Wirtschaftswissenschaftliche Fakultät

Lehrstuhl für Betriebswirtschaftslehre mit Schwerpunkt

Management Science / Operations and Supply Chain Management

Master Seminar

in Management Science

Competitive facility location and design problem

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Passau, Abgabedatum:	19.07.2024

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Contents

1. Introduction

In today's increasingly competitive marketplace, particularly in the retail, healthcare, and logistics sectors, facility planners face more complex decisions, especially when many factors influence them. These decisions are compounded by limited available capital. Among those factors, the location and characteristics of facilities can significantly impact market share, customer satisfaction, and operational efficiency. More specifically, a customer's preference for a facility will be influenced by the distance between them and this facility's attractiveness, which can be determined by quantitative (e.g., size, product quality, quantity) and qualitative (e.g., customer service, pricing) characteristics.

Therefore, the Competitive Facility Location and Design Problem (CFLDP) has become an essential problem for several companies. Solving it with a model that is effective, applicable in many cases, and highly scalable becomes essential. In 2007, R. Aboolian, O. Berman, and D. Krass published two papers in the European Journal of Operational Research: "Competitive Facility Location and Design Problem" and "Competitive Facility Location Model with Concave Demand". The authors introduced a general spatial interaction model that overcomes some of the limitations that models from previous studies did not address (or had shortcomings). From now on, the term R. Aboolian et al. (2007) refers to both papers mentioned above.

The CFLDP involves questions of a company about the number, location, and characteristics of new facilities that intend to be opened in an area where facilities from competitors as well as the company itself already exist. It can be formulated as a spatial interaction model. R. Aboolian et al. (2007) pointed out that in the standard spatial interaction model, customer demand and the attractiveness of the facilities are assumed to be fixed. This assumption implies that the sole determining factor is the location itself. However, the capacity to modify the attractiveness of the facility enables CFLDP to capture the design-location trade-offs. Given a restricted amount of capital, opening more facilities allows for closer proximity to customers, but this reduces the overall level of attractiveness. The model in R. Aboolian et al. (2007) incorporates the concept of variable expenditures derived from Berman and Krass (1998). It allows for considering elastic demand, permitting both cannibalization and market expansion effects. The introduction of new facilities will not only result in a shift in customer demand from existing facilities but also an increase in the total level of attractiveness, leading to overall growth in demand. However, the demand for an individual facility will not increase when a new facility is opened, which means that the cannibalization effect always overcomes the market expansion effect at the individual level. Nevertheless, this does not mean the total demand for all the company's facilities will not increase. Adding new facilities can still be beneficial from the company's perspective.

R. Aboolian et al. (2007) consider a concave, non-decreasing demand function in their model. The non-linearity poses difficulties in solving the problem. Therefore, the authors have proposed two approaches to solve the problem more optimally in terms of computing time: Tangent-Line Approximation Procedure (TLA) and two heuristic algorithms. One of the heuristics is the Adapted Weighted Greedy Heuristic. These methods effectively reduce the computation time required, especially in large-scale instances, while maintaining an acceptable level of relative errors between the approximated and optimal solutions.

This thesis will analyze models of CFLDP and the approximation methods given in R. Aboolian et al. (2007). In Chapter 2, the mathematical model of the problem is presented and explained in detail. Furthermore, the impact of parameters within the objective function on the direction of the solution will be discussed. In Chapter 3, the Tangent Line Approximation Procedure (TLA) will be analyzed, explained, and compared with the Adapted Weighted Greedy Heuristic. Finally, in Chapter 4, computational results will be presented to compare the effectiveness of the TLA approach and the exact model, as well as illustrate the solution changes when adjusting the values of parameters.

2. Mathematical formulation of CFLDP

$N = \{1,..,n\}$	Set of customer nodes
$P \subset N$	Set of potential facility locations, including existing facilities
$C \subset P$	Set of nodes for competitive facilities
$S = P - C$	Set of nodes for locating the company's facilities
u_{ij}	The utility of customers $i \in N$ for a facility at location $j \in P$
$U_i(S)$	The total utility for the customers i from our company
$U_i(C)$	The total utility for the customers i from competitive companies
K	Number of attractiveness attributes
A_j	The attractiveness of facility $j \in P$
α_j	The basic attractiveness of facility $j \in P$
f_j	Fixed cost to open facility j
$c_j(y_{jk})$	Cost of improving the attribute k of the basic design to the level y_jk>0

d_{ij}	Distance between customer $i \in N$ and facility $j \in P$
B	The budget
$x_j \in \{0,\ 1\}$	Decision variable for the opening of facility $j \in P$
$y_{jk} \in [0, y_k^{max}]$	Level of improvement over the basic design attribute k for facility j

It is assumed that each node $i \in N$ will include ω_i customers who have identical spending behavior and make the same facility choices. Companies can select locations in $P \subset$ N for the opening of facilities. It is assumed that each node $j \in P$ can have at most one facility open. However, without loss of generality, the presence of two factories at the same node can be achieved by creating a copy of that node with the same location parameters. In our model, information about the location and design of existing facilities, both from our company and the competitors, is known in advance. With budget B, the company can place its facilities at nodes in subset $S = P - C$. Each node within the considered area is placed within a metric space, wherein the distance between any two nodes i and j, is represented by d_{ij}. The value of d_{ij} may be either the Euclidean distance or the shortest path between two nodes.

