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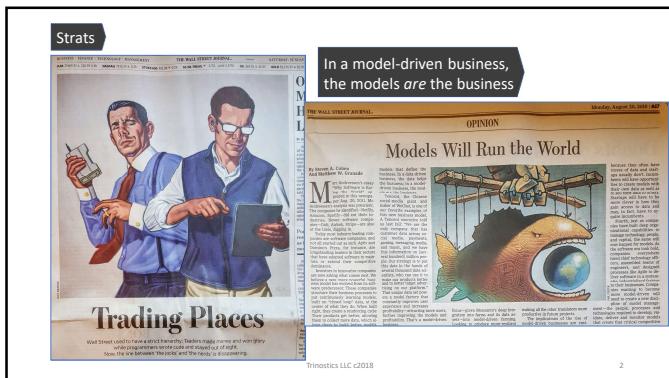
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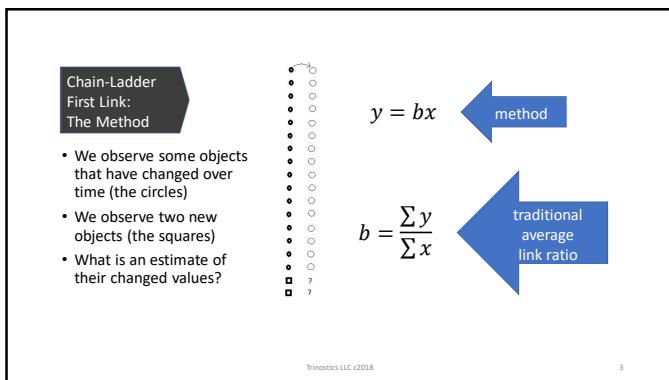
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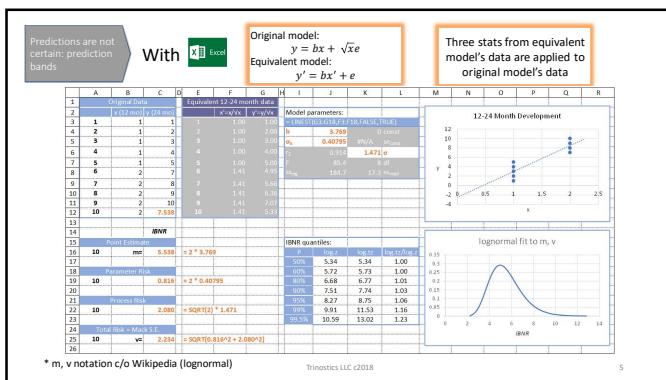
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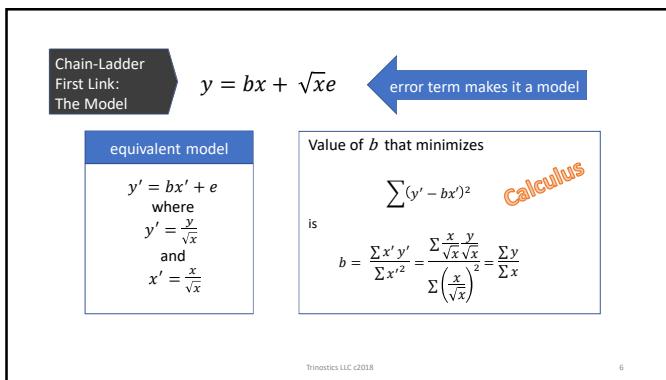
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Chain-Ladder  
First Link:  
An Example

- Apply the model

$y = bx + \sqrt{xe}$

to this skinny triangle

	x	y
1	129.28	218.24
2	135.47	255.51
3	94.53	232.66
4	77.33	165.16
5	130.29	296.19
6	9.10	35.77
7	131.50	233.45
8	86.19	114.70
9	85.79	112.39
10	54.03	161.14
11	94.19	165.68
12	190.87	416.01
13	118.53	263.72
14	126.01	244.73
15	62.47	150.62
16	140.85	385.98
17	77.33	
18	131.50	

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7

12-24 Month  
Development  
Experience

$b = \frac{\sum y}{\sum x} = 2.074$ 

- 2.074 = slope of the line through origin
- prediction of new initial observations:

