

Chapter 3: Intensity Transformations and Spatial Filtering

Enhancement with Averaging Operations

When images are displayed (or printed), they often have suffered from noise and interferences from several sources including:

- electrical sensor noise,
- photographic grain noise, and
- channel errors.



Noisy image

These noise effects can be removed by simple ad hoc “noise-cleaning” techniques or more advanced methods, including deep (machine) learning) applied to local neighborhoods of input pixels.

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Enhancement with Averaging Operations

Consider a (simplified) noisy image:

$$g(x, y) = f(x, y) + \eta(x, y)$$

where the second term is noise,

We assume that the noise is *uncorrelated* with the input and has *zero mean*. Then, averaging K different noisy images:

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^K g_i(x, y)$$

produces an output image with

$$E[\bar{g}(x, y)] = f(x, y) \quad \text{and} \quad \sigma_{\bar{g}}^2 = \frac{1}{K} \sigma_{\eta}^2$$

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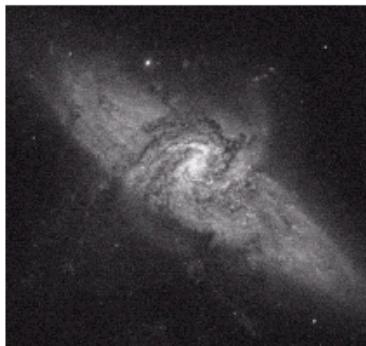
Enhancement with Averaging Operations

original



noisy image, $N(0,64^2)$

result of
averaging
8 noisy
images



16 noisy images

64 noisy
images



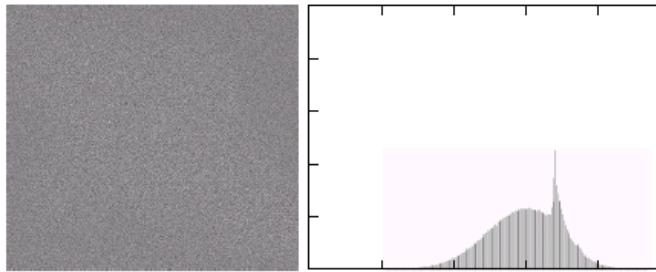
a b
c d
e f

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

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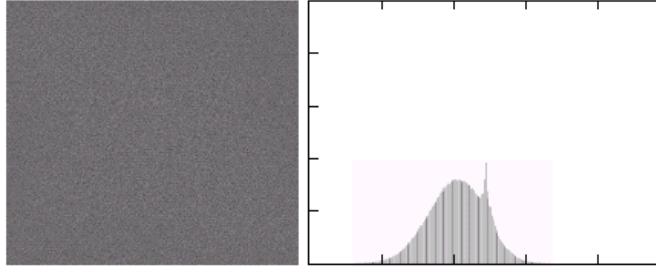
Enhancement with Averaging Operations

$K=8$



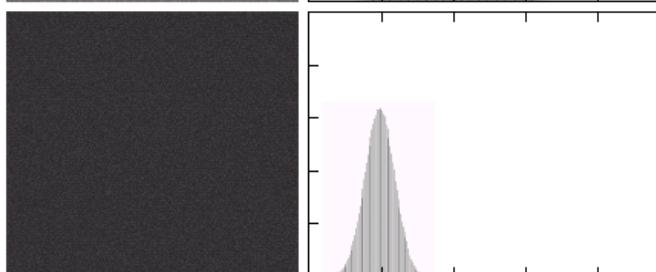
a

$K=16$

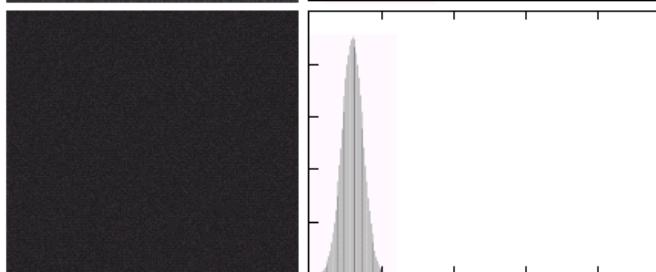


b

$K=64$



$K=128$



a

b

FIGURE 3.31

(a) From top to bottom:
Difference images
between
Fig. 3.30(a) and
the four images in
Figs. 3.30(c)
through (f),
respectively.
(b) Corresponding
histograms.

Remarks:

1. Notice how the noise is
 - smoothed, and its
 - variance is decreasing with increasing K .
2. The operation is costly in computation and memory.

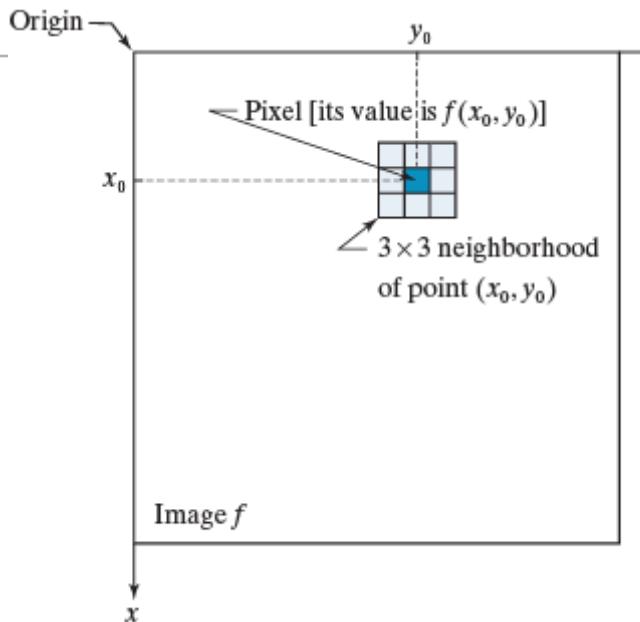
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Alternative solution: Spatial Domain Operation:

$$g(x,y) = T[f_{N(x,y)}(x,y)]$$

$f_{N(x,y)}(x,y)$ is the original image $f(x,y)$ defined over the neighborhood $N(x,y)$ of (x,y) ; $g(x,y)$ is the output image and $T[.]$ is an operator on $f(x,y)$ defined over a neighborhood $N(x,y)$ of (x,y) .

FIGURE 3.1
A 3×3 neighborhood about a point (x_0, y_0) in an image. The neighborhood is moved from pixel to pixel in the image to generate an output image. Recall from Chapter 2 that the value of a pixel at location (x_0, y_0) is $f(x_0, y_0)$, the value of the image at that location.



Special case: when the neighborhood $N(x,y)$ contains 1 pixel, then $T[.]$ is called **intensity transformation function**.

