

CS 517 Midterm I

Due March 19, 1159 PM

March, 17 2025

Please answer all the questions. Each question is worth 20 points. There will be partial credits, try to solve each problem independently, if you take the right approach, I will give you points even if you cannot solve it till the very end. How much you get will depend on how well you tried.

Please write your name and A # in your submission. If you forget, I will not be able to assign a grade to you.

Problem 1

Prove the following: (each one is worth 5 points)

1. Show that $(P \rightarrow Q) \vee (Q \rightarrow P)$ is a tautology
2. Prove that $P \rightarrow Q$ and $\neg P \vee Q$ are logically equivalent.
3. Suppose that P is false and $P \vee \neg Q$ is true. What can you say about the truth value of Q .
4. Write the negation of the following statement ensuring that only simple statements are negated. "Calvin is not home or Bonzo is at the movies". Please show your work.

Problem 2 [10 + 10]

1. Prove that the sum of squares of the first n natural numbers is given by $\frac{n(n+1)(2n+1)}{6}$
2. Prove that for any natural number n , $\sum_{i=0}^{n-1} 2^i = 2^n - 1$

Problem 3

A convex polygon is a polygon such that all its interior angles are less than 180 degrees. Given a convex polygon, an elementary triangulation of the polygon is a way of connecting the vertices with lines such that no two lines intersect and the polygon is converted into a set of triangles. Prove that every elementary triangulation of a polygon with n vertices requires $n - 3$ lines.

Problem 4

Use a truth table to test the validity of the following arguments. You should conclude whether the argument is valid or invalid and show your work. Each one is worth 10 points.

1. If I enter the poodle den, then I will carry my electric poodle prod or my can of mace. I am carrying my electric poodle prod but not my can of mace. Therefore, I will enter the poodle den.
2. I will buy a new goat or a used Yugo. If I buy both a new goat and a used Yugo, I will need a loan. I bought a used Yugo and I don't need a loan. Therefore, I didn't buy a new goat.

Problem 5

Consider the following recurrence relation:

$$T(n) = 2T(n/2) + n, T(1) = 100$$

Solve the above recurrence *without* using the formula given in the slides assuming that $n = 2^k$.