Statistics – Hypothesis Testing

Hypothesis Testing

Hypothesis Testing – an example

The ABC Company produces screws with a target length of 100 millimeters (mm).

The length of the screws follows a Normal Distribution with a (population) standard deviation of 2 mm.

The machines need to be cleaned and recalibrated once a week. After the cleaning/recalibration process, ABC produces a sample of 20 screws to check whether the machines are correctly calibrated (mean length = 100 mm).

After the most recent calibration you suspect that the machines are incorrectly calibrated. Based on the drawn sample (sample size = 20) with sample mean 100.929 mm, test on a 2% level of significance, whether the machine is correctly calibrated or corrupted (two-tailed). Calculate the z-statistic and the p-value of your test.

Null Hypothesis and alternative Hypothesis

What you (actually) want to assess:

"you suspect that the machines are incorrectly calibrated"

But: You cannot really prove anything with statistics!

Solution: Reject the opposite statement / hypothesis:

"the machines are correctly calibrated"

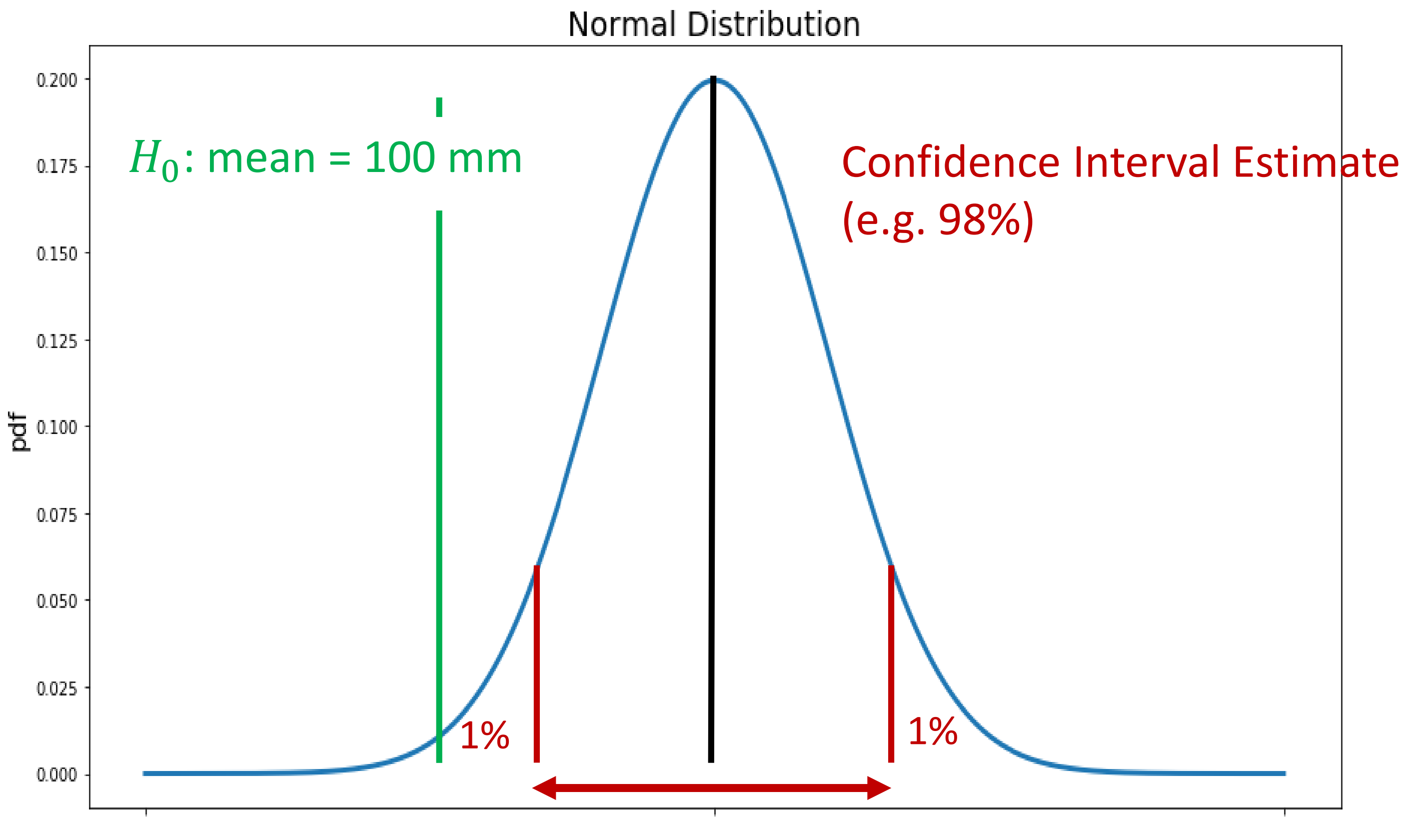
The ABC Company produces screws with a target length of 100 millimeters (mm). The length of the screws follows a Normal Distribution with a (population) standard deviation of 2 mm.
The machines need to be cleaned and recalibrated once a week. After the cleaning/recalibration process, ABC produces a sample of 20 screws to check whether the machines are correctly calibrated (mean length = 100 mm).
After the most recent calibration you suspect that the machines are incorrectly calibrated. Based on the drawn sample (sample size = 20) with sample mean 100.929 mm, test on a 2% level of significance, whether the machine is correctly calibrated or corrupted (two-tailed). Calculate the z-statistic and the p-value of your test.