Now we consider the information from larger neighborhoods

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Localized Histogram Equalization

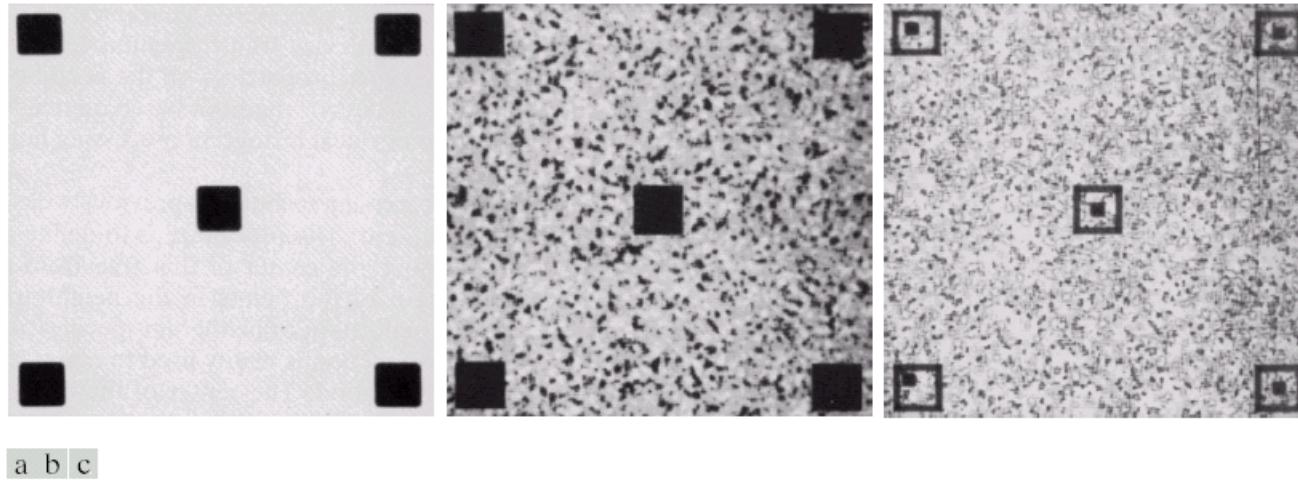


FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

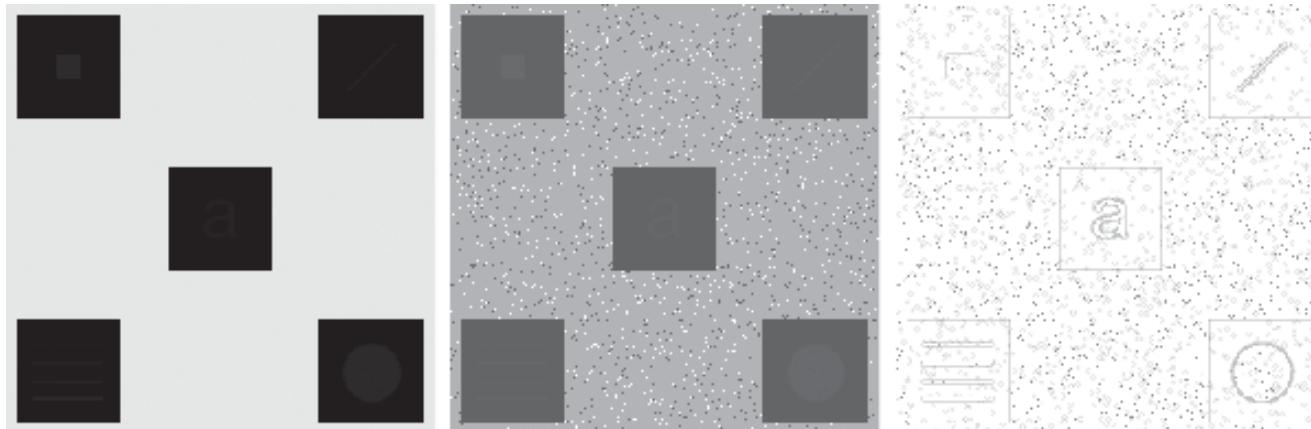


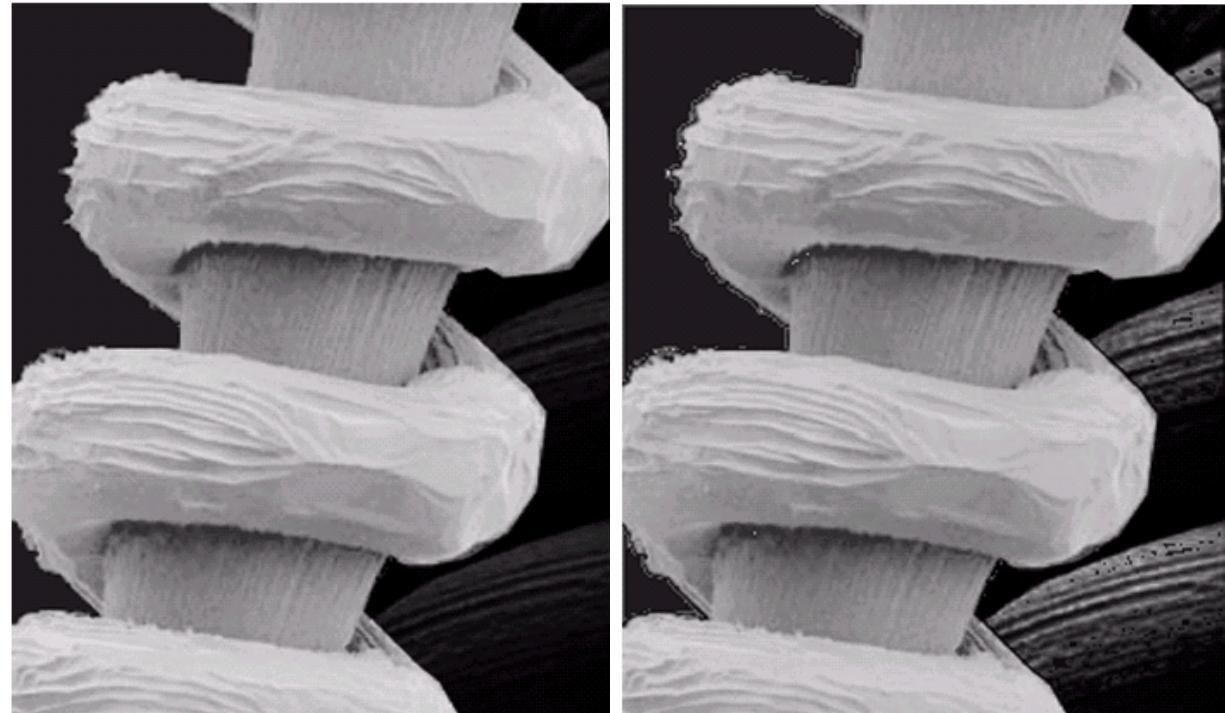
FIGURE 3.32
(a) Original
image. (b) Result
of global
histogram
equalization.
(c) Result of local
histogram
equalization.

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Local Histogram Statistics

How to obtain this?

FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately $130\times$. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).



There is another filament behind the front one, but it's too dark to see. Can we use local information to highlight regions of low-intensity while preserving the visible ones as they are?

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Local Histogram Statistics

Normalized histogram values can be treated as probabilities.

We can then compute global statistics over the whole image:

$$\text{Mean: } m = \sum_{i=0}^{L-1} r_i p(r_i) \quad \text{Variance: } \sigma^2 = \sum_{i=0}^{L-1} [r_i - m]^2 p(r_i)$$

Local statistics can be computed in neighborhoods of the image:

$$\text{Mean: } m_{S_{xy}} = \sum_{(s,t) \in S_{xy}} r_{s,t} p(r_{s,t})$$

$$\text{Variance: } \sigma^2_{S_{xy}} = \sum_{(s,t) \in S_{xy}} [r_{s,t} - m_{S_{xy}}]^2 p(r_{s,t})$$

(index i omitted here for clarity)

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Local Histogram Statistics

We can use local statistics to selectively apply intensity transforms.

