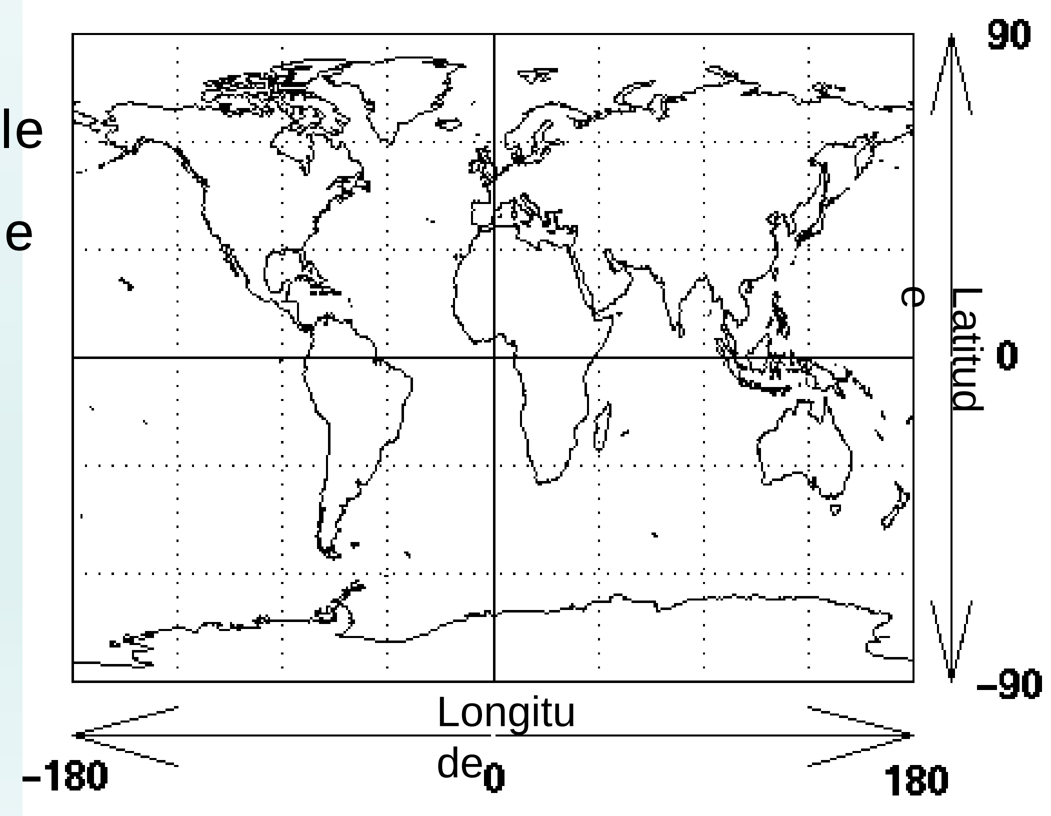


What is the average intensity per square meter of the Earth’s surface?

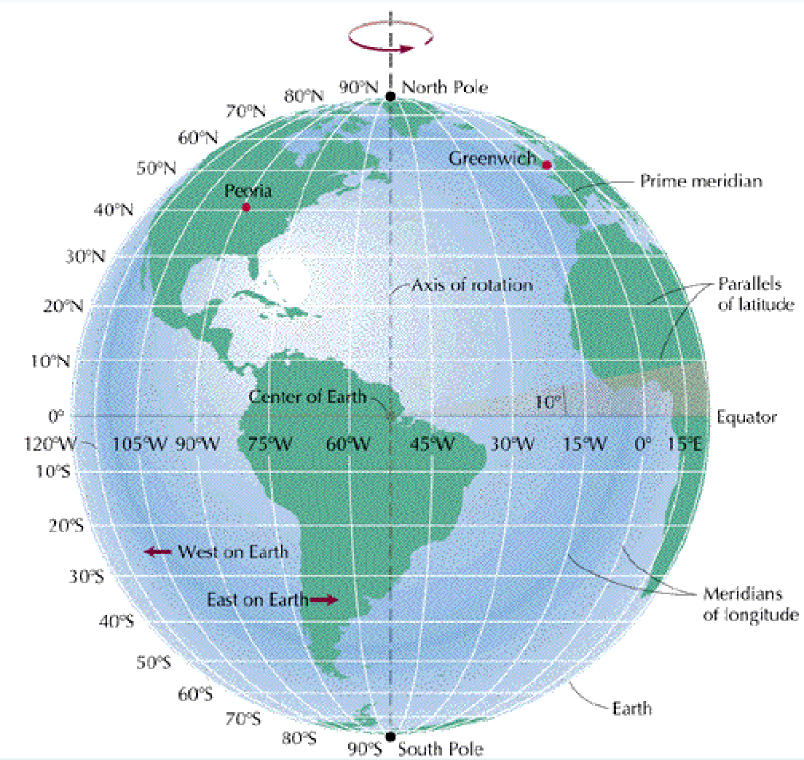
1- Latitude φ

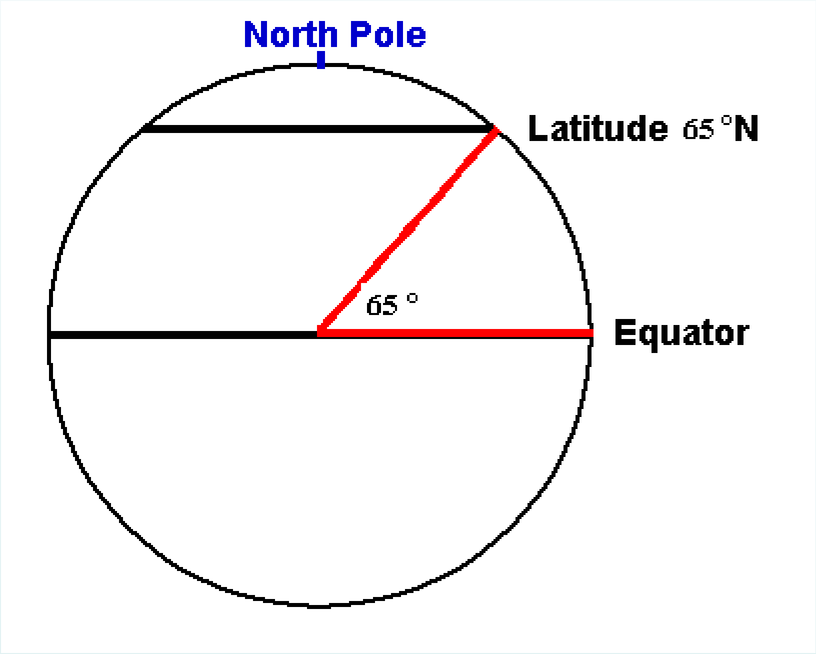
latitude is used to state how far north or south you are, relative to the equator.

- If you are on the equator your latitude is zero.
- If you are near the north pole or the south pole your latitude is nearly 90 degrees.



1- Latitude φ





Latitude is the angle measured at the centre of the Earth, between the Equator plane and where you are. It is expressed either north or south, and varies from 0° to 90°.

2- Longitude 'L'

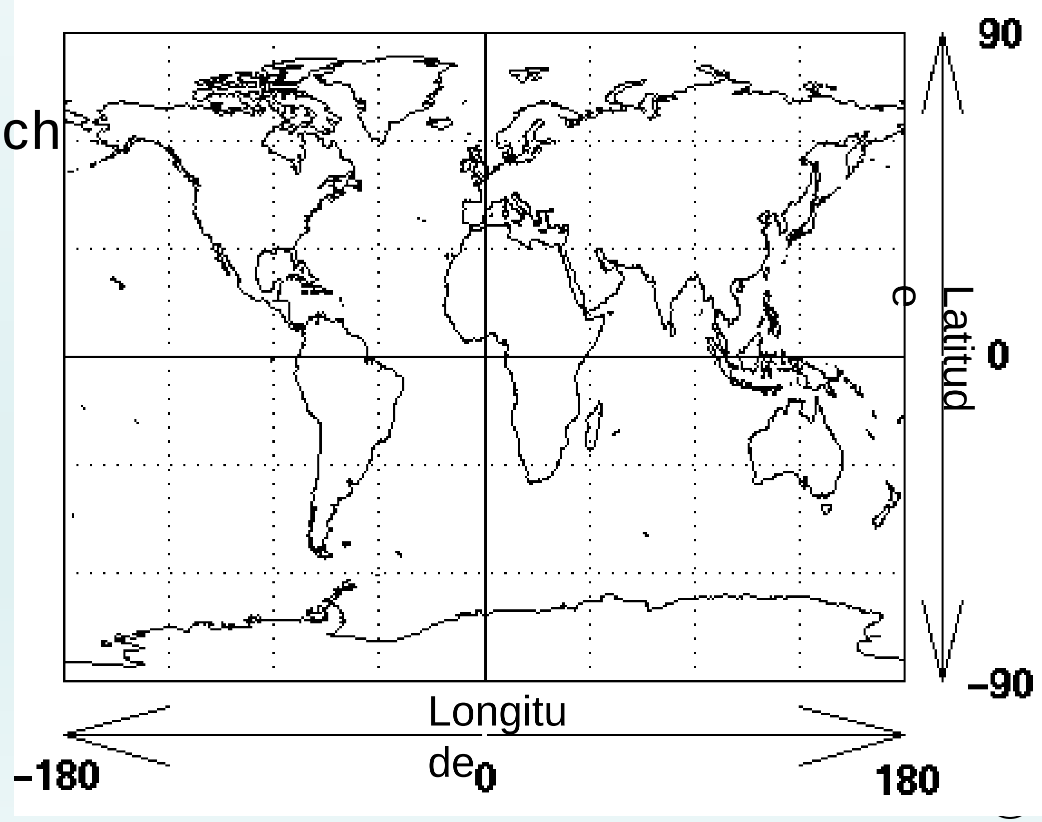
Longitude shows your location in an east or west direction, relative to the Greenwich meridian.

