CS 517 Midterm 2

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Please answer all the questions. Each question is worth 25 points. There will be partial credits, try to solve each problem independently, if you take the right approach, I will give you points even if you cannot solve it till the very end. How much you get will depend on how well you tried.

Please write your name and A # in your submission. If you forget, I will not be able to assign a grade to you.

Problem 1

1. Design an algorithm to find all the common elements in two sorted lists of numbers. For example, for the lists $[2, 5, 5, 5]$ and $[2, 2, 3, 5, 5, 7]$ the output should be $2, 5, 5$ .What is the maximum number of comparisons your algorithm makes if the lengths of the two given lists are m and n, respectively? [15 Points]

2. Design an algorithm for computing $\lfloor \sqrt{n} \rfloor$ for any positive integer n. Besides assignment and comparison, your algorithm may only use the four basic arithmetical operations. [10 Points]

Problem 2

1. Write pseudocode for an algorithm for finding real roots of equation $ax^2 + bx + c = 0$ for arbitrary real coefficients a, b, and c. (You may assume the availability of the square root function $sqrt(x)$). [15 Points]

2. Consider the following algorithm for finding the distance between the two closest elements in an array of numbers.

```
ALGORITHM MinDistance(A[0.....n − 1])
//Input: Array A[0.....n − 1] of numbers
//Output: Minimum distance between two of its elements
dmin = ∞
for i = 0 to n − 1 do
    for j = 0 to n − 1 do
        if i ≠ j and |A[i] − A[j]| < dmin
        dmin = |A[i] − A[j]|
return dmin
```

Make as many improvements as you can in this algorithmic solution to the problem. If you need to, you may change the algorithm altogether; if not, improve the implementation given. [10 Points]

Problem 3

1. Write an algorithm to find the "magic index" as defined below in a **sorted** array **A** of distinct integers.

   Given a sorted array **A**, we say that index j is the magic index if $A[j] = j$. For this problem, assume that the arrays are "0" indexed, that is the array elements start at $A[0]$ and go up to $A[n-1]$. Clearly write the algorithm and provide a pseudo code for your algorithm. [5 + 5 = 10 Points]

2. Consider the definition-based algorithm for adding two $n \times n$ matrices. What is its basic operation? How many times is it performed as a function of the matrix order n? As a function of the total number of elements in the input matrices? [7 Points]

3. Answer the same questions for the definition based algorithm for matrix multiplication. [8 Points]

Problem 4

1. Prove the following statements using definitions of O, Ω, Θ. Note that this is a question that is on the harder side and hence I will allow proofs that are argumentative but I need them to be intuitively correct. [5+5+5 = 15 Points]

   (a) $3n^2 + 7n + 3 \in O(n^2)$.

   (b) $n^2 + n + 9 \notin O(n)$.

   (c) $25n^3 - n^2 \in \Theta(n^3)$.

2. Prove or disprove the following statements. [5+5 = 10 Points]

   (a) $f \in O(g) \rightarrow f \in \Theta(g)$

   (b) $n^{0.01} \in O((\log n)^2)$