In selecting a design for a facility, K characteristics will be considered. Each characteristic $k \in K$ will be assigned a basic level of attractiveness, which will be associated with a fixed cost. The total for all K characteristics will result in the basic attractiveness level α_j, with a fixed cost f_j of the facility at node j. From this basic level, we can improve the level of each attribute further by value y_{jk} at a cost $c_j(y_{jk})$. As a characteristic can be either qualitative or quantitative, the level of improvement y_{jk} can vary within a certain interval $[0, y_k^{max}]$ or a discrete set. The attractiveness term A_j will be calculated as follows:

$$A_j = \alpha_j \prod_{k \in K} \left(1 + y_{jk}\right)^{\theta_k}$$

The parameter θ_k control the level of impact on improving a characteristic. With $0 \leq \theta_k \leq 1$ and $y_{jk} \geq 0$, the function is concave, non-decreasing in y_{jk} with a minimum value α_j. As $\theta_k \to 0$ and y_{jk} increases, the marginal impact of these improvements is reduced, and vice versa.

R. Aboolian et al. (2007) use a gravity-type spatial interaction model in which the customers split their demand between available facilities in the considered area. The probability of choosing a facility increases with the utility that this customer derives from it, whose value is a function of the distance between them and the facility's attractiveness. In particular, they adopt the product form of the utility functions, whereby the value will decrease in distance and increase in attractiveness:

$$u_{ij} = \frac{F(A_j)}{H(d_{ij})} = A_j\left(d_{ij} + 1\right)^{-\beta}$$

In order to prevent a facility from automatically capturing all customers' utility at the same location, regardless of its attractiveness, we add 1 to the distance. The parameter β represents the distance sensitivity of customers. As $\beta \to 0$, customer demand depends less on distance, so investments in attractiveness will become more effective. Therefore, we tend to open fewer facilities that are more attractive. However, R. Aboolian et al. (2007) pointed out that the decision also depends on the relationship between fixed costs and costs for improvement, as well as the level of demand elasticity.

Then, the utilities are aggregated at the company level. That means utility derived by the customer from a company is equal to utility derived by the customer from all of its facilities:

$$U_i(S) = \sum_{j \in S} u_{ij} \ ; \ U_i(C) = \sum_{j \in C} u_{ij}$$

It also shows that our company's market share derived by a customer is the proportion of utilities from our company in the total utility:

$$MS_i = \frac{U_i(S)}{U_i(S) + U_i(C)} = \frac{U_i(S)}{U_i} = 1 - \frac{U_i(C)}{U_i}$$

R. Aboolian et al. (2007) assume that the total demand of customers is elastic and develop the exponential demand function $g(U_i)$, a total utility function whose value will range from 0 to 1.

$$0 \le g(U_i) = 1 - \mathrm{e}^{-\lambda U_i} \le 1$$

This function represents the proportion of the maximal available demand ω_i captured by all companies. If the demand elasticity parameter λ is very big ($\lambda \to \infty$), the demand of customers becomes very inelastic. The value of the demand function will be very close to one. Therefore, the total demand of customers tends to equal their demand weight regardless of the value of total utility. Therefore, it is essential to capture as much market share as possible by locating new facilities near competitors' facilities. The opposite is indicated when $\lambda \to 0$. The value of the demand function is small ($g(U_i) \to 0$). Therefore, many customer demands have not been satisfied. The market expansion effect becomes crucial, so new facilities should be opened where few facilities are presented.

After defining all the components, R. Aboolian et al. (2007) constructed the MIP formulation for CFLDP:

$$\begin{aligned} &\textit{maximize} \quad \textstyle\sum_{i\in N} \ \omega_i \, g(U_i)\, MS_i \\ &\textit{subject to} \quad \textstyle\sum_{j\in S}\left[\sum_{k\in K} c_j\left(y_{jk}\right) + f_j x_j\right] \le B && \textit{(1)} \\ &\qquad\qquad \textstyle\sum_{k\in K} y_{jk} \le M x_j, \quad j \in S && \textit{(2)} \\ &\qquad\qquad x_j \in \{0,\ 1\},\ \ y_{jk} \in [0, y_k^{max}] && \textit{(3, 4)} \end{aligned}$$

The objective function is to maximize the total demand from all customers captured by the company. The constraints (1) avoid total costs exceeding our limited budget. The set of constraints (2) restricts the values of the design variables y_{jk} equal to 0 if there is no new facility opened at node j. Constraint sets (3) and (4) are used to define decision variables. Binary variables x_j represent the considerations of whether we open a new facility at node j. Positive continuous variables y_{jk} represent the level of improvement over the basic design for the design characteristic k of facility j.

In many real-world situations, the options for facility design are discrete rather than continuous. For example, a facility may have a limited number of potential sizes, layouts, or product varieties. The number of staff, machines, and many other factors can take discrete, specifically integer, values. Therefore, R. Aboolian et al. (2007) introduced a formulation for CFLDP with discrete design scenarios. In this formulation, only a finite number of facility designs are available, and one must be chosen when opening a new facility.

