Introduction to Quality factor (Q–value) and inductor datasheet data

Learning outcomes: Student

+ knows the general definition to the Quality factor (Q–value),
+ knows how particular expressions are derived e.g. for series RLC –circuit,
+ knows how the Q–value is related to the bandwidth BW,
+ knows about typical datasheet data of inductors.

Quality factor (Q–value)

First the definition:

Quality factor is defined as a measure of the loss of a resonator

$$Q(\omega) = \omega \times \frac{\max\{W_m, W_e\}}{\text{power loss}}, \tag{1}$$

where ω is the angular frequency at which the stored energy and power loss are measured, W_m the magnetic energy, and W_e the electric energy.

Remarks,

- The energy stored in a resonator consists of the magnetic energy W_m and the electric energy W_e. At the resonance(s) it holds that $W_m = W_e$, elsewhere one should use the greater of the two as stated in the definition, i.e. $\max\{W_m, W_e\}$.
- Quality factor is a unitless quantity.
- The eqn. 1 holds for all resonators (incl. dielectric and transmission line resonators), not only to resonant components and circuits.
- Based on the eqn. 1 , lower loss implies a higher Q, i.e. better quality in terms of losses.
- Some further material is available on Wikipedia, see Quality factor (Q–value).

<u>Question:</u>
How could you apply eqn. 1 to a series RLC –circuit (see Fig. 1)?

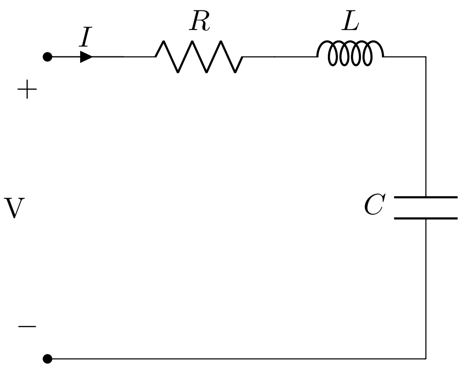

Figure 1: Series RLC circuit

SOLUTION:

Let us first determine the total *impedance* of the circuit which is

$$Z(\omega) = R + j\omega L + \frac{1}{j\omega C}.$$

Fig. 2 shows the impedance as function of frequency.

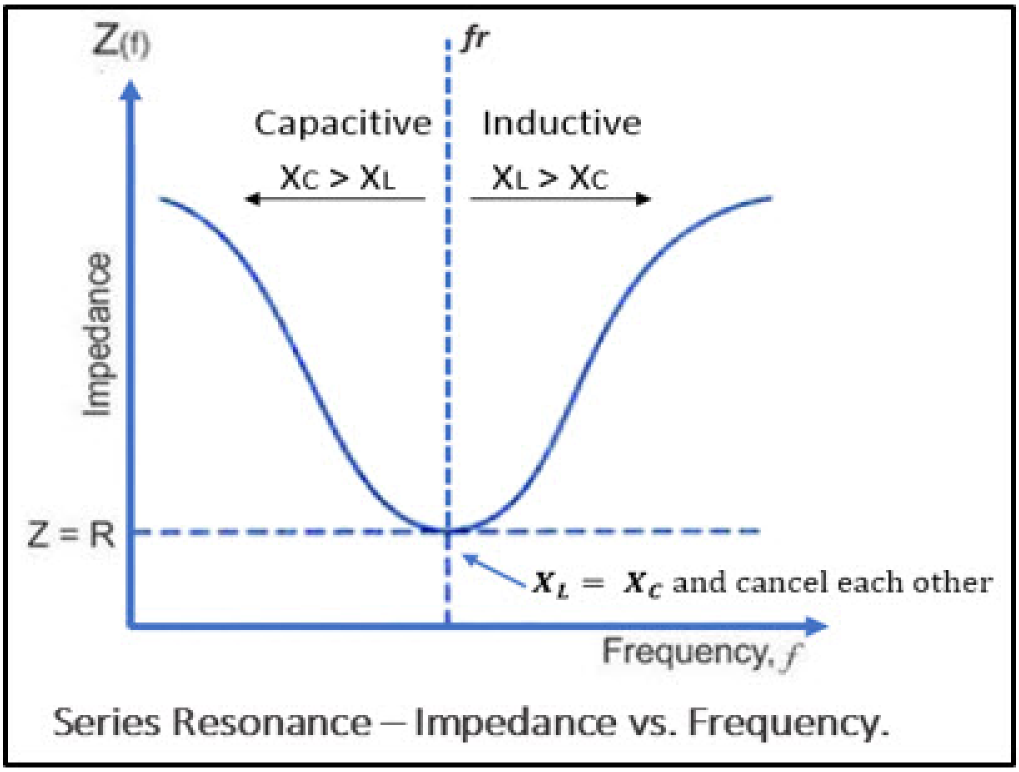

Figure 2: Impedance of the series RLC –circuit as the function of frequency. Picture source `https://www.etcourse.com/news-blog/bandwidth-resonant-circuits`.