•Places to the east of Greenwich have longitude angles up to 180 degrees east.

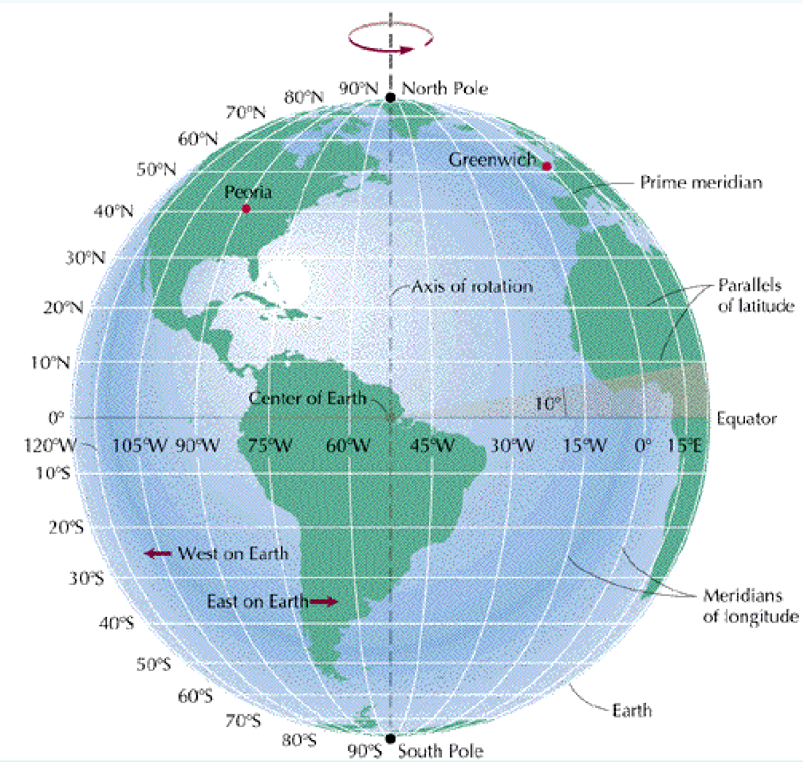
•Places to the west of Greenwich

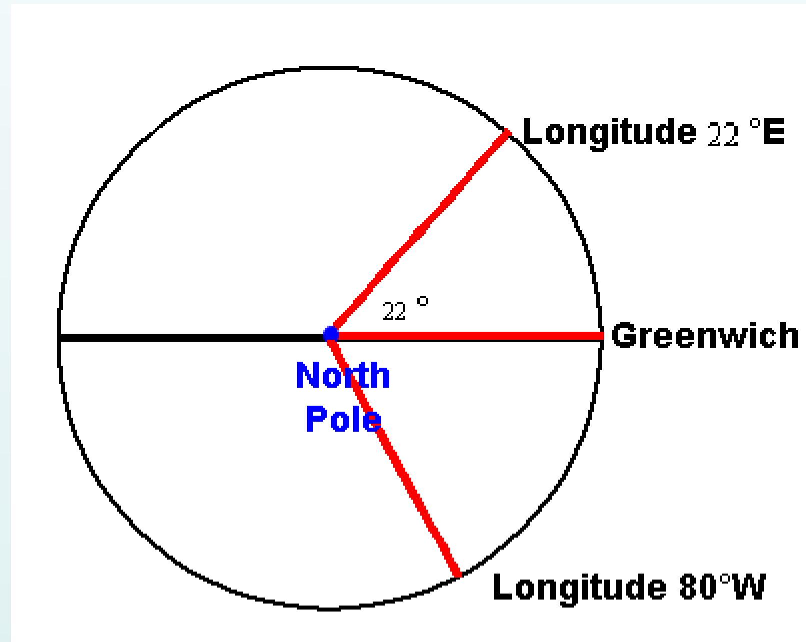
have negative angles up to

180 deg west.



2- Longitude ‘L’





Longitude is the angle at centre of the Earth, between where you are and Greenwich. It can be measured either east or west and varies from 0° to 180°.

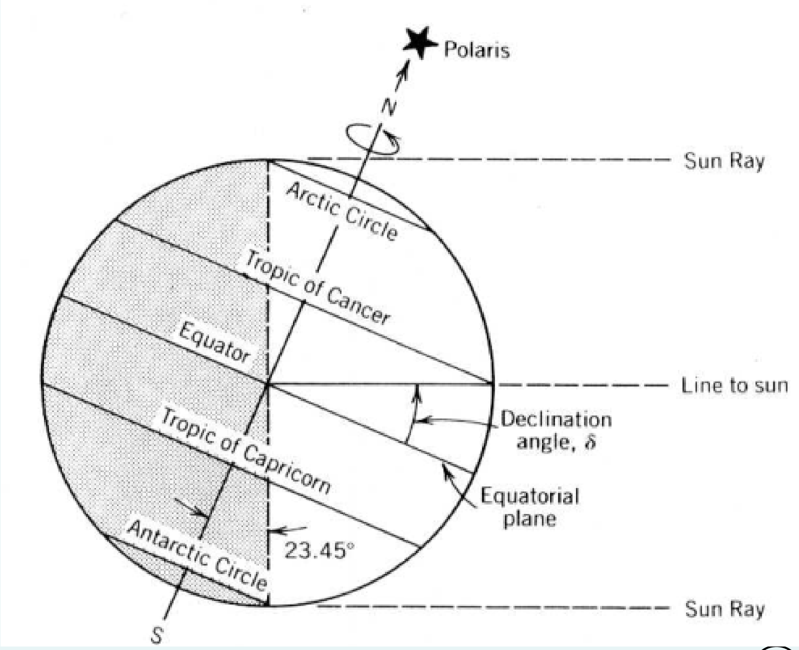
3- Declination δ

Declination is the angle made between the plane of the equator and the line joining the two centres of the earth and the sun:

$$\delta \approx 23.45 \cdot \sin\left(360 \cdot \frac{284+n}{365}\right)$$

Table 1.2: Day numbers and standard mean day of the month

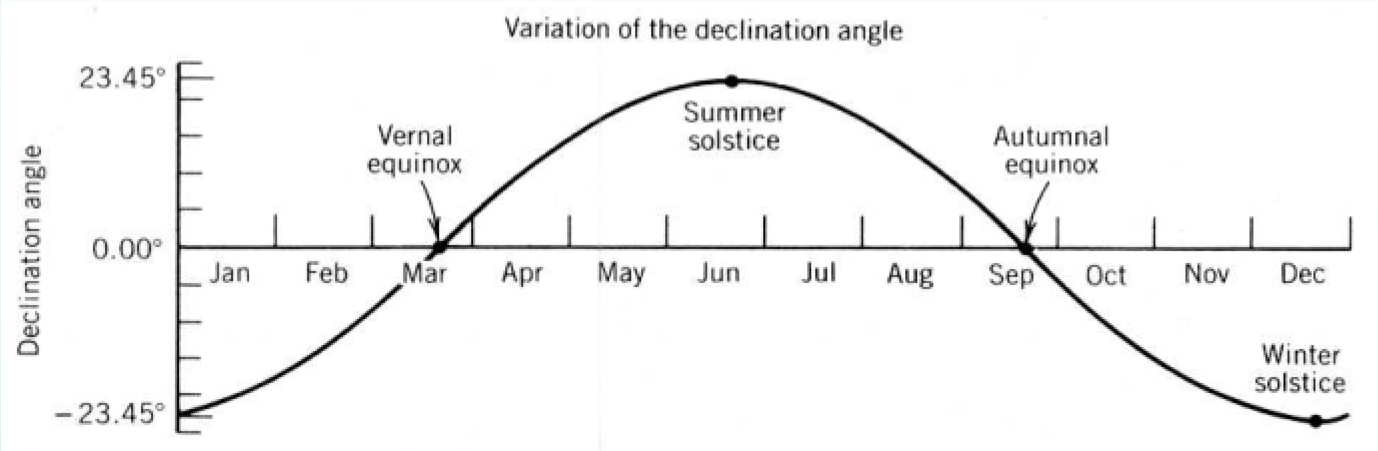
Month	n for i:th Day of Month	For the Average Day of the Month		
		Date	n, Day of Year	δ, Declination
January	i	17	17	-20.9°
February	31 + i	16	47	-13.0°
March	59 + i	16	75	-2.4°
April	90 + i	15	105	9.4°
May	120 + i	15	135	18.8°
June	151 + i	11	162	23.1°
July	181 + i	17	198	21.2°
August	212 + i	16	228	13.5°
September	243 + i	15	258	2.2°
October	273 + i	15	288	-9.6°
November	304 + i	14	318	-18.9°
December	334 + i	10	344	-23.0°



3- Declination δ

The Declination varies between $-23.45° \leq \delta \leq 23.45°$

and is positive during summer and negative during winter

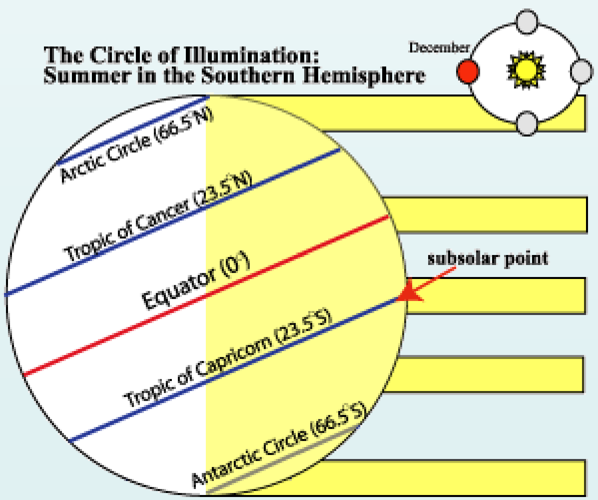


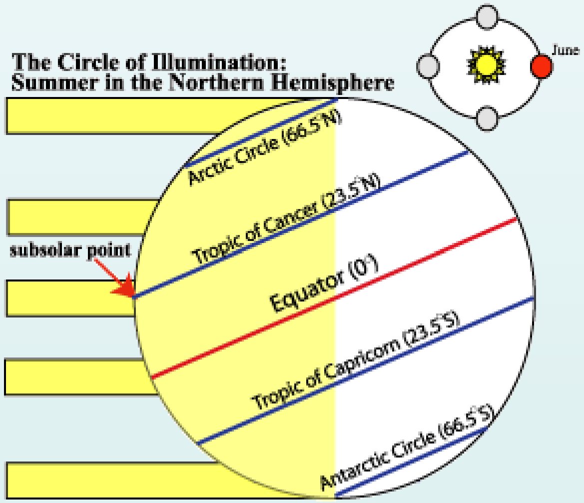
On the same day the declination is equal everywhere on the Earth

What is the declination angle today?

