Teorema 5: (AturanJumlah)

Jika f dan g duafungsidengan f'(x) dan g'(x) ada, h fungsi yang didefinisikansebagai h(x) = f(x) + g(x), maka h'(x) = f'(x) + g'(x).

Secarasimbolik:

h(x) = f(x) + g(x) danf'(x),g'(x) ada$\Rightarrow$ h'(x) = f'(x) + g'(x)

Bukti: h(x) = f(x) + g(x) $\Rightarrow$ h(x+h) = f(x+h) + g(x+h)

$$h'(x) = \lim_{h\to 0} \frac{h(x+h)-h(x)}{(x+h)-x}$$

$$= \lim_{h\to 0} \frac{f(x+h)+g(x+h)-(f(x)+g(x))}{(x+h)-x}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

= f'(x) + g'(x) ■

$\therefore$h(x) = f(x) + g(x) $\Rightarrow$ h'(x) = D(h(x)) = f'(x) + g'(x)

Teorema 6: (AturanSelisih)

Jika f dan g duafungsidengan f'(x) dan g'(x) ada, h fungsi yang didefinisikansebagai h(x) = f(x) - g(x), maka h'(x) = f'(x) - g'(x).

Secarasimbolik:

h(x) = f(x) - g(x) $\wedge$f'(x),g'(x) ada$\Rightarrow$ D(h(x)) = f'(x) - g'(x)

Bukti: h(x) = f(x) - g(x) $\Rightarrow$ h(x+h) = f(x+h) - g(x+h)

$$h'(x) = \lim_{h\to 0} \frac{f(x+h)-g(x+h)-(f(x)-g(x))}{h}$$

$$= \lim_{h\to 0} \frac{(f(x+h)-(f(x)) - (g(x+h)-g(x))}{h}$$

$$= \lim_{h\to 0} \frac{(f(x+h)-f(x))}{h} - \lim_{h\to 0} \frac{(g(x+h)-g(x))}{h}$$

= f'(x) - g'(x) ■

$\therefore$h(x) = f(x) - g(x) $\Rightarrow$ h'(x) = D(h(x)) = f'(x) - g'(x)

Contoh 1: Diketahui f(x) = g(x) + h(x), jika g(x) = 3 x^2dan h(x) = 10x. Tentukan f'(x).

Penyelesaian: g(x) = 3 $x^2$$\Rightarrow$ g'(x) = 6x

h(x) = 10x $\Rightarrow$ h'(x) = 10

f(x) = 3 x^2 + 10x, berdasaraturanjumlahdiperoleh

$f'(x) = 6x + 10$

$\therefore f(x) = 3x^2 + 10x \Rightarrow f'(x) = 6x + 10$

Contoh 2: Diketahui f(x) = g(x) - h(x), jika $g(x) = 3x^3$dan $h(x) = 15x^2$. Tentukan f'(x).

Penyelesaian: $g(x) = 3x^3 \Rightarrow g'(x) = 9x^2$

$h(x) = 15x^2 \Rightarrow h'(x) = 30x$

$f(x) = 3x^3 - 15x^2$, berdasaraturanselisihdiperoleh

$f'(x) = 9x^2 - 30x$

$\therefore f(x) = 3x^3 - 15x^2 \Rightarrow f'(x) = 9x^2 - 30x$

Teorema 7: (Aturanhasil kali)

Jika f dan g duafungsidengan f'(x) dan g'(x) ada, h fungsi yang didefinisikansebagai h(x) = f(x).g(x), maka h'(x) = f(x)g'(x) + g(x)f'(x).

Secarasimbolik:

$h(x) = f(x).g(x) \wedge f'(x), g'(x)$ ada $\Rightarrow D(h(x)) = f(x)g'(x) + g(x)f'(x)$.

