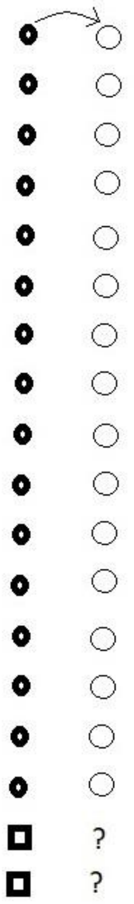
Mack/Murphy Unchained

CLRS 2018
Anaheim CA
Daniel Murphy
Trinostics

Chain-Ladder: First link

- We observe some objects that have changed over time
- We observe two new objects
  - What is our estimate of their changed values?



$$y = bx$$

method

$$b = \frac{\sum x}{\sum y}$$

traditional average link ratio

Chain-Ladder: First link

- The traditional weighted average link ratio solves this model via MSE
- Because it also solves the equivalent model

$$y = bx + \sqrt{x}e$$

model

$$b = \frac{\sum x}{\sum y}$$

solves model

equivalent model

$$\frac{y}{\sqrt{x}} = b\sqrt{x} + \mathrm{e}$$

$$or$$

$$Y = bX + e$$

$$b = (X`X)^{-1}X`Y$$

$$minimizes$$

$$\sum (Y - bX)^2$$

Chain-Ladder: First link

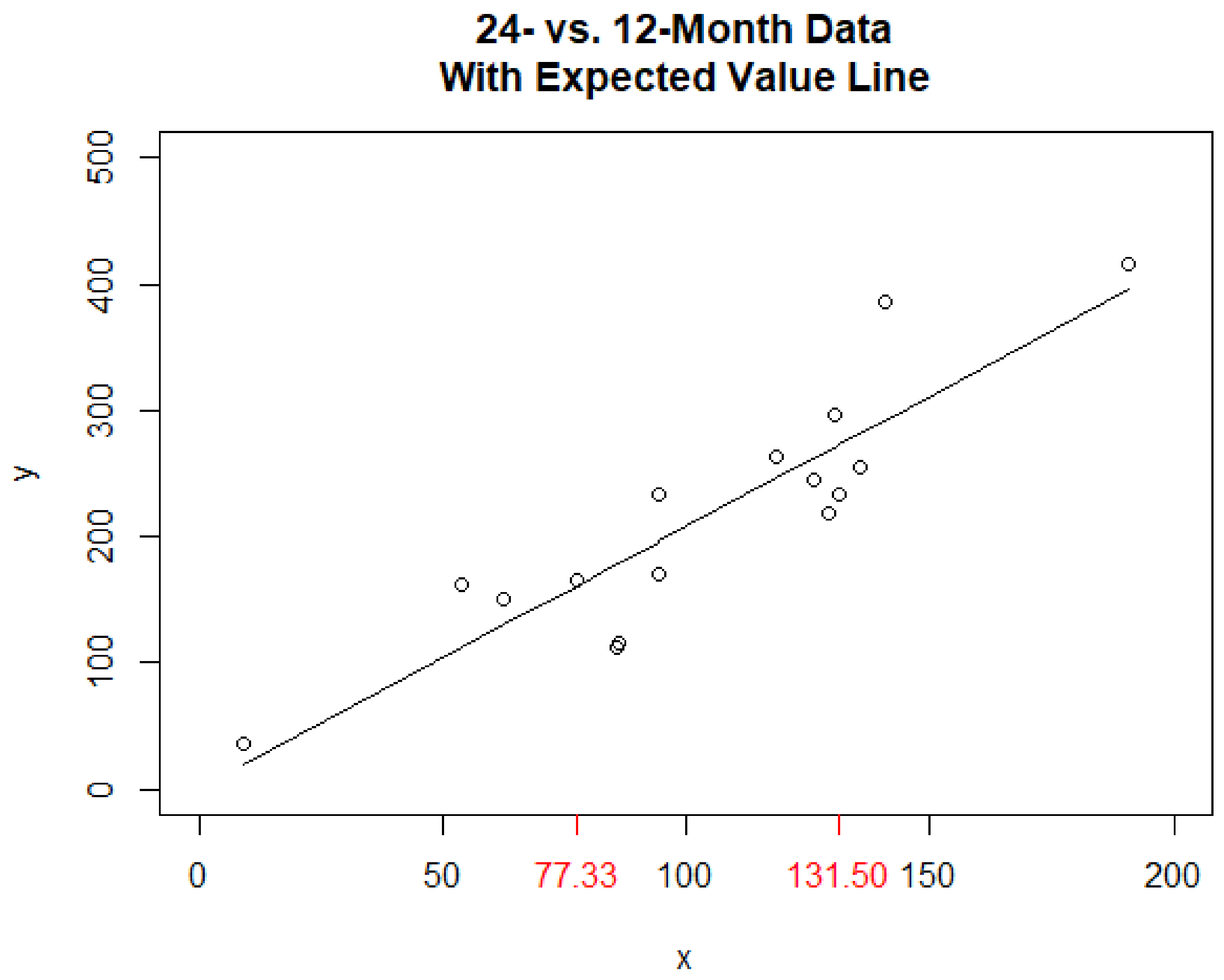
- Graph $y = bx + \sqrt{x}e$ for this triangle

	x	y
1	129.28	218.24
2	135.47	255.51
3	94.53	232.66
4	77.33	165.16
5	130.29	296.19
6	9.10	35.77
7	131.50	233.45
8	86.19	114.70
9	85.79	112.39
10	54.03	161.14
11	94.19	169.68
12	190.87	416.01
13	118.53	263.72
14	126.01	244.73
15	62.47	150.62
16	140.85	385.98
17	77.33	
18	131.50	