The number of scenarios can be defined as $R = \prod_{k\in K} r_k$ with r_k is the number of possible values for characteristic $k \in K$. The cost c_{jr} and attractiveness k_{ijr} of each scenario $r \in \{1, .., R\}$ with respect to customer i are known in advance. Therefore, the value of y_{jk} and x_j in the general formulation can be integrated into binary variables x_{jr} in the new formulation, which has the value of 1 if a new facility will be opened at node j with design scenario r and 0 otherwise. With the new definitions, CFLDP with Discrete Design Scenarios can be formulated as follows:

$$\begin{aligned} &\textit{maximize} \quad \textstyle\sum_{i\in N} \ \omega_i \ g\left(\sum_{j\in S} \sum_{r=1}^{R} \ k_{ijr} x_{jr} + U_i(C)\right)\left(\frac{\sum_{j\in S} \sum_{r=1}^{R} k_{ijr} x_{jr}}{\sum_{j\in S} \sum_{r=1}^{R} k_{ijr} x_{jr} + U_i(C)}\right) \\ &\textit{subject to} \quad \textstyle\sum_{j\in S} \sum_{r=1}^{R} \ c_{jr} x_{jr} \le B && (5) \\ &\qquad\qquad \textstyle\sum_{r=1}^{R} \ x_{jr} \le \ 1 \text{ for } j \in S && (6) \end{aligned}$$

$$x_{jr} \in \{0,1\} \text{ for } r = 1, \dots, R;\ j \in S \tag{7}$$

The objective function and the first constraint set are based on the same concept as the general CFLDP formulation. The set of constraints (6) ensures that a facility is located with only one design scenario. From now on, the methods developed in the upcoming chapter will be based on this problem formulation, called P.

3. Solution approaches

With $\omega_i(U_i) = g_i(U_i)MS_i = g_i(U_i)\left(1 - \frac{U_i(C)}{U_i}\right)$ for all $i \in N$, we derive the first derivative and the second derivative of $\omega_i(\widetilde{U})$:

$$\omega_i'(U_i) = g'(U_i)\left(1 - \frac{U_i(C)}{U_i}\right) + g_i(U_i)\frac{U_i(C)}{U_i^2} \geq 0$$

$$\omega_i''(U_i) = g''(U_i)\left(1 - \frac{U_i(C)}{U_i}\right) + g_i'(U_i)\frac{2U_i(C)}{U_i^2} - g_i''(U_i)\frac{2U_i(C)}{U_i^3} \leq 0$$

Therefore $\omega(U)$ is a twice differentiable, non-decreasing concave function such that $\omega(0) = 0$ for $i \in N$, which is separable to the components $\omega_i(U_i)$ for $i \in N$.

The non-linearity and non-convexity from the objective function make the formulated Integer Programming challenging to solve, even with modern solvers. To improve this problem, R. Aboolian et al. (2007) proposed two approaches: Tangent-Line Approximation (TLA) procedure and the Adapted Weighted Greedy Heuristic (AWGH) algorithm. Both have been proven to offer excellent performance regarding solution quality and computational time.

3.1. Tangent-Line Approximation (TLA)

Based on the particular structure of $\omega_i(\phi_i)$, R. Aboolian et al. (2007) proposed a concave, piece-wise linear over-approximator $\omega_i^{\alpha}(\phi_i)$ for a given $\epsilon \in (0,1)$.

Here, $\omega_i^{\alpha}(\phi_i) \geq \omega_i\,(\phi_i)$, and ϵ takes the pre-specified value that represents the maximum value of the relative error between $\omega_i(\phi_i)$ and $\omega_i^{\alpha}(\phi_i)$. The parameter of the function is ϕ_i with $0 \leq \phi_i = U_i(S) \leq \bar{\phi}_i$. The value $\bar{\phi}_i = \max\,(U_i(S))$ can be achieved when we ignore the limited budget and open new facilities at all possible nodes with the best design. As $\omega_i^{\alpha}(\phi_i)$ is a piece-wise linear function, it can be constructed from multiple linear segments. The number of segments is $l_i \in \{1, \dots, L\}$ with $L = 1 + \left\lceil \frac{1}{\epsilon} \right\rceil$, which depends on our choice of ϵ. We can choose a very small ϵ and almost achieve the optimal solution with the approximator. Still, the number of segments will be huge, especially when we have to determine the segments that make up the function for each customer.

To sum up, R. Aboolian et al. (2007) stated that the TLA procedure converges in finitely many steps to a piece-wise linear function $\omega_i^\alpha(\phi_i)$ such that $\omega_i^\alpha(0) = 0$ for every $i \in N$ with the features mentioned above. Let $\omega^\alpha(\phi) = \sum_{i\in N} \omega_i^\alpha(\phi_i)$. Then $\omega^\alpha(\phi)$ is a concave, piece-wise linear, non-decreasing function that is separable for the vector ϕ with components ϕ_i. The following statement can be made:

$$\omega^\alpha(\phi) \geqslant \omega\,(\phi)\ for\, \phi \in [0, \bar{\phi}];\ \max_{\phi} \left\{ \frac{\omega^\alpha(\phi) - \omega\,(\phi)}{\omega\,(\phi)} \middle| 0 \le \phi \le \bar{\phi} \right\} \leqslant \epsilon \in (0,1)$$

In order to construct $\omega^\alpha(\phi)$, we define each segment of $\omega^\alpha(\phi)$ by l, with $l \in \{1, .., L\}$. Each segment can be identified by: c_l and c_{l+1} are the starting and endpoint of segment l projected in the x-axis (ϕ); b_l is the slope of segment l; $a_l = c_{l+1} - c_l$ is the length of the segment projected in the x-axis (ϕ).