Local mean compared to global mean indicates whether the area is dark or light.

Difference between local and global variance indicates contrast.

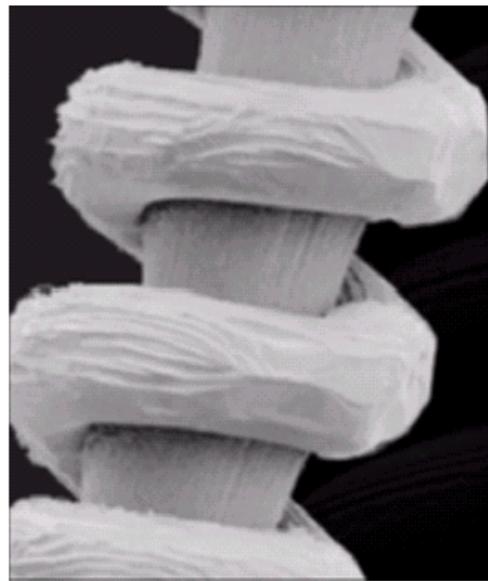
By setting thresholds we obtain a selective transformation:

$$g(x, y) = \begin{cases} C \cdot f(x, y) & \text{if } k_0 m_G < m_{S(x,y)} < k_1 m_G \text{ AND } k_2 \sigma_G < \sigma_{S(x,y)} < k_3 \sigma_G \\ f(x, y) & \text{otherwise} \end{cases}$$

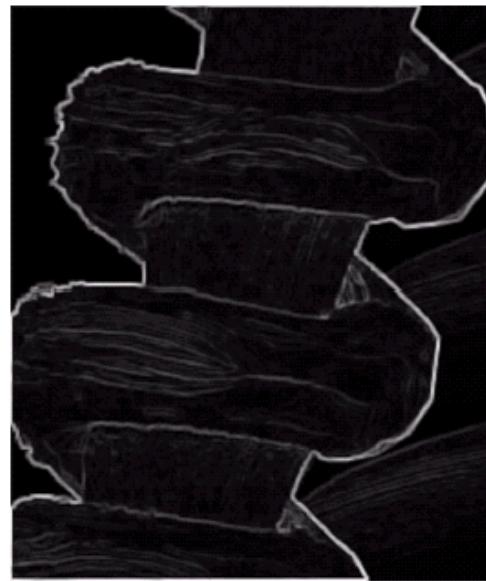
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Local Histogram Statistics

$m_{S_{xy}}$



$\sigma^2_{S_{xy}}$



$(1, C)$



a b c

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

Here all the neighborhoods of size 3x3 are used.

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Local Histogram Statistics



FIGURE 3.26

Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

The following values are used:

$$C = 4.0, \\ k_0 = 0, \quad k_1 = 0.4, \\ k_2 = 0.02, \quad k_3 = 0.4$$

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Local Histogram Statistics

Same technique used to obtain this result

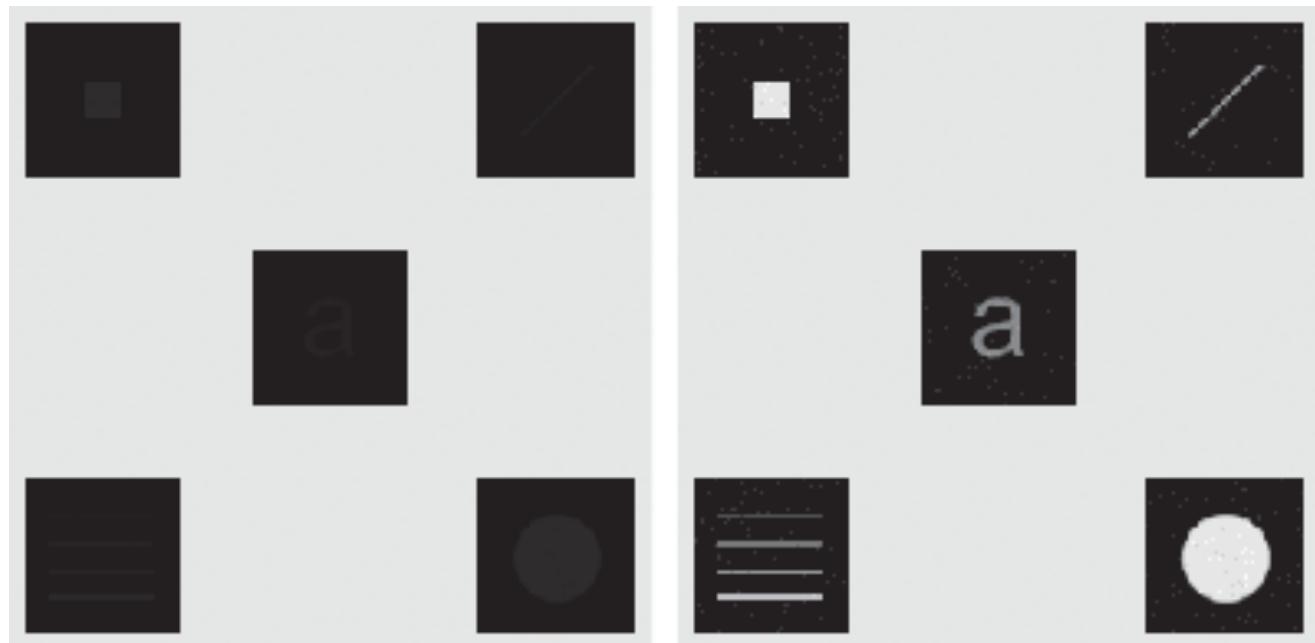
$$C = 22.8,$$

$$k_0 = 0, \quad k_1 = 0.1,$$

$$k_2 = 0, \quad k_3 = 0.1$$

a b

FIGURE 3.33
(a) Original image. (b) Result of local enhancement based on local histogram statistics. Compare (b) with Fig. 3.32(c).