3- Declination δ

Changing this angle causes the seasons, how is that?





4- Hour Angle ω

The hour angle is the sun's angular deviation from south

$$\omega = 15^\circ \cdot (\text{Solar Time} - 12)$$

$-180^\circ \leq \omega \leq 180^\circ$, negative before Solar Noon

4- Hour Angle ω

Two corrections must be applied

$$\text{Solar Time} = \text{Standard time} + \frac{\boxed{4 \cdot (L_{st} - L_{loc})} + \boxed{E}}{60}$$

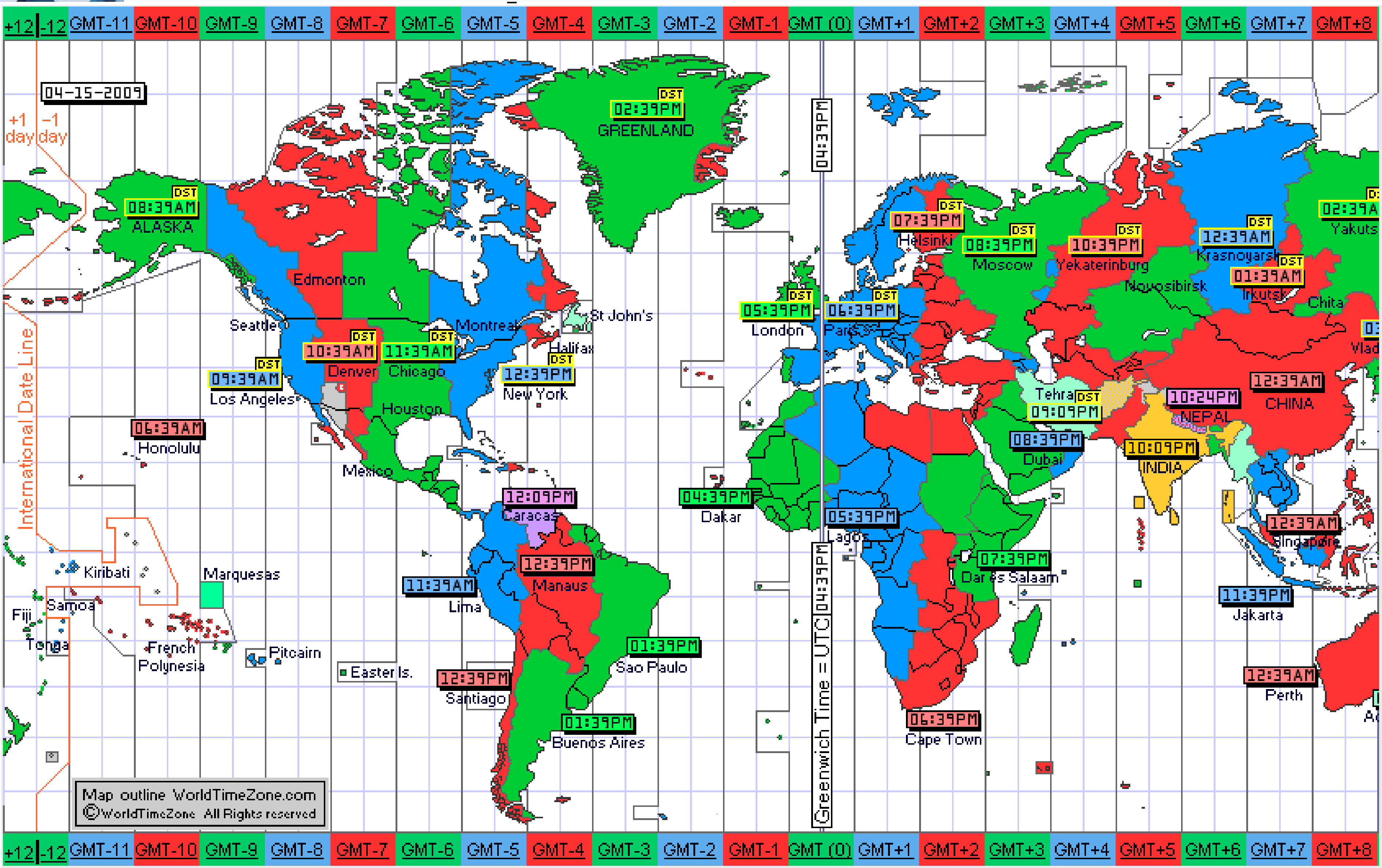
If the longitude is different from the current time zone meridian of the location.

because the earth's orbit and rate of rotation are subject to small fluctuations.

$$E = 229.2 \cdot (0.000075 + 0.001868 \cos B - 0.032077 \sin B - 0.014615 \cos 2B - 0.04089 \sin 2B)$$

B = (n-1)360/365

What is the hour angle in Luleå at 3 PM?


+12
-12
GMT-11
GMT-10
GMT-9
GMT-8
GMT-7
GMT-6
GMT-5
GMT-4
GMT-3
GMT-2
GMT-1
GMT (0)
GMT+1
GMT+2
GMT+3
GMT+4
GMT+5
GMT+6
GMT+7
GMT+8
04-15-2009
+1 day
-1 day
International Date Line
DST
02:39PM
GREENLAND
04:39PM
DST
08:39AM
ALASKA
Edmonton
Seattle
DST
09:39AM
Los Angeles
DST
10:39AM
Denver
DST
11:39AM
Chicago
Houston
Montreal
St John's
Halifax
DST
12:39PM
New York
06:39AM
Honolulu
Mexico
12:09PM
Caracas
12:39PM
Manaus
11:39AM
Lima
01:39PM
Sao Paulo
12:39PM
Santiago
01:39PM
Buenos Aires
Kiribati
Marquesas
Samoa
Fiji
Tonga
French Polynesia
Pitcairn
Easter Is.
DST
05:39PM
London
DST
06:39PM
Paris
DST
07:39PM
Helsinki
DST
08:39PM
Moscow
DST
10:39PM
Yekaterinburg
Novosibirsk
DST
12:39AM
Krasnoyarsk
DST
01:39AM
Irkutsk
Chita
02:39A
Yakuts
Vlad
DST
09:09PM
Tehran
08:39PM
Dubai
10:24PM
NEPAL
10:09PM
INDIA
12:39AM
CHINA
12:39AM
Singapore
11:39PM
Jakarta
12:39AM
Perth
04:39PM
Dakar
05:39PM
Lagos
07:39PM
Dar es Salaam
06:39PM
Cape Town
04:39PM
Greenwich Time = UTC
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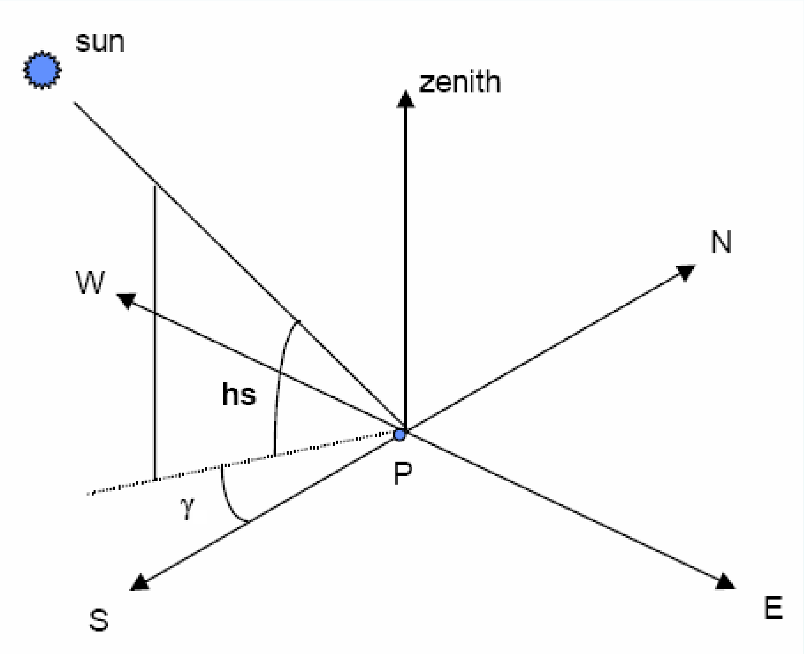
5- Sun altitude hs

Angle between horizontal plane and line joins the site with the centers of the sun (sun elevation).