Development experience

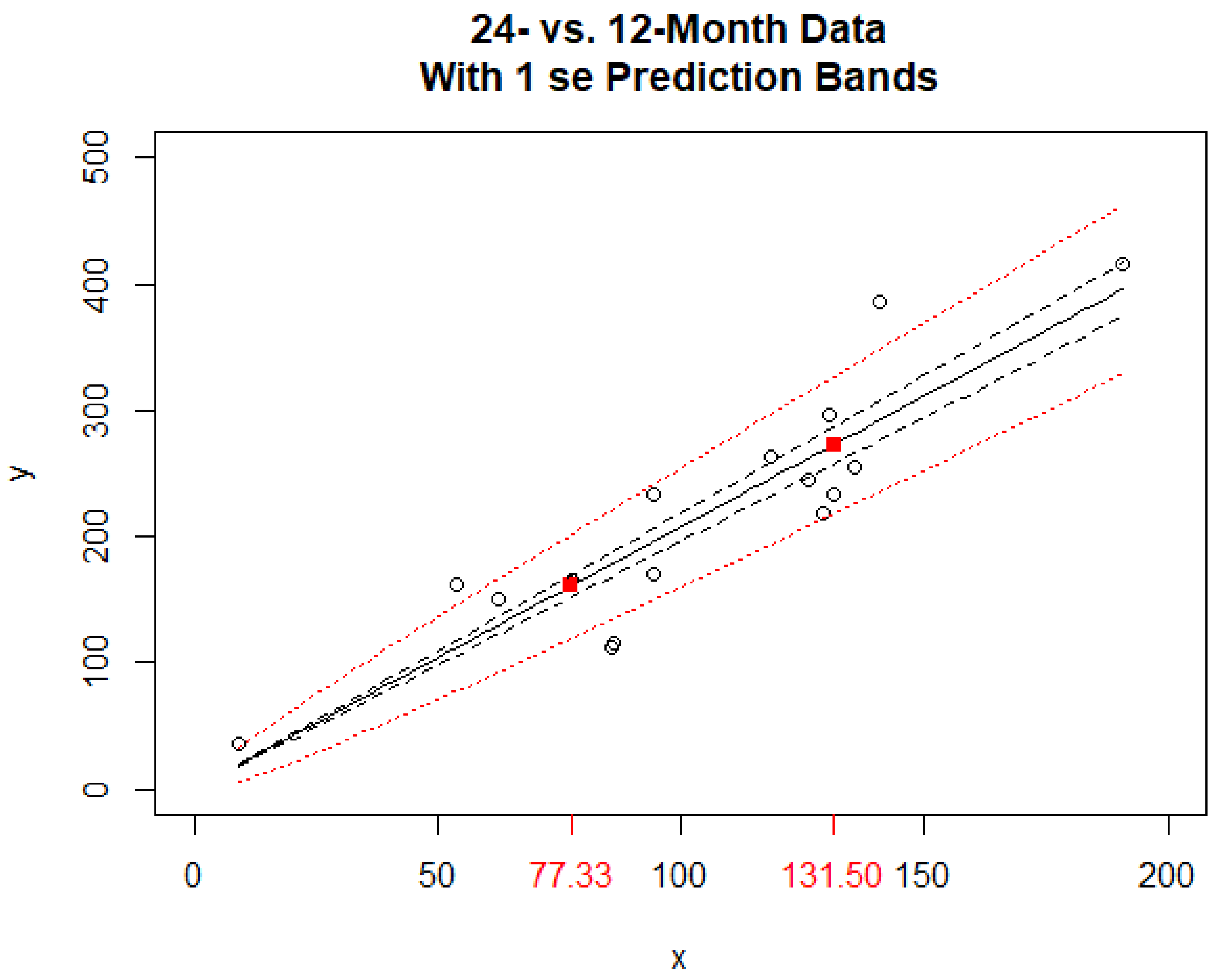
- slope of the line = 2.037 = $\dfrac{\sum x}{\sum y}$

- new points awaiting blastoff



Predicted experience with “confidence levels”

- --- parameter risk
  ~ confidence interval
  for estimated mean
- ······ total risk
  ~ prediction interval
  for estimated outcome



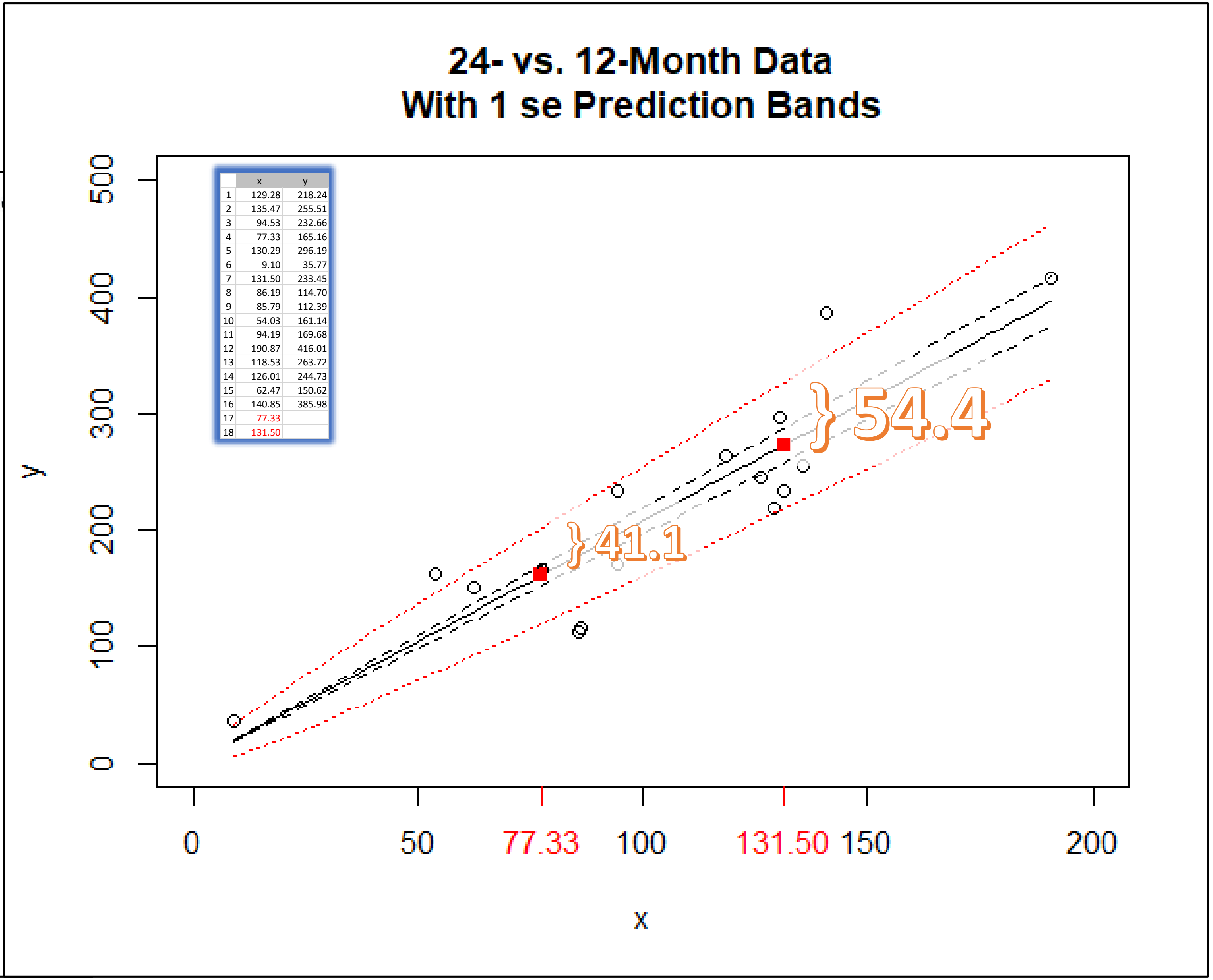
Predicted experience with "confidence levels"

