

Statistics – Student's t-Distribution



Student's
t-Distribution

Student's t vs. Normal
Distribution? What to use when?

Confidence Intervals

To create a **Confidence Interval Estimate** for the **population mean**, we require:

- Point Estimate for the mean



sample mean

- Standard Error



~~$\frac{\text{population std}}{\sqrt{n}}$~~

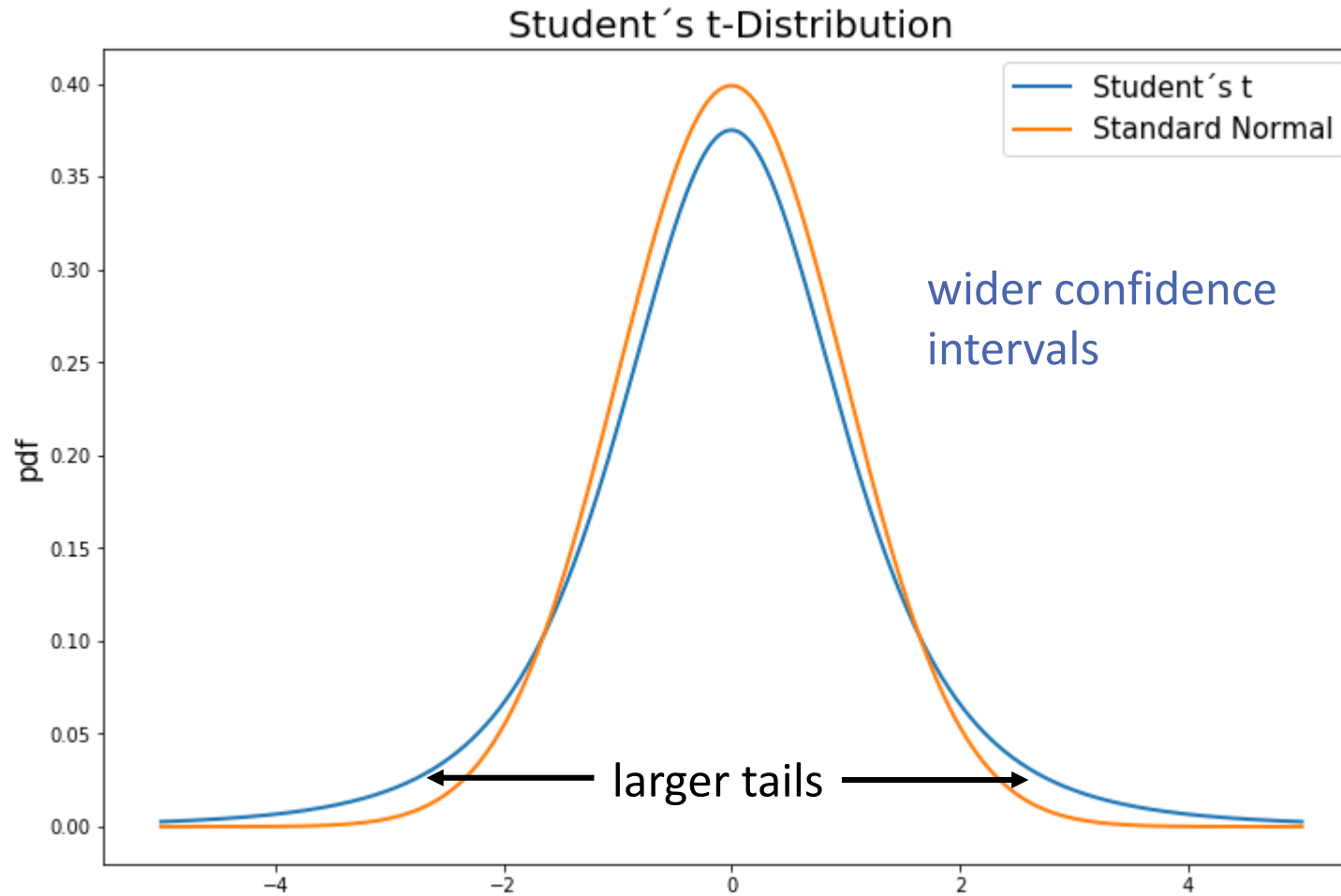
- Point Estimate for the std



sample std

→ estimates are less reliable/precise (in particular for small sample sizes)
→ Normal Distribution not appropriate, we need a Distribution with more probabilities in the tails (more conservative)

