4.) $tr(F^TF - I) = tr\left(\begin{pmatrix} f_{11} & f_{21} \\ f_{12} & f_{22} \end{pmatrix} \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right)$

$= tr\left(\begin{pmatrix} f_{11}f_{11} + f_{21}f_{21} & f_{11}f_{22} + f_{21}f_{22} \\ f_{12}f_{11} + f_{22}f_{21} & f_{12}f_{12} + f_{22}f_{22} \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right)$ | Matrix Mult

$= tr\left(\begin{pmatrix} f_{11}f_{11} + f_{21}f_{21} - 1 & f_{11}f_{22} + f_{21}f_{22} \\ f_{12}f_{11} + f_{22}f_{21} & f_{12}f_{12} + f_{22}f_{22} - 1 \end{pmatrix} \right)$ | Matrix Sub

$= f_{11}f_{11} + f_{21}f_{21} - 1 + f_{12}f_{12} + f_{22}f_{22} - 1$ | trace def.

$= f_{11}f_{11} + f_{21}f_{21} + f_{12}f_{12} + f_{22}f_{22} - 2$

$= \left(f_{11}^2 + f_{21}^2 + f_{12}^2 + f_{22}^2 \right) - 2$

$= \left(\sum_{i=1}^{2} \sum_{j=1}^{2} f_{ij}^2 \right) - 2 = \sqrt{\sum_{i=1}^{2} \sum_{j=1}^{2} f_{ij}^2}^{\,2} - 2$

$= \|F\|_F^2 - 2$ | Frobenius norm def.

□