Introduction to diodes and small signal analysis

Learning outcomes: Student

+ knows basics about junction diodes and is able to describe the Shockley equation,
+ knows how the "local linearization principle" is applied in the small signal analysis,
+ is able to explain steps inherent in small signal analysis of diodes,
+ is able to determine the dynamic resistance of a diode at a chosen operation points,
+ is able to simulate and measure the dynamic resistance of a diode.

Diode

Diode is a nonlinear two–terminal device, the terminals are called the anode and the cathode (see Fig. 1). In this course we consider briefly semiconductor (junction) diodes. They consist of a p-type and an n–type semiconductor materials. The next paragraphs focus on their (external) modeling which has similarities to modeling of semiconductor transistors (which will be our next topic).

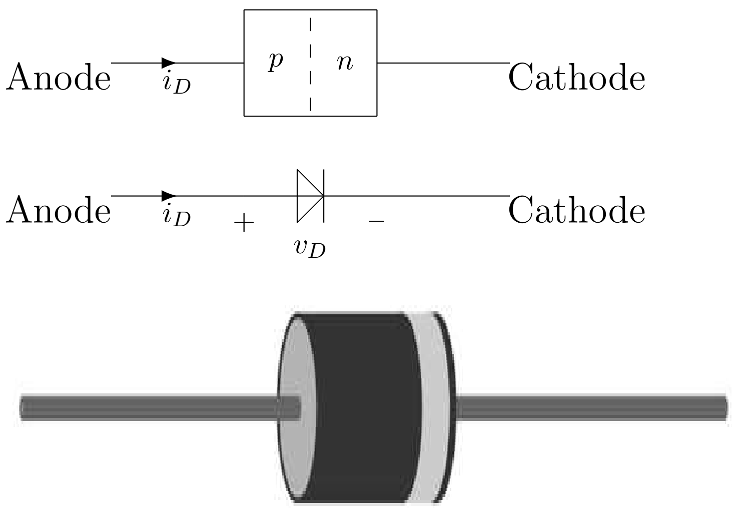


Figure 1: **Top**: simplified physical structure of semiconductor diodes, p represents a semiconductor material with excess holes; n represents a semiconductor material with excess electrons. **Middle**: circuit symbol for a diode. **Bottom**: typical markings in a diode, 'stripe' denotes the cathode.