**77.33 -> 160.4**  
**131.5 -> 272.7**

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8

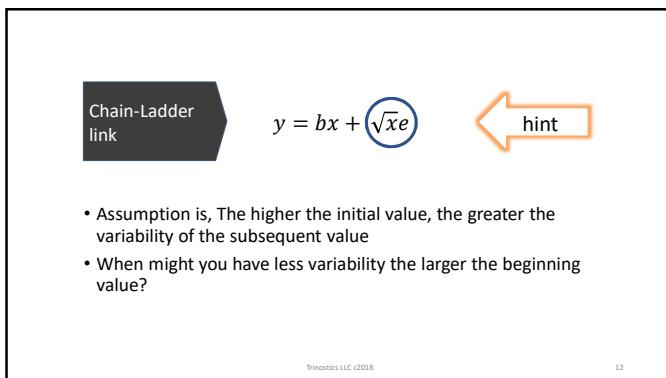
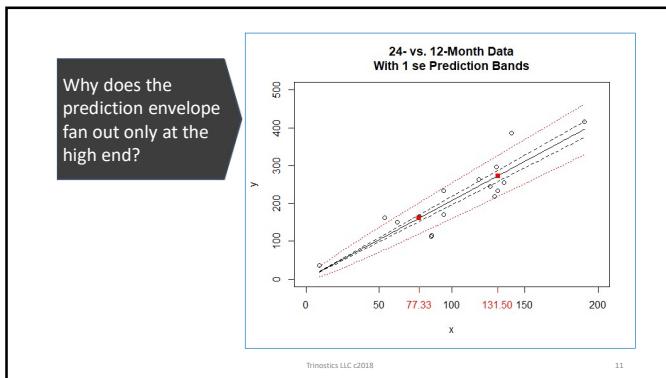
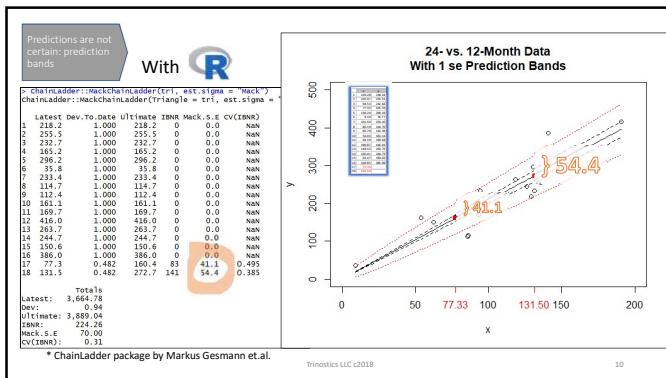
Predictions are not  
certain: prediction  
bands

- --- Parameter risk  $\Delta$   
Variability of estimated mean
- Process risk  $\Gamma$   
Variability around theoretical mean
- .... Total risk =  $\sqrt{\Delta^2 + \Gamma^2}$   
Variability of a predicted outcome

\* notation by Ali Majidi

9

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## How do prediction bands look under different models?

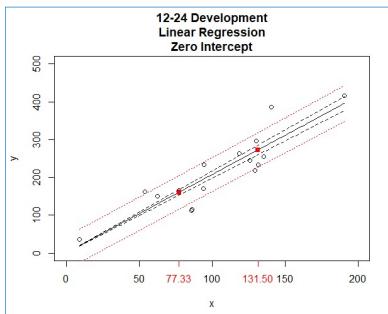
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13

Prediction bands  
without  
square-root-o-  
skedasticity

$$y = bx + e$$

- parameter risk
- ... total risk



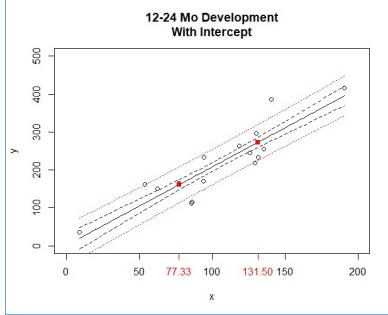
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14

Prediction bands  
when there's an  
intercept

$$y = a + bx + e$$

- parameter risk
- ... total risk



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15

## Homework

1. What would the graph of the model look like if the simple average is the optimal link ratio?
  - Does the answer change if “optimal” is a matter of actuarial judgment?
2. What could be drivers of a non-zero intercept?
3. How to model the BF method within the Chain-Ladder paradigm?
4. How to model the first column within the Chain-Ladder paradigm?
5. Prove that our game satisfies the assumptions of the model

$$y = bx + \sqrt{x}e$$

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16

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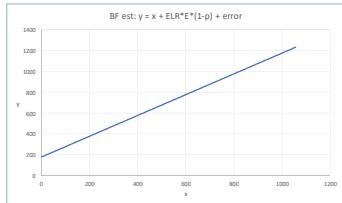
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## Bornhuetter-Ferguson

- What is the slope of the line?
- What is the intercept?



