

Misal $(2x - 4) = u$ $\frac{du}{dx} = 2$ atau $du = 2 dx$, sehingga
 $\int (2x - 4) dx = \frac{1}{2} \int (2x - 4) 2dx = \frac{1}{2} \int u du$
 $= \frac{1}{12} (2x - 4)^2 + C$

h. $\int (x + 2)(3x) dx = ?$

Misal $(x + 2) = u$ $\frac{du}{dx} = (3x)$ atau $du = (3x) dx$,
 sehingga

$$\int (x + 2)(3x) dx = \int u du = \frac{1}{3} u^2 + C$$

$$= \frac{1}{3} ((x + 2)^3) + C$$

i. $\int 3x^2 dx = ?$

Misal $1 - 2x = u$, sehingga $du = (-2) dx$, maka

$$\int 3x^2 dx = \int (-3/4)(-4x^2) dx \text{ atau}$$

$$-3/4 \int (-4x^2) dx = -3/4 \int u du$$

$$= -3/4 (2/3) u^2 + C \text{ atau}$$

$$= - (1/2) (1 - 2x)^2 + C$$

j. $\int x^2 dx = ?$

$$\int x^2 dx = \int (1 - x^2) dx = \int 1 dx - \int x^2 dx$$

$$= x + C' + \frac{1}{3} x^3 + C'' \text{ atau}$$

$$= \frac{1}{3} x^3 + x + C$$

2. $\int \ln^3 u^3 + C$

Contoh :

a. $\int \ln^3 x dx = ?$

Misal $x + 2 = u$, sehingga $du = d(x + 2)$, maka

$$\int \ln^3 x dx = \int \ln^3 u du = \ln^3 x + 2^3 + C$$

b. $\int \ln^2 x dx = ?$

Misal $2x + 3 = u$, maka $du = 2 dx$, sehingga

$$\int \ln^2 x dx = \int \ln^2 u du = \frac{1}{2} \int \ln^2 u du$$

$$= 1/2 \ln^3 u^3 + C \text{ atau}$$

$$= 1/2 \ln^3 (2x + 3)^3 + C$$

c. $\int \frac{1}{x^3} dx = ?$

Misal $x^{-1} = u$ dan $du = 2x dx$, sehingga

$$\int \frac{1}{x^3} dx = \int \frac{1}{u^3} du = 1/2 \int \frac{1}{u^3} du = 1/2 \ln^3 u^3 + C \text{ atau}$$

$$= 1/2 \ln^3 x^{-1} + C$$

d. $\int \frac{1}{1-2x^3} dx = ?$

Misal $1 - 2x^3 = u$ dan $du = (-6x^2) dx$, sehingga

$$\int \frac{1}{1-2x^3} dx = \int \frac{1}{u} du = -1/6 \int \frac{1}{u} du \text{ atau}$$

$$= -1/6 \int \frac{1}{u} du = -1/6 \ln^3 u^3 + C \text{ atau}$$

$$= -1/6 \ln^3 (1 - 2x^3) + C$$

e. $\int \frac{1}{x^3 + 1} dx = \int \frac{1}{(x+1)(x^2-x+1)} dx$

$$= \int \frac{1}{x+1} dx + \int \frac{1}{x^2-x+1} dx$$

$$= \ln|x+1| + \ln^3 x + 1^3 + C$$

3. $\int \frac{1}{a^x} dx = \frac{1}{a} + C$, $a > 0$ dan $a \neq 1$

Contoh :

a. $\int \frac{1}{a^x} dx = \frac{1}{a} + C$, $a > 0$ dan $a \neq 1$

b. $\int \frac{1}{a^{2x}} dx = ?$

Misal $2x = u$, maka $du = 2 dx$, sehingga

$$\int \frac{1}{a^{2x}} dx = 1/2 \int \frac{1}{a^u} du = 1/2 \left(\frac{1}{a^u} \right) + C \text{ atau}$$