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Spatial Filtering Introduction

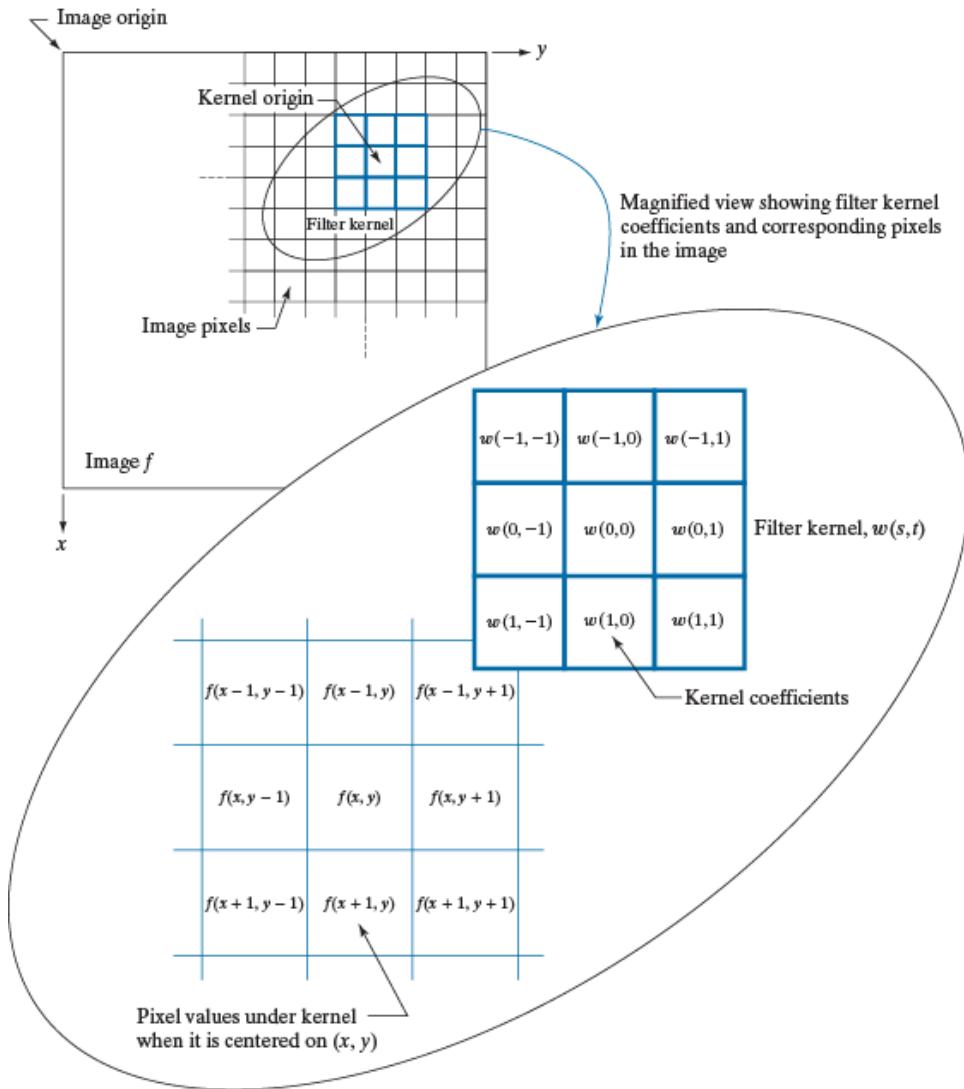


FIGURE 3.34
The mechanics of linear spatial filtering using a 3×3 kernel. The pixels are shown as squares to simplify the graphics. Note that the origin of the image is at the top left, but the origin of the kernel is at its center. Placing the origin at the center of spatially symmetric kernels simplifies writing expressions for linear filtering.

Each pixel is replaced by a weighted sum of its neighbourhood pixels in a sliding window

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Correlation and Convolution

Correlation and convolution are two similar operations with weighted sums on sliding windows.

Correlation:
$$(w \star f)(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t)f(x + s, y + t)$$

Convolution:
$$(w * f)(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t)f(x - s, y - t)$$

Linear filtering uses convolution unless explicitly said otherwise.

Convolution operates like correlation, but the kernel is rotated 180°.
(no difference for center-symmetric filters)

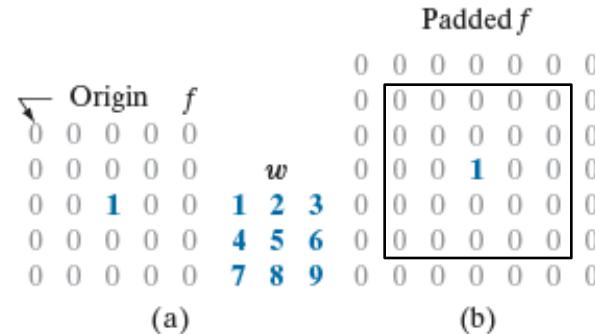
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Correlation and Convolution in 1-D

	<i>Correlation</i>	<i>Convolution</i>
(a)		
(b)		
(c)		
(d)		
(e)		
(f)		
(g)	Correlation result $0 \ 8 \ 2 \ 4 \ 2 \ 1 \ 0 \ 0$	
(o)		Convolution result $0 \ 1 \ 2 \ 4 \ 2 \ 8 \ 0 \ 0$

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Correlation and Convolution in 2-D



Initial position for w	Correlation result	Full correlation result
1 2 3	0 0 0 0 0	0 0 0 0 0 0 0 0
4 5 6	0 0 0 0 0	0 0 0 0 0 0 0 0
7 8 9	0 0 0 0 0	0 0 9 8 7 0 0 0
0 0 0 1 0	0 6 5 4 0	0 0 6 5 4 0 0 0
0 0 0 0 0	0 3 2 1 0	0 0 3 2 1 0 0 0
0 0 0 0 0	0 0 0 0 0	0 0 0 0 0 0 0 0
0 0 0 0 0	0 0 0 0 0	0 0 0 0 0 0 0 0
(c)	(d)	(e)

Rotated w	Convolution result	Full convolution result
9 8 7	0 0 0 0 0	0 0 0 0 0 0 0 0
6 5 4	0 0 0 0 0	0 0 0 0 0 0 0 0
3 2 1	0 0 0 0 0	0 0 1 2 3 0 0 0
0 0 0 1 0	0 4 5 6 0	0 0 4 5 6 0 0 0
0 0 0 0 0	0 7 8 9 0	0 0 7 8 9 0 0 0
0 0 0 0 0	0 0 0 0 0	0 0 0 0 0 0 0 0
0 0 0 0 0	0 0 0 0 0	0 0 0 0 0 0 0 0
(f)	(g)	(h)

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Difference between Correlation and Convolution

Convolution with an impulse signal produces the filter itself, which does not hold for correlation. The importance of this will be noted in the next chapter.

Properties of correlation and convolution differ.

TABLE 3.5
Some fundamental properties of convolution and correlation. A dash means that the property does not hold.

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \star (g + h) = (f \star g) + (f \star h)$

Note: the order of linear spatial filters does not matter!

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Examples of Linear Filter Kernels

a b

FIGURE 3.37

Examples of smoothing kernels:
(a) is a *box* kernel;
(b) is a *Gaussian* kernel.