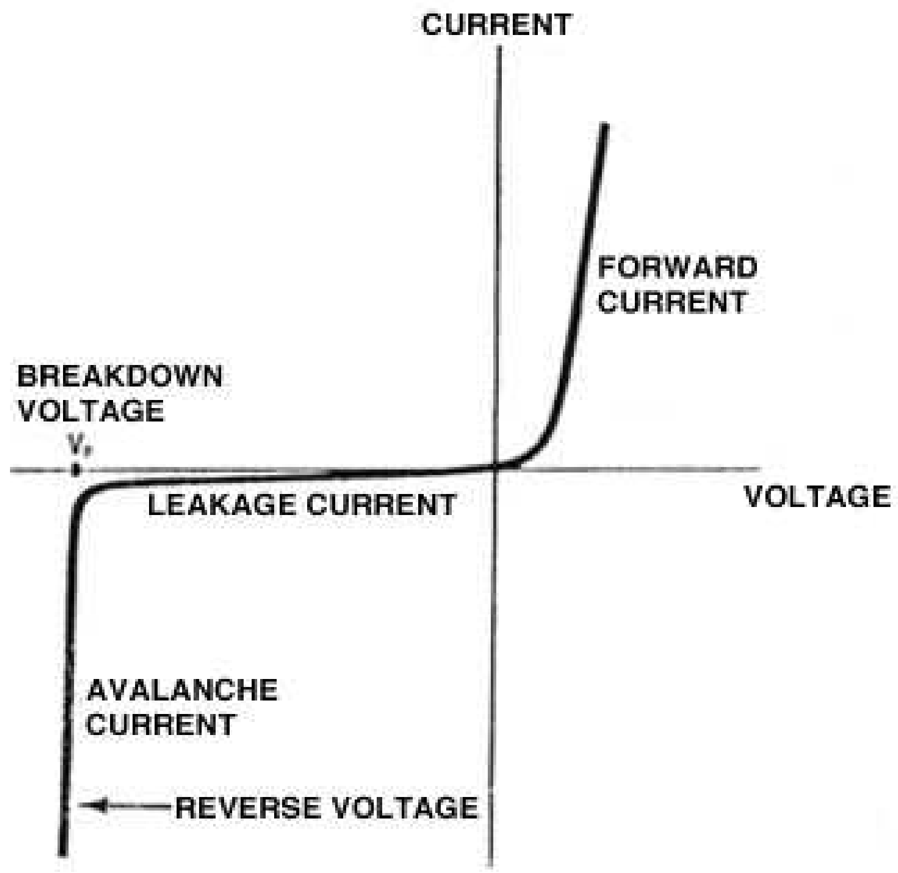


Figure 2: Typical current–voltage characteristic for a junction diode.

Fig. 2 shows a typical current–voltage characteristic for a junction diode. The figure illustrates the following properties:

In the forward region (the first quadrant) forward current starts to rise rapidly beyond voltage $v_f \approx 0.7$V.

In the reverse region so called breakdown voltage v_b is defined, for example value of v_b can be -100V (note, however, for some diode types v_b can have positive value).

Some types of diodes are intended to operate in the forward region while others in the reverse region (Zener diodes).

Shockley equation

To analyze circuits that include junction diodes, we need to be able to model the relationship between voltage and current for the junction diode. We utilize an analytical model (so called Shockley equation) that is based on empirical studies. See Fig. 3 for an example.

The Shockley equation relates the diode *current* i_D and the diode *voltage* v_D as

$$i_D = I_S \left[e^{\frac{v_D}{nV_T}} - 1 \right] \tag{1}$$

where

I_S is called the reverse saturation current (typically of order 10^{-10}A),

n is called emission coefficient ($1 \leq n \leq 2$, for silicon for instance about 1.3). Its value can be adjusted to improve accuracy of the model for different diodes.

$V_T = \frac{kT}{q}$ is called thermal voltage, at room temperature $V_T \approx 25$ mV.