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## Chain-Ladder Second Link: Add Another Column

	x (12 mo)	y (24 mo)	z (36 mo)
1	129.28	218.24	330.88
2	135.47	255.51	359.34
3	94.53	232.66	267.56
4	77.33	165.16	200.61
5	130.29	296.19	309.08
6	9.10	35.77	9.53
7	131.50	233.45	337.82
8	86.19	114.70	127.00
9	85.79	112.39	159.52
10	54.03	161.14	86.60
11	94.19	169.68	145.21
12	190.87	416.01	514.95
13	118.53	263.72	
14	126.01	244.73	
15	62.47	150.62	
16	140.84	385.98	
17	77.33		
18	131.50		

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18

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Chain-Ladder  
Second Link:  
Add Another  
Column

- $b_y = 1.181$
- $\sigma_{b_y} = 0.083$
- $\sigma_{y} = 4.1$

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Chain-Ladder predicts the future recursively

- Orange projections are products of a scalar and an estimated parameter **Which is which?**
  - Formulas for Parameter Risk and Process Risk can be found in slides above
- Red projections are products of an estimate and an estimated parameter
  - Formulas for Parameter Risk and Process Risk are derived from the **Law of Total Variance**

	x (12 mo)	y (24 mo)	z (36 mo)
1	129.28	218.24	330.88
2	135.47	255.51	350.34
3	94.53	232.66	267.56
4	77.33	165.16	200.61
5	130.29	296.19	309.08
6	9.10	35.77	9.08
7	131.50	233.45	337.82
8	86.19	114.70	127.00
9	85.70	112.39	159.52
10	54.03	161.14	86.60
11	94.19	169.68	145.21
12	190.87	416.01	514.95
13	118.53	263.72	311.46
14	126.01	244.73	289.03
15	62.47	150.62	177.08
16	140.84	385.98	455.84
17	77.33	160.38	189.41
18	131.50	272.73	322.10
b	2.074	1.181	

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**Law of Total Variance**

• Wikipedia:  $Var(Y) = E[Var(Y|X)] + Var(E[Y|X])$

• "In actuarial science, specifically credibility theory, the first component is called the expected value of the process variance (**EV<sub>PV</sub>**) and the second is called the variance of the hypothetical means (**V<sub>HM</sub>**)."

- Retrieved June 25, 2015

• See Majidi and Bardis formula derivations, "A Family of Chain-Ladder Models," *Variance*, Vol 6, Issue 2, pp. 157-158

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## Recursive projections with statistics – complete many squares

- Expected Value
  - Parameter Risk –  $\Delta$ 

$$\Delta_y^2 = x^2 \cdot \widehat{\sigma_b}^2$$

$$\Delta_z^2 = \hat{y}^2 \cdot \widehat{\sigma_b}^2 + \hat{b}^2 \cdot \Delta_y^2 + \widehat{\sigma_b}^2 \cdot \Delta_y^2$$
  - Process Risk –  $\Gamma$ 

$$\Gamma_y^2 = x \cdot \widehat{\sigma_e}^2$$

$$\Gamma_z^2 = \hat{y} \cdot \widehat{\sigma_e}^2 + \hat{b}^2 \cdot \Gamma_y^2$$

Data	x	y	z
1	xxx	yyy	zzz
2	xxx	yyy	
3			

Parameter estimates			
	b <sub>x</sub>	b <sub>y</sub>	
$\hat{b}_x$			$\hat{b}_y$
$\hat{b}_y$			$\hat{b}_x$