$$= 1/2 \left(\frac{1}{a^{2x}} \right) + C$$

c. $\int \frac{1}{a^{4x+4}} dx = ?$

Misal $x^2 + 2x + 1 = u$, maka $du = (2x + 2) dx$, sehingga

$$\int \frac{1}{a^{4x+4}} dx = 2 \int \frac{1}{a^{2x+2}} dx = 2 \int \frac{1}{a^u} du$$

$$= 2 \left(\frac{1}{a^u} \right) + C \text{ atau}$$

$$= 2 \left(\frac{1}{a^{x^2+2x+1}} \right) + C$$

$$4. \int e^u du = e^u + C$$

Contoh :

$$a. \int e^t dt = e^t + C$$

$$b. \int e^{2x} dx = ?$$

Misal $x = u$, maka $du = 2 dx$, sehingga

$$\begin{aligned} \int e^{2x} dx &= \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C \quad \text{atau} \\ &= \frac{1}{2} e^{2x} + C \end{aligned}$$

$$c. \int e^{4x+6} dx = ?$$

Misal $x + 3x + 6 = u$, maka $du = (2x + 3) dx$, sehingga

$$\begin{aligned} \int e^{4x+6} dx &= 2 \int e^u (2x + 3) dx \\ &= 2 \int e^u du = 2 e^u + C \quad \text{atau} \\ &= 2 e^{4x+6} + C \end{aligned}$$

$$5. \int \ln^3 x^3 = \ln^3 x^3 + C$$

Contoh :

$$a. \int \ln^3 x^3 = \ln^3 x^3 + C$$

$$b. \int \ln^3 x^3 = \frac{1}{2} \int \ln^3 x^3 = \ln^3 x^3 + C$$

$$c. \int \ln^3 x^3 = \int \ln^3 x^3 = \ln^3 x^3 + C$$

$$6. \int \ln^3 x^3 = \ln^3 x^3 + C$$

Contoh :

$$a. \int \ln^3 x^3 = \ln^3 x^3 + C$$

$$\text{b. } \int \frac{1}{4} i^3 = \frac{1}{4} i^3 = \ln^3 i^3 + C$$

$$\text{c. } \int i^3 = i^3 = \ln^3 i^3 + C$$

$$7. \int i = \ln(u + \quad) + C$$

Contoh :

$$\text{a. } \int i = \ln(t + \quad) + C$$

$$\text{b. } \int i = \frac{1}{2} i = \frac{1}{2} (\ln(s + \quad)) + C$$

$$\begin{aligned} \text{c. } \int i &= 4 i \\ &= 4 (\ln(p + 1) + \quad) + C \end{aligned}$$

$$8. \int i = \ln^3 u + \quad^3 + C$$

Contoh :

$$\text{a. } \int i = \ln^3 t + \quad^3 + C$$

$$\text{b. } \int i = \frac{1}{2} i = \frac{1}{2} (\ln(r + \quad)) + C$$

$$\begin{aligned} \text{c. } \int i &= 4 i \\ &= 4 (\ln(p + 1) + \quad) + C \end{aligned}$$

$$9. \int i \sin u \, du = -\cos u + C$$

Contoh :

$$\text{a. } \int i \sin t \, dt = -\cos t + C$$

$$\text{b. } \int i \sin 3x \, dx = \frac{1}{3} \int i \sin 3x \, 3 \, dx = -\frac{1}{3} \cos 3x + C$$

$$\begin{aligned} \text{c. } \int \sin x (\cos x \, dx) &= \int \sin x \, d(\sin x) \\ &= \frac{1}{3} \sin^3 x + C \end{aligned}$$

$$10. \int \cos u \, du = \sin u + C$$

Contoh :

$$\text{a. } \int \cos t \, dt = \sin t + C$$

$$\begin{aligned} \text{b. } \int \cos \frac{1}{3} x \, dx &= \int \cos \frac{1}{3} x \left(3 \cdot \frac{1}{3} \right) dx \\ &= 3 \int \cos \frac{1}{3} x \left(\frac{1}{3} dx \right) \\ &= 3 \sin \frac{1}{3} x + C \end{aligned}$$

$$\begin{aligned} \text{c. } \int \cos x (\sin x \, dx) &= - \int \cos x \, d(\sin x) \\ &= - \frac{1}{3} \cos^3 x + C \end{aligned}$$