With R

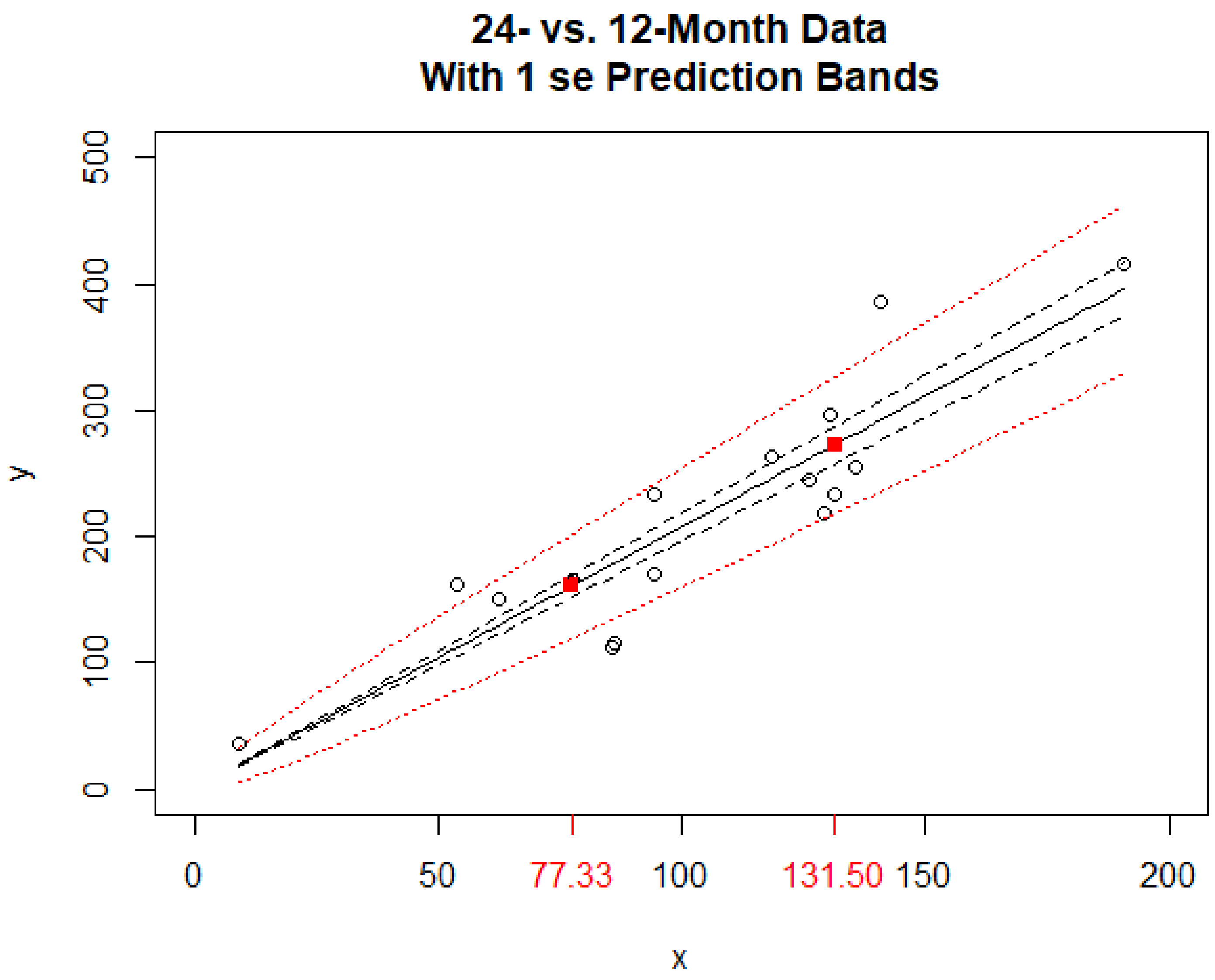
```
> ChainLadder::MackChainLadder(tri, est.sigma = "Mack")
ChainLadder::MackChainLadder(Triangle = tri, est.sigma = "

   Latest Dev.To.Date Ultimate IBNR Mack.S.E CV(IBNR)
1   218.2       1.000    218.2    0      0.0      NaN
2   255.5       1.000    255.5    0      0.0      NaN
3   232.7       1.000    232.7    0      0.0      NaN
4   165.2       1.000    165.2    0      0.0      NaN
5   296.2       1.000    296.2    0      0.0      NaN
6    35.8       1.000     35.8    0      0.0      NaN
7   233.4       1.000    233.4    0      0.0      NaN
8   114.7       1.000    114.7    0      0.0      NaN
9   112.4       1.000    112.4    0      0.0      NaN
10  161.1       1.000    161.1    0      0.0      NaN
11  169.7       1.000    169.7    0      0.0      NaN
12  416.0       1.000    416.0    0      0.0      NaN
13  263.7       1.000    263.7    0      0.0      NaN
14  244.7       1.000    244.7    0      0.0      NaN
15  150.6       1.000    150.6    0      0.0      NaN
16  386.0       1.000    386.0    0      0.0      NaN
17   77.3       0.482    160.4   83     41.1    0.495
18  131.5       0.482    272.7  141     54.4    0.385

              Totals
Latest:    3,664.78
Dev:           0.94
Ultimate:  3,889.04
IBNR:        224.26
Mack.S.E      70.00
CV(IBNR):      0.31
```



Why does the prediction envelope fan out only at the high end?



