1장. 행렬(Matrix)

행렬과 행렬의 연산

행렬(Matrix)

	월	화	수	목	금	토	일
국어	2	3	2	4	1	4	2
영어	0	3	1	4	3	2	2
수학	4	1	3	1	0	0	2

2	3	2	4	1	4	2
0	3	1	4	3	2	2
4	1	3	1	0	0	2

$$\begin{bmatrix} 2 & 3 & 2 & 4 & 1 & 4 & 2 \\ 0 & 3 & 1 & 4 & 3 & 2 & 2 \\ 4 & 1 & 3 & 1 & 0 & 0 & 2 \end{bmatrix}$$

행렬(Matrix)

행렬(Matrix)

- $(n \times m)$ 행렬: nm개의 실수를 직사각형 형태로 나열한 배열, $\boldsymbol{A} = [a_{ij}]_{n\times m} = [a_{ij}]$

$[\boldsymbol{A}]_{n\times m}$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

행(Row): 가로줄
열(Column): 세로줄
행렬의 크기(차원): $n \times m$
a_{ij}: 원소, (i, j)의 성분
스칼라(Scalar)
$(\boldsymbol{A})_{ij}$
i: 행 번호, j: 열 번호
행 행렬(벡터): $1 \times m$
열 행렬(벡터): $n \times 1$

n차 정사각행렬(n-Square Matrix, 정방행렬)

- $(n \times m)$ 행렬 $\boldsymbol{A} = [a_{ij}]$에서 행과 열의 수가 같을 때$(m = n)$

$$[\boldsymbol{A}]_{n\times n} = [\boldsymbol{A}]_n = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$(n \times n)$ 정사각행렬 $\boldsymbol{A}$
$(n \times n)$ 정방행렬 $\boldsymbol{A}$

대각행렬(Diagonal Matrix)

$\forall x$: 모든 x에 대하여

- $(n \times n)$ 정사각행렬 $\boldsymbol{D} = [d_{ij}]$, $\forall i \forall j \{i \neq j\} \Rightarrow (d_{ij} = 0)$

d_{ii}는 0일 수 있음

$$\boldsymbol{D} = \begin{bmatrix} d_{11} & 0 & \cdots & 0 \\ 0 & d_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & d_{nn} \end{bmatrix}$$

d_{ii}: 대각 원소(Diagonal Element)
대각합(Trace): 대각 원소들의 합

단위행렬(Unit Matrix, Identity Matrix): $\boldsymbol{I}$, $\boldsymbol{E}$

대각행렬에서 대각 원소가 모두 1인 행렬, $\forall i \{a_{ii} = 1\}$

$$\boldsymbol{I} = [\boldsymbol{I}]_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} = \boldsymbol{0} = [\boldsymbol{0}]_n$$

영 행렬(Zero Matrix): $\forall i \forall j (a_{ij} = 0)$

행렬(Matrix)

- **행렬의 예**

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 0 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

행렬(Matrix)

행렬의 상등(Equality)

- $A = [a_{ij}]_{n\times m}$ $B = [b_{ij}]_{s\times t}$
  - $\{(n = s) \wedge (m = t)\} \wedge \{\forall i \forall j (a_{ij} = b_{ij})\} \Leftrightarrow (A = B)$
    - 행렬 A의 크기와 행렬 B의 크기가 같음
    - 행렬 A의 (i, j) 성분(a_{ij})과, 행렬 B의 (i, j) 성분(b_{ij})이 같아야 함

$(A = B) \Leftrightarrow (a_{ij} = b_{ij})$
$(A = B) \Leftrightarrow (A)_{ij} = (B)_{ij}$

전치행렬(Transpose Matrix): A^T

- $(n \times m)$ 행렬 $A = [a_{ij}]$에서 행과 열을 바꾼 $(m \times n)$ 행렬(A^T)을 전치행렬이라 함

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \qquad A^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ a_{1m} & a_{2m} & \cdots & a_{nm} \end{bmatrix}$$

$(A)_{ij} = (A^T)_{ji}$

대칭행렬(Symmetric Matrix)

- $(n \times m)$ 행렬 $A = [a_{ij}]$의 전치행렬이 행렬 A와 같을 때($A^T = A$) 행렬 A를 대칭행렬이라 함
  $\forall i \forall j \{a_{ij} = a_{ji}\}$

($A^T = -A$): 반대칭(Skew-Symmetric) 행렬(대각 원소는 모두 0)

- 행렬 $A = [a_{ij}]$가 대칭행렬이면 행렬 A는 정사각행렬임

A, B: 대칭행렬	AA^T
$A + B$	$A + A^T$
$A - B$	$A - A^T$

분할된 행렬(Partitioned Matrix), 부분행렬(Submatrix)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} F & B \\ C & D \end{bmatrix} \quad A = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \quad A = [\, c_1 \quad c_2 \quad c_3 \,]$$

행렬(Matrix)

행렬의 연산

행렬의 덧셈과 뺄셈

행렬 $\boldsymbol{A}$의 크기와 행렬 $\boldsymbol{B}$의 크기가 같아야 함

$$\boldsymbol{A} = [a_{ij}]_{n\times m} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \qquad \boldsymbol{B} = [b_{ij}]_{n\times m} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{bmatrix}$$

$$\boldsymbol{A} + \boldsymbol{B} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1m}+b_{1m} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2m}+b_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1}+b_{n1} & a_{n2}+b_{n2} & \cdots & a_{nm}+b_{nm} \end{bmatrix}$$

$$\boldsymbol{A} + \boldsymbol{B} = [a_{ij} + b_{ij}]_{n\times m}$$

$$(\boldsymbol{A} + \boldsymbol{B})_{ij} = (\boldsymbol{A})_{ij} + (\boldsymbol{B})_{ij}$$

$$\boldsymbol{A} - \boldsymbol{B} = \begin{bmatrix} a_{11}-b_{11} & a_{12}-b_{12} & \cdots & a_{1m}-b_{1m} \\ a_{21}-b_{21} & a_{22}-b_{22} & \cdots & a_{2m}-b_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1}-b_{n1} & a_{n2}-b_{n2} & \cdots & a_{nm}-b_{nm} \end{bmatrix}$$

$$\boldsymbol{A} - \boldsymbol{B} = [a_{ij} - b_{ij}]_{n\times m}$$

$$(\boldsymbol{A} - \boldsymbol{B})_{ij} = (\boldsymbol{A})_{ij} - (\boldsymbol{B})_{ij}$$

행렬의 스칼라 곱

$$k\boldsymbol{A} = k\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} & \cdots & ka_{1m} \\ ka_{21} & ka_{22} & \cdots & ka_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ ka_{n1} & ka_{n2} & \cdots & ka_{nm} \end{bmatrix}$$

$$\boldsymbol{A}k = k\boldsymbol{A} = [ka_{ij}]_{n\times m}$$

$$(-1)\boldsymbol{A} = -\boldsymbol{A}$$

행렬(Matrix)