Null Hypothesis (H_0)	Alternative Hypothesis (H_a)
mean length = 100 mm	mean length ≠ 100 mm

→ When H_0 is rejected, the implication is that H_a is a valid statement!

we are almost there…

Sampling Distribution

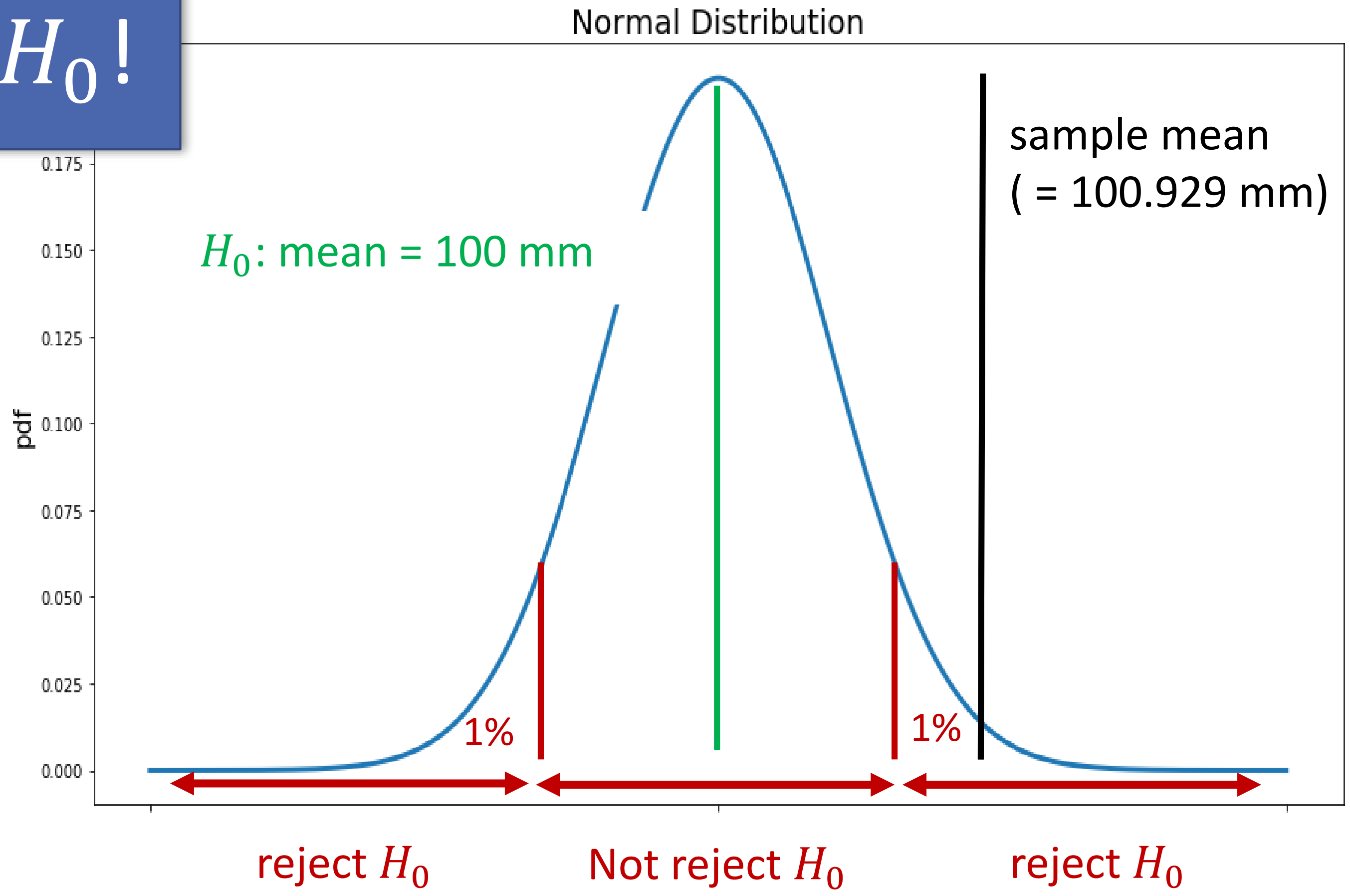


sample mean
(= 100.929 mm)

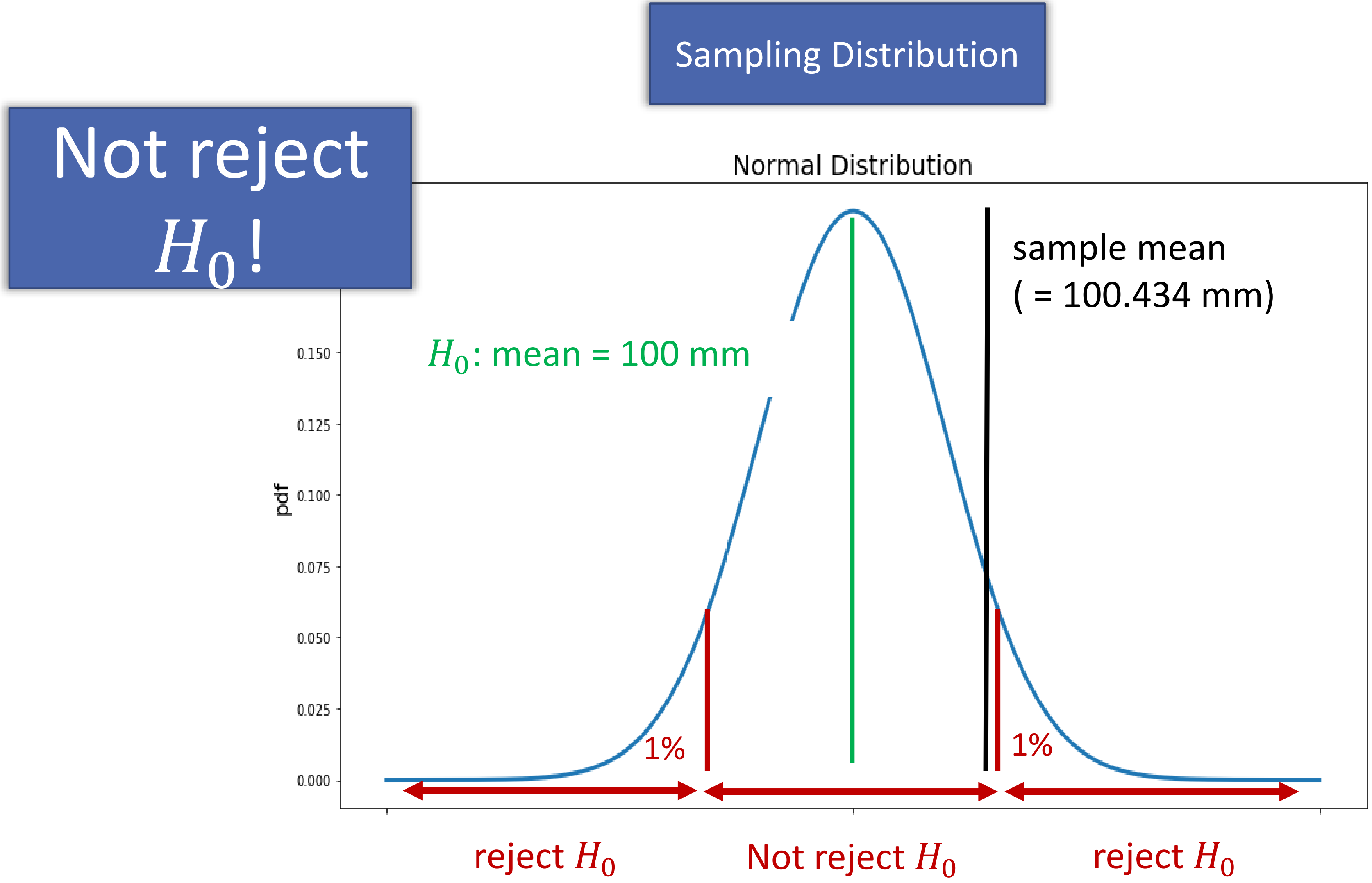
…just a shift

Sampling Distribution

Reject H_0!



