IBNR

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Prediction is hard. Especially of the future.
~ Old Danish Proverb

It is in the public interest for insurance companies to have sufficient assets to make all claim payments owed in the future on policies purchased in the past. The estimated total future value owed on behalf of policyholders as of any given financial statement date is called the company's "Liability for Claims" (technically that's not the NAIC's word for it, but close enough). Estimating this liability, usually the largest on a company's balance sheet, falls under the purview of two company departments: Claims and Actuarial.

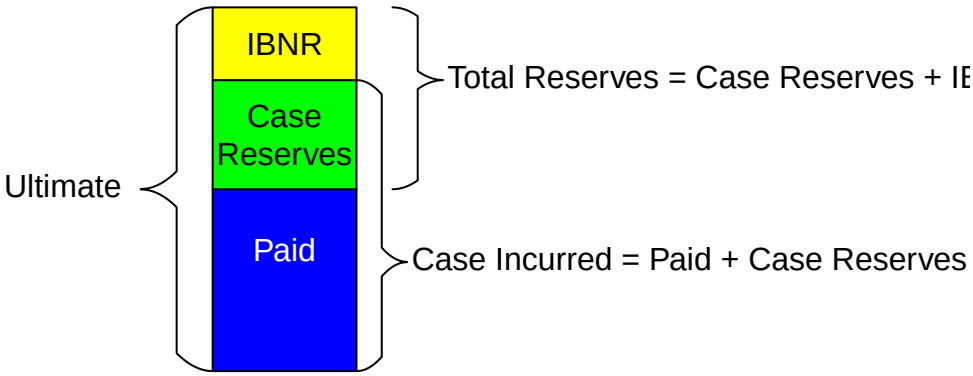
When a policyholder files a claim, it is the responsibility of the Claims Department to estimate the total amount likely to be paid out. These claim-level estimates are called "case reserves." Case reserves reflect the value of a claim from the company's standpoint. But the total value of case reserves usually falls short of the company's true Liability for Claims for two reasons:

```
Updated claims: as a claim stays open and new facts are discovered, values tend to rise

B. Delayed claims: new, as-yet-unknown claims are often filed after a financial statement date
```

The estimate of the total shortfall (A+B) is called the "Incurred But Not Reported" (IBNR) reserve. It is the responsibility of the Actuarial Department to estimate IBNR.

Most actuarial techniques for estimating IBNR actually estimate the ultimate value of claims first, then subtract paid-to-date to get total reserves, and finally subtract case reserves to get IBNR.



Actuaries have many techniques for estimating the ultimate value of claims, and thus IBNR. One of the simplest techniques is called the Loss Development Method.

The Loss Development Method

In a nutshell, the Loss Development Method multiplies the total current value of a cohort of claims by a factor.

called a "development factor," the product of which is called the "ultimate" value of total Liability for Claims

the Chain Ladder method looks at ratios of total claim values as of The purpose of this paper is to investigate the mathematics behind the Chain Ladder Method.

We start this investigation with summarized claim data as displayed in triangular format in Schedule P of the U.S. *Annual Statement*.

Annual Statement: Schedule P

For the benefit of US regulators, as of every December 31st every insurance company must file a set of reports called the *Annual Statement*. This report, a.k.a the "Yellow Book," is a thick compendium of statistics describing all aspects of an insurance company's business: from premium to claims to investments and well beyond deep into the bowels of an insurance company. One of the key aspects monitored in the yellow book is the company's Liability for Claims. Not only is the size of that liability important, but changes in that liability are important too because changes can directly impact the company's bottom line. We will see how that works with two tables in **Schedule P** of the yellow book, **Part2** and **Part3**.

Changes in that liability are closely scrutinized because they directly flow through to the bottom line.

year over year directly impact the company's net income that year. over time is illuminated from various perspectives. The data that we will poke and prod in this paper is called GenIns.