행렬의 연산

- 행렬의 덧셈과 뺄셈, 스칼라 곱셈

$$\boldsymbol{A}=\begin{bmatrix}1&2&0\\2&1&3\\0&1&1\end{bmatrix}\qquad \boldsymbol{B}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\qquad \boldsymbol{A}+\boldsymbol{B}=\begin{bmatrix}2&2&0\\2&2&3\\0&1&2\end{bmatrix}\qquad \boldsymbol{A}-\boldsymbol{B}=?$$

$$\boldsymbol{A}=\begin{bmatrix}1\\2\\0\end{bmatrix}\qquad \boldsymbol{B}=\begin{bmatrix}1\\2\\3\end{bmatrix}\qquad \boldsymbol{A}+\boldsymbol{B}=\begin{bmatrix}2\\4\\3\end{bmatrix}\qquad \boldsymbol{A}-\boldsymbol{B}=?$$

$$\boldsymbol{A}=\begin{bmatrix}1&0&3\end{bmatrix}\qquad \boldsymbol{B}=\begin{bmatrix}1&0&3\end{bmatrix}\qquad \boldsymbol{A}+\boldsymbol{B}=\begin{bmatrix}2&0&6\end{bmatrix}\qquad \boldsymbol{A}-\boldsymbol{B}=?$$

$$\boldsymbol{A}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\qquad \boldsymbol{B}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\qquad \boldsymbol{A}+\boldsymbol{B}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\qquad \boldsymbol{A}-\boldsymbol{B}=?$$

$$\boldsymbol{A}=\begin{bmatrix}1&2&0\\2&1&3\\0&1&1\end{bmatrix}\qquad 3\boldsymbol{A}=\begin{bmatrix}3&6&0\\6&3&9\\0&3&3\end{bmatrix}\qquad (-2)\boldsymbol{A}=\begin{bmatrix}-2&-4&0\\-4&-2&-6\\0&-2&-2\end{bmatrix}$$

행렬(Matrix)

행렬의 연산

행렬의 곱셈

- $(m \times r)$ 행렬 $\boldsymbol{A}$와 $(r \times n)$ 행렬 $\boldsymbol{B}$의 곱셈의 결과는 $(m \times n)$ 행렬임
- $\boldsymbol{A} \times \boldsymbol{B}$ 또는 $\boldsymbol{AB}$

행렬 $\boldsymbol{A}$의 열의 개수와 행렬 $\boldsymbol{B}$의 행의 개수가 같아야 함

- $\boldsymbol{AB} = [c_{ij}] = [c_{ij}]_{m \times n}$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{ir}b_{rj} = \sum_{k=1}^{r} a_{ik}b_{kj}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mr} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{r1} & b_{r2} & \cdots & b_{rn} \end{bmatrix} = \begin{bmatrix} c_{11} & \boxed{c_{12}} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

- $(1 \times n)$ 행렬 $\boldsymbol{v}$, $(n \times n)$ 행렬 $\boldsymbol{M}$
  - $\boldsymbol{vM} = \boldsymbol{v}'$
- $(n \times 1)$ 행렬 $\boldsymbol{v}$, $(n \times n)$ 행렬 $\boldsymbol{M}$
  - $\boldsymbol{Mv} = \boldsymbol{v}'$

$$[\, v_1 \quad v_2 \quad \cdots \quad v_n \,]$$

벡터(Vector)

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$\boldsymbol{A} = [a_{ij}]_{2\times3}$ $\quad \boldsymbol{B} = [a_{ij}]_{3\times2}$

$\boldsymbol{AB} = [c_{ij}]_{2\times2}$

$\boldsymbol{BA} = [d_{ij}]_{3\times3}$

$\boldsymbol{AB} \neq \boldsymbol{BA}$

- $\boldsymbol{AB} = \boldsymbol{A}[\boldsymbol{b}_1 \;\; \boldsymbol{b}_2 \;\; \cdots \;\; \boldsymbol{b}_n] = [\boldsymbol{Ab}_1 \;\; \boldsymbol{Ab}_2 \;\; \cdots \;\; \boldsymbol{Ab}_n]$
- $\boldsymbol{AB} = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \vdots \\ \boldsymbol{a}_m \end{bmatrix} \boldsymbol{B} = \begin{bmatrix} \boldsymbol{a}_1\boldsymbol{B} \\ \boldsymbol{a}_2\boldsymbol{B} \\ \vdots \\ \boldsymbol{a}_m\boldsymbol{B} \end{bmatrix}$

$\boldsymbol{AA} = \boldsymbol{A}^2$

$\boldsymbol{AAA} = \boldsymbol{A}^3$

$\boldsymbol{AA} \cdots \boldsymbol{A} = \boldsymbol{A}^n$

$\boldsymbol{A}^0 = \boldsymbol{I}$

행렬(Matrix)

행렬의 연산

- 행렬의 곱셈

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix} \quad AB = \begin{bmatrix} 9 & 4 & 9 \\ 2 & 10 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 3 \end{bmatrix} \quad AB = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad AB = \begin{bmatrix} 10 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 1 & 0 & 3 \end{bmatrix} \quad AB = \begin{bmatrix} 1 & 5 & 4 \\ 5 & 4 & 8 \\ 1 & 2 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad AB = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

행렬(Matrix)

행렬 연산의 성질

- $\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{B}+\boldsymbol{A}$
- $\boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C})=(\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C}$
- $\boldsymbol{A}+\mathbf{0}=\mathbf{0}+\boldsymbol{A}=\boldsymbol{A} \quad \boldsymbol{A}-\mathbf{0}=\boldsymbol{A} \quad \mathbf{0}-\boldsymbol{A}=-\boldsymbol{A}$
- $\boldsymbol{A}+(-\boldsymbol{A})=\boldsymbol{A}-\boldsymbol{A}=(-\boldsymbol{A})+\boldsymbol{A}=\mathbf{0}$
- $(-1)\boldsymbol{A}=-\boldsymbol{A}$
- $k(\boldsymbol{A}+\boldsymbol{B})=k\boldsymbol{A}+k\boldsymbol{B} \quad k(\boldsymbol{A}-\boldsymbol{B})=k\boldsymbol{A}-k\boldsymbol{B}$
- $(m+n)\boldsymbol{A}=m\boldsymbol{A}+n\boldsymbol{A} \quad (m-n)\boldsymbol{A}=m\boldsymbol{A}-n\boldsymbol{A}$
- $(mn)\boldsymbol{A}=m(n\boldsymbol{A})=n(m\boldsymbol{A})$
- $(\boldsymbol{AB})\boldsymbol{C}=\boldsymbol{A}(\boldsymbol{BC})=\boldsymbol{ABC}$
- $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})=\boldsymbol{AB}+\boldsymbol{AC} \quad \boldsymbol{A}(\boldsymbol{B}-\boldsymbol{C})=\boldsymbol{AB}-\boldsymbol{AC}$
- $(\boldsymbol{B}+\boldsymbol{C})\boldsymbol{A}=\boldsymbol{BA}+\boldsymbol{CA} \quad (\boldsymbol{B}-\boldsymbol{C})\boldsymbol{A}=\boldsymbol{BA}-\boldsymbol{CA}$
- $k(\boldsymbol{AB})=(k\boldsymbol{A})\boldsymbol{B}=\boldsymbol{A}(k\boldsymbol{B})$
- $\boldsymbol{IA}=\boldsymbol{A}=\boldsymbol{AI}$
- $0\boldsymbol{A}=\mathbf{0} \quad \boldsymbol{A}\mathbf{0}=\mathbf{0}\boldsymbol{A}=\mathbf{0}$
- $\boldsymbol{A}^m\boldsymbol{A}^n=\boldsymbol{A}^{m+n}=\boldsymbol{A}^n\boldsymbol{A}^m \qquad \boldsymbol{A}^0=\boldsymbol{I}$
- $(\boldsymbol{A}^m)^n=\boldsymbol{A}^{mn}$