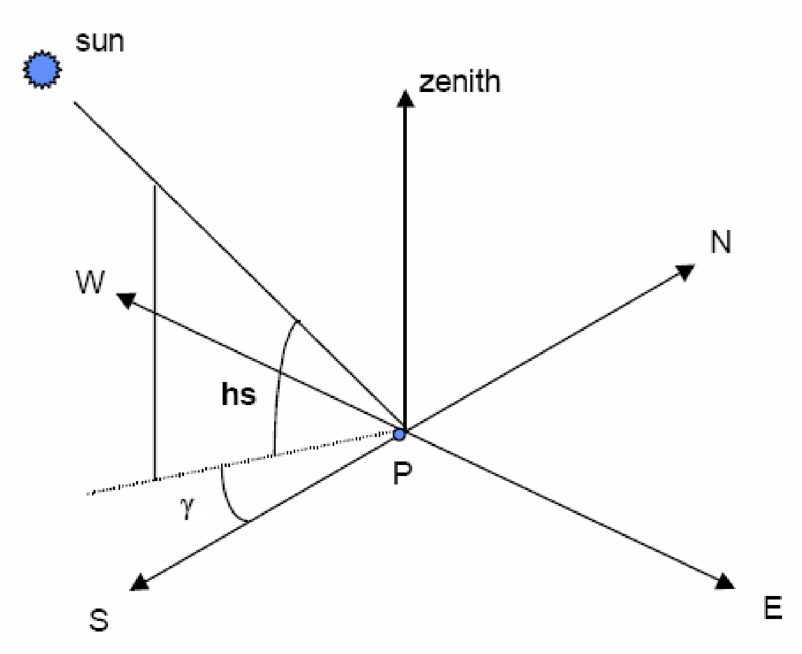
$$hs = Arc\sin\left[\cos(\varphi).\cos(\delta).\cos(\omega)+\sin(\varphi).\sin(\delta)\right]$$

What is the Sun altitude in Luleå at 10 AM?

6- Solar azimuth γ

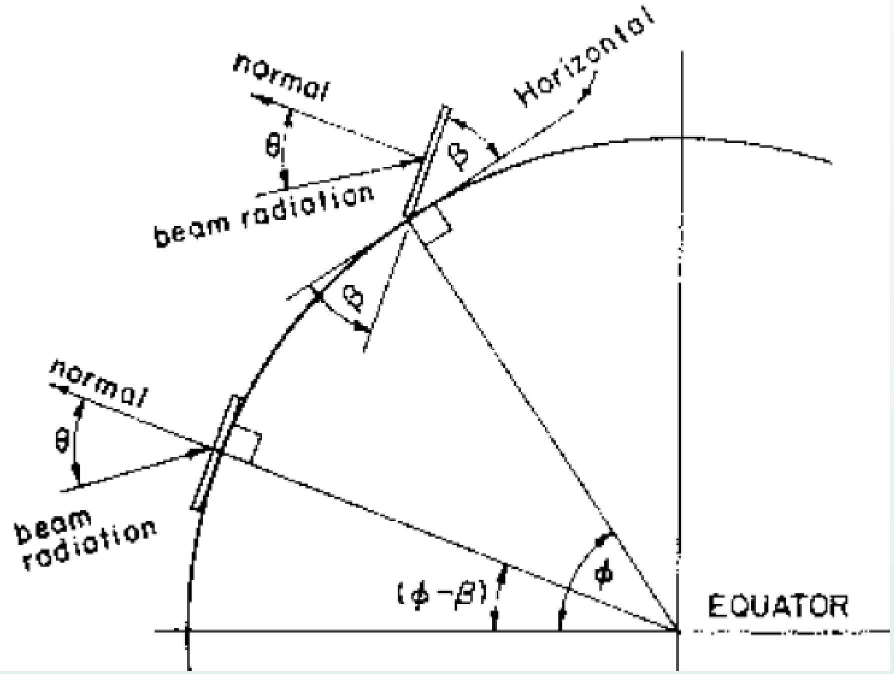
Angle between the projection of the straight line joins the site with the centers of the sun on the horizontal plane and due south.

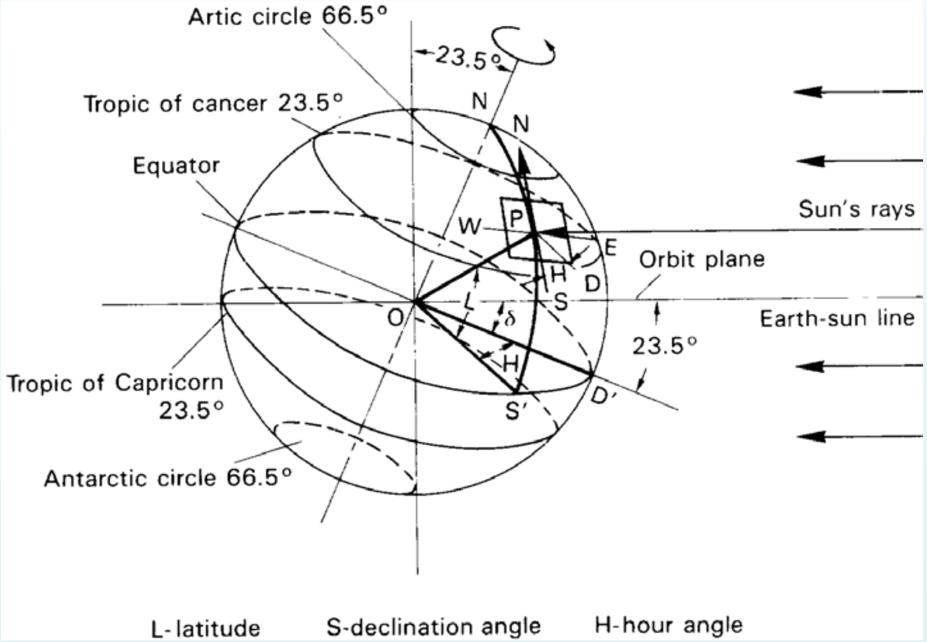
$$\gamma = Arc\sin\left[\frac{\cos(\delta).\sin(\omega)}{\cos(hs)}\right]$$



What is the Solar azimuth in Luleå at 3 PM?

7- Incident Beam Radiation θ





8- Incidence Angle on Sloped Plane

The angle of tilted surface 'β' is angle between the surface and the horizontal.

Incidence angle on a surface of tilt β and azimuth γ on the latitude φ at a time when the declination is δ and the hour angle is ω, is:

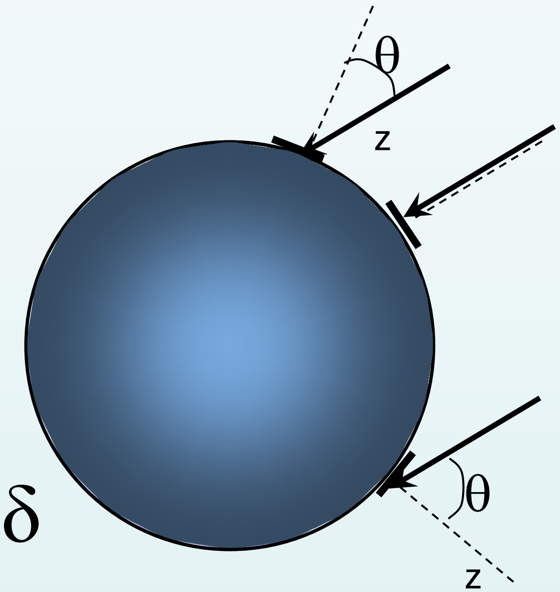
$$\cos\theta = \sin\delta\cdot\sin\varphi\cdot\cos\beta - \sin\delta\cdot\cos\varphi\cdot\sin\beta\cdot\cos\gamma + \cos\delta\cdot\cos\varphi\cdot\cos\beta\cdot\cos\omega$$
$$+\cos\delta\cdot\sin\varphi\cdot\sin\beta\cdot\cos\gamma\cdot\cos\omega + \cos\delta\cdot\sin\beta\cdot\sin\gamma\cdot\sin\omega$$

What is the incidence angle on a sloped surface in Luleå $\beta = 45°$?

9- Zenith Angle θz

Zenith Angle, θ_z is incidence angle of sunbeam on a horizontal surface

It is found by inserting $\beta=0$ in incidence angle equation



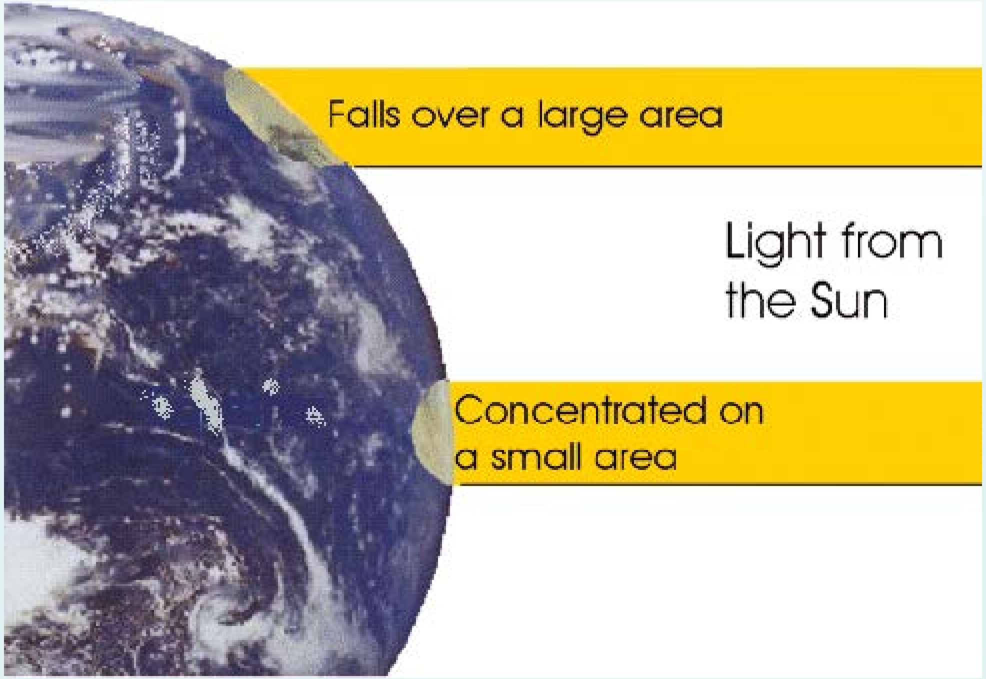
$$\cos\theta_z = \cos\varphi \cdot \cos\delta \cdot \cos\omega + \sin\varphi \cdot \sin\delta$$

$$0^\circ \leq \theta z \leq 90^\circ$$

What is the relationship between zenith angle and Sun altitude?