Chain-Ladder error term

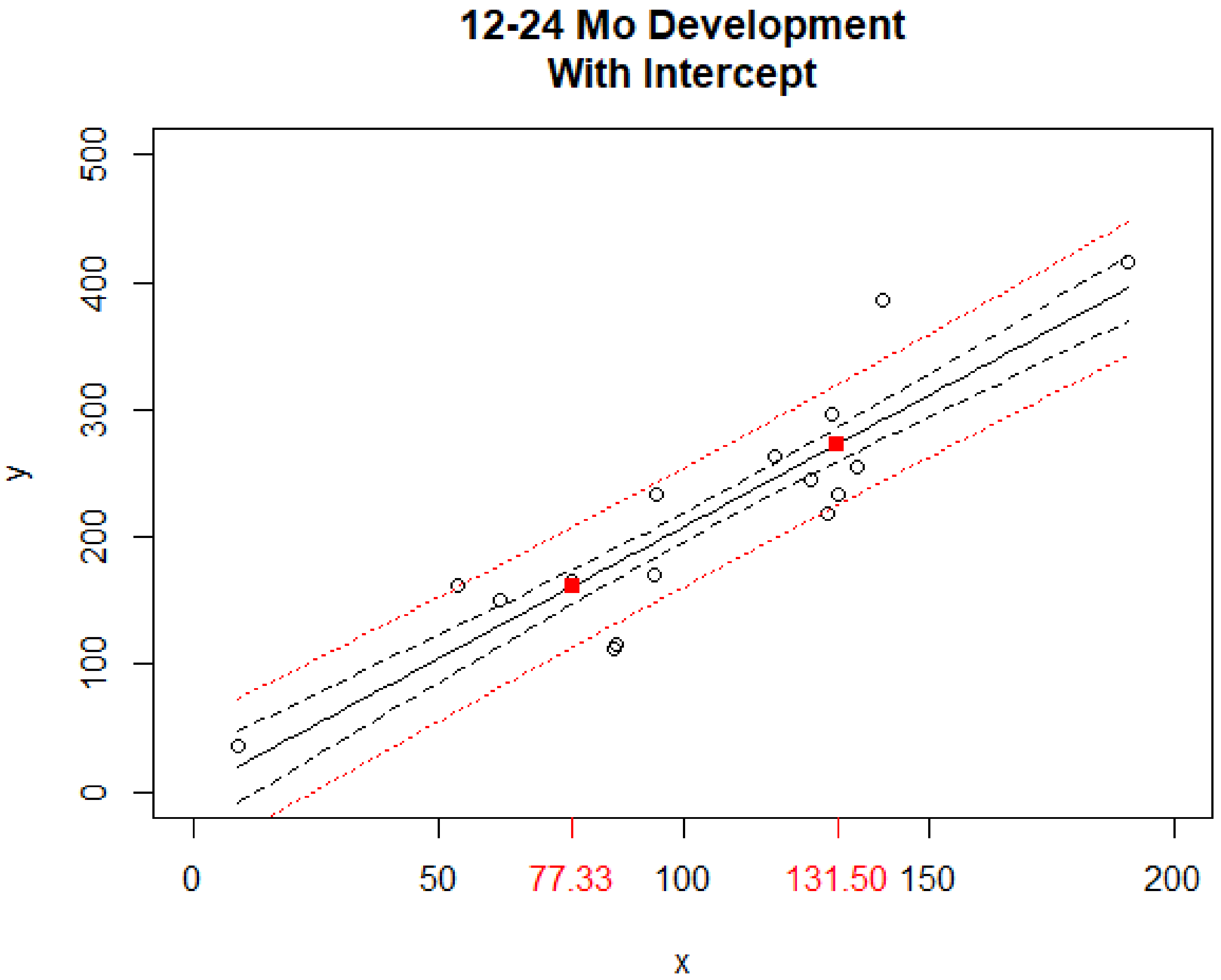
$$y = bx + \sqrt{x}e$$

model

- Assumption: The higher the initial value, the greater the variability of the subsequent value
- For what types of situations might this assumption not hold?
  - When might you have less variability the larger the beginning value and more variability the smaller the beginning value?

Prediction bands providing for an intercept

- --- parameter risk
- ······ total risk



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and variance:

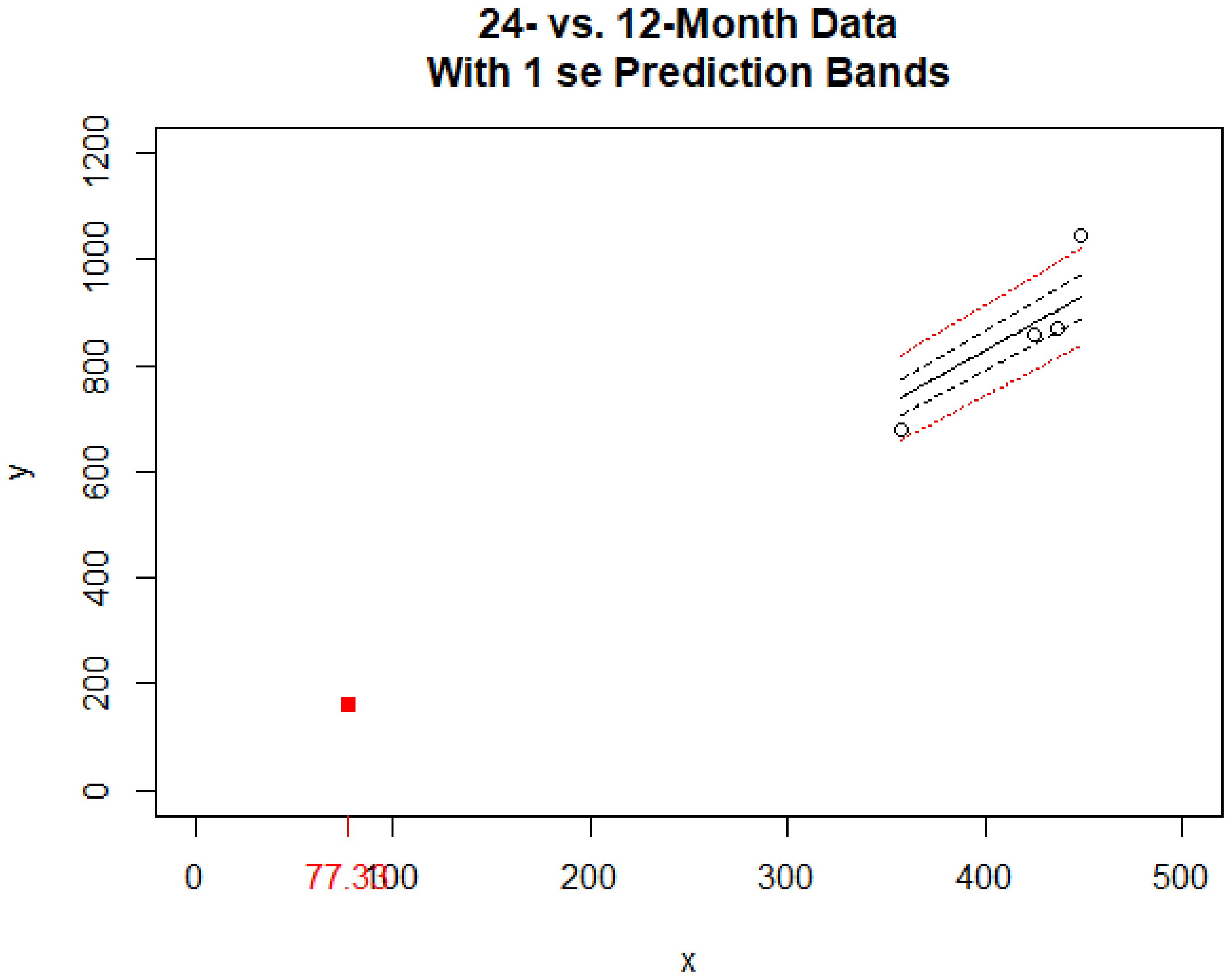
$$Var(w) = Var\left[Y_{n+1} - \hat{\alpha} - \hat{\beta}(x_{n+1} - \bar{x})\right] \overset{IND}{=} Var(Y_{n+1}) + Var(\hat{\alpha}) + (x_{n+1} - \bar{x})^2 Var(\hat{\beta})$$

$$= \sigma^2 + \frac{\sigma^2}{n} + \frac{(x_{n+1} - \bar{x})^2 \sigma^2}{\Sigma (x_i - \bar{x})^2}$$

$$= \sigma^2 \left[1 + \frac{1}{n} + \frac{(x_{n+1} - \bar{x})^2}{\Sigma (x_i - \bar{x})^2}\right]$$

- The estimated intercept is “not significantly different from zero”
- But rather than take the extra step and assume it equals zero, why not hedge your bet and use the uncertainty estimates that allows for that possibility?