GenIns

The ChainLadder package describes GenIns as a "run off triangle of accumulated general insurance claims data." GenIns is a real, not simulated, cumulative paid loss was created by the Australian actuary Greg Taylor.

egins with its representation in Schedule P - Part 3, meticulously named "Cumulative Paid Net Loss and Defense and Cost Containment Expenses Reported at Year End ($000 omitted)":

```
#setwd("C:/Users/Dan/Desktop/GenIns")
library(knitr)
library(kableExtra)
library(ChainLadder)
```

```
##
## Welcome to ChainLadder version 0.2.5
##
## Type vignette('ChainLadder', package='ChainLadder') to access
## the overall package documentation.
##
## See demo(package='ChainLadder') for a list of demos.
##
## More information is available on the ChainLadder project web-site:
## https://github.com/mages/ChainLadder
##
## To suppress this message use:
## suppressPackageStartupMessages(library(ChainLadder))
```

```
library(mondate)
```

```
##
## Attaching package: 'mondate'
```

```
## The following object is masked from 'package:base':
##
##     as.difftime
```

```
df <- as.data.frame(GenIns, na.rm = TRUE)
df$ay <- as.numeric(df$origin) + 2000 # give labels to the origin's
df$Accident_Year <- as.numeric(df$origin) + 2007 # give labels to the origin's
df$Evaluation_YearEnd <- df$ay + df$dev - 1
df[["Evaluation_Date"]] <- mondate.ymd(df$Evaluation_YearEnd)
tri <- as.triangle(df, origin = "Accident_Year",
                   dev = "Evaluation_YearEnd", value = "value")
tri
```

```
##              Evaluation_YearEnd
## Accident_Year   2001    2002    2003    2004    2005    2006    2007
##          2008 357848 1124788 1735330 2218270 2745596 3319994 3466336
##          2009     NA  352118 1236139 2170033 3353322 3799067 4120063
##          2010     NA      NA  290507 1292306 2218525 3235179 3985995
##          2011     NA      NA      NA  310608 1418858 2195047 3757447
##          2012     NA      NA      NA      NA  443160 1136350 2128333
##          2013     NA      NA      NA      NA      NA  396132 1333217
##          2014     NA      NA      NA      NA      NA      NA  440832
##          2015     NA      NA      NA      NA      NA      NA      NA
##          2016     NA      NA      NA      NA      NA      NA      NA
##          2017     NA      NA      NA      NA      NA      NA      NA
##              Evaluation_YearEnd
## Accident_Year    2008    2009    2010
##          2008 3606286 3833515 3901463
##          2009 4647867 4914039 5339085
##          2010 4132918 4628910 4909315
##          2011 4029929 4381982 4588268
##          2012 2897821 3402672 3873311
##          2013 2180715 2985752 3691712
##          2014 1288463 2419861 3483130
##          2015  359480 1421128 2864498
##          2016      NA  376686 1363294
##          2017      NA      NA  344014
```

```
round(tri)
```

```
##              Evaluation_YearEnd
## Accident_Year   2001    2002    2003    2004    2005    2006    2007
##          2008 357848 1124788 1735330 2218270 2745596 3319994 3466336
##          2009     NA  352118 1236139 2170033 3353322 3799067 4120063
##          2010     NA      NA  290507 1292306 2218525 3235179 3985995
##          2011     NA      NA      NA  310608 1418858 2195047 3757447
##          2012     NA      NA      NA      NA  443160 1136350 2128333
##          2013     NA      NA      NA      NA      NA  396132 1333217
##          2014     NA      NA      NA      NA      NA      NA  440832
##          2015     NA      NA      NA      NA      NA      NA      NA
##          2016     NA      NA      NA      NA      NA      NA      NA
##          2017     NA      NA      NA      NA      NA      NA      NA
##              Evaluation_YearEnd
## Accident_Year    2008    2009    2010
##          2008 3606286 3833515 3901463
##          2009 4647867 4914039 5339085
##          2010 4132918 4628910 4909315
##          2011 4029929 4381982 4588268
##          2012 2897821 3402672 3873311
##          2013 2180715 2985752 3691712
##          2014 1288463 2419861 3483130
##          2015  359480 1421128 2864498
##          2016      NA  376686 1363294
##          2017      NA      NA  344014
```

```r
noquote(format(round(tri), big.mark = ","))
```

```
