# Master Paper Note

Tri Huynh

April 2017

# 1 Current Goal

Create new methods and tools to visualize volumetric space-time data:

- Create interactive 3D tool to visualize and inspect the data: e.g. showing the tracked path with MIP at each time step, allowing users to browse through data by interacting with the tracked path, ...
- Propose some methods to make new visualizations as the summary of the space-time data: e.g. extend curved planar reformation to 4D data, ...

# 2 Technical Note

## 2.1 Coordinate Frame

### 2.1.1 Frenet-Serret Frame

The Frenet–Serret formulas describe the kinematic properties of a particle moving along a continuous, differentiable curve in three-dimensional Euclidean space. The formulas describe the derivatives of the so-called tangent, normal, and binormal unit vectors in terms of each other.
The tangent, normal, and binormal unit vectors, often called T, N, and B, or collectively the Frenet–Serret frame or TNB frame, together form an orthonormal basis spanning $R^3$ and are defined as follows:

- T is the unit vector tangent to the curve, pointing in the direction of motion.
- N is the normal unit vector, the derivative of T with respect to the arclength parameter of the curve, divided by its length.
- B is the binormal unit vector, the cross product of T and N.

Suppose that the curve is given by r(t), where the parameter t need no longer be arclength. Then the unit tangent vector T may be written as

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}.$$

The normal vector N takes the form

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|} = \frac{\mathbf{r}'(t) \times (\mathbf{r}''(t) \times \mathbf{r}'(t))}{\|\mathbf{r}'(t)\| \, \|\mathbf{r}''(t) \times \mathbf{r}'(t)\|}.$$

The binormal B is then

$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t) = \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}.$$

[Todo: Add sample image of the frame]

#### 2.1.2 Parallel Transport

#### 2.1.3 PCA Pseudo Side Vector

While the Frenet-Serret Frame can fit well to curves with high curvature, e.g. helix, it may not be desirable in many situations. For a simple example, since the N(t) vector is the second derivative at time t, the Frenet-Serret Frame is not well defined along a straight line segment of the curve.
For our data, the trajectory of the tracked path mainly lies inside a plane, which suggests we can use the direction orthogonal to the plane as one pseudo side vector for the local coordinate frame. Here, we propose to find that direction based on the Principle Components of the tracked points. We perform PCA on the set of tracked points, the principle component with lowest eigenvalue is then the pseudo side vector for our coordinate frame.

### 2.2 Curved Planar Reformation

#### 2.2.1 In 3D Spacial Data

#### 2.2.2 Extension to 4D Space-Time Data

### 2.3 Volume Rendering

#### 2.3.1 Simple MIP

Maximum Intensity Projection (MIP) is the classic method to render 3D volume data, where the rays passing each image pixel are projected through the volume (either by orthographic or perspective projection). The maximum value along the ray is then assigned to the corresponding pixel value.

#### 2.3.2 RFP-Isosurface-Bounded MIP

MIP has limitations in case there are many objects in the volume, in which they can overlap from each other, causing difficulties to interpret the data. Based on the nature of our data and specific interest in investigating the neurons, we propose to use RFP channel to limit the region of interest when doing projection on GFP channel. The RFP channel captures the neurons in zebrafish, therefore its isosurfaces can be used as the boundary of the neurons. Based on this information, we perform projection on GFP channel (where the nuclei of the cells are captured) within the found boundary surface.

### 2.3.3 Newton-based Maxima

MIP doesn't always generate desirable images due to noise and the nature of the operation to project everything along the ray, including objects that are not of our interest. Therefore, an analytical approach to extract the object of interest would help clean up the visualization substantially. Observing the data, we see that the image of each cell approximately follows a Gaussian form, with the highest value in the center of the cell. The farther away from the cell center, the voxel value decreases gradually. Based on this knowledge of data, we propose to use a method to find the local maxima and visualize the cell based on the distance to the maxima. [Todo: add Taylor series formula, add images of the 3 rendering methods for comparison]

## 2.4 Capabilities of The 3D Interactive System

### 2.4.1 Building Components

- Teem
- Hale
- CUDA

### 2.4.2 Functionalities