$$\boldsymbol{A}-\boldsymbol{B}\neq\boldsymbol{B}-\boldsymbol{A}$$
$$\boldsymbol{A}-\boldsymbol{B}=\boldsymbol{A}+(-\boldsymbol{B})$$
$$\boldsymbol{B}-\boldsymbol{A}=\boldsymbol{B}+(-\boldsymbol{A})$$

$$\boldsymbol{AB}\neq\boldsymbol{BA}$$

$$(k\boldsymbol{A}=\mathbf{0})\Rightarrow(k=0)\vee(\boldsymbol{A}=\mathbf{0})$$

$$(\boldsymbol{AD}=\mathbf{0})\nRightarrow(\boldsymbol{A}=\mathbf{0})\vee(\boldsymbol{D}=\mathbf{0})$$

$$(\boldsymbol{B}=\boldsymbol{C})\Rightarrow(\boldsymbol{AB}=\boldsymbol{AC})$$
$$(\boldsymbol{AB}=\boldsymbol{AC})\nRightarrow(\boldsymbol{B}=\boldsymbol{C})$$

$$\boldsymbol{A}=\begin{bmatrix}0&1\\0&2\end{bmatrix}\ \boldsymbol{B}=\begin{bmatrix}1&1\\3&4\end{bmatrix}\ \boldsymbol{C}=\begin{bmatrix}2&5\\3&4\end{bmatrix}\ \boldsymbol{D}=\begin{bmatrix}1&1\\0&0\end{bmatrix}$$
$$\boldsymbol{AD}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$
$$\boldsymbol{AB}=\boldsymbol{AC}=\begin{bmatrix}3&4\\6&8\end{bmatrix}$$

행렬(Matrix)

행렬 연산의 성질

- $\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{B}+\boldsymbol{A}$

  ① 행렬 $(\boldsymbol{A}+\boldsymbol{B})$의 크기와 행렬 $(\boldsymbol{B}+\boldsymbol{A})$의 크기가 같음
  $\boldsymbol{A}$가 $(m\times n)$ 행렬이라고 가정하면 덧셈 $(\boldsymbol{A}+\boldsymbol{B})$이 성립하려면 $\boldsymbol{B}$가 $(m\times n)$ 행렬이어야 함
  $(\boldsymbol{A}+\boldsymbol{B})$는 $(m\times n)$ 행렬이고 $(\boldsymbol{B}+\boldsymbol{A})$도 $(m\times n)$ 행렬임

  ② 행렬 $(\boldsymbol{A}+\boldsymbol{B})$와 행렬 $(\boldsymbol{B}+\boldsymbol{A})$의 대응 원소가 같음
  $(\boldsymbol{A}+\boldsymbol{B})_{ij}=(\boldsymbol{A})_{ij}+(\boldsymbol{B})_{ij}=(\boldsymbol{B})_{ij}+(\boldsymbol{A})_{ij}=(\boldsymbol{B}+\boldsymbol{A})_{ij}$

- $(\boldsymbol{B}=\boldsymbol{C})\rightarrow(\boldsymbol{AB}=\boldsymbol{AC})$

  ① 행렬 $\boldsymbol{AB}$의 크기와 행렬 $\boldsymbol{AC}$의 크기가 같음
  $\boldsymbol{A}$가 $(m\times r)$ 행렬이고 $\boldsymbol{B}$와 $\boldsymbol{C}$가 $(r\times n)$ 행렬이라고 가정하면 $\boldsymbol{AB}$와 $\boldsymbol{AC}$는 $(m\times n)$ 행렬임

  ② 행렬 $\boldsymbol{AB}$와 행렬 $\boldsymbol{AC}$의 대응 원소가 같음
  $(\boldsymbol{AB})_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+\cdots+a_{ir}b_{rj}=a_{i1}c_{1j}+a_{i2}c_{2j}+\cdots+a_{ir}c_{rj}=(\boldsymbol{AC})_{ij}$

- $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})=\boldsymbol{AB}+\boldsymbol{AC}$

  ① 행렬 $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})$의 크기와 행렬 $(\boldsymbol{AB}+\boldsymbol{AC})$의 크기가 같음
  $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})$가 $(m\times n)$ 행렬이라고 가정하면 $\boldsymbol{A}$가 $(m\times r)$ 행렬이고 $(\boldsymbol{B}+\boldsymbol{C})$가 $(r\times n)$ 행렬임
  $(\boldsymbol{B}+\boldsymbol{C})$가 $(r\times n)$ 행렬이면 $\boldsymbol{B}$와 $\boldsymbol{C}$가 $(r\times n)$ 행렬임
  $\boldsymbol{AB}$와 $\boldsymbol{AC}$는 $(m\times n)$ 행렬이고 $(\boldsymbol{AB}+\boldsymbol{AC})$는 $(m\times n)$ 행렬임

  ② 행렬 $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C})$와 행렬 $(\boldsymbol{AB}+\boldsymbol{AC})$의 대응 원소가 같음
  $(\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C}))_{ij}=a_{i1}(b_{1j}+c_{1j})+a_{i2}(b_{2j}+c_{2j})+\cdots+a_{ir}(b_{rj}+c_{rj})$
  $=a_{i1}b_{1j}+a_{i1}c_{1j}+a_{i2}b_{2j}+a_{i2}c_{2j}+\cdots+a_{ir}b_{rj}+a_{ir}c_{rj}$
  $=(a_{i1}b_{1j}+a_{i2}b_{2j}+\cdots+a_{ir}b_{rj})+(a_{i1}c_{1j}+a_{i2}c_{2j}+\cdots+a_{ir}c_{rj})$
  $=(\boldsymbol{AB})_{ij}+(\boldsymbol{AC})_{ij}$

행렬(Matrix)

행렬 연산의 성질

- $(\boldsymbol{B} = \boldsymbol{C}) \rightarrow (\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{A} + \boldsymbol{C})$

  ① 행렬 $(\boldsymbol{A} + \boldsymbol{B})$의 크기와 행렬 $(\boldsymbol{A} + \boldsymbol{C})$의 크기가 같음
  $\boldsymbol{B}$가 $(m \times n)$ 행렬이라고 가정하면 $\boldsymbol{C}$도 $(m \times n)$ 행렬이다.
  덧셈 $(\boldsymbol{A} + \boldsymbol{B})$와 $(\boldsymbol{A} + \boldsymbol{C})$가 성립하려면 $\boldsymbol{A}$가 $(m \times n)$ 행렬이어야 함
  $(\boldsymbol{A} + \boldsymbol{B})$는 $(m \times n)$ 행렬이고 $(\boldsymbol{A} + \boldsymbol{C})$도 $(m \times n)$ 행렬임

  ② 행렬 $(\boldsymbol{A} + \boldsymbol{B})$와 행렬 $(\boldsymbol{A} + \boldsymbol{C})$의 대응 원소가 같음
  $(\boldsymbol{A} + \boldsymbol{B})_{ij} = (\boldsymbol{A})_{ij} + (\boldsymbol{B})_{ij} = (\boldsymbol{A})_{ij} + (\boldsymbol{C})_{ij} = (\boldsymbol{A} + \boldsymbol{C})_{ij}$

- $(\boldsymbol{B} = \boldsymbol{C}) \rightarrow (\boldsymbol{B} - \boldsymbol{A} = \boldsymbol{C} - \boldsymbol{A})$

- $(\boldsymbol{X} + \boldsymbol{A} = \boldsymbol{B})$를 만족하는 행렬 $\boldsymbol{X}$는 유일하고 $(\boldsymbol{X} = \boldsymbol{B} - \boldsymbol{A})$이다.