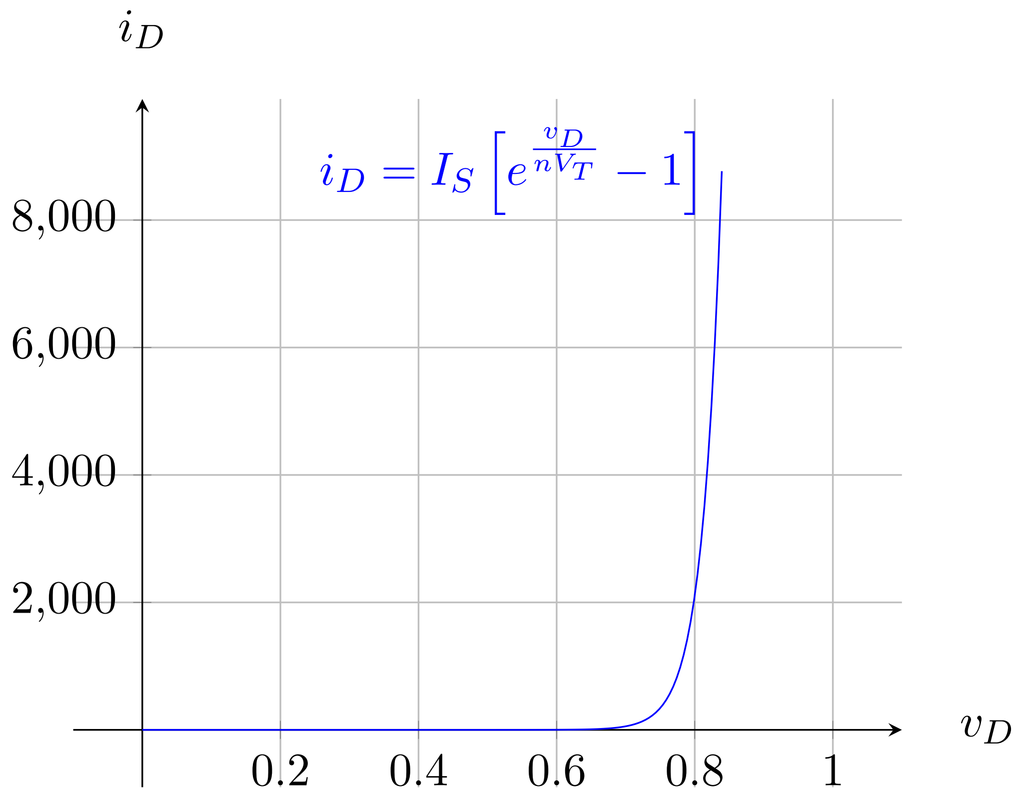


Figure 3: Plot of the Shockley equation for a diode ($I_S = 0.49$nA, $n = 1.1$). Observe the similarity at the first quadrant to the curve shown in Fig. 2 (recall also the examples in the pretask videos).

Note that if $v_D \gg 25$ mV, the exponential term $e^{\frac{v_D}{nV_T}}$ in eqn. 1 is far greater than one and thus eqn. 1 can be simplified to

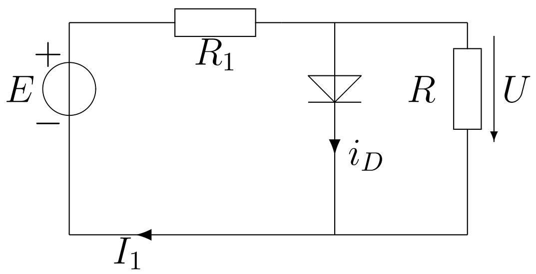
$$i_D \approx I_S \left[e^{\frac{v_D}{nV_T}} \right]. \tag{2}$$

Example for self study

Find resistance R in parallel with the diode such that voltage over the diode is $U = v_D = 0.7$ V. Use the Shockley equation, but check whether you could simplify it (i.e. whether you can use eqn. 2).

Let $E = 1.5$ V, $R_1 = 1250\ \Omega$, $I_S = 0.49$ nA, and $nV_T = 50$ mV.

HINT: Find first i_D and then use KVL to find R.



Solution:

First notice that $e^{\frac{v_D}{nV_T}} = e^{14} \gg 1$, so that

$$i_D \approx I_S \left[e^{\frac{v_D}{V_T}} \right]$$

Then using KVL:

$$\begin{aligned} -E + R_1 I_1 + U &= 0 \\ \Leftrightarrow -E + R_1(i_D + U/R) + U &= 0 \\ \Leftrightarrow i_D + \frac{U}{R} &= \frac{E-U}{R_1} \\ \Rightarrow \frac{U}{R} &\approx \frac{E-U}{R_1} - I_S \left[e^{\frac{v_D}{V_T}} \right] \\ \Rightarrow R &= \frac{U}{\frac{E-U}{R_1} - I_S \left[e^{\frac{v_D}{V_T}} \right]} = 13800\ \Omega, \end{aligned}$$

and $i_D = 589\ \mu$A, $I_1 = 0.64$ mA.

Example: Tangent line of a polynomial function

The blue curve shows the second order polynomial.

The red line shows its tangent line at point $x_0 = 4$.

Near point (x_0, y_0) the blue curve and the red line are close to each other, see page 14 for more detailed example.

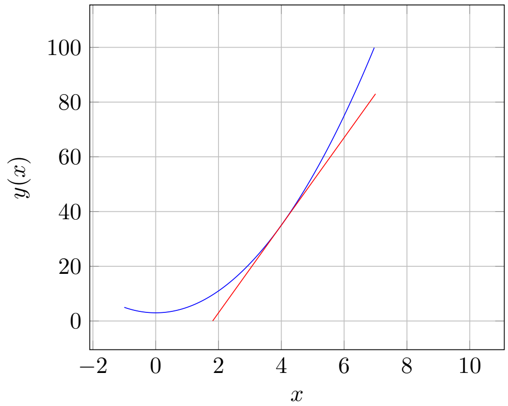


Figure 4: The blue curve shows the second order polynomial, the red line shows its tangent line at point $x_0 = 4$. The tangent line is obtained via the derivative of the polynomial $\left(\frac{\mathrm{d}y(x)}{\mathrm{d}x}\right)_{x_0=4}$.

