Name:

Closed book. Use only pen and calculator and formulas included here. Don't use a communications device as calculator. Correct solution and answer is spaces provided is required for full points. Every problem requires only short calculations. If you start long calculations you may be on the wrong track. When numerical answers are appropriate the answer must be numerical. You <u>must</u> include the correct units in your answer.

	Ta	ble with the del operator in cartesian, cylindrica	al and spherical coordinates
Operation	Cartesian coordinates (x, y, z)	Cylindrical coordinates (ho, ϕ, z)	Spherical coordinates $(r, heta, \phi)$, where $ heta$ is the polar angle and ϕ is azimuthal
A vector field A	$A_x\hat{\mathbf{x}}+A_y\hat{\mathbf{y}}+A_z\hat{\mathbf{z}}$	$A_{ ho}\hat{oldsymbol{ ho}} + A_{arphi}\hat{oldsymbol{arphi}} + A_{z}\hat{oldsymbol{z}}$	$A_{r}\hat{f r}+A_{ heta}\hat{m heta}+A_{arphi}\hat{m arphi}$
Gradient ∇f	$rac{\partial f}{\partial x}\hat{\mathbf{x}} + rac{\partial f}{\partial y}\hat{\mathbf{y}} + rac{\partial f}{\partial z}\hat{\mathbf{z}}$	$rac{\partial f}{\partial ho}\hat{oldsymbol{ ho}} + rac{1}{ ho}rac{\partial f}{\partial arphi}\hat{oldsymbol{arphi}} + rac{\partial f}{\partial z}\hat{f z}$	$rac{\partial f}{\partial r}\hat{\mathbf{r}} + rac{1}{r}rac{\partial f}{\partial heta}\hat{m{ heta}} + rac{1}{r\sin heta}rac{\partial f}{\partial arphi}\hat{m{arphi}}$
Divergence ∇ · A	$rac{\partial A_x}{\partial x} + rac{\partial A_y}{\partial y} + rac{\partial A_z}{\partial z}$	$rac{1}{ ho}rac{\partial\left(ho A_{ ho} ight)}{\partial ho}+rac{1}{ ho}rac{\partial A_{arphi}}{\partialarphi}+rac{\partial A_{z}}{\partial z}$	$rac{1}{r^2}rac{\partial \left(r^2A_r ight)}{\partial r}+rac{1}{r\sin heta}rac{\partial}{\partial heta}\left(A_ heta\sin heta ight)+rac{1}{r\sin heta}rac{\partial A_arphi}{\partial arphi}$
Curl $ abla imes \mathbf{A}$	$\begin{split} & \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) \hat{\mathbf{x}} \\ & + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) \hat{\mathbf{y}} \\ & + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \hat{\mathbf{z}} \end{split}$	$egin{aligned} \left(rac{1}{ ho}rac{\partial A_z}{\partial arphi}-rac{\partial A_arphi}{\partial z} ight)\!\hat{ ho} \ +\left(rac{\partial A_ ho}{\partial z}-rac{\partial A_z}{\partial ho} ight)\!\hat{oldsymbol{arphi}} \ +rac{1}{ ho}\left(rac{\partial \left(ho A_arphi ight)}{\partial ho}-rac{\partial A_ ho}{\partial arphi} ight)\!\hat{oldsymbol{z}} \end{aligned}$	$egin{aligned} &rac{1}{r\sin heta}\left(rac{\partial}{\partial heta}\left(A_{arphi}\sin heta ight)-rac{\partial A_{ heta}}{\partialarphi} ight)\hat{\mathbf{r}}\ &+rac{1}{r}\left(rac{1}{\sin heta}rac{\partial A_{r}}{\partialarphi}-rac{\partial}{\partial r}\left(rA_{arphi} ight) ight)\hat{oldsymbol{ heta}}\ &+rac{1}{r}\left(rac{\partial}{\partial r}\left(rA_{ heta} ight)-rac{\partial A_{r}}{\partial heta} ight)\hat{oldsymbol{arphi}} \end{aligned}$
Differential displacement dl	$dx\hat{f x}+dy\hat{f y}+dz\hat{f z}$	$d ho\hat{oldsymbol{ ho}} + hodarphi\hat{oldsymbol{arphi}} + dz\hat{f z}$	$dr\hat{f r} + rd heta\hat{m heta} + r\sin hetadarphi\hat{m arphi}$
Differential normal area dS	$dydz\hat{ extbf{x}}\ +dxdz\hat{ extbf{y}}\ +dxdy\hat{ extbf{z}}$	$egin{aligned} hodarphidz\hat{oldsymbol{ ho}}\ &+d hodz\hat{oldsymbol{arphi}}\ &+ hod hodarphi\hat{oldsymbol{z}} \end{aligned}$	$egin{aligned} r^2 \sin heta d heta d\hat{arphi} \hat{\mathbf{r}} \ + r \sin heta dr darphi \hat{eta} \ + r dr d heta \hat{oldsymbol{arphi}} \end{aligned}$
Differential volume dV	dxdydz	hod hodarphidz	$r^2 \sin heta dr d heta darphi$

