CS 6375

PSET 3

Student: TRI M. CAO

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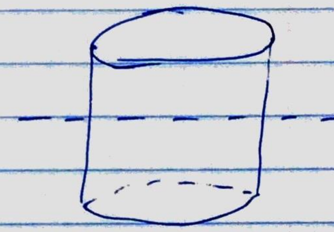
Problem 1:

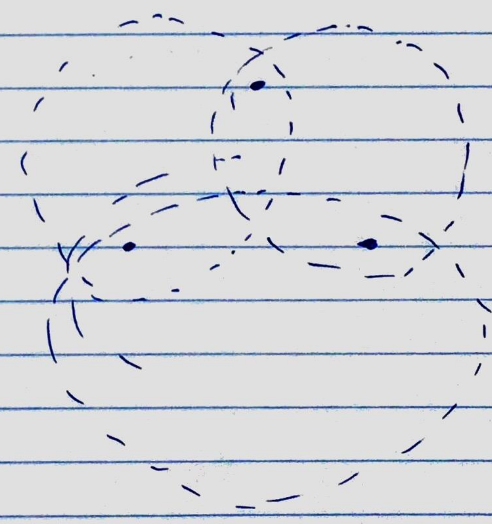
(1.) cylinders of a fixed radius $r > 0$ and fixed length $l > 0$.

- If we ~~slide~~ slice the cylinder in the direction of the dashed line on the right, we will have a circle ~~in~~ in 2-D with radius r.



We want to argue that the VC dimension of a circle is 3.

Given 3 points in $\mathbb{R}^2$, it's clear that we will always find a circle that cover all + points. For example, let ~~all~~ the 3 points be:

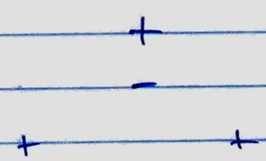


So the VC dimension ≥ 3.

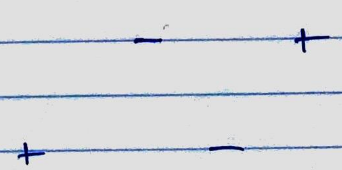
- If we have 4 points, then a circle will not be able to cover all the + points.

We have two cases:

Case 1:



Case 2:



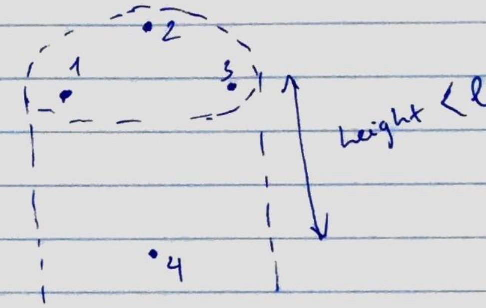
In both cases, we cannot find a circle that covers the + points.

Also, if we have more than 4 points, then we can always find a group of 4 points that looks like one ~~of~~ of the cases above

$\Rightarrow$ VC dimension of circle is 3.

- Now we will argue that the VC dimension of a cylinder in $\mathbb{R}^3$ is 4.

Given 4 points like this:
the 3 points at the top will make up a plane p. The point below will not be on plane p, but on a plane that is lower in height from plane p.
The height difference is less than l.