Bukti: $h(x) = f(x).g(x) \Rightarrow h(x+h) = f(x+h).g(x+h)$

$h'(x) = \lim\limits_{h \to 0} \frac{f(x+h).g(x+h) - f(x).g(x) - f(x+h).g(x) + f(x+h).g(x)}{h}$ ***ruas suatu persamaan tidak berubah nilainya bila ditambah dengan nol*** $(f(x+h).g(x) - f(x+h).g(x))$

$$= \lim_{h \to 0} \frac{f(x+h).g(x+h) - f(x+h).g(x) + f(x+h).g(x) - f(x).g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h).g(x+h) - f(x+h).g(x)}{h} + \lim_{h \to 0} \frac{f(x+h).g(x) - f(x).g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)(g(x+h) - g(x)}{h} + \lim_{h \to 0} \frac{g(x)\left(f(x+h) - f(x)\right)}{h}$$

$$= \lim_{h \to 0} f(x+h). \lim_{h \to 0} \frac{(g(x+h) - g(x)}{h} + \lim_{h \to 0} g(x). \lim_{h \to 0} \frac{(f(x+h) - f(x)}{h}$$

$= f(x).g'(x) + g(x) . f'(x)$ ■

$\therefore h(x) = f(x)g(x) \Rightarrow h'(x) = f(x)g'(x) + g(x)f'(x)$

Contoh 1: Diketahui f(x) = g(x) . h(x), jika g(x) = 2x - 1 dan $h(x) = 3x^2$. Tentukan f'(x),

Penyelesaian: $f(x) = (2x - 1)(3x^2)$ dengancarabiasa

$f(x) = 6x^3 - 3x^2$

$f'(x) = 6 . 3x^2 - 3 . 2x$

$= 18x^2 - 6x$

Cara 2: (Aturanhasil kali)

$f(x) = (2x - 1)(3x^2)$

$g(x) = 2x - 1$, maka $g'(x) = 2$

$h(x) = 3x^2$, maka $h'(x) = 6x$

$f'(x) = (2x - 1)6x + 3x^2 \cdot 2$

$= 12x^2 - 6x + 6x^2$

$= 18x^2 - 6x$

Contoh 2: Diketahui $g(x) = (x^2 - 4x + 4)$ dan $h(x) = (4x - 7)$ dan $f(x) = g(x) \cdot h(x)$. Tentukan $f'(x)$.

Penyelesaian: $g(x) = (x^2 - 4x + 4)$, maka $g'(x) = 2x - 4$

$h(x) = (4x - 7)$, maka $f'(x) = 4$

$f'(x) = (x^2 - 4x + 4)(4) + (4x - 7)(2x - 4)$

$= 4x^2 - 16x + 16 + 8x^2 - 30x + 28$

$= 12x^2 - 46x + 44$

Apa yang terjadi jika h(x) = g(x)?

Jika $f(x) = g(x).h(x)$, turunan pertamanya yaitu $f'(x) = g(x).h'(x) + h(x)\, g'(x)$. Jika $h(x) = g(x)$ didapat $f(x) = g(x).\, g(x)$ atau

$f(x) = (g(x))^2$, bagaimana dengan $f'(x)$?

Dengan mengganti huruf h dengan g di dapat

$f'(x) = g(x).g'(x) + g(x)\, g'(x)$ atau

$= 2\, g(x).g'(x)$

$\therefore f(x) = (g(x))^2 \Rightarrow f'(x) = 2\, g(x).g'(x)$

Sekarang, kitaperhatikan

$f(x) = (g(x))^3$

$=(g(x))^2\, (g(x))$

Selanjutnya, ditentukanturunandari f(x) berdasaraturanperkalian, didapat

$f'(x) = (g(x))^2 .\, g'(x) + g(x).\{(g(x))^2\}'$

$= (g(x))^2 .\, g'(x) + g(x).2\, g(x).g'(x)$ $[\because \{(g(x))^2\}' = 2\, g(x).g'(x)]$

$= (g(x))^2 .\, g'(x) + 2\, (g(x))^2 .\, g'(x)$

$= 3\, (g(x))^2 .\, g'(x)$

$\therefore$ f(x) = $(g(x))^3 \Rightarrow$ f '(x) = 3 $(g(x))^2$. g'(x)

Bilahaliniditeruskanuntukf(x) = $(g(x))^4$, didapatf '(x) = $4(g(x))^3$. g'(x).