Expected Values			
	x	y	z
1			
2			$z^A = yyy^Ab_y$
3			$y^A = xxx^Ab_x$

Parameter Risk			
	x	y	z
1			
2			$\Delta_z$
3		$\Delta_y$	$\Delta_x$

Process Risk			
	x	y	z
1			
2			$\Gamma_z$
3		$\Gamma_y$	$\Gamma_x$

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## Tricks for Risk Estimates for the Total/sum Row

Expected Values		
	X	Y
1		
2	YYY	$Z^A = YYY^*B_0$
3	$Y^A = XXX^*B_0$	$Z^A = Y^A * B_0$
future sum	$Y^A$	$Z^A = (YYY + Y^A)^*B_0$

Parameter Risk		
	X	Y
1		
2		$\Delta_A$
3	$\Delta_A$	$\Delta_A$
future sum	$\Delta_Y$	same formula

Process Risk		
	X	Y
1		
2		$\Gamma_z$
3	$\Gamma_y$	$\Gamma_z$
future sum	$\Gamma_x$	$\Gamma_x^2 = \Gamma_x^2 + \Gamma_z^2$

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| Why not directly estimate the 12-36 month link ratio?

- What if you learned  $b_{xz} = \text{sum}(z) / \text{sum}(x) = 2.337$
  - Why not say the expected 36-month value of  $x = 77.33$  is  
 $77.33 * 2.337 = 180.7$  (se 44.7)  
rather than  
 $77.33 * 2.074 * 1.181 = 189.4$  (se 72.1 see above)

rather than

$$77.33 * 2.074 * 1.181 = 189.4 \text{ (see 72.1 see above)}$$

Trainee 11C - 2011

Can you ignore y or not?

- There is important information in the 24-month value
  - The path to ultimate is important
  - *It's the journey*

	$x$	$[x] \text{ mol}$	$[y] \text{ mol}$	$[z] \text{ mol}$
1	139.28	218.24	330.88	
2	135.47	255.51	359.34	
3	94.53	232.66	267.56	
4	77.33	165.16	200.61	
5	130.29	296.19	309.08	
6	9.10	35.77	9.53	
7	131.50	233.45	337.82	
8	85.19	114.70	127.00	
9	85.79	112.39	159.52	
10	54.03	161.14	86.60	
11	94.19	169.68	145.21	
12	190.87	416.00	514.95	
13	164.01	263.77	277.03	
14	45.61	100.00	56.31	
15	62.47	150.62	146.00	-4.61
16	140.94	385.98	329.18	56.89
17	77.33		180.74	103.41
18	131.50		307.35	175.85

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26

Continue the Journey –  
Chain to Ultimate

- Recursive estimates are carried forward to the last pair of development columns
  - Technical considerations
    - What to do when there are not enough observations to get a good estimate of sigma (zero degrees of freedom)
    - What to do with a tail
  - Mack has recommendations for handling these technicalities
  - The ChainLadder package's MackChainLadder function includes Mack's recommendations, as well as others
  - Let's see some examples