The complex power delivered to circuit is defined as

$$S_{in} = \frac{1}{2}VI^* = \frac{1}{2}Z|I|^2 = \frac{1}{2}|I|^2\left(R + j\omega L + \frac{1}{j\omega C}\right). \tag{2}$$

Power loss at the resistor (power dissipated by the resistor) is

$$P_{loss} = \frac{1}{2}|I|^2R, \tag{3}$$

the time average magnetic energy stored in the inductor L is

$$W_m = \frac{1}{4}|I|^2L, \tag{4}$$

and the time average electric energy stored in the capacitor C is

$$W_e = \frac{1}{4}|V_C|^2C = \frac{1}{2}|I|^2\frac{1}{\omega^2 C}. \tag{5}$$

At the resonant frequency $\omega_0 = 2 * \pi f_0$ the total impedance Z is real

$$Z(\omega_0) = R$$

because it holds that

$$\omega_0 L + \frac{1}{j\omega_0 C} = 0,$$

from where

$$\omega_0 = \frac{1}{\sqrt{LC}}, \tag{6}$$

whose SI–unit is rad/s.

At the resonant frequency holds also that $W_e = W_m$ and consequently

$$\begin{aligned} Q_0 =&\omega_0\frac{W_m + W_e}{P_{loss}} = 2 * \omega_0\frac{W_m}{P_{loss}} \\ =&2 * \omega_0\frac{\frac{1}{4}L|I|^2}{\frac{1}{2}R|I|^2} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}. \end{aligned} \tag{7}$$

- If we are using the eqn. 1 not at the resonant frequency, we should use larger of W_e and W_m to obtain the *maximum energy stored.*

- Recall that real inductors and capacitors act as resonant circuits due to the parasitic properties and thus Quality factor is used as a parameter also for them.
  Real inductors and capacitors have "self-resonant" frequency and losses that are characterized by the "parasitic R".
- It can be shown that
  $$BW \sim \frac{f_0}{Q},$$
  where BW is the half–power bandwidth.
  This expression relates BW and Q and it follows that as R increases then Q decreases and consequently BW increases.

Inductor datasheet parameters

The following sections give examples of typical inductor parameters, see the related www-page.

Quality Factor (Q)

The Quality Factor is the ratio of the reactance of the inductor to its resistance as was derived in eq. 7. Inductors are not purely inductive, and their resistance causes energy loss which can limit its performance. Hence, the higher the Q factor, the lower the rate of energy loss.

Self-Resonant Frequency or SRF

The low distributed capacitance between inductor terminals causes inductors to have a self-resonant frequency (SRF). The SRF is the frequency at which an inductor stops working as an inductor, because its reactance becomes capacitive. For most applications designers should select an inductor with an SRF that is higher than the operating frequency by a safe margin. However, as RF choke we would like to use inductors whose SRF is close to the operating frequency.

Saturation Current

The saturation current is the current value at which the conductor can not take in any more magnetic flux. At this point, the inductance will drop at a specified value. Like the SRF, the saturation current must be significantly higher than the maximum current that will pass through the inductor.

DC Resistance

The DC resistance (DCR) is the resistance component of the inductor. A high DCR will cause high losses, so it is always better to select an inductor with low DCR value. It is important to consider the acceptable DCR of the inductor according to your application. The DC wire resistance is an important parameter in transformer and general inductor design because it contributes to the impedance of the component.

Equivalent series resistance (ESR)

If not otherwise specified, the ESR is always an AC resistance, which means it is measured at specified frequencies. Unfortunately, it is not often clearly indicated whether e.g. a datasheet is referring to DCR and ESR are. Fig. 3 shows an example of ESR vs. f.

Tolerance

Tolerance indicates how much the inductance can vary from the nominal value. This information is usually found in the component's datasheet. Tolerances in general are not desired, but are inversely proportional with cost. Therefore, this parameter should be ultimately selected based on your application. Recall that the tolerance is due to the manufacturing, not because of the parasitics.

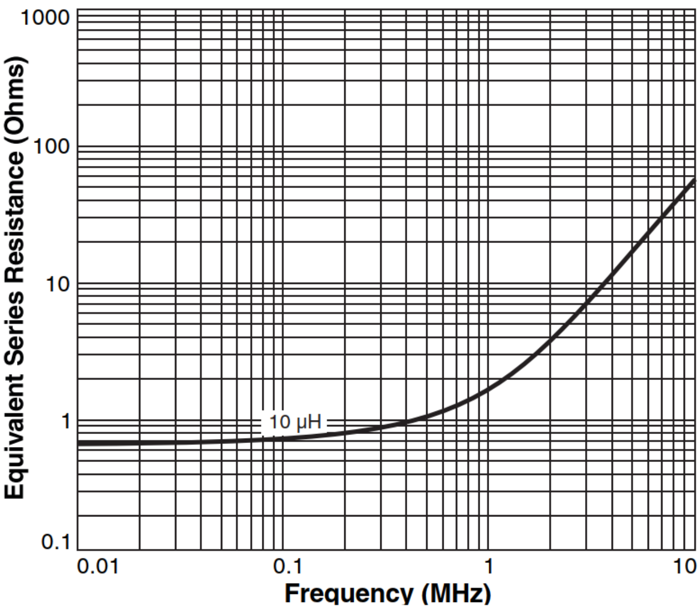

Figure 3: This picture shows how the ESR i.e. AC resistance increases as the frequency increases.