9- Zenith Angle θz

During the same day, zenith angel determines amount of radiation received by surface



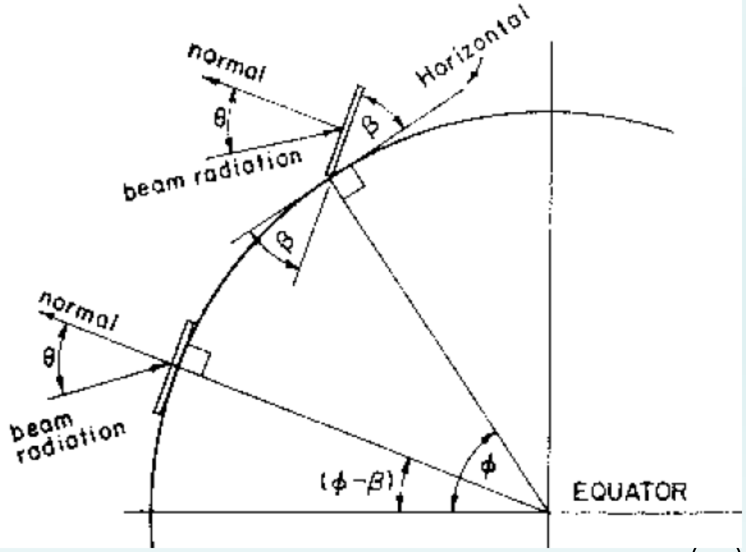
What is the zenith angle today in Luleå at 10 AM?

Incidence Angle on Sloped Plane

The incidence angle on surfaces with slope β due north or south at latitude φ is equivalent to the Zenith Angle at an artificial latitude (φ -β) for the northern hemisphere, or (φ +β) for the southern hemisphere.

$$\cos\theta = \cos(\varphi - \beta)\cdot\cos\delta\cdot\cos\omega + \sin(\varphi - \beta)\cdot\sin\delta$$

$$\cos\theta = \cos(\varphi + \beta)\cdot\cos\delta\cdot\cos\omega + \sin(\varphi + \beta)\cdot\sin\delta$$

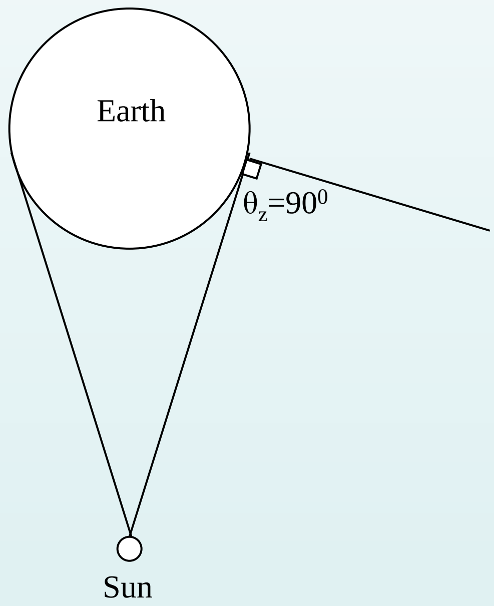


Sunset, Sunrise & Day Length

- The sunset hour angle is the hour angle when $\theta z = 90°$

$$\cos\theta_z = \cos\varphi \cdot \cos\delta \cdot \cos\omega + \sin\varphi \cdot \sin\delta$$

$$\cos\omega_s = -\tan\varphi \cdot \tan\delta$$



- Since the hour angle increases 15°/hour, ω_s can be used to define the day length through the relation

$$N = \frac{2}{15} \cdot Arcos(-\tan\varphi \cdot \tan\delta)$$

How long does the sun shine in Luleå today?

Sunset Hour Angle of Sloped Surface

- The sunset hour angle for a plane tilted towards the south is defined as the hour angle when $\theta = 90°$:

$$\cos\theta = \cos(\varphi - \beta)\cdot\cos\delta\cdot\cos\omega + \sin(\varphi - \beta)\cdot\sin\delta$$

$$\cos\omega_{st} = -\tan(\varphi - \beta)\cdot\tan\delta$$

Slope surface

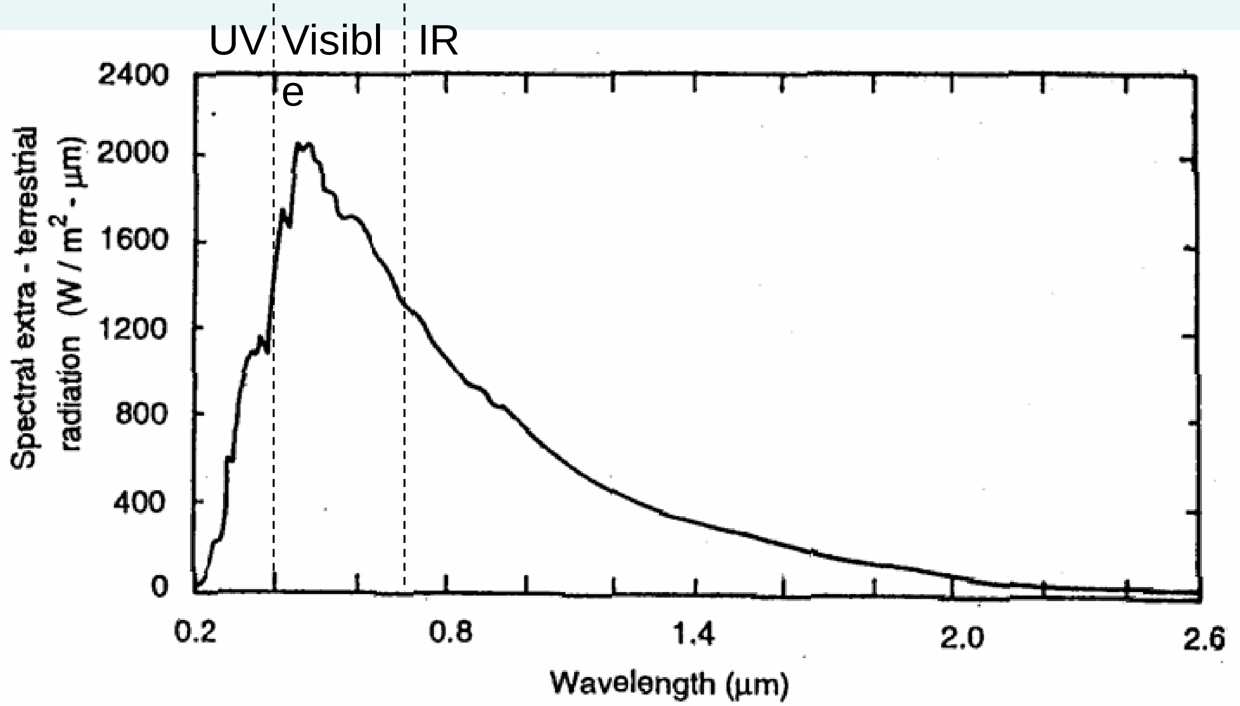
Increasing β is decreases θ this is positive effect simultaneously it decreases day long which it is negative effect.

Solar Radiation

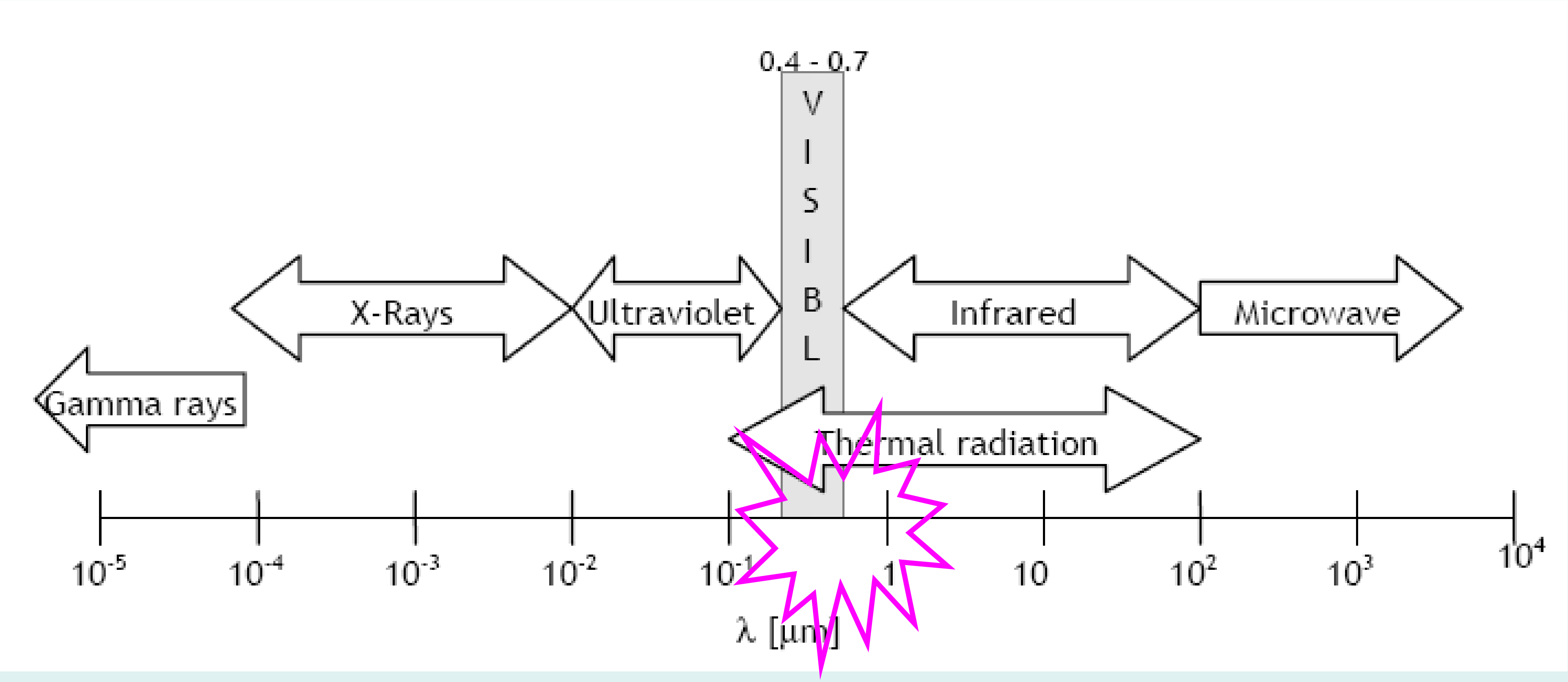
- Extraterrestrial Solar Radiation
- Atmosphere influence on Solar Radiation
- Solar Radiation on ground surface (horizontal plane), clear day

