MA30060 PS4: The sawtooth map
Feedback hand-in: Friday 5 Nov 2021, 4:00pm

1. P2 orbits on odd functions

Consider $x_{n+1} = F(x_n)$ where F is an odd function about $x = 0$. Prove that if x^* satisfies $-x^* = F(x^*)$, then x^* is part of a period-2 orbit.

2. Sawtooth map and orbits

Consider the sawtooth map $x_{n+1} = 2x_n \bmod 1$.

   i Investigate the forward orbits of the initial conditions $x_0 = 1/7$ and $x_0 = 3/7$. Hence, or otherwise, write down the binary expansions corresponding to these points.

   ii Using the results of part (a), describe the behaviour of the forward orbits of all initial conditions of the form $x_0 = p/(7 \times 2^k)$ for integers $1 \le p \le 6$ and $k \ge 0$.

3. Sawtooth map and binary expansions

For the sawtooth map, consider the possible binary expansions to show that there are

   i exactly three distinct 4-cycles (orbits with least period 4)

   ii exactly six distinct 5-cycles.

Compute, as a fraction, a point that lies on each cycle and find its forward orbit under the map to demonstrate that the cycles you have found are distinct.