Use Student's t-Distribution



properties and t-tables

Properties

- **symmetrical** around the mean
- completely **defined** and described by the “degrees of freedom df ” (**sample size** – 1).
- more probabilities in the tails (“**fatter tails**”)
- for **large sample sizes**, it approaches the **standard normal distribution**
- **t-scores** can be converted to **probabilities** and vice versa (depending on df) → t-table / scipy.stats

Numbers in each row of the table are values on a t -distribution with (df) degrees of freedom for selected right-tail (greater-than) probabilities (p).



| df/p | 0.40 | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
|------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 0.324920 | 1.108032 | 1.960954 | 2.231817 | 2.706137 | 3.077744 | 3.182446 |
| 2 | 0.288675 | 0.958962 | 1.677955 | 1.885917 | 2.179891 | 2.575829 | 2.689529 |
| 3 | 0.274777 | 0.917241 | 1.637716 | 1.847714 | 2.131847 | 2.547554 | 2.669269 |
| 4 | 0.266927 | 0.896080 | 1.612736 | 1.821438 | 2.109815 | 2.524949 | 2.648690 |
| 5 | 0.261835 | 0.880214 | 1.599231 | 1.807017 | 2.099879 | 2.517654 | 2.638693 |
| 6 | 0.258193 | 0.869580 | 1.589329 | 1.795886 | 2.093022 | 2.511929 | 2.631667 |
| 7 | 0.255570 | 0.860814 | 1.581179 | 1.786264 | 2.087564 | 2.507093 | 2.626420 |
| 8 | 0.253639 | 0.853815 | 1.574582 | 1.778359 | 2.083140 | 2.503143 | 2.622575 |
| 9 | 0.252240 | 0.848187 | 1.569044 | 1.772033 | 2.079713 | 2.500000 | 2.619694 |
| 10 | 0.251189 | 0.843601 | 1.564267 | 1.766888 | 2.077043 | 2.497514 | 2.617514 |
| 11 | 0.250353 | 0.839732 | 1.560121 | 1.762793 | 2.075001 | 2.495693 | 2.615979 |
| 12 | 0.249689 | 0.836458 | 1.556598 | 1.759639 | 2.073576 | 2.494327 | 2.614893 |
| 13 | 0.249166 | 0.833668 | 1.553599 | 1.757298 | 2.072592 | 2.493315 | 2.614097 |
| 14 | 0.248753 | 0.831262 | 1.551016 | 1.755670 | 2.071967 | 2.492638 | 2.613521 |
| 15 | 0.248429 | 0.829141 | 1.548839 | 1.754400 | 2.071554 | 2.492187 | 2.613125 |
| 16 | 0.248174 | 0.827206 | 1.547043 | 1.753427 | 2.071289 | 2.491841 | 2.612841 |
| 17 | 0.247978 | 0.825458 | 1.545521 | 1.752681 | 2.071067 | 2.491589 | 2.612625 |
| 18 | 0.247839 | 0.823888 | 1.544259 | 1.752093 | 2.070884 | 2.491391 | 2.612459 |
| 19 | 0.247748 | 0.822487 | 1.543141 | 1.751611 | 2.070736 | 2.491237 | 2.612325 |
| 20 | 0.247696 | 0.821246 | 1.542151 | 1.751216 | 2.070619 | 2.491117 | 2.612217 |
| 22 | 0.247625 | 0.819965 | 1.541187 | 1.750879 | 2.070528 | 2.491026 | 2.612131 |
| 23 | 0.247589 | 0.819000 | 1.540511 | 1.750641 | 2.070467 | 2.490975 | 2.612080 |
| 24 | 0.247561 | 0.818241 | 1.540000 | 1.750478 | 2.070421 | 2.490931 | 2.612041 |
| 25 | 0.247539 | 0.817580 | 1.539582 | 1.750341 | 2.070387 | 2.490897 | 2.612009 |
| 26 | 0.247522 | 0.816999 | 1.539241 | 1.750229 | 2.070362 | 2.490871 | 2.611983 |
| 27 | 0.247509 | 0.816480 | 1.538962 | 1.750137 | 2.070344 | 2.490852 | 2.611962 |
| 28 | 0.247498 | 0.816014 | 1.538731 | 1.750061 | 2.070331 | 2.490839 | 2.611946 |
| 29 | 0.247489 | 0.815594 | 1.538534 | 1.749999 | 2.070322 | 2.490830 | 2.611935 |
| 30 | 0.247482 | 0.815212 | 1.538367 | 1.749950 | 2.070316 | 2.490824 | 2.611928 |
| z | 0.253347 | 0.674490 | 1.281552 | 1.644854 | 1.95996 | 2.32635 | 2.57583 |
| CI | —— | —— | 80% | 90% | 95% | 98% | 99.9% |