Solar Radiation

- Radiation is a composition of many wavelengths
- Waves of different length carry different amount of energy
- 98% of SR is carried by wave of length $0.3 \leq \lambda \leq 3$ μm



Solar Spectrum



Ultraviolet:

$0.2\ \mu m < \lambda < 0.38\ \mu m$

Visible light:

$0.38\ \mu m < \lambda < 0.78\ \mu m$

Infrared:

$\lambda > 0.78\ \mu m$

Solar Spectrum

1- What is the fraction of the solar radiation in:

- the ultraviolet ($\lambda<0.38\mu m$),
- the visible ($0.38\mu m<\lambda<0.78\mu m$), and
- the infrared ($\lambda>0.78\mu m$).

Solar Spectrum

Table 1.3.1a Extraterrestrial Solar Irradiance (The WRC Spectrum) in Increments of Wavelength[a]

λ μm	$G_{sc,\lambda}$ W/m^2 μm	$f_{0-\lambda}$	λ μm	$G_{sc,\lambda}$ W/m^2 μm	$f_{0-\lambda}$	λ μm	$G_{sc,\lambda}$ W/m^2 μm	$f_{0-\lambda}$
0.250	13.8	0.002	0.520	1820.9	0.243	0.880	965.7	0.621
0.275	224.5	0.005	0.530	1873.4	0.257	0.900	911.9	0.635
0.300	542.3	0.012	0.540	1873.3	0.271	0.920	846.8	0.648
0.325	778.4	0.023	0.550	1875.0	0.284	0.940	803.8	0.660
0.340	912.0	0.033	0.560	1841.1	0.298	0.960	768.5	0.671
0.350	983.0	0.040	0.570	1843.2	0.311	0.980	763.5	0.683
0.360	967.0	0.047	0.580	1844.6	0.325	1.000	756.5	0.694
0.370	1130.8	0.056	0.590	1782.2	0.338	1.050	668.6	0.720
0.380	1070.3	0.065	0.600	1765.4	0.351	1.100	591.1	0.743
0.390	1029.5	0.071	0.620	1716.4	0.377	1.200	505.6	0.783
0.400	1476.9	0.079	0.640	1693.6	0.401	1.300	429.5	0.817
0.410	1698.0	0.092	0.660	1545.7	0.424	1.400	354.7	0.846
0.420	1726.2	0.104	0.680	1492.7	0.447	1.500	296.6	0.870
0.430	1591.1	0.117	0.700	1416.6	0.468	1.600	241.7	0.890
0.440	1837.6	0.129	0.720	1351.3	0.488	1.800	169.0	0.921
0.450	1995.2	0.143	0.740	1292.4	0.507	2.000	100.7	0.941
0.460	2042.6	0.158	0.760	1236.1	0.526	2.500	49.5	0.968
0.470	1996.0	0.173	0.780	1188.7	0.544	3.000	25.5	0.981
0.480	2028.8	0.187	0.800	1133.3	0.561	3.500	14.3	0.988
0.490	1892.4	0.201	0.820	1089.0	0.577	4.000	7.8	0.992
0.500	1918.3	0.216	0.840	1035.2	0.593	5.000	2.7	0.996
0.510	1926.1	0.230	0.860	967.1	0.607	8.000	0.8	0.999

[a] $G_{sc,\lambda}$ is the average solar irradiance over the interval from the middle of the preceding wavelength interval to the middle of the following wavelength interval. For example, at 0.600 μm, 1765.4 W/m^2 μm is the average value between 0.595 and 0.610 μm.

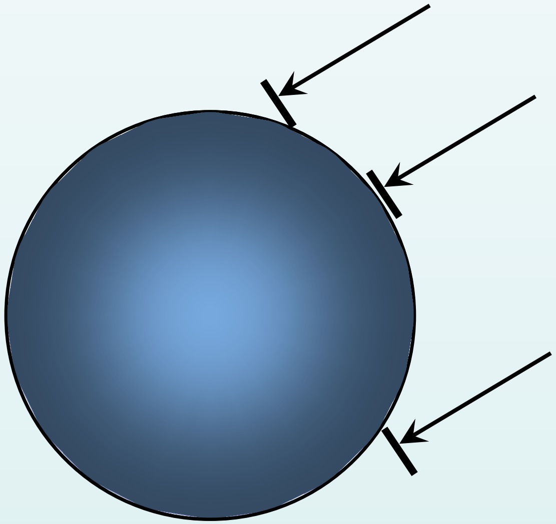
Solar Spectrum

Conclusion

- About 6.5% of the total energy is contained in the ultraviolet region ($\lambda < 0.38\ \mu m$);
- another 47.9% is contained in the visible region ($0.38\ \mu m < \lambda < 0.78\ \mu m$); and
- 45.6% is contained in the infrared region ($\lambda > 0.78\ \mu m$).

Solar Constant

Solar Constant, G_{sc}: is radiation received per square meter of the atmosphere at incidence angle $\theta = 0$ when the earth is at the mean distance from the sun



Solar Constant: 1367 W/m^2

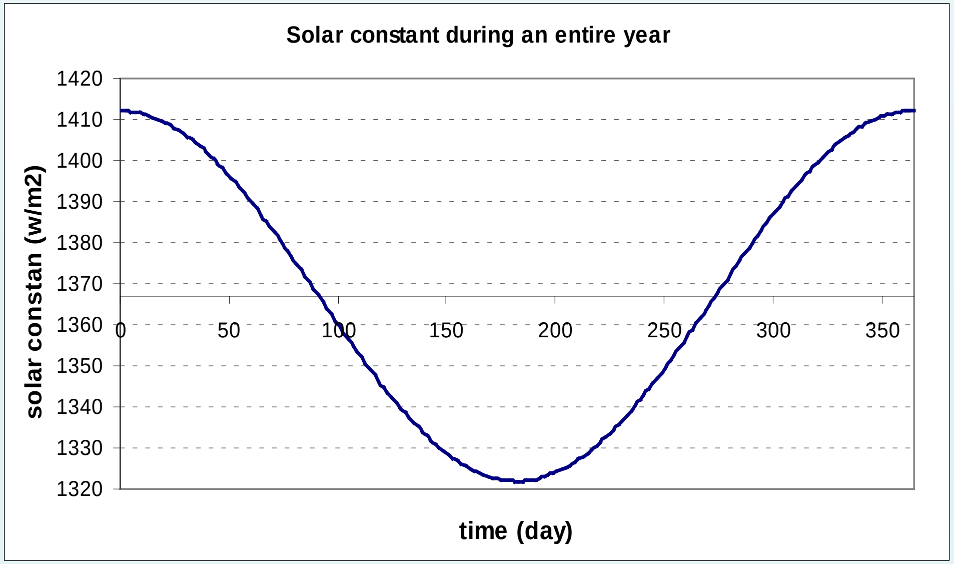
The solar constant is a mean value:

- the earth's orbit is elliptical
  - the distance between the sun and earth varies by 3.3%
    - Solar irradiation must vary approximately over the year

Solar Constant

The exact irradiation incident on a surface of ***$\theta = 0$*** just outside the atmosphere, G_{on} [W/m^2], is calculated from the solar constant, G_{sc}, and the day number as follows:

$$G_{on} = G_{sc} \cdot \left(1 + 0.033 \cdot \cos \frac{360 \cdot n}{365} \right)$$



- Solar irradiation varies approximately ±45 W/m2 over the year

Extraterrestrial Radiation

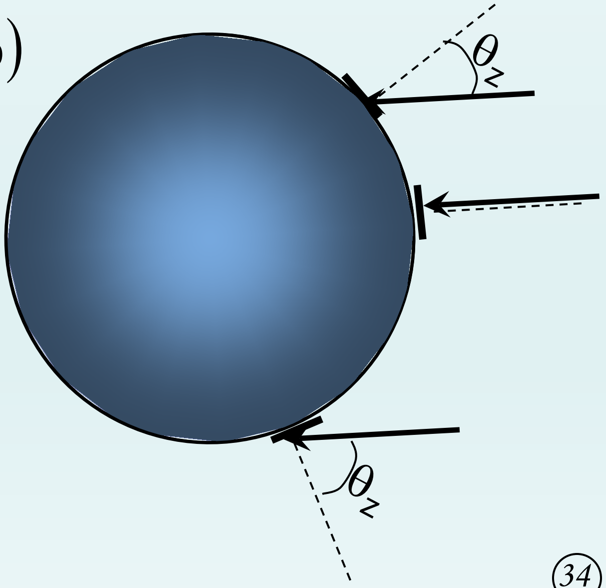
Extraterrestrial radiation G_o [W/m2] is the radiation incident on the surface ***tangent*** to the outer surface of the atmosphere.

It is function of zenith angle, θz, thus, it is function of:

latitude, time during the day and number of the day

ER is given by:

$$G_o = G_{on} \cdot \cos\theta_z = G_{on} \cdot (\cos\varphi \cdot \cos\delta \cdot \cos\omega + \sin\varphi \cdot \sin\delta)$$



2- What is the ET radiation, G_o, at 2 PM in Luleå?