The authors also proved that to keep the number of segments to a minimum, we can choose the starting point of each segment c_l such that $\frac{\omega^\alpha(c_l) - \omega\,(c_l)}{\omega\,(c_l)} = \epsilon$, except for the first segment, where $c_1 = 0$, and the relative error can be set to 0. Then, the TLA procedure can be performed through the following steps:

Step 1: Start with $l = 1$ by setting $(\phi,\ \omega^\alpha(\phi)) = (c_1,\ \omega^\alpha(c_1)) = (0,\ 0)$ as the starting point of the first segment. The slope of the first segment $b_1 = \omega_i'(0)$. The segment is tangent to the graph of $\omega(.)$ at 0.

Step 2: Find the endpoint of the current segment l by locating a point $\omega^\alpha(c_{l+1})$ on the ray originating at c_l and with slope b_l such that the relative error $\frac{\omega^\alpha(c_l) - \omega\,(c_l)}{\omega\,(c_l)} = \epsilon$, using bisection method. The endpoint of segment l is the starting point of the next segment $l + 1$. Set $l = l + 1$.

Step 3: Find the ray originating at $(c_l,\ \omega^\alpha(c_l))$, that tangent to the graph of $\omega(.)$, by finding the tangent point using the bisection method. Repeat step 2 to find the endpoint of this segment.

Step 4: Repeat step 2 to find the endpoint of the current segment. If $\omega^\alpha(\phi)$ is defined for all points in $[0, \bar{\phi}]$, stop the procedure.

Note that in step 1, if $\frac{\omega^{\alpha}(\bar{\phi})-\omega(\bar{\phi})}{\omega(\bar{\phi})} < 1$, which means the relative error is less than its upper bound at the point $\bar{\phi}$, we can extend the first segment to $\bar{\phi}$ and achieve the approximation goal. In Step 3, we stop if the tangent line cannot be constructed on the interval $[c_l, \bar{\phi}]$. In this situation, the current segment is extended to point $(\bar{\phi}, \omega(\bar{\phi}))$ and set the slope to 0. By controlling exceptions, we ensure that the relative error can not exceed ϵ.

Given a continuous function $f(x)$ varies in an interval $[a, b]$, $f(a).f(b) < 0$, the bisection method stated in Step 2 and Step 3 can be formulated as follows:

(1) Calculate midpoint $c = (a + b)/2$

(2) Find the function value at the midpoint $f(c)$

(3) If $f(c) = 0$ or c is very close to a, return c and stop.

(4) If $f(c).f(a) > 0$, set $a = c$. Else, set $b = c$. Then repeat (1).

The TLA procedures are proved to converge in finitely many steps to a piece-wise linear non-decreasing concave function. R. Aboolian et al. (2007) also show that, although the upper bound on the number of segments is very large, especially for small ϵ, the number of segments is relatively small and acceptable in practice. They show that 12 segments for a function are sufficient to reduce the relative error to below 1%.

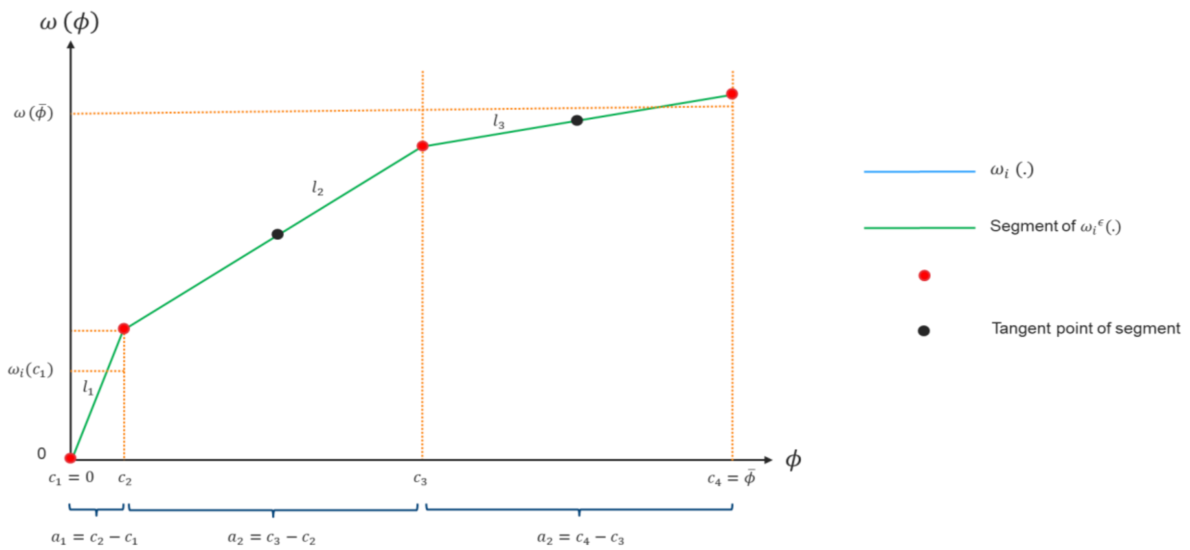

Figure 1 Example for a function with 3 segments

After identifying all the segments, we can transform the piece-wise linear, concave, and separable objective function into a linear one by introducing new continuous decision variables y_i^l for each linear segment in the objective function. The function can be formulated as follows:

$$\omega_i^\alpha(U) = \sum_{l=1}^{L^i(u)} a_l^i b_l^i y_l^i$$

for $U \in [0, \bar{U}]$, $L_i(U) = \max\{l: c_l \le U\}$, and $y_i^l = \begin{cases} 1 & if\ l < L_i(U) \\ \frac{U - c_l^i}{a_l^i} & if\ l = L_i(U) \end{cases}$.

We can formulate new linear integer programming for a new problem called P^ϵ:

$$\begin{aligned}
&maximize && \textstyle\sum_{i \in N} \sum_{l=1}^{L^i} w_i a_l^i b_l^i y_l^i && \\
&subject\ to && \textstyle\sum_{j \in S} k_{ijr} x_{jr} = \sum_{l=1}^{L^i} a_l^i y_l^i,\ \ for\ i \in N && (8) \\
& && \textstyle\sum_{r \in R} x_{jr} \le 1 && (9) \\
& && \textstyle\sum_{j \in S} \sum_{r=1}^{R} c_{jr} x_{jr} \le B && (10) \\
& && 0 \le y_l^i \le 1\ \text{ for } i \in N,\ l = 1, \dots, L^i && (11) \\
& && x_{jr} \in \{0,1\}\ for\ j \in S,\ for\ r \in R && (12)
\end{aligned}$$

The constraints (8) ensure that the lengths of $\omega_i^\alpha(U)$ and $\omega^\alpha(U)$ are equal when they are projected on the x-axis. The constraints (11) define new decision variables y_l^i and their bounds. Give x^* is the optimal solution for the original problem P with objective $Z(.)$, and x^ϵ is the optimal solution for the problem P^ϵ with objective Z^ϵ. R. Aboolian et al. (2007) show that:

$$Z(x^*) \le Z^\epsilon(x^\epsilon) \le (1+\epsilon) Z(x^*) \ \ and\ \frac{1}{1+\epsilon} Z(x^*) \le Z(x^\epsilon) \le Z(x^*)$$

So we can use x^ϵ as a $(1-\epsilon)$-optimal solution for the original problem. Furthermore, $Z(x^\epsilon)$ can be used as an upper bound for $Z(x^*)$, speeding up the calculation process while still providing the optimal solution.

3.2. Adapted Weighted Greedy Algorithm

R. Aboolian et al. (2007) proposed the Adapted Weighted Greedy Heuristic algorithm with the simple idea: starting with an empty set L, we identify the potential pair of node and design scenario (j, r) that provides the highest increase in the objective function per unit cost. Then we add (j, r) to the set L if the budget still allows, and there is no other pair with the same value j in L. The algorithm stops when no node-design pairs are available for improvement within the given budget.

We define the annotation $\rho_{jr}(L)$, which represents the improvement in the objective value if the location-design pair (j, r) is added to set L:

$$\rho_{jr}(L) = Z(L \cup \{j, r\}) - Z(L)$$

To avoid a node appearing more than once in set L, we introduce set T, which contains pairs that would be excluded from further consideration. The algorithm can be formulated as follows:

Step 1: Set $L^0 = \emptyset, T^0 = \emptyset, t = 1$

Step 2: Let $\big(j(t), r(t)\big) = \text{argmax}_{(j,r)\notin t^{t-1}} \left\{\frac{\rho_{jr}(L^{\prime -1})}{c_{j,r}}\right\}$,

If $\sum_{(j,r)\in L^{t-1}} c_{jr} + c_{j(t),r(t)} \leq B$:

Set $L^t = L^{t-1} + \big\{\big(j(t), r(t)\big)\big\},\ T^t = T^{t-1} + \{(j(t), r), r \in \{1, .., R\}\}$

Set $t = t + 1$

Return to step 2.

Else: Go to Step 3.

Step 3: If $Z(L^{t-1}) \geq Z(\{(j(t), r(t))\})$:

$L^H = L^{t-1}$ is the adapted greedy solution with value $Z^H = Z(L^{t-1})$

Else:

Set $L^1 = \{(j(t), r(t))\}, T^1 = \{(j(t), r), r \in \{1, \dots, R\}\}, t = 2$

Return to Step 2.

Stop

Step 2 ends when the selected node-design pair cannot be added to L. In Step 3, we check if the current node-design pair alone outperforms the current solution L. If it does, we reset the current solution to this location-design pair and restart Step 2; otherwise, the algorithm terminates. The authors have pointed out that, although the worst-case performance of the heuristic algorithm above can be quite bad: 65% or worse in comparison to the optimal solution, nevertheless, the computational experiments show that the algorithm tends to produce reasonably accurate solutions.