  $\boldsymbol{X} + \boldsymbol{A} = \boldsymbol{B}$

  $\boldsymbol{X} + \boldsymbol{A} - \boldsymbol{A} = \boldsymbol{B} - \boldsymbol{A}$

  $\boldsymbol{X} + \boldsymbol{0} = \boldsymbol{B} - \boldsymbol{A}$

  $\boldsymbol{X} = \boldsymbol{B} - \boldsymbol{A}$

  $\boldsymbol{Y} + \boldsymbol{A} = \boldsymbol{B} = \boldsymbol{X} + \boldsymbol{A}$

  $\boldsymbol{Y} + \boldsymbol{A} = \boldsymbol{X} + \boldsymbol{A}$

  $\boldsymbol{Y} + \boldsymbol{A} - \boldsymbol{A} = \boldsymbol{X} + \boldsymbol{A} - \boldsymbol{A}$

  $\boldsymbol{Y} + \boldsymbol{0} = \boldsymbol{X} + \boldsymbol{0}$

  $\boldsymbol{Y} = \boldsymbol{X}$

행렬(Matrix)

행렬 연산의 성질

- $(\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C} = \boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C})$
  $((\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C})_{ij} = (\boldsymbol{A}+\boldsymbol{B})_{ij}+(\boldsymbol{C})_{ij} = \{(\boldsymbol{A})_{ij}+(\boldsymbol{B})_{ij}\}+(\boldsymbol{C})_{ij} = (\boldsymbol{A})_{ij}+(\boldsymbol{B}+\boldsymbol{C})_{ij} = (\boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C}))_{ij}$
- $k(\boldsymbol{A}+\boldsymbol{B}) = k\boldsymbol{A}+k\boldsymbol{B}$
  $(k(\boldsymbol{A}+\boldsymbol{B}))_{ij} = k(\boldsymbol{A}+\boldsymbol{B})_{ij} = k\{(\boldsymbol{A})_{ij}+(\boldsymbol{B})_{ij}\} = k(\boldsymbol{A})_{ij}+k(\boldsymbol{B})_{ij}$
- $(\boldsymbol{AB})\boldsymbol{C} = \boldsymbol{A}(\boldsymbol{BC}) = \boldsymbol{ABC}$

  ① 행렬 $(\boldsymbol{AB})\boldsymbol{C}$의 크기와 행렬 $\boldsymbol{A}(\boldsymbol{BC})$의 크기가 같음
  $(\boldsymbol{AB})\boldsymbol{C}$가 $(m\times n)$ 행렬이라고 가정하면 $\boldsymbol{AB}$가 $(m\times r)$ 행렬이고 $\boldsymbol{C}$가 $(r\times n)$ 행렬임
  $\boldsymbol{AB}$가 $(m\times r)$ 행렬이면 $\boldsymbol{A}$가 $(m\times s)$ 행렬이고 $\boldsymbol{B}$가 $(s\times r)$ 행렬임
  $\boldsymbol{BC}$는 $(s\times n)$ 행렬이고 $\boldsymbol{A}(\boldsymbol{BC})$는 $(m\times n)$ 행렬임

  ② 행렬$(\boldsymbol{AB})\boldsymbol{C}$와 행렬 $\boldsymbol{A}(\boldsymbol{BC})$의 대응 원소가 같음
  $(\boldsymbol{AB}=\boldsymbol{M})$, $(\boldsymbol{BC}=\boldsymbol{N})$이라고 가정
  $(\boldsymbol{M})_{pq} = m_{pq} = a_{p1}b_{1q}+a_{p2}b_{2q}+\cdots+a_{ps}b_{sq} = \sum_{k=1}^{s} a_{pk}b_{kq}$
  $(\boldsymbol{N})_{pq} = n_{pq} = b_{p1}c_{1q}+b_{p2}c_{2q}+\cdots+b_{pr}c_{rq} = \sum_{k=1}^{r} a_{pk}b_{kq}$
  $((\boldsymbol{AB})\boldsymbol{C})_{ij} = (\boldsymbol{MC})_{ij} = m_{i1}c_{1j}+m_{i2}c_{2j}+\cdots+m_{ir}c_{rj}$
  $= (a_{i1}b_{11}+a_{i2}b_{21}+\cdots+a_{is}b_{s1})c_{1j}+(a_{i1}b_{12}+a_{i2}b_{22}+\cdots+a_{is}b_{s2})c_{2j}+\cdots$
  $+(a_{i1}b_{1r}+a_{i2}b_{2r}+\cdots+a_{is}b_{sr})c_{rj}$
  $= a_{i1}b_{11}c_{1j}+a_{i2}b_{21}c_{1j}+\cdots+a_{is}b_{s1}c_{1j}+a_{i1}b_{12}c_{2j}+a_{i2}b_{22}c_{2j}+\cdots+a_{is}b_{s2}c_{2j}+\cdots+a_{is}b_{sr}c_{rj}$
  $(\boldsymbol{A}(\boldsymbol{BC}))_{ij} = (\boldsymbol{AN})_{ij} = a_{i1}n_{1j}+a_{i2}n_{2j}+\cdots+a_{is}n_{sj}$
  $= a_{i1}(b_{11}c_{1j}+b_{12}c_{2j}+\cdots+b_{1r}c_{rj})+a_{i2}(b_{21}c_{1j}+b_{22}c_{2j}+\cdots+b_{2r}c_{rj})+\cdots$
  $+a_{is}(b_{s1}c_{1j}+b_{s2}c_{2j}+\cdots+b_{sr}c_{rj})$
  $= a_{i1}b_{11}c_{1j}+a_{i1}b_{12}c_{2j}+\cdots+a_{i1}b_{1r}c_{rj}+a_{i2}b_{21}c_{1j}+a_{i2}b_{22}c_{2j}+\cdots+a_{i2}b_{2r}c_{rj}+\cdots+a_{is}b_{sr}c_{rj}$
- $[\boldsymbol{I}]_n[\boldsymbol{A}]_{n\times m} = [\boldsymbol{A}]_{n\times m} = [\boldsymbol{A}]_{n\times m}[\boldsymbol{I}]_m$
  $(\boldsymbol{IA})_{kj} = i_{k1}a_{1j}+i_{k2}a_{2j}+\cdots+i_{kk}a_{kj}+\cdots+i_{kn}a_{nj} = a_{kj} = (\boldsymbol{A})_{kj}$

$$\boxed{i_{pq} = \begin{cases} 1 & (p=q) \\ 0 & (p\neq q) \end{cases}}$$

행렬(Matrix)

행렬 연산의 성질

- $(\boldsymbol{A}^T)^T = \boldsymbol{A}$
- $\boldsymbol{I}^T = \boldsymbol{I}$
- $(k\boldsymbol{A})^T = k\boldsymbol{A}^T$