Now pretend data come from 4 companies and aggregate into 4 AYs



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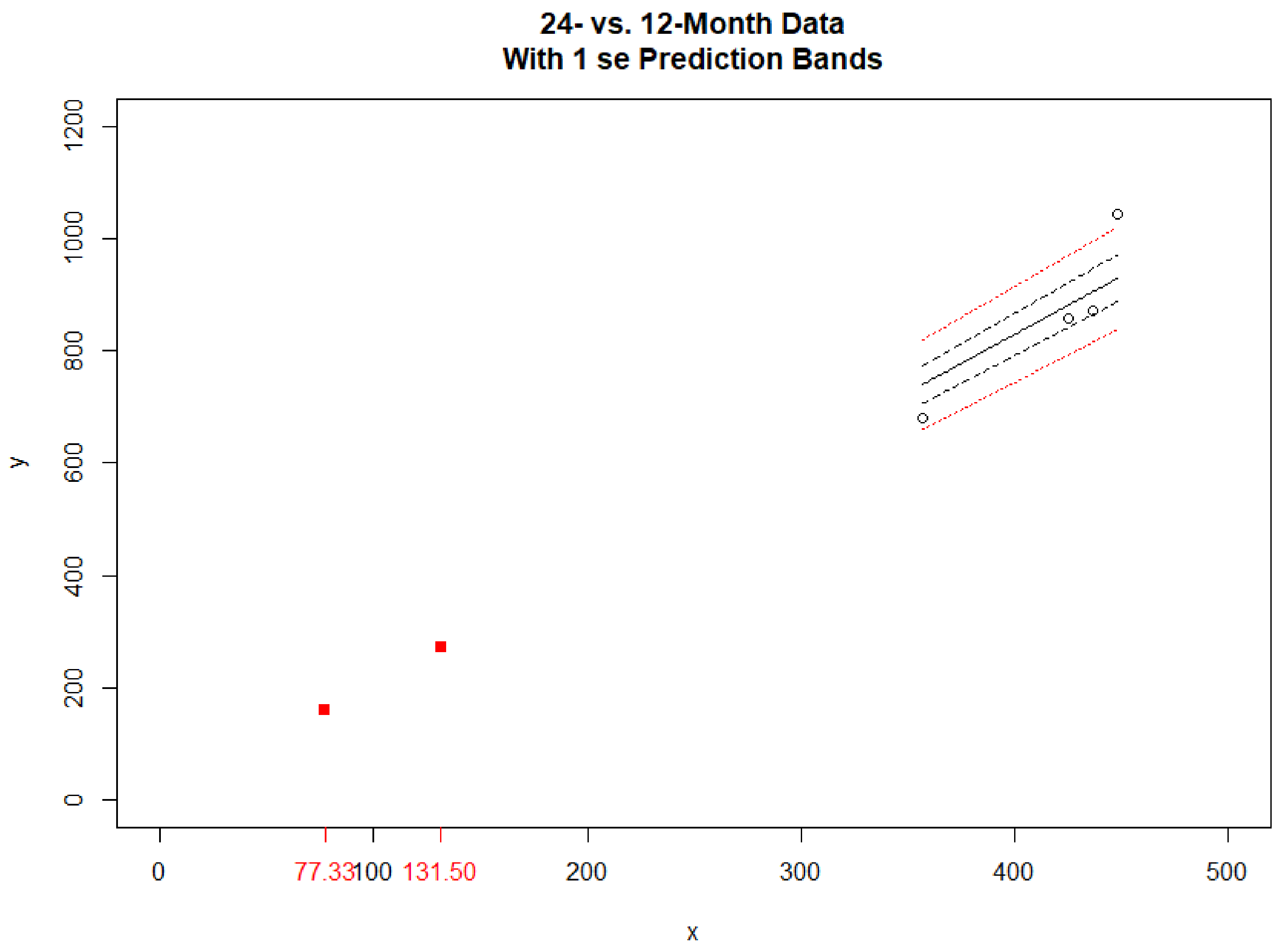