Berdasarpola yang ada, f(x) = $(g(x))^n$didapatf '(x) = n $(g(x))^{n-1}$. g'(x)

$\therefore$ f(x) = $(g(x))^n \Rightarrow$ f '(x) = n $(g(x))^{n-1}$. g'(x) . . . (*)

Teorema 8: (Aturanhasilbagi)

Jika f dan g duafungsidengan f'(x) dan g'(x) ada, h fungsi yang didefinisikansebagaih(x) = f(x)/g(x), maka h'(x) = $\frac{g(x).\ f\ '(x) - f(x).g'(x)}{(g(x)^2}$

Secarasimbolik:

h(x) = f(x)/g(x) $\wedge$ f'(x),g'(x) ada$\Rightarrow$ h'(x) = $\frac{g(x).\ f\ '(x) - f(x).g'(x)}{(g(x)^2}$

Bukti: h(x) = f(x)/g(x) $\Rightarrow$ h(x+h) = f(x+h)/(g(x+h)

$$h(x+h) - h(x) = \frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}$$

$$= \left(\frac{g(x).f(x+h) - f(x)g(x+h)}{g(x+h).g(x)}\right)$$

$$\frac{h(x+h) - h(x)}{h} = \left(\frac{g(x).f(x+h) - f(x)g(x+h)}{h.g(x+h).g(x)}\right)$$

$$= \left(\frac{g(x).f(x+h) - f(x)g(x+h) - (f(x)g(x) + f(x)g(x)}{h.g(x+h).g(x)}\right)$$ ***ruas suatu persamaan tidak berubah nilainya bila ditambah dengan nol*** $(f(x).g(x) - f(x).g(x))$

$$h'(x) = \lim_{h\to 0} \left(\frac{g(x).f(x+h) - f(x)g(x+h) - (f(x)g(x) + f(x)g(x)}{h.g(x+h).g(x)}\right)$$

$$= \lim_{h\to 0} \left(\frac{g(x).f(x+h) - f(x)g(x) - ((f(x)g(x+h) - f(x)g(x))}{h.g(x+h).g(x)}\right)$$

$$= \lim_{h\to 0} \left(\frac{g(x).f(x+h) - f(x)g(x)}{h.g(x+h).g(x)}\right) - \lim_{h\to 0} \left(\frac{(f(x)g(x+h) - f(x)g(x)}{h.g(x+h).g(x)}\right)$$

$$= \lim_{h\to 0} \left(\frac{g(x)\{f(x+h) - f(x)\}}{h.g(x+h).g(x)}\right) - \lim_{h\to 0} \left(\frac{(f(x)\{g(x+h) - g(x)\}}{h.g(x+h).g(x)}\right)$$

$$= \lim_{h\to 0} \frac{g(x)}{g(x+h).g(x)} \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} - \lim_{h\to 0} \frac{f(x)}{g(x+h)g(x)} \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \frac{g(x)}{g(x).g(x)}.\ f\ '(x) - \frac{f(x)}{g(x)g(x)}\ .\ g\ '(x) \text{ atau}$$

$$= \frac{g(x).\ f\ '(x) - f(x).g'(x)}{(g(x)^2}$$

$\therefore h(x) = f(x)/g(x) \Rightarrow h'(x) = \dfrac{g(x).\ f'(x) - f(x).g'(x)}{(g(x)^2}$

Contoh 1: Diketahui f(x) = f(x)/g(x), dengan g(x) = 3 - 2x dan h(x) = 3 + 2x. Tentukan f'(x).

Penyelesaian: g(x) = 3 - 2x, maka g'(x) = -2

h(x) = 3 + 2x, maka h'(x) = 2

$f'(x) = \dfrac{(3-2x).2-(3+2x).-2}{(3-2x)^2}$

$= \dfrac{(6-4x)+6+4x}{(3-2x)^2}$

$= \dfrac{12}{(3-2x)^2}$

Contoh 2: Diketahui $f(x) = \dfrac{g(x)}{h(x)}$, dengan $g(x) = x^2 + 10$ dan $h(x) = 3x^2$. Tentukan f'(x).