  $((k\boldsymbol{A})^T)_{ij} = (k\boldsymbol{A})_{ji} = k(\boldsymbol{A}_{ji})$

  $(k\boldsymbol{A}^T)_{ij} = k(\boldsymbol{A}^T)_{ij} = k(\boldsymbol{A}_{ji})$
- $(\boldsymbol{A}+\boldsymbol{B})^T = \boldsymbol{A}^T + \boldsymbol{B}^T = \boldsymbol{B}^T + \boldsymbol{A}^T$

  $((\boldsymbol{A}+\boldsymbol{B})^T)_{ij} = (\boldsymbol{A}+\boldsymbol{B})_{ji} = (\boldsymbol{A})_{ji} + (\boldsymbol{B})_{ji} = (\boldsymbol{A}^T)_{ij} + (\boldsymbol{B}^T)_{ij} = (\boldsymbol{A}^T + \boldsymbol{B}^T)_{ij} = (\boldsymbol{B}^T + \boldsymbol{A}^T)_{ij}$

  $\boldsymbol{A}$와 $\boldsymbol{B}$가 대칭행렬이면 $(\boldsymbol{A}+\boldsymbol{B})^T$는 대칭행렬이다: $(\boldsymbol{A}+\boldsymbol{B})^T = \boldsymbol{A}+\boldsymbol{B}$

대칭행렬: $\boldsymbol{A}^T = \boldsymbol{A}$

- $(\boldsymbol{A}-\boldsymbol{B})^T = \boldsymbol{A}^T - \boldsymbol{B}^T$

  $((\boldsymbol{A}-\boldsymbol{B})^T)_{ij} = (\boldsymbol{A}-\boldsymbol{B})_{ji} = (\boldsymbol{A})_{ji} - (\boldsymbol{B})_{ji} = (\boldsymbol{A}^T)_{ij} - (\boldsymbol{B}^T)_{ij} = (\boldsymbol{A}^T - \boldsymbol{B}^T)_{ij}$
- $(\boldsymbol{AB})^T = \boldsymbol{B}^T\boldsymbol{A}^T$

  ① 행렬 $(\boldsymbol{AB})^T$의 크기와 행렬 $\boldsymbol{B}^T\boldsymbol{A}^T$의 크기가 같음

  $\boldsymbol{A}$가 $(m \times r)$ 행렬이고 $\boldsymbol{B}$가 $(r \times n)$ 행렬이라고 가정하면 $\boldsymbol{AB}$가 $(m \times n)$ 행렬이고 $(\boldsymbol{AB})^T$가 $(n \times m)$ 행렬임

  $\boldsymbol{B}^T$가 $(n \times r)$ 행렬이고 $\boldsymbol{A}^T$가 $(r \times m)$ 행렬이고 $\boldsymbol{B}^T\boldsymbol{A}^T$가 $(n \times m)$ 행렬임

  ② 행렬 $(\boldsymbol{AB})^T$와 행렬 $\boldsymbol{B}^T\boldsymbol{A}^T$의 대응 원소가 같음

  $((\boldsymbol{AB})^T)_{ij} = (\boldsymbol{AB})_{ji} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jr}b_{ri}$

  $(\boldsymbol{B}^T = \boldsymbol{M})$이라고 하면 $(m_{ij} = b_{ji})$이고 $(\boldsymbol{A}^T = \boldsymbol{N})$이라고 하면 $(n_{ij} = a_{ji})$이다.

  $(\boldsymbol{B}^T\boldsymbol{A}^T)_{ij} = (\boldsymbol{MN})_{ij} = m_{i1}n_{1j} + m_{i2}n_{2j} + \cdots + m_{ir}n_{rj}$

  $= b_{1i}a_{j1} + b_{2i}a_{j2} + \cdots + b_{ri}a_{jr}$

  $= a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jr}b_{ri}$

행렬(Matrix)

행렬의 연산

행렬의 일차 결합(Linear Combination)

- 크기가 같은 행렬 $\boldsymbol{A}_1, \boldsymbol{A}_2, \ldots, \boldsymbol{A}_n$과 실수 $c_1, c_2, \ldots, c_n$이 주어질 때 다음을 일차 결합이라 함
  $c_1\boldsymbol{A}_1 + c_2\boldsymbol{A}_2 + \cdots + c_n\boldsymbol{A}_n$
- $(m \times n)$ 행렬 $\boldsymbol{A}$, $(n \times 1)$ 행렬 $\boldsymbol{x}$
  - 행렬의 곱 $\boldsymbol{Ax}$는 행렬 $\boldsymbol{A}$의 열벡터와 $\boldsymbol{x}$의 원소들의 일차 결합으로 표현할 수 있음

$$\boldsymbol{Ax} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

$$= x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

$$\boxed{\boldsymbol{A} = [\, \boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \cdots \quad \boldsymbol{a}_n \,]}$$

$$= x_1\boldsymbol{a}_1 + x_2\boldsymbol{a}_2 + \cdots + x_n\boldsymbol{a}_n$$

$$\boxed{\begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & -1 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -12 \\ -2 \end{bmatrix} = 2\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix}}$$

- $\boldsymbol{A}(k\boldsymbol{x}) = k(\boldsymbol{Ax})$
  $\boldsymbol{A}(k\boldsymbol{x}) = (kx_1)\boldsymbol{a}_1 + (kx_2)\boldsymbol{a}_2 + \cdots + (kx_n)\boldsymbol{a}_n = k(x_1\boldsymbol{a}_1 + x_2\boldsymbol{a}_2 + \cdots + x_n\boldsymbol{a}_n) = k(\boldsymbol{Ax})$
- $\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{y}) = \boldsymbol{Ax} + \boldsymbol{Ay}$
  $\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{y}) = (x_1 + y_1)\boldsymbol{a}_1 + (x_2 + y_2)\boldsymbol{a}_2 + \cdots + (x_n + y_n)\boldsymbol{a}_n$
  $= (x_1\boldsymbol{a}_1 + x_2\boldsymbol{a}_2 + \cdots + x_n\boldsymbol{a}_n) + (y_1\boldsymbol{a}_1 + y_2\boldsymbol{a}_2 + \cdots + y_n\boldsymbol{a}_n) = \boldsymbol{Ax} + \boldsymbol{Ay}$
- $\boldsymbol{A}(k_1\boldsymbol{x}_1 + k_2\boldsymbol{x}_2 + \cdots + k_n\boldsymbol{x}_n) = k_1\boldsymbol{Ax}_1 + k_2\boldsymbol{Ax}_2 + \cdots + k_n\boldsymbol{Ax}_n$

행렬(Matrix)

행렬의 연산

행렬의 일차 결합(Linear Combination)

- 행렬의 일차 결합으로 행렬의 곱을 표현하기
  - $(m \times r)$ 행렬 $\boldsymbol{A}$, $(r \times s)$ 행렬 $\boldsymbol{B}$, $(s \times 1)$ 행렬 $\boldsymbol{x}$