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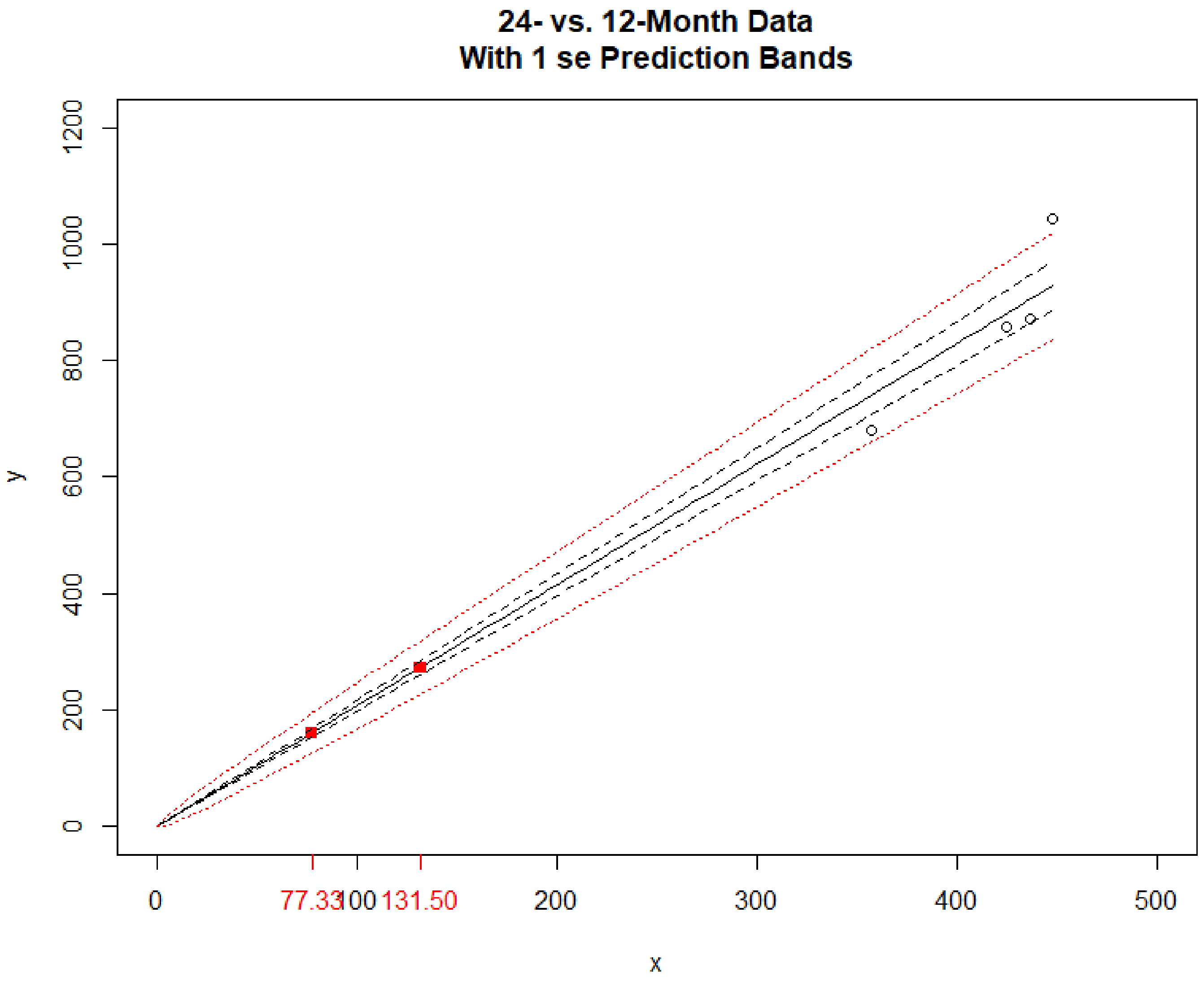
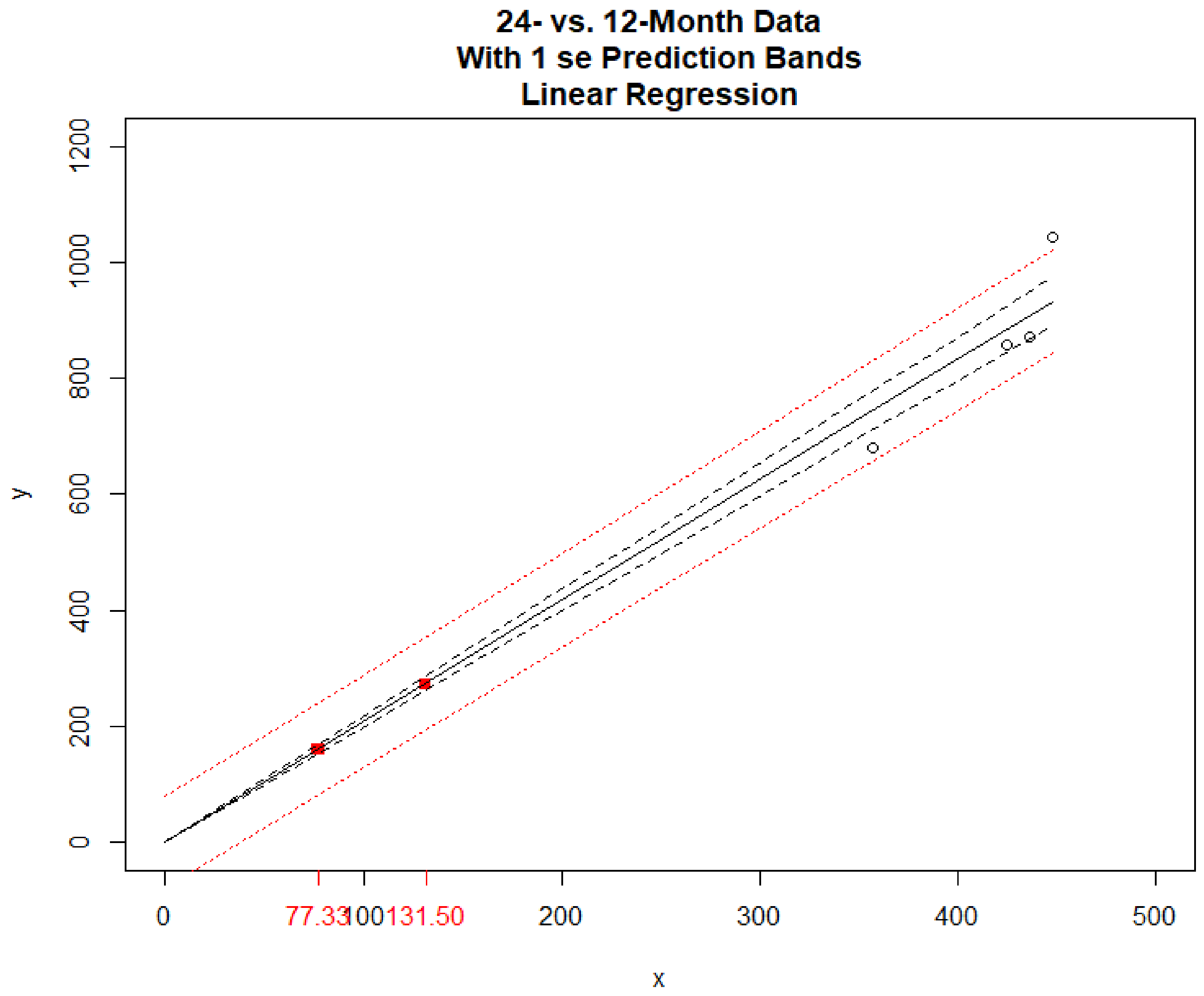


Linear regression standard error influence not diminished with small x



Predicted experience with "confidence levels"