Extraterrestrial Radiation

Integrating ET over a specified time, usually an **hour** or a **day**, we obtain hourly or daily radiation:

$$I_o = \frac{12 \cdot 3600}{\pi} \cdot G_{sc} \cdot \left(1 + 0.033 \cdot \cos\frac{360 \cdot n}{365}\right) \cdot \left(\cos\varphi \cdot \cos\delta \cdot (\sin\omega_2 - \sin\omega_1) + \frac{\pi \cdot (\omega_2 - \omega_1)}{180} \sin\varphi \cdot \sin\delta\right)$$

$$H_o = \frac{24 \cdot 3600}{\pi} \cdot G_{sc} \cdot \left(1 + 0.033 \cdot \cos\frac{360 \cdot n}{365}\right) \cdot \left(\cos\varphi \cdot \cos\delta \cdot \sin\omega_s + \frac{\pi \cdot \omega_s}{180} \sin\varphi \cdot \sin\delta\right)$$

Solar constant:	G_{sc}	[W/m^2]
• Hourly radiation:	I_0	[J/m^2]
• Daily radiation:	H_0	[J/day. m^2]

Extraterrestrial Radiation

3- What is the daily solar radiation on a horizontal surface in the absence of the atmosphere today in Luleå?

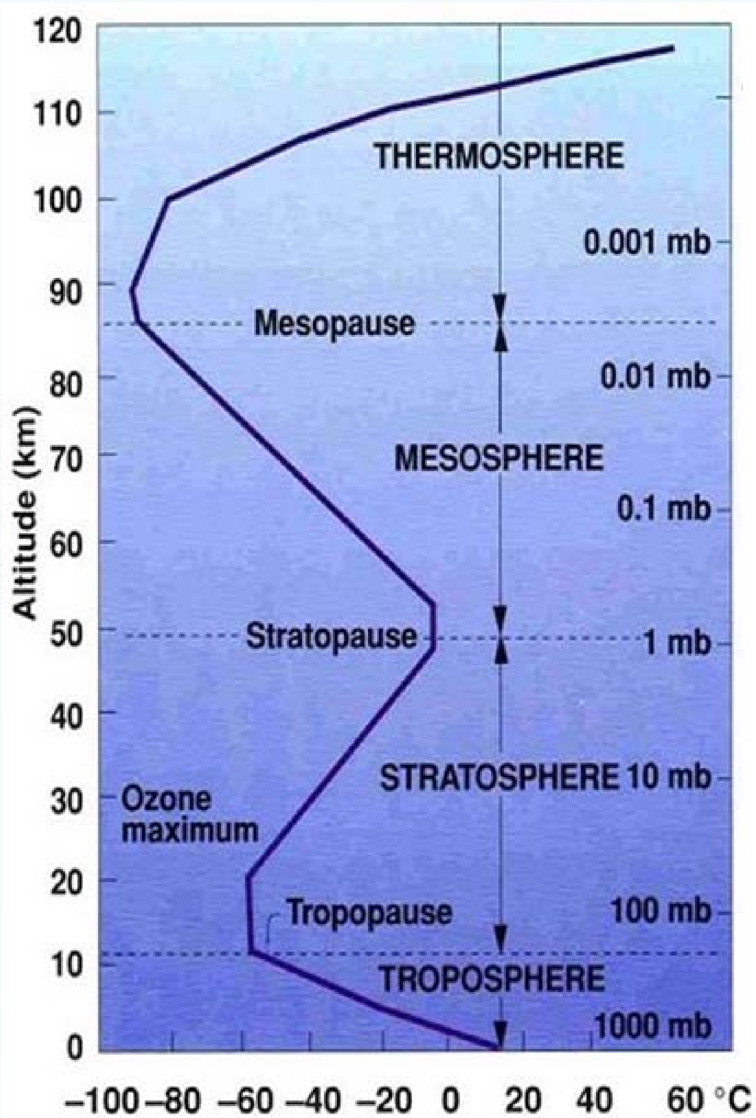
4- What is the solar radiation on a horizontal surface in the absence of the atmosphere today in Luleå between the 13 &14?

Atmosphere Influence on Solar Radiation

- The atmosphere is ~100-500 km thick and consists of gas molecules, particles and dust of different concentration
    - The composition varies with height, weather, location and number of pollution sources

- The density of the atmosphere decreases with the altitude
    - The atmosphere has no sharp border to space
    - 75% of the atmosphere's mass below 10 km

Atmosphere Influence on Solar Radiation



Thermosphere ("Upper Atmosphere")

- Temperature > 1700°C due to solar radiation but the particles are so few that heat transfer is low
- Meteors and shooting stars burn up while passing

Mesosphere

- Altitude 50-85 km above ground
- Temperature falls to -93°C

Stratosphere

- Altitude from 10-50 km
- Includes the ozone layer (19-30 km)
- Temperature increases due to UV-absorption

Troposphere ("Lower Atmosphere")

- About 10 km high
- Temperature drops from 15°C to about -55°C

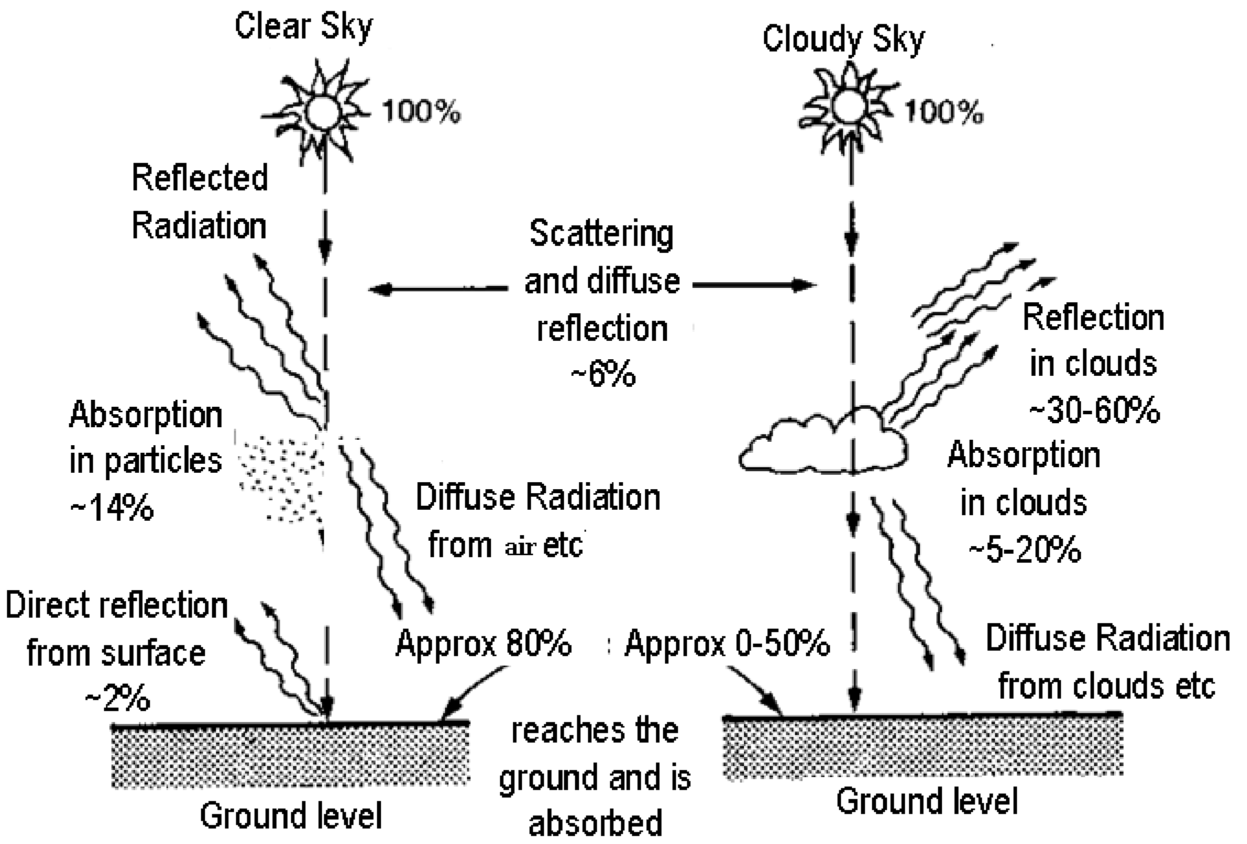
Atmosphere Influence on Solar Radiation

- Significant amount of solar radiation is attenuated as it travels through the atmosphere.
- This attenuation is due to:
  - *absorption of solar radiation by different particles in the atmosphere*
  - *backward scattering and reflection of solar radiation by air particles, water vapor, dust*...