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_s \end{bmatrix}$$

$$\boldsymbol{AB} = [c_{ik}]$$

$$c_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + \cdots + a_{ir}b_{rk} = \textstyle\sum_{j=1}^{r} a_{ij}b_{jk}$$

$$\boldsymbol{A} = [\,\boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \cdots \quad \boldsymbol{a}_r\,] \qquad \boldsymbol{B} = [\,\boldsymbol{b}_1 \quad \boldsymbol{b}_2 \quad \cdots \quad \boldsymbol{b}_s\,]$$

$$\boldsymbol{a}_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} \ \boldsymbol{a}_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} \cdots \ \boldsymbol{a}_r = \begin{bmatrix} a_{1r} \\ a_{2r} \\ \vdots \\ a_{mr} \end{bmatrix} \qquad \boldsymbol{b}_1 = \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{r1} \end{bmatrix} \ \boldsymbol{b}_2 = \begin{bmatrix} b_{12} \\ b_{22} \\ \vdots \\ b_{r2} \end{bmatrix} \cdots \ \boldsymbol{b}_s = \begin{bmatrix} b_{1s} \\ b_{2s} \\ \vdots \\ b_{rs} \end{bmatrix}$$

$$\boldsymbol{AB} = [\,\boldsymbol{Ab}_1 \quad \boldsymbol{Ab}_2 \quad \cdots \quad \boldsymbol{Ab}_s\,]$$

$$\boldsymbol{Ab}_k = b_{1k}\boldsymbol{a}_1 + b_{2k}\boldsymbol{a}_2 + \cdots + b_{rk}\boldsymbol{a}_r$$

$$= b_{1k}\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + b_{2k}\begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + b_{rk}\begin{bmatrix} a_{1r} \\ a_{2r} \\ \vdots \\ a_{mr} \end{bmatrix} = \begin{bmatrix} a_{11}b_{1k} + a_{12}b_{2k} + \cdots + a_{1r}b_{rk} \\ a_{21}b_{1k} + a_{22}b_{2k} + \cdots + a_{2r}b_{rk} \\ \vdots \\ a_{m1}b_{1k} + a_{m2}b_{2k} + \cdots + a_{mr}b_{rk} \end{bmatrix} = \begin{bmatrix} c_{1k} \\ c_{2k} \\ \vdots \\ c_{mk} \end{bmatrix}$$

$$\boldsymbol{Bx} = x_1\boldsymbol{b}_1 + x_2\boldsymbol{b}_2 + \cdots + x_s\boldsymbol{b}_s$$

$$\boldsymbol{A}(\boldsymbol{Bx}) = (\boldsymbol{AB})\boldsymbol{x} = \boldsymbol{A}(x_1\boldsymbol{b}_1 + x_2\boldsymbol{b}_2 + \cdots + x_s\boldsymbol{b}_s) = x_1(\boldsymbol{Ab}_1) + x_2(\boldsymbol{Ab}_2) + \cdots + x_s(\boldsymbol{Ab}_s)$$

행렬(Matrix)

역행렬(Inverse Matrix): A^{-1}

- 정사각행렬 $\boldsymbol{A}$에 대해 $\boldsymbol{AB} = \boldsymbol{BA} = \boldsymbol{I}$를 만족하는 행렬 $\boldsymbol{B}$를 행렬 $\boldsymbol{A}$의 역행렬이라 함

  $\boldsymbol{AA^{-1}} = \boldsymbol{A^{-1}A} = \boldsymbol{I}$

- **가역행렬(Invertible Matrix 또는 Nonsingular Matrix)**
  - 역행렬이 존재하는 행렬을 가역행렬이라 함(행렬이 가역이다)
- **특이행렬(Singular Matrix)**
  - 역행렬이 존재하지 않는 행렬을 특이행렬 또는 비가역행렬이라 함
- 역행렬이 존재하면 유일(Unique)하다.
  - 행렬 $\boldsymbol{B}$가 행렬 $\boldsymbol{A}$의 역행렬이고 행렬 $\boldsymbol{C}$가 행렬 $\boldsymbol{A}$의 역행렬이면 $(\boldsymbol{B} = \boldsymbol{C})$이다.

    행렬 $\boldsymbol{B}$가 행렬 $\boldsymbol{A}$의 역행렬: $\boldsymbol{BA} = \boldsymbol{I}$
    행렬 $\boldsymbol{C}$가 행렬 $\boldsymbol{A}$의 역행렬: $\boldsymbol{AC} = \boldsymbol{I}$
    $\boldsymbol{C} = \boldsymbol{IC} = (\boldsymbol{BA})\boldsymbol{C} = \boldsymbol{B}(\boldsymbol{AC}) = \boldsymbol{BI} = \boldsymbol{B}$

- $(\boldsymbol{AB} = \boldsymbol{I}) \Rightarrow (\boldsymbol{B} = \boldsymbol{A}^{-1})$, $(\boldsymbol{BA} = \boldsymbol{I}) \Rightarrow (\boldsymbol{A} = \boldsymbol{B}^{-1})$

  $(\boldsymbol{AB} = \boldsymbol{I}) \Rightarrow \boldsymbol{A}$ 와 $\boldsymbol{B}$는 서로의 가역행렬

  A^{-1}가 존재하는 경우

  $\boldsymbol{AB} = \boldsymbol{I} \quad \boldsymbol{A}^{-1}(\boldsymbol{AB}) = \boldsymbol{A}^{-1}\boldsymbol{I} \quad (\boldsymbol{A}^{-1}\boldsymbol{A})\boldsymbol{B} = \boldsymbol{A}^{-1}\boldsymbol{I} \quad (\boldsymbol{I})\boldsymbol{B} = \boldsymbol{A}^{-1}\boldsymbol{I} \quad \boldsymbol{B} = \boldsymbol{A}^{-1}$

  $\boldsymbol{BA} = \boldsymbol{I} \quad (\boldsymbol{BA})\boldsymbol{A}^{-1} = \boldsymbol{IA}^{-1} \quad \boldsymbol{B}(\boldsymbol{AA}^{-1}) = \boldsymbol{IA}^{-1} \quad \boldsymbol{B}(\boldsymbol{I}) = \boldsymbol{IA}^{-1} \quad \boldsymbol{B} = \boldsymbol{A}^{-1}$

  A^{-1}가 존재하는가?

- **2차 정사각행렬의 역행렬**

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\boldsymbol{A}^{-1} = \frac{1}{(a_{11}a_{22} - a_{12}a_{21})}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$\boldsymbol{AA}^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\frac{1}{(a_{11}a_{22} - a_{12}a_{21})}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

행렬(Matrix)

- **역행렬(Inverse Matrix): A^{-1}**

$$\boldsymbol{A} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$\boldsymbol{AB} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{BA} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{A}^{-1} = \frac{1}{(2*3-(-5*-1))}\begin{bmatrix} 3 & -(-5) \\ -(-1) & 2 \end{bmatrix} = \frac{1}{(6-1)}\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$\boldsymbol{A}^{-1}$가 존재하는가?

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} = [\,\boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \boldsymbol{0}\,] \qquad \boldsymbol{BA} = \boldsymbol{B}[\,\boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \boldsymbol{0}\,] = [\boldsymbol{Ba}_1 \quad \boldsymbol{Ba}_2 \quad \boldsymbol{0}\,] \neq \boldsymbol{I}$$

$$c_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + \cdots + a_{ir}b_{rk} = \textstyle\sum_{j=1}^{r} a_{ij}b_{jk}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

행렬 $\boldsymbol{A}$의 i번째 행이 $[\,0 \quad \cdots \quad 0\,]$이면 $\boldsymbol{AB}$의 i번째 행 $[\,0 \quad \cdots \quad 0\,]$이다.