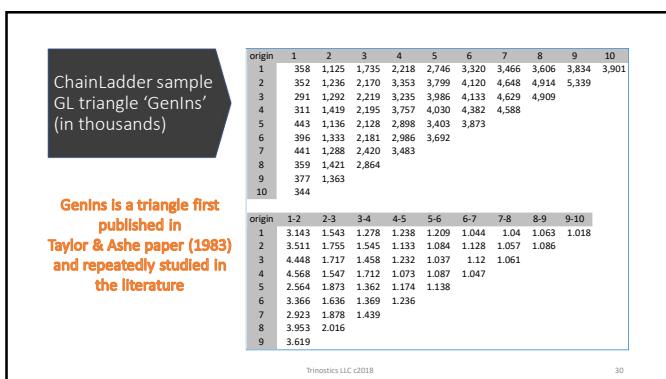
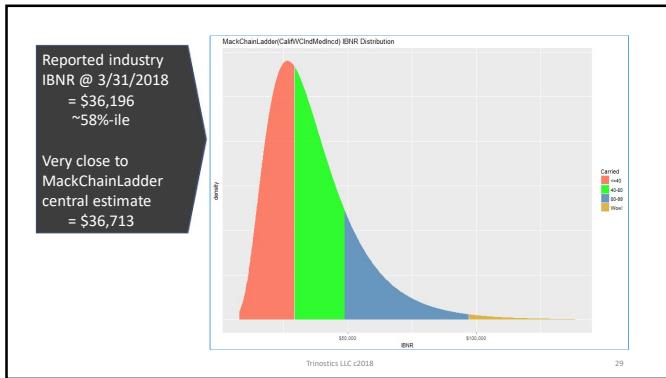
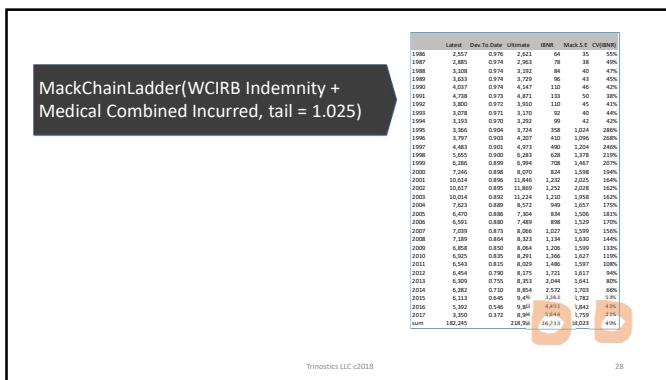
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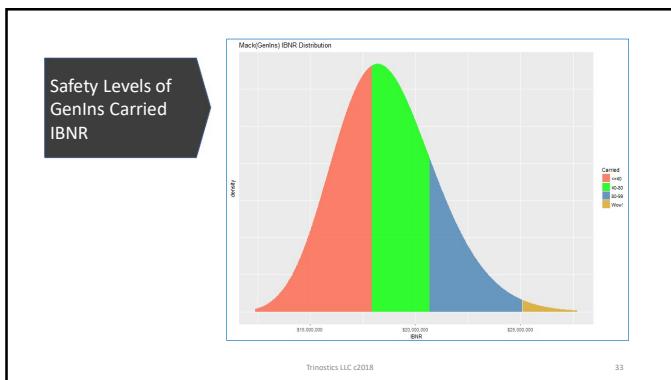
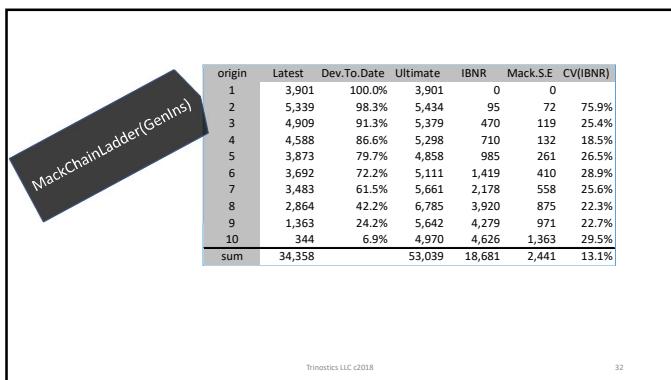
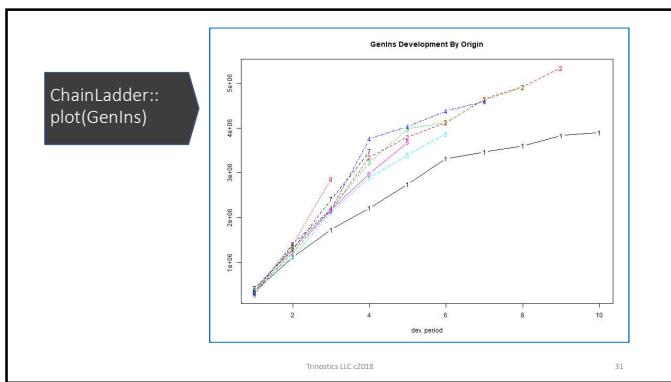
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California WCIRB Agenda June 2018  
Combined Indemnity and Medical Incurred

\* triangle creation approach thanks to Dave Bellusci; data entry thanks to Connan Houser  
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- Wrap-up: What are possible uses of an IBNR distribution?
- Rather than a distribution, can Mack/Murphy be used in predictive analytics?

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34

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What happens when Mack/Murphy is run on detail data?

- Suppose  $x$  and  $y$  are actually observations from 4 companies in 4 accident years
- Will link ratios from aggregated data and detail data always be the same?
- What about the risk statistics?