Small signal approximation

In many applications diodes are *biased* to a certain DC voltage. This DC voltage and the corresponding DC current constitute the so called *DC operating point*, or, quiescent point (Q-point) of the device.

Upon the DC voltage a small AC signal can be superimposed. The AC is a (usually weak) perturbation of voltage and current around the DC operating point. Consequently, the voltage over diode can be expressed as

$$v_D(t) = V_{DQ} + v_d(t),$$

where V_{DQ} is DC voltage and $v_d(t)$ is the small AC signal ($V_{DQ} \gg |v_d|$, recall again the examples in the videos).

Note that also the current can be expressed as

$$i_D(t) = I_{DQ} + i_d(t),$$

where $i_D(t) = I_{DQ}$ when no AC signal is present ($v_d(t) = 0 \Rightarrow i_d(t) = 0$).

In such a case, diodes (and many other electronic devices) in electric circuits can be analyzed in two steps:

1. Appropriate DC operating point I_{DQ}, V_{DQ} is chosen and set by DC–analysis.
2. Using tangent line approximation we can obtain a linear model of a diode (device) near the Q–point. The tangent line is obtained via derivative of the function (aka local linearization, see Fig. 4 and page 14).

For the second step we find the tangent line and write

$$i_d(t) \approx \left(\frac{\mathrm{d}i_D}{\mathrm{d}v_D}\right)_Q v_d(t) = \frac{1}{r_d} v_d(t),$$

where r_d is so called *dynamic resistance* which relates the AC voltage and the AC current

$$\frac{1}{r_d} = \left(\frac{\mathrm{d}i_D}{\mathrm{d}v_D}\right)_Q . \tag{3}$$

NOTE: The *dynamic resistance* r_d is very central in AC analysis, since it characterizes diode's AC behaviour.

Using the Shockley model and taking derivative with respect to v_D we obtain

$$\frac{\mathrm{d}i_D(v_D)}{\mathrm{d}v_D} = \frac{\mathrm{d}\left(I_S\left[e^{\frac{v_D}{nV_T}} - 1\right]\right)}{\mathrm{d}v_D} = I_S \frac{1}{nV_T} e^{\frac{v_D}{nV_T}}. \tag{4}$$

In most cases current at the Q–point $i_D = I_{DQ} \approx I_S e^{\frac{v_{DQ}}{nV_T}}$ (recall eqn. 2) so that eqn. (4) can be written as

$$\left(\frac{\mathrm{d}i_D}{\mathrm{d}v_D}\right)_Q \approx \frac{I_{DQ}}{nV_T},$$

which results (recall eqn 3)

$$\boxed{r_d \approx \frac{nV_T}{I_{DQ}}.}$$

As a result we obtain an approximation of the Shockley equation about V_{DQ} as

$$\begin{aligned} i_D(t) = I_{DQ} + i_d(t) &\approx I_{DQ} + \left(\frac{\mathrm{d}i_D}{\mathrm{d}v_D}\right)_Q * v_d(t) \\ &= I_{DQ} + \frac{1}{r_d} v_d(t). \end{aligned}$$

Example

Let us consider the case shown shown in Fig. 5. Voltage source V_C is used to bias the diode, and the capacitor (DC block) act as open circuit for the DC signal. Recall that DC blocks should be chosen so that at the AC signal frequency they act as shortcuts.

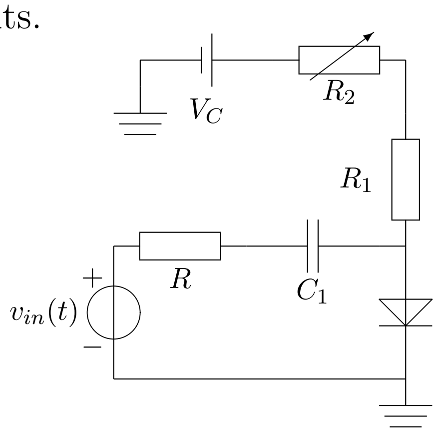
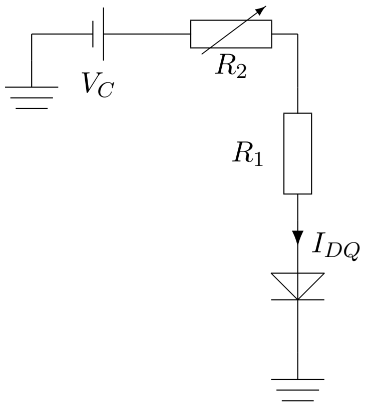


Figure 5: Simple circuit with a diode. Source voltage of V_C is assumed fixed and hence variable resistor R_2 is used to control the bias point.