행렬 $\boldsymbol{A}$의 k번째 열이 $\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$이면 $\boldsymbol{AB}$의 k번째 열이 $\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$이다.

행렬(Matrix)

역행렬(Inverse Matrix)의 성질

- $(A^{-1} = B) \Leftrightarrow (AB = I)$
- A가 가역행렬이면 A^{-1}는 가역행렬이고 $(A^{-1})^{-1} = A$이다.

  $A^{-1}A = I = AA^{-1}$

- $I^{-1} = I$

  $I^2 = I = I^2$

- A와 B가 가역행렬이면 AB는 가역행렬이고 $(AB)^{-1} = B^{-1}A^{-1}$이다.

  $(AB)(B^{-1}A^{-1}) = ABB^{-1}A^{-1} = A(BB^{-1})A^{-1} = A(I)A^{-1} = AA^{-1} = I$

  $(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}(I)B = B^{-1}B = I$

- $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
- A가 가역행렬이면 A^k는 가역행렬이고 $(A^k)^{-1} = (A^{-1})^k$이다.

  $A^k\{(A^{-1})^k\} = (AA\cdots AA)(A^{-1}A^{-1}\cdots A^{-1}A^{-1}) = I$

- $(n \geq 2)$인 모든 자연수 n에 대하여 $(A^n)^{-1} = (A^{-1})^n$이다.

  $(A^n)^{-1} = (AAA\cdots A)^{-1} = A^{-1}A^{-1}A^{-1}\cdots A^{-1} = (A^{-1})^n$

  $A^{-1}A^{-1} = (A^{-1})^2 = A^{-2}$
  $A^{-1}A^{-1}A^{-1} = (A^{-1})^3 = A^{-3}$
  $A^{-1}A^{-1}\cdots A^{-1} = (A^{-1})^n = A^{-n}$

  (수학적 귀납법)
  ① $(k = 2)$일 때 성립: $(A^2)^{-1} = (AA)^{-1} = A^{-1}A^{-1} = (A^{-1})^2$
  ② $(k = n-1)$일 때 성립한다고 가정: $(A^{n-1})^{-1} = (A^{-1})^{n-1}$
  ③ $(k = n)$일 때 성립함을 증명: $(A^n)^{-1} = (AA^{n-1})^{-1} = (A^{n-1})^{-1}A^{-1} = (A^{-1})^{n-1}A^{-1} = (A^{-1})^n$

- A가 가역행렬이면 kA는 가역행렬이고 역행렬은 $(kA)^{-1} = k^{-1}A^{-1} = \frac{1}{k}A^{-1}$이다.

  $(kA)(k^{-1}A^{-1}) = k^{-1}(kA)A^{-1} = k^{-1}k(A)A^{-1} = I = kk^{-1}(A^{-1})(A) = k(k^{-1}A^{-1})(A) = (k^{-1}A^{-1})(kA)$

행렬(Matrix)

- **퀴즈(Quiz)**
  - 행렬 $\boldsymbol{B}$가 가역행렬일 때 $(\boldsymbol{A}\boldsymbol{B}^{-1} = \boldsymbol{B}^{-1}\boldsymbol{A})$는 $(\boldsymbol{A}\boldsymbol{B} = \boldsymbol{B}\boldsymbol{A})$와 동치이다.
    - $(\boldsymbol{A}\boldsymbol{B}^{-1} = \boldsymbol{B}^{-1}\boldsymbol{A})$이면 $(\boldsymbol{A}\boldsymbol{B} = \boldsymbol{B}\boldsymbol{A})$이다.
    - $(\boldsymbol{A}\boldsymbol{B} = \boldsymbol{B}\boldsymbol{A})$이면 $(\boldsymbol{A}\boldsymbol{B}^{-1} = \boldsymbol{B}^{-1}\boldsymbol{A})$이다.
  - $(\boldsymbol{A}^4 = \boldsymbol{0})$이면 $(\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \boldsymbol{A}^3$이다.
  - $(\boldsymbol{A}^{n+1} = \boldsymbol{0})$이면 $(\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \cdots + \boldsymbol{A}^n$이다.

행렬(Matrix)

역행렬(Inverse Matrix)의 성질

- $\boldsymbol{A}$가 대칭행렬이면 $\boldsymbol{A}^T$이 대칭행렬이다.
- $\boldsymbol{A}$가 가역행렬이면 $\boldsymbol{A}^T$는 가역행렬이고 $(\boldsymbol{A}^T)^{-1} = (\boldsymbol{A}^{-1})^T$이다.

  $A^T(A^{-1})^T = (A^{-1}A)^T = I^T = I$, $(A^{-1})^T A^T = (AA^{-1})^T = I^T = I$

  $(A^{-1})^T$는 A^T의 역행렬이다($(A^{-1})^T = (A^T)^{-1}$).

- $\boldsymbol{A}$가 가역 대칭행렬이면 $\boldsymbol{A}^{-1}$이 대칭행렬이다.

  $(A^{-1})^T = (A^T)^{-1} = A^{-1}$

- $\boldsymbol{AA}^T$와 $\boldsymbol{A}^T\boldsymbol{A}$는 대칭행렬이다.

  $(AA^T)^T = (A^T)^T A^T = AA^T$

  $(A^T A)^T = A^T (A^T)^T = A^T A$

- $\boldsymbol{A}$가 가역행렬이면 $\boldsymbol{AA}^T$와 $\boldsymbol{A}^T\boldsymbol{A}$는 가역행렬이다.

  A가 가역행렬이면 A^T도 가역행렬이다. 즉, A^{-1}와 $(A^T)^{-1}$가 존재한다.

  $A^{-1}(A^T)^{-1} = (A^T A)^{-1}$

  $(A^T)^{-1}A^{-1} = (AA^T)^{-1}$

- $\boldsymbol{A}$와 $\boldsymbol{B}$가 대칭행렬일 때, $\boldsymbol{AB}$가 대칭행렬인 것은 $(\boldsymbol{AB} = \boldsymbol{BA})$와 동치이다.

  A와 B가 대칭행렬 $\Rightarrow A^T = A$, $B^T = B$

  ① $(AB = BA) \Rightarrow AB$가 대칭행렬

  $(AB)^T = B^T A^T = BA = AB$

  ② AB가 대칭행렬 $\Rightarrow (AB = BA)$

  $(AB)^T = AB$

  $AB = (AB)^T = B^T A^T = BA$

$(\boldsymbol{AB})^T = \boldsymbol{B}^T\boldsymbol{A}^T$

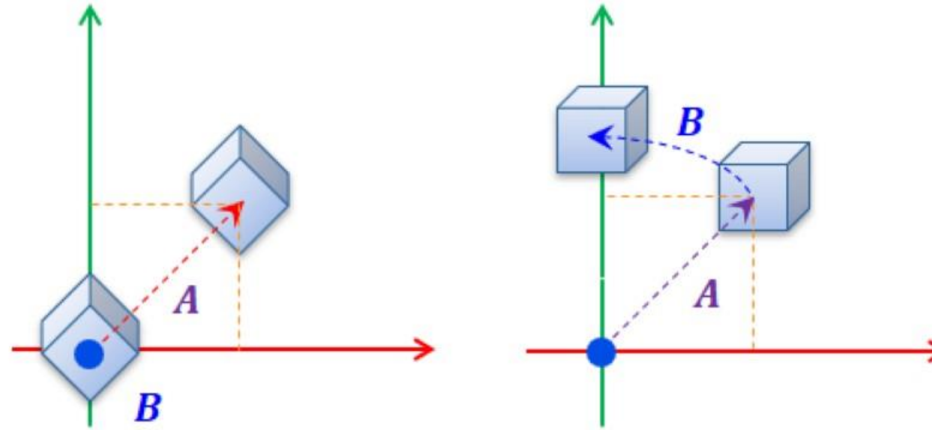
$\boldsymbol{B}^T\boldsymbol{A}^T = (\boldsymbol{AB})^T$