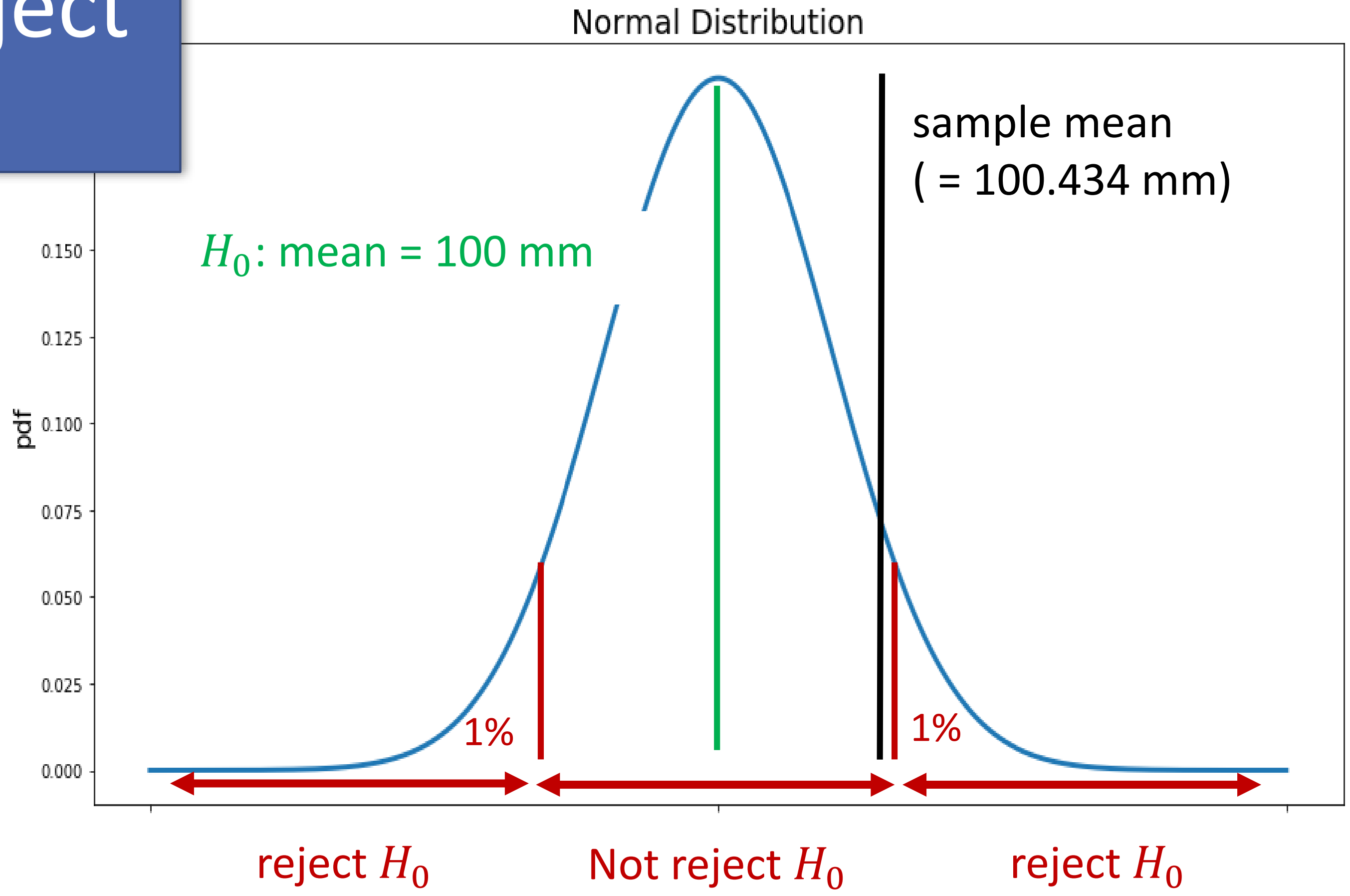
…another example



…it depends on the…

Sampling Distribution

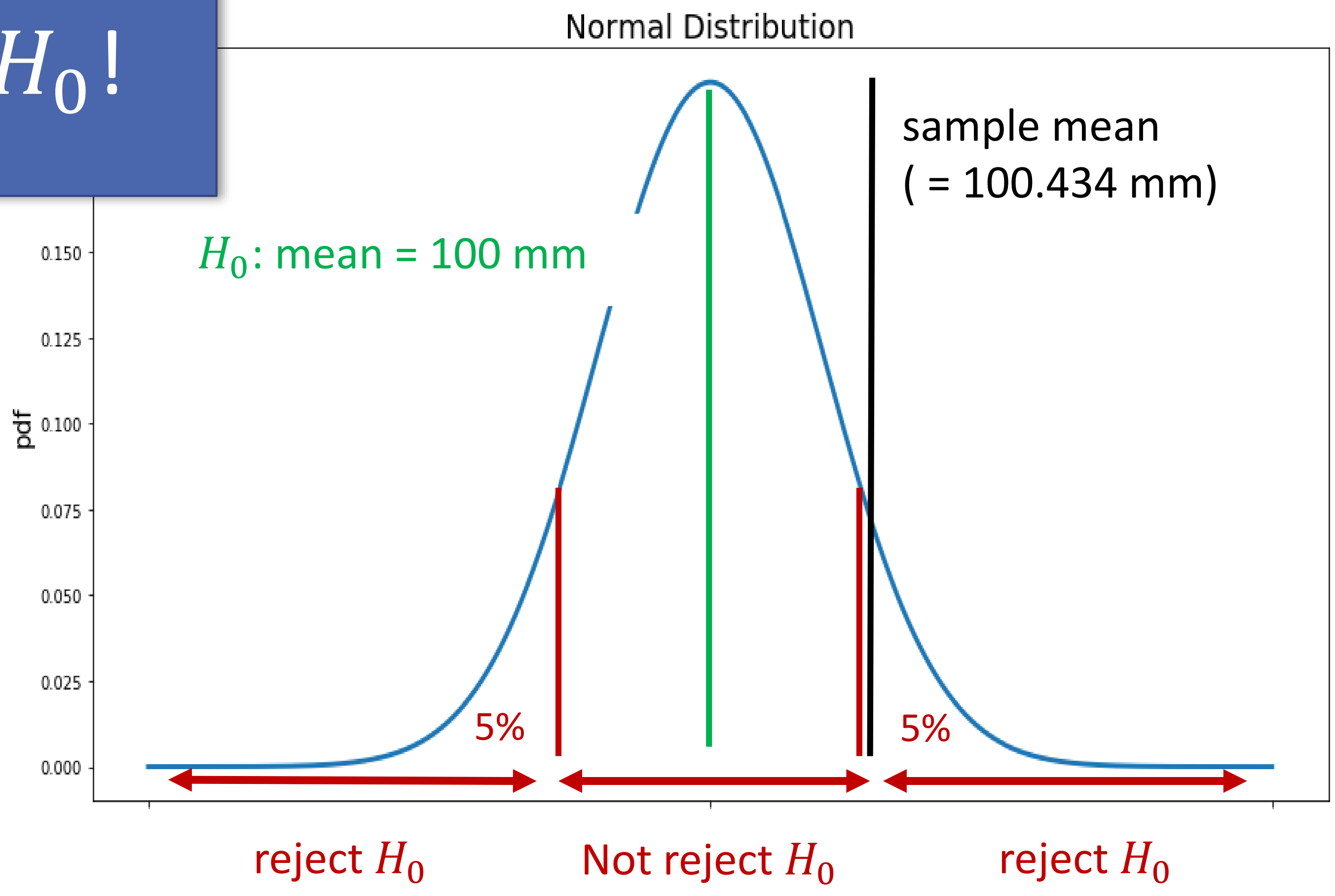
Not reject H_0!



...Level of Significance

Sampling Distribution

Reject H_0!



Choosing the Level of Significance (α)

Intuition behind the Significance Level (α)

- Probability of rejecting H_0 when it is true. (Type I Error)
- Decreasing α lowers probability of Type I Error…
- …but increases the probability of Type II Error (not rejecting H_0 when it s false)…
- …and therefore it´s getting harder to prove that H_a is a valid statement.
- The right α? it depends:
  - Research: between 5% and 1% … or even less
  - Business: it depends on the action:
    - Producing too long / too short screws vs.
    - Stopping the machines to recalibrate

  → economic analysis / business decision