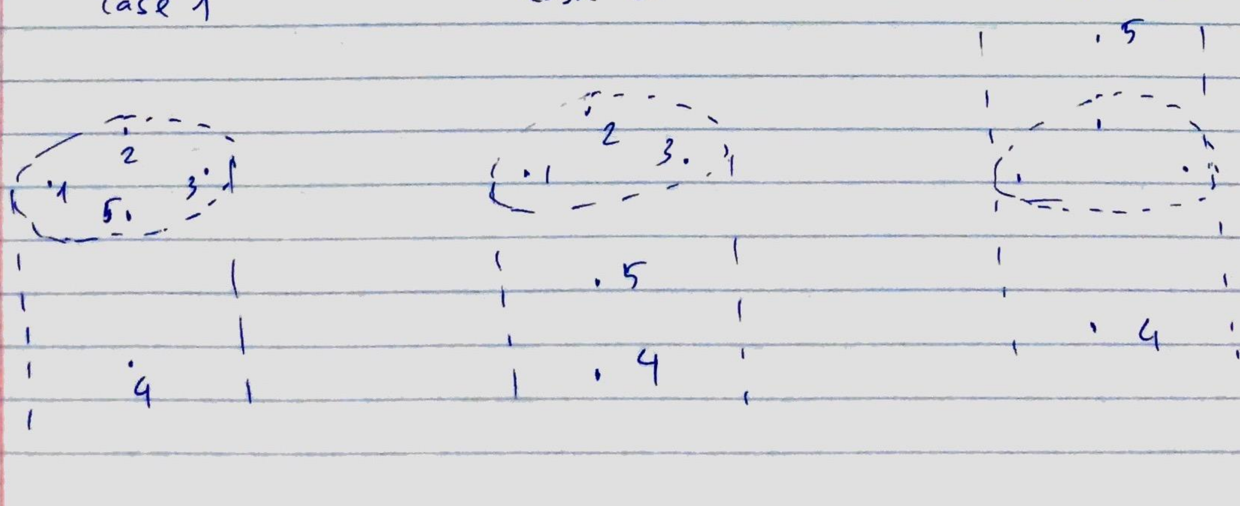
we choose those 4 points such that we ~~do~~ can classify the 3 points on plane p with a circle, and that circle can extend the ~~to~~ length to also cover the lower point (point 4).

It's clear that we can classify the 3 points on plane p. For point 4, if it is + then we move the cylinder down to cover ~~the~~ it; if it is − then we move the cylinder up to avoid it. Either way, we can classify all 4 points.

$$\Rightarrow \underline{VC(H) \geq 4}$$

- when we add one more point ~~the~~ to the 4 points above, we have 3 cases:

case 1 case 2 case 3



Case 1: point 5 is on the same plane as points 1, 2, 3.
In this case we cannot classify 4 points on the same plane with a circle so we fail to classify 5 points.

Case 2: point 5 ~~has a height~~ is not on plane p, but has a height that makes it between plane p and point 4.
If point 4 is + and point 5 is −, then it's impossible for a cylinder to cover all + points.

Case 3: point 5 is higher than plane p.
If height of 5 − height of 4 > l, then we cannot classify the case when point 5 and 4 are both +.
If height of 5 − height of 4 < l, then we cannot classify the case when both 5 and 4 are −.

1a) Sample complexity

$$M \geq \frac{1}{\epsilon}\left(4 \ln \frac{2}{\delta} + 8 \cdot VC(H) \ln \frac{13}{\epsilon}\right)$$

We have $\epsilon = 0.2$, $\delta = 0.05$, $VC(H) = 4$

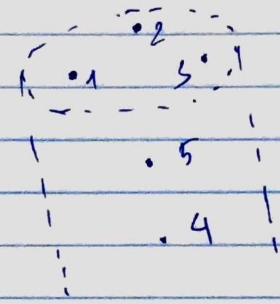
$$\Rightarrow M \geq \frac{1}{0.2}\left(4 \ln \frac{2}{0.05} + 8 \cdot 4 \ln \frac{13}{0.2}\right)$$

$$\boxed{M \geq 5\left(4 \ln(40) + 32 \cdot \ln(65)\right)}$$

1b) If we also have cylinders that cover − points then we will have VC dimension of 5.

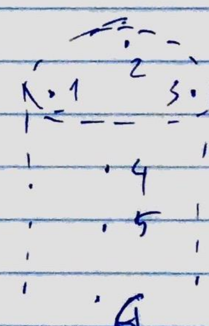
The new hypothesis space will solve case 2 in the discussion before.

