Course Code: PGDS103

MAX MARKS: 60 **TIME: 2 Hr 30 Min.**

Note: (1) All questions are compulsory

(2) Figures to the right indicate marks.

(3) Mixing of sub-questions is not allowed.

Q1. Answer the following. (Any TWO) **12**

1)

i. Let $A = \begin{pmatrix} 4 & 1+i & 2+5i \\ 1-i & -3 & 7+3i \\ 2-5i & 7-3i & -6 \end{pmatrix}$. Find the conjugate transpose of the matrix A. Is A a Hermitian matrix? Justify. **03**

ii. Let $A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 7 & 9 \\ 11 & 3 & -2 \end{pmatrix}$. Evaluate the determinant of A and deduce whether the matrix is singular or not. **03**

2) Determine the value of real number k for which the following system has no solution. **06**

$$x - 2y = 1$$
$$x - 2y + kz = -2$$
$$ky + 4z = 6$$

3) Write a program in python to do the following: **06**

i. Find the transpose of the matrix of M.

ii. Find the scalar multiplication of M with a scalar α.

4) Find the eigen values and bases of corresponding eigen spaces of the matrix $A = \begin{pmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{pmatrix}$. **06**

Q2. Answer the following. (Any TWO) **12**

1) Define the following: **06**

i. Vector space over a field K.

ii. Linearly dependent set of vectors.

iii. Sparse vectors.

iv. Basis of a vector space.

2) For the vector space $\mathbb{R}^3$ over $\mathbb{R}$, express the vector, $(2,3,1)$ as a linear combination of $(1,0,0)$, $(0,2,-1)$ and $(1,0,-1)$. **06**

3)

i. Check whether $\left\{\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 5 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}\right\}$ is linearly independent. **03**

ii. Write a program in python to enter two vectors u and v as a n −list and find the linear combination of u and v. **03**

4) Define a linear function. Determine whether $T: \mathbb{R}^2 \to \mathbb{R}$, defined by $T(x, y) = 2x + y$ is a linear function. **06**

Q3. Answer the following. (Any TWO) **12**

1) Find an orthonormal basis of $\mathbb{R}^3$ corresponding to $\{(1,0,0), (0,2,1), (-1,0,1)\}$ using Gram-Schmidt orthogonalization process with respect to usual dot product. **06**

2) If u, v and w are any three vectors in an inner product space V and k is any scalar then show that **06**

i. $\langle u, 0\rangle = 0 = \langle 0, u\rangle$

ii. $\langle u, v+w\rangle = \langle u, v\rangle + \langle u, w\rangle$

iii. $\langle u, k.v\rangle = k.\langle u, v\rangle$

3) i. Verify Cauchy-Schwarz inequality for the vectors $x = (-1,-1,0,2)$ and $y = (2,-3,1,-2)$ with respect to the inner product $\langle x, y\rangle = x_1y_1 + 2x_2y_2 + x_3y_3 + 4x_4y_4$. **03**

ii. Find the value of k for which the vectors $x = (1,4,k)$ and $y = (0,-1,k)$ are orthogonal. **03**

4) i. Find the orthogonal projection of the vector $v = (1,-2,0,3)$ along $u = (1,0,2,-1)$ with respect to usual dot product. **03**

ii. Define orthogonal set. Check whether $S = \{(2,3,-1),(0,1,3),(1,0,0)\}$ is orthogonal set of vectors? Justify. **03**

Q4. Answer the following. (Any TWO) **12**

1) Explain the problem of least-squares. Further give a solution to the least-square problem. **06**

2) Find the least-square solution of $Ax = b$, where $A = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$. What quantity is being minimized? **06**

3) Find the line that best approximates the data points, $(2,1), (-1,0)$ and $(0,-1)$. **06**

4) Find the linear function that best approximated the following data: **06**

x	y	$f(x,y)$
0	1	-5
1	0	2
1	1	-3
2	-1	9

Q5. Answer the following. (Any TWO) **12**

1) Define a matrix. Explain any three types of matrices with example. **06**

2) Check whether the set $V = \left\{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in \mathbb{R}\right\}$ forms a vector space over $\mathbb{R}$. **06**

3) Write a program in python to do the following: **06**

i. Find the dot product of two vectors u and v.

ii. Find the orthogonal projection of v along u.

4) Find the least-square solution of $Ax = b$, where $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 6 \end{pmatrix}$. **06**