##              Evaluation_YearEnd
## Accident_Year 2001      2002      2003      2004      2005      2006
##          2008   357,848 1,124,788 1,735,330 2,218,270 2,745,596 3,319,994
##          2009        NA   352,118 1,236,139 2,170,033 3,353,322 3,799,067
##          2010        NA        NA   290,507 1,292,306 2,218,525 3,235,179
##          2011        NA        NA        NA   310,608 1,418,858 2,195,047
##          2012        NA        NA        NA        NA   443,160 1,136,350
##          2013        NA        NA        NA        NA        NA   396,132
##          2014        NA        NA        NA        NA        NA        NA
##          2015        NA        NA        NA        NA        NA        NA
##          2016        NA        NA        NA        NA        NA        NA
##          2017        NA        NA        NA        NA        NA        NA
##              Evaluation_YearEnd
## Accident_Year 2007      2008      2009      2010
##          2008 3,466,336 3,606,286 3,833,515 3,901,463
##          2009 4,120,063 4,647,867 4,914,039 5,339,085
##          2010 3,985,995 4,132,918 4,628,910 4,909,315
##          2011 3,757,447 4,029,929 4,381,982 4,588,268
##          2012 2,128,333 2,897,821 3,402,672 3,873,311
##          2013 1,333,217 2,180,715 2,985,752 3,691,712
##          2014   440,832 1,288,463 2,419,861 3,483,130
##          2015        NA   359,480 1,421,128 2,864,498
##          2016        NA        NA   376,686 1,363,294
##          2017        NA        NA        NA   344,014
```

```r
tri2 <- as.triangle(df, origin = "Accident_Year",
                    dev = "Evaluation_Date", value = "value")
tri2
```

```
##              Evaluation_Date
## Accident_Year 12/31/2001 12/31/2002 12/31/2003 12/31/2004 12/31/2005
##          2008     357848    1124788    1735330    2218270    2745596
##          2009         NA     352118    1236139    2170033    3353322
##          2010         NA         NA     290507    1292306    2218525
##          2011         NA         NA         NA     310608    1418858
##          2012         NA         NA         NA         NA     443160
##          2013         NA         NA         NA         NA         NA
##          2014         NA         NA         NA         NA         NA
##          2015         NA         NA         NA         NA         NA
##          2016         NA         NA         NA         NA         NA
##          2017         NA         NA         NA         NA         NA
##              Evaluation_Date
## Accident_Year 12/31/2006 12/31/2007 12/31/2008 12/31/2009 12/31/2010
##          2008    3319994    3466336    3606286    3833515    3901463
##          2009    3799067    4120063    4647867    4914039    5339085
##          2010    3235179    3985995    4132918    4628910    4909315
##          2011    2195047    3757447    4029929    4381982    4588268
##          2012    1136350    2128333    2897821    3402672    3873311
##          2013     396132    1333217    2180715    2985752    3691712
##          2014         NA     440832    1288463    2419861    3483130
##          2015         NA         NA     359480    1421128    2864498
```

```
##           2016          NA          NA          NA      376686     1363294
##           2017          NA          NA          NA          NA      344014
```

```
round(tri2)
```

```
##              Evaluation_Date
## Accident_Year 12/31/2001 12/31/2002 12/31/2003 12/31/2004 12/31/2005
##           2008     357848    1124788    1735330    2218270    2745596
##           2009          NA     352118    1236139    2170033    3353322
##           2010          NA          NA     290507    1292306    2218525
##           2011          NA          NA          NA     310608    1418858
##           2012          NA          NA          NA          NA     443160
##           2013          NA          NA          NA          NA         NA
##           2014          NA          NA          NA          NA         NA
##           2015          NA          NA          NA          NA         NA
##           2016          NA          NA          NA          NA         NA
##           2017          NA          NA          NA          NA         NA
##              Evaluation_Date
## Accident_Year 12/31/2006 12/31/2007 12/31/2008 12/31/2009 12/31/2010
##           2008    3319994    3466336    3606286    3833515    3901463
##           2009    3799067    4120063    4647867    4914039    5339085
##           2010    3235179    3985995    4132918    4628910    4909315
##           2011    2195047    3757447    4029929    4381982    4588268
##           2012    1136350    2128333    2897821    3402672    3873311
##           2013     396132    1333217    2180715    2985752    3691712
##           2014          NA     440832    1288463    2419861    3483130
##           2015          NA          NA     359480    1421128    2864498
##           2016          NA          NA          NA     376686    1363294
##           2017          NA          NA          NA          NA     344014
```