When comparing the proposed approaches, the adapted weighted greedy heuristic algorithm outperforms both the TLA procedure and the exact model regarding computation time. However, we cannot control the relative error between the result of this heuristic and the optimal solution or determine the gap between them. In contrast, the TLA procedure provides the relative error control. Therefore, it is more suitable for application in a highly competitive market, especially when decisions on CFLDP are often important and costly.

4. Computational experiments

4.1. Performance of TLA procedure

In this section, we test the correctness and effectiveness of the TLA procedure in solving CFLDP with discrete design scenarios. Computational experiments will be performed to compare the results of two TLA approaches ($\epsilon = 0.05$ and $\epsilon = 0.01$) with the exact model in terms of relative error and computational time. We will divide the computational time for the TLA procedure into two parts: constructing segments and solving the IP model. These experiments will be performed on a computer with a CPU configuration model: Intel(R) Core(TM) i7-4910MQ CPU @ 2.90GHz, 4 physical cores, 8 logical processors using Python 3.10 with gurobipy package (Gurobi 11.0.1).

Instances used for experiments will have the following configuration: Customer nodes will be randomly selected in a 100×100 map. The number of nodes for customer N ranges from 80 to 400. Set of node $P \subset N$ with $|P| = \left\lceil \frac{|N|}{3} \right\rceil$ are used for potential facility locations. Competitive facilities are placed on nodes belonging to set $C \subset P$ with $|C| = \left\lceil \frac{|P|}{3} \right\rceil$. The remaining nodes $S = P - S$ are potential nodes for our company's facilities. The locations of these nodes and the selection of subset elements will be chosen randomly. The distance between nodes is calculated using Euclidean distance. The value of the parameters will be set to $\beta = \lambda = \theta = 1$. We randomly select the demand weight of each customer as an integer value between 1 and 5. $K = 2$ attractiveness attributes are considered for new facilities. Each attribute can take two values, corresponding to basic and improved levels. Each attribute has two value levels, corresponding to basic and improved, and has the corresponding value $s \in \{0,1\}$. Therefore we have $2^2 = 4$ design scenarios that can be chosen. With $\theta = 1$, attractiveness value can take values of $A_j \in \{1, 2, 4\}$, corresponding to cost $c_j \in \{1, 2, 3\}$. Attractiveness values of competitive facilities are chosen as random integer values between 3 and 5. This setup helps us describe scenarios without losing the generality and scalability of the model. We have five instances created with five different seeds for each set of setup values. Each group's results (objective values and computational times) are the average of the instances belonging to that group. With all the instances in place, we perform two experiments by setting budget $B = 9$ and $B = 15$.

Customer	Relative Error		Num.TLA.Opt		Time.TLA.Segs		Time.TLA.IP		Total CPU Time		
	TLA 0.05	TLA 0.01	TLA 0.05	TLA 0.01	TLA 0.05	TLA 0.01	TLA 0.05	TLA 0.01	TLA 0.05	TLA 0.01	Exact Model
80	0.0007	0.0001	3	4	2.66	5.89	0.13	0.18	2.79	6.07	24.53
90	0.0011	0.0011	3	3	3.63	8.55	0.18	0.26	3.81	8.82	25.69
100	0.0000	0.0000	5	5	4.51	10.14	0.17	0.25	4.68	10.39	19.94
110	0.0000	0.0000	5	5	5.43	12.54	0.20	1.43	5.64	13.97	26.69
120	0.0000	0.0000	5	5	6.74	15.23	0.29	0.35	7.02	15.57	28.50
130	0.0000	0.0000	5	5	7.81	17.58	0.39	0.40	8.20	17.98	25.34
140	0.0000	0.0000	5	5	9.22	20.85	0.34	0.49	9.57	21.34	37.76
160	0.0015	0.0000	4	5	12.05	27.21	0.35	0.51	12.40	27.72	18.48
180	0.0000	0.0000	5	5	15.95	35.95	0.47	0.64	16.43	36.59	22.95
200	0.0000	0.0000	5	5	19.63	44.17	0.60	0.79	20.23	44.95	22.53
250	0.0000	0.0000	5	5	30.66	69.10	0.60	0.75	31.26	69.85	12.43
300	0.0000	0.0000	5	5	44.84	101.49	0.86	1.08	45.70	102.57	19.48
350	0.0000	0.0000	5	5	61.42	138.70	0.64	0.85	62.06	139.55	25.03
400	0.0000	0.0000	5	5	80.40	183.84	0.76	1.01	81.16	184.84	25.90

Tabelle 1 Experiment with Budget =9

Customer	Relative Error		Num.TLA.Opt		Time.TLA.Segs		Time.TLA.IP		Total CPU Time		
	TLA 0.05	TLA 0.01	TLA 0.05	TLA 0.01	TLA 0.05	TLA 0.01	TLA 0.05	TLA 0.01	TLA 0.05	TLA 0.01	Exact Model
80	0.0011	0.0004	3	4	2.64	5.91	0.32	0.43	2.97	6.34	109.86
90	0.0001	0.0001	4	4	3.79	8.29	0.40	0.58	4.19	8.87	132.17
100	0.0000	0.0000	5	5	4.50	10.15	0.38	0.51	4.88	10.66	97.29
110	0.0000	0.0000	5	5	5.50	12.53	0.54	0.63	6.04	13.16	161.17
120	0.0000	0.0031	5	4	6.90	15.38	0.58	0.72	7.48	16.10	244.69
130	0.0000	0.0000	5	5	7.85	17.67	0.64	0.99	8.49	18.66	172.96
140	0.0004	0.0004	4	4	9.28	20.94	0.75	1.04	10.03	21.98	277.97
160	0.0000	0.0000	5	5	12.45	27.48	0.81	1.00	13.26	28.48	266.07
180	0.0011	0.0000	3	5	16.21	36.23	0.98	1.31	17.19	37.54	191.94
200	0.0000	0.0000	5	5	19.81	44.63	1.20	1.88	21.01	46.51	233.23
250	0.0000	0.0000	5	5	31.32	70.16	1.42	2.09	32.74	72.25	232.76
300	0.0000	0.0000	5	5	45.60	102.61	1.55	2.59	47.15	105.20	245.60
350	0.0001	0.0000	4	5	62.32	139.24	2.01	2.35	64.33	141.59	158.08
400	0.0000	0.0000	5	5	82.52	185.20	1.75	2.43	84.27	187.63	245.52

Tabelle 2 Experiment with Budget = 15

The “Relative Error” columns indicate the relative errors between the solutions of the TLA procedure and the exact model. The “Num.TLA.OPT” gives the number of instances solved optimally. Columns “Time.TLA.Segs” and “Time.TLA.IP” correspond to the computational time needed to identify the segments and solve the linear IP model. The last column represents the total computational times of each approach.

The two tables show that TLA procedures always give results within the selected error gaps. Large numbers of instances are given optimal results. When decreasing the value of ϵ from 0.05 to 0.01, both the accuracy of the results and the number of optimal solutions are improved. However, the computational time also significantly increases when reducing ϵ to 0.01. The $\epsilon = 0.05$ can be an acceptable threshold.

The central computational time of the TLA procedure is used to identify segments. The time spent solving the linear IP is relatively tiny and strictly outperforms the exact model. Therefore, refining the construction of segments is important, especially for the bisection methods. Limiting the number of loops or setting an acceptable threshold for the midpoint c can be considered.

Point to note, the exact model handles instances with small budgets very well (here $B = 9$) and outperforms the TLA procedure in some instances, especially for large-scale instances. However, when the budget is increased (here $B = 15$), the exact model's time performances decrease significantly and worse than the TLA procedure. A test has shown that the exact model cannot provide the optimal solution for instances with 400 customer nodes and budget 30 within 3600 seconds. Meanwhile, the TLA procedure is not affected by these changes.

4.2. Sensitivity analysis for parameters

We perform comparisons between solution instances when changing the parameters $\beta \in \{0.1, 2\}$, $\theta \in \{0.1, 0.9\}$, or $\lambda \in \{0.0000001, 10\}$ while keeping other values equal to 1. The analysis will be done through instances with 50 customer nodes, budget $B = 5$.

The gray, red, and green nodes correspond to customers, competitive facilities, and our facilities. The size of a customer node represents the demand weight of that node. The size of the facilities' nodes represents the attractiveness level.

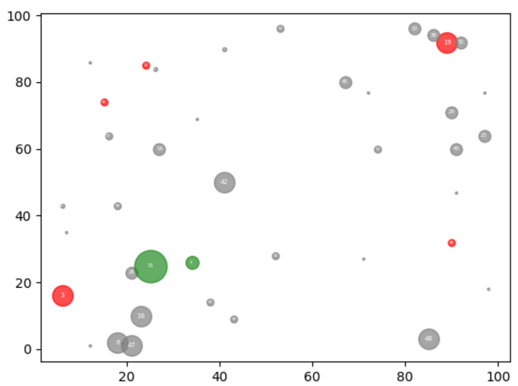

Figure 2: λ=1, β=0.1, θ=1

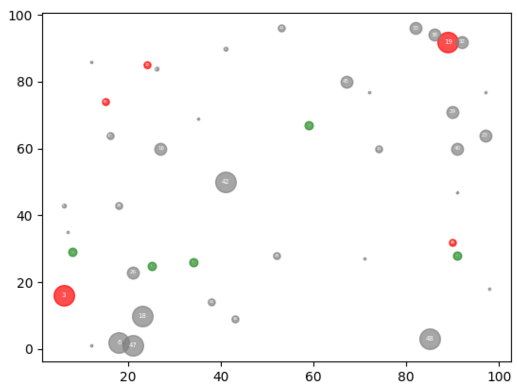

Figure 3: λ=1, β=2, θ=1

We can observe that, with small $\beta = 0.1$, the distance between facilities and customers becomes less important, so the optimal solution is to open two facilities, one with attractiveness 4 (cost 3) and one with attractiveness 2 (cost 2). As customers are more sensitive to distance (here $\lambda = 2$), we focus on increasing the number of facilities so that the company is closer to customers. Figure 3 shows we should open five facilities with attractiveness levels of 1.

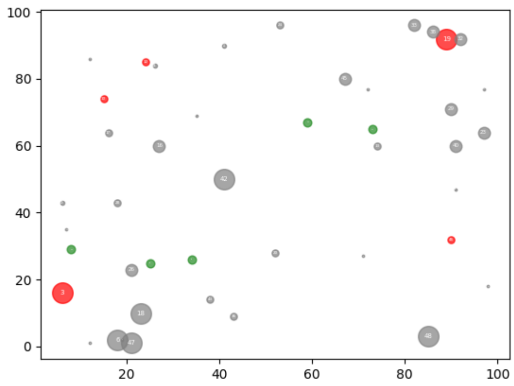

Figure 4: λ=1, β=1, θ=0.1

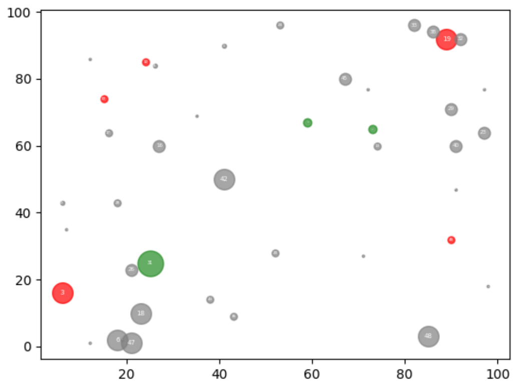

Figure 5: λ=1, β=1, θ=0.9

There is a contrast in the number of new facilities opened in Figures 4 and 5. With $\theta = 0.1$ (small), it becomes ineffective to focus on improving the characteristics of facilities because the marginal benefit is low. The optimal decision is to open as many facilities as possible (5 facilities), even with minimal attractiveness. Conversely, having facilities with a high level of attractiveness in areas with a large concentration of customers becomes effective when the value of θ is high ($\theta = 0.9$).

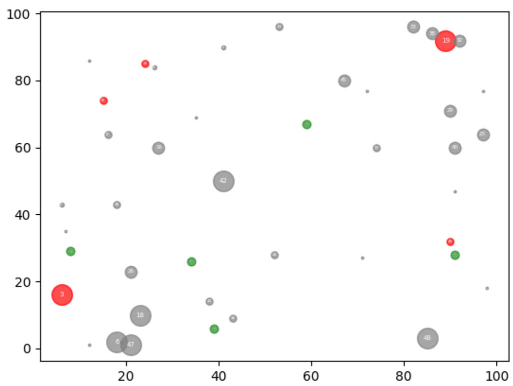

Figure 6: λ=0.0000001, β=1, θ=0.1

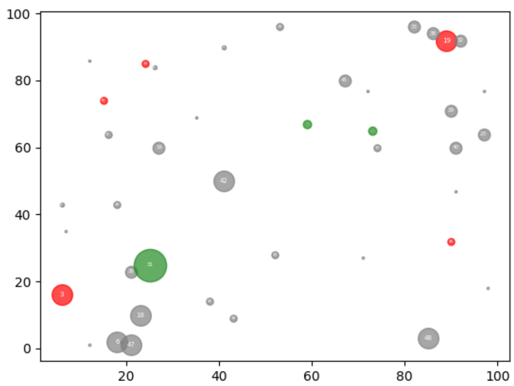

Figure 7: λ=10, β=1, θ=1

The influence of λ on the result depends heavily on each given instance. The distribution of customers and competitive facilities plays an important role. However, a specific effect that can be observed is that when $\lambda -> 0$, the demand is very elastic, which makes both the improvement in design characteristics and the expansion of the number of facilities less effective. However, the fixed cost is equal to the improvement cost in our setting, meaning the improvement is relatively expensive. Therefore, in Figure 6, five basic design-level facilities are opened.

The models and results for all experiments can be found at https://github.com/trikieu293/Competitive-Facility-Location-and-Design-Problem.

5. Conclusion

We have given the definition and motivations for the Competitive Facility Location and Design Problem, in which the decision maker must determine the locations, quantity, and designs for some facilities planned to be opened in a specific area where competitors are present and have a limited budget. We analyze the work of R. Aboolian et al. (2007), introducing a general spatial interaction mathematical model for CFLDP. The objective function of this model is investigated, showing that the model has the potential to be applied in many variants of CFLDP that can be observed in practice. Besides, the mathematical model for CFLDP with discrete design scenarios is also mentioned.

Two approximation approaches from R. Aboolian et al. (2007) are studied to compete with the non-linear nature of the objective function in the original model: the Tangent-Line Approximation procedure and the Adapted Weighted Greedy Heuristic algorithm. Through computational experiments, TLA has been proved to effectively reduce calculation time while controlling errors at a pre-determined allowable level. Furthermore, the influence of changing parameters on the direction of decision and solution is also stated with illustrative examples.

However, some points mentioned in R. Aboolian et al. (2007) have not yet been considered: computational experiment for using the results of TLA as the lower-bound in exact model and the implementation of heuristic algorithms. Using the product form of utility function also creates room for expansion. For example, other factors that negatively affect utility can be considered part of the utility function with negative exponents.

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Github-Repo: https://github.com/trikieu293/Competitive-Facility-Location-and-Design-Problem