	X	Y	X	Y
1	129.28	218.24	436.61	871.57
2	135.47	255.51		
3	94.53	232.66		
4	135.35	116.15		
5	130.29	296.19	357.09	680.11
6	9.10	35.77		
7	131.50	233.45		
8	86.19	114.70		
9	85.79	113.39	424.88	859.22
10	140.49	151.14		
11	94.19	169.68		
12	190.87	416.01		
13	118.53	263.72		
14	126.01	244.73	447.86	1045.05
15	62.47	150.62		
16	140.85	383.98		
17				
18	133.50			
	1666.44	3455.95	1666.44	3455.95
	b <sub>Detail</sub> =	2.074	b <sub>Aggregated</sub> =	2.074

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### GenIns at the Claim Level

- Hai You generated simulations of over 6000 synthetic claims whose accident year aggregation is “close in shape” to GenIns
  - We pegged the 13% cv as the primary measurement of similarity
- Hai’s claim-characteristic choices included:
  - Frequency distribution
  - Severity distribution
  - Distribution for the number of payments per claim
  - Report lag and payment lag
- The purpose of this exercise was to compare the Mack results on the claim detail with the statistics from the aggregate triangle

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36

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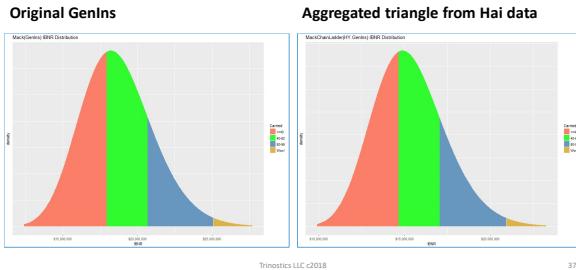


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IBNR distributions from aggregated triangles are very similar



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GenIns at the Claim Level:  
Claim detail sample in triangle format

	Latest	Ultimate	IBNR	Mack.S.E	CV(IBNR)
GenIns	34,358	53,039	18,681	2,441	13.1%
HY.GenIns	32,556	47,866	15,310	2,127	13.9%
HY detail	32,556	40,909	8,353	695	8.3%

# Why the IBNR drop? Why the CV drop?

\* simulated claims by Hai You

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Is the weighted average development factor app

Dev Factor	12-24
GenIns	3.491
HY.GenIns	3.413
HY.detail	1.288

## What happened to the 12-24 factor from the claim detail?!?

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The 12-24 month relationship from the claim detail

- Are Chain-Ladder assumptions violated by the detailed data?

```
linear regression in R:  
lm(y~x)  
Coefficients:  
(Intercept) x  
1330.6 0.96
```

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The 12-24 month relationship from the GenIns triangle

- Are Chain-Ladder assumptions violated by the aggregate data?

Is that the line you would draw through that data?

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The 12-24 month relationship from the GenIns triangle

- Rhetorical question:  
Why should this model not be considered for projecting the 12-month value?

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What's next?

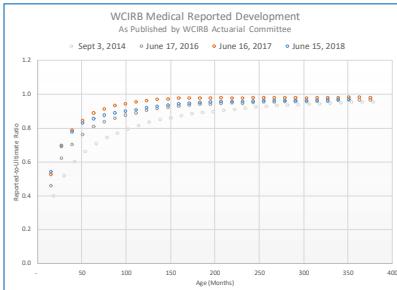
- How to model serial correlation?
    - ARMA
    - Michael Wacek, "The Path of the Ultimate Loss Ratio Estimate", *eForum*
  - Growth curves
    - Sherman; Clark; Guszczak
  - Bayes
  - Wüthrich
    - Individual Claim Development with Machine Learning (2017)
    - Neural Networks Applied to Chain-Ladder Reserving (2018)



Tripartite IUC c2018

43

## Can InsureTech jump the curve?



\* graphics by Kirsten Singer

## Summary

- Despite all its problems, the Chain-Ladder Mack/Murphy *model* is useful
    - The regression tale of development is easy to understand
    - Distributions help our principals make decisions
  - Exciting actuarial analysis in the future
    - Combining methods mid-stream
    - AI modeling of the path to ultimate
  - Stories/models with clarity sell best
    - Everybody likes pictures



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**Q&A**

Thank you for coming!  
Dan Murphy  
[dmurphy@trinostics.com](mailto:dmurphy@trinostics.com)

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46

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