```
noquote(format(round(tri2/1000), big.mark = ","))
```

```
##              Evaluation_Date
## Accident_Year 12/31/2001 12/31/2002 12/31/2003 12/31/2004 12/31/2005
##           2008   358      1,125      1,735      2,218      2,746
##           2009    NA        352      1,236      2,170      3,353
##           2010    NA         NA        291      1,292      2,219
##           2011    NA         NA         NA        311      1,419
##           2012    NA         NA         NA         NA        443
##           2013    NA         NA         NA         NA         NA
##           2014    NA         NA         NA         NA         NA
##           2015    NA         NA         NA         NA         NA
##           2016    NA         NA         NA         NA         NA
##           2017    NA         NA         NA         NA         NA
##              Evaluation_Date
## Accident_Year 12/31/2006 12/31/2007 12/31/2008 12/31/2009 12/31/2010
##           2008 3,320      3,466      3,606      3,834      3,901
##           2009 3,799      4,120      4,648      4,914      5,339
##           2010 3,235      3,986      4,133      4,629      4,909
##           2011 2,195      3,757      4,030      4,382      4,588
##           2012 1,136      2,128      2,898      3,403      3,873
##           2013   396      1,333      2,181      2,986      3,692
##           2014    NA        441      1,288      2,420      3,483
```

```
##          2015    NA         NA        359      1,421      2,864
##          2016    NA         NA         NA        377      1,363
##          2017    NA         NA         NA         NA        344
```

```
m <- round(tri2/1000)
names(dimnames(m))[2L] <- "Evaluation_Date ($000 omitted)"
noquote(format(m, big.mark = ","))
```

```
##              Evaluation_Date ($000 omitted)
## Accident_Year 12/31/2001 12/31/2002 12/31/2003 12/31/2004 12/31/2005
##          2008   358      1,125      1,735      2,218      2,746
##          2009    NA        352      1,236      2,170      3,353
##          2010    NA         NA        291      1,292      2,219
##          2011    NA         NA         NA        311      1,419
##          2012    NA         NA         NA         NA        443
##          2013    NA         NA         NA         NA         NA
##          2014    NA         NA         NA         NA         NA
##          2015    NA         NA         NA         NA         NA
##          2016    NA         NA         NA         NA         NA
##          2017    NA         NA         NA         NA         NA
##              Evaluation_Date ($000 omitted)
## Accident_Year 12/31/2006 12/31/2007 12/31/2008 12/31/2009 12/31/2010
##          2008 3,320      3,466      3,606      3,834      3,901
##          2009 3,799      4,120      4,648      4,914      5,339
##          2010 3,235      3,986      4,133      4,629      4,909
##          2011 2,195      3,757      4,030      4,382      4,588
##          2012 1,136      2,128      2,898      3,403      3,873
##          2013   396      1,333      2,181      2,986      3,692
##          2014    NA        441      1,288      2,420      3,483
##          2015    NA         NA        359      1,421      2,864
##          2016    NA         NA         NA        377      1,363
##          2017    NA         NA         NA         NA        344
```

```
mf <- format(m, big.mark = ",")
nams <- names(dimnames(m))
dimnms <- dimnames(m)
mfp <- paste("   ", mf)
dim(mfp) <- dim(mf)
dimnames(mfp) <- dimnms
noquote(mfp)
```

```
##              Evaluation_Date ($000 omitted)
## Accident_Year 12/31/2001 12/31/2002 12/31/2003 12/31/2004 12/31/2005
##          2008       358      1,125      1,735      2,218      2,746
##          2009        NA        352      1,236      2,170      3,353
##          2010        NA         NA        291      1,292      2,219
##          2011        NA         NA         NA        311      1,419
##          2012        NA         NA         NA         NA        443
##          2013        NA         NA         NA         NA         NA
##          2014        NA         NA         NA         NA         NA
##          2015        NA         NA         NA         NA         NA
##          2016        NA         NA         NA         NA         NA
##          2017        NA         NA         NA         NA         NA
```