Penyelesaian: $g(x) = x^2 + 10$, maka $g'(x) = 2x$

$h(x) = 3x^2$ maka $h'(x) = 6x$

$f'(x) = \dfrac{(3x^2.2x-(x^2+10).6x}{(3x^2)^2}$

$= \dfrac{6x^3-6x^3-60x}{(3x^2)^2}$

$= \dfrac{-60x}{(3x^2)^2}$

3.6. TurunanFungsiTrigonometri

3.6.1. TurunanFungsiSinus

Jika f(x) = sin x, maka f'(x) = cos x

Bukti: $f(x) = \sin x \Rightarrow f(x+h) = \sin(x+h)$

f(x+h) - f(x) = sin (x+h) - sin x

= 2 cos 1/2 ((x+h) + x) .sin 1/2 (x+h) - x)

= 2 cos(x + 1/2 h) .sin 1/2h

$f'(x) = \lim_{h \to 0} \dfrac{2\cos(x + 1/2\,h).\sin 1/2h}{h}$

$= \lim_{h \to 0} \dfrac{2\cos(x + 1/2\,h).\ \sin 1/2h}{2.\ (\frac{1}{2}h)}$

$= \lim_{h \to 0} \dfrac{\cos(x + 1/2\,h).\ \sin 1/2h}{(\frac{1}{2}h)}$

$= \lim_{h \to 0} \left(\cos\left(x + \frac{1}{2}h\right)\right) . \lim_{h \to 0} \frac{\sin 1/2h}{\frac{1}{2}h}$

= cosx . 1

= cos x ■

$\therefore f(x) = \sin x \Rightarrow f'(x) = D(\sin x) = \cos x$

Contoh :Tentukanturunanpertamadari:

a. $f(x) = (x + 3) + 2 \sin x$

b. $f(x) = \sin^2 x$

c. $f(x) = x \sin x$

d. $f(x) = \frac{x}{\sin x}$

Penyelesaian: a. $f(x) = (x + 3) + 2 \sin x$

Misal $g(x) = (x + 3) \Rightarrow g'(x) = 1$

Misal $h(x) = 2 \sin x \Rightarrow h'(x) = 2 \cos x$

$\therefore f'(x) = 1 + 2 \cos x$

b. $f(x) = \sin^2 x = \sin x . \sin x$

$f'(x) = \sin x . \cos x + \sin x . \cos x$

$\therefore f'(x) = 2 \sin x \cos x$

c. $f(x) = x \sin x$

Misal $g(x) = x \Rightarrow g'(x) = 1$

Misal $h(x) = \sin x \Rightarrow h'(x) = \cos x$

$f'(x) = x . \cos x + \sin x . 1$

$\therefore f'(x) = x \cos x + \sin x$

d. $f(x) = \frac{x}{\sin x}$

Misal $g(x) = x \Rightarrow g'(x) = 1$

Misal $h(x) = \sin x \Rightarrow h'(x) = \cos x$

$f'(x) = \frac{\sin x . 1 - x.\cos x}{\sin x^2}$

$\therefore f'(x) = \frac{\sin x - x.\cos x}{\sin x^2}$

3.6.2. TurunanFungsiCosinus

Jika $f(x) = \cos x$, maka $f'(x) = -\sin x$

Bukti: $f(x) = \cos x \Rightarrow f(x+h) = \cos(x+h)$

$f(x+h) - f(x) = \cos(x+h) - \cos x$

$= -2 \sin 1/2 ((x+h) + x) . \sin 1/2 (x+h) - x)$

$= -2 \sin(x + 1/2\,h) . \sin 1/2h$

$$f'(x) = \lim_{h \to 0} \frac{-2 \sin(x + 1/2\,h) . \sin 1/2h}{h}$$

$$= \lim_{h \to 0} \frac{-2 \sin\left(x + \frac{1}{2}h\right) . \sin 1/2h}{2.(1/2h)}$$

$$= \lim_{h \to 0} -\sin(x + 1/2\,h) \lim_{h \to 0} \frac{\sin 1/2h}{\frac{1}{2}h}$$