1. DC analysis:
   (a) Draw the circuit for DC analysis, i.e. simplify the circuit to what it appears at DC.
   (b) Then find the current I_{DQ}. Estimate the voltage over the diode as 0.7 V. Let $V_C = 5$ V, $n = 1$, $V_T = 25$ mV, $R_1 = 230\ \Omega$, $R_2 = 1\ \text{k}\Omega$, $R = 10\ \Omega$, and $C_1 = 220\ \mu$F.
   (c) Find value for the dynamic resistance.
2. AC analysis (use the same component values as above):
   (a) Draw the circuit for AC analysis, i.e. simplify the circuit to what it appears at AC.
   (b) Look at the other set instructions to measure the dynamic resistance of a diode.

SOLUTION:

1. Circuit for DC analysis at the operating point is shown in Fig. 6. Note that the capacitors *block* undesirable DC currents from source or from load from affecting the DC operating point.



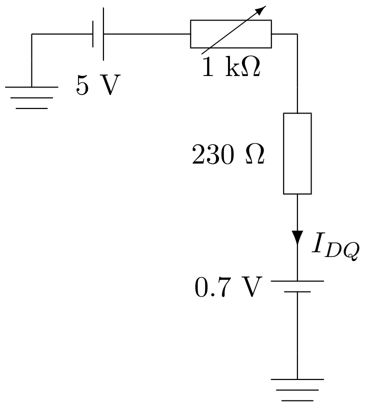
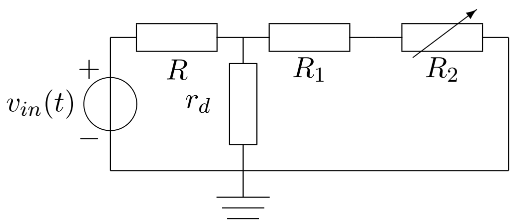


Figure 6: Left: Simplified circuit for DC analysis at the operating point.
Right: Numerical values and ideal diode model substituted to the simplified circuit.

From the circuit on right of Fig. 6 we obtain $I_{DQ} = \frac{5\ \mathrm{V} - 0.7\ \mathrm{V}}{1\ \mathrm{k\Omega} + 230\ \Omega} \approx 3.5$ mA. The dynamic resistance is then

$$r_d \approx \frac{nV_T}{i_{DQ}} = \frac{1 * 25\ \mathrm{mV}}{3.5\ \mathrm{mA}} \approx 7.152\ \Omega.$$

2. Circuit for the small signal AC analysis near the operating point is shown in Fig. 7. In particular, the diode is characterized by the dynamic resistance. Note also that V_C can be replaced by a short circuit in AC analysis because, by definition, there is absolutely no AC voltage across a DC source. The $v_{in}(t)$ represents AC voltage source with small amplitude.



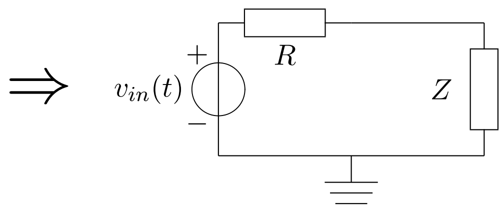


Figure 7: Left: Circuit for the small signal AC analysis near the operating point.
Right: From the impedance measurement directly we obtain Z, not r_d.