If point 5 is − and point 4 is +, we will use a cylinder to cover all − points, that way we can ignore ~~the~~ point 4.
In other cases of point 5, we can still cover all + points.



~~I~~ Now let's try to add one more point, then we would see case 1 and case 3 again, and we cannot ~~solve~~ solve those cases. look at case 2, we would see something like this

If point 4 is −, point 5 is +, point 6 is −, then we cannot ~~find~~ classify those points.



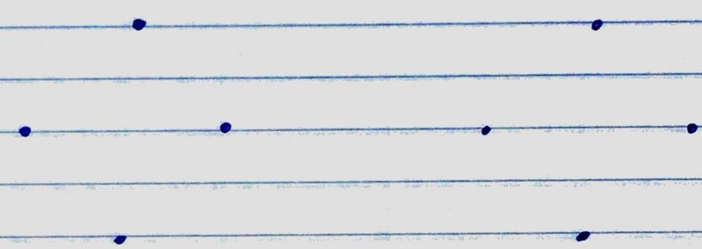
Note that if we add more than one point we still have the 3 cases above.

$\Rightarrow$ $\boxed{VC(H) = 5}$

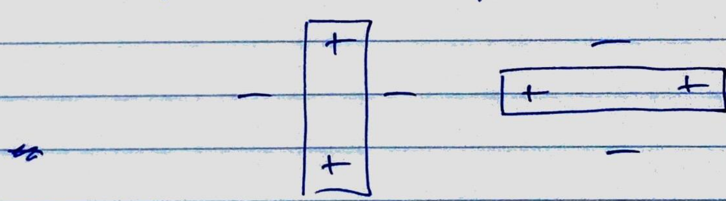
2 Pair of axis aligned rectangles.

- From the lecture, we have VC dimension of a single axis aligned rectangle is 4.
- We want to argue that two rectangles will have VC dimension of 8.

Given 8 points



For any labeling, a single axis aligned rectangle can classify the group of 4 points on the left, and the remaining rectangle will handle the group on the right. For example:

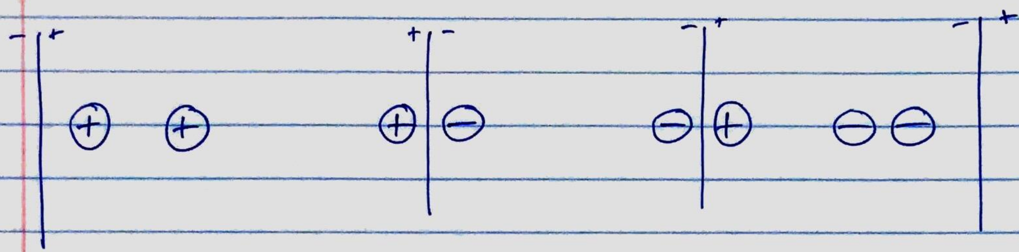


- If we add one or more points, then we no longer have two groups of 4 points. There will be at least one group of 5 or more points, which a single rectangle cannot classify.

$\Rightarrow \boxed{VC(H) = 8}$

Problem 1.3

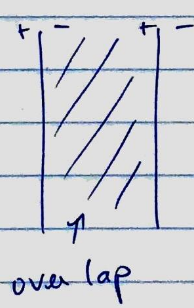
We have 3 linear classifiers.
let's try to classify two points:

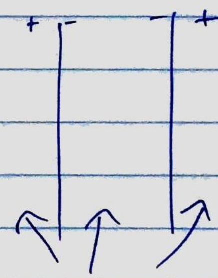


So we need 4 different linear separators to classify two points.
$\Rightarrow VC(H) < 2$

- We can always classify one point with 3 linear separators
  $\Rightarrow VC(H) \geq 1$
  The reason is, if we have two distinct linear separators, then there will be a space where the predictions of the two linear separators overlap.

Examples:





- Therefore I guess the min and max VC dimension of 3 distinct linear separators are both 1