$\frac{1}{9} \times$	<table border="1"><tr><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	1	1	1	1	1	1	1	1	1	$\frac{1}{4.8976} \times$	<table border="1"><tr><td>0.3679</td><td>0.6065</td><td>0.3679</td></tr><tr><td>0.6065</td><td>1.0000</td><td>0.6065</td></tr><tr><td>0.3679</td><td>0.6065</td><td>0.3679</td></tr></table>	0.3679	0.6065	0.3679	0.6065	1.0000	0.6065	0.3679	0.6065	0.3679
1	1	1																			
1	1	1																			
1	1	1																			
0.3679	0.6065	0.3679																			
0.6065	1.0000	0.6065																			
0.3679	0.6065	0.3679																			

Note: filter values sum to 1 to preserve average intensity of the original image

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Smoothing (Averaging) Filters



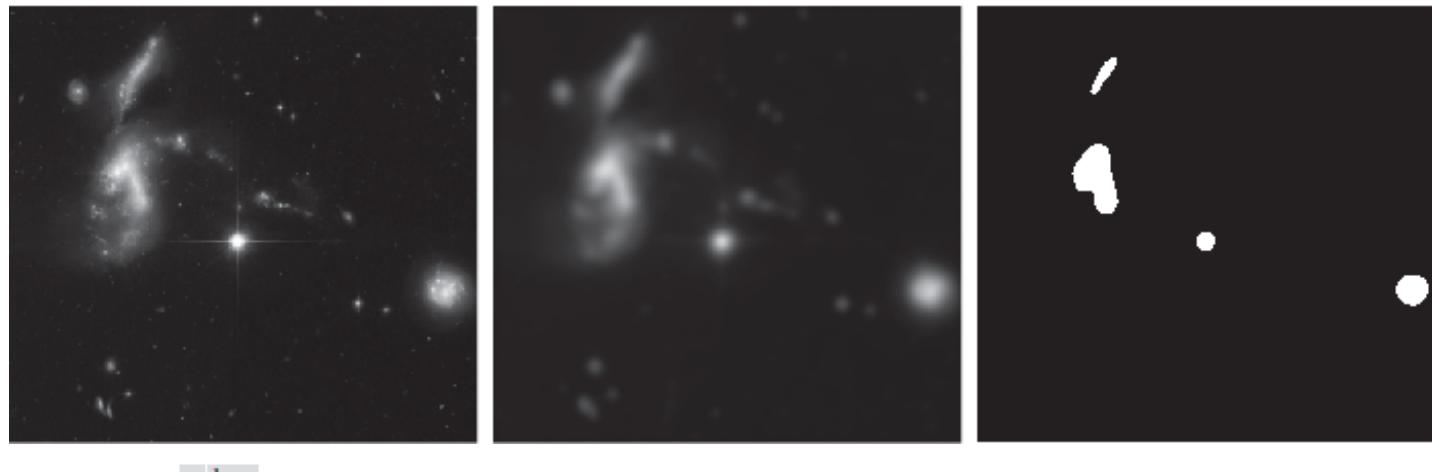
a
b
c
d

FIGURE 3.39

(a) Test pattern of size 1024×1024 pixels.
(b)-(d) Results of lowpass filtering with box kernels of sizes 3×3 , 11×11 , and 21×21 , respectively.

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Averaging Filter Application



a b c

FIGURE 3.47 (a) A 2566×2758 Hubble Telescope image of the *Hickson Compact Group*. (b) Result of lowpass filtering with a Gaussian kernel. (c) Result of thresholding the filtered image (intensities were scaled to the range $[0, 1]$). The Hickson Compact Group contains dwarf galaxies that have come together, setting off thousands of new star clusters. (Original image courtesy of NASA.)

Image from the Hubble space telescope in orbit around the Earth.

Here, we want to blur the image in order to only see large objects.

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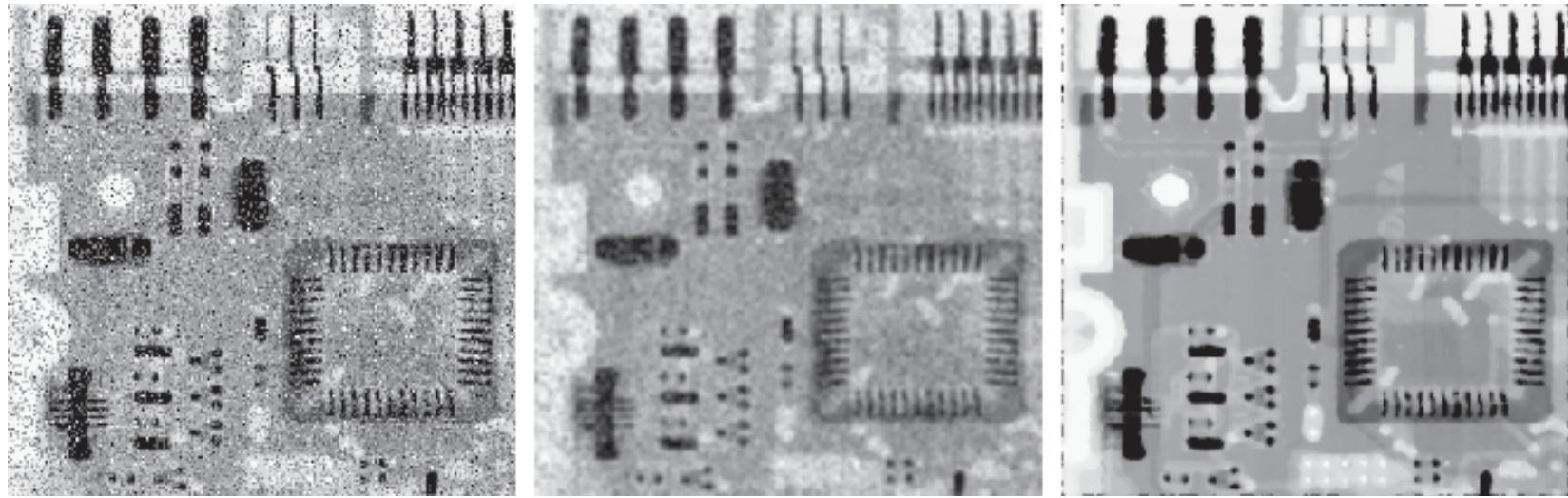
Q: What happens when important details must be preserved or the noise is non-Gaussian?

A: may consider a number of techniques:

- median and order statistics (min, max) filtering,
- sharpening spacial filters
 - directional filtering,
- hybrid combinations

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Nonlinear (Order-Statistic) Filtering



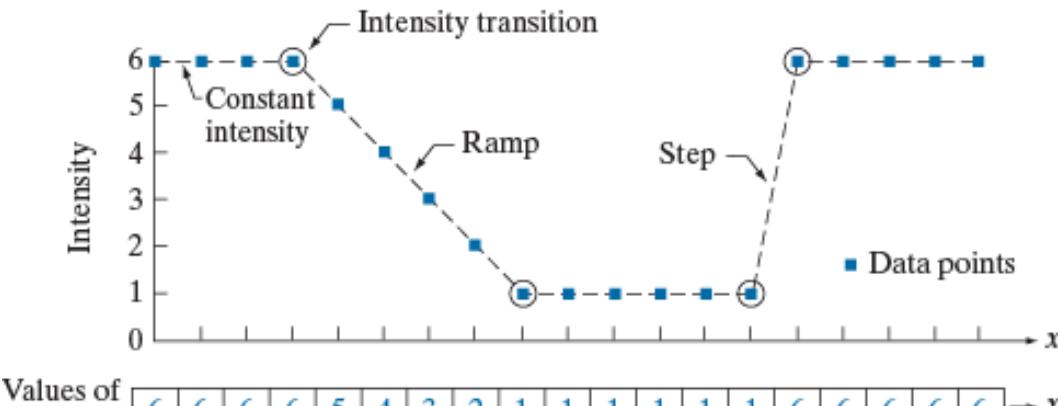
a b c

FIGURE 3.49 (a) X-ray image of a circuit board, corrupted by salt-and-pepper noise. (b) Noise reduction using a 19×19 Gaussian lowpass filter kernel with $\sigma = 3$. (c) Noise reduction using a 7×7 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