대칭행렬: $\boldsymbol{A}^T = \boldsymbol{A}$

$(\boldsymbol{AB} = \boldsymbol{BA})$: 교환 가능(Commute)

행렬(Matrix)

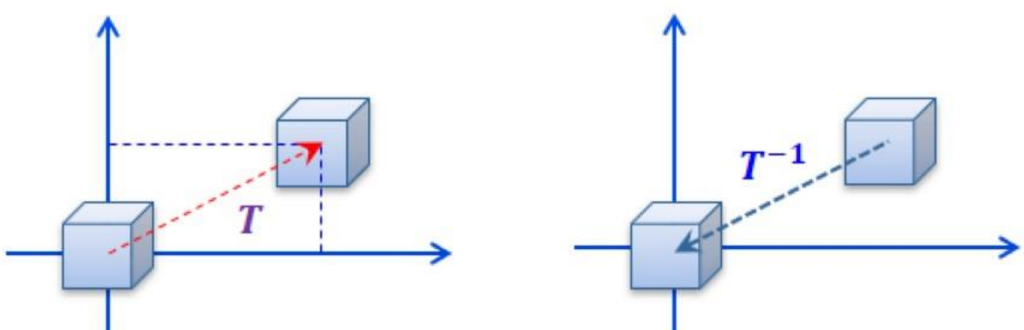
- **행렬의 연산**
  - 행렬의 곱
    - $A\,v_1 = v_2$
  - 행렬의 곱의 교환법칙
    - $AB \neq BA$

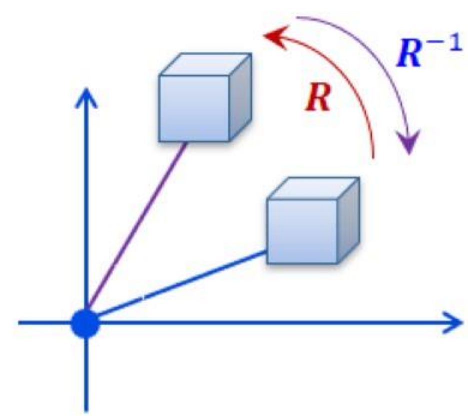


  - 역행렬(Inverse Matrix)
    역행렬은 **역변환**을 의미
    - $A\,v_1 = v_2,\ (A^{-1})v_2 = (A^{-1})(A\,v_1) = (A^{-1}A)v_1 = v_1$

$$\boxed{A\,A^{-1} = A^{-1}A = I}$$

  - 평행이동 변환과 역행렬
  - 회전 변환과 역행렬





행렬(Matrix)

- **행렬의 연산**
  - 행렬의 곱의 결합법칙
    - $A(BC) = (AB)C$

$$
\begin{aligned}
&\boldsymbol{A}_1\boldsymbol{v}_{i1} = \boldsymbol{v}_{i2}\\
&\boldsymbol{A}_2\boldsymbol{v}_{i2} = \boldsymbol{v}_{i3}\\
&\boldsymbol{A}_3\boldsymbol{v}_{i3} = \boldsymbol{v}_{i4}\\
&\cdots\\
&\boldsymbol{A}_n\boldsymbol{v}_{i(n-1)} = \boldsymbol{v}_{in}
\end{aligned}
$$

벡터와 행렬의 곱:15번 연산

$n = 5, i = 10{,}000$
$(15 * 5) * 10{,}000 = 750{,}000$

$n = 10, i = 10{,}000$
$(15 * 10) * 10{,}000 = 1{,}500{,}000$

$$
\begin{aligned}
&\boldsymbol{A}_1\boldsymbol{v}_{i1} = \boldsymbol{v}_{i2}\\
&\boldsymbol{A}_2(\boldsymbol{A}_1\boldsymbol{v}_{i1}) = \boldsymbol{v}_{i3}\\
&\boldsymbol{A}_3(\boldsymbol{A}_2(\boldsymbol{A}_1\boldsymbol{v}_{i1})) = \boldsymbol{v}_{i4}\\
&\cdots\\
&\boldsymbol{A}_n(\boldsymbol{A}_{(n-1)}\cdots\boldsymbol{A}_3((\boldsymbol{A}_2(\boldsymbol{A}_1\boldsymbol{v}_{i1}))))) = \boldsymbol{v}_{in}\\
&(\boldsymbol{A}_n\boldsymbol{A}_{(n-1)}\cdots\boldsymbol{A}_3\boldsymbol{A}_2\boldsymbol{A}_1)\boldsymbol{v}_{i1} = \boldsymbol{v}_{in}\\
&\boldsymbol{M} = \boldsymbol{A}_n\boldsymbol{A}_{(n-1)}\cdots\boldsymbol{A}_3\boldsymbol{A}_2\boldsymbol{A}_1\\
&\boldsymbol{M}\,\boldsymbol{v}_{i1} = \boldsymbol{v}_{in}
\end{aligned}
$$

행렬과 행렬의 곱:45번 연산

$n = 5, i = 10{,}000$
$(45 * 4) + 15 * 10{,}000 = 150{,}180$

$n = 10, i = 10{,}000$
$(45 * 9) + 15 * 10{,}000 = 150{,}405$

행렬(Matrix)

퀴즈(Quiz)

- (10×1) 열벡터 $\boldsymbol{x}$와 $\boldsymbol{y}$가 주어질 때 $\boldsymbol{v}$를 최소의 곱셈과 덧셈 연산을 사용하여 계산하기 위한 방법은?

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{10} \end{bmatrix} \quad \boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{10} \end{bmatrix}$$

$$\boldsymbol{v} = \boldsymbol{x}\boldsymbol{x}^T(\boldsymbol{I} + \boldsymbol{y}\boldsymbol{y}^T)\boldsymbol{x}$$

- $(n \times n)$ 행렬 $\boldsymbol{A}$가 가역행열이고, $(n \times 1)$ 열벡터 $\boldsymbol{b}$, $\boldsymbol{c}$에 대하여 $\boldsymbol{c}^T\boldsymbol{A}^{-1}\boldsymbol{b} \neq -1$일 때 다음이 성립함을 증명하라.

$$(\boldsymbol{A} + \boldsymbol{b}\boldsymbol{c}^T)^{-1} = \boldsymbol{A}^{-1} - \frac{\boldsymbol{A}^{-1}\boldsymbol{b}\boldsymbol{c}^T\boldsymbol{A}^{-1}}{1 + \boldsymbol{c}^T\boldsymbol{A}^{-1}\boldsymbol{b}}$$