$= -\sin x . 1$

$= -\sin x$ ■

$\therefore f(x) = \cos x \Rightarrow D(\cos x) = -\sin x$

Contoh :Tentukanturunanpertamadari:

a. $f(x) = (3x + 3) + 2 \cos x$

b. $f(x) = \cos^2 x$

c. $f(x) = 2x \cos x$

d. $f(x) = \frac{\cos x}{x^2}$

Penyelesaian: a. $f(x) = (3x + 3) + 2 \cos x$

Misal $g(x) = (3x + 3) \Rightarrow g'(x) = 3$

Misal $h(x) = 2 \cos x \Rightarrow h'(x) = -2 \sin x$

$\therefore f'(x) = 3 - 2 \sin x$

b. $f(x) = \cos^2 x = \cos x . \cos x$

$f'(x) = (\cos x . -\sin x) + (\cos x . -\sin x)$

$\therefore f'(x) = -2 \sin x \cos x$

c. $f(x) = 2x \cos x$

Misal $g(x) = 2x \Rightarrow g'(x) = 2$

Misal $h(x) = \cos x \Rightarrow h'(x) = -\sin x$

$f'(x) = 2x . -\sin x + \cos x . 2$

$\therefore$f'(x) = - 2x sin x + 2 cos x

d. f(x) = $\frac{\cos x}{x^2}$

Misal g(x) = cos x ⇒ g'(x) = - sin x

Misal h(x) = x^2⇒ h'(x) = 2x

f '(x) = $\frac{x^2.-\sin x - \cos x \;.\; 2x}{x^4}$

$\therefore$f'(x) = $\frac{-x^2 \sin x - 2x \cos x}{x^4}$

3.6.3. TurunanFungsiTangen

Jika f(x) = tan x, maka f'(x) = $\sec^2$ x

Bukti:

f(x) = tan x ⇒ f(x+h) = tan (x + h) =$\frac{\tan x+\tan h}{\text{1- tan xtan h}}$

f(x+h) - f(x) = $\frac{\tan x+\tan h}{\text{1- tan xtan h}}$ - tan x

$= \frac{\tan x+\tan h - \tan x\ (1-\tan x+\tan h)}{\text{1- tan xtan h}}$

$= \frac{\tan x+\tan h - \tan x\ (1-\tan x \tan h)}{\text{1- tan xtan h}}$

$= \frac{\tan x+\tan h - \tan x+ \tan x.\tan x \tan h)}{\text{1- tan xtan h}}$

$= \frac{\tan h+ \tan^2 x \tan h}{\text{1- tan xtan h}}$

f '(x) = $\lim_{h \to 0} \frac{\tan h(1+ \tan^2 x)}{h\ \text{(1- tan xtan h)}}$

f'(x) = $\lim_{h \to 0} \frac{\tan h \sec^2 x}{h\ \text{(1- tan xtan h)}}$

$= \lim_{h\to 0} \frac{\tan h}{h} \cdot \lim_{h\to 0} \sec^2 x \lim_{h\to 0} \frac{1}{\text{(1- tan xtan h)}}$

= 1 .$\sec^2$ x . 1

= $\sec^2$ x■

$\therefore$f(x) = tan x ⇒ f'(x) = D(tan x) = $\sec^2$ x

Cara lain untukmemperolehturunan f(x) = tan x, yaitudenganmenerapkanteorema 8

$f(x) = \tan x \iff f(x) = \frac{\sin x}{\cos x}$

$f'(x) = \frac{\cos x . \cos x - (\sin x . -\sin x)}{(\cos x)^2}$

$= \frac{1}{\cos^2 x}$

$= (\frac{1}{\cos x})^2$

$= \sec^2 x$ ■

$\therefore f(x) = \tan x \Rightarrow f'(x) = D(\tan x) = \sec^2 x$

Contoh :Tentukanturunanpertamadari:

a. $f(x) = (x^2 - 2x + 3) + \tan x$

b. $f(x) = \tan^2 x$

c. $f(x) = 2x \tan x$

d. $f(x) = \frac{x^2}{\tan x}$

Penyelesaian: a. $f(x) = (x^2 - 2x + 3) + \tan x$

Misal $g(x) = (x^2 - 2x + 3) \Rightarrow g'(x) = 2x - 2$

Misal $h(x) = \tan x \Rightarrow h'(x) = \sec^2 x$

$\therefore f'(x) = 2x - 2 + \sec^2 x$

b. $f(x) = \tan^2 x = \tan x . \tan x$

$f'(x) = \tan x . \sec^2 x + \tan x . \sec^2 x$

$\therefore f'(x) = 2 \tan x \sec^2 x$

c. $f(x) = x \tan x$

Misal $g(x) = 2x \Rightarrow g'(x) = 2$

Misal $h(x) = \tan x \Rightarrow$L $h'(x) = \sec^2 x$

$f'(x) = 2x . \sec^2 x + \tan x . 2$

$\therefore f'(x) = 2x \sec^2 x + 2 \tan x$

d. $f(x) = \frac{x^2}{\tan x}$

Misal $g(x) = x^2 \Rightarrow g'(x) = 2x$

Misal $h(x) = \tan x \Rightarrow h'(x) = \sec^2 x$

$f'(x) = \frac{\tan x . 2x + x^2 . \sec^2 x}{(\tan x)^2}$

$\therefore f'(x) = \frac{\tan x.2x+x^2.\ \sec^2 x}{(\tan x)^2}$

3.6.4. TurunanFungsiCotangen

Jika f(x) = cot x, maka f'(x) = -csc^2 x

Bukti: f(x) = cotx $\Rightarrow$ f(x) = -csc^2 x

f (x) = cot x =$\frac{\cos x}{\sin x}$

f '(x)= $\frac{\sin x.-\sin x-\cos x.\cos x}{\sin^2 x}$

$= \frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x}$

$= \frac{-1}{\sin^2 x}$

=- $(\frac{1}{\sin x})^2$

= - csc^2 x ■

$\therefore$ f(x) = cot x $\Rightarrow$ f'(x) = D(cot x) = -csc^2 x

Contoh : a. f(x) = (x^2 + 2x + 3) + cot x

b. f(x) = cot^2 x

c. f(x) = x^{3}cot x

d. f(x) = $\frac{2x}{\cot x}$

Penyelesaian : a. f(x) = (x + 2x + 3) + cot x

Misal g(x) = (x + 2x + 3) $\Rightarrow$ g'(x) = 2x + 2

Misal h(x) = cot x $\Rightarrow$ h'(x) = - csc^2 x

$\therefore$f'(x) = 2x + 2 – csc^2 x

b. f(x) = cot^2 x = cot x . cot x

f'(x) = cot x . –csc^2 x + cot x . –csc^2 x

$\therefore$f'(x) = - 2 cot x.csc^2 x

c. f(x) = x cot x

Misal g(x) = x^3$\Rightarrow$ g'(x) = 3 x^2

Misal h(x) = cot x $\Rightarrow$ h'(x) = - csc^2 x

f '(x) = cot x . 3 x^2 + x . (- csc^2 x)

$\therefore$f'(x) = 3 x^{2}cot x - x csc^2 x

d. $f(x) = \frac{2x}{\cot x}$

Misal $g(x) = 2x \Rightarrow g'(x) = 2$

Misal $h(x) = \cot x \Rightarrow g'(x) = -\csc^2 x$

$f'(x) = \frac{\cot x.2 - 2x.-\csc^2 x}{(\cot x)^2}$

$\therefore f'(x) = \frac{2(\cot x + x \csc^2 x)}{(\cot x)^2}$

3.6.5. TurunanFungsiSecant

Jika $f(x) = \sec x$, maka $f'(x) = \sec x .\tan x$

Bukti: $f(x) = \sec x$ atau $f(x) = \frac{1}{\cos x}$

$f'(x) = \frac{\cos x .0 - 1.-\sin x}{(\cos x)^2}$

$= \frac{\sin x}{(\cos x)^2}$

$= \frac{\sin x}{(\cos x)(\cos x)}$

$= \frac{\sin x}{\cos x} \frac{1}{\cos x}$

$= \tan x.\ \sec x$atau

$= \sec x.\tan x$ ■

$\therefore f(x) = \sec x \Rightarrow f'(x) = D(\sec x) = \sec x\ .\ \tan x$

Contoh :Tentukanturunanpertamadari:

a. $f(x) = (x^3 + 2x^2 + 12) + \sec x$

b. $f(x) = \sec^2 x$

c. $f(x) = x^5 \sec x$

d. $f(x) = \frac{\sec x}{3x}$

Penyelesaian : a. $f(x) = (x^3 + 2x^2 + 12) + \sec x$

Misal $g(x) = (x^3 + 2x^2 + 12) \Rightarrow g'(x) = 3x^2 + 4x$

Misal $h(x) = \sec x \Rightarrow h'(x) = \sec x \tan x$

$\therefore f'(x) = 3x^2 + 4x + \sec x \tan x$

b. $f(x) = \sec^2 x = \sec x \cdot \sec x$

$f'(x) = \sec x (\sec x \tan x) + \sec x (\sec \tan x)$

$\therefore f'(x) = 2 \sec^2 x \tan x$

c. $f(x) = x^5 \sec x$

Misal $g(x) = x^5 \Rightarrow g'(x) = 5 x^4$

Misal $h(x) = \sec x \Rightarrow h'(x) = \sec x \tan x$

$f'(x) = x^5 (\sec x \tan x) + 5 x^4 \sec x$

$\therefore f'(x) = x^4 \sec x (x \tan x + 5)$

d. $f(x) = \frac{\sec x}{3x}$

Misal $g(x) = \sec x \Rightarrow g'(x) = \sec x \tan x$

Misal $h(x) = 3 x \Rightarrow h'(x) = 3$

$f'(x) = \frac{3x.\sec x \tan x - 3.\sec x}{(3x)^2}$

$\therefore f'(x) = \frac{\sec x (x.\tan x - 1)}{3x^2}$

3.6.6. TurunanFungsiCosecant

Jika $f(x) = \csc x$, maka $f'(x) = -\csc x . \cot x$

Bukti: $f(x) = \csc x$ atau $f(x) = \frac{1}{\sin x}$

$f'(x) = \frac{\sin x .0 - 1.\cos x}{(\sin x)^2}$

$= \frac{-\cos x}{\sin x.\sin x}$

$= \frac{-1}{\sin x} . \frac{\cos x}{\sin x}$

$= -\csc x . \cot x$ ■

$\therefore f(x) = \csc x \Rightarrow f'(x) = -\csc x . \cot x$

Contoh :Tentukanturunanpertamadari:

a. $f(x) = (x^3 + 2x + 10) + 5 \csc x$

b. $f(x) = 3 \csc^2 x$

c. $f(x) = x^4 \csc x$

d. $f(x) = \frac{15}{\csc x}$

Penyelesaian : a. $f(x) = (x\ \ + 2x + 10) + 5 \csc x$

Misal $g(x) = (x^2\ + 2x + 10) \Rightarrow g'(x) = 2x + 2$

Misal $h(x) = 5 \csc x \Rightarrow h'(x) = - 5 \csc x \cot x$

$\therefore f'(x) = (2x + 2) - 5 \csc x \cot x$

b. $f(x) = 3 \csc^2\ x = (3 \csc x)(\csc x)$

$f'(x) = (3 \csc x)(- \csc x .\cot x) + \csc x (- 3 \csc x . \cot x)$

$\therefore f'(x) = - 6 \csc^2\ x \cot x$

c. $f(x) = x^4 \csc x$

Misal $g(x) = x^4 \Rightarrow g'(x) = 4 x^3$

Misal $h(x) = \csc x \Rightarrow h'(x) = - \csc x \cot x$

$f '(x) = x^4 (- \csc x \operatorname{ctg} x) + \csc x (4 x^3)$

$\therefore f'(x) = x^3 \csc x (4 - x \cot x)$

d. $f(x) = \frac{15}{\csc x}$

Misal $g(x) = 15 \Rightarrow g'(x) =0$

Misal $h(x) \Rightarrow h'(x) = - \csc x \cot x$

$f '(x) = \frac{\csc\ x\ .\ 0 - 15 . - \csc x \cot x}{(\csc x)^2}$

$\therefore f'(x) = \frac{15 \cot x}{\csc x}$