4. (1) $C\frac{dV}{dt} = -\frac{V}{R} + Q\sum_{k=-\infty}^{+\infty}\delta(t-kT)$

$$\frac{d}{dt}\left(Ve^{\frac{t}{RC}}\right) = \frac{Q}{C}\sum_{k=-\infty}^{+\infty}\delta(t-kT)$$

$$V(t) = e^{-\frac{t}{RC}}\frac{Q}{C}\int_{-\infty}^{t}\sum_{k=-\infty}^{+\infty}\delta(t-kT)\,e^{\frac{t}{RC}}\,dt$$

when $mT \le t < (m+1)T$

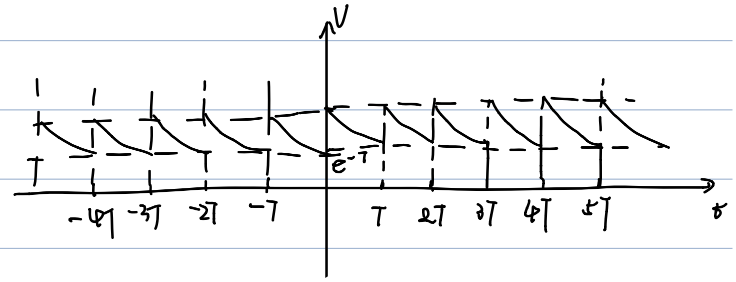
$$V(t) = e^{-\frac{t}{RC}}\frac{Q}{C}\sum_{k=-\infty}^{m} e^{\frac{kT}{RC}} = e^{-\frac{t}{RC}}\frac{Q}{C}\, e^{\frac{mT}{RC}}\frac{1}{1-e^{-\frac{T}{RC}}} = \frac{Q}{C}\frac{1}{1-e^{-\frac{T}{\tau}}}\,e^{\frac{mT-t}{\tau}}$$

const $\triangleq \frac{Q}{C}\frac{1}{1-e^{-\frac{T}{\tau}}}$. $\tau \triangleq RC$

WLOG 令 $RC=1$ $\quad \frac{Q}{C}\cdot\frac{1}{1-e^{-T}} = 1$

$$V(t) = e^{mT-t}$$

t	$V(t)$
$[-2T,-T)$	e^{-2T-t}
$[-T,0)$	e^{-T-t}
$[0,T)$	e^{-t}
$[T,2T)$	e^{T-t}
$[2T,3T)$	e^{2T-t}



so: for every $t=kT$. V has a sudden change.

the membrane protential change periodically. if V does not reach the V_{th}

(2) if the neuron can fire spikes. so, at an exact time, $V(t) > V_{th}$ and then V reset to 0, then the charge start to cross the membrane again we need to count the number of charge from I. that means. $I = Q\sum_{k=0}^{\infty}\delta(t-kT)$

Solve the equation, I assume that. when $t=0$. the first charge come in

$$C\frac{dV}{dt} = -\frac{V}{R} + I = -\frac{V}{R}\,Q\sum_{k=0}^{\infty}\delta(t-kT)$$

$$V = e^{-\frac{t}{RC}}\frac{Q}{C}\int_{-\infty}^{t}\sum_{k}e^{\frac{t}{RC}}\delta(t-kT)\,dt$$

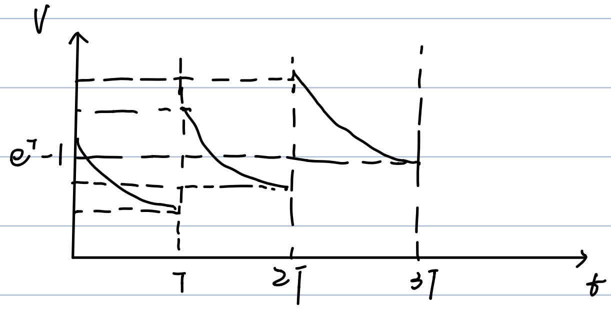
$$V(t) = e^{-\frac{t}{RC}} \frac{Q}{C} \sum_{k=0}^{m} e^{\frac{kT}{RC}} = e^{-\frac{t}{RC}} \frac{Q}{C} \frac{1-e^{(m+1)T/RC}}{1-e^{\frac{T}{RC}}}$$

$$= e^{-\frac{t}{RC}} \frac{Q}{C} \frac{1}{e^{T/RC}-1} \left[e^{\frac{(m+1)T}{RC}} - 1\right]$$

WLOG. $\tau = RC = 1$ $\quad \frac{Q}{C} \frac{1}{e^{T/RC}-1} = 1$

so : $V(t) = e^{-t}\left[e^{(m+1)T} - 1\right]$

k	t	$V(t)$	$V(kT+0)$	$V[(k+1)T-0]$
0	$[0,T)$	$e^{-t}(e^{T}-1)$	$e^{T}-1$	$1-e^{-T}$
1	$[1,2)$	$e^{-t}(e^{2T}-1)$	$e^{T}-e^{-T}$	$1-e^{-2T}$
2	$[2,3)$	$e^{-t}(e^{3T}-1)$	$e^{T}-e^{-2T}$	$1-e^{-3T}$



$\max\{V(kT+0)\} = e^{T}$. so when $e^{T} > V_{th}$

the neuron has a chance to fire spikes.

I assume that when $t=kT$,

$V(kT) \geq V_{th} > V[(k-1)T]$, if $k=0$. $V_{th} < e^{T}-1$. $T_{fire} = T$. $f = \frac{1}{T}$

i.e. $e^{T} - e^{-kT} \geq V_{th} > e^{T} - e^{-(k-1)T}$

so $\quad T_{fire} = (k+1)T \quad f = \frac{1}{(k+1)T}$

denote : $e^{T} - V_{th} = \tilde{V} \in (0,1)$

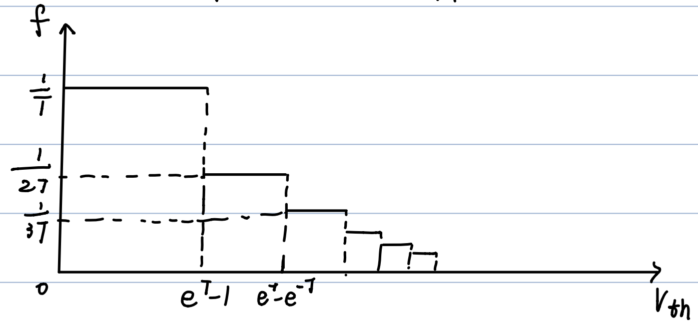
$e^{-kT} \leq \tilde{V} < e^{-(k-1)T}$

$-k \leq \ln \tilde{V}/T < -(k-1)$

$-\ln \tilde{V}/T \leq k < -\ln \tilde{V}/T + 1 \quad$ so. $k = \lceil -\ln \tilde{V}/T \rceil$

for given T:

$$f = \begin{cases} \frac{1}{T}. & V_{th} < e^{T} - 1 \\ \frac{1}{(k+1)T} & e^{T} - e^{-(k-1)T} < V_{th} \le e^{T} - e^{-kT} \end{cases}$$



because. the smaller the V_{th} is.

then. there will be much easier for neuron to reach V_{th}.

then the larger the f is