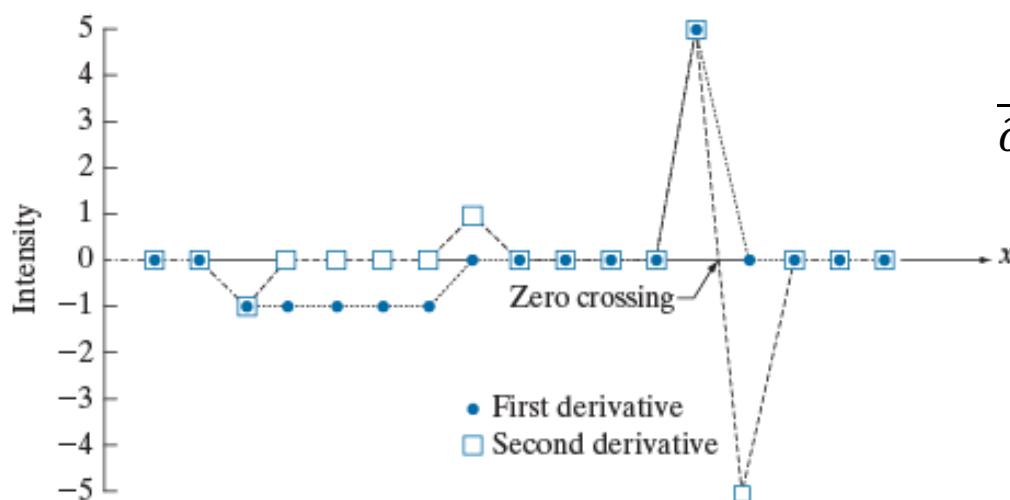
Median or order statistics filters may perform much better in the presence of *non-Gaussian* noise. (More details in Ch. 5)

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Sharpening Spatial Filters



1st derivative	0	0	-1	-1	-1	-1	0	0	0	0	0	5	0	0	0	0
2nd derivative	0	0	-1	0	0	0	0	1	0	0	0	0	5	-5	0	0



First derivative:

$$\frac{\partial f}{\partial x} = f(x + 1) - f(x)$$

Second derivative:

$$\frac{\partial^2 f}{\partial^2 x^2} = f(x + 1) + f(x - 1) - 2f(x)$$

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Sharpening Spatial Filters

- **First order derivative**
 - produces thicker edges because the 1st derivative is non-zero along a "ramp"
 - has strong response to grey-level steps
- **Second order derivative**
 - has a much stronger response to details (e.g. edges and sharp changes)
 - produces a double response at step changes in grey level
 - has a stronger response to a thin line than to a step and to an isolated point than to a thin line.
- **Conclusion**
 - Second derivative is more useful to enhance image fine details (but be careful about noise!!)

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Sharpening Filters: Laplacian

- **Isotropic** 2nd order derivative (Laplacian)

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x^2} + \frac{\partial^2 f}{\partial^2 y^2}$$

- In digital form:

$$\frac{\partial^2 f}{\partial^2 x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

in the x-direction and in the y-direction:

$$\frac{\partial^2 f}{\partial^2 y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

- Can be implemented using the mask in the next slide

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Sharpening Filters: Laplacian

isotropic for rotations in increments of:

Note: these sum to zero!

90°			45°		
0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

a	b
c	d

FIGURE 3.39 3.37

(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4).

(b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

negatives of the above two masks

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Sharpening Filters: Laplacian

- Typical use: emphasizing the edges on the image
- Laplacian image is added on top of the original
- Standard enhancement with Laplacian:

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the mask center coeff is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the mask center coeff is positive} \end{cases}$$

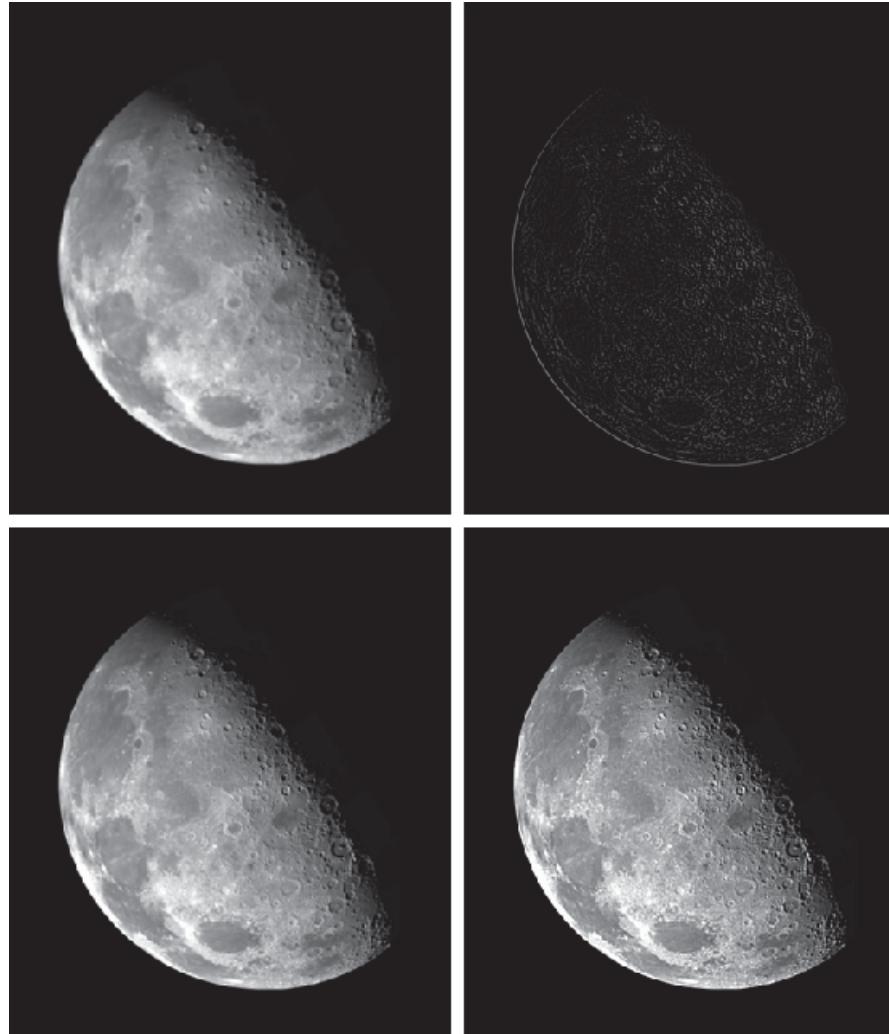
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Laplacian Filtering Example

a b
c d

FIGURE 3.52

- (a) Blurred image of the North Pole of the moon.
- (b) Laplacian image obtained using the kernel in Fig. 3.51(a).
- (c) Image sharpened using Eq. (3-63) with $c = -1$.
- (d) Image sharpened using the same procedure, but with the kernel in Fig. 3.51(b).
(Original image courtesy of NASA.)