```
##              Evaluation_Date ($000 omitted)
## Accident_Year 12/31/2006 12/31/2007 12/31/2008 12/31/2009 12/31/2010
##          2008     3,320      3,466      3,606      3,834      3,901
##          2009     3,799      4,120      4,648      4,914      5,339
##          2010     3,235      3,986      4,133      4,629      4,909
##          2011     2,195      3,757      4,030      4,382      4,588
##          2012     1,136      2,128      2,898      3,403      3,873
##          2013       396      1,333      2,181      2,986      3,692
##          2014        NA        441      1,288      2,420      3,483
##          2015        NA         NA        359      1,421      2,864
##          2016        NA         NA         NA        377      1,363
##          2017        NA         NA         NA         NA        344
```

```
mfp[is.na(m)] <- "     XXX"
noquote(mfp)
```

```
##              Evaluation_Date ($000 omitted)
## Accident_Year 12/31/2001 12/31/2002 12/31/2003 12/31/2004 12/31/2005
##          2008       358      1,125      1,735      2,218      2,746
##          2009      XXX         352      1,236      2,170      3,353
##          2010      XXX        XXX         291      1,292      2,219
##          2011      XXX        XXX        XXX         311      1,419
##          2012      XXX        XXX        XXX        XXX         443
##          2013      XXX        XXX        XXX        XXX        XXX
##          2014      XXX        XXX        XXX        XXX        XXX
##          2015      XXX        XXX        XXX        XXX        XXX
##          2016      XXX        XXX        XXX        XXX        XXX
##          2017      XXX        XXX        XXX        XXX        XXX
##              Evaluation_Date ($000 omitted)
## Accident_Year 12/31/2006 12/31/2007 12/31/2008 12/31/2009 12/31/2010
##          2008     3,320      3,466      3,606      3,834      3,901
##          2009     3,799      4,120      4,648      4,914      5,339
##          2010     3,235      3,986      4,133      4,629      4,909
##          2011     2,195      3,757      4,030      4,382      4,588
##          2012     1,136      2,128      2,898      3,403      3,873
##          2013       396      1,333      2,181      2,986      3,692
##          2014      XXX         441      1,288      2,420      3,483
##          2015      XXX        XXX         359      1,421      2,864
##          2016      XXX        XXX        XXX         377      1,363
##          2017      XXX        XXX        XXX        XXX         344
```

```
mfp2 <- rbind(Prior = c("     000", rep("", 9)), mfp)
print(noquote(mfp2))
```

```
##       12/31/2001 12/31/2002 12/31/2003 12/31/2004 12/31/2005 12/31/2006
## Prior      000
## 2008        358      1,125      1,735      2,218      2,746      3,320
## 2009       XXX         352      1,236      2,170      3,353      3,799
## 2010       XXX        XXX         291      1,292      2,219      3,235
## 2011       XXX        XXX        XXX         311      1,419      2,195
## 2012       XXX        XXX        XXX        XXX         443      1,136
## 2013       XXX        XXX        XXX        XXX        XXX         396
## 2014       XXX        XXX        XXX        XXX        XXX        XXX
```

```
## 2015       XXX        XXX        XXX        XXX        XXX        XXX
## 2016       XXX        XXX        XXX        XXX        XXX        XXX
## 2017       XXX        XXX        XXX        XXX        XXX        XXX
##       12/31/2007 12/31/2008 12/31/2009 12/31/2010
## Prior
## 2008      3,466      3,606      3,834      3,901
## 2009      4,120      4,648      4,914      5,339
## 2010      3,986      4,133      4,629      4,909
## 2011      3,757      4,030      4,382      4,588
## 2012      2,128      2,898      3,403      3,873
## 2013      1,333      2,181      2,986      3,692
## 2014        441      1,288      2,420      3,483
## 2015        XXX        359      1,421      2,864
## 2016        XXX        XXX        377      1,363
## 2017        XXX        XXX        XXX        344
```