With R

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6    35.8       1.000     35.8    0      0.0      NaN
7   233.4       1.000    233.4    0      0.0      NaN
8   114.7       1.000    114.7    0      0.0      NaN
9   112.4       1.000    112.4    0      0.0      NaN
10  161.1       1.000    161.1    0      0.0      NaN
11  169.7       1.000    169.7    0      0.0      NaN
12  416.0       1.000    416.0    0      0.0      NaN
13  263.7       1.000    263.7    0      0.0      NaN
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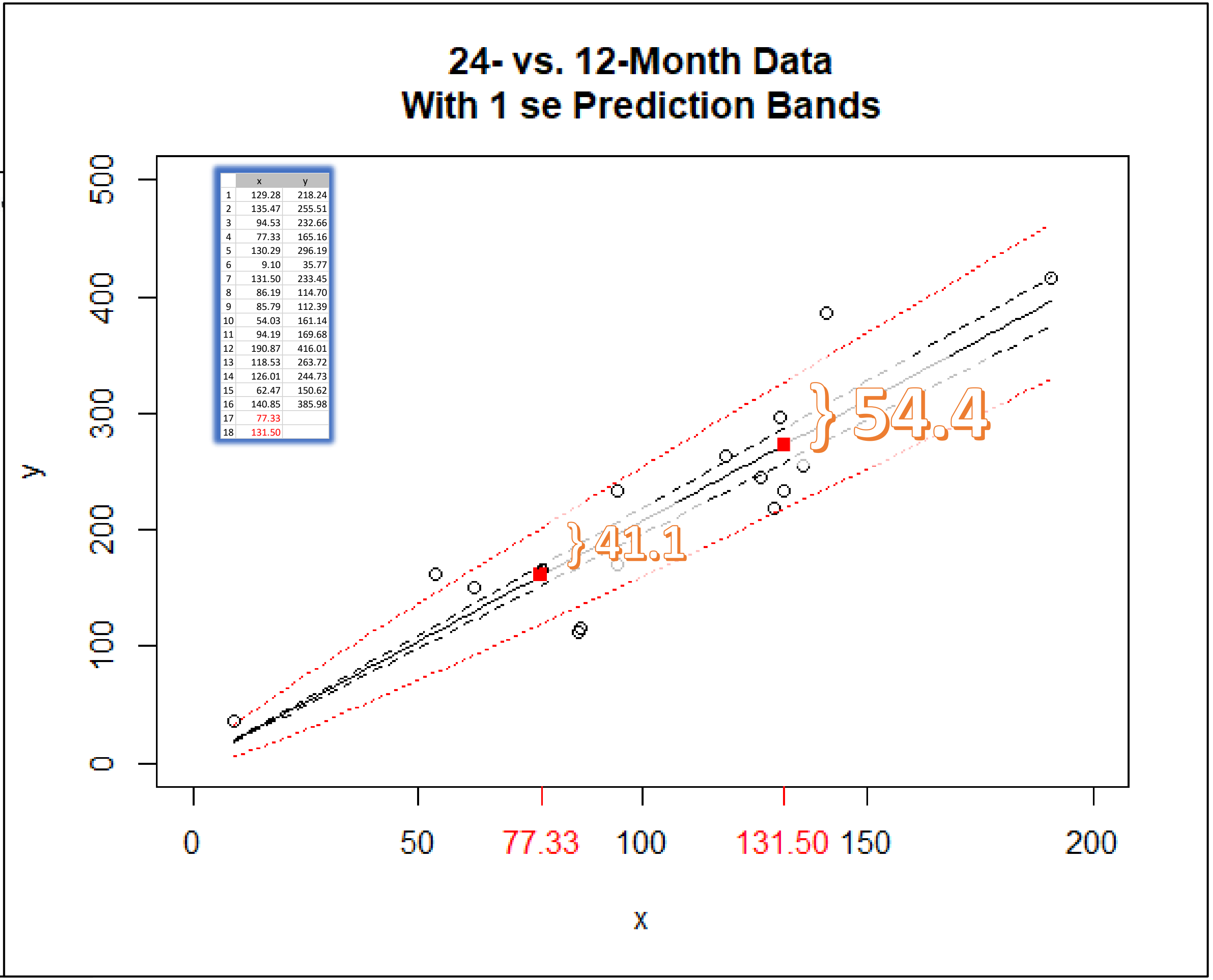

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7	131.50	233.45
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9	85.79	112.39
10	54.03	161.14
11	94.19	169.68
12	190.87	416.01
13	118.53	263.72
14	126.01	244.73
15	62.47	150.62
16	140.85	385.98
17	77.33	
18	131.50	

Let’s regroup

	x	y
1	129.28	218.24
2	135.47	255.51
3	94.53	232.66
4	77.33	165.16
5	130.29	296.19
6	9.10	35.77
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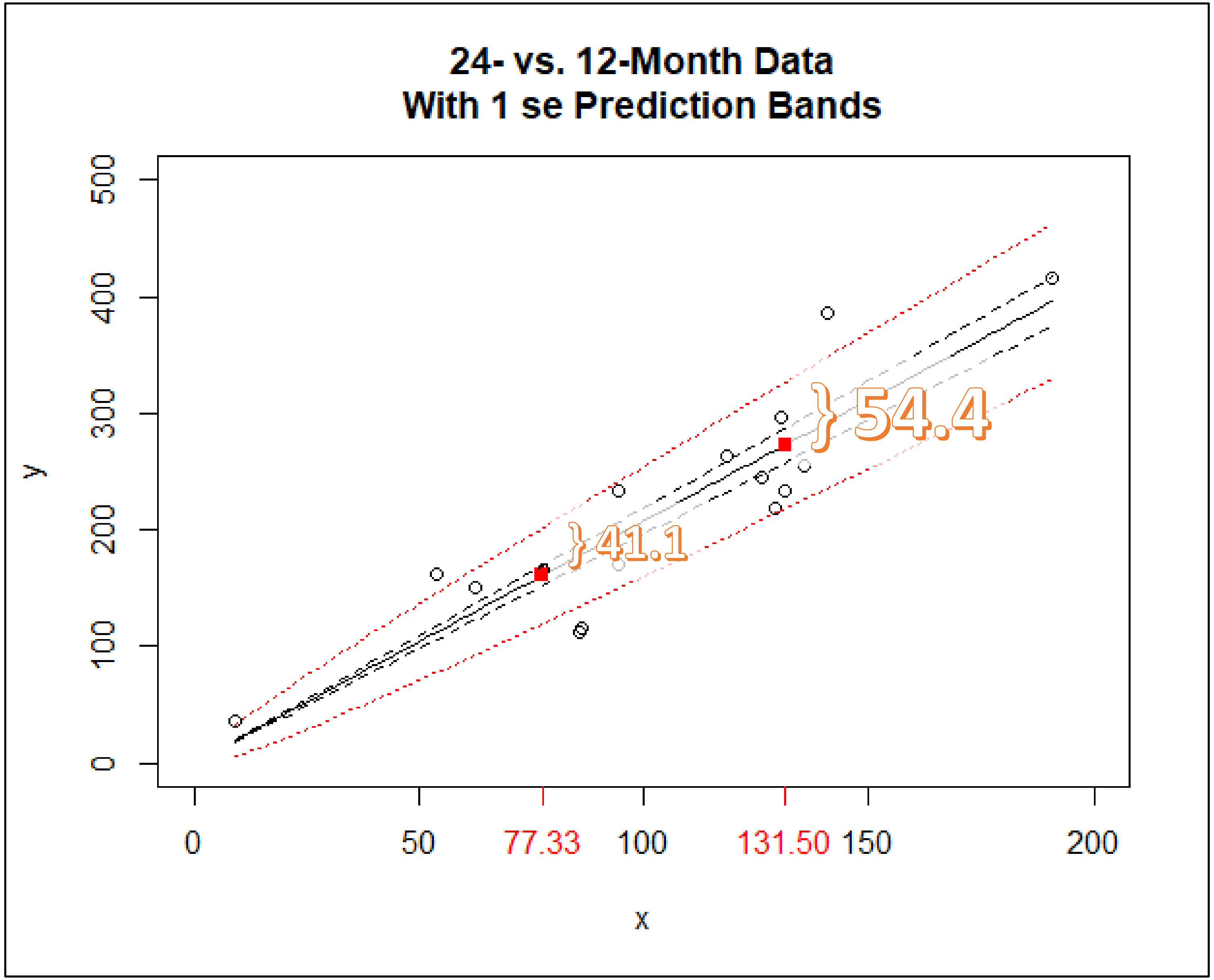
14	126.01	244.73
15	62.47	150.62
16	140.85	385.98
17	77.33	
18	131.50	



In Excel

	Original Data	
	x (12 mo)	y (24 mo)
1	129.2764	218.24
2	135.4733	255.51
3	94.5348	232.66
4	77.3251	165.16
5	130.2944	296.19
16	140.845	385.98

