

Exercises 5

Exercise 1

Simulate artificial regression data using the following R code:

```
n <- 30
set.seed(19062017)      ## to always obtain the same numbers
eps <- rnorm(n, sd=0.5)
x1 <- rnorm(n); x2 <- rnorm(n); x3 <- rnorm(n)
y <- 1 + x1 + eps
```

Estimate the following linear models:

```
lm1 <- lm( y ~ x1 )
lm2 <- lm( y ~ x1 + x2 )
lm3 <- lm( y ~ x1 + x3 )
lm4 <- lm( y ~ x1 + x2 + x3 )
```

- (a) Which of the coefficients in which of the models are significant at level 5%?
- (b) Which of the models could be compared by F tests? Do these tests in R and comment the results.
- (c) Draw a scatterplot of x_1 and y and plot the regression line corresponding to model $lm1$. Do also calculate the R^2 values for all models (how could you obtain the value from the model in R?).
If you look at the R code above that generated the data: Which of the parameters should be changed such that the R^2 values for all four models do increase? (How does this change the plot?)

Exercise 2

Consider a data set with an explanatory variable X and a dependent variable Y having the following values:

x_i	a	a	a	b	b	b	b	c	c	c
y_i	5	7	5	4	5	5	6	4	4	3

- (a) Use R to fit a linear model to these data. Which possibilities do you have to code the variable X ? Try to write the possible design matrices first on paper, then check with R.
- (b) Could you calculate the estimated coefficients using a pocket calculator, i.e. without using R? What are the predicted values \hat{y}_i ?

Exercise 3

The cdf of the standard logistic distribution is given by $F(x) = \frac{1}{1 + e^{-x}}$.

- (a) What are the properties of a cdf? Explain why F fulfills these.
- (b) Calculate the pdf $f(x)$.
- (c) Use the R function `rlogis` to generate pseudo-random numbers for the logistic distribution. (The standard logistic has `location=0` and `scale=1.`) Simulate samples from the standard logistic distribution and illustrate that its expectation is 0 and the variance equals $\frac{\pi^2}{3}$ ist.

Exercise 4

We consider again the standard logistic distribution (see previous exercise).

- (a) Plot the curves for the pdf and the cdf.
- (b) Add the corresponding curves for the Gaussian (standard normal) to your plots. How would you describe the differences between both distributions?
- (c) Which parameters of the normal distribution could you choose in order to have a distribution that resembles the standard logistic? Do also compare the pdf and cdf curves.