$$11. \int \tan u \, du = \ln |\sec u| + C$$

Contoh :

$$\text{a. } \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = ?$$

Misal $\cos x = u$, maka $du = -\sin x \, dx$, sehingga

$$\begin{aligned} \int \frac{\sin x}{\cos x} \, dx &= - \int \frac{1}{u} \, du = - \ln |u| + C \quad \text{atau} \\ &= - \ln |\cos x| + C \quad \text{atau} \\ &= \ln |\sec x| + C \end{aligned}$$

$$\text{b. } \int \tan 5x \, dx = \int \frac{\sin 5x}{\cos 5x} \, dx = ?$$

Misal $\cos 5x = u$, maka $du = -5 \sin 5x \, dx$, sehingga

$$\begin{aligned} \int \frac{\sin 5x}{\cos 5x} \, dx &= - \frac{1}{5} \int \frac{1}{u} \, du = - \frac{1}{5} \ln |u| + C \quad \text{atau} \\ &= - \frac{1}{5} \ln |\cos 5x| + C \quad \text{atau} \\ &= \frac{1}{5} \ln |\sec 5x| + C \end{aligned}$$

$$\begin{aligned} \text{c. } \int \frac{1}{(x^2 + 6x + 9)(x + 3)} dx \\ = \int \frac{1}{(x + 3)^3} dx = ? \end{aligned}$$

Misal $\cos(x^2 + 6x + 9) = u$, maka

$du = -\sin(x^2 + 6x + 9)(2x + 6) dx$, sehingga

$$\begin{aligned} \int \frac{1}{(x + 3)^3} dx &= \\ &= -\frac{1}{2} \int \frac{1}{(2x + 6)^3} dx \\ &= -\frac{1}{2} \int \frac{1}{u^3} du = -\frac{1}{2} \ln^3 u^3 + C \quad \text{atau} \\ &= -\frac{1}{2} \ln^3 \cos(x^2 + 6x + 9)^3 + C \quad \text{atau} \\ &= \frac{1}{2} \ln^3 \sec(x^2 + 6x + 9)^3 + C \end{aligned}$$

$$12. \int \cot^3 u \, du = \ln^3 \sin u^3 + C$$

Contoh :

$$\text{a. } \int \cot^3 x \, dx = \int \frac{1}{\sin^3 x} dx = ?$$

Misal $\sin x = u$, maka $du = \cos x \, dx$, sehingga

$$\begin{aligned} \int \frac{1}{\sin^3 x} dx &= \int \frac{1}{u^3} du = \ln^3 u^3 + C \quad \text{atau} \\ &= \ln^3 \sin x^3 + C \end{aligned}$$

$$\text{b. } \int \cot^3 2x \, dx = \int \frac{1}{\sin^3 2x} dx = ?$$

Misal $\sin 2x = u$, maka $du = \cos 2x \cdot 2 \, dx$, sehingga

$$\begin{aligned} \int \frac{1}{\sin^3 2x} dx &= \frac{1}{2} \int \frac{1}{u^3} du = \frac{1}{2} \ln^3 u^3 + C \quad \text{atau} \\ &= \frac{1}{2} \ln^3 \sin 2x^3 + C \end{aligned}$$

$$\begin{aligned} \text{c. } \int \cot^3 (x^2 + 8x + 16) dx \\ = \int \frac{1}{(x + 4)^3} dx = ? \end{aligned}$$

Misal $\sin(x^2 + 8x + 16) = u$, maka

$du = \cos(x^2 + 8x + 16)(2x + 8) dx$, sehingga

$$\begin{aligned} \int \frac{1}{(x + 4)^3} dx \\ = \frac{1}{2} \int \frac{1}{(2x + 8)^3} dx = \frac{1}{2} \int \frac{1}{u^3} du \end{aligned}$$

$$= \frac{1}{2} \ln^3 u^3 + C \quad \text{atau}$$

$$= \frac{1}{2} \ln^3 \sin x + 8x + 16^3 + C$$