Enhanced
image using
the 90° mask

Laplacian
image using
the 90° mask

Enhanced
image using
the 45° mask

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Composite Mask for Laplacian Filtering

- The previous expression :

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the mask center coeff is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the mask center coeff is positive} \end{cases}$$

- The two terms in the summation can be combined and thus simplified:

$$g(x, y) = 5f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)]$$

- This operation can be implemented using the mask in the next slide.

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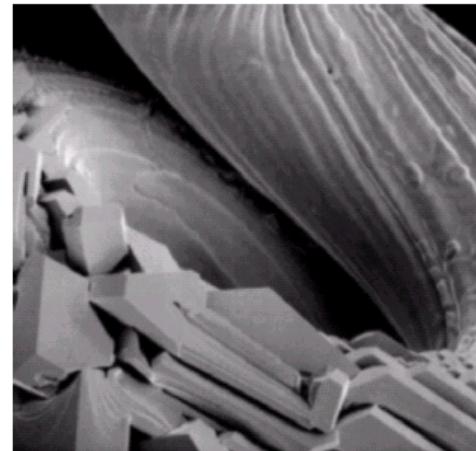
Composite Mask for Laplacian Filtering

0	-1	0
-1	5	-1
0	-1	0

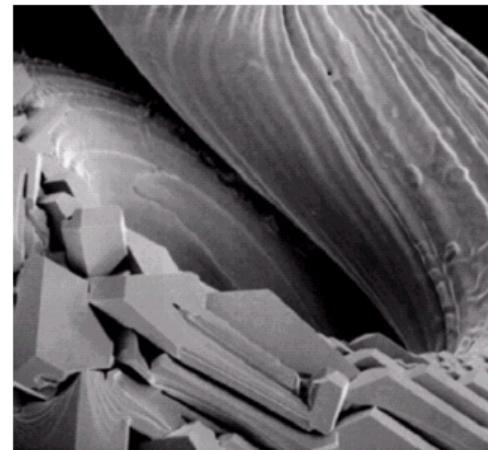
90°

-1	-1	-1
-1	9	-1
-1	-1	-1

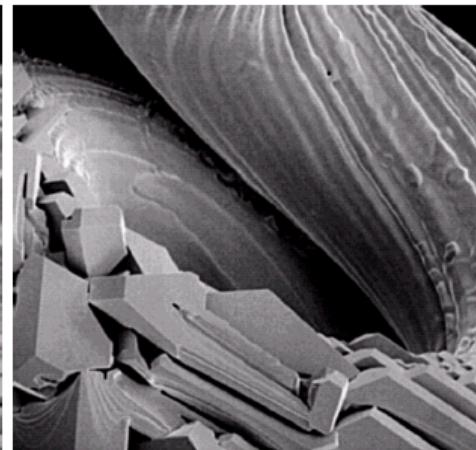
45°



SEM image



results for the 90°



results for the 45°

Notice how much sharper it is!

a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

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Laplacian with High-Boost Filtering

0	-1	0
-1	$A + 4$	-1
0	-1	0

-1	-1	-1
-1	$A + 8$	-1
-1	-1	-1

a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

This mask can give better results if the original image is darker than desired, see example in next slide.

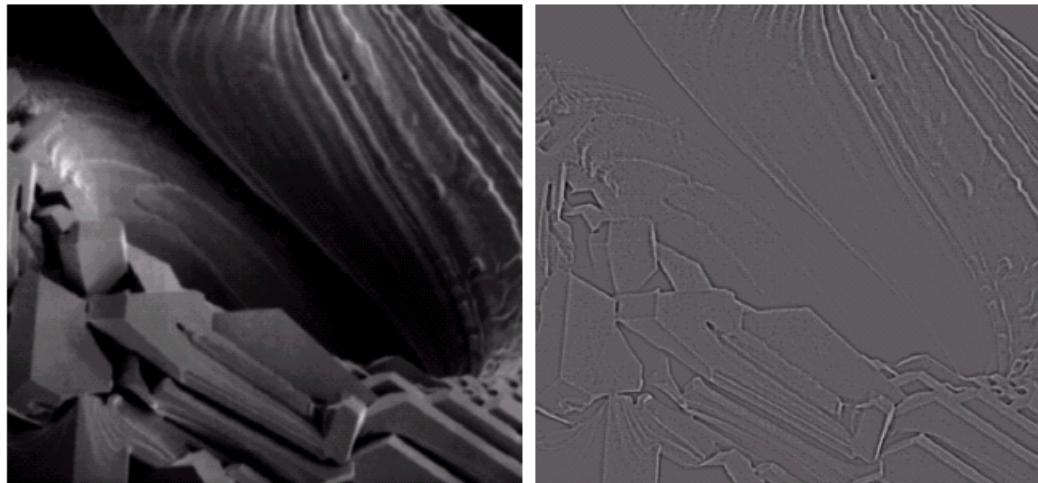
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Laplacian with High-Boost Filtering

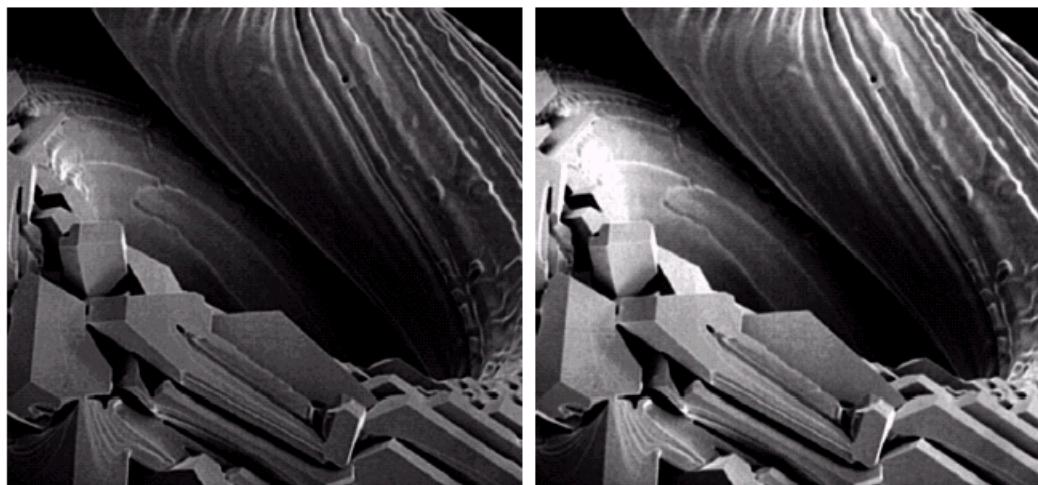
a b
c d

FIGURE 3.43

- (a) Same as Fig. 3.41(c), but darker.
(a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$.
(c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.



$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$



$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9.7 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

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Sharpening Filters with First-Order Derivatives

- First derivatives are implemented using the magnitude of the gradient. The gradient is given by:

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- The magnitude of the gradient is given by:

$$\nabla f = \text{mag}(\nabla \mathbf{f}) = [G_x^2 + G_y^2]^{1/2} = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

Note that components of the gradient are linear operators, but the magnitude is not! Also the partial derivatives are not isotropic, but the magnitude is!

- The magnitude can be (for faster implementation) approximated by:

$$\nabla f \approx |G_x| + |G_y|$$

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Sharpening Filters with First-Order Derivatives

+

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$f(x,y)$

$$\nabla f = [(z_8 - z_5)^2 + (z_6 - z_5)^2]^{\frac{1}{2}}$$

-1	0	0	-1
0	1	1	0

}

Roberts cross-gradient operator

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

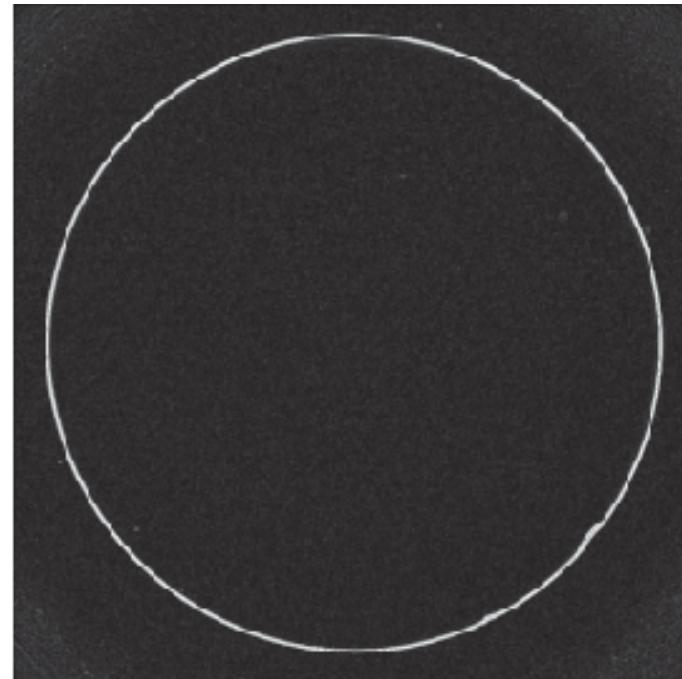
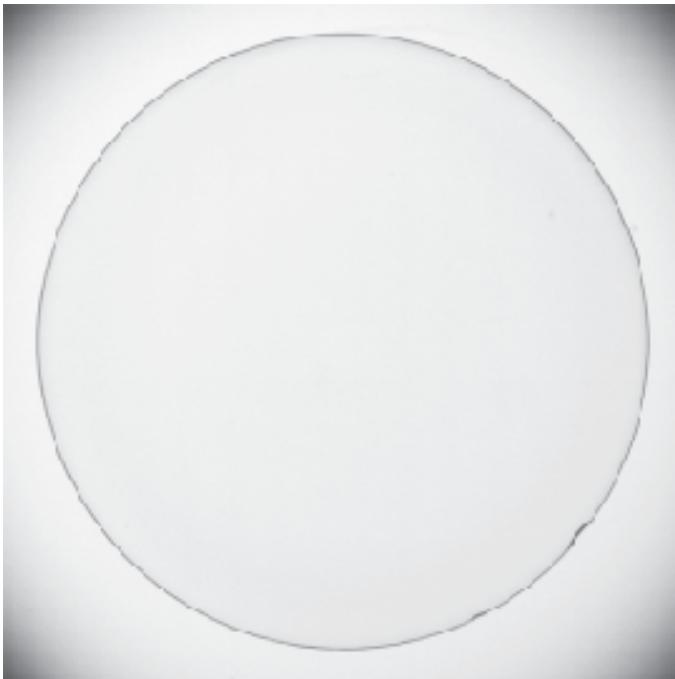
}

Sobel operators

$$\nabla f \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

Chapter 3: Intensity Transformations and Spatial Filtering

Gradient Sharpening Filter Example



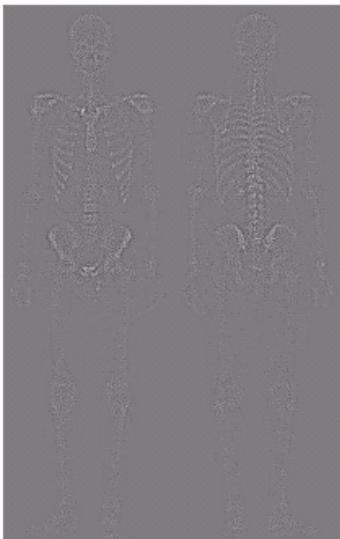
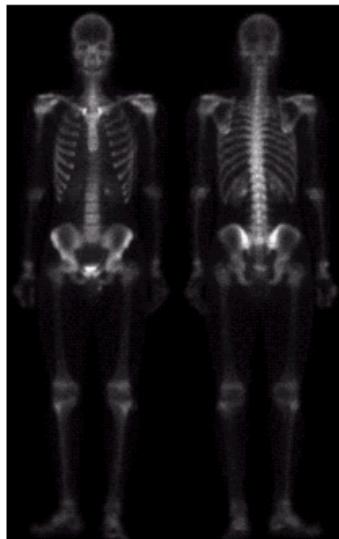
a b

FIGURE 3.57
(a) Image of a contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient. (Original image courtesy of Perceptics Corporation.)

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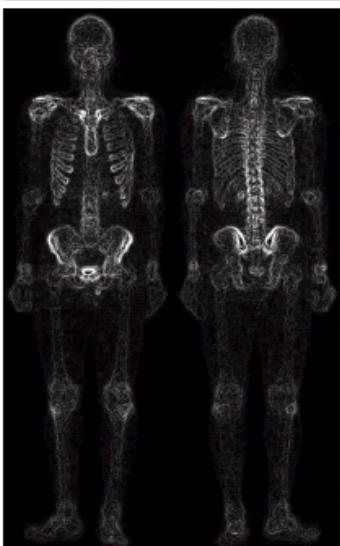
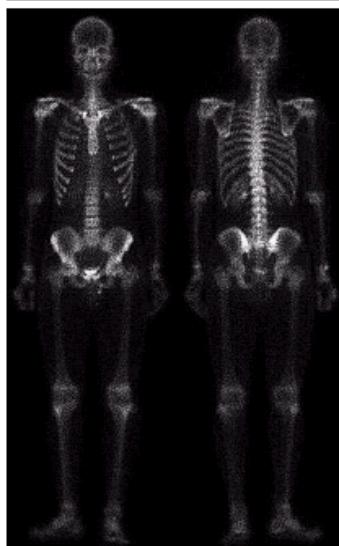
Combining Spatial Enhancement Methods

Original



Laplacian of original

original+
Laplacian



Sobel of orig

a
b
c
d

- use Laplacian to highlight details,
- gradient to enhance edges,
- grey-level transformation to increase dynamic range

FIGURE 3.63
(a) Image of whole body bone scan.
(b) Laplacian of (a).
(c) Sharpened image obtained by adding (a) and (b).
(d) Sobel gradient of image (a). (Original image courtesy of G.E. Medical Systems.)

Problem:

Nuclear whole body scan want to detect diseases, e.g. bone infection and tumors

Challenge:

- Single technique may not produce desirable results
- Must thus devise a strategy for the given application at hand

Strategy:

Chapter 3: Intensity Transformations and Spatial Filtering

Combining Spatial Enhancement Methods