$$\oint_{S} \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_{0}} \int_{V} \rho \, dv$$

$$\oint_{S} \vec{B} \cdot d\vec{s} = 0$$

$$\oint_{L} \vec{B} \cdot d\vec{l} = \int_{S} \left[\mu_{0} \vec{J} + \mu_{0} \epsilon_{0} \frac{\partial \vec{E}}{\partial t} \right] \cdot d\vec{s}$$

$$\oint_{L} \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_{S} \vec{B} \cdot d\vec{s}$$

$$d\vec{B} = \frac{\mu_{0}}{4\pi} \frac{I \, d\vec{l} \times \hat{R}}{R^{2}}$$

$$\oint_{S} \vec{J} \cdot d\vec{s} = -\int_{V} \frac{\partial \rho}{\partial t} \, dv$$

$$e = 1.602 \times 10^{-19} \, \text{C}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\vec{F} = q \left[\vec{E} + \vec{v} \times \vec{B} \right]$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \frac{F}{m}$$

Name:

EE 332 Electricity and Magnetism Spring 2022 Exam 1 21 February 2022

1. A spherically symmetric charge distribution consists of a constant charge density $\rho = 1 \mu \text{C/m}^3$ inside a radius $r_0 = 1 \, \text{cm}$ as well as a constant surface charge $\sigma = 1 \mu \text{C/m}^2$ at $r_1 = 2 \, \text{cm}$. Compute the electric field everywhere and make a plot of it.

$$ec{E} = \left\{
ight.$$

2. A proton, with charge q=e and mass $m=1.6\times 10^{-27}\,\mathrm{kg}$ has a velocity $\vec{v}=1000\,\hat{x}\,\mathrm{m/s}$. There is an electric field $\vec{E}=1\,\hat{z}\,\mathrm{V/m}$ and a magnetic field $\vec{B}=1\,\hat{z}\,\mathrm{mT}$ (a) what is the force, \vec{F} on the proton at that moment? (b) sketch the motion that the particle will follow over an extended period of time.

 $\vec{F} =$ _____

3. Compute and plot the spherically symmetric charge distribution, $\rho\left(r\right)$, which will result in a radial electric field

$$E_r = egin{cases} E_0 rac{r}{r_0} & r < r_0 \ E_0 rac{r_0}{r^2} & r_0 \leq r \end{cases}$$

where $E_0=10^3\,\mathrm{V/m}$ and $r_0=1\,\mathrm{cm}.$

$$ho(r) = \left\{$$

4. Compute the magnetic field created by the cylindrically symmetric current distribution

$$J_z = egin{cases} 1\,\mathrm{A/m^2} &
ho < 1\,\mathrm{cm} \ 0 & \mathrm{otherwise} \end{cases}$$