Atmosphere Influence on Solar Radiation

30% is reflected.
17% is absorbed by the atmosphere.
53% reaches the earth surface:

- ***31%*** *direct radiation*
- ***22%*** *diffuse radiation .*



Atmosphere Influence on Solar Radiation

Beam Radiation G_{cb}

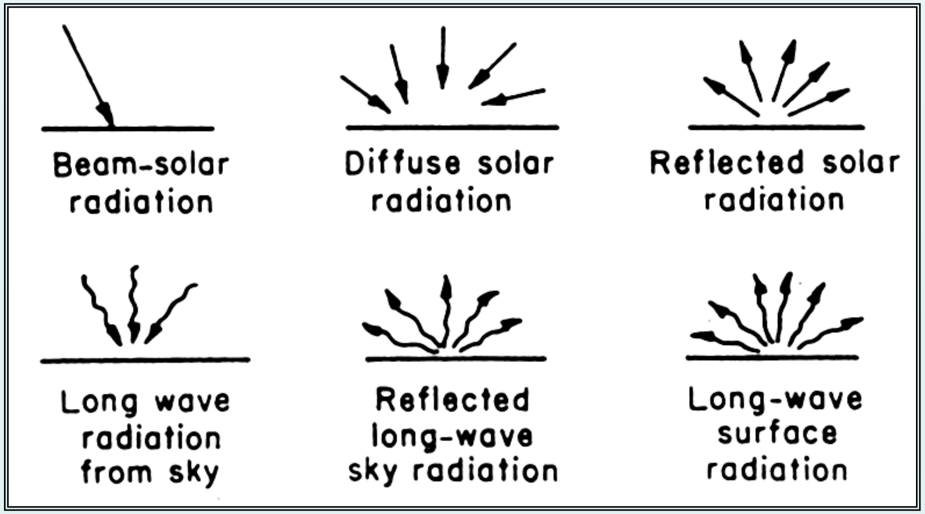
- Solar rays reaching the ground without change in direction

Diffuse Radiation G_{cd}

- Solar rays reaching the ground after a change in direction by particles in the atmosphere

Reflected Radiation

- Solar rays reflected from surrounding
- Albedo, ρ_g, is ground reflectance of both beam and diffuse radiation
- <u>Only sloped surfaces receives reflected radiation from the surrounding</u>

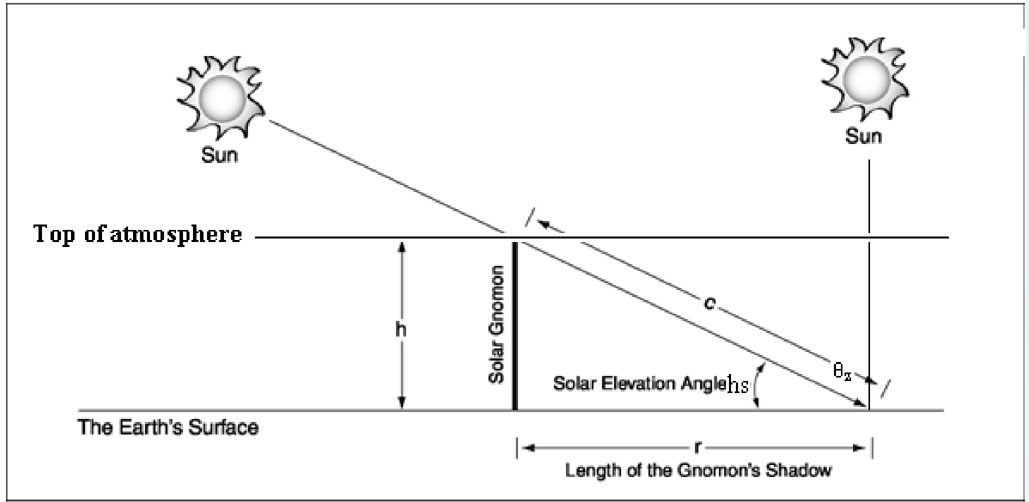


Atmosphere Influence on Solar Radiation

Air Mass 'm'

The ratio of the path length, which beam radiation passes through the atmosphere, to the path it would pass through if the Sun were at the zenith, i.e. directly overhead.

$$m = \frac{C}{h} = \frac{1}{\cos\theta_z}$$



Radiation on Ground Surface Clear day

Total radiation received on ***horizontal*** surface at ground surface is:

$$G_c = G_{cb} + G_{cd}$$

Radiation on Ground Surface Clear day

1- Beam Radiation G_{cb}

The atmospheric transmittance for beam is the ratio of the transmitted direct radiation to the total radiation incident at the top of atmosphere:

$$\tau_b = \frac{G_{cb}}{G_o} = a_o + a_1 \cdot e^{(-k/\cos\theta_z)}$$

$$G_{cb} = \tau_b \cdot G_{on} . (\cos\varphi \cdot \cos\delta \cdot \cos\omega + \sin\varphi \cdot \sin\delta)$$

Radiation on Ground Surface Clear day

Where $a_{0,}$ $a_1$and k are constant calculated suing next equations:

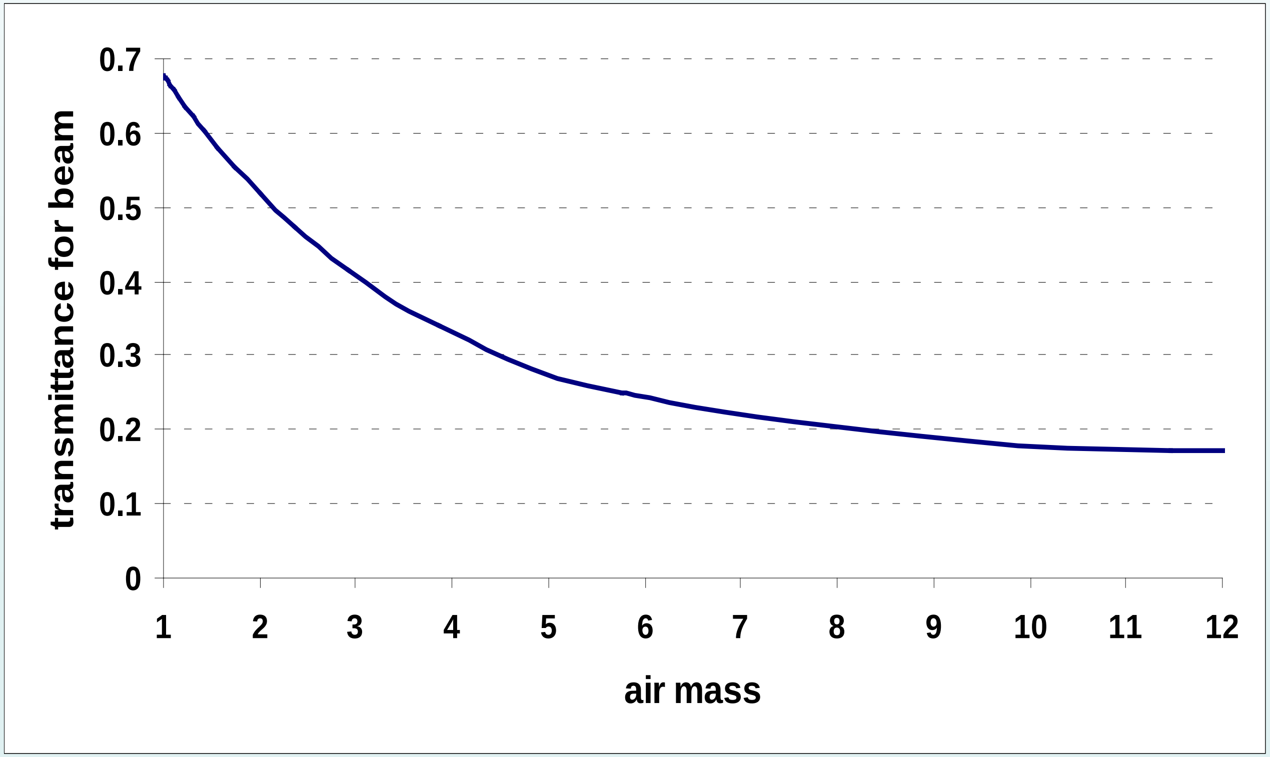
$$a_0 = r_0 \cdot \left(0.4237 - 0.00821 \cdot (6 - A)^2\right)$$

$$a_1 = r_1 \cdot \left(0.5055 + 0.00595 \cdot (6.5 - A)^2\right)$$

$$k = r_k \cdot \left(0.2711 + 0.01858 \cdot (2.5 - A)^2\right)$$

Climate Type	r_0	r_1	r_k
Tropical	0.95	0.98	1.02
Midlatitude summer	0.97	0.99	1.02
Subarctic summer	0.99	0.99	1.01
Midlatitude winter	1.03	1.01	1.00

Radiation on Ground Surface Clear day



Radiation on Ground Surface Clear day

5- Calculate the transmittance for beam radiation of the standard clear atmosphere at Luleå (altitude 0 m) today at 1:30 PM solar time. Estimate the intensity of beam radiation at that time and its component on a horizontal surface.

Radiation on Ground Surface Clear day

2- Diffuse Radiation G_{cd}

The atmospheric transmittance for diffuse radiation is the ratio of the transmitted diffuse radiation to the total radiation incident at the top of atmosphere

The transmission coefficient for diffuse radiation is:

$$\tau_d = \frac{G_{cd}}{G_o} = 0.271 - 0.294 \cdot \tau_b$$

$$G_{cd} = \tau_d \cdot G_{on} . (\cos\varphi \cdot \cos\delta \cdot \cos\omega + \sin\varphi \cdot \sin\delta)$$

Radiation on Ground Surface Clear day

6- Estimate the standard clear-day radiation on a horizontal surface for Luleå today at 13:30 PM solar time

Radiation on Ground Surface Clear day

Total radiation received on horizontal surface at ground surface is:

$$G_c = G_{cb} + G_{cd}$$

$$\boxed{G_c = (\tau_b + \tau_d) \cdot G_{sc} \cdot \left(1 + 0.033 \cdot \cos\frac{360 \cdot n}{365}\right) \cdot (\cos\varphi \cdot \cos\delta \cdot \cos\omega + \sin\varphi \cdot \sin\delta)}$$

Radiation on Ground Surface Clear day

The hourly radiation on a horizontal surface is written

$$\boxed{I = I_b + I_d}$$

b: Beam component

d: Diffuse component

$$I_c = \frac{12 \cdot 3600}{\pi} \cdot (\tau_b + \tau_d) \cdot G_{sc} \cdot \left(1 + 0.033 \cdot \cos\frac{360 \cdot n}{365}\right) \cdot \left(\cos\varphi \cdot \cos\delta \cdot (\sin\omega_2 - \sin\omega_1) + \frac{\pi \cdot (\omega_2 - \omega_1)}{180} \sin\varphi \cdot \sin\delta\right)$$

7- What is the solar radiation on a horizontal surface today in Luleå between the 13 &14?