And the kable version:

```
kable(mfp2, align = 'r') %>%
  kable_styling(font_size = 6)
```

	12/31/2001	12/31/2002	12/31/2003	12/31/2004	12/31/2005	12/31/2006	12/31/2007	12/31/2008	12/31/2009	12/31/2010
Prior	000									
2008	358	1,125	1,735	2,218	2,746	3,320	3,466	3,606	3,834	3,901
2009	XXX	352	1,236	2,170	3,353	3,799	4,120	4,648	4,914	5,339
2010	XXX	XXX	291	1,292	2,219	3,235	3,986	4,133	4,629	4,909
2011	XXX	XXX	XXX	311	1,419	2,195	3,757	4,030	4,382	4,588
2012	XXX	XXX	XXX	XXX	443	1,136	2,128	2,898	3,403	3,873
2013	XXX	XXX	XXX	XXX	XXX	396	1,333	2,181	2,986	3,692
2014	XXX	XXX	XXX	XXX	XXX	XXX	441	1,288	2,420	3,483
2015	XXX	XXX	XXX	XXX	XXX	XXX	XXX	359	1,421	2,864
2016	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX	377	1,363
2017	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX	344

```
class(tri2) <- "matrix"
kable(tri2)
```

	12/31/2001	12/31/2002	12/31/2003	12/31/2004	12/31/2005	12/31/2006	12/31/2007	12/31/2008	1
2008	357848	1124788	1735330	2218270	2745596	3319994	3466336	3606286	
2009	NA	352118	1236139	2170033	3353322	3799067	4120063	4647867	
2010	NA	NA	290507	1292306	2218525	3235179	3985995	4132918	
2011	NA	NA	NA	310608	1418858	2195047	3757447	4029929	
2012	NA	NA	NA	NA	443160	1136350	2128333	2897821	
2013	NA	NA	NA	NA	NA	396132	1333217	2180715	
2014	NA	NA	NA	NA	NA	NA	440832	1288463	
2015	NA	NA	NA	NA	NA	NA	NA	359480	
2016	NA	NA	NA	NA	NA	NA	NA	NA	
2017	NA	NA	NA	NA	NA	NA	NA	NA	

Then plain ol' GenIns by kable:

```
G <- GenIns
class(G) <- "matrix"

kable(G, row.names = TRUE,
      format.args = list(big.mark = ','),
      caption = "GenIns in triangle format")
```

Table 1: GenIns in triangle format

	1	2	3	4	5	6	7	8	9	10
1	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
2	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	NA
3	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315	NA	NA
4	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268	NA	NA	NA
5	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311	NA	NA	NA	NA
6	396,132	1,333,217	2,180,715	2,985,752	3,691,712	NA	NA	NA	NA	NA
7	440,832	1,288,463	2,419,861	3,483,130	NA	NA	NA	NA	NA	NA
8	359,480	1,421,128	2,864,498	NA	NA	NA	NA	NA	NA	NA
9	376,686	1,363,294	NA	NA	NA	NA	NA	NA	NA	NA
10	344,014	NA	NA	NA	NA	NA	NA	NA	NA	NA