	Equivalent 12-24 month data	
	x'=x/√x	y'=y/√x
1	11.37	19.19
2	11.64	21.95
3	9.72	23.93
4	8.79	18.78
5	11.41	25.95
16	11.87	32.52

=LINEST(G3:G18,F3:F18,FALSE,TRUE)

b	2.074	0	const
σ_b	0.11184	#N/A	se_{const}
r_2	0.958	4.566	σ
F	343.8	15	df
ss_{reg}	7167.1	312.7	ss_{resid}

Point Estimate			
17	77.33	160.4	= 77.33 * 2.074
18	131.50	272.7	
Parameter Risk			
17	77.33	8.6	= 77.33 * 0.11184
18	131.50	14.7	
Process Risk			
17	77.33	40.15	= sqrt(77.33) * 4.566
18	131.50	52.36	
Total Risk = Mack S.E.			
17	77.33	41.1	= sqrt(8.65^2 + 41.07^2)
18	131.50	54.4	

Model:

$$y = bx + \sqrt{x}e$$

Table

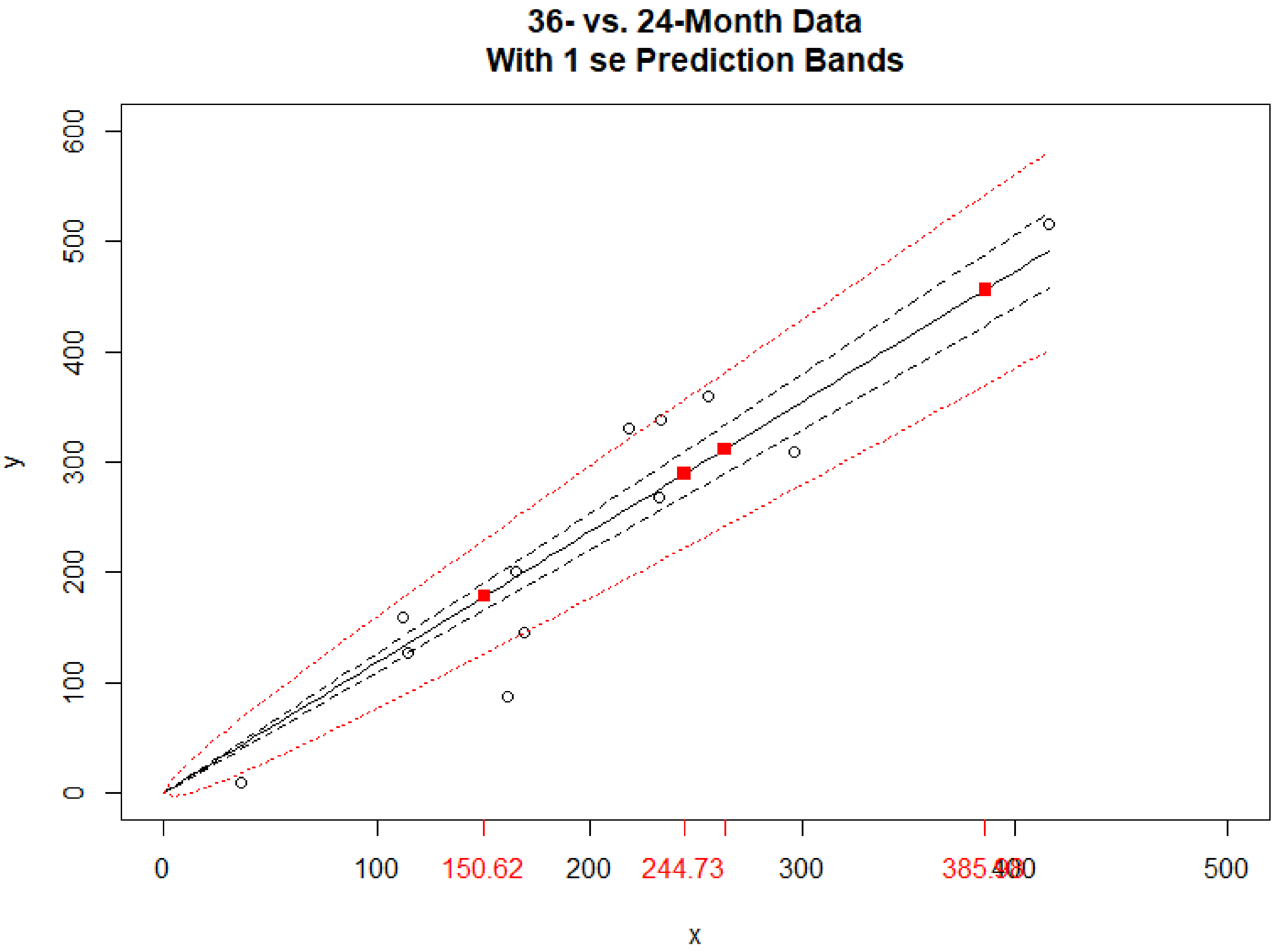
	IBNR	Mack.S.E	Parameter	Process	b	sigma_b	sigma
Agg	224.25	58.8	19.6	55.4	2.074	0.094	3.8
Det	224.26	70.0	23.3	66.0	2.074	0.112	4.6

Add another column

	x (12 mo)	y (24 mo)	z (36 mo)
1	129.28	218.24	330.88
2	135.47	255.51	359.34
3	94.53	232.66	267.56
4	77.33	165.16	200.61
5	130.29	296.19	309.08
6	9.10	35.77	9.53
7	131.50	233.45	337.82
8	86.19	114.70	127.00
9	85.79	112.39	159.52
10	54.03	161.14	86.60
11	94.19	169.68	145.21
12	190.87	416.01	514.95
13	118.53	263.72	
14	126.01	244.73	
15	62.47	150.62	
16	140.84	385.98	
17	77.33		
18	131.50		

Add another column

- b hat = 1.181
- sigma_b = 0.083
- sigma = 4.1



Recursive projection statistics

- Orange projections are products of a scalar and an estimated parameter
  - See above for risk formulas
- Red projections are products of an estimate and an estimated parameter
- Parameter Risk:
  Law of product of two independent r.v.'s
- Process Risk:
  Law of total variance

	x (12 mo)	y (24 mo)	z (36 mo)
1	129.28	218.24	330.88
2	135.47	255.51	359.34
3	94.53	232.66	267.56
4	77.33	165.16	200.61
5	130.29	296.19	309.08
6	9.10	35.77	9.53
7	131.50	233.45	337.82
8	86.19	114.70	127.00
9	85.79	112.39	159.52
10	54.03	161.14	86.60
11	94.19	169.68	145.21
12	190.87	416.01	514.95
13	118.53	263.72	**311.45**
14	126.01	244.73	**289.03**
15	62.47	150.62	**177.88**
16	140.84	385.98	**455.84**
17	77.33	160.38	**189.41**
18	131.50	272.73	**322.10**
b	**2.074**	**1.181**	