We shall shortly test this diode circuit with Multisim and myDAQ. For measuring the dynamic resistance of a diode we exploit the "good–old" impedance measurement where few additional quantities are needed:

> From the impedance measurement we obtain Z, not r_d and we need to do some extra calculations. Firstly, we need to know the sum of R_1 and R_2 (see Fig. 5).
>
> – Measure R_1 before installing it into the circuit.
>
> – Measure voltage V_1 over R_1 to find $I_1 = V_1/R_1$
>
> – Measure voltage V_3 over both R_1 and R_2.
>
> – Let R_3 the total resistance of R_1 and R_2 (see the dashed box in Fig. 8). Find R_3 from V_3 and I_1.
>
> From the impedance measurement we obtain Z (resistive) which is the total parallel resistance of r_d and R_3 (see Figs. 7 & 8). Finally,
>
> $$1/r_d = 1/|Z| - 1/R_3.$$

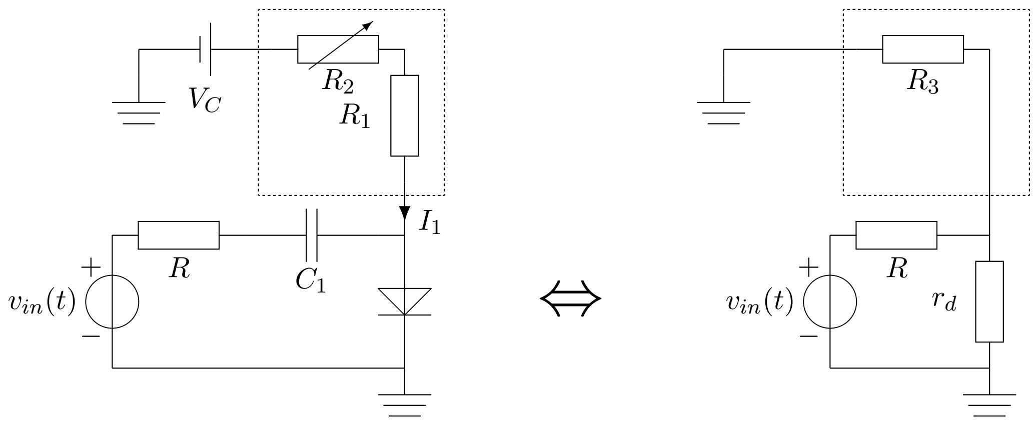


Figure 8: Interpreting the circuit for measuring the dynamic resistance of a diode.

Example for self study

Let us consider the case shown shown in Fig. 9. Voltage source V_C is used to bias the diode, and the two capacitors (DC blocks) act as open circuits for the DC signal. Recall that they should be chosen so that for the AC signal they would act as shortcuts.

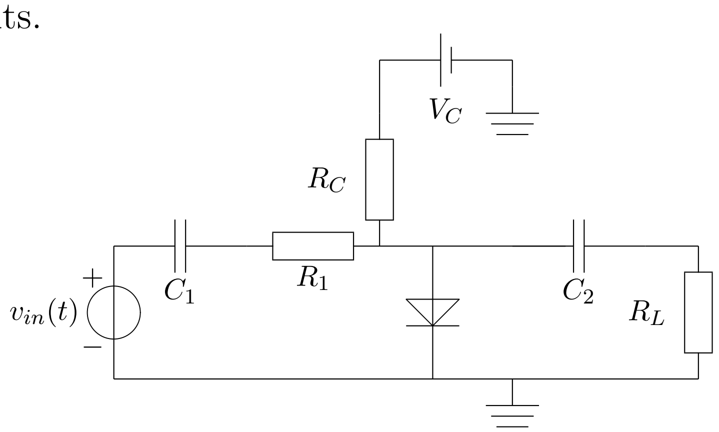


Figure 9: Simple circuit with a diode.

1. DC analysis:
   (a) Draw the circuit for DC analysis, i.e. simplify the circuit to what it appears at DC.
   (b) Then find the current I_{DQ}. Estimate the voltage over the diode as 0.7 V. Let $V_C = 5$ V, $n = 1$, $V_T = 25$ mV, $R_C = 2$ kΩ, $R_L = 2$ kΩ, and $R_1 = 100$ Ω.
   (c) Find value for the dynamic resistance.
2. AC analysis (use the same component values as above):
   (a) Draw the circuit for AC analysis, i.e. simplify the circuit to what it appears at AC.
   (b) Let voltage over R_L be the output of the circuit. Find the voltage gain

$$A_v = \frac{v_{out}}{v_{in}}.$$

SOLUTION:

1. Circuit for DC analysis at the operating point is shown in Fig. 10. Note that the capacitors prevent undesirable DC currents from source or load from affecting the choice of operating point.

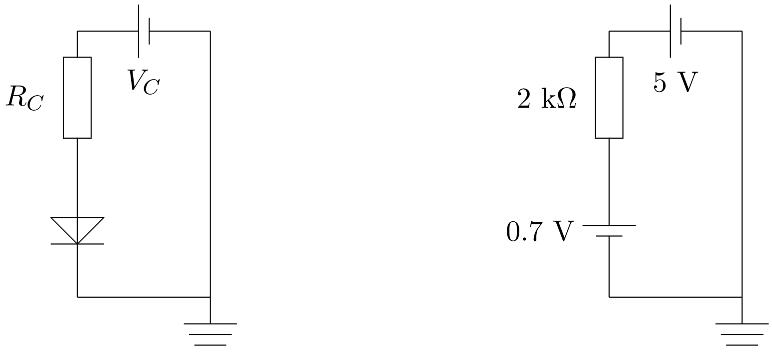


Figure 10: Left: Simplified circuit for DC analysis at the operating point.
Right: Numerical values and ideal diode model substituted to the simplified circuit.

From the circuit on right of Fig. 10 we obtain $I_{DQ} = \frac{5\text{ V}-0.7\text{ V}}{2\text{ k}\Omega} \approx 2.15$ mA. The dynamic resistance is

$$r_d \approx \frac{nV_T}{i_{DQ}} = \frac{1 * 25\text{ mV}}{2.15\text{ mA}} \approx 11.6\ \Omega.$$

2. Circuit for the small signal AC analysis near the operating point is shown in Fig. 11. In particular, the diode is characterized by the dynamic resistance. Note also that V_C can be replaced by a short circuit in AC analysis because, by definition, there is absolutely no AC voltage across a DC source. The $v_{in}(t)$ represents AC voltage source with small amplitude.

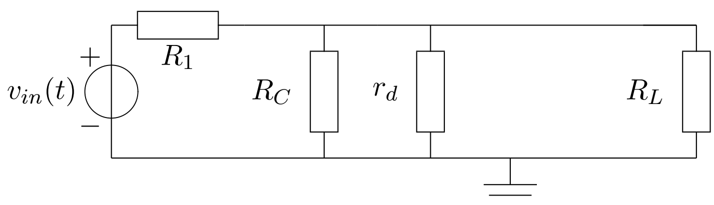


Figure 11: Circuit for the small signal AC analysis near the operating point.

Consequently, the output (voltage over R_L) is obtained using voltage division; let $R_p = R_C||r_d||R_L$ then

$$A_v = \frac{v_{out}}{v_{in}} = \frac{R_p}{R_1 + R_p}.$$

With the values provided $R_P \approx 11.5\ \Omega$ and $A_v \approx 0.10$.

Example*: How to find tangent line of a curve (an application of Taylor series)

The *tangent line approximation* (aka *local linearization*) is based on the Taylor series [1]. The Taylor series of a real-valued function $f(x)$ about a point x_0 reads

$$f(x_0 + \Delta x) \approx f(x_0) + \frac{f'(x_0)}{1!}\Delta x + \frac{f''(x_0)}{2!}(\Delta x)^2 + \cdots .$$

If only the terms $f(x_0)$ and $\frac{f'(x_0)}{1!}\Delta x$ are used (the series is truncated), the approximation is called as the local linearization which can be expressed as

$$f(x_0 + \Delta x) \approx f(x_0) + \left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{x_0} \Delta x, \tag{5}$$

where Δx is small change near x_0.

Fig. 12 shows a curve and its tangent line. The curve is a second order polynomial $y = f(x) = 2x^2 + 3$.

1. Explain how to find the tangent line of f at point $x_0 = 4$.

   Solution: $f(x_0) = f(4) = 35$ and $f'(x) = 2 * 2 * x + 0 = 4x$, so $\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{x_0} = 4x_0 = 4 * 4 = 16$.

2. Derive the equation for the tangent line, shown in Fig. 12.

   Lets gather the terms: $f(x_0 + \Delta x) \approx f(x_0) + \left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{x_0} \Delta x = 35 + 16\Delta x$.

An example of the values obtained near $x_0 = 4$ by the original function $f(x) = 2x^2 + 3$ and the approximation $f(x_0 + \Delta x) \approx 35 + 16\Delta x$ are given in Table 1.

x	3.5	3.9	4	4.1	5	7
Δx	-0.5	-0.1	0	0.1	1	3
$f(x) = 2x^2 + 3$	27.5	33.42	35	36.62	53	101
$f(x_0 + \Delta x) \approx 35 + 16\Delta x$	27	33.4	35	36.6	51	83

Table 1: An example of tangent line (use of the Taylor series) near point $x_0 = 4$.

References

[1] Edwards, Charles Henry and Penney, David E, *Calculus: Early Transcendentals: Matrix version*, Prentice Hall, 2002.

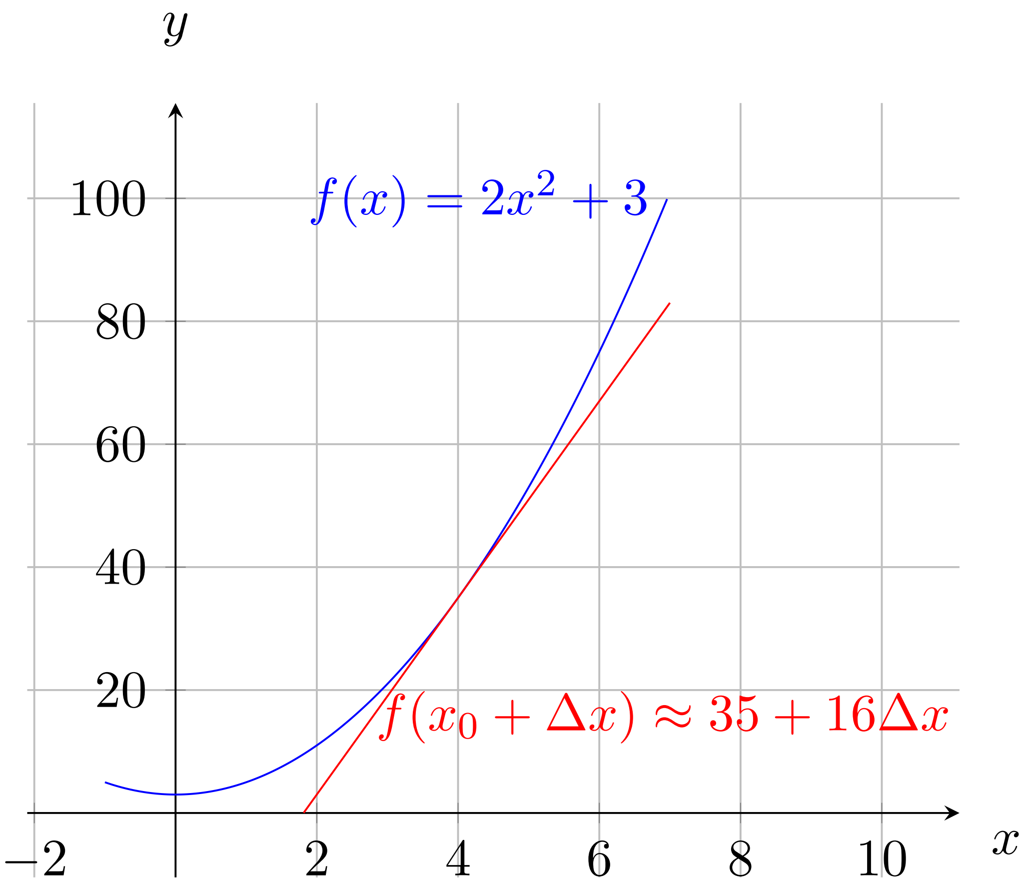


Figure 12: The blue curve shows the second order polynomial, the red line shows its tangent line at point $x_0 = 4$.