Name: ______________________________

Closed book. Use only pen and calculator and formulas included here. Don't use a communications device as calculator. Correct solution and answer is spaces provided is required for full points. Every problem requires only short calculations. If you start long calculations you may be on the wrong track. When numerical answers are appropriate the answer must be numerical. You must include the correct units in your answer.

Table with the del operator in cartesian, cylindrical and spherical coordinates

Operation	Cartesian coordinates (x, y, z)	Cylindrical coordinates (ρ, φ, z)	Spherical coordinates (r, θ, φ), where θ is the polar angle and φ is azimuthal
A vector field $\mathbf{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_\rho\hat{\boldsymbol{\rho}} + A_\varphi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\theta\hat{\boldsymbol{\theta}} + A_\varphi\hat{\boldsymbol{\varphi}}$
Gradient ∇f	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}$
Divergence $\nabla\cdot\mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$
Curl $\nabla\times\mathbf{A}$	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}}$ $+\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}}$ $+\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{\rho}}$ $+\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\varphi}}$ $+\frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\mathbf{z}}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right)\hat{\mathbf{r}}$ $+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\boldsymbol{\theta}}$ $+\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\varphi}}$
Differential displacement $d\boldsymbol{\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$d\rho\,\hat{\boldsymbol{\rho}} + \rho\,d\varphi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\varphi\,\hat{\boldsymbol{\varphi}}$
Differential normal area $d\mathbf{S}$	$dy\,dz\,\hat{\mathbf{x}}$ $+\,dx\,dz\,\hat{\mathbf{y}}$ $+\,dx\,dy\,\hat{\mathbf{z}}$	$\rho\,d\varphi\,dz\,\hat{\boldsymbol{\rho}}$ $+\,d\rho\,dz\,\hat{\boldsymbol{\varphi}}$ $+\,\rho\,d\rho\,d\varphi\,\hat{\mathbf{z}}$	$r^2\sin\theta\,d\theta\,d\varphi\,\hat{\mathbf{r}}$ $+\,r\sin\theta\,dr\,d\varphi\,\hat{\boldsymbol{\theta}}$ $+\,r\,dr\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume dV	$dx\,dy\,dz$	$\rho\,d\rho\,d\varphi\,dz$	$r^2\sin\theta\,dr\,d\theta\,d\varphi$

$$\oint_S \vec{E}\cdot d\vec{s} = \frac{1}{\epsilon_0}\int_V \rho\,dv$$

$$\oint_S \vec{B}\cdot d\vec{s} = 0$$

$$\oint_L \vec{B}\cdot d\vec{l} = \int_S \left[\mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right]\cdot d\vec{s}$$

$$\oint_L \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{s}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{R}}{R^2}$$

$$\oint_S \vec{J}\cdot d\vec{s} = -\int_V \frac{\partial\rho}{\partial t}\,dv$$

$$e = 1.602\times 10^{-19}\,\mathrm{C}$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\vec{F} = q\left[\vec{E} + \vec{v}\times\vec{B}\right]$$

$$\mu_0 = 4\pi\times 10^{-7}\,\frac{\mathrm{H}}{\mathrm{m}}$$

$$\epsilon_0 = 8.854\times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}}$$

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EE 332 Electricity and Magnetism Spring 2022
Exam 1
21 February 2022

1. **A spherically symmetric charge distribution consists of a constant charge density $\rho = 1\mu\mathrm{C/m^3}$ inside a radius $r_0 = 1\,\mathrm{cm}$ as well as a constant surface charge $\sigma = 1\mu\mathrm{C/m^2}$ at $r_1 = 2\,\mathrm{cm}$. Compute the electric field everywhere and make a plot of it.**

$$\vec{E} = \begin{cases} \\ \\ \\ \end{cases}$$

2. **A proton, with charge $q = e$ and mass $m = 1.6 \times 10^{-27}$ kg has a velocity $\vec{v} = 1000\,\hat{x}$ m/s. There is an electric field $\vec{E} = 1\,\hat{z}$ V/m and a magnetic field $\vec{B} = 1\,\hat{z}$ mT (a) what is the force, $\vec{F}$ on the proton at that moment? (b) sketch the motion that the particle will follow over an extended period of time.**

$\vec{F} =$ ________________________

3. Compute and plot the spherically symmetric charge distribution, $\rho(r)$, which will result in a radial electric field

$$\boldsymbol{E_r} = \begin{cases} \boldsymbol{E_0 \frac{r}{r_0}} & \boldsymbol{r < r_0} \\ \boldsymbol{E_0 \frac{r_0^2}{r^2}} & \boldsymbol{r_0 \le r} \end{cases}$$

where $\boldsymbol{E_0 = 10^3}$ V/m and $\boldsymbol{r_0 = 1}$ cm.

$$\boxed{\rho(r) = \begin{cases} \\ \\ \end{cases}}$$

4. Compute the magnetic field created by the cylindrically symmetric current distribution

$$\boldsymbol{J_z} = \begin{cases} \mathbf{1\,A/m^2} & \boldsymbol{\rho} < \mathbf{1\,cm} \\ \mathbf{0} & \textbf{otherwise} \end{cases}$$

$$\vec{B} = \begin{